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Getting off the ground with AP Calculus

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ABSTRACT

"The study of mathematics, like the Nile, begins in minuteness but ends in magnificence." – Charles Caleb Colton

Caught in a downward spiral, American mathematical students continue to be outperformed by their international peers as reported by the TIMSS. This has left educators scrambling to find an instructional strategy that will slow this decline in mathematical literacy. This thesis reports on the framework that a new teacher could use to develop and establish a successful AP Calculus program, while increasing mathematical literacy, equity in the classroom, and student achievement through the integration of educational technology into current instructional trends. This thesis speaks to the advantages of educational technology in supplementing the classroom transformation from a teacher-centered classroom to a student-centered classroom. Such technology can offer many potential advantages to educators, by providing instantaneous feedback, tutorial assistance and increased practice opportunities for each and every assigned problem, affording students the opportunity to develop content mastery, and freeing teachers to teach.

Herein lays the true novelty of this thesis. In the appendix, new AP teachers are provided a suitcase, equipped with everything they need to get AP Calculus off the ground. In this suitcase, they will find the instructor's resource kit, a collection of lecture notes, student-centered activities, thought provoking projects, helpful hints and other instructional resources, and MyMathLab online system by Pearson Education serving as the online component. The combination of MyMathLab and the instructor's resource kit act as a conceptual compass, helping the teacher maintain a steady bearing while allowing students time to stop and study concepts in further detail.

CHAPTER 1 EDUCATIONAL VISION

1.1 Prologue

Imagine that you are organizing an expedition to a foreign land for a group of travelers that have a limited working knowledge of both the local culture and language of that land. How would you prepare? What tools and/or resources would you want to have on your journey? Whom could you turn to for support if you had questions? These are some of the questions that may flow through one's mind upon assignment to teach a new course, such as AP Calculus[1].

AP Calculus is not a foreign land, well not actually, though it does possess similar traits. The travelers are the members of an educational alliance formed in the classroom between the students and the teacher, with each having specific roles. First, consider the students, with diverse backgrounds, skills and interests, who have a limited knowledge of the world of mathematics; where there is both a distinct culture, that encompasses a set of higher order thinking skills, (i.e. logic), and a distinct language of symbols and notations that evolved and continues to evolve over time. Their role is to explore this mathematical world, acclimating themselves to the language and culture; thus increasing their level of mathematical literacy. Now consider the teacher, charged with the responsibility of helping the student explore the mathematical world while providing them opportunities to delve deeper into concepts that help broaden their capacity to think for themselves. For the new teacher, all of this must be accomplished while they, themselves, are taking the same journey as an adult learner. So what tools and/or resources would a new teacher choose to cling onto during this journey? What preparation is offered or is expected? Moreover, who will be there along the journey for support if the teacher has questions?

[1] Advanced Placement Program ® and AP® are registered trademarks of the College Entrance Examination Board.

As for tools and/or resources, the new teacher can take some comfort in the textbook, but it is not the sole resource needed, as it is rarely adequate for what is needed by the new teacher undertaking this journey with the students. Unfortunately, outside the standard textbook, limited amounts of resources are readily available in the average high school classroom. New teachers may expend a significant portion of time trying to develop the most efficient way to teach the material while trying to maintain a certain level of rigor and relevance in the classroom. Without adequate resources, this can become a time consuming job as the new teacher stumbles through the different strategies and methods.

The new teacher may feel capable to charge ahead, armed with their own education and the right tools in the classroom; but as with inadequate resources, the lack of proper preparation can lead the new teacher to spin their wheels needlessly, expending valuable time and energy while sorting through the mist. Having the right training equips the new teacher with the insight needed to be able to help the student traveler acquire the language skills necessary to process information while navigating the course.

Surely, the new teacher will have questions and will need to know where to turn for answers. To have a successful AP Calculus program it is important to work on establishing a support network within the school. This can be accomplished by making sure that the math department is setup with vertical teaming, allowing for inter-department collaboration, as well as, launching cross-curricular activities with other teachers in the school. Integration into the school culture is vital to the success of any program and AP Calculus is no exception. Henry Ford captured this when he said, “Coming together is a beginning. Keeping together is progress. Working together is success.” Dr. David Bressoud (2004) echoed this emphasis when he shared this caution:

> The movement of calculus into the high school is not necessarily bad, but it does require the efforts of the mathematical community – individuals, departments, and professional associations – to prepare and support those who will teach it and to resist the pressures that would weaken it.

Recognizing that course sequencing can have a major impact on the success of an AP Calculus student is not the focus of this paper and will be assumed that students have met the prerequisites established by the local institutions to be eligible for AP Calculus. Another noteworthy point, AP Calculus is not merely an educational vehicle that enables students to get college credit in high school; but is a preparatory environment focused on getting students college ready by engaging students in mathematics, transforming them into refined thinkers. This paper answers these questions and proposes an instructional foundation, created to help new and experienced teachers get off the ground with teaching AP Calculus.

In the upcoming chapters, the reader will become familiar with how to get an effective AP Calculus program off the ground. In Chapter 2, an overview of what is calculus will be provided as a review of the conceptual and educational history of calculus along with a discussion of the instructional makeup of teaching AP Calculus. Chapter 3 will discuss the planning and preparation needed to create the online component, MyMathLab. I will provide information into what is MyMathLab and why it was chosen as the online component of my course. Later on into the chapter, I will discuss the development and implementation of MyMathLab in an AP Calculus course and highlight important factors that need to be considered when implementing MyMathLab. Chapter 4 will elaborate on the instructional resource kit and any related training/resources that are available to teachers. Finally, in Chapter 5, I will reflect on my experiences, lessons learned, and will give an outlook on my plans concerning MyMathLab and the use of the instructional resource kit in helping new teachers not only get off the ground but staying off the ground as they soar to new heights with AP Calculus.

1.2 Educational Philosophy

Every new recruit into the armed forces goes through two stages of training, basic training and advanced individual training. These two stages are vital to the success of the recruit as they fulfill their tour of duty. As a mathematics teacher, I believe that every college bound student should complete his or her mathematical basic training by taking a calculus course in high school. This high school course paints with a broad brush, permitting students to become proficient with all the "how-to's" of calculus and preparing them for the next stage at college. Again, let us note that the goal of a high school calculus course is not college credit or beefing up a transcript for college admissions, but the goal must be a focus on laying a mathematical foundation for the encouragement of future studies. Thus permitting students to then pursue their advanced individual training in college, where the college is to assume that all incoming students have been instructed in the basic procedures of calculus and can design a course that is leaner and deeper by focusing on the "why's" and "what if's" of calculus. This fact was captured by Francis Worrell (1962) when he made the following statement regarding mathematics education:

> Some criticism, I think, fairly be directed at the teaching of mathematics, for it has often tended at the lower levels (secondary schools) to emphasize the master of rules, while at the higher levels the subject has at times been presented as a beautiful system of logic, but somewhat divorced from reality.

Before proceeding, allow me to elaborate on my educational philosophy. As a progressivist,[2] I feel that the world of knowledge is forever changing and as such needs citizens that are prepared to embrace these changes. To help students adapt to these different fluxes in society and to be more able to positively contribute to society, schools must take a more active

[2] The Progressive education philosophy was established in America from the mid 1920s through the mid 1950s. John Dewey was its foremost proponent. One of his tenets was that the school should improve the way of life of our citizens through experiencing freedom and democracy in schools.

role in helping expand the knowledge of the student body by increasing their problem solving capabilities. I am concerned that one of the major problems that face students today is that they are inept to think for themselves when solving a problem. More and more often, the students want all of the answers given to them either on a formula sheet or in some other manner. Students who can learn to think on their own will be empowered with the knowledge and capability to solve any problem that they face in this ever-changing world of ours. For a progressivist, knowledge is not a concrete list of things that students must learn but is a wide gamut of problem solving skills that the student must have in their educational arsenal. The true value of knowledge is not in what to think but in how to think.

The purpose of schooling is to provide a concentrated atmosphere where students may learn problem-solving skills that will help them navigate through the different challenges of life. All subject matters provided in the education system should be directed to the single goal of providing the student with these important problem-solving skills. For example, in math the student learns how to read a problem, analyze the problem, postulate a solution to the problem and verify its solution. In life, these steps are invaluable to the student. Other subjects provide likewise steps to problem solving but continue to allow the student the freedom to create their own steps as their knowledge of how to think increases.

In this educational process, the teacher should not be a robot that dispenses a set curriculum in a set manner each day, but should break from this mundane routine and help facilitate student's learning. This is accomplished by providing stimulating projects and engaging activities that help build up the students' problem solving skills. For this reason, progressivists feel that the role of a teacher is that of a facilitator and not a taskmaster. In other words, the teacher should be a "guide on the side and not a sage on the stage." It is important to

guide the students along their educational journey, allowing them to stop and smell the roses to gain a deeper knowledge of the problem they are solving. This process of being able to allow students to discover solutions at their own rate will help develop a set of skills that will free them from the dependence of being told how to solve something.

For a teacher to be able to facilitate properly the learning of a student, there are some fundamental things that the teacher must know about the student. It is important to know the student's educational and social background. This information would help the teacher choose projects that correlate to the student's interests and experiences. When a student is interested in the topic being discussed, they will be more engaged in learning more about that particular topic. For example, if a student lives in Louisiana and is interested in weather patterns, then the teacher can create a project that allows the student to study the predictability of the path of a hurricane. This topic would allow the student to apply topics from class into a real-world application and actually see real results that relate to their own lives. The idea of linking a project to a student's interest will increase the level of student participation.

One roadblock that may impede this type of instruction is standards-based instruction. Some of the standards that are put out by administrators fail to realize that not all students are at the same level or have the same learning capacity. Student interests cover a much wider scope than what is covered by these one size fits all standards. Despite this potential roadblock, teachers should be able to incorporate the standards into the different instructional activities. To be an effective educator, the teacher must create a type of melting pot of standards, interests, curriculum and other educational requirements to produce projects that allow the students to discover how to think for themselves. To do this, the teacher must develop some good pedagogical practices.

The foremost pedagogical practice for a progressive is allowing the flow of the lesson to go in the direction that best corresponds to the students' interests. To do this, the teacher needs to be prepared to accommodate the various interests that may arise from a certain activity. One student may wish to write a paper on the lesson, while another wants to conduct an analytical experiment on the concept being learned. The teacher needs to be able to tie together these various projects to maintain class decorum. The best strategy for a teacher is to know the habits, interests and experiences of the students so when teaching a lesson, the teacher can make a good estimate as to what student will like which project. This will limit the randomness of the projects and help in limiting the time spent grading the different projects. However, the teacher needs to be able to adapt to other scenarios that may not have been incorporated in their original lesson plan or project outline. For an educator to be effective in and out of the classroom they need to prepare for the many changes that occur in life. The most important area that they need to be well educated on is the subject that they teach. How can a teacher be able to incorporate everything that is required and still create engaging projects for their students if they themselves have not stayed abreast of all the ins and outs of their subject matter?

Another aspect in teaching that will help facilitate a good learning environment is how the classroom is organized. Provided the dimensions of the classroom permit it, the classroom should be set up in such a way to focus on the front of the classroom. With limited distractions, the students will be able to maximize their learning and the teacher will be able to instruct more efficiently. On the outlying edges of the classroom can be a computer terminal for in-class research for that student that wants to do a deeper search on a topic that was presented in class. Also on the outer edges of class can be a small area to accommodate a group of students that might want to discuss a topic in more detail. Again, the teacher needs to be flexible in how the

students arrange their desks so they might be able to meet in small groups, one large group circle or the standard rows of desks. Allowing the students to work on the computer, meet in small groups, or do individual research does create some management problems.

To handle these problems, the teacher needs to have a system in place. For the computer, the teacher can give the students specific time limits to allow more students to access this valuable resource. By having a time limit, the students research should be more focused and on task. Walking around the class, monitoring the students' activities and facilitating group discussions will help the class stay on target and maintain class decorum. If there are any behavior problems, they can be addressed on a one to one basis. Students should be allowed to defend their actions but in the end must be held accountable for their own behavior. Allowing the students to help create the class rules and the consequences gives them a sense of ownership and decreases the feeling of unfairness that is oftentimes associated with behavior management.

To motivate students, a reward system can be created to counter the punishment system. When students do well on an assignment, it is appropriate to tell them that they have done a good job and to do so publicly. However, when a student does poorly, it is imperative that this information be kept between the teacher and the student. The old saying is true, "Criticize in private and praise in public." To encourage participation, bonus points can be assigned for activities that go beyond the teacher's expectations of what the student should be accomplishing in the classroom. These bonus points are a good way to disguise a lesson that the teacher wants the students to seriously attempt.

All in all, the teacher is a tour guide that must be flexible in what exhibit the students want to explore next. Proper decorum can and must be kept, while actively allowing the students to determine their own educational experiences based on their interests and experiences. This is

a principle that holds true in real life. If in real life we are able to determine what activities we wish to partake in based on our interests then students should be allowed to do the same in school, since school is supposed to be preparing them for real life. Again, the most important skill that a student can obtain from schooling is the ability to think for themselves.

1.3 Rationale

How can teachers, as professionals, effectively prepare to establish a successful AP Calculus program in their school enabling students to become better thinkers? This question was derived through a discussion with other educators about the global problems/concerns facing today's education and sparked my interest in this topic. Along with my personal experience of this topic, the expected outcome of this research is discussed before this section is summed up.

According to the Third International Mathematics and Science Survey (TIMSS), American students are not age-mates with their international counterparts. In fact, on the math/science literacy assessment administered to high school seniors, American students average 18.1 years of age while other industrial countries average 19-22 years of age (Bracey, 2000). This means that of the sample tested, American students are 1-3 years behind their international peers. The results of this type of research have caused alarm in the educational community. Over the years, the media has reported on the downward spiral in which American students have found themselves. One reporter, Debra Viadero of *Education Week* (1998), was quoted as saying, "American high school seniors – even the best and brightest among them – score well below the average for their peers participating in TIMSS." This epidemic has caused many in the world of education to wonder what can be done to reverse this detrimental trend. Increased standards, common assessments, equity in the classroom and teacher preparation have been the answer from many educators.

With the increased demands on the classroom, new teachers may become overwhelmed with the wide range of instructional issues that they must face on a daily basis. One such issue is how to effectively utilize the allotted classroom time to maximize teaching, thus resulting in an increase in student achievement and literacy in the core content areas. My own school's administration has recognized and addressed this issue by asking teachers to maximize instructional time by teaching "bell to bell", which helps increase time-on-task, which by definition is the amount of time that students are paying attention and trying to learn (Stuck & White, 1992).

This request coupled with my discussion with other educators caused me to reflect on my own experiences in the classroom. Fresh out of school and new to the realm of teaching I was given the great privilege of having AP Calculus among the courses I would be teaching. Being a recent mathematics graduate, I knew the content well and felt ready for the challenge that lay before me. Upon entering the classroom the first day, I realized how wrong I was. Armed with only a textbook and my content knowledge, I felt like I had brought a water gun to a five-alarm blaze. I left the class that day determined never to feel like that again. I went home and found as many supplemental resources I could get my hands on. Over the years, I have continued this by going to AP summer institutes, conferences, and collaborating with a network of other AP Calculus teachers. Now presented with the opportunity to help others, I have formally organized all these resources into a user-friendly instructor's resource kit that accompanies an online portion of the class to help facilitate in assessing the students through homework, quizzes and tests. The expected outcome is new teachers will be able to utilize the resources as a springboard into their teaching career, establishing a successful AP Calculus program that prepares students to go deeper in their collegiate studies.

1.4 Literature Review

After sifting through numerous articles, studies and research, the same reoccurring variables of "Mathematical Literacy", "Equity", "Instructional Trends", "Educational Technology" and "Technology Integration" were noted over and over. Each variable in relation to how new teachers can effectively prepare to establish a successful AP Calculus program in their school, enabling students to become better thinkers and better equipped for college, is presented in each of the following sub-sections with corresponding reviews of selected literature to substantiate each variable.

1.4.1 Mathematical Literacy

When it comes to mathematics in the world of education, the foremost concern is the mathematical literacy of the students. It is no longer accepted practice to allow students to simply scrape by with a basic knowledge of repetitious algorithms, but they "need to learn to reason and communicate using mathematical ideas" (Schoenfeld, 2002). Robert Moses (2001) states that those that are mathematical and technological literate will be afforded economic opportunities that would otherwise be withheld:

> Sixty percent of new jobs will require skills possessed by only 22 percent of the young people entering the job market now. These jobs require the use of a computer and pay about 15 percent more than jobs that do not. And those jobs that do not are dwindling. Right now, the Department of Labor says, 70 percent of all jobs require technology literacy; by the year 2010[3] all jobs will require significant technical skills. And if that seems unimaginable, consider this: The Department of Labor says that 80 percent of those future jobs do not yet exist. (pp. 8-9)

In the article, "Making Mathematics Work for All Children: Issues of Standards, Testing, and Equity," Schoenfeld (2002) discusses several issues as they relate to mathematical literacy. In the beginning of the article, Schoenfeld clarifies the importance of mathematics in helping

[3] As of this thesis, 2010 data was not available on the Department of Labor website.

prepare to take part of the opportunities that are available in society by stating, “Course work in mathematics has traditionally been a gateway to technological literacy and to higher education.” He continues to outline the state of mathematics instruction in the late 20^{th} century noting that one could receive full credit on any given mathematics problem by completing the algorithm and writing an answer in a box (Schoenfeld, 2002). Over the past few decades, there have been many changes to the way educational information is disseminated to the learner. These changes have caused concern amongst opponents of this educational reform, as they feared that the curriculum would become watered down with algorithms taking precedence over content; thus, decreasing the mathematical competencies of the learner (Schoenfeld, 2002).

The article continues by discussing how national entities, such as the National Council of Teachers of Mathematics (NCTM), set out to battle the decline in mathematical literacy by erecting a common set of standards, which was published in a work entitled “Curriculum and Evaluation Standards for School Mathematics” in 1989. This work addressed “what it means to be mathematically literate” (National Council of Teachers of Mathematics, 1989). Later in 2000, NCTM published a refined version entitled, “Principles and Standards.” This latest document helped show the commitments of its authors to mathematical literacy. This commitment was outlined with the following set of principles: “equity (high expectations and strong support for all students); coherent curricula rather than disconnected sets of activities; teacher professionalism, including knowledge of curricula and learning; and the effective use of assessment and technology in the service of mathematics learning” (Schoenfeld, 2002). This work broke from tradition and called for a core curriculum that not only prepared them for higher education but would also prepare them to enter the workfore with a mathematical background for quantitative literacy.

