1-1-1981

Null Faraday rotation-a clean method for determination of relaxation times and effective masses in MIS and other systems

R. F. O'Connell
Louisiana State University

G. L. Wallace
Louisiana State University

Recommended Citation

O'Connell, R., & Wallace, G. (1981). Null Faraday rotation-a clean method for determination of relaxation times and effective masses in MIS and other systems. *Solid State Communications, 38* (5), 429-432. https://doi.org/10.1016/0038-1098(81)90272-6

Solid State Communications, Vol. 38, pp. 429–432.
Pergamon Press Ltd. 1981. Printed in Great Britain.

0038/1098/81/170429–04 $02.00/0

NULL FARADAY ROTATION – A CLEAN METHOD FOR DETERMINATION OF RELAXATION TIMES AND EFFECTIVE MASSES IN MIS AND OTHER SYSTEMS

R.F. O'Connell and G.L. Wallace

Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, U.S.A.

(Received 20 October 1980 by A.G. Chynoweth)

We determine the relation between photon, cyclotron, collision, and plasma frequencies which ensures a null Faraday rotation for electromagnetic wave propagation in a free-carrier magnetoplasma. This provides a *clean* determination (in the sense that it is independent of multiple reflections and the length of the plasma along the beam direction) of scattering times and effective masses in MIS and other systems.

FARADAY ROTATION is one of the most useful and extensively used techniques for the determination of effective masses in semi-conductors [1–6]. It is generally considered not to be a sensitive method for the determination of relaxation times τ ($\tau = \nu^{-1}$ where ν is the collision frequency). For thin samples the analysis is greatly complicated by the fact that multiple reflections can play a significant role [7].

Our purpose here is to point out that *null* Faraday rotation measurements can provide a *clean* way of determining not only effective masses but also relaxation times in many systems. Our emphasis will be on MOS (metal–oxide–semi-conductor) systems but it will be apparent that many of the results will hold for a wider variety of materials.

An electric field at a semiconductor-insulator interface quantizes the electron motion normal to the interface, resulting in a quasi-two-dimensional system. This system is of interest for both technological and basic scientific reasons [8–10].

The possibility of using Faraday rotation measurements for gleaning information about MOS systems has been discussed by Chiu *et al.* [11] and by Piller [12]. In these systems, the metallic layers on the two outer surfaces produce strong multiple internal reflections [11–13].

A theoretical calculation of the Faraday rotation θ is complicated not only by the presence of multiple reflections but also because of the varying density of charge across the thickness of the charge layer. What we wish to point out is that such complications and uncertainties can be eliminated simply by adjusting the various parameters (such as photon frequency ω, the magnetic field B which is perpendicular to the surface or the gate voltage V) to achieve a zero reading for θ. Thus we are motivated to seek a relation between ω, ω_c, ν and ω_p (the photon, cyclotron, collision and plasma frequencies, respectively) which ensures a null Faraday rotation for electromagnetic propagation of linearly-polarized radiation.

In general, the rotation per unit path length is given by

$$\frac{d\theta}{dl} = \frac{\omega}{2c}(n_+ - n_-) \tag{1}$$

where c is the velocity of light and where n_+ and n_- are the real parts of the refractive indices of the right and left circularly polarized components of the linearly-polarized wave. The quantities $n_\pm$ are obtained from the dielectric constants $\epsilon_\pm$ as follows:

$$\epsilon_\pm \equiv \epsilon'_\pm + i\epsilon''_\pm = (n_\pm + i\kappa_\pm)^2, \tag{2}$$

where $\kappa_\pm$ denotes the imaginary part of the refractive index. It follows that

$$n_\pm^2 = \tfrac{1}{2}\{[(\epsilon'_\pm)^2 + (\epsilon''_\pm)^2]^{1/2} + \epsilon'_\pm\}. \tag{3}$$

It is clear that null Faraday rotation is obtained whenever

$$n_+ = n_-. \tag{4}$$

Hence, making use of equations (3) and (4), we obtain, after some algebra, the general result

$$\{(\epsilon''_+)^2 - (\epsilon''_-)^2\}^2 = 4(\epsilon'_+ - \epsilon'_-)\{\epsilon'_+(\epsilon''_-)^2 - \epsilon'_-(\epsilon''_+)^2\}. \tag{5}$$

To proceed further we must make a specific choice for ϵ.

