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Weak localisation theory for lightly doped semiconductor quantum wires

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Abstract. A weak localisation theory for a semiconductor quantum wire, which has a width of the order of the Fermi wavelength, is presented. In our model the electronic motion is essentially one-dimensional and the localisation length L_l is much larger than the mean free path l, so that, in contrast to conventional theories a non-localised quantum wire with a total length $L < L_l$ but much larger than l is possible. For the static properties, we study the temperature dependence and the subbands effect of the weak localisation. We find that when (the phase coherent length) $L_\phi > L$, the conductance of the quantum wire depends on L instead of L_ϕ, implying a temperature independent behaviour. Our theory explains recent experiments which found temperature independent transport behaviour at very low temperature for narrow AlGaAs/GaAs quantum wire. In studying the AC conductivity, our calculation predicts that, for the quantum wire with $L > L_l$, there exists a critical value of the frequency above which the system is delocalised and the AC conductivity $\sigma(\omega)$ rises as ω^2.

1. Introduction

In recent years the study of lightly doped semiconductor quantum wires (Skocpol *et al* 1982, Dean and Pepper 1984, Skocpol 1988, Hiramoto *et al* 1989) has attracted much attention. A quantum wire is defined here to have a very narrow width $W \sim \lambda_F$ (Fermi wavelength) so that the electron states are essentially quantised laterally. In this way it is different from both the usual effective one-dimensional (1D) system (Thouless 1977, 1980) where $W \gg \lambda_F$ (no lateral quantisation), and the strictly 1D systems (Mott and Twose 1961, Landauer 1970) where there is no lateral degree of freedom. Thus, the influence of lateral quantisation on weak localisation of the quantum wire is a subject that needs to be explored. The purpose of this paper is to carry out such an exploration and we present a weak localisation theory for the semiconductor quantum wire which complements the existing weak localisation theory (Lee and Ramakrishnan 1985, Altshuler and Aronov 1985) for the 1D and effective 1D systems. Our focus here will be a discussion of the temperature dependence, the subband effect and the low frequency behaviour of the weak localisation of the semiconductor quantum wire. Our theory was developed in our study of the electric field effect on weak localisation (Hu and O'Connell 1989), where we found that the physics of the electrons in a semiconductor quantum wire is better described by a sudden reversal picture (see discussion below) in contrast to the diffusive picture.

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1.1. Diffusive picture

A decade ago, Thouless (1977, 1980) showed that below some critical temperature T_c, the electrons in the effective 1D system are free to diffuse over a distance L_l (the localisation length), but can then go no further until a phonon or another electron causes a transition to a new state. Thouless's idea forms the foundation for the modern theory of weak localisation (Abrahams *et al* 1979). Weak localisation is a quantum effect (Lee and Ramakrishnan 1985, Altshuler and Aronov 1985) caused by the coherent back scattering (CBS), where an electron with initial momentum $\boldsymbol{k}$ is finally scattered into the opposite state $-\boldsymbol{k}$ elastically. According to the *diffusive picture*, in a real system the CBS is realised through coherent scattering sequences (fan diagram), where an electron of Fermi momentum k_F moves in a diffusive way such that its momentum gradually changes to $-k_F + q$ (with $q/k_F \ll 1$). The average distance (the phase coherent length L_ϕ) over which the electron diffuses during these sequences, is estimated to be $\sqrt{D\tau_\phi}$, where D is the diffusion constant and τ_ϕ is the phase coherent time, the average time for a CBS process. This diffusive description of the electron motion serves as the basis of almost all the theoretical treatments of the quantum correction to the conductivity in the metallic regime.

