1-1-2012

Quantum scalar field in quantum gravity with spherical symmetry

Rodolfo Gambini
Universidad de la Republica Instituto de Fisica

Jorge Pullin
Louisiana State University

Saeed Rastgoo
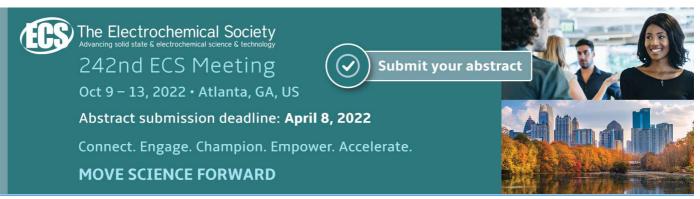*Universidad de la Republica Instituto de Fisica*

Quantum scalar field in quantum gravity with spherical symmetry

To cite this article: Rodolfo Gambini *et al* 2012 *J. Phys.: Conf. Ser.* **360** 012005

View the article online for updates and enhancements.

Quantum scalar field in quantum gravity with spherical symmetry

Rodolfo Gambini[1], Jorge Pullin[2], Saeed Rastgoo[1]

1. Instituto de Física, Facultad de Ciencias, Iguá 4225, esq. Mataojo, Montevideo, Uruguay.
2. Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803-4001

E-mail: `rgambini@fisica.edu.uy,saeed@fisica.edu.uy,pullin@lsu.edu`

Abstract. This is a summary of the talk presented by one of us in Loops 2011. We discuss the application of the uniform discretization approach to spherically symmetric gravity coupled to a spherically symmetric scalar field.

1. Introduction
Canonical loop quantum gravity is being developed in detail in scenarios of increasing complexity. The first detailed calculations were carried out for homogeneous cosmologies [1], constituting what is know called loop quantum cosmology. Further work has been developed in spherical symmetry in vacuum [2], Gowdy cosmologies [3] and parameterized field theory [4]. For various reasons in all those examples one could ignore the "problem of dynamics" of canonical quantum gravity: the fact that the constraint algebra is not a Lie algebra. In homogeneity there is only one constraint and a trivial algebra. In vacuum spherical symmetry one can choose special gauges where the problem is not present. The Gowdy case has been tackled through a hybrid quantization that bypasses the problem. The parameterized field theory has special properties that also allows to ignore the problem.

However, even in relatively simple model like spherically symmetric gravity coupled to a spherically symmetric scalar field there is currently no known method of avoiding having to deal with a constraint algebra that has structure functions. As such, we do not know how to treat the problem using the Dirac quantization procedure. To handle the situation we decided to use the uniform discretization approach [5]. In it, one discretizes the theory one wishes to study and constructs the master constraint [6] and studies its quantum spectrum. If the quantum spectrum contains the zero eigenvalue, then the continuum limit is achieved. If it does not, then one has a theory with a fundamental level of discreteness that will approximate the theory of interest at wavelengths long compared to the size of the minimum eigenvalue.

2. Polymerized spherical gravity coupled to an unpolymerized scalar field
We started our study by polymerizing and doing a loop quantization for the gravitational part of the variables whereas we quantized the scalar field in a traditional fashion [7]. We did this for simplicity and because we wanted to probe the regime close to flat space, where we expect the usual Fock treatment for the scalar field should emerge. It should be noted that the model

we are studying can in principle exhibit a very rich dynamics including the formation of black holes, critical phenomena in gravitational collapse, quasinormal ringing, etc. Our goals initially are very modest: we would be happy to complete the quantization in a regime where the fields are very weak and space-time is close to flat.

Unfortunately, even in such a regime, carrying out the uniform discretization program for a theory as complex as the one we wish to study here in closed form does not appear feasible at present. The master constraint is a quite complicated operator to work out its spectrum. To deal with this shortcoming we decided to pursue a variational technique. We posited a trial state dependent on parameters. The choice we made was to consider a state that was a product of Gaussians in the gravitational variables at each lattice site, centered around flat space, times a Fock-like vacuum for the scalar field. We polymerized the gravitational variables but not the scalar field in a first study. We then proceeded to find the minimum for the master constraint in terms of the free parameters, the widths of the Gaussians and the lattice spacing. As such we are computing a "ground state". It turns out the master constraint's minimum occurs for a lattice spacing different from zero. This is not surprising, the model we are considering has a scalar field that contributes a zero point field whose energy diverges when the lattice spacing goes to zero. So one has to live with a lattice that will be larger than Planck scale, but is very small compared to, say, particle physics lengths. The minimum we find for the master constraint is non-zero. We are therefore in a situation where the discrete quantum theory we find will not approximate in the semiclassical regime general relativity at all scales, but only at scales large

3. Polymerizing the field
3.1. Validating the approximation
We then went on to remove the restriction we made on the scalar field and polymerize it [8]. In turn, one would like to validate that the calculations described in the previous section were justified. To this end we took the quantum master constraint with all variables polymerized and we computed its expectation value on the ground state we discussed above. We found that the expectation value differs very little from the one computed before when we had not polymerized the field. Given that the approximation is validated, we continue to use the same ground state to study the polymerized theory.