If certain conditions are fulfilled (see discussion below) the two-dimensional gas, which constitutes the surface charge layer of a MOS system, may be treated in a way analogous to the treatment of intraband effects in semiconductors. Thus, we may write

$$\epsilon_\pm = \epsilon_l\left\{1 - \frac{\omega_P^2}{\omega(\omega \pm \omega_c + i\nu)}\right\}, \tag{6}$$

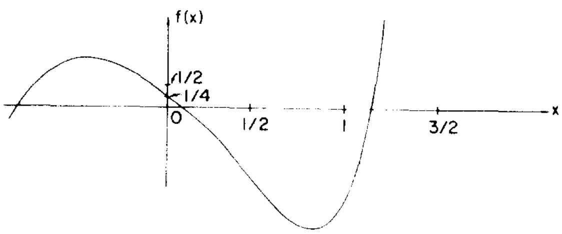

Fig. 1. Plot of $f(x)$ vs $x \equiv (\omega/\Omega)^2$, for $(\omega_c/\Omega) = (3/5)$, $(\nu/\Omega) = (4/5)$, and $(\omega_P/\Omega) = 0.5$. A null Faraday rotation is associated with the largest positive root, which occurs at $x = 1.1320$ ($\omega/\Omega = 1.0640$).

where ϵ_l, which is the dielectric constant of the lattice, is real, homogeneous, and isotropic. In addition

$$\omega_P^2 = 4\pi \frac{ne^2}{m^* \epsilon_l}, \tag{7}$$

where m^* is the effective mass and we take [12, 14, 15] $n = N^{3/2}$ where N is the surface concentration or, alternatively [10], $n = N/l$, where l is the thickness of the charge layer. It follows that

$$(\epsilon_\pm/\epsilon_l) = \left\{1 - \frac{\omega_P^2(\omega \pm \omega_c)}{\omega[(\omega \pm \omega_c)^2 + \nu^2]}\right\} + i\,\frac{\nu\omega_P^2}{\omega[(\omega \pm \omega_c)^2 + \nu^2]}. \tag{8}$$

The ω_P appearing in this model should not be confused with the plasma frequency for the *two*-dimensional gas.

Substituting this result into equation (5), and after much tedious algebra, we obtain the condition for null Faraday rotation in the form of a quintic equation for $x \equiv (\omega/\Omega)^2$:

$$f(x) \equiv 4x^5 + \left\{8\left[2\left(\frac{\nu}{\Omega}\right)^2 - 1\right] - 3\left(\frac{\omega_P}{\Omega}\right)^2\right\}x^4 - 8\left(\frac{\omega_P}{\Omega}\right)^2\left[2\left(\frac{\nu}{\Omega}\right)^2 - 1\right]x^3 - 2\left\{4\left[2\left(\frac{\nu}{\Omega}\right)^2 - 1\right] + 3\left(\frac{\omega_P}{\Omega}\right)^2\right\}x^2 - 4x + \left(\frac{\omega_P}{\Omega}\right)^2 = 0, \tag{9}$$

where

$$\Omega = (\omega_c^2 + \nu^2)^{1/2}. \tag{10}$$

The introduction of the frequency Ω is extremely useful, as will be apparent shortly. For now we simply note that $\epsilon_+' = \epsilon_-'$ (but $\epsilon_+'' \neq \epsilon_-''$) when $\omega = \Omega$. Thus, instead of treating ω, ω_P, ω_c, and ν as our four basic variables we found it more convenient to choose ω, ω_P, Ω, ν (the choice ω, ω_P, Ω, ω_c would be equally good). It is also useful to re-write equation (9) in the form:

$$f(x) \equiv 4x^5 + 8\left[2\left(\frac{\nu}{\Omega}\right)^2 - 1\right]x^4 - 8\left[2\left(\frac{\nu}{\Omega}\right)^2 - 1\right] - \left(\frac{\omega_P}{\Omega}\right)^2\left\{3x^4 + 8\left[2\left(\frac{\nu}{\Omega}\right)^2 - 1\right]x^3 + 6x^2 - 1\right\} = 0. \tag{11}$$

The fact that only an ω_P^2 term occurs in $f(x)$ is a notable feature of this equation. We have investigated this quintic equation in x algebraically, graphically, and numerically, with the following conclusions:

(1) For fixed ν, ω_P, and ω_c the equation has two complex roots, one negative root, and two positive roots. We present a typical plot of $f(x)$ vs x in Fig. 1. Of course, only the two positive roots are of potential physical significance but it turns out that only the larger of the two positive roots gives $\theta = 0$ when inserted in equation (1). We conclude that $f(x)$ has only one physically significant root for which $\theta = 0$.