1.2. Sudden reversal picture

While the diffusive picture for the CBS is illuminating and correct for most of the weakly localised systems, it certainly does not rule out other possible ways for electrons to achieve CBS in some peculiar systems. The strong localisation of the 1D system (including the system with finite width $W < \lambda_F$) mentioned earlier is one example, where the CBS process happens one-dimensionally at a length scale of l. Another picture, the *sudden reversal picture* for the CBS of electrons, proposed by us (Hu and O'Connell 1989), is such that the CBS process basically exhibits a 1D behaviour but at a length scale much larger than l. In other words, our emphasis is on providing a mechanism (the sudden reversal picture) for treating semiconductor wires having a width $W \sim \lambda_F$, a situation where lateral quantisation is playing a decisive role and for which the Thouless diffusive picture is not applicable. As in the diffusive picture, the CBS is realised by the coherent scattering sequence which has a total momentum transfer of $-2k_F + q$ ($q/k_f \ll 1$) for electrons near the fermi surface. The difference is that instead of diffusing elastically through many different states gradually to achieve the CBS (as in the diffusive picture), the electrons are now assumed to be scattered by impurities into only two kinds of states. One is a small momentum transfer forward process which essentially does not change the velocity of electrons, the other is a large momentum transfer ($\sim 2k_F$) process which makes the electron moves essentially in the reversed direction. In addition, the assumption that the system is lightly doped makes the probability of the reversal scattering much less than the forward scattering. (The opposite case, i.e. when the reversal scattering dominates, corresponds to the 1D case). In this way an electron will experience many forward scatterings with little change in its original speed. Eventually it will experience a reversal scattering. This is illustrated schematically in figure 1. Thus in our picture an electron will travel a distance $L_\phi \sim v_F\tau_\phi$ in a CBS process, as distinct from the result $L_\phi \sim \sqrt{D\tau_\phi}$ in the diffusive picture. We stress that the Thouless diffusive picture is applicable to effective one-dimensional systems where $L_\phi > W \gg \lambda_F$ whereas the sudden reversal picture may be used to analyse semiconductor quantum wires for which $L_\phi > W \sim \lambda_F$ so that the electron states are quantised laterally. Also, the L_ϕ in our

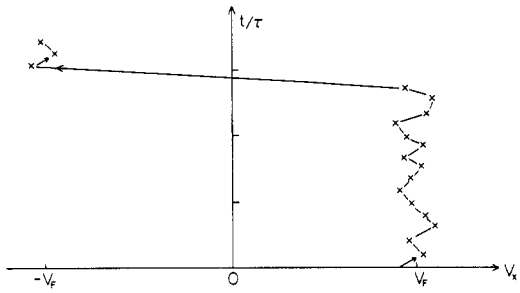

Figure 1. Schematic picture of the velocity (v_x) evolution (time in units of the momentum relaxation time (τ)) of an electron in a lightly doped semiconductor quantum wire.

picture is much larger than l, in contrast to the 1D case (no lateral dimension of freedom) in the diffusive picture where $L_\phi \sim l$. A direct consequence of the sudden reversal picture is that when the electric field E exceeds a critical value $E_c = \hbar v_F / eL_\phi^2$, it introduces a new cut-off length $L_s = (E_c/E)^{1/2} L_\phi$ (Hu and O'Connell 1989) and delocalises the system. This is in better agreement with the experiments of Hiramoto *et al* (1989) than the Mott–Kaveh theory (1981) which also has a cut-off length which is electric field dependent so that again the system is delocalised for a sufficiently large E value. Incidentally, we note that, in the diffusive picture, the possible effect of the electric field on weak localisation is a controversial issue (Lee and Ramakrishnan 1985, Hu and O'Connell 1988) and is known to be different for systems with different dimensionalities (Kirkpatrick 1986).

The sudden reversal picture of the CBS proposed here is applicable to many of the semiconductor lightly doped quantum wires available recently through advances in microfabrication technology (Skocpol 1988, Hiramoto *et al* 1989). The width of these thin wires is comparable to the Fermi wavelength ($\sim 10^3$ Å), which makes the motion of electrons in them basically one-dimensional in a quantum mechanical way. On the other hand, the presence of a finite cross section also makes them totally different from the strictly 1D systems. Physically, due to the relatively large value of the Fermi wavelength ($\sim 10^3$ Å) of the semiconductor, the dilute impurities in the quantum wire cannot individually block the way of the moving electrons and hence ensures that the reversal scattering has a small probability (roughly proportional to the ratio of the size of the impurity to the width of the wire). At the same time, the lateral quantisation of the sample restricts the motion of the electrons essentially in a 1D fashion and thus makes the other possible way of impurity scattering, the forward scattering, the dominant process.

