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Fraction proficiency and the number line

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ABSTRACT

The Common Core State Standards place special emphasis on developing fraction proficiency and the use of the number line, especially in grades three through five, whereas the previously mandated “Louisiana Comprehensive Curriculum” put lower priority on fractions and gave little attention to the number line as a model for fractions. The present study was performed in several 6^{th}-grade math classrooms in rural Louisiana. We piloted fraction proficiency tests that were intended to check basic vocabulary and student access to various fraction models, as expected in the Common Core. Some strong error patterns were observed. They might be related to difference between the curricula. Recent work independent of this thesis discovered remarkably similar error patterns in California 6^{th}-graders. The net outcome of this work is a step toward the design of better tests of fraction proficiency.

INTRODUCTION

Fraction proficiency is vital for success in mathematics. For many years, researchers have studied difficulties with fractions. Students who are proficient in fractions ultimately have a better chance of being successful in high school and beyond. Fractions are somewhat difficult to grasp, and this creates a huge challenge in mathematics education. Fraction concepts appear first in elementary school and culminate during the middle grades. If a student does not understand the concept early on, it then creates a phobia for future learning.

Louisiana education is currently moving from using the Louisiana Comprehensive Curriculum to using Common Core State Standards. The Common Core State Standards focus more on the quality and less on the quantity of skills. The Common Core also places special attention on the teaching and learning of fractions. Fractions are first seen in grade 2. As the grades progress, so do the expectations of fraction proficiency. By the time students reach grade 6, they are expected to understand all the fraction operations. This presents a problem with the current teaching and learning of fractions.

The number line is a fundamental tool in the learning of fractions. In Common Core State Standards, students begin seeing and working with number lines as early as grade 3. In the Louisiana Comprehensive Curriculum, teachers between grades 3 and 6 are currently using area models to teach fractions; therefore they have no understanding of the number line and are unable to place fractions on it.

In my five years of teaching, I have always noticed that most of my students cringed when they heard the word “fractions”. Students have only learned how to solve fraction problems using rote operations such as, finding common denominators, changing the numerator, and reducing to lowest terms, etc. If asked the question “why”, they would not have an answer.

Usually I would get a shoulder shrug, or a mumble of "That is what I was told to do". Knowing that Louisiana is transitioning to Common Core State Standards, and that fractions and the number line play an important part for future success in mathematics, I decided to assess students' current knowledge of the latter.

In this study, I examined the current understanding of fractions and fractions as points on a number line of my sixth grade students. Dr. James J. Madden and I met numerous times to discuss the current issues I was facing teaching fractions. We also discussed the direction in which the teaching and learning of fractions was headed. During these meetings, we looked at past researchers' findings on fractions, especially previous MNS theses, and their suggestions for future investigations. Through these discussions, we decided to create several different assessments to gauge students' fraction competency, aiming to be able to compare it to what they optimally need. In order to create these assessments, we again looked at other researchers' assessments of fractions, and particularly looked at Lauren Lejeune's (LaMSTI 2010) fraction test. Each assessment was created based on a guess as to what might give a clear picture of current knowledge. When these tests were administered to my sixth-graders, I found that their understanding of fractions was limited in some specific ways. My overall goal in this study is to take a step toward developing tests that will be useful indicators of what students needs to know to meet the goals of the Common Core.

In the present thesis, Chapter 1 contains a literature review. First, I describe the Common Core expectations for grades three through six. After this, I review the book *A Focus on Fractions*, which emphasizes the importance of the mental models people use to understand fractions. Next, I describe the high-stakes tests that Louisiana has been using at the end of Grade 5 It appears that these tests have not given much attention to fractions, and have not expected

the use of the number line model for fractions (though this may be changing). I close this chapter with a description of other fraction tests that I have been able to review.

In chapter 2, I describe each of the seven assessments that I created for this project. Chapter 3 describes my selection sample and how all assessments were administered to students.

In Chapter 4, I describe the findings for each assessment. Assessments 1, 2, and 3 were considered to be easy, and most students answered these assessments correctly. Assessments 4 and 5 were considered more difficult and showed the most errors. I describe two specific kinds of error that occurred frequently and I present some possible hypotheses concerning why they might occur. Students did well on Assessments 6 and 7, and no interesting error patterns were noticed.

I summarize my findings from Chapter 4, with further discussion of the recognizable error patterns. I also relate my findings to an unrelated independent study done in California. I then make suggestions for future research.

CHAPTER 1. LITERATURE REVIEW

1.1 The Common Core State Standards and Fractions

Education policy in Louisiana is constantly evolving. Most recently, Louisiana has decided to start using Common Core State Standards for the 2013-2014 school term. The Common Core Standards focus more on the quality and less on the quantity of what is being taught. One of the most focused aspects of the Common Core Standards is its treatment of fractions. By the end of Grade 6, students should be able to understand any rational number as a point on the number line and to be able to perform and apply all arithmetic operations with rational numbers, including negative rational numbers (CCSS, p. 39-45).

According to the Common Core State Standards as explained by Professor Hung-Hsi Wu, students' learning of fractions can be divided into roughly two stages; the first stage being in grade 3 and some in grade 4. During this time, students are exposed to numerous ways in which fractions are used and how simple computations can be made on the foundation of uncomplicated analogies and intuitive reasoning. Students learn to depict fractions with fraction strips (mostly made of paper or plain drawings), fraction bars, rectangles, number lines and other manipulatives. Even in the beginning and investigational stage, teachers help students form good habits; such as always paying special attention to a fixed unit (the whole) throughout an entire discussion, and always being as precise as practical. Students must understand at the onset that the actual <u>shape</u> of a rectangle is not the "whole"; its area is. In comparison to the old pizza as area models, rectangle models are superior. There is little to no flexibility in dividing the area of a pizza (circle) into equal parts except by using sectors. Also, discs divided by radii cannot be used in modeling in the multiplication of fractions (Wu, 2011).

The second stage begins in the fourth grade with the formal mathematical development of fractions. Students in grade 4 begin to learn the fact that a fraction is an actual *number,* and that fact begins to assume overriding importance on account of the numerous computations students make with fractions at this point in the school curriculum. Students are required to learn how to add, subtract, multiply, and divide fractions and use these operations to solve complex problems. In order to be able to come to terms with all of the arithmetic operations, students need a clear-cut *definition* of a fraction as a number. This eventually shifts the emphasis from multiple models of a fraction in the initial stage to an almost exclusive conceptualization of a fraction as a point on the number line. This shift, implicit in the Common Core State Standards, is done gradually beginning in grade 4. Once complete, then other interpretations of fractions can be coordinated with the number model. For example, m/n, in addition to being the collection of m parts when the whole is partitioned into n equal parts, "is also the number obtained when 'm is divided by n', where the last phrase must be carefully explained with the help of the number line" (Wu, 2011).

Grade by grade, the Common Core proceeds as follows. In Grade 3, students start developing the ideas of fractions, building upon previously-learned language of partitioning the whole into equal parts. Students in Grade 3 begin with unit fractions (fractions with 1 as the numerator) by taking a whole and partitioning that whole into equal parts and then taking one of those parts. From here, students build other fractions from unit fractions, "seeing the numerator 3 in the fraction ¾ as saying that ¾ is the quantity you get by putting three ¼'s together" (Common Core Writing Team, 2011). Once students can read any fraction this way, there is no need to introduce "proper" or "improper" fractions. 5/3 would be the quantity someone would get when combining five parts when the whole has been divided into three equal parts (Common Core Standards Writing Team, 2011).

Number line diagrams are the most important representations of fractions for students as they develop an understanding of fractions as numbers. In second grade, students divide rectangles and circles into equal parts and learn to call these “halves”, “thirds”, etc., but this is only a minor contribution to the concept of fraction as number developed in grades 3 and beyond. As an image, the number line resembles an endless ruler. When students start using this image, they must grasp that that the relevant <u>whole</u> is the unit interval (i.e., the interval from 0 to 1). Researchers have observed that some students have trouble with this. When locating a particular fraction in a picture of (a piece of) the number line, they may assume that the complete portion of the number line shown is the whole. This error stood out in the SESM Fractions study (Kerslake 1986).

> The confusion between measuring of a fraction of the line and placing a point that represents a fraction was present in 14 of the 15 children in the case of the fractions ½, and in 12 of the 15 children for the fraction 2/3. This appears to be another example of the children perceiving the line as a ‘whole’ of which the fraction is a part. (page 17-18).

Also in grade 3, students begin learning about equivalent fractions and comparing fractions. As the students experiment with number line diagrams, they soon learn that many fractions have the same name, and are therefore equal; i.e, equivalent fractions. Also, students in grade 3, as having previously learned in grade 2 how to compare lengths using a standard measurement unit, they begin to build on the idea of comparing fractions with the same denominator. Once they figure out that the fractions have the same denominator and that the underlying unit fractions are the same size, they can determine that the fraction with the greater numerator is the greater fraction because it is comprised of more unit fractions (Common Core Writing Team, 2011).

When students enter grade 4, they learn the fundamental properties of equivalent fractions. Students begin "multiplying the denominator of a fraction by the same non-zero whole number results in a fraction that represents the same number as the original fraction" (Common Core Writing Team, 2011). By learning this property, students will acquire the core for much of the other skills taught in Grade 4, including comparing, adding, and subtracting fractions and the beginnings of finite decimals. During grade 4, number line representations are a critical part in making sense of order and operations with fractions.

By the end of Grade 4, students are expected to be able to add and subtract fractions with different denominators in the special case where one of the denominators is a divisor of the other, i.e. where only one of the fractions needs to be changed. In Grade 5, students extend this reasoning to situations where it becomes necessary to re-write both fractions in terms of a new denominator. By this time, students understand the process of expressing both denominators in terms of the same fraction so they can be added or subtracted. Students soon learn two fractions can be added or subtracted by subdividing the unit fractions in one using the other denominator. They also find out it is unnecessary to find a least common denominator, as the process for finding it distracts from understanding the algorithm (Common Core Writing Team, 2011).

Also, in Grade 5, students are supposed to connect fractions with the concept of division, grasping that $5 \div 3 = 5/3$. Students should be able to explain this by working with their prior understanding of division as "equal sharing", but Kerslake (1986) points out that relating fractions and division is another big deficit. Although the aspect that "the fraction **a/b** can be interpreted as '**a**' things shared between '**b**' people" appears in textbooks, students seem to be very reluctant to acknowledge the connection between **a/b** and **a÷b**.

According to the Common Core Progressions Documents, the skills described above are supposed to strengthen students' abilities to work with fractions in a more conceptual way, thus preparing them for rational numbers. In grade 6, students will divide one fraction by another. Since students previously have worked with positive rational numbers, now the whole rational number line is a domain for arithmetic and order.

1.2. *A Focus on Fractions*

The book *A Focus on Fractions* emphasizes the importance of multiple models in understanding teaching fractions. Models are the "means of mathematics, not the end" (Post, 1981, Clements, 1999). Using models to support the teaching of fractions is said to help students build an understanding of fractional concepts. We will discover the different ways models are supposed to aid in the conceptualization of fractions by focusing on using models for the understanding of fractions, inappropriate whole number reasoning, the actual whole, and partitioning. By focusing on these concepts, teachers should be able to take the ideas and further the use of models in the teaching of comparing, ordering, adding, subtracting, multiplying, and dividing fractions.

"Models are mental maps mathematicians use as they solve problems or explore relationships. For example, when mathematicians are thinking about a number, they may have a number line in mind. They think about where the numbers are in relation to one another on this line, and they imagine moving back and forth along the line" (Fosnot & Dolk, 2002, p. 73). Models are the images, mental or physical, they have already established. Students, on the other hand, are constantly developing their understanding of concepts, and they will try and physically construct models to try and solve their problems. As they continue on in their education, they

should move away from always needing to construct models and begin carrying that mental image of the model.

> Using models, regular probing, and asking students to explain their thinking or demonstrate their models, should play a key role in instruction as students are solving problems and building their understanding of part to whole relationships, the relative magnitude of fractions, or fraction operations (Petit, Laird, Marsden, 2010).

In addition, the use of models should pervade teaching, not just be a coincidental practice, but as a way of thinking and acquiring a new understanding for students (Petit, Laird, Marsden, 2010).

There are three types of models that we will explore in hopes of understanding fractions – area models (regions), set models (sets of objects), and the number line. Petite, Laird, and Marsden recognized that students should be allowed to interact with these models in terms of problem solving and using them to generalize fraction concepts.

Area models involve thinking about the part to whole relationship. The area models that students usually interact with during school include objects and illustrations such as geoboards, folding paper, grids, circles, and pattern blocks. **Set models** involve inquiry about a fractional part of a set of discrete objects. The types of set models that students usually use during instruction include a collection of things found in a typical classroom such as erasers, marbles, candy, pencils, etc. **Number lines** involve thinking about how far something travels along a line or the position of a point on number lines, rulers, tape measure, or other measurement tools (Petit, Laird, Marsden, 2010).

If teachers only use one type of model, it limits the student's experiences with exploring and utilizing the other models. The most ideal situation involves a balance of models that "differ in perceptual features, causing students continuously to rethink the concept (and not to overgeneralize on the strength of one model)" (Dienes, cited in Post & Reys, 1979). When

students are actually involving themselves with the perceptual features of models, they are learning different aspects and characteristics of the models (Petit, Laird, Marsden, 2010).

> Once teachers have understood the importance of using a variety of models with different perceptual features, they report that their instruction changes. For example, teachers in the OGAP 2005 Exploratory Study reported:
> - using a greater variety of models in their instruction;
> - making explicit links between modeling;
> - providing more opportunities for students to use models to solve problems;
> - an increase in the use of number lines.
>
> These instructional changes appear to be reflected in student work in the OGAP 2005 Study. Some 30 percent ($^{39}/_{128}$) of grade 4 student pre- and post-assessments were analyzed for the use of models to solve problems (VMP OGAP [2005] [Grade 4 pre- and post-assessment] Unpublished raw data.).
> - In the pre-assessment only 23.1 percent ($^{9}/_{39}$) of the students effectively used one or more models to solve problems.
> - In the post-assessment 79.5 percent ($^{31}/_{39}$) of the students effectively used one or more models to solve problems (Petit, Laird, Marsden, 2010).

Another issue to be explored is how students inappropriately use whole number reasoning when it comes to fractions. Many students will see a fraction as two complete whole numbers. For example, if we take the fraction $^{3}/_{4}$, some students will see this as just a 3 and a 4. This leads to inappropriately using whole number reasoning instead of reasoning with a fraction as a single quantity. When students misuse whole number reasoning, it often results in the students making errors when it they try to:

- "Locate numbers on the number line;
- Compare fractions;
- Identify fractional parts of wholes;
- Estimate the magnitude of fractions;
- Operate with fractions" (VMP OGAP (2005). [Grade 4 pre-assessment.] Unpublished raw data, as cited in Petite, Laird, & Marsden, 2010).

The following are some examples of student work where the student inappropriately uses whole number reasoning. The first example shows how the student used the magnitude of the numerator and denominator to compare fractions:

> Question: There are some candies in a dish. 2/5 of the candies are chocolate, and 3/10 of the candies are peppermint. Are there more chocolate or peppermint candies in the dish?
>
> Student's Response: I think there are more peppermint than chocolate because 10 is higher than 5 and 3 is also higher than 2 so I thought my answer was peppermint (Petit, Laird, Marsden, 2010).

The second example shows how the student used inappropriate whole number reasoning when they added numerators and denominators to find the sum

> Question: The sum of 1/12 and 7/8 is closest to
>
> A. 20
> B. 8
> C. ½
> D. 1
>
> Explain your answer.
>
> Student's Response: 1/12 + 7/8 = 8/20 is close to 10/20 which is half (Petit, Laird, Marsden, 2010).

To help remedy these misconceptions, Petite, Laird, and Marsden recommend that teachers should place a greater emphasis on comparing and ordering fractions and on the use of a number line (Petit, Laird, Marsden, 2010). Additionally, teachers should:

> recognize a situation in which they may have been inadvertently reinforcing inappropriate whole number reasoning by only providing opportunities for students to solve part-to-whole relationship problems in which the fraction in the problem has a denominator which equals the number of objects in the whole (in a set model) or the number of parts in the whole, as in the case of an area model (VMP OGAP, personal communication, 2005 cited in [Petit, Laird, Marsden, 2010].).

Therefore, teachers need to be aware of the examples they are giving, and make sure the students are not misusing the whole number concept.

Teachers also need to look at "the whole". "The concept of the whole underlies the concept of a fraction" (Behr & Post, 1992, p. 13). They should know that a fraction should

always be understood in relation to a specific whole. Finding the whole is easy for students when they are given a square cake pan, and the area model shows that $1/_4$ has been removed. They understand quickly there is $3/_4$ of the cake still in the pan. As the students develop their understanding of part-to-whole relationships, finding a fractional part of a whole is the beginning in getting students to understand that fractions actually only have meaning in terms of wholes. As the students develop other fractional concepts, such as relative magnitude, operations, and equivalence, they will use the understandings to solve part-to-whole problems with larger wholes or more complex fractions (i.e. finding $7/_{12}$ of 144) (Petit, Laird, Marsden, 2010).

