1985

Modeling the Failure Time Distribution for Manufacturing and Retail Corporations Using Survival Analysis (Bankruptcy, Cox Model, Binary Response, Business).

George S. Karamessinis
Louisiana State University and Agricultural & Mechanical College

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MODELING THE FAILURE TIME DISTRIBUTION FOR MANUFACTURING AND RETAIL CORPORATIONS USING SURVIVAL ANALYSIS

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A Dissertation
Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Business Administration

by
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B.S. University of Patras, Greece, 1978
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August 1985
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It is not practical to explicitly cite here all who assisted and encouraged me in this effort. Consequently, this acknowledgement is incomplete by design. My sincere thanks to all that helped.

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G. S. Karamessinis
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ABSTRACT

In the past the problem of financial distress has been investigated mainly through discriminant analysis and conditional response (logit, probit) techniques. With the use of such models, inference is made about the future status of a company as failure or non-failure conditional upon its observed financial attributes. Although response models (and discriminant models under certain assumptions) can be used to estimate the probability of failure of a firm as a function of its observed characteristics, neither group of techniques can provide estimates of the failure rate (hazard) of a population as a function of time.

In certain situations, when the time to failure is an important determinant of the payoffs, knowing the failure rate over time becomes critical. Expected payoffs, under different investment or lending decision policies, can be estimated when a model of the evolution of failures over time is available.

This study provides a functional method of modeling the empirical survivor function of a corporation over a period of at least five years, conditional upon the corporation's observed financial characteristics. The survivor function $S(t, z)$ (which provides the probability that a firm of $z$ financial attributes will survive for at least $t$ years) was estimated through the proportional hazards model. The covariates employed in the formation of the hazard function were
chosen from accounting variables and financial ratios constructed from
the information contained in the annual statement of publicly traded
manufacturing and retail companies.

The survivor function leads to the estimation of the
probabilities of failure by time intervals of interest, inside the
study period of five years. The significance of this feature is that
one does not need to be confined to the probability of binary response
(i.e., failure on non-failure) within the whole study period; the
probabilities of failure over finer time segments are provided. This
is in contrast to the information provided by discriminant analysis
and other binary response models which by themselves provide little
insight into the way explanatory variables affect survival.
CHAPTER I

1. INTRODUCTION

1.1 Statement of the Problem and General Approach

In an economic system, the continuous entry and exit of firms, in and out of the productive arena, is viewed as part of the mechanism that operates to increase the efficiency of the economy. Through competition, inefficient members are eliminated in favor of the more efficient ones. A firm's exit from the above system is manifested by a liquidation or bankruptcy. However, this process of elimination is not free of cost to the parties associated with a troubled entity. A failure and death of a company results in investors' loss of equity and dividends, creditors' loss of principal/return, and employees' loss of jobs. The death of a large company is also felt indirectly by many other companies and in many other sectors of the economy through the chain effects operating in a dynamic environment. Furthermore, a firm involved in bankruptcy proceedings incurs direct as well as indirect costs. These include tangible costs such as fees for bankruptcy filing, legal and other fees for professional services and intangible costs such as managerial time expended for liquidation or reorganization and most important, loss of sales due to perceived potential bankruptcy.
From the standpoint of the troubled firm and the parties associated with it, we can see the benefits of recognizing a risky state early in order to minimize the losses and remedy the situation if possible. This task of recognition calls for modeling the company's conditional life distribution as a function of its own financial characteristics and the external risk due to market conditions. A probability model of this nature, by putting the company in perspective with other companies in the economy, can signal impending financial trouble and alert its management to take corrective actions. Potential uses of such a model go beyond the boundaries of the company itself. Regulatory agencies, for instance, need to assess the riskiness of regulated firms. Estimating the financial risk is also important for an investor in capital stock, a purchaser of bonds, or a banker making a commercial loan decision. Moreover, under the failing company doctrine, a firm in distress can be exempted from certain antitrust prohibitions and be allowed to merge with another firm, if high probability for its failure can be established.

Building effective models that detect risky states of firms has become increasingly important in recent years. The main reason for this is the fact that the number of business bankruptcy filings has increased dramatically in the last decade. Since 1974 the average number of bankruptcy filings\(^1\) per year has not fallen below 20,744; reaching the record of 47,414 in 1981 which was later surpassed in 1982 by more than 10,000. The last decade also saw a substantial increase in average liability of a failing firm\(^2\) as compared to the
preceding decade. The average liability\(^3\) has remained uniformly above the $300,000 level since 1974, and this is twice the highest average liability experienced in any year between 1960 and 1970.\(^4\)

Researchers, in their endeavors to detect bankruptcy, have been building models for the last two decades. Unfortunately, faced with such a complicated task, the models are not free of trade-offs. Consequently, the adequacy of a model is determined on the basis of its intended use. The majority of the models are designed to classify firms into either a future state of failure or a state of a non-failure. For some decisions, however, such a dichotomous classification is not adequate without the simultaneous estimation of the probability that a firm will be routed to one of the above two states. Moreover, for these transition probabilities to be most useful, one needs to express them as functions of time. Formulation of decision rules for setting risk premia or taking investment actions become more accessible when one has the distribution of failure times within the period of study. It appears, therefore, that estimating the survivor function of publicly traded companies will be welcomed as an alternative to previous efforts in modeling bankruptcy probabilities. In summary, this research estimates the survivor function of publicly traded manufacturing and retail firms using the Cox proportional hazard model. The survivor function \(S(t, z)\) provides the probability that a company with characteristics vector \(z\) will survive for at least \(t\) years into the future. The estimation of the survivor function is in contrast to other studies that classify companies as potential failures or non-failures.
Due to the great importance of the subject "corporate failure", the literature is very extensive. Fortunately, there are several reviews of the literature available, including those by Altman (1983), Scott (1981), and Zavgren (1983). The vast majority of the studies design models to predict whether a company will fail within a period of one to five years following the observation of its financial characteristics. In the sequel, in order to summarize the literature on corporate failure prediction models in the most convenient form, the existing studies on the topic will be grouped by the statistical techniques that they used and then the following points will be discussed in each group:

1. statistical and theoretical findings that are suggestive for this research, and
2. weaknesses and limitations.

1.2 Literature Review: General Considerations

The great number of papers on the topic of bankruptcy and the continuing search for a better model are symptoms of the difficulties inherent in the subject. Such difficulties arise from the following:

1. There is not an explicit, well-defined theory of the bankruptcy process.
2. The underlying conditions of the economy that relate to the bankruptcy phenomenon are hard to estimate and predict.
3. The inherent limitations of the statistical models.
4. A researcher cannot obtain variables in the economic sense that describe the characteristics of a company. The best available alternative is to use surrogate accounting data.

5. Most of the variables that one can get from financial statements are correlated either on a one-to-one basis or in the form of linear compounds.

6. The small number of corporate bankruptcies for which one has data within a given year.

7. Policy, law and regulations affecting the bankruptcy process change over time.

The most popular corporate failure prediction models can be classified into four major groups: (a) single variate discriminant, (b) multivariate discriminant, (c) models based on the gambler's ruin stochastic process, and (d) conditional response models.

In developing the models of the above groups, the researcher obtains a sample of failed and non-failed firms which is divided into two sub-samples. One of these sub-samples is used for fitting the model whereas the other (hold out) sample is used to estimate the model's classification accuracy. Companies in the hold-out sample are classified as potential failures or non-failures on the basis of their observed attributes and the obtained accuracy over this sample is assumed to be a good estimate of any ex ante prediction performance. In some cases the model is revalidated over samples taken from future periods [Altman (1983, p. 136); Altman and McGough (1974); Moyer (1977)].

The limited theoretical knowledge about the mechanism of failure
led to the construction of the vast majority of models using the empirical approach of choosing ten to twenty variables and then reducing the number on the basis of their predictive ability. An exception to this approach was taken by Wilcox (1971, 1973) who, using the framework of the "gambler's ruin" theory, made a more conscientious attempt to incorporate a dynamic process of failure into the development of his model.

The explanatory variables used in the construction of the existing failure prediction models are obtained from the financial statements which contain accounting information. Using accounting information obscures the comparability of different firms because the measurement of accounting variables is not consistent across companies. Better information, however, is not available.

Ideally, one would like to incorporate into the models those economic variables which influence the failure rate of corporations. However, the association between economic variables and failure rate is intricate and therefore hard to model; moreover, it is probably non-stationary over time. As a result, the absence of economic variables from failure models is widespread. One could argue that the absence of market and economic variables from the bankruptcy prediction models is the result of being pragmatic and functional. Economic and market variables, more often than not, are hard to forecast and in some cases the current observation of such a variable is the best forecast for its future value. Consequently, incorporating economic variables into the models would not necessarily increase the accuracy of an ex ante prediction as this accuracy would
be subject to the precision of the forecasted economic variables. On the other hand, a model with economic variables has the potential of providing, subject to any non-stationarity between economic variables and failure rate, the benefit of sensitivity analysis, by allowing the incorporation of one's personal judgment about the future economic conditions into the model.

Another point that needs to be discussed is the definition of failure in the different studies in the literature. Some researchers consider a company as failed when it files under Chapter X, XI, or 11 of the Bankruptcy Act. More general definitions include liquidation, bond default, overdrawn bank account and non-payment of preferred stock dividend. One consequence of this is that the comparability of different studies is somewhat obscured.

Another point that interferes with the comparability of the existing studies is that they use non-random samples from different time periods to estimate the performance of their models. The latter makes the task of interstudy comparison difficult as the error rates depend on the underlying diversity of the hold out sample. The more uniform the hold out sample within each subgroup (e.g. failed, non-failed), the easier is the task of discrimination and prediction. In the following sections each group of techniques will be discussed in more detail.

1.3 Single Covariate Models

The most representative research done in this group is that of
Beaver (1966). In contrast to most subsequent researchers, Beaver investigated a broader group of failure including bankruptcies, bond defaults, overdrawn bank accounts and omission of preferred dividends.

Beaver examined 30 financial ratios and observed that the following three ratios performed best as predictors of failure: cash flow/total debt, net income/total debt, and cash flow/total assets. The procedure that he followed is rather subjective. Through observation of the distribution of these ratios he determined cut-off points so that companies with ratios below these points were classified as potential failures. In a subsequent study, Beaver (1968) conducted another test to see if the stock market could predict failure before accounting ratios did and found that the stock market performed slightly better.

Most subsequent researchers felt that these single ratio models are too simplistic to capture the complicated nature of the bankruptcy process. However, Beaver's studies are quite informative on the behavior of ratios of companies close to bankruptcy. In particular he showed empirically that the average ratios of the bankrupt group have more distinct trends over time than do those of the non-bankrupt group.

1.4 Multiple Discriminant Models

Most of the available bankruptcy prediction models belong to this group. The models that have received greatest publicity are those of Altman (1968); Altman, Haldeman, Narayanan (1977); Blum (1974); and

Researchers in this group use multivariate discriminant models arguing that single variable models are too simplistic to capture the complexity of the financial failure process.

In most of the papers in this group, the independent variables are financial ratios. Ratios based on stocks and flows and ratios closely related to corporate earnings are the ones most commonly used.

One assumption in using discriminant classification methods is that the underlying populations (e.g., bankrupt, non-bankrupt) are described by different probability distributions. In order to determine which population a given individual comes from, the researcher constructs a linear or a non-linear index that best discriminates among populations. Conceptually though, when an alive company is observed and inference is attempted for its future status (e.g., failure or non-failure), the company is definitely in the group of live companies. Consequently, discriminant analysis implicitly tries to make a statement about the path of the process that will classify the company in either one of the two populations. This process is a dynamically evolving one and hence calls for the time of classification as well as the probability that a company will be routed to one or the other population. Discriminant analysis techniques cannot address explicitly either one of these two requirements. What has been done in most studies, as an alternative, is to take a sample of failed companies one year before failure and a matched sample of healthy firms from the same time span, and construct the discriminant function. Then for a company whose profile resembles
the profile of the bankrupt group, it is inferred that the company will fail in one year. Similarly, for bankruptcy inference in k years, a sample of bankrupt firms k years before failure is taken and the same inference technique is used. Evidently, the time of the event does not enter the model explicitly. However, the success of these models in ex post discrimination suggests that the financial ratios do contain information useful in determining the conditional probability (or transition probability) that a company will move from a non-bankrupt state to a bankrupt one. Consequently, the task of this paper is to estimate the survivor function of corporations using such financial information.

Discrimination techniques, apart from their conceptual shortcomings, suffer from other difficulties in model implementation. In summary these are:

1. Lack of normality of independent variables (this will be expanded upon further in the sequel).

2. Unequal dispersion matrices of different populations.

3. Establishment of the classification rule, which is complicated by the lack of normality and inappropriate selection of a priori probabilities and costs of misclassification.

