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Sustained Flow Index: A Stochastic Measure of Freeway Performance

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SUSTAINED FLOW INDEX:
A STOCHASTIC MEASURE OF FREEWAY PERFORMANCE

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Civil and Environmental Engineering

by

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December 2017

To

My Parents

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ABSTRACT

The capacity of a road addresses its quantitative traffic carrying ability. The estimation of capacity as a parameter to assess traffic flow performance on freeway facilities has received considerable attention in the literature. Research into the traffic operation at high volumes reveals that the capacity of freeways is not a fixed number, but rather a random variable. Thus, in a stochastic approach to freeway capacity of estimation, the capacity is treated as a random variable generated from a population of flow observations, stemmed from a certain distribution function. Since the type of capacity distribution function is generally not known with certainty, it needs to be modeled. The Normal and the Weibull distributions have been among the most common function types that were suggested for freeway capacity.

In this research, different capacity distribution types were tested for freeway facilities by applying the models for censored data on empirical observations of United States (U.S.) freeways. Based on the findings of this research and the results of previous studies on German freeways, it was suggested that the capacity distribution function may be characterized with left-skewedness.

Since traditional operational performance measures for the analysis of traffic flow on freeways typically disregard the randomness of capacity, new approaches to make use of the concept of randomness within freeway operation analysis are necessary. To address this need, this research introduces a new indicator of freeway performance based solely on a stochastic approach to capacity estimation. This new indicator, the Sustained Flow Index (SFI), was defined as the product of the traffic volume and the probability of survival of this volume (as the probability that the acceptable traffic operation can be sustained).

By maximizing the SFI, the optimum volume that can be carried by a freeway over prolonged time periods was derived from parameters of different capacity distribution functions.

The breakdown probability (the probability that the acceptable traffic operation fails) corresponding to the optimum volume may be used as a benchmark to select a single value from the capacity distribution function. To validate the optimum volumes as design capacity values, an empirical comparison was made between the conventional capacity estimates and optimum volumes for 19 freeway sections in the U.S. The results show that, on average, optimum volumes obtained by maximizing the SFI corresponded well to conventional capacity values. To illustrate the application of the SFI, a ramp metering algorithm was modified to enhance performance of a freeway section.

CHAPTER 1: INTRODUCTION

It is recognized that traffic congestion results in economic and environmental losses to society. According to the Urban Mobility Report (Schrank et al., 2012), the cumulative societal costs of traffic congestion totals more than \$121 billion per year on the United States (U.S.) economy and annual time losses in the U.S. average about 38 hours per person. And, while economic and population growth continue to generate even more traffic, efforts to “build our way out of congestion” are constrained by budgets, environmental concerns, and societal limitations. With this growing imbalance between traffic, congestion, and the ability to add system capacity, the trend of growing congestion will increase. To counter the challenges created by traffic congestion, research has emerged to better understand and utilize the existing transportation infrastructure.

Today, a considerable focus of exploratory investigation in traffic engineering is on freeway capacity. Capacity is the maximum attainable flow (veh/h) on a road. In conventional analyses, the capacity of a freeway is treated as a constant value. As such, operational capacity in a traditional sense can be empirically estimated by identifying the apex volume of the fundamental diagram. This approach, however, disregards the stochastic nature of capacity. In fact, the apex volume is just one possible estimate for road capacity, because the capacity of a freeway segment can change constantly as the behavior of individual drivers also change from moment to moment. If used for capacity analysis, the conventional approach has limitations as it does not take into account breakdown phenomena. Fundamentally, uncongested and congested traffic states represent different capacities due to the capacity drop (Hall and Agyemang-Duah, 1991; Banks, 1991; Cassidy and Bertini, 1999; Wu, 2004; Mamdoohi et al, 2014). Thus, fitting the fundamental diagram through both uncongested and congested observations can bias the capacity estimation results.

In past decades, several research articles have reported variability in capacity, even under similar prevailing conditions (Elefteriadou et al., 1997; Minderhoud et al., 1997; Persaud et al., 1998; Lorenz and Elefteriadou, 2000; and Elefteriadou and Lertworawanich, 2003; Brilon et al., 2005; Kondyli, 2009; Geistefeldt, 2009; Danpeng, 2013). These articles suggest that capacity is stochastic and can be treated as a random variable in traffic flow analysis. Research has also proposed methods to estimate stochastic capacity. In general, the observed flow just before a traffic breakdown can be considered as (momentary) capacity. Breakdowns can be detected by a sudden and drastic reduction in traffic speed that demarcates the transition from an uncongested to a congested state. Over a prolonged period of time (generally up to one year) these observed transition points along with the other non-congested observations can be used to estimate a reliable distribution function for capacity of a facility. Using this distribution function, the probability of a breakdown, the likelihood that traffic state will change from fluid to stop and go conditions, can be estimated for any given volume representing the capacity of a road section, and vice versa.

It is interesting to note, however, that despite the now-general acceptance of the concept of stochastic capacity and different methods of establishing its value, there has been considerably less effort to create a quantitative measure of freeway performance based on it. A freeway performance measurement is a standard metric that can be used to evaluate traffic conditions at different times and locations. It can also be used to evaluate the effectiveness of strategies and analyze alternative investments options.

In the past, various qualitative and quantitative measures have been used to evaluate freeway performance. These measures have been applied to assess traffic conditions at specific locations, corridors, and subsection areas of a network. From a qualitative perspective, the most well recognized and long accepted measurement scale is the level of service (LOS) rating system

of the Highway Capacity Manual (HCM, 1965; HCM, 2016). To assess traffic flow on freeways, the LOS ratings rely on quantitative parameters like traffic density, speed, and volume, which are routinely collected by data surveillance systems. Most recently, other useful and descriptive quantitative performance measures like “highway efficiency” and “network productivity” have also been proposed to better assess and optimize freeway system performance (Brilon, 2000; Geroliminis and Daganzo, 2007; Geroliminis and Daganzo, 2008).

Based on the evolving need for a freeway performance measure that can quantitatively evaluate flow characteristics on the basis of stochastic capacity, this research introduces the concept of a “Sustained Flow Index” (SFI) to evaluate traffic operations on freeways based on the capacity distribution function. The SFI was defined as the product of the traffic volume and the “probability of survival” (the complement of the breakdown probability) at that volume. For example, if the probability of breakdown for a freeway segment is 0.1 at a volume of 2,000 veh/h/lane, then the probability of survival is 0.9 (one minus ten percent) and, as a result, the SFI is $2,000 \cdot 0.90 = 1,800$ veh/h/lane. In this sense, SFI is the theoretical average flow that can be sustained over large number of observations given the uncertainty of capacity. That is, if a flow of 2,000 veh/h/lane was observed on a given segment for a large number of independent observations, then the average sustained volume would converge to the SFI value, given that in 10 percent of the observations a breakdown has occurred.

It is important to note that freeways are not designed based on all of the observed capacity values. Rather, a single, representative capacity value is typically used for analysis and freeway design in practice. In addition to serving as a new, more descriptive metric to evaluate freeway performance, the SFI is also an objective measure that can be used to select a single volume from the capacity distribution function. In fact, it can be used to calculate the optimal volume that will

likely result in the highest theoretical flow over prolonged periods. While higher flows can be realized on any link during a given time interval, this new measure can be used to compute the maximum volume that has the highest reliability over all time intervals.

To investigate the validity of the optimum volumes as reasonable capacity estimate, a thorough comparison between optimum volumes and conventional capacity estimates was conducted based on a large sample of U.S. freeway data. The conventional (deterministic) capacity estimates, capacity distribution functions and optimum volumes were estimated and compared for 19 U.S. freeway bottleneck sections. In addition, a new procedure was developed to estimate the Wald and Bootstrap confidence intervals for the optimum volumes and determine their stability. This research also proposes new mathematical derivations which suggests the probability of breakdown at the optimum volume may solely depend on the shape parameter of the capacity distribution function. This finding indicates that probability of breakdown at the capacity may remain constant for roads with different numbers of lanes. A comparison of these methods showed that the optimum volumes obtained by maximizing the SFI were, on average, roughly equivalent to the conventional capacity estimates. It was also observed that the optimum volumes were stable as their confidence intervals were fairly small. These findings also provided a practical method for estimating the capacity distribution function for sections in which conventional capacity has already been estimated or for circumstances in which a reliable capacity distribution function could not be estimated.

The SFI quantifies the optimal flow of a randomized system, opening the door to strategies such as ramp metering that can be implemented to both maximize revenue and minimize travel delays of users. Thus, for a freeway segment located in California, the San Diego Ramp Metering System (SDRMS) was modified to meter the freeway onramp based on the optimum volume and

occupancy of the mainline section. It was observed that under the modified metering algorithm, the overall network performance was improved.

In the chapters that follow, a review of literature is included to provide background on the development of emerging and useful traffic flow performance measures, capacity estimation methods, and concept of stochastic capacity. Next, a description of the conventional and stochastic methodologies used to estimate capacity values in this research is provided and the procedure to derive the new SFI metric, the optimum volumes, and the confidence intervals for the optimum volumes is introduced. Then, to demonstrate an application of the experimental method, the proposed methodologies are applied to the selected freeway bottlenecks to investigate similarities between the optimum volumes and the conventional capacity values and modify the SDRMS algorithm subsequently. The research is concluded with the summary of the findings and recommendations for future application of this work.

CHAPTER 2: LITERATURE REVIEW

In this research, SFI was introduced as a *freeway performance measure* that is used to select a *single capacity value* from the *capacity distribution function*. As a result, the literature review is divided into three parts to provide a better background on each of the three key components used in this definition. The first part of the literature review provides a short background on some of the best-known freeway performance measures. The second part provides a description of the conventional methods used to estimate a single freeway capacity value. The third part provides a background on the stochastic concept of freeway capacity and the methods used to estimate its distribution function.

2.1. Freeway Performance Measures

Since the second publication of the HCM in 1965, the conventional assessment of roadway traffic quality has been based on LOS analysis. Here, the quality of traffic service is assessed based on observations of capacity and demand during distinct peak hours. The LOS is classified into six categories designated by letters A (free flow) through F (oversaturated flow). In the HCM, density is used as Measure of Effectiveness (MOE) to classify the LOS categories for basic freeway segments. This MOE is preferred to speed since speed is relatively insensitive to changes in traffic conditions in non-congested flow. Here, once the input data from operational and geometric characteristics of the section under study are collected, given the ideal base characteristics, adjustments are made for the current section and the actual demand flow rate and the density are computed. Having the actual demand flow rate and the density, the LOS is determined graphically in the last step.

Figure 2.1 Speed-flow curves and different LOS (HCM, 2016).

Currently, the method suggested by HCM is used in most traffic studies and LOS remains the most popular freeway performance measure. Nevertheless, it is important to note that despite the popularity of the LOS, determination of the threshold densities between different LOS categories still depends on the subjective judgment of experts. In fact, in the beginning, the HCM committee intended to define five LOS (A through E) instead of six, and the discussion over the number of LOS between the committee members intensified to the point that Howard Hanna stated “It appears the Committee believes that, in the Beginning, God created the Heavens, the Earth, and Five Levels of Service!” (Kittelson, 2000). As a result, the researchers also introduced quantitative performance measures to evaluate performance of the freeway objectively.

Brilon (2000) proposed a quantitative performance measure based on the analogy between parameters of traffic flow and mechanical systems. In his method, mechanical power, the product

of force and distance over time, was used synonymously for the efficiency of a freeway, the product of the total number of vehicles and the distance traveled over time (i.e., veh-km/hour). Table 2.1 shows this analogy in a side-by-side comparison for a single trip and for multiple trips over a roadway segment. In this analogy, force (mass times acceleration) was considered synonymous with a vehicle or a set of vehicles on a road segment. Work, the product of force and distance, was then computed for a single trip as vehicle-kilometers traveled. Finally, work done over time was defined as mechanical power, as such, the power of a vehicle or set of vehicles is the vehicle-kilometers traveled per hour. Brilon proposed that power of a roadway segment is essentially the segments efficiency and by increasing the power, the segment would be operating more efficiently.

Table 2.1 Comparison of parameters from mechanics and traffic flow (Brilon, 2000).

Mechanical System			Single Vehicle Trip			All Vehicles on a Road Segment		
Parameter	Symbol	Units	Parameter	Symbol	Units	Parameter	Symbol	Units
Force	P	N	Vehicle	veh	veh	M ¹	M	veh
Distance	S	m	Distance	S	m	Length ²	S	km
Time	t	s	Time	t	s	Duration ³	T	h
Velocity	v	m/s	Velocity	v	m/s	velocity	V _T	km/h
Work	W = P·S	N.m	Work	W = veh · S	veh. m	Traffic Work	W = M·S	veh· km
Power	E = W/t	N. m /s	Power	E = W/t	veh. m/s	Power (efficiency)	E = W/T	veh·km /h

1) Set of vehicles passing the section within duration T of the time interval

2) Length of the highway section under consideration

3) Duration of the time interval under consideration

From this analogy, Brilon demonstrated that roadway efficiency is highly sensitive to the speed-flow relationship. To calculate real values for efficiency of an observed roadway segment, the speed-flow relationship must be established from empirical observations. Using a well-defined speed-flow relationship, the efficiency of a road segment can be calculated according to Equation(2.1).

$$E = q \cdot V_t \cdot T \quad (2.1)$$

where

E = traffic efficiency (veh.km/h)

q = flow rate (veh/h)

V_t = speed obtained for the real speed-flow relationship (km/h)

T = time interval (h)

By deriving the flow-efficiency relationship based on empirical data, the point with maximum efficiency was found to be roughly 90% of the deterministically estimated capacity, which is the threshold between LOS D and E in the German Highway Capacity Manual (FGSV, 2015). Figure 2.2 shows the speed-flow curve (in red) and flow-efficiency curve (in blue) for a section of German freeways. Brilon and Zurlinden (2003) applied the concept of variability of capacity into the efficiency and derived an equation to estimate the expected efficiency at any given volume based on its corresponding probability of breakdown.

Figure 2.2 Speed-flow and flow-efficiency curves (Brilon, 2000).

Geroliminis and Daganzo (2007, 2008) also developed a network-wise measure of performance called “productivity.” They introduced a macroscopic fundamental diagram (MFD) linking the space-mean flow, density, and speed based on observations in urban networks (e.g., Yokohama, Japan and San Francisco, California). Based on this, the Network productivity was defined as the total vehicle-miles traveled in a certain time interval over the network. Therefore, the term productivity may be seen as aggregated efficiency over an entire network. It was also suggested by the authors that productivity of the network is maximized when the network achieves its highest veh-miles traveled during a certain time interval.

2.2. Conventional Concept of Capacity

Traditionally, freeway capacity is treated as a single constant value under the same prevailing conditions. In fact, it is viewed as the maximum volume that can be sustained by the facility. Due to its versatile applications, many of the traffic engineering guidelines around the world, such as the HCM, use a static value to estimate capacity. To estimate the capacity of basic freeway segments, the HCM delivers a set of base capacities dependent upon the free-flow speed. Although

these capacity values may found to be fairly representative for U.S. freeways, local conditions may significantly affect the accuracy of the capacity estimates. Thus, many researchers have suggested different functions to locally determine the relationship between speed, volume, and density. Given the empirical observations of speed, flow, and density, parameters of the pre-determined model are calibrated and the apex volume of the speed-flow diagram is considered as the capacity of the segment. Hence, selecting an appropriate model plays a key role to estimate realistic capacity values.

Greenshields (1935) proposed the first, and perhaps the best-known, mathematical function, to model the relationship between macroscopic traffic variables (i.e., speed, density, and volume). This functional relationship between the macroscopic traffic variables is named fundamental diagram, which can be used to predict operation of the facility under different circumstances. Once the model is calibrated and the relationship is established, the apex volume of the speed-flow diagram is determined and regarded as the capacity of the facility.

Greenshields (1935) collected traffic data using a 16-mm simplex movie camera equipped with an electric motor which allowed photographs to be taken at a constant time interval (Kühne, 2011). As shown in Equation (2.2), he assumed a linear relationship between traffic speed and density, and used the Ordinary Least Squares (OLS) method to calibrate the model parameters. He also assumed that the three macroscopic variables are related to each other according to Equation (2.3). Thus, by replacing the density from Equation (2.2) in Equation (2.3), the linear relationship between speed and density would convert into a parabolic relationship between speed and volume shown in Figure 2.3. As a result, the two regimes of stable and unstable traffic flow are separated by the apex volume of the speed-flow diagram (i.e., capacity) which occurs at a speed equal to 50

percent of the free-flow speed. This simple linear relationship is called “univariate model” as both congested and non-congested regimes are determined with the same formula (Kühne, 2011).

$$s = s_f - \left(\frac{s_f}{d_j} \right) \cdot d \quad (2.2)$$

$$v = s \cdot d \quad (2.3)$$

where

s = traffic speed (km/h)

v = traffic flow rate (vph)

d = traffic density (km/veh)

Figure 2.3 The first fundamental diagram (Greenshields, 1935).