3.2. Effects of the polymerization of the field
We decided to study the polymerized scalar field as an interacting quantum field theory treating the polymerization parameter as a "coupling". That is, in the limit in which the polymerization parameter goes to zero one has a "free" theory which is just an ordinary massless scalar field. The effect of the addition of the polymerization is viewed as the addition of an "interaction" for the field with itself, and can be studied with standard Feynman diagrammatics.

There is a choice one faces in this example. Because in the lattice in $1 + 1$ dimensions both the scalar field and its canonical momentum are scalars one can choose to polymerize the field or polymerize the momentum. We analyzed both cases and proceeded to compute the interacting propagator to lowest order in perturbation theory. The result is that one recovers the usual propagator for the scalar field on a lattice. In the case in which the field is polymerized the former acquires a mass proportional to the square of the polymerization parameter. The resulting propagators violate Lorentz invariance due to the presence of the lattice.

3.3. Lorentz invariance
Since one is dealing with a field theory with spatial degrees of freedom we can probe if the final result makes contact with Lorentz invariant quantum field theory. As we mentioned in the previous subsection, the resulting propagators violate Lorentz invariance. This runs the danger of falling in the class of examples considered by Collins *et al.* [9] in which they argue that the

Lorentz violations generate catastrophic experimental signatures when one considers interacting quantum field theory. We have however argued [10, 8] that the situation here differs because one is dealing with a generally covariant theory and therefore one should ask questions about Dirac observables. Propagators can be turned into Dirac observables if one introduces real clocks and rods to label the points of space-time. But real clocks and rods come with fundamental limitations in the accuracy of measurement. The limitations are significantly larger than the Planck scale (or in our case, of the lattice spacing). This has the practical effect in calculations of cutting off frequencies considerably lower than Planck's. The net result is that the resulting Lorentz violations in terms of propagators that are observable are suppressed [8].

4. Summary
We have studied quantum spherically symmetric gravity coupled to a spherical quantum scalar field using the uniform discretization technique. We minimized the master constraint using a variational technique. We studied the polymerized scalar field treating its polymerization parameter as a coupling constant and computed the physical propagators. We argued that the resulting theory violates Lorentz invariant but in magnitudes that are too small to be experimentally detectable.

5. Acknowledgments

[1] See A. Ashtekar, "Singularity Resolution in Loop Quantum Cosmology: A Brief Overview," arXiv:0812.4703 [gr-qc] for a recent review.
[2] R. Gambini and J. Pullin, Phys. Rev. Lett. **101**, 161301 (2008) [arXiv:0805.1187 [gr-qc]].
[3] L. J. Garay, M. Martin-Benito, G. A. Mena Marugan, Phys. Rev. **D82**, 044048 (2010). [arXiv:1005.5654 [gr-qc]].
[4] A. Laddha, M. Varadarajan, Phys. Rev. **D83**, 025019 (2011). [arXiv:1011.2463 [gr-qc]].
[5] M. Campiglia, C. Di Bartolo, R. Gambini and J. Pullin, Phys. Rev. D **74**, 124012 (2006) [arXiv:gr-qc/0610023].
[6] T. Thiemann, Class. Quant. Grav. **23**, 2249 (2006) [arXiv:gr-qc/0510011] and references therein.
[7] R. Gambini, J. Pullin, S. Rastgoo, Class. Quant. Grav. **26**, 215011 (2009). [arXiv:0906.1774 [gr-qc]].
[8] R. Gambini, J. Pullin, S. Rastgoo, "Quantum scalar field in quantum gravity: the propagator and Lorentz invariance in the spherically symmetric case," [arXiv:1105.0667 [gr-qc]].
[9] J. Collins, A. Perez, D. Sudarsky, L. Urrutia and H. Vucetich, Phys. Rev. Lett. **93**, 191301 (2004) [arXiv:gr-qc/0403053].
[10] R. Gambini, S. Rastgoo, J. Pullin, Class. Quant. Grav. **28**, 155005 (2011). [arXiv:1106.1417 [gr-qc]].