1985

Loan and Deposit Rate Setting, Risk Aversion, Uncertainty and the Theory of Depository Financial Intermediaries.

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LOAN AND DEPOSIT RATE SETTING, RISK AVERSION, UNCERTAINTY AND THE THEORY OF DEPOSITORY FINANCIAL INTERMEDIARIES

A Dissertation
Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Doctor of Philosophy
in
The Department of Finance

by
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B.S., Louisiana State University, 1973
M.B.A., Louisiana State University, 1975
August, 1985
ACKNOWLEDGEMENTS

I would like to express my gratitude to the members of my dissertation committee: Dr. David T. Crary, Professor of Finance; Dr. William R. Lane, Associate Professor of Finance and Acting Associate Dean of the College of Business Administration; Dr. William F. Staats, Louisiana Bankers Association Professor of Banking; Dr. David M. Cordell, Assistant Professor of Finance; Dr. Robert E. Martin, Associate Professor of Economics; and Dr. Paul F. Paskoff, Associate Professor of History.

I remain indebted to Dr. Robert E. Martin whose valuable comments and ideas were a source of encouragement and provided me with the guidance and direction necessary to complete this dissertation.

I also want to thank Dr. David M. Cordell who closely monitored the progress of my work and gave me his support throughout the whole experience.

Finally, I am grateful to my family and especially to my father who has made a considerable investment in my education. Without his moral and economic support, this dissertation could not have been completed.
## CONTENTS

<table>
<thead>
<tr>
<th>Table/Chapter/Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF TABLES.</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF FIGURES.</td>
<td>vi</td>
</tr>
<tr>
<td>ABSTRACT.</td>
<td>vii</td>
</tr>
<tr>
<td>Chapter</td>
<td></td>
</tr>
<tr>
<td>1. INTRODUCTION.</td>
<td>1</td>
</tr>
<tr>
<td>2. REVIEW OF ALTERNATIVE APPROACHES TO THE THEORY OF DEPOSITORY</td>
<td>4</td>
</tr>
<tr>
<td>FINANCIAL INTERMEDIARIES.</td>
<td></td>
</tr>
<tr>
<td>Notation</td>
<td>5</td>
</tr>
<tr>
<td>Portfolio Theory Approach</td>
<td>7</td>
</tr>
<tr>
<td>A Portfolio Model of the Intermediary</td>
<td>9</td>
</tr>
<tr>
<td>Firm-Theoretic Approach</td>
<td>15</td>
</tr>
<tr>
<td>A Firm-Theoretic Model of the Intermediary</td>
<td>17</td>
</tr>
<tr>
<td>Extensions and Recent Models of the Intermediary</td>
<td>21</td>
</tr>
<tr>
<td>Sealey's Model</td>
<td>22</td>
</tr>
<tr>
<td>Mason's Model</td>
<td>27</td>
</tr>
<tr>
<td>Conclusions from the Review</td>
<td>30</td>
</tr>
<tr>
<td>3. A MODEL OF A DEPOSITORY FINANCIAL INTERMEDIARY.</td>
<td>34</td>
</tr>
<tr>
<td>Structure of the Balance Sheet</td>
<td>37</td>
</tr>
<tr>
<td>The Intermediary's Loan Market</td>
<td>39</td>
</tr>
<tr>
<td>The Intermediary's Deposit Market</td>
<td>40</td>
</tr>
<tr>
<td>Federal Funds Borrowing/Lending and the Timing of Decisions</td>
<td>41</td>
</tr>
<tr>
<td>The Intermediary's Objective Function</td>
<td>42</td>
</tr>
</tbody>
</table>
TABLES

Table | Page
---|---
1. Effects of a Change in $c_L$ on $r_L$ | 70
2. Effects of a Change in Expected Profit on the Risk Premium | 72
3. Effects on $r_L$ from a Change in $c_D$ | 75
4. Effect of a Change in $r$ on $r_L$ | 82
5. Effects on the Risk Premium From a Change in $r$ When $D > L$ | 84
6. Effects on the Risk Premium From a Change in $r$ When $D < L$ | 85
7. Effects of a Mean Preserving Spread of $\mu$ on Loan Rate | 89
8. Loan Uncertainty Case: Comparative Statics Results | 91
9. Effects of a Change in $c_D$ on $r_D$ | 102
10. Effects of a Change in $c_L$ on $r_D$ | 107
11. Effects on $r_D$ from a Change in $r$ | 111
12. Effects on the Risk Premium From a Change in $r$ When $D > L$ | 113
13. Effects on the Risk Premium From a Change in $r$ When $L > D$ | 113
14. Effects of a Mean Preserving Spread of $\varepsilon$ on $r_D$ | 117
15. Deposit Uncertainty Case: Comparative Statics Results | 119
# FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Risk Premiums and Attitudes Towards Risk</td>
<td>44</td>
</tr>
<tr>
<td>2. Decreasing Absolute Risk Aversion</td>
<td>46</td>
</tr>
<tr>
<td>3. Certainty versus Uncertainty Comparisons for $r_L$ and $r_D$ (Loan Uncertainty)</td>
<td>61</td>
</tr>
<tr>
<td>4. Certainty versus Uncertainty Comparisons for $r_L$ and $r_D$ (Deposit Uncertainty)</td>
<td>95</td>
</tr>
</tbody>
</table>
ABSTRACT

This dissertation examines the effects of loan and deposit quantity uncertainty and risk aversion on: 1) the spread between the risk averse loan and deposit rates, and 2) the behavior of the intermediary in the Federal funds market. The intermediary's major function is to provide transaction and liquidity services to its customers. Theoretical results indicate that quantity uncertainty from either loans or deposits reduces the spread to a level below that of a profit maximizing intermediary. The pure effects of quantity uncertainty on the intermediary's spread are consistent and mutually reinforcing regardless of whether uncertainty comes from loans or deposits. Comparative statics results reveal that, under decreasing absolute risk aversion, the management of the spread may be significantly different than it is under risk neutrality. The model's results imply that deposit variability is an increasing function of size and of the degree of liquidity in the intermediary's asset portfolio. On the other hand, deposit variability appears to have no effect on the intermediary's loan rate. The results presented in this dissertation have a number of important implications for several aspects of the theory of depository financial intermediaries.
Chapter 1

INTRODUCTION

Corporate financial theory has long been recognized as inapplicable to the modeling of depository intermediaries. This notion stems from the assumption of perfect capital markets which underlies most of the corporate financial theory. Yet, for the most part, depository financial intermediaries are known to operate in imperfect capital markets. In such markets, the appropriate mode of behavior is that of rate setting in most assets and liabilities of these institutions. Essentially, rate setting creates a liquidity problem for depository intermediaries. They must face uncertainty as to the quantities of assets and liabilities they will have on their balance sheets ex post. In light of this fact, quantity variability is certainly more important to depository institutions than rate variability.

Until now, the theory of depository financial intermediaries has not been successful in resolving the problem of quantity uncertainty in depository institutions. Models of the intermediary behavior have generally followed either a portfolio theory or a firm-theoretic approach. Whereas portfolio models of the intermediary cannot deal with the


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liquidity problem of these institutions, firm-theoretic models have mainly been restricted to consideration of linear risk preferences.

Sealey\(^2\) was the first to consider rate setting, uncertainty, and risk aversion simultaneously in a model of a depository intermediary. His model incorporated rate-setting on deposits while assuming quantity setting in the loan market. Although the assumption of a perfectly competitive loan market is defensible, some recent empirical evidence\(^3\) suggests that loan markets are characterized by imperfect competition, and rate setting behavior is prevalent.

This dissertation will systematically explore the simultaneous loan and deposit rate-setting behavior of depository financial intermediaries under alternative risk preferences. In essence, the model depicted here assumes that the institution's management acts as a "spread manager" operating to maximize the expected utility of profits on behalf of the owners.

The structure of the model is relatively simple and abstract. It assumes a balance sheet constraint of loans, deposits, and Federal funds purchased and sold. The


intermediary's management is an active participant in the market for Federal funds using this market to make balance sheet adjustments resulting from uncertainty in loans and deposits. Thus, the mechanism used by the institution in dealing with quantity uncertainty depends entirely on the existence of a perfectly competitive Federal funds market. This view is consistent with a renewed emphasis on markets for immediately available liquid funds.

The results indicate that quantity uncertainty in the intermediary's balance sheet and risk aversion generally tend to reduce the size of the intermediary's spread. The theoretical results support the contention that depository intermediaries are highly risk averse, and willing to sacrifice profits for reduced risks.

Comparative statics developed in the discussion demonstrate that the degree of risk aversion (absolute sense) is a critical determinant of the behavior of the intermediary's spread in response to changes in the cost parameters of the model.

These results have important implications for several aspects of the theory of depository intermediaries.

Chapter 2 reviews alternative approaches used to model the depository intermediary. Chapter 3 presents the structure of the model, its solution, and the comparative statics results. Finally, chapter 4 details the implications for the theory of depository intermediaries.
Chapter 2

REVIEW OF ALTERNATIVE APPROACHES TO THE THEORY OF
DEPOSITORY FINANCIAL INTERMEDIARIES

This chapter synthesizes the various approaches used to model and explain the behavior of depository financial intermediaries.\(^1\) In recent times, a substantial amount of literature has emerged aimed at developing a theoretical framework appropriate for modeling the process of financial intermediation. Thus far, this literature remains in a fragmented and heterogeneous state. A great deal of controversy exists over the type of framework in which to model the macroeconomic process of financial intermediation. This controversy has led some authors to question the mere existence of financial intermediaries as viable firms in the financial markets.\(^2\) At the microeconomic level, the published literature has offered a multitude of models of the intermediary based on several competing approaches.

The largest portion of the existing literature on financial intermediaries has concentrated on specific problems of asset selection. For the most part, the

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\(^1\)Hereinafter, the terms "intermediary", "financial intermediary", and "institution" will be used to refer only to a depository financial intermediary.


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emphasis has been on issues pertaining to the management of
liquidity and reserves by commercial banks.\(^3\) With few
exceptions, most microeconomic models of the intermediary
have been of partial equilibrium in nature and have dealt
exclusively with the selection of the intermediary's assets
while assuming the size and structure of its liabilities to
be fixed and exogenously given. However, a smaller segment
of the literature on financial intermediaries has focused on
complete models of the intermediary; models in which all the
traditional decisions of the firm are considered.\(^4\) In
contrast to portfolio choice models, complete models of the
intermediary's behavior address the joint determination of
asset/liability structure, their interrelationship if any,
and the total size or scale of the intermediary.

Before proceeding with the examination of alternative
frameworks, the first part of this chapter will introduce
some of the notation necessary in order to facilitate
comparison between models to be discussed.

**Notation**

Currently, no standard set of symbols is used in the
literature dealing with financial intermediaries.

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\(^3\)See for example William Poole, "Commercial Bank
Reserve Management in an Uncertain World: Implications for
Monetary Policy," *Journal of Finance*, 23:769-91, December,
1968.

\(^4\)For a review of this literature see Ernst
Baltensperger, "Alternative Approaches to the Theory of the
Banking Firm," *Journal of Monetary Economics*, 6:1-37,
Nevertheless, it is important to establish at this point some basis notation system to be employed throughout this review and in forthcoming chapters as well.

The following list defines the variables most frequently used in the text (additional variables and symbols will be defined as needed):

- $D =$ deposits
- $L =$ loans
- $S =$ securities
- $R =$ reserves or money market borrowing/lending
- $r_L =$ loan rate
- $r_D =$ deposit rate
- $r_f =$ risk-free rate
- $\lambda =$ market price of risk
- $\Pi =$ profits
- $U(\Pi) =$ utility of profit function
- $\text{Var}(\Pi) =$ variance of profit
- $\text{Cov} =$ covariance

This review will concentrate on alternative frameworks used to formulate complete models of the intermediary. The underlying assumption here is that most important aspects of the theory of financial intermediaries are best examined using complete models of these firms. Accordingly, this chapter will proceed with a three-part review of the different frameworks used in the literature to model the behavior of depository intermediaries: 1) portfolio theory approach, 2) firm-theoretic approach, 3) extensions and
recent models of the intermediary.

**Portfolio Theory Approach**

The principles of portfolio selection developed by Markowitz\(^5\) and later extended by Tobin\(^6\) have served as a framework for a number of models of the intermediary. Models using this particular framework view the intermediary as a risk averse investor attempting to select a portfolio of securities in a way that maximizes the expected utility of the intermediary's terminal wealth.\(^7\)

The most notable portfolio-based model in the literature, Pyle's\(^8\), concentrated on developing a set of conditions under which the process of financial intermediation is more likely to take place. Pyle found that the incentive for financial intermediation to occur is greater (1) the smaller the risk premium on deposits and the larger the risk premium on loans, (2) the greater the positive dependence between loan and deposit yields, and (3) the larger the standard deviation of deposit yields and smaller


\(^7\)Since portfolio theory does not address the problem of liabilities, they are simply viewed as negative positions on assets.

the standard deviation of loan yields. Furthermore, he showed that the asset and liability choices of the intermediary are generally interdependent. In the context of his model, Pyle concluded that the degree of correlation between asset and liability yields is the most important determinant of the process of financial intermediation.

Hart and Jaffee provided another important portfolio-based model of the intermediary. Using a general model of the intermediary subject to reserve requirements, they established several important results. They derived the mathematical properties of the opportunity set for the intermediary and demonstrated the existence of a separation theorem which did not require equal borrowing and lending by the institution. With respect to the opportunity set, they found the efficient frontier to be a linear function in the expected value-standard deviation space. Both the comparative statics properties as well as the proof of the separation theorem were found to be quite general and only dependent on the assumption of a zero net worth component of total liabilities of the intermediary.


With respect to empirical studies, few have attempted to test portfolio-based models of the intermediary. Parkin was the first to use a portfolio framework to examine empirically the behavior of the intermediary.11 He found that a large portion of the changes in portfolio and debt accounts of discount houses in the United Kingdom could be explained in terms of a simple portfolio model of these institutions.

Other existing portfolio models of the intermediary have been of partial equilibrium in nature. As such, their usefulness has been limited to the study of more specific asset management problems.

Next to be reviewed is the formulation of portfolio models of the intermediary. Major assumptions required will be detailed first. This discussion will be followed by a simple model of the intermediary within the context of portfolio theory framework.

A Portfolio Model of the Intermediary

In general, portfolio models of the intermediary require a number of restrictive assumptions: (1) a single decision period, (2) risk aversion, (3) perfectly competitive capital markets, (4) existence of riskless security, (5) borrowing and lending at the riskless rate by all economic units, and (6) management of the intermediary

acting in the best interests of shareholders. Additionally, portfolio-based models disregard the existence of resource costs or solvency considerations attached to the management of the intermediary's assets and liabilities.

These assumptions have several implications. Of particular interest is the way in which these restrictions have usually been incorporated into models of the financial intermediary. Assumption (2), for example, restricts the form and properties of the utility function being maximized by the intermediary to those that are strictly concave in profits. Moreover, the assumption implies that shareholders' preferences can be completely specified by indifference curves defined solely in terms of the first two moments of the distribution of returns from all portfolios of securities available for investment.

Assumption (3) is perhaps the most restrictive assumption required by portfolio models of the intermediary. With perfectly competitive capital markets, the intermediary faces perfectly elastic demand and supply functions for its assets and liabilities. The intermediary thus behaves essentially as a quantity setter selecting the desired quantities of the assets and liabilities it wishes to hold on the bases of expected market prices and yields on these securities. With quantity setting as the appropriate behavioral mode, uncertainty is assumed to enter into these models via random rates of return or random yields on the securities held by the institution at the start of its
decision period. Two factors presumably contribute to rate uncertainty. First, the inability of these institutions to match perfectly the maturities of their asset and liability portfolios with the length of their decision or holding period. The second factor is related to the possibility of default on the intermediary's asset portfolio and to the renewal cost of maturing liabilities. Generally, for most intermediaries, the maturity structure of their liabilities is less than that of their asset portfolio and all portfolio models of the intermediary have been developed within a single period framework.

Given the assumptions previously listed, a simple portfolio model of the intermediary will now be described.

Consider a depository intermediary with two assets to choose from at the start of its decision period: loans \((L)\) and U.S. Government securities \((S)\). On the liability side, the intermediary can issue one type of claim: deposits \((D)\). The balance sheet identity is given by

\[ L + S = D \]  

(2-1)

with \(L, S,\) and \(D\) traded in perfectly competitive markets.

The intermediary's profit function, \(\Pi\) is given by

\[ \Pi = r_L L + r_f S - r_D D \]  

(2-2)

where \(r_L, r_f,\) and \(r_D\) are the rates of return on loans, securities, and deposits respectively. It is assumed that \(r_L\) and \(r_D\) are exogenous and stochastic. The rate of return
on securities, \( r_f \), is known with certainty and equal to the risk-free rate of return.

The intermediary's problem amounts to the selection of a portfolio of loans, securities, and deposits in a way that maximizes the utility of the institution. This optimization problem can be stated as follows:

\[
\text{maximize } U(\Pi, \text{Var}(\Pi)) \\
\text{subject to } L + S = D \\
L > 0, S > 0, D > 0,
\]

where \( U(\Pi, \text{Var}(\Pi)) \) is a single period utility function defined in terms of the mean (\( \Pi \)) and variance (\( \text{Var}(\Pi) \)) of profit. It will be assumed that \( U(\Pi, \text{Var}(\Pi)) \) has properties consistent with increasing marginal utility, \( U' > 0 \), and risk aversion \( U'' < 0 \).

The mean and variance of profit are given by

\[
\Pi = r_L L - r_D D + r_f S
\]
\[
\text{Var}(\Pi) = L^2 \text{Var}(r_L) + D^2 \text{Var}(r_D) + 2DL \text{Cov}(r_L, r_D),
\]

where \( r_L \) and \( r_D \) are the expected rates of return on loans and deposits; \( \text{Var}(r_L) \), \( \text{Var}(r_D) \), and \( \text{Cov}(r_L, r_D) \) are the variances of \( r_L \), \( r_D \), and the covariance between \( r_L \) and \( r_D \).

Next, consider one particular solution to the optimization problem given by (2-3). In particular, consider Parkin's\(^\text{12}\) assumption with respect to the utility

\(^\text{12}\)Parkin, op. cit., pp. 471-73.
function being maximized by the intermediary. Parkin assumed III to be normally distributed and U(III) to be given by

\[ U(\Pi) = a - b \exp(-\alpha \Pi), \]  

(2-6)

where a, b and \( \alpha \) are parameters and \( b > 0, \alpha > 0 \).

Lintner\(^{13}\) has shown that if profits are normally distributed, maximizing the expected value of (2-6) is equivalent to maximizing the certainty equivalent of random profit, \( \hat{\Pi} \), given by

\[ \hat{\Pi} = \Pi - \frac{a}{2} \text{Var}(\Pi). \]  

(2-7)

Thus, the problem given by (2-3) can be rewritten as

\[ \text{maximize} \quad \hat{\Pi} - \frac{a}{2} \text{Var}(\Pi) \]  

(2-8)

subject to \( L + S = D \)

\[ L > 0, \quad S > 0, \quad D > 0. \]

Using the definitions of \( \bar{\Pi} \) and \( \text{Var}(\Pi) \) given by (2-4) and (2-5), and incorporating the balance sheet constraint into (2-8), the following unconstrained problem is obtained:

\[ \text{maximize} \quad \bar{\Pi}L - \bar{\Pi}D + r_f(D - L) - \frac{a}{2} \left[ L^2 \text{Var}(r_L) + \ight. \\
\left. D^2 \text{Var}(r_D) + 2DL \text{Cov}(r_L, r_D) \right]. \]  

(2-9)

After (2-9) is solved for \( L \), the optimal amount of loan extensions, \( L^* \), is given by

\[
L^* = \frac{(\bar{r}_L - r_f) \text{Var}(r_D) - (\bar{r}_D - r_f) \text{Cov}(r_L, r_D)}{\alpha[\text{Var}(r_L) \text{Var}(r_D) - [\text{Cov}(r_L, r_D)]^2]}.
\]  
(2-10)

Similarly, solving for \( D \), the optimal amount of deposits that the intermediary should accept is equal to:

\[
D^* = \frac{(\bar{r}_L - r_f) \text{Cov}(r_L, r_D) - (\bar{r}_D - r_f) \text{Var}(r_L)}{\alpha[\text{Var}(r_L) \text{Var}(r_D) - [\text{Cov}(r_L, r_D)]^2]}.
\]  
(2-11)

The parameter \( \alpha \) in (2-10) and (2-11) is equal to the absolute value of the ratio of marginal utility of profit to the variance of profit. As long as risk aversion is assumed on the part of the intermediary, the value of \( \alpha \) will always be positive.

The model presented above is nearly identical to the models developed by Parkin\(^{14}\) and Pyle.\(^{15}\) Parkin's model was aimed at empirically studying the behavior of discount houses in the United Kingdom. Pyle's analysis, on the other hand, attempted to derive a set of conditions under which intermediation would be more likely to occur.

In terms of importance, Pyle's results are perhaps the most significant among the literature dealing with portfolio models of the intermediary.

\(^{14}\)Parkin, loc. cit.

\(^{15}\)Pyle, loc. cit.
Pyle demonstrated that asset and liability decisions of the intermediary were generally not independent. Referring to expressions (2-10) and (2-11), it is clear that the optimal lending and deposit decisions are each a function of a combination of factors including the expected rates on loans and deposits, the variances of $r_L$ and $r_D$, and the covariance between $r_L$ and $r_D$. Furthermore, Pyle found the correlation between loan and deposit rates to be a key determinant of the process of intermediation. In particular, if the rates on loans and deposits are uncorrelated, the necessary and sufficient conditions for financial intermediation to take place are 1) a positive risk premium on loans, and 2) a negative risk premium on deposits. With positive correlation between the rates on loans and deposits, Pyle concluded that intermediation would be more likely to take place under the following conditions: 1) the larger the difference between expected rates of return on loans and deposits, 2) the greater the positive correlation between loan and deposit rates, and 3) the larger the standard deviation of deposit rates and the smaller the standard deviation of loan rates.

The next section examines a second approach to the modeling of depository financial intermediaries. Rooted in the microeconomic theory of the firm, this will be termed the firm-theoretic approach.

**Firm-Theoretic Approach**

Another segment of the literature focuses on the
optimizing behavior of depository financial intermediaries in a framework that closely resembles the neoclassical microeconomic theory of the firm. Models following this approach have taken one of two directions. On the one hand, one particular subset of these models is an attempt to analyze the behavior of the intermediary under imperfect capital markets. Klein\textsuperscript{16} was among the first to recognize the need to develop models of the intermediary that incorporate market imperfections. His model considered a bank facing a downward-sloping demand function for loans and an upward-sloping supply function for deposits under conditions of risk neutrality.

A second segment of this literature has emphasized the real resource costs and production aspects associated with the financial services provided by intermediaries of the depository type. Models of this kind, such as the ones provided by Pesek,\textsuperscript{17} Saving,\textsuperscript{18} and Sealey and Lindley\textsuperscript{19} represent pure production costs models of the intermediary.


attempting to explain asset and liability structures of the intermediary on the basis of the resource costs incurred by the institution in maintaining a certain level of financially related services such as check clearing, book-keeping, and deposit withdrawals. In contrast to the former type of models which have emphasized the imperfect nature of the intermediary's markets, the latter type of models have been formulated under perfectly competitive markets.

A Firm-Theoretic Model of the Intermediary

For the most part, firm-theoretic models of the intermediary have departed from the following assumptions:

1) a single decision period, 2) risk neutrality, 3) imperfect asset and liability markets, and 4) a perfectly-elastic supply of U.S. Government securities at an exogenously given rate of return. Given these assumptions, the intermediary is viewed as a private economic unit attempting to maximize the expected value of end-of-the-period profit.

Firm-theoretic models differ from portfolio-type models in two significant aspects. First, most firm-theoretic models have been developed under conditions of risk neutrality with the exception of Ratti's model. Second, in

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20 In addition, it is assumed that assets and liabilities have equal maturities of the same length as the decision period.


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contrast to the perfect markets assumption used in portfolio models, firm-theoretic models rely on the existence of imperfect capital markets.

The workings of the firm-theoretic approach can be illustrated with a simple model. In essence, the formulation of the model that follows is similar in structure to Klein's model.²²

Consider an intermediary with three kinds of assets: loans (L), cash reserves (R), and Government securities (S). The institution is allowed to issue one type of claim, namely deposits (D). The balance sheet constraint is given by

\[ L + R + S = D. \] (2-12)

The intermediary will be assumed to be a monopolist in its loan market. The loan demand is a downward-sloping function of the rate of interest on loans chosen by the intermediary:

\[ r_L = r_L(L); r_L'(L) < 0. \] (2-13)

The demand function given by (2-13) simply states that the average return from loans is a downward-sloping function of the amount L.

The Government securities market is assumed to be perfectly competitive and the intermediary faces an upward-sloping supply of deposits given as a function of the rate

²²Klein, loc. cit.
paid on deposits,
\[ D = D(r_D); \quad D'(r_D) > 0. \] (2-14)

Uncertainty affects the intermediary's decisions in the following manner. The model assumes that during the decision period the institution experiences a net random deposit outflow. The intermediary enters its decision period with a given stock of cash reserves and with full knowledge of the probability density function of the net deposit outflow that might occur during the period. In the event of a cash reserve deficiency (defined as a situation in which the deposit outflow is larger than the initially held amount of cash reserves), the intermediary has to pay a penalty cost of \( r \) percent per dollar of the shortfall.