The article cited above demonstrates how mathematical literacy is an important issue, one in which needs to be a main focus when establishing any mathematics course that will help students become higher academic achievers as they prepare to enter the workplace. In later chapters, I explain how combinations of educational resources, that I use, are implemented to help raise the mathematical literacy of the learner.

1.4.2 Equity

In the Declaration of Independence, our forefathers stated that "all men are created equal" and we have "unalienable Rights" that include the "pursuit of Happiness." Although they were referencing the political issues of the time, these same statements are applicable to the realm of education. Each individual student, regardless of background, race, creed, gender, etc. has the right to equitable access to educational resources that will afford them opportunities to participate in the pursuit of happiness. This concept of equity has been an issue that has advanced to the forefront over the past 50-60 years.

In the article, "So Much for the Theory That Blacks Can't Do Mathematics," the author of the article (JBHF Foundation, 1999) discusses a program created by Professor Phillip Uri Treisman of the University of Texas at Austin to help increase the equity among his students and decrease the achievement gap between the minority and non-minority students (JBHE Foundation, 1999). The author of the article points out that even though our forefathers believed that all men were created equal, Thomas Jefferson thought differently when it came to the education capabilities of blacks. In a letter to a friend, Jefferson wrote, "I have not yet found one of them [Negros] who could solve the geometrical problems of Euclid" (JBHE Foundation, 1999). These biases barred minority students from having equitable access to the same educational opportunities afforded to white students. Through the tests of time, these biases have

been whittled down as the world of education develops more equitable policies, opening up endless possibilities to the students of today. One such opportunity was created by Professor Treisman of the University of Texas at Austin.

Professor Treisman had noticed that there was a signifcant difference in the success rates of black students and non-black students in his classroom and refused to subscribe to the philosophy that black students can not do math. Upon looking into why black students were outperformed by non-black students, Professor Treisman noted that black students tend to study alone while the non-black students tend to study in groups. Professor Treisman referred to these study groups as “academic fraternities” (JBHE Foundation, 1999). The author of the article states that Professor Treisman speculated that by studying alone, black students would become discouraged when unable to arrive at a correct solution; whereas, non-black students would be able to see that others had difficulties as well and would help each other in uncovering the solution to a problem. To combat this issue, Professor Treisman established the Emerging Scholars Program (ESP) to help minority students increase their success rate through organized study sessions led by teaching assistants (JBHE Foundation, 1999).

His findings were that ESP participants earned up to one full letter grade higher than those minority students who did not particpate in ESP. Not only were the academic grades affected by this program but graduation rates and student majoring in mathematics increased as well with graduation rates increasing to twenty points higher than non participating minorities, equaling the success rate of white students, and the number of minority math majors jumping from six to one hundred and fifty (JBHE Foundation, 1999).

In the report, “Minority Student Success: The Role of Teachers in Advanced Placement Program (AP) Courses,” Burton et al. (2002) reports on how successful teachers of minority

students are good teachers for all groups by expressing a high opinion of students and holding them to high standards. The authors of the report state that AP is a program with high academic standards that improves students' skills for succeeding in college and instills confidence in their ability to succeed (Burton et al, 2002). However, students from minority groups are commonly underrepresented in higher education because they experience one or more barriers to a college education. The AP program is a chance for these underrepresented minority groups to overcome these educational obstacles. The authors of the report point out that "this chance is not appropriate for all students, since AP courses are challenging even for the well-prepared students" (Burton et al, 2002). Even though AP is not appropriate for all, it is open to all that have a desire to accept the challenge of a rigorous course. To make this point clear, the College Board issued a formal statement on Equity that reads:

> The College Board and the Advanced Placement Program® encourage teachers, AP Coordinators, and school administrators to make equitable access a guiding principle for their AP programs. The College Board is committed to the principle that all students deserve an opportunity to participate in rigorous and academically challenging courses and programs. All students who are willing to accept the challenge of a rigorous academic curriculum should be considered for admission to AP courses. The Board encourages the elimination of barriers that restrict access to AP courses for students from ethnic, racial, and socioeconomic groups that have been traditionally underrepresented in the AP Program. Schools should make every effort to ensure that their AP classes reflect the diversity of their student population. (Equity Policy Statement, 2002)

Burton et al. (2002) continue by outlining how the teacher plays a significant role in the AP program. What teachers know and can do is important and many organizations have recognized this fact by placing a higher emphasis on higher-order thinking skills as a part of the educational reform. The authors of the report point out that "the NCTM standards suggest that rote memorization and repeated practice on low-level computational skills be replaced with open-ended problems that put emphasis on conceptual understanding through the making and

testing of hypotheses and the communication of ideas" (Burton et al., 2002). Throughout the article, Burton et al. (2002) make reference to various research that demonstrate that teachers need to understand their subject matter deeply and flexibly so they can help students make cross-curricular and real-life connections, emphasizing how the material is applicable to the students' lives. Teachers also need to infuse their personality into their teaching, by including humor, personal experience, and stories while delivering the content with enthusiasm (Burton et al., 2002).

These two articles help illustrate how new teachers and experienced teachers alike have to face issues of equity. Each teacher needs to ensure that equitable practices are being employed when recruiting students to join their class, thus ensuring that the class is a true cross-section of the student body. The new teacher must also bear in mind that the ideal student is not necessarily one that will ace the AP exam, but instead is the student with a desire to learn that will benefit from exposure of calculus in high school.

1.4.3 Instructional Trends

For many years, American students were graduating from school prepared to fulfill the needs of the job market. With the advances that have been made in technology, there has been a shift in the demand for highly qualified workers, which can enter into the workplace ready to help keep the country at the forefront of economic competition. However, traditional education has not been able to meet that demand, resulting in a need to transform the classroom from a teacher-centered environment to one that is student-centered. New teachers must enter the profession with a solid understanding of the various instructional strategies that can be employed to help students become college ready. Even then, however, it must be recognized that they are beginners who will take years to evolve into full-fledged professionals (Schoenfeld, 2002).

To make better use of available technology, teachers need to transition from whole-class instruction towards smaller group projects and activities that are conducive to active, engaged learning and student interactions. Unfortunately, this is not a move that all teachers are prepared to make in their classroom (Ryan, & Cooper, 2004). A majority of today's mathematics teachers learned the traditional curriculum in the traditional way and have no point of reference when it comes to teaching in the ways that would best facilitate their students' development of mathematical understanding (Schoenfeld, 2002). According to Ryan and Cooper (2004), many teachers like to remain in control of their classroom and desire the entire class to learn the same material together. Smaller group work would mean that there is the possibility of students learning different things at different times. Integrating technology into their teaching can change the way teachers deliver content to their classes; and if properly used, technology can increase academic achievement through the facilitation of instruction (Ryan & Cooper, 2004).

One recommended instructional strategy for increasing student attention according to Stuck and White (1992) is to ask a question before calling on a student to answer it. If students think that they might be called on to answer, they are more likely to pay attention to all of the questions asked instead of only to those questions specifically asked to them. Ensuring that teachers adequately employ wait-time[4] is another way to increase student participation, thus decreasing the amount of instructional time lost to answering questions multiple times in one class. Metzker (2003) suggests increasing student engagement and learning by having teachers foster student motivation through a repertoire of interesting, innovative, and thought-provoking instructional endeavors rather than offering activities as repetitive seatwork.

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[4] Wait-time is the amount of time (normally 3-5 seconds) that the teacher should wait before eliciting a response from a student.

In the report, "Key Building Blocks for Student Achievement in the 21^{st} Century: Assessment, alignment, accountability, access and analysis," Rogers et al. (2001) reports on several issues pertinent to instructional strategies. A large focus of the report was centered on an 8^{th} grade math class. Pre- and Post-Assessments were given to measure the growth of the students throughout the study. It was shown that through technology integration, the students were able to gain a one-third grade level increase. The report also described the instructional strategies that can be used to ensure that teachers are effectively integrating technology into their curriculum. The authors conclude that the technology must align to the educational standards and support the best practices, which are already in use in the classroom. They also state that there needs to be equity of access for all students and that teachers need to base their classroom management of this resource on rotational system to ensure that all students remain engaged throughout the allotted classroom time in order to maximize instructional time.

In the study, "Teaching with Technology: Creating Student-Centered Classrooms (Apple Classrooms of Tomorrow [ACOT])," Sandholtz, Ringstaff, and Dwyer (1997) researched a collaborative effort between Apple Computer, Inc., the National Science Foundation, and multiple universities. 20,000 entries, composed of email journals, unstructured audiotape reports of teachers, observations of classrooms, and in-depth interviews were used to collect data during this study (Sandholtz, Ringstaff, and Dwyer, 1997). Their intervention provided computers to each student and teacher at the participating institutions. The study was centered on how routine use of computers and technology influence teaching and learning. The authors of the study postulated that as teachers became more comfortable with the technology, the students became more collaborative. Using computers, the authors were able to collect electronic mailings and other documents that detailed how technology was influencing the teaching and learning at the

participating universities. The authors of the study found this to be true as teachers demonstrated an understanding of the student learner and altered their teaching strategies to transform the classroom from the traditional lecture style to a form of constructivism (Sandholtz, Ringstaff, and Dwyer, 1997).

The two studies cited above demonstrate how different instructional strategies can be employed positively to integrate technology into the classroom. Later, I explain how learning logs are used as one of my instructional strategies to increase the dialogue between student and teacher and how integrating MyMathLab and Blackboard helped students extend the classroom experience beyond the standard period.

1.4.4 Educational Technology

According to Steckroth, Driskell, and Rider (2006), the mathematics education community has been struggling for many years with the challenge of incorporating multiple representations of function concepts. Advances in technology have brought about the development of several user-friendly programs that can be employed in helping compare and contrast the various representations of a function and much, much more. Such programs include, the TI-Smart View Emulator, the Mathematical Visualization Toolkit, Mathematica, Matlab and others.

Given the fact that our school uses the TI-84 Plus calculator, the TI-Smart View is the program that is used by the math teachers to help increase student comprehension. The teacher can use the software in conjunction with a projector to demonstrate how to use a graphing utility to solve different problems. The Smart Views ability to provide students with three different simultaneous representations of the function being analyzed aids students in gaining a deeper conceptual understanding of functions and their different characteristics. Not only does this

increase instructional time, but the key stroke history allows the teacher to displays the various key strokes used, so the teacher does not have to spend large amounts of the class helping each student punch the correct key. If the Smart-View is used in conjunction with an interactive board, the teacher gains more flexibility in presenting lessons rich in these different representations. The multiple representation capability of SmartView makes it a pedagogically valuable tool (Steckroth, Driskell, & Rider, 2006).

Metzker (2003) reminds us that learning does not need to stop at the end of the academic day. He reports that higher achieving students spend more time in structured learning activities outside school, demonstrating the importance of technology. The teacher can have students spending time outside class learning by implementing a classroom technology that allows for the extension of the classroom outside the normal boundaries of a school day.

New teachers can choose such web-based programs such as Moodle, Blackboard, Blogs, WebCT and others to help extend the learning experience. The chosen program for my district is Blackboard, which is a program that provides students the opportunity to access notes, presentations, assignments and even virtual chat with their teacher or fellow student. Being able to continue the learning outside of the classroom is one way to keep the students actively interested in your content.

Recently, our mathematics department has sought for more ways to help our students reach a higher level of mathematical proficiency. To accomplish this, different web-based applications, such as MyMathLab and WebWorks, were considered. MyMathLab was selected, as it provides students with a series of web-based applications that supplement the mathematical curriculum. Further discussion of MyMathLab is included in Chapter 3.

With new technological advances come different pitfalls of which both teachers and students must be aware (Harper & Driskell, 2006). For example, a graphing calculator can only display a function as accurately as the screen resolution permits. Therefore, graphs are not smooth and may appear to have parts added to them. Although there are technological pitfalls in commonly used technology in the mathematics classroom, teachers remain strongly committed to the use of these technologies (Harper & Driskell, 2006). These technologies provide a means of reaching a diverse student population and aids in motivating the student population. The use of this technology can lead to a deeper understanding of the underlying mathematics, which should be the aim of any mathematics course (Harper & Drirkell, 2006).

In the study, "Case Studies of High-Performing, High-Technology Schools: Final research report on schools with pre-dominantly low-income, African-American, or Latino students," (Sweet et al., 2004), the issue of educational technology as it pertains to closing the problematic age gap was addressed. The purpose of this study was to answer two questions, "To what extent can education technology help schools close achievement gaps?" and "What types of educational technology can administers, teachers, and students use to help close achievement gaps?" In order to address these questions, Sweet et al. conducted a study with 19 participating schools that have predominately low-income, African-American, or Latino student populations. This study used open-ended interviews, classroom observations, and surveys to gather data during the course of their study. The authors of the study also collected a copy of the school improvement plan and technology plans (Sweet et al., 2004).

During two on-site visits, classroom observations and interviews with the administration and a handful of teachers were conducted. Teachers were selected to have a true sample of the teacher population of the school. Upon completion of the site visits, the authors of the study

would administer the survey to the principal and the teachers. The study found that the technology that was being integrated at the 19 schools was supporting the overall learning environment (Sweet et al., 2004). Sweet et al. noted that the technology integration into the classroom motivated the students and the open access to technology contributed to the positive learning environment of the school, which plays a role in producing high-performing students. They concluded that, “Access to technology for teachers and students is an important part of the physical learning environment in most, but not all, of the schools positive attitudes toward technology often contribute to the challenging and cohesive quality of the learning environments in these schools” (Sweet et al., 2004).

In the study, “Does it compute? The relationship between education technology and student achievement in mathematics,” Wenglinsky (1998) does a national study of the effectiveness of technology by analyzing the 1996 National Assessment of Education Progress (NAEP) in mathematics. He used four factors (frequency of use, access, professional development, and high-order usage) when analyzing the NAEP to demonstrate the effect of technology on achievement. Data was mapped out in charts to quantify the findings and to develop a better understanding of the data collected. It was shown that technology does have an effect on the academic achievement level of students. This affect is positive if the teacher is well trained in the usage of the technology, and uses the best instructional practices effectively to integrate technology into the classroom. They conclude that technology can be a useful tool in increasing motivation, decreasing absenteeism, and positively changing the atmosphere of the classroom, but only if teachers apply it effectively (Wenglinsky, 1998).

The two studies cited above have shown that educational technology and effective technology integration does help support the students by creating an environment in which

students can thrive and excel. Evidence is provided as to the importance of educational technology in creating a positive learning environment for the students, thus resulting in increased student achievement levels. Based on this finding, educational technology is another intervention that will be employed in helping new teachers continue to reach new heights as they soar with AP Calculus.

1.4.5 Technology Integration

Throughout my experience, both as a student and an educator, technological advances have produced many different tools that can be used in the classroom to help enhance teaching and learning. Many of today's classrooms are equipped with the basics, namely a desktop computer, a television, a VCR, and an overhead projector. With the desktop computer, the teacher has word processing software, spreadsheets, and different presentation software to create presentations that allow for the quick dissemination of information through handouts versus having the students copy everything from the board. Other available technologies include, but are not limited to, an interactive board, a digital projector, graphing calculators, different software packages, distance learning classes and online class forums.

Each year brings more technological advances that are readily embraced by society. With society embracing the use of these new technologies, should the classroom be an exception? No. Many hold to the opinion that the main mission of schools is to prepare students to be productive and active participants of society. For this reason, technology integration has gained more attention in the educational realm. Teachers often wonder what their students are thinking, and technology is providing some innovative ways to understand the cognitive processes of students. Besides enabling a pathway of understanding into the cognitive processes of students, technology integration helps create and amplify interdisciplinary connections. For

this reason, Louisiana created the Louisiana K-12 Educational Technology Standards (Louisiana Department of Education, 2008), an adaptation of the technology standards from the National Educational Technology Standards (International Society for Technology in Education (ISTE), 2010); where performance indicators are outlined to help the teacher know if their students are operating on grade-level.

Robertson (2001) points out "As teachers, we often search for new methods of instruction, new skills to implement in the classroom to better meet the needs of our students." Teachers are constantly striving to improve in subject matter, pedagogy, and educational technology (Robertson, 2001). Technology gives teachers the tools necessary to enhance the education of their students. Robertson (2001) continues to point out that the tools are not the focus of instruction, but are imbedded in the facilitation of the learning process.

In the article, "Technology and Achievement: The bottom line," Wenglinsky (2006) discussed the effects of technology integration into the classroom and what affect it has had on the achievement level of students. To support his discussion, he references the 1998 NAEP study in History.

Wenglinsky points out that the quality of computer work was more important than the quantity. The study pointed out that in-school usage was not beneficial but its lack of benefits was not attributed to the technology, instead; it was attributed to the fact that many teachers ineffectively used the technology. Students could receive a substantial benefit, no benefit, or even a negative benefit from working with computers in the classroom according to the study. The study continues to point out that using computers to help students work through complex problems, thus tapping higher-order thinking skills, produced greater benefits than using computers to drill students on a set of routine tasks. In fact, it was noticed that when teachers

used the computers to promote higher-order thinking skills the computers were most effective in the classroom (Wenglinsky, 2006). Wenglinsky concludes that teachers should not plan their lesson around technology but should assume that technology will be utilized during each lesson.

Many schools and teachers have been slow to discover the real potential of new technologies, but some new trends are emerging. Differences in location, size, budget, composition of populations, and graduation requirements of many districts have created a gap between the resources used from one district to another. Although the same technology is available to all the districts, some districts simply cannot afford to provide the quality and variety of courses offered by larger or more affluent districts (Kellough & Carjuzaa, 2006). Quoting Hokanson and Hooper (2004):

> Computer integration occurs along a spectrum of effectiveness and involvement. At one end, computers may be available to assist with various tasks, but their impact on educational activities is minimal. At the other end, a lively curriculum may fully integrate their use as part of normal growth and change. In practice, most teachers fall somewhere in between.

