

8-19-2016

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Recommended Citation

Campiglia, M., Gambini, R., Olmedo, J., & Pullin, J. (2016). Quantum self-gravitating collapsing matter in a quantum geometry. *Classical and Quantum Gravity*, 33 (18) <https://doi.org/10.1088/0264-9381/33/18/18LT01>

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Quantum self-gravitating collapsing matter in a quantum geometry

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The problem of how space-time responds to gravitating quantum matter in full quantum gravity has been one of the main questions that any program of quantization of gravity should address. Here we analyze this issue by considering the quantization of a collapsing null shell coupled to spherically symmetric loop quantum gravity. We show that the constraint algebra of canonical gravity is Abelian both classically and when quantized using loop quantum gravity techniques. The Hamiltonian constraint is well defined and suitable Dirac observables characterizing the problem were identified at the quantum level. We can write the metric as a parameterized Dirac observable at the quantum level and study the physics of the collapsing shell and black hole formation. We show how the singularity inside the black hole is eliminated by loop quantum gravity and how the shell can traverse it. The construction is compatible with a scenario in which the shell tunnels into a baby universe inside the black hole or one in which it could emerge through a white hole.

There has been recent progress in loop quantum gravity with spherical symmetry. The key ingredient is the realization that through suitable combinations and rescalings the constraint algebra can be made a Lie algebra, both at a classical level and when quantized using loop quantum gravity techniques [1]. In vacuum the theory can be solved in closed form including finding the space of physical states annihilated by the constraints and suitable self-adjoint Dirac observables characterizing the physics. The singularity that is present inside black holes in the classical theory is replaced in the quantum theory with a region with large curvatures, large fluctuations of the curvature and high sensitivity to Planck scale microstructure, so it cannot be well approximated by a semiclassical geometry. The theory is well defined there and eventually one reaches a region inside the black hole where space-time is again semiclassical. This analysis has also been generalized to the electrovac case [2]. Test (non-backreacting) shells were also studied in this framework [3]. In this letter we would like to analyze the case of a self-gravitating shell. This is significant since it is the first full quantum treatment of a midsuperspace model including quantum matter in loop quantum gravity. It would also provide a model for the emergence of a shell from a white hole, as conjectured in several heuristic scenarios. It could also provide the starting point for studying the backreaction of Hawking radiation on a black hole, which cannot be properly studied with the previously existing solutions, which were time independent [4].

In a previous paper [3] we studied the motion of quantized null shells in a fixed quantum geometry background. In this work we would like to consider the full backreaction of the shells on the quantum geometry. We start from the Hamiltonian considered in our previous manuscript. The context is spherically symmetric gravity written in terms of Ashtekar variables. There are two pairs of canonical variables E^φ , K_φ and E^x , K_x , that are related to the traditional canonical variables in spherical symmetry $ds^2 = \Lambda^2 dx^2 + R^2 d\Omega^2$ by $\Lambda = E^\varphi/\sqrt{|E^x|}$, $P_\Lambda = -\sqrt{|E^x|}K_\varphi$, $R = \sqrt{|E^x|}$ and

$P_R = -2\sqrt{|E^x|}K_x - E^\varphi K_\varphi/\sqrt{|E^x|}$ where P_Λ, P_R are the momenta canonically conjugate to Λ and R respectively, x is the radial coordinate and $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$. We will take the Immirzi parameter and Newton's constant to be unity.

The total Hamiltonian (after a rescaling and an integration by parts) is given by

$$\begin{aligned}
 H_T = & \int dx \left[-N' \left(-\sqrt{|E^x|} (1 + K_\varphi^2) \right. \right. \\
 & + \frac{[(E^x)']^2 \sqrt{|E^x|}}{4(E^\varphi)^2} + F(r)p \Theta(x-r) + 2M \left. \right) \\
 & + N^x \left[-(E^x)' K_x + E^\varphi K_\varphi' - p \delta(x-r) \right] \\
 & + N_- M + \frac{N_+}{2} [F(r)p + M], \tag{1}
 \end{aligned}$$

