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Conformal loop quantum gravity coupled to the Standard Model

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We argue that a conformally invariant extension of general relativity coupled to the Standard Model is the fundamental theory that needs to be quantized. We show that it can be treated by loop quantum gravity techniques. Through a gauge fixing and a modified Higgs mechanism particles acquire mass and one recovers general relativity coupled to the Standard Model. The theory suggests new views with respect to the definition of the Hamiltonian constraint in loop quantum gravity, the semi-classical limit and the issue of finite renormalization in quantum field theory in quantum space-time. It also gives hints about the elimination of ambiguities that arise in quantum field theory in quantum space-time in the calculation of back-reaction.

General relativity is not conformally invariant. This is due to the presence of a dimensionful constant in the action, Newton's constant. However, there exists a conformal extension of general relativity that is locally conformally invariant and that is equivalent to general relativity, as one can show by fixing a certain family of gauges. We will show that the theory can be coupled to matter and defines a gravitational extension of the Standard Model that is conformally invariant. Again one recovers the usual Standard Model coupled to general relativity through a gauge fixing. It is reasonable to think that a conformal extension of general relativity is the fundamental theory that should be quantized. A strong suggestion stems from the fact that it has been observed that the high energy limit of all non-trivial renormalizable field theories is conformally invariant [1].

But there are additional reasons to consider a conformal theory: a) The quantization of gravity in terms of loops has encountered important obstructions at the level of finding a quantization without quantum anomalies. Particularly problematic is the implementation of the Hamiltonian constraint that up to now has only allowed for an ultralocal implementation [2]. The main obstacle to a full quantization is that the kinematics of loop quantum gravity is quite limited at the time of implementing the constraints and making contact with the usual non-diffeomorphism invariant semiclassical physical picture of gravity. There have been some extensions proposed to deal with this but none is still widely accepted (see for instance [3–6]).

b) Related to the last point, it has also been observed that the existence of a Planck scale imposes restrictions at the process of going to the continuum limit. The idea is that if one adds additional points to the spin network in order to refine it, the continuum approximation of volumes and areas does not improve and one just adds volume to the space-time, as there is a minimum eigenvalue for areas. Among other things, this makes it difficult to take the various limits involved in the definition of the Hamiltonian constraint in a non-trivial way. In a conformally invariant theory one does not have a length scale and there is a chance to improve the situation concerning these problems, as we shall see.

c) Studies of quantum field theory in quantum space-times defined by loop quantum gravity have suggested that although the discreteness of the quantum space-time makes the quantum field theory finite, a finite renormalization is needed to remove dependence of the lower energy physics with respect to the Planck scale degrees of freedom [7, 8]. Unfortunately, it appears that the amount of renormalization depends on the state of the quantum space-time background, in particular on the spacing of the vertices of its spin networks. This is a potential problem, although we do not know whether it will be present once the full interacting theory of gravitating quantum fields will be fully quantized.

Conformal invariance can help with points a), b) and c): In a conformal theory of gravity conformal spin networks can be defined that can be indefinitely refined approximating the conformal geometry with arbitrary precision. This suggests that conformal loop quantum gravity will admit a formulation that may not present the problems of the usual quantization. In particular one has excellent perspectives that it will have a local Hamiltonian that is non-trivial as is typical in discrete models. And it opens possibilities for the renormalization problems in quantum field theory in quantum space-time since no counterterms are needed in the renormalization that depend on spacing.

Let us outline the proposed theory. It is well known that the Brans–Dicke theory with coupling $\omega = -3/2$ is conformally invariant [9]. The action is given by,

$$S = 3 \int d^4x \sqrt{-g} \left[\frac{\phi^2 R}{6} + g^{ab} \partial_a \phi \partial_b \phi \right]. \quad (1)$$

In the gauge $\phi(x) = \kappa^{-1/2}$, with $\kappa = 8\pi G$ the theory is identical to Einstein's gravity. The combination $g_{ab}^{(c)} := \phi^2 g_{ab}$, is conformally invariant. In terms of $g_{ab}^{(c)}$, the action (1) takes the manifestly conformally invariant form: $S = \frac{1}{2} \int d^4x \sqrt{-g^{(c)}} R^{(c)}$. It should be noted that the conformal metric $g_{ab}^{(c)}$ has different dimensionality than the usual metric. If one computes invariant intervals with the conformal metric the result is dimensionless.