In the study, "Time for Technology: Successfully integrating technology in elementary school classrooms," English (2002) addressed the issue of technology integration and the time needed to use said technology adequately. The main focus of the study was to investigate the correlation between the time teachers spent learning how to use the technology and the amount of success that teachers have with the integration of technology. To address this focus, English (2002) included a sample of second, third, and fourth grade teachers from her local district. These surveys were collected and the data was compiled to provide evidence to support her position. Pertinent technology information from the different participants was gathered using a survey and this data was displayed with various charts and graphs (English, 2002).

English (2002) found that the more technology that was present in a classroom, the more involved the teacher was in the lesson. The fact that technology is simply a tool to be used in the classroom was noted. She points out that there is no hard evidence that shows the correlation between the integration of technology, but did mention that current studies are indicating that technology integration is helping to increase student achievement.

From this study, I was able to see that technology integration alone is not the answer to increasing student achievement, but with the proper time spent implementing the technology as English (2002) notes in her study, teachers will be able to successfully integrate technology into the classroom, creating a positive learning environment, increasing instructional time and increasing student achievement.

With the added pressures from society, technology integration is necessary. Teachers have an overabundance of material and a shortage of instructional time. Without the added benefits of technology, teachers will continue to struggle to produce graduates that are equipped with the necessary skills to be productive and active participants in society.

In the study, "Effects of Using Instructional Technology in Elementary and Secondary Schools: What controlled evaluation studies say," Kulik (2003) focuses on several questions, "Are schools doing a better job because of their embrace of technology?", "Can schools improve their teaching effectiveness by investing more heavily in technology?", and " How can schools best use technology?" To answer these questions during this study, Kulik (2003) chose to evaluate 27 controlled studies to extract the data needed. The 27 studies were grouped into subject areas and were analyzed separately. In the course of answering the previously stated questions, it was noted that of the 27 studies that looked at technology integration in a mathematics classroom, none of them reported a negative effect of this integration. In fact, the

reports demonstrated that a positive affect was the case. The overall outcome of this study showed that instructional technology is growing increasingly effective in elementary and secondary school applications (Kulik, 2003).

The three studies cited above demonstrate that when technology is integrated correctly and effectively into the classroom, student motivation increases, maximized instructional time is achieved and student achievement increases. To be able to integrate technology effectively into the classroom, the new teacher would have to be sufficiently trained in the best practices that accompany the technology that is being integrated.

Certainly new technologies are not the cure all for the classroom, but they offer tools that can aid in the transformation of the classroom from a teacher-centered environment to a more cooperative and student-centered environment (Ryan & Cooper, 2004). Students can learn to use technological tools in the same way that they will most likely be in their future careers and not as just expensive toys. All teachers, especially new teachers, should have the opportunity to gain and improve upon their educational technology skills (Ryan & Cooper, 2004). According to Li (2005), "Research studies demonstrated that new roles, responsibilities and technologies are developing and need to be mastered by teachers." Teachers continually need to ask themselves how technology will affect their classroom and where technology can enhance what they do. If the use of technology is not accomplishing an increase in instructional time, then teachers need to ask themselves why they are using it (Harper & Driskell, 2006).

In this section, the reoccurring variables of mathematical literacy, equity, instructional trends, educational technology and technology integration were discussed and followed by studies that substantiate the importance of each variable. To increase academic achievement, educators have to find ways to decrease the amount of allotted time that is used for non-

instructional activities/tasks. To do this, new teachers will need to employ multiple instructional strategies that are based on best practices with the aid of educational technology that is effectively integrated into the classroom and subsequent activities. Using this formula for success will help new teachers guide their students on their journey to new educational heights while gaining a firmer mathematical foundation preparatory to a leaner and deeper study of mathematics in college.

CHAPTER 2 OVERVIEW OF CALCULUS

2.1 History of Calculus

Before the new teacher can mobilize the resources for the excursion, which are addressed in further detail in chapters 3 and 4, it is necessary to take a pause in order to understand the history of both calculus and mathematics education. Derived from Latin, calculus means "a method of computation or calculation in a special notation (as of logic or symbolic logic)" (Calculus, 2009). "The history of the development of the calculus is significant because it illustrates the way in which mathematics progresses. Ideas are first grasped intuitively and extensively explored before they become fully clarified and precisely formulated even in the minds of the best mathematicians" (Kline, 1985). Rheticus[5], the man who prepared for publication the major work of Copernicus, echoed this idea that mathematics is discovered as part of a journey when he said:

> The mathematician…is surely like a blind man who, with only a staff to guide him, must make a great, endless, hazardous journey that winds through innumerable desolate places. What will be the result? Proceeding anxiously for a while and groping his way with his staff, he will at some time, leaning upon it, cry out in despair to heaven, earth, and all the gods to aid him in his misery. God will permit him to try his strength for a period of years, that he may in the end learn that he cannot be rescued from threatening danger by his staff. Then God compassionately stretches forth His hand to the despairing man, and with His hand conducts him to the desired goal.

It is important to realize that students may not fully understand the origins of mathematics, with some feeling that mathematics was something that burst forth from the minds of some ancient mind. It is easy to overlook that mathematics has been molded by the problems of society for the past 4000 years. Great mathematicians have looked to their understanding of

[5] Rheticus, also known as Georg Joachim von Lauchen (16 February 1514 – 4 December 1574), was the sole student of Copernicus and is best known for his trigonometric tables. He also was one of the first people to adopt and spread the heliocentric theory of Copernicus.

mathematics to solve some of the social, political and otherwise practical problems of the day. Proclus Diadochus[6] had the following to say about the evolution of mathematics:

> This, therefore, is Mathematics: she reminds you of the invisible forms of the soul; she gives life to her own discoveries; she awakens the mind and purifies the intellect; she brings to light our intrinsic ideas; she abolishes oblivion and ignorance which are ours by birth…

Early mathematicians in Egypt and Babylonia did not look at mathematics as a topic of intellectual study, but rather thought of it as a tool to organize and understand the data of the events that were occurring around them. The Egyptians had a firm grasp on the math used to erect their pyramids and the math used to predict the flooding of the Nile, but did not delve deeper into the equations, that they were using, to find any underlying connections to other areas of life. Similarly, the Babylonians were able to construct very detailed calendars based on the placement of the stars in the heavens, but did not begin to question the properties that governed the movement of the stars.

Despite the lack of deeper investigation by the Egyptians and Babylonians into the mathematical methods being used, others would take that journey to understand mathematics in detail. P.S. Laplace[7] once said, "All the effects of nature are only mathematical results of a small number of immutable laws" and it was the search for these immutable laws that was the driving force for the Greeks. The Greeks were able to take and expound upon the observations of the Egyptians and the Babylonians. Greek mathematicians were not interested in just the practical applications of mathematics but were interested in the intellectual aspects as well. One of the most famous of these Greek mathematicians is Archimedes of Syracuse[8].

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[6] Proclus Diadochus (411-485) was a Greek philosopher, who became the head of Plato's Academy, and is important mathematically for his commentaries on the work of other mathematicians.
[7] Pierre-Simon LaPlace (23 March 1749 – 5 March 1827) had earned the title of "the Newton of France" with his publication of the five-volume *Traite de mecanique celeste*.
[8] Archimedes of Syracuse (c. 287 BC – c. 212 BC) is generally considered the greatest mathematician of antiquity.

The study of mathematics has not been one that has been developed overnight. In fact, Xenophanes[9] had said, “The gods did not reveal all things to men at the start; but as time goes on, by searching, they discover more and more.” It has taken mathematicians many years to search out mathematics and connect ideas together to produce statements that are more precise. Robert Hooke[10] summed up the need for the precise mathematical statements when he said, “More laws are vain where less will serve.” These concise mathematical laws were the result of studies that spanned time, continents and fields of study by mathematicians such as Euclid, Fibonacci, Vieté, Descartes, Kepler, Galilei, Fermat, Pascal, Euler, Gauss, Laplace, Lagrange, Cauchy, Riemann, and many others[11].

Although calculus was developed over the course of many years and had been used to solve area, volume, related rates, optimization, position/velocity/acceleration problems, it was Newton[12] and Leibniz[13] that were accredited with creating calculus because they joined together the building blocks of the past in a manner to be the first to state, understand and effectively use the Fundamental Theorem of Calculus, making the math applicable to real-life applications. Due to this discovery, they were able to deepen our understanding of calculus faster and farther than previously done by others. They were able to accomplish this by drawing on a large amount of material in differential and integral calculus. (Bressoud D. , Calculus Before Newton and Leibinz: Part I, 2008)

In the late 1700’s education in France was undergoing some changes, due largely to the French Revolution. Education in France did not become stabilized until 1794 when the École

[9] Xenophanes (c. 570 BC – c. 475 BC) was a Greek philosopher
[10] Robert Hooke (18 July 1635 – 3 March 1703) is best known for his law of elasticity, also known as Hooke’s law
[11] For further insight into the lives and contributions of these mathematicians, please consult Men of Mathematics – The Lives and Achievements of the Great Mathematicians from Xeno to Poincare by Bell, E.
[12] Sir Isaac Newton (4 January 1643 – 31 March 1727)
[13] Gottfried Wilhelm Leibniz (1 July 1646 – 14 November 1716)

Polytechnique was established. During this time there was much discussion on the amount of mathematics that would be included in the curriculum. Specific engineering applications were slighted for a more general area of application. Through this system, France was too reach a level of educational dominance that inspired other nations to initiate their own educational systems modeled after the French. One such endeavour was the restructuring of West Point in the United States of America. With this new focus on how mathematics can be taught in institutions of higher education, different professional publications emerged, helping propel mathematics education into the future (Grattan-Guinness, 1997).

With the advent of technologies such as the printing press, telegraph, radio and internet, the time it takes to disseminate information to the public has rapidily declined. As such, education has had to keep up with the growth of knowledge readily available to the student.

In the 1800's calculus was a fixture in the mathematical curriculum of most universities. Rosensten (2008) pointed out that students in 1830 were not able to take calculus at Harvard until their third year. Over time, societal needs and other pressures would necessitate a migration of calculus to the first year of college and even into the secondary education. The Civil War was one of these events that furthered the spread of education. Through the Morrill Act of 1862[14], more institutions of higher learning were being formed and the curriculum expanded. With this expansion, calculus moved from a core requirement to a specialty requirement (Rosensten, 2008). Over time, society has required more of college graduates. To help higher education meet this demand, incoming students need a stronger college prepartory curriculum. It was from this need that Advanced Placement was born.

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[14] The Morrill Act of 1862 (A.K.A. Land Grant College Act) was a major boost to higher education in America as it established institutions in each state that would educate people in agriculture, home economics, mechanical arts, and other professions that were practical at the time. Changing the course of higher education, shifting the purpose from the classical studies to the applied studies, the act ensured that graduates would be prepared to enter into the workforce prepared.

2.2 What Is Advanced Placement (AP)?

The 1950's saw a change in how education was regarded in the United States of America. No longer a side note in mainstream conversation, it had become vital to the survival of the American way of life as Democracy battled Communism. However, there was a problem. High schools and colleges seemed to be teaching overlapping material, slowing down the process of producing the graduates needed to stay at the educational forefront.

To help rectify this problem, some elite high schools (Andover, Lawrenceville, etc.) and colleges (Harvard, Princeton, Yale, etc.) formed an agreement to help superior students seek advanced independent studies in high school to be eligible for advanced placement in college (Rothschild, 1999). During the next four decades, the original advanced placement program, created by the elitists of the 1950's, would take root and grow into what is now known as the Advanced Placement (AP) program run by the College Board[15] with placement exams created by the Educational Testing Service (ETS).

The AP program has not been immune to turbulent times, as it faced a period of unpopularity during the 1960's. However, with the release of some reports (A Nation at Risk, High School, etc.) on the status of America's education helped rekindle educators' resolve to provide the best educational opportunities to their students (Rothschild, 1999). Advanced Placement has evolved since its initial conception in the early 1950's, growing from 10 subjects to more than 30 subjects and from 959 exams issued to over 1 million.

Today, AP continues to strive for excellence in education by applying best practices and a rigorous curriculum to the high school classroom. The College Board (A Brief History of Advanced Placement Program, 2003) reports that "according to TIMSS, AP students who

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[15] The College Board, a non-profit member association who is composed of more than 5,400 schools, colleges, universities, and other educational organizations, was founded in 1900 with the mission to connect students to college success and opportunity. (About Us, 2008)

received a 3, 4, or 5 on AP Examinations in Physics and Calculus outperformed other physics and advanced math students from both the United States and abroad."

2.3 Role of AP Calculus in High Schools

> The good Christian should beware of mathematicians and all those who make empty prophecies. The danger already exists that the mathematician have made a covenant with the devil to darken the spirit and to confine man in the bonds of Hell – St. Augustine, Bishop of Hippo in Africa

Making it to the land of calculus, the students may begin to develop the same sentiments as St. Augustine when it comes to mathematics. To avoid this pitfall and to make the journey a rememberable one, it is important for students to have an understanding of the various activities that are planned for their exploration of this foreign land.

AP Calculus[16] is setup in a way that allows in-depth flexibility for the class, as a whole or individually, to investigate the topics to increase both the knowledge and understanding of the learner. To find these topics of investigation, the new teacher can reference the AP Calculus Course Description, which includes a topic outline (See Appendix A). Although the topic online is intended to indicate the scope of the course, teachers may change the order and enrich their courses with additional topics (Course Description, 2008).

High school curriculums are setup so that a student must take their mathematics courses in a succeeding order. It is normal for a student, who wishes to take calculus to be required to have taken and passed two years of algebra, a year of geometry and a year of trigonometry. More crucial than the prerequisite work is the possession of a strong desire to participate actively in this course. Active participation would include "a willingness to work both in and out of class, a willingness to collaborate with classmates to foster mutual understanding, and a sincere

[16] Calculus AB is a course in single-variable calculus that includes techniques and applications of the derivative, techniques and applications of the definite integral, and the Fundamental Theorem of Calculus. Emphasis is placed on being able to represent a problem algebraically, numerically and graphically. The material taught is equivalent to a semester of freshmen calculus at most colleges and universities. (About Us, 2008)

intent to place out of the first semester of college calculus rather than repeat it" (About Us, 2008).

After getting students anxiously engaged in taking calculus, teachers need to reflect on what the purpose of teaching calculus is and what deeper effects should take place in the classroom. The role of AP Calculus in high schools should not be to pad the transcript for college admissions, or to help students skip ahead in college by placing out of freshman college calculus. Rather, the role of AP Calculus in high schools should be focused on helping students establish a firm understanding of the algorithmic process and a conceptual understanding of assumption and conclusion, providing students with a sense that mathematics is not dead but is alive. When reviewing theorems more emphasis needs to be placed on the "if" and less on the "then". For example, when students are asked to state the Mean Value Theorem, most are ready to state the formula $f'(c) = \frac{f(b) - f(a)}{b - a}$, but fail to make any reference to the fact that the function has to be differentiable on the closed interval and c has to be included in the interval (a, b). As a guide, teachers must help students come to an appreciation that mathematics still contains more questions than answers, as not all things can be quantified, and that it was these questions that caught the interest of the great minds since the beginning of time.

CHAPTER 3 PLANNING AND PREPARING THE ONLINE COMPONENT

3.1 Introduction

Before discussing, what MyMathLab is and how it is used to help the new teacher meet the needs of guiding students through the conceptual jungles of calculus, it is necessary to explain how it was chosen as the web-based application for my AP Calculus course. I was first introduced to the MyMathLab system in the summer of 2008 as I went through a workshop on using it in connection with a dual enrollment opportunity in college algebra/trigonometry with Southeastern Louisiana University. I received further exposure to MyMathLab over the course of the Master of Natural Sciences program at Louisiana State University as it was used to complete two courses that used the online tutorial and assessment system. The use of this program, coupled with traditional lectures is a requirement of all College Algebra and College Trigonometry students at Louisiana State University and for all first and second year students in the Master of Natural Sciences program.

My course is setup so that prior to working through problems and assessments online, students attend formal lectures that discuss the process and procedures needed to successfully complete assignments. Students are expected to spend a specific amount of time in a supervised environment while completing the work, and using additional time outside of class as needed. For students in the Master of Natural Sciences program, similar requirements are also in place in the first and second year (summer) when Algebra and Calculus are emphasized respectively. The opportunity to use MyMathLab had a great impact on my teaching and leaning approaches for the future. The more I worked with the MyMathLab system, the more I was convinced that having online access was another step in the right direction for reaching AP Calculus students

and helping new teachers establish productive AP Calculus programs. With this idea in mind, my goal for the 2009-2010 school year was to:

- develop a MyMathLab based course for my own students, and
- create an Instructor's Resource Kit for new teachers

3.2 What Is MyMathLab?

MyMathLab is an innovative and user-friendly web-based system from Pearson Education, which includes homework, tutorials, study plans and assessments – all geared towards increasing mathematical ability and understanding. Algorithmic generated exercises and/or custom-built exercises, from which the teacher may select to create the homework and assessments. Each exercise is equipped to provide immediate feedback to the students, guided solutions, and access to a supplemental multimedia and tutorial exercise system.

3.3 Why Have an Online Component?

Teachers are faced with the challenge of differentiating instruction and providing ample exercises to meet the individual needs of the students. Textbooks and worksheets only provide a finite set of static problems and take time to create, grade, and identify weaknesses. MyMathLab is designed to overcome this first issue with the design of its algorithmic generated exercises. Each exercise has multiple iterations, providing the student with ample exercises to meet the individual needs of the students. MyMathLab also lends itself well to the differentiated instruction model, allowing students to be self-paced, as they are able to access the system anywhere via the web. As students work through the exercises, they are able to utilize several different helps, each designed to cater to the individual needs of the students, through the push of a button. These different helps include being guided systematically through the exercise,

working with an example, referencing the textbook, watching an instructional video or even asking the instructor.

With traditional paper and pencil homework, the student would toil away at solving a set of given problems, turn it in and wait for it to be graded only to discover that more practice is needed. With MyMathLab, this cycle is streamlined by providing immediate feedback and similar iterations for any additional practice that might be deemed necessary. The available immediate feedback and tutorial assistance motivates students to do more homework, hence spending more time doing math and preparing for quizzes and tests. Stuck and White (1992) highlighted the benefits of spending more time doing math when they said, "The most economical and feasible way to increase student's school achievement is to maximize the time teachers spend teaching the content and the time students spend paying attention and trying to learn." To study these events, John B. Carroll's model of school learning, which described a relationship between time and learning such that the degree of learning is a function of the time spent learning. In mathematical terms, Carroll's formula is

$$\text{Degree of Learning} = \frac{\text{Time Spent Learning}}{\text{Time Needed for Learning}}$$ (Stuck and White, 1992).

