1985

Cyclotron Radiation From Magnetic Cataclysmic Variables (Polarization, Plasmas, Magnetized, Stars, Herculis, Puppis).

Paul Everett Barrett

Louisiana State University and Agricultural & Mechanical College

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CYCLOTRON RADIATION FROM
MAGNETIC CATAclySMIC VARIABLES

A Dissertation
Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy
in
The Department of Physics and Astronomy

by
Paul E. Barrett
B.S., Georgia Institute of Technology, 1979
May 1985
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V. Conclusion

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<th>Description</th>
<th>Symbol</th>
<th>Description</th>
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</thead>
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<tr>
<td>A</td>
<td>area of emitting region</td>
<td>V</td>
<td>circular polarization intensity</td>
</tr>
<tr>
<td>B</td>
<td>magnetic field (induction)</td>
<td>W_s</td>
<td>complex plasma dispersion function</td>
</tr>
<tr>
<td>D</td>
<td>electric induction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>electric field</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F_s</td>
<td>imaginary part of complex dispersion function</td>
<td>a</td>
<td>polarization coefficient</td>
</tr>
<tr>
<td>G</td>
<td>gravitational constant</td>
<td>c</td>
<td>speed of light</td>
</tr>
<tr>
<td>H</td>
<td>magnetic field</td>
<td>e</td>
<td>electric charge</td>
</tr>
<tr>
<td>H_s</td>
<td>real part of complex dispersion function</td>
<td>f</td>
<td>polar cap fraction</td>
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<tr>
<td>I</td>
<td>intensity</td>
<td>g</td>
<td>distribution function</td>
</tr>
<tr>
<td>I_s</td>
<td>modified Bessel function of integer order s</td>
<td>h</td>
<td>postshock height</td>
</tr>
<tr>
<td>J</td>
<td>emissivity</td>
<td>h</td>
<td></td>
</tr>
<tr>
<td>J_s</td>
<td>Bessel function of integer order s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>luminosity</td>
<td>m</td>
<td>mass</td>
</tr>
<tr>
<td>M</td>
<td>mass of star</td>
<td>m_v</td>
<td>stellar magnitude in V band</td>
</tr>
<tr>
<td>N</td>
<td>number density</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>period</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>linear polarization intensity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>radius of star</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>polarization intensity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td></td>
<td></td>
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</table>

**Symbols:**
- A: area of emitting region
- B: magnetic field (induction)
- D: electric induction
- E: electric field
- F_s: imaginary part of complex dispersion function
- G: gravitational constant
- H: magnetic field
- H_s: real part of complex dispersion function
- I: intensity
- I_s: modified Bessel function of integer order s
- J: emissivity
- J_s: Bessel function of integer order s
- L: luminosity
- M: mass of star
- N: number density
- P: period
- Q: linear polarization intensity
- R: radius of star
- T: temperature
- U: polarization intensity
- V: circular polarization intensity
- W_s: complex plasma dispersion function
- a: polarization coefficient
- c: speed of light
- e: electric charge
- f: polar cap fraction
- g: distribution function
- h: postshock height
- h: Plank constant
- i: inclination
- j: current density
- k: Boltzmann constant
- k: wave vector
- l: length
- m: mass
- m_v: stellar magnitude in V band
- m: accretion rate
- n: index of refraction
- p: momentum
- q: integer index
- r: radius
- s: harmonic number
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>( t )</td>
<td>time</td>
</tr>
<tr>
<td>( u )</td>
<td>((\omega/\omega_B))</td>
</tr>
<tr>
<td>( v )</td>
<td>velocity</td>
</tr>
<tr>
<td>( w )</td>
<td>((\omega_p^2/\omega^2))</td>
</tr>
<tr>
<td>( z_s )</td>
<td>argument of complex dispersion function</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>Gaunt factor</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>dimensionless plasma parameter ( (= \omega_p^2/\omega_B c))</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>summation</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>absorption coefficient</td>
</tr>
<tr>
<td>( \beta )</td>
<td>(v/c)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Lorentz factor ( \left[= (1 - \beta^2)^{1/2}\right])</td>
</tr>
<tr>
<td>( \delta_{ij} )</td>
<td>Kronecker delta ( (= 1; \text{for } i = j; = 0; \text{for } i \neq j))</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>wavelength</td>
</tr>
<tr>
<td>( \rho )</td>
<td>mass density</td>
</tr>
<tr>
<td>( \mu )</td>
<td>mean molecular particle number</td>
</tr>
<tr>
<td>( \nu )</td>
<td>frequency</td>
</tr>
<tr>
<td>( \sigma_{ij} )</td>
<td>conductivity tensor</td>
</tr>
<tr>
<td>( \sigma_T )</td>
<td>Thomson scattering cross section</td>
</tr>
<tr>
<td>( \tau )</td>
<td>optical depth</td>
</tr>
<tr>
<td>( \chi )</td>
<td>argument of Bessel function</td>
</tr>
<tr>
<td>( \phi )</td>
<td>orbital phase</td>
</tr>
<tr>
<td>( \omega )</td>
<td>frequency</td>
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<tr>
<td>( \varepsilon_{ij} )</td>
<td>complex dielectric tensor</td>
</tr>
<tr>
<td>( (\varepsilon_0)_{ij} )</td>
<td>dielectric permittivity tensor</td>
</tr>
<tr>
<td>( \eta )</td>
<td>(mc^2/kT)</td>
</tr>
<tr>
<td>( \theta )</td>
<td>angle between ( k ) and ( B )</td>
</tr>
<tr>
<td>( )</td>
<td></td>
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ABSTRACT