4. Sampling problems of (a) pooling data across periods, (b) dropping from the sample companies that do not have data for several years, (c) ignoring companies that merged, were acquired, etc.

5. Ex post validation of the models (using contemporary data).

From the theoretical point of view, this group of models deserve the attention of the reader. The stochastic process is based on the following simple scenario. A gambler enters a game with k dollars. At each bet he either loses a dollar with probability of p or wins one with probability 1-p. In financial applications the firm is viewed as a gambler that has k amount of "worth". Each year this "worth" increases or decreases by amount s with probability of p or 1-p, respectively.

This approach also has several shortcomings: (1) The transition probability p is considered fixed, despite the fact that it is probably a function of the state of the company and the economy and (2) Each year the company's worth can change by s dollars only.

This model is important because it does address the dynamic nature of a company's evolutionary process. The above shortcomings, however, make the model subject to criticisms. Scott (1981, p.323), who compliments the model for its dynamic nature, says the following:

"The attempts to apply this model have been disappointing, perhaps because the version of the theory used is too simple, assuming as it does that cash flows result from a series of independent trials, without the benefit of an intervening management action. Although the theory specified a functional form for the probability of ultimate ruin, Wilcox found that this probability was not meaningful empirically ... Nevertheless Wilcox's work is
notable as the first attempt to use explicit theory to predict bankruptcy."

This model does have some nice characteristics. It is descriptive, yet simple. Another useful characteristic is its power to provide probabilities for the different states.

1.6 Conditional Probability Response Models

In recent years a few studies have employed conditional probability response models to estimate the probabilities of business failures. Corporate failure studies using such models include those by Ohlson (1980), Zavgren (1982), and White and Turnbull's (1975) unpublished analyses of the probability of failure of industrial firms. Conditional response models have also been used by Martin (1977) to study the probability of bank failure and by Chesser (1974) to study commercial loan non-compliance.

Dichotomous (polytomous) conditional probability response models are used to relate a set of attributes of an individual to the probability that the individual will be associated with one of two (several) mutually exclusive states; this association is commonly referred to as the observed response. The most popular models in this class are logit and probit. The logit model is based on the assumption that the given explanatory variables relate to the response according to the logistic distribution, whereas the probit model assumes that this relationship follows the normal distribution.
Alternate model specifications arise by postulating other probability distributions for the response variable, conditional upon the observed covariates.

The most general procedure for estimating the regression coefficients of the covariates in the response models is that of maximum likelihood. If the distributional assumptions are correct, the estimated regression coefficients are asymptotically normally distributed and they enjoy the associated properties of maximum likelihood estimates; namely, consistency and asymptotic efficiency.

The major advantage of the response models over those of discriminant analysis is that they provide an estimate of the probability of occurrence of a qualitative outcome (response) based on the observed explanatory variables. Consequently, they are not confined to a dichotomous (polytomous) prediction. Discriminant scores can also be associated with probabilities of occurrence, if the populations of failed and non-failed companies are assumed multivariate normal with respect to the independent variables. Martin (1977) discusses some findings in regard to the quality of probabilities that can be provided by a variant of discriminant analysis. Collins and Green (1982) provided simulation results of a comparison between the logistic model and the discriminant model.

Both discriminant analysis models and the qualitative response models provide little insight into the way the explanatory variables affect the survival of individuals over time. The reduction of a state space, however, to a binary response is useful when the survival of each individual is easily classified as either very short or very
1.7 Evidence of Lack of Normality

One of the problems with the discriminant analysis studies is the assumption that the distribution of independent variables is multivariate normal. To quote Eisenbeis (1977, p. 875):

"In practice, deviations from the normality assumption, at least in Economics and Finance, appear more likely to be the rule rather than the exception."

This qualitative statement has been quantitatively substantiated by Deakin (1976) who studied eleven ratios from asset turnover, liquid asset and debt equity groups. He found that the distribution of ten out of these eleven ratios were significantly different from the normal. Even square root or lognormal transformations failed to approximate normality in the majority of ratios. Another study by Lachenbruch, Sneeringer and Revo (1973) investigated the robustness of both linear and quadratic classification procedures for lognormal, logit normal and inverse hyperbolic sine normal distributions. The authors concluded that the standard linear procedures may be quite sensitive to lack of multivariate normality. In addition, they found that attempts to correct for inequality of dispersion matrices by using quadratic classification techniques did not significantly improve the results, and in many cases even made them worse. The latter phenomenon is observed in the Altman et al (1977) study, where the authors used a quadratic discriminant function and observed lower
classification accuracy.

Violation of the normality assumption may bias the tests of significance of the independent variables, the establishment of classification rules, and the estimates of the error rates [Eisenbeis (1977)]. In addition, if the linear discriminant index is viewed as the percentile from the normal distribution which describes the probability of failure, then this becomes non-descriptive as the underlying distribution deviates from normality.
CHAPTER II

2. PURPOSE AND METHODOLOGY OF THIS DISSERTATION

2.1. Definition of Hazard

The purpose of this dissertation is to model the failure distribution over five years of non-financial, publicly traded manufacturing and retail firms, conditional upon their financial characteristics. The pursued model provides the probability that a particular company of certain financial characteristics will survive for a certain number of years into the future.

The modeling is done in the context of survival analysis regression models. These models can be implemented in parametric or semi-parametric modes. The existing lack of knowledge about the distributional characteristics of the underlying corporate failure rate in conjunction with the small number of responses (failures) makes the semi-parametric version a very useful tool. Semi-parametric survival analysis techniques are rather recently developed and have commanded great attention in applications in reliability and medicine. Recently they have been applied to business problems such as modeling payments by insurance companies (Leavit and Olsen, 1974), as well as in other cases where the variable to be modeled assumes positive non-decreasing values (e.g. time, ability, etc.).
One of the differences between survival analyses and other statistical techniques is their ability to handle censored observations, that is, those firms which drop out of the study due to some cause other than business failure. This particular phenomenon (censoring) is present in modeling the life distribution of firms since we cannot observe all firms until they die; it is also present in cases where firms live up to a certain time and then they are acquired by or merge with another company. This ability to handle censoring allows for the utilization of information up to the point where the company changes identity or until the end of the time period under study.

In the survival analysis context the main concern is to model the hazard rate; one can then use this rate to form the survivor function. The hazard rate is defined as:

\[
b(t) = \lim_{\Delta t \to 0} \frac{P(t < T < t + \Delta t \mid T > t)}{\Delta t},
\]

where \( T \) is a continuous random variable representing the failure time and \( t \) is a certain time value.

The above hazard provides the failure rate, per unit of time, which is experienced in period \((t, t + \Delta t)\) of all companies that survived up to time \( t \). In human life distributions, the hazard rate is the age specific mortality rate.

Defined as above for the continuous case, the hazard in terms of the p.d.f. and c.d.f. is:

\[
b(t) = \frac{f(t)}{1-F(t)},
\]
where \( f \) is the probability density function (p.d.f.) of \( T \) and \( F \) is the cumulative distribution function (c.d.f.).

For the discrete case, the hazard is defined as

\[
h_j = P(T=x_j/T > x_j),
\]

where \( x_j \) is the \( j \)th time at which the process can be observed.

The survival function (the probability that an individual will live beyond time \( t \)) is given by

\[
S(t) = \exp[-\int_0^t h(u)du] \quad \text{and} \quad S(t) = \pi(1-h(u)),
\]

for the continuous and discrete case, respectively.

The proportional hazard (regression) model introduced by Cox (1972) assumes the following hazard

\[
h(t) = k(\bar{\beta}'\bar{z})h_0(t), \tag{3}
\]

where \( \bar{z} \) is the vector containing the concomitant information and \( h_0(t) \) is the hazard function of the "average" individual having \( \bar{z}=0 \). The function \( k \) can be any positively valued function. The most popular one uses \( k(\bar{\beta}'\bar{z}) = \exp(\bar{\beta}'\bar{z}) \) where \( \bar{\beta} \) is the vector of the regression coefficients. The baseline hazard function \( h_0 \) does not need to be specified or estimated until after the regression coefficients have been estimated (Kalbfleish and Prentice 1980, p. 84). The regression coefficients are estimated using non-linear optimization techniques and they are asymptotically normally distributed when \( \bar{z} \) is not time dependent.

Using the proportional hazard model, as in formula (3), the
following two assumptions are made:

1) Any company can fail due to external reasons, such as market or economic conditions, or due to other parameters that are not included explicitly in the vector of the concomitant information.

2) The function of concomitant information $k(\beta'z)$ affects the hazard in a multiplicative way.

If the actual failure distribution is Weibull, then use of the proportional hazard function allows for an alternative assumption regarding the way that the function $k(\beta'z) = \exp(\beta'z)$ affects the failure rate. In particular, the alternative assumption is that of an accelerated life scenario whereby the function $k(\beta'z)$ is viewed to affect multiplicatively the time to failure rather than the hazard, i.e., $T^1 = k(\beta'z)T$ where $T^1$ is the observed failure time and $T$ is the failure time of an individual with $z = 0$. The significance of this is that the proportional hazard model is appropriate under both the assumptions of accelerated life and the above proportionality assumptions, when the actual failure distribution of the individual with $z = 0$ is Weibull. This latter property makes the proportional hazard model robust especially in view of the wide applicability of the Weibull distribution to life data.

2.2 Model Conventions

In view of the complicated nature of the bankruptcy process, certain pragmatic conventions, as explained below, have to made in
order to assure a mathematically tractable form for the survivor function to be estimated.

The dynamics of the bankruptcy process can be summarized by the following transition graph.

Management confronted with the attribute state $z_k$ of the company makes decisions $d(t_k)$ at time $t_k$. The decisions $d(t_k)$ together with the state $z_k$ of the company are translated through the stochastic filter "economy" into the new attribute state $z_{k+1}$, which in its turn may lead to a bankrupt or a non-bankrupt condition. If the company remains solvent, the process repeats itself. For theoretical reasons,
it would be desirable to define the state information $z_k$ in a way that it would depict the actual evolutionary mechanism of the above dynamic process. Such a representation, however, is obscured by the following conditions:

1) The available information on the companies is based on accounting data, not on economic variables.

2) Failure is not determined only by the attributes $z_k$ of a company. In most cases, the failure of a company is determined on the basis of its future cash flow potential, given its financial attributes and other conditions of the business environment external to the company. The final determination of a bankruptcy condition is usually a subject of a court decision.

Consequently, defining the state information $z_k$ in a way that depicts actual transitions from one attribute state to another probably would not provide a means of predicting bankruptcy in a parsimonious and functional manner. Wilcox (1971), who used accounting information to define the state of a company in a way which uniquely depicted failure/survival, found his results disappointing. Due to lack of data, he could not construct his state variable for many companies; furthermore, companies that were alive at the time of the data collection were found to be depicted as failed according to his constructed variable. Following the above, the convention to use financial ratios based on accounting data is made in this dissertation.

The second convention to be discussed is that of how to use the
concomitant information \( z \), agreed above to be financial ratios, in the estimation of the survivor function. There are two possibilities:

1) Define the hazard to be

\[ h(t) = h(z(t), t), \]

in which case the concomitant information is treated as time dependent, or

2) Define the hazard rate to be

\[ h(t) = h(z_k, t), \]

where \( k \) is time of observation, in which case the concomitant information is treated as fixed over the period of the study.

Following the definition of the survivor function:

\[ S(t_0) = \exp \left[ - \int_0^{t_0} h(t) \, dt \right] \]

and treating \( z \) as time dependent requires the modeling of the multivariable stochastic process of \( z \), which is also non-stationary, assuming changing pertinent economic conditions.

In light of the difficulty in modeling this process using surrogate accounting information, it is assumed that observed information \( z_k \) at time \( k \) is the best estimate for the concomitant information \( z \) values over the period of study. This assumption leads to the convention of treating concomitant information as fixed, which allows the estimation of the probability that a company with characteristics \( z_k \) at time \( k \) will survive for more than \( t_0 \) years in the future, without having to model the stochastic process.
of $z$. It will be shown in the sequel that having made this assumption does not appear to render the fitted proportional hazards model inappropriate.

2.3 Mathematical Details of the Model

The proportional hazards model of Cox (1972) defines the hazard in the following form

$$h(t,z) = \exp(\beta'z) h_0(t)$$

where: $\beta$ is the vector of unknown regression coefficients,
$t$ is the time,
$h_0(t)$ is the hazard of an individual with $z = 0$ at time $t$,
and $z$ is the vector of covariates.

There are two assumptions that are peculiar to this hazard model:

1) The effect of the covariate vector $z$ on the hazard is given by the function $\exp(\beta'z)$.