MyMathLab also helps overcome the grading and identifying weaknesses issue since it automatically grades online homework, quiz assignments, and tracks all student results. This allows the instructor to shift from spending time grading and focus more on the instruction of the material. Through the grade book, the teacher can monitor the progress of individual students or the whole class and at a glance, identify areas that need to be re-taught. This system is user-friendly and is easily customized to suit student needs.

Pearson Education, one of the world's largest publishers, offers a wealth of textbooks to customize specific MyMathLab courses. Pearson companies like Prentice Hall, Addison

Wesley, Scott Foresman, and Longman have over 200 textbooks with MyMathLab links from which to choose. This list of calculus books (See Appendix B) allowed me to select a textbook that would be best aligned to the needs of an AP Calculus class. The textbook that I selected was Calculus: Early Transendentals by William Briggs and Lyle Cochran

3.4 Development and Implementation of MyMathLab in AP Calculus

It is important for students to be able to progress through the different levels of being able to "do" math. First, students need to master the skills (the how-to's). Next, comes the mastery of the concepts (the whys). Finally, when these two have been used to lay down a solid foundation, the students are prepared to work on the real-life applications (the what if's). To help guide the student through these different levels, the teacher must use a variety of instructional approaches.

The MyMathLab system allows me, as the teacher, to maximize my instruction and allows my students to receive personalized and immediate feedback as they work on assignments online, in school, at home, or wherever internet is accessible. In my own classroom, I use a combination of traditional teaching (lecture), group work, and projects to investigate the material as needed. To help students progress to the higher-order thinking skills needed beyond my classroom, I start by helping the students establish a firm foundation of the facts, rules, formulas, theorems, procedures and the ability to communicate effectively graphically, numerically and algebraically. Coupling MyMathLab with my traditional teaching methods allows me to help my students construct a firm foundation that will be used as we traverse the material and explore the many real-life applications that exist.

3.5 Planning for the Use of MyMathLab as a Part of Teaching AP Calculus

The MyMathLab system allowed me to select a textbook that would best be aligned with the needs of an AP Calculus class. It also allowed me to customize the assignments and quizzes. The assignment section not only had problems to access and work, but also gave opportunities to have available personalized resource sections. After choosing the appropriate text, matching each section of the Briggs Cochran text with the Houghton Mifflin Calculus (7^{th} ed.) by Ron Larson, Robert Hostetler, and Bruce Edwards text, the book adopted by the local school system, was the next step (See Appendix C).

Once done, I began naming section topics and selecting questions appropriate for assignments. Then the decision would be made about quizzes for students. I also chose to create quizzes aligned with the assignments. I set the quizzes up in a manner that each student would be able to take the quiz up to ten times with no time limits on the length of each working session. In other words, a student could work on an assignment or a quiz an unlimited time just as long as the work was complete by the assigned deadline. Students would be allowed to work on the assigned material any place and anytime. All of this may sound “easy”, but is in fact a significant amount of work since all problems chosen do have to be checked carefully and put in the proper places. A complete list of the assignments and quizzes as a part of this project is provided in Appendix D.

CHAPTER 4 INSTRUCTIONAL RESOURCE KIT

4.1 Introduction

The Instructor's Resource Kit is the result of my experiences as a brand new teacher. Recalling my first days as a brand new teacher, when I felt I was equipped with a water gun to battle a five-alarm blaze, I vowed never to be without adequate resources again or let anyone else feel like that, if I could help it. To be better equipped and to help others effectively teach AP Calculus, I collected information, notes, activities and other educational issues as they pertain to AP Calculus from going to AP institutes, conferences, and collaborating with a network of other experienced AP Calculus teachers. I then compiled this collection into a resource kit. In addition to the lecture notes and activities, the new teacher can find information on administrative issues, the AP Audit, the AP Exam, recruitment, public relations, training, and more.

4.2 Administrative/Public Relations Issues

Before you can teach students calculus, you first need to have students enrolled. A major obstacle for many programs is finding a sufficient number of students that are both prepared and motivated to take AP Calculus. In my experience, students, parents and administrators shy away from AP Calculus because they are ignorant as to the true nature of AP Calculus. Many have the misconception that calculus is only for the little Einsteins and is beyond the ability of mortal students. To help battle this mentality, an AP informational night was organized to help dispel these misconceptions.

This AP information night is open to all that would like to find out more about AP, but potential AP students are identified and invited with the aid of the AP Potential Report and teacher recommendations. The AP Potential Report is a free, web-based tool that allows teachers

to generate a list of students who are statistically likely to score a 3 or higher on a given AP Exam. This report was designed to increase access to AP and to ensure that no student, who has the potential of succeeding in AP, is overlooked by drawing on the strong correlations between the PSAT/NMSQT scores and the AP Exam results, which are stronger predictors of success in AP than the more traditional factors. However, teachers need to be cautioned never to use the AP Potential report as a gatekeeper, since it does not account for such variables as student motivation, parental support and teacher efficacy.

4.3 AP Audit

Each institution that desires to have an AP program needs to complete the AP Audit process. Without the completion of the audit, schools are not allowed to utilize the title advanced placement in their course catalog, student transcripts or in any other manner. The AP Audit was created to provide clear guidelines on curricular and resource requirements that must be in place for AP Courses. This audit process also provides colleges with the confidence that AP Courses are designed to meet the same criteria across all participating high schools. Schools are required to renew this audit on an annual basis and must submit a new audit application in the event that the teacher, textbook, or syllabus changes.

4.4 AP Exam

At the end of the academic year, usually in May, students have the opportunity to take an exam in the given discipline. These exams represent the culmination of college-level work in a secondary school setting. Each exam tests the students' ability to perform at a college level and is rigorously developed by committees of college and AP high school faculty, who meet throughout the year to create and review new exams. Each exam contains a free-response

section and a section of multiple-choice questions. In AP Calculus, the sections are split into a calculator and non-calculator portion.

So that students might better appreciate the connections among various representations, it is assumed that they will have access to graphing calculators in class and on homework. To that end, and to shift the emphasis from mere computation to a deeper understanding of concepts, graphing calculators are required on portions of the AP Examination (About Us, 2008).

It is interesting to note that the multiple-choice questions are scored by computer, while the free-response questions are scored by hand. Annually during the summer, a team of college professors and high school teachers meet to score the constructed response questions. The involvement of college faculty at all levels of exam development and scoring ensures that the AP Exams truly reflect college-level achievement. Students who perform well can receive course credit and/or advanced standing at thousands of universities worldwide.

Although the goal for students should be to learn calculus rather than to succeed on a single assessment, the desire to do well on the AP Examination can be a strong motivation for students and an exploitable resource for teachers. It is recommended that teachers obtain copies of old AP Examinations (from colleagues or from the College Board), learn how they are scored, and hold students to that level of performance throughout the course. (Kennedy, 2008)

4.5 Professional Development Opportunities

Prerequisites for the teacher include a good understanding of calculus, a willingness to teach and learn from good students, and (if possible) the ability to attend a College Board® AP workshop or Summer Institute to communicate the goals of the course. (About Us, 2008)

Throughout the course of the year, a new teacher can find AP Institutes being offered by various educational entities. These professional development opportunities are designed to focus

on providing support to AP teachers with a wide range of experience and exposure to teaching AP courses. These sessions provide teachers with the opportunity to interact with experienced AP teachers while highlighting program development, curriculum, the exam, technology, and best practices. Additional topics that can be addressed are development of successful lesson plans, syllabi, assignments, AP Course Audit, test development and reading, pedagogical concerns and the AP exam.

Another professional development opportunity that is available to AP teachers is the AP Annual Conference, which is the largest professional development gathering of the Advanced Placement Program and Pre-AP communities. This annual gathering promises to engage, inspire and promote innovation in schools. The conference provides the opportunity to participate in workshops and sessions focused on instructional strategies for new and experienced teachers and increasing educational access and equity. The AP Annual Conference truly is a forum for all to exchange experiences, strengthen professional ties, and gain a better sense of how they can help their students to prepare for college success.

4.6 Lecture Notes and Activities

Whoever said, “Those that fail to plan, plan to fail,” could have been talking about educators. It is inadvisable to enter into a classroom without a clear and concise written plan of attack. These plans need to include the appropriate educational lesson plan structure, consisting of measureable objectives, a bellringer, an anticipatory set or hook, lesson, guided practice, independent practice and assessment. To avoid creating a stagnant learning environment, the teacher needs to infuse their personality into the lesson while using a variation of activities, projects, and instructional models to help the students explore the world of calculus. The

resource kit includes lecture notes and corresponding activities that I created and a sample of these lecture notes is included in Appendix E.

CHAPTER 5 REFLECTIONS

At the beginning of this thesis project, I had set out to help other teachers learn how to bring a great educational opportunity to their students by establishing an AP Calculus program in their school. It was my goal to help them become experienced tour guides, or at least well prepared tour guides, so they could blaze the mathematical trail and lead students to stretch their minds and acquire a thirst for the "what ifs" of math. However, I had not expected myself to be one of the big benefactors of this thesis. I knew I would learn, but I had underestimated how much I would learn.

During the literature review, I was able to gain a deeper understanding of the need to help students develop a higher level of mathematical literacy. I learned that this not solely rooted in the rote memorization and algorithmic processes of mathematics but greatly lies in the search of why something works and how it can be manipulated.

In the movie "The Karate Kid," Mr. Miyagi had Daniel wax the car, paint the house, scrub the floor, and paint the fence. When challenged on his instructional methods, Mr. Miyagi showed Daniel that he had indeed learned karate. Through these repetitive tasks, his muscles had been trained to respond instinctively with power. I learned that many of our students are like Daniel when it comes to performing the same algorithmic tasks repeatedly. It is our job to help them see the bigger picture beyond task set before them. Helping students train their own mathematical muscles to respond instinctively will help them gain further insight, thus transforming them into capable thinkers.

Technology has been propelling humanity forward at an increasing rate. For example, cellular devices now hold the computational power to not only operate as phone, but also operate as a P.D.A., internet access, mp3 player, camera, and G.P.S. Students literally hold the world's

information at their fingertips. I learned that through the application of technology, educators could begin help students to study mathematics in a new manner. Instead of discussing volume by slicing with a two-dimensional drawing, teachers can access three-dimensional digital images that can be manipulated, thus bringing the mathematics alive.

In the planning and preparation of the online component, I learned that it is not as simple as clicking on some random problem sets. Each question has to be vetted against the course objectives, level of depth needed, and course continuity. The textbook had to be aligned with existing course requirements and instructional notes needed to be prepared. This process taught me about the different layers that make up curriculum design.

In the end, the development of the resource kit dredged up so many lessons. I learned to remember to continually learn from my past and move to the future. Being able to collaborate with teachers has taught me that there still is work to do, and only by working together will we be able to accomplish that work.

REFERENCES

Equity Policy Statement. (2002). Retrieved June 20, 2010, from www.apcentral.collegeboard.com: http://apcentral.collegeboard.com/apc/members/repository/ap03_ap_equity_policy_22057.pdf

A Brief History of the Advanced Placement Program. (2003). Retrieved June 12, 2010, from www.collegeboard.com: http://www.collegeboard.com/prod_downloads/about/news_info/ap/ap_history_english.pdf

About Us. (2008). Retrieved April 25, 2009, from www.collegeboard.com: http://www.collegeboard.com/about/index.html

Course Description. (2008). Retrieved April 25, 2009, from www.apcentral.collegeboard.com: http://apcentral.collegeboard.com/apc/public/repository/ap08_calculus_coursedesc.pdf

Calculus. (2009). Retrieved May 6, 2009, from Merriam-Webster Online Dictionary: http://www.merriam-webster.com/dictionary/calculus

Bell, E. (2008). *Men of Mathematics - The Lives and Achievements of the Great Mathematicians from Zeno to Poincare.* New York: Touchstone.

Boyer, C. (1959). *The History of the Calculus and its Conceptual Development.* New York: Dover Publications.

Bracey, G. W. (2000). The TIMSS "Final Year" Study and Report: A Critique. *Educational Researcher, 99*(4), pp. 4-10.

Bressoud, D. (2004). The Changing Face of Calculus: First-Semester Calculus as a High School Course. *MAA Focus, 24*(6), 6-8.

Bressoud, D. (2008). *Calculus Before Newton and Leibinz: Part I.* Retrieved April 25, 2009, from www.apcentral.collegeboard.com: http://apcentral.collegeboard.com/apc/members/features/2015.html

Bressoud, D. (2008). *Calculus Before Newton and Leibinz: Part II.* Retrieved April 25, 2009, from www.apcentral.collegeboard.com: http://apcentral.collegeboard.com/apc/members/features/9118.html

Bressoud, D. (2008). *Calculus Before Newton and Leibinz: Part III.* Retrieved April 25, 2009, from www.apcentral.collegeboard.com: http://apcentral.collegeboard.com/apc/members/features/10812.html

Burton, N. W., & et al. (2002). *Minority Students Success: The Role of Teachers in Advanced Placement Program (AP) Courses.* New York: College Entrance Examination Board.

English, S. J. (2002). *Time for Technology: Successfully integrating technology in elementary school classrooms.* Retrieved June 12, 2010, from www.eric.ed.gov: http://eric.ed.gov/ERICDocs/data/ericdocs2sql/content_storage_01/0000019b/80/1a/e4/14.pdf

Fauvel, J., & Gray, J. (Eds.). (1987). *The History of Mathematics: A Reader.* London: MacMilan Press.

Grattan-Guiness, I. (1997). *The Norton History of the Mathematical Sciences.* New York: W.W. Norton and Comany.

Harper, S. R., & Driskell, S. (Eds.). (2006). Teacher's technology class continues discussion of pitfalls. *Mathematics Teacher, 99*(5), pp. 340-342.

Hokanson, B., & Hooper, S. (2004). Integrating technology in classrooms: We have met the enemy and he is us. *Association for Educational Communications and Technology, 27*(5).

International Society for Technology in Education (ISTE). (2010). *National Educational Technology Standards.* Retrieved June 26, 2010, from www.iste.org: https://www.iste.org/AM/Template.cfm?Section=NETS

JBHE Foundation. (1999). So Much for the Theory That Blacks Can't Do Mathematics. *The Journal of Blacks in Higher Education, 25,* pp. 48-49.

Kellough, R. D., & Carjuzza, J. (2006). *Teaching in the middle and secondary schools* (8th ed.). New Jersey: Pearson Prentice Hall.

Kennedy, D. (2008). *Calculus AB Course Perspective.* Retrieved April 25, 2009, from www.apcentral.collegeboard.com: http://apcentral.collegeboard.com/apc/public/courses/descriptions/4347.html

Kline, M. (1985). *Mathematics for the Nommathematician.* New York: Dover Publications.

Kulik, J. A. (2003). *Effects of Using Instructional Technology in Elementary and Secondary Schools: What controlled evaluation studies say.* Retrieved June 12, 2010, from www.sri.com: http://www.sri.com/policy/csted/reports/sandt/it/Kulik_ITinK-12_Main_Report.pdf

Li, Q. (2005). Infusing technology into a mathematics methods course: any impact? *Educational Research, 47*(2), pp. 217-233.

Louisiana Department of Eduction. (2008). *Leadership and Technology: Educational Technology Standards.* Retrieved June 26, 2010, from www.doe.state.la.us: http://www.doe.state.la.us/lde/lcet/324.html

Metzker, B. (2003). *Time and Learning.* Retrieved June 12, 2010, from www.eric.ed.gov: http://www.eric.ed.gov/ERICDocs/data/ericdocs2sql/content_storage_01/0000019b/80/1a/dd/9d.pdf

Moses, R. P. (2001). *Radical Equations: Math literacy and civil rights.* Boston: Beacon Press.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Robertson, B. (2001). Integrating Technology into Instruction. *MultiMedia School, 7*(2), p. 34.

Rogers, T. W., & et al. (2001). *Key Building Blocks fro Student Achievement in the 21st Century: Assessment, Alignment, Accountability, Access and Analysis.* Retrieved June 12, 2010, from www.ceoforum.org: http://www.ceoforum.org/downloads/report4.pdf

Rosenstein, G. M. (2008). *One Hundred and Fifty Years of Teaching Calculus.* Retrieved April 25, 2009, from www.apcentral.collegeboard.com: http://apcentral.collegeboard.com/apc/public/repository/ap02_calc_aptalk_fina_19516.pdf

Rothschild, E. (1999). Four Decades of the Advanced Placement Program. *The History Teacher, 32*(2), pp. 175-206.

Ryan, K., & Cooper, J. M. (2004). *Those who can, teach* (10th ed.). New York: Houghton Mifflin Company.

Sandholtz, J. H., Ringstaff, C., & Dwyer, D. C. (1997). *Teaching with Technology: Creating student-centered classrooms.* New York: Teachers College Press.

Sawyer, W. (1961). *What Is Calculus About?* New York: Random House.

Schoenfeld, A. H. (2002). Making Mathematics Work for All Children: Issues of Standards, Testing, and Equity. *Educational Researcher, 31*(1), 13-25.

Steckroth, J. J. (2006). TI-Smartview: revolutionizing representations in mathematics. (S. Driskell, & R. Rider, Eds.) *Mathematics Teacher, 100*(2), pp. 146-150.

Stuck, G. B., & White, K. P. (1992). *Maximizing Time to Teach and Time to Learn.* Retrieved June 12, 2010, from www.eric.ed.gov:

http://www.eric.ed.gov/ERICDocs/data/ericdocs2sql/content_storage_01/0000019b/80/13/4a/48.pdf

Sweet, J. R., & et. al. (2004). *Case Studies of High-Performing, High-Technology Schools: Final research report on schools with predominatly low-income, African-American, or Latino students.* Learning Point Associates/ North Central Regional Educational Laboratory (NCREL).

Viadero, D. (1998). U.S. Seniors Near Bottom in World Test. *Education Week*, p. 1.