The absorption coefficient for the ordinary and extraordinary modes of wave propagation are calculated for cyclotron radiation from hot magnetized plasmas ($kT < 50$ keV). Two relativistic methods are used to calculate the absorption coefficients: the dielectric formulation and the single particle formulation. A nonrelativistic approximation which includes the effects of inverse-bremsstrahlung and Thomson scattering (collisions) is also made. The equations of radiative transfer for a homogeneous plasma, with large Faraday rotation, are solved, and simple analytic expressions for the Stokes parameters $Q$ and $V$ are derived in terms of the optical depths in both modes. The results are applied to the accretion columns of AM Herculis binaries.

The inclusion of collisional effects in the nonrelativistic approximation reduces the amount of fractionally polarized light to levels which agree better with the observations. For small viewing angles with the magnetic field, the circular polarization does not approach 100% as is observed in the relativistic calculations without the effects of collisions. The polarization approaches a value much less than 100%. This result may provide a qualitative explanation of the standstill which is observed in some AM Herculis binaries.

Comparisons of theoretical and observational circular polarization curves for AM Herculis give surprisingly good agreement for a magnetic field of $2.7 \times 10^7$ gauss, temperature of 0.2 keV, and plasma slab thickness of $2.6 \times 10^8$ cm.

The detection of cyclotron lines in the optical spectrum is limited to a small parameter space in magnetic fields $(2-10) \times 10^7$ gauss,
plasma temperature (< 15 keV), and direction of the accretion column (nearly perpendicular to the line of sight for extended periods of time; ~10 minutes). Theoretical spectra confirm the conclusion by Wickramasinghe and Megyitt (1982) that the broad lines in VV Puppis are due to cyclotron emission, but dispute their conclusion that the addition of an unpolarized component of radiation in the blue and UV spectrum is required. A best fit to the data of VV Puppis yields a polar magnetic field of 3.15 x 10^7 gauss, a postshock temperature of 8.7 keV, and a dimensionless plasma parameter of ~10^6.
CHAPTER 1