2) This function $\exp(\beta'z)$ operates on the underlying hazard $h_0(t)$ multiplicatively (the proportionality assumption) to provide the hazard of the individual with characteristics $z$.

A convenient feature of this model is that the underlying hazard $h_0(t)$ does not need to be specified in any parametric form.

The above two assumptions have the implication that the relative risk of two individuals with characteristics $z_1$ and $z_2$ is given by the ratio of $\exp(\beta'z_1)$ and $\exp(\beta'z_2)$. Moreover, this relative risk does not depend on time when $z_1$ and $z_2$ are not time dependent. Any dependence of the hazard on time is assumed to be through the
underlying baseline hazard. In the particular scenario of bankruptcies, this translates into saying that the economic and market conditions do not affect the relative risk between two companies, this risk being affected only by their characteristics. The company specific risk (hazard), however, is assumed to be affected by economic and market conditions through the baseline hazard function.

The regression coefficients are estimated by maximizing the partial likelihood as follows. Let \( t_1 < t_2 < \ldots < t_k \) be \( k \) distinct times when deaths were observed. Assuming one death per instance, the conditional probability that an individual with characteristics \( z_i \) dies at time \( t_i \), given that at time \( t_i \) there were \( R_i \) individuals at risk, is the ratio of the hazards:

\[
\exp(\beta'z_i)/\sum_{j \in R_i} \exp(\beta'z_j).
\]

Multiplying these probabilities over the \( k \) failure times gives the partial likelihood (Cox, 1975):

\[
L(\beta) = \prod_{i=1}^{k} \left( \frac{\exp(\beta'z_i)}{\sum_{j \in R_i} \exp(\beta'z_j)} \right).
\]

Maximization of this partial likelihood function leads to maximum likelihood estimators of the regression coefficients \( \beta \), which have the property of asymptotic normality when the covariates are not time dependent [(Miller (1981, p. 132); Tsiatis (1981)]. When there are ties among the death times the maximized partial likelihood is modified as suggested by Breslow (1974):
L(\theta) = \prod_{i=1}^{k} \left[ \exp(\theta' s_i) \left/ \left( \prod_{j \in R_i} \exp(\theta' z_j) \right) \right] \right]^{m_i},

where \( m_i \) is the number of deaths at \( t_i \) and \( s_i \) is the vector sum of the variables of the \( m_i \) expired individuals.

2.4 Sample Considerations

In most bankruptcy discrimination studies, because of the small number of corporate failures, a retrospective sampling procedure is followed. Bankrupt companies from different time periods are pooled together, their financial characteristics are recorded one or two years before the failure and a certain model is fitted.

To what extent this retrospective sampling interferes with the estimation of the different models is not known. Pooling observations over different time periods can present a problem if the relationship among the explanatory variables is not stationary or, if the underlying failure rate and/or the economic significance of particular explanatory variables varies over time. Johnson (1970), Eisenbeis (1977), Altman and Eisenbeis (1978), and Altman (1981, p. 259) have discussed this potential non-stationarity problem. In the sequel a prospective sampling procedure is described which is more naturally suited to the data under consideration, and which can be accommodated by the Cox model employed in this study.

In the survival analysis context, in order to design the sample, one has to define how the time to failure is measured. For this, three measurement attributes have to be agreed upon: (a) the time origin when an individual enters the study, (b) a scale of measuring
the passage of time, and (c) what constitutes failure.

(a) The time origin. A major consideration in determining the time origin is that a specific value of an explanatory variable, other conditions being equal, should have a comparable impact on the failure time across individuals in the study. This condition may be violated if observations are pooled over different periods, since certain financial characteristics might have different effect on failure at different calendar times. Another consideration in determining the origin is to keep the underlying stress conditions, which are not accounted for by the covariates, similar for all individuals in the population of study. According to Cox and Oakes (1984, p.4):

"It is ... desirable that, subject to any known differences on explanatory variables, all individuals should be as comparable as possible at their time origin...".

In order to accommodate the above premises, all samples used in this study were taken so that all firms in a sample had their attributes recorded in the same calendar year. Thus, the origin of entry for all companies in a sample was the same randomly selected calendar year.

(b) Unit of measurement. For each company in the sample, the time in the study was defined to be the time, in months, between the month of the fiscal year end (in the year of entry) and the time, defined by month and year, when the company failed or the time when it was censored. Month was chosen as the unit of measurement in order to minimize ties among the failure times, which can present a computational difficulty when estimating the regression
coefficients. A small number of ties does not create any major problems [Breslow (1974)].

(c) **Definition of failure.** A company was defined as failed when it entered bankruptcy proceedings under Chapter X, XI, 11 or when a liquidation was announced. The information on failure was obtained from the Compustat Research files, the Wall Street Journal Index, and the S and F Index of Corporate Changes.

### 2.5 Prospective Samples

Three samples, extending over five years each, were analyzed, starting with entry years 1973, 1975 and 1977, respectively. In order to construct a sample, a calendar year was selected, hereafter called the entry year, and the explanatory variables of all manufacturing and retail companies that had adequate information in this year were recorded. The companies in the sample were traced for failure prospectively for the following five years. Companies that were alive at the end of the sample period were censored at the end of five years; those that were lost to follow up before the end, due to reasons other than failure, were censored at the time of loss. For each company, the time on the study, defined to be the time between the entry and the time of failure or censorship, was also recorded. Then the values of the explanatory variables from the entry year were used as covariates to model the hazard function. The model with the same variables was estimated over the three samples starting with the years 1973, 1975 and 1977 in order to examine the changes of the
regression coefficient over time. The samples were taken two years apart, assuming that the model would be updated every other year.

Each sample contained approximately 1,400 manufacturing and retail companies, including 2-3 percent failed entities. The information for the explanatory variables was taken from the Primary-Supplementary-Tertiary Industrial Annual and Research Compustat files of 1982.

This prospective sampling plan has certain desirable features, both in terms of theory and in terms of practice. In particular, one of the major benefits of adopting this plan is that it allows the estimation of the underlying failure rate; this would have not been possible had observations been pooled over different time periods on a non-random basis. Another benefit of this plan is that it provides a means of taking a sample which is uniformly closer to a future period of inference. The latter would be beneficial, even under the undesirable scenario in which the impact of the covariates changes over time according to some trend. In such a case, a more recent observation of the changes would be advantageous in the sense of being more up to date. Altman (1983 p. 125), realizing the benefit of having data more representative of the business environment, makes the following comment on his 1968 Z-score model:

"Ideally, we would like to develop a bankruptcy prediction model utilizing a homogenous group of bankrupt companies and data as near to the present as possible."

Observing the explanatory variables in the same year also facilitates the collection of data in light of changing accounting
practices, thereby making the model easy to update. It also makes it possible to compare the company specific covariates to the average values of the same covariates in the sample.

Arguing in the favor of the above sampling plan, one can also point out that this plan more closely resembles the process of a decision scenario. A decision maker, faced with a population of companies whose future life is critical to the decision, has to decide on a policy for lending money or setting risk premia based on information that is available in the year of the decision. The policy is implemented for the respective companies under consideration in the same year, but the payoffs are experienced in accordance with the realized failure rate in the future. The experience of the failure rate is prospective as is this sampling plan.

The sample period of five years used in this study was selected (1) as a likely period for a decision horizon, (2) in order to secure an adequate amount of data, and (3) because this is the longest horizon that other bankruptcy studies have used.

An advantage of the above sample scheme in terms of estimation is that the economic stress conditions are controlled (kept common) for all individuals in a particular sample. Controlling the stress conditions accommodates the aforementioned premise that all individuals, at the entry point, should be as comparable as possible, subject to any differences in explanatory variables. Keeping the economic conditions common also facilitates the investigation of the impact (expressed by the regression coefficients) of the explanatory variables on failure time, as this impact is expected to be more
uniform across companies within the same period of time than it would be otherwise.
CHAPTER III

3. EMPIRICAL RESULTS

3.1 Selection of Variables: Criteria

The literature is rich with bankruptcy prediction studies which utilize ratios and other accounting variables in their models. Which set of variables provides the best combination of predictors remains an open question. The surrogate nature of accounting data, the intricate complications resulting from multicollinearity and the small number of failed firms in the different samples obscure the determination of the undisputable winners. Fortunately, however, several different sets of variables [see Zavgren (1983) or Altman (1983)] lead to comparable results, suggesting that it might well be that the predictive quality of a model is reasonably robust across a variety of combinations of financial ratios.

Confronted with the absence of an explicit theory of bankruptcy and the lack of definite rules in determining the importance of financial variables from other studies, one is left with the recourse of empirical search. In this study a large set of financial variables was searched and a set of five variables was selected. No claim is made as to whether this set constitutes the best set of variables for the proportional hazards model; it might be that another set performs as well or better. It is quite likely that much can be
gained by improving the data quality, however, this is a very difficult task when one is confronted with such a large data base.

Several considerations had to be made in choosing the final set of variables. These can be summarized in terms of two main directions: (a) avoiding introduction of bias, and (b) adhering to the assumptions of the model.

As far as bias is concerned, preference was given to variables that did not lead to any considerable reduction of the sample size. Lack of availability of data can reduce the sample size, thereby potentially introducing bias. Such a problem is prominent when one attempts to use financial variables requiring observation over several periods. Variance of earnings or trend variables are typical examples of variables that require observation over several years. Altman et al. (1977) used the variance of earnings over ten years as a surrogate measure of business risk. This led to the exclusion of companies from the sample that did not have data for such a length of time. However, if younger companies are more susceptible to failure than older, more established, entities, this exclusion can lead to an underestimation of the failure risk. In the case of Altman et al. (1977), one could argue that the age related risk is prominent in the initial few years of a corporation's life and, therefore, such an exclusion has no effect on a sample of publicly traded corporations that have already lived for several years before entering a stock exchange. A problem remains, however, in that one does not know how to evaluate a company which does not have enough financial statements that are available publicly.
Another consideration in adopting variables was that of adhering to the proportionality assumption which underlies the Cox model. When a choice had to be made among ratios that provided similar information, the one that appeared to comply with the proportionality assumption was preferred to the others. The proportionality assumption was checked using the generalized residuals [Kay (1977)].

Other more standard considerations in adopting a variable included statistical significance, predictive ability, and simplicity to obtain. The latter is especially important because it makes updating a model easier and minimizes the exclusion of companies due to lack of data.

3.2 Selection of Variables: The Final Set

The variables of the model were selected sequentially, one at a time, on the basis of their contribution to the maximization of the partial likelihood function and the other criteria set forth in the previous section. Employment of automated stepwise selection procedures was ineffective because these procedures often came to a halt due to overflow problems.\(^\text{14}\)

The final five variables included in the model are the following:

1. \( V_1 \) (size): Natural log of total assets. This variable was found to be significant in all runs of the model. Other studies that have found this variable important include those by Altman et al (1977) and Ohlson (1980). The importance of the variable can be attributed to the information it contains about a company's accessibility to
financial markets and its age and also because a larger value for size suggests a broader base of assets which the company can partially liquidate in difficult times to meet current obligations. It is also possible that a larger company can be more successful in an attempted merger in order to avert a formal bankruptcy situation.

(2) $V_2$: Total debt divided by total invested capital. This ratio was found to be one of the more useful variables in maximizing the likelihood function. Ratios involving debt or total liabilities divided either by capital or total assets have appeared in many other bankruptcy studies including Beaver (1966), Deakin (1972), Ohlson (1980) and Zavgren (1982). The importance of this ratio is attributed to the information it provides about the obligation of a company relative to its total invested capital.

(3) $V_3$: Operating income before depreciation divided by tangible assets. Operating income was found to be preferable to other measures of income such as sales, earnings before interest and taxes, and operating income after depreciation. This ratio imputes such factors as the profitability of the company and a measure of income from operation that is available to serve debt. Ratios of return have been popular in all major studies of bankruptcy, and this one was found to be very important in maximizing the likelihood function in this study.

(4) $V_4$: Capital surplus divided by tangible assets. Capital surplus represents the excess value of capital stock over its par or stated value. More specifically, it is calculated as the cumulative amount that a company was able to acquire in the financial markets by selling common stock, over and above the par value of the stock sold, which is
usually issued at the face value of one dollar per unit. In terms of this definition, the ratio of capital surplus to assets can be viewed both as a surrogate measure of the performance of the stock of a company of a certain size over its life, and as a surrogate measure of the accessibility that a company has to the financial markets. \( V_4 \) was introduced into the model as a surrogate for the performance of a company over its lifetime. Another candidate considered as an indicator of the cumulative performance of a firm over time was the ratio of retained earnings to tangible assets. The latter variable was found insignificant, for all samples, despite that fact that Altman et al (1977) found the latter variable to be their most "important" variable.