Wenglinsky, H. (1998). *Does it Compute? The relationship between education technology and student achievement in mathematics.* Retrieved June 12, 2010, from www.eric.ed.gov: http://eric.ed.gov/ERICDocs/data/ericdocs2sql/content_storage_01/0000019b/80/17/11/aa.pdf

Wenglinsky, H. (2006). Technology and Achievement: The bottom line. *Educational Leadership, 63*(4), pp. 29-32.

Worrell, F. T. (1962). What is Calculus About? *American Journal of Physics, 30*(12), pp. 938-939.

APPENDIX A: AP CALCULUS AB TOPIC OUTLINE

1. Functions, Graphs, and Limits	
1.1 Analysis of graphs	With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.
1.2 Limits of functions (including one-sided limits)	• An intuitive understanding of the limiting process • Calculating limits using algebra • Estimating limits from graphs or tables of data
1.3 Asymptotic and unbounded behavior	• Understanding asymptotes in terms of graphical behavior • Describing asymptotic behavior in terms of limits involving infinity • Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)
1.4 Continuity as a property of functions	• An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.) • Understanding continuity in terms of limits • Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem
2. Derivatives	
2.1 Concept of the derivative	• Derivative presented graphically, numerically, and analytically • Derivative interpreted as and instantaneous rate of change • Derivative defined as the limit of the difference quotient • Relationship between differentiability and continuity
2.2 Derivative at a point	• Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents. • Tangent line to a curve at a point and local linear approximation • Instantaneous rate of change as the limit of average rate of change • Approximate rate of change from graphs and tables of values
2.3 Derivate as a function	• Corresponding characteristivs of graphs of f and f' • Relationship between the increasing and decreasing behavior of f and the sign of f' • The Mean Value Theorem and its geometric interpretation • Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.
2.4 Second derivatives	• Corresponding characteristics of the graphs of f, f', and f'' • Relationship between the concavity of f and the sign of f'' • Points of inflection as places where concavity changes

2.5 Application of derivatives	• Analysis of curves, including the notions of monotonicity and concavity • Optimization, both absolute (global) and relative (local) extrema • Modeling rates of change, including related rates problems • Use of implicit differentiation to find the derivative of an inverse function • Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration • Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations
2.6 Computation of derivatives	• Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions • Derivative rules for sums, products, and quotients of functions • Chain rule and implicit differentiation
3. Integrals	
3.1 Interpretations and properties of definite integrals	• Definite integral as a limit of Riemann sums • Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: $\int_a^b f(x)dx = F(b) - F(a)$ • Basic properties of definite integrals (examples include additivity and linearity)
3.2 Application of integrals	• Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change.
3.3 Fundamental Theorem of Calculus	• Use of the Fundamental Theorem to evaluate definite integrals • Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined
3.4 Techniques of antidifferentiation	• Antiderivatives following directly from derivatives of basic functions • Antiderivatives by substitution of variables (including change of limits for definite integrals)

3.5 Applications of antidifferentiation	• Finding specific antiderivatives using initial conditions, including applications to motion along a line • Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth)
3.6 Numerical approximation to definite integrals	• Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

APPENDIX B: CALCULUS TEXTBOOKS AVAILABLE ON MYMATHLAB

Author	Book Title	Ed	copyright	ISBN (text/student access)
Adams	Calculus: A Complete Course (HB) Pearson Canada	7	2010	0-13-704400-3
Barnett/ Ziegler/ Byleen	Calculus for Business, Economics, Life Sciences & Social Sciences (HB)	11	2008	0-13-513149-9
Briggs/ Cochran	Calculus (HB)	1	2011	TBA
Briggs/ Cochran	Calculus: Early Transcendentals (HB)	1	2011	TBA
Finney/ Demana/ Waits/ Kennedy	Calculus: Graphical, Numerical, Algebraic (HB)	3	2007	N/A
Goshaw	Concepts of Calculus with Applications (HB)	1	2008	0-321-58427-9
Hass/Weir/ Thomas	University Calculus (HB)	1	2007	0-321-40931-0
Hass/Weir/ Thomas	University Calculus: Alternate Edition (HB)	1	2008	0-321-51243-X
Hass/Weir/ Thomas	University Calculus: Elements with Early Transcendentals (HB)	1	2009	0-321-57057-X
Lial/ Greenwell/ Ritchey	Calculus with Applications (HB)	9	2008	0-321-44873-1
Thomas/ Weir/Hass	Thomas' Calculus, Early Transcendentals (HB)	12	2010	0-321-64842-0
Thomas/ Weir/Hass/ Giordano	Thomas' Calculus (HB)	12	2010	0-321-61469-0
Thomas/ Weir/Hass/ Giordano	Thomas' Calculus, Early Transcendentals (HB)	11	2006	0-321-26759-1
Varberg/ Purcell/ Rigdon	Calculus (HB)	9	2007	0-13-230811-8
Varberg/ Purcell/ Rigdon	Calculus Early Transcendentals (HB)	1	2007	0-13-225856-0

http://www.mymathlab.com/titles-available-category/171#table

APPENDIX C: TOPIC ALIGNMENT CHART

<u>Calculus: Early Transendentals</u> by William Briggs and Lyle Cochran	**<u>Calculus</u> (7th ed.)** by Ron Larson, Robert Hostetler, and Bruce Edwards

TOPIC: FUNCTIONS

-	P.1 – Graphs and Models
-	P.2 – Linear Models and Rates of Change
1.1 – Review of Functions	P.3 – Functions and Their Graphs
1.2 – Representing Functions	P.4 – Fitting Models to Data
1.3 – Inverse, Exponential, and Logarithmic Functions	-
1.4 – Trigonometric Functions and Their Inverses	Appendix D.3 – Review of Trigonometric Functions

TOPIC: LIMITS

2.1 – The Idea of Limits	1.1 – A Preview of Calculus
-	1.2 – Finding Limits Graphically and Numerically
2.2 – Definitions of Limits	1.3 – Evaluating Limits Analytically
2.3 – Techniques for Computing Limits	
2.4 – Infinite Limits	1.5 – Infinite Limits
2.5 – Limits at Infinity	3.5 – Limits at Infinity
2.6 – Continuity	1.4 – Continuity and One-Sided Limits
2.7 – Precise Definition of Limits	-

TOPIC: DIFFERENTIATION

3.1 – Introducing the Derivative	2.1 – The Derivative and the Tangent Line Problem
3.2 – Rules of Differentiation	2.2 – Basic Differentiation Rules and Rates of Change
3.3 – The Product and Quotient Rules	2.3 – The Product and Quotient Rules and Higher-Order Derivatives
3.4 – Derivatives of Trigonometric Functions	-
3.5 – Derivatives as Rates of Change	-
3.6 – The Chain Rule	2.4 – The Chain Rule
3.7 – Implicit Differentiation	2.5 Implicit Differentiation
3.8 – Derivatives of Logarithmic and Exponential Functions	5.1 – The Natural Logarithmic Function: Differentiation
3. 9 – Derivatives of Inverse Trigonometric Functions	5.3 – Inverse Functions
3.10 – Related Rates	2.6 – Related Rates

TOPIC: APPLICATIONS OF DIFFERENTIATION

4.1 – Maxima and Minima	3.1 – Extrema on an Interval
4.2 – What Derivatives Tell Us	3.3 – Increasing and Decreasing Functions and the First Derivative Test
4.2 – What Derivatives Tell Us	3.4 – Concavity and the Second Derivative Test
4.3 – Graphing Functions	3.6 – A Summary of Curve Sketching
4.4 – Optimization Problems	3.7 – Optimization Problems
4.5 – Linear Approximations and Differentials	3.9 – Differentials
4.6 – Mean Value Theorem	3.2 – Rolle's Theorem and the Mean Value Theorem
4.7 – L'Hôpital's Rule	7.7 – Indeterminate Forms and L'Hôpital's Rule
4.8 – Antiderivatives	4.1 – Antiderivatives and Indefinite Integration

TOPIC: INTEGRATION

5.1 – Approximating Areas Under Curves	4.1 – Antiderivatives and Indefinite Integration
	4.2 – Area
5.2 – Definite Integrals	4.3 – Riemann Sums and Definite Integrals
5.3 – Fundamental Theorem of Calculus	4.4 – The Fundamental Theorem of Calculus
5.4 – Working with Integrals	
5.5 – Substitution Rule	4.5 – Integration by Substitution

TOPIC: APPLICATIONS OF INTEGRATION

6.1 – Velocity and Net Change	-
6.2 – Regions Between Curves	6.1 – Area of a Region Between Two Curves
6.3 – Volume by Slicing	6.2 – Volume: Disk Method
6.4 – Volume by Shells	6.3 – Volume: Shell Method
6.5 – Length of Curves	6.4 – Arc Length and Surfaces of Revolution
6.6 – Physical Applications	6.5 – Work
6.7 – Logarithmic and Exponential Functions Revisited	5.1 – The Natural Logarithmic Function: Differentiation
	5.2 – The Natural Logarithmic Function: Integration
	5.3 – Inverse Functions
	5.4 – Exponential Functions: Differentiation and Integration
	5.5 – Bases Other Than *e* and Applications
	5.7 – Differential Equations: Separation of Variables
	5.8 – Inverse Trigonometric Functions: Differentials

	5.9 – Inverse Trigonometric Functions: Integration
	5.10 – Hyperbolic Functions
6.8 – Exponential Models	5.6 – Differential Equations: Growth and Decay
7.1 – Integration by Parts	7.2 – Integration by Parts
7.6 – Numerical Integration	3.8 – Newton's Method
	4.6 – Numerical Integration

APPENDIX D: MYMATHLAB HOMEWORK AND QUIZ TOPICS

MyMathLab Homework Assignment Topics
Created by David Woods

Order	Ch.	Assignment Name
1	0	0.1 Answering Exercises Orientation
3	GR	GR.1: Review the Basics
4	GR	GR.2: Review of Algebra I
5	GR	GR.2: Review of Algebra II
6	GR	GR.2: Review of Algebra III
7	1	1.1: Review of Functions
8	1	1.2: Representing Functions
10	GR	GR.3: Review of Exp.and Logarithmic Functions I
11	GR	GR.3: Review of Exp.and Logarithmic Functions II
12	1	1.3: Inverse, Exponential, Logarithmic Functions
13	GR	GR.4: Review of Complex Numbers
14	GR	GR.4: Review of Trigonometric Functions I
15	GR	GR.4: Review of Trigonometric Functions II
16	GR	GR.4: Review of Trigonometric Functions III
17	1	1.4: Trigonometric Functions and Their Inverses
19	GR	GR.5: Getting Ready for Limits
20	2	2.1: The Idea of Limits
21	2	2.2: Definitions of Limits
23	2	2.3: Techniques for Computing Limits
25	2	2.4: Infinite Limits
26	2	2.5: Limits at Infinity
28	2	2.6: Continuity
29	2	2.7: Precise Definitions of Limits
32	GR	GR.6: Getting Ready for Derivatives
33	3	3.1: Introducing the Derivative
35	3	3.2: Rules of Differentiation
37	3	3.3: The Product and Quotient Rules
39	3	3.4: Derivatives of Trigonometric Functions
41	3	3.5: Derivatives as Rates of Change
43	GR	GR.7: Getting Ready for the Chain Rule
44	3	3.6: The Chain Rule
46	3	3.7: Implicit Differentiation
48	3	3.8: Derivatives of Logarithmic and Exp. Functions
49	3	3.9: Derivatives of Inverse Trigonometric Function
51	GR	GR.8: Getting Ready Related Rates and Max/Min Apps
52	3	3.10: Related Rates
55	4	4.1: Maxima and Minima

56	4	4.2: What Derivatives Tell Us
58	4	4.3: Graphing Functions
60	4	4.4: Optimization Problems
62	4	4.5: Linear Approximation and Differentials
63	4	4.6: Mean Value Theorem
65	4	4.7: L'Hôpital's Rule
66	4	4.8: Antiderivatives
69	5	5.1: Approximating Areas under Curves
71	5	5.2: Definite Integrals
72	5	5.3: Fundamental Theorem of Calculus
74	5	5.4: Working with Integrals
75	5	5.5: Substitution Rule
78	6	6.1: Velocity and Net Change
80	6	6.2: Regions Between Curves
82	6	6.3: Volume by Slicing
83	6	6.4: Volume by Shells
85	6	6.5: Length of Curves
86	6	6.6: Physical Applications
88	6	6.7: Logarithmic and Exp. Functions Revisited
89	6	6.8: Exponential Models
92	7	7.1: Integration by Parts
93	7	7.4: Partial Fractions
94	7	7.6: Numerical Integration

MyMathLab Quizzes/Tests with Corresponding Section Numbers
Created by David Woods

Order	**Ch.**	**Assignment Name**
2	1-6	Pre-Test
9	1	Quiz #1 (1.1 - 1.2)
18	1	Quiz #2 (1.3 - 1.4)
22	2	Quiz #3 (2.1 - 2.2)
24	2	Quiz #4 (2.3)
27	2	Quiz #5 (2.4 - 2.5)
30	2	Quiz #6 (2.6 - 2.7)
31	2	Test #1 (Ch. 1-2)
34	3	Quiz #7 (3.1)
36	3	Quiz #8 (3.2)
38	3	Quiz #9 (3.3)
40	3	Quiz #10 (3.4)
42	3	Quiz #11 (3.5)
45	3	Quiz #12 (3.6)
47	3	Quiz #13 (3.7)
50	3	Quiz #14 (3.8 - 3.9)
53	3	Quiz #15 (3.10)
54	3	Test #2 (Ch. 3)
57	4	Quiz #16 (4.1 - 4.2)
59	4	Quiz #17 (4.3)
61	4	Quiz #18 (4.4)
64	4	Quiz #19 (4.5 - 4.6)
67	4	Test #3 (Ch. 4)
68	4	Fall Semester Exam
70	5	Quiz #20 (5.1)

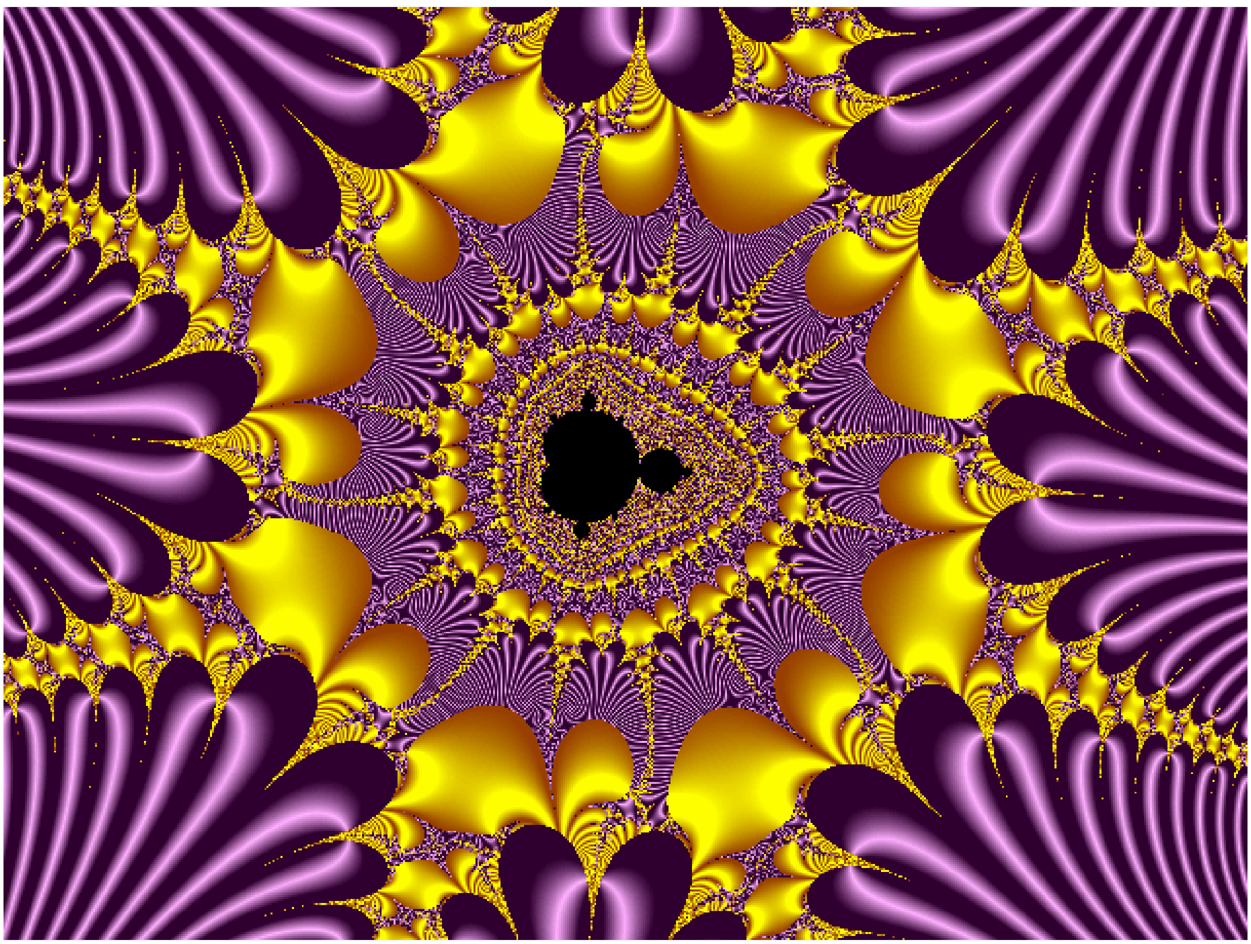
73	5	Quiz #21 (5.2 - 5.3)
76	5	Quiz #22 (5.4 - 5.5)
77	5	Test #4 (Ch. 5)
79	6	Quiz #23 (6.1)
81	6	Quiz #24 (6.2)
84	6	Quiz #25 (6.3 - 6.4)

87	6	Quiz #26 (6.5 - 6.6)
90	6	Quiz #27 (6.7 - 6.8)
91	6	Test #5 (Ch. 6)
95	7	Quiz #28 (7.6)
96	1	Spring Semester Final

APPENDIX E: SAMPLE FROM RESOURCE KIT

The following documents are selected excerpts from the resource kit. Those wishing to review the entire instructor's resource kit may do so online at http://cid-4460ce8b61530556.skydrive.live.com/redir.aspx?resid=4460CE8B61530556!123&Bpub=SDX.Docs&Bsrc=GetSharingLink. This link will be available until May 2012.

