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An introduction to spherically symmetric loop quantum gravity black holes

Rodolfo Gambini
Universidad de la Republica Instituto de Fisica

Jorge Pullin
Louisiana State University

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Rodolfo Gambini[1], Jorge Pullin[2]
1. Instituto de Física, Facultad de Ciencias, Iguá 4225, esq. Mataojo, 11400 Montevideo, Uruguay.
2. Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803-4001

We review recent developments in the treatment of spherically symmetric black holes in loop quantum gravity. In particular, we discuss an exact solution to the quantum constraints that represents a black hole and is free of singularities. We show that new observables that are not present in the classical theory arise in the quantum theory. We also discuss Hawking radiation by considering the quantization of a scalar field on the quantum spacetime.

I. INTRODUCTION

Spherically symmetric minisuperspaces are an interesting arena to test ideas of quantum gravity. Classically the system's dynamics is very rich, including the possibility of gravitational collapse and the formation of black holes and the critical phenomena discovered by Choptuik. Quantum mechanically they include the possibility of black hole evaporation and the issue of information loss and the recently discussed subject of firewalls.

Spherically symmetric vacuum space-times were first quantized by Kastrup and Thiemann [1] using the original (complex) version of Ashtekar's variables. Through a series of variable changes and gauge fixings they were able to complete the quantization. The system is reduced to one degree of freedom, the mass at infinity, that does not evolve. Quantum mechanically on has wavefunctions that are functions of the mass. The resulting quantum states represent superpositions of black holes with different masses. A year later Kuchař [2] completed the quantization using traditional metric variables and essentially reached the same results. More recently, Campiglia et al. [3] completed the quantization of the exterior of a black hole using the modern version of the Ashtekar variables and obtained similar results. In all these treatments the singularity present in classical black holes was not eliminated by the quantum treatment. Other groups [4] pursued a different strategy for quantization. Using the fact that the interior of a Schwarzschild black hole is isometric to a Kantowski–Sachs cosmology, they quantized the system using the techniques of loop quantum cosmology. As is typically the case in loop quantum cosmology, the singularity was resolved. There was therefore a tension between these different treatments. It appeared as in the first set of treatments one had reduced the theory too much before quantizing for the techniques of loop quantum gravity to be able to do much about the singularity. Very recently we [9] completed the quantization of the complete space-time using modern loop quantum gravity techniques. We found that the singularity is resolved and remarkably, one could find in closed form the physical space of states. More recently, we used the resulting quantum space-time to study a scalar field living on it and compute Hawking radiation for the quantum black hole. This paper presents a brief summary of the latter two sets of results.

II. SPHERICALLY SYMMETRIC CANONICAL GRAVITY

The framework for studying spherically symmetric space-time with the modern version of the Ashtekar variables was laid down by Bojowald and Swiderski [5]. Choosing triads and connections adapted to spherical symmetry and after a few changes of variables it is observed that the Gauss law constraint is eliminated and one is left with a diffeomorphism constraint along the radial direction and a Hamiltonian constraint. There are two pairs of canonical variables, one associated to the radial direction and one to the transverse direction. The diffeomorphism and Hamiltonian constraints satisfy the same algebra as in the general non-spherically symmetric case. In particular, they do not form a Lie algebra. This presents problems at the time of attempting a Dirac quantization. We [9] noted that one can do a rescaling and combination of the constraints that yield a Hamiltonian constraint that has an Abelian algebra with itself and the usual algebra with the diffeomorphism and therefore the total system of constraints does not contain structure functions in its algebra: it is a Lie algebra. This opened the door for a Dirac quantization.

The kinematical quantum arena of loop quantum gravity is given by states based on spin networks. These are graphs with "colors" in their edges corresponding to representations of $SU(2)$. In each edge there is an open holonomy constructed with the Ashtekar connection in a given representation of $SU(2)$ and at the vertices joining edges the holonomies are contracted with $SU(2)$ intertwiners. There is a natural diffeomorphism invariant innner product on this space which essentially states that two spin network states are orthogonal unless their graphs are equivalent under diffeomorphisms to each other and the colors on the corresponding edges are the same. On this space of states the holonomy of the Ashtekar connection and the (smeared) triads have well defined actions. An important observation is that although the holonomy is a well defined operator, the connection is not. This requires rewriting the equations of the theory in terms of holonomies and leads to quantum behaviors that may differ from the ones one sees in the classical theory. This is at the center of the resolution of the singularities observed, for instance, in loop quantum cosmology.