The penalty cost function is given by \( P(X) \):
\[
P(X) = \begin{cases} 
  r(X-R) & \text{if } X > R \\
  0 & \text{if } X < R,
\end{cases} \] (2-15)

where \( X \) stands for the random net deposit outflow.

Profits at the end of the period can be written as
\[
\Pi = r_L L + r_F S - r_D D - P(X). \] (2-16)

The expected profits are given by
\[
\bar{\Pi} = r_L L + r_F S - r_D D - \int_{-\infty}^{\infty} P(X)f(X)dX, \] (2-17)

where \( f(X) \) is the probability density function of \( X \).

The intermediary's objective is to maximize (2-17).
subject to the balance sheet constraint given by (2-12). This problem can be stated as

\[
\text{maximize } \int_{r_L}^{r_D} r L + r_f S - r_D D - \int_{R}^{\infty} P(X)f(X)dX
\]

subject to

\[L + R + S = D.\]

The solution to the optimization problem given by (2-18) can be easily obtained through the use of the Lagrange multiplier method. It turns out that the optimal values for \(r_L\) and \(r_D\) are given implicitly in the following expressions:

\[MRL = r\] \hspace{1cm} (2-19)

and

\[MCD = r\] \hspace{1cm} (2-20)

where MRL and MCD are the marginal revenue and marginal cost on loans and deposits respectively. The interpretation of (2-19) and (2-20) is as follows. The intermediary in this simple model will set its rate on loans by equating the marginal revenue from loans to the marginal cost of cash reserves. Similarly, the intermediary determines the interest rate to be paid on deposits by equating the marginal cost of deposits to the exogenous rate \(r\). In terms of the size of the loan portfolio, the intermediary extends loans up to the point where the marginal revenue from loans equals the expected rate of return on Government securities. The same also holds true for deposits: the intermediary accepts deposits until their marginal cost equals the risk-free rate.
Based on a model very similar to the one presented in this section, Klein concluded that the intermediary's asset selection could be considered independent of its liability decisions.\textsuperscript{23} Pringle questioned the generality of Klein's results attributing these conclusions to the restrictive nature of the assumptions Klein used in formulating his model.\textsuperscript{24}

The next section of this chapter will examine recent contributions to the theory of depository financial intermediaries.

\textbf{Extensions and Recent Models of the Intermediary}

In recent years, a number of models of the intermediary have attempted to combine or extend the two major approaches previously discussed. Sealey\textsuperscript{25} developed a model of the intermediary which blends together the major characteristics of the Markowitz-Tobin framework and the firm-theoretic approach. Sealey's model allows for simultaneous consideration of nonlinear risk preferences, deposit rate-setting behavior, and liquidity cost considerations all under uncertainty. Another notable contribution to the theory of

\textsuperscript{23}Klein, op. cit., pp. 213-15.


depository financial intermediaries, Mason's\textsuperscript{26}, can be viewed as an extension of the Markowitz-Tobin portfolio approach to a context of capital asset pricing under uncertainty.

Sealey's Model

Sealey's model requires the following assumptions about the intermediary: 1) a single decision period, 2) a perfectly competitive loan market, 3) a stochastic supply of deposits to the intermediary, 4) an objective of the intermediary to maximize the expected value of a utility function that defines shareholders' attitudes towards risk, 5) resource costs associated with the production and servicing of loan and deposit contracts.

The intermediary's balance sheet identity is given by

\[ L = D + Z, \quad (2-21) \]

where L and D stand, as before, for loans and deposits; and Z is a borrowing/lending variable measuring the difference between money market borrowing and lending.

This model assumes that the intermediary operates in a perfectly competitive loan market with the rate of return on loans \( r_L \) assumed stochastic with a known subjective probability density function. Furthermore, \( r_L \) has a range over the interval between zero and infinity.

Following Leland\textsuperscript{27} the deposit supply function to the intermediary will be assumed an implicit function $D^*$:

$$D^*(D, r_D, u) = 0,$$  \hspace{1cm} (2-22)

where $D^*$ is assumed to have continuous partial derivatives; $D$ is the quantity of deposits supplied; $u$ is a random parameter unknown ex-ante but with a known probability distribution defined over the half-closed interval $[\beta, \infty)$; $r_D$ is the contract rate of interest on deposits.

For a deposit rate setter, (2-22) can be rewritten as

$$D = D(r_D, u).$$  \hspace{1cm} (2-23)

The supply of deposits function given by (2-23) has two restrictions: 1) for any $u$, the relationship between $D$ and $r_D$ is upward-sloping, or $\partial D / \partial r_D > 0$, and 2) $D(r_D, u)$ is an increasing function of $u$.

The timing of decisions is as follows: at the start of its decision period, the institution selects the rate of interest to be paid on deposits and the quantity of loans to be extended. Hence, $r_D$ and $L$ are termed ex-ante controls. After a small interval of time has elapsed, the value of the random parameter $u$ becomes known and thus the amount of deposits. At almost the same time, the return on loans becomes known, and the intermediary must either borrow ($Z > 0$) or lend ($Z < 0$), in order to balance its assets and claims.

The intermediary's objective is assumed to be maximization of expected utility of profits. Denote the preference function by $U(\Pi)$ with $U'(\Pi) > 0$, and $U''(\Pi) < 0$ depending on the assumed risk preferences of the intermediary's shareholders.

The mathematical optimization problem is given by

$$\max_{r_0, L, Z} \{ \max_{r_0, L} \mathbb{E}[U(\Pi)] \}$$

subject to

$$L = D + Z.$$  

The intermediary's random profit, $\Pi$, is given by

$$\Pi = r_L L - r_D D - C_L(L) - C_D(D) - C(Z), \quad (2-25)$$

where $C_L(L)$ and $C_D$ are resource costs in servicing loans and deposits; $C(Z)$ is the liquidity cost from having to borrow ($Z > 0$) or the revenue from lending excess funds ($Z < 0$) at the end of the decision period. It is assumed that $C_L$ and $C_D$ are increasing functions of $L$ and $D$ respectively; or $C'_L > 0$ and $C'_D > 0$. Moreover, economies of scale are ignored. This is equivalent to assuming that both $C_L$ and $C_D$ increase at an increasing rate.

The liquidity cost function $C(Z)$ is given by

$$C(Z) = rZ; \quad \text{for all } Z = 0, \quad (2-26)$$

where $r$ is both the borrowing and lending rate.

Incorporating these cost functions and the balance sheet constraint into (2-25) yields the following expression for profits:
\[ \Pi = r_L L - r_D D(r_D, u) - C_L(L) - C_D(D) - r[L - D(r_D, u)]. \quad (2-27) \]

Using (2-27), the unconstrained problem in terms of ex-ante controls and parameters of the model becomes

\[ \max_{r_D, L} U[r_L L - r_D D(r_D, u) - C_L(L) - C_D(D) - r[L - D(r_D, u)]] \quad (2-28) \]

Using (2-28), Sealey derived two sets of optimal solutions for \( r_D \) and \( L \). First, he looked at a risk-neutral intermediary (i.e., \( U''(\Pi) = 0 \)). Letting \( r^n_D \) and \( L^n \) stand for the optimal deposit rate and loan position under risk neutrality, he defined these as follows:

\[ \bar{r}_L = r + C_L'(L^n) \quad (2-29) \]

\[ r^n_D = \bar{r}_L - D/D' - C_D'(D) - C_L'(L^n). \quad (2-30) \]

Expression (2-29) implicitly defines the optimal loan position under risk neutrality. The risk-neutral intermediary extends loans up to the point that expected marginal revenue from loans equals the sum of the marginal resource and liquidity cost. The intermediary sets the optimal deposit rate under risk neutrality as the difference between the marginal return on loans and the sum of the marginal interest and resource cost of deposits and the marginal resource cost on loans (2-30). Note that in the absence of resource costs on loans and deposits, these results are identical to the ones obtained by Klein.\(^\text{28}\)

Under risk aversion (\( U''(\Pi) < 0 \)), Sealey found the optimal

\(^\text{28}\)Klein, loc. cit.
risk averse loan position \( (L^a) \) to be given by

\[
\bar{r}_L = r + C'_L(L^a) - \frac{\text{Cov}(U'(\Pi), r_L)}{E[U'(\Pi)]}. \tag{2-31}
\]

The last term in (2-31) is the covariance between the marginal utility of profit and the random loan return. Sealey\(^{29}\) demonstrated that the risk premium demanded by a risk averse intermediary is an increasing function of loan extensions.

Sealey's results indicated that the risk-averse intermediary tends to operate at a lower loan position than it would under risk neutrality.

The effects of risk aversion on deposit rate-setting decisions obtained by Sealey were far less conclusive. In most cases, the effects of non-linear risk preferences tend to depend on the functional form of the deposit supply function. For a general deposit supply function, the effects turned out to be inconclusive.

Using a model similar to the one discussed in this section, Sealey\(^{30}\) examined the capital structure decisions and shareholders' unanimity for a depository financial intermediary. He demonstrated that unanimity with respect to capital structure decisions among the intermediary's shareholders is possible if the intermediary's returns are

\(^{29}\)Sealey, op. cit., pp. 1144-45.

independent of investors' payment flows, and the market is fully spanned in all states.

Koppenhaver extended Sealey's model to include CD futures trading. He compared the hedging performance of this model against a portfolio-based model for four different hedging strategies. Koppenhaver concludes that if banks maximize expected utility of profit and exhibit constant relative risk aversion, then hedges using CD's or Treasury bill futures are equally effective.

**Mason's Model**

Mason has provided a model of a commercial banking firm that assumes maximization of value rather than expected utility. Using the familiar capital asset pricing model, he lets the value of the bank be given by

\[ V = \frac{1}{1 + r_f} (\bar{\pi} - \text{Cov}(\pi, r_m)), \]  
(2-32)

where \( \bar{\pi} \) and \( \pi \) now stand for rates of return rather than dollars, and \( \lambda \) is the market price of risk or \( (\bar{r}_m - r_f)/\text{Var}(r_m) \). Furthermore, \( \bar{r}_m \) and \( r_m \) are the expected rate and rate of return on a market index portfolio, such as S & P 500 stock index; \( \text{Var}(r_m) \) is the variance of this market index portfolio.

Mason assumed the bank faces a downward-sloping demand
curve for loans and an upward-sloping supply curve for deposits. These can be written as

\[ L = L(r_L,u) ; \quad \frac{\partial L}{\partial r_L} < 0 \]  
\[ D = D(r_D,v) ; \quad \frac{\partial D}{\partial r_D} > 0. \]  

(2-33) (2-34)

The random terms \( u \) and \( v \) are assumed to have finite means and variances. The demand and supply functions in (2-33) and (2-34) are also assumed to have finite means and variances.

The bank's balance sheet identity is given by

\[ L = D + K, \]  

(2-35)

where \( L, D \) and \( K \) are defined as proportions (expected) of loans, deposits and equity to the total amount of funds.

The results from maximizing \( V \) (given by (2-32)) subject to (2-35) are given by the following equilibrium pricing relationship

\[ r_L(1 + 1/e_L) - \frac{\lambda}{(\partial L/\partial r_L)} \cdot \frac{\partial \text{Cov}(\Pi, r_m)}{\partial r_L} = \]  
\[ r_D(1 + 1/e_D) + \frac{\lambda}{(\partial D/\partial r_D)} \cdot \frac{\partial \text{Cov}(\Pi, r_m)}{\partial r_D} \]  

(2-36)

where \( e_L \) and \( e_D \) are the mean elasticity of loans and deposits.

Using equilibrium relationships such as the one described in (2-36), Mason\(^3\) pointed out the significance of quantity variations as opposed to interest rate variations.

for the commercial banking firm. He then contrasted this relationship (2-36) to the one developed for a mutual fund in trying to establish modeling differences for these two types of intermediaries.

Chateau\textsuperscript{34} employed the same framework as Mason for the analysis of rate differentials in correlated-uncorrelated multideposit markets. He showed that under some conditions excess liability capacity may be optimal for an intermediary which must meet capital adequacy tests with attached penalties, or enforceable balance-sheet growth limitations.

O'Hara\textsuperscript{35} developed a dynamic model of a commercial bank which incorporates the functions of brokerage and risk transformation. The model is novel in the sense that it explicitly considers the agency costs incurred due to separation of management from the ownership of the institution.

A substantial amount of literature has focused on the development of a theory of bank loan commitments. Thakor\textsuperscript{36} examined the characteristics of fixed versus variable rate loan commitments using a contingent-claims framework. Based on a multiperiod model of information asymmetry, Thakor

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attempted to provide an explanation of the trend toward the substitution of fixed rate for variable rate loan commitments. More recently, Deshmukh, Greenbaum and Kanatas\(^3\) have focused on the intermediary's choice between operating as a broker or as an asset-transformer. They have shown that an increase in interest rate uncertainty leads the intermediary toward a reduction in the asset transformation function and to an increase in brokerage services. Their model is based on balance-sheet duration mismatching.

Conclusions form the Review

This chapter has reviewed the major approaches employed to model the depository financial intermediary. Its scope has been limited to models that consider both asset and liability decisions as well as the determination of the size or scale of the intermediary. This chapter will now address some of the issues raised by modeling differences. This discussion will be followed by an examination of the issues and results arising from these models and approaches.

Modeling differences can be explained in terms of at least three key assumptions. The first concerns the competitive nature of the loan and deposit markets in which the intermediary operates. In the loan and deposit markets that are assumed perfectly competitive, the intermediary

behaves as a quantity setter or rate taker. That is, it has no control over the rates to be paid on loans or on the yields provided to depositors. These rates are essentially exogenous to the institution and cannot be determined within the model. Two immediate consequences of this assumption are (1) uncertainty enters via random rates of return on assets and liabilities, and (2) only a portfolio decision has to be made to specify completely the balance sheet of the institution.

Such is the case of portfolio models. If on the other hand, the loan and deposit markets are assumed imperfect in nature, the intermediary becomes a rate-setter in these markets and exerts complete control over rate decisions. With this type of behavioral mode, the intermediary faces quantity uncertainty and must balance assets and liabilities before and after rates are announced. The problem becomes more cumbersome in that the possibility exists that assets might fall short of liabilities. It is this possibility that gives rise to a liquidity decision that must be made at the start of the period: enough liquid assets must be on hand or the intermediary must have access to funds that can be borrowed in order to meet liquidity deficiencies. Such is the case of firm-theoretic models.

A second assumption that distinguishes firm-theoretic from portfolio models concerns the risk preferences of the intermediary's shareholders. In contrast to portfolio models, firm-theoretic models assume linear risk preferences.
or risk neutrality on the part of the institution. This implies that the intermediary does not consider the variability of its income stream as a factor in its objective function. Presumably, a risk-neutral intermediary would be willing to accept a rate of return on loans equal to the risk-free rate regardless of the perceived risk of the loan.

The third and final distinguishing assumption relates to the existence of resource costs. Classified as firm-theoretic, production cost models have emphasized the presence of real resource costs incurred by the intermediary in providing depositors with such services as safekeeping, check clearing, and bookkeeping. However, propounders of these type of models have, for the most part, neglected to consider such important aspects of the theory of intermediaries as nonlinear risk preferences, imperfect markets, and liquidity adjustment costs.

In terms of significant results, the majority of the earlier work done on the theory of depository intermediaries tended to concentrate on issues concerning the separability of the intermediary's asset and liability decisions. Most notably Pyle, using a portfolio model, concluded that asset and liability portfolios of the intermediary generally could not be chosen independently. Additionally, his statement concerning the necessary conditions for intermediation to occur still remains as one of the most important results.

\[^{38}\text{Pyle, loc. cit.}\]
in this area.

Klein\textsuperscript{39} provided another significant contribution to this area of finance. Basing his results on a firm-theoretic model of the intermediary, Klein concluded that asset and liability decisions of the intermediary were independent.

More recently, Sealey\textsuperscript{40} attempted to show that the conclusions reached by Pyle lacked generality and were restricted to cases where the intermediary was assumed to follow a quantity-setting behavioral mode. Sealey also investigated the effects of risk aversion on the intermediary's loan position and deposit rate decision. His conclusions were mostly restricted by the functional form of the deposit supply function used in his model.

Chapter 3 will introduce a rate-setting model of the intermediary under uncertainty. The model is used to examine the effects of quantity uncertainty and risk aversion on the intermediary's management of its spread between loan and deposit rates.

\footnote{\textsuperscript{39}Klein, loc. cit.}

\footnote{\textsuperscript{40}Sealey, op. cit., pp. 1150-51.}
Chapter 3

A MODEL OF A DEPOSITORY FINANCIAL INTERMEDIARY

The approaches employed in modeling the behavior of financial intermediaries have been the subject of considerable controversy.\(^1\) The issues have ranged from the effects of regulation (such as capital adequacy) to problems dealing with the management of specific assets and liabilities. One aspect of the theory of financial intermediaries has received far less attention in the literature: the rate-setting behavior of depository intermediaries in an uncertain environment. Sealey\(^2\) was among the first to consider the effects of uncertain deposit supplies on the ex-ante deposit rate paid by a risk averse intermediary. More recently, Chateau\(^3\) considered rate-setting strategies under correlated-uncorrelated multi-deposit markets using a model of an intermediary within the CAPM framework.


Up to date, most have focused on extending the theory of depository intermediaries to consider deposit rate setting under uncertainty and risk aversion. As Mason\textsuperscript{4} has pointed out, depository intermediaries are, for the most part, ex-ante rate setters in the loan and deposit markets. Ex-ante rate setting coupled with a downward-sloping demand for loans and an upward-sloping supply of deposits causes the intermediary's balance sheet to be uncertain. One important effect of this uncertainty is on the liquidity that these institutions must maintain in order to deal with unexpected changes in assets and liabilities.

For more than a decade depository institutions have attempted to manage their balance sheets by focusing on the liability side. One liability management practice which has found widespread use among depository institutions is the so-called "spread management".\textsuperscript{5} Spread management emerged during the early 1970's when depository institutions experienced increasing loan demand coupled with declining growth in deposits. Institutions such as banks turned to non-deposit sources of funds to meet their needs while maintaining an adequate spread between lending and borrowing rates. The mechanism by which institutions determine and


control the spread is yet to be understood.\textsuperscript{6}

The purpose of this dissertation is to explore systematically the simultaneous loan and deposit rate-setting behavior of depository financial intermediaries under alternative risk preferences. This chapter describes the proposed model and its solution.

The proposed model is that of a rate-setting depository financial intermediary under conditions of uncertainty. In essence, the view of the intermediary proposed in this study is analogous to that of a spread manager attempting to maximize the expected utility of profits subject to a balance sheet constraint. Accordingly, the spread is taken to be the difference between the rate charged on loans and the yield offered on deposit liabilities.

The framework employed is a single period or static model. Uncertainty enters the model in two ways: random loan demand and random deposit supplies at the start of the decision period.

The intermediary is assumed to hold assets consisting of loans and reserves in the form of investments in the Federal funds market. Only a single type of deposit liability is considered. In addition, the model assumes that the intermediary has access to the Federal funds market as a source of nondeposit financing and that equity capital

\textsuperscript{6}Two reasons that perhaps account for this are 1) lack of support for this practice from the finance discipline, 2) absence of models of the intermediary which emphasize this behavior.
equals zero.\textsuperscript{7} The intermediary incurs real resource costs in the production and maintenance of services associated with loan and deposit contracts. Production (and thus cost functions of the intermediary) are assumed separable.\textsuperscript{8} Finally, the model recognizes the possibility of liquidity adjustment costs associated with random deposit supplies and uncertain loan demand.

This chapter is organized as follows. It first describes the model and its assumptions. The model's solution, structured into four parts, then follows: 1) Certainty Equivalent Case, 2) Loan Uncertainty Case, 3) Deposit Uncertainty Case, 4) Loan and Deposit Uncertainty Case.

\textbf{Structure of the Balance Sheet}

The depository intermediary described in this model can acquire two kinds of assets: loans (private obligations of individuals and firms) and loans in the Federal funds market (lending excess funds). The intermediary issues two types

\textsuperscript{7}One of the distinguishing characteristics of financial intermediaries is the small amount of capital held in relation to total liabilities. In most cases, capital regulation sets a minimum ratio of net worth to deposits for these firms. See for example, Oliver D. Hart and Dwight M. Jaffee, "On the Application of Portfolio Theory to Depository Financial Intermediaries," Review of Economic Studies, 41:129-47, January 1974.

of liabilities: deposits and borrowings in the Federal funds market.

The balance sheet constraint takes the following form:

\[ L^* + R = D^* \]  \hspace{1cm} (3-1)

where:
\[ L^* = \text{ex-ante loans}. \]
\[ R = \text{Federal funds lending/borrowing}. \]
\[ D^* = \text{ex-ante deposits}. \]

Note that R is a composite variable representing the net lending and borrowing in the Federal funds market. An \( R > 0 \) implies that the intermediary is a net lender of Federal funds. If \( R = 0 \), the intermediary does not lend or borrow Federal funds. Finally, an \( R < 0 \) implies the intermediary is a net borrower in the Federal funds market.

The model assumes that the intermediary holds no capital, an assumption adopted in several places in the literature. For example, Hart and Jaffee\(^{10}\) justified this assumption by pointing out that capital constitutes a small portion of the intermediary's total claims. Furthermore, in most instances, capital remains fixed relative to deposit liabilities. The model also abstracts from legal reserve requirements and other regulatory constraints usually imposed on financial intermediaries.

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\(^9\)Note that a single type of homogeneous, interest bearing liability is assumed.

\(^{10}\)Hart and Jaffee, op. cit., pp. 130-32.
The Intermediary's Loan Market

Loans are private contracts issued by the intermediary and held by individual economic units. The average maturity of a loan contract is assumed to coincide with the length of the intermediary's decision period. All borrowers are viewed as homogeneous by the intermediary and with equal risk of default characteristics. All nonprice terms on loans, such as collateral provisions and other protective covenants are assumed identical for every borrower.

The intermediary faces a loan market characterized by market imperfections and demand uncertainty. Under an imperfect loan market, the intermediary sets its rate on loans and confronts a random demand curve for loans. The model assumes that the intermediary's loan demand function contains a random error term which is unknown at the time rate decisions are made. In its most general form, the intermediary's loan demand can be written as

\[ L^* = L^*(r_L, \mu), \]

where \( L^* \) is the amount of loans demanded, \( r_L \) is the rate of interest on loans, and \( \mu \) is a random term having a known probability density function \( g(\mu) \). The random function in (3-2) simply states that for each set of values for \( r_L \) and \( \mu \), the intermediary knows the amount of loans that will be demanded. But, since \( \mu \) is unknown at the start of the

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decision period, the intermediary is unable to forecast the exact value of \( L^* \) in advance and must wait until a particular value of \( \mu \) obtains. It is assumed that the size of \( \mu \) cannot be altered by changing \( r_L \). One additional assumption concerns the functional form of the random loan demand given by (3-2): \( \mu \) affects the quantity of loans demanded in an additive fashion.\(^{12}\) Thus, the loan demand function is written as

\[
L^* = L(r_L) + \mu, \tag{3-3}
\]

where \( L(r_L) \) is the certain loan demand and \( \mu \) has an expected value equal to zero over its domain \((-L, \infty)\). In addition, \( L(r_L) \) is assumed to be a downward-sloping function of \( r_L \); or \( L'(r_L) < 0 \).

The intermediary incurs resource costs in connection with the maintenance and service of loan contracts. Costs include those resulting from the evaluation and control of collateral, appraisal of credit risks, bookkeeping costs, and so on. The total administrative costs on loans is given by the product of a constant marginal administrative cost on loans, \( c_L \), times the total amount of loans, \( L^* \).

The Intermediary's Deposit Market

The model will consider a single homogeneous type of deposit liability. The intermediary acts as a monopsonist

in the deposit market, setting the rate of interest on deposits and confronting an upward-sloping supply function of deposits. Uncertainty in the deposit market is modeled through the use of an additive random term $\varepsilon$ with known probability density function $h(\varepsilon)$. The deposit supply function is given by

$$D^* = D(r_D) + \varepsilon,$$

where $D(r_D)$ is the certain deposit supply; $\varepsilon$ is a random disturbance term with expected value of zero over its domain $(-D, \infty)$. $D(r_D)$ is an upward-sloping function of $r_D$; $D'(r_D) > 0$.

The marginal administrative cost incurred in the maintenance of deposit contracts is assumed constant. The total administrative cost of deposits is given as the product of the marginal administrative cost of deposits, $c_D$, and the amount of deposits, $D^*$.

**Federal Funds Borrowing/Lending and the Timing of Decisions**

The depository intermediary is assumed to have access to the borrowing and lending of funds in the Federal funds market. Through its choice of $r_L$ and $r_D$, the intermediary will also be selecting whether it wants to be an expected net lender ($R > 0$), expected net borrower ($R < 0$), or neither a lender nor a borrower ($R = 0$). The ultimate realized value of $R$ depends on rate relationships and conditions in the loan and deposit markets.

The intermediary will not be assumed to be a storer of liquidity, that is, one keeping a certain amount of liquid
assets on stock to meet liquidity needs. Instead, the firm uses the Federal funds market as a source or as a use of funds in order to make needed balance-sheet adjustments after resolution of loan and deposit uncertainties.

The timing of decisions is as follows. At the start of its decision period, the intermediary makes decisions concerning the rates on loans and deposits. Assumed known at this point are 1) the probability density functions of $U$ and $e$, and 2) $L(r_L)$ and $D(r_D)$ at the selected rates $r_L$ and $r_D$. Following the selection of $r_L$ and $r_D$, the uncertainty surrounding loan demand is revealed. Immediately following this, the supply of deposits at the posted rate, $r_D$, is realized. At this point, the intermediary makes the necessary balance-sheet adjustments by using the Federal funds market. This eventually determines the value of profit at the end of the period.