with Θ the Heaviside function, r the position of the shell and p its canonical momentum. Following Louko et al. [5] we took the coordinates x, r to have the $(-\infty, \dots, \infty)$ range and assumed that the variables have the usual fall-offs in asymptotic radial coordinates. We are including the usual boundary term at spatial infinity with N_+ the lapse there N_- the lapse at the other end of the manifold extended beyond where the singularity in the classical theory used to be and, as we will see, is removed by loop quantum gravity like in the static case [1]. We are allowing the possibility of a pre-existing black hole of mass M . Also, $F(r) = \sqrt{|E^x|} \left(\eta (E^x)' (E^\varphi)^{-2} + 2K_\varphi (E^\varphi)^{-1} \right) |_{x=r}$. N is a rescaled lapse, $N = N_{\text{orig}} E^\varphi / (E^x)'$ with N_{orig} the original unrescaled lapse and the shift is also changed $N^x = N_{\text{orig}}^x + 2N_{\text{orig}} K_\varphi \sqrt{|E^x|} / (E^x)'$ with prime denoting derivative with respect to the radial coordinate x . The parameter $\eta = \pm 1$ is the sign of the momentum, depending on it one will have shells that are either outgoing or ingoing if one is outside the black hole.

Notice that $m \equiv F(r)p/2$ is a Dirac observable, which can be verified by direct calculation of its Poisson brackets with the constraints. Also, taking into account the

falloff of the gravitational variables at spatial infinity the constraint indicates that the ADM mass of the space-time is $M + m$ (if there is no pre-existing black hole in the space time and all the mass is provided by the shell we have $M = 0$). The lapse at infinity N_+ may be written as usual [6] in terms of the proper time at infinity τ as $N_+ = \dot{\tau}$. It can also be verified that the Hamiltonian for gravity coupled to a shell has an Abelian algebra with itself. One can also check that the Poisson brackets with the diffeomorphism constraint are the usual ones.

There also exists a Dirac observable, $V \equiv -\int_r^\infty dy [-2F^{-1}(y) + \eta(1 + 2(M + m)/y)] + \tau - \eta[r + 2(M + m)\ln(r/(2(M + m)))]$, and it can be checked that it has vanishing Poisson brackets with all the constraints and is canonically conjugate to m . The observable V is associated with the Eddington–Finkelstein coordinate v of an observer at scri from which the shell is incoming or exiting.

For the quantization, we choose the same Hilbert space as in the test shell case, which is a direct product of the Hilbert space of vacuum spherically symmetric gravity and L^2 functions for the shell. As in previous papers for the gravitational part we consider linear combinations of products of cylindrical functions of the form,

$$T_{g, \vec{k}, \vec{\mu}}(K_x, K_\varphi) = \prod_{e_j \in g} \exp\left(\frac{i}{2} k_j \int_{e_j} dx K_x(x)\right) \times \prod_{x_j \in g} \exp\left(\frac{i}{2} \mu_j K_\varphi(x_j)\right), \quad (2)$$

where the label $k_j \in \mathbb{Z}$ is the valence associated with the edge e_j , and $\mu_j \in \mathbb{R}$ the valence associated with the vertex x_j (usually called ‘‘coloring’’).

We adopt a representation for the point holonomies as quasi-periodic functions (for an alternative choice see [7]), so that the labels μ_j belong to a countable subset of the real line with equally displaced points. We take $\mu_j = 2\rho(l_j + \delta_j)$ with l_j an integer, δ_j between 0 and 1 and ρ is the polymerization parameter. The kinematical Hilbert space associated with a given graph g is, $\mathcal{H}_{\text{kin}}^g = \mathcal{H}_{\text{kin}}^M \otimes \mathcal{H}_{\text{kin}}^{\text{sh}} \otimes \left[\bigotimes_{j=1}^n \ell_j^2 \otimes \ell_{\delta_j}^2\right]$, where ℓ_j^2 and $\ell_{\delta_j}^2$ denote the space of square summable functions of k_j and μ_j , respectively, $\mathcal{H}_{\text{kin}}^M$ is the Hilbert space of square summable functions of the ADM mass and $\mathcal{H}_{\text{kin}}^{\text{sh}}$ is the Hilbert space associated with the shell variables, square integrable functions of r .