INTRODUCTION
I. HISTORICAL SYNOPSIS

Novae are the oldest and most familiar members of a class of stars called cataclysmic variables (CVs). Today, CVs are divided into many subclasses: novae or classical novae, recurrent novae, dwarf novae, novalike CVs, magnetic CVs, and some symbiotic and Mira variables. The classification of a CV depends on its outburst properties (See Table 1.1; Robinson 1983). Classical novae, by definition, have been seen only once during outburst with an increase in brightness of 9-15 magnitudes. The recurrent novae - as the name suggests - have been seen more than once. The period of recurrence is usually 10-100 years with an increase in brightness less than that of the classical novae (5-9 mag). The dwarf novae have the shortest recurrence times (one week to several months) and the smallest increase in brightness (2-6 mag). The other subclasses have not been seen during outburst, but their optical characteristics - such as emission lines and short term (~ minutes) variability or flickering - are similar to dwarf novae. Apparently, the change in brightness of these systems, at least for novae, is directly proportional to the length of time between outbursts (Cordova and Mason 1983).

The first major step in the understanding of CVs resulted from the observations of the dwarf nova SS Cygni (Joy 1956) and the novalike CV AE Aquarii (Joy 1954). These stars were identified as spectroscopic binaries. This identification led Crawford and Kraft (1956) to suggest that the properties of CVs are due to mass transfer from a normal companion star to a white dwarf star in a close binary (orbital period $P_{\text{orb}} < 1$ day). The various types of novae may then be a result of the different rates of mass transfer: the classical novae have the lowest
TABLE 1

PROPERTIES OF CATACLYSMIC VARIABLES

<table>
<thead>
<tr>
<th>Subclass</th>
<th>Outburst amplitude (mag)</th>
<th>Recurrence time (yr)</th>
<th>Orbital Period (hr)</th>
<th>Cause of outburst</th>
<th>Type of stars</th>
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<tr>
<td>Classical novae</td>
<td>9 - 14</td>
<td>$10^3$ - $10^5$</td>
<td>3.3-16.4</td>
<td>Thermo-nuclear runaway</td>
<td>Red dwarf</td>
</tr>
<tr>
<td></td>
<td>or greater</td>
<td>seen only once</td>
<td></td>
<td></td>
<td>White dwarf</td>
</tr>
<tr>
<td>Recurrent novae</td>
<td>7 - 9</td>
<td>10 - 100</td>
<td>1.2-227 days</td>
<td>Thermo-nuclear runaway</td>
<td>Red giant</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>or Red dwarf</td>
<td>White dwarf</td>
</tr>
<tr>
<td>Dwarf</td>
<td>2 - 6</td>
<td>0.02 - 3</td>
<td>1.6-14.6</td>
<td>Change in mass rate or disk</td>
<td>Red dwarf</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>White dwarf</td>
</tr>
<tr>
<td>Novalike variables</td>
<td>&lt; 2</td>
<td>Irregular outbursts</td>
<td>3.2-9.9</td>
<td>Change in mass rate or disk</td>
<td>Red dwarf</td>
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<td></td>
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<td>Magnetic</td>
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<td>White dwarf</td>
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<tr>
<td>Polars</td>
<td>&lt; 2</td>
<td>Irregular outbursts</td>
<td>1.3-3.6</td>
<td>Change in mass rate or pattern</td>
<td>Red giant</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>White dwarf</td>
</tr>
</tbody>
</table>

Comments: There is an absence of CVs with periods between 2 and 3 hrs and a minimum period of 80 mins.

References: Robinson (1976b); Cordova and Mason (1983); Trimble 1984.
rates and the dwarf novae have the highest rates. Models explaining the mass transfer were finally presented by Warner and Nather (1971) and Smak (1971). Their explanation for the cause of the mass transfer is attributed to the overflow of the Roche lobe of the companion star (see Fig. 1.1). The mass of the companion star has been determined for a few CVs (see Payne-Gaposchkin 1977; Cordova and Mason 1983), and is found to be less than 2 $M_\odot$ with the majority of them less than 1 $M_\odot$ ($M_\odot = 1.99 \times 10^{33}$ y is the mass of the sun). Therefore, the combined low mass ($M_{WD} + M_{LMC} < 3.5 M_\odot$) and short orbital period of these binaries gives an orbital radius $r_{\text{orb}} \sim 10^{11}$ cm. This value is of order of the radius of the low mass companion star ($R_{LMC} \sim 5 \times 10^{10}$ cm). The Roche lobe surface of the white dwarf may then reach the companion star's surface. When this occurs, matter is transferred through the inner Lagrangian point into the Roche lobe surrounding the white dwarf.

