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Quantum space-time of a charged black hole

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We quantize spherically symmetric electrovacuum gravity. The algebra of Hamiltonian constraints can be made Abelian via a rescaling and linear combination with the diffeomorphism constraint. As a result the constraint algebra is a true Lie algebra. We complete the Dirac quantization procedure using loop quantum gravity techniques. We present explicitly the exact solutions of the physical Hilbert space annihilated by all constraints. The resulting quantum space-times resolve the singularity present in the classical theory inside charged black holes and allows to extend the space-time through where the singularity used to be into new regions. We show that quantum discreteness of space-time may also play a role in stabilizing the Cauchy horizons, though back reaction calculations are needed to confirm this point.

I. INTRODUCTION

Charged black holes are not expected to play a significant role in astrophysics, but they are a good laboratory to test important ideas in black hole physics. Unlike neutral Schwarzschild black holes, charged Reissner-Nordstrom black holes share elements in common with rotating black holes, like the appearance of Cauchy horizons. Vacuum Schwarzschild black holes have been recently treated using loop quantum gravity techniques [1]. Key to being able to quantize these systems was the realization that one can linearly combine the Hamiltonian and diffeomorphism constraints into constraints that satisfy a Lie algebra. This allows the completion of the Dirac quantization program. Perhaps more surprising, the physical space of states was found in closed form. New observables that do not have a classical counterpart appear in the quantum theory. The metric of space-time can be written as an operator associated with a parametrized Dirac observable acting on the space of physical states. Analyzing the metric, it was found that the singularity is resolved by quantum effects and one tunnels into another region of space-time through a region where the singularity used to be in the classical theory where quantum effects are not negligible.

The purpose of this paper is to show that the above results can be extended to the case of charged spherically symmetric black holes. We will see that the singularity is again resolved by the quantum theory. In addition to that, new perspectives on the stability of Cauchy horizons arise.

II. SPHERICALLY SYMMETRIC ELECTROVAC GRAVITY: THE CLASSICAL THEORY

The treatment of spherically symmetric space-times with Ashtekar-type variables was pioneered by Bengtsson [2] and in more modern language discussed in detail by Bojowald and Swiderski [3]. We will follow here the notation of our previous paper [4] and we refer the reader to them and to Bojowald and Swiderski for more details.

Ashtekar-like variables adapted to the symmetry of the problem, after some work, lead to two pairs of canonical variables E^φ , K_φ and E^x , K_x , that are related to the traditional canonical variables in spherical symmetry $ds^2 = \Lambda^2 dx^2 + R^2 d\Omega^2$ by $\Lambda = E^\varphi / \sqrt{|E^x|}$, $P_\Lambda = -\sqrt{|E^x|} K_\varphi$, $R = \sqrt{|E^x|}$ and $P_R = -2\sqrt{|E^x|} K_x - E^\varphi K_\varphi / \sqrt{|E^x|}$ where P_Λ, P_R are the momenta canonically conjugate to Λ and R respectively, x is the radial coordinate and $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$. We consider a spherically symmetric electromagnetic field $\mathbf{A} = \Gamma dr + \Phi dt$ parameterized by two configuration variables Γ, Φ and their canonically conjugate momenta, P_Γ, P_Φ . We assume a trivial bundle for the electromagnetic field implying the absence of monopoles. In the canonical treatment it is found that Φ operates as a Lagrange multiplier, and can be dropped as a canonical variable [5].

The constraints of the theory are given by the Hamiltonian, diffeomorphism and electromagnetic Gauss law constraints,

$$H = -\frac{E^\varphi}{2\sqrt{|E^x|}} - 2K_\varphi \sqrt{|E^x|} K_x - \frac{E^\varphi K_\varphi^2}{2\sqrt{|E^x|}} + \frac{((E^x)')^2}{8\sqrt{|E^x|} E^\varphi} - \frac{\sqrt{|E^x|} (E^x)' (E^\varphi)'}{2(E^\varphi)^2} + \frac{\sqrt{|E^x|} (E^x)''}{2E^\varphi} + G \frac{E^\varphi}{2(E^x)^{3/2}} P_\Gamma^2, \quad (1)$$