The importance of the \( V_4 \) is hard to assess given the existing multicollinearity of the data. It was found to be significant at an \( \alpha \) level of less than .001 in the sample of 1973-78, insignificant in the 1975-80 sample and marginally significant [\( \alpha = .1 \)] in the 1977-82 sample when the likelihood ratio test was used. The Wald test for the same variable in the latter sample led to a \( p \)-value of 0.3045. It is possible that a shift of importance of the \( V_4 \) variable occurred over time. The sign of the regression coefficient of \( V_4 \) is as expected for all samples.

(5) \( V_5 \): Market value of common stock divided by total invested capital. A similar capitalization ratio, common equity divided by total capital, was used by Altman et al (1977) in their model. The numerator of \( V_5 \) is computed using market value in order to introduce a measure of the evaluation of a company by the market.
In fitting the model, the value of each covariate is defined to be the observed value of the respective variable minus its mean over the entire population in a given sample period, thereby facilitating a comparison between the population average and a company specific value. Subtracting the mean of each variable from the observed values also facilitates the estimation of the regression coefficients using the Newton-Raphson procedure.

3.3 Estimation Results

In order to observe the stability and performance of the model over a long period of time, three data sets were analyzed in this study. The data sets were obtained according to the sampling plan described in the preceding chapter. In summary, the covariates for each company were calculated for the entry year of each sample and the failure status was recorded over the following five calendar years. The first data set has entry year 1973, the second starts with 1975 and the final one starts with 1977. The data sets were taken to begin two years apart, under the assumption that the model would be updated every two years.

The proportional hazards model containing $V_1-V_5$ was estimated using the data from each of the three samples, resulting in three models (73, 75, 77) of the same form but with different estimated regression coefficients. The remainder of this section will be dedicated to reporting the results of the estimation of the model for the three data sets. These results are presented in Tables 1 through...
3; details on the computational aspect of the presented estimates can be found in Appendix B.

Each of Tables 1 through 3 contains the following information:

1. The estimates of the regression coefficients $\hat{\beta}$ used in the hazard function. A positive coefficient for a covariate indicates a higher risk of failure whereas a negative one has the opposite interpretation. Given that each covariate was defined to be the observed value of a variable minus its mean over the sample population (see Section 3.2), the regression coefficients should be interpreted as depicting the effect on the hazard that results from one unit of positive deviation of a variable from its mean.

2. The standard errors of the estimated regression coefficients.

3. The logarithm of the maximized partial likelihood function and the chi-square value resulting from the score function test. The global chi-square statistic tests the hypothesis that all regression coefficients are identically equal to zero. The model was found significant for all samples at an $\alpha$ level of $10^{-4}$ or less.

4. The significance of the individual regression coefficients was tested using the Wald test statistic, which is based on the asymptotic normality property of the maximum likelihood estimates. The Wald statistic is asymptotically chi-square distributed. The obtained chi-square values are reported for the regression coefficient of each variable.

5. The estimated values of the baseline survival function $\hat{S}_0(t)$ are reported for $t=1,2,3,4,5$. These estimates provide the probability that an individual with covariate vector $\mathbf{z} = 0$ will survive at least $t$
years into the future. An estimate for the survival function of a company with covariate vector $z_1$ can be easily obtained in terms of the baseline survival function $\hat{S}_0(t)$ using the following formula:

$$\hat{S}(t, z_1) = [\hat{S}_0(t)]^{\exp(z'z_1)}.$$  

(6) The estimated asymptotic correlation matrix of the regression coefficients. This matrix is provided because it is used to construct tests on the regression coefficients using the Wald statistic.

3.4 Testing the Proportionality Assumption

The most commonly used technique for testing the goodness of fit of the Cox model is based on the following idea. If $T_1$ has survivor function $S(t_1, z_1, \theta)$ with $\theta$ parameter vector, then $S(t_1, z_1, \theta)$ is uniformly distributed and $-\log S(t_1, z_1, \theta)$ has a unit exponential distribution [Cox and Oakes (1984, p. 89)]. Therefore, if one defined the residual of the individual that died or was censored at time $T_1$ to be

$$e_i = -\log \hat{S}(T_1, z_1, \hat{\theta}),$$

then the residuals obtained in this manner should behave as a random sample of censored unit exponential variates. Hence, the cumulative hazard of the residuals, $\hat{H}(e)$, when plotted against the ordered residuals, should look like a straight line of slope one through the origin [see Cox and Snell (1968), Kay (1977), Link (1979), and Lagakos (1981)].

Figures 1 to 3 show the plots of the cumulative hazard of the residuals against the residuals themselves for the three samples of
this study. From these figures it can be seen that the plots resemble straight lines of slope one and the proportional hazards model does not appear to be inappropriate.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Regression Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>-.5795</td>
<td>.1381</td>
</tr>
<tr>
<td>V2</td>
<td>1.5764</td>
<td>.2873</td>
</tr>
<tr>
<td>V3</td>
<td>-5.7096</td>
<td>1.9312</td>
</tr>
<tr>
<td>V4</td>
<td>-2.6805</td>
<td>.7616</td>
</tr>
<tr>
<td>V5</td>
<td>-.6901</td>
<td>.3937</td>
</tr>
</tbody>
</table>

Log likelihood = -289

Global Chi-square = 215.4  p-value = .0000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Chi-square</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>17.6</td>
<td>.0000</td>
</tr>
<tr>
<td>V2</td>
<td>30.1</td>
<td>.0000</td>
</tr>
<tr>
<td>V3</td>
<td>8.74</td>
<td>.0031</td>
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<tr>
<td>V4</td>
<td>12.39</td>
<td>.0004</td>
</tr>
<tr>
<td>V5</td>
<td>3.07</td>
<td>.0796</td>
</tr>
</tbody>
</table>

Survival

\[
\hat{S}_0(t) = \begin{array}{cccccc}
.999 & .9975 & .9955 & .9934 & .991 \\
\end{array}
\]

Estimated Asymptotic Correlation Matrix

\[
\begin{array}{cccccc}
V_1 & V_2 & V_3 & V_4 & V_5 \\
V_1 & 1.00 &  &  &  &  \\
V_2 & -.22 & 1.00 &  &  &  \\
V_3 & -.21 & .44 & 1.00 &  &  \\
V_4 & .17 & -.55 & .19 & 1.00 &  \\
V_5 & .09 & -.18 & -.35 & -.60 & 1.00 \\
\end{array}
\]
### TABLE 2

**ESTIMATION RESULTS: MODEL 75**

Sample 75 (1975-1980)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Regression Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
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<td>$V_2$</td>
<td>.8451</td>
<td>.2735</td>
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<td>$V_3$</td>
<td>-6.6894</td>
<td>1.5050</td>
</tr>
<tr>
<td>$V_4$</td>
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<td>$V_5$</td>
<td>-1.3715</td>
<td>.6969</td>
</tr>
</tbody>
</table>

Log likelihood = -182

Global Chi-square = 78.61 p-value = .0000

<table>
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<tr>
<th>Variable</th>
<th>Chi-square</th>
<th>p-value</th>
</tr>
</thead>
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</tr>
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<td>$V_2$</td>
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<tr>
<td>$V_3$</td>
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<td>.0000</td>
</tr>
<tr>
<td>$V_4$</td>
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<td>.7403</td>
</tr>
<tr>
<td>$V_5$</td>
<td>3.87</td>
<td>.0491</td>
</tr>
</tbody>
</table>

Survival

<table>
<thead>
<tr>
<th>$t$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{S}_0(t)$</td>
<td>.9991</td>
<td>.9981</td>
<td>.9959</td>
<td>.9945</td>
<td>.9892</td>
</tr>
</tbody>
</table>

**Estimated Asymptotic Correlation Matrix**

<table>
<thead>
<tr>
<th></th>
<th>$V_1$</th>
<th>$V_2$</th>
<th>$V_3$</th>
<th>$V_4$</th>
<th>$V_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_2$</td>
<td>-.25</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_3$</td>
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<td>.32</td>
<td>1.00</td>
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<td></td>
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<tr>
<td>$V_4$</td>
<td>.27</td>
<td>-.02</td>
<td>-.03</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$V_5$</td>
<td>.01</td>
<td>.03</td>
<td>.03</td>
<td>-.03</td>
<td>1.00</td>
</tr>
</tbody>
</table>
### TABLE 3

**ESTIMATION RESULTS: MODEL 77**

Sample 77 (1977-1981)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Regression Variable</th>
<th>Standard Error</th>
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<tr>
<td>V₁</td>
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<td>.1569</td>
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<tr>
<td>V₂</td>
<td>.9537</td>
<td>.2694</td>
</tr>
<tr>
<td>V₃</td>
<td>-4.8350</td>
<td>1.4730</td>
</tr>
<tr>
<td>V₄₁₈</td>
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<td>1.0276</td>
</tr>
<tr>
<td>V₅</td>
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<td>.5212</td>
</tr>
</tbody>
</table>

Log likelihood = -180

Global chi-square = 59.89  \( p\)-value = .0000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Chi-square</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>V₂</td>
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<td>.0004</td>
</tr>
<tr>
<td>V₃</td>
<td>10.77</td>
<td>.0010</td>
</tr>
<tr>
<td>V₄</td>
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</tr>
<tr>
<td>V₅</td>
<td>2.70</td>
<td>.1003</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>t</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
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<tbody>
<tr>
<td>( S₀(t) )</td>
<td>.991</td>
<td>.9972</td>
<td>.995</td>
<td>.9919</td>
<td>.9875</td>
</tr>
</tbody>
</table>

**Estimated Asymptotic Correlation Matrix**

\[
\begin{array}{ccccc}
V₁ & V₂ & V₃ & V₄ & V₅ \\
V₁ & 1.00 & .01 & .04 & .32 & .05 \\
V₂ & .01 & 1.00 & 1.00 & .29 & -.12 \\
V₃ & .04 & 1.00 & 1.00 & .24 & -.12 \\
V₄ & .32 & .00 & .24 & 1.00 & .00 \\
V₅ & .05 & -.12 & .00 & -.12 & 1.00 \\
\end{array}
\]
FIGURE 1
CUMULATIVE HAZARD FUNCTION OF RESIDUALS, SAMPLE 73
FIGURE 2

CUMULATIVE HAZARD FUNCTION OF RESIDUALS, SAMPLE 75
FIGURE 3
CUMULATIVE HAZARD FUNCTION OF RESIDUALS, SAMPLE 77
4. IMPLEMENTATION

4.1 Estimating Expected Payoffs

In order to use the results of a discriminant or other binary response model in making a decision, it has to be assumed that a dichotomous partition of the payoff space, in terms of failure or non-failure, is adequate. Such an assumption, however, may be an oversimplification in cases where the time to failure is one of the critical determinants of the payoffs. The use of the survivor function can facilitate the computation of the expected payoffs in such cases. This facility can be shown using the scenario of a loan that requires \( k \) annual payments. The loan process can be depicted schematically as follows:

\[
\begin{align*}
\text{Year 1} & \quad \text{Year 2} & \quad \ldots & \quad \text{Year } k \\
+ \text{Failure } -L_1 & \quad + \text{Failure } -L_2 & \quad + \text{Failure } -L_k \\
P(T=1) & \quad P(T=2/T>1) & \quad P(T=k/T>k-1) \\
\text{Survival } P_1 & \quad \text{Survival } P_2 & \quad \text{Survival } P_k \\
P(T>1) & \quad P(T>2/T>1) & \quad P(T>k/T>k-1)
\end{align*}
\]

where:  
\( T \) is time of failure,  
\( L_t \) is the amount of loss resulting from failure in year \( t \),  
\( P_t \) is the payment on the loan in year \( t \).
A lender gives a loan to a Company A which loan requires annual payments $P_t$, $t = 1, 2, \ldots, k$. If the company fails in year $t$, the lender will lose amount $L_t$ on principal outstanding; else payment $P_t$ is collected. This process continues until $k$ annual payments are received.