Due to the large orbital angular momentum imparted to the matter by the companion star, the matter does not fall directly onto the white dwarf. Instead, it forms a ring about 0.4 $r_{\text{orb}}$ from the white dwarf (Robinson 1976). Viscosity then broadens the ring into a disk (see Fig. 1.1) and heats the disk to temperatures of $\sim 3000$ to $100,000$ K (Patterson 1984). The luminosities of the disk and "hot spot" (the area where the mass stream from the companion star intersects the disk) usually exceed the luminosities of both stars in the visible spectrum. Occasionally, the luminosity of the companion star dominates at red and infrared wavelengths and the luminosity from the inner disk may dominate at far ultraviolet wavelengths (Cordova and Mason 1983).

The advent of X-ray astronomy (Giacconi et al. 1971) produced
Fig. 1.1.—Illustration (drawn to scale) of a cataclysmic variable with an accretion disk. It has been suggested that the accretion stream intercepts the accretion disk and forms a "hot spot". The dashed line circling the white dwarf is the Roche lobe potential which has the value of the gravitational potential at the inner Lagrangian point.
another major step in the understanding of CVs. An ultrasoft ($h\nu < 0.2\text{ keV}$) X-ray source was identified with - again - the dwarf nova SS Cygni (Kappaport et al. 1974). The suggestion by many people (Saslaw 1968; Warner 1972; Kraft 1972; McClintock 1973) was that dwarf novae may be soft X-ray sources. This suggestion is consistent with the theory that CVs are close binary systems in the process of mass transfer. The soft X-rays are emitted by bremsstrahlung radiation in a high temperature plasma near the white dwarf surface. The electrons and ions acquire large kinetic energies as they fall freely in the strong gravitational potential near the white dwarf. The kinetic energy is then transformed into thermal energy through collisions, heating the plasma to high temperatures and emitting X-rays.

A new subclass, the magnetic CVs, was added to the list of CVs in 1977. Beginning with an analysis of a spectrogram of the star AM Herculis (AM Her), Bond and Tifft (1974) found a very blue continuum with several sharp emission lines and no absorption lines. These characteristics are similar to a type of novalike CV of which U Geminorum is the prototype. They suggested that, in fact, AM Her had been incorrectly classified as a RW Aurigae variable. This suggestion was supported by photometric observations of AM Her by Bery and Duthie (1977). In addition, Bery and Duthie (1977) suggested that AM Her may be the optical candidate for the weak X-ray source 3U 1809+60 (Giacconi et al. 1974) and a soft X-ray source detected by SAS-3 (Hearn, Richardson, and Clark 1976; see Fig. 1.2). Hearn and Richardson (1977) quickly confirmed their suggestion by identifying a common 3.1 hour period (presumably the orbital period; see Fig. 1.3) in 3U 1809+60 and AM Her.
Fig. 1.2.--Plot of positions for the X-ray source 3U 1809+50, as given by: 3U 1809+50 (Giacconi et al. 1974); Ariel-5 (Ricketts 1976); and SAS-3 (Hearn, Richardson, and Clark 1976) (2-sigma errors). The position of AM Her is also shown. From Hearn, Richardson and Clark 1976.
Fig. 1.3.--The average X-ray light curve of AM Her, in the energy range 0.1-0.3 keV. The same data has been plotted for two cycles. Zero phase is chosen to be that of maximum linear optical polarization (Tapia 1977). A rate of one count per second corresponds to $\sim 1.1 \times 10^{-16}$ ergs cm$^{-2}$ s$^{-1}$ in this energy range. Approximately twelve 3.1 hr cycles were observed. Phase 0.05-0.10 received relatively little exposure. From Hearn and Richardson 1977.
Contemporaneous with the SAS-3 observations, Tapia (1977) observed AM Her in the visible spectrum and found strong linear and circular polarization (~10%; see Figs. 1.4 and 1.5). The strong polarization suggested the presence of cyclotron radiation from hot electrons in a magnetic field $B \sim 2 \times 10^8$ gauss (Ingham, Brecher, and Wasserman 1976). Krzeminski and Serkowski (1977) observed AN Ursae Majoris (AN UMa) and found strong circular polarization (as much as -35%; see Figs. 1.6 and 1.7). The similarity of AN UMa to AM Her prompted Krzeminski and Serkowski to propose that this new type of object be called a "polar", because of the strong polarization.