$$C = -(E^x)' K_x + E^\varphi (K_\varphi)' - G \Gamma P_\Gamma', \quad (2)$$

$$\mathcal{G} = P_\Gamma', \quad (3)$$

where we have chosen the Immirzi parameter to one. We proceed to rescale the Lagrange multipliers, $N_r^{\text{old}} = N_r^{\text{new}} - 2N^{\text{old}} \frac{K_\varphi \sqrt{|E^x|}}{(E^x)'}$ and $N^{\text{old}} = N^{\text{new}} \frac{(E^x)'}{E^\varphi}$, and from now onwards we will drop the “new” subscripts for brevity.

This leads to a total Hamiltonian,

$$H_T = \int dx \left\{ -N \left[\left(-\sqrt{E^x} (1 + K_\varphi^2) + \frac{((E^x)')^2 \sqrt{E^x}}{4(E^\varphi)^2} + 2GM \right)' + G \frac{(E^x)'}{2(E^x)^{3/2}} P_\Gamma^2 + 2G \frac{K_\varphi}{E^\varphi} \Gamma P_\Gamma' \right] + N_r [-(E^x)' K_x + E^\varphi (K_\varphi)' - \Gamma P_\Gamma'] + \lambda' (P_\Gamma + Q) \right\}, \quad (4)$$

with the Lagrange multipliers N , the lapse, N_r the shift and λ the parameter of Gauss law. The GM and Q terms are constants of integration that arise from an examination of the theory at spatial infinity. This is standard so we refer the reader to previous papers for it [6, 7]. The rescaling makes the Hamiltonian constraint have an Abelian algebra with itself, and the usual algebra with the diffeomorphism constraint and Gauss law. We had already noted this in vacuum [1], here we point out that it also holds with the inclusion of an electromagnetic field.

We are interested in partially fixing the electromagnetic gauge to $\Gamma = 0$, which is natural for static situations. This determines the Lagrange multiplier λ and also turns the Gauss law into a strong constraint $P_\Gamma = -Q$. This leads to a total Hamiltonian of the form,

$$H_T = \int dx \left\{ -N \left(-\sqrt{E^x} \left(1 + K_\varphi^2 + \frac{GQ^2}{E^x} \right) + \frac{((E^x)')^2 \sqrt{E^x}}{4(E^\varphi)^2} + 2GM \right)' + N_r [-(E^x)' K_x + E^\varphi (K_\varphi)'] \right\}, \quad (5)$$

where we identify the contribution of the electromagnetic field to the mass function, proportional to Q^2 .

Notice that if one were to choose the gauge $E^x = x^2$ and $K_\varphi = 0$ the preservation of the gauge conditions requires that $N_r = 0$ and one would get the Reissner-Nordstrom metric in Schwarzschild form,

$$ds^2 = - \left(1 - \frac{2GM}{x} + \frac{GQ^2}{x^2} \right) dt^2 + \frac{1}{1 - \frac{2GM}{x} + \frac{GQ^2}{x^2}} dr^2 + x^2 d\Omega^2. \quad (6)$$

III. QUANTIZATION: KINEMATICS

We now proceed to quantize. We start by recalling the basis of spin network states in one dimension (see [4] for details). One has graphs g consisting of a collection of edges e_j connecting the vertices v_j . It is natural to associate the variable K_x with edges in the graph and the variable K_φ with vertices of the graph. For bookkeeping purposes we will associate each edge with the vertex to its left. One then constructs the ‘‘holonomies’’ (only K_x is a true connections, so the ‘‘holonomies’’ associated with K_φ are ‘‘point’’ holonomies),