Taking into consideration that the survivor function of a company with a Risk Index $I$ (to be defined later) is defined as $S^I_o(k) = P_I(T>k)$, the above probabilities are expressed as follows:

$$P_I(T=1) = 1 - S^I_o(t)$$

$$P_I(T>1) = S^I_o(1)$$

$$P_I(T=k/T<k-1) = \frac{(S^I_o(k-1) - S^I_o(k))}{S^I_o(k-1)}$$

$$P_I(T>k/T<k-1) = \frac{S^I_o(k)}{S^I_o(k-1)}$$

where $S^I_o(k)$ is the survivor probability of an individual with risk index $I = 1$, and $I$ is the risk index defined as $\exp(\beta'z)$ for a company with covariate vector $z$. Following this, the expected monetary value of the loan is:

$$EMV = -(1-S^I_o(1))L_1 + S^I_o(1)P_1 + \sum_{\ell=2}^{k} [-L_\ell (S^I_o(\ell-1) - S^I_o(\ell)) + P_\ell S^I_o(\ell)].$$

Rearranging the terms, we get

$$EMV = -L_1 + (L_1 + P_1 - L_2) S^I_o(1) + \ldots + (L_k + P_k) S^I_o(k), \quad (4)$$
which is a monotonically decreasing function with respect to \( I \) since
\[
L_t + P_t > L_{t+1}.
\]
Therefore, given \( L_t \) and \( P_t \) one can solve for \( I_0 \) and use this value to screen out loan applicants that are associated with an expected monetary value less than a specified value \( EMV_0 \).

Alternatively, if one assumed equal annual payments and expressed the losses \( L_t \) as a function of the payments \( P \), then by substituting the survivor probabilities in (4) he could solve for the required annual payment \( P \) that would maintain a certain level of desired expected profit. Moreover, if the payments \( P \) were expressed in terms of the interest charge, the decision maker could calculate the interest charge that would lead to the desired expected level of profit.

### 4.2 Setting the Cutoff Point

Following their development, the models of corporate failure are typically evaluated in terms of their accuracy of prediction on another sample\(^{19}\), usually referred to as the holdout sample. To use a model for prediction, a cutoff point is selected so that a company is classified as failure or non-failure, depending on whether its risk index lies above or below this cutoff point. The risk index may be a statistical score indicative of the risk of failure, as it is in discriminant analysis models, or it may be the estimated probability of failure, as it is in probabilistic models. In most previous bankruptcy prediction studies, this cutoff point was set subjectively so that it minimized some combination or average of the rate of
occurrence of the two misclassification errors: the error of misclassifying a failed company (Type I) and the error of misclassifying a non-failed company (Type II).

In setting the cutoff point there is a trade off between Type I and Type II errors. If one classified all companies that have probability of failure greater than zero as future failures, he would eliminate Type I error but maximize Type II error; on the other hand, declaring failures to be only those companies that have probability of failure equal to one (which happens only when failure occurs) would eliminate Type II error but would predict no failures.

If the costs of misclassification associated with the two types of error were equal, setting the cutoff point in order to minimize the average of the rate of occurrence of the two errors (Type I and II) would be appropriate; however, this is not the case in reality. The cost of misclassifying a non-failed company is considerably lower than the cost of misclassifying a company that fails in the future. For instance, let us define the Type I error cost to be the loss of principal and interest on a loan resulting from bankruptcy of the misclassified failed company, and the Type II error cost to be the opportunity cost of not giving a loan to a company that does not fail. The former cost is likely to be substantially higher than the latter cost; therefore, setting the cutoff point in order to minimize the overall misclassified proportion of companies is not appropriate in a profit maximizing environment. A more realistic procedure for setting the cutoff score would take into consideration the cost differential of the two errors.
Certain recent studies have incorporated costs in setting the cutoff points. Such studies include the studies by Altman et al. (1977), Diamond (1976) and Meyer and Pifer (1970). A plausible method of assessing Type I and II error costs can be based on the above mentioned loan scenario whereby Type I cost is defined as the loss on a loan given to a company that fails, whereas the alternate cost is defined in terms of the opportunity cost of not giving a loan to a company that does not fail. There is a difficulty, however, in determining the costs of misclassification due to the unavailability of relevant data on loan losses and opportunity costs. The available information on bad loans is not detailed enough to provide precise estimates of the costs of the two errors. One can obtain overall statistics about the recovery rates on charged off portions of loans; however, no detailed information is available on factors influencing the recovery rate on the amount loaned. Such factors include the time between the loan date and the bankruptcy, the terms of the loan, the size of the debtor and creditor, the size of the loan and the length of the recovery period. Altman (1983, pp. 185-188) discusses some of the factors that affect the recovery and collection rates on loans, such as size of bank and size of loan. It appears that the cost of gathering the information and its confidential nature are the main reasons for the unavailability of precise data [see Makeever (1984)].

As described above, the corporate failure prediction models are evaluated on the basis of their performance in correctly classifying companies as future failures or non-failures. In theory, however, if
a risk neutral individual had a precise rule for estimating the expected payoffs associated with certain decisions, there would be no reason to preclude a company of a certain risk level from an investment or lending consideration if the risk could be compensated by higher expected returns. The implemented rule in such a case would be judged on the basis of the resulting profits. A creditor or investor willing to base a decision on estimated expected payoffs can use the expression (4) in Section 4.1 to determine his course of action. Evaluation of the potential performance of an expected payoff model such as (4) is confounded by the lack of information on incurred losses as a function of the elapsed time from the loan date and the lack of information on interest charges that companies are likely to accept, given their risk status.

The potential of the survivor function, however, can be evaluated on the same basis that other models have been evaluated on; that is, on the basis of the performance in discriminating between high and low risk individuals. In order to be able to perform such an evaluation the following scenario was created. A company applying for a three year balloon maturity loan was considered as a "high risk" individual if it did not meet certain profit expectations of a lender, which are described below. A three year balloon maturity loan was defined to be a loan that calls for repayment of the principal and interest at the end of three years from the loan date. This repayment structure was chosen in order to obtain a dichotomous partition of the payoff space as is done in other failure prediction studies.

The determination of the cutoff point was based on the premise
that a loan should be extended if the expected profit from the loan lies above a certain "hurdle" level. Therefore, the cutoff point corresponds to the risk index of a company that has expected profit equal to the hurdle rate; above this cutoff point the probability of failure becomes greater and the corresponding expected profit becomes less than the hurdle rate. The expected profit expression is given by

\[ H = -L [1 - S^I_0(3)] + P S^I_0(3), \]

where:  
\( L \) is the loss on the loan in case of bankruptcy,  
\( P \) is the return on the loan,  
\( S^I_0(3) \) is the probability that a company survives for more than three years,  
\( I \) is the risk index given by \( \exp(\beta'z) \), representing the relative risk between a particular company with attributes \( z \) and a company with attributes equal to population averages,  
\( H \) is the "hurdle" level.

The following assumptions were made in calculating the parameters of the expected profit expression:

1. The recovery rate on the amount of principal outstanding at the time of failure is 37.5%, [Altman (1983, p. 185)].
2. A company applying for the above loan that has a risk index greater than or equal to the cutoff point is willing to pay 13.5% annually compounded interest.
3. In case of failure any interest outstanding is lost.
4. The hurdle rate was set at 44%, calculated as approximately
the expected profit from a loan given to a company that has attributes equal to population averages (i.e. $\mu = 0$), and which is charged 13.0% interest on the above loan.

(5) $S_0(3)$, the probability that a company with $\mu=0$ survives for more than three years, was set equal to .995 which is the estimate $S_0(3)$ obtained from the 1977-82 sample.

Using the above assumptions, the cutoff point was set at 2.86. More details on the calculations of the cutoff point and justifications of the assumptions are presented in Appendix D.

The determined cutoff point is by no means universal since the determination was based on the above assumptions; however, it is realistic. Furthermore in the neighborhood of 2.86 the classification results are robust, in the sense that fluctuation of the cutoff point around 2.86 will not change the results substantially. When the companies in the different samples were ranked with respect to their calculated risk indices, the number of companies that ranked between 2.86 and 3.0 was in the vicinity of one percent.

4.3 Classification Results

The cutoff point of 2.86 which was selected as explained in the preceding section was used to obtain the classification results for the three models (section 3.3) on the samples of 1972, 1973, 1975 and 1977 entry years, respectively. The sample of 1972 was created in addition to the other three in order to have a sample that did not contain any failed companies in common with the sample of entry year
54

In column (6) of Table 9 the same information is provided as in
1973, 1975, and 1977, and by using the model based on the 1977
cutoff point, the same model was used for all samples in order to
assure comparability among the classification performances of the
different models.

The cumulative accuracy of correctly classified failed companies is
reported by distance from the data year. This accuracy is calculated by
combining the data from the three models of Table 8, Table 6, and 9
constructed in that order. In this format, the cumulative accuracy of
screening failed companies is given by time periods which start with
the entry year of the respective sample and successively extend by one
calendar year. Since the cumulative accuracy is calculated from the
cumulative accuracy of correctly classified failed companies in a given
period, the accuracy for failed companies is presented as the ratio of the
total number of the correctly classified failures over the total number
of failed companies in the respective sample. The cumulative accuracy for
screening a fixed number of years prior to the failure event and then
successively extends the effectiveness of such models in predicting failure.
In order to present our results in this format, the attributes of the
companies were used for reasons that will be explained below.

Customarily, failure classification studies evaluate the
classification performance of their models by observing the
attributes of the companies a fixed number of years prior to the
failure event and then assess the effectiveness of such models in
predicting failure. In order to present our results in this format,
Tables 8 and 9 were constructed. In column (6) of Table 8 the
accuracy of correctly classified failed companies is reported by
distance from the data year. This accuracy is calculated by
combining the data from the three models of Table 8, Table 6, and 9
constructed in that order. In this format, the cumulative accuracy of
screening failed companies is given by time periods which start with
the entry year of the respective sample and successively extend by one
calendar year. Since the cumulative accuracy is calculated from the
cumulative accuracy of correctly classified failed companies in a given
period, the accuracy for failed companies is presented as the ratio of the
total number of the correctly classified failures over the total number
of failed companies in the respective sample. The cumulative accuracy for
screening a fixed number of years prior to the failure event and then
successively extends the effectiveness of such models in predicting failure.
In order to present our results in this format, the attributes of the
companies were used for reasons that will be explained below.
calculated by using the samples of 1972, 1973, and 1975; elements of the 1977 sample were excluded in order to minimize the dependence of the classification estimates on the 1977 sample on which the final model was based. Surprisingly, the results for Type I error by distance from data year are slightly better in Table 9 than in Table 8 even though we minimized the dependence on Sample 77. This improvement can perhaps be attributed to the fact that the estimates of Type I error in Table 9 depend less on the failures of the years 1980 to 1982 of the last recession, where accelerated deterioration of companies may have been present.

From Table 8 we see that the Type I error rate is 13.9%, 14.8%, 23.5%, 38.7% and 26% using data 1, 2, 3, 4, and 5 years prior to failure, respectively. The corresponding Type I error figures from Table 9 are 14.8%, 14.3%, 18.5%, 36%, and 27.3%. The Altman et al. (1977) model, which appears to be the most commonly used reference of comparison, resulted in corresponding Type I errors of 7%, 15.1%, 25.5%, 32%, and 30%. In this study, the Type I error rate using data four years before failure appears to be relatively higher than the error rate for years 1, 2, 3 and 5 prior; this perhaps may be due to business cycle dependency. In computing the error rate in question (four years prior), twelve failures came from 1976, seven from 1977, six from 1979, and six from 1981 for the Table 8 entry, whereas the Table 9 figure was computed using the same failures with the exception of the failures in 1981.

In this study the Type II error rate of misclassifying non-failed companies is calculated as the cumulative misclassification of such companies over a horizon of five years. This measurement is
comparable to the estimate that other studies report as "misclassification of non-failed companies using data five years prior to observed non-failure". It is our contention that Type II error based on a random sample of non-failed companies should not deteriorate as the prediction horizon increases; this contention is based on the argument that nothing of relevance to non-failure takes place to alter the profile of an average non-failed company as the prediction horizon increases. Certain corporate failure studies have reported increasing Type II error as the prediction horizon increases; we can offer no explanations for this phenomenon except for the following:

1. The population of the non-failed companies in such studies may have contained a large number of relatively young companies and as the information on which prediction was based was observed at earlier times, the profiles of the younger companies deteriorated due to the fact that the data came from the volatile early years of their life. This is certainly not the case in this study where the sample comes from publicly traded companies.

2. The sample of the non-failed companies in such studies was not random in the sense that the companies were chosen on the basis of good health in order to separate the population of failed and non-failed individuals. Under this scenario the profile of the average firm will justifiably deteriorate as one observes such companies a few years back and the average company in the sample resembles more the profile of the average individual in a random sample.

In order to support the position taken with regard to Type II
error, the misclassification of the companies that were alive in 1982 was calculated using the three models and covariate information from years 1972, 1973, 1975, and 1977. The results are presented in Table 10. Even though there is a variation from one year to another, no consistent deterioration is observed as the prediction horizon increases. It can also be noted that the reported accuracies of the classification of the non-failed companies in Tables 4 to 7, despite a slight deterioration due to the inclusion of the acquired, merged, and delisted companies, are in agreement with the figures in Table 10.