AP CALCULUS AB RESOURCE KIT: INSTRUCTIONAL NOTES TO ACCOMPANY F10 – S11 AP CALCULUS LSUCRP



By: David Woods

Tara High School

PREFACE

This instructional resource kit has been designed to support the instruction of AP Calculus in the high school environment. My approach is based on my experience of teaching AP Calculus AB to a diverse student population using the best teaching practices that I know. Teachers are encouraged to adapt this kit to the needs of their individual situation. This kit is not intended to tell teachers "how" to teach, but give new teachers a road map of "what" to teach. The overall goal was not to write a reference book for a shelf, but to provide a user-friendly source of suggestions and activities for any teacher of calculus within a calculus curriculum.

PEDAGOGICAL FEATURES

Each instructional unit includes pedagogical features to provide insight into the material. Lessons are designed to help the teacher guide the students on their exploration of calculus and aid in the development of higher-order thinking skills. The included pedagogical features include:

- Chapter Preview
- Section and Concept Introductions
- Corresponding College Board Objectives for AP Calculus AB
- Concept Objectives
- Bellringers
- Review of Vocabulary
- Guided Practice
- Independent Practice
- Homework
- Support Notes
- Additional Activities, Projects and Resources

ONLINE SUPLLEMENTS

MyMathLab® Online Course (access coded required)
Mathematical Visualization Toolkit
AP Central
TI-SmartView

METHODOLOGY

I have been teaching Advanced Placement Calculus AB since 2005. I recall that my first year teaching calculus was not as smooth as I would have liked. Fresh out of school and new to the realm of teaching I was given the great privilege of having AP Calculus among the courses I would be teaching. Being a recent mathematics graduate, I knew the content well and felt ready for the challenge that lay before me. Upon entering the classroom the first day, I realized how wrong I was. Armed with only a textbook and my content knowledge, I felt like I had brought a water gun to a five-alarm blaze. I left the class that day determined never to feel like that again. I went home and found as many supplemental resources I could get my hands on. Over the years, I have continued this by going to AP summer institutes, conferences, and

collaborating with a network of other AP Calculus teachers. Now presented with the opportunity to help others, I have formally organized all these resources into a user-friendly instructor's resource kit that accompanies an online portion of the class to help facilitate in assessing the students through homework, quizzes and tests. The expected outcome is new teachers will be able to utilize the resources as a springboard into their teaching career, establishing a successful AP Calculus program that prepares students to go deeper in their collegiate studies.

The material in this kit is presented to correspond to the order of the material in the Briggs and Cochran Textbook being used for this course. Additional activities and resources are added to the lessons to provide for deeper investigations and an increased level of rigor. Each student is issued a TI-84 Plus for the duration of this course. We use graphing calculators daily to explore, discover, and reinforce the concepts of calculus and the relationship between graphical representation and the various functions. Class participation is expected from all students, and an emphasis on proper use of language and mathematical notation is stressed.

Students are given plenty of time to work in class in small groups on a variety of problems and projects. Students are given homework on MyMathLab and are expected to complete all assignments before the designated due date.

The course is structured in such a way that students are allowed to connect ideas and explore the various applications of the content. AP-type questions are used throughout the course to expose students to the type of questions that they may see on the AP Calculus exam. These questions are also used to draw connections between different concepts and preview future concepts both in AP Calculus AB and future collegiate studies.

EVALUATION

Students are given informal assessments throughout the lesson. Formal assessments (quizzes) are given every Friday and chapter tests are given at the conclusion of a chapter. The students also take a semester exam in December and May. In addition to the informal and formal assessments, students also participate in two AP practice exams – one sponsored by the mathematics department at Louisiana State University and the other sponsored by the school.

SCHOOL ENVIRONMENT

Tara High School is a public high school in the East Baton Rouge Parish School System. School enrollment ranges from 900 – 1200 in any given year, with a high level of socio-economically disadvantaged students. The school uses a semester system and offers the standard selection of high school courses. To graduate, students are required to have 24 units of credit with 4 units in math, science, English, and social studies, 2 units in a foreign language, 2 units in physical/health education, 1 unit in the arts, and the remaining 3 units coming from electives. In addition, the standard curriculum, Tara offers 4 college-level Advanced Placement courses and 4 dual enrollment courses, permitting students to earn college credit for their work. The school year is divided into six 6-week grading periods. Students are required to take seven classes for the duration of the school year. The average class size is 30 students. The school day, from 7:05 a.m. to 2:23 p.m., which includes seven 53-minute academic periods plus lunch and five minute class changes. Study sessions are held as needed after school from 3:00 p.m. to 5:00 p.m. and during my planning period.

INSTRUCTIONAL GOALS BY 6-WEEK GRADING PERIOD

Generally, Monday and Wednesday are used for instruction of new material and Tuesday and Thursday, students are given the opportunity to work on MyMathLab homework in the computer lab. Fridays are also used for lab-time, if additional instructional time is not needed. Group projects are normally substituted for lab-time as needed.

First 6-weeks

SECTION	TOPIC	DATE TAUGHT	DATE ASSESSED
1.1	Review of Functions		
1.2	Representing Functions		
1.3	Inverse, Exponential, and Logarithmic Functions		
1.4	Trigonometric Functions and Their Inverses		
2.1	The Idea of Limits		
2.2	Definitions of Limits		
2.3	Techniques for Computing Limits		
2.4	Infinite Limits		
2.5	Limits at Infinity		
2.6	Continuity		
2.7	Precise Definition of Limits		

Second 6-weeks

SECTION	TOPIC	DATE TAUGHT	DATE ASSESSED
3.1	Introducing the Derivative		
3.2	Rules of Differentiation		
3.3	The Product and Quotient Rules		
3.4	Derivatives of Trigonometric Functions		
3.5	Derivatives as Rates of Change		
3.6	The Chain Rule		
3.7	Implicit Differentiation		
3.8	Derivatives of Logarithmic and Exponential Functions		
3.9	Derivatives of Inverse Trigonometric Functions		
3.10	Related Rates		

Third 6-weeks

SECTION	TOPIC	DATE TAUGHT	DATE ASSESSED
4.1	Maxima and Minima		
4.2	What Derivatives Tell Us		
4.3	Graphing Functions		

4.4	Optimization Problems		
4.5	Linear Approximation and Differentials		
4.6	Mean Value Theorem		
4.7	L'Hôpital's Rule		
4.8	Antiderivatives		

Fourth 6-weeks

SECTION	TOPIC	DATE TAUGHT	DATE ASSESSED
5.1	Approximating Areas Under Curves		
5.2	Definite Integrals		
5.3	Fundamental Theorem of Calculus		
5.4	Working with Integrals		
5.5	Substitution Rule		
2.7	Precise Definition of Limits		

Fifth 6-weeks

SECTION	TOPIC	DATE TAUGHT	DATE ASSESSED
6.1	Velocity and Net Change		
6.2	Regions Between Curves		
6.3	Volume by Slicing		
6.4	Volume by Shells		
6.5	Length of Curves		
6.6	Physical Applications		
6.7	Logarithmic and Exponential Functions Revisited		
6.8	Exponential Models		
7.1	Integration by Parts		
7.6	Numerical Integration		

Sixth 6-weeks

SECTION	TOPIC	DATE TAUGHT	DATE ASSESSED
	Review		

CHAPTER 2
"LIMITS"

Chapter Preview

All of calculus is based on the idea of a limit. Not only are limits important in their own right, but they underlie the two fundamental operations of calculus: differentiation (calculating derivatives) and integration (evaluating integrals). Derivatives enable us to talk about the instantaneous rate of change of a function, which, in turn, leads to concepts such as velocity and acceleration, population growth rates, marginal costs, and flow rates. Integrals enable us to compute areas under curves, surface areas, and volumes. Because of the incredible reach of this single idea, it is essential to develop a solid understanding of limits.

Section 1: The Idea of Limits

SECTION INTRODUCTION: This section illustrates how limits arise in two seemingly unrelated problems: finding the instantaneous velocity of a moving object and finding the slope of a line tangent to a curve. These two problems provide important insights into limits, and they reappear in various forms throughout the book.

COLLEGE BOARD OBJECTIVE (S): **Analysis with Graphs –** With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits) – An intuitive understanding of the limiting process is sufficient for this course.

- Calculating limits using algebra
- Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior
- Describing asymptotic behavior in terms of limits involving infinity
- Comparing relative magnitudes of functions and their rates of change (For example, contrasting exponential growth, polynomial growth, and logarithmic growth)

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to curve at a point and local linear approximation
- Instantaneous rate of change as the limit of average rate of change
- Approximate rate of change from graphs and tables of values

Concept 1: Average Velocity

OBJECTIVE(S): TSWBAT

- o calculate average velocity (GLE 11/12: N/A)

BELLRINGER(S):

- o Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 meters above the ground. Find the average velocity between $t = 2\sec$ and $t = 5\sec$.
    - ▪ Solution:
        - To find the position of the ball at time *t*, use the function for projectile motion $s(t) = \frac{1}{2}at^2 + v_0 t + s_0$, where *a* is acceleration due to gravity, v_0 is initial velocity, and s_0 is initial position.

- Average velocity = $\dfrac{\text{change in distance}}{\text{change in time}}$
- $\dfrac{s(5)-s(3)}{5-3}$

  39.2 m/s down
- In your own words, what is calculus?
  - Answers will vary

LESSON:

- Concept Introduction: Calculus takes the topics previously covered (tangent lines, slopes, areas, velocities, and acceleration) and applies the limit process to them. It truly is the mathematics of change.

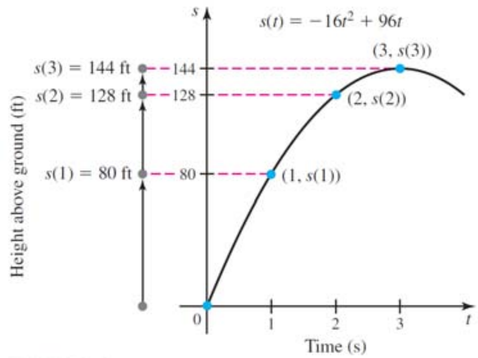
WITHOUT CALCULUS	WITH CALCULUS
Slope of a line	Slope of a curve at a given point
Average rate of change over time	Instantaneous rate of change
Height of a curve at a given point	Maximum height of a curve on a given interval
Direction of motion along a straight line	Direction of motion along a curved line
Area of a rectangle	Area under a curve

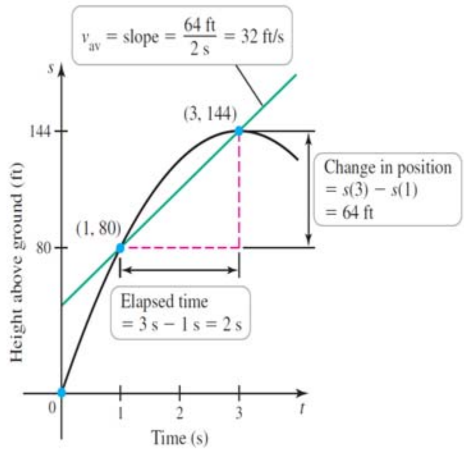
- Review of Vocabulary:
  - Average velocity → The velocity over a given time interval. Change of distance (displacement) divided by the interval of time, given by $v_{av} = \dfrac{\Delta s}{\Delta t}$. Another notation used for average velocity is $\overline{v}$.
  - Instantaneous velocity → The velocity at a given instant in time. This instantaneous rate of change is found by applying the limit to average velocity, given by $v = \lim\limits_{t \to 0} \dfrac{\Delta s}{\Delta t}$.
  - Secant[17] line → A line joining two points, (t_1, s_1) and (t_2, s_2), on a curve with a slope given by $m_{\sec} = \dfrac{s_2 - s_1}{t_2 - t_1}$

- Guided Practice:
  - A rock is launched vertically upward from the ground with a speed of 96 ft/s. Neglecting air resistance, we can use the position formula for projectiles, given by $s(t) = \dfrac{1}{2}at^2 + v_0 t + s_0$. Find the average velocity of the rock between each pair of times.
    - $t = 1s$ and $t = 3s$
    - $t = 1s$ and $t = 2s$

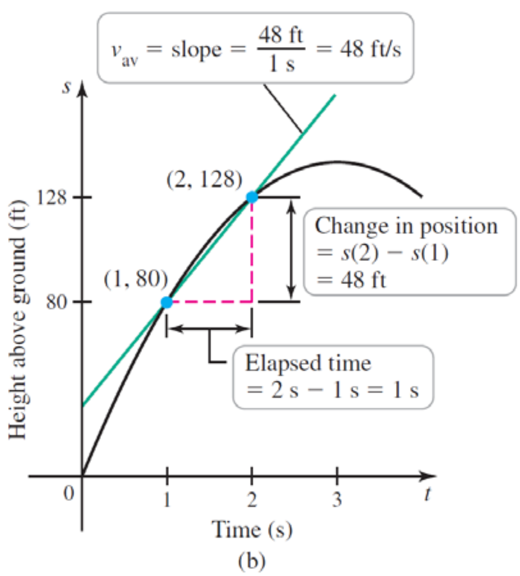
[17] This use of the word secant comes from the Latin secare, meaning to cut, and is not a reference to the trigonometric function of the same name.

- Solution:
    - Graphing the function $s(t) = -16t^2 + 96t$ gives us





$$\bar{v} = \frac{\Delta s}{\Delta t}$$
$$\bar{v} = \frac{s(3) - s(1)}{3 - 1}$$
$$\bar{v} = \frac{144\,ft - 80\,ft}{3s - 1s}$$
$$\bar{v} = \frac{64\,ft}{2s}$$
$$\bar{v} = 32\ ft/s$$



$$\bar{v} = \frac{\Delta s}{\Delta t}$$
$$\bar{v} = \frac{s(2) - s(1)}{2 - 1}$$
$$\bar{v} = \frac{128\,ft - 80\,ft}{2s - 1s}$$
$$\bar{v} = \frac{48\,ft}{1s}$$
$$\bar{v} = 48\ ft/s$$

    - Notice that when looking at a position-time graph, the average velocity is the equivalent of the slope of the secant line.

- Independent Practice:

- A rock breaks loose from the top of a cliff. What is its average velocity in feet per second during the first 2 seconds of fall? (use $s(t)=16t^2$) 32 ft/s

- *Concept 2: Instantaneous Velocity*

OBJECTIVE(S): TSWBAT
- calculate instantaneous velocity (GLE 11/12: N/A)

BELLRINGER(S):
- Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 meters above the ground. Fill in the table to find the instantaneous velocity at $t = 2\sec$. (Hint: use $s(t) = -4.9t^2 + 450$ and $\bar{v} = \dfrac{s(t)-s(2)}{t-2}$)

x	1.75	1.9	1.99	1.999	2	2.001	2.01	2.1	2.25
$\bar{v}$	?	?	?	?	?	?	?	?	?

  - Solution:

x	1.75	1.9	1.99	1.999	2	2.001	2.01	2.1	2.25
$\bar{v}$	-18.375	-19.11	-19.551	-19.5951	?	-19.6049	-19.649	-20.09	-20.825

    - So the instantaneous velocity at $t = 2\sec$ is -19.6 m/s

LESSON:
- Concept Introduction: It is natural to think of change as change with respect to time, but other variables can be treated in the same way. For example, a physician may want to know how change in dosage affects the body's response to a drug. An economist may want to study how the cost of producing steel varies with the number of tons produced.

- Review of Vocabulary:
  - NONE

- Guided Practice
  - Besides telling how fast an object is moving, velocity tells the direction of motion. When the object is moving in the direction of reference (up, forward, to the right on the real number line, etc.), the velocity is positive; when the object is moving opposite the direction of reference (down, backward, to the left on the number line, etc.), the velocity is negative.
  - A rock is launched vertically upward from the ground with a speed of 96 ft/s. Neglecting air resistance, we can use the position formula for projectiles, given by $s(t) = \frac{1}{2}at^2 + v_0 t + s_0$. Estimate the instantaneous velocity at the single point $t = 3s$
    - Solution:
      - Using the table method that was used in the bellringer we have

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$\overline{v}$	67.09	66.649	66.605	?	66.595	66.551	66.11

- So the instantaneous velocity at $t = 3\sec$ is 66.6 ft/s

- Independent Practice
  - A rock breaks loose from the top of a cliff. What is its instantaneous velocity in feet per second at $t = 2\sec$? (use $s(t) = 16t^2$) -64 ft/s

- *Concept 3: Slope of the Tangent Line*

OBJECTIVE(S): TSWBAT
- calculate slopes of secant and tangent lines (GLE 11/12: N/A)

BELLRINGER(S):
- Given the function $f(x) = x^2$, find the slope between the points $(2,4)$ and $(3,9)$.
  - Solution

$$\frac{f(2)-f(3)}{2-3} = \frac{4-9}{2-3} = \frac{-5}{-1} = 5$$

- How many points are needed to find the slope of a line?
  - *TWO*
- Is it possible to find the slope if given only one point?
  - *Not without using calculus*

LESSON:

- Concept Introduction: In mechanics, the tangent determines the direction of a body's motion at every point along its path. In geometry, the tangents to two curves at a point of intersection determine the angle at which the curves intersect. In optics, the tangent determines the angle at which a ray of light enters a curved lens. The problem of how to find a tangent to a curve became the dominant mathematical problem of the early seventeenth century and it is hard to overestimate how badly the scientists of the day wanted to know the answer. In 1637, Descartes went so far as to say that the problem was the most useful and most general problem not only that he knew but that he had any desire to know.

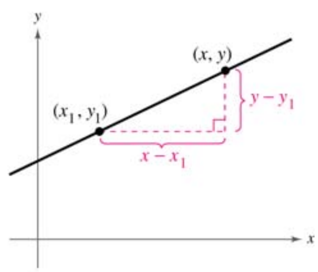
The dynamic approach to tangency, invented by Pierre de Fermat (1601 – 1665) in 1629, proved to be one of the seventeenth century's major contributions to calculus.