(2) In general, we see by inspection of equations (9) or (11) that

$$f(\pm\infty) = \pm\infty, \tag{12}$$

$$f(0) = \left(\frac{\omega_P}{\Omega}\right)^2 > 0, \tag{13}$$

$$f'(0) = -4 < 0, \tag{14}$$

where the prime denotes differentiation with respect to x, and

$$f(1) = -16\left(\frac{\nu}{\Omega}\right)^2\left(\frac{\omega_P}{\Omega}\right)^2 < 0. \tag{15}$$

It thus follows that *the desired root always occurs for* $\omega > \Omega$ and

$$f(1) \to 0 \quad \text{if} \quad \omega_P \ll \Omega, \tag{16}$$

i.e. *the smaller ω_P is with respect to Ω the closer the zero in $f(x)$ approaches the point $\omega = \Omega$.*

It has been generally accepted that $\theta = 0$ occurs at the position of the cyclotron resonance [1]. What we have shown is that this is true only if $\omega_c \gg \nu$, and if $\Omega^2 \gg 4\nu\omega_P$ (which here is equivalent to $\omega^2 \gg 4\nu\omega_P$).

Furthermore, it is clear that if $\nu \gg \omega_c$ and $\Omega \gg \omega_P$ we get $\theta = 0$ in the vicinity of $\omega = \nu$. This result has clear implications with respect to an experimental determination of ν.

(3) Since

$$f(x \gg 1) = 4x^5 - 3\left(\frac{\omega_P}{\Omega}\right)^2 x^4 + 0(x^4), \tag{17}$$

it follows that $f = 0$ when $x = \frac{3}{4}(\omega_P/\Omega)^2$. In other words, we obtain $\theta = 0$ if

$$\omega = 0.886\omega_P \gg \Omega. \tag{18}$$

Such a relation may be used to investigate the effects of gate voltage V, temperature, etc. on the surface concentration N. Alternatively, since the variation of N with V is generally understood, it provides a means of investigating the possible variation of m^* with ω [16].

(4) If we set

$$x = (1+t)^2 \approx 1 + 2t, \quad t \ll 1 \tag{19}$$

in equation (9), and solve to lowest order in t, we obtain

$$t = \frac{\omega_P^2}{4\Omega^2}, \quad \omega_P \ll \Omega. \tag{20}$$

In other words, we obtain $\theta = 0$ if

$$\omega = \Omega\left(1 + \frac{\omega_P^2}{4\Omega^2}\right), \quad \omega_P \ll 2\Omega \quad (\text{i.e. } \omega_P \ll 2\omega). \tag{21}$$

Thus equation (21) provides us with an improvement over the result given in equation (16). A notable feature of this result is that ν does not appear as an independent variable. Inverting this equation, and using equation (10), we obtain

$$\nu^2 = \omega^2\left(1 - \frac{\omega_P^2}{2\omega^2}\right) - \omega_c^2, \quad \omega_P \ll \omega. \tag{22}$$

Hence, if ω or ω_c is adjusted to give $\theta = 0$, for a fixed $\omega_P \ll \omega$, we obtain a value for ν from equation (22). We also emphasize the weak dependence of ν on ω_P in this equation.

For arbitrary values of ω_P we must resort to either numerical or graphical (as in Fig. 1) means to obtain the value of (ω/Ω) for which $f = 0$. It is apparent that many useful investigations can thus be carried out. For example, one could determine how ν depends on the temperature or gate voltage by simply finding where the zero of θ occurs for different values of these quantities and then making use of equation (10) to obtain ν. The possible ω dependence of ν, which has been explored by various investigators [16], could also be obtained by, for example, obtaining a zero for a particular ω, then varying ω with a resultant $\theta \neq 0$ and next re-adjusting ω_c to bring θ back to zero.

We turn now to a discussion of the justification of the choice of equation (6) and, to be specific, we focus our remarks on electron inversion layers on Si as these have been extensively investigated. At the Si(100) surface, the energy levels in the potential V can be grouped into two distinct sets of overlapping subbands [17]. We will not dwell on the confusion which exists as to whether or not we are dealing with two distinct masses in such circumstances [15], but remark that some investigators [15, 18] have considered a plasma consisting of two different types of degenerate electrons, characterized by different effective masses and interacting with one another. Thus it should be of interest to calculate the relations which ensue for such a system when $\theta = 0$. Such a calculation is presently under study.