Upon comparing the classification results of this study with those of other studies, one can observe that certain studies have reported misclassification error rates for bankrupt companies based on data one year prior to failure which are somewhat below those obtained in this and other studies. One might be inclined to attribute this misclassification inconsistency to differences in model characteristics and to the covariates that were used in the various studies, which admittedly is a possibility. However, such reasoning becomes tenuous when one considers that other studies which have used similar variables and models, or even the same variables and models but on different samples, have yielded much higher misclassification error rates than those models which had high accuracy. For instance, Altman and McGough (1974) have used a sample from the seventies to check the predictive accuracy of Altman's 1968 model. The results indicated 18% misclassification error for bankrupt companies using data one year prior to failure and a Type I error rate of 42% two
years prior, whereas the corresponding misclassification errors for the same model on the original 1968 sample were in the vicinity of 6% for one year and 18% for two years prior to failure.

Moyer (1977) also reexamined Altman's 1968 model using data from the 1965-75 period and the average error rate reported by Moyer for this model was no less than 25%. Reestimation of the parameters of the 1968 model, using the 1965-75 sample, yielded a corresponding error rate of 10% on the same sample of 54 companies where the model was reestimated.

Deakin (1972) reported an overall misclassification error rate for one year prior to failure of 3%; but trying the same model on a random sample led to an overall misclassification rate of 22% using one year prior data.

The only two studies in the seventies that report a Type I error of less than 10% are those by Altman et al. (1977) and Diamond (1976). Diamond reports Type I error rates of 3% and 21% based on data one and two years prior to failure, respectively; the deterioration in the error rate from the first to the second year is substantial. The coefficients of the variables for the 1977 model of Altman et al. are not reported since the model is commercially marketed, and cross validation on different samples is, therefore, not possible.

All of these examples perhaps suggest that the reported low misclassification results using data one year prior to failure are sample dependent. Ohlson (1980) also expresses his reservations about
the reported low misclassification results for one year prior to
failure and, alluding to the possible sample dependency he suggests
that the data used (from one statement prior to bankruptcy) may have
been too close to the actual occurrence of the failure event. It is
possible that certain ratios, when observed slightly before the event
of failure, are strong predictors (or rather indicators) of such an
impending event. However, whether such variables are useful in
predicting failure beyond a realistic lead time might be subject to
question.

In the next section we will discuss certain peculiarities of this
study which should be taken into account when comparing the classi­
fication results of this study to those of other models.

4.4 Considerations in Comparing Classification Results

In comparing classification accuracies from different studies,
several considerations have to be taken into account: (a) the
randomness of the sample used in fitting the model, (b) the prediction
horizon, (c) the randomness and composition of the validation sample
(i.e., ratio of failed to non-failed), and (d) the purpose and the
information content of the different models. The latter consideration
is relevant to such issues as setting the cutoff points and the
usefulness of the models in different decision situations.

Any differences in the above conditions will render any
comparisons ineffectual. As Ohlson suggests (1980, p. 124), a
comparison of the predictive power of a set of models at minimum requires a complete specification of a common decision problem including a preference structure defined over the appropriate state space.

There are definite differences between the samples of this study and those of other existing bankruptcy studies and, therefore, the classification results of this study should be judged in light of these differences. One of the differences is that each sample of this study was taken prospectively from one calendar year (the entry year) onwards in order to simulate the experience under a decision scenario as explained in section 2.4. Following this prospective sampling scheme, all manufacturing and retail companies which had published financial statements in the entry year of a sample were included regardless of their future status. This led to the inclusion of companies that were subsequently acquired by or merged with another company or even companies that were delisted from a stock exchanges for various reasons. These "irregular" companies were treated as non-failed up to the point when they were lost to follow up, at which point they were censored. This censoring mechanism is realistic from the statistical point of view and is preferable to excluding all these companies from the sample; however, it may not result in the best classification results for the reasons that will be explained below.

An acquisition/merger arrangement can be consummated for various reasons, one of which is the aversion of formal bankruptcy. This creates a problem in that censorship in this case would be correlated
with failure and censoring companies in this category would underestimate the failure rate. However, defining a company as one that averted bankruptcy through merger is not easy, as the conditions that lead to a merger are not required to be disclosed. Accounting for the correlation between censorship and survival or death is a statistical problem still in the infancy of its resolution. Merged/acquired companies present a problem during the validation stage of a model as it is not clear how the acquired or merged company (which might be close to bankruptcy) should be classified. An easy alternative, which avoids dealing with the above difficulties without solving the problem, is to exclude all these "irregular" companies from the sample. This latter recourse is statistically questionable, and it leads to loss of information.

It is not known whether excluding an irregular company or keeping it in the sample is the lesser evil. In this study, including these merger/acquired and delisted companies was felt to provide the more conservative approach in that such an inclusion tests the performance of the model under the most adverse conditions. One can also argue that including these "irregular" companies is more realistic since a decision maker has to deal with them at the time of the decision when their future status is not known.

Another peculiarity of the samples of study is that the information on the companies was taken from the Annual Primary-Supplementing-Tertiary file and the corresponding Research file of the Compustat Library. The results of this study, therefore, pertain to
companies that are included in these files. These files include all companies traded in the New York and American Stock exchanges, as well as those companies traded over the counter which are used in the computation of the Standard and Poor 400 and 500 indexes. The failure rate among such companies is probably lower than it is among smaller firms. The failure rate observed in this study is in the vicinity of six per thousand per year. Such a small failure rate makes the estimation task harder because it requires a sensitive model that is able to detect a very small group of failed individuals within a larger population.

Apart from the idiosyncrasies of the sample scheme followed in this study, differences among the samples of this and other studies with regard to the time period from which the data were taken may inhibit a direct comparison. The major corporate failure studies that have obtained the majority of their data from the seventies include those by Altman and McGough (1974), Altman et al. (1977), Diamond (1976), Moyer (1977), Ohlson (1980) and Zavgren (1983). Most of these studies utilize data up to 1976 with the exception of the study by Zavgren that used failure data up to 1978. The current study is the only one known to us, as of this writing, that has investigated data up to 1982.
### TABLE 4

**CLASSIFICATION RESULTS**

**CUMULATIVE ACCURACY BY PERIOD**

1972 Sample

<table>
<thead>
<tr>
<th>Period</th>
<th>Model 73 Correctly Classified/Total</th>
<th>Model 75 Correctly Classified/Total</th>
<th>Model 77 Correctly Classified/Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Failed/Non-Failed</td>
<td>Failed/Non-Failed</td>
<td>Failed/Non-Failed</td>
</tr>
<tr>
<td>72-73</td>
<td>11/13</td>
<td>11/13</td>
<td>12/13</td>
</tr>
<tr>
<td>72-74</td>
<td>16/20</td>
<td>14/20</td>
<td>17/20</td>
</tr>
<tr>
<td>72-75</td>
<td>23/27</td>
<td>20/27</td>
<td>24/27</td>
</tr>
<tr>
<td>72-76</td>
<td>30/39</td>
<td>27/39</td>
<td>31/39</td>
</tr>
<tr>
<td>72-77</td>
<td>35/46</td>
<td>1151/1431</td>
<td>31/46</td>
</tr>
</tbody>
</table>

The cumulative accuracy in a particular period is represented by the ratio of the correctly classified failed (non-failed) companies over the total number of the failed (non-failed) companies present in the sample within the respective period.

Merged, acquired, and delisted companies are included among the non-failed companies.
### TABLE 5

#### CLASSIFICATION RESULTS

**CUMULATIVE ACCURACY BY PERIOD**

1973 Sample

<table>
<thead>
<tr>
<th>Period</th>
<th>Model 73 Correctly Classified/Total</th>
<th>Model 75 Correctly Classified/Total</th>
<th>Model 77 Correctly Classified/Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Failed</td>
<td>Non-Failed</td>
<td>Failed</td>
</tr>
<tr>
<td>73-75</td>
<td>14/17</td>
<td>15/17</td>
<td>15/17</td>
</tr>
<tr>
<td>73-76</td>
<td>21/28</td>
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<td>22/28</td>
</tr>
<tr>
<td>73-77</td>
<td>27/35</td>
<td>27/35</td>
<td>27/35</td>
</tr>
<tr>
<td>73-78</td>
<td>34/43</td>
<td>1129/1498</td>
<td>33/43</td>
</tr>
</tbody>
</table>

See footnotes in Table 4.
**TABLE 6**

**CLASSIFICATION RESULTS**

**CUMULATIVE ACCURACY BY PERIOD**

1975 Sample

<table>
<thead>
<tr>
<th>Period</th>
<th>Failed</th>
<th>Non-Failed</th>
<th>Model 73 Correctly Classified/Total</th>
<th>Failed</th>
<th>Non-Failed</th>
<th>Model 75 Correctly Classified/Total</th>
<th>Failed</th>
<th>Non-Failed</th>
<th>Model 77 Correctly Classified/Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>75-76</td>
<td>2/3</td>
<td>1148/1525</td>
<td>2/3</td>
<td>9/11</td>
<td>1239/1525</td>
<td>25/33</td>
<td>2/3</td>
<td>1210/1525</td>
<td></td>
</tr>
<tr>
<td>75-77</td>
<td>9/11</td>
<td>1239/1525</td>
<td>16/20</td>
<td>21/26</td>
<td>1210/1525</td>
<td>25/33</td>
<td>21/26</td>
<td>1210/1525</td>
<td></td>
</tr>
<tr>
<td>75-80</td>
<td>25/33</td>
<td></td>
<td>25/33</td>
<td></td>
<td></td>
<td>25/33</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

See footnotes in Table 4.
TABLE 7

CLASSIFICATION RESULTS
CUMULATIVE ACCURACY BY PERIOD

1977 Sample

<table>
<thead>
<tr>
<th>Period</th>
<th>Correctly Classified/Total</th>
<th>Failed</th>
<th>Non-Failed</th>
<th>Model 73</th>
<th>Model 75</th>
<th>Model 77</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Correctly Classified/Total</td>
<td></td>
<td></td>
</tr>
<tr>
<td>77-78</td>
<td>9/9</td>
<td>8/9</td>
<td>8/9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>77-79</td>
<td>13/15</td>
<td>12/15</td>
<td>13/15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>77-80</td>
<td>17/22</td>
<td>16/22</td>
<td>17/22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>77-81</td>
<td>20/28</td>
<td>19/28</td>
<td>20/28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>77-82</td>
<td>24/33</td>
<td>21/33</td>
<td>24/33</td>
<td>1267/1589</td>
<td>1349/1589</td>
<td>1298/1589</td>
</tr>
</tbody>
</table>

See footnotes in Table 4.
TABLE 8
CLASSIFICATION ACCURACY BY DISTANCE FROM DATA YEAR

Model 77

Samples 1972, 1973, 1975 and 1977

<table>
<thead>
<tr>
<th>Sample Entry Years</th>
<th>1972</th>
<th>1973</th>
<th>1975</th>
<th>1977</th>
<th>Prior To Bankruptcy</th>
<th>Accuracy</th>
<th>%</th>
<th>Error</th>
<th>%</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
<td>(10)</td>
</tr>
<tr>
<td>a₁/b₁</td>
<td>12/13</td>
<td>9/11</td>
<td>2/3</td>
<td>8/9</td>
<td>31/36</td>
<td>1</td>
<td>86.1</td>
<td>13.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a₂/b₂</td>
<td>17/20</td>
<td>15/17</td>
<td>9/11</td>
<td>13/15</td>
<td>54/63</td>
<td>2</td>
<td>85.2</td>
<td>14.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a₃/b₃</td>
<td>24/27</td>
<td>22/28</td>
<td>17/20</td>
<td>17/22</td>
<td>80/97</td>
<td>3</td>
<td>76.5</td>
<td>23.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a₄/b₄</td>
<td>31/39</td>
<td>27/35</td>
<td>21/26</td>
<td>20/28</td>
<td>99/128</td>
<td>4</td>
<td>61.3</td>
<td>38.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a₅/b₅</td>
<td>36/46</td>
<td>34/43</td>
<td>25/33</td>
<td>24/33</td>
<td>119/155</td>
<td>5</td>
<td>74.0</td>
<td>26.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Columns (1), (2), (3), and (4) are taken from Tables 4, 5, 6, 7 and they represent the cumulative accuracies (by period) of classifying correctly failed companies with the use of Model 77 in the samples of 72, 73, 75, and 77, respectively.