2. Formulation

Next, we implement the physical idea of the sudden reversal picture of CBS into a quantitative evaluation of the associated quantum corrections to the electric conductivity of the quantum wire (with a width of the order of the Fermi wavelength and containing dilute impurities). For a simple discussion of the problem, the electron–electron interaction will be neglected in this paper.

Obviously, to formulate the sudden reversal picture, we can no longer adopt the conventional technique of the fan diagram calculation in the Kubo formalism, the basis of the diffusive picture. Instead one must perform a calculation, from first principles,

which will include the high-order impurity scattering effect self-consistently. We achieve this by a centre of mass formulation of the electron system and by solving the equation of motion of relative electrons self-consistently (Hu and O'Connell 1987, 1988). This formulation enables us to study the macroscopic quantum mechanic effects directly. The main approximation involved is to assume that the total number of electrons N in the system is much larger than one. Such an approximation is certainly good for a realistic semiconductor quantum wire obtained from a two-dimensional system. For instance, $N \sim 10^3$ for a typical system $1 \times 0.1\ \mu m^2$ with $n_s \sim 10^{12}\ cm^{-2}$.

Our calculation scheme is a generalised Langevin equation (GLE) approach for the centre of mass electrons, which we have developed in a series of papers (Hu and O'Connell 1987, 1988). For completeness, we repeat a few steps (equations (1), (2) and part of (3)) in the following. First, we recall that the dynamical conductivity (Hu and O'Connell 1987) may be written as

$$\sigma(\omega) = (ine^2/m)/(\omega + i\mu(\omega)/M) \tag{1}$$

where n is the electron density, e the electron charge, m the effective mass, $M = Nm$, and $\mu(\omega)$, the Fourier transform of the memory function in the GLE, contains all the information concerning the effect of the heat bath (the relative electrons and phonons) on the transport properties of the quantum particle (centre of mass electrons). Secondly, a self-consistent expression for the memory function, obtained by solving the Heisenberg equation for the density operator of the relative electrons (Hu and O'Connell 1987), is

$$\mu(\omega) = \mu^{(0)}(\omega) + \sum_{kq} P_{kq}(\omega)\mu_{kq}(\omega) + \sum_{kq} Q_{kq}(\omega)\mu_{kq}(\omega) \tag{2}$$

where the detailed expressions for $\mu^{(0)}(\omega)$, $P_{kq}(\omega)$ and $Q_{kq}(\omega)$ can be found in Hu and O'Connell (1987), and

$$\mu(\omega) = \sum_{q} \mu_q(\omega) = \sum_{kq} \mu_{kq}(\omega).$$

The $\mu^{(0)}(\omega)$ in (2) is the lowest order impurity contribution to the memory function, while the other terms are due to higher order contributions from impurity scattering. Also, the approximations used in obtaining (2) are the use of a random impurity distribution and a cumulant decoupling scheme for higher order scattering terms.

Equation (2) can be further simplified when applied to the semiconductor quantum wire. First, the structure of $Q_{kq}(\omega)$ shows that it is an on-site high order contribution, which arises from the repeated forward scattering from the same impurity site. It is not related to the dominant quantum interference term (which arises from multi-site scattering) and will be neglected. Secondly, it is straightforward to show that in the $\omega \ll 1/\tau$ limit (τ is the momentum relaxation time), the $P_{kq}(\omega)$ in (2) is independent of k and the CBS events make the dominant contribution (Hu and O'Connell 1988). For a quantum wire, obtained from a two-dimensional electron gas by lateral confinement, it is

$$P_{kq}(\omega) \approx -2n_i U^2 \sum_{q'} \sum_{n,n'=0}^{m} \frac{\theta(\varphi_c - \varphi_n)}{\omega^2 - \omega^2_{q'q_F}(nn')} \tag{3}$$

where q' is the wave vector along the wire (x direction), $\theta(x)$ is a step function which we will discuss later, n,n' are subband indices due to the lateral quantisation, and m