Many models were presented for AM Her (Szkody and Brownlee 1977; Crampton and Cowley 1977; Fabian et al. 1977; Channugam and Wayner 1977; Priedhorsky and Krzeminski 1978). Channugam and Wayner (1977) argued that if $B \sim 10^8$ gauss, then the magnetospheric radius $r_A$ is $\sim 10^{11}$ cm or $r_A > r_{orb}$. Outside the magnetospheric surface, the flow of matter can be spherical, radial, etc., but inside, the matter is channelled along the magnetic field lines. Therefore, Channugam and Wayner (1977) proposed that the formation of an accretion disk in AM Her may not be possible. Instead, the matter forms an accretion column above one or both magnetic poles (see Fig. 1.8). In addition, the strong magnetic field interacts with the companion star forcing the white dwarf to rotate synchronously (Joss et al. 1979; Channugam and Dulk 1983; Lamb et al. 1983; Campbell 1983; see section II of this chapter for a more detailed discussion of this model).

Theoretical studies of cyclotron radiation from hot magnetized plasmas by Channugam and Dulk (1981) and Meggitt and Wickramasinghe (1982) have been presented. Applications of these calculations to the
Fig. 1.4.—Linear polarization observations (upper) and photometry (lower) of AM Her obtained on 1976 August 16 (JD 2,443,006.5). Small and large symbols indicate sample times of 30 and 60 s, respectively. Dots and circles represent the V and U bands, respectively. One standard deviation error bars have been added to the polarization observed in the V band. From Tapia 1977.
Fig. 1.5.—Circular polarization of AM Her observed in the V band. The data, obtained on 1976 September 17 (dots) and 18 (bars), have been plotted in terms of the phase of the linear polarization pulse. Vertical and horizontal bars represent measurements made with the polarimeter at two perpendicular orientations. Error bars typical of two standard deviations are also indicated. From Tapia 1977.
Fig. 1.6.--The normalized Stokes parameters describing the linear polarization of AN UMa in the blue light. Each symbol is based a 1 min integration. From Krzeminski and Serkowski 1977.
Fig. 1.7.--Circular polarization of AN UMa in the U, B, and V bands as a function of phase from the maxima of linear polarization. From Krzeminski and Serkowski 1977.
Fig. 1.8.—Illustration (drawn to scale) of a cataclysmic variable with an accretion column. The magnetic field of the white dwarf is sufficiently strong to prevent the formation of an accretion disk. Instead, an accretion column forms. A standing shock(s) forms at the magnetic pole(s) of the white dwarf. The dashed line represents the Roche lobe potential (see Fig. 1.1). The dot-dashed lines represent the lines of force of a magnetic dipole which is inclined at an angle $\delta$ to the axis of rotation of the white dwarf.
accretion column are able to explain the linear polarization pulse and the periodic behavior of the circular polarization. The polarized radiation is a result of cyclotron emission at harmonic numbers $s \sim 5-10$ in a plasma at a temperature $kT = 1$ keV (Chanmugam and Dulk 1981) or with $s \sim 15$ and plasma temperature $kT = 20$ keV (Meyyitt and Wickramasinghe 1982). These calculations imply that $B \sim 2 \times 10^7$ gauss, an order of magnitude less than the initial estimate. This result is consistent with the values of $2 \times 10^7$ gauss obtained for AM Her (Schmidt, Stockman, and Margon 1981; Latham, Liebert, and Steiner 1981; Hutchings, Crampton, and Cowley 1981) and CW L103+254 (Schmidt, Stockman and Grandi 1983) using Zeeman spectroscopy and of $3 \times 10^7$ gauss for VV Puppis (VV Pup) from observations of optical cyclotron lines (Visvanathan and Wickramasinghe 1979; Wickramasinghe and Meyyitt 1982).