Students often encounter problems when trying to identify the whole. Especially if there is more than one part to the whole, or they are given just a part of the whole. When there is more than one part in the whole students may use an out of parts strategy, but not equal parts. In a student example, Tom was given 4 hearts. He was then asked to circle $1/_2$ of them. In his answer, Tom only circled $1/_2$ of each heart. While this showed he understands what a half is, he might not fully understand it in term of the whole, which in this case was 4. Sonia, on the other hand, was given the same question, and she correctly circled 2 of the 4 hearts (VMP OGAP, student work, 2005). "The inference from this student work and research suggests that instruction should focus on strategies to help students see the whole and to use an 'out of equal parts' strategy" (Petit, Laird, Marsden, 2010). Another approach, suggested by Lamon (1999), says that, "students may have an easier time identifying the whole and subsequently will make fewer partitions if they have an opportunity to first visualize the whole from a distance." This means that teachers should begin projecting images at a distance, and then help the students visualize different partitions as a class. In the case where students are just given part of the

whole, they generally understand how to complete the whole if, for example, they are given $1/7$ of a candy bar and are asked the draw the whole candy bar. Students get confused when they are given, for example, $4/5$ of a candy bar and are asked to draw the whole candy bar. More time should be taken to help students visualize the actual whole (Petit, Laird, Marsden, 2010).

The biggest area in which models are key in the conceptualization of fractions is partitioning. Partitioning is the act of dividing. When we partition, we separate our models into sections that do not overlap one another. Also, if each section is the same size, then each represents the same fractional part. "Partitioning is a 'fundamental mechanism for building up fraction concepts' and is key to understanding and generalizing concepts related to fractions such as:

- identifying fair shares;
- identifying fractional parts of an object;
- identifying fractional parts of sets of objects;
- comparing and ordering fractions;
- locating fractions on number lines;
- understanding the density of rational numbers;
- evaluating whether two fractions are equivalent;
- operating with fractions;
- measuring (Lamon, 1999).

Researchers also suggest that "early experiences with physically partitioning objects or sets of objects may be as important to a child's development of fraction concepts as counting is to their development of whole number concepts" (Behr & Post, 1992, p. 14). The actual goal is to get students to have those early experiences with partitioning wholes so that as they solve problems and generalize fraction concepts, the impact will ultimately be desirable (Behr & Post, 1992). The Common Core State Standards are consistent in this aspect.

Students will develop an understanding of the properties of fractions by creating their own partitions within models, which may be either drawings or manipulatives. As they reach a

higher level of understanding, they will be able to apply partitioning in solving problems, even if the models and partitions are not explicit. There are some problems students may encounter with partitioning. For example, some students will not consider the size of the partitioned parts, but only the number of partitioned parts. Teachers need to be mindful when teaching/reviewing partitioning for this reason (Petit, Laird, Marsden, 2010).

We have looked at the basics of using models to help aid in the conceptualization of fractions. Teachers do need to take into consideration all the pros and cons of using models. Again, models are the "means to mathematics, not the end" (Post, 1981, Clements, 1989).

1.3 Assessing Fraction Competency

Even though there is a significant amount of research on fraction competency, very little research has been done on how to assess fraction competency. Researchers agree that fractions are an area of conceptual weakness, and, as a result, many tests have been created to explore these weaknesses. However, these tests are rarely used as a basis for promotion to the next grade.

In Louisiana, students enrolled in any public school system are required to pass, standardized tests in order to be promoted to the next grade level. Since fraction competency is an important component of mathematics, it is rational to inquire if the assessments students are required to take is actually measuring this. The assessments mentioned below only focus on students in grade 5.

1.3.1 *integrated* Louisiana Educational Assessment Program (iLEAP)

In 2006, Louisiana developed an educational assessment for public school students in grades 3, 5, 6, and 7 called the iLEAP. This assessment was developed as comprehensive test that judges a varied set of skills. The skills tested on the iLEAP came from the Grade Level

Expectations (GLE's) given by the state, according to the No Child Left Behind Act. The *integrated* Louisiana Educational Assessment Program (iLEAP) test is given to all 5th graders and is a prereqresite for promotion to grade 6. According to the iLEAP Assessment Guide, from Spring 2006- Spring 2012, the 5th grade iLEAP was composed of 33% of questions that related to the "number and number systems" strand of the curriculum. (This includes fractional operations, as well as decimals and percents.) (See Figure 1.1.)

Strands	% of Total Points
Number and Number Relations	33
Algebra	10
Measurement	16
Geometry	18
Data Analysis, Probability, and Discrete Math	15
Patterns, Relations, and Functions	8
Total	**100**

Figure.1.1. Percentage of problems tested on the 5th grade iLeap from 2006-2012.

Figure 1.1 (iLEAP Assessment Guide, Grade 5) shows that in 5th grade the iLEAP does not treat fraction competency as a separate domain. A student may pass the iLEAP, be promoted to grade 6, and still be incompetent when working with fractions.

1.3.2 Transition Common Core Assessments

The state of Louisiana is currently going through the transition from Louisiana Comprehensive Curriculum to Common Core State Standards, thus needing to change state assessments. Beginning in Spring 2013, and culminating in Spring 2014, Louisiana's standardized tests will be considered "transitional". These transitional tests are supposed to begin incorporating Common Core skills and questions with great ease. Teachers were also given a

revised version of the Assessment Guide, which gives information about what content and skills would be tested. (See Figure 1.2) The transitional tests are still being called iLEAP.

Reporting Category	% of Multiple-Choice Points
Number and Operations	26
Fractions	50
Measurement, Data, and Geometry	24
Total	**100**

Figure 1.2. Percentage of problems tested on the 5th grade iLEAP from 2013-2014.

Included in the 2012-2013 iLEAP exam, the "fraction" section made up 50% of the questions as Figure 1.2 (iLEAP Assessment Guide, Grade 5) shows. (This is the most recent data I was able to maintain from the Louisiana Department of Education website). This is a large transition from Spring 2012 where fractions were included in the general standard of "number and number systems". The new Assessment Guide also included a selection of sample test items, which are supposed be Common-Core-like. However, upon examination of these sample test items, only 20% of the 30 questions ask students to operate with fractions. After further investigation, the six questions that deal with fractions are not aligned with the new Common Core Standards. Two of the six questions would typically be seen during grade 3, and the other four would be seen during grade 4. In conclusion, according to Assessment Guide, Louisiana does not test fraction proficiency separately.

1.3.3 Other Fraction Tests

Kerslake (1986) studied the mathematical errors and strategies in children. She used tests from the Concepts in Secondary Mathematics and Science project, in which four tests on the operations of fractions were given to children between the ages of 12-14. The items on the tests were designed to prompt children's understanding of a topic, rather than their ability to apply

skills. The first two tests consisted of items that were shown in problem or diagrammatic form; the other two tests consisted of items that were shown with no words or diagrams, strictly computational.

Coretta Thomas (LaMSTI 2008) talked about a fraction proficiency test published by Silver, Burdett, and Ginn. This test consists of 24 questions addressing all the basic operations with fractions. It is supposed to show how much core fraction understanding the student has conquered. The purpose of this test is to evaluate a students mastery of fraction manipulation, and to indicate which skills a student lacks. The test is in open response format instead of multiple choice. The questions are computational in nature, and require student work to be shown. Roughly 40% of the test is in word problem format, but are still single-step and only address one basic skill at a time. This test is usually given to students at the end of their seventh grade year, and is not appropriate for students going through the transition from fifth to sixth grade.

Lauren Lejeune (LaMSTI 2010) created a test to determine where middle school kids stood with fractions. Her test consisted of 12 questions that students had to analyze and answer. She referred to the Common Core State Standards to develop the questions. The questions on the test ask students to do the following: explain or illustrate fractions, order fractions on the number line, compare equivalent fractions, compare fractions with different denominators, adding fractions, and multiplying fractions.

CHAPTER 2. ASSESSMENTS

The work reviewed in the Chapter I provided very little to go on in terms of test designs. The book *A Focus on Fractions* mentioned the OGAP study, but the assessments used in this study could not be obtained. Using the tests and results from the MNS theses mentioned in the previous chapter, I made a series of seven tests that focused on basic vocabulary and ability to interpret and use area and number-line models. The details of each assessment were largely based on informed guesses about what might give the clearest picture. In this chapter, I will describe each of the seven assessments.

The first assessment was created to see what students knew about the "whole" and partitioning. One whole bar was created to show an actual "whole" that had not been divided. Subsequent bars were then partitioned into halves, thirds, quarters, fifths, and sixths. The bars were arranged randomly and students were required to name what each bar had been partitioned into. Students were also given a word bank to choose from. According to Common Core Standards, the knowledge tested in this assessment would be seen during grade 2. See Attached Assessment in Appendix A.

The second assessment was created to see if students could name various fractions and understand fraction language. Ten fractions were lined up in two columns with blanks beside them. The fractions consisted of proper and improper fractions. Students were asked to select the correct word or phrase and write it next to the fraction it was describing. A word bank was available to select from. Some phrases included "the denominator is four", " two halves", and "four thirds". See Attached Assessment in Appendix A.

The third assessment was to see if students understood equal parts of a whole. This assessment was broken up into two parts. The first part was multiple choice with justification of response. Students were given three bars that had been divided into parts. Two of the possible answers had been divided into the same number of parts, but one of those two had been divided evenly while the other had not been divided evenly. The other possible answer had been divided into a different number of equal parts. The students were first asked which bar had been divided correctly into a specified number of parts. Once they chose their answer, they were then asked to justify why the other two options were not correct. The second part required bars to be partitioned. Students were given bars that had either not been divided, or had been divided into various equal parts. Then they were asked to cut those bars again into specified parts. This was done to see if students would appropriately cut the bars into equal parts. Again, according to Common Core Standards, the knowledge tested in this assessment would be seen in grade 2. See attached Assessment in Appendix A.

The fourth assessment was designed to test students' knowledge of fractions as points on the number line. For this assessment students were given number lines with various units of length. Each number line was then partitioned into various specific units of length from 0 to 1. Every number line started at 0, and each whole number was labeled. The segments between each whole number were purposefully left unlabeled so that students would have to figure out which unit of length was being used. An "X" was marked above a random unit of length on the number line. Students were asked to name the number that was marked by the "X". The knowledge tested on this assessment would be seen during grade 3 using the Common Core Standards, however, the denominators are larger than what would be typically used in grade 3. See Attached Assessment in Appendix A.

Assessment 5 was designed to learn if students understood the idea of using a unit, and subdivisions of it, to measure a length. In this assessment students were shown two bars in each question. The top bar was intended to show what one whole unit was supposed to be. It was labeled, “one whole unit”. The bottom bar was divided into segments of length equal to the unit, and each unit part was further subdivided into equal length; some of those subdivided units had been shaded. Below the bars, a paragraph was written to summarize the picture above. In each sentence of the paragraph, a blank was intentionally left so students would have to fill it in with their interpretation. See below.

> In the picture above, a unit is divided into__________ equal parts. Each of the small equal parts is called a/an _________ of a unit. The number of small equal parts that has been shaded is __________. The length of the shaded part of the bar is____________________________.

The questions were arranged purposely for the degree of difficulty to increase with each problem. The first two problems dealt with proper fractions that were smaller than 1, while the last three problems dealt with fractions that were larger than 1. This assessment tests fraction knowledge in grade 4 according to Common Core Standards, but possibly the denominators used in this assessment might be bigger than the fourth grade expectations. However, I used denominators of eighths, twelfths, and sixteenths because these are commonly seen on rulers. See Attached Assessment in Appendix A.

The sixth assessment was developed to see if students could identify multiple points on a single number line, and if they could associate an improper fraction with the equivalent mixed number. For this assessment, students were given a single number line marked at whole numbers, halves, quarters, eighths, and sixteenths, as these units are seen on a tape measure and ruler. Some numbers were labeled with letters. Students were then asked to name the number for each letter, both as a fraction, proper or improper, and, if appropriate, as a mixed number.

This assessment, according to Common Core Standards, would typically assess knowledge of students in grade 4. See Attached Assessment in Appendix A.

The last assessment was designed to determine if students could use given number labels on a number line to infer the number labels of other positions. For this assessment students were given several versions of a number line. Each number line had been marked with numbers at a few positions. Some versions of the number line were marked with only whole numbers, where students had to fill in missing whole numbers. Other versions were marked with mixed numbers. Once again, according to Common Core Standards, this assessment would typically test the knowledge of students in grade 4. See Attached Assessment in Appendix A.

Also, during the creation of these assessments, the unit on fractions was in the process of being taught. The content of the unit included comparing fractions, as well as adding and subtracting fractions. These assessments did not address any of this content.

CHAPTER 3. ADMINISTERING ASSESSMENTS

The data collected for this thesis was obtain at the Iberville Parish Mathematics, Science, and Art Academy-West (MSA-W), a K-12 public magnet program for the children residing in Iberville Parish, in Plaquemine, Louisiana. MSA-W is located in south Plaquemine off of Highway 1. Students are accepted into MSA-W by an application and lottery process. Once accepted, students are required to maintain 2.50 GPA, which has recently been changed to 2.75 beginning Fall 2013. Students must also maintain a clean discipline record, as well as pass all state standardized tests to remain at the Academy. Students failing to uphold these requirements are subject to dismissal from the Academy. The ethnic background for MSA-W is 48% White, 48% African American, and 4% Hispanic, Asian, or Pacific Islander. In terms of socioeconomic level, the school is predominantly made up of low-income, single household families with approximately 80% of students on Free or Reduced Lunch. During the 2012-2013 school year, approximately 102 students were enrolled in my 6^{th} grade math class. Of those 102 students, I tested approximately 52. Those 52 students were enrolled in my 1^{st} – 3^{rd} period classes. I did not test my 4^{th} or 6^{th} period classes due to extremely large numbers of absences.

I taught the unit on fractions from the 6^{th} grade Louisiana Comprehensive Curriculum (transitional version) in the time period from December 13, 2012 to February 7, 2013. The fraction assessments were administered during Thursday classes in my classroom during regular class time. Students typically got about 40 minutes to complete them.

On the first day I had a brief discussion with my students. During this discussion, I stated they would be taking a series of assessments to gauge their proficiency in fractions. They were also told they could take the entire class period, but no longer. Some students asked if they could ask me questions, and I informed them they could not.

After the discussion, each student was got an identifying number according to his or her class period and location on the roll. This was done in order to keep results anonymous. On the subsequent assessment days, there was no discussion. Students used the same identification numbers as the first assessment. Data collection was completed on Thursday, February 7, 2013.

CHAPTER 4. FINDINGS

After I collected and graded all seven assessments, I looked for major patterns in the results. I found that Assessments 1, 2, and 3 (2nd and 3rd grade level according to CCSS) were easy; the class did very well, and there were very few mistakes. Assessments 4 and 5 were found to be challenging in that there were interesting patterns of error that will be described in detail below. Students, overall, did ok on Assessment 6, as they were able to name mixed numbers. The first four problems on Assessment 7 were considered to be easy, and students did very well on these; the last three problems were much harder and most students difficulties in answering them correctly.

4.1 Assessment 4

As described in Chapter 2, assessment 4 required students to name various points on the number line. There were seven questions total on this assessment. Each question was graded as correct or incorrect. Incorrect responses were then analyzed to determine if there were common errors. I will only discuss incorrect responses that had recognizable error patterns. Itemized graphs for each question's answers can be found in Appendix B.

Question 1 is shown in Figure 4.1. The correct answer for question 1 is 2½. Of the 52 students taking this assessment, 37 got the answer correct, and 15 got the answer incorrect. In analyzing the incorrect answers, there were no recognizable patterns.

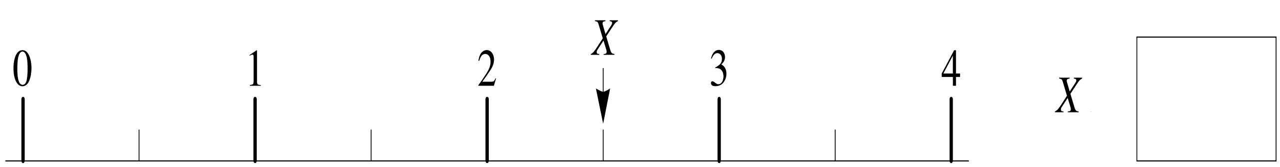


Figure 4.1. Assessment 4 Question 1

Question 2 is shown in Figure 4.2. The correct answer for this question is 1 2/3. Only 7 out of 52 students got the answer correct. The 45 incorrect answers varied. 8 of the 45 put 1½ as the correct answer. Students possibly counted the tick marks between the two numbers instead of the spaces, which explains the denominator choice. A total of 19 students either put 1 ¾, 1/3, or 1 1/3 as the correct answer. There is no hypothesis concerning these causes. 5 students thought the correct answer was 1 2/2; this answer is possibly explained by the students, again, counting the tick marks instead of the spaces. The 2 students that answered 5/9 possibly thought that the picture is equal to the full unit, meaning they counted 9 tick marks and saw that the "X" was over the fifth tick mark.

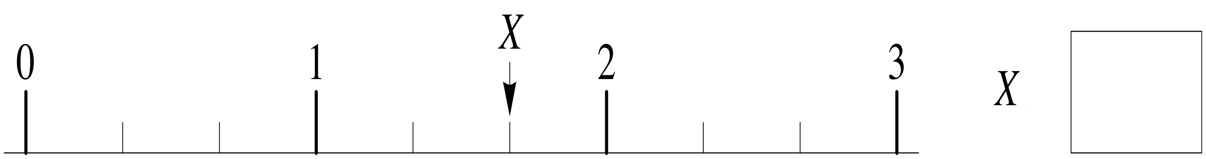


Figure 4.2. Assessment 4 Question 2

Question 3 is show in Figure 4.3. The correct answer for this question is 2 2/5. Once again, only 7 out of 52 students got this answer correct, leaving 45 students to get the answer incorrect. Amazingly, over half of the students who answered this question incorrectly put the same answer. 23 students thought the correct answer should have been 2 ½. I hypothesize that these students simply counted four tick marks between two numbers, and saw that the X was placed over the second one tick mark, so they automatically assumed ½.