represents the highest populated subband. Also n_i and U are the impurity density and the impurity scattering potential respectively. In addition,

$$\omega_{q'q_F}(nn') = v_F(q' \cos \varphi_n + k_{n'} \sin \varphi_n) \tag{4}$$

where $k_{n'}$ is the wave number of the electrons in the n'th subband, and φ_n is the angle between the Fermi wave vector of the nth subband and the x axis, i.e., $\sin \varphi_n = k_n/k_F$. Also φ_c is defined by the relation $\tan \varphi_c = W/l$ ($l > W$). The magnitude of φ_n relative to φ_c determines the way that the electrons are being scattered. Due to the narrow width of the wire ($W < l$), those electrons with $\varphi_n > \varphi_c$ will be mainly scattered by boundaries, while those electrons with $\varphi_n < \varphi_c$ will be scattered by impurities before they hit the boundary. The step function $\theta(x)$ in (3) is introduced to take account of this effect, by excluding the contributions to the back scattering events due to those electrons (with $\varphi_n > \varphi_c$) having the boundary reflection as their main scattering events because such boundary reflections lead to a random change in phase.

Substituting (3) into (2) and neglecting the last term on the RHS of (2), after some algebra we obtain a closed form expression for the memory function:

$$\mu(\omega) = \mu^{(0)}(\omega)\left(1 + \frac{2m}{N\pi\hbar\tau}\sum_{n,n'=0}^{m}\sum_{q'}\frac{\theta(\varphi_c - \varphi_n)}{q_\omega^2 - \omega_{q'q_F}^2(nn')/v_F^2}\right)^{-1} \tag{5}$$

where $q_\omega = \omega/v_F$, $\hbar/\tau \equiv 2\pi n_i U^2 N(\varepsilon_F)$, $N(\varepsilon_F)$ is the density of states per spin at the Fermi energy, and the factor 2 in the last term takes account of the spin degeneracy of q'. We note that the sum over q' in (4) is carried out by the standard continuum approximation and by the introduction of an upper and lower cut-off for q', $1/l$ and $1/L_\phi$ ($1/L$ if $L_\phi > L$) respectively. Here L_ϕ is the phase coherent length which in our (sudden reversal) picture is proportional to the phase coherent time τ_ϕ, as mentioned in the introduction. Equations (1) and (5) are used to evaluate the electric conductivity for the quantum wire in the following.

3. Static conductivity

The static conductivity is obtained from (1) and (5) by putting $\omega = 0$. Thus, we obtain (using $\sigma^0 = ne^2\tau/m$, and $\mu^0(0) = M/\tau$) the conductivity of a quantum wire, constructed from a 2D electron system by lateral confinement, in the form:

$$\sigma(\omega = 0) = \sigma^0[1 - (\alpha/\pi)(L - l)/l] \qquad (L_\phi > L) \tag{6a}$$

$$\sigma(\omega = 0) = \sigma^0[1 - (\alpha/\pi)(L_\phi - l)/l] \qquad (L_\phi < L) \tag{6b}$$

where $\alpha = 2\gamma/k_F W$, and

$$\gamma = \sum_{n,n'=0}^{m}\frac{\theta(\varphi_c - \varphi_n)}{(\cos \varphi_n + k_{n'}L \sin \varphi_n)(\cos \varphi_n + k_{n'}l \sin \varphi_n)} \tag{7}$$

is a factor which takes account of the lateral quantisation into subbands, and the σ^0 is the bulk two-dimensional conductivity. We note that (6*a*) and (6*b*) are the same as the well known diffusive picture results of the 1d perturbation theory namely $\sigma(\omega = 0) = \sigma_{1d}^0[1 - (1/\pi)(L - l)/l]$ if we take $\sigma_{1d}^0 = W\sigma^0$ and $\alpha = 1$ (i.e. $\gamma = 1$ and $W = 2/k_F$).