The full Hilbert space is equipped with the inner product $\langle g, \vec{k}, \vec{\mu}, M, r | g', \vec{k}', \vec{\mu}', M', r' \rangle = \delta(M - M') \delta(r, r') \delta_{\vec{k}, \vec{k}'} \delta_{\vec{\mu}, \vec{\mu}'} \delta_{g, g'}$, where $\delta_{g, g'}$ is equal to the unit if $g = g'$ or zero otherwise, and similarly for \vec{k} and $\vec{\mu}$.

On this space we have several well defined basic operators. The mass and triads act multiplicatively and the operators associated with the connection variables are holonomies in the case of K_x and point holonomies in the case of K_φ . Explicit expressions are given in [7].

We write the Hamiltonian constraint as, $\tilde{H}(x) = H_g(x) + F(r)p\Theta(x - r)$. The operator \hat{E}^φ only acts at

the vertices as the point holonomies for K_φ only have support there. This leads us to consider a Hamiltonian constraint that only acts at the vertices, just like in the full theory,

$$\begin{aligned} \hat{H}(x_j) &:= \mathbf{H}_j^g + \frac{1}{2}(\boldsymbol{\theta}_j \widehat{F(r)p} + \widehat{F(r)p} \boldsymbol{\theta}_j) \\ &= \mathbf{H}_j^g + \frac{1}{2} \sum_i \mathbf{F}_i(\boldsymbol{\theta}_j \mathbf{X}_i + \mathbf{X}_i \boldsymbol{\theta}_j), \end{aligned} \quad (3)$$

where $\boldsymbol{\theta}_j$ and \mathbf{X}_j are operators acting on the wavefunction of the shell and defined as,

$$\boldsymbol{\theta}_j \psi(r) := \int_0^{\epsilon_j} d\epsilon \theta(x_j + \epsilon - r) \psi(r), \quad (4)$$

$$\boldsymbol{\delta}_j \psi(r) := \int_0^{\epsilon_j} d\epsilon \delta(x_j + \epsilon - r) \psi(r), \quad (5)$$

$$\mathbf{X}_j := \frac{1}{2}(\boldsymbol{\delta}_j \hat{p} + \hat{p} \boldsymbol{\delta}_j), \quad \hat{p} \psi(r) := -i \partial_r \psi(r), \quad (6)$$

and ϵ_j is the spacing of the vertices $\epsilon_j = x_{j+1} - x_j$. The gravitational and shell parts of the Hamiltonian are,

$$\mathbf{H}_j^g = \widehat{H_g}(x_j) = \hat{b}_j \left(-1 - \widehat{K_\varphi^2}(x_j) + \hat{a}_j^2 [\widehat{1/E^\varphi}]^2(x_j) + 2\hat{M} \right), \quad (7)$$

$$\mathbf{F}_j = \widehat{F}(x_j) = 2 \hat{b}_j \left(\hat{a}_j [\widehat{1/E^\varphi}]^2(x_j) + [\widehat{K_\varphi/E^\varphi}](x_j) \right). \quad (8)$$

They can all be written in terms of the elementary operators,

$$\begin{aligned} \widehat{K_\varphi^2}(x_j) &= \frac{\sin(\widehat{\rho K_\varphi}(x_j))}{\rho} \hat{E}^\varphi(x_j) \frac{\sin(\widehat{\rho K_\varphi}(x_j))}{\rho} \hat{E}^\varphi(x_j)^{-1}, \\ [\widehat{K_\varphi/E^\varphi}](x_j) &= \frac{\sin(\widehat{\rho K_\varphi}(x_j))}{\rho} \cos(\widehat{\rho K_\varphi}(x_j)) \hat{E}^\varphi(x_j)^{-1}, \\ [\widehat{1/E^\varphi}]^2(x_j) &= \cos(\widehat{\rho K_\varphi}(x_j)) \hat{E}^\varphi(x_j)^{-1} \cos(\widehat{\rho K_\varphi}(x_j)) \hat{E}^\varphi(x_j)^{-1}, \\ \hat{a}_j &= \frac{\eta}{2} \left(\hat{E}^x(x_j) - \hat{E}^x(x_{j-1}) \right), \quad \hat{b}_n = \sqrt{|\hat{E}^x(x_j)|}, \end{aligned}$$