- Review of Vocabulary:

- NONE

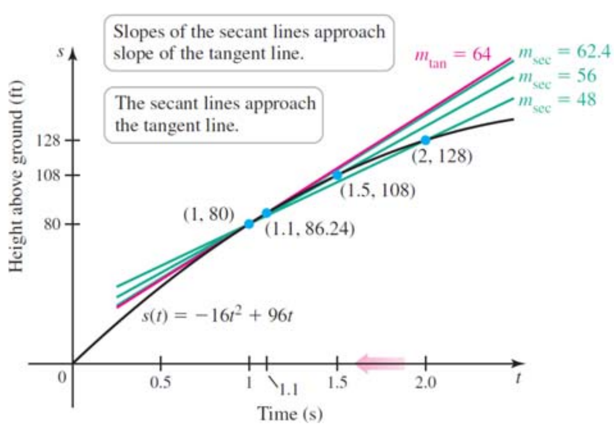
- Guided Practice
  - The word tangent is derived from the Latin word tangens, which means “touching.” Thus, a tangent to a curve is a line that touches the curve. We define tangent lines carefully in section 3.1. For the moment, imagine zooming in on a point P on a smooth curve. As you zoom in, the curve appears more and more like a line passing through P. This line is the tangent line at P.

  - *Using MVT, show how we can determine the tangent line by finding the slopes of secant lines.*
    - Revisiting the lesson on slope, we know that $m = \dfrac{y - y_1}{x - x_1}$ (See the graph)



    - We could also write, $m = \dfrac{f(b) - f(a)}{b - a}$, which is the $\dfrac{\text{change in output}}{\text{change in input}}$.

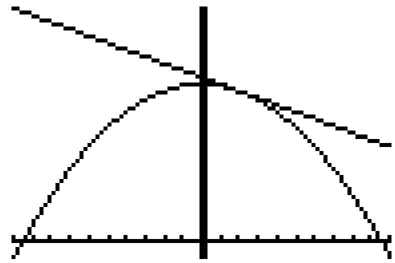
      To get the slope of the secant lines to approach the slope of the tangent line, we need the “change in input” to approach zero. To do this we use calculus.



  - The point-slope form of the line is used to write the equation of the tangent line at point P (x_p, y_p), where the slope is the instantaneous rate of change at the point P and given by $y - y_p = m_p(x - x_p)$.
  - Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 meters above the ground. From previous study of this problem

we know that the slope is -19.6 m/s at $t = 2\sec$. Write the equation of the tangent line at $t = 2\sec$.

- Solution:
  - $(x_p, y_p) = (2, 430.4)$ and $m_p = -19.6$
  - The equation is $y - 430.4 = -19.6(x - 2)$



- Independent Practice
  - What do we know about the point (x_p, y_p) if $m_p = 0$? The point is a critical value – either an extrema or a point of inflection.

- HOMEWORK
  - MML
    - GR 5: Getting Ready for Limits
    - Section 2.1: The Idea of Limits

- Support Notes: Ask student to compute an average velocity for a simple situation (e.g. a car travels 110 miles in 2 hours; what is its average velocity?). Increase the complexity of the problem by providing a plausible position function for an object, and ask students to compute the average velocity over some time interval. Students should be able to come up with the formula for average velocity.

  Point out that average velocities are just slopes of secant lines on a position curve

  Show how average velocity can be used to approximate instantaneous velocity

  Then show how shrinking the time intervals leads to better approximations of instantaneous velocity

  Introduce the idea of a limit in moving from average to instantaneous velocity

- Additional Activities, Projects and Resources:
  - *Guided Projects* found on MML under Instructor Resources
    - *Local Linearity* is a guided project that fits well with this section. It can be used in class as a launching point for a discussion about tangent lines.

Section 2: Definitions of Limits

SECTION INTRODUCTION: Computing tangent lines and instantaneous velocities are just two of the many important calculus problems that rely on limits. We now put these two problems aside until chapter 3 and begin with a preliminary definition of the limit of a function.

COLLEGE BOARD OBJECTIVE (S): **Analysis with Graphs –** With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits) – An intuitive understanding of the limiting process is sufficient for this course.

- Calculating limits using algebra
- Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior
- Describing asymptotic behavior in terms of limits involving infinity
- Comparing relative magnitudes of functions and their rates of change (For example, contrasting exponential growth, polynomial growth, and logarithmic growth)

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to curve at a point and local linear approximation
- Instantaneous rate of change as the limit of average rate of change
- Approximate rate of change from graphs and tables of values

Concept 1: Definition of a Limit (Preliminary)

OBJECTIVE(S): TSWBAT

- estimate a limit using a numerical or graphical approach (GLE 11/12: N/A)

BELLRINGER(S):

- Given $f(x)=\dfrac{x^2-3x+2}{x-2}$, fill in the table

x	1.75	1.9	1.99	1.999	2	2.001	2.01	2.1	2.25
$f(x)$	?	?	?	?	?	?	?	?	?

Solution

x	1.75	1.9	1.99	1.999	2	2.001	2.01	2.1	2.25
$f(x)$	.75	.9	.99	.999	?	1.001	1.01	1.1	1.25

LESSON:

- Concept Introduction: Limits help us to advance our study of change. No longer confined to intervals of time, we can now investigate change at a specific point in time.

- Review of Vocabulary:
  - Limit → Suppose the function *f* is defined for all *x* near *a* except possibly at *a*. If $f(x)$ is arbitrarily close to *L* (as close to *L* as we like) for all *x* sufficiently close (but not equal) to *a*, we write $\lim_{x\to a} f(x) = L$ and say the limit of $f(x)$ as *x* approaches *a* equals *L*.

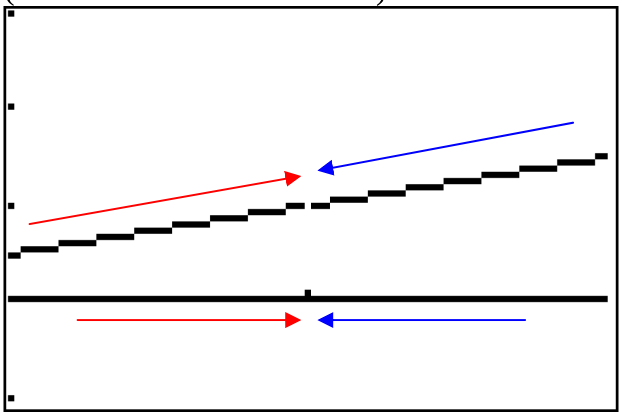
- Guided Practice:
  - Looking at the bellringer, what happens when we let $x = 2$?
    - Since the function is undefined when $x = 2$, we can ask what happens around $x = 2$? We do this by asking what does $f(x)$ approach as we let *x* approach 2 from the right and as we let *x* approach 2 from the left? This process is called evaluating the limit of $f(x)$ as *x* approaches 2. (ie. As *x* gets really close to 2, we want to know to what value of $f(x)$ is getting close?)
    - This process has a special notation $\lim_{x\to c} f(x) = L$. This means the limit of $f(x)$ as *x* approaches *c* is equal to some value *L*.
  - There are three different ways to evaluate a limit.
    - Numerical approach → Construct a table of values
    - Graphical approach → Draw a graph by hand or using technology
    - Analytical approach → Use algebra or calculus
    - **It is important to note that we will be using all three methods throughout this course*
  - In all three ways, it is important to note that the limit exists if and only if…
    - The limit from the right exists $\lim_{x\to c^+} f(x) = L$
    - The limit from the left exists $\lim_{x\to c^-} f(x) = L$
    - The limit from the right equals the limit from the left $\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x)$

****THIS IS THE FIRST STORY IN OUR HOUSE OF CALCULUS!!***

  - Solve $\lim_{x\to 2} \dfrac{x^2 - 3x + 2}{x - 2}$
    - (NUMERICAL APPROACH)
      In the bell ringer, we used a numerical approach to solve $\lim_{x\to 2} f(x)$. From the left half of the table we saw that as *x* approaches 2 from the left, $f(x)$ approaches 1. And from the right half of the table we saw that as we let *x* approaches 2 from the right, $f(x)$ approaches 1. So we can say that the limit exists from the right, the limit exists from the left, and right-hand

limit equals the left-hand limit. Therefore, we can say that the limit of $f(x)$ exists at $x = 2$ and is equal to 1.

- (GRAPHICAL APPROACH)



Notice as x approaches 2 from the left (red), $f(x)$ is approaching 1; therefore, the limit exists from the left. And as x approaches 2 from the right (blue), $f(x)$ is approaching 1; therefore the limit exists from the right. Since they are both approaching the same number, the limit exists.

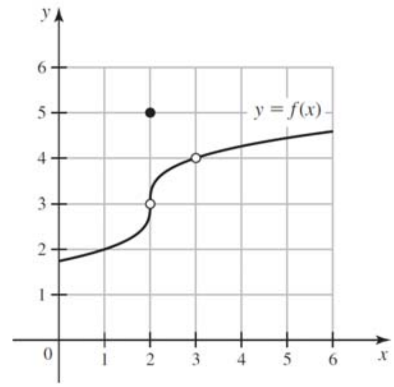
- (ANALYTICAL APPROACH)

$$\lim_{x\to 2}\frac{x^2-3x+2}{x-2}$$
$$=\lim_{x\to 2}\frac{(x-1)(x-2)}{x-2}$$
$$=\lim_{x\to 2}(x-1)$$
$$=2-1$$
$$=1$$

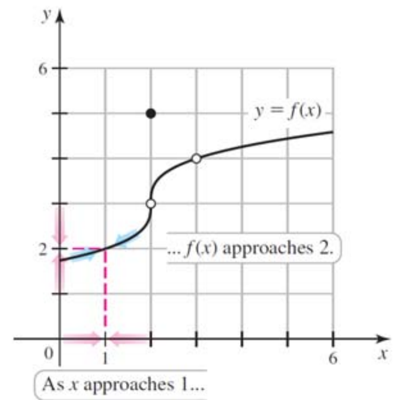
- Does the function have to be defined at a given *x-value* for the limit to exist?
  - No we saw in the bell ringer that the function does not need to be defined at a given *x-value* for the limit to exist.
- Using a numerical approach, evaluate the limit of $f(x)=\begin{cases}1, & x\neq 2\\ 0, & x=2\end{cases}$ as x approaches 2.
  - Notice that the limit from the right and the limit from the left both equal 1. The fact that $f(2)=0$ has no bearing on the existence or value of the limit as x approaches 2.
  - **The important thing to remember that the limit is not asking what happens at a given value, but instead is asking what happens as we approach a given value from the right and left.*

➢ Independent Practice:

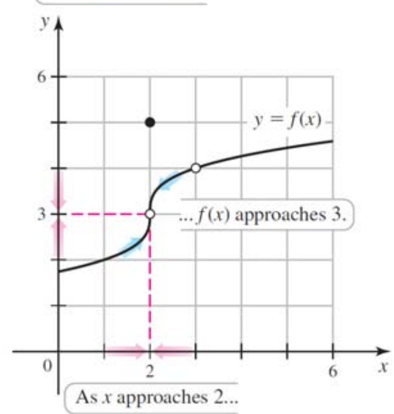
From this graph, determine the following values, if possible.



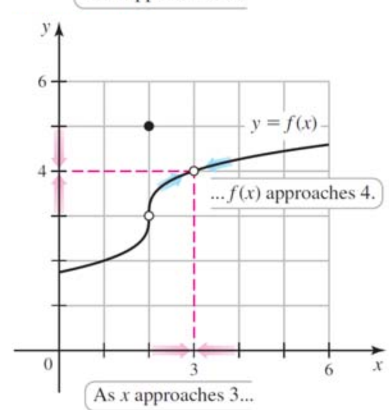
- $f(1)$ and $\lim_{x\to 1} f(x)$
  - $f(1) = 2$
  - $\lim_{x\to 1} f(x) = 2$



- $f(2)$ and $\lim_{x\to 2} f(x)$
  - $f(2) = 5$
  - $\lim_{x\to 2} f(x) = 3$



- $f(3)$ and $\lim_{x\to 3} f(x)$
  - $f(3) = \mathit{undefined}$
  - $\lim_{x\to 3} f(x) = 4$



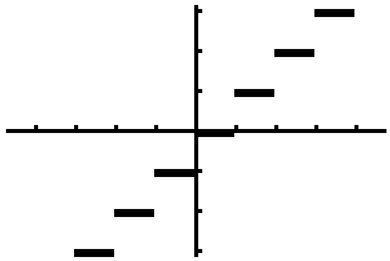
- *Concept 2: One-Sided Limits***Error! Bookmark not defined.**

OBJECTIVE(S): TSWBAT

- analyze limits (GLE 11/12: N/A)
- determine one-sided limits and continuity on a closed interval (GLE 11/12: N/A)

BELLRINGER(S):

- Find the limit of the greatest integer function $f(x) = [\![x]\!]$ as x approaches 0 from the left and from the right. Does the limit exist at $x = 0$?



Limit from the right

- $\lim_{x \to 0^+} [\![x]\!] = 0$

Limit from the left

- $\lim_{x \to 0^-} [\![x]\!] = -1$

RHS $\neq$ LHS, $\lim_{x \to 0} [\![x]\!] = DNE$

LESSON:

- Concept Introduction: Sometimes the values of a function *f* tend to different limits as *x* approaches a number *c* from opposite sides. When this happens, we call the limit of *f* as *x* approaches *c* from the right the right-hand limit of *f* at *c* and the limit of *f* as *x* approaches *c* from the left the left-hand limit of *f* at *c*.

- Review of Vocabulary:
  - NONE

- Guided Practice
  - When we discussed the three things that determined the existence of a limit, we discussed but did not define one-sided limits.
    - $\lim_{x \to c^+} f(x)$ is the notation for the one-sided limit from the right
    - $\lim_{x \to c^-} f(x)$ is the notation for the one-sided limit from the left
    - Of course a limit exists if the right-hand limit exists, the left-hand limit exists and they are equal
  - Sometimes it is necessary to discuss the one-sided limit.
    - Find the limit of $f(x) = \sqrt{4 - x^2}$ at $x = 1$
      - Solution:
        - We know that by the definition of limits we need to verify three things.
          - $\lim_{x \to c^+} f(x)$ exists **Check*
          - $\lim_{x \to c^-} f(x)$ exists **Check*
          - $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)$ **Check*

- $\lim_{x \to 1^+} f(x) = \sqrt{3}$
- $\lim_{x \to 1^-} f(x) = \sqrt{3}$
- $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x)$

- So the limit of $f(x) = \sqrt{4 - x^2}$ at $x = 1$ exists.

- Independent Practice
  - Find the limit of $f(x) = |x|$ at $x = 0$
    - Solution:
      - We know that by the definition of limits we need to verify three things.
        - $\lim_{x \to c^+} f(x)$ exists **Check*
        - $\lim_{x \to c^-} f(x)$ exists **Check*
        - $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)$ **Check*
      - $\lim_{x \to 0^+} f(x) = 0$
      - $\lim_{x \to 0^-} f(x) = 0$
      - $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$
    - So the limit of $f(x) = |x|$ at $x = 0$ exists.

- HOMEWORK
  - MML
    - Section 2.2: Definitions of Limits
    - Quiz #3 (2.1-2.2)

- Support Notes: Ask student to compute an average velocity for a simple situation (e.g. a car travels 110 miles in 2 hours; what is its average velocity?). Increase the complexity of the problem by providing a plausible position function for an object, and ask students to compute the average velocity over some time interval. Students should be able to come up with the formula for average velocity.

  Point out that average velocities are just slopes of secant lines on a position curve

  Show how average velocity can be used to approximate instantaneous velocity

  Then show how shrinking the time intervals leads to better approximations of instantaneous velocity

  Introduce the idea of a limit in moving from average to instantaneous velocity

- Additional Activities, Projects and Resources:
  - *Guided Projects* found on MML under Instructor Resources

- *Local Linearity* is a guided project that fits well with this section. It can be used in class as a launching point for a discussion about tangent lines.

Student: ______________________ **Date:** ______________________ **Time:** ______________________	**Instructor:** David Woods **Course:** F10-S11 AP Calculus LSUCRP - COORDINATOR **Book:** Briggs Cochran: Calculus Early Transcendentals	**Assignment:** 2.2: Definitions of Limits

1. When $\lim_{x \to a} f(x)$ exists, it always equals f(a). State whether this statement is true or false.

   Choose the correct answer below.

   ◯ True

   ◯ False

2. Explain the meaning of $\lim_{x \to b^+} h(x) = L$.

   Choose the correct answer below.

   ◯A. As x approches b from either side, the value of h(x) approaches L.

   ◯B. As x approaches b from the right, the value of h(x) approaches L.

   ◯C. As x approaches b from the left, the value of h(x) approaches L.

3. If $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = M$, where L and M are finite real numbers, then what must be true about L and M in order for $\lim_{x \to a} f(x)$ to exist?

   Choose the correct answer below.

   ◯A. $L > M$

   ◯B. $L = M$

   ◯C. $L < M$

   ◯D. $L \neq M$

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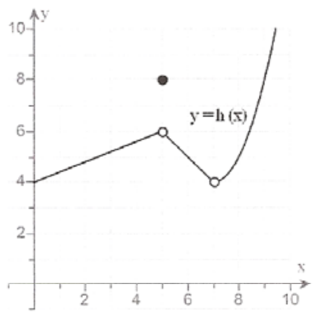
4. Use the graph of h in the given figure to find the following values, if they exist.

(a) h(5) **(b)** $\lim_{x \to 5} h(x)$ **(c)** h(7)
(d) $\lim_{x \to 7} h(x)$ **(e)** $\lim_{x \to 8} h(x)$



(a) Find h(5). Select the correct choice below, and fill in the answer box if necessary.

○A. h(5) = ☐
(Type an integer or a decimal.)

○B. The value h(5) is undefined.

(b) Find $\lim_{x \to 5} h(x)$. Select the correct choice below, and fill in the answer box if necessary.

○A. $\lim_{x \to 5} h(x) =$ ☐
(Type an integer or a decimal.)

○B. The limit does not exist.

(c) Find h(7). Select the correct choice below, and fill in the answer box if necessary.

○A. h(7) = ☐
(Type an integer or a decimal.)

○B. The value h(7) is undefined.

(d) Find $\lim_{x \to 7} h(x)$. Select the correct choice below, and fill in the answer box if necessary.

○A. $\lim_{x \to 7} h(x) =$ ☐
(Type an integer or a decimal.)

○B. The limit does not exist.

(e) Find $\lim_{x \to 8} h(x)$. Select the correct choice below, and fill in the answer box if necessary.