$$T_{g, \vec{k}, \vec{\mu}}(K_x, K_\varphi) = \left\langle K_x, K_\varphi \left| \begin{array}{c} \mu_i \qquad \mu_{i+1} \\ \begin{array}{c} k_{i-1} \quad k_i \quad k_{i+1} \\ \bullet \qquad \bullet \\ v_i \qquad v_{i+1} \end{array} \end{array} \right. \right\rangle \\ = \prod_{e_j \in g} \exp \left(\frac{i}{2} k_j \int_{e_j} K_x(x) dx \right) \prod_{v_j \in g} \exp \left(\frac{i}{2} \mu_j \gamma K_\varphi(v_j) \right) \quad (7)$$

with e_j the edges of the spin network g and v_j its vertices and the integer k_j is the (integer) valence associated with the edge e_j and the integer number μ_j the ‘‘valence’’ associated with the vertex v_j . Notice that since we gauge fixed the electromagnetic field, the kinematical states are the same as those for vacuum gravity.

On these states the triads act multiplicatively,

$$\hat{E}^x(x) T_{g, \vec{k}, \vec{\mu}} = \ell_{\text{Planck}}^2 k_i(x) T_{g, \vec{k}, \vec{\mu}} \quad (8)$$

$$\int_I \hat{E}^\varphi(x) T_{g, \vec{k}, \vec{\mu}} = \frac{\gamma \ell_{\text{Planck}}^2}{4\pi} \sum_{v_j \in I} \mu_j T_{g, \vec{k}, \vec{\mu}}, \quad (9)$$

where I is an interval, and $k_i(x)$ is the valence of the edge that contains the point x .

The problem has two global variables, the mass and the charge. Each of them is associated with a Hilbert space of square integrable functions. So the complete kinematical Hilbert space is given by, functions, the kinematical Hilbert space is given by,

$$H_{\text{kin}} = H_{\text{kin}}^M \otimes H_{\text{kin}}^Q [\otimes_{j=1}^V l_j^2 \otimes l_j^2] \quad (10)$$

with l_j^2 the space of square integrable functions associated with the vertex v_j and V the number of vertices and H_{kin}^M and H_{kin}^Q are the Hilbert spaces associated with the mass and charge. We have chosen periodic functions in K_φ with period π/ρ with ρ a real constant. As discussed in [8] an equivalent quantization can be constructed choosing a Bohr compactification. Notice that we are working with a fixed number of vertices. This will be justified later on by noticing that the diffeomorphism and Hamiltonian constraints do not change the number of vertices.

The Hilbert space is endowed with an inner product,

$$\langle g, \vec{k}, \vec{\mu}, q, M | g', \vec{k}', \vec{\mu}', q', M' \rangle = \delta_{\vec{k}, \vec{k}'} \delta_{\vec{\mu}, \vec{\mu}'} \delta_{g, g'} \delta(M - M') \delta(Q - Q') \quad (11)$$

where we are not assuming the charge to be quantized.

On this space the kinematical momentum operators are multiplicative,

$$\hat{M} |g, \vec{k}, \vec{\mu}, Q, M\rangle = M |g, \vec{k}, \vec{\mu}, Q, M\rangle, \quad (12)$$

$$\hat{Q} |g, \vec{k}, \vec{\mu}, Q, M\rangle = Q |g, \vec{k}, \vec{\mu}, Q, M\rangle, \quad (13)$$

$$\hat{E}^x(x) |g, \vec{k}, \vec{\mu}, Q, M\rangle = \ell_{\text{Planck}}^2 k_j(x) |g, \vec{k}, \vec{\mu}, Q, M\rangle, \quad (14)$$

$$\int_I dx \hat{E}^\varphi(x) |g, \vec{k}, \vec{\mu}, Q, M\rangle = \sum_{v_j \in I} \ell_{\text{Planck}}^2 \mu_j |g, \vec{k}, \vec{\mu}, Q, M\rangle, \quad (15)$$

and the holonomies act as,

$$\exp\left(\frac{in}{2} \int_{e_j} dx K_x(x)\right) |g, \vec{k}, \vec{\mu}, Q, M\rangle = |g, k_1, \dots, k_j + n, \dots, k_V, \vec{\mu}, Q, M\rangle, \quad (16)$$

$$\exp\left(\pm \frac{in}{2} \rho K_\varphi(v_j)\right) |g, \vec{k}, \vec{\mu}, Q, M\rangle = |g, \vec{k}, \mu_1, \dots, \mu_j \pm n, \dots, \mu_V, Q, M\rangle. \quad (17)$$

We are restricting the action of the holonomy of K_φ to vertices since acting elsewhere it would create a new vertex and we are only interested in situations with a fixed number of vertices.