The major problem with these earlier cyclotron calculations is that the models predict, under certain conditions, fractional polarization of $\sim 100\%$, whereas the observed circular polarization from AM Her binaries is less than $\sim 40\%$ (1114+182, Bierman et al. 1982; AN UMa, Krzeminski and Serkowski 1977).

In this thesis, improvements are made to these earlier calculations in order to take account of bremsstrahlung and Thomson scattering which were previously ignored. In addition, of the ten known AM Her binaries (see Table 1.2), only one, VV Pup, on a rare occasion has exhibited cyclotron lines in its spectrum. Calculations are made to determine the conditions under which distinct cyclotron lines are observable in these binaries (Visvanathan and Wickramasinghe 1979; Meyyitt and Wickramasinghe 1982).
<table>
<thead>
<tr>
<th>STAR (X-ray source)</th>
<th>P (min)</th>
<th>m1/low (%)</th>
<th>U/I</th>
<th>V/I</th>
<th>W/I (oey)</th>
<th>I (oey)</th>
<th>B (mb)</th>
<th>Comments/References</th>
</tr>
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<tr>
<td>EF Eridani (ZA 0311-227)</td>
<td>61.02</td>
<td>14-15</td>
<td>9</td>
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<td>70</td>
<td>7.5</td>
<td>X-rays</td>
<td>1, 2, 3</td>
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<td>E114+142</td>
<td>89.80</td>
<td>17-21</td>
<td>+10/-35</td>
<td>X-rays</td>
<td>4</td>
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<tr>
<td>WV Puppis (0612-169)</td>
<td>100.44</td>
<td>14-16</td>
<td>15</td>
<td>+10/-4</td>
<td>75</td>
<td>145</td>
<td>M4-5 comp.</td>
<td>10,11,14</td>
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<tr>
<td>E140S-451</td>
<td>101.52</td>
<td>15-16</td>
<td>10</td>
<td>0/-30</td>
<td>X-rays</td>
<td>10,11,14</td>
<td></td>
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<tr>
<td>PG1550+191</td>
<td>113.68</td>
<td>15-16</td>
<td>5</td>
<td>N/A</td>
<td>0/12</td>
<td>C</td>
<td>X-rays</td>
<td>11,13,14</td>
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<tr>
<td>M139-68</td>
<td>113.68</td>
<td>15-16</td>
<td>20</td>
<td>0.45</td>
<td>10</td>
<td>+15/-9</td>
<td>X-rays</td>
<td>11,13,14</td>
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<tr>
<td>M139-68</td>
<td>113.89</td>
<td>15-17</td>
<td>16</td>
<td>0.7</td>
<td>0/20</td>
<td>75</td>
<td>14d</td>
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</tr>
<tr>
<td>AM Ursae Maioris</td>
<td>114.64</td>
<td>15-17</td>
<td>9</td>
<td>N/A</td>
<td>0/-19</td>
<td>6b</td>
<td>Z</td>
<td>X-rays</td>
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<tr>
<td>AM Herculis (4U 1814+30)</td>
<td>125.46</td>
<td>12-14</td>
<td>7</td>
<td>0.4</td>
<td>3</td>
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<td>X-rays</td>
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<td>E2043+225</td>
<td>222.51</td>
<td>-15</td>
<td>8/0</td>
<td>X-rays</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