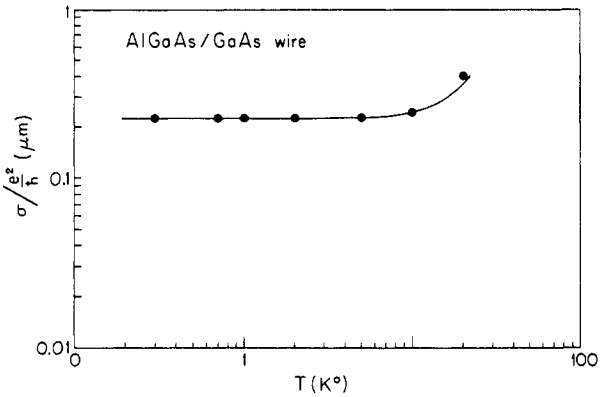

Figure 2. Conductivity $\sigma/(e^2/\hbar)$ of a AlGaAs/GaAs quantum wire ($L = 1.9\,\mu$m, $W = 0.09\,\mu$m, $l = 0.48\,\mu$m) as a function of temperature T. Full circles are data extracted from the experiments of Hiramoto *et al* (1989) and the full line is the theoretical result from (6*a*) and (6*b*) with $L_\phi = 1.9\,T^{-1/3}$ and $\alpha = 0.51$.

In fact, when $\alpha < 1$ our result of $\sigma(\omega = 0)$ in (6) is essentially different from that of the diffusive picture ($\alpha = 1$). From (6), we can estimate the quantum wire will remain non-localised as long as the length of the system satisfies

$$l \ll L < (1 + \pi/\alpha)l \qquad \alpha < 1.$$

Thus, our theory can be consistently used for the quantum wire having a phase coherence length much larger than l. Also, because of $L_\phi \sim T^{-n}$ (n depends on the mechanism for the inelastic scattering), σ is independent of T in the low temperature region where L_ϕ exceeds L (see (6*a*)), and σ increases with increase of T in the high temperature region where $L_\phi < L$ (see (6*b*)). Experimentally, Hiramoto *et al* (1989) have measured the conductivity of AlGaAs/GaAs quantum wires and obtained some puzzling results for the phase coherence length from a fitting to the existing weak localisation theory. Here we show that their results can be better understood by fitting (6*a*) and (6*b*) to the values of conductivity extracted from figure 3(*a*) of their paper. The extracted data of σ for a AlGaAs/GaAs quantum wire (sample number 44, $L = 1.9\,\mu$m $W = 0.09\,\mu$m, $l = 0.48\,\mu$m) from the Hiramoto *et al* paper is shown in figure 2 by full circles. As can be seen from this figure, the temperature dependence of σ is just what is expected from (6*a*) and (6*b*) (constant σ at low T and increasing σ at high T). First, by (6*a*) the temperature independence of σ at low T is because $L_\phi > L$ in that region. A fitting of (6*a*) to the constant part of σ at low T ($\sigma/(e^2/\hbar) = 0.25\,\mu$m, $L = 1.9\,\mu$m, $l = 0.48\,\mu$m) gives $\alpha = 0.51$. Secondly, we assume the electron–electron interaction determines the phase coherence length with $L_\phi = CT^{-1/3}\,\mu$m (C is a fitting parameter, Altshuler and Aronov 1985), and use (6*b*) to fit the temperature dependent part of σ in figure 2. The theoretical fitting curve of σ by (6*a*) and (6*b*) is shown by the full line in figure 2 where $C = 1.9$ is used.

Next, we turn to a study of the α appearing in (6*a*) and (6*b*), the subband effect on the weak localisation. For this purpose, we introduce the relative change of resistance in the weak localisation limit, which can be deduced directly from (6*b*) ($L_\phi < L$, $L_\phi \gg l$) as

$$\Delta R/R = \alpha L_\phi/\pi l. \tag{8}$$

We note that the relative change of resistance was first studied by Thouless using a