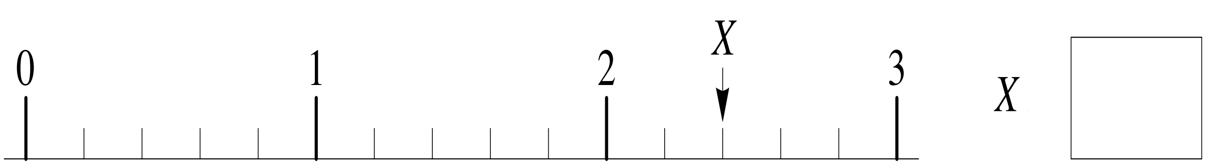


Figure 4.3. Assessment 4 Question 3

Question 4 is show in Figure 4.4, where the correct answer is 3 3/8. 9 out of the 52 students answered this question successfully. Of the 43 students that answered the question incorrectly, 15 of them thought 3 3/7 was correct. This mistake was the most common among all the other incorrect responses. Students choosing this answer simply have counted only the tick marks between numbers 3 and 4, not the units of length between 3 and 4.

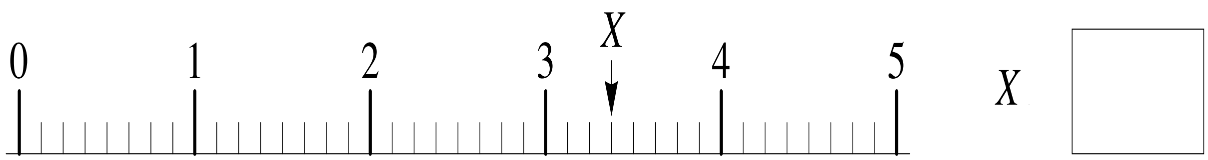


Figure 4.4. Assessment 4 Question 4

Question 5 is shown in Figure 4.5, where the correct answer is 4¼ (4 2/8). Thirteen students successfully named this point on the number line. This time, out of the 39 students who incorrectly named this point, 15 of them named it 4 2/7. Once again, as seen in questions 3 and 4, students are not appropriately counting unit segments between each whole number, they are only accounting for the number of tick marks that occur between each number.

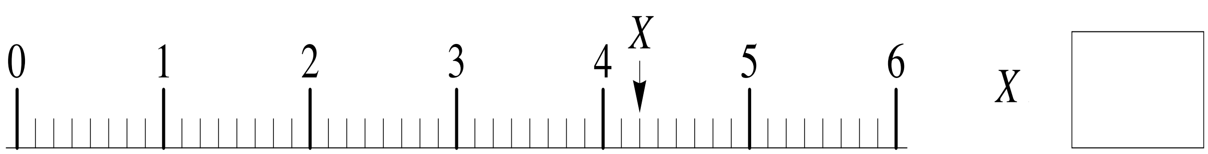


Figure 4.5. Assessment 4 Question 5

Many of the incorrect answers to questions 6 (Figure 4.6), and 7 (Figure 4.7) may be explained in the same way. In question 6, only 10 students correctly named the point 3 4/10 (3 2/5), while another 13 students incorrectly named the point 3 4/9. In question 7, only 7 students

got the answer of 3¾ (3 9/12) correct, and 13 students incorrectly put an answer of 3 9/11.

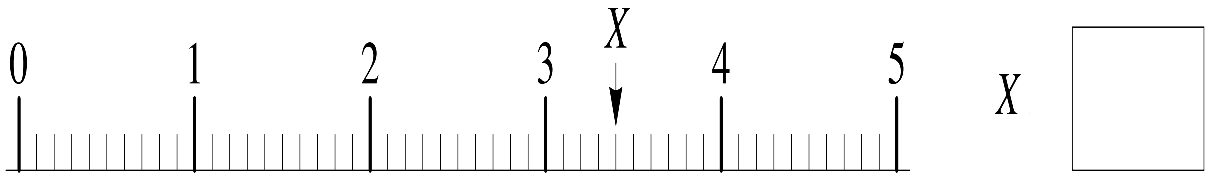


Figure 4.6. Assessment 4 Question 6

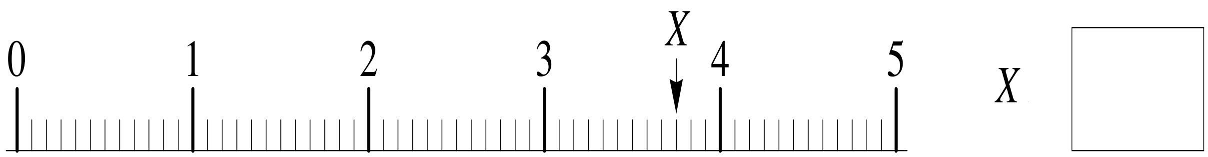


Figure 4.7. Assessment 4 Question 7

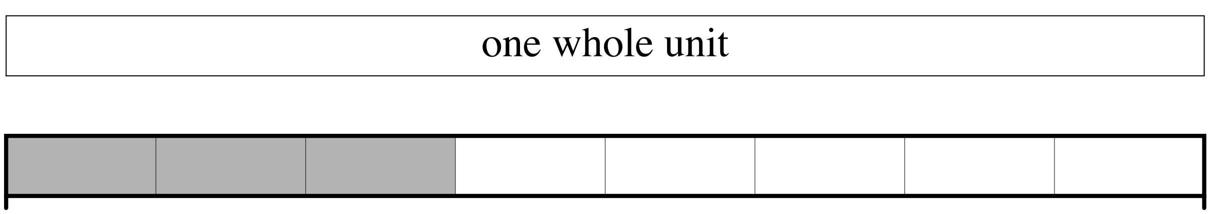
About a quarter of all answers gives clear evidence of the error of counting tick marks instead of counting the unit spaces.

4.2 Assessment 5

Assessment 5, as described in Chapter 2, required students to answer questions concerning various fraction models by filling in the blanks of a paragraph. There were a total of five questions, with each question asking for the same four elements in each paragraph. Each blank for each question was graded as correct or incorrect. Incorrect responses were then analyzed to see if there were patterns in the errors. I will only discuss incorrect responses that had recognizable error patterns. Graphs showing the distribution of errors can be found in Appendix B.

Question 1 is shown in Figure 4.8. The correct answer for Blank 1 is 8. Forty-four of my students got this answer correct, while 8 students got it incorrect. Among the 8 that got the incorrect answer, there were no significant error patterns. The correct response for Blank 2 was

"eighth". Only 13 students got this blank correct, leaving 39 students giving incorrect answers. Of those 39 students, 8 of them thought the answer should have been "whole", 6 of them thought it should be "half", 4 of them thought it should be "third", and another 4 thought it should be "part". The students who answered "whole" or "part" may simply have not paid attention to detail by reading the sentence carefully. The students answering "half", may have compared the shaded bar to the whole unit, noticing that the shaded region is almost half. The students that answered "thirds", may have done something similar, or may have counted three shaded areas and automatically thought thirds. Thirty-three of my students correctly filled in Blank 3 with an answer of 3. The 19 students filling in Blank 3 with 3/8, probably just misread the question. For Blank 4, the correct answer is 3/8; 28 students answered correctly, leaving 24 giving the incorrect answer. Although there were a significant number of students incorrectly answering blank 4, there was only one pattern found in the errors. Five students thought the answer should have been 3. This may have occurred due to students only seeing the words "shaded part", in which they counted 3.

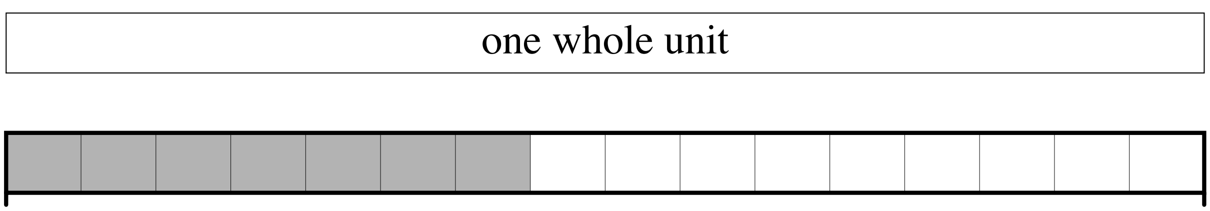


In the picture above, a unit is divided into__________ equal parts. Each of the small equal parts is called a/an _________ of a unit. The number of small equal parts that has been shaded is __________. The length of the shaded part of the bar is _____________________.

Figure 4.8. Assessment 5 Question 1

Question 2 is shown in figure 4.9. The correct answer for Blank 1 is 16, which 42 of my students got correct. The 10 students with incorrect responses showed no patterns. For Blank 2,

the correct answer is “sixteenth”. Only 14 students answered correctly, leaving 38 students with the incorrect answer. As in question 1, a group of students (11 total) incorrectly thought the answer should have been “whole” (6) or “half” (5). This may be due to lack of attention to detail by reading the question carefully. I also still attribute students incorrect response of “half” to them glancing quickly at the shaded bar and seeing it as almost half of the whole bar. Thirty-eight of my students correctly filled in Blank 3 with the correct answer, 7. There were no recognizable patterns in the incorrect answers of the 14 other students. The correct answer for Blank 4 is 7/16; 25 students answered correctly. Of the 27 students who got it incorrect, there was only one common pattern error. Six of the 27 students thought the correct answer was 7. This is similar to the error we saw in question 1. Students counted the shaded rectangles, but did not take the length of each into account.

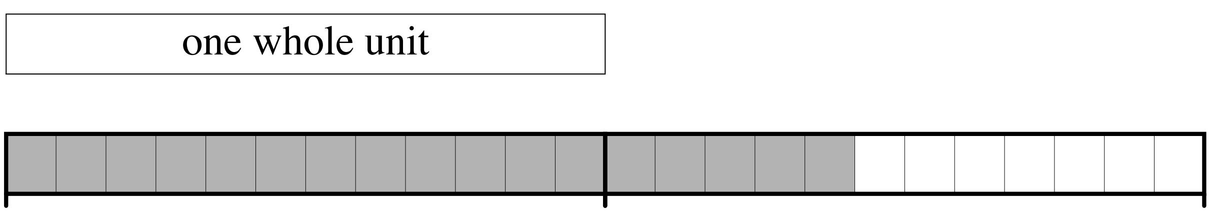


In the picture above, a unit is divided into__________ equal parts. Each of the small equal parts is called a/an _________ of a unit. The number of small equal parts that has been shaded is __________. The length of the shaded part of the bar is _____________________.

Figure 4.9. Assessment 5 Question 2

Question 3 is shown in Figure 4.10. The correct answer for Blank 1 is 12. Out of 52 students, only 8 of them got the blank correct. Of the remaining 44 students, astonishingly 34 of them thought the correct answer was 24. Clearly these students paid no attention to the sample unit that was given above the shaded bar. They also ignored the darker and longer marks on the shaded bar, which help to indicate whole units. The students who missed Blank 1, are likely to

get Blanks 2 and 4 incorrect as well. Blank 2's correct answer should be "twelfth". Looking back upon how many students got Blank 1 incorrect, it is not surprising that only 2 students got Blank 2 correct, leaving 50 students with incorrect answers. Eleven of the 50 students incorrectly thought "twenty-fourth" would be the correct answer, and given the results of blank one, this answer is understandable. Another common error in Blank 2 is the answer "whole" (6), or "half" (5) which 11 students gave, with this the third time these two answers have been mentioned, a new hypothesis concerning them arises. It is possible that these students have no concept of what a portion of a whole unit is called. These students might think every small part is considered a "whole" or a "half" of a unit. Blank 3 has 36 students answering correctly with an answer of 17. Among the 16 students answering Blank 3 incorrectly, we see no recognizable pattern. Based on prior assumptions as seen above, only 3 students correctly answered blank 4 with 1 5/12 (17/12). Nineteen of the students incorrectly answered this blank with 17/24, which again, is reasonable considering the possible thought process behind blank 1. All other versions of the incorrect answer showed no common pattern error.

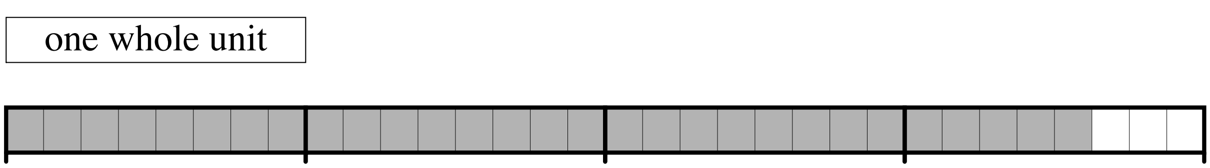


In the picture above, a unit is divided into__________ equal parts. Each of the small equal parts is called a/an _________ of a unit. The number of small equal parts that has been shaded is __________. The length of the shaded part of the bar is ______________________.

Figure 4.10. Assessment 5 Question 3

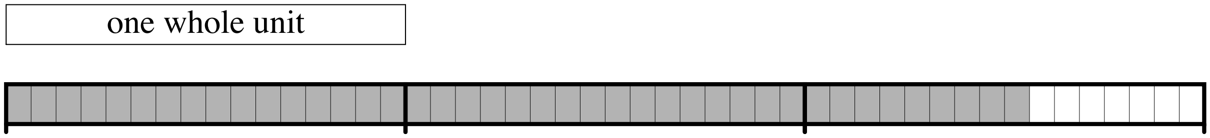
Very similar results occurred in both questions 4 (Figure 4.11) and 5 (Figure 4.12). The correct answer for Blank 1 in question 4 should have been 8. While only 8 students got this

correct, 35 of the 44 students who got it incorrect thought the answer should have been 32. In question 5 Blank 1, again, only 8 students put the correct answer of 16, with 29 of the 44 students incorrectly putting 48, and 4 of those incorrect students putting 47 probably due to counting errors. Blank 2's results for both questions 4 and 5 are, again, very similar to the results in question 3. In question 4, 4 students correctly answered "eighth", leaving 48 students to answer incorrectly. Of those 48 incorrect answers, 11 students thought the answer should have been "thirty-seconds". In question 5, only 3 students correctly answered "sixteenth", leaving 49 students answering incorrectly. Of those 49 incorrect answers, 10 students thought the answer should have been "forty-eighth". These responses are reasonable considering their earlier responses in Blank 1. Also in Blank 2, there were 11 students who incorrectly answered either "whole" (6) or "half" (5), for both questions 4 and 5, once again strengthening my earlier hypothesis of students not having any concept of a portion of a unit. Most students answered Blank 3 correctly with 29 (question 4) and 41 (question 5). The students answering incorrectly quite simply miscounted the number of shaded boxes. Blank 4's answer for question 4 should have been 3 5/8 (29/8). Only 3 students answered correctly. Of the 49 students responding incorrectly, 20 answered with 29/32. Blank 4's answer for question 5 should have been 2 9/16 (41/16). Again, only 3 students answered correctly. This time, of the 49 students responding incorrectly, 19 answered with 41/48.



In the picture above, a unit is divided into__________ equal parts. Each of the small equal parts is called a/an _________ of a unit. The number of small equal parts that has been shaded is __________. The length of the shaded part of the bar is ______________________.

Figure 4.11. Assessment 5 Question 4



In the picture above, a unit is divided into__________ equal parts. Each of the small equal parts is called a/an _________ of a unit. The number of small equal parts that has been shaded is __________. The length of the shaded part of the bar is _____________________.

Figure 4.12. Assessment 5 Question 5

As you can see, the error patterns shown in this assessment are very revealing. These results show that students do not think about units in the way that we might expect or at least hope. Most students ignored the top part of the question where a whole unit was given for them to reference. They continued by just counting how many "blocks" there were in all, then reference how many of those were shaded. This concept of units can easily be compared back to the results of Assessment 4. If students cannot use a picture model to understand whole units, and/or unit length, then they most certainly cannot be expected to perform tasks with fractions on a number line.

CONCLUSIONS

Fraction proficiency is necessary in order to be successful in middle school mathematics and beyond. In this study, I tested, with limitations, students' current understanding of fractions in many different ways by the use of different types of assessments. Seven assessments were created and given over a two-month period. One limitation I faced during this period was after two weeks of giving the students the assessments, I was on maternity leave, and a substitute teacher finished giving the assessments. Therefore students may have been uncomfortable with an unfamiliar face and answered some assessments poorly. Another limitation was that some students, already having negative attitudes towards fractions, may have gotten frustrated with the nature of the assessments and therefore did not answer questions honestly and to the best of their ability.

Once all seven assessments were collected and analyzed, only two produced a clear picture of missing concepts. I discovered, that a large number of students have misunderstandings concerning the use of fractional units to identify or name a fraction on a number line. Assessment 4 shows that students counted the tick marks between two points instead of counting the spaces (which are the fractional units). Assessment 5 shows that students did not recognize the intended unit and thought the entire model was the whole unit. One conjectures that these deficits have their origin in gaps in elementary mathematics courses between grades 3 and 5. The Common Core should address these missing concepts. This discovery puts those students currently in middle school mathematics at a large disadvantage as they begin to take on higher mathematics courses where the curriculum will follow the Common Core State Standards.