**The Intermediary's Objective Function**

The intermediary's objective is to maximize the expected utility of profits subject to a balance sheet constraint. Let $U(\Pi)$ be a von Neumann-Morgenstern utility function defined in terms of end-of-period profits, $\Pi$, with properties such that $U'(\Pi)>0$, and $U''(\Pi)>0$, depending on whether the intermediary is risk seeker, risk neutral, or risk averse.

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13 The Federal funds market is assumed to be perfectly competitive with a perfectly elastic supply of funds at an exogenous rate of $r\%$. 

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The optimization problem is given by

$$\text{Maximize } \mathcal{U} = \int_{-L}^{\infty} \int_{D}^{\infty} U(\pi) h(\varepsilon) g(\mu) d\varepsilon d\mu$$  \hspace{1cm} (3-5)$$

subject to:

$$L^* + R = D^*,$$

where $\mathcal{U}$ is the expected value of $U(\pi)$.

Profits can be expressed as the difference between revenues and costs:

$$\Pi = r_L L^* - c_L L^* - r_D D^* - c_D D^* + rR.$$  \hspace{1cm} (3-6)

The constrained optimization problem given by (3-5) can be transformed into an unconstrained objective function. Solving for $R$ out of (3-1), and using the definitions of $L^*$ and $D^*$ given by (3-3) and (3-4) respectively, (3-5) can be stated as:

$$\text{Maximize } \int_{-L}^{\infty} \int_{D}^{\infty} \left[r_L (r_L \mu + \mu) - c_L (r_L \mu + \mu) ight.$$

$$\left. - r_D (r_D \varepsilon + \varepsilon) - c_D (r_D \varepsilon + \varepsilon) + r[D(r_D \varepsilon + \varepsilon) ight.$$

$$\left. - L(r_L \mu + \mu)]h(\varepsilon)g(\mu)d\varepsilon d\mu. \right]$$  \hspace{1cm} (3-7)$$

Risk Premiums and Measures of Risk Aversion

Before considering the solution of the model, it is useful at this point to review the notions of risk premiums and measures of risk aversion as they have been applied to individuals. Risk premium is defined as the maximum amount of expected income a risk averse individual is willing to forego in order to avoid a risky prospect. Risk premium can also be interpreted as the maximum amount of expected income a risk preferring individual is willing to pay in order to obtain a risky prospect.
Figure 1(a) illustrates the case of an individual with a risk averse utility function. Suppose this individual is faced with a random game which could increase his wealth to the level $\Pi_1$, or lower it down to $\Pi_0$. The individual's expected wealth would be given by $\hat{\Pi}$. The certainty equivalent of the game, $\hat{\Pi}$, would then be defined by

$$\hat{\Pi} = \Pi - \theta^a$$

where $\theta^a$ is the risk premium for the risk averse utility function. It is easily verified that $\theta^a > 0$, since $\Pi > \hat{\Pi}$, as shown in Figure 1(a). Figures 1(b) and 1(c) illustrate the risk neutral and risk preferring cases. From 1(b) it is clear that if the utility function is linear in $\Pi$ (risk neutral case), the risk neutral premium, $\theta^0$, equals zero. On the other hand, if the individual prefers risk (convex utility function), the risk premium is less than zero. The risk preferring premium, denoted by $\theta^p$, is defined by

$$\hat{\Pi} = \Pi - \theta^p,$$
where $\hat{\Pi}$ is the certainty equivalent of $\Pi$ for the risk preferring case. Figure 1(c) shows $\hat{\Pi}$ lies to the right of $\Pi$. Consequently, $\theta^P < 0$, since the individual is willing to pay in order to engage in a risky gamble. Note that as risk aversion decreases, the size of the risk premium decreases as well, and $\theta^a > \theta^n > \theta^P$. It should also be noted that given risk aversion, the risk premium is always greater than zero. Furthermore, for a given risky prospect, the greater the risk premium, the more risk averse an individual is. Thus, the size of the risk premium measures the degree or strength of risk aversion exhibited by a utility function for a given risky gamble.

As a measure of risk aversion, the risk premium depends on the utility function and on the distribution of the random prospect being considered. For a given risky gamble, the size of the premium demanded depends upon the individual's initial wealth. If the size of the risk premium demanded for a given risky prospect decreases as the individual's wealth increases then his behavior is consistent with decreasing absolute risk aversion (DARA). This case is illustrated below in Figure 2.

Figure 2 shows the risk premiums, $\theta_0$ and $\theta_1$, demanded by a risk averse individual for the same risky gamble at two different levels of profit, $\Pi_0$ and $\Pi_1$. The expected values of $\Pi_0$ and $\Pi_1$ are given by $\Pi_0$ and $\Pi_1$ respectively. With decreasing absolute risk aversion, if one offered the same risky prospect to a risk averse individual at $\Pi_0$ and then at
\( \Pi_1 \), the size of the risk premium demanded by the individual would decrease as his/her level of wealth (or profit in this case) increased. As shown in Figure 2, \( \theta_0 > \theta_1 \) since \( \Pi_1 > \Pi_0 \). If the risk premium demanded by the individual increased with his/her wealth, then his/her behavior would be consistent with increasing absolute risk aversion (IARA). Finally, if the risk premium demanded by the individual for a given risky prospect remains unchanged as his/her wealth changes, then the individual's behavior is consistent with constant absolute risk aversion (CARA).

An important measure of risk aversion in the absolute sense was first developed by Pratt\(^{14}\) and independently by Arrow.\(^{15}\) The Arrow-Pratt index of absolute risk aversion is

\[ \theta \]


defined as:

\[ R_a(\Pi) = -\frac{U''(\Pi)}{U'(\Pi)} \]

where \( U'(\Pi) \) and \( U''(\Pi) \) are the first and second derivatives of the utility function with respect to profit. \( R_a(\Pi) \) measures absolute risk aversion at a given level of wealth (in terms of profit or income), and for a particular utility function. Pratt\(^1\) demonstrated that \( R_a(\Pi) \) is proportional to the risk premium demanded by a risk averse individual.

Given DARA, the size of the risk premium is inversely related to the level of wealth, and \( R_a(\Pi) \) is a decreasing function of \( \Pi \). Under IARA, the risk premium increases with the level of wealth for a given risky prospect, and \( R_a(\Pi) \) is an increasing function of \( \Pi \). Finally, if the risk premium demanded for a given gamble remains constant as the level of profit changes, then \( R_a(\Pi) \) is a constant. This is the case of constant absolute risk aversion (CARA). In general, the most widely accepted hypothesis about the relationship between wealth and risk taking behavior is that of decreasing absolute risk aversion (DARA).\(^2\)

The remainder of this chapter will focus on the solution of the model. The certainty equivalent or risk neutral case will be considered first.

\(^1\)Pratt, op. cit., p. 118.
The Certainty Equivalent Case

Objective Function

Under the certainty equivalent case, the intermediary maximizes the expected value of unconstrained profits. The objective function is given by

\[
\max \pi = r_L L(r_L) - c_L L(r_L) - r_D D(r_D) - c_D D(r_D) + r[D(r_D) - L(r_L)] ,
\]

where \( \pi \) is the expected value of profits with the balance sheet constraint incorporated into it. The random terms \( y \) and \( \varepsilon \) disappear since \( E[y] = E[\varepsilon] = 0 \).

First and Second Order Conditions

The first order conditions are obtained by taking the partials of \( \pi \) with respect to \( r_L \) and \( r_D \) and setting them equal to zero:

\[
\pi_L = L(r_L) + L'(r_L)[r_L - c_L - r] = 0 . \tag{3-9}
\]
\[
\pi_D = -D(r_D) + D'(r_D)[r - r_D - c_D] = 0 . \tag{3-10}
\]

Inspection of (3-9) reveals that an interior solution for \( r_L \) requires that \( r_L - c_L - r > 0 \), since \( L(r_L) > 0 \) and \( L'(r_L) < 0 \) by assumption. In the same manner, an interior solution for \( r_D \) requires that \( r - r_D - c_D > 0 \), since \( D'(r_D) > 0 \) by assumption and \( -D(r_D) < 0 \). Thus, combining the two conditions, an interior solution for \( r_L \) and \( r_D \) requires that

\[
r_L - r_D > c_L + c_D , \tag{3-11}
\]
or that the spread between \( r_L \) and \( r_D \) be strictly greater than the sum of the marginal administrative cost of loans and deposits.

The second order conditions for a maximum are given by

\[
\pi_{LL} = [2L'(r_L) + L''(r_L)[r_L - c_L - r]] < 0 \quad (3-12)
\]

\[
\pi_{DD} = [-2D'(r_D) + D''(r_D)[r - r_D - c_D]] < 0 \quad (3-13)
\]

\[
D = \pi_{LL}\pi_{DD} - (\pi_{LD})^2 > 0. \quad (3-14)
\]

Since \( \pi_{LD} = 0 \), sufficient conditions for a maximum are that \( L''(r_L) \leq 0 \) and \( D''(r_D) \leq 0 \).

Using the first order conditions, an expression for the certainty equivalent spread may be obtained. Solving for \( r \) out of (3-10), substituting into (3-9), and rearranging yields

\[
r_L - r_D = \frac{D(r_D) - L(r_L)}{L'(r_L)} \quad (3-15)
\]

noting that \( \frac{D(r_D)}{D'(r_D)} = \frac{r_D}{e_D} \) and \( \frac{L(r_L)}{L'(r_L)} = \frac{r_L}{e_L} \); where \( e_D \) and \( e_L \) are the rate elasticities of the deposit supply and loan demand respectively. Hence (3-15) can be rewritten as

\[\text{These sufficient conditions together with the assumptions of downward-sloping loan demand and upward-sloping supply of deposits imply that: 1) loan demand is a decreasing function of } r_L \text{ at an increasing rate, and 2) deposit supply is an increasing function of } r_D \text{ at a decreasing rate.}\]
\[ r_L - r_D = c_L + c_D + \frac{r_D}{e_D} + \frac{r_L}{e_L} \quad (3-16) \]

From (3-16), the certainty equivalent spread appears to be a function of the marginal administrative cost of loans and deposits, the elasticities of loan demand and supply of deposits, and the rates on loans and deposits. Note that the assumption of imperfect loan and deposit markets is of foremost importance with respect to the size of the spread. As the loan and deposit markets become perfectly competitive, \( e_D \) and \( e_L \) approach infinity. The last two terms in (3-16) vanish, and the spread narrows. As would be expected, imperfect loan and deposit markets allow the intermediary to operate at a wider spread than it would otherwise under perfectly competitive markets.

The certainty equivalent spread in (3-16) can be shown to be identical to the spread set by the intermediary under conditions of risk neutrality.\(^{19}\)

**Comparative Statics**

A number of comparative static results can be established with respect to the certainty equivalent or risk neutral solution. For example, how would a change in the marginal administrative cost of loans affect the rates chosen by the intermediary? What would be the overall

\(^{19}\)This can be easily shown by finding the first order condition of the problem given by (3-7) and using the assumption of risk neutrality; that is, \( U(\Pi) \) is linear and \( U'(\Pi) = \text{constant} \). Once this assumption is incorporated, then it will follow that \( U_L = \tilde{\Pi}_L \) and \( U_D = \tilde{\Pi}_D \).
impact of this change on the risk neutral spread?

Implicit differentiation of the first order conditions with respect to $c_L$ yields

$$\frac{\partial r_L}{\partial c_L} = -D^{-1}[\Pi_{Lc_L} \cdot \Pi_{DD}] \quad (3-17)$$

$$\frac{\partial r_D}{\partial c_L} = -D^{-1}[\Pi_{LL} \cdot \Pi_{Dc_L}] \quad (3-18)$$

From (3-17), $\Pi_{Lc_L} = -L'(r_L) > 0$. Since $\Pi_{DD} < 0$ and $D < 0$ by the second order conditions, $\frac{\partial r_L}{\partial c_L} > 0$. The certainty equivalent rate on loans is an increasing function of the marginal administrative cost of loans. $\Pi_{Dc_L} = 0$ in (3-18) implies that $\frac{\partial r_D}{\partial c_L} = 0$. Thus, the certainty equivalent rate on deposits is invariant to changes in $c_L$. After combining the two effects (i.e., $\frac{\partial r_L}{\partial c_L} > 0$ and $\frac{\partial r_D}{\partial c_L} = 0$), the certainty equivalent spread will be an increasing function of the marginal administrative cost of loans.

In similar fashion, implicit differentiation of the first order conditions with respect to $c_D$ yields

$$\frac{\partial r_L}{\partial c_D} = -D^{-1}[\Pi_{Lc_D} \cdot \Pi_{DD}] \quad (3-19)$$

$$\frac{\partial r_D}{\partial c_D} = -D^{-1}[\Pi_{LL} \cdot \Pi_{Dc_D}] \quad (3-20)$$

Since $\Pi_{Lc_D} = 0$ in (3-19), $\frac{\partial r_L}{\partial c_D} = 0$, and the certainty equivalent rate on loans is invariant to changes in the marginal administrative cost of deposits. From (3-20),
\[ \Pi_{DCD} = -D'(r_D) < 0, \quad \Pi_{LL} < 0, \quad \text{and} \quad D > 0 \] by the second order conditions, thus \( \frac{\partial r}{\partial c_D} < 0 \). The certainty equivalent rate on deposits is a decreasing function of the marginal administrative costs of deposits. An increase (decrease) in the marginal administrative costs of deposits will tend to lower (raise) the rate paid on deposits while having no effect on the loan rate. Thus, the certainty equivalent or risk neutral spread will be an increasing function of the marginal administrative cost of deposits as well.

Finally, how would the rates on loans and deposits change in response to changes in the cost of liquidity adjustments, \( r \)? Implicit differentiation of the first order conditions with respect to \( r \) yields

\[
\frac{\partial r_L}{\partial r} = -D^{-1}[\Pi_{LR} - \Pi_{DD}] > 0, \quad (3-21)
\]

and

\[
\frac{\partial r_D}{\partial r} = -D^{-1}[\Pi_{LL} - \Pi_{Dr}] > 0. \quad (3-22)
\]

Both, \( \frac{\partial r_L}{\partial r} \) and \( \frac{\partial r_D}{\partial r} \) are strictly positive. This follows from fact that \( \Pi_{LR} = -L'(r_L) > 0 \) and \( \Pi_{Dr} = D'(r_D) > 0 \).

The certainty equivalent loan and deposit rates are both increasing functions of \( r \).

The overall effect on the certainty equivalent spread from a change in \( r \) is not clear. With both, since the effects on loan and deposit rates from a change in \( r \) are in the same direction, the resulting change in the size of the
spread cannot be determined. Subtracting (3-22) from (3-21) yields the following expression for the change in the spread from a change in \( r \):

\[
\frac{\Delta}{\Delta r} (r_L - r_D) = \frac{L'(r_L)}{\Pi_{LL}} + \frac{D'(r_D)}{\Pi_{DD}}.
\]  

(3-23)

The first term in (3-23) can be explained as the marginal loan demand created by a change in \( r_L \) from the certainty equivalent rate. The second one is the marginal supply of deposits resulting from a change in \( r_D \) from the optimal certainty equivalent rate. The magnitude of each of these terms depends on the rate elasticities of loan demand and deposit supply.

Consider now the effects of changes in the administrative cost of loans and deposits on the intermediary's participation in the Federal funds market. How would changes in \( c_L \) and \( c_D \) affect the volume of Federal funds purchased or sold by the institution? The certainty equivalent borrowing/lending of Federal funds is given by:

\[
R = D(r_D) - L(r_L).
\]  

(3-24)

Differentiation of (3-24) with respect to \( c_L \) and \( c_D \) yields

\[
\frac{\partial R}{\partial c_L} = D' \frac{\partial r_D}{\partial c_L} - L' \frac{\partial r_L}{\partial c_L},
\]

(3-25)

and

\[
\frac{\partial R}{\partial c_D} = D' \frac{\partial r_D}{\partial c_D} - L' \frac{\partial r_L}{\partial c_D}.
\]  

(3-26)
Inspection of (3-25) reveals that \( \frac{\partial R}{\partial c_L} > 0 \), since \( \frac{\partial D}{\partial c_L} = 0 \) and \( \frac{\partial L}{\partial c_L} < 0 \). Thus, as the administrative cost per loan increases, the intermediary shifts deposit funds from the loan market into the Federal funds market.

From (3-26), \( \frac{\partial R}{\partial c_D} < 0 \), since \( \frac{\partial D}{\partial c_D} < 0 \) and \( \frac{\partial L}{\partial c_D} = 0 \). As the marginal administrative cost of deposits increases, the intermediary tends to reduce the volume of lending in the Federal funds market and/or begins to borrow Federal funds.

Finally, what would be the effect of a change in the Federal funds rate on the borrowing/lending volume of the intermediary? Differentiating (3-24) with respect to \( r \) yields

\[
\frac{\partial R}{\partial r} = D' \frac{\partial D}{\partial r} - L' \frac{\partial L}{\partial r}.
\]

It is easily verified that \( \frac{\partial R}{\partial r} > 0 \) since \( \frac{\partial D}{\partial r} > 0 \), and \( \frac{\partial L}{\partial r} > 0 \). Therefore, as the Federal funds rate increases, the intermediary increases the amount of lending in Federal funds (or, borrows less Federal funds).

This section has presented the solution to the certainty equivalent model and the comparative statics results. Also examined were the effects of changes in cost parameters of the model on the intermediary's borrowing/lending in Federal funds. The results can be summarized as follows: 1) \( r_L \) is an increasing function of \( c_L \), invariant with respect to \( c_D \), and increasing in \( r \); 2) \( r_D \) is a...
decreasing function of \( c_D \), invariant to \( c_L \), and increasing in \( r \). The spread between \( r_L \) and \( r_D \) increases (decreases) as the marginal administrative cost of loans and deposits increases (decreases). The effect on the spread from a change in the Federal funds rate depends on the rate elasticities of deposit supply and loan demand. With respect to the intermediary's participation in the Federal funds market, any change in the cost of either loans or deposits (such as change in \( c_L \) or \( c_D \)) causes the intermediary to shift towards the constant cost (return) source (use) of the perfectly competitive Federal funds market. For instance, an increase in administrative expenses of deposits raises the cost of this particular source of funds to the intermediary. As a result, the intermediary will use the Federal funds market as an alternative source of funding and its borrowings from the Federal funds market will increase. Finally, an increase in the Federal funds rate increases the opportunity cost to the intermediary. Consequently, the intermediary will increase the amount of lending of Federal funds.

The results indicate that the loan and deposit pricing decisions are made independently by the intermediary.

One final note with respect to the certainty equivalent solution of the model. Equations (3-9) and (3-10) yield

\[
\begin{align*}
 r_L + \frac{L}{L'} &= c_L + r \\
 r &= c_D + r_D + \frac{D}{D'}
\end{align*}
\]
respectively. Recall that $L' = \frac{\partial L}{\partial r_L}$. Hence, $r_L + \frac{L}{L'} = r_L + L \cdot \frac{\partial r_L}{\partial L}$, which is marginal revenue from loans, say $MR_L$. In addition, the firm can lend funds in the Federal funds market. The marginal revenue per dollar lent in the Federal funds market is $r$, since the firm is a perfect competitor in the Federal funds market. Similarly, $D' = \frac{\partial D}{\partial r_D}$ and $c_D + r_D + \frac{D}{D'} = c_D + r_D + D \cdot \frac{\partial r_D}{\partial D}$, which is the marginal cost of deposits, say $MC_D$. Therefore, the intermediary chooses $r_L$ and $r_D$ such that

$$MR_L = c_L + MC_D.$$  

The firm equates the marginal revenue from loans with the marginal cost of loans, which includes the marginal administrative cost of loans plus the marginal cost of deposits. If the firm lends funds in the Federal funds market ($R > 0$) as well as makes customer loans, it has two sources of revenue. One dollar lent to a customer cannot be lent in the Federal funds market. Thus, the opportunity cost of the last dollar lent to a customer is the marginal revenue given up in the Federal funds market; that is, $MR_L = c_L + r$.

The Loan Uncertainty Case

This section will consider the effects of uncertain loan demand on the rates set by the intermediary under different attitudes toward risk. For this particular case, deposits are assumed to be known with certainty. All of the other assumptions introduced at the beginning of this chapter remain in effect for the analysis of this section.
Comparative statics exercises link the Arrow-Pratt$^{20}$ index of absolute risk aversion to the effects on loan and deposit rates arising from changes in the marginal administrative and liquidity costs of the intermediary.

**Objective Function**

The objective function of the intermediary is given by

$$\text{Maximize } \int_{0}^{\infty} U(\Pi) g(\mu) d\mu, \quad (3-29)$$

where $U(\Pi)$ is a von Neumann-Morgenstern utility of profits function with $U'(\Pi) > 0$ and $U''(\Pi) \leq 0$, depending on whether the intermediary is risk averse, risk neutral, or risk seeking.

The intermediary's profit$^{21}$ ($\Pi$) is given by

$$\Pi = r_{L}[L(r_{L}) + \mu] - c_{L}[L(r_{L}) + \mu] - r_{D}D(r_{D}) - c_{D}D(r_{D})$$

$$+ r[D(r_{D}) - L(r_{L}) - \mu]. \quad (3-30)$$

**First and Second Order Conditions**

The first order conditions can be obtained by differentiating (3-29) with respect to $r_{L}$ and $r_{D}$. After replacing the integral sign with the expected value operator, $E$, the

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$^{21}$This profit function assumes deposits are certain (i.e., $\varepsilon = 0$).
first order conditions are

\[ \bar{U}_L = E\{U'(\Pi)[L(r_L) + \mu + L'(r_L)(r_L - c_L - r)]\} = 0 \]  
(3-31)

\[ \bar{U}_D = \{E\ U'(\Pi)[-D(r_D) + D'(r_D)(r - r_D - c_D)]\} = 0, \] (3-32)

where \( \bar{U}_L = \frac{\partial E[U(\Pi)]}{\partial r_L} \) and \( \bar{U}_D = \frac{\partial E[U(\Pi)]}{\partial r_D} \). Rewriting \( \bar{U}_L \) as

\[ \bar{U}_L = E\{U'(\Pi)[L(r_L) + \mu]\} + E\{U'(\Pi)L'(r_L)(r_L - c_L - r)\} = 0. \] (3-33)

\[ E\{U'(\Pi)[L(r_L) + \mu]\} > 0 \] since \( U'(\Pi) > 0 \) for all \( \Pi \), and \( L(r_L) + \mu \geq 0 \) for all \( \mu \). Thus, it follows that a necessary condition for an optimal value of \( r_L \) is that \( r_L - c_L - r > 0 \). Similarly \( \bar{U}_D \) can be rewritten as

\[ \bar{U}_D = E\{U'(\Pi)[-D(r_D) + D'(r_D)(r - r_D - c_D)]\} = 0. \] (3-34)

Eliminating \( E\{U'(\Pi)\} \) from (3-34), and recognizing \( D'(r_D) > 0 \), a necessary condition for an optimum value of \( r_D \) is that \( r - r_D - c_D > 0 \). Thus, just as in the certainty equivalent case, a necessary condition for an optimum value requires that the spread be strictly greater than the sum of the marginal administrative cost of loans and deposits.

The second order conditions for a maximum are

\[ \bar{U}_{LL} = E\{U'(\Pi)[2L'(r_L) + L''(r_L)(r_L - c_L - r)] \]
\[ + U''(\Pi)(\Pi_L)^2\} < 0, \] (3-35)

\[ \bar{U}_{DD} = E\{U'(\Pi)[-2D'(r_D) + D''(r_D)(r - r_D - c_D) \]
\[ + U''(\Pi)(\Pi_D)^2\} < 0 \]
and
\[ D = U_{LL}U_{DD} - (U_{LD})^2 > 0, \]
where \( U_L \) and \( U_D \) are the partial derivatives of \( U \) with respect to \( r_L \) and \( r_D \). Similarly, \( U_{LL} \) and \( U_{DD} \) are the second partials of the first order conditions with respect to \( r_L \) and \( r_D \). \( U_{LD} \) is given by
\[
U_{LD} = \frac{\partial^2 U}{\partial r_L \partial r_D} = E(HD) - E(U'(U)(U_L)). \tag{3-36}
\]
However, \( U_{LD} = 0 \) since \( E[H] = 0 \) by (3-32). Inspection of (3-35) reveals that sufficient conditions for a maximum in the case of the risk averse intermediary \( U''(U) < 0 \) are \( D''(r_D) < 0 \) and \( L''(r_L) < 0 \). For \( U''(U) > 0 \), the second order conditions may not be satisfied.

Before proceeding with the comparative statics of the model, it is possible to compare the rates on loans and deposits under loan demand uncertainty and different attitudes toward risk to the rates that would prevail under certainty.