diffusive picture. As we stated earlier the main difference between our sudden reversal picture and the diffusive picture is the way the CBS is achieved. Actually, when $\gamma = 1$ (only the lowest subband occupied), from (8) we obtain $\Delta R/R = 2L_\phi/\pi W k_F l$ which agrees with (up to a numerical factor) the result $\Delta R/R \sim L_\phi/Wk_F l$ from the Thouless' theory (if we extend it to the effective 1D system from a 2D system). For the multi-subband case, the subband factor γ (see (6)) depends on the details of the confinement potential. In case of a square well confinement potential, $k_n = n\pi/W$

$$\gamma = \sum_{n,n'=0}^{m} \frac{\beta^2 \theta(\varphi_c - \varphi_n)}{(\sqrt{\beta^2 - n^2} + nk_{n'}L)(\sqrt{\beta^2 - n^2} + nk_{n'}l)} \tag{9}$$

where $\beta = Wk_F/\pi$, m is the largest integer smaller than β. Also, the step function $\theta(x)$ in (9) restricts the n's which contributes to γ. Now, from the condition $\tan \varphi_n$ $(= k_n/\sqrt{k_F^2 - k_n^2} = n/\sqrt{\beta^2 - n^2}) < \tan \varphi_c$ $(= W/l)$, the $\theta(x)$ can be transformed to the inequality

$$\beta^2 - n^2 > (nl/W)^2 \tag{10}$$

which must be satisfied for the β and n in (9). Equation (10) indicates that for a very narrow quantum wire ($l \gg W$), only the electrons in the lowest subbands ($n = 0$) contribute to the quantum corrections of the resistance, and from (9) one easily obtains $\gamma = 1$ and $\alpha = 2/\pi\beta$ follows. The sample number 44 in the experiments of Hiramoto *et al* (1989) which we studied above ($L = 1.9\,\mu$m, $l = 0.48\,\mu$m, $W = 0.09\,\mu$m) should be one example. As a check, we mention that the $\alpha = 0.51$ we used in our previous discussion for that sample would imply a theoretical value of $\beta = 1.25$, which corresponds to a not untypical value $n_s \approx 10^{12}$ cm^{-2} for that sample. This estimate consistently supports our theory presented in this section. Furthermore, for samples with a width $W \leqslant l$ at large density, from (9) and (10) we expect more subbands will add to the contribution to γ and so α will decrease slower than W^{-1}. These facts are all very consistent with experimental results (Skocpol 1988). Also, the finding that only the electrons in the lowest subbands or so, contributes to the quantum correction to the resistance is good evidence in support of the sudden reversal picture presented in section 1 for weak localisation in the semiconductor quantum wire.

4. AC conductivity

The dynamical conductivity $\sigma(\omega)$ in both the $\omega\tau \ll 1$ and $\omega\tau \gg 1$ region can be obtained analytically from (1) and (5), as we will now show.

In the $\omega\tau \gg 1$ region, our results simply reduce to the well-known Drude model, with the use of $\mu^0(\omega) \simeq M/\tau$,

$$\sigma(\omega) = \sigma^0/\omega^2\tau^2 \qquad (\omega\tau \gg 1) \tag{11}$$

where σ^0 is the same as that in (6). The sample length, L, independence of $\sigma(\omega)$ at $\omega\tau \gg 1$ seen above, reflects the delocalisation effect of the high frequency field.

Let us consider the low frequency regime where, as we shall see, new results emerge. In general, $\omega\tau \ll 1$, by using (5), (1) is reduced to

$$\sigma(\omega) = \sigma^0\{1 - (\alpha/2\pi q_\omega l)\ln[|(1 - q_\omega l)(1 + q_\omega L_\phi)/(1 + q_\omega l)(1 - q_\omega L_\phi)|]\} \qquad (\omega\tau \ll 1). \tag{12}$$

The above result reveals two different behaviours in two different frequency regions roughly separated by the dephasing rate (τ_ϕ^{-1}).