IV. QUANTIZATION: DYNAMICS

To deal with the Hamiltonian constraint one needs to polymerize it and choose a factor ordering. We start with the classical expression and integrate by parts,

$$H(N) = \int dx N' \left[\sqrt{E^x} \left(1 + K_\varphi^2 + \frac{GQ^2}{E^x} \right) - 2GM - \frac{((E^x)')^2 \sqrt{E^x}}{4(E^\varphi)^2} \right]. \quad (18)$$

This expression can be factorized,

$$H(N) = \int dx N' H_+ H_-, \quad (19)$$

with

$$H_\pm = \sqrt{\sqrt{E^x} \left(1 + K_\varphi^2 + \frac{GQ^2}{E^x} \right) - 2GM} \pm \frac{(E^x)' (E^x)^{1/4}}{2E^\varphi}. \quad (20)$$

We now absorb one of the two factors into the lapse and have and rescaling by a factor of $4(E^\varphi)^2$,

$$H(\bar{N}) = \int dx \bar{N} \left(2E^\varphi \sqrt{\sqrt{E^x} \left(1 + K_\varphi^2 + \frac{GQ^2}{E^x} \right) - 2GM} - (E^x)' (E^x)^{1/4} \right). \quad (21)$$

This expression is readily quantized choosing a factor ordering,

$$\hat{H}(\bar{N})|\Psi_g\rangle = \int dx\bar{N} \left(2 \left[\sqrt{\widehat{E^x} \left(1 + \frac{\sin^2(\rho K_\varphi)}{\rho^2} + \frac{GQ^2}{\hat{E^x}} \right)} - 2GM \right] \hat{E}^\varphi - \widehat{(E^x)'} (\widehat{E^x})^{1/4} \right) |\Psi_g\rangle. \quad (22)$$

The term involving a sine, although readily realizable, implies a finite translation in $\vec{\mu}$ leading to an equation in finite differences, that is not easy to solve. It turns out that it is much more convenient to study the action of the Hamiltonian constraint in a mixed representation, where we use the connection representation in K_φ and the loop representation in K_x ,

$$|\Psi_g\rangle = \int_0^\infty dM \int_{-\infty}^\infty dQ \prod_{v_j \in g} \int_0^{\pi/\rho} dK_\varphi(v_j) \sum_{\vec{k}} |g, \vec{k}, \vec{K}_\varphi, M, Q\rangle \psi(M, Q, \vec{k}, \vec{K}_\varphi), \quad (23)$$

where \vec{K}_φ is a vector that has as i -th component $K_\varphi(v_i)$. On these states $\hat{E}^\varphi = -i\ell_{\text{Planck}}^2 \partial/\partial K_\varphi$.

We will assume that the function ψ is factorizable, i.e.,

$$\psi(M, Q, \vec{k}, \vec{K}_\varphi) = \prod_j \psi_j(M, Q, k_j, k_{j-1}, K_\varphi(v_j)). \quad (24)$$

This does not imply loss of generality as the operator has the form of a sum of operators each acting non-trivially only on a given vertex.

$$4i\ell_{\text{Planck}}^2 \frac{\sqrt{1 + m_j^2 \sin^2(y_j)}}{m_j} \partial_{y_j} \psi_j + \ell_{\text{Planck}}^2 (k_j - k_{j-1}) \psi_j = 0. \quad (25)$$

where $y_j = \rho K_\varphi(v_j)$ and

$$m_j^2 = \rho^{-2} \left(1 - \frac{2GM}{\sqrt{\ell_{\text{Planck}}^2 k_j}} + \frac{GQ^2}{\ell_{\text{Planck}}^2 k_j} \right). \quad (26)$$

This equation can be readily solved,

$$\psi_j(M, Q, k_j, k_{j-1}, K_\varphi(v_j)) = \exp\left(\frac{i}{4} m_j (k_j - k_{j-1}) F(\rho K_\varphi(v_j), im_j)\right), \quad (27)$$

with F a function of two variables given by,

$$F(\phi, K) = \int_0^\phi \frac{dt}{\sqrt{1 + K^2 \sin^2 t}}, \quad (28)$$

with m_j complex inside the black hole between the horizons. The states are normalizable with respect to the kinematical inner product. For a lengthier discussion of normalizability, we refer the reader to [8].