○A. $\lim_{x \to 8} h(x) =$ ☐
(Type an integer or a decimal.)

○B. The limit does not exist.

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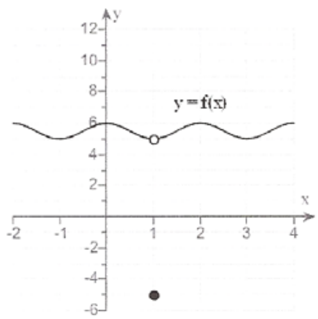
5. Use the graph of f(x) in the given figure to find the following values, if they exist.

(a) f(1) **(b)** $\lim_{x \to 1} f(x)$

(c) f(0) **(d)** $\lim_{x \to 0} f(x)$



(a) Find f(1). Select the correct choice below, and fill in the answer box if necessary.

A. f(1) = ☐ (Type an integer or a decimal.)

B. f(1) is undefined.

(b) Find $\lim_{x \to 1} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

A. $\lim_{x \to 1} f(x) =$ ☐ (Type an integer or a decimal.)

B. The limit does not exist.

(c) Find f(0). Select the correct choice below, and fill in the answer box if necessary.

A. f(0) = ☐ (Type an integer or a decimal.)

B. f(0) is undefined.

(d) Find $\lim_{x \to 0} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

A. $\lim_{x \to 0} f(x) =$ ☐ (Type an integer or a decimal.)

B. The limit does not exist.

Student: ______ Date: ______ Time: ______	Instructor: David Woods Course: F10-S11 AP Calculus LSUCRP - COORDINATOR Book: Briggs/Cochran: Calculus Early Transcendentals	Assignment: 2.2: Definitions of Limits

6. Let $f(x) = \dfrac{x^2 - 25}{x - 5}$. **(a)** Calculate f(x) for each value of x in the following table. **(b)** Make a conjecture about the value of $\lim\limits_{x \to 5} \dfrac{x^2 - 25}{x - 5}$.

(a) Calculate f(x) for each value of x in the following table.

x	4.9	4.99	4.999	4.9999
$f(x) = \dfrac{x^2 - 25}{x - 5}$	☐	☐	☐	☐
x	5.1	5.01	5.001	5.0001
$f(x) = \dfrac{x^2 - 25}{x - 5}$	☐	☐	☐	☐

(Type an integer or decimal rounded to four decimal places as needed.)

(b) Make a conjecture about the value of $\lim\limits_{x \to 5} \dfrac{x^2 - 25}{x - 5}$.

$\lim\limits_{x \to 5} \dfrac{x^2 - 25}{x - 5} =$ ☐ (Type an integer or a decimal.)

7. Complete the table and use the results to find the indicated limit.

If $h(x) = \dfrac{\sqrt{x} - 4}{x - 1}$, find $\lim\limits_{x \to 1} h(x)$.

x	.9	.99	.999	1.001	1.01	1.1
h(x)						

Fill in the missing table values.

x	.9	.99	.999	1.001	1.01	1.1
h(x)	☐	☐	☐	☐	☐	☐

(Round to four decimal places.)

What is the limit? Select the correct choice below and fill in any answer boxes in your choice.

○ A. $\lim\limits_{x \to 1} h(x) =$ ☐ (Simplify your answer.)

○ B. The limit does not exist.

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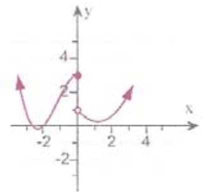
8. Use the graph to find the following limits and function value.

a. $\lim_{x \to 0^-} f(x)$

b. $\lim_{x \to 0^+} f(x)$

c. $\lim_{x \to 0} f(x)$

d. $f(0)$



a. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

○ A. $\lim_{x \to 0^-} f(x) = \square$ (Type a whole number.)

○ B. The limit does not exist.

b. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

○ A. $\lim_{x \to 0^+} f(x) = \square$ (Type a whole number.)

○ B. The limit does not exist.

c. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

○ A. $\lim_{x \to 0} f(x) = \square$ (Type a whole number.)

○ B. The limit does not exist.

d. Find the function value. Select the correct choice below and fill in any answer boxes in your choice.

○ A. $f(0) = \square$ (Type a whole number.)

○ B. The answer is undefined.

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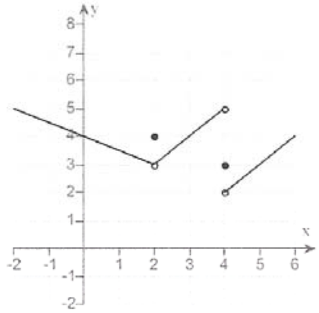
9. Use the graph of f to complete parts **(a)** through **(l)**. If a limit does not exist, explain why.



(a) Find f(2). Select the correct choice below, and fill in the answer box if necessary.

- A. $f(2) = \square$ (Type an integer or a fraction.)
- B. The value of f(2) is undefined.

(b) Find $\lim_{x \to 2^-} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

- A. $\lim_{x \to 2^-} f(x) = \square$ (Type an integer or a fraction.)
- B. The limit does not exist because f(x) is not defined for all $x < 2$.

(c) Find $\lim_{x \to 2^+} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

- A. $\lim_{x \to 2^+} f(x) = \square$ (Type an integer or a fraction.)
- B. The limit does not exist because f(x) is not defined for all $x > 2$.

(d) Find $\lim_{x \to 2} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

- A. $\lim_{x \to 2} f(x) = \square$ (Type an integer or a fraction.)
- B. The limit does not exist because $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$.

(e) Find f(4). Select the correct choice below, and fill in the answer box if necessary.

9.

(cont.)

A. $f(4) = \square$
(Type an integer or a fraction.)

B. The value of f(4) is undefined.

(f) Find $\lim_{x \to 4^-} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

A. $\lim_{x \to 4^-} f(x) = \square$
(Type an integer or a fraction.)

B. The limit does not exist because f(x) is not defined for all $x < 4$.

(g) Find $\lim_{x \to 4^+} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

A. $\lim_{x \to 4^+} f(x) = \square$
(Type an integer or a fraction.)

B. The limit does not exist because f(x) is not defined for all $x > 4$.

(h) Find $\lim_{x \to 4} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

A. $\lim_{x \to 4} f(x) = \square$
(Type an integer or a fraction.)

B. The limit does not exist because $\lim_{x \to 4^-} f(x) \neq \lim_{x \to 4^+} f(x)$.

(i) Find f(3). Select the correct choice below, and fill in the answer box if necessary.

A. $f(3) = \square$
(Type an integer or a fraction.)

B. The value f(3) is undefined.

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Time: ____________________

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9.

(cont.)

(j) Find $\lim_{x \to 3^-} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

- A. $\lim_{x \to 3^-} f(x) = \square$
  (Type an integer or a fraction.)
- B. The limit does not exist because f(x) is not defined for all $x < 3$.

(k) Find $\lim_{x \to 3^+} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

- A. $\lim_{x \to 3^+} f(x) = \square$
  (Type an integer or a fraction.)
- B. The limit does not exist because f(x) is not defined for all $x > 3$.

(l) Find $\lim_{x \to 3} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

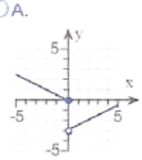
- A. $\lim_{x \to 3} f(x) = \square$
  (Type an integer or a fraction.)
- B. The limit does not exist because $\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$.

10. Sketch a possible graph of a function that satisfies the conditions below.

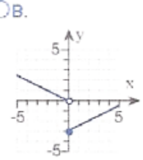
$$f(0) = -3; \ \lim_{x \to 0^-} f(x) = 0; \ \lim_{x \to 0^+} f(x) = -3$$

Choose the correct graph below.

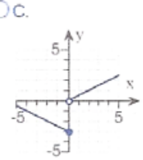
A.



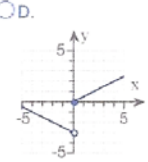
B.



C.



D.



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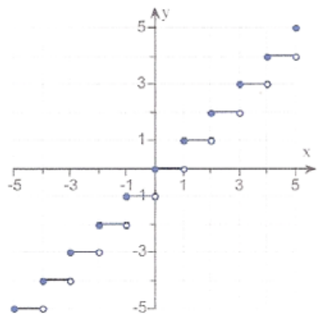
11. For any real number x, the floor function (or greatest integer function), $\lfloor x \rfloor$, is defined to be the greatest integer less than or equal to x (see figure). Answer parts **(a)** through **(e)**.



(a) Compute $\lim_{x \to -2^-} \lfloor x \rfloor$, $\lim_{x \to -2^+} \lfloor x \rfloor$, $\lim_{x \to 3^-} \lfloor x \rfloor$, and $\lim_{x \to 3^+} \lfloor x \rfloor$.

Compute $\lim_{x \to -2^-} \lfloor x \rfloor$.

Select the correct choice below, and fill in the answer box if necessary.

○A. $\lim_{x \to -2^-} \lfloor x \rfloor = \square$

○B. $\lim_{x \to -2^-} \lfloor x \rfloor$ does not exist.

Compute $\lim_{x \to -2^+} \lfloor x \rfloor$.

Select the correct choice below, and fill in the answer box if necessary.

○A. $\lim_{x \to -2^+} \lfloor x \rfloor = \square$

○B. $\lim_{x \to -2^+} \lfloor x \rfloor$ does not exist.

Compute $\lim_{x \to 3^-} \lfloor x \rfloor$.

Select the correct choice below, and fill in the answer box if necessary.

○A. $\lim_{x \to 3^-} \lfloor x \rfloor = \square$

○B. $\lim_{x \to 3^-} \lfloor x \rfloor$ does not exist.

Compute $\lim_{x \to 3^+} \lfloor x \rfloor$.

Select the correct choice below, and fill in the answer box if necessary.

○A. $\lim_{x \to 3^+} \lfloor x \rfloor = \square$

○B. $\lim_{x \to 3^+} \lfloor x \rfloor$ does not exist.

Student: ______________ **Date:** ______________ **Time:** ______________	**Instructor:** David Woods **Course:** F10-S11 AP Calculus LSUCRP - COORDINATOR **Book:** Briggs/Cochran: Calculus Early Transcendentals	**Assignment:** 2.2: Definitions of Limits

11.
(cont.)

(b) Compute $\lim\limits_{x \to 3.3^-} \lfloor x \rfloor$, $\lim\limits_{x \to 3.3^+} \lfloor x \rfloor$, and $\lim\limits_{x \to 3.3} \lfloor x \rfloor$.

Compute $\lim\limits_{x \to 3.3^-} \lfloor x \rfloor$.

Select the correct choice below, and fill in the answer box if necessary.

○A. $\lim\limits_{x \to 3.3^-} \lfloor x \rfloor = \square$

○B. $\lim\limits_{x \to 3.3^-} \lfloor x \rfloor$ does not exist.

Compute $\lim\limits_{x \to 3.3^+} \lfloor x \rfloor$.

Select the correct choice below, and fill in the answer box if necessary.

○A. $\lim\limits_{x \to 3.3^+} \lfloor x \rfloor = \square$

○B. $\lim\limits_{x \to 3.3^+} \lfloor x \rfloor$ does not exist.

Compute $\lim\limits_{x \to 3.3} \lfloor x \rfloor$.

Select the correct choice below, and fill in the answer box if necessary.

○A. $\lim\limits_{x \to 3.3} \lfloor x \rfloor = \square$

○B. $\lim\limits_{x \to 3.3} \lfloor x \rfloor$ does not exist.

(c) In general, for an integer a, state the values of $\lim\limits_{x \to a^-} \lfloor x \rfloor$ and $\lim\limits_{x \to a^+} \lfloor x \rfloor$.

Select the correct choice below, and fill in the answer box if necessary.

○A. $\lim\limits_{x \to a^-} \lfloor x \rfloor = \square$

(Type an expression using a as the variable.)

○B. $\lim\limits_{x \to a^-} \lfloor x \rfloor$ does not exist.

11.
(cont.)

Select the correct choice below, and fill in the answer box if necessary.

A. $\lim_{x\to a^+} \lfloor x \rfloor = \square$
(Type an expression using a as the variable.)

B. $\lim_{x\to a^+} \lfloor x \rfloor$ does not exist.

(d) In general, if a is not an integer, state the values of $\lim_{x\to a^-} \lfloor x \rfloor$ and $\lim_{x\to a^+} \lfloor x \rfloor$.

Choose the correct choice below.

A. $\lim_{x\to a^-} \lfloor x \rfloor = \lfloor a \rfloor$

B. $\lim_{x\to a^-} \lfloor x \rfloor = -\lfloor a \rfloor$

C. $\lim_{x\to a^-} \lfloor x \rfloor$ does not exist.

Choose the correct choice below.

A. $\lim_{x\to a^+} \lfloor x \rfloor = -\lfloor a \rfloor$

B. $\lim_{x\to a^+} \lfloor x \rfloor = \lfloor a \rfloor$

C. $\lim_{x\to a^+} \lfloor x \rfloor$ does not exist.

(e) For what values of a does $\lim_{x\to a} \lfloor x \rfloor$ exist? Explain.
Select the correct choice below.

A. $\lim_{x\to a} \lfloor x \rfloor$ exists only if a is an integer.

B. $\lim_{x\to a} \lfloor x \rfloor$ exists only if a is not an integer.

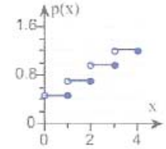
Student: ____________ **Date:** ____________ **Time:** ____________	**Instructor:** David Woods **Course:** F10-S11 AP Calculus LSUCRP - COORDINATOR **Book:** Briggs/Cochran: Calculus Early Transcendentals	**Assignment:** 2.2: Definitions of Limits

12. A courier service's rates are \$0.46 for the first ounce and \$0.25 for each additional ounce (or fraction thereof). In the graph of this function shown to the right, the horizontal axis represents ounces and the vertical axis represents dollars. Evaluate each of the following functions or limits.

$p(x) = \$0.46, \text{ if } 0 < x \le 1,$

$p(x) = \$0.71, \text{ if } 1 < x \le 2,$

$p(x) = \$0.96, \text{ if } 2 < x \le 3.$



a. $p(0.5)$ **b.** $\lim_{x \to 0.5} p(x)$

c. $p(1)$ **d.** $\lim_{x \to 1} p(x)$

a. $p(0.5) = \$$ ☐

(Type an integer or decimal rounded to the nearest hundredth as needed.)

b. Select the correct choice below and fill in any answer boxes in your choice.

○A. $\lim_{x \to 0.5} p(x) = \$$ ☐

○B. The limit does not exist.

c. $p(1) = \$$ ☐

(Type an integer or decimal rounded to the nearest hundredth as needed.)

d. Select the correct choice below and fill in any answer boxes in your choice.

○A. $\lim_{x \to 1} p(x) =$ ☐

○B. The limit does not exist.

Student: ______________ **Date:** ______________ **Time:** ______________	**Instructor:** David Woods **Course:** F10-S11 AP Calculus LSUCRP - COORDINATOR **Book:** Briggs/Cochran: Calculus Early Transcendentals	**Assignment:** 2.2: Definitions of Limits

13. Evaluate $\frac{\sin x^2}{x^2}$ for x = 1, 0.75, 0.5, 0.1, 0.01. Based on the value given by the calculator, propose a value for $\lim_{x \to 0^+} \frac{\sin x^2}{x^2}$. Does the proposed value for the limit agree? Explain.

Evaluate $\frac{\sin x^2}{x^2}$ for x = 1, 0.75, 0.5, 0.1, 0.01. Complete the following table.

x	1	0.75	0.5	0.1	0.01
$\frac{\sin x^2}{x^2}$	☐	☐	☐	☐	☐

(Type an integer or decimal rounded to two decimal places as needed.)

Based on the value given by the calculator, propose a value for $\lim_{x \to 0^+} \frac{\sin x^2}{x^2}$.

$\lim_{x \to 0^+} \frac{\sin x^2}{x^2} = \square$

Does the proposed value for the limit agree?

○ Yes

○ No

Student: ____________ **Date:** ____________ **Time:** ____________	**Instructor:** David Woods **Course:** F10-S11 AP Calculus LSUCRP - COORDINATOR **Book:** Briggs/Cochran: Calculus Early Transcendentals	**Assignment:** 2.2: Definitions of Limits

1. the second choice

2. B

3. B

4. A, 8
A, 6
B
A, 4
A, 5

5. A, -5
A, 5
A, 6
A, 6

6. 9.9
9.99
9.999
9.9999
10.1
10.01
10.001
10.0001
10

7. 30.5132
300.5013
3000.5001
-2999.5001
-299.5012
-29.5119
B

8. A, 3
A, 1
B
A, 3

Student: ____________ **Date:** ____________ **Time:** ____________	**Instructor:** David Woods **Course:** F10-S11 AP Calculus LSUCRP - COORDINATOR **Book:** Briggs/Cochran: Calculus Early Transcendentals	**Assignment:** 2.2: Definitions of Limits

9. A, 4
A, 3
A, 3
A, 3
A, 3
A, 5
A, 2
B
A, 4
A, 4
A, 4
A, 4

10. B

11. A, -3
A, -2
A, 2
A, 3
A, 3
A, 3
A, 3
A, $a-1$
A, a
A
B
B

12. 0.46
A, 0.46
0.46
B

13. 0.84
0.95
0.99
1
1
1
the first choice

VITA

David Frank Woods was born in Fort Ord, California, the son of Billy Ray Woods and Barbara Molinari Fuller. He graduated from South Lafourche High School in May 1996. The following August, he entered Louisiana State University and in December 2004 earned a Bachelor of Science degree in mathematics with a concentration in applied mathematics/communications engineering and a Bachelor of Arts degree in French. In June 2005, he entered Southeastern Louisiana University and graduated with a Master of Arts in Teaching in May 2008. He entered the Louisiana State University Graduate School in June 2008 and is a candidate for the Master of Natural Sciences degree. He currently teaches Advanced Mathematics, Advanced Mathematics Dual Enrollment, Advanced Placement Calculus AB, Advanced Placement Calculus BC and Advanced Placement Physics C: Mechanics. He also coaches varsity girls' soccer at Tara High School in Baton Rouge, Louisiana, where he currently serves as the mathematics department chair. He is married to Catherine Johnson Woods and is the father of one daughter, Ainsly, and two sons, Dawson and Liam.