For the loop quantization of this theory, we will con-

sider the Holst version of the conformal action,

$$S_H^{(c)} = \int \frac{1}{2} e^{(c)} e_I^{a(c)} e_J^{b(c)} \left(\Omega_{ab}^{IJ(c)} + \frac{1}{\gamma} * \Omega_{ab}^{IJ(c)} \right), \quad (2)$$

with γ the Immirzi parameter and $\Omega_{ab}^{IJ(c)}$ the curvature of the $SL(2, C)$ connection $\omega_a^{IJ(c)}$. One can write this action in terms of the geometrical triad and connection by substituting

$$e_a^{I(c)} = \phi e_a^I, \quad (3)$$

$$e_I^{a(c)} = \frac{e_I^a}{\phi}, \quad (4)$$

$$\omega_a^{IJ(c)} = \omega_a^{IJ} - 2\phi^{-1} \partial_b \phi e^{b[I} e_a^{J]}, \quad (5)$$

in (2). We now describe the relation between conformal and geometrical variables at the canonical level.

First, we can express the theory in Hamiltonian form in terms of (conformally invariant) triad and extrinsic curvature canonical pair:

$$\{K_a^{i(c)}(x), E_j^{b(c)}\} = \delta_a^b \delta_j^i \delta^3(x, y), \quad (6)$$

with a Hamiltonian, diffeomorphisms and Gauss constraints. Since the action is independent of the field ϕ , its conjugated momentum vanishes. That is, we have the additional canonical pair and constraint:

$$\{\phi(x), \pi^{(c)}(y)\} = \delta^3(x, y), \quad (7)$$

$$\pi^{(c)} \approx 0. \quad (8)$$

The transformation to geometrical variables associated to the action (1) can be achieved through the canonical transformation:

$$E_i^a = \phi^{-2} E_i^{a(c)}, \quad (9)$$

$$K_a^i = \kappa \phi^2 K_a^{i(c)}, \quad (10)$$

$$\pi = \pi^{(c)} - 2\phi^{-1} K_a^{i(c)} E_i^{a(c)}, \quad (11)$$

with ϕ unchanged. The new nonzero Poisson brackets are

$$\{K_a^i(x), E_j^b\} = \kappa \delta_a^b \delta_j^i \delta^3(x, y), \quad \{\phi(x), \pi(y)\} = \delta^3(x, y). \quad (12)$$

The constraint (8) becomes, after a rescaling by ϕ , the conformal constraint:

$$\phi \pi^{(c)} = \frac{2}{\kappa} K_a^i E_i^a + \pi \phi =: \mathcal{S}. \quad (13)$$

Finally, in terms of Ashtekar variables $A_a^i = \gamma K_a^i + \Gamma_a^i$, the remaining Hamiltonian, diffeomorphisms and Gauss constraints can be written as:

$$H = \frac{\phi^2}{2} \epsilon_i^{lm} E_l^a E_m^b [F_{ab}^i - (\gamma^2 + (\kappa \phi^2)^{-2}) K_a^j K_b^k \epsilon_{jk}^i] - E^{ai} E_i^b \partial_a \phi \partial_b \phi + 2E^{ai} E_i^b (\nabla_a \partial_b \phi) \phi, \quad (14)$$

$$\mathcal{C}_a = \frac{1}{\gamma} F_{ab}^i E_i^b + \pi \partial_a \phi, \quad (15)$$

$$\mathcal{G}^i = \partial_a E_i^a + \epsilon_{ijk} A_a^j E^{ak}. \quad (16)$$

It is clear that this theory is ready for a loop quantization.

We start by constructing the kinematical Hilbert space. As usual we start by building variables that are gauge invariant given by parallel transports of the conformally invariant connection $A_a^{i(c)}$,

$$U(A^{(c)}, \eta) = P \exp \int A_a^{(c)} dy^a, \quad (17)$$

with η a path and $A_a^{(c)}$ the conformally invariant connection

$$A_a^{(c)} = \Gamma_a^{i(c)} + \gamma K_a^{i(c)}, \quad (18)$$

with $\Gamma_a^{i(c)}$ the conformal spin connection defined through its usual relation with $E_i^{a(c)}$ (in particular, it has the same $SU(2)$ transformation properties as the Ashtekar connection A_a^i).

Under gauge transformations the holonomies transform as

$$U(A^{(c)}, \eta) \rightarrow \Lambda(x_f^\eta) U(A^{(c)}, \eta) \Lambda^{-1}(x_i^\eta) \quad (19)$$

with x_f^η and x_i^η the ending and starting points of the path η and Λ 's are finite gauge transformation matrices. The conformal holonomy has vanishing Poisson bracket with the conformal constraint. In terms of them one can define conformally invariant spin networks in the usual way. The cylindrical conformal functions are defined as,

$$\psi_S(A^{(c)}) = \otimes_l R^{(j_l)} \left(U(A^{(c)}, \eta) \right) \otimes_n i_n, \quad (20)$$

where $R^{(j_l)}$ is a representation of the group, in the case of pure gravity $SU(2)$, of dimension $2j + 1$, l is the label of the path, and i_n are the intertwiners associated to the vertex v_n . These spin networks are trivially annihilated by the quantum version of Gauss' law, and the conformal constraint (13). Through group averaging can be made diffeomorphism invariant. We will not expand on that since it is the same construction as in the usual theory. In particular the orthogonality properties of the conformally invariant spin knots $|S^{(c)}\rangle$ are as in the usual case. One can define conformal operators in the basis $|S^{(c)}\rangle$ that characterize properties of a conformal manifold, in particular angular properties are invariant under conformal transformations.