V. OBSERVABLES

There are several immediately identified Dirac observables. To begin with one has the mass and charge, which are observables both at a classical and quantum level. But in addition to them one has observables that do not have a simple classical counterpart. The first such observable is the number of vertices. The implementation of the Hamiltonian constraint we chose does not change the number of vertices when acting on states of the kinematical Hilbert space. The states of the physical space of states, annihilated by the constraint, can be chosen all with the same number of vertices.

An additional observable can be hinted from the fact that (non-singular) diffeomorphisms in one dimension will not alter the order of the vertices. Therefore the tower of values of \vec{k} is diffeomorphism invariant and unchanged by the Hamiltonian constraint. Therefore one can readily construct an observable associated with this property. Consider a parameter z in the interval $[0, 1]$. We define,

$$\hat{O}(z)|\Psi\rangle_{\text{phys}} = \ell_{\text{Planck}}^2 k_{\text{Int}(Vz)} |\Psi\rangle_{\text{phys}}, \quad (29)$$

where $\text{Int}(Vz)$ is the integer part of the product of z times the number of vertices. As z sweeps from zero to one, it will produce as a result the components of \vec{k} in an ordered way. This observable may sound artificial, but it actually can be used to capture the gauge invariant portion of E^x . The latter is not diffeomorphism invariant. However, if we consider a function of the real line into the $[0, 1]$ interval $z(x)$ we can define,

$$\hat{E}^x(x)|\Psi\rangle_{\text{phys}} = \hat{O}(z(x))|\Psi\rangle_{\text{phys}}. \quad (30)$$

The result is a parametrized Dirac observable (or “evolving constant of the motion”). It is a Dirac observable, but its value is only well defined if one specifies a (functional) parameter $z(x)$. Specifying the parameter is tantamount to fixing the gauge (diffeomorphisms) in the radial direction. This is a known mechanism [9] for representing gauge dependent quantities on the space of physical states, where only Dirac observables are well defined naturally.

Defining \hat{E}^x on the space of physical states has interesting physical quantities as it allows us to define the metric as an operator on such space. Classically its components are given by,

$$g_{tx} = -\frac{K_\varphi (E^x)'}{2\sqrt{E^x} \sqrt{(1 + K_\varphi^2) - \frac{2GM}{\sqrt{E^x}} + \frac{GQ^2}{E^x}}}, \quad (31)$$

$$g_{xx} = \frac{((E^x)')^2}{4E^x \left((1 + K_\varphi^2) - \frac{2GM}{\sqrt{E^x}} + \frac{GQ^2}{E^x} \right)}, \quad (32)$$

$$g_{tt} = -\left(1 - \frac{2GM}{\sqrt{E^x}} + \frac{GQ^2}{E^x} \right). \quad (33)$$

These expressions can be readily promoted to (parametrized) Dirac observables acting on the space of physical states. One replaces $E^x \rightarrow \hat{E}^x$, $M \rightarrow \hat{M}$ and $Q \rightarrow \hat{Q}$. The quantity K_φ remains classical, it is a (functional) parameter on which the observable depends (it also depends on $z(x)$ through \hat{E}^x). The parameter K_φ is associated with the slicing. This can be directly seen in g_{tx} . A choice $K_\varphi = 0$ yields $g_{tx} = 0$, that is, a manifestly static slicing. With nonzero K_φ one can accommodate slicings that are horizon penetrating like Painlevé–Gullstrand or Kerr–Schild.