(5) \[ a₅ = a₁ + a₂ + a₃ + a₄ \]
(5) \[ b₅ = b₁ + b₂ + b₃ + b₄ \]
(6) Absolute difference of successive numerators of column (5)/absolute difference of successive denominators of column (5)
<table>
<thead>
<tr>
<th>Years Prior To Bankruptcy</th>
<th>Accuracy</th>
<th>% Error</th>
<th>Type I Error %</th>
<th>Type II Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>23/27</td>
<td>23/27</td>
<td>85.2</td>
<td>14.8</td>
<td></td>
</tr>
<tr>
<td>41/48</td>
<td>18/21</td>
<td>85.7</td>
<td>14.3</td>
<td></td>
</tr>
<tr>
<td>63/75</td>
<td>22/27</td>
<td>81.5</td>
<td>18.5</td>
<td></td>
</tr>
<tr>
<td>79/100</td>
<td>16/25</td>
<td>64.0</td>
<td>36.0</td>
<td></td>
</tr>
<tr>
<td>95/122</td>
<td>16/22</td>
<td>72.7</td>
<td>27.3</td>
<td>19.8</td>
</tr>
</tbody>
</table>

See footnotes in Table 8.

(5) Calculated as in Table 9 by using Columns (1), (2), and (3)
### TABLE 10

**MISCLASSIFICATION OF NON-FAILED COMPANIES ALIVE IN 1982**

**BY MODEL AND YEAR OF DATA**

<table>
<thead>
<tr>
<th>Data Year</th>
<th>Error Type II</th>
<th>Percent</th>
<th>Data Year</th>
<th>Error Type II</th>
<th>Percent</th>
<th>Data Year</th>
<th>Error Type II</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>236/1225</td>
<td>19.2</td>
<td>1972</td>
<td>135/1225</td>
<td>11.0</td>
<td>1972</td>
<td>187/1225</td>
<td>15.2</td>
</tr>
<tr>
<td>1973</td>
<td>293/1242</td>
<td>23.5</td>
<td>1973</td>
<td>196/1242</td>
<td>15.7</td>
<td>1973</td>
<td>249/1242</td>
<td>20.0</td>
</tr>
<tr>
<td>1975</td>
<td>322/1239</td>
<td>25.9</td>
<td>1975</td>
<td>254/1239</td>
<td>20.5</td>
<td>1975</td>
<td>289/1239</td>
<td>23.3</td>
</tr>
</tbody>
</table>

Using data from a particular year, companies alive in 1982 are classified as future failures or non-failures in the period between the data year and 1982. The ratio of the misclassified non-failed companies over the total number of non-failed companies in a sample provides the Type II error.
CHAPTER V

5. CONCLUSIONS AND FUTURE EXTENSIONS.

5.1 Conclusions

The major contribution of this study is that it provides a functional method of modeling the empirical survivor function of a corporation over a period of at least five years, conditional upon the corporation's observed financial characteristics. This is in contrast to the information provided by discriminant analysis and other binary response models which by themselves can shed little light on the way the independent variables affect the survival of companies over time.

The survivor function was estimated through the hazard function, which was specified to have the form of the proportional hazards model. The covariates employed in the formation of the hazard function were chosen from accounting variables and financial ratios constructed from the information contained in the annual statements of publicly traded manufacturing and retail companies.

The survivor and hazard functions in parametric form provide two mathematically equivalent ways of specifying the distribution of failure times. In semiparametric form, as in this study, the survivor function leads to the estimation of the probabilities of failure by time intervals of interest, inside the study period of five years. The significance of this feature is that one does not need to be
confined to the probability of binary response (i.e., failure or non-failure) within the whole study period; the probabilities of failure over finer time segments are estimable, thereby providing an adequate approximation of the distribution of failure times. Knowledge of the distribution of failure times lends itself to the estimation of the expected monetary payoffs under different decision policies when the time to failure is a determinant of the payoff structure.

The estimation procedure used here provides a systematic way of estimating the distribution of failure times, thereby avoiding a priori specifications of the failure and survival probabilities. Specifying such a priori probabilities is commonly used in setting the cutoff point in studies of corporate failure employing discriminant analysis methods. Eisenbeis (1977) and other authors have discussed the consequences of using a priori probabilities in such models.

Another difficulty in using discriminant analysis and logit models is the requirement of certain distributional assumptions for the independent variables in the former and the cumulative distribution of the response variable in the latter model. The proportional hazards model does not require any particular assumption with regard to the distribution of the covariates. Moreover, the non-parametric estimation of the baseline hazard rate adds to the applicability of the model of this study in light of the difficulties in finding the exact parametric form of such a hazard. A parametric specification of the underlying hazard is complicated, given the lack of theoretical guidance, the small number of failures in the samples,
and the possibility that the underlying hazard changes over time. Admittedly, knowledge of the exact parametric form of the baseline hazard can lead to better precision in estimating the regression coefficients and the survivor function; a misspecification of the hazard, on the other hand, can lead to a biased survivor function and greater variance for the estimated regression coefficients. The proportional nature of the hazard function, the only major assumption of the Cox model, did not appear to be inappropriate for the data considered here.

Reliability/survival analysis models, besides providing estimates of the survivor function, also provide a means of exploring the functional form of the effect of the independent variables on the failure distribution. This feature can contribute to the theoretical understanding of the failure process of corporations. In this study, the proportional hazards model was explored for its adequacy in modeling such a failure process. Alternate specifications of the hazard function can help in exploring other assumptions besides proportionality and the performance of other covariates in modeling the survival of corporations. Gradually more information will be gained about the failure process. Discussion on alternate model specification is postponed until the next section.

Direct comparison of the classification results of this study with those of other studies are tenuous because of substantial differences in the populations underlying the samples, the ways in which the cut-off points are set, and the differences in the sampling
schemes used to obtain the data. Subject to all of the conditions interfering with the comparability of the different studies, the screening accuracy of the model in this study appears to be in agreement with the accuracy of other studies; one could even go a little further in suggesting that the classification accuracy in this study is more consistent over longer periods than it is in some other studies which report higher accuracy in the first year but lower accuracies than those in this study in the later years. This study has adhered to random sampling techniques and has examined the performance of the model over samples from different periods of time, in contrast to other studies. The interested reader should compare classification accuracies conditional upon the sample differences. Lack of comparability of the samples among different studies is the most pervasive difficulty in comparing the classification results of different models.

Even though models are customarily compared in terms of their ability to classify companies as future failures or non-failures, such a dichotomous partition of the state space is rarely adequate. For an investor in bonds or stock, an estimate of the probability of failure over specific time intervals may allow better decisions with consequent higher payoffs. A lender knowing the distribution of failure times can investigate the choice of different policies in regard to setting premia and examining collateral covenants. In theory, if a risk neutral individual had an exact model for the distribution of failure times, there would be no reason to preclude a
company in a certain risk group from an investment or lending consideration if the risk could be compensated for by higher profit expectations or higher premia charges and the person could diversify adequately to conform to the principles of expectations. In practice, however, because of the inherent deficiencies in empirical models, a prudent user would employ an empirical model as an exploratory device rather than as an exact rule. The investigation of the performance of a model over time in the context of its use is more appropriate than a mere comparison of its classification results with those of other models. However, if the survival times could be easily classified as being either very short or very long, then a binary response model such as a discriminant or logit model would be adequate and the classification results for random samples would be representative of its performance.

Empirical issues that interfere with the modeling of corporate failure distributions include the small number of failed companies, the multicollinearity among the attribute variables, the surrogate nature of accounting information, the lack of theoretical guidance in specifying the failure distributions, and the variability introduced by market conditions which cannot be deduced without observing the individual company's business environment. Certain researchers in the corporate failure area, including Ohlson (1980) and Zavgren (1983) have suggested that the prediction accuracy of corporate failure models based on accounting information may have approached an asymptotic limit. Further substantial improvement of prediction
results may not be possible without the utilization of information indicative of the future cash flow potential of corporations, and obtaining such information would require a closer analysis of the competitive business environment of an entity. Whether better information indicative of the potential of a firm can be found and incorporated into a general purpose prediction model remains an open question.

5.2 Future Research

Survival analysis models can be used in various applications in finance where the time to the occurrence of an event of interest is an important determinant of the pay-off structure. Examples of potential areas of application include consumer credit rating analysis, bond investments, modeling the time to a merger/acquisition, and the analysis of expected payoffs under different loan policies.

In regard to corporate failure modeling, several aspects need to be investigated further, including:

(1) The possibility that certain ratios (independent variables) are strong determinants of failure in the short run but lose their information content with regard to potential failure after the passage of a relatively short time. Using such transient effect variables may lead to spuriously good short run predictions even though using the same variables for long run inference may bias or distort the
results. Liquidity ratios indicating temporary difficulties in meeting cash obligations or sudden irregular fluctuations in a company's stock price could be examples of variables that have transient effects.

(2) Certain circumstances may accelerate the onset of failure in some individuals while leaving the remainder unchanged, even though after the passage of a certain time the accelerated onset individuals will have the same hazard as the unaffected ones. Certain economic conditions, such as interest rates, probably increase the vulnerability of companies in certain industries more so than in others.

(3) Another situation that needs to be investigated is the aversion of bankruptcy through a merger or acquisition.

Items (1) and (2) can be explored via models of "transient effects" and "accelerated onset" (see Cox and Oakes 1984 pp. 74-76). The aversion of bankruptcy through a merger can be investigated through competing risk models [David and Moeschberger (1978)].

Competing risk models can also be used to model loan problems in a more pragmatic way by accounting for the different types of loan related losses. A problematic loan does not always culminate in bankruptcy but can result in a loss for a creditor through an unfavorable settlement.

The application of the above suggested models, however, is not easily accomplished because these models are still under development and testing their adequacy requires a large amount of data. Moreover,
specification of a parametric form is limited by the lack of knowledge about the underlying mechanism that leads to failure.
FOOTNOTES

1. Number of firms that petitioned courts to liquidate or to reorganize under the protection of bankruptcy laws, U. S. Administrative Bankruptcy Courts, Washington, D.C.

2. Failure defined as in Business Failure Records by Dunn and Bradstreet. (Failures include businesses that ceased operations following a bankruptcy, voluntary liquidation leaving unpaid obligation, or companies involved in bankruptcy court proceedings.)

3. Average liability figures do not include publicly held debt.

4. For more detailed data see also Altman, 1983.

5. See Appendix A.


7. These models are also referred to as qualitative choice models. For more details on response models see also McFadden (1973) and Theil (1978).

8. In their common form.

9. Concomitant variables should be fixed.


11. Period of study not to exceed 5 years.

12. Set of indices of individuals at risk at time $t_i$.

13. See Appendix C

14. Maximization of the partial likelihood function is sought using the Newton-Raphson procedure, which in the process of search can lead to overflow in the exponential part of the hazard function. This problem is alleviated when the variables are introduced into the model one at a time and a good starting point for the Newton-Raphson is selected.

15. These samples of this study and that of Altman et al (1977) differ considerably. It should also be noted that Altman et al built their model using data one year prior to failure only.

16. See Section 2.3 for the mathematical form of the postulated hazard function.

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17. For more information on tests see Appendix B and Cox and Oakes (1984, p. 35 and 97).

18. Significant at \( \alpha = .1 \) using the likelihood ratio test.

19. Usually contemporary to the sample used for fitting the model.


21. Naturally, companies associated with high probabilities of failure would be excluded from consideration since normal changes would not lead to acceptable levels of expected profit.

22. As in the hazard function.

23. Given by the ratio of the differences of the numerators divided by the difference of the corresponding denominators of the two successive cumulative accuracy ratios.

24. Ratio of correctly classified non-failed firms over the total number of non-failed companies.

25. Defined as the percentage of misclassified failed companies over the total number of failed companies.

26. Given by \( 1 - \) (classification accuracy of non-failed companies).

27. Using data one year prior to failure.

28. Average of Type I and Type II error.

29. Whenever information was found that a delisted company filed for bankruptcy or was liquidated subsequent to its delisting, it was treated as a failed company.
REFERENCES


Bankruptcy and Reorganization Law: Highlights and Definitions

Several terms are used in the literature to describe troubled firms. Unfortunately these terms are often used interchangeably and the distinctions are not clear. The terms most commonly used are failure, insolvency, and bankruptcy.

In the first part of this Appendix the distinctions among the different aforementioned terms are highlighted; in the second part, the same thing is done for the distinctions among the different Chapters of Bankruptcy Code that apply to corporate failures.

Definitions

The term failure is used to depict the situation in which a firm's realized rate of return on invested capital is continually below the rate on similar investments and is expected to remain so in the future. A firm can be a failure in the above sense and yet continue to operate if it can meet its monetary obligations.

The term failure is used by Dun and Bradstreet (D & B) in a slightly more restrictive manner. This leading supplier of statistics on business failures defines an entity to be a failure if:

(a) a legal action was taken by the creditors,
(b) an entity withdrew from operation leaving unpaid obligations,
(c) an entity voluntarily compromised with its creditors.