After completing my study, I found a research article in which the study's results turned out to be the identical to mine. The study was to teach the addition of fractions by using a video game called *Save Patch*. The hope of the study was that students would apply concepts of underlying rational number addition in order to help the character, Patch, which was in an unsafe place, to safety by bouncing over obstacles to reach his home. In order to do this correctly, students must place trampolines at assorted locations along a grid. Students then had to drag coils (which were labeled with various fraction length) onto the trampoline to make it bouncy. The distance Patch would bounce is the sum of all the coils added to the trampoline. For example, if a student placed two 1/4 coils on the trampoline, Patch would bounce 1/2 of a unit (Kerr and Chung, 2010).

The first error they encountered directly correlates with my results of Assessment 5. The article states "students who made unitizing errors failed to pay attention to the red lines that indicated a unit of length. Instead, these students appeared to see the entire grid as one unit" (Kerr and Chung, 2012). The other error that was discovered correlates to my results from Assessment 4 by saying, "students who made partitioning errors determined the denominator by counting the dots between red lines rather than counting the spaces" (Kerr and Chung, 2012). Our studies were completely unrelated in nature, but produced the same results. This only amplifies the fact that something needs to be done.

Although there are no current recommendations for how this problem should be handled, I have come up with my own. I believe that elementary teachers teaching grade 3 through 5 should be required to attend focused sustained professional development regarding the teaching of fractions. These teachers need a clear idea of what is expected of students, not only in their particular grade, but in future grades as well. These teachers should also be given the materials

needed to teach fractions in the way the students are expected to learn them. These teachers should also be properly trained on how to create good tests involving fraction concepts. It is my hope, that one day I can develop this particular training.

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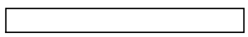
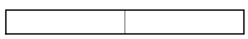
APPENDIX A
ASSESSMENTS

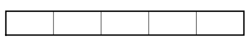
Fraction Quiz **Name:**__________________________

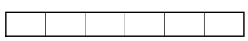
Select the correct word and write it in the blank.

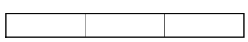
"halves" **"thirds"** **"quarters"** **"fifths"** **"sixths"**

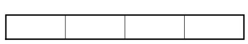
 This is a whole bar. It is not divided.

 This whole bar is divided into ____________________

 This whole bar is divided into ____________________

 This whole bar is divided into ____________________

 This whole bar is divided into ____________________

 This whole bar is divided into ____________________

Fraction Quiz **Name:**__________________________

Select the correct word or phrase and write it next to the fraction.

"one half" **"two thirds"** **"four thirds"** **"one third"**

"three eighths" **"the denominator is five"** **"eight thirds"**

"the denominator is four" **"two halves"** **"the numerator is five"**

$\frac{2}{5}$ ________________________ $\frac{3}{8}$ ________________________

$\frac{2}{2}$ ________________________ $\frac{1}{3}$ ________________________

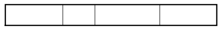
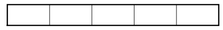
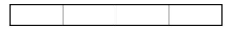
$\frac{2}{3}$ ________________________ $\frac{1}{2}$ ________________________

$\frac{1}{4}$ ________________________ $\frac{8}{3}$ ________________________

$\frac{4}{3}$ ________________________ $\frac{5}{7}$ ________________________

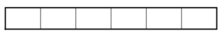
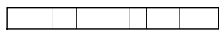
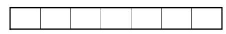
Fraction Quiz **Name:**________________________

Which bar is divided into fourths? Why are the other two **not** divided into fourths?

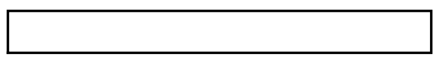
A:  **B:**  **C:** 

Please write your response here in full sentences:

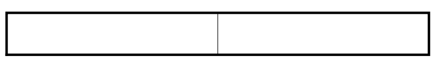
Which bar is divided into sixths? Why are the other two **not** divided into sixths?

A:  **B:**  **C:** 

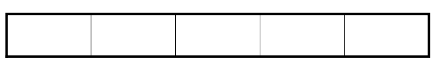
Please write your response here in full sentences:

Divide this bar into thirds: 

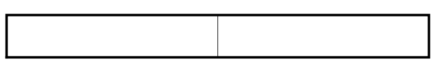
This bar is already divided in half. Show where to cut it to divide it into quarters:



This bar is already divided into fifths. Show where to cut it to divide it into tenths:

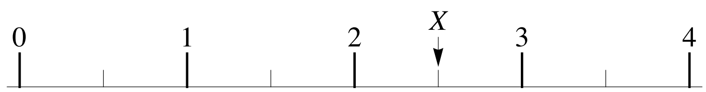


This bar is already divided into halves. Show where to cut it to divide it into sixths:

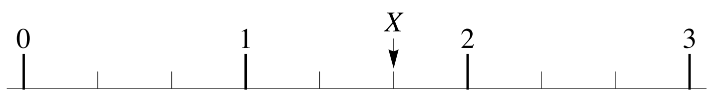


Fraction Quiz **Name:**________________________

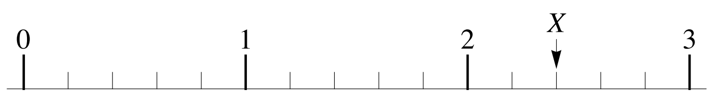
Name the number that is marked by the X:



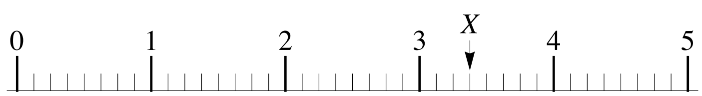
$X =$ ☐



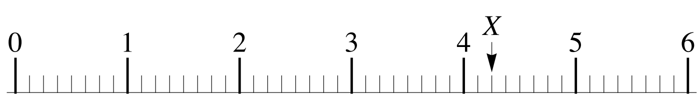
$X =$ ☐



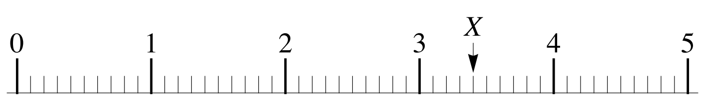
$X =$ ☐



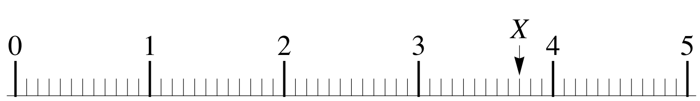
$X =$ ☐



$X =$ ☐

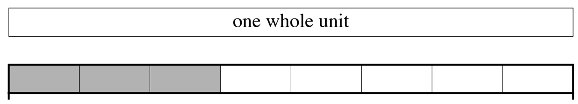


$X =$ ☐

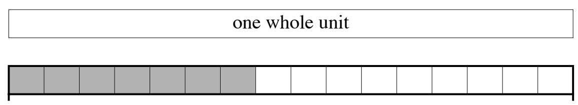


$X =$ ☐

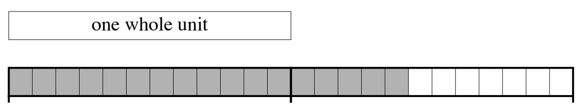
Fraction Quiz **Name:**____________________



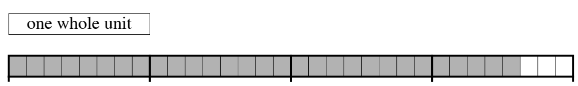
In the picture above, a unit is divided into __________ equal parts. Each of the small equal parts is called a/an __________ of a unit. The number of small equal parts that has been shaded is __________. The length of the shaded part of the bar is ____________________.



In the picture above, a unit is divided into __________ equal parts. Each of the small equal parts is called a/an __________ of a unit. The number of small equal parts that has been shaded is __________. The length of the shaded part of the bar is ____________________.



In the picture above, a unit is divided into __________ equal parts. Each of the small equal parts is called a/an __________ of a unit. The number of small equal parts that has been shaded is __________. The length of the shaded part of the bar is ____________________.

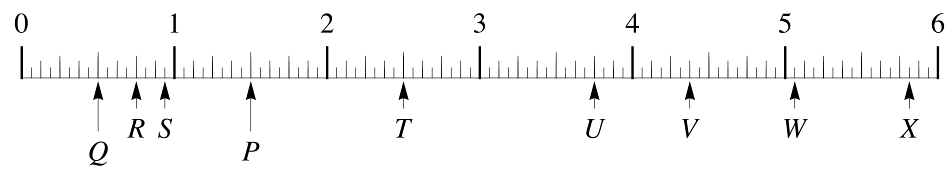


In the picture above, a unit is divided into __________ equal parts. Each of the small equal parts is called a/an __________ of a unit. The number of small equal parts that has been shaded is __________. The length of the shaded part of the bar is ____________________.

In the picture above, a unit is divided into __________ equal parts. Each of the small equal parts is called a/an __________ of a unit. The number of small equal parts that has been shaded is __________. The length of the shaded part of the bar is ____________________.

Fraction Quiz **Name:**__________________________

Here is a piece of the number line, marked at whole numbers, halves, quarters, eighths and sixteenths. Some numbers are labeled with the letters $P, Q, R, \ldots, X$. For example, P is $3/2$ (as an improper fraction), or $1\frac{1}{2}$ (as a mixed number).



In the spaces below, name the number for each letter, both as a fraction–proper or improper–and, if appropriate, as a mixed number.

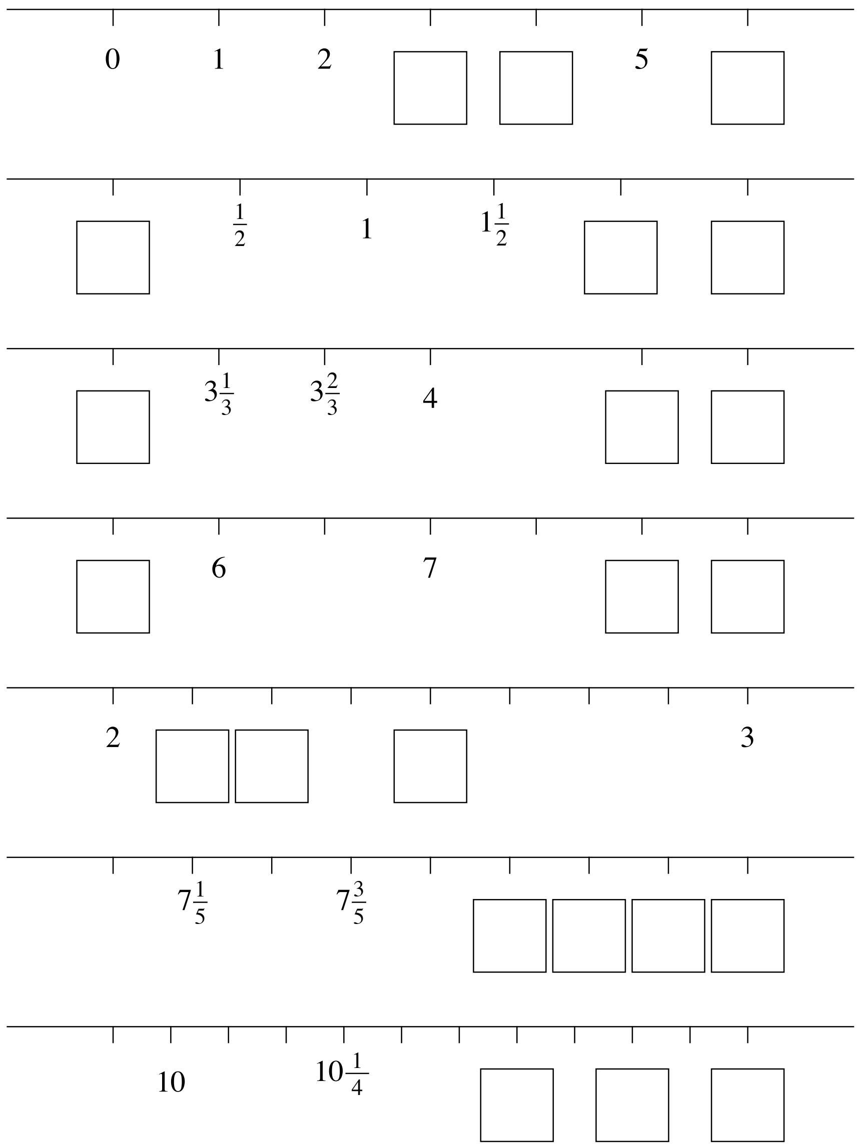
$P =$ __$3/2$__ $=$ __$1\frac{1}{2}$__ $Q =$ __$1/2$__ $=$ __(*proper*)__ $R =$ ________ $=$ ________

$S =$ ________ $=$ ________ $T =$ ________ $=$ ________ $U =$ ________ $=$ ________

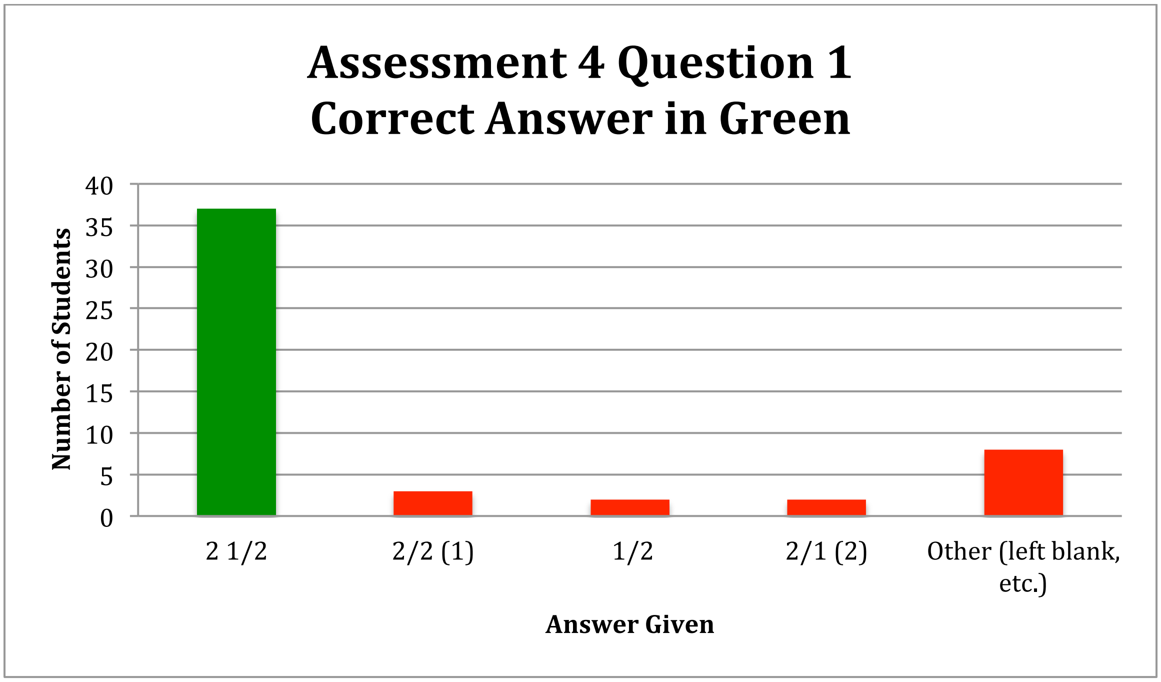
$V =$ ________ $=$ ________ $W =$ ________ $=$ ________ $X =$ ________ $=$ ________

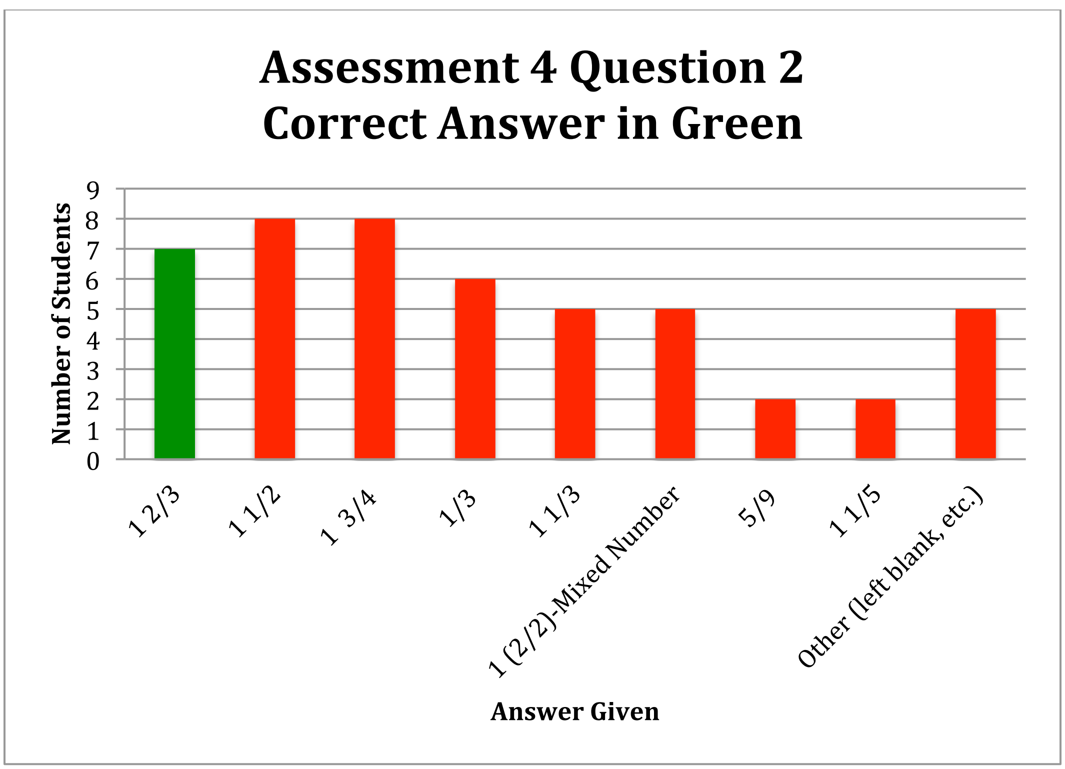
Fraction Quiz **Name:**__________________________