Van Aerde (1995) proposed a function based on a simple car following model which assumes density to be a function of the current speed (s), the free-flow speed (s_f), and three parameters (c_1, c_2, c_3). As a continuous three-regime traffic flow model, the Van Aerde formula has the capability to accurately estimate the capacity independent of configurations of the congested

and non-congested regimes. Also, the formula is a generalized form of Greenshields' model in the sense that once the constant terms c_1 and c_3 in Equation (2.4) are set to zero, the relationship between speed and density will become linear. Although the suggested model is still continuous, the assumption of a linear relationship between speed and density is relaxed and, as a result, the model is capable of describing different traffic states with more accuracy.

$$\hat{d}_i = \frac{1}{h} = \frac{1}{c_1 + \frac{c_2}{s_f - \hat{s}_i} + c_3 \hat{s}_i} \quad \forall i \quad (2.4)$$

$$\hat{v}_i = \hat{d}_i \cdot \hat{s}_i \quad \forall i \quad (2.5)$$

where

h = distance headway between consecutive vehicles (km/veh)

d_i = traffic density (veh/km)

c_1 = fixed distance headway constant (km)

c_2 = first variable distance headway constant (km²/h)

c_3 = second variable distance headway constant (h⁻¹)

s_f = free-speed (km/h)

s_c = speed at capacity (km/h)

s_i = prevailing speed associated with headway h (km/h)

v_i = flow rate of traffic traveling at speed s (vph)

Van Aerde and Rakha (1995) suggested multivariate calibration of the speed-density scatterplot as an unbiased method when it is not clear which variable (i.e., speed, density, or volume) is the dependent and which is the independent one. In order to calibrate the model parameters, initial values of free-speed (s_f), speed-at-capacity (s_c), flow at capacity (v_c) and jam density (d_j) are input into Equations (2.10) to (2.13) and the values of c_1 , c_2 , c_3 and k are calculated.

Once the initial model parameters are computed, the sum of squared orthogonal error can be calculated according to Equation (2.6). Next, by iteratively changing the values of free-speed (s_f), speed-at-capacity (s_c), flow at capacity (v_c) and jam density (d_j), and applying a hill climbing technique, the set of model parameters that minimize the sum of squared orthogonal errors can be selected as the final estimates.

$$\text{Min } E = \sum_i \left\{ \left(\frac{s_i - \hat{s}_i}{\tilde{s}} \right)^2 + \left(\frac{v_i - \hat{v}_i}{\tilde{v}} \right)^2 + \left(\frac{d_i - \hat{d}_i}{\tilde{d}} \right)^2 \right\} \quad (2.6)$$

$$\text{S.T. } \hat{d}_i = \frac{1}{h} = \frac{1}{c_1 + \frac{c_2}{s_f - \hat{s}_i} + c_3 \hat{s}_i} \quad \forall i \quad (2.7)$$

$$\hat{v}_i = \hat{d}_i \cdot \hat{s}_i \quad \forall i \quad (2.8)$$

$$\hat{v}_i, \hat{d}_i, \hat{s}_i \geq 0 \quad \forall i \quad (2.9)$$

$$k = \frac{2s_c - s_f}{(s_f - s_c)^2} \quad (2.10)$$

$$c_2 = \frac{1}{d_m \left(k + \frac{1}{s_f} \right)^2} \quad (2.11)$$

$$c_1 = k c_2 \quad (2.12)$$

$$c_3 = \frac{-c_1 + \frac{s_c}{v_c} - \frac{c_2}{(s_f - s_c)}}{s_c} \quad (2.13)$$

where

h = distance headway between consecutive vehicles (km/veh)

d = traffic density (veh/km)

c_1 = fixed distance headway constant (km)

c_2 = first variable distance headway constant (km²/h)

c_3 = second variable distance headway constant (h⁻¹)

s_f = free-speed (km/h)

s_c = speed at capacity (km/h)

s = prevailing speed associated with headway h (km/h)

v = flow rate of traffic traveling at speed s (vph)

v_c = flow at capacity (vph)

d_j = jam density (veh/km)

k = dimensionless constant to set the speed at capacity (-)

Since the apex volume of the speed-flow diagram is only one possible capacity estimate and freeway facilities may experience saturation at higher or lower volumes, other procedures were developed to estimate the probability of breakdown occurrence as a function of the traffic volume.

2.3. Stochastic Concept of Capacity

Freeway capacity is suggested to be stochastic not only due to changes in prevailing and environmental conditions of the facility, but also due to the variability in individual driving behaviors (Minderhoud et al., 1997). Research has shown that even under similar road, traffic, and control conditions, the empirical observation of capacity, irrespective of its definition in different research articles, fluctuates from time to time (Hall and Agyemang-Duah, 1991; Banks, 1991; Persaud et al., 1998; Lorenz and Elefteriadou, 2000; and Elefteriadou and Lertworawanich, 2003).

Minderhoud et al. (1997) reviewed 10 methods of capacity estimation and described their individual characteristics. They also mentioned the stochastic nature of capacity and suggested that its variability stemmed from differences in individual driver behavior, and variable road and weather conditions. From different methods, they selected Product Limit Method (PLM) as the best method to estimate capacity due to its theoretical advantages and the fact that it estimates a

distribution function for capacity rather than just a single deterministic value. They noticed that presuming a normal distribution for capacity could be a reasonable first assumption.

Table 2.2 Capacity estimation methods and their characteristics (Minderhoud et al, 1997).

Method	Data Needs				Traffic State		Capacity		Type	Validity
	h	q	v	k	{Q}	{C}	q _c	F(q)		(--, -, o, +, ++)
Headway Models	Yes				Yes		Yes		1	-
Bimodal Distribution		Yes			Yes	Yes		Yes	m	o
Selected Maxima		Yes			Yes	Yes		Yes	m	o
Direct Probability		Yes				Yes	Yes		d	-
Asymptotic		Yes				Yes	Yes		d	--
Empirical Distribution		Yes	Yes			Yes		Yes	2	+
Product Limit		Yes	Yes		Yes	Yes		Yes	m	++
Selection		Yes	Yes		Yes	Yes	Yes		m	-
Fundamental Diagram		Yes	Yes	Yes	Yes		Yes		d	+
Online Procedure		Yes	Yes	Yes	Yes		Yes		1	o

{Q} represents free flow intensities, {C} represents congested flow intensities

Type 1 denotes a capacity value estimation representing the maximum free flow intensity, Type 2 denotes a capacity value estimation representing the maximum congested flow intensity, m stands for type 1 and type 2 mixed into one capacity estimate and d stands for the dependency with the study set up (either type 1 or 2 is possible)

Lorenz and Elefteriadou (2000) collected and analyzed speed and flow rate data in two bottleneck sites at highway 401 located in Toronto, Canada. During a twenty-day period, they observed 40 breakdowns at each site and found a threshold speed of 90 km/h between the congested and free flow states. Based on preliminary models of breakdown probability shown in Figure 2.4, the researchers found that the probability of breakdown increases as the flow rate increases. They also concluded that given the same equivalent hourly flow rate, probability of breakdown increases as the observation duration increases. The authors further provided a probabilistic definition of freeway capacity as “...the rate of flow (expressed in pcphpl and specified for a particular time interval) along a uniform freeway segment corresponding to the expected probability of breakdown deemed acceptable under prevailing traffic and roadway conditions in a specified direction”.

Figure 2.4 Probability of breakdown for different time intervals (Lorenz and Elefteriadou, 2000).

Elefteriadou and Lertworawanich (2003) also focused on the bottleneck sites located in highway 401, Toronto, Canada. As shown in Figure 2.5, these researchers defined breakdown flow as the flow rate observed immediately prior to breakdown, maximum pre-breakdown flow as the maximum flow observed at the site prior to the occurrence of congestion, and maximum discharge flow as the maximum flow observed at the site after the occurrence of breakdown and prior to recovery to non-congested conditions. They examined these three flow parameters and discovered that they were approximately normally distributed for both sites and both aggregation intervals (5 and 15-minute) under investigation. It was also observed that the maximum pre-breakdown flow and maximum queue discharge flow were greater than breakdown flow rate at both sites. Hence, the authors proposed breakdown flow rate to be used in the definition of capacity.

Figure 2.5 Observed flow rates with 5-minute aggregation (Elefteriadou and Lertworawanich, 2003).

Van Toorenburg (1986) estimated the capacity distribution function based on the analogy drawn between incomplete lifetime observations and freeway capacity. In order to derive the capacity distribution function, observations of both congested and non-congested regimes were included. Brilon et al. (2005, 2007) used a slightly modified approach and considered only the non-congested observations for the analysis because observations under congested traffic conditions provide no information about the capacity before a breakdown. They used Maximum Likelihood (ML) estimation method to estimate the parametric distribution function, and PLM with the analogy to the statistics of lifetime data, to estimate the non-parametric distribution function. The researchers also employed different parametric distributions to fit the empirical observations collected from German freeways and found that the Weibull distribution provides the best fit to the data. They also found that the wet road surface decreased the capacity by 11%, while

speed limit increased the capacity by 3%, and darkness had no effect on freeway capacity. Table 2.3 shows the analogy drawn by Brilon et al. between lifetime data analysis and capacity analysis.

Table 2.3 Analogy between lifetime data analysis and capacity analysis (Brilon et al, 2005).

	Analysis of Lifetime Data	Capacity Analysis
Parameter	Time t	Traffic volume q
Failure Event	Death at time t	Breakdown at volume q
Lifetime variable	Lifetime T	Capacity c
Censoring	Lifetime T is longer than the duration of the experiment	Capacity c is greater than traffic demand
Survival function	$S(t)=1-F(t)$	$S_c(q)=1-F_c(q)$
Probability density function	$f(t)$	$f_c(q)$
Probability distribution function	$F(t)$	$F_c(q)$

Geistefeldt and Brilon (2009) compared the distribution functions estimated based on the models for censored data to those obtained from direct breakdown probability estimation models. The two approaches were compared for accuracy, consistency of results, and applicability. Using a simulation model, the authors found that the capacity estimates based on the models for censored data were more consistent than those obtained through the direct breakdown probability estimation models. Moreover, to overcome hardships in detecting breakdown flow rates due to fluctuations in speed, in addition to the simple two-step criterion, a five-step criterion was also introduced by the authors. The paper also discussed the influences of daylight/darkness, weather condition, accidents and incidents, share of heavy vehicles, commuter/recreational traffic, and work zones on capacity of freeway. Figure 2.6 shows the capacity distribution functions estimated with different methods for a section of German freeways.

Figure 2.6 Estimated capacity distribution functions for a section of German freeways (Geistefeldt and Brilon, 2009).

Kim et al. (2010) examined the probability of breakdown occurrence during rainy and clear weather situations for five U.S. freeway sections. They used ML and PLM to estimate the parametric and non-parametric empirical distributions of breakdown, and found that rainy conditions significantly reduced the capacity of the sections under investigation. They compared Weibull, Normal, and Logistic distributions based on their Log-Likelihood value, for both rainy and clear weather conditions, and found Normal distribution as the one that best fitted the observed data. Furthermore, by applying semi-parametric Cox Proportional Hazard (PH) model, it was found that rainy weather conditions during the breakdown increased the duration of breakdown (defined as the period between the occurrence of breakdown and recovery) from 34.8% to 43.8%.

Figure 2.7 Probability of breakdown under rain and no rain conditions (Kim et al, 2010).

Geistefeldt (2008) applied the method implemented by Brilon et al. (2005) to estimate the capacity distribution function and superimposed the Van Aerde speed-flow diagrams over the estimated distribution function for 27 German freeway sections. It was concluded that for 5-minute aggregation intervals, the average probability of breakdown corresponding to the apex volume of the Van Aerde model is nearly 3 percent. Modi et al (2014) performed the same analysis and concluded that the peak volume corresponds to nearly 4 percent breakdown probability for U.S. freeways. It was also found that for the sections under investigation the capacities provided by the HCM were generally larger than those estimated with other methods.

Figure 2.8 Relation between the nominal capacity and the capacity distribution functions (Geistefeldt, 2008).

Elefteriadou et al. (2009) incorporated the concept of randomness of capacity in ramp metering. In their research, the authors modified two ramp metering algorithms by determining the maximum acceptable upstream volume as a function of metering rates and the acceptable probability of breakdown. As shown in Figure 2.9, for each metering rate, a unique capacity distribution function was estimated based on the real-world observations and the upstream volume was estimated as a function of the breakdown probability. Also, a VISSIM simulation model was calibrated to replicate the real-world speed-flow scatterplot under the current ramp metering algorithm. Next, given a certain breakdown probability (e.g., 20 percent), upstream mainline volumes corresponding to each of the metering rates were estimated, and the VISSIM was run for the new set of volumes. By repeatedly changing probability of breakdown and running the VISSIM model for the corresponding new upstream volumes, the probability of breakdown that provided

the best balance between onramp queue lengths and postponing the congestion was selected to modify the current ramp metering algorithm.

Figure 2.9 Maximum acceptable breakdown volumes for a 20% breakdown probability threshold (Elefteriadou et al., 2009).

From the literature review, it is apparent that despite the advancement in traffic flow analysis, except for the research conducted by Brilon and Zurlinden (2003), few, if any, freeway performance measures have been proposed which can account for the stochastic nature of freeway capacity. As a result, this research aims to 1) investigate type of the capacity distribution function for U.S. freeways, 2) propose SFI as a new measure to assess freeway performance based on stochastic approach to freeway capacity, 3) derive the optimum volume that provides the best compromise between the probability of breakdown and unutilized capacity of freeway, 4) empirically compare the optimum volumes with capacities obtained in the speed-flow diagram by applying the Van Aerde model to investigate the reasonableness of the optimum volumes as appropriate capacity estimates, 5) provide confidence intervals for the parameters of the Weibull

function, and the optimum volumes to investigate their variability, 6) develop a method to transform a conventional capacity estimate into an entire capacity distribution function, and 7) investigate potential application of the SFI to modify a ramp metering algorithm.

CHAPTER 3: METHODOLOGY

The research methodology is presented in three parts. The first part discusses the Van Aerde model as a conventional approach to estimate capacity. The second part describes the methods for censored data to estimate capacity distribution function as well as the Sustained Flow Index (SFI) as a method to select a single capacity value from the capacity distribution function. This part also gives an explanation of the methods used to determine confidence intervals for the model parameters and the optimum volumes. The third part provides a description of the method used to calibrate a VISSIM model used to make modification on a ramp metering algorithm. Subsequently, the results of the application of the presented methodology to data collected from the freeway bottleneck sections are discussed in the next chapter.

3.1. Deterministic Capacity Estimation

To estimate conventional (deterministic) capacities, the Van Aerde (1995) model, as a function capable of describing the speed-flow-density relationship based on a simple car following equation, was applied in this study. In this model (Equation (3.1)), the distance headway between consecutive vehicles (h) depends on the free-flow speed (s_f), the current speed (s), and three parameters (c_1, c_2, c_3). As a continuous traffic flow model, the Van Aerde formula has the capability to accurately estimate the capacity independent of configurations of the congested and non-congested regimes.

$$d = \frac{1}{h} = \frac{1}{c_1 + \frac{c_2}{s_f - s} + c_3 \cdot s} \quad (3.1)$$

where

d = density (veh/km)

h = distance headway between consecutive vehicles (km)

s_f = free flow speed (km/h)

c_1 = fixed distance headway parameter (km)

c_2 = first variable headway parameter (km²/h)

c_3 = second variable distance headway parameter (h-1)

s = speed (km/h)

To estimate the model parameters, reasonable starting values for the key traffic flow variables (i.e., capacity, free-flow speed, speed at capacity, and jam density) were assumed and a starting set of parameters (c_1, c_2, c_3, s_f) were calculated. Next, using a non-linear regression in the speed-density-volume plot, an iterative approach was implemented to estimate the model parameters which minimized the sum of squared errors of the model with respect to dependent variable. Thus, the choice of the dependent variable (i.e., speed, volume, or density) affects the calibration of the parameters and, as a result, the capacity value. As it is not always clear which variable should be chosen as dependent and which as independent, the orthogonal sum of the squared errors was minimized as an unbiased compromise using multivariate calibration (Van Aerde and Rakha, 1995; Rakha and Arafeh, 2010). Once the parameters were calibrated, the capacity was calculated as the apex volume of the speed-flow diagram.

3.2. Stochastic Capacity Estimation Based on Models for Censored Data

Van Toorenburg (1986) first estimated the capacity distribution functions based on models for censored data. In his methodology, observations of traffic are divided into “censored” and “uncensored” intervals, where both congested and non-congested intervals are included. In contrast, Brilon et al. (2005), only included observations of non-congested flow in the capacity estimation due to the hypothesized difference between capacity in congested and non-congested flow, which is also known as capacity drop. This suggests that only the intervals with the speeds above the threshold speed were considered for analysis and the rest of the intervals were disregarded.

The estimation of capacity distribution functions in this research was based on the method proposed by Brilon et al. (2005, 2007). To estimate the capacity distribution function, traffic breakdowns, i.e., the transitions from the uncongested to the congested state, needed to be detected. To identify traffic breakdowns, a threshold speed, as the boundary between fluid and congested traffic, was determined by analyzing the speed and flow rate time series. Once the threshold speed was determined, a set of three criteria were applied to detect breakdowns in interval (i) based on 5-minute observations:

- 1- If the average speed in the i^{th} time interval was above the threshold speed, but dropped below the threshold speed in the next $(i+1)^{\text{th}}$ time interval and remained below for at least 15 minutes (i.e., three consecutive 5-minute intervals), then the i^{th} interval was considered as uncensored, i.e., the flow rate in the i^{th} interval represented the momentary capacity of the facility.
- 2- If the average speed in time the i^{th} interval was above the threshold speed and remained above the threshold speed in the next time the $(i+1)^{\text{th}}$ interval, then the i^{th} interval was

considered as censored, i.e., the momentary capacity was greater than the observed flow rate.

- 3- If the i^{th} interval did not satisfy the above conditions, it was not considered for further analysis.