**Certainty versus Uncertainty Comparisons**

It is possible to show that \( U'(U) \) and \( [L(r_L) + \mu + L'(r_L)[r_L - c_L - r]] \) in (3-31) are oppositely, independently, or similarly ordered in \( \mu \) depending on whether the intermediary is risk averse, risk neutral or risk seeking.
Since

\[ \frac{\partial U'(\Pi)}{\partial \mu} = U''(\Pi)[r_L - c_L - r] \leq 0 \text{ as } U''(\Pi) \leq 0 \quad (3-37) \]

and, \( \partial [\cdot] / \partial \mu = 1 \), where \( [\cdot] = [L(r_L) + \mu + L'(r_L)[r_L - c_L - r]] \). As a result, \( \text{Cov}(U'(\Pi), [\cdot]) < 0 \), for all \( U''(\Pi) < 0 \), and with \( E[U'(\Pi)] > 0 \), then \( E[L(r_L) + \mu + L'(r_L)[r_L - c_L - r]] > 0 \). Therefore, by theorem 236 in Hardy, Littlewood and Polya,\(^{22}\) it follows that

\[ [L(r_L) + L'(r_L)[r_L - c_L - r]] \geq 0 \text{ as } U''(\Pi) \leq 0. \quad (3-38) \]

Let \( r_L^n \) and \( r_D^n \) denote the risk neutral rates on loans and deposits which define the locus of points in the \((r_L, r_D)\) space such that \( \Pi_L[r_L, r_D] = 0 \). Defining \( r_L \) as an implicit function of \( r_D \), and differentiating \( \Pi_L[r_L(r_D), r_D] \) with respect to \( r_D \) yields

\[ \Pi_{LL} \frac{\partial r_L}{\partial r_D} + \Pi_{LD} = 0, \quad (3-39) \]

which implies that \( \partial r_L / \partial r_D = 0 \) by the second order condition, \( \Pi_{LD} = 0 \). Thus, \( \Pi_L = 0 \) if and only if \( r_L = r_L^n \). For \( \Pi_L > 0 \), \( r_L < r_L^n \) (see Figure 3).

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Define $r_L^a$ and $r_L^p$ as the rates charged on loans by a risk averse and a risk preferring intermediary. As demonstrated previously $\Pi_L(r_L^a) = [L(r_L^a) + L'(r_L^a)[r_L^a - C_L - r]] > 0$, which implies that $r_L^a < r_L^n$. For the risk preferring intermediary, $\Pi(r_L^p) < 0$, and $r_L^p > r_L^n$.

Letting $r_D^a$ and $r_D^p$ denote the rates paid on deposits by a risk averse and a risk preferring intermediary. Then, dividing (3-34) by $E[U'(\Pi)]$ yields:

$$[-D(r_D) + D'(r_D)[r - r_D - c_D]] = 0,$$

which implies that $\Pi_D = \Pi_D = 0$. Therefore, $r_D^a = r_D^n = r_D^p$.

Hence, the risk averse intermediary selects a loan rate that is lower than the risk neutral loan rate. If the institution prefers risk, it will charge a higher loan rate than that of the risk neutral intermediary. On the other
hand, deposit rates of the risk averse and risk preferring intermediary are the same as the risk neutral deposit rate. Consequently, loan demand uncertainty leads the risk averse intermediary to operate at a smaller spread than the risk neutral spread. In contrast, the risk preferring institution sets a spread that is larger than the risk neutral's.

One possible explanation of the effects of loan demand uncertainty and risk aversion on the size of the spread is in terms of the mean and variance of the intermediary's profits. It can be demonstrated that the variance of profits is an increasing function of \( r_L \). Under risk aversion, the intermediary dislikes dispersion or variability of profits and is willing to sacrifice mean income (from lower \( r_L \)) in exchange for the reduction in the variance of profits. Since the deposit rate remains at the risk neutral level, the reduction in \( r_L \) causes the spread under risk aversion to be smaller than the risk neutral spread.

Other questions arise. Will the risk averse firm tend to be a lender or a borrower in the Federal funds market? How does risk preference influence the intermediary's lending or borrowing behavior? Suppose the loan demand, deposit supply, liquidity cost and administrative expense conditions are such that the risk neutral firm expects to be

\[ \text{The variance of II is } \text{Var}(\Pi) = (r_L - c_L - r)^2 \text{Var}(\mu), \]

\[ \frac{\partial \text{Var}(\Pi)}{\partial r_L} = 2(r_L - c_L - r)\text{Var}(\mu) > 0. \]
neither a lender nor a borrower; that is, expected $R^n = D(r^n_D) - L(r^n_L) = 0$. Our results, $r^A_L < r^n_L < r^P_L$ and $r^A_D = r^n_D = r^P_D$, imply that expected $R^A < 0$ and expected $R^P > 0$. The risk averse intermediary chooses $r_L$ and $r_D$ such that it expects to be a borrower in the Federal funds market. The risk preferring firm chooses $r_L$ and $r_D$ such that it expects to be a lender in the Federal funds market. At first, these conclusions may appear highly counter intuitive. However, the lower $r_L$ chosen by the risk averse intermediary raises expected loans as it reduces the variance of random profit. Hence, the reduction in profit "risk" is obtained at the cost of borrowing in the Federal funds market.

**Comparative Statics**

The discussion now moves to the effects of changes in the marginal administrative and liquidity costs on the intermediary's spread. Inasmuch as real resource and liquidity costs are significant factors in the pricing of loans and deposits, it is important to consider the effects of changes in these costs on the optimal spread set by the intermediary. For example, are increases in the marginal administrative cost of loans reflected in a larger spread? Is the behavior of the risk-averse spread indistinguishable from the certainty equivalent case? Is the loan rate independent of the rate and costs of deposits?

Turning now to the comparative statics of the model, consider first the impact of a change in the marginal administrative cost of loans on the optimal loan rate as
chosen by the intermediary at the start of the period. Implicit differentiation of the first order conditions with respect to $c_L$ yields\(^{24}\)

$$\frac{\partial r_L}{\partial c_L} = -L'E[U'(\Pi)] - \frac{E[U''(\Pi)(L+\mu)[L+\mu+L'(r_L-c_L-r)]]}{U''L}$$  \hspace{1cm} (3-40)

The first term in the numerator of (3-40) is positive since $-L' > 0$ and $E[U'(\Pi)] > 0$. After one adds and subtracts $L'(r_L - c_L - r)$ inside the parenthesis at $(L + \mu)$, the second term in the numerator of (3-40) can be rewritten as

$$-E[U''(\Pi)(L + \mu)[L + \mu + L'(r_L - c_L - r)] =$$

$$-E[U''(\Pi)[L + \mu + L'(r_L - c_L - r)]^2]$$

$$+ E[U''(\Pi)[L + \mu + L'(r_L - c_L - r)]L'(r_L - c_L - r)].$$  \hspace{1cm} (3-41)

The first term in (3-41) is clearly positive for $U''(\Pi) < 0$.

**Proposition 1**: Given decreasing absolute risk aversion (DARA) in the Arrow-Pratt sense, the sign of $\frac{\partial r_L}{\partial c_L}$ is ambiguous.

**Proof**: In order to verify that $\frac{\partial r_L}{\partial c_L}$ is ambiguous under DARA, the sign of the second term in (3-41) must be shown to be negative.

Let $\mu^0$ be the value of $\mu$ such that $L + \mu^0 + L'(r_L - c_L - r) = 0$, and let $\Pi^0$ be the value of $\Pi$ when $\mu = \mu^0$. The Arrow-Pratt index of absolute risk aversion is defined as

\(^{24}\)Note that the arguments of $L$ and $D$ will be omitted in order to simplify the various expressions. The same omission applies to their derivatives.
\[ R_a(\Pi) = \frac{U''(\Pi)}{U'(\Pi)} \]  

(3-42)

\( R_a(\Pi) \) is decreasing under DARA, thus

\[ \frac{U''(\Pi)}{U'(\Pi)} < \frac{U''(\Pi^0)}{U'(\Pi^0)} \quad \text{for all } \mu \geq \mu^0, \]  

(3-43)

since \( \frac{\partial \Pi}{\partial \mu} = (r_L - c_L - r) > 0 \). In addition,

\[ U'(\Pi)[L + \mu + L'(r_L - c_L - r)] \geq 0 \quad \text{for all } \mu \geq \mu^0 \]  

(3-44)

by construction. Therefore, from (3-43) and (3-44) it follows that for all \( \mu \geq \mu^0 \),

\[ \frac{U''(\Pi)}{U'(\Pi)} \leq \frac{U''(\Pi^0)}{U'(\Pi^0)} \cdot U'(\Pi)[L + \mu + L'(r_L - c_L - r)]. \]  

(3-45)

Now, consider the case when \( \mu < \mu^0 \). Then,

\[ \frac{U''(\Pi)}{U'(\Pi)} > \frac{U''(\Pi^0)}{U'(\Pi^0)} \]  

(3-46)

by DARA, since \( \frac{\partial \Pi}{\partial \mu} > 0 \). Similarly,

\[ U'(\Pi)[L + \mu + L'(r_L - c_L - r)] < 0 \quad \text{for all } \mu < \mu^0 \]  

(3-47)

by construction. Hence, from (3-46) and (3-47),

\[ \frac{U''(\Pi)}{U'(\Pi)} \leq \frac{U''(\Pi^0)}{U'(\Pi^0)} \cdot U'(\Pi)[L + \mu + L'(r_L - c_L - r)]. \]  

(3-48)
By (3-48) one concludes that (3-45) holds for all values of \( \mu \). Therefore, taking expected values of both sides of (3-45) gives

\[
- \mathbb{E}\{U''(\Pi)[L + \mu + L'(r_L - c_L - r)]\} < - \frac{U''(\Pi)}{U'(\Pi)} \cdot \mathbb{E}\{U'(\Pi)[L + \mu + L'(r_L - c_L - r)]\} \]

\( = 0 \). (3-49)

by the first order condition for \( r_L \). From (3-49) it follows that,

\[
\mathbb{E}\{U''(\Pi)[L + \mu + L'(r_L - c_L - r)]\} > 0. \quad (3-50)
\]

Multiplying both sides of (3-50) by \( L'(r_L - c_L - r) \) yields the desired result that

\[
\mathbb{E}\{U''(\Pi)[L + \mu + L'(r_L - c_L - r)] L'(r_L - c_L - r)\} < 0.
\]

(3-51)

Hence, the sign of (3-41) cannot be determined, and \( \partial r_L / \partial c_L \) is ambiguous under DARA.

**Proposition 2):** Given constant absolute risk aversion (CARA), the optimal loan rate is an increasing function of the marginal administrative cost of loans.

**Proof:** The proof that \( \partial r_L / \partial c_L > 0 \) under CARA requires that (3-37) be signed as positive.

Since the first term in (3-41) is always positive for \( U''(\Pi) < 0 \), it remains to show that the second term is either positive or zero. Under CARA, the second term of (3-41) can be expressed as
\[ \mathbb{E}\{U''(\Pi)[L + \mu + L'(r_L - c_L - r)]\} L'(r_L - c_L - r) = \]
\[ \mathbb{E}\{U''(\Pi) \frac{U'(\Pi)}{U'(\Pi)} \cdot [L + \mu + L'(r_L - c_L - r)]\} L'(r_L - c_L - r). \]
\[ (3-52) \]
Since \( \frac{U''(\Pi)}{U'(\Pi)} \) is constant under CARA, (3-52) can be rewritten as
\[ \mathbb{E}\{U''(\Pi) \frac{U'(\Pi)}{U'(\Pi)} \cdot [L + \mu + L'(r_L - c_L - r)]\} L'(r_L - c_L - r) = \]
\[ \frac{U''(\Pi)}{U'(\Pi)} \cdot \mathbb{E}\{U'(\Pi)[L + \mu + L'(r_L - c_L - r)]\} L'(r_L - c_L - r) = 0 \]
\[ (3-53) \]
by the first order condition for \( r_L \). Thus, the numerator of (3-40) is positive since (3-41) has been shown to be positive. With \( U_{LL} < 0 \) by the second order conditions, it follows that \( r_L \) is an increasing function of \( c_L \) under CARA.

**Proposition 3:** Given increasing absolute risk aversion (IARA), the optimal loan rate is an increasing function of the marginal administrative cost of loans.

**Proof:** In order to unambiguously sign \( \partial r_L / \partial c_L \) under IARA, the second term in (3-41) must be shown to be positive or zero.

Let \( \mu^0 \) be the value of \( \mu \) such that \( L + \mu^0 + L'(r_L - c_L - r) = 0 \), and \( \Pi^0 \) be the value of \( \Pi \) when \( \mu = \mu^0 \). Under IARA, \( R_a(\Pi) \) is an increasing function of \( \Pi \), thus
\[ \frac{U''(\Pi)}{U'(\Pi)} \geq \frac{U''(\Pi^0)}{U'(\Pi^0)} \] for all \( \mu \geq \mu^0 \)
\[ (3-54) \]
by IARA, since \( \frac{\partial \Pi}{\partial \mu} > 0 \). Furthermore,
\[ U'(\Pi)[L + \mu + L'(r_L - c_L - r)] \geq 0, \] for all \( \mu \geq \mu^0 \)
\[ (3-55) \]
by construction. From (3-54) and (3-55), it follows that, for all $\mu \geq \mu^0$,

$$-
\frac{U''(\pi)}{U'(\pi)} \cdot U'(\pi)[L + \mu + L'(r_L - c_L - r)] \geq
\frac{U''(\pi^0)}{U'(\pi^0)} \cdot U'(\pi^0)[L + \mu + L'(r_L - c_L - r)].$$

(3-56)

Consider the case when $\mu < \mu^0$. Then,

$$-
\frac{U''(\pi)}{U'(\pi)} < -\frac{U''(\pi^0)}{U'(\pi^0)} \quad \text{for all } \mu < \mu^0$$

(3-57)

and

$$U'(\pi)[L + \mu + L'(r_L - c_L - r)] < 0, \quad \text{for all } \mu < \mu^0$$

(3-58)

by construction. Hence, from (3-57) and (3-58), it follows that for all $\mu < \mu^0$,

$$-
\frac{U''(\pi)}{U'(\pi)} \cdot U'(\pi)[L + \mu + L'(r_L - c_L - r)] >
\frac{U''(\pi^0)}{U'(\pi^0)} \cdot U'(\pi^0)[L + \mu + L'(r_L - c_L - r)].$$

(3-59)

Inspection of (3-59) reveals that (3-56) holds for all values of $\mu$. Therefore, taking the expected value of both sides of (3-56) yields

$$-
\frac{U''(\pi)}{U'(\pi)} \cdot E[U'(\pi)[L + \mu + L'(r_L - c_L - r)] >
\frac{U''(\pi^0)}{U'(\pi^0)} \cdot E[U'(\pi^0)[L + \mu + L'(r_L - c_L - r)] = 0$$

(3-60)

by the first order condition for $r_L$. From (3-60), it is clear that, under IARA,
$$E(U''(\Pi)[L + \mu + L'(r_L - c_L - r)]) < 0. \quad (3-61)$$

Since $L'(r_L - c_L - r) < 0$, the second term of $(3-41)$ is strictly positive. Therefore, $r_L$ is an increasing function of $c_L$ under IARA.

To determine the full impact on the intermediary's spread from a change in the administrative cost of loans, one must consider the effect of a change in $c_L$ on the optimal deposit rate.

Proposition 4): The optimal deposit rate is invariant with respect to changes in the marginal administrative cost of loans.

Proof: Implicit differentiation of the first order conditions with respect to $c_L$ yields

$$\frac{\partial r_D}{\partial c_L} = - \frac{E\{U''(\Pi)(-L-\mu)[-D + D'(r - r^D - c_D)]\}}{U_{DD}}. \quad (3-62)$$

The numerator of $(3-62)$ can be expressed as

$$E\{U''(\Pi)(-L-\mu)[-D + D'(r - r^D - c_D)]\} =$$

$$E\{-D + D'(r - r^D - c_D)\} E\{U''(\Pi)(-L-\mu)\} = 0. \quad (3-63)$$

Since $E[-D + D'(r - r^D - c_D)] = 0$ by the first order condition, $r^D$ is invariant to changes in $c_L$.

The results from Propositions 1), 2), and 3) are summarized in Table 1. Given CARA or IARA, the effect is the same as in the certainty equivalent model.
Table 1

<table>
<thead>
<tr>
<th></th>
<th>DARA</th>
<th>CARA</th>
<th>IARA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\partial r_L/\partial c_L$</td>
<td>$&lt; 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
</tr>
</tbody>
</table>

Given CARA or IARA, the intermediary will pass increased loan administration expenses on to the consumer in the form of higher loan rates. Proposition 4) implies that $\partial r_D/\partial c_L = 0$ for all attitudes toward risk. Therefore, the spread between $r_L$ and $r_D$ increases as $c_L$ increases given CARA or IARA. Interestingly, the intermediary characterized by DARA (the most empirically significant type of risk aversion) may actually reduce its spread between $r_L$ and $r_D$ in response to an increase in administrative expenses. This interesting result demands further exploration.

From the earlier discussion of attitudes towards risk and risk premiums, recall that the risk premium, $\theta$, decreases as the agent's risk aversion decreases; i.e., $\theta^A > \theta^N > \theta^P$. Similarly, the section dealing with certainty versus uncertainty comparisons for $r_L$ (see Figure 3), demonstrated that $r_L^A < r_L^N < r_L^P$ and $r_D^A = r_D^N = r_D^P$. Hence, greater risk aversion leads the intermediary to reduce its spread between loan and deposit rates. In general, one can conclude that the optimal loan rate and the risk premium relate inversely. As the risk premium increases, $\Delta \theta > 0$; then, the optimal loan rate decreases, $\Delta r_L < 0$. 

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The risk premium demanded by the intermediary depends on its attitudes towards risk and the characteristics of the random prospect. Hence, $\Theta$ depends on expected profit and the riskiness of a particular prospect. Although it is an imperfect measure of risk, the variance of random profit, $\text{Var}(\Pi)$, is used as a proxy for risk. Then, the risk premium demanded by the intermediary is some function of expected profit and the variance (risk) of random profit. Thus, $\Theta = \Theta (\Pi, \text{Var}(\Pi))$. Furthermore, $\Pi$ and $\text{Var}(\Pi)$ are both functions of $c_L$. Therefore,

$$
\frac{d\Theta}{dc_L} = \frac{\Theta}{\Pi} \frac{\partial \Pi}{c_L} + \frac{\Theta}{\text{Var}(\Pi)} \frac{\partial \text{Var}(\Pi)}{c_L}.
$$

(3-64)

It follows immediately that

$$
\frac{\partial \Pi}{\partial c_L} = -L(r_L) < 0
$$

and

$$
\frac{\partial \text{Var}(\Pi)}{\partial c_L} = -2(r_L - c_L - r)\text{Var}(\mu) < 0.
$$

An increase in the loan administrative cost reduces the expected profitability and reduces the "risk" of random profit. Other things equal, the risk premium for a risk averse agent increases as the riskiness of a prospect increases, $\frac{\partial \Theta}{\partial \text{Var}(\Pi)} > 0$. The effect of an incremental reduction in risk on $r_L$ is positive since $\frac{\partial \Theta}{\partial \text{Var}(\Pi)} \frac{\partial \text{Var}(\Pi)}{c_L} < 0$ for all risk averse intermediaries and since $r_L$ and $\Theta$ are inversely related. The ambiguity in the overall effect of a
change in $c_L$ on $r_L$ occurs because of $\frac{d\theta}{d\Pi}$. Table 2 follows from the prior discussion of DARA, CARA and IARA.

Table 2

<table>
<thead>
<tr>
<th>$\frac{d\theta}{d\Pi}$</th>
<th>DARA</th>
<th>CARA</th>
<th>IARA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$&gt; 0$</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

In the case of either CARA or IARA, $\frac{d\theta}{dc_L} < 0$, and $\frac{d\theta}{dc_L} > 0$. However, DARA implies $\frac{d\theta}{d\Pi} \frac{d\Pi}{dc_L} > 0$ and $\frac{d\theta}{dc_L} < 0$.

If the effect of an increase in $c_L$ on expected profit dominates the effect of $c_L$ on "risk," the intermediary characterized by DARA will reduce $r_L$. If the DARA intermediary is compensated for the reduction in expected profit caused by an increase in $c_L$, the firm will increase its spread between $r_L$ and $r_D$ as $c_L$ increases. Alternatively, if risk is held constant, the DARA intermediary will lower the spread between $r_L$ and $r_D$ as $c_L$ increases. By lowering $r_L$ the firm increases the volume of expected loans. Other things equal, an increase in $c_L$ lowers the net return on the average loan. The reduction in average expected profit per loan can be partially offset by an increase in loan volume.

Consider now the effect of a change in $c_L$ on the net borrowing/lending of Federal funds. Differentiating $R =$
Given DARA, $\frac{\partial R}{\partial c_L}$ is ambiguous since $\frac{\partial r_L}{\partial c_L}$ is indeterminate under DARA. For either CARA or IARA, $\frac{\partial r_L}{\partial c_L} > 0$, and the intermediary increases lending of Federal funds as the marginal administrative costs of loans increases (i.e., $\frac{\partial R}{\partial c_L} > 0$). Interestingly, the DARA intermediary may actually decrease the amount of lending in the Federal funds market as a result of an increase in $c_L$. This would be contrary to intuition.

The next set of propositions explores the relationship between the optimal risk averse loan rate and the marginal administrative cost of deposits. What effect, if any, do changes in the administrative cost of deposits have on the spread between $r_L$ and $r_D$? Differentiation of the first order conditions with respect to $c_D$ yields

$$\frac{\partial r_L}{\partial c_D} = \frac{-D \cdot E[U''(\Pi)[L + \mu + L'(r_L - c_L - r)]]}{\partial c_D} \quad \frac{U_{LL}}{U_{LL}}$$

Proposition 5): Under DARA, the optimal risk averse loan rate is a decreasing function of the marginal administrative cost of deposits.

Proof: As previously demonstrated by (3-42) through (3-50), $E[U''(\Pi)[L + \mu + L'(r_L - c_L - r)]] > 0$ given DARA. Thus, with $-D < 0$ and $U_{LL} < 0$ by the second order conditions, it follows that the optimal loan rate is a decreasing function of $c_D$.  

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Proposition 6: Under CARA, the optimal risk averse loan rate is invariant with respect to changes in the marginal administrative cost of deposits.

Proof: This proposition can be easily verified since 
\[ E(U''(W)[L + \mu + L'(r_L - c_L - r)]) = 0 \] 
under CARA, as shown by (3-52) and (3-53).

Proposition 7: Given IARA, the optimal risk averse loan rate is an increasing function of the marginal administrative cost of deposits.

Proof: \[ E(U''(W)[L + \mu + L'(r_L - c_L - r)]) < 0 \] 
given IARA, as established by (3-54) through (3-61). Therefore, \( r_L \) is an increasing function of \( c_D \) under IARA.

Proposition 8): The optimal deposit rate is a decreasing function of the marginal administrative cost of deposits.

Proof: Differentiating implicitly the first order conditions with respect to \( c_D \),

\[
\frac{\partial r_D}{\partial c_D} = \frac{-D' \cdot E[U'(W)] + E(U''(W)(-D)(r - r_D - c_D))}{UD_D}.
\] (3-66)

Inspection of the numerator of (3-66) reveals that the first term is negative since \(-D' < 0\) and \(E[U'(W)] > 0\). The second term in the numerator can be rewritten as

\[ E(U''(W)(-D)[ -D + D'(r - r_D - c_D)]) \]

\[ = E(U''(W)D) \cdot E[-D + D'(r - r_D - c_D)] = 0, \] (3-67)

by the first order condition for \( r_D \).

Table 3 presents the results from Propositions 5), 6) and 7). The risk averse loan rate is a decreasing, an
invariant or an increasing function of the marginal administrative cost of deposits under DARA, CARA and IARA respectively. Proposition 8) indicates that $\partial r_D / \partial c_D < 0$ irrespective of the degree of absolute risk aversion. Thus, given CARA or IARA, the spread between $r_L$ and $r_D$ is an increasing function of $c_D$. This result is identical to the risk neutrality case. Under DARA, the effect on the spread $(r_L - r_D)$ from a change in $c_D$ is ambiguous in sign. The DARA intermediary lowers its loan rate in response to an increase in the administrative cost of deposits. Since $\partial r_D / \partial c_D < 0$ for all degrees of risk aversion, an increase in $c_D$ means also a reduction in $r_D$. As a result, the spread may increase, decrease or stay the same following an increase in $c_D$. The ambiguity of this result under DARA suggests the possibility that the intermediary could in fact reduce the size of the spread in response to an increase in the administrative cost of deposits.

The propositions in Table 3 can be given a risk premium interpretation. As discussed in the previous section, the risk premium demanded by the intermediary in connection with its random profit is a function of the expected profit and the variance or risk of profit. The total rate of change in the risk premium ($\Theta$) as $c_D$ changes is given by

<table>
<thead>
<tr>
<th>$\partial r_L / \partial c_D$</th>
<th>DARA</th>
<th>CARA</th>
<th>IARA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; 0$</td>
<td>0</td>
<td>$&gt; 0$</td>
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</table>

Table 3
Effects on $r_L$ from a Change in $c_D$
\[
\frac{d\Theta}{dc_D} = \frac{\partial\Theta}{\partial\bar{\Pi}} \frac{\partial\bar{\Pi}}{\partial c_D} + \frac{\partial\Theta}{\partial \text{Var}(\Pi)} \frac{\partial \text{Var}(\Pi)}{\partial c_D},
\]  

(3-68)

where

\[
\frac{\partial\bar{\Pi}}{\partial c_D} = -D(r_D) < 0
\]

and

\[
\frac{\partial \text{Var}(\Pi)}{\partial c_D} = 0.
\]

Therefore, a change in the administrative cost of deposits has an effect only on expected profit. An increase in \(c_D\) reduces expected profit but has no effect on the variance or risk of profit. From this, it follows that the second term in (3-68) vanishes, and the effect on the risk premium depends solely on the mean income effect, i.e., \(\frac{\partial\Theta}{\partial\bar{\Pi}} \left(\frac{\partial\bar{\Pi}}{\partial c_D}\right)\). Effects of a change in expected profit on the risk premium under DARA, CARA and IARA are shown in Table 2 in the preceding section. Under DARA, \(\frac{\partial\Theta}{\partial\bar{\Pi}} < 0\), and since the mean profit effect is always negative, it follows that \(\frac{d\Theta}{dc_D} > 0\). As established previously, an increasing risk premium implies a decreasing loan rate. Hence, under DARA, \(\frac{\partial r_L}{\partial c_D} < 0\). Similarly, given CARA, \(\frac{\partial\Theta}{\partial\bar{\Pi}} = 0\). A change in \(\bar{\Pi}\) has no effect on the risk premium under CARA. From this, it follows that \(\frac{d\Theta}{dc_D} = 0\), and \(r_L\) is invariant with respect to changes in the marginal administrative cost of deposits under CARA. Finally, IARA implies that \(\frac{d\Theta}{dc_D} < 0\), since \(\frac{\partial\Theta}{\partial\bar{\Pi}} > 0\). Thus, it
follows from the relationship between \( \theta \) and \( r_L \) (inversely related), \( \partial r_L / \partial c_D > 0 \). Given IARA, the intermediary's loan rate is an increasing function of \( c_D \).