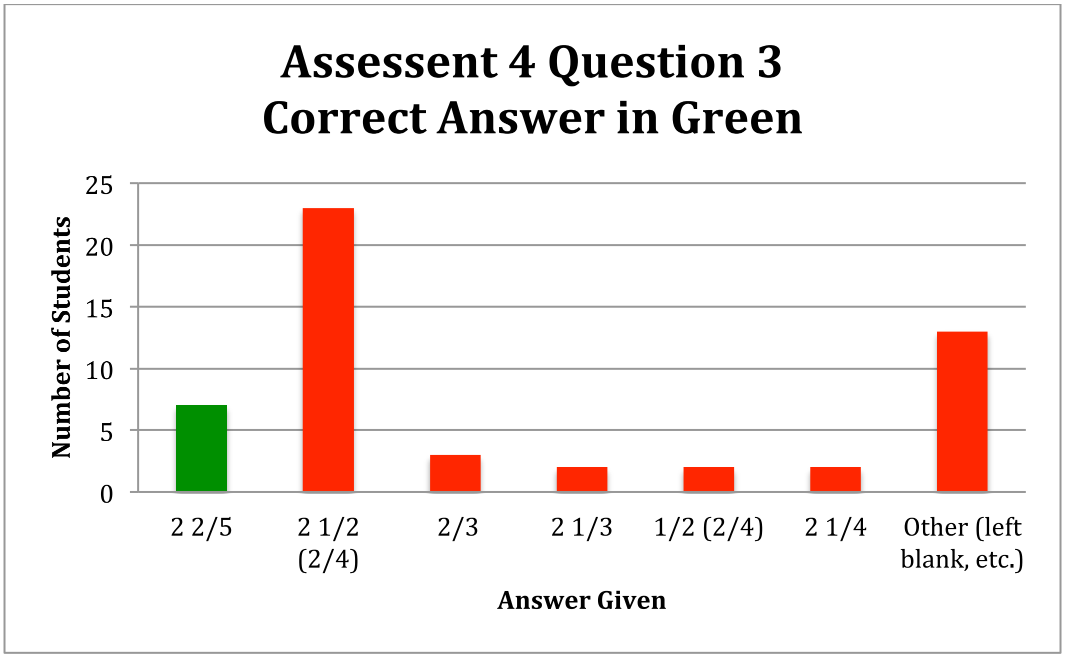
Put the appropriate labels in the boxes:

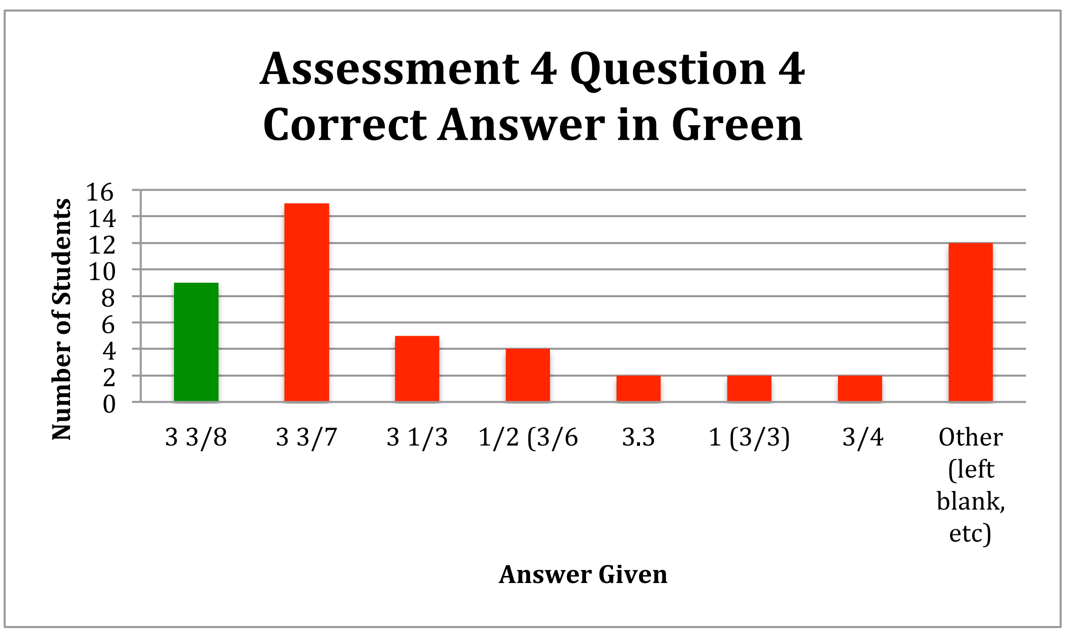


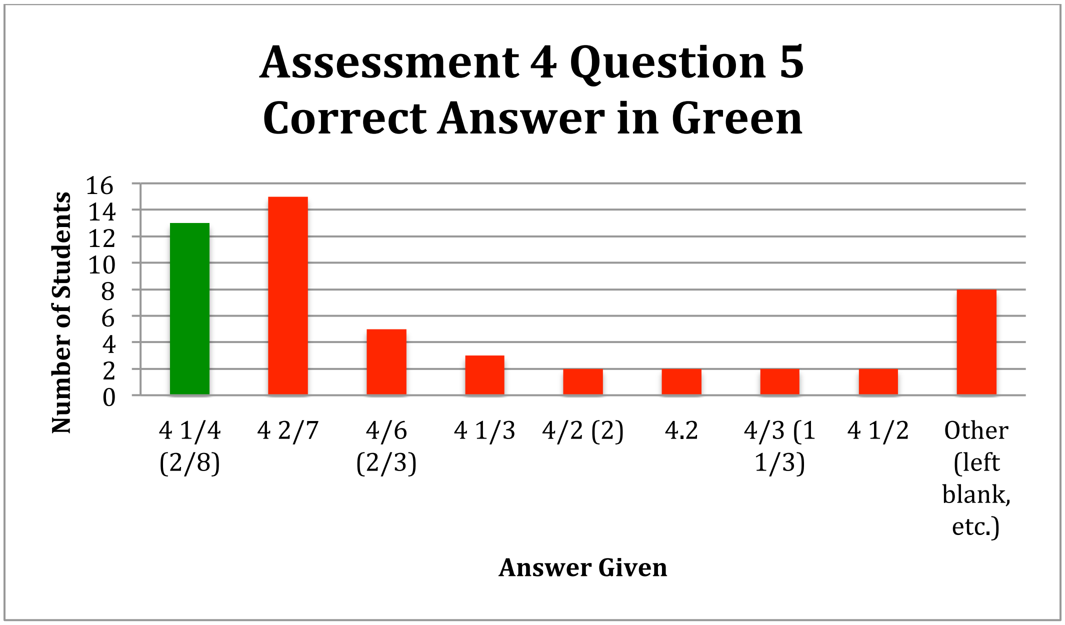
APPENDIX B
ITEMIZED GRAPHS OF STUDENT ANSWERS

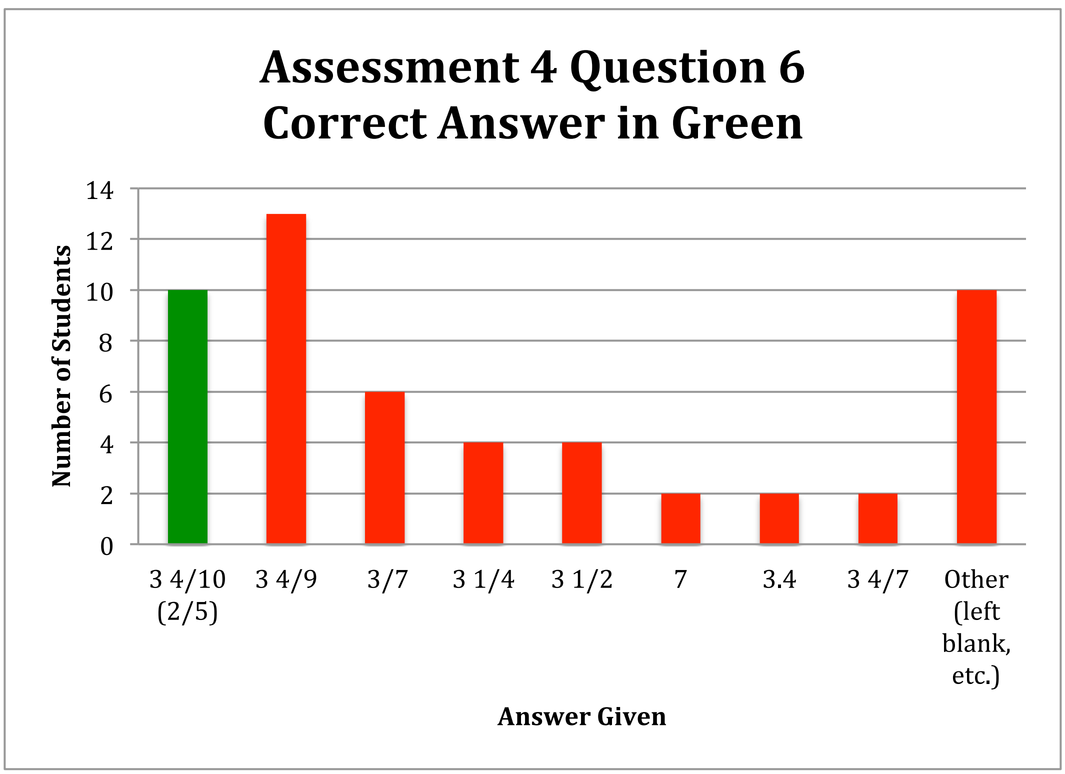


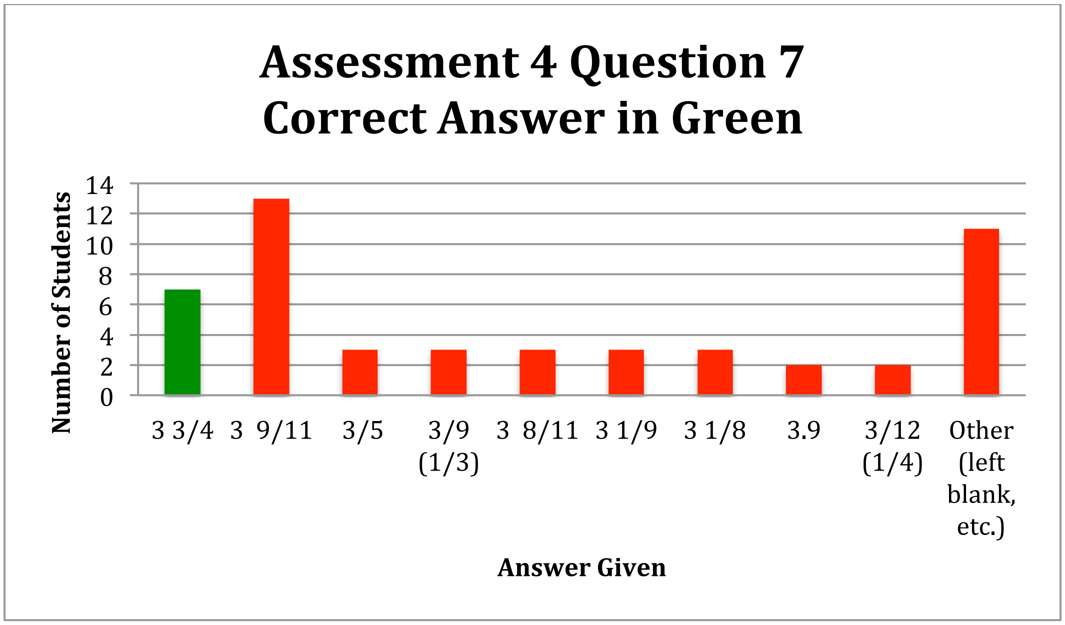


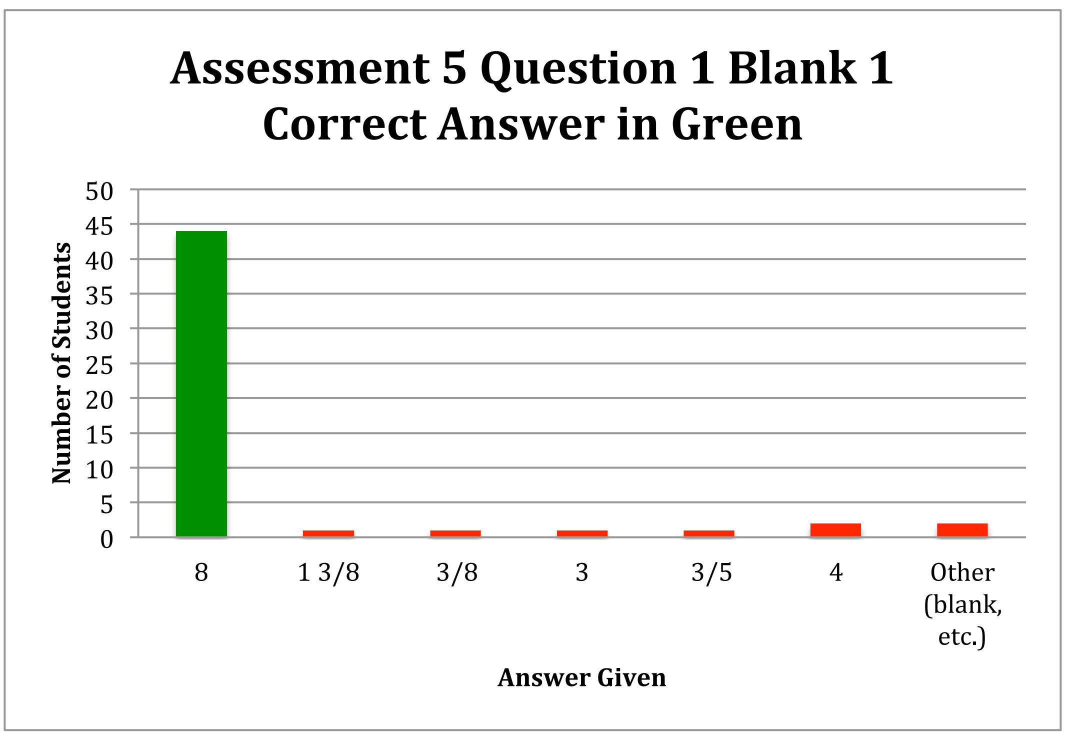

Assessent 4 Question 3
Correct Answer in Green
Number of Students
25
20
15
10
5
0
2 2/5
2 1/2 (2/4)
2/3
2 1/3
1/2 (2/4)
2 1/4
Other (left blank, etc.)
Answer Given

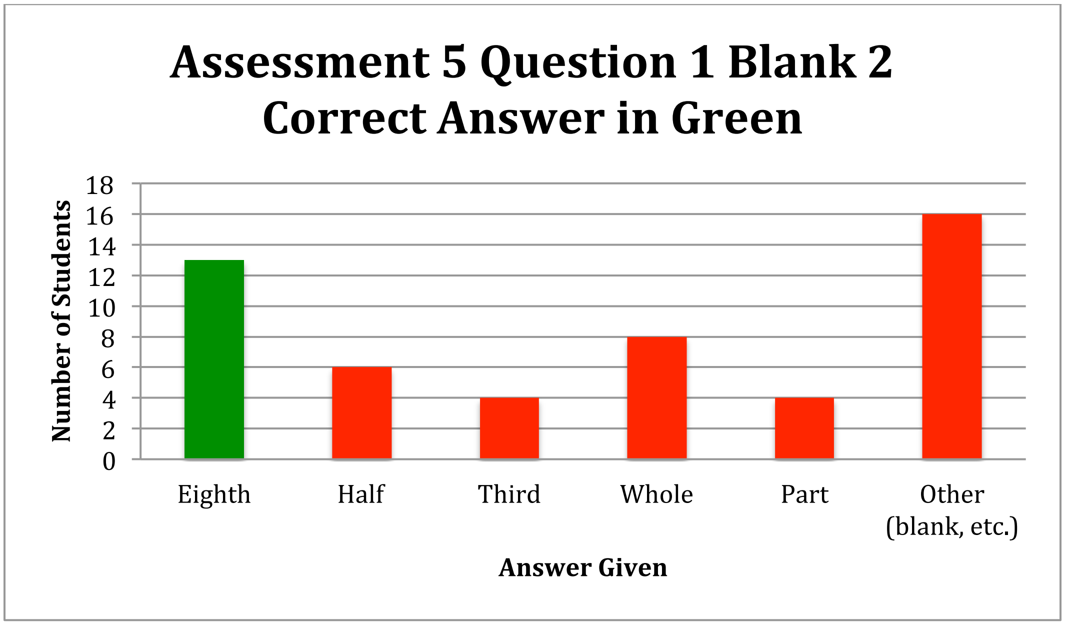

Assessment 4 Question 4
Correct Answer in Green
Number of Students
16
14
12
10
8
6
4
2
0
3 3/8
3 3/7
3 1/3
1/2 (3/6
3.3
1 (3/3)
3/4
Other (left blank, etc)
Answer Given