According to recent applications of the stochastic capacity estimation technique, the capacity distribution function was estimated in 5-minute intervals. In addition, since the HCM (2016) defined the pre-breakdown flow rate as “the 15-minute average flow rate immediately prior to the breakdown event,” the capacity distribution was also estimated in 15-minute intervals to receive results comparable with the HCM. Because only short time intervals were appropriate to detect the speed drops, breakdowns were detected based on 5-minute intervals (as above), but the average of the three consecutive 5-minute flow rates before a breakdown was considered as the uncensored (pre-breakdown) observation. Similarly, the average flow rates of every three 5-minute intervals between a recovery and the following breakdown were considered as censored observations. If the number of 5-minute intervals between the recovery and the breakdown was not an integer multiplier of three, the first one or two 5-minute intervals following the recovery were disregarded.

Once the pre-breakdown (uncensored) and the other non-congested (censored) observations were determined, both non-parametric and parametric approaches were used to estimate the capacity distribution function.

3.2.1. Non-parametric Estimation Method

The Kaplan-Meier (KM) estimator was used to estimate the non-parametric distribution function of capacity. In its basic form, it was written as Equation (3.2) (Lawless, 2003):

$$S(t) = \prod_{i:t_i \leq t} \left(\frac{n_i - d_i}{n_i} \right) \quad (3.2)$$

where

$S(t)$ = estimated survival function

n_i = number of individuals with a lifetime $T \geq t_i$

d_i = number of deaths at time t_i

Brilon et al. (2005) drew an analogy between parameters of lifetime data and the road capacity (Table 2.3). Equation (3.3) shows the transformed version of the PLM that was applied Brilon et al. (2005) to estimate a non-parametric capacity distribution function.

$$F_c(q) = 1 - S_c(q) = 1 - \prod_{i:q_i \leq q} \left(\frac{k_i - d_i}{k_i} \right) \quad (3.3)$$

where

$F_c(q)$ = capacity distribution function

$S_c(q)$ = capacity survival function

q = traffic volume (veh/h)

q_i = traffic volume in interval i (veh/h)

k_i = number of intervals with traffic volume $q_i \leq q$

d_i = number of breakdowns at volume q_i

The function reaches 1, only if the maximum observed volume is followed by breakdown. As a result, a complete distribution function is rarely reached with the PLM method. Even if a

complete distribution function is reached, it might not be so reliable in higher volumes unless a large sample size is collected.

3.2.2. Parametric Estimation Method

The ML estimation technique was employed to estimate the parametric capacity distribution. Criterion that ML uses to estimate the parameters, states that given the observed data, “desired probability distribution is the one that makes the observed data “most likely”, which means “one should find the PDF, among all the probability densities the model prescribes, that is most likely to have produced the data” (Myung, 2002). To do this, a priori assumption about the capacity distribution type was made and the parameters that maximize the Likelihood value were selected as the calibrated parameters. Equation (3.4) shows the likelihood function¹ that was applied for capacity analysis (Brilon et al., 2005 and 2007):

$$L = \prod_{i=1}^n f_c(q_i)^{\delta_i} \cdot [1 - F_c(q_i)]^{1-\delta_i} \quad (3.4)$$

where

$f_c(q_i)$ = density function of capacity

$F_c(q_i)$ = cumulative distribution function of capacity

n = number of intervals

δ_i = 1, if the interval i is uncensored

δ_i = 0, if the interval i is censored

¹- The likelihood is proportional to the probability of observing the data under the proposed model.

Because the log-likelihood is a monotone function of the likelihood, the same estimates are reached by maximizing the log-likelihood function as well. As a result, for easier calculation, the logarithm is taken from both sides of likelihood function and different distribution functions are compared based on their log-likelihood values (as shown in Equation (3.5)).

$$\ln(L) = \sum_{i=1}^n \{ \delta_i \cdot \ln[f_c(q_i)] + (1 - \delta_i) \cdot \ln[1 - F_c(q_i)] \} \quad (3.5)$$

In this research the Weibull, Normal, Log-Normal, Logistic, Smallest Extreme Value (SEV), Largest Extreme Value (LEV), and Fréchet distributions were considered as possible capacity distribution functions and their log-likelihood values were compared to select the distribution function that best fit the data collected from the 19 bottleneck sections.

It is important to note that the Weibull distribution was suggested by previous research (Brilon et al., 2005 and 2007; and Modi et al., 2014) as the function type that best represented the capacity distribution for freeways. As will be shown later in the results chapter, this is a good suggestion once the objective is to select the optimum volume as a single, representative capacity value from the capacity distribution function. Equation (3.6) shows the formula for Weibull distribution.

$$F_c(q) = 1 - e^{-\left(\frac{q}{\beta}\right)^\alpha} \quad (3.6)$$

where

$F_c(q)$ = capacity distribution function

q = traffic volume (veh/h)

α = shape parameter

β = scale parameter (veh/h)

3.2.3. Sustained Flow Index

The Sustained Flow Index (SFI) introduced in this research provides a measure of freeway performance that accounts for the stochastic nature of capacity, represented by the capacity distribution function. The SFI was defined as the product of the traffic volume (q_i) and the value of the survival function at this volume ($S_c(q_i)$), which is the complement of the breakdown probability or, in other words, the probability that the traffic volume can be sustained without a breakdown. That is, the SFI is the theoretical average volume that can be sustained in fluid traffic given the uncertainty of capacity. Its general form is given in Equation (3.7).

$$SFI = q_i \cdot S_c(q_i) = q_i \cdot [1 - F_c(q_i)] \quad (3.7)$$

where

SFI = sustained flow index (veh/h)

$S_c(q_i)$ = probability of survival at volume q_i

$F_c(q_i)$ = cumulative distribution function of capacity

= probability of breakdown at volume q_i

q_i = traffic volume in interval i (veh/h)

Figure 3.1 displays a capacity distribution function of a 4-lane carriageway. The distribution function was assumed to be Weibull. Three volumes are shown (7,000, 8,000, and 9,000 veh/h) along with their corresponding survival and breakdown probabilities. While the probability of survival is still very high at a volume of 7,000 veh/h, it decreases to 0.93 at 8,000 veh/h and drops significantly to 0.5 at 9,000 veh/h.

Figure 3.1 Capacity distribution function for a 4-lane freeway cross section.

The example above shows the existence of a trade-off between unused capacity and the probability of survival. At volumes of 7,000, 8,000, and 9,000 veh/h, SFIs of 6,930, 7,440, and 4,500 veh/h were estimated, respectively. This suggests that among the three volumes, 8,000 veh/h provided the best compromise. While at a volume of 7,000 veh/h the likelihood of survival was high, the low volume suggested that a significant amount of capacity was not used. On the other hand, a volume 9,000 veh/h used this capacity, it decreased the probability of survival, significantly. As a result, the freeway was likely to experience congestion more often, resulting in a reduction in the overall number of vehicles that could be carried.

3.2.4. Derivation of the Optimum Volume

There will always be trade-off between allowing additional vehicles on road and the probability that these vehicles will cause a breakdown. It is desirable to increase both the probability of

survival and the traffic volume for a given freeway section. But, since any increase of volume necessarily leads to a decrease of the survival probability and vice versa, the SFI (as the product of the two) provides a joint performance measure. Thus, the volume that leads to the maximum SFI can be regarded as the best compromise between maximizing the throughput and minimizing the risk of a traffic breakdown. Using the general formulation of the SFI, the optimal volume can be obtained as a solution to Equation (3.8).

$$\frac{\partial}{\partial q} (q \cdot (1 - F_c(q))) = 1 - (F_c(q) + q \cdot f_c(q)) = 0 \quad (3.8)$$

where

$F_c(q)$ = cumulative distribution function of capacity

$f_c(q)$ = density function of capacity

q = traffic volume (veh/h)

In case of the Weibull distribution, the optimum volume can be derived as a function of the distribution parameters:

$$\frac{\partial}{\partial q} \left(q \cdot e^{-\left(\frac{q}{\beta}\right)^\alpha} \right) = 1 - \left(1 - e^{-\left(\frac{q}{\beta}\right)^\alpha} + q \cdot \frac{\alpha e^{-\left(\frac{q}{\beta}\right)^\alpha} \left(\frac{q}{\beta}\right)^{\alpha-1}}{\beta} \right) = 0 \quad (3.9)$$

$$q_{\text{opt}} = \beta \left(\frac{1}{\alpha} \right)^{\frac{1}{\alpha}} \quad (3.10)$$

Since the second derivative at the optimum point is always negative and any volume higher or lower than the optimum volume reduces the value of the SFI, the optimum volume is a global maximum. Previous research (Wemple et al., 1991; Minderhoud et al., 1997; Brilon et al., 2005; Kim et al., 2010; Jia, 2013) suggested other distribution functions as an appropriate assumption

for freeway capacity. The SFI optimal value formulation is independent of this assumption and the global optimum can be found regardless of distribution type. Table 3.1 shows the capacity distribution functions for various assumed distribution types and their optimal volume solution based on the SFI. For the Normal and Log-Normal distributions, the optimum volume estimate is obtained numerically because no closed form expression for these distribution functions exist.

Table 3.1 Derived optimum volumes for different capacity distribution functions.

Distribution Type	$F_c(q)$	Parameters	Optimum volume
Weibull	$1 - e^{-\left(\frac{q}{\beta}\right)^\alpha}$	α =shape β =scale	$\beta\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha}}$
Logistic	$\frac{1}{1 + e^{-\frac{\mu - q}{\sigma}}}$	μ =location σ =scale	$\sigma(W^2(e^{\frac{\mu}{\sigma}-1})+1)$
SEV	$1 - e^{-e^{\left(\frac{q-\mu}{\sigma}\right)}}$	μ =location σ =scale	$\sigma W(e^{\frac{\mu}{\sigma}})$
LEV	$e^{-e^{\left(\frac{q-\mu}{\sigma}\right)}}$	μ =location σ =scale	$1 - e^{-e^{\left(\frac{\mu-x}{\sigma}\right)}} - \frac{x \cdot e^{\frac{\mu-x}{\sigma}} - e^{\left(\frac{\mu-x}{\sigma}\right)}}{\sigma}$
Fréchet	$e^{-e^{\left(\frac{\log(x)-\mu}{\sigma}\right)}}$	μ =location σ =scale	$1 - e^{-e^{\left(\frac{\mu-\log(x)}{\sigma}\right)}} - \frac{e^{\frac{\mu-\log(x)}{\sigma}} - e^{\left(\frac{\mu-\log(x)}{\sigma}\right)}}{\sigma}$
Normal	$\frac{1}{2} \left(1 + \operatorname{erf}\left(\frac{q - \mu}{\sigma\sqrt{2}}\right)\right)$	μ =location σ =scale	found numerically
Log-Normal	$\frac{1}{2} \left(1 - \operatorname{erf}\left(\frac{(\ln(q) - \mu)}{\sigma\sqrt{2}}\right)\right)$	μ =location σ =scale	found numerically

According to the statistics theory, if a variable is Weibull distributed, its natural logarithm is SEV distributed. Therefore, the natural logarithm of the p-quantile of the Weibull distribution can be written as (Meeker et al., 2017):

$$\ln(t_p) = \mu + \sigma \Phi_{SEV}^{-1}(p) \quad (3.11)$$

² Lambert function

where

t_p = p-quantile = time at which the proportion p of the population fails

μ = location parameter = $\ln(\beta)$

σ = scale parameter = $1/\alpha$

p = probability of failure

$\Phi_{SEV}(z) = 1 - e^{(-e^z)} =$ smallest extreme value distribution

Replacing the p-quantile (t_p) in Equation (3.11) with the optimum volume (q_{opt}) from Equation (3.10), the probability of breakdown at the optimum volume (P_{opt}) was calculated as:

$$P_{opt} = 1 - e^{-\frac{1}{\alpha}} \quad (3.12)$$

As can be seen in Equation (3.12), the breakdown probability at the optimum volume depends only on the value of the shape parameter of the Weibull distribution. As the shape parameter increases, the probability of breakdown at the optimum volume decreases. Hence, control strategies such as variable speed limits that increase the shape parameter (i.e., reduce variance of the distribution function, cf. Geistefeldt, 2011) decrease the probability of breakdown.

3.2.5. Confidence Intervals

Estimated values for the parameters of the capacity distribution function and the optimum volume strongly depend on the collected data sample in the sense that another set of parameters will be estimated if another sample is collected. Thus, it is important to estimate the confidence intervals for these parameters to address sample to sample variations.

Assuming a Weibull distribution for the capacity of a freeway, Wald confidence intervals could be estimated for both the parameters of the capacity distribution function and the optimum

volume. However, if another assumption is made for the capacity distribution function (e.g., normal distribution), the optimum volume may only be derived with numeric calculations and the Wald confidence intervals cannot be easily computed. Thus, this research has provided Bootstrap confidence intervals, as the confidence intervals computed numerically, to compare the proximity of the estimation results for future reference in case only the Bootstrap confidence intervals are applicable.

3.2.5.1. Wald Confidence Intervals

Once the scale parameter of the Weibull-type capacity distribution function was calibrated, its standard error was estimated with the Taylor series approximation and the Wald confidence interval was calculated according to Equation (3.13). The same method was applied to estimate the Wald confidence interval for the shape parameter.

$$\hat{\beta} \pm z_{(1-\alpha/2)} \hat{se}_{\hat{\beta}} \quad (3.13)$$

where

$\hat{\beta}$ = scale parameter estimate (veh/h)

$\hat{se}_{\hat{\beta}}$ = estimate of standard error of the scale parameter

$z_{(1-\alpha/2)}$ = standard score at $\alpha\%$ significance level

To estimate the Wald confidence interval for the optimum volume, a hand calculation was needed to estimate its standard error. Thus, the p-quantile (t_p) in Equation (3.11) was replaced with the optimum volume (q_{opt}) from Equation (3.10) and its variance was estimated with Equation (3.14) using the Taylor series approximation.

$$\widehat{\text{Var}}_{\ln(\widehat{q_{\text{opt}}})} = \left[\frac{\partial \ln(q_{\text{opt}})}{\partial \mu} \right]^2 \cdot \widehat{\text{Var}}(\hat{\mu}) + \left[\frac{\partial \ln(q_{\text{opt}})}{\partial \sigma} \right]^2 \cdot \widehat{\text{Var}}(\hat{\sigma}) + 2 \cdot \frac{\partial \ln(q_{\text{opt}})}{\partial \mu} \cdot \frac{\partial \ln(q_{\text{opt}})}{\partial \sigma} \cdot \widehat{\text{Cov}}(\hat{\mu}, \hat{\sigma}) \quad (3.14)$$

Since $\frac{\partial \ln(q_{\text{opt}})}{\partial \hat{\mu}} = 1$, $\frac{\partial \ln(q_{\text{opt}})}{\partial \hat{\sigma}} = -\ln(\alpha)$, $\widehat{\text{VAR}}(\hat{\mu})$, $\widehat{\text{VAR}}(\hat{\sigma})$, and $\widehat{\text{COV}}(\hat{\mu}, \hat{\sigma})$ were provided by the software, the variance and, as a result, the standard error of the natural logarithm of the optimum volume ($\ln(q_{\text{opt}})$) could be calculated. Thus, the confidence interval for the natural logarithm of the optimum volume were estimated with Equation (3.15).

$$\ln(\widehat{q_{\text{opt}}}) \pm Z_{(1-\alpha)/2} \widehat{\text{se}}_{\ln(\widehat{q_{\text{opt}}})} \quad (3.15)$$

Next, an antilog was taken from the upper and lower bounds of Equation (3.15) to estimate the Wald confidence interval for the optimum volume (q_{opt}).

3.2.5.2. Bootstrap Confidence Intervals

Bootstrap confidence intervals are approximate confidence intervals that are built based on sampling with replacement. In this method, once the sample data (of size n) is collected, B Bootstrap samples of the same size (also called resamples) are taken from the initial sample with replacement. This procedure suggests that in a single Bootstrap sample some observations may be repeated whereas some others may not be selected even once. Thus, new parameters are estimated for each of the Bootstrap samples. If a sufficient number (usually between 2,000 and 5,000) of Bootstrap samples is selected, the confidence intervals delivered for the parameters by this method are quite similar to those delivered by exact methods. Also, Bootstrap confidence intervals have the advantage of relaxing the assumption of the underlying distribution of the data that is used to build the conventional confidence intervals (Meeker et al., 2017).

3.3. Modifying a Ramp Metering Algorithm to Meter Based on the Optimum Volume

It has been shown that vehicle platoons entering a freeway create turbulence which has the potential to cause traffic breakdowns. Ramp management strategies have been shown to reduce this turbulence by metering the traffic at onramps through signalization. The number of vehicles prescribed to enter the freeway is usually calculated by an online adaptive traffic control that attempts to optimize the freeway performance by striking a balance between allowable turbulence of the mainline freeway and the vehicles waiting at the onramp (Elefteriadou et al., 2009).

In order to modify a ramp metering algorithm to meter the onramps based on the optimum volume, real-world data were collected from a freeway section located at San Diego, California. The reason behind selection of the section was that the current (in use) ramp metering algorithm at the section was kindly provided by the authorities. Also, the ramp metering algorithm implemented in the section under study was San Diego Ramp Metering System (SDRMS) whose vehicle actuated programing (VAP) logic was provided by VISSIM in their training example files.

Figure 3.2 Freeway section under study (I5-N, San Diego, CA).

Figure 3.2 shows the section under investigation. As can be seen, the section has two onramp lanes, consisting of a regular lane and an HOV lane that is rarely used. Due to the low volume of the HOV lane, influence of the vehicles entering from the HOV lane on probability of breakdown of the downstream mainline section could not be directly measured. Thus, throughout the study, the HOV volume was added to the ordinary onramp volume, and the total ramp volume was considered for analysis.