The implications of the propositions discussed in this section can be summarized as follows. Given either risk neutrality or nondecreasing absolute risk aversion, the spread between \( r_L \) and \( r_D \) is an increasing function of the marginal administrative cost of deposits. Under DARA, however, an increase in \( c_D \) could lead to a reduction in the size of the spread. Invariably, an increase in \( c_D \) causes a reduction in the expected size of the intermediary irrespective of attitudes toward risk.

Finally, it is easily demonstrated that an increase in the administrative cost of deposits causes the intermediary to become a net borrower in the Federal funds market under either DARA or CARA. The expected amount of borrowing/lending in Federal funds is given by

\[
R = D(r_D) - L(r_L).
\]

The effect of a change in \( c_D \) on \( R \) is given by

\[
\frac{\partial R}{\partial c_D} = D' \frac{\partial r_D}{\partial c_D} - L' \frac{\partial r_L}{\partial c_D}.
\]

For all degrees of risk aversion, \( \partial r_D / \partial c_D < 0 \). From Table 3, \( \partial r_L / \partial c_D < 0 \) under DARA. Thus, it follows that \( \partial R / \partial c_D < 0 \). Similarly, under CARA, \( \partial r_L / \partial c_D = 0 \) which fact implies \( \partial R / \partial c_D < 0 \) under CARA.
Hence, given either DARA or CARA, the intermediary tends to reduce the amount of lending in Federal funds (becomes a net borrower) as the administrative cost of deposits increases. An increase in the administrative cost of deposits essentially increases the marginal resource cost of this source of funds. This causes an input substitution effect to take place whereby deposits are substituted by Federal funds as a source of funds to the intermediary. Under IARA, $\frac{\partial R}{\partial c_D} > 0$ since $\frac{\partial r_L}{\partial c_D} > 0$ (Table 3). Thus, contrary to intuition, an increase in the administrative cost of deposits could result in the intermediary lending more in the Federal funds market (or borrowing less).

The next set of propositions examines the effects of a change in the Federal funds rate on the intermediary's spread between $r_L$ and $r_D$, as well as the firm's lending/borrowing in the Federal funds market. In general, most market rates tend to follow the movements in the Federal fund rate. Although the rate on Federal funds is determined by aggregate supply and demand under more or less perfectly competitive conditions, the Federal Reserve exerts considerable influence on this rate through its open market operations.

Consider the effect on the intermediary's loan rate from a change in the Federal funds rate. Implicit differentiation of the first order conditions with respect to $r$ yields

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\[
\frac{\delta r_L}{\hat{U}_{LL}} = \frac{-L'\cdot E[U'(\Pi)] + E[U''(\Pi)(D L - \mu)[L + \mu + L'(r_L - c_L - r)]]}{E[U''(\Pi)(D L - \mu)[L + \mu + L'(r_L - c_L - r)]]}.
\]

(3-69)

The first term in the numerator of (3-69) is clearly positive. The second term can be rewritten as

\[
E[U''(\Pi)(D L - \mu)[L + \mu + L'(r_L - c_L - r)]] = \\
D\cdot E[U''(\Pi)[L + \mu + L'(r_L - c_L - r)]] \\
- E[U''(\Pi)(L + \mu)[L + \mu + L'(r_L - c_L - r)]].
\]

(3-70)

Substituting (3-41) for the last term in (3-70), and factoring similar terms yields

\[
E[U''(\Pi)(D L - \mu)[L + \mu + L'(r_L - c_L - r)]] = \\
\frac{D}{2} E[U''(\Pi)[L + \mu + L'(r_L - c_L - r)]^2] \\
+ E[U''(\Pi)[L + \mu + L'(r_L - c_L - r)][D + L'(r_L - c_L - r)]].
\]

(3-71)

Inspection of (3-71) reveals that the sign of the first term is positive for \(U''(\Pi) < 0\). With \(U_{LL} < 0\) by the second order conditions, the sign of \(\delta r_L/\delta r\) depends on the sign of the last term in (3-71).

**Proposition 9**: Given DARA, a sufficient condition for \(\delta r_L/\delta r > 0\) is \(D > L\).

**Proof**: \(E[U''(\Pi)[L + \mu + L'(r_L - c_L - r)] > 0\) under DARA, as shown in (3-42) - (3-50). From (3-38), it is clear that

\[
L'(r_L - c_L - r) > -L.
\]

(3-72)
Adding $D$ to both sides of (3-72) gives

$$L'(r_L - c_L - r) + D > D - L.$$  \hspace{1cm} (3-73)

Clearly, $D + L'(r_L - c_L - r) > 0$ provided that $D > L$. Hence, provided that the firm is not an expected borrower ($D > L$), the sign of the last term in (3-71) will be positive and $\partial r_L / \partial r > 0$ under DARA.

Proposition 10): Given CARA, the optimal risk averse loan rate is an increasing function of the rate at which the intermediary can borrow and lend Federal funds.

Proof: Under CARA, $E[U''(\Pi)(L + \mu + L'(r_L - c_L - r))] = 0$, as demonstrated previously by (3-52) and (3-53). Thus, the last term of (3-71) is positive. It follows from this that the numerator of (3-69) is clearly positive. Since $U_{rL} < 0$ by the second order conditions, $\partial r_L / \partial r > 0$ given CARA.

Proposition 11): Under IARA, the sign of $\partial r_L / \partial r$ is ambiguous.

Proof: Under IARA, $E[U''(\Pi)(L + \mu + L'(r_L - c_L - r))] < 0$, as demonstrated earlier in (3-54) - (3-61). Since $[D + L'(r_L - c_L - r)] < 0$, $\partial r_L / \partial r$ cannot be signed under IARA.

In general, the sign of $\partial r_L / \partial r$ is ambiguous under IARA. However, it can be demonstrated that a sufficient condition for $\partial r_L / \partial r > 0$ implies that $D < L$.

Corollary 1): A sufficient condition for $\partial r_L / \partial r > 0$ under IARA implies that the intermediary must be an expected net borrower in the Federal funds market.

Proof: Since $E[U''(\Pi)(L + \mu + L'(r_L - c_L - r))] < 0$ under IARA, a sufficient condition for $\partial r_L / \partial r > 0$ is $D + L'(r_L - c_L - r) < 0$. Thus, (3-73) yields $D < L$. 

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Propositions 9) - 11) and Corollary 1) reveal that the impact of a change in the Federal funds rate on the firm's choice of an optimal loan rate depends on both the firm's attitude towards risk and whether the firm is an expected lender or borrower. These results closely parallel the role played by the Federal funds market in this model. In Proposition 9), \( \frac{\partial r^*_L}{\partial r} > 0 \) under DARA if \( D > L \). It is interesting to note that if the firm is characterized by DARA and it is a net borrower \( (D < L) \), then, the optimal loan rate may be a decreasing function of \( r \).

Consider now the effect of a change in the Federal funds rate on the optimal deposit rate.

**Proposition 12**: The optimal deposit rate is an increasing function of the borrowing and lending rate on Federal funds.

**Proof**: Implicit differentiation of the first order conditions with respect to \( r \) gives

\[
\frac{\partial r_D}{\partial r} = \frac{E[U'(\Pi)D'] + E[U''(\Pi)(D-L-\mu)[-D+D'(r-r_D-c_D)]]}{U_D'}.
\]

The first term of the numerator of (3-74) is positive since \( D' > 0 \) by assumption. The second term can be shown to be equal to zero. Rewriting the second term in the numerator of (3-74) as

\[
E[U''(\Pi)(D-L-\mu)[-D+D'(r-r_D-c_D)] = E[U''(\Pi)(D-L-\mu) \cdot E[-D+D'(r-r_D-c_D)] = 0, (3-75)]
\]
since \( E[-D + D'(r - r_D - c_D)] = 0 \) by the first order condition for \( r_D \). Hence, \( \partial r_D / \partial r > 0 \), since \( \bar{U}_{DD} < 0 \) by the second order conditions.

Table 4 shows the results from Propositions 9), 10), and 11).

Table 4

<table>
<thead>
<tr>
<th>Effect of a Change in ( r ) on ( r_L )</th>
<th>DARA</th>
<th>CARA</th>
<th>IARA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \partial r_L / \partial r )</td>
<td>&gt;0*</td>
<td>&gt;0</td>
<td>&gt;0</td>
</tr>
</tbody>
</table>

\* Requires \( D \geq L \)

Given either DARA or CARA, the intermediary's loan rate is an increasing function of the Federal funds rate. But note that under DARA this result requires that \( D \geq L \). The sign of \( \partial r_L / \partial r \) is ambiguous under IARA. Proposition 12) states that the deposit rate increases as a function of the Federal funds rate for all degrees of risk aversion. The effect on the spread between \( r_L \) and \( r_D \) is indeterminate under DARA, CARA or IARA. Thus, a change in the Federal funds rate could increase, decrease or leave unchanged the size of the spread between \( r_L \) and \( r_D \).

Intuitively, the qualitative behavior of the risk premium, \( \Theta \), in response to a change in the Federal funds rate can help explain the results obtained in these propositions. Recall that \( \Theta \) depends on at least the first two moments of the distribution of random profit. Thus, \( \Theta = \Theta(\bar{\Pi}, \text{Var}(\Pi)) \). Then, it follows that
\[
\frac{d\Theta}{dr} = \frac{\partial \Pi}{\partial r} + \frac{\partial \Theta}{\partial \text{Var}(\Pi)} \frac{\text{Var}(\Pi)}{\partial r}.
\]

(3-76)

Furthermore,

\[
\frac{\partial \Pi}{\partial r} = D - L
\]

and

\[
\frac{\partial \text{Var}(\Pi)}{\partial r} = -2(r_L - c_L - r) \text{Var}(\mu) < 0.
\]

The effect on the variance of profit from a change in \( r \) is negative. As the variance of profit increases (decreases) the risk premium increases (decreases), \( \partial \Theta / \partial \text{Var}(\Pi) > 0 \). From this, it follows that the second term inside the brackets in (3-76) is negative for all degrees of risk aversion. The effect on mean profit from a change in \( r \) depends on \( D \) and \( L \) since \( \partial \Pi / \partial r = D - L > 0 \) as \( D > L \). If the intermediary sets \( r_L \) and \( r_D \) such that it expects to be a net lender in the Federal funds market \( (D > L) \), the effect on mean profit from a change in \( r \) is positive. On the other hand, if the intermediary expects to be a net borrower in the Federal funds market \( (D < L) \), \( \partial \Pi / \partial r < 0 \). Finally, if \( D = L \), the institution is neither an expected borrower nor an expected lender in Federal funds, and \( \Pi \) is invariant with respect to changes in \( r \). Table 5 below shows the effects on
the risk premium from a change in \( r \) under DARA, CARA and IARA for the case when \( D > L \).

<table>
<thead>
<tr>
<th></th>
<th>DARA</th>
<th>CARA</th>
<th>IARA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Profit Effect</td>
<td>&lt;0</td>
<td>=0</td>
<td>&gt;0</td>
</tr>
<tr>
<td>Variance Effect</td>
<td>&lt;0</td>
<td>&lt;0</td>
<td>&lt;0</td>
</tr>
</tbody>
</table>

The mean and variance effects shown in Table 5 correspond to the first and second terms inside the brackets in (3-76). As Table 2 showed, under DARA, \( \frac{\partial \Theta}{\partial \pi} < 0 \). Thus, the mean profit effect on \( \Theta \) for the case when the intermediary is an expected net lender in Federal funds is negative under DARA, since \( \frac{\partial \pi}{\partial r} > 0 \). The mean profit effect on \( \Theta \) from an increase in \( r \) is negative under DARA. This fact is mutually reinforced by the variance effect on \( \Theta \) which is also negative. Recall that a decreasing risk premium implies increasing \( r_L \). Therefore, \( \frac{\partial r_L}{\partial r} > 0 \) under DARA. Given CARA, no mean effect on \( \Theta \) arises from a change in \( r \). The variance effect under CARA is negative. Thus, it follows that \( \frac{\partial r_L}{\partial r} > 0 \) since a decline is the risk premium implies an increase in \( r_L \). The ambiguity under IARA in the sign of \( \frac{\partial r_L}{\partial r} \) is due to conflicting signs in the mean and variance effects on the risk premium. In the case when the intermediary sets \( r_L \) and \( r_D \) such that the intermediary is an expected net borrower in Federal funds (\( D < L \)), the effects...
on the risk premium from a change in the Federal funds rate are shown in Table 6. In this particular case, the mean and variance of profit effects have conflicting signs under DARA. Thus, the DARA intermediary which is an expected net borrower in the Federal funds market could possibly lower \( r_L \) in response to an increase in \( r \). This action would reduce the size of the spread between \( r_L \) and \( r_D \). If the intermediary sets \( r_L \) and \( r_D \) such that \( D = L \), there is no mean effect on the risk premium since \( \frac{\partial \Pi}{\partial r} = 0 \). Since the variance effect on \( \theta \) is negative, \( \frac{\partial r_L}{\partial r} > 0 \) for all degrees of risk aversion.

Consider now the effect on the amount of Federal funds borrowing/lending by the intermediary from a change in \( r \). The amount of Federal funds borrowing/lending is given by

\[
R = D - L.
\]

Differentiating \( R \) with respect to \( r \) yields

\[
\frac{\partial R}{\partial r} = \frac{\partial r_D}{\partial r} - \frac{\partial r_L}{\partial r}.
\]
It is easily verified that given nonincreasing absolute risk aversion, \( \frac{\partial R}{\partial r} > 0 \) provided that \( D > L \). Thus, if the intermediary is an expected net lender in the Federal funds market, an increase in \( r \) increases the amount of lending in the Federal funds market under either DARA or CARA.

Before examining the case of uncertain deposit supplies, it is of interest to consider the effects on the loan and deposit rates from a mean preserving spread in the distribution of \( \mu \). Rothschild and Stiglitz\(^{25}\) introduced the concept of a mean preserving spread in the distribution of a random variable as a tool for defining increasing risk. Essentially, a mean preserving spread redistributes the weight of a distribution from the center to the tails while keeping the mean of the distribution unchanged. Thus, a mean preserving spread increases the variability or dispersion of the distribution around the constant mean.

Define a mean preserving spread in the distribution of \( \mu \) by

\[
\mu^* = \gamma \mu, \tag{3-77}
\]

where \( \gamma \) is a shift parameter. Now, writing the first order conditions ((3-31) and (3-32)) in terms of \( \mu^* \), and differentiating implicitly with respect to \( \gamma \) (evaluating at \( \gamma = 1 \)) yields

\[
\frac{\partial r_L}{\partial y} = \frac{E[U'(\Pi)\mu] + E[U''(\Pi)(r_L - c_L - r)(\mu)[L + \mu + L'(r_L - c_L - r)]]}{ULL}. \tag{3-78}
\]

The first term in the numerator of (3-78) can be expressed as
\[
E[U'(\Pi)\mu] = E[U'(\Pi)]\cdot E[\mu] + \text{Cov}(U'(\Pi),\mu). \tag{3-79}
\]

Since \( E[\mu] = 0 \), the sign of (3-79) depends on the sign of \( \text{Cov}(U'(\Pi),\mu) \). Note that \( \partial \Pi / \partial \mu = (r_L - c_L - r) > 0 \), and \( \partial U'(\Pi) / \partial \mu = U''(\Pi)(r_L - c_L - r) < 0 \) for \( U''(\Pi) < 0 \). These facts imply that \( \text{Cov}(U'(\Pi),\mu) < 0 \), and thus the first term in (3-78) is negative. Consider now the second term in the numerator of (3-78). Adding and subtracting \((L + L'(r_L - c_L - r))\) simultaneously inside the parenthesis at \( \mu \) results in the following expression for the second term in the numerator of (3-78):
\[
E[U''(\Pi)(r_L - c_L - r)(\mu)[L + \mu + L'(r_L - c_L - r)]] = \\
E[U''(\Pi)[L + \mu + L'(r_L - c_L - r)]^2](r_L - c_L - r) \\
- E[U''(\Pi)[L + \mu + L'(r_L - c_L - r)]](r_L - c_L - r) \\
+ L'(r_L - c_L - r)^2. \tag{3-80}
\]

The first term in (3-80) is clearly negative for \( U''(\Pi) < 0 \).

**Proposition 13**: Given DARA, the effect on the risk averse loan rate from a mean preserving increase in \( \mu \) is negative.

**Proof**: To verify this proposition it must be shown that (3-80) is negative in sign. As previously demonstrated in (3-50)
under DARA. From the first order condition, \([L + L'(r_L - c_L - r)] > 0\). Multiplying both sides of this inequality by \((r_L - c_L - r)\) gives

\[
[L(r_L - c_L - r) + L'(r_L - c_L - r)^2] > 0.
\]

Hence, the term (3-80) is negative and \(\frac{\partial r_L}{\partial \gamma} \big|_{\gamma=1} < 0\).

Proposition 14): Given CARA, the effect on the risk averse loan rate from a mean preserving increase in \(\mu\) is negative.

Proof: Under CARA, \(E\{U''(\Pi)(L + \mu + L'(r_L - c_L - r))\} = 0\) as shown earlier in (3-53). The last term of (3-80) is zero, and (3-80) is negative. Thus, it follows that \(\frac{\partial r_L}{\partial \gamma} \big|_{\gamma=1} < 0\).

Proposition 15): Under IARA, the effect of a mean preserving increase in \(\mu\) on the risk averse loan rate is ambiguous.

Proof: Recall that under IARA,

\(E\{U''(\Pi)(L + \mu + L'(r_L - c_L - r))\} < 0\)

as shown in (3-61). Thus, the sign of (3-80) cannot be determined, and \(\frac{\partial r_L}{\partial \gamma} \big|_{\gamma=1}\) is ambiguous given IARA.

Proposition 16): A mean preserving increase in the distribution of \(\bar{\mu}\) has no effect on the optimal deposit rate.

Proof:

\[
\frac{\partial r_D}{\partial \gamma} \bigg|_{\gamma=1} = - \frac{E\{U''(\Pi)(\bar{\mu})(r_L - c_L - r)[-D + D'(r - r_D - c_D)]\}}{\bar{\mu}D'}.
\]

(3-81)
Noting that

\[ E\{U'(\mu)(r_L - c_L - r)[-D + D'(r - r_D - c_D)]\} = \]

\[ (r_L - c_L - r)E[-D + D'(r - r_D - c_D)]E\{U'(\mu)\} = 0, \quad (3-82) \]

by the first order condition for \( r_D \). Therefore, \( \partial r_D / \partial \gamma \) \( \gamma = 1 \) = 0.

Table 7 shows the results from a mean preserving spread (MPS) of the distribution of \( \mu \) on the loan rate.

| Effect of a Mean Preserving Spread of \( \mu \) on Loan Rate |
|-------------|-------------|-------------|
| \( \partial r_D / \partial \gamma \) \( \gamma = 1 \) | DARA | CARA | IARA |
| \( \partial r_L / \partial \gamma \) \( \gamma = 1 \) | <0 | <0 | >0 |

A mean preserving spread of \( \mu \) has only one effect on the distribution of the intermediary's profit: it increases the variability or dispersion around the mean profit. However, a mean preserving spread does not change the expected value of profit. Increased variability with constant mean profit implies increasing riskiness of profit. Intuitively, as the risk of profit increases, the size of the risk premium also increases irrespective of the degree of risk aversion. Recall that as the risk premium increases, the loan rate declines. Hence, one should expect a mean preserving spread of \( \mu \) to lower the intermediary's loan rate irrespective of the degree of risk aversion. Except for the
ambiguity under IARA, the signs of the effect of a mean preserving spread of \( \mu \) on the loan rate agree with the increasing risk premium interpretation.

Proposition 16 implies that a mean preserving spread of \( \mu \) has no effect on \( r_D \). Therefore, given nonincreasing absolute risk aversion, the size of the spread between \( r_L \) and \( r_D \) is reduced by a mean preserving spread of \( \mu \).

Finally, how does the amount of borrowing and lending of Federal funds change in response to a mean preserving spread of \( \mu \)? Differentiating \( R = D - L \) with respect to \( Y \) (evaluated at \( Y = 1 \)) yields

\[
\frac{\partial R}{\partial Y}
= \frac{D'\partial r_D}{\partial Y} - \frac{L'\partial r_L}{\partial Y}.
\]

Given nonincreasing absolute risk aversion, \( \frac{\partial R}{\partial Y}
|_{Y=1} < 0. \) This implies that a mean preserving spread of \( \mu \) reduces the amount of lending in the Federal funds market (or increases borrowing of Federal funds). As profit "risk" increases, the firm lowers \( r_L \) which increases the volume of expected loans. The increase in expected loans is funded by an increase in borrowing (reduced lending) in the Federal funds market. On balance, the increased risk leads the firm to accept a lower profit margin per dollar invested in consumer loans and to compensate partially by increasing the volume of expected loans.

Table 8 presents the results from the comparative statics derived in this section.
<table>
<thead>
<tr>
<th>Effects on Loan Rate</th>
<th>DARA</th>
<th>CARA</th>
<th>IARA</th>
<th>Effects on Deposit Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a r_L/\alpha L$</td>
<td>$&lt;0$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$a r_D/\alpha L = 0$</td>
</tr>
<tr>
<td>$a r_L/\alpha D$</td>
<td>$&lt;0$</td>
<td>$=0$</td>
<td>$&gt;0$</td>
<td>$a r_D/\alpha D &lt; 0$</td>
</tr>
<tr>
<td>$a r_L/\alpha r$</td>
<td>$&gt;0^{*}$</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
<td>$a r_D/\alpha r &gt; 0$</td>
</tr>
<tr>
<td>MPS**</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
<td>$&gt;0$</td>
<td>MPS** = 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Effects on Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a s/\alpha L$</td>
</tr>
<tr>
<td>$a s/\alpha D$</td>
</tr>
<tr>
<td>$a s/\alpha r$</td>
</tr>
<tr>
<td>MPS**</td>
</tr>
</tbody>
</table>

*Requires $D \geq L$

**Mean Preserving Spread of $\mu$
Deposit Uncertainty Case

The preceding section investigated the rate setting behavior of the intermediary confronted with a random loan demand and a certain supply of deposits. This section will determine the properties of the model when deposit supplies are random and the intermediary knows its loan demand with certainty.

Objective Function

The intermediary's objective function can be expressed as

\[
\text{Maximize } \int_{-D}^{\infty} U(\Pi)h(\epsilon)d\epsilon, \quad (3-83)
\]

where \( U(\Pi) \) denotes the utility of profits function with \( U'(\Pi) > 0 \) and \( U''(\Pi) < 0 \), depending on whether the intermediary is risk averse, risk neutral, or risk seeking. The function \( h(\epsilon) \) represents the probability density of the random deposit flows, \( \epsilon \). The domain of the random disturbance \( \epsilon \) is such that \(-D \leq \epsilon < \infty\). The intermediary's profit \( (\Pi) \) may now be stated as

\[
\Pi = r_L L(r_L) - c_L L(r_L) - r_D[D(r_D) + \epsilon] - c_D[D(r_D) + \epsilon] \\
+ r[D(r_D) + \epsilon - L(r_L)]. \quad (3-84)
\]

Note that the above expression of the intermediary's profits assumes that loan demand is known with certainty, and the value of \( \mu \) is zero.
First and Second Order Conditions

The first order conditions for an interior optimum are

$$U_L = E\{U'(\pi)[L + L'(r_L - c_L - r)]\} = 0 \quad (3-85)$$

$$U_D = E\{U'(\pi)[-D - e + D'(r - r_D - c_D)]\} = 0, \quad (3-86)$$

where $$U_L = \frac{\partial E[U(\pi)]}{\partial r_L}$$ and $$U_D = \frac{\partial E[U(\pi)]}{\partial r_D}$$. $$U_L$$ can be rewritten as

$$U_L = E[U'(\pi)]L + E[U'(\pi)]L'(r_L - c_L - r) = 0. \quad (3-87)$$

Hence, $$[L + L'(r_L - c_L - r)] = 0$$. From (3-87), a clear necessary condition for an optimal value of $$r_L$$ is that $$r_L - c_L - r > 0$$. This follows from the fact that $$L' < 0$$ and $$E[U'(\pi)] > 0$$.

In similar fashion, $$U_D$$ can be rewritten as

$$U_D = E[U'(\pi)(-D - e)] + E[U'(\pi)]D'(r - r_D - c_D) = 0. \quad (3-88)$$

The first term in (3-88) is negative since $$U'(\pi) > 0$$ and $$-(D + e) \leq 0$$ for $$D + e \geq 0$$. Therefore, it follows that a necessary condition for an optimal value of $$r_D$$ is that $$r - r_D - c_D > 0$$. Again, this follows from $$D' > 0$$ by assumption, and $$E[U'(\pi)] > 0$$.