where as usual we have polymerized the K_φ variable but also introduced a holonomy correction in E^φ such that the change is equivalent to a canonical transformation at the classical level. There one has that $\{K_\varphi(x), E^\varphi(y)\} = \delta(x - y)$ and also that $\left\{ \frac{\sin(\rho K_\varphi(x))}{\rho}, \cos(\rho K_\varphi(y))^{-1} E^\varphi(y) \right\} = \delta(x - y)$. So we replace K_φ by the sines and E^φ by itself times the inverse cosine in all expression, that is the canonical transformation. This polymerization preserves the constraint algebra and leads to the original classical theory when $\rho \rightarrow 0$. Interestingly it can also be extended to matter fields, like scalar fields and it preserves the constraint algebra. In a rather lengthy but straightforward calculation it can be shown that for the case of the shell the constraint has an Abelian commutator algebra with itself also at the quantum level.

Unlike the static vacuum case, we do not at the moment know how to find in closed form the space of physical states associated with the constraints we defined. However, with the constraints represented by well defined operators, we can recognize the quantum observables of the model $\widehat{O}(z(x))$, \widehat{M} , $\widehat{\tau}$, \widehat{Fp} and \widehat{V} ($z(x)$ is a function of the radial variable that takes values in the interval $[-1, 1]$ that enters the definition of \widehat{E}^x , we give more details below, for additional details see [7]). The former is present in the vacuum theory already and corresponds to the diffeomorphism invariant content of \widehat{E}^x , or in other words that diffeomorphisms in one dimension cannot change the order of the vertices of the spin network. The middle ones are the ADM mass and its canonically conjugate momentum, the time at infinity. The two latter ones are associated with the shell. They have the commutator, $[\widehat{m}, \widehat{V}] = i\hbar$. In terms of them we would like to express dynamical variables of the problem as parameterized Dirac observables (evolving constants of the motion) and study the physics in the quantum regime. It should be emphasized that we do not have at the moment an explicit self adjoint implementation of the observables associated with the shell. We will assume one exists and, given their algebra, we can characterize the eigenstates of the complete set of observables and define parameterized Dirac observables based on them.

Following similar steps to the ones we carried out in the vacuum case [8] leads to explicit expressions for the components of the metric as functions of the Dirac observables and classical (functional) parameters K_φ and $z(x)$. We are interested in studying the whole space-time so we choose the horizon penetrating Eddington–Finkelstein coordinates [5]. For that purpose we consider that E^x is time-independent, what fixes $N^x = 0$, and a relationship between K_φ and E^x given by

$$K_\varphi = \frac{R_S [\Theta(x) - \Theta(-x)]}{\sqrt{|E^x|} \sqrt{1 + \frac{R_S}{\sqrt{|E^x|}}}}, \quad (9)$$

where

$$R_S = 2M + 2m \left[\Theta(x) \Theta \left(\sqrt{|E^x|} + (t - V) \right) + \Theta(-x) \Theta \left(\sqrt{|E^x|} - (t - V) \right) \right]. \quad (10)$$