3.3.1. Estimating Capacity Distribution Functions for Different Ramp Volumes

To develop different capacity distribution functions for different ramp volumes, as the first step, five years of speed, volume, and occupancy data were collected for the upstream section. For the onramps, only volume data were collected. Next, different breakdown probability models were estimated for different ramp volume categories. To do this, in addition to the mainline volumes and occupancies, ramp volumes that had been grouped into different categories (e.g., < 600 veh/h, $600-720$ veh/h, $720-900$ veh/h, and > 900 veh/h) were also considered for analysis and a unique probability distribution function was estimated for each category.

To verify the statistical difference of the capacity distribution functions estimated for different ramp volume categories from one another, a log-rank test was performed. The log-rank test is a statistical test used for comparing the distribution functions of different categories (i.e., different ramp volume categories in this study). Under the null hypothesis, the log-rank test assumes that different distribution functions are not statistically different from one another. Thus, one should use the log-rank test to evaluate whether the selected ramp volume categories provide distribution functions that are statistically different from one another, and if not, consider other categories (c.f. Elefteriadou et al., 2009). Figure 3.3 shows the set of capacity distribution functions

developed for four different ramp volume categories by Elefteriadou et al. (2009) for a section of freeway.

Figure 3.3 Set of breakdown probability models by ramp demand (Elefteriadou et al., 2009).

After the capacity distribution functions for different ramp volume categories were estimated, the mean value of the ramp volume categories were calculated and interpolation/extrapolation was used to estimate the capacity distribution function corresponding to any desired ramp volume (in between or outside of the categories). Next, an SFI was estimated for each capacity distribution function and its unique optimum volume (and occupancy) was calculated for each ramp volume.

3.3.2. Applying the SDRMS in VISSIM

As mentioned before, the algorithm of the SDRMS was provided by VISSIM and only minor modifications were made to make the SDRMS work for the section under study. The original SDRMS algorithm was designed for a three lane freeway section, but the section under study has

four lanes. Thus, a lane was added to the original algorithm code to reflect this. Furthermore, since the ramp metering system is active from 5am-11am and 2pm-7pm in real-world, the start and end times were also added to the algorithm to only meter the onramps between those times.

The current in use SDRMS algorithm for the section under investigation is shown in Table 3.2. As can be seen in the table, corresponding to each mainline volume and occupancy is a specific maximum permitted onramp volume. As soon as the mainline volume or occupancy, whichever that is more restrictive, reaches its threshold, the SDRMS allows its corresponding ramp volume to enter the freeway. If the upstream mainline volume/occupancy becomes greater than the maximum number allowed (i.e., 36.1 veh/ln/59.1seconds or 20.5 percent), the minimum ramp volume will still be allowed to enter the facility. This suggests that irrespective of the value of upstream mainline volume/occupancy, the number of ramp vehicles allowed to enter the freeway is never less than 5.45 veh/min.

The SDRMS algorithm considers the minimum and maximum occupancies and volumes for the upstream mainlines. By subtracting the minimum values from the maximum and dividing the result to the difference between their rate code values, the average value of increase in volume and occupancy per increase in rate code is calculated. These are called “delta volume” and “delta occupancy”, respectively. For example, as can be seen in Table 3.2, the upstream mainline volumes for rate code number one and number fifteen are 28.73 and 36.08 veh/ln/59.1seconds, respectively. Thus, by dividing the difference between the volumes to the difference between the rate codes (i.e., 7.37 divided by 14), delta volume of 0.5253 veh/ln/59.1seconds is obtained. By following the same steps for occupancy, delta occupancy of 0.28³ is obtained. It can also be seen that the delta volume for the total ramp volume is 0.35 veh/ln/min. This means that the upstream mainline volumes and

³ Please note that in .vap logic provided by VISSIM this value is multiplied by ten and used in the algorithm.

occupancies, and their respective ramp volumes increase linearly (i.e., using constant increments of delta volume and delta occupancy) in this algorithm.

Table 3.2 Currently used SDRMS algorithm for the section under study.

Rate Code	Upstream Occupancy (percent)	Upstream Volume (veh/ln/59.1seconds)	Total Onramp Volume (veh/min)
1	16.20	28.73	10.35
2	16.48	29.25	10
3	16.76	29.78	9.65
4	17.04	30.31	9.3
5	17.33	30.83	8.95
6	17.61	31.36	8.6
7	17.89	31.88	8.25
8	18.17	32.41	7.9
9	18.45	32.93	7.55
10	18.74	33.46	7.2
11	19.02	33.98	6.85
12	19.30	34.51	6.5
13	19.58	35.03	6.15
14	19.86	35.56	5.8
15	20.15	36.08	5.45

3.3.3. Calibration of the VISSIM Model

After the SDRMS algorithm was coded into VISSIM, the real-world observations of volume and speed were collected for a single day from the PeMS website⁴ and input into the VISSIM model. Before modifying the SDRMS algorithm, the VISSIM model needed to be calibrated first to reflect the real-world volumes and speeds. Thus, by trial and error for different VISSIM driving behavior and lane changing parameter values, those that minimized the difference between the observed and simulated speeds and volumes were selected as the calibrated parameters.

To calibrate traffic volumes, the GEH⁵ statistic, named after its inventor Geoffrey E. Havers, was used as a measure of effectiveness (MOE). The GEH statistic is a well-known empirical

⁴ <http://pems.dot.ca.gov>

⁵ Named after its inventor Geoffrey E. Havers.

measure that is widely used in transportation studies to calibrate hourly traffic volumes. Its formula is given in Equation (3.16).

$$GEH = \sqrt{\frac{2(m - c)^2}{m + c}} \quad (3.16)$$

where

m = output traffic volume from the simulation model (veh/h)

c = input traffic volume (veh/h)

It is important to note that the GEH formula can only be used for hourly volumes. Because calibrating the hourly volumes for the whole day was of interest in this study, the percentage of hourly volumes with an acceptable GEH value was regarded as an appropriate calibration target. According to the FHWA (2004), the GEH statistic for individual link flows is acceptable only if GEH is less than 5 for a least 85 percent of the all cases (Dowling et al., 2004). This means that for an observation period with 24 different GEH values (i.e., one GEH for each hourly volume), at least 21 of GEH values should have a value less than 5.

The root mean squared percentage error (RMSPE) was used as an MOE to calibrate the speeds. The RMSPE is a measure that is usually applied to calculate the deviation of the simulation speeds from the observed speeds. Therefore, as a first step, the Van Aerde model was fitted to the real-world 5-minute data collected for the day under investigation. After the model was fitted to the data, for every simulated volume, the difference between its simulated speed and the observed speed (i.e., the speed that is estimated for the simulated volume from the Van Aerde model) was measured and the RMSPE was calculated according to Equation (3.17). As a rule of thumb, RMSPE of 5 percent is usually regarded as an appropriate calibration target for the speeds.

$$\text{RMSPE} = \sqrt{\frac{1}{N} \cdot \sum_{n=1}^N \left(\frac{x_n^{\text{sim}} - x_n^{\text{obs}}}{x_n^{\text{obs}}} \right)^2} \quad (3.17)$$

where

x_n^{sim} = n^{th} simulated speed (mph)

x_n^{obs} = n^{th} observed speed (mph)

Once the current SDRMS ramp metering algorithm was coded in the VISSIM and the VISSIM model was calibrated to reflect the real-world situation, the SDRMS metering algorithm was modified to meter the onramp vehicles as a function of the mainline optimum volumes and occupancies. Thus, the estimated optimum occupancies and volumes corresponding to each ramp volume were used instead of the values shown in the first and second columns of Table 3.2. As the null hypothesis (H_0), it was assumed that the current (in use) SDRMS algorithm would perform better than the modified algorithm. As a result, as the alternative hypothesis (H_1), it was assumed that the modified SDRMS algorithm would perform better than the current algorithm. As suggested in the literature, different performance measures such as 1) average delay over the entire network, 2) average speed over the entire network, 3) total distance travelled over the entire network, and 4) total travel time over the entire network, were used to make a conclusion about the null hypothesis.

CHAPTER 4: EMPIRICAL RESULTS

This chapter is divided in five parts to report the results of application of the methodology discussed in previous chapter on the data collected from sections under investigation. The first part provides a description of the data collection points and the preparation of the data. The second part discusses the results of fitting different distribution functions to the observed 5- and 15-minute data to find which function provided the best fit. To investigate the validity of the optimum volumes as reasonable capacity estimates, the third part compares the estimated optimum volumes with the conventional capacity values for the sections under study. To determine stability of the estimated optimum volumes, the fourth part provides the results of the Wald and Bootstrap confidence intervals estimated for the optimum volumes. This part also discusses the method that can be applied to estimate a capacity distribution function based on the conventional capacity estimate. The fifth part provides the results of modifying the SDRMS in VISSIM to meter an onramp as a function of the mainline optimum volume.

4.1. Data Description and Preparation

To investigate the topic numerically and quantitatively calibrate model parameters, 19 urban bottleneck freeway sections located in California, U.S., were selected for the case study. The reason behind selection of sections located in California was simply availability of extensive and detailed data provided freely by the Caltrans Performance Measurement System (PeMS).

Sections selected in this study were distinct bottlenecks, meaning that there were minimal effects from the downstream sections on their performance. Although one cannot always guarantee absence of downstream queue backup in a macro level analysis, long contours of speed (i.e., the aggregated average speed over different sections of the freeway over the year) were used to select the sections minimally affected from downstream. Thus, only section that the speed in their

immediate downstream section was high but in their immediate upstream section was low, were selected as potential distinct bottlenecks for further analysis. Figure 4.1 shows long contours of speed for I8-E for the year 2011 as well as a bottleneck location shown in white dashed line.

Figure 4.1 Long contours of speed for I8-E during 2011 (PeMS).

If a detector at any given section did not report the real-time data accurately, the PeMS imputed those values based on historic observations or nearby detectors. As a result, the bottlenecks detected using the above criterion were not reliable unless the percentage of data observed (at the bottlenecks and their downstream and upstream sections) were checked to guarantee that the speeds reported in the long contours were actually observed throughout the year and not simply imputed.

Once the bottleneck sections were selected, the speed and flow rate data, which have been measured and aggregated into 5-minute intervals by PeMS, were collected for each of the 19 sections under investigation for a minimum period of one year. The reason to collect a minimum of one year of data at each section was the fact that the PLM is a data-intensive method and a

reliable distribution function could not be estimated unless a relatively large sample was collected. Also, since it was desirable to separate the commuter and recreational traffic, only workdays were considered for analysis.

It is important to note that all sections under study were merge bottlenecks. In some cases, the detectors were located upstream of the gore point where the onramp and mainline intersected. Here, the onramp volumes were added to the upstream mainline volumes for capacity estimation. The speed at the upstream mainline section was assumed to be the same as the speed of the gore point. On the other hand, in some other cases the detectors were located at the gore point (i.e., immediately after the lane drop). In these cases, only the data from the detectors located on the gore point were collected and used for capacity estimation.

4.2. Estimation of the Capacity Distribution Function

To investigate the type of capacity distribution function for U.S freeways and estimate the SFI and optimum volumes in the next step, different distribution functions including the SEV, Weibull, Logistic, Normal, Log-normal, LEV, and Fréchet were fitted to the data collected in 5-minute intervals. Thus, for each section, the log-likelihood values of different distribution types were compared and the one with the maximum log-likelihood value was selected as the capacity distribution type of the section. Table A.1 to Table A.7 show the calibrated parameters of different distribution functions as well as their log-likelihood values. Table 4.1 provides a summary of the previous tables and ranks the distribution functions based on their log-likelihood values for each study section. As can be seen from the table, in most cases, the Logistic distribution provided the best fit to the data. The Logistic distribution resembles the Normal distribution in the sense that it is a symmetrical distribution (i.e., not skewed to the right or left). Unlike the Normal distributions, the Logistic distribution has heavy tails (greater kurtosis).

In some cases, the SEV, Weibull, and Normal distributions also provided the best fit to the observed data. However, for most cases the Log-normal, LEV and Fréchet distributions did not provide an acceptable fit to the data. From these results, it was concluded that the capacity distribution function may not be characterized with right-skewness.

Table 4.1 Rankings of different distribution functions (5-minute intervals).

Section	Distribution Type						
	SEV	Weibull	Logistic	Normal	Lognormal	LEV	Fréchet
1	I	III	II	IV	V	VI	VII
2	VI	III	IV	I	II	V	VII
3	II	III	I	IV	V	VI	VII
4	III	II	I	IV	V	VI	VII
5	III	II	I	IV	V	VI	VII
6	II	III	I	IV	V	VI	VII
7	III	II	I	IV	V	VI	VII
8	III	II	I	IV	V	VI	VII
9	III	II	I	IV	V	VI	VII
10	I	III	II	IV	V	VI	VII
11	III	II	I	IV	V	VI	VII
12	II	III	I	IV	V	VI	VII
13	I	II	III	IV	V	VI	VII
14	III	II	I	VII ⁶	IV	V	VI
15	I	III	II	IV	V	VI	VII
16	III	II	I	IV	V	VI	VII
17	II	III	I	IV	V	VI	VII
18	II	I	III	IV	V	VI	VII
19	I	III	II	IV	V	VI	VII

In recent applications of the stochastic capacity estimation technique, the capacity distribution function was estimated in 5-minute intervals. The HCM (2016), however, defined the pre-breakdown flow rate as the 15-minute average flow rate immediately prior to the breakdown event and suggested the volume corresponding to the 15 percent probability of breakdown as the benchmark to select a single capacity value from the capacity distribution function. Thus, to compare the 15 percent probability of breakdown suggested by HCM with those corresponding to

⁶ In this case, for the Normal distribution, the maximum number of iterations was reached but the algorithm did not converge.

the optimum volumes, capacity distribution functions were estimated in 15-minute intervals as well. Table A.8 to Table A.14 show the calibrated parameters of the distribution functions estimated based on 15-minute data along with their log-likelihood values. Table 4.2 provides a summary of the previous tables and ranks the distribution functions based on their log-likelihood values for each study section. As can be seen from the table, in most cases, the SEV distribution provided the best fit to the data, followed by the Logistic, Weibull, Normal, Log-normal, LEV, and Fréchet distributions respectively. The SEV distribution is a left-skewed distribution that is sometime used to model the “strength” of materials. The left-skewedness of the SEV distribution suggests that in few cases the roadway reaches its capacity on lower volumes, but in majority of the cases the roadway capacity is reached at higher volumes. The estimation results for 15-minutes intervals also indicated that right-skewed distributions, such as LEV and Fréchet, may not be appropriate functions to characterize the freeway capacity distribution.

Table 4.2 Rankings of different distribution functions (15-minute intervals).

Section	Distribution Type						
	SEV	Weibull	Logistic	Normal	Log-normal	LEV	Fréchet
1	I	II	III	IV	V	VI	VII
2	III	II	I	IV	V	VI	VII
3	I	III	II	IV	V	VI	VII
4	I	III	II	VI	IV	V	VII
5	I	III	II	IV	V	VI	VII
6	I	II	III	IV	V	VI	VII
7	I	III	II	IV	V	VI	VII
8	II	III	I	VI	IV	V	VII
9	I	II	III	IV	V	VI	VII
10	I	II	III	IV	V	VI	VII
11	I	II	III	IV	V	VI	VII
12	I	III	II	IV	V	VI	VII
13	I	II	III	IV	V	VI	VII
14	II	III	I	VI	IV	V	VII
15	I	II	III	IV	V	VI	VII
16	I	III	II	IV	V	VI	VII
17	I	II	III	IV	V	VI	VII
18	III	II	I	IV	V	VI	VII
19	I	III	II	IV	V	VI	VII

4.3. Comparing the Optimum Volumes and the Conventional Capacities

To calculate the optimum volumes by maximizing the SFI, the parameter estimates of the Weibull distribution functions were used. Despite the fact that the Logistic and the SEV distributions provided the best fit to the observed data for 5- and 15-minute intervals, the Weibull distribution was assumed to represent the capacity for three reason:

- 1- The formula of optimum volume (shown on Table 3.1) was easy to apply for the Weibull distribution.
- 2- The difference between the optimum volumes obtained assuming the Weibull distribution and other distribution types (i.e., SEV and Logistic distributions for 5- and 15-minute intervals respectively) was very small. In fact, for the 5-minute intervals, assuming a

Weibull distribution instead of a Logistic distribution resulted in RMSPE of 0.0017 between the optimum volumes (see Table A.15). For the 15-minute intervals, assuming a Weibull distribution instead of a SEV distribution resulted in RMSPE of 0.0032 between the optimum volumes (see Table A.16).

- 3- Previous literature about parameters of the Weibull distribution for capacity analysis has enabled researchers to interpret them. As will be shown later in Figure 4.6, interpretation of the Weibull parameters played a key role in application of the optimum volume (i.e., obtaining the capacity distribution function given the conventional capacity estimate).

Using the calibrated parameters of the Weibull distribution from Table A.2 and Table A.9, optimum volumes and their corresponding breakdown probabilities were calculated according to Equations (3.10) and (3.12), respectively. Figure 4.2 shows the parametric and non-parametric capacity distribution functions and SFI's estimated based on 5- and 15-minute observations for a bottleneck section located at Interstate 5. As can be seen, for 5- and 15-minute observation intervals, the freeway section reached its maximum SFI's at volumes of 7,630 veh/h and 7,185 veh/h respectively. These optimum volumes correspond to 4.6 percent and 4.5 percent probability of breakdown.