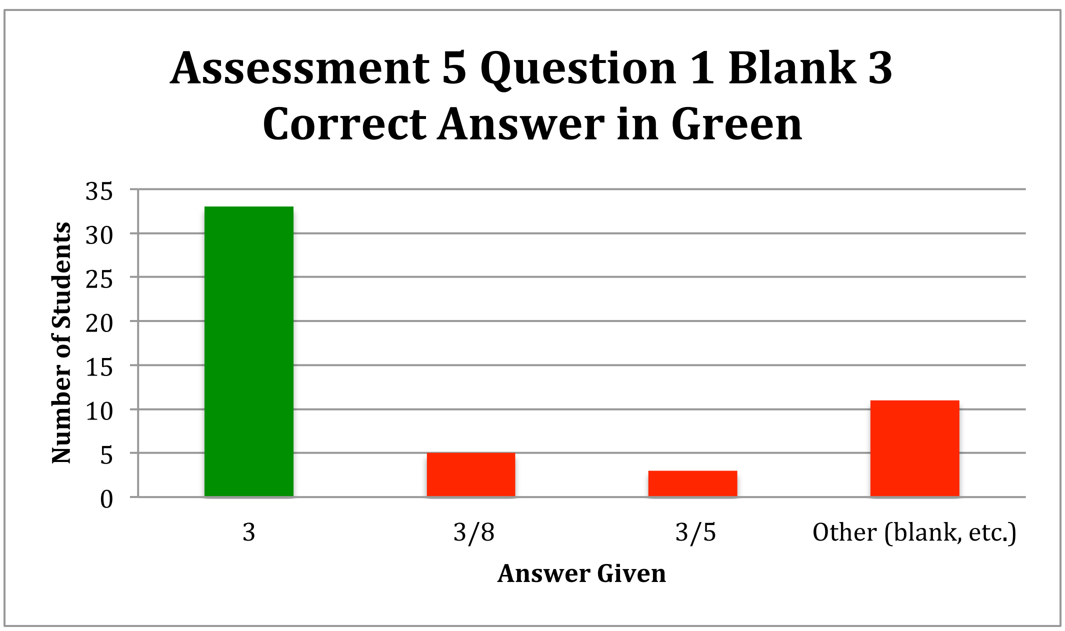

Assessment 4 Question 5
Correct Answer in Green
Number of Students
16
14
12
10
8
6
4
2
0
4 1/4 (2/8)
4 2/7
4/6 (2/3)
4 1/3
4/2 (2)
4.2
4/3 (1 1/3)
4 1/2
Other (left blank, etc.)
Answer Given

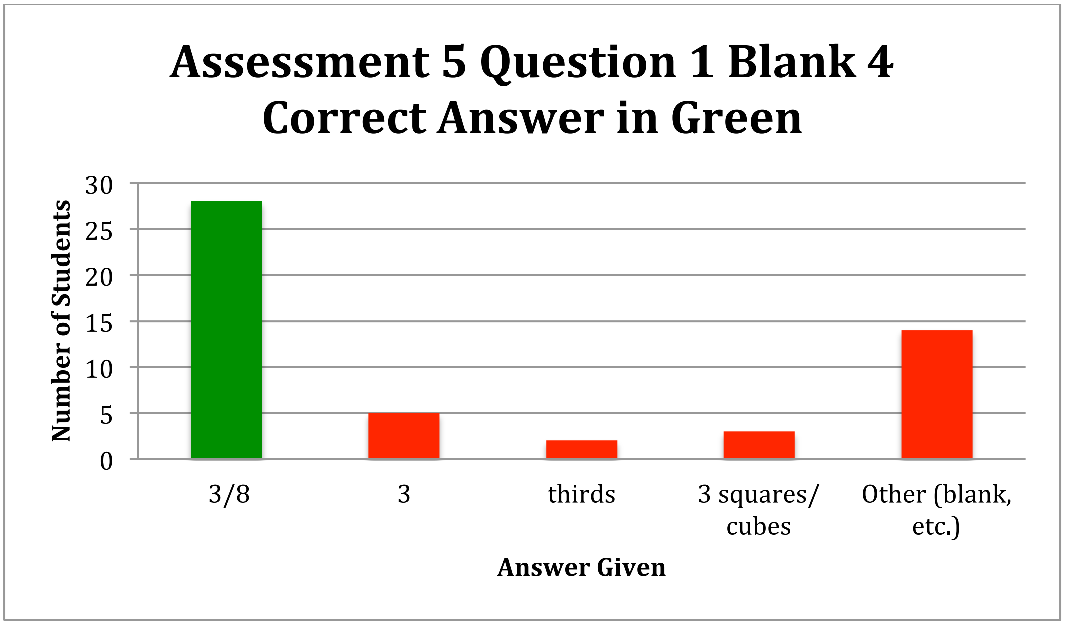

Assessment 4 Question 6
Correct Answer in Green
Number of Students
14
12
10
8
6
4
2
0
3 4/10 (2/5)
3 4/9
3/7
3 1/4
3 1/2
7
3.4
3 4/7
Other (left blank, etc.)
Answer Given

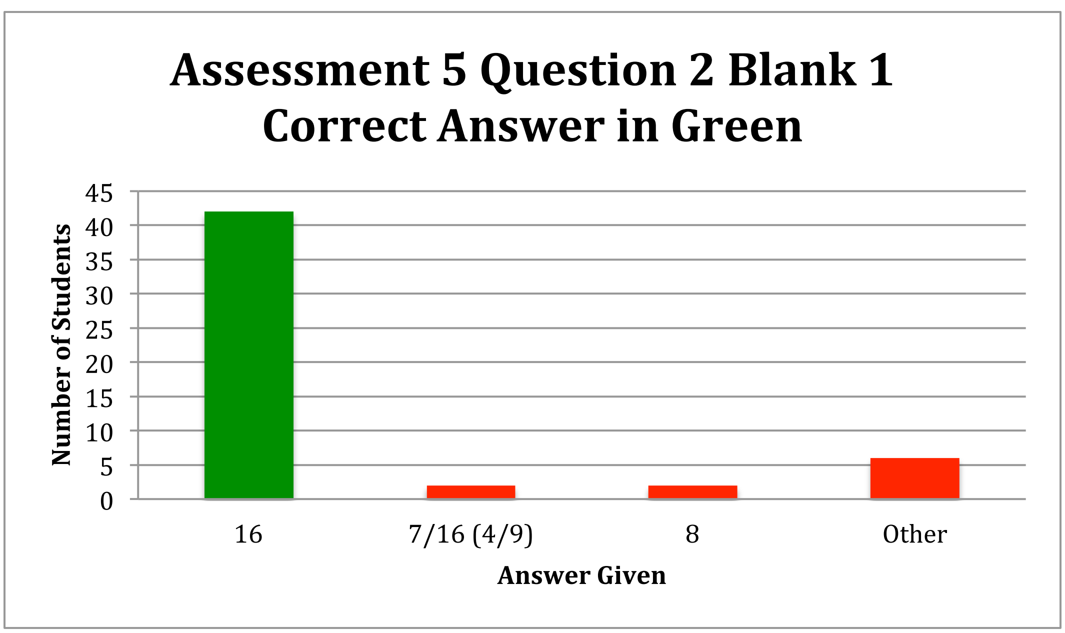

Assessment 4 Question 7
Correct Answer in Green
Number of Students
14
12
10
8
6
4
2
0
3 3/4
3 9/11
3/5
3/9 (1/3)
3 8/11
3 1/9
3 1/8
3.9
3/12 (1/4)
Other (left blank, etc.)
Answer Given

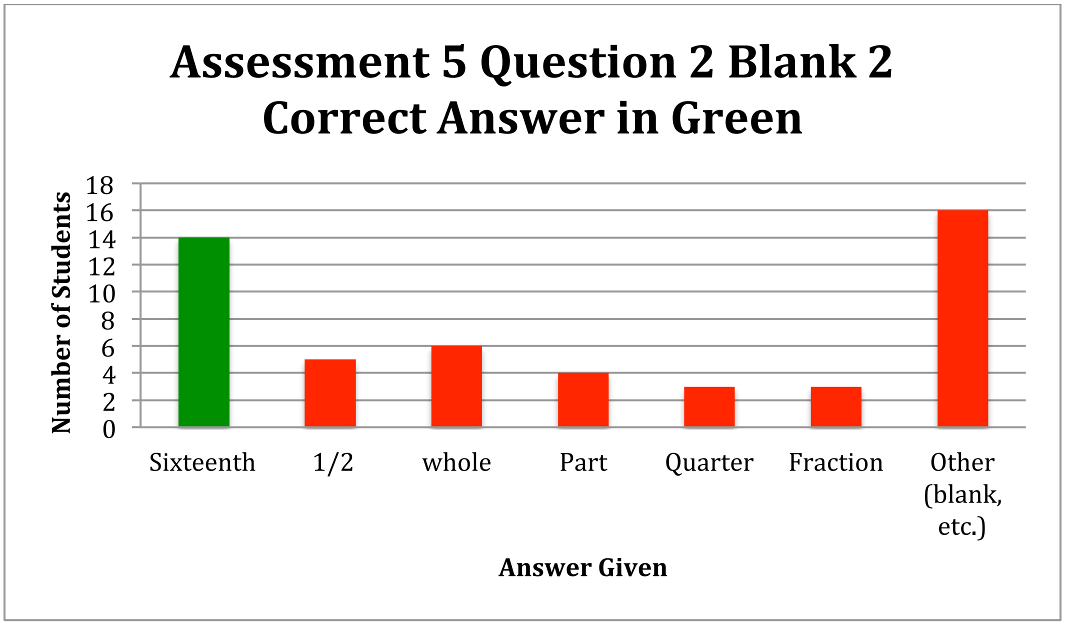

Assessment 5 Question 1 Blank 1
Correct Answer in Green
Number of Students
50
45
40
35
30
25
20
15
10
5
0
8
1 3/8
3/8
3
3/5
4
Other (blank, etc.)
Answer Given

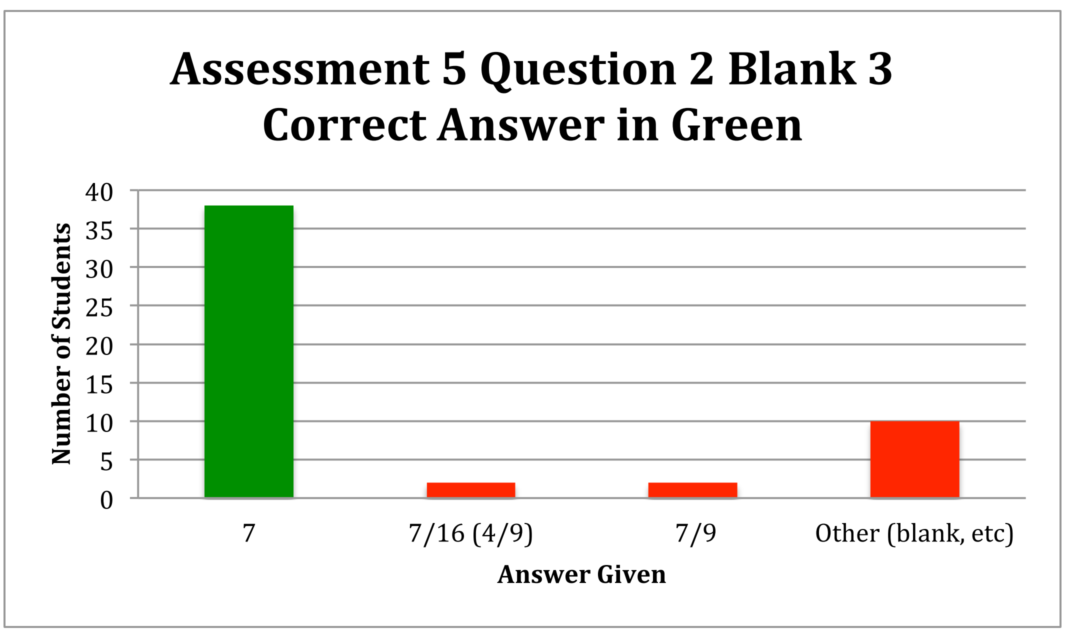

Assessment 5 Question 1 Blank 2
Correct Answer in Green
Number of Students
18
16
14
12
10
8
6
4
2
0
Eighth
Half
Third
Whole
Part
Other (blank, etc.)
Answer Given

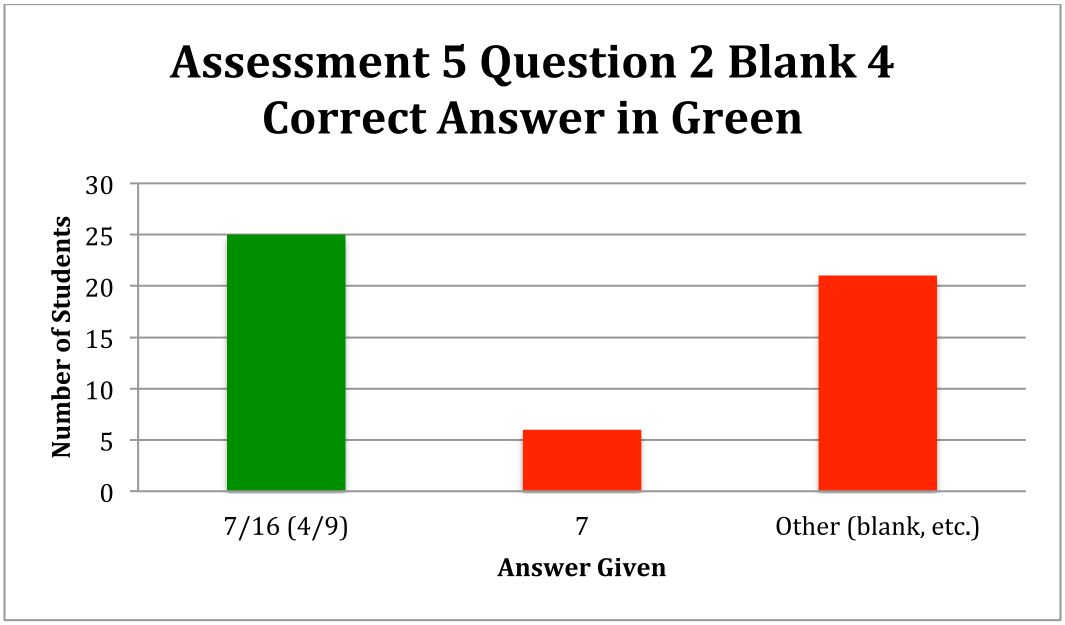

Assessment 5 Question 1 Blank 3
Correct Answer in Green
Number of Students
35
30
25
20
15
10
5
0
3
3/8
3/5
Other (blank, etc.)
Answer Given

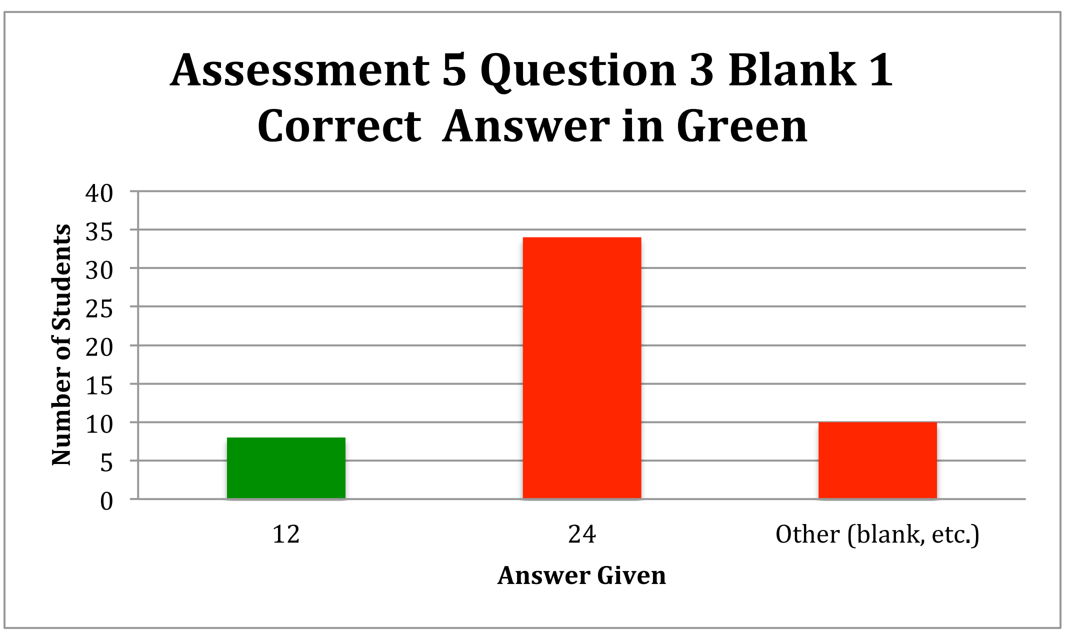

Assessment 5 Question 1 Blank 4
Correct Answer in Green
Number of Students
30
25
20
15
10
5
0
3/8
3
thirds
3 squares/ cubes
Other (blank, etc.)
Answer Given