One wishes the metric to be a self-adjoint operator. Given the square root this would be violated if one allowed a component of \vec{k} to vanish. Fortunately, since the action of the constraints does not connect states with vanishing values of components of \vec{k} with other states, that means we can simply exclude such states and the operators remain well defined and are self-adjoint. Remarkably, this implies that $r = 0$ is excluded from the treatment, therefore removing the singularity. This is similar to what we observed in vacuum. One can then consider extending the geometry to negative values of x , continuing it through the region where the singularity used to be into a new region of space-time. The resulting Penrose diagram is similar to the one obtained by analytic extensions [10].

VI. CAUCHY HORIZONS AND DISCRETE SPACE-TIME

Recalling that $E^x = R^2$, with R the radius of the spheres of symmetry, the fact that the eigenvalues of \hat{E}^x are discrete imposes a constraint on the minimum increment in the value of R as one goes from a vertex of the spin network to the next, equal to $\ell_{\text{Planck}}^2/(2R)$. That means that in the exterior of a black hole the maximum spacing one can have occurs close to the horizon and is given by $\ell_{\text{Planck}}^2/(4GM)$. This fundamental level of discreteness has implications when one studies the propagation of waves on the quantum space-time. It implies that transplanckian modes of very high frequencies are eliminated. The finest lattice one can have, determined by the spin network and the condition of the quantization of E^x , will be a non-uniform lattice that gets progressively coarser towards the horizon. However, propagation of waves on non-uniform lattices involves a series of phenomena, like attenuation and reflection of waves. If one studies the propagation of waves on a black hole geometry in the exterior of the black hole, the natural coordinate to use is the tortoise coordinate $r = 2GM + \ln(r/(2GM) - 1)$, since in such coordinate one is left with a wave equation with a potential that can be readily analyzed. In such coordinates, the condition for the quantization of the areas implies that the lattice points get progressively more and more separated as one approaches the horizon [9]. So the propagation of wavepackets gets more and more disrupted as one approaches the horizon, exhibiting attenuation and reflection. In ordinary radial coordinates this can also be seen, there it would be the by-product of the progressive blueshifting of the incoming modes.

This non standard propagation due to the quantum space-time may have implications for the stability of the Cauchy horizons present in the interior of Reissner–Nordstrom black hole [11]. The heuristic argument for instability of such horizons is as follows. Suppose one has two observers in the exterior and one of them decides to enter the black

hole. The external observer remains static and shines a flashlight on the infalling observer. By the time the infalling observer reaches the inner Cauchy horizon, the observer in the outside reaches i^+ . That means the exterior observer had a chance of shining an infinite amount of energy on the infalling observer in what, from the point of view of the latter, is a finite amount of time. This suggests an instability can occur. This has been confirmed in classical general relativity using perturbation theory and numerical analysis.

In a quantum space-time the above argument gets modified by the reflections and backscatters that are implied by the quantization of space-time that we discussed above. To begin with, not all light enters the horizon to reach the infalling observer. Some is backscattered outside the black hole towards $scri^+$. Some light crosses the horizon and backscattering continues in the interior towards the Cauchy horizon. At this heuristic level this is not enough to argue that the Cauchy horizon is stabilized, but it clearly suggests that a rethinking of the situation in a quantum space-time is in order. This however, significantly exceeds the scope of this paper, as it would require studying back reaction of perturbations at a quantum level, something that is not possible in loop quantum gravity today, though it may become feasible in a relatively near future. Since the backscattering starts only very close to the horizon, the backscattered light would become visible only in the remote future to external observers, so it will not conflict with black hole observations.

VII. SUMMARY

We have showed that one can complete the Dirac quantization procedure using loop quantum gravity techniques for spherically symmetric electrovacuum space-times. The space of physical states can be found in closed form. Dirac observables can be identified and the physical states labeled with their eigenvalues. The singularity is resolved due to quantum effects as had been observed in the vacuum case. The fundamental discreteness of space-time opens new possibilities in analyzing the stability of the Cauchy horizon inside the Reissner-Nordstrom black hole.

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