Figure 4.2 Estimated capacity distribution functions and SFI's for a 4-lane freeway cross section.

To compare the capacity values delivered with the conventional and stochastic methods discussed above, an empirical analysis was performed based on traffic flow data in 5-minute intervals collected from sections under investigation. A same sample size of one year was used to estimate both deterministic and stochastic capacities for every section under investigation.

In order to calculate the conventional capacity values, parameters of the Van Aerde model were estimated by applying multivariate calibration. Thus, the SPD_CAL software was used to minimize the orthogonal sum of squared errors of speed, volume, and density using nonlinear regression (Rakha and Arafeh, 2010). The capacity obtained as the volume at the apex of the Van Aerde model is referred to as c_{VAM} in the following.

For better illustration, Figure 4.3 shows the SFI superimposed over the Van Aerde model for the same freeway section (shown on Figure 4.2). In this case, the capacity estimated by

multivariate calibration of the Van Aerde model was 7,472 veh/h. Once both the conventional capacity values and the optimum volumes⁷ were estimated, they could be compared based on the percentage difference between the results delivered by the two methods. Thus, for the section shown in Figure 4.3, the optimum volume is 2 percent greater than the conventional capacity estimate.

Figure 4.3 Conventional capacity estimate vs. optimum volume for a 4-lane freeway cross section.

Table 4.3 and Table 4.4 show the results of applying the above procedure to all sections under investigation. As can be seen in Table 4.3, the average probability of breakdown corresponding to the optimum volume was 3.9 percent for the 15-minute optimum volume. The 3.9 percent probability of breakdown found in this research was not consistent with the 15 percent probability of breakdown recommended by the HCM 6th edition to select a single value from the

⁷ Throughout this dissertation, the “optimum volume” referred to 5-minute optimum volume unless otherwise stated. The 15-minute optimum volumes were only used for comparison with the 15 percent volumes suggested by HCM.

15-minute capacity distribution function. The last column of Table 4.3 shows the volumes corresponding to the 15 percent probability of breakdown for the capacity distribution function estimated based on 15-minute observations. As can be seen, all of the 15th percentile volumes were greater than their corresponding 15-minute optimum volumes (on average, 5.7 percent greater). Based on these results, it was concluded that the 15th percentile method suggested by HCM overestimates the capacity.

Table 4.3 Estimated 15-minute optimum volumes, their probabilities of breakdown, and 15th percentile volumes for the sections under investigation.

Section	Lanes	15-Minute Optimum Volume q_{opt-15} (veh/h)	$F_{c,15}(q_{opt-15})$	15 th Percentile Volume
1	2	3,363	4.1	3,559
2	2	3,653	3.9	3,864
3	2	3,670	3.6	3,876
4	2	3,611	3.5	3,812
5	3	5,253	4.2	5,562
6	3	4,977	3.9	5,263
7	3	5,450	4.2	5,770
8	3	5,678	3.7	5,999
9	4	7,921	2.7	8,316
10	4	7,960	3.6	8,408
11	4	7,138	4.1	7,554
12	4	8,085	4.5	8,568
13	4	7,185	4.5	7,614
14	4	7,596	3.6	8,022
15	4	6,897	3.5	7,281
16	4	7,543	4.6	7,995
17	5	8,550	4.4	9,059
18	5	7,591	3.9	8,029
19	5	9,509	3.9	10,056
Average Probability			3.9	

According to Table 4.4, the average probabilities of breakdown corresponding to the (5-minute) optimum volumes and the capacity values estimated with the Van Aerde model were 4.5 and 3.7 percent respectively. The 3.7 percent probability was quite close to the 4 percent probability of breakdown obtained by Modi et al. (2010) for U.S. freeways. The results also

revealed that the standard deviation of the probability of breakdown at the optimum volume was less than that of the conventional capacity: $\text{stdev}(F_{c,5}(q_{\text{opt}})) = 0.4 < \text{stdev}(F_{c,5}(c_{\text{VAM}})) = 1.2$. This indicated that the probability of breakdown at the optimum volume varies less than the probability of breakdown corresponding to the volume at the apex of the Van Aerde curve.

The average percentage difference between the optimum volumes and the capacities estimated with the Van Aerde model was 1.1 percent and the correlation between them was more than 99 percent. These findings suggested that the optimum volumes obtained by maximizing the SFI were on average within a reasonable range of the conventional capacity estimates and could therefore be considered as suitable capacity estimates.

Table 4.4 Estimated capacity values and the optimum volumes as well as their corresponding probabilities of breakdown for the sections under investigation.

Section	Lanes	Optimum Volume q_{opt} (veh/h)	$F_{c,5}(q_{\text{opt}})$	Capacity c_{VAM} (veh/h)	$F_{c,5}(c_{\text{VAM}})$
1	2	3,611	4.8	3,516	2.9
2	2	3,899	4.7	3,942	5.8
3	2	3,880	4.4	3,762	2.3
4	2	3,815	4.4	3,735	2.7
5	3	5,639	4.6	5,574	3.6
6	3	5,286	5.1	5,363	6.6
7	3	5,800	4.9	5,696	3.4
8	3	5,990	4.3	5,994	4.4
9	4	8,254	3.4	8,306	4.1
10	4	8,393	4.4	8,378	4.2
11	4	7,494	4.4	7,486	4.3
12	4	8,635	4.8	8,416	2.9
13	4	7,630	4.6	7,472	3
14	4	7,841	4.1	7,684	2.5
15	4	7,273	3.9	7,154	2.6
16	4	8,098	5.1	7,826	2.7
17	5	9,151	4.3	9,249	5.5
18	5	8,155	4.3	8,001	2.8
19	5	10,004	4.2	9,995	4.1
Average Probability			4.5		3.7

As can be seen in Table 4.3 and Table 4.4, the estimation results suggested that there was no evident empirical relationship between the number of lanes and the probability of breakdown at the optimum volume for the distribution functions estimated based on 5- and 15-minute observations. This was reasonable because, according to Equation (3.12), the probability of breakdown at the optimum volume only depends on the shape parameter of the Weibull distribution and previous research did not find a meaningful relationship between the number of lanes and the shape parameter either. Therefore, it was concluded that the probability of breakdown at the optimum volume for freeways with different number of lanes remains nearly constant.

4.4. Estimating the Confidence Intervals for the Optimum Volumes

Based on the calibrated parameters of the Weibull distribution, Bootstrap confidence intervals were calculated. For this, 10,000 Bootstrap samples were drawn (with replacement) from the initial sample used to estimate parameters of the distribution function. For each of the Bootstrap samples, a unique set of parameters and, as a result, the optimum volumes were estimated. After estimating the parameters and the optimum volumes, their confidence intervals at a significance level of 5% were calculated. Figure 4.4 shows the Bootstrap-based parameter values along with the set of parameters estimated based on initial sample.

Figure 4.4 Parameters estimated using Bootstrapping technique.

To estimate the Wald confidence intervals for the estimated parameters and optimum volumes, Equations (3.13) and (3.15) were applied respectively. Both Bootstrap-based and Wald confidence intervals estimated at a significance level of 5 percent are shown in Table 4.5. The results delivered by both methods were very similar, which meant that applying the Bootstrapping technique provides a suitable approximation of confidence intervals for the optimum volume in case that another function type is assumed for the capacity distribution (whose optimum volume can only be estimated numerically). Also, since the estimated 95% confidence intervals were relatively small, it was inferred that the optimum volume is a fairly stable indicator of freeway capacity.

Table 4.5 Estimated Wald and Bootstrap confidence intervals for the parameters and optimum volumes.

Section Number		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Shape Parameter		20.2	20.7	22.0	22.5	21.3	19.2	20.1	22.8	28.6	22.2	22.4	20.1	21.1	23.9	25.1	19.0	22.6	22.7	23.1
Wald CI	Lower Bound	19.4	19.6	20.9	21.1	19.2	17.9	18.7	21.4	26.9	20.5	21.5	19.4	20.0	22.2	23.4	18.0	21.9	21.1	21.7
	Upper Bound	21.0	21.8	23.2	24.0	23.5	20.6	21.6	24.4	30.6	24.4	23.4	21.0	22.2	26.0	27.2	20.1	23.4	24.7	24.7
Bootstrap CI	Lower Bound	19.2	19.7	20.5	21.0	19.2	17.9	18.5	21.0	26.0	19.9	21.4	19.1	19.9	22.0	22.8	17.4	21.4	21.0	20.9
	Upper Bound	21.2	21.7	23.7	24.1	23.8	20.8	21.8	24.8	32.3	24.8	23.5	21.2	22.3	26.0	27.9	21.1	23.9	24.8	25.5
Scale Parameter		4,190	4,515	4,465	4,382	6,509	6,164	6,735	6,871	9,281	9,649	8,609	10,023	8,818	8,954	8,269	9,457	10,504	9,357	11,460
Wald CI	Lower Bound	4,165	4,481	4,435	4,349	6,419	6,110	6,661	6,809	9,226	9,548	8,567	9,961	8,753	8,882	8,211	9,361	10,463	9,288	11,381
	Upper Bound	4,216	4,549	4,496	4,415	6,621	6,227	6,811	6,934	9,336	9,751	8,652	10,087	8,883	9,026	8,327	9,554	10,545	9,426	11,539
Bootstrap CI	Lower Bound	4,162	4,482	4,427	4,346	6,414	6,104	6,655	6,796	9,187	9,543	8,564	9,947	8,756	8,881	8,205	9,279	10,447	9,289	11,364
	Upper Bound	4,221	4,550	4,508	4,421	6,626	6,232	6,827	6,953	9,373	9,780	8,657	10,108	8,888	9,039	8,343	9,623	10,573	9,429	11,578
Optimum Volume		3,611	3,899	3,880	3,815	5,639	5,286	5,800	5,990	8,254	8,393	7,494	8,635	7,630	7,841	7,273	8,098	9,151	8,155	10,004
Wald CI	Lower Bound	3,598	3,880	3,861	3,793	5,593	5,252	5,760	5,959	8,211	8,330	7,463	8,598	7,589	7,791	7,223	8,056	9,118	8,082	9,950
	Upper Bound	3,624	3,918	3,898	3,837	5,686	5,321	5,841	6,021	8,298	8,457	7,526	8,673	7,671	7,891	7,324	8,140	9,184	8,228	10,059
Bootstrap CI	Lower Bound	3,598	3,881	3,863	3,797	5,594	5,257	5,761	5,958	8,221	8,338	7,468	8,596	7,594	7,801	7,235	8,024	9,123	8,102	9,962
	Upper Bound	3,625	3,919	3,898	3,835	5,696	5,319	5,846	6,027	8,293	8,448	7,521	8,676	7,669	7,881	7,316	8,175	9,179	8,213	10,050

The formulation of the optimum volume and its correspondence with conventional capacity (i.e., $q_{opt} = \beta(1/\alpha)^{1/\alpha}$ and $c \approx q_{opt}$ thus $c \approx \beta(1/\alpha)^{1/\alpha}$) implies that a relationship exists between the shape and scale parameters of the capacity distribution function for any capacity value. Thus, different combinations of the shape and the scale parameters may lead to the same capacity value. Since the relationship between the shape and scale parameter is true for any capacity value, a set of indifference curves (one for each capacity value) was created.

Given a conventional capacity value, e.g., the design capacity obtained from the HCM (10), and a reasonable assumption for the shape parameter, the scale parameter can be calculated and the capacity distribution function can be estimated. To assume a reasonable value for the shape parameter, its confidence interval was calculated by estimating another equation with regressing the scale parameters against the conventional capacity values. As shown on Figure 4.5, once the equation for the line was estimated, the two equations (i.e., $c = 0.8717\beta - 86.717$ and $c \approx \beta(1/\alpha)^{1/\alpha}$) were set equal to each other. Next, by taking a derivative with respect to the scale parameter and solving for the shape parameter, an average value of $\alpha = 22.7$ was estimated for the shape parameter. By following the same steps for the lower and upper confidence limits of the slope of the regressed line, the 95% confidence limits for the shape parameter was calculated as [15.78, 35.18].

Figure 4.5 Scale parameters (β) of the Weibull distribution against the conventional capacity values.

After estimating the confidence interval for the shape parameter, as shown in Figure 4.6, the indifference curves were drawn for the shape parameters located within the confidence limits. For standard conditions, practitioners may assume a shape parameter of $\alpha = 22$ (according to the average value given in Table 4.5) and estimate the corresponding scale parameter. However, given the fact that the shape parameter is proportional to the variance of the capacity distribution function, in case it is believed that the capacity variance is lower or higher than for standard conditions, a smaller or greater shape parameter may be selected. For example, it seems reasonable to assume a greater shape parameter if control strategies such as variable speed limits that reduce the capacity variance (cf. Geistefeldt, 2011) are implemented in the segment under investigation. This method allows practitioners to estimate a capacity distribution function for segments whose conventional capacity has already been estimated or can be taken from the guidelines in case that an empirical estimation of the capacity distribution function is not feasible.

Figure 4.6 Indifference curves for different capacity values.

4.5. Applying the SFI to Modify the SDRMS Ramp Metering Algorithm

This section briefly discusses the results of applying the SFI to modify the SDRMS algorithm. In previous chapter, it was hypothesized that using the SFI for demand management could possibly lead to better performance of the entire network. To test this hypothesis, as the first step, different capacity distribution functions were estimated for different ramp volumes. Thus, by trial and error, four different ramp volume categories (i.e., < 331 veh/h, 331-435veh/h, 435-518 veh/h, 518-538veh/h) whose corresponding capacity distribution functions were statistically different from one another were selected for analysis. In fact, the significance level of the log-rank test for the capacity distribution functions estimated based on the upstream mainline volume and occupancy

was 0.0019 and 0.0001. This suggests that, in both cases, the capacity distribution functions estimated for different ramp volume categories were significantly different from each other.

In addition to the non-parametric capacity distribution functions, different parametric capacity distribution functions were also fitted to the data. For the capacity distribution functions estimated based on the upstream mainline volume and occupancy, the logistic and normal distribution types provided the best fit to the observed data. Figure 4.7 and Figure 4.8 show the capacity distribution functions estimated for different ramp categories based on the upstream mainline volume and mainline occupancy.

Figure 4.7 Capacity distribution functions estimated for different ramp volume categories based on upstream mainline volume.

Figure 4.8 Capacity distribution functions estimated for different ramp volume categories based on upstream mainline occupancy.

After estimation of the capacity distribution functions for different ramp volume categories, the mean values of the four ramp volume categories were calculated. Next, capacity distribution functions corresponding to desired ramp volumes (located between the ramp volume categories) were estimated using the interpolation technique. To estimate the capacity distribution functions corresponding to desired ramp volumes located outside the ramp volume categories, the extrapolation technique was used. Once the capacity distribution functions for individual ramp volumes were estimated, their corresponding SFI's, optimum volumes, and optimum occupancies were estimated as well. Figure 4.9 and Figure 4.10 show the capacity distribution functions and the SFI's developed for individual ramp volumes based on the upstream mainline volume and mainline occupancy.

Figure 4.9 Capacity distribution functions and the SFI's developed for individual ramp volumes based on the upstream mainline volume.

Figure 4.10 Capacity distribution functions and the SFI's developed for individual ramp volumes based on the upstream occupancy volume.

Table 4.6 shows the upstream mainline optimum volumes and occupancies that have been estimated for different onramp volumes based on SFI's. As required by the SDRMS, the upstream volumes were stated in terms of vehicles per lane per 59.1 seconds and the total onramp volumes were stated in terms of vehicles per minute⁸. However, it is important to note that the optimum volumes shown in Table 4.6, weren't ready to be coded into SDRMS logic. As seen in Table 3.2, the difference between upstream volumes (and occupancies) was a constant increment named "delta volume" in the SDRMS algorithm.

Table 4.6 Optimum volumes and occupancies for different ramp volumes.

Rate Code	Upstream Occupancy (percent)	Upstream Volume (veh/lane/59.1seconds)	Total Onramp Volume (veh/min)
1	12.99	28.016	10.35
2	13.03	28.082	10
3	13.08	28.146	9.65
4	13.13	28.214	9.3
5	13.18	28.283	8.95
6	13.23	28.454	8.6
7	13.31	28.747	8.25
8	13.39	29.045	7.9
9	13.45	29.161	7.55
10	13.49	29.186	7.2
11	13.53	29.213	6.85
12	13.57	29.241	6.5
13	13.61	29.264	6.15
14	13.66	29.267	5.8
15	13.87	29.282	5.45

To use the estimated upstream mainline optimum volumes and occupancies in the SDRMS algorithm, they had to be slightly modified to make the increments between them a constant number. Thus, by subtracting the minimum upstream volume from the maximum upstream volume and dividing the result by the difference between their corresponding rate codes, the value of delta volume was calculated (i.e., $(29.282-28.016)/14=0.09$). Next, to calculate the upstream optimum

⁸ Instead of vehicles per hour

volume corresponding to any rate code, the rate code's number minus one was multiplied by the delta volume and this result was added to the minimum upstream volume. For example, for rate code 10, the mainline upstream volume was calculated as: $(10-1) * 0.09 + 28.016 = 28.83$. The same steps were followed for the occupancy as well. Table 4.7 shows the modified SDRMS algorithm that was coded into VISSIM for the section under investigation.

Table 4.7 Modified SDRMS algorithm for the section under study.