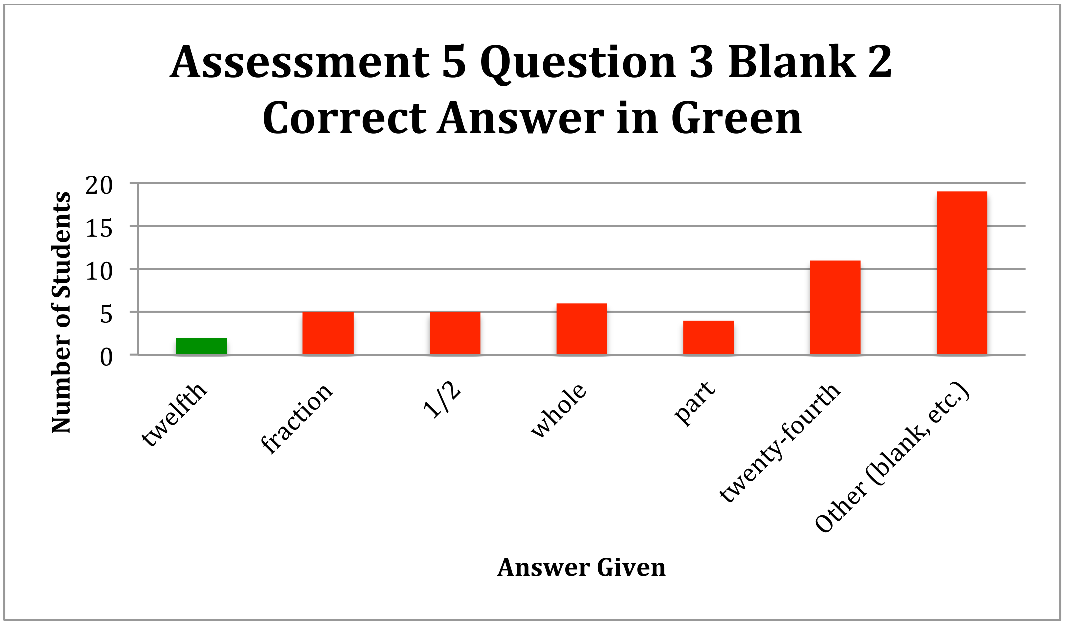

Assessment 5 Question 2 Blank 1
Correct Answer in Green
Number of Students
45
40
35
30
25
20
15
10
5
0
16
7/16 (4/9)
8
Other
Answer Given

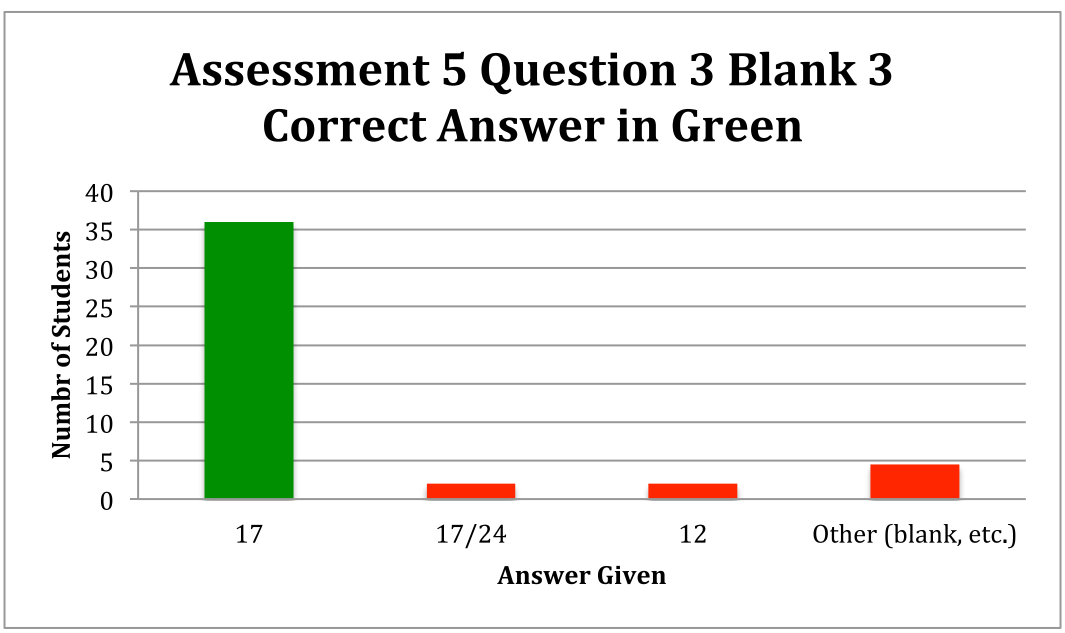

Assessment 5 Question 2 Blank 2
Correct Answer in Green
Number of Students
18
16
14
12
10
8
6
4
2
0
Sixteenth
1/2
whole
Part
Quarter
Fraction
Other (blank, etc.)
Answer Given

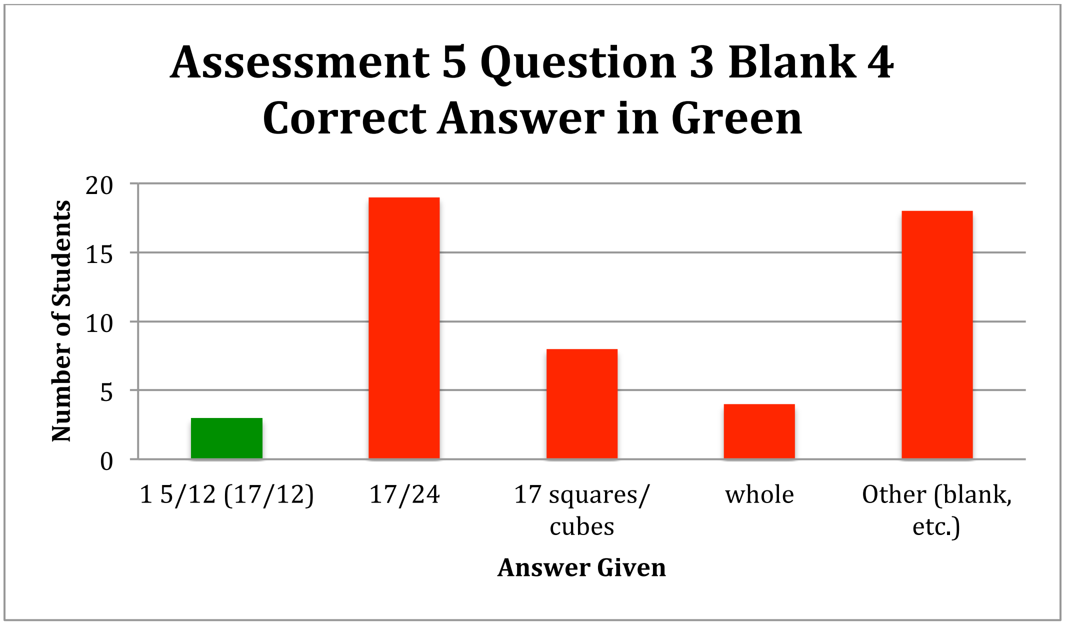

Assessment 5 Question 2 Blank 3
Correct Answer in Green
Number of Students
40
35
30
25
20
15
10
5
0
7
7/16 (4/9)
7/9
Other (blank, etc)
Answer Given


Assessment 5 Question 2 Blank 4
Correct Answer in Green
Number of Students
30
25
20
15
10
5
0
7/16 (4/9)
7
Other (blank, etc.)
Answer Given

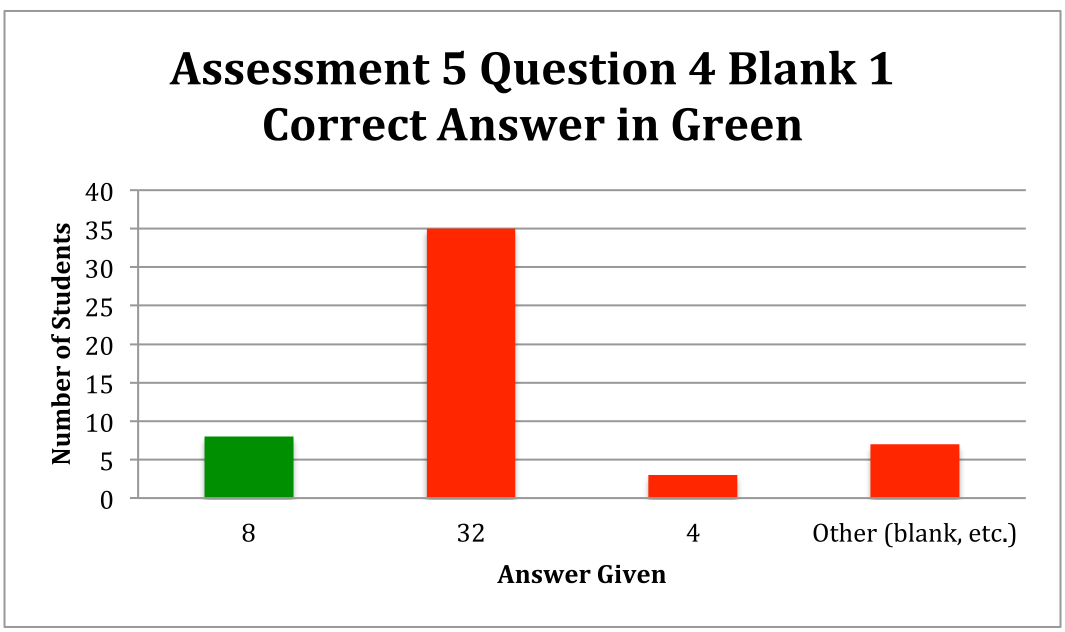

Assessment 5 Question 3 Blank 1
Correct Answer in Green
Number of Students
40
35
30
25
20
15
10
5
0
12
24
Other (blank, etc.)
Answer Given


Assessment 5 Question 3 Blank 2
Correct Answer in Green
Number of Students
20
15
10
5
0
twelfth
fraction
1/2
whole
part
twenty-fourth
Other (blank, etc.)
Answer Given

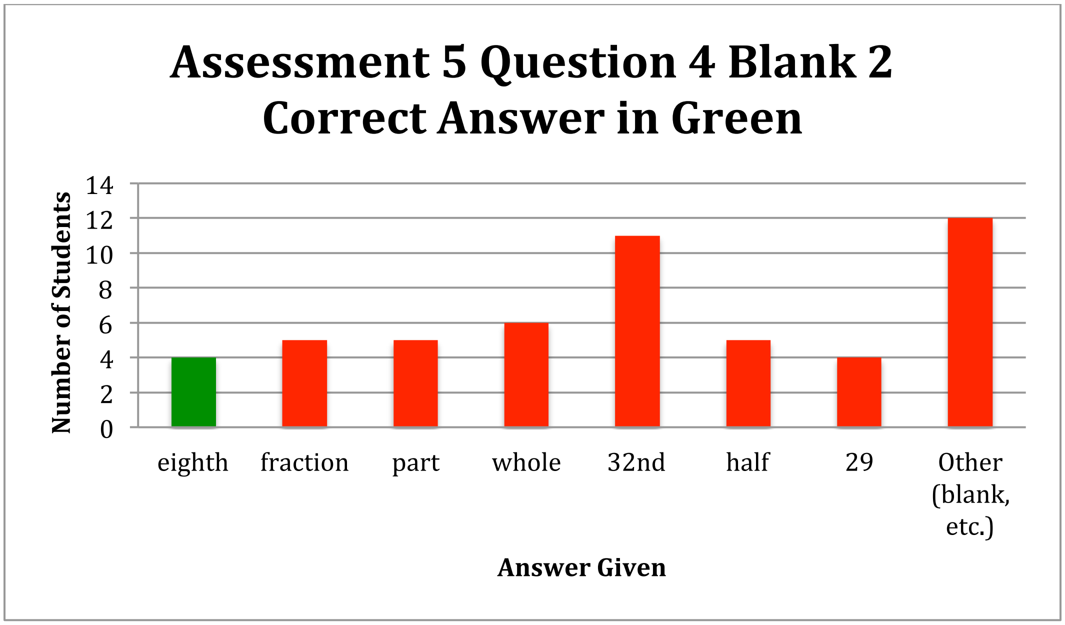

Assessment 5 Question 3 Blank 3
Correct Answer in Green
Numbr of Students
40
35
30
25
20
15
10
5
0
17
17/24
12
Other (blank, etc.)
Answer Given

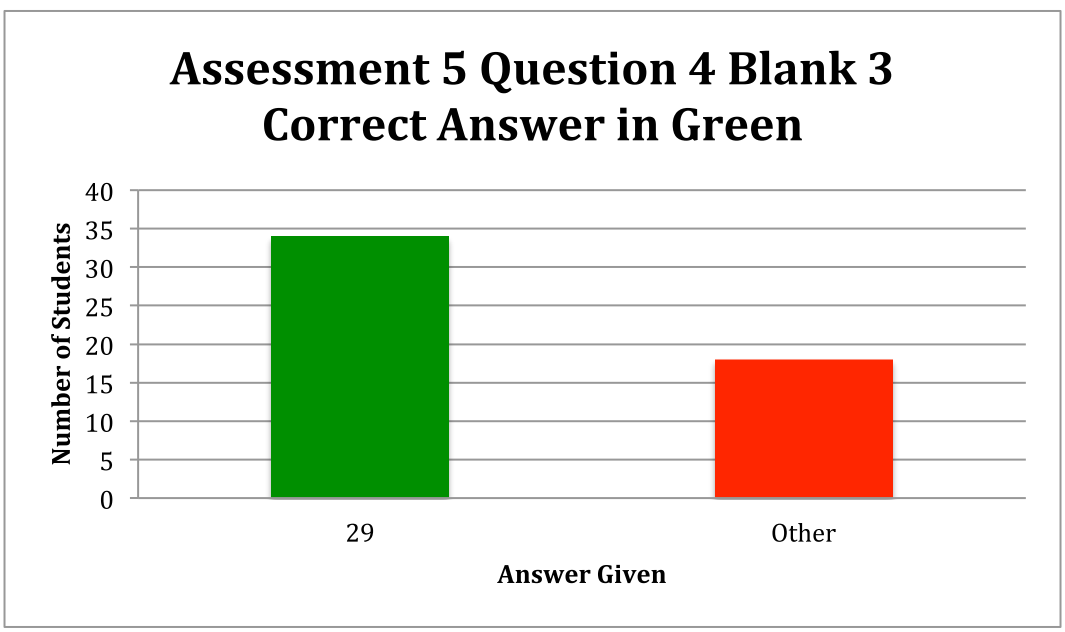


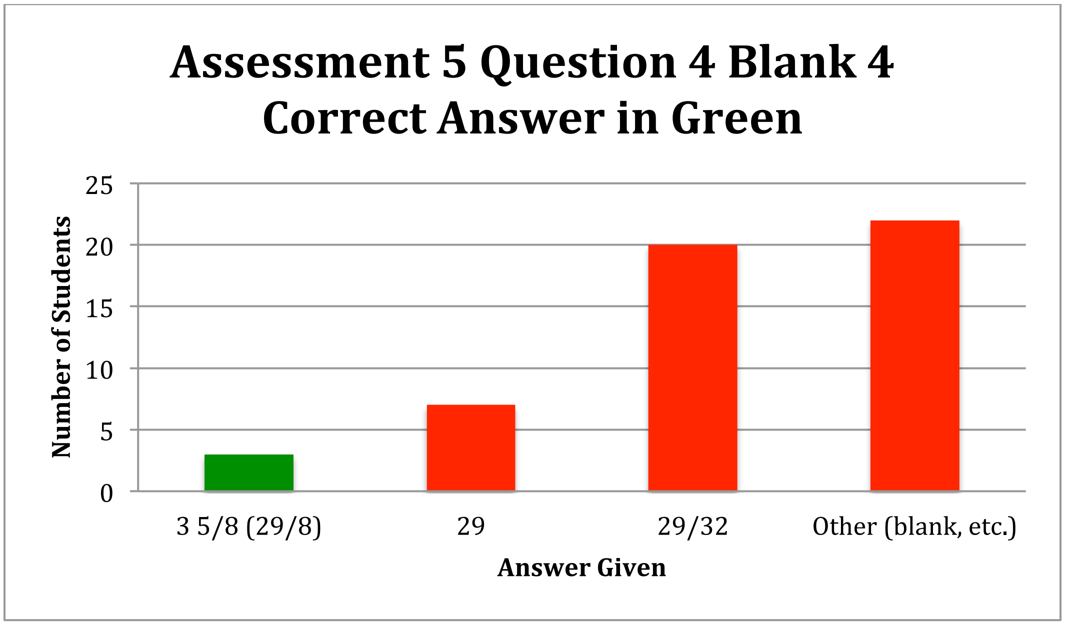
Assessment 5 Question 3 Blank 4
Correct Answer in Green
Number of Students
20
15
10
5
0
1 5/12 (17/12)
17/24
17 squares/ cubes
whole
Other (blank, etc.)
Answer Given