In the *very low frequency* limit ($\omega \ll v_F/L_\phi$), (12) is reduced to

$$\sigma(\omega) = \sigma(\omega = 0) - \alpha(e^2/3\hbar\pi)\omega^2\tau^2(L_\phi^3/l^3 - 1) \qquad (\omega \ll \tau_\phi^{-1}) \qquad (13)$$

where $\sigma(\omega = 0)$ is defined by (6). For a localised system, ($L > L_l$), $\sigma(\omega = 0)$ cannot be further reduced and (13) tells us the system will still be localised at $\omega \ll \tau_\phi^{-1}$. For a non-localised system ($L < L_l$), (6) and (13) show that the system stays non-localised and the $\sigma(\omega)$ will decrease from $\sigma(\omega = 0)$ when ω increases in the region $\omega \ll \tau_\phi^{-1}$.

In the *intermediate low frequency* region ($\tau_\phi^{-1} \ll \omega \ll \tau^{-1}$), (12) is reduced to

$$\sigma(\omega) = \sigma_0[1 + \alpha/\pi - \alpha v_F/\pi\omega^2\tau L_\phi + (\alpha/3\pi)\omega^2\tau^2]. \qquad (14)$$

Equation (14) is a particular interesting result for the system which is originally localised (i.e., $L > (1 + \pi/\alpha)l$). The term which is proportional to $-\omega^{-2}$ in (14), keeps the $\sigma(\omega)$ localised at low frequencies up to a frequency ω_c estimated by the condition $1 + \alpha/\pi = \alpha V_F/(\pi\omega^2\tau L_\phi)$. Once $\omega > \omega_c$, the system is delocalised and the last term in (14) will determine the frequency behaviour of $\sigma(\omega)$. In other words, (14) predicts that for a weakly localised quantum wire, there exists a critical frequency $\omega_c = (\alpha v_F/(\pi + \alpha)L_\phi\tau)^{1/2}$ above which the system will be delocalised and will have a $\sigma(\omega) \sim \omega^2$ behaviour. It is interesting to note that a $\sigma(\omega) \sim \omega^2$ delocalisation behaviour has previously been shown for the strongly localised system (Mott and Twose 1961), a property due to the hopping conductivity where the system has a localisation length essentially equal to the mean free path. On the other hand, a clear estimate of the critical frequency ω_c for the delocalisation has not been achieved as we did here. Our finding of $\sigma(\omega) \sim \omega^2$ for the weakly localised quantum wire (similar to the behaviour for a strongly localised strictly 1D system) suggests that the $\sigma(\omega) \sim \omega^2$ behaviour should not be used as a criteria for judging whether the system is strongly localised. Instead, one may use the clear different temperature dependence of the weakly localised ($\sigma \sim T^{-n}$, where n depends on the inelastic scattering mechanism) and the strongly localised ($\sigma \sim \exp(aT^{-n})$) systems. We note that existing experimental data (Skocpol 1988, Hiramoto *et al* 1989) for the lightly doped semiconductor quantum wires, indicate a power law dependence for the conductance of these systems, in support of the weak localisation description. Unfortunately, to our knowledge, no AC conductance measurements on those systems have been performed up to now. According to our theory, the $\sigma(\omega)$ measurement will give a clue to the dynamics of the CBS process.

In the hope of stimulating experimental work, in figure 3, we have plotted the conductivity $\sigma(\omega)$ calculated by (1) and (5) in the whole range of ω for different values of α and τ_ϕ/τ. We note that near $\omega\tau \sim 1$, (5), is no longer a very good approximation and the memory function $\mu(\omega)$ in (1) is expected to have a more complicated form. For this reason, the results for $\sigma(\omega)$ near $\omega\tau \sim 1$ are plotted in figure 3 by broken lines. As can be seen from that figure, the main features of $\sigma(\omega)$ for our quantum wire are: (i) there exists a critical frequency $\omega_c = (\alpha v_F/(\pi + \alpha)L_\phi\tau)^{1/2}$ below which the system is localised; (ii) when $\omega \geqslant \omega_c$, the system is delocalised and $\sigma(\omega) \sim \omega^2$, (iii) at $\omega\tau \gg 1$, $\sigma \sim \omega^{-2}$, i.e. it shows the classical Drude behaviour.