Rate Code	Upstream Occupancy (percent)	Upstream Volume (veh/ln/59.1seconds)	Total Onramp Volume (veh/min)
1	12.99	28.016	10.35
2	13.05	28.107	10
3	13.11	28.197	9.65
4	13.18	28.288	9.3
5	13.24	28.378	8.95
6	13.30	28.468	8.6
7	13.37	28.559	8.25
8	13.43	28.649	7.9
9	13.49	28.740	7.55
10	13.55	28.830	7.2
11	13.62	28.921	6.85
12	13.68	29.011	6.5
13	13.74	29.102	6.15
14	13.81	29.192	5.8
15	13.87	29.282	5.45

To use the above table to modify the SDRMS algorithm, the VISSIM model was calibrated first. Thus, the current SDRMS algorithm (i.e., Table 3.2) was coded in VISSIM and the car following and lane changing parameters were changed to minimize the difference between simulated and real-world volumes and speeds. To model the ramp and merge sections, Wiedemann 74 was selected as the car following model since it provided a more realistic representation of the traffic flow. The calibrated car following and lane changing parameters are shown in Figure A.1 and Figure A.2.

As mentioned in previous chapter, the GEH statistic was used to calibrate the traffic volumes. Thus, for each simulation hour a unique GEH statistic was calculated. From the total of 24 simulation hours, 23 hours had a GEH value less than 5. This suggests that 95.8 percent of all cases had a GEH value above 5, which was well above the 85 percent threshold. It is also interesting to note that the only hour with GEH value greater than 5, had a marginally acceptable GEH value of 5.42.

To calibrate the speeds, the RMSPE was used as an appropriate MOE. Thus, the Van Aerde model was fitted to the real-world data and the differences between the simulated and the observed speeds were measured and used to calculate the RMSPE. It was observed that an RMSPE of 3.7 percent existed between the real and simulated speeds. Since this value was less than 5 percent, calibration accuracy was considered acceptable. Figure 4.11 shows the simulated and real-world speed-flow scatterplots for the section under study.

Figure 4.11 Comparison of speed-flow scatterplots between observed and simulated data.

Once the VISSIM was calibrated, the random seed was changed and the VISSIM was run for 30 times, and the model outputs (under the current SDRMS algorithm) were saved. Next, the modified SDRMS algorithm (i.e., Table 4.7) was coded into VISSIM and, once more, the model was run for 30 times and the outputs were saved for comparison. Table 4.8 shows the vehicle network performance evaluation results before and after enhancement of the SDRMS algorithm. The table shows that all performance measures have improved after the modification. The 2.5 percent reduction in the average network delay was perhaps the most important and most significant of all. Thus, according to the results shown in Table 4.8, the null hypothesis was rejected in favor of the alternative hypothesis and it was concluded that the modified SDRMS algorithm performs better than the current SDRMS algorithm.

Table 4.8 Comparison of different network performance measures before and after enhancement of the SDRMS algorithm.

Scenario	Average Delay (seconds)	Average Speed (mph)	Total Travel Time (seconds)	Total Distance Traveled (miles)
Modified SDRMS Algorithm	92.31	46.68	40,369,180	523,062
Current SDRMS Algorithm	94.70	46.31	40,680,223	523,056

The above results have shown that the modified SDRMS algorithm provided a better network-wise performance compared to the current SDRMS algorithm. Nevertheless, these results cannot guarantee that metering the onramps as a function of the optimum volume of the downstream section will deliver the best possible solution. In fact, if solely performance of the freeway mainline (without the onramps) is of concern, then metering up to the optimum volume may optimize freeway performance. Since the optimum volume derived in the previous chapter provides the best compromise between probability of breakdown and the unused capacity of the section, it is a good indicator for the maximum reliable volume that can be traversed by freeway. On the other hand, if the performance of the entire freeway facility (i.e., freeway and the onramps) is of interest, then metering to the optimum volume may lead to a suboptimal solution (especially at higher onramp volumes) due to the increased onramp queue. The optimum volume, however, reveals an important piece of information in this case: the minimum acceptable probability of breakdown is the one corresponding to the optimum volume. This suggests if an overall downstream volume (i.e., sum of mainline and onramp volumes) less than the optimum volume is selected for metering, then the capacity of road is not used sufficiently and an extra delay is imposed to the onramp vehicles.

CHAPTER 5: CONCLUSION

In the absence of available funding for new roadway construction projects, transportation engineers must look to develop strategies to enhance the efficiency of the existing road network and therefore need sophisticated methods to assess freeway traffic performance. One emerging concept in this field is stochastic capacity, the idea that the capacity of a roadway is not a fixed value but varies, even under the similar prevailing conditions. The stochastic variability of freeway capacity has evolved to become a widely accepted concept in traffic engineering and although it has yet to be applied in practice, it has been proposed as a method to enhance the performance of ramp metering strategies.

Previous research conducted in different environments has suggested different distribution functions for capacity. To investigate the distribution type that best fits observations of flow in the U.S., in this research, different capacity distribution types were tested for 19 freeway bottleneck sections by applying the models for censored data. The results of this research showed that the Logistic and SEV distributions provided the best fit to the 5- and 15-minute data respectively.

Previously, different methods have been proposed and applied to estimate the capacity distribution function. In light of this emerging area of research, however, there has been little work to identify freeway performance measurements that reflect the random nature of capacity. As an initial step to address this problem, the sustained flow index (SFI) was introduced to approximate the degree of trade-off between the probability of survival and the unused capacity. The SFI was defined as the product of the traffic volume and the value of the survival function at this volume; where survival in this content was defined as the non-congested traffic operation. Thus, the SFI is the theoretical average volume that can be sustained in non-congested traffic given the variability of capacity.

Despite traffic flow assessment procedures given in guidelines like the HCM are still mostly based on conventional (deterministic) design capacities, only limited research has been carried out on how to select a single, representative, capacity value from the estimated capacity distribution function. At the most fundamental level, the SFI is a tool that can be applied to more accurately and reliably calculate the capacity of a freeway under constantly changing conditions. Thus, the SFI was used to estimate the optimum volume and pick a single capacity value from the capacity distribution function of the roadway. Also, to verify the reasonableness of the results obtained by maximizing the SFI, in this study, optimum volumes were compared with the conventional capacities based on data collected for 19 U.S. freeway bottleneck sections. Despite the application of two completely different methodologies, the obtained average differences between the estimated capacity values were small (nearly 1.2 percent), and a very strong correlation between the capacity estimates existed. The empirical results revealed that the optimum volume estimated in 5- and 15-minute intervals were roughly equivalent to a 5 and 4 percent probability of breakdown respectively. Therefore, the optimum volume obtained by maximizing the SFI did not correspond with the capacity according to the HCM, which proposes a 15 percent probability of breakdown for the capacity estimation from field data. This study also showed that the probability of breakdown at the optimum volume solely depends on the shape parameter of the Weibull-type capacity distribution and it can be expected to be relatively constant for freeways with different numbers of lanes.

To investigate the stability of the estimated optimum volumes, a new procedure was developed to calculate the 95% confidence intervals with the Wald and Bootstrap methods. The results obtained by the two methods were very similar and suggested that the confidence intervals of the optimum volumes were relatively small. Hence, it was concluded that the optimum volume

is a fairly stable indicator of freeway capacity. Furthermore, based on the correspondence between the optimum volume and the conventional capacity, a practical method to estimate the entire capacity distribution function given a conventional capacity estimate was developed.

Based on the presented findings, it is suggested that the optimum volume obtained by maximizing the SFI may be considered as a preferred approach to select a single capacity value from the capacity distribution function. Compared with the conventional approach of estimating the capacity in the speed-flow diagram, the optimum volume solely relies on uncongested flow observations and therefore best represents the pre-breakdown capacity. The application of the proposed approach is simple as the optimum volume can be calculated from the parameters of the capacity distribution function by a single equation.

Finally, the SFI was applied to modify the San Diego Ramp Metering System (SDRMS) that is currently used in a section of freeway located in San Diego, California. To do this, different capacity distribution functions and SFI's were developed for different ramp volumes, and their corresponding optimum volumes and occupancies were estimated and used to modify the SDRMS algorithm. By calibrating a VISSIM model to reflect the real-world volumes and speeds, the current and modified SDRMS algorithms were coded into VISSIM and compared with each other in terms of different network performance measures (such as average delay over the entire network, average speed over the entire network, total distance travelled over the entire network, and total travel time over the entire network). It was observed that using the modified SDRMS algorithm for demand management resulted in better performance of the entire network.

One advantage of using the SFI as a benchmark to define freeway capacity is its flexibility since it applies the capacity distribution function, which reflects different characteristics of the section, to define the acceptable probability of breakdown. This suggests that different factors

which affect the capacity distribution function also influence the SFI and, as a result, the optimum volume. For instance, previous research by Geistefeldt and Brilon (2009) has suggested that daylight/darkness, weather condition, accidents and incidents, share of heavy vehicles, commuter/recreational traffic, and work zones may influence capacity distribution function of a freeway. As such, these factors will also influence the SFI and the optimum volumes as a single capacity estimate. Therefore, it is suggested to apply the SFI in future research to estimate optimum volumes under different prevailing and environmental conditions.

Another application of the SFI could be system optimal traffic assignment. If the SFI is estimated for a number of links in the network, then vehicles from the links operating above the optimum volume can be routed to the links with lower volumes. Since the SFI is a quantitative measure, the overall SFI of the network will then be the sum of the SFIs of the individual links. If the SFI for the overall network is maximized, the system is running at its optimum efficiency. Application of SFI in vehicle routing becomes increasingly important during emergency evacuation in which demand is significantly higher than the capacity and the authorities have the power to regulate the traffic.

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APPENDIX: SUPPLEMENTAL RESULTS

Table A.1 Calibration results for SEV distribution (5-minute intervals).

Section	SEV Distribution			
	Location Parameter	Scale Parameter	Expected value	-2Loglikelihood
1	4,159.58	178.58	4,056.50	14,450.43
2	4,490.40	191.64	4,379.78	8,473.46
3	4,441.19	178.07	4,338.40	8,137.60
4	4,366.91	175.31	4,265.72	6,001.31
5	6,449.51	259.82	6,299.53	2,596.85
6	6,122.35	277.75	5,962.03	6,111.47
7	6,686.26	287.61	6,520.25	4,299.87
8	6,828.54	264.24	6,676.02	6,164.56
9	9,274.03	302.65	9,099.33	5,222.91
10	9,583.79	375.04	9,367.32	3,728.73
11	8,583.63	343.99	8,385.08	12,150.40
12	9,981.31	434.69	9,730.40	11,503.26
13	8,774.35	364.48	8,563.96	7,071.52
14	8,917.71	336.03	8,723.75	5,393.47
15	8,244.27	296.11	8,073.36	4,455.90
16	9,405.45	433.86	9,155.02	8,747.57
17	10,491.18	422.02	10,247.59	17,048.05
18	9,345.92	377.90	9,127.79	3,805.81
19	11,389.25	431.59	11,140.13	7,811.34

Table A.2 Calibration results for Weibull distribution (5-minute intervals).

Section	Weibull Distribution			
	Location Parameter	Scale Parameter	Expected value	-2Loglikelihood
1	4,190.40	20.2	4,080.35	14,475.89
2	4,514.70	20.7	8,446.87	8,450.87
3	4,465.27	22.0	4,356.80	8,151.89
4	4,381.95	22.5	4,277.55	5,992.18
5	6,508.99	21.3	6,346.45	2,596.45
6	6,164.40	19.2	5,995.14	6,111.90
7	6,735.42	20.1	6,557.42	4,297.87
8	6,870.95	22.8	6,709.41	6,163.23
9	9,280.78	28.6	9,104.42	5,212.97
10	9,649.33	22.2	9,417.47	3,737.26
11	8,609.23	22.4	8,403.90	12,137.83
12	10,023.46	20.1	9,759.65	11,515.09
13	8,817.72	21.1	8,594.84	7,079.01
14	8,953.76	23.9	8,752.61	5,392.39
15	8,268.69	25.1	8,091.35	4,469.90
16	9,457.03	19.0	9,193.97	8,727.85
17	10,503.82	22.6	10,255.37	17,061.11
18	9,356.68	22.7	9,136.14	3,805.03
19	11,460.08	23.1	11,194.32	7,856.00

Table A.3 Calibration results for Logistic distribution (5-minute intervals).

Section	Logistic Distribution			
	Location Parameter	Scale Parameter	Expected value	-2Loglikelihood
1	4,114.66	167.02	4,114.66	14,453.76
2	4,427.99	175.53	4,427.99	8,448.36
3	4,382.64	161.66	4,382.64	8,117.82
4	4,303.39	156.96	4,303.39	5,982.19
5	6,396.12	247.52	6,396.12	2,594.61
6	6,047.60	256.97	6,047.60	6,105.42
7	6,594.70	264.75	6,594.70	4,290.64
8	6,737.47	239.98	6,737.47	6,142.99
9	9,133.83	257.67	9,133.83	5,193.17
10	9,497.23	352.47	9,497.23	3,728.73
11	8,453.97	307.16	8,453.97	12,116.58
12	9,794.83	387.27	9,794.83	11,463.99
13	8,671.98	340.23	8,671.98	7,079.45
14	8,817.88	306.76	8,817.88	5,382.13
15	8,155.17	268.70	8,155.17	4,461.65
16	9,187.11	376.41	9,187.11	8,701.79
17	10,299.48	361.72	10,299.48	16,991.77
18	9,203.09	331.86	9,203.09	3,809.57
19	11,289.29	402.66	11,289.29	7,822.08

Table A.4 Calibration results for Normal distribution (5-minute intervals).

Section	Normal Distribution			
	Location Parameter	Scale Parameter	Expected value	-2Loglikelihood
1	4,268.65	407.43	4,268.65	14,616.14
2	4,518.66	384.11	4,518.66	8,423.17
3	4,534.63	399.23	4,534.63	8,239.55
4	4,387.32	345.37	4,387.32	6,002.05
5	6,627.22	601.27	6,627.22	2,605.44
6	6,210.50	583.89	6,210.50	6,140.21
7	6,826.88	641.38	6,826.88	4,321.81
8	6,981.78	592.00	6,981.78	6,188.61
9	9,288.44	584.48	9,288.44	5,233.47
10	9,808.24	863.69	9,808.24	3,777.15
11	8,612.20	684.08	8,612.20	12,191.79
12	10,132.34	939.97	10,132.34	11,633.28
13	8,893.96	792.43	8,893.96	7,149.62
14	9,970.94	1,321.22	9,970.94	8,446.87
15	8,345.35	645.72	8,345.35	4,551.70
16	9,642.61	961.47	9,642.61	8,747.76
17	10,553.80	862.89	10,553.80	17,312.89
18	9,287.95	691.17	9,287.95	3,836.14
19	11,722.43	1,047.65	11,722.43	8,000.99

Table A.5 Calibration results for Log-normal distribution (5-minute intervals).

Section	Log-normal Distribution			
	Location Parameter	Scale Parameter	Expected value	-2Loglikelihood
1	8.38	0.120	4,407.07	14,721.32
2	8.43	0.101	4,599.36	8,431.42
3	8.44	0.111	4,671.54	8,331.06
4	8.40	0.092	4,456.23	6,026.01
5	8.82	0.113	6,827.81	2,617.11
6	8.75	0.115	6,365.46	6,170.64
7	8.85	0.118	7,045.00	4,351.62
8	8.87	0.106	7,181.44	6,228.96
9	9.14	0.072	9,376.80	5,254.21
10	9.21	0.110	10,084.67	3,812.27
11	9.07	0.094	8,749.79	12,264.04
12	9.25	0.118	10,471.46	11,768.68
13	9.11	0.109	9,109.76	7,207.19
14	9.12	0.089	9,130.60	5,436.71
15	9.04	0.093	8,489.40	4,598.77
16	9.21	0.129	10,056.78	8,786.56
17	9.28	0.099	10,764.92	17,505.50
18	9.14	0.084	9,370.47	3,855.63
19	9.40	0.115	12,117.47	8,114.24

Table A.6 Calibration results for LEV distribution (5-minute intervals).

Section	LEV Distribution			
	Location Parameter	Scale Parameter	Expected value	-2Loglikelihood
1	4,377.95	714.32	4,790.26	14,851.02
2	4,527.83	600.96	4,874.71	8,462.00
3	4,695.56	754.25	5,130.93	8,451.95
4	4,432.03	576.43	4,764.75	6,087.13
5	6,795.46	1,055.91	7,404.95	2,634.52
6	6,279.47	968.80	6,838.68	6,218.14
7	7,012.65	1,139.27	7,670.25	4,390.98
8	7,221.57	1,102.26	7,857.81	6,290.66
9	9,375.03	992.03	9,947.64	5,325.05
10	10,100.02	1,600.15	11,023.65	3,860.92
11	8,718.31	1,170.79	9,394.11	12,431.99
12	10,540.29	1,764.62	11,558.85	11,929.67
13	9,083.28	1,382.25	9,881.14	7,298.14
14	9,137.00	1,200.05	9,829.68	5,504.28
15	8,507.47	1,176.60	9,186.62	4,688.74
16	9,970.48	1,718.91	10,962.66	8,827.17
17	10,791.60	1,569.10	11,697.32	17,835.71
18	9,240.36	1,071.94	9,859.10	3,912.20
19	12,193.80	2,069.10	13,388.12	8,239.57

Table A.7 Calibration results for Fréchet distribution (5-minute intervals).