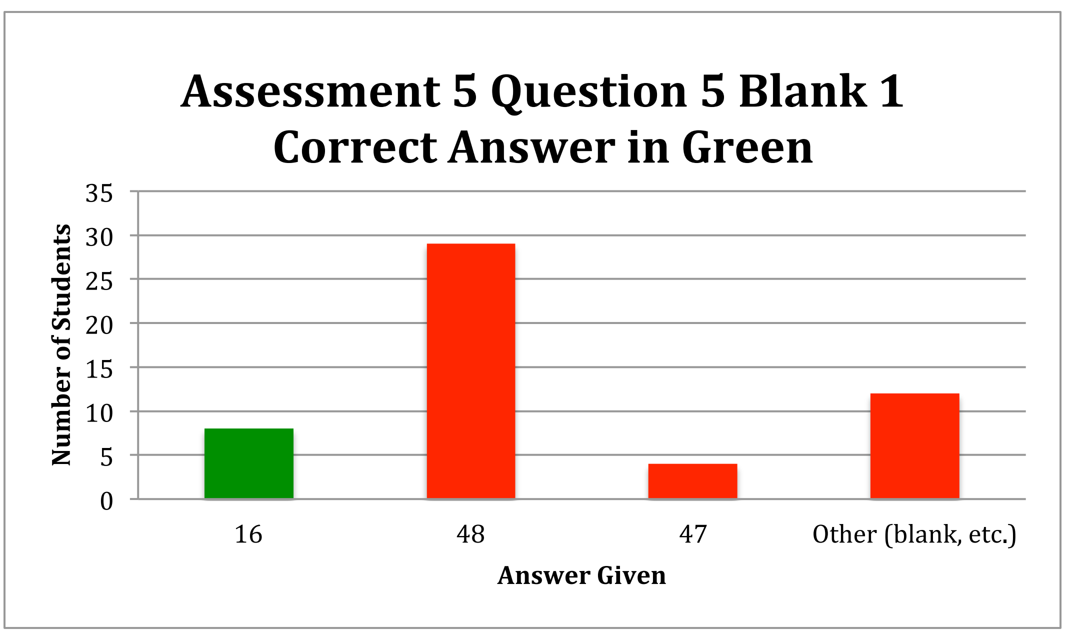


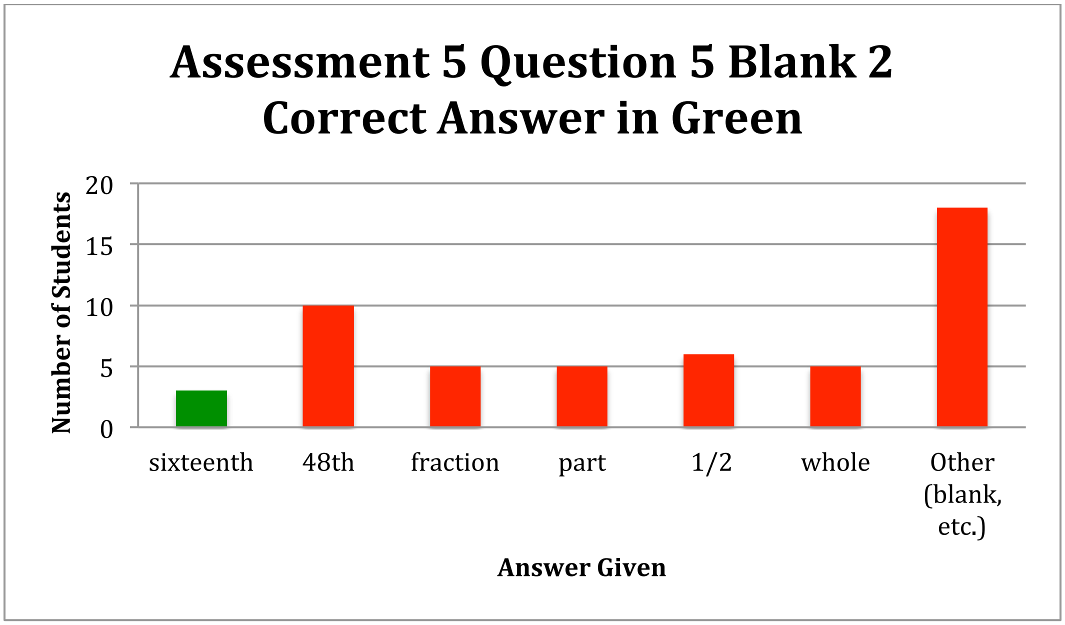
Assessment 5 Question 4 Blank 1
Correct Answer in Green
Number of Students
40
35
30
25
20
15
10
5
0
8
32
4
Other (blank, etc.)
Answer Given

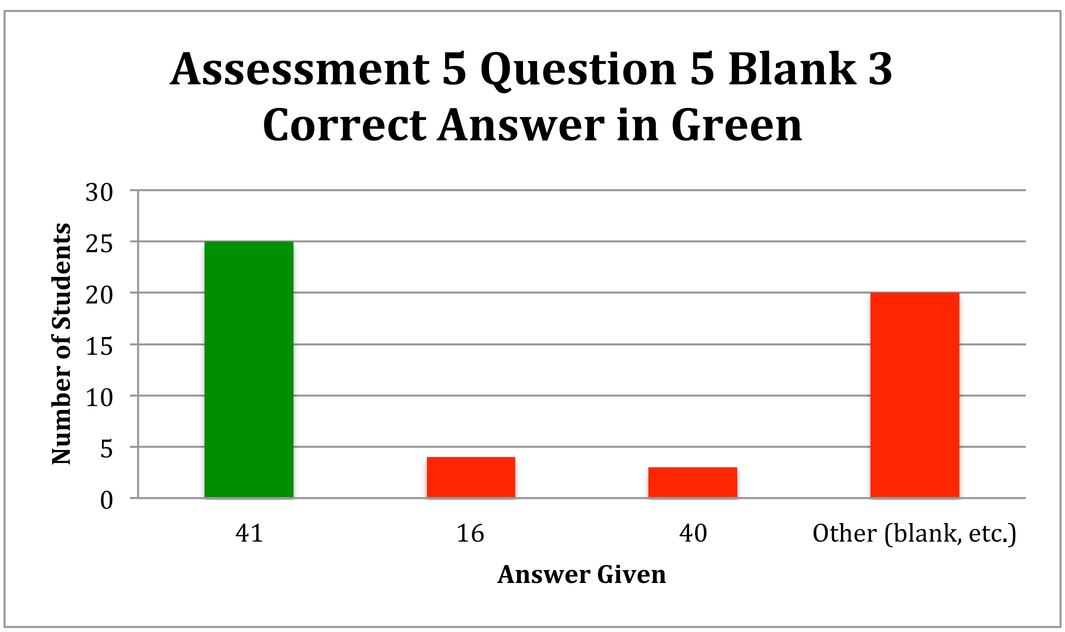

Assessment 5 Question 4 Blank 2
Correct Answer in Green
Number of Students
14
12
10
8
6
4
2
0
eighth
fraction
part
whole
32nd
half
29
Other (blank, etc.)
Answer Given

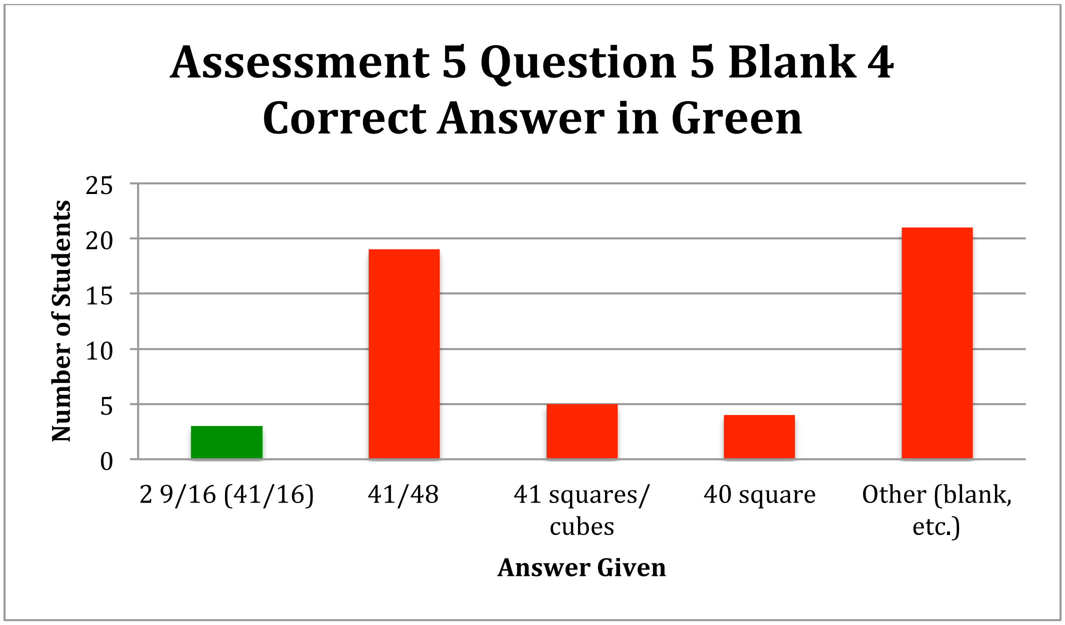

Assessment 5 Question 4 Blank 3
Correct Answer in Green
Number of Students
40
35
30
25
20
15
10
5
0
29
Other
Answer Given


Assessment 5 Question 4 Blank 4
Correct Answer in Green
Number of Students
25
20
15
10
5
0
3 5/8 (29/8)
29
29/32
Other (blank, etc.)
Answer Given


Assessment 5 Question 5 Blank 1
Correct Answer in Green
Number of Students
35
30
25
20
15
10
5
0
16
48
47
Other (blank, etc.)
Answer Given


Assessment 5 Question 5 Blank 2
Correct Answer in Green
Number of Students
20
15
10
5
0
sixteenth
48th
fraction
part
1/2
whole
Other (blank, etc.)
Answer Given


Assessment 5 Question 5 Blank 3
Correct Answer in Green
Number of Students
30
25
20
15
10
5
0
41
16
40
Other (blank, etc.)
Answer Given


Assessment 5 Question 5 Blank 4
Correct Answer in Green
Number of Students
25
20
15
10
5
0
2 9/16 (41/16)
41/48
41 squares/ cubes
40 square
Other (blank, etc.)
Answer Given

APPENDIX C
IRB FORMS

Application for Exemption from Institutional Oversight

Unless qualified as meeting the specific criteria for exemption from Institutional Review Board (IRB) oversight, ALL LSU research/projects using living humans as subjects, or samples, or data obtained from humans, directly or indirectly, with or without their consent, must be approved or exempted in advance by the LSU IRB. This Form helps the PI determine if a project may be exempted, and is used to request an exemption.

LSU
Institutional Review Board
Dr. Robert Mathews, Chair
131 David Boyd Hall
Baton Rouge, LA 70803
P: 225.578.8692
F: 225.578.6792
irb@lsu.edu
lsu.edu/irb

– Applicant, Please fill out the application in its entirety and include the completed application as well as parts A-E, listed below, when submitting to the IRB. Once the application is completed, please submit two copies of the completed application to the IRB Office or to a member of the Human Subjects Screening Committee. Members of this committee can be found at http://www.lsu.edu/screeningmembers.shtml

– A Complete Application Includes All of the Following:
- **(A)** Two copies of this completed form and two copies of part B thru E.
- **(B)** A brief project description (adequate to evaluate risks to subjects and to explain your responses to Parts 1&2)
- **(C)** Copies of all instruments to be used.
  - *If this proposal is part of a grant proposal, include a copy of the proposal and all recruitment material.
- **(D)** The consent form that you will use in the study (see part 3 for more information.)
- **(E)** Certificate of Completion of Human Subjects Protection Training for all personnel involved in the project, including students who are involved with testing or handling data, unless already on file with the IRB. Training link: (http://phrp.nihtaining.com/users/login.php.)
- **(F)** IRB Security of Data Agreement: (http://www.lsu.edu/irb/IRB%20Security%20of%20Data.pdf)

1) Principal Investigator: Jeanne' E. Bass **Rank:** Student

Dept: Math/MNS/LaMSTI **Ph:** 225.573.3399 **E-mail:** Jbass3184@yahoo.com

2) Co Investigator(s): please include department, rank, phone and e-mail for each

James Madden Professor
madden@math.lsu.edu 225.978.3525

IRB# E5562 LSU Proposal #
- Complete Application
- Human Subjects Training

3) Project Title: Teaching Fractions + Decimals Together Using Pictures To Enhance Understanding of Decimals + Their Expansions

Study Exempted By:
Dr. Robert C. Mathews, Chairman
Institutional Review Board
Louisiana State University
203 B-1 David Boyd Hall
225-578-8692 | www.lsu.edu/irb
Exemption Expires: 7-17-2014

4) Proposal? (yes or no) **If Yes, LSU Proposal Number**

Also, if YES, either
- ○ **This application completely matches the scope of work in the grant**

OR
- ○ **More IRB Applications will be filed later**

5) Subject pool (e.g. Psychology students) Math students in the school I work in.

*Circle any **"vulnerable populations" to be used:** (children <18; the mentally impaired, pregnant women, the ages, other). Projects with incarcerated persons cannot be exempted.

6) PI Signature Jeanne E Bass **Date** 6.30.11 (no per signatures)

**** I certify my responses are accurate and complete.** If the project scope or design is later changes, I will resubmit for review. I will obtain written approval from the Authorized Representative of all non-LSU institutions in which the study is conducted. I also understand that it is my responsibility to maintain copies of all consent forms at LSU for three years after completion of the study. If I leave LSU before that time the consent forms should be preserved in the Departmental Office.

Screening Committee Action: Exempted ✓ Not Exempted ____ Category/Paragraph 1

Reviewer Mathews **Signature** [signature] **Date** 7/18/11

Study Exempted By:
Dr. Robert C. Mathews, Chairman
Institutional Review Board
Louisiana State University
203 B-1 David Boyd Hall
225-578-8692 | www.lsu.edu/irb
Exemption Expires: 7/17/2014

Parental Consent Form

Project Title: Teaching fractions as points on the number to help enhance fractional understanding.

Performance Site: Iberville Mathematics, Science and Arts Academy-West

Investigators: The following investigator is available for questioning, Monday – Friday 7:00 a.m.-2:30 p.m.
Jeanne Bass
Mathematics Dept. Faculty
Master's of Natural Science Candidate at LSU
225.687.6845
jeannebass@ipsb.net

Purpose of Study: The purpose of this study is to collect data for my Master's of Natural Science thesis project. It is also to help develop a teaching strategy to enhance students' long term understanding of fractions.

Inclusion Criteria: All 6th grade math students attending MSA during the 2012-2013 school year and in Ms. Bass's class.

Exclusion Criteria: None

Description of the Study: In all of my sixth grade classes this year I plan to teach fractions as points on the number line in hopes of enhancing long term fractional understanding. In the LCC, fractions are first taught in Unit 3: Fractions, Decimals and Parts and continue on in Unit 4: Operations with Fractions. Before beginning these units, I plan to give students a pre-test evaluating their current understanding of fractions. The test will consist of questions asking students to identify fractions using models, locate fractions on the number line, rename fractions, and locate missing fractions on a number line. This pre test will be graded but will not go into the investigators grade book. Results will only be used as data. I will then begin teaching the units using the number line and constantly referring everything back to a fraction equivalent. At the end of the two units, I plan to give a post-test with exactly the same questions. The post-test will be graded and the grades will be entered into the investigator's grade book. Also, I will evaluate and analyze the results.

Benefits: Subjects will have the opportunity to better understand fractions and their expansions, therefore, retaining the skill for higher grades without the need for remediation.

Risks: There are no known risks.

Right to Refuse: Participation in this study is completely voluntary, and a child will become part of the study only if both child and parent agree to the child's participation. At any time, either the subject may withdraw from the study or the subject's parent may withdraw the subject from the study without penalty or loss of any benefit to which they might otherwise be entitled. If a subject or subject's parent refuse to participate in the study, the subject will still be required to complete all work involved, but results will not be used in the study.

Privacy: The school records of participants in this study may be reviewed by investigators. Results of the study may be published, but no names or identifying information included for publication. Subject identity will remain confidential unless disclosure is required by law.

Financial Information: There is no cost for participation in this study, nor is there any compensation to the subjects for participation.

Signatures:

The study has been discussed with me and all my questions have been answered. I may direct additional questions regarding study specifics to the investigator. If I have questions about subjects' rights or other concerns, I can contact Robert C. Matthews, Chairman, Institutional Review Board, (225) 578-8692, irb@lsu.edu, www.lsu.edu/irb. I will allow my child to participate in the study described above and acknowledge the investigator's obligation to provide me with a signed copy of this consent form.

Parent's Signature:____________________________ Date:________________

The parent/guardian has indicated to me that he/she is unable to read. I certify that I have read this consent form to the parent/guardian and explained that by completing the signature line above he/she has given permission for the child to participate in the study.

Signature of Reader:____________________________ Date:________________

Study Exempted By:
Dr. Robert C. Mathews, Chairman
Institutional Review Board
Louisiana State University
203 B-1 David Boyd Hall
225-578-8692 | www.lsu.edu/irb
Exemption Expires: 7/17/2014

Child Assent Form

I, ________________________, agree to participate in a study that will possibly help me understand fractions more completely. I will have to do special school work for the teacher in my classroom. I will have to take a pre-test and a post-test. I have to follow all classroom rules at all times. I can decide to stop being in the study at any time without getting in trouble. I also realize if I decide to stop participating, I still have to complete all work involved, but that my work and results will not be included in the data.

Child's Signature:__________________________ Age: _______ Date:________________

Witness*: ______________________________ Date:___________________

*(N.B. Witness must be present for the assent process, not just the signature by the minor.)

VITA

Jeanne Elizabeth Bass was born in Opelousas, LA, to Joe and Rebecca Bass. She is the youngest child with an older sister, and is the mother of Braydin James Kenly. She graduated from Nicholls State University with a Bachelor of General Studies, with a minor in Culinary Arts in December 2007. She received her alternative certification in middle school mathematics through Louisiana Resource Center for Education (LRCE). Upon completion from LRCE she taught Sixth Grade Mathematics for one year at Crescent Middle and Elementary School in Iberville Parish. She then began teaching at the Iberville Mathematics, Science, and Arts Academy-West in Iberville Parish and continues there teaching Sixth and Seventh Grade Mathematics.