Bojowald and Swiderski [5] constructed the kinematical quantum states for spherically symmetric gravity. The states are one dimensional spin networks. They are constituted by a "graph" which is a collection of one dimensional edges joined by vertices and each edge carries a color, labeled by an integer. At the vertices there is an additional real variable associated with the "transverse" canonical pair. As in the full case, the holonomies and smeared triads are well defined operators. In terms of them one can promote the Hamiltonian constraint to a well defined quantum operator.

III. THE QUANTUM BLACK HOLE

Remarkably, considering superpositions of spin networks one can relatively straightforwardly show what kind of combinations solve the Hamiltonian constraint exactly. The resulting states depend on a graph, the mass of the space-time (which is a Dirac observable) and a vector constituted by the colors of the edges in the spin network. These states are annihilated by the Hamiltonian constraint but not by the diffeomorphism constraint. But one can construct states that solve all the constraints using the group averaging procedure. Essentially one takes the above states and superposes them with all the states related by diffeomorhisms. The resulting superposition is invariant under diffeomorphisms.

The fact that one can solve the constraints with states with a well defined number of vertices and a given vector of colors implies that associated with them are Dirac observables. The total number of vertices and the vector of colors are Dirac observables. Notice that these observables do not have any simple classical counterpart. In the classical theory the only Dirac observable was the mass at infinity, which is also a Dirac observable at the quantum level.

In gauge theories it is often of interest to study certain gauge dependent variables evaluated in a given gauge. Gauge dependent variables in a well defined gauge are gauge invariant information. For instance one may be interested in studying the metric of the spacetime associated with the quantum state we discussed above, in a particular coordinate system. However, one cannot write the metric as an operator acting on the space of physical states, as it does not commute with the constraints and therefore as an operator the metric would map us out of the space of physical states. A technique for capturing the gauge invariant information present in a gauge dependent variable evaluated in a particular gauge is the one that Rovelli calls "evolving constants of motion". These are Dirac observables dependent on a (functional) parameter. Different choices of the parameter correspond to different choices of gauge. It turns out that one can write the metric of the spacetime as an evolving constant of motion. In this case the functional parameters specify a system of coordinates. The gauge invariant information present in it is carried by the vector of colors of the spin network and the mass of the space-time. The freedom in choosing coordinates allows to discuss a complete treatment of the space-time, that is, one can choose the parameters in such a way that the resulting metric does not have coordinate singularities at the horizon. As a quantum operator, the evolving constant of the motion associated with the metric, is distributional in space-time. It is concentrated at the vertices of the spin network. One therefore is approximating a classical geometry via a set of distributions. If one demands that the evolving constant of the motion be self adjoint, one notices that one needs to restrict the vector of values of the colors of the spin network. The value zero and perhaps some other small range of values has to be excluded. This truncation of the Hilbert space is consistent: the Hamiltonian and diffeomorphism constraints do not connect the excluded sector with the remaining one. But the truncation has a profound consequence: it implies that the region of space-time where the classical singularity would have been present is excluded. Since the geometry was distributional to begin with, excluding a small region does not cause a problem. The geometry can be extended through the region where the singularity used to be into another region of space-time, isometric to the exterior. In that region is a Cauchy horizon, but since it is isometric to the exterior horizon, it is likely to be stable. The region is to the future of the space-time, so this cannot be considered a wormhole. However, the presence of such a region could have significant implications. For instance, it could mean that the missing information present in black hole evaporation went into the new region, if the structure we found here survived black hole evaporation.