We would also like to emphasize that our results are applicable to many *three*-dimensional charge systems viz. those for which equation (6) is applicable, where ω_P now refers to the conventional plasma frequency for a quasi-free three-dimensional charge system (we refer to the extensive reviews of Palik and Furdyna [1] and Piller [4] for details of such systems).

Acknowledgements – One of us (RFO'C) would like to thank Dr H. Piller for his valuable assistance with the Faraday rotation literature and also for many useful conversations. This research was partially supported by the Department of Energy under Contract No. DE-AS05-79ER10459.

REFERENCES

1. E.D. Palik & J.K. Furdyna, *Rep. Prog. Phys.* **33**, 1193 (1970).
2. B. Lax & J.G. Mavroides, *Semiconductors and Semimetals* (Edited by R.K. Willardson and A.C. Beer), Vol. 3, p. 321. Academic Press, New York (1967).
3. E.D. Palik & G.B. Wright, *Semiconductors and Semimetals* (Edited by R.K. Willardson and A.C. Beer), Vol, 3, p. 421. Academic Press, New York (1967).
4. H. Piller, *Semiconductors and Semimetals* (Edited by R.K. Willardson and A.C. Beer), Vol. 8, p. 103. Academic Press, New York (1972).
5. J.K. Furdyna, *Appl. Optics* **6**, 675 (1967).
6. Y.I. Ukhanov, *Sov. Phys.–Usp.* **16**, 236 (1973).
7. B. Donovan & T. Medcalf, *Brit. J. Appl. Phys.* **15**, 1139 (1964).
8. S.M. Sze, *Physics of Semiconductor Devices*, J. Wiley, New York (1969).
9. F. Stern, *Phys. Rev.* **B5**, 4891 (1972).
10. B.D. McCombe, *Far Infrared Optical and Magneto-Optical Studies of Si Space Charge Layers* (to be published) gives a very useful view of the burgeoning interest in this area.
11. K.W. Chiu, T.K. Lee & J.J. Quinn, *Surf. Sci.* **58**, 182 (1976).
12. H. Piller, *J. Vac. Sci. Technol.* **16**, 2096 (1979).
13. T.A. Kennedy, R.J. Wagner, B.D. McCombe & J.J. Quinn, *Solid State Commun.* **18**, 275 (1976).
14. R. Freedman, *Phys. Rev.* **B18**, 2482 (1978).
15. J. Appel & A.W. Overhauser, *Phys. Rev.* **B18**, 758 (1978).
16. C.S. Ting, S.C. Ying & J.J. Quinn, *Phys. Rev. Lett.* **37**, 215 (1976); *Phys. Rev.* **B16**, 5394 (1977); A.K. Ganguly & C.S. Ting, *Phys. Rev. Lett.* **37**, 3541 (1977).
17. F. Stern & W.E. Howard, *Phys. Rev.* **163**, 816 (1967).
18. C.S. Ting (to be published).

Notes added in proof:

(i) In the case of a charged plasma with varying density we thus have the complication of varying plasma frequency. For example, in the MOS system the density profile through the inversion layer must be calculated theoretically (F. Stern and W.E. Howard, *Phys. Rev.* **163**, 816 (1967)). Hence the calculation of a *non-zero* value for the Faraday rotation θ is more involved and uncertain. However, if we select our parameters so that equation (21) holds we see that the condition for *null* Faraday rotation is only weakly dependent on ω_p. Furthermore, we have varified numerically that, for ω_p values close to the value for which null Faraday rotation is obtained ($\omega_p^{(0)}$ say), θ is very small and proportional to $(\omega_p^{(0)} - \omega_p)$. Thus, the optimum accuracy is achieved by selecting $\omega_p^{(0)}$ to be a suitably weighted average of the plasma frequencies and also by taking $(\omega_p^{(0)}/\Omega)$ to be as small as possible.

(ii) There is another contribution to Faraday rotation, apart from the charged plasma contributions we have discussed above. This is the so-called *polar reflection Faraday effect* due to each reflection from the metal gate. It is relatively small (E.A. Stern *et al.*, *Phys. Rev.* **135**, A1306 (1964) quote a value of about 10^{-4} deg for a field of 10^{-3} G) and can be easily obtained independently and subtracted out. More details on both of the above points will be given elsewhere (G.L. Wallace, Louisiana State University thesis 1981, in preparation).