5. Summary

In summary, we have presented a new (sudden reversal) picture of the weak localisation theory for lightly doped semiconductor quantum wires, which have a cross section

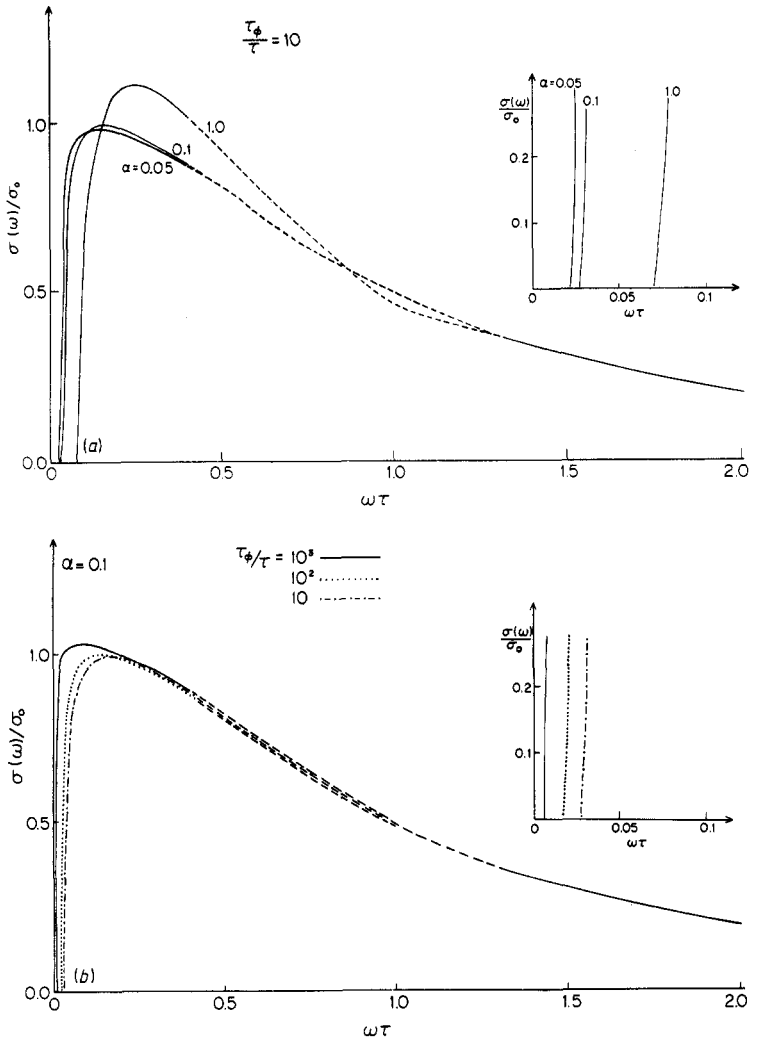

Figure 3. The conductivity (in units of $\sigma_0 = ne^2\tau/m$) as a function of frequency (in units of the inverse of the momentum relaxation time (τ)) for a quantum wire in the sudden reversal picture evaluated by (1) and (5): (*a*) at three different inverse cross section parameters α = 0.05, 0.1, 1; (*b*) at three different inelastic scattering times $\tau_\phi/\tau = 10, 10^2, 10^3$. Broken lines centred near $\omega\tau \sim 1$ indicate our formalism is not fully applicable in this region.

comparable to the Fermi wavelength. The electronic motion in our picture is essentially one-dimensional and the localisation length of the system is much larger than the mean free path. This physical picture survives even when multi-subbands are involved. In the static limit, our result (6) is consistent with that of the diffusive picture in the 1D case ($\alpha = 1$) as well as in the effective 1D case ($\alpha \sim W^{-1}$). Our theory explains well the recent experiments of Hiramoto *et al* who found temperature independent transport behaviour at very low temperature for narrow AlGaAs/GaAs quantum wire, and has a very good fit to the conductivity data extracted from their experiments. In the calculation of the dynamical conductivity, our result (12) shows that for the quantum wire systems we

studied, there is a critical value of the frequency above which the system is delocalised and the AC conductivity rises as ω^2. We hope our study will stimulate more experimental interests in the study of the frequency dependence of the conductivity of the lightly doped semiconductor quantum wire.

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