We will concentrate from now on in the case without a pre-existing black hole, i.e. $M = 0$. The expression for K_φ reduces to ordinary Eddington–Finkelstein outside the shell in $x > 0$ and satisfies $\widehat{K}_\varphi = \{K_\varphi, H_T\}$. Besides, one can check that $N = 1/2$ and

$$E^\varphi = \frac{(E^x)'}{2} \sqrt{1 + \frac{R_S}{\sqrt{|E^x|}}}. \quad (11)$$

These expressions take values for $x < 0$ that allow to extend the parameterized observables to the region beyond the singularity. Since generically we will be interested in considering superpositions of states of different

eigenvalues of \widehat{m} in order to approximate semiclassical space-times, it is convenient to promote the above expression to a quantum identity so it automatically adjusts to each state. This gauge choice extends the kind of gauge fixings up to now considered where K_φ was a c-number. We now consider a choice that also depends on the observables. The conservation of the gauge fixing ensures the consistency of the choice. The resulting expressions for the metric are (for reasons of space we give only one component, the others are similar in nature),

$$g_{tx} = -\frac{R_S (E^x)'}{2 |E^x|} [\Theta(x) - \Theta(-x)]. \quad (12)$$

All the relevant quantities can be readily turned into well defined quantum operators. The operator associated with \widehat{E}^x is

$$\begin{aligned} \widehat{E}^x(x_j) |g, \vec{k}, \vec{\mu}, r\rangle &= \widehat{O}(z(x_j)) |g, \vec{k}, \vec{\mu}, r\rangle \\ &\equiv \ell_{\text{Pl}}^2 k_{\text{Int}(z(x_j)v)} |g, \vec{k}, \vec{\mu}, r\rangle, \end{aligned} \quad (13)$$

where Int refers to the integer part and $z(x)$ is an arbitrary function taking values in $[-1, 1]$ whose choice determines the radial coordinate chosen, $n = 2v + 1$ is the number of vertices of the spin network considered (we work in a finite domain of length L to avoid dealing with asymptotic issues involving spin nets). We recall that x_j is the radial coordinate of the j -th point in the spin network. So the Dirac observables are specified in terms of two parameters, one of them functional, $z(x)$ and t . Notice that the resulting metric is time-dependent. To understand this time dependence we recall that in Eddington–Finkelstein coordinates time is related to the position of the shell by $t + x_r = V$ with x_r the radius of the shell, $E^x(r) = x_r^2$, and V constant along the shell. The resulting picture of the geometry is that of an ingoing shell that forms a black hole traversing its event horizon, going towards $x = 0$ and continuing beyond to the region of $x < 0$ as we observed in the case of test shells [3].

The above physical description of the metric is qualitative, based on looking at the classical expressions of the metric as Dirac observables. Let us discuss how this vision can be implemented in detail in the quantum theory. We will consider a situation with states $|k, m\rangle$ which are eigenstates of \widehat{O} and \widehat{m} , constituting a basis, and form a wavepacket centered in m_0, V_0 of width σ_m . We proceed to construct a semiclassical solution based on a spin network with n vertices, $i = -v \dots v$. Although there are infinitely many possible choices for the position of the vertices we will choose a simple one that leads to a good semiclassical behavior. We will place the vertices of the spin network at radial positions $x_i = (i + 1)\Delta$ and Δ a spacing bounded below by the quantization of areas $\Delta > \ell_{\text{Planck}}^2 / (2x_r)$. We have for $i \geq 0$ that $x_i \in [\Delta, L]$ with $L = \Delta(v + 1)$. Also, $z(x) = x/L$ and $k_i = \text{Int}(x_i^2 / \ell_{\text{Planck}}^2)$. In addition $(E_i^x)' = \ell_{\text{Planck}}^2 \frac{(k_i - k_{i-1})}{\Delta} \sim (2i + 1)\Delta$. On the other hand, for $i < 0$ we have that $k_i = -\text{Int}(i^2 \Delta^2 / \ell_{\text{Planck}}^2)$