Section	Fréchet Distribution			
	Location Parameter	Scale Parameter	Expected value	-2Loglikelihood
1	8.43	0.219	5,420.38	14,999.41
2	8.44	0.163	5,194.99	8,495.99
3	8.51	0.224	5,900.06	8,596.28
4	8.42	0.164	5,113.40	6,142.94
5	8.87	0.208	8,317.19	2,655.88
6	8.78	0.200	7,541.03	6,271.90
7	8.90	0.223	8,762.02	4,442.36
8	8.93	0.209	8,899.22	6,362.34
9	9.16	0.126	10,368.59	5,363.78
10	9.27	0.218	12,540.39	3,917.49
11	9.10	0.173	10,178.39	12,589.38
12	9.33	0.239	13,584.95	12,138.28
13	9.15	0.202	11,013.12	7,393.78
14	9.15	0.171	10,666.34	5,568.06
15	9.08	0.179	9,992.14	4,759.85
16	9.26	0.240	12,786.94	8,888.68
17	9.32	0.191	12,878.01	18,120.88
18	9.14	0.137	10,271.13	3,947.20
19	9.47	0.243	15,700.84	8,395.09

Table A.8 Calibration results for SEV distribution (15-minute intervals).

Section	SEV Distribution			
	Location Parameter	Scale Parameter	Expected value	-2Loglikelihood
1	3,829.24	144.49	3,745.84	10,790.32
2	4,148.20	152.92	4,059.93	6,787.35
3	4,136.54	138.78	4,056.43	5,726.16
4	4,065.49	136.56	3,986.67	4,085.65
5	5,982.61	228.01	5,851.00	2,046.43
6	5,646.13	206.45	5,526.96	4,474.34
7	6,224.61	241.16	6,085.41	2,976.74
8	6,405.14	219.24	6,278.60	4,681.55
9	8,731.11	223.92	8,601.86	4,135.54
10	8,987.61	309.10	8,809.19	2,847.03
11	8,138.20	309.97	7,959.28	7,185.80
12	9,307.55	389.44	9,082.76	8,587.00
13	8,272.94	345.29	8,073.63	5,559.75
14	8,549.45	286.43	8,384.11	3,813.01
15	7,765.98	260.29	7,615.74	3,746.53
16	8,640.85	352.10	8,437.61	7,090.23
17	9,823.26	402.50	9,590.93	9,257.96
18	8,640.29	331.46	8,448.97	3,206.62
19	10,772.00	380.41	10,552.42	5,622.41

Table A.9 Calibration results for Weibull distribution (15-minute intervals).

Section	Weibull Distribution			
	Location Parameter	Scale Parameter	Expected value	-2Loglikelihood
1	3837.76	24.13	3,752.24	10,818.17
2	4152.23	25.22	4,063.48	6,778.33
3	4145.91	27.03	4,062.81	5,770.51
4	4069.42	27.80	3,989.98	4,093.28
5	6017.25	23.11	5,877.65	2,053.18
6	5652.01	25.47	5,532.22	4,477.26
7	6237.63	23.32	6,094.17	2,986.09
8	6420.76	26.72	6,290.66	4,690.64
9	8738.18	36.68	8,606.94	4,147.49
10	8992.82	27.01	8,812.41	2,861.05
11	8147.06	24.05	7,964.96	7,196.14
12	9313.08	21.79	9,084.98	8,600.67
13	8274.96	21.82	8,072.55	5,574.15
14	8572.13	27.39	8,402.43	3,817.65
15	7768.03	28.05	7,617.64	3,753.37
16	8705.55	21.35	8,488.30	7,108.22
17	9832.88	22.17	9,595.91	9,297.95
18	8632.89	25.05	8,447.09	3,201.41
19	10802.73	25.35	10,572.84	5,671.48

Table A.10 Calibration results for Logistic distribution (15-minute intervals).

Section	Logistic Distribution			
	Location Parameter	Scale Parameter	Expected value	-2Loglikelihood
1	3,789.66	132.82	3,789.66	10,846.95
2	4,086.32	132.15	4,086.32	6,773.46
3	4,098.99	126.02	4,098.99	5,766.94
4	4,012.61	117.17	4,012.61	4,087.14
5	5,942.99	218.00	5,942.99	2,052.19
6	5,570.60	178.88	5,570.60	4,478.04
7	6,138.68	215.45	6,138.68	2,983.98
8	6,322.89	192.16	6,322.89	4,667.32
9	8,659.40	198.03	8,659.40	4,151.02
10	8,870.07	263.28	8,870.07	2,863.12
11	8,024.95	274.66	8,024.95	7,207.15
12	9,123.51	335.13	9,123.51	8,593.66
13	8,132.28	302.97	8,132.28	5,589.65
14	8,466.41	259.49	8,466.41	3,809.12
15	7,668.09	221.97	7,668.09	3,766.11
16	8,583.63	337.64	8,583.63	7,103.13
17	9,673.33	353.14	9,673.33	9,305.67
18	8,461.19	250.18	8,461.19	3,194.26
19	10,677.35	345.86	10,677.35	5,663.14

Table A.11 Calibration results for Normal distribution (15-minute intervals).

Section	Normal Distribution			
	Location Parameter	Scale Parameter	Expected value	-2Loglikelihood
1	3,853.83	296.56	3,853.83	11,008.62
2	4,118.71	269.01	4,118.71	6,800.20
3	4,195.89	323.38	4,195.89	5,982.27
4	4,343.98	483.85	4,343.98	4,328.16
5	6,111.16	522.02	6,111.16	2,081.78
6	5,618.43	375.69	5,618.43	4,527.29
7	6,263.04	500.45	6,263.04	3,046.99
8	7,098.14	891.80	7,098.14	4,961.11
9	8,756.01	468.70	8,756.01	4,259.95
10	8,999.23	637.26	8,999.23	2,955.77
11	8,103.36	592.35	8,103.36	7,305.76
12	9,296.05	753.33	9,296.05	8,745.91
13	8,234.76	665.82	8,234.76	5,693.75
14	9,415.00	1,218.42	9,415.00	4,009.61
15	7,706.41	475.35	7,706.41	3,829.68
16	8,850.42	806.02	8,850.42	7,178.04
17	9,867.86	831.01	9,867.86	9,541.29
18	8,478.36	484.01	8,478.36	3,215.03
19	10,937.73	917.11	10,937.73	5,866.68

Table A.12 Calibration results for Log-normal distribution (15-minute intervals).

Section	Log-Normal Distribution			
	Location Parameter	Scale Parameter	Expected value	-2Loglikelihood
1	8.27	0.090	3,906.75	11,097.77
2	8.33	0.072	4,144.07	6,820.20
3	8.36	0.094	4,272.69	6,087.29
4	8.32	0.075	4,108.82	4,211.06
5	8.73	0.104	6,247.67	2,098.54
6	8.64	0.075	5,657.25	4,553.11
7	8.76	0.096	6,377.96	3,087.83
8	8.79	0.085	6,567.12	4,811.25
9	9.08	0.060	8,810.00	4,298.74
10	9.11	0.085	9,112.47	3,004.28
11	9.01	0.084	8,180.34	7,367.70
12	9.15	0.097	9,460.92	8,850.94
13	9.03	0.095	8,351.16	5,765.65
14	9.07	0.079	8,713.24	3,880.75
15	8.95	0.068	7,742.83	3,857.47
16	9.11	0.112	9,075.10	7,222.69
17	9.21	0.101	10,043.15	9,667.33
18	9.05	0.061	8,498.34	3,228.60
19	9.32	0.105	11,166.63	5,969.77

Table A.13 Calibration results for LEV distribution (15-minute intervals).

Section	LEV Distribution			
	Location Parameter	Scale Parameter	Expected value	-2Loglikelihood
1	3,889.40	501.04	4,178.61	11,293.33
2	4,087.58	400.29	4,318.64	6,903.16
3	4,293.74	625.47	4,654.77	6,263.22
4	4,116.42	485.61	4,396.73	4,326.50
5	6,226.17	917.97	6,756.03	2,125.49
6	5,592.54	597.83	5,937.62	4,639.07
7	6,402.84	928.59	6,938.84	3,159.91
8	6,635.16	901.74	7,155.66	4,926.45
9	8,832.21	879.07	9,339.62	4,440.36
10	9,122.54	1,236.93	9,836.51	3,103.18
11	8,121.37	988.30	8,691.83	7,534.87
12	9,509.49	1,368.85	10,299.61	9,042.36
13	8,344.31	1,160.36	9,014.09	5,908.53
14	8,756.68	1,105.08	9,394.55	3,958.87
15	7,643.62	752.51	8,077.98	3,956.35
16	8,994.88	1,380.78	9,791.89	7,288.42
17	10,014.52	1,459.27	10,856.84	9,875.17
18	8,356.40	704.29	8,762.93	3,303.15
19	11,132.87	1,760.69	12,149.17	6,112.29

Table A.14 Calibration results for Fréchet distribution (15-minute intervals).

Section	Fréchet Distribution			
	Location Parameter	Scale Parameter	Expected value	-2Loglikelihood
1	8.29	0.159	4,445.18	11,425.45
2	8.32	0.111	4,433.55	6,945.44
3	8.40	0.193	5,121.53	6,402.51
4	8.34	0.148	4,659.57	4,397.80
5	8.77	0.191	7,408.15	2,149.64
6	8.64	0.125	6,145.60	4,685.32
7	8.80	0.192	7,637.26	3,219.82
8	8.83	0.183	7,863.39	5,020.37
9	9.10	0.119	9,685.07	4,501.37
10	9.14	0.175	10,602.40	3,169.68
11	9.02	0.150	9,190.51	7,650.03
12	9.19	0.190	11,349.61	9,201.01
13	9.06	0.179	9,780.73	6,019.08
14	9.10	0.166	10,144.82	4,020.24
15	8.95	0.112	8,284.67	3,998.58
16	9.14	0.201	10,832.71	7,358.10
17	9.24	0.186	11,795.22	10,033.52
18	9.03	0.094	8,906.67	3,333.02
19	9.35	0.212	13,525.96	6,242.58

Table A.15 RMSPE between Weibull and Logistic optimum volumes (5-minute intervals).

5-Minute Optimum Volumes				
Section	Weibull Distribution	Logistic Distribution	Percentage Error	RMSPE
1	3,611.3	3,609.3	0.0005	0.0017
2	3,899.2	3,893.1	0.0016	
3	3,879.7	3,875.9	0.0010	
4	3,815.0	3,809.4	0.0015	
5	5,638.6	5,633.7	0.0009	
6	5,285.7	5,283.5	0.0004	
7	5,799.4	5,790.3	0.0016	
8	5,990.2	5,975.8	0.0024	
9	8,253.3	8,248.9	0.0005	
10	8,392.1	8,394.9	-0.0003	
11	7,494.3	7,485.9	0.0011	
12	8,631.8	8,611.4	0.0024	
13	7,629.4	7,629.3	0.0000	
14	7,841.0	7,836.1	0.0009	
15	7,272.7	7,278.8	-0.0008	
16	8,098.2	8,052.2	0.0057	
17	9,150.5	9,145.7	0.0005	
18	8,154.8	8,154.4	0.0001	
19	10,013.6	10,012.0	0.0002	

Table A.16 RMSPE between Weibull and SEV optimum volumes (15-minute intervals).

15-Minute Optimum Volumes				
Section	Weibull Distribution	SEV Distribution	Percentage Error	RMSPE
1	3,363.4	3,374.0	-0.0031	0.0032
2	3,653.5	3,662.5	-0.0024	
3	3,669.8	3,681.6	-0.0032	
4	3,610.7	3,618.0	-0.0021	
5	5,252.6	5,266.7	-0.0027	
6	4,977.3	4,988.6	-0.0023	
7	5,449.7	5,471.7	-0.0040	
8	5,677.9	5,691.2	-0.0023	
9	7,920.8	7,932.3	-0.0015	
10	7,959.6	7,982.6	-0.0028	
11	7,138.0	7,168.9	-0.0044	
12	8,084.9	8,124.5	-0.0049	
13	7,184.6	7,223.0	-0.0053	
14	7,596.4	7,610.0	-0.0018	
15	6,897.5	6,912.4	-0.0022	
16	7,542.7	7,561.0	-0.0024	
17	8,550.4	8,591.3	-0.0048	
18	7,591.3	7,602.0	-0.0014	
19	9,509.5	9,546.1	-0.0024	

Driving Behavior

No.: 6 Name: Merge Sections

Following Lane Change Lateral Signal Control

Look ahead distance

min.: 0.00 ft

max.: 200.00 ft

2 Observed vehicles

Look back distance

min.: 0.00 ft

max.: 200.00 ft

Temporary lack of attention

Duration: 0.00 s

Probability: 0.00 %

☒ Smooth closeup behavior

☐ Standstill distance for static obstacles: 1.64 ft

Car following model

Wiedemann 74

Model parameters

Average standstill distance: 6.56

Additive part of safety distance: 2.00

Multiplic. part of safety distance: 3.00

OK Cancel

Figure A.1 Calibrated car following parameters.

Driving Behavior

No.: 6 Name: Merge Sections

Following Lane Change Lateral Signal Control

General behavior: Free lane selection

Necessary lane change (route)

	Own	Trailing vehicle
Maximum deceleration:	-20.00 ft/s ²	-20.00 ft/s ²
- 1 ft/s ² per distance:	1000.00 ft	1000.00 ft
Accepted deceleration:	-10.00 ft/s ²	-10.00 ft/s ²

Waiting time before diffusion: 600.00 s

Min. headway (front/rear): 1.64 ft

To slower lane if collision time is above. 11.00 s

Safety distance reduction factor: 0.20

Maximum deceleration for cooperative braking: -20.00 ft/s²

☒ Cooperative lane change

Maximum speed difference: 10.00 mph

Maximum collision time: 10.00 s

☐ Lateral correction of rear end position

Maximum speed: 1.86 mph

Active during time period from 1.00 s until 10.00 s after lane change start

☐ Overtake reduced speed areas

☒ Advanced merging

☐ Consider subsequent static routing decisions

OK Cancel

Figure A.2 Calibrated lane changing parameters.

The current SDRMS algorithm as coded in VISSIM

PROGRAM Ramp_Meter;

/* Constant Definition */

CONST

DF1 = 201, /* Freeway Detectors */
DF2 = 202,
DF3 = 203,
DF4 = 204, /* new for added lane */

PCW1 = 26, /* 1-minute Period */

Ramp1SignalHead = 1, /* Ramp Signal Heads */
Ramp2SignalHead = 2,

Ramp1Green = 2,
Ramp2Green = 2,

GreenOffsetTime = 3,

DeltaCyclesPerMinute = 0.35, /* Updated from table */
DeltaVolume = 0.525333333, /* Updated from table */
DeltaOccupancy = 2.8, /* Updated from table */

MinOccupancy = 162.0,
MinVolume = 28.72916667, /* Updated from table */

MaxOccupancy = 201.5,
MaxVolume = 36.08383333,

MinCyclesPerMinute = 5.45, /* Updated from table */
MaxCyclesPerMinute = 10.35, /* Updated from table */

BeginOccupancy1 = 162.0,
EndOccupancy1 = 162.0,
BeginVolume1 = 28.72916667,
EndVolume1 = 28.72916667,

StartTime1 = 18000,
EndTime1 = 39600,
StartTime2 = 50400,
EndTime2 = 68400;

SUBROUTINE Compute_Expressions;

/****** Lane Data *****/

/*** 1 second interval ***/

six_sec:=six_sec+1;
SimuTime:=SimuTime+1;

/*** Lane 1 ***/

FlowRate1:=presence(DF1); /* flow rate determined by sampling of detector presence */
FlowRate61:=FlowRate61+FlowRate1;
Occupancy1:=30*occup_rate(DF1); /* occupancy determined by sampling of detector occupancy rate */
Occupancy61:=Occupancy61+Occupancy1; /* (multiplied by 30 to match Caltrans controller sampling rate */

/*** Lane 2 ***/

```

FlowRate2:=presence(DF2);
FlowRate62:=FlowRate62+FlowRate2;
Occupancy2:=30*occup_rate(DF2);
Occupancy62:=Occupancy62+Occupancy2;

/** Lane 3 */
FlowRate3:=presence(DF3);
FlowRate63:=FlowRate63+FlowRate3;
Occupancy3:=30*occup_rate(DF3);
Occupancy63:=Occupancy63+Occupancy3;

/** Lane 4 new for added lane */
FlowRate4:=presence(DF4);
FlowRate64:=FlowRate64+FlowRate4;
Occupancy4:=30*occup_rate(DF4);
Occupancy64:=Occupancy64+Occupancy4;

/** 6 second interval */
If six_sec=6 THEN
    six_sec:=0;

    /** Total Freeway Average */
    FlowRate6:=(FlowRate61+FlowRate62+FlowRate63+FlowRate64)/4;
    Occupancy6:=(Occupancy61+Occupancy62+Occupancy63+Occupancy64)/4; /* updated for 4 lanes*/
    FlowRate61:=0;
    FlowRate62:=0;
    FlowRate63:=0;
    FlowRate64:=0; /* updated for 4 lanes*/

    Occupancy61:=0;
    Occupancy62:=0;
    Occupancy63:=0;
    Occupancy64:=0; /* updated for 4 lanes*/

    /** Average Running Values */

    NRA_Flow_1:=FlowRate6+((256-PCW1)*(NRA_Flow_1/256));
    NRA_Occ_1:=Occupancy6+((256-PCW1)*(NRA_Occ_1/256));

    /** Display in VISSIM */
    record_value(201,NRA_Flow_1);
    record_value(203, NRA_Occ_1);

    /** BEGIN & END LOGIC */
    IF RateCode THEN
        IF (NRA_Flow_1<EndVolume1) AND (NRA_Occ_1<EndOccupancy1) THEN
            RateCode:=0;
        END;
    ELSE
        IF (NRA_Flow_1>BeginVolume1) OR (NRA_Occ_1>BeginOccupancy1) THEN
            RateCode:=1;
        END;
    END;

    /** Maximum Control */
    IF (NRA_Flow_1>MaxVolume) THEN
        NRA_Flow_1:=MaxVolume;
    END;

    IF (NRA_Occ_1>MaxOccupancy) THEN
        NRA_Occ_1:=MaxOccupancy;
    END;