Failures in the economic sense which ceased operations without having any legally enforceable debt obligations are not included under such a
definition.

Insolvency, also referred to as technical insolvency, depicts the situation in which a firm cannot meet its current liabilities because of lack of liquidity.

Bankruptcy is the condition in which a firm's total liabilities exceed the market value of its total assets; another term used for this condition is that of insolvency in bankruptcy sense (definition appears in Section 101, Clause 26, of the Bankruptcy Reform Act of 1978). Insolvency in bankruptcy sense is hard to determine because of the required fair valuation of a company's assets. The latter is usually left to the court proceedings which are initiated by a petition for liquidation or reorganization. If a firm's capitalized future value is estimated to be above its current liquidation value, then the firm is allowed to reorganize. This is legally referred to as bankruptcy reorganization. If the capitalized value of the firm is deemed to be below its liquidation value, the company is declared bankrupt and is liquidated.

Bankruptcy and Reorganization Law Highlights

The Bankruptcy Act of 1898 was first passed by the U.S. Congress to regulate the intricacies of bankruptcies. Since then several amendments and revisions of the Bankruptcy Act have taken place. In 1933 and 1934 another Act was passed which was comprehensively revised in 1938 to become the Chandler Act. Finally, in 1978, Congress enacted the latest Bankruptcy Reform Act which is in effect at present.

For the purpose of this study, the two Chapters of major interest
regarding bankruptcy and reorganization, are the Chapter X and Chapter XI of the Chandler Act. These two Chapters were revised and integrated into Chapter 11 of the Bankruptcy Reform Act of 1978.

The three different Chapters, X, XI, and 11, which will be briefly described in the sequel, differ in regard to the following functions:

1. initiation of proceedings,
2. custody of property,
3. creditor protection,
4. preparation, review and acceptance of the Reorganization plan.

Chapter X proceedings applied to corporations with secured creditors, publicly held corporations (except railroads) and those companies with complex debt structure that could not be processed through the Chapter XI proceedings. Filing under Chapter X required justifications as to why a case could not be handled under the simpler Chapter XI proceedings.

From the management's point of view, Chapter X was less desirable than Chapter XI because an independent and disinterested trustee was automatically appointed for all cases with indebtedness of more than $250,000 to assume control of the company for the duration of the proceedings.

The Securities and Exchange Commission (SEC) played an important advisory role, under the Chandler Act, in all cases exceeding $3 million in liabilities. The SEC's role under the 1978 bankruptcy Reform Act is minimized.

Chapter XI proceedings applied to unsecured creditors of
companies. Chapter XI arrangements were much simpler in nature, usually required much less time and the plan of action was required to be approved only by the unsecured creditors. The Court had the power to appoint a trustee to manage the company during the proceeding; however, this was not done automatically as under Chapter X arrangements. Under Chapter XI the debtor could borrow new funds that had priority over the unsecured debt.

For both Chapters X and XI, the assets were put under the protection of the court. Also, under the provisions of both Chapters X and XI, a plan for reorganization was proposed: if accepted, the company was reorganized, otherwise, the company was liquidated. The premise on which a company was allowed to reorganize was whether the capitalized value of its future earnings, discounted for risks, exceeded the liquidation value of its assets. If the capitalized value exceeded that of liquidation, then the company was reorganized, otherwise it was liquidated.

The Bankruptcy Reform Act of 1978 was enacted in order to revise and make the older Code more efficient. The text of the new Code is available in Bankruptcy Law Reports, No. 389, October 26, Part II, published by the Commerce Clearing House, Chicago, Illinois. A review of the new Code is also provided by Duberstein ("A Broad View of the New Bankruptcy Code" Brooklyn Barrister, April 1979) and by Altman (1983).

Chapter 11 of the 1978 Reform Act deals with procedural aspects of business reorganization. Chapter 11 integrated Chapters XI and X and a portion of Chapter XII on real property arrangements.
The procedure for rehabilitation of corporations under Chapter 11 provides for the following:

1. Bankruptcy proceedings can be initiated voluntarily by the debtor or involuntarily by three or more creditors with claims above a specified level;

2. The court may or may not appoint a trustee;

3. The creditors are represented by the largest seven creditors and any others approved by the court;

4. The debtor has to propose (unless an extension is granted) a plan within 120 days. If any specified deadline is not met, any interested party can submit a plan;

5. The court holds hearings on the plan and if deemed fair, creditors vote for its approval;

6. If the plan is approved, it is put forward under the supervision of a trustee or of the court.

For more detailed discussion on Chapter 11 the interested reader is referred to Weintraub (1980), "What Every Credit Executive Should Know About Chapter 11 of the Bankruptcy Code," New York: National Association of Credit Management.
(1) **Newton Raphson Algorithm:**

The Newton Raphson algorithm, which is used to maximize the partial likelihood, searches for the optimal values of the coefficients $\theta$ by changing the parameter estimates at each iteration using

$$
\Delta \hat{\theta} = I^{-1}(\hat{\theta}) U(\hat{\theta}),
$$

where: $U(\hat{\theta})$ and $I(\hat{\theta})$ represent the vector of the first derivatives and the negative of the matrix of the second partial derivatives of the (partial) log likelihood with respect to $\theta$, respectively [see Miller (1981, p. 124); BMDP (1983, p. 684)].

The N-R algorithm stops when the absolute value of

$$
[\ln L(\hat{\theta}_j) - \ln L(\hat{\theta}_{j-1})]/\ln L(\hat{\theta}_j)
$$

falls below $10^{-5}$, where $L(\hat{\theta}_k)$ represents the partial likelihood function at the kth iteration.

(2) **Asymptotic Covariance (Correlation) Matrix:**

The asymptotic covariance matrix is calculated by the inversion of the $I(\hat{\theta})$ matrix. The correlation matrix is obtained by dividing the elements of the covariance matrix by the corresponding standard errors.
(3) Tests of Hypotheses:

Hypotheses of the form \( H_0 : \beta^* = 0 \) can be tested using the following statistics:

(a) The Wald statistic, given by

\[
W = \hat{\beta}^* '\left[ \text{COV}(\hat{\beta}^*) \right]^{-1} \hat{\beta}^*,
\]

where \( \hat{\beta}^* \) is the vector of the regression coefficients considered for elimination and \( \hat{\beta}^* \) is the corresponding vector of estimates.

(b) The likelihood ratio test, defined by

\[
L = 2 \left[ \ln L(\hat{\beta}) - \ln L(\hat{\beta}_R) \right],
\]

where \( \hat{\beta}_R \) are the estimated coefficients using the restricted model.

(c) The score statistic, given by

\[
U'(\hat{\beta}_o) I^{-1}(\hat{\beta}_o) U(\hat{\beta}_o),
\]

where \( \hat{\beta}_o \) is the vector having zero for the coefficients considered for elimination and the estimates from the restricted model in the remaining positions.

The Global chi-square statistic is given by the score statistic when \( \hat{\beta}_o = 0 \).

All three statistics are compared with the Chi-square distribution with \( q \) degrees of freedom, where \( q \) equals the number of coefficients considered for elimination.
(4) The estimated cumulative hazard function $\hat{S}(t,z)$ is calculated using the method of Link (1979):

$$\hat{S}(t,z) = \exp(\hat{\beta}, z) \left[ \sum_{i=1}^{k} (t_i - t_{i-1}) \hat{h}_{oi} + (t - t_1) \hat{h}_{ok+1} \right],$$

where

$$\hat{h}_{oi} = \hat{m}_i (t_i - t_{i-1})^{-1} (\prod_{j \in R_i} \exp(\hat{\beta}, z_j))^{-1},$$

$R_i$ is the set of indices of the individuals at risk at the time of the $i$th failure,

$$t_k < t \leq t_{k+1},$$

$$t_0 = 0.$$

(5) The residuals are calculated as

$$e_i = \exp(\hat{\beta}, z_i) \hat{S}(t,0).$$

(6) The cumulative hazard function $\hat{H}(e)$ of the sorted residuals is calculated as

$$\hat{H}(e) = \sum_{j=1}^{k} \hat{m}_i (\#R_j)^{-1} + (e - e_k) \hat{m}_{k+1} (\#R_{k+1})^{-1},$$

where $e_k < e \leq e_{k+1}, e_0 = 0$ and $\#R_j$ denotes the number of cases at risk at time $j$. 
Appendix C

Set of Variables

The following variables in the original or transformed form were considered as covariates:

1. \((\text{Cash and Equivalents + Receivables})/\text{Current Liabilities}\).
2. \((\text{Income before Extraordinary Items and Discontinued Operations + Depreciation})/\text{Sales}\).
3. Operating Income before Depreciation/\text{Sales}.
4. Operating Income before Depreciation/\text{Tangible Assets}.
5. \(\text{Current Assets}/\text{Current Liabilities}\).
6. \((\text{Current Assets} - \text{Current Liabilities})/\text{Tangible Assets}\).
7. \text{Sales}/\text{Tangible Assets}.
8. \((\text{Income before Extraordinary Items and Discontinued Operations + Taxes})/\text{Sales}\).
9. \((\text{Income before Extraordinary Items and Discontinued Operations + Taxes})/\text{Tangible Assets}\).
10. \((\text{Income before Extraordinary Items and Discontinued Operations + Taxes + Interest})/\text{Interest}\).
11. \((\text{Income before Extraordinary Items and Discontinued Operations + Taxes + Interest})/(\text{Interest + Minimum rental in one year.})\).
12. Operating Income after Depreciation/\text{Tangible Assets}.
13. Retained Earnings/\text{Tangible Assets}.
14. Capital Surplus/\text{Tangible Assets}.
15. Natural log of total assets.
16. Total Debt/Total Invested Capital.
18. (Market value + Preferred Stock at Liquidating Value)/Total Debt.
21. Trend in Primary Earnings
22. Index: equal to 1 if trend in Primary Earnings is positive, -1 otherwise.
APPENDIX D

Calculation of the Cutoff Point

1. Following the assumptions of section 4.2, the cutoff point is the value of $I$ that solves the following expression:

$$-L (1 - S_o^I (3)) + P (S_o^I (3)) = 0.44$$

Let 0.625 be the amount lost per dollar of principal outstanding at the time of failure, and let 0.462 be the return on a dollar resulting from 13.5% interest compounded annually on a three year balloon loan. Then by substitution the above expression becomes:

$$-1.087 (1-.995^I) + .462 (.995)^I = 0.44$$

$$I = \ln \left( \frac{1.527}{1.549} \right) \ln^{-1}(0.995) = 2.86$$

2. The assumption that a company with a risk index $I$ greater than or equal to the cutoff point is willing to pay 13.5% interest on the above loan was based on the following information:

At the time when the prime rate was 12.5%, the following interest rates were considered to be realistic for three year balloon maturity loans: 12.7, 12.8, 12.9, 13.1, 13.3 and 13.5 for companies carrying a bond rating of Aaa, Aa, A, Bbb, Bb and B, respectively. Therefore, taking into consideration that loans given to companies rated below B are considered speculative, the 13.5% interest rate was deemed as a plausible charge to be used for separating high and low risk individuals.
3. The assumed charge of 13% to a company that has characteristics equal to population averages was considered plausible, taking into account that an "average" company's rating will fall in the interval between A and Bbb.

Even though the calculated cutoff point is realistic, it is based on certain assumptions that may not suit everyone's preferences. For this reason, in the sequel, classification results are also presented for certain alternative cutoff points.
### Type I and II Errors by Cutoff Point*

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<th>Years</th>
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<th>Cutoff Point 4</th>
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<td>Type II %</td>
<td>Type I %</td>
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*Errors were calculated the same way as in Table 8, using Model 77 and samples of 1972, 1973, 1975 and 1977.
## Type I and II Errors by Cutoff Point

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<th>Cutoff Point 2.75 Error</th>
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VITA

George Sarantis Karamessinis was born on August 29, 1953. He graduated from the University of Patras in June of 1978 with a Bachelor of Science degree in Mathematics.

In August of 1978 he began graduate studies in Quantitative Methods at Louisiana State University where he received his Master of Science degree in December of 1979. In January of 1980 he enrolled in the program for the Ph.D. degree in the department of Quantitative Business Analysis where he is now a candidate for the degree of Doctor of Philosophy.
DOCTORAL EXAMINATION AND DISSERTATION REPORT

Candidate: George Sarantis-Karamessinis

Major Field: Business Administration (Quantitative Business Analysis)

Title of Dissertation: Modeling the Failure Time Distribution for Manufacturing and Retail Corporations Using Survival Analysis

Approved:

[Signature]
Major Professor and Chairman

[Signature]
Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:
April 19, 1985