and $(E_0^x)' = \ell_{\text{Planck}}^2 (k_0 - k_{-1}) / \Delta = 2\Delta$ and it is non-vanishing i 's where k_i , i.e. E^x , changes sign at $x = 0$. The allowed spin networks need to exclude the value $k_0 = 0$ in order to ensure that the description is singularity free. The fact that both the Hamiltonian constraint and the observables are independent of K_x (conjugate to E^x) ensures that this condition is a consistent restriction on the physical Hilbert space. Notice that K_x is only present in the diffeomorphism constraint that does not change the sequence of values \vec{k} . With these assumptions the result of the quantum construction is essentially a discretization of the above classical expressions of the metric on a lattice determined by a given spin network. They are extremely well approximated by their classical counterparts in regions away from where the classical singularity used to be. Close to it, such expressions are sensitive to the lattice chosen. However, to have good semiclassical behavior in the regions far away from the singularity, it is best to consider superpositions of spin networks with different values of \vec{k} and m . This will imply that there are states providing a description of the region close to where the singularity was where there will be large fluctuations of the metric and therefore the singularity is replaced by a highly quantum but regular region that is not well approximated by a semiclassical space-time.

Because we are not including fields, and therefore

Hawking radiation, one ends up with a black hole that resembles the eternal black hole we discussed in the past [1], with an interior region (which can be viewed as a “baby universe”) beyond where the singularity was that is disconnected from the exterior. In a more realistic approximation involving Hawking radiation, the black hole will evaporate. Presumably in that case the region where the shell emerges will coincide with the exterior of the black hole. It is likely that we could approximate large portions of such space-time with the results of this paper, including the emergence of a shell from the highly quantum region that has been conjectured in scenarios such as that of [9–12]. We cannot however provide a precise picture, in particular, the time in which the shell emerges cannot be computed in this approach, so generically for the outgoing shell one would have $-x_r + t + t_0 = V$ with t_0 a constant.

Summarizing, we have provided a detailed model for quantum collapse of matter and formation of a black hole in a quantum geometry. It provides insights into possible scenarios of black hole formation and evaporation.

This work was supported in part by Grant No. NSF-PHY-1305000, ANII FCE-1-2014-1-103974, funds of the Hearne Institute for Theoretical Physics, CCT-LSU, and Pedeciba. J.O also acknowledges the projects MICINN/MINECO FIS2011-30145-C03-02 and FIS2014-54800-C2-2-P (Spain).

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- [1] R. Gambini and J. Pullin, Phys. Rev. Lett. **110**, no. 21, 211301 (2013) [arXiv:1302.5265 [gr-qc]].
 - [2] R. Gambini, E. M. Capurro and J. Pullin, Phys. Rev. D **91**, no. 8, 084006 (2015) [arXiv:1412.6055 [gr-qc]].
 - [3] R. Gambini and J. Pullin, Class. Quant. Grav. **32**, no. 3, 035003 (2015) [arXiv:1408.4635 [gr-qc]].
 - [4] N. Barrios, R. Gambini, J. Pullin, Proceedings of the 14th Marcel Grossmann Meeting, arXiv:1512.04766 [gr-qc].
 - [5] J. Louko, B. F. Whiting and J. L. Friedman, Phys. Rev. D **57**, 2279 (1998) [gr-qc/9708012].
 - [6] K. V. Kuchar, Phys. Rev. D **50**, 3961 (1994) [gr-qc/9403003].
 - [7] R. Gambini, J. Olmedo and J. Pullin, Class. Quant. Grav. **31**, 095009 (2014) [arXiv:1310.5996 [gr-qc]].
 - [8] R. Gambini and J. Pullin, Class. Quant. Grav. **31**, 115003 (2014) [arXiv:1312.3595 [gr-qc]].
 - [9] A. Ashtekar and M. Bojowald, Class. Quant. Grav. **22**, 3349 (2005) [gr-qc/0504029].
 - [10] A. Barrau, C. Rovelli and F. Vidotto, Phys. Rev. D **90**, no. 12, 127503 (2014) [arXiv:1409.4031 [gr-qc]].
 - [11] R. Gambini and J. Pullin, PoS FFP **14**, 038 (2015) [arXiv:1408.3050 [gr-qc]].
 - [12] C. Barceló, R. Carballo-Rubio and L. J. Garay, arXiv:1511.00633 [gr-qc].