```

```

/** Metering Start and End Times */
IF (SimuTime < StartTime1) THEN
    RateCode:=0;
END;

IF (SimuTime >= EndTime2) THEN
    RateCode:=0;
END;

IF (SimuTime >= EndTime1) AND (SimuTime < StartTime2) THEN
    RateCode:=0;
END;

IF RateCode THEN
    /** Look-up Table VOLUME */
    IF (NRA_Flow_1<MinVolume) THEN
        VolumeRateCode:=1;
    ELSE

        IF (NRA_Flow_1<(MinVolume+DeltaVolume)) THEN
            VolumeRateCode:=2;
        ELSE

            IF (NRA_Flow_1<(MinVolume+(2*DeltaVolume))) THEN
                VolumeRateCode:=3;
            ELSE

                IF (NRA_Flow_1<(MinVolume+(3*DeltaVolume))) THEN
                    VolumeRateCode:=4;
                ELSE

                    IF (NRA_Flow_1<(MinVolume+(4*DeltaVolume))) THEN
                        VolumeRateCode:=5;
                    ELSE

                        IF (NRA_Flow_1<(MinVolume+(5*DeltaVolume))) THEN
                            VolumeRateCode:=6;
                        ELSE

                            IF (NRA_Flow_1<(MinVolume+(6*DeltaVolume))) THEN
                                VolumeRateCode:=7;
                            ELSE

                                IF (NRA_Flow_1<(MinVolume+(7*DeltaVolume))) THEN
                                    VolumeRateCode:=8;
                                ELSE

                                    IF (NRA_Flow_1<(MinVolume+(8*DeltaVolume))) THEN
                                        VolumeRateCode:=9;
                                    ELSE

                                        IF (NRA_Flow_1<(MinVolume+(9*DeltaVolume))) THEN
                                            VolumeRateCode:=10;
                                        ELSE

                                            IF (NRA_Flow_1<(MinVolume+(10*DeltaVolume))) THEN
                                                VolumeRateCode:=11;
                                            ELSE

                                                IF (NRA_Flow_1<(MinVolume+(11*DeltaVolume))) THEN

```



```

        VolumeRateCode:=12;
ELSE

IF (NRA_Flow_1<(MinVolume+(12*DeltaVolume))) THEN
    VolumeRateCode:=13;
ELSE

IF (NRA_Flow_1<(MinVolume+(13*DeltaVolume))) THEN
    VolumeRateCode:=14;
ELSE
    VolumeRateCode:=15;

END;
END;
END;
END;
END;
END;
END;
END;
END;
END;
END;
END;
END;
END;
END;

/**** Look-up Table OCCUPANCY ****/
IF (NRA_Occ_1<MinOccupancy) THEN
    OccupancyRateCode:=1;
ELSE

IF (NRA_Occ_1<(MinOccupancy+DeltaOccupancy)) THEN
    OccupancyRateCode:=2;
ELSE

IF (NRA_Occ_1<(MinOccupancy+(2*DeltaOccupancy))) THEN
    OccupancyRateCode:=3;
ELSE

IF (NRA_Occ_1<(MinOccupancy+(3*DeltaOccupancy))) THEN
    OccupancyRateCode:=4;
ELSE

IF (NRA_Occ_1<(MinOccupancy+(4*DeltaOccupancy))) THEN
    OccupancyRateCode:=5;
ELSE

IF (NRA_Occ_1<(MinOccupancy+(5*DeltaOccupancy))) THEN
    OccupancyRateCode:=6;
ELSE

IF (NRA_Occ_1<(MinOccupancy+(6*DeltaOccupancy))) THEN
    OccupancyRateCode:=7;
ELSE

IF (NRA_Occ_1<(MinOccupancy+(7*DeltaOccupancy))) THEN
    OccupancyRateCode:=8;
ELSE

IF (NRA_Occ_1<(MinOccupancy+(8*DeltaOccupancy))) THEN

```



```

/** Ramp 1 Control */
RedOK1 := t_green(Ramp1SignalHead) >= Ramp1Green;
GreenOK1 := (Phase2Timer+1) >= (CycleLength - Ramp1Green);

/** Ramp Offset Control */
OffsetOK2 := Phase3Timer >= GreenOffsetTime;

/** Ramp 2 Control */
RedOK2 := t_green(Ramp2SignalHead) >= Ramp2Green;
GreenOK2 := (Phase5Timer+1) >= (CycleLength - GreenOffsetTime - Ramp2Green);

/** Concurrent Ramp Check */
ConcurrentOK1 := (GreenOK1 AND GreenOK2) OR (RateCode=0).

SUBROUTINE Ramp1;
/** Ramp 1 Green Timer */
IF (t_green(Ramp1SignalHead)) THEN
    IF (RateCode AND RedOK1) THEN
        set_sg_direct(Ramp1SignalHead,red);
        Ring1ActivePhase:=2;
        start(Phase2Timer);
    END;
END;

/** Ramp 1 Red Timer */
IF (Ring1ActivePhase=2) THEN
    IF ConcurrentOK1 THEN
        stop(Phase2Timer);
        reset(Phase2Timer);
        set_sg_direct(Ramp1SignalHead,green);
        Ring1ActivePhase:=1;
    END;
END.

SUBROUTINE Ramp2;
/** Ramp 2 Offset Timer */
IF (Ring2ActivePhase=3) THEN
    IF OffsetOK2 THEN
        stop(Phase3Timer);
        reset(Phase3Timer);
        set_sg_direct(Ramp2SignalHead,green);
        Ring2ActivePhase:=4;
    END;
END;

/** Ramp 2 Green Timer */
IF (t_green(Ramp2SignalHead)) THEN
    IF (RateCode AND RedOK2) THEN
        set_sg_direct(Ramp2SignalHead,red);
        Ring2ActivePhase:=5;
        start(Phase5Timer);
    END;
END;

/** Ramp 1/2 Concurrent Control */

```

```
IF (Ring2ActivePhase=5) THEN
    IF ConcurrentOK1 THEN
        stop(Phase5Timer);
        reset(Phase5Timer);
        Ring2ActivePhase:=3;
        start(Phase3Timer);
    END;

END.

/***** MAIN PROGRAM *****/
GOSUB Compute_Expressions;
GOSUB Ramp1;
GOSUB Ramp2.
```

The modified SDRMS algorithm coded in VISSIM

PROGRAM Ramp_Meter;

/* Constant Definition */

CONST

DF1 = 201, /* Freeway Detectors */
DF2 = 202,
DF3 = 203,
DF4 = 204, /* new for added lane */

PCW1 = 26, /* 1-minute Period */

Ramp1SignalHead = 1, /* Ramp Signal Heads */
Ramp2SignalHead = 2,

Ramp1Green = 2,
Ramp2Green = 2,

GreenOffsetTime = 3,

DeltaCyclesPerMinute = 0.35, /* Updated from table */
DeltaVolume = 0.090438244, /* Updated from table */
DeltaOccupancy = 0.630170353, /* Updated from table */

MinOccupancy = 129.8731208,
MinVolume = 28.01627292, /* Updated from table */

MaxOccupancy = 138.6955058,
MaxVolume = 29.28240833,

MinCyclesPerMinute = 5.45, /* Updated from table */
MaxCyclesPerMinute = 10.35, /* Updated from table */

BeginOccupancy1 = 129.8731208,
EndOccupancy1 = 129.8731208,
BeginVolume1 = 28.01627292,
EndVolume1 = 28.01627292,

StartTime1 = 18000,
EndTime1 = 39600,
StartTime2 = 50400,
EndTime2 = 68400;

SUBROUTINE Compute_Expressions;

/****** Lane Data *****/

/*** 1 second interval ***/

six_sec:=six_sec+1;
SimuTime:=SimuTime+1;

/*** Lane 1 ***/

FlowRate1:=presence(DF1); /* flow rate determined by sampling of detector presence */
FlowRate61:=FlowRate61+FlowRate1;
Occupancy1:=30*occup_rate(DF1); /* occupancy determined by sampling of detector occupancy rate */
Occupancy61:=Occupancy61+Occupancy1; /* (multiplied by 30 to match Caltrans controller sampling rate */

/*** Lane 2 ***/

```

FlowRate2:=presence(DF2);
FlowRate62:=FlowRate62+FlowRate2;
Occupancy2:=30*occup_rate(DF2);
Occupancy62:=Occupancy62+Occupancy2;

/** Lane 3 */
FlowRate3:=presence(DF3);
FlowRate63:=FlowRate63+FlowRate3;
Occupancy3:=30*occup_rate(DF3);
Occupancy63:=Occupancy63+Occupancy3;

/** Lane 4 new for added lane */
FlowRate4:=presence(DF4);
FlowRate64:=FlowRate64+FlowRate4;
Occupancy4:=30*occup_rate(DF4);
Occupancy64:=Occupancy64+Occupancy4;

/** 6 second interval */
If six_sec=6 THEN

    six_sec:=0;

    /** Total Freeway Average */

    FlowRate6:=(FlowRate61+FlowRate62+FlowRate63+FlowRate64)/4;

    Occupancy6:=(Occupancy61+Occupancy62+Occupancy63+Occupancy64)/4; /* updated for 4 lanes*/

    FlowRate61:=0;
    FlowRate62:=0;
    FlowRate63:=0;
    FlowRate64:=0; /* updated for 4 lanes*/

    Occupancy61:=0;
    Occupancy62:=0;
    Occupancy63:=0;
    Occupancy64:=0; /* updated for 4 lanes*/

    /** Average Running Values */
    NRA_Flow_1:=FlowRate6+((256-PCW1)*(NRA_Flow_1/256));
    NRA_Occ_1:=Occupancy6+((256-PCW1)*(NRA_Occ_1/256));

    /** Display in VISSIM */

    record_value(201,NRA_Flow_1);
    record_value(203, NRA_Occ_1);

    /** BEGIN & END LOGIC */
    IF RateCode THEN
        IF (NRA_Flow_1<EndVolume1) AND (NRA_Occ_1<EndOccupancy1) THEN
            RateCode:=0;
        END;
    ELSE
        IF (NRA_Flow_1>BeginVolume1) OR (NRA_Occ_1>BeginOccupancy1) THEN
            RateCode:=1;
        END;
    END;

    /** Maximum Control */
    IF (NRA_Flow_1>MaxVolume) THEN

```

```

        NRA_Flow_1:=MaxVolume;
END;

IF (NRA_Occ_1>MaxOccupancy) THEN
    NRA_Occ_1:=MaxOccupancy;
END;

/**/ Metering Start and End Times /**/
If (SimuTime < StartTime1) THEN
    RateCode:=0;
END;
If (SimuTime >= EndTime2) THEN
    RateCode:=0;
END;
If (SimuTime >= EndTime1) AND (SimuTime < StartTime2) THEN
    RateCode:=0;
END;

IF RateCode THEN

    /**/ Look-up Table VOLUME /**/
    IF (NRA_Flow_1<MinVolume) THEN
        VolumeRateCode:=1;
    ELSE

        IF (NRA_Flow_1<(MinVolume+DeltaVolume)) THEN
            VolumeRateCode:=2;
        ELSE

            IF (NRA_Flow_1<(MinVolume+(2*DeltaVolume))) THEN
                VolumeRateCode:=3;
            ELSE

                IF (NRA_Flow_1<(MinVolume+(3*DeltaVolume))) THEN
                    VolumeRateCode:=4;
                ELSE

                    IF (NRA_Flow_1<(MinVolume+(4*DeltaVolume))) THEN
                        VolumeRateCode:=5;
                    ELSE

                        IF (NRA_Flow_1<(MinVolume+(5*DeltaVolume))) THEN
                            VolumeRateCode:=6;
                        ELSE

                            IF (NRA_Flow_1<(MinVolume+(6*DeltaVolume))) THEN
                                VolumeRateCode:=7;
                            ELSE

                                IF (NRA_Flow_1<(MinVolume+(7*DeltaVolume))) THEN
                                    VolumeRateCode:=8;
                                ELSE

                                    IF (NRA_Flow_1<(MinVolume+(8*DeltaVolume))) THEN
                                        VolumeRateCode:=9;
                                    ELSE

                                        IF (NRA_Flow_1<(MinVolume+(9*DeltaVolume))) THEN
                                            VolumeRateCode:=10;
                                        ELSE

                                            IF (NRA_Flow_1<(MinVolume+(10*DeltaVolume))) THEN

```

```

        VolumeRateCode:=11;
ELSE

IF (NRA_Flow_1<(MinVolume+(11*DeltaVolume))) THEN
    VolumeRateCode:=12;
ELSE

IF (NRA_Flow_1<(MinVolume+(12*DeltaVolume))) THEN
    VolumeRateCode:=13;
ELSE

IF (NRA_Flow_1<(MinVolume+(13*DeltaVolume))) THEN
    VolumeRateCode:=14;
ELSE
    VolumeRateCode:=15;
END;
END;
END;
END;
END;
END;
END;
END;
END;
END;
END;
END;
END;
END;
END;

/** Look-up Table OCCUPANCY **/
IF (NRA_Occ_1<MinOccupancy) THEN
    OccupancyRateCode:=1;
ELSE

IF (NRA_Occ_1<(MinOccupancy+DeltaOccupancy)) THEN
    OccupancyRateCode:=2;
ELSE

IF (NRA_Occ_1<(MinOccupancy+(2*DeltaOccupancy))) THEN
    OccupancyRateCode:=3;
ELSE

IF (NRA_Occ_1<(MinOccupancy+(3*DeltaOccupancy))) THEN
    OccupancyRateCode:=4;
ELSE

IF (NRA_Occ_1<(MinOccupancy+(4*DeltaOccupancy))) THEN
    OccupancyRateCode:=5;
ELSE

IF (NRA_Occ_1<(MinOccupancy+(5*DeltaOccupancy))) THEN
    OccupancyRateCode:=6;
ELSE

IF (NRA_Occ_1<(MinOccupancy+(6*DeltaOccupancy))) THEN
    OccupancyRateCode:=7;
ELSE

IF (NRA_Occ_1<(MinOccupancy+(7*DeltaOccupancy))) THEN
    OccupancyRateCode:=8;
ELSE

```



```

        record_value(207,ratecode);
    END;
END;

/** Ramp 1 Control **/
RedOK1 := t_green(Ramp1SignalHead) >= Ramp1Green;
GreenOK1 := (Phase2Timer+1)>= (CycleLength - Ramp1Green);

/** Ramp Offset Control **/
OffsetOK2 := Phase3Timer >= GreenOffsetTime;

/** Ramp 2 Control **/
RedOK2 := t_green(Ramp2SignalHead) >= Ramp2Green;
GreenOK2 := (Phase5Timer+1)>= (CycleLength - GreenOffsetTime - Ramp2Green);

/** Concurrent Ramp Check **/
ConcurrentOK1 := (GreenOK1 AND GreenOK2) OR (RateCode=0).

SUBROUTINE Ramp1;

/** Ramp 1 Green Timer **/
IF (t_green(Ramp1SignalHead)) THEN
    IF (RateCode AND RedOK1) THEN
        set_sg_direct(Ramp1SignalHead,red);
        Ring1ActivePhase:=2;
        start(Phase2Timer);
    END;
END;

/** Ramp 1 Red Timer **/
IF (Ring1ActivePhase=2) THEN
    IF ConcurrentOK1 THEN
        stop(Phase2Timer);
        reset(Phase2Timer);
        set_sg_direct(Ramp1SignalHead,green);
        Ring1ActivePhase:=1;
    END;
END.

SUBROUTINE Ramp2;

/** Ramp 2 Offset Timer **/
IF (Ring2ActivePhase=3) THEN
    IF OffsetOK2 THEN
        stop(Phase3Timer);
        reset(Phase3Timer);
        set_sg_direct(Ramp2SignalHead,green);
        Ring2ActivePhase:=4;
    END;
END;

/** Ramp 2 Green Timer **/
IF (t_green(Ramp2SignalHead)) THEN
    IF (RateCode AND RedOK2) THEN
        set_sg_direct(Ramp2SignalHead,red);
        Ring2ActivePhase:=5;
        start(Phase5Timer);
    END;
END;

```

```

/*** Ramp 1/2 Concurrent Control ***/
IF (Ring2ActivePhase=5) THEN
    IF ConcurrentOK1 THEN
        stop(Phase5Timer);
        reset(Phase5Timer);
        Ring2ActivePhase:=3;
        start(Phase3Timer);
    END;
END.

/***** MAIN PROGRAM *****/

GOSUB Compute_Expressions;
GOSUB Ramp1;
GOSUB Ramp2.

```

VITA

Siavash Shojaat was born in the city of Tehran, Iran in July 1985 to Aliasghar Shojaat and Talat Tavana. He earned his diploma from Fajre Danesh high school in Mathematics. He received his Bachelor of Science degree in Civil Engineering in 2009 and a Master of Science degree in Civil Engineering in 2012 from Islamic Azad University. He will receive his Doctorate degree in Civil Engineering from Louisiana State University in 2017. He also earns a PhD Minor in Applied Statistics from Louisiana State University in 2017.