Combination of the necessary conditions for $$r_L$$ and $$r_D$$ reveals that an interior optimum requires that the spread between $$r_L$$ and $$r_D$$ be strictly greater than the sum of the administrative costs of loans and deposits.

The second order conditions for a maximum are

$$U_{LL} = E[U'(\pi)[2L' + L''(r_L - c_L - r)] + U''(\pi)(\pi_L)^2] < 0 \quad (3-89)$$
\[ U_{DD} = E[U'(\Pi)[-2D' + D''(r-r_D-c_D)] + U''(\Pi)(\Pi_D)^2] < 0 \quad (3-90) \]

and

\[ D = U_{LD}U_{DD} - (U_{LD})^2 > 0, \]

where \[ U_{LD} = \frac{\partial^2 U}{\partial r_L \partial r_D} = E[\Pi_L] \cdot E[U''(\Pi)\Pi_D] = 0. \]

Examination of the second order conditions reveals that sufficient conditions for a maximum in the case of the risk averse intermediary are \[ D'' < 0 \] and \[ L'' < 0. \] For the risk preferring intermediary, the second order conditions may not hold.

Certainty versus Uncertainty Comparisons

Rewriting the first order condition for \( r_D, \bar{U}_D \) as

\[ \bar{U}_D = E[U'(\Pi)] \cdot E[-D - \varepsilon + D'(r - r_D - c_D)] \\
+ \text{Cov}(U'(\Pi), \cdot) = 0, \quad (3-91) \]

where \( \cdot = [-D - \varepsilon + D'(r - r_D - c_D)]. \)

It can be demonstrated that \( U'(\Pi) \) and \( \cdot \) are similarly, independently, or oppositely ordered in \( \varepsilon \) depending on whether the intermediary is risk averse, risk neutral, or risk seeking. Since

\[ \frac{\partial U'(\Pi)}{\partial \varepsilon} = U''(\Pi)(r - r_D - c_D) < 0 \text{ as } U''(\Pi) < 0 \quad (3-92) \]

and, \[ \partial [\cdot]/\partial \varepsilon = -1. \] Consequently, \( \text{Cov}(U'(\Pi), \cdot) > 0, \) for all \( U''(\Pi) < 0. \) With \( E[U'(\Pi)] > 0, \) then \( E[-D - \varepsilon + D'(r - r_D - c_D)] < 0. \) Hence, by theorem 236 in Hardy, Littlewood and

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Polya,\textsuperscript{26} it follows that
\[ [-D + D'(r - D - c_D)] \leq 0, \text{ as } U''(\Pi) \leq 0. \]  
(3-93)

Denote by \( r^n_L \) and \( r^n_D \) the risk neutral rates on loans and deposits. Then, define the locus of points in the \((r_L, r_D)\) space such that \( \Pi_D[r_L, r_D] = 0 \). Defining \( r_D \) as an implicit function of \( r_L \), and differentiating \( \Pi_D[r_L, r_D(r_L)] \) with respect to \( r_L \) gives
\[ \Pi_{DD} \frac{\partial r_D}{\partial r_L} + \Pi_{LD} = 0, \]  
(3-94)
which implies that \( \partial r_D / \partial r_L = 0 \) by the second order condition, \( \Pi_{LD} = 0 \). Therefore, \( \Pi_D = 0 \) if and only if \( r_D = r^n_D \).

For all \( r_D > r^n_D \), \( \Pi_D < 0 \) (see Figure 4).

\begin{figure}[h]
\centering
\begin{tikzpicture}
\draw[->] (0,0) -- (4,0) node[right] {\( x_L \)};
\draw[->] (0,0) -- (0,4) node[above] {\( x_D \)};
\draw (0,0) -- (4,4) node[right, above] {\( \Pi_L > 0 \)};
\draw (0,0) -- (0,4) node[left] {\( \Pi_D > 0 \)};
\draw (0,0) -- (4,0) node[below] {\( \Pi_D > 0 \)};
\draw (0,0) -- (4,4) node[above] {\( \Pi_D < 0 \)};
\end{tikzpicture}
\caption{Certainty versus Uncertainty Comparisons for \( r_L \) and \( r_D \) (Deposit Uncertainty)}
\end{figure}

Let \( r^a_D \) and \( r^p_D \) denote the rates paid on deposits by a risk averse and a risk preferring intermediary. As

\textsuperscript{26} Hardy, Littlewood and Polya, loc. cit.
demonstrated earlier, \( \overline{\Pi}_D(r_D^a) < 0 \), which implies that \( r_D^a > r_D^n \). For the risk seeking intermediary, \( \overline{\Pi}_D(r_D^p) > 0 \) and \( r_D^p < r_D^n \).

Denote by \( r_L^a \) and \( r_L^p \) the rates on loans charged by a risk averse and a risk preferring intermediary. Dividing (3-80) by \( E[U'(\Pi)] \) yields \( [L + L'(r_L - c_L - r)] = 0 \), which implies that \( \overline{\Pi}_L = \Pi_L = 0 \). Therefore, it is clear that \( r_L^a = r_L^p = r_L^n \).

Under deposit supply uncertainty, the risk averse intermediary selects a deposit rate strictly higher than the risk neutral deposit rate. If the intermediary prefers risk, it will pay a rate on deposits lower than the risk neutral rate. Loan rates, on the other hand, will be unaffected by random deposit supplies. Under uncertain deposit supply, the risk averse intermediary will set a spread smaller than the risk neutral spread.

When deposits are random, the variability of profit relates inversely to the rate on deposits. Essentially, the risk averse intermediary can lower the risk or dispersion of profit by raising the rate it pays on deposits. In essence, a higher deposit rate implies a greater expected volume of deposits and, consequently, a larger expected size or scale for the risk averse intermediary in comparison to the risk neutral firm. Since the

\[ \text{Var}(\Pi) = (r - r_D - c_D)^2 \text{Var}(e), \text{ and } \frac{\text{Var}(\Pi)}{\partial r_D} = -2(r - r_D - c_D) \text{Var}(e) < 0. \]
loan rate remains unaffected by random deposit supplies, the spread between \( r_L \) and \( r_D \) becomes smaller as \( r_D \) is increased.

Consider now the amount of borrowing/lending of Federal funds by the risk averse intermediary in comparison to the risk neutral case. Assume that the risk neutral intermediary sets \( r_L \) and \( r_D \) such that \( R^n = D(r^n_D) - L(r^n_L) = 0 \). Then, \( R^a = D(r^a_D) - L(r^a_L) > 0 \) and \( R^p = D(r^p_D) - L(r^p_L) < 0 \). This follows from the fact that \( r^a_D > r^n_D > r^p_D \), and \( r^a_L = r^n_L = r^p_L \) as demonstrated in this section. Hence, the risk averse firm expects to be a net lender in Federal funds, and the risk preferring firm a net borrower in Federal funds.

Comparative Statics

This section first considers the effects on the optimal loan and deposit rates from changes in cost parameters of the model. Then it establishes the behavior of the intermediary's spread between \( r_L \) and \( r_D \) in response to changes in the return/risk characteristics of the supply and demand for funds to the firm.

First, consider the effect of a change in the administrative cost of deposits on the optimal deposit rate. Implicit differentiation of the first order conditions with respect to \( c_D \) yields

\[
\frac{\partial \varphi}{\partial c_D} = \frac{-E[U'(\Pi)]D' + E[U''(\Pi)(-D-\epsilon)[-D-\epsilon + D'(r-r_D-c_D)]}{\partial c_D}.
\]

(3-95)

The first term in the numerator of (3-95) is negative since
\[ E[U'(\Pi)] > 0 \] and \( D' > 0 \) by assumption. After adding and subtracting \( D'(r - r_D - c_D) \) inside the parenthesis at \( (-D - \epsilon) \), the second term in the numerator of (3-95) can be rewritten as

\[ E\{U''(\Pi)(-D - \epsilon + D'(r - r_D - c_D))\} = E\{U''(\Pi)[-D - \epsilon + D'(r - r_D - c_D)]\} - D'(r - r_D - c_D) \cdot E\{U''(\Pi)[-D - \epsilon + D'(r - r_D - c_D)]\}. \] (3-96)

The first term in (3-96) is clearly negative for \( U''(\Pi) < 0 \).

**Proposition 17:** Given DAR, the effect of an increase in \( c_D \) on \( r_D \) is ambiguous.

**Proof:** In order to verify the ambiguity of this result under DAR, it must be shown that the last term in (3-96) is positive. Let \( \epsilon^0 \) be the value of \( \epsilon \) such that \([-D - \epsilon^0 + D'(r - r_D - c_D)] = 0 \), and let \( \Pi^0 \) be the value of \( \Pi \) when \( \epsilon = \epsilon^0 \). The Arrow-Pratt index of absolute risk aversion is defined by

\[ R_a(\Pi) = -\frac{U''(\Pi)}{U'(\Pi)}. \]

\( R_a(\Pi) \) is a decreasing function of \( \Pi \) under DAR, thus

\[ \frac{U''(\Pi)}{U'(\Pi)} \leq \frac{U''(\Pi^0)}{U'(\Pi^0)} \quad \text{for all} \quad \epsilon \geq \epsilon^0, \] (3-97)

since \( \partial\Pi/\partial\epsilon = (r - r_D - c_D) > 0 \) by the first order condition for \( r_D \). Furthermore, consider all values of \( \epsilon \) such that \( \epsilon \geq \epsilon^0 \). Multiplying both sides of this inequality by \((-1)\) and adding \(-D + D'(r - r_D - c_D)\) to both sides yields

\[ [-D - \epsilon + D'(r - r_D - c_D)] \leq [-D - \epsilon^0 + D'(r - r_D - c_D)] = 0. \] (3-98)

From (3-98) it follows that
by construction. Therefore, from (3-97) and (3-99) it follows that for all \( \varepsilon \geq \varepsilon^0 \),

\[
- U''(\Pi)(-D - \varepsilon + D'(r - r_D - c_D)] > \frac{U''(\Pi^0)}{U'(\Pi^0)} \cdot U'(\Pi)(-D - \varepsilon + D'(r - r_D - c_D)]. \tag{3-100}
\]

Consider the case when \( \varepsilon < \varepsilon^0 \). Then,

\[
- \frac{U''(\Pi)}{U'(\Pi)} > - \frac{U''(\Pi^0)}{U'(\Pi^0)} \tag{3-101}
\]

by DARA since \( \frac{\partial \Pi}{\partial \varepsilon} > 0 \). Similarly,

\[
U'(\Pi)(-D - \varepsilon + D'(r - r_D - c_D)] > 0 \text{ for all } \varepsilon < \varepsilon^0 \tag{3-102}
\]

by construction. Hence, from (3-101) and (3-102)

\[
- \frac{U''(\Pi)}{U'(\Pi)} > - \frac{U''(\Pi^0)}{U'(\Pi^0)}. \tag{3-103}
\]

From (3-103), it is apparent that (3-100) holds for all values of \( \varepsilon \). Taking expectations of both sides of (3-100) yields

\[
- \mathbb{E}\{U''(\Pi)(-D - \varepsilon + D'(r - r_D - c_D)]} \mathbb{E}\{U'(\Pi)(-D - \varepsilon + D'(r - r_D - c_D)]} = 0 \tag{3-104}
\]

by the first order condition for \( r_D \). Thus, it follows from (3-104) that under DARA,

\[
\mathbb{E}\{U''(\Pi)(-D - \varepsilon + D'(r - r_D - c_D)]} < 0. \tag{3-105}
\]
Multiplying (3-105) by \(-D'(r-r_D-c_D)\) establishes the sign of the last term of (3-96) as positive. With \(U_D < 0\) by the second order condition, \(\partial r_D/\partial c_D > 0\) under DARA.

**Proposition 18:** Given CARA, the optimal risk averse deposit rate is a decreasing function of the marginal administrative cost of deposits.

**Proof:** This proposition follows from the fact that under CARA the last term of (3-96) vanishes. Rewriting the last term of (3-96) as

\[
-D'(r-r_D-c_D)E\{ U''(\Pi)[-D-\varepsilon + D'(r-r_D-c_D)] \} =
\]

\[
-D'(r-r_D-c_D)\cdot U''(\Pi)\cdot E\{ U'(\Pi)[-D-\varepsilon + D'(r-r_D-c_D)] \} = 0 
\]

(3-106)

by the first order condition for \(r_D\), and since \(U''(\Pi)/U'(\Pi)\) is constant under CARA. Hence, (3-96) is negative, and \(\partial r_D/\partial c_D < 0\) under CARA.

**Proposition 19:** Given IARA, the optimal risk averse deposit rate is a decreasing function of the marginal administrative cost of deposits.

**Proof:** In order to show the validity of this proposition, the last term in (3-96) must be shown to be negative in sign. Let \(\varepsilon^0\) be the value of \(\varepsilon\) such that 

\[-D-\varepsilon^0 + D'(r-r_D-c_D) \]  =  0,

and let \(\Pi^0\) be the value of \(\Pi\) when \(\varepsilon = \varepsilon^0\). Under IARA, \(R_a(\Pi)\) is increasing with \(\Pi\), thus,

\[
\frac{U''(\Pi)}{U'(\Pi)} \geq \frac{U''(\Pi^0)}{U'(\Pi^0)} \quad \text{for all} \quad \varepsilon \geq \varepsilon^0, \tag{3-107}
\]

since \(9\Pi/9\varepsilon > 0\). Furthermore, from (3-99) it follows that

\[
U'(\Pi)[-D-\varepsilon + D'(r-r_D-c_D)] \leq 0 \quad \text{for all} \quad \varepsilon \geq \varepsilon^0 \tag{3-108}
\]
by construction. From (3-107) and (3-108), it follows that for all \( \epsilon \geq \epsilon^0 \),

\[
- U''(\pi)(-D-\epsilon + D'(r-r_D-C_D)) \leq - \frac{U''(\pi^0)}{U'(\pi^0)} U'(\pi)(-D-\epsilon + D'(r-r_D-C_D)). \tag{3-109}
\]

Now, consider the case when \( \epsilon < \epsilon^0 \). Then,

\[
- \frac{U''(\pi)}{U'(\pi)} < - \frac{U''(\pi^0)}{U'(\pi^0)} \text{ for all } \epsilon < \epsilon^0, \tag{3-110}
\]

and

\[
U'(\pi)(-D-\epsilon + D'(r-r_D-C_D)) > 0, \text{ for all } \epsilon < \epsilon^0 \tag{3-111}
\]

by construction. Hence, from (3-110) and (3-111), it follows that for all \( \epsilon < \epsilon^0 \),

\[
- U''(\pi)(-D-\epsilon + D'(r-r_D-C_D)) < - \frac{U''(\pi^0)}{U'(\pi^0)} U'(\pi)(-D-\epsilon + D'(r-r_D-C_D)). \tag{3-112}
\]

It is clear from (3-112) that (3-109) holds for all values of \( \epsilon \). Taking expected values of both sides of (3-109) yields

\[
- \mathbb{E}\{U''(\pi)(-D-\epsilon + D'(r-r_D-C_D))\} < - \frac{U''(\pi^0)}{U'(\pi^0)} \mathbb{E}\{U'(\pi)(-D-\epsilon + D'(r-r_D-C_D))\} = 0 \tag{3-113}
\]

by the first order condition for \( r_D \). Hence, under IARA,

\[
\mathbb{E}\{U''(\pi)(-D-\epsilon + D'(r-r_D-C_D))\} > 0. \tag{3-114}
\]
From (3-114) it follows that the last term of (3-96) is negative since \(-D'(r-r_D-c_D) < 0\), and \(\frac{\partial r_L}{\partial c_D} < 0\).

In order to determine the impact of a change in the marginal administrative cost of deposits on the intermediary's spread, one must consider the effect of a change in \(c_D\) on \(r_L\). Implicit differentiation of the first order conditions with respect to \(c_D\) yields

\[
\frac{\partial r_L}{\partial c_D} = \frac{E[U''(\Pi)(-D-e)]E[L + L'(r_L-c_L-r)]}{U_L}.
\]  

(3-115)

Proposition 20): The optimal loan rate is invariant with respect to changes in the marginal administrative costs of deposits.

Proof: Since \(E[L + L'(r_L-c_L-r)] = 0\), by the first order condition for \(r\). Thus \(\frac{\partial r_L}{\partial c_D} = 0\).

Table 9 summarizes the results from Propositions 17), 18) and 19). Given CARA or IARA, the effect of a change in \(c_D\) on \(r_D\) is the same as in the risk neutral case. Under nondecreasing absolute risk aversion, the intermediary will transmit to depositors any changes in the administrative cost of deposits through changes in the rate of interest paid on deposits. Increases in the administrative cost of
deposits will result in a decrease in the rate of interest paid to depositors under CARA or IARA. Under DARA, an increase in $c_D$ could result in a higher interest rate paid to depositors. Proposition 20) states that the optimal loan rate is invariant with respect to changes in the administrative cost of deposits for all degrees of risk aversion. Thus, given nondecreasing absolute risk aversion, the spread between $r_L$ and $r_D$ is an increasing function of the marginal administrative cost of deposits. Note that the DARA intermediary may actually lower the size of the spread in response to an increase in the cost of administering deposits. The results presented in Table 9 deserve further explanation.

Recall from the earlier discussion of risk premiums that the intermediary's risk premium ($\Theta$) depends on the mean and variance of profit, i.e., $\Theta = \Theta(\bar{\Pi}, \text{Var}(\Pi))$. The effect on the risk premium demanded by the intermediary from a change in $c_D$ is given by

$$
\frac{d\Theta}{dc_D} = \left[ \frac{\partial \Theta}{\partial \bar{\Pi}} \frac{\partial \bar{\Pi}}{\partial c_D} + \frac{\partial \Theta}{\partial \text{Var}(\Pi)} \frac{\partial \text{Var}(\Pi)}{\partial c_D} \right].
$$

Furthermore, the effects on the mean and variance of profit from a change in $c_D$ are given by

$$
\frac{\partial \bar{\Pi}}{\partial c_D} = -D(r_D) < 0,
$$

and

$$
\frac{\partial \text{Var}(\Pi)}{\partial c_D} = -2(r - r_D - c_D) \text{Var}(e) < 0.
$$
An increase in the administrative cost of deposits lowers the mean and variance of the intermediary's profit. The first term in (3-116) captures the mean income effect on $\Theta$ from a change in $c_D$. The second term inside the brackets in (3-116) measures the variance or "risk" impact on $\Theta$ from a change in the administrative cost of deposits. For this particular case, the second term in (3-116) is always negative since $\partial \Theta / \partial \text{Var}(\Pi) > 0$ for all degrees of risk aversion, and $\partial \text{Var}(\Pi) / \partial c_D < 0$. Given IARA, the mean profit effect on $\Theta$ from a change in $c_D$ is negative since $\partial \Theta / \partial \Pi > 0$. That is, as $\Pi$ increases, the intermediary becomes more risk averse under IARA. Hence, under IARA both effects (mean and variance) on $\Theta$ are negative, and $d \Theta / d c_D < 0$. Recall from the certainty versus uncertainty comparisons for $r_D$ and attitudes toward risk that $r_D^a > r_D^n > r_D^p$. Therefore, an increasing risk premium implies an increasing deposit rate. Given IARA, $\partial r_D / \partial c_D < 0$ since $d \Theta / d c_D < 0$. Furthermore, under CARA the mean and variance effects on $\Theta$ are mutually reinforcing in sign (zero and negative), and $r_D$ is a decreasing function of $c_D$. Finally, the ambiguity under DARA results from conflicting signs for the mean and variance effects on $\Theta$. Since $\Theta$ decreases as $\Pi$ increases, the mean profit effect on $\Theta$ is positive. The overall impact on $\Theta$, and thus on $r_D$, will depend on which effect dominates in magnitude following a change in $c_D$.

What effect does a change in the administrative cost of deposits have on the borrowing/lending of Federal funds?
Differentiating $R = D - L$ with respect to $c_D$ yields

$$\frac{\partial R}{\partial c_D} = D' \frac{\partial d}{\partial c_D} - L' \frac{\partial l}{\partial c_D}.$$  \hspace{1cm} (3-117)

From Proposition 20), $\frac{\partial r}{\partial c_D} = 0$. Thus, $\frac{\partial R}{\partial c_D} < 0$ for either CARA or IARA since $\frac{\partial r}{\partial c_D} < 0$. An increase in the administrative cost of deposits reduces the intermediary's loans in the Federal funds market. Thus, as part of the resource cost of deposits increases, the intermediary tends to become more of a net borrower in the Federal funds market. One could interpret this as an input substitution effect. The institution will shift from one source of funds to another as the input cost characteristics of each change.

Consider now the effects of a change in the administrative cost of loans on the intermediary's spread between $r_L$ and $r_D$. First, examine the effect of a change in $c_L$ on $r_D$. Recall that in the certainty equivalent case the deposit rate was invariant with respect to changes in the administrative cost of loans. The risk neutral intermediary essentially manages loans and deposits independently of one another. Can this result be extended to the case where deposit supplies are random and the intermediary is risk averse? Implicit differentiation of the first order conditions with respect to $c_L$ gives

$$\frac{\partial r_D}{\partial c_L} = - \frac{L \cdot E[U''(\Pi) [D - e + D'(r - r_D - c_D)]]}{U''}.$$  \hspace{1cm} (3-118)

Proposition 21): Given DARA, the optimal risk averse deposit rate is an increasing function of the marginal administrative cost of loans.
Proof: Under DARA, \( E\{U''(\Pi)[-D-\epsilon+D'(r-r_D-c_D)]\} < 0 \) as demonstrated previously in (3-105). Since \( L > 0 \) and \( U_{DD} < 0 \) (second order condition), then \( \partial r_D/\partial c_L > 0 \) under DARA.

Proposition 22): Under CARA, the optimal risk averse deposit rate is invariant with respect to changes in \( c_L \).

Proof: This proposition follows from the fact that \( E\{U''(\Pi)[-D-\epsilon+D'(r-r_D-c_D)]\} = 0 \) under CARA, as shown in (3-106).

Proposition 23): Given IARA, the optimal risk averse deposit rate is a decreasing function of the marginal administrative cost of loans.

Proof: Under IARA, \( E\{U''(\Pi)[-D-\epsilon+D'(r-r_D-c_D)]\} > 0 \) as shown in (3-114). From this fact, it follows that \( \partial r_D/\partial c_L < 0 \).

Turn now to the optimal loan rate. Differentiating the first order conditions implicitly with respect \( c_L \) yields

\[
\frac{\partial r_L}{\partial c_L} = \frac{-L'E\{U'(\Pi)\} + E\{U'(\Pi)(-L)E[L+L'(r_L-c_L-r)]\}}{U_{LL}} .
\]

Proposition 24): The optimal loan rate is an increasing function of the marginal administrative cost of loans.

Proof: Inspection of (3-119) reveals that the numerator is positive since \( E[L+L'(r_L-c_L-r)] = 0 \) by the first order condition for \( r_L \), and \( -L'E\{U'(\Pi)\} > 0 \). Thus, \( \partial r_L/\partial c_L > 0 \) since \( U_{LL} < 0 \) by the second order condition.

Table 10 presents the results from Propositions 21), 22) and 23). The risk averse deposit rate is an increasing
function of $c_L$ under DARA, and a decreasing function of $c_L$ under IARA. Given CARA, the behavior of the risk averse deposit rate is independent of $c_L$ as in the certainty equivalent case. Proposition 24) implies that the optimal loan rate is an increasing function of the marginal administrative cost of loans for all degrees of risk aversion. Given nondecreasing absolute risk aversion, the spread between $r_L$ and $r_D$ is an increasing function of $c_L$. Interestingly, that under DARA the intermediary increases the deposit rate in response to an increase in $c_L$. Since the loan rate also increases with $c_L$, the overall effect on the size of the spread is uncertain under DARA.

Propositions 21) - 23) can be explained by again appealing to the risk premium interpretation. The effect on the risk premium demanded by the intermediary from a change in $c_L$ is given by

$$\frac{d\theta}{dc_L} = \left[\frac{\alpha \Pi}{\alpha \Pi} + \frac{\alpha \theta}{\alpha \Pi} \frac{\alpha \text{Var}(\Pi)}{\alpha c_L}\right].$$

(3-120)

Furthermore, $\frac{\alpha \Pi}{\alpha c_L} = -L < 0$ and $\frac{\alpha \text{Var}(\Pi)}{\alpha c_L} = 0$. Hence, the second term inside the brackets in (3-120) vanishes and a change in $c_L$ has only a mean income effect on the risk premium. This effect is positive under DARA since $\frac{\alpha \theta}{\alpha \Pi} < 0$.

<table>
<thead>
<tr>
<th>$\alpha r_D/\alpha c_L$</th>
<th>DARA</th>
<th>CARA</th>
<th>IARA</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 0</td>
<td>= 0</td>
<td>&lt; 0</td>
<td></td>
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Table 10
Effects of a Change in $c_L$ on $r_D$. 

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(as \( \Pi \) increases \( \theta \) decreases). Recall that as the risk premium increases, the deposit rate also increases. Therefore, \( \partial \sigma_D / \partial c_L > 0 \).