IV. BLACK HOLE EVAPORATION

Hawking radiation arises because in a curved space-time there is not a unique definition of the vacuum of a quantum field. The construction of a vacuum is coordinate dependent. This corresponds to different observers seeing different number of particles according to their state of acceleration, a phenomenon already present in flat space-time. Several vacua have been discussed in the literature associated with black holes. The Boulware vacuum is obtained using a coordinate system that covers only the exterior of the black hole. The modes that are used to construct this vacuum get infinitely blue shifted as they approach the horizon, as in these coordinates the modes do not penetrate the horizon. This leads to the development of singularities in the vicinity of the (past and future) horizon in physical quantities computed in this vacuum, as the expectation value of the stress tensor. The Hartle–Hawking vacuum is obtained in coordinates that cover the whole space-time. It includes ingoing and outgoing modes, so it does not represent an evaporating black hole but rather an evaporating black hole with incoming radiation. It does not lead to the type of singularities seen in the Boulware vacuum. The Unruh vacuum is also derived in a set of coordinates that cover the exterior. The Cauchy slices are such that into the past they get deformed asyptotically into the union of the past horizon and the past null infinity. This vacuum has singularity issues at the past horizon as well. Hawking radiation can be computed comparing quantities like the number operator in the Boulware and Unruh vacua. As is well known one ends up with a purely thermal spectrum with a temperture inversely proportional to the mass of the black hole. This implies that the black hole becomes hotter with time and eventually would appear to radiate all its mass. Of course this cannot be analyzed in detail because the calculations of Hawking radiation are done assuming a fixed background, a hypothesis that does not hold when the black hole is shrinking rapidly. This has led to decades of discussion of what exactly is the fate of the black hole. In particular, given that Hawking's radiation is purely thermal, one can ask what happened to all the information that went into the creation of the black hole. Also one could consider forming a black hole by collapsing a pure quantum state. Since at the end one would allegedly be only left with thermal radiation, which is in a mixed state, it appears unitarity is being violated by the process. These issues have led to the recent proposal of replacing the smooth horizon of black holes by a "firewall" that separates the interior and the extrior [6]. Having a quantum theory of gravity raises the hopes that these issues could be eventually clarified.

In a recent piece of work [**?**] we studied a quantum scalar field theory on the background given by the quantum black hole. The approach was to consider the scalar field on a fixed background and ignore the back reaction. The construction is based on considering quantum states that are a direct product of the quantum states of vacuum gravity we discussed above and the states of matter. One takes the Hamiltonian constraint and evaluates its expectation value on the gravitational states. The gravitational part of the Hamiltonian constraint vanishes and the matter part of the Hamiltonian constraint becomes an operator acting on the matter variables. For this construction it is key to write the matter part of the Hamiltonian as an evolving constant of the motion of the vacuum theory, otherwise it would not be well defined as a quantum operators acting on the vacuum states. What remains of the matter part of the Hamiltonian after taking the expectation values is the quantum field theory on a quantum space time we wish to study. The main effect of the quantum space time on the quantum field theory is due to the discrete nature of the quantum geometry. What would have been the partial differential equations of a scalar field on a curved space time now become difference equations.

The discreteness has important implications for the quantum scalar field. As expected, the quantum geometry acts as a natural regulator of the matter theory. It implies the existance of a maximum frequency for the quantum modes. One can choose the level of discreteness present by choosing the state of vacuum gravity used to take the expectation value of the Hamiltonian constraint. Naturally, in order to approximate well a smooth space-time, one would like to choose a small spacing between the vertices of the spin network. One would also like to choose the vector of colors of the spin network in such a way that adjacent colors do not differ by much. Since the colors are associated with the eigenvalues of the metric and these with the values of the areas of the surfaces of symmetry, if one choose colors that differ abruptly there would be jumps in the geometry. The quantization of the area in loop quantum gravity in turn imposes a lower bound on how close the vertices of the spin network can be, it leads to a limit of $\ell_{\rm Planck}/(2r)$. Notice that this number is, for macroscopic black holes, considerably smaller than Planck's length, so this implies the resulting discrete quantum theory provides an excellent approximation to the continuum theory. So most results concerning black hole evaporation hold as a consequence. However, the presence of discreteness brings an important difference. Since there is a maximum frequency, the problems that originated the singularities in the Boulware and Unruh vacuum are absent.

The discrete quantum theory naturally implements the cutoff in the propagators discuss by [17] and lead to the same formula for the Hawking radiation discussed in that reference. At least for macroscopic black holes and for typical frequencies, it implies a very small correction to the formula of Hawking's temperature.

V. SUMMARY

Spherically symmetric vacuum space times can be quantized in loop quantum gravity. The physical space of states can be found in closed form. The metric can be realized as a parameterized Dirac observable. As a function of space time it is distributional, taking values only at the vertices of the one-dimensional spin network. For the metric to be self adjoint, the region where classically one would have a singularity is absent. Space-time can be continued to another region isometric to the region exterior to where the singularity used to be. The presence of the new region could have implications for the black hole information paradox.

On the quantum space time one can study a quantum scalar field theory. The main effect of the quantum background space-time is to discretize the equations of the field theory due to the discreteness of space in loop quantum gravity. The discrete equations approximate ordinary quantum field theory very well and most usual results hold. However, the presence of a maximum frequency due to the discreteness eliminates the singularities associated with some of the quantum vacua. Hawking radiation can be computed and one ends up with a formula that differs little from the ordinary for macroscopic black holes and typical frequencies.

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