For many situations the conformally invariant spin nets can be defined on a compact spatial manifold. For instance if one considers asymptotically flat space-times infinity can be brought to a finite boundary via a conformal transformation. The compact manifold is a Penrose diagram.

The geometric operators corresponding to the area of a surface and the volume of a region can be extended to conformal operators substituting in the expressions that define the geometric operators g, E, A with their conformally invariant counterparts $g^{(c)}, E^{(c)}, A^{(c)}$. The spectrum of the conformally invariant areas and volumes

are the same as the usual ones with the exception that the factors involving the Planck area and volume do not appear. These operators define the conformal geometry associated to the spin net $|S^{(c)}\rangle$.

Something different from the usual theory is that in the conformal case one can indefinitely add vertices to the spin network to refine it without any limitations in order to approximate a given conformal geometry. In the non-conformal case if one adds vertices the volume of the region increases and therefore the geometry changes. This limits how well one can approximate a given geometry.

It is possible to couple massless matter to conformal gravity. In fact one can couple the complete Standard Model through a mechanism in which the Higgs boson acquires mass in the gauge fixed conformal theory. When one gauge fixes, the Planck scale becomes determined and from there one generates the Higgs mass and indirectly the masses of all known elementary particles. We will see that the relation between their masses and the Planck mass will be given by the dimensionless constants of the theory.

It will be possible to rewrite the total Lagrangian as follows,

$$\mathcal{L}_T = \mathcal{L}_{\text{GR}}(g^{(c)}) + \mathcal{L}_M\left(g^{(c)}, \frac{\Psi^M}{\phi^d}\right), \quad (21)$$

where the Ψ^M are the matter fields and d is a suitable power of ϕ that is introduced to ensure conformal invariance of the matter fields, as we shall see. The equations of motion imply that the stress tensor of the matter fields is traceless. With this one can incorporate all the particles in the Standard Model, without mass. The only thing that requires special discussion is the Higgs field, which endows all other fields with mass via the Higgs mechanism.

Let us recall that in the Standard Model one considers a Higgs boson in a given representation, we will take here the fundamental one where $H^\alpha(x)$ is a doublet, with $\alpha = 1, 2$. Let us focus on the portion of the Lagrangian that involves $H^\alpha(x)$,

$$\mathcal{L}_H = \left[-\frac{1}{2} (\partial_a H^{\dagger\alpha} + g A_a^i \tau_\beta^{i\alpha} H^{\dagger\beta}) (\partial^a H^\alpha + g A^{ak} \tau_{k\gamma}^\alpha H^\gamma) - V(H) \right] \sqrt{-g}, \quad (22)$$

with the potential given by,

$$V(H) = \frac{\lambda}{4} (H^{\dagger\alpha} H^\alpha)^2 - \mu^2 H^{\dagger\alpha} H^\alpha + \text{const.} \quad (23)$$

which corresponds to a term of mass $-\mu^2$ that gives rise to a Mexican hat potential. In the above expression we assume that A_a^i is a Yang–Mills connection associated with the weak interactions τ are the generators of $SU(2)$, λ is a dimensionless coupling constant. For simplicity, we are not including in (22) the $U(1)$ boson field.

One can construct loop invariants that include matter in terms of $\Psi^{(c)} = \Psi/\phi^{3/2}$ $H^{\alpha(c)} = H^\alpha/\phi$. The fermions

can be included at the ends of open paths, the scalar bosons can be included anywhere. The spin networks including matter will have valences in each path not only associated with the representation of the $SU(2)$ associated with gravity, but also with the representations of all the vector bosons [10]

To consider the loop framework it will be good to write the action in terms of conformal variables. In particular, for the portion involving the Higgs field, it reads,

$$S_{\text{Higgs}} = \int d^4x \sqrt{-g^{(c)}} \left[-g^{ab(c)} D_a H^{(c)\dagger} D_b H^{(c)} - \frac{\lambda}{4} (H^{(c)\dagger} H^{(c)} - \alpha^2)^2 + \frac{\lambda'}{4} + \mathcal{L}_{SM}(g^{(c)}, \Psi^{(c)}, A_a) \right]. \quad (24)$$

For brevity we do not list the Standard Model terms explicitly, but one can easily show that the standard action for the Dirac fields in curved space time may be written without any change of form in terms of the conformal invariant combinations as in (21).