The DARA intermediary increases the deposit rate as the administrative cost of loans increases. A higher deposit rate implies a larger dollar amount of expected deposits and thus an increase in the size of the firm. Furthermore, an increase in \( c_L \) increases the intermediary's loan rate. A higher loan rate implies a smaller amount of loan extensions and a greater participation as a net lender in the Federal funds market. Note that under DARA, a possibility exists that an increase in \( c_L \) could actually lead to a smaller spread between \( r_L \) and \( r_D \). Under CARA and LARA, the mean income effect on \( \theta \) from a change in \( c_L \) is zero and negative respectively. This explains the signs of Propositions 22) and 23).

Finally, how does a change in \( c_L \) affect the amount of borrowing/lending in the Federal funds market? Differentiating \( R (= D - L) \) with respect to \( c_L \) gives

\[
\frac{\partial R}{\partial c_L} = D' \frac{\partial \sigma_D}{\partial c_L} - L' \frac{\partial \sigma_D}{\partial c_L}.
\]

The second term in (3-121) is positive since \( \partial \sigma_D / \partial c_L > 0 \) and \( L' < 0 \). Given DARA, \( \partial \sigma_D / \partial c_L > 0 \) (Proposition 21)), and \( \partial R / \partial c_L > 0 \). Under DARA the intermediary increases the
amount of lending (expected) in Federal funds. For CARA the 
first term in (3-121) is zero since $\partial r_D/\partial c_L = 0$ under CARA. 
As a result, $\partial R/\partial c_L > 0$ under CARA. Finally, under IARA the 
effect of a change in $c_L$ on $R$ is not clear. Since $\partial r_Q/\partial c_L$ 
$< 0$, $\partial R/\partial c_L < 0$ under IARA.

Consider now the effect of a change in the Federal 
funds rate on the intermediary's spread between $r_L$ and $r_D$, and on the amount of borrowing/lending of Federal funds. Implicit differentiation of the first order conditions with 
respect to $r$ yields

$$\frac{\partial r_D}{\partial r} = \frac{E[U'(\Pi)]D' + E[U''(\Pi)(D+\epsilon-L)[-D-\epsilon+D'(r-r_D-c_D)]]}{U_{\Pi D}}$$

(3-122)

The first term in the numerator of (3-122) is positive, 
$E[U'(\Pi)]D' > 0$, since $D' > 0$ and $E[U'(\Pi)] > 0$. The second 
term in the numerator of (3-122) can be rewritten as

$$E[U''(\Pi)(D+\epsilon-L)[-D-\epsilon+D'(r-r_D-c_D)] =$$

$$- E[U''(\Pi)[-D-\epsilon+D'(r-r_D-c_D)]^2]$$

$$+ (D'(r-r_D-c_D) - L)E[U''(\Pi)[-D-\epsilon+D'(r-r_D-c_D)]]$$

(3-123)

The first term in (3-123) is clearly positive for $U''(\Pi) < 0$.

Proposition 25): Given DARA, a sufficient condition 
for the optimal risk averse deposit rate to be an increasing 
function of the Federal funds rate is that $L > D$.

Proof: Under DARA, $E[U''(\Pi)[-D-\epsilon+D'(r-r_D-c_D)] < 0$ as 
shown in (3-105). Therefore, the sign of the last term of 
(3-123) depends on the sign of $(D'(r-r_D-c_D) - L)$. From (3-
\[ [-D + D'(r-r_D-c_D)] < 0 \text{ which implies that } D'(r-r_D-c_D) - L < D - L. \text{ Thus, a sufficient condition for (3-123) to be positive is that } L > D. \text{ Consequently, given } L > D, \frac{\partial r_D}{\partial r} > 0 \text{ under DARA.}

**Proposition 26):** Under CARA, the optimal risk averse rate is an increasing function of the Federal funds rate.

Proof: Given CARA, \( E[U''(\Pi)[-D-D'+D'(r-r_D-c_D)]] = 0 \) as shown in (3-106). Thus, (3-123) is positive, and \( \frac{\partial r_D}{\partial r} > 0 \) under CARA.

**Proposition 27):** Under IARA, \( \frac{\partial r_D}{\partial r} \) is ambiguous in sign.

Proof: Given IARA, \( E[U''(\Pi)[-D-D'+D'(r-r_D-c_D)]] > 0 \) as demonstrated previously in (3-114). The sign of the last term in (3-123) depends on the sign of \( (D'(r-r_D-c_D) - L) \). From (3-93), \(-D + D'(r-r_D-c_D) < 0\). Subtracting \( L \) from both sides yields

\[ -L + D'(r-r_D-c_D) < D - L. \]

If \( D \leq L \), \( \frac{\partial r_D}{\partial r} \) is ambiguous. For \( D \geq L \), the sign of (3-123) is indeterminate, and \( \frac{\partial r_D}{\partial r} \geq 0 \).

To determine the overall effect on the intermediary's spread between \( r_L \) and \( r_D \) from a change in the Federal funds rate, examine the effects of a change in \( r \) on \( r_L \). Implicit differentiation of the first order conditions with respect to \( r \) yields

\[
\begin{align*}
\frac{\partial r_L}{\partial r} &= -L'E[U''(\Pi)] + E[L + L'(r_L-c_L-r)]E[U''(\Pi)(D+e-L)] - L' E[U'(\Pi)] + E[L + L'(r_L-c_L-r)]E[U''(\Pi)(D+e-L)] \frac{\partial^2 U}{\partial L^2} \\
&= -L'L'E[U'(\Pi)] + E[L + L'(r_L-c_L-r)]E[U''(\Pi)(D+e-L)] \frac{\partial^2 U}{\partial L^2}.
\end{align*}
\]

(3-124)
Proposition 28): The optimal loan rate is an increasing function of the Federal funds rate.

Proof: The numerator of (3-124) is clearly positive since $E[L + L'(r_L - c_L - r)] = 0$, and $-L'E[U'(x)] > 0$. Therefore, $\delta r_L/\delta r > 0$. Table 11 below summarizes the results from Propositions 25), 26) and 27). Given either

<table>
<thead>
<tr>
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<th>DARA</th>
<th>CARA</th>
<th>IARA</th>
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<tbody>
<tr>
<td>$\delta r_D/\delta r$</td>
<td>$&gt;0^*$</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
</tr>
</tbody>
</table>

*Requires $L \geq D$.

DARA or CARA, the intermediary's deposit rate is an increasing function of the Federal funds rate. In the case of DARA, this result requires that the intermediary be an expected net borrower in the Federal funds market, i.e., $L \geq D$. The sign of $\delta r_D/\delta r$ cannot be determined under IARA. Proposition 28) implies that the loan rate is an increasing function of the Federal funds rate irrespective of the degree of risk aversion. The overall impact from a change in $r$ on the spread between $r_L$ and $r_D$ is unclear. A change in the Federal funds rate could increase, decrease or leave unchanged the size of the spread.

An intuitively appealing explanation of the results derived here can be given in terms of the behavior of the risk premium with respect to changes in the Federal funds rate.
The effect on the risk premium demanded by the intermediary (θ) from a change in r given by

\[
\frac{d\theta}{dr} = \left[ \frac{\partial \theta}{\partial \Pi} \frac{\partial \Pi}{\partial r} + \frac{\partial \theta}{\partial \text{Var}(\Pi)} \frac{\partial \text{Var}(\Pi)}{\partial r} \right].
\] (3-125)

In addition,

\[
\frac{\partial \Pi}{\partial r} = D - L
\]

and

\[
\frac{\partial \text{Var}(\Pi)}{\partial r} = 2(r - r_D - c_D)\text{Var}(\varepsilon) > 0.
\]

The effect on the variance of profit from a change in r is positive and implies that the second term inside the brackets in (3-125) is positive for all degrees of risk aversion. On the other hand, the mean profit effect from a change in r depends on whether D > L. If the intermediary is an expected net lender in the Federal funds market (D > L), the mean profit effect from a change in r is positive. If the intermediary is an expected net borrower in the Federal funds market (D < L), then \( \frac{\partial \Pi}{\partial r} < 0 \). Finally, if the institution is neither an expected lender nor an expected borrower (D = L), then \( \Pi \) is invariant with respect to changes in the Federal funds rate.

Table 12 shows the effects on the risk premium as a result of a change in r under DARA, CARA and IARA, for the case when D > L. The signs of the two effects shown in
Table 12 are mutually reinforcing only for either CARA or IARA. Notice that the variance effect on the risk premium from a change in $r$ is always positive. The ultimate effect on $\Theta$ and thus on $r_D$ from a change in $r$ depends on the size of the mean profit effect. It is clear from Table 12 that

<table>
<thead>
<tr>
<th>Mean Profit Effect</th>
<th>DARA</th>
<th>CARA</th>
<th>IARA</th>
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<tbody>
<tr>
<td>Variance Effect</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
</tr>
</tbody>
</table>

if the intermediary is an expected net lender in the Federal funds market, nondecreasing risk aversion (absolute) implies $r_D$ increases with $r$.

Table 13 below illustrates the effects on the risk premium from a change in $r$ for the case when the intermediary is an expected net borrower in the Federal funds market ($L > D$).

<table>
<thead>
<tr>
<th>Mean Profit Effect</th>
<th>DARA</th>
<th>CARA</th>
<th>IARA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance Effect</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
</tr>
</tbody>
</table>

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Table 13 helps to explain the actual effects on $r_D$ from a change in $r$ as obtained in Propositions 25 - 27). Note that if the intermediary is an expected net borrower of Federal funds, the mean income effect on the risk premium from a change in $r$ is positive under DARA. This point is reinforced by the fact that the variance or "risk" effect on $\Theta$ is also positive. Recall that $\Theta$ increases as $r_D$ increases. Hence, it follows that under DARA, $d\Theta/dr > 0$, implies $\partial r_D/\partial r > 0$, provided that $L > D$. This result also holds for the case when $L = D$ since the mean effect is then equal to zero, and the variance effect is still positive.

Finally, how would the amount of the intermediary's borrowing/lending of Federal funds change in response to a change in $r$? Differentiating $R$ with respect to $r$ yields

$$\frac{\partial R}{\partial r} = \frac{D' \partial r_D}{\partial r} - \frac{L' \partial r_L}{\partial r}. \quad (3-126)$$

The second term in (3-126) is always positive since $L' < 0$, and $\partial r_L/\partial r > 0$ for all degrees of risk aversion. Given that the intermediary is an expected net borrower in the Federal funds market, $\partial R/\partial r > 0$ under nonincreasing absolute risk aversion. If the intermediary is an expected net lender in the Federal funds market, $\partial R/\partial r > 0$ holds true only for the CARA intermediary. Note that as $r$ increases, the intermediary's size or scale will be expected to increase under either DARA or CARA for the net borrower intermediary.
The next set of comparative static propositions focuses on the effects on the risk averse deposit rate and spread between $r_L$ and $r_D$ from a mean preserving increase in the distribution of $\varepsilon$. Recall that a mean preserving spread of a random variable increases the variance or spread around the mean of the distribution without affecting the size of the expected value of the random variable. Thus, a mean preserving spread of $\varepsilon$ increases the variance or risk of profit without altering the expected profit. Define a mean preserving spread of $\varepsilon$ as

$$\varepsilon^* = \Gamma \varepsilon.$$  \hfill (3-127)

Writing (3-85) and (3-86) in terms of $\varepsilon^*$, and differentiating with respect to $\Gamma$ (evaluating at $\Gamma = 1$)

$$\left. \frac{\partial r_D}{\partial \Gamma} \right|_{\Gamma = 1} = -E\{U'(\Pi)\varepsilon\} - (r-r_D-c_D)E\{U''(\Pi)(-\varepsilon)\} \left[ -D-\varepsilon + D'(r-r_D-c_D) \right].$$  

(3-128)

Note that the first term in the numerator of (3-128) is positive, $-E\{U'(\Pi)\varepsilon\} > 0$, since $\partial U'(\Pi)/\partial \varepsilon < 0$ for $U'(\Pi) < 0$, and $\partial \Pi/\partial \varepsilon > 0$. After one adds and subtracts $-D + D'(r-r_D-c_D)$ inside the parenthesis at $(-\varepsilon)$, the second term of the numerator of (3-128) can be expressed as

$$-(r-r_D-c_D)E\{U''(\Pi)(-\varepsilon)\} \left[ -D-\varepsilon + D'(r-r_D-c_D) \right] =$$

$$-(r-r_D-c_D)E\{U''(\Pi)(-\varepsilon)\} \left[ -D-\varepsilon + D'(r-r_D-c_D) \right]^2$$

$$-(r-r_D-c_D)(D-D'(r-r_D-c_D))E\{U''(\Pi)(-\varepsilon)\} \left[ -D-\varepsilon + D'(r-r_D-c_D) \right].$$  \hfill (3-129)
Proposition 29): Given DARA, the effect of a mean preserving spread of $e$ on the optimal risk averse deposit rate is positive.

Proof: The first term in (3-129) is positive since $(r-r_D-c_D) > 0$ by the first order condition for $r_D$. The second term in (3-129) is also positive, since $E\{U''(\Pi) [-D-e+D'(r-r_D-c_D)]\} < 0$ under DARA, and $[D-D'(r-r_D-c_D)] > 0$. Thus, (3-129) is positive and $\frac{\partial r_D}{\partial \Pi} \bigg|_{\Pi=1} > 0$.

Proposition 30): Given CARA, a mean preserving spread of $e$ increases the optimal risk averse deposit rate.

Proof: Under CARA, the second term in (3-129) is zero since $E\{U''(\Pi) [-D-e+D'(r-r_D-c_D)]\} = 0$ for CARA (see equation (3-106)). Therefore, (3-129) is strictly positive and (3-128) is also greater than zero.

Proposition 31): Given IARA, the effect of a mean preserving spread of $e$ on $r_D$ is ambiguous in sign.

Proof: This result follows from the fact that under IARA, $E\{U''(\Pi) [-D-e+D'(r-r_D-c_D)]\} > 0$ as shown in (3-114). This implies that the last term in (3-129) is negative and thus (3-128) cannot be signed.

Finally, it is necessary to investigate the effect of a mean preserving increase in $e$ on the intermediary's loan rate. Implicit differentiation of the first order conditions with respect to $\Pi$ (at $\Pi = 1$) yields

$$\frac{\partial r_L}{\partial \Pi} \bigg|_{\Pi=1} = -\frac{(r-r_D-c_D)E[L+L'(r_L-c_L-r)]E\{U''(\Pi)e\}}{U_{LL}}. \quad (3-130)$$

Proposition 32): A mean preserving increase in the distribution of $e$ has no effect on the intermediary's loan rate.
Proof: This proposition follows from the fact that $E[L + L'(r_L - c_L - r)] = 0$ by the first order condition for $r_L$.

Table 14 summarizes the results from Propositions 29), 30) and 31). A mean preserving spread of the distribution

| Table 14 |
| Effects of a Mean Preserving Spread of $e$ on $r_D$. |

<table>
<thead>
<tr>
<th>DARA</th>
<th>CARA</th>
<th>IARA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\partial r_D/\partial r'_I</td>
<td>_{r=1}$</td>
<td>$&gt; 0$</td>
</tr>
</tbody>
</table>

of $e$ has only a variance or risk of profit effect on the intermediary's risk premium. As the variability of $e$ increases, the size of the risk premium increases as well. Recall that as the risk premium demanded by the intermediary increases, the deposit rate also increases. Hence, one should expect an increase in the deposit rate as the distribution of $e$ becomes more volatile (keeping the mean constant). As shown in Table 14, the signs of the effect of a mean preserving spread of $e$ on $r_D$ agree with an intuitive risk premium explanation except for the case of IARA. Proposition 32) implies that a mean preserving spread of $e$ has no effect on $r_L$. Therefore, a mean preserving spread of $e$ reduces the size of the intermediary's spread under nonincreasing absolute risk aversion.

One final issue to be addressed in this section concerns the behavior of the intermediary in the market for
Federal funds following a mean preserving spread of $\varepsilon$. The effect on $R$ from a mean preserving spread in $\varepsilon$ is given by

$$\frac{\partial R}{\partial \varepsilon} \bigg|_{\varepsilon=0} = D \frac{\partial \varepsilon}{\partial \varepsilon} \bigg|_{\varepsilon=0}. \quad (3-131)$$

From (3-131), the behavior of $R$ in response to a mean preserving spread of $\varepsilon$ clearly depends strictly on the effect of the mean preserving spread of $\varepsilon$ on $r_D$. Given nonincreasing absolute risk aversion, a mean preserving spread of $\varepsilon$ increases the amount of lending in the Federal funds market. It is interesting to note that as the distribution of random deposits becomes more risky (in the sense of a mean preserving spread), the intermediary's size (expected) tends to increase.

Table 15 on the following page depicts the comparative static results developed in this section.

**Loan and Deposit Uncertainty Case**

What about the intermediary's rate-setting decisions when loans and deposits are both uncertain? Unfortunately, comparative static results cannot be derived for both sources of uncertainty appearing simultaneously. However, some partial equilibrium comparative static results can be obtained, for instance, the effects of risk aversion on the optimal loan rate when deposit rates are fixed. This mode of behavior is observed on the part of financial intermediaries subject to constraints on the rates that they are allowed to pay to depositors. The last part of this section
Table 15  
Deposit Uncertainty Case  
Comparative Statics Results  
\(S = r_L - r_D\)

<table>
<thead>
<tr>
<th>Effects on Loan Rate</th>
<th>Effects on Deposit Rate</th>
<th>DARA</th>
<th>CARA</th>
<th>IARA</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha r_L/\alpha c_L &gt; 0)</td>
<td>(\alpha r_D/\alpha c_L)</td>
<td>&gt; 0</td>
<td>= 0</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>(\alpha r_L/\alpha c_D = 0)</td>
<td>(\alpha r_D/\alpha c_D)</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>(\alpha r_L/\alpha r &gt; 0)</td>
<td>(\alpha r_D/\alpha r)</td>
<td>&gt; 0*</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>MPS** = 0</td>
<td>MPS**</td>
<td>&gt; 0</td>
<td>&gt; 0</td>
<td>&gt; 0</td>
</tr>
</tbody>
</table>

Effects on Spread

<table>
<thead>
<tr>
<th>(\alpha S/\alpha c_L)</th>
<th>(\alpha S/\alpha c_D)</th>
<th>(\alpha S/\alpha r)</th>
<th>MPS**</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 0</td>
<td>&gt; 0</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>&lt; 0</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
</tr>
</tbody>
</table>

* Requires that \(L \geq D\)

** MPS = mean preserving spread of \(\varepsilon\).

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examines the effects of an increase in risk aversion on the intermediary's spread between $r_L$ and $r_D$. The text shows that an increase in risk aversion tends to lower the intermediary's spread under constant absolute risk aversion.

**Objective Function**

The intermediary's objective function is given by

$$\text{Maximize } \int_{r_L}^{r_D} \int_{-L}^{-D} U(\Pi) g(\mu) h(\varepsilon) d\varepsilon d\mu, \quad (3-132)$$

where $\Pi$ is defined as

$$\Pi = r_L[L+\mu] - c_L[L+\mu] - r_D[D+\varepsilon] - c_D[D+\varepsilon] + r[D+\varepsilon - L + \mu]. \quad (3-133)$$

**First and Second Order Conditions**

The first order conditions are

$$\bar{U}_L = \mathbb{E}\{U'(\Pi)[L+\mu+L'(r_L - c_L - r)]\} = 0 \quad (3-134)$$

$$\bar{U}_D = \mathbb{E}\{U'(\Pi)[-D-r+D'(r-r_D - c_D)]\} = 0. \quad (3-135)$$

The second order conditions are given by

$$\bar{U}_{LL} = \mathbb{E}\{U'(\Pi)[2L + L''(r_L - c_L - r)] + U''(\Pi)(\Pi_L)^2\} < 0 \quad (3-136)$$

$$\bar{U}_{DD} = \mathbb{E}\{U'(\Pi)[-2D + D''(r-r_D - c_D)] + U''(\Pi)(\Pi_D)^2\} < 0$$

$$\bar{D} = \bar{U}_{LL}\bar{U}_{DD} - (\bar{U}_{LD})^2 > 0.$$

For the risk averse intermediary ($U''(\Pi) < 0$), sufficient conditions for the second order conditions to be satisfied are that $L'' < 0$ and $D'' < 0$.

**Certainty versus Uncertainty Comparisons**

In order to determine the effects of risk aversion on
the optimal loan rate, rearrange the first order condition for $r_L$ as follows:

$$MR_L = c_L + r - \frac{\text{Cov}(U'(\Pi), \mu)}{L'E[U'(\Pi)]}.$$  \hfill (3-137)

Under linear risk preferences, the last term of (3-137) vanishes, and the intermediary chooses its optimal loan rate by equating the expected marginal revenue on loans to the sum of the administrative and liquidity costs. When risk preferences are nonlinear, the last term in (3-137) is nonzero, and the intermediary sets its optimal loan rate by equating the expected marginal revenue on loans to a quantity different from just the sum of marginal costs. How does the optimal rate on loans under risk aversion compare to the risk neutral loan rate? The following propositions explore this question.

**Proposition 33**: Assuming $r_\theta$ fixed, and $\mu$ and $\epsilon$ independently distributed with additive loan demand in terms of $\mu$, then $r_L^a < r_L^n$.

**Proof**: In order to verify this proposition it must be shown that the risk premium, $\theta$, that yields equality between $U(\Pi - \theta) = E[U(\Pi)]$ is an increasing function of $r_L^a$. Differentiating $U(\Pi - \theta)$ with respect to $r_L$ and evaluating at $r_L = r_L^a$ yields

$$\frac{\partial}{\partial r_L} U(\Pi - \theta) = U'(\Pi - \theta)[L + L'(r_L^a - c_L - r)] - \frac{\partial}{\partial r_L^a} \theta = 0. \hfill (3-138)$$

From the first order condition for $r_L$, it follows that

$$L + L'(r_L^a - c_L - r) = -\frac{\text{Cov}(U'(\Pi), \mu)}{E[U'(\Pi)]}. \hfill (3-139)$$
Substituting (3-122) into (3-121), it follows that

$$\frac{\partial \theta}{\partial r_L^a} = - \frac{\text{Cov}(U'(\Pi), \mu)}{E[U'(\Pi)]}$$ (3-140)

From (3-123), $\theta$ will be an increasing (decreasing) function of $r_L^a$ as $\text{Cov}(U'(\Pi), \mu) < (>) 0$. Since, $[\partial U'(\Pi)/\partial \mu] = U''(\Pi)(r_L - c_L - r) < 0$, for $U''(\Pi) < 0$, then $\text{Cov}(U'(\Pi), \mu) < 0$, and $\partial \theta / \partial r_L^a > 0$. The risk premium ($\theta$) is an increasing function of the risk averse loan rate, $r_L^a$.

Proposition 34: For $\mu$ and $\epsilon$ jointly normally distributed with $\mu$ and $\epsilon$ positively correlated ($\rho > 0$) and $r_D$ fixed at the optimal risk neutral level $r_D^n$, $r_L^a < r_L^n$.

Proof: This proof requires that $\text{Cov}(U'(\Pi^n), \mu) < 0$, where $\Pi^n$ is the value of $\Pi$ evaluated at $r_D = r_D^n$. Using Taylor series expansion of $U'(\Pi^n)$ about the expected value of $\Pi^n$:

$$\text{Cov}(U'(\Pi^n), \mu) = U''(\Pi_c) \text{Cov}(\Pi^n, \mu)$$ (3-141)

where $\Pi_c$ is a constant that exists by the Law of the Mean. The $\text{Cov}(\Pi^n, \mu)$ can be expressed as

$$\text{Cov}(\Pi^n, \mu) = U''(\Pi_c)((r_L - c_L - r) \text{Var}(\mu) + (r - r_D - c_D) \text{Cov}(\mu, \epsilon)).$$ (3-142)

Substitution of (3-142) into (3-141) yields:

$$\text{Cov}(U'(\Pi^n), \mu) = U''(\Pi_c)((r_L - c_L - r) \text{Var}(\mu) + (r - r_D - c_D) \text{Cov}(\mu, \epsilon))$$ (3-143)

The $\text{Cov}(\mu, \epsilon) > 0$ by assumption ($\rho > 0$) and $U''(\Pi_c) < 0$ for the risk averse intermediary. Thus, it follows that (3-143)
is strictly negative and $r_L^a < r_L^n$.

**Corollary 2:** For $\mu$ and $\varepsilon$ jointly normally distributed with $\rho < 0$ and $r_D^a$ fixed at the risk neutral optimal level $r_D^n$, $r_L^a > r_L^n$. This follows from the fact that (3-143) cannot be signed for $\rho < 0$.

Propositions 33) and 34) suggest that the risk averse intermediary will choose a rate on loans that is below the risk neutral level. Given independent or positively correlated loan and deposit levels, the risk or variability of profit is an increasing function of the loan rate. Thus, the risk averse intermediary will reduce its loan rate to a level below the risk neutral rate, trading off part of its profits in order to reduce the risk or dispersion of profit.

**Proposition 35:** Assuming $r_L^a$ fixed, and $\mu$ and $\varepsilon$ independently distributed with additive deposit supply in terms of $\varepsilon$, then $r_D^a > r_D^n$.