Since the above action is written entirely in terms of variables that are conformally invariant, the term involving one sixth the Higgs field squared times the scalar curvature is not needed to enforce conformal invariance [11] If added it would lead to a slightly different particle physics only in situations where gravity is important. When curvature is negligible with the gauge fixing $\phi = \kappa^{-1/2}$ it coincides with the usual Standard Model.

If one considers a gauge fixing $\phi(x) = \phi_0 = \text{constant}$ one can write the dimensionful parameters in terms of ϕ_0 ,

$$G = \frac{1}{8\pi\phi_0^2}, \quad (25)$$

$$\frac{\Lambda}{16\pi G} = \frac{\lambda'\phi_0^4}{4}, \quad (26)$$

with Λ the cosmological constant. This is reminiscent of how Newton's constant emerges in string theory [12]. If one now carries out the Higgs mechanism for the resulting theory, one gets for the mass and the expectation value of the Higgs,

$$\langle H^\dagger H \rangle = \alpha^2 \phi_0^2, \quad (27)$$

$$m_{\text{Higgs}}^2 = \lambda \alpha^2 \phi_0^2. \quad (28)$$

A similar construction can be done for all the other massive particles in the Standard Model. Notice that all the masses and expectation values are determined in terms of the Planck scale and dimensionless parameters, no matter what choice of gauge fixing, implying that the physics is gauge invariant.

Physics beyond the Standard Model, like neutrino oscillations, could also be incorporated in the conformal context through mechanisms like the one in [13].

Given that the gauge fixed theory is the usual one, it could be asked what has been gained by introducing

the conformal theory. First of all, the quantization of a gauge fixed theory is usually inequivalent to the Dirac quantization of the full theory. However, the most interesting answer is that in a quantum theory the constants are expected to be running coupling constants. If one chooses the Planck scale G fixed, the relations of the masses of particles in terms of the Planck mass could therefore change. In the low energy world we live in the particle masses have a fixed relation to the Planck mass. This would change at high energies where quantum gravitational effects are important. At that level the best way to depict things is conformally invariant and therefore this suggests that the conformal theory is the correct theory to quantize. An alternative proposal is the one by 't Hooft [14]. He noted that in conformally invariant theories there is a contribution from dimensional regularization that depends on the dilaton field, which suggests that to keep conformal invariance at a quantum level the dimensionless constants are determined by requiring that the beta function of the renormalization group vanishes. In either case the theory makes non-trivial predictions.

Another advantage of the conformal theory arises when one considers the quantization of a field theory living in a quantum space-time as for instance in [15] in which the background is provided by a loop quantization of the space-time. The divergent terms of the stress energy tensor depend on the quantum state that represents the background space-time [7, 8]. That is clearly not acceptable since to have low energy physics be independent of such state one would have to include counterterms that are state-dependent. This is not possible in the theory with fixed G . It is possible in the conformal theory. In that theory one can absorb the terms that would give divergences in the cosmological constant and the curvature in the action in the continuum theory simply by changing the gauge. This also suggests a solution to the problem noted by Wald [16] that an ambiguity appears in the renormalization for the terms quadratic in the curvature. They depend on constants not determined by the theory and therefore there is ambiguity in computing back reaction. In the conformal theory the terms quadratic in the curvature cannot get corrections since they are in-

dependent of Newton's constant. Thus, if one assumes that these terms vanish in the original action, in a finite theory like the one stemming from a loop quantization, they would emerge from the terms that diverge logarithmically in the continuum. These would just provide very small corrections to the action that can be viewed as originating in quantum gravity effects. Since these terms are not reabsorbed, the conformal (trace) anomaly of quantum field theory in curved space time would not seem to appear.

As we have introduced an extra degree of freedom and an extra constraint, one may think that one has introduced a spurious invariance that does not add anything to the usual description [17]. However, at the quantum level, [1, 14] as we have already noticed, the conformal treatment could have deep consequences in the analysis of quantum anomalies even ignoring its implications on the loop quantization of gravity. The introduction of the dilaton also opens the possibility of having matter fields only coupled to gravity in a conformal invariant way. For instance one could consider spinors coupled to the dilaton via Yukawa couplings, which would allow to have purely gravitating fermions of arbitrary mass. These could be dark matter candidates.

We have presented a conformally invariant theory of gravity coupled to the Standard Model that is amenable to the quantization techniques of loop quantum gravity. The values of the dimensionful parameters are all determined in terms of the Planck scale and dimensionless parameters. Kinematically its Hilbert space is given by conformal spin networks with edges labeled with the representations of all the gauge groups involved in gravity and the Standard Model. Dynamically one has the expectation that there could be improvements in the treatment of the Hamiltonian constraint. Because the theory admits infinite refinements of the spin networks this opens the possibility of finding a Hamiltonian constraint whose action would not be ultralocal.

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