Proof: This proposition can be verified by showing that the risk premium ($\Theta$) that yields equality between $U(\Pi - \Theta)$ and $E[U(\Pi)]$ is a decreasing function of $r_D^a$. Differentiating $U(\Pi - \Theta)$ with respect to $r_D$ and evaluating at $r_D^a$ yields

$$\frac{\partial}{\partial r_D} U(\Pi - \Theta) = U'(\Pi - \Theta) \left[ -D + D'(r - r_D^a - c_D) \right] - \frac{\partial \Theta}{\partial r_D^a} = 0. \quad (3-144)$$

From the first order condition for $r_D$,

$$-D + D'(r - r_D^a - c_D) = \frac{\text{Cov}(U'(\Pi), \varepsilon)}{E[U'(\Pi)]}. \quad (3-145)$$

Substituting (3-145) into (3-144) gives
Hence, \( \theta \) will be an increasing (decreasing) function of \( r_D^a \) as \( \text{Cov}(U'(\Pi), \varepsilon) > (\leq) 0 \). Since, \( \varepsilon U'(\Pi)/\varepsilon = U''(\Pi)(r-r_D-c_D) \leq 0 \) for \( U''(\Pi) < 0 \), and \( \varepsilon \Pi/\varepsilon > 0 \), \( \text{Cov}(U'(\Pi), \varepsilon) < 0 \) and \( \theta \) is a decreasing function of the optimal risk averse deposit rate, \( r_D^a \).

**Proposition 36**: Assuming \( \mu \) and \( \varepsilon \) normally distributed with \( \mu > 0 \) and \( r_L \) fixed at its optimal risk neutral level \( r_L^n \), then \( r_D^a > r_D^n \).

**Proof**: This proposition requires that \( \text{Cov}(U'(\Pi^n), \varepsilon) < 0 \). Taylor series expansion of \( \text{Cov}(U'(\Pi^n), \varepsilon) \) about \( \Pi^n \) yields

\[
\text{Cov}(U'(\Pi^n), \varepsilon) = U''(\Pi_c)\text{Cov}(\Pi^n, \varepsilon), \tag{3-147}
\]

where \( \Pi_c \) is a constant that exists by the Law of the Mean. The \( \text{Cov}(\Pi^n, \varepsilon) \) is given by

\[
\text{Cov}(\Pi^n, \varepsilon) = \{(r-r_D-c_D)\text{Var}(\varepsilon)
\]

\[+(r_L^n-c_L-r)\text{Cov}(\mu, \varepsilon)\}. \tag{3-148}
\]

Substituting (3-149) into (3-148) yields

\[
\text{Cov}(U'(\Pi^n), \varepsilon) = U''(\Pi_c)\{(r-r_D-c_D)\text{Var}(\varepsilon)
\]

\[+(r_L^n-c_L-r)\text{Cov}(\mu, \varepsilon)\}. \tag{3-149}
\]

With \( \text{Cov}(\mu, \varepsilon) > 0 \) by assumption, \( U''(\Pi_c) < 0 \) and \( (r-r_D-c_D) > 0 \), \( (r_L^n-c_L-r) > 0 \) by the first order conditions, \( \text{Cov}(U'(\Pi^n), \varepsilon) < 0 \). Therefore, \( r_D^a > r_D^n \).
Corollary 3): For $\mu$ and $\epsilon$ jointly normally distributed with $\rho < 0$ and $r_L$ fixed at $r_L^n$, $r_D^a > r_D^n$. In this case (3-149) is ambiguous in sign for $\rho < 0$.

When loan and deposit levels are either independent or positively correlated, the risk averse deposit rate is greater than the risk neutral rate. With the loan rate fixed at its optimal risk neutral level, the risk of profit is a decreasing function of the risk averse deposit rate. The risk averse intermediary lowers the dispersion of profit by raising its deposit rate above the risk neutral level.

Effects of Increase in Risk Aversion

This section will examine the effects of an increase in risk aversion on the optimal loan and deposit rates. To allow for consideration of both loan and deposit uncertainty, the following function is assumed to describe the intermediary's utility function:

$$U(\Pi) = -\exp(-\alpha \cdot \Pi).$$  \hspace{1cm} (3-150)

where $\alpha$ is a positive constant which equals the Arrow-Pratt index of absolute risk aversion. That is, $R_a(\Pi) = \alpha$. Thus, (3-150) is consistent with behavior exhibited under constant absolute risk aversion.

Hildreth has shown that when $\Pi$ is normally distributed, maximizing $E[U(\Pi)]$ is tantamount to maximizing: $\Pi - \alpha/2 \ Var(\Pi)$.

---

Hence, the intermediary’s optimization problem becomes

\[
\text{Maximize } \Pi - \frac{\alpha}{2} \operatorname{Var}(\Pi).
\]  \hfill (3-151)

The expected value and variance of profits are given by

\[
\Pi = (r_L - c_L - r)L + (r_D - c_D)D
\]  \hfill (3-152)

\[
\operatorname{Var}(\Pi) = (r_L - c_L - r)^2 \operatorname{E}[^2] + (r_D - c_D)^2 \operatorname{E}[e]^2
\]
\[
+ 2(r_D - c_D)(r_L - c_L - r) \operatorname{Cov}(\mu, \epsilon), \hfill (3-153)
\]

where \( \operatorname{E}[\mu]^2 = \operatorname{Var}(\mu) \) and \( \operatorname{E}[e]^2 = \operatorname{Var}(e) \), since \( \operatorname{E}[^] = \operatorname{E}[e] = 0 \). Incorporating (3-152) and (3-153) into (3-151) results in

\[
F = (r_L - c_L - r)L + (r_D - c_D)D - \frac{\alpha}{2}(r_L - c_L - r)^2 \operatorname{Var}(\mu)
\]
\[- \frac{\alpha}{2}(r_D - c_D)^2 \operatorname{Var}(e) - \alpha(r_L - c_L - r)(r_D - c_D) \operatorname{Cov}(\mu, \epsilon).
\]  \hfill (3-154)

where \( F \) denotes \( \{\Pi - \frac{\alpha}{2} \operatorname{Var}(\Pi)\} \).

If \( \mu \) and \( \epsilon \) are independently distributed, \( \operatorname{Cov}(\mu, \epsilon) = 0 \).

The first order conditions are given by:

\[
F_L = \{L + L'(r_L - c_L - r) - \alpha(r_L - c_L - r) \operatorname{Var}(\mu)\} = 0 \hfill (3-155)
\]

\[
F_D = \{-D + D'(r_D - c_D) + \alpha(r_D - c_D) \operatorname{Var}(e)\} = 0. \hfill (3-156)
\]

Second order conditions are

\[
F_{LL} = \{L' + [L' - \alpha \operatorname{Var}(\mu)] + L''(r_L - c_L - r)\} < 0 \hfill (3-157)
\]

\[
F_{DD} = \{-D' - [D' + \alpha \operatorname{Var}(e)] + D''(r_D - c_D)\} < 0,
\]
and

\[ D = F_{LL}F_{DD} - (F_{LD})^2 > 0 \]

\[ F_{LD} = \text{Cov}(\mu, \varepsilon) \overset{>}{\underset{<}{\approx}} 0 \text{ as Cov}(\mu, \varepsilon) \overset{>}{\underset{<}{\approx}} 0. \]

Inspection of the first order conditions reveals that necessary conditions for an interior optimum are that \( r_L - c_L - r > 0 \) and \( r - r_D - c_D > 0 \). For \( U''(\Pi) < 0 \), the second order conditions will be satisfied, provided that \( L'' < 0 \) and \( D'' < 0 \).

Differentiating the first order conditions with respect to \( \alpha \) yields:

\[ \frac{\partial r_L}{\partial \alpha} = -D^{-1}[F_{L\alpha} \cdot F_{DD}] \quad (3-158) \]

\[ \frac{\partial r_D}{\partial \alpha} = -D^{-1}[F_{D\alpha} \cdot F_{LL}], \quad (3-159) \]

where \( F_{L\alpha} = - (r_L - c_L - r)\text{Var}(\mu) < 0 \), and \( F_{D\alpha} = (r - r_D - c_D)\text{Var}(\varepsilon) > 0 \). Therefore, \( \frac{\partial r_L}{\partial \alpha} < 0 \), and \( \frac{\partial r_D}{\partial \alpha} > 0 \). An increase in the coefficient of risk aversion \( (\alpha) \), increases \( r_D \) and decreases \( r_L \). Hence, given uncorrelated loan and deposit levels, increases in risk aversion tend to reduce the size of the controllable rate spread. With \( \text{Cov}(\mu, \varepsilon) \neq 0 \), the signs of \( \frac{\partial r_L}{\partial \alpha} \) and \( \frac{\partial r_D}{\partial \alpha} \) are not determinate a priori.
Chapter 4

CONCLUSIONS AND IMPLICATIONS FOR THE THEORY OF DEPOSITOR FINANCIAL INTERMEDIARIES

Chapter 3 developed a microeconomic model of a depository financial intermediary under uncertainty. The model's formulation emphasized the behavior of depository intermediaries as rate setters in the loan and deposit markets. Several key assumptions underlay the model, including imperfect loan and deposit markets, real resource cost considerations, and nonlinear risk preferences. The approach employed to model the intermediary's optimizing behavior was that of expected utility maximization of profits subject to a balance sheet constraint. Profits were assumed random due to uncertainty over the quantity of loans and deposits. This chapter will review the theoretical results derived in the preceding chapter and examine their bearing on the theory of depository financial intermediaries.

In the risk neutral or certainty equivalent case, the intermediary determines the optimal loan rate by equating the marginal revenue from loans to the sum of the marginal administrative cost of loans plus the rate on Federal funds. Similarly, the optimal risk neutral deposit rate is set at a level at which the sum of the marginal interest cost of 128
deposits and their marginal administrative cost just equals
the Federal funds rate. Under risk neutrality, the loan and
deposit rate-setting decisions are clearly separable. The
optimal risk neutral loan rate is independent both of the
rate to be paid on deposits and of the marginal administra-
tive costs incurred in providing services to depositors.
Independence between the optimal loan and deposit rate
decisions under risk neutrality is consistent with the
findings of Klein.\footnote{M. A. Klein, "A Theory of the Banking Firm," \textit{Journal of
Money Credit and Banking}," 3:205-18, May, 1971.} The optimal scale of operation of the
risk neutral intermediary is determined by equating the
marginal revenue from loans to the sum of both the marginal
administrative cost of loans plus the marginal cost of
deposits. Comparative statics results indicate that the
optimal risk neutral loan rate is an increasing function of
the marginal administrative cost of loans and independent of
the marginal administrative cost of deposits.

On the deposit side, the optimal risk neutral deposit
rate is a decreasing function of the marginal administrative
cost of deposits and invariant with respect to changes in
the marginal administrative cost of loans. Furthermore, the
risk neutral loan and deposit rates are increasing functions
of the Federal funds rate. With respect to the
intermediary's spread between $r_L$ and $r_D$, an increase in the
administrative cost of either loans or deposits increases
the size of the intermediary's spread. In addition, an increase in the marginal administrative cost of loans increases the amount of lending done by the intermediary in the Federal funds market. As $c_L$ increases, the institution tends to become more of a net lender in the Federal funds market. On the other hand, an increase in the marginal administrative cost of deposits reduces the amount of lending in Federal funds. The intermediary tends to become a net borrower of Federal funds as $c_D$ increases. Finally, an increase in the Federal funds rate leads the intermediary to raise both $r_L$ and $r_D$. Consequently, the effect on the risk neutral spread from a change in the Federal funds rate is indeterminate. The effect on the spread between $r_L$ and $r_D$ from a change in the Federal funds rate depends on both the rate elasticities of the loan demand and deposit supply functions.

Relaxing the assumption of risk neutrality has significant effects on the behavior of the depository intermediary confronted with quantity uncertainty on either side of its balance sheet. Under uncertain loan demand, the variance of the intermediary's profit is an increasing function of the risk averse loan rate. As a result, the risk averse intermediary sets a loan rate that is below the risk neutral rate in an effort to reduce the dispersion of profit. Thus, the risk averse institution is willing to accept a reduction in expected profit in exchange for less variability around the expected value of profit. This
result is quite opposite to the conclusion reached by Pringle.\(^2\) The results derived here imply that loan demand uncertainty and risk aversion lead to a reduction in the size of the intermediary's spread between \(r_L\) and \(r_D\). Given uncertain loan demand, the size of the intermediary's spread decreases as risk aversion increases.

The model also has some implications concerning the borrowing/lending position of the intermediary in the Federal funds market. The risk averse intermediary under loan demand uncertainty sets \(r_L\) and \(r_D\) (and thus the spread) such that the intermediary expects to be a net borrower in the Federal funds market. On the other hand, if the intermediary prefers risk, it will set \(r_L\) and \(r_D\) such that it expects to be a net lender of Federal funds.

Comparative static results of the loan uncertainty case indicate that the behavior of the risk averse spread in response to changes in \(c_L\), \(c_D\) and \(r\) is identical to the certainty equivalent case under CARA and IARA. Given nondecreasing absolute risk aversion, an increase in the administrative cost of either loans or deposits leads to an increase in the intermediary's spread. The effect of a change in the Federal funds rate on the spread is ambiguous under risk neutrality as well as for all degrees of risk aversion.

Under DARA, the effects on the spread from changes in $c_L$, $c_D$ and $r$ are inconclusive. This ambiguity is important in that it raises the possibility that spread management behavior of the DARA intermediary may differ significantly from the behavior of the intermediary under CARA or IARA. Recall that DARA represents the most empirically significant hypothesis concerning the relationship between wealth and risk taking behavior. One effect on the spread, whose direction is unambiguously clear under DARA, is that of a mean preserving spread of the distribution of $\mu$. A mean preserving spread of $\mu$ tends to reduce the size of the intermediary's spread under either DARA or CARA. Recall that a mean preserving spread of $\mu$ increases the variability of the distribution of $\mu$ without affecting the mean of the distribution. Note that a mean preserving spread of $\mu$ has no effect on the optimal deposit rate. Hence, as loan demand variability increases (mean preserving spread), the DARA and CARA intermediaries reduce the size of their spread by lowering their loan rates. The effect of a mean preserving spread of $\mu$ on the spread is ambiguous under IARA.

Comparative static results of the model reveal several insights into the behavior of the risk averse intermediary in the market for federal funds. Given CARA or IARA, an increase in the administrative cost of making a loan induces the intermediary to lend more funds in the Federal funds market. Under DARA, the intermediary could actually end up
borrowing more in the Federal funds market as a result of a change in \(c_L\). Note that the Federal funds market serves as a source as well as use of funds for the intermediary. Thus, it represents a constant cost source of funds alternative to deposits and a constant return-earning asset as well. It follows that any changes which affect the return or cost of loans and deposits will cause a change in the amount of borrowing/lending of Federal funds by the intermediary. For example, an increase in the administrative cost of deposits increases the total resource cost of using deposits as a source of funds. An increase in \(c_D\) will lead the intermediary to borrow more Federal funds.

Perceiving the relationship between the amount of borrowing/lending of Federal funds and the behavior of the spread under different degrees of risk aversion helps one to understand the process of liability management.

The results of the loan uncertainty model have significant implications for the interaction between loan and deposit rate decisions. Comparative static results indicate that the optimal risk averse loan rate is a decreasing function of the marginal administrative cost of deposits under DARA, invariant under CARA, and increasing for IARA. Thus, whereas the risk neutral loan rate is invariant with respect to changes in the administrative cost of deposits, the optimal risk averse loan rate depends on the parameters of the deposit supply function. It appears that the existence of resource cost considerations, loan
demand uncertainty, and risk aversion provide a link between the optimal loan and deposit rate decisions.

When uncertainty exists on the deposit side, the risk averse intermediary selects a deposit rate which is strictly higher than the risk neutral deposit rate. Since the loan rate is unaffected by random deposit supplies, the risk averse spread between \( r_L \) and \( r_D \) is smaller than the risk neutral spread. When deposit supplies are random, the variance of the intermediary's profit is a decreasing function of the risk averse deposit rate. The risk averse intermediary is willing to forego some expected profit (higher deposit rate) in order to reduce the risk or variability of profit. Consequently, it sets the risk averse deposit rate at a level above the risk neutral rate. Thus, the introduction of either loan or deposit uncertainty coupled with risk aversion leads to a reduction in the size of the intermediary's spread from the risk neutral level. A smaller risk averse spread results in the reduction of profits and of their variability as well.

This result provides theoretical support for Heggestad's empirical findings. Examining the influence of market structure on bank profitability, his study concluded that banks with monopoly power use it to reduce the risks they take rather than to increase their level of profits.

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Note that one of the implications of deposit uncertainty under risk aversion is an increase in the expected size of the intermediary. Furthermore, the risk averse intermediary facing deposit supply uncertainty sets \( r_L \) and \( r_D \) such that it is an expected net lender in the Federal funds market. Thus, if the institution has considerable uncertainty as to the amount of deposit funding, it will set \( r_L \) and \( r_D \) such that it expects to lend funds in the Federal funds market.

Comparative static results of the deposit uncertainty case reveal that the effects on the intermediary's spread from changes in \( c_L \), \( c_D \) and \( r \) are fully consistent with the effects under loan demand uncertainty. Again, just as in the case of loan demand uncertainty, these results imply that the behavior of the spread under DARA may differ significantly from the spread under CARA or IARA. For the most part, the effects on the amount of borrowing/lending of Federal funds from changes in \( c_L \), \( c_D \) and \( r \) are consistent with the effects obtained for the loan demand uncertainty case.

Finally, Pyle\(^4\) provides one of the most important contributions to the theory of depository financial intermediaries. His analysis explored the conditions under which the process of financial intermediation would be enhanced. Using a portfolio model of a depository financial intermediary, he concluded that an increase in the correlation

between loan and deposit yields would tend to facilitate the process of financial intermediation.

Pyle's model viewed the depository intermediary as a quantity setter in both loan and deposit markets. In such a setting, the degree of correlation between uncertain yields on assets and liabilities becomes a key determinant of the process of financial intermediation.

The issue raised here is whether Pyle's results can be extended to the case of a rate setting depository intermediary. However, under the assumption of rate-setting on both loans and deposits, it is no longer appropriate to focus on the correlation between loan and deposit yields. Rather, the issue becomes whether an increase in the correlation between loan and deposit levels enhances the process of financial intermediation.

When both loans and deposits are uncertain, and the intermediary's utility function is exponential, the objective function as given by (3-137) is

\[ F = (r_L - c_L - r)L + (r - r_D - c_D)D - \alpha/2 (r_L - c_L - r)^2 \text{Var}(\mu) \]

- \( \alpha/2 (r - r_D - c_D)^2 \text{Var}(\varepsilon) - \alpha(r_L - c_L - r)(r - r_D - c_D) \text{Cov}(\mu, \varepsilon). \) \( (4-1) \)

The first order conditions are

\[ F_L = (r_L - c_L - r)L' + L - \alpha(r_L - c_L - r) \text{Var}(\mu) - \alpha(r - r_D - c_D) \text{Cov}(\mu, \varepsilon) = 0 \]

\[ F_D = (r - r_D - c_D)D' - D + \alpha(r - r_D - c_D) \text{Var}(\varepsilon) \]

\[ + \alpha(r_L - c_L - r) \text{Cov}(\mu, \varepsilon) = 0. \] \( (4-2) \)
Necessary conditions for optimal values of \( r_L \) and \( r_D \) are \( r_L - c_L - r > 0 \) and \( r - r_D - c_D > 0 \). The second order conditions will be satisfied, provided that \( L'' < 0 \) and \( D'' < 0 \).

In order to determine the impact of a change in the correlation between \( \mu \) and \( \varepsilon \) on the optimal loan and deposit rates, the first order conditions can be implicitly differentiated with respect to \( \rho \):

\[
\frac{\partial r_L}{\partial \rho} = -D^{-1} \begin{bmatrix} F_{L \rho} & F_{LD} \\ F_{D \rho} & F_{DD} \end{bmatrix}
\]

\[
= -D^{-1}[F_{L \rho} F_{DD} - F_{LD} F_{D \rho}],
\]  

(4-4)

and

\[
\frac{\partial r_D}{\partial \rho} = -D^{-1} \begin{bmatrix} F_{LL} & F_{LP} \\ F_{DL} & F_{DP} \end{bmatrix}
\]

\[
= -D^{-1}[F_{LL} F_{DP} - F_{LD} F_{LP}],
\]  

(4-5)

where \( F_{L \rho} \) and \( F_{D \rho} \) are given by

\[
F_{L \rho} = -\alpha (r - r_D - c_D) \sigma_\mu \sigma_\varepsilon
\]  

(4-6)

\[
F_{D \rho} = \alpha (r_L - c_L - r) \sigma_\mu \sigma_\varepsilon
\]  

(4-7)

\( F_{LL} < 0 \), \( F_{DD} < 0 \) by the second order conditions, and

\[ D = F_{LL} F_{DD} - (F_{LD})^2 > 0. \]
If the signs of (4-4) and (4-5) are such that an increase in \( \rho \) results in a larger spread between \( r_L \) and \( r_D \), then one can conclude that an increase in the correlation between loan and deposit levels facilitates the process of financial intermediation. If an increase in the correlation coefficient between loans and deposits reduces the size of the spread, then such an increase will more than likely make financial intermediation less attractive.

Returning to expressions (4-4) and (4-5), the term \( F_{LD} \) is given by

\[
F_{LD} = \alpha \text{Cov}(\mu, \epsilon), \quad \alpha > 0.
\]  

(4-8)

The sign of \( F_{LD} \) depends on whether \( \mu \) and \( \epsilon \) are positively, independently, or negatively correlated. Thus, \( F_{LD} > 0 \) as \( \rho > 0 \).

When loan and deposit levels correlate negatively, then \( F_{LP} < 0 \) and \( F_{DP} > 0 \). This implies that \( \partial r_L / \partial \rho < 0 \) and \( \partial r_D / \partial \rho > 0 \). Hence, for negatively correlated loan and deposit levels, an increase in \( \rho \) tends to reduce the size of the spread. A smaller spread means lower profits which make the intermediation process less attractive.

With positively correlated loan and deposit levels, the signs of (4-4) and (4-5) are indeterminate. Thus, Pyle's conclusions regarding the conditions that would enhance the process of financial intermediation do not generally hold when extended to the rate-setting depository intermediary. It has been demonstrated here that with loan and deposit...
levels negatively correlated, an increase in the correlation coefficient between loan and deposits will actually tend to lessen the attractiveness of the intermediation process by reducing the intermediary's profits.

A final question arises. What are the major implications of this dissertation for the theory of depository financial intermediaries?

1) The overall impact of quantity uncertainty and risk aversion is the reduction of the intermediary's spread from the risk neutral (expected profit maximizer level). This result has policy implications concerning the relationship between profitability (prices), market structure, and risk aversion and suggests that risk averse depository institutions use their monopoly or market power to reduce the risks they take rather than to increase their profits.

2) The optimal net position in the Federal funds market adopted by a risk averse intermediary will depend on the sources of uncertainty that dominate its balance sheet constraint. If the intermediary's loan demand uncertainty dominates other sources of uncertainty in its balance sheet, then the intermediary will set $r_L$ and $r_D$ such that it expects to be a net borrower in the Federal funds market. If, on the other hand, deposit supply uncertainty dominates, then the intermediary will set $r_L$ and $r_D$ such that the firm is an expected net lender in the Federal funds market. These results are conditional upon the risk neutral
intermediary's selecting a neutral net position in the Federal funds market.

3) Quantity uncertainty and risk aversion will tend to increase the scale of operation of the intermediary to a level about the profit maximizing or risk neutral level. In the case of loan uncertainty, the growth in size takes place through the purchase of Federal funds. On the other hand, deposit uncertainty implies that the increase in size is funded by deposits.

4) The effects of quantity variability on the liquidity of the intermediary's asset portfolio are asymmetrical. An increase in the variability of loan demand will reduce the liquidity of the asset portfolio. In contrast, an increase in deposit variability enhances the liquidity of the intermediary's asset portfolio.

5) Comparative statics results imply a positive relationship between deposit variability and the size or scale of operation of the intermediary. For a partial review of the literature on this particular relationship see Kaufman. Furthermore, the results presented here indicate that deposit variability and the liquidity of the intermediary's asset portfolio are positively related, thus

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supporting the conclusions of Hester\(^6\) and Kane and Malkiel.\(^7\)

6) The results from the model imply that the intermediary's loan rate is independent of deposit variability. Essentially, the intermediary neutralizes the effects of deposit variability on the loan rate by the use of the Federal funds market. As deposit variability increases, the intermediary increases its lending of Federal funds.

7) Comparative statics results indicate that deposit variability relates inversely to the intermediary's profitability.

8) Behavior of the spread in response to changes in administrative costs could differ significantly for the DARA intermediary. The effects of quantity uncertainty in either loans or deposits are consistent and mutually reinforce the behavior of the spread. The degree of risk aversion appears to be an important factor in determining spread management behavior.

9) The dissertation showed that when loan and deposit levels are correlated negatively, an increase in the correlation coefficient between \(\mu\) and \(\varepsilon\) may actually lessen

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the attractiveness of the process of financial intermediation. This was shown for the case when the intermediary's utility function exhibits properties consistent with CARA. Therefore, Pyle's\(^8\) conclusions cannot be extended to rate-setting depository financial intermediaries. However, if loan and deposit levels correlate positively, an increase in the correlation between \(\mu\) and \(\sigma\) could lead to an increase in the intermediary's spread. This trend would enhance the process of financial intermediation.

This dissertation has provided a workable model of a depository financial intermediary under uncertainty. The model yields a number of intuitively appealing results (theoretical), and has some significant implications for several aspects of the published literature on the theory of depository financial intermediaries. A number of extensions could prove useful in any empirical attempts to test the model's implications. The inclusion of institutional factors, such as reserve requirements and deposit insurance would be one of these possible extensions.

\(^8\)Pyle, loc. cit.
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VITA

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Major Field: Finance

Title of Dissertation: Loan and Deposit Rate Setting, Risk Aversion, Uncertainty, and the Theory of Depository Financial Intermediaries

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Date of Examination:

May 2, 1985