References:
1. Bailey et al. 1962
2. Cropper 1965
3. Griffiths et. al. 1979
4. Bierman et. al. 1942
5. Liebert et. al. 1970
6. Visvanathan and Wickramasinghe 1979
7. Liebert and Stockman 1979
8. Barrett and Chamunday 1985
9. Barent and Lamb 1964
10. Patterson et. al. 1963
11. Mason et. al. 1963
12. Tapia 1962
13. Liebert et. al. 1962
14. Schmidt et. al. 1965
15. Agraawal et. al. 1965
16. Picares and Visvanathan 1965
17. Stockman et. al. 1968
18. Schmidt et. al. 1968
19. Krzeminski and Serekowski 1977
20. Liebert et. al. 1982
22. Wickramasinghe, Visvanathan, and Touhy 1964
23. Tapia 1977
24. Pannehosty et. al. 1976
25. Schmidt et. al. 1961
26. Latham et. al. 1961
27. Rothschild et. al. 1980
28. Noeke et. al. 1983

N/A = Not applicable
The model of AM Her binaries has stimulated the idea that polars might be, as the proverbial phrase goes, 'just the tip of the iceberg' for magnetic CVs, and that a large number of CVs with a smaller or larger magnetic field than the field strength in AM Her binaries might also exist. CVs with $B < 10^7$ gauss may contain both an accretion column inside and an accretion disk outside the magnetosphere. The accretion disk would apply torques near the magnetospheric surface forcing the white dwarf to rotate asynchronously (i.e. at the orbital period of the inner edge of the accretion disk). Warner (1983) introduced the classification "intermediate polar" to describe these binaries and suggested that the establishment of their identities might not be as direct as the AM Her binaries. As a result, the method of classifying a CV as an intermediate polar is due to the manifestation of two or more periods (the orbital period, the rotational period of the white dwarf, and usually, the beat period between the two) in the observed light curve of CVs.

The polars and intermediate polars combine to form the subclass, magnetic CVs. Recently, an objection was made against the classification "intermediate polar" (Patterson 1984). First, no intermediate polar conclusively shows any polarization. Therefore, they are not polars in the strict sense of the word. Second, there is only one type of polar. Thus, what is the intermediate polar intermediate to? Therefore, the term intermediate is vague. Because these objections are valid and no better term has gained approval, the terms polar and intermediate polar will not be used in this thesis. Henceforth, the more widely accepted usage of AM Herculis (AM Her) binary (see Table 1.2) and DQ Herculis (DQ Her) binary (see Table 1.3)
<table>
<thead>
<tr>
<th>Star</th>
<th>$P_{\text{orb/amp}}$ ($\text{min}/%$)</th>
<th>$m_V$</th>
<th>$P_{\text{beat/amp}}$ ($\text{min}/%$)</th>
<th>$P_{\text{spin/amp}}$ ($\text{min}/%$)</th>
<th>$P$</th>
<th>$V_p$ ($\text{MG}$)</th>
<th>Comments/References</th>
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<td>EX Hydrae</td>
<td>98.2, 45</td>
<td>13.5</td>
<td>67</td>
<td>4.5x10^{-11}</td>
<td>0.1-6</td>
<td>&gt;M2 comp</td>
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<tr>
<td>1A U729+103</td>
<td>194.2, 8</td>
<td>14.5</td>
<td>15.22</td>
<td>&lt; 2x10^{-10}</td>
<td>0.3-4</td>
<td>1,7</td>
<td></td>
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<tr>
<td>TT Arietis</td>
<td>194.0, 10.6</td>
<td>13.24</td>
<td>12.50</td>
<td>&lt; 5x10^{-11}</td>
<td>0.5-5</td>
<td>1,11,12</td>
<td></td>
</tr>
<tr>
<td>V1223 Sayittart (4U 1849-31)</td>
<td>202.8, 3-15</td>
<td>13.4</td>
<td>13.24</td>
<td>&lt; 5x10^{-11}</td>
<td>0.5-5</td>
<td>1,11,12</td>
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</tr>
<tr>
<td>AO Piscium (H2752-335)</td>
<td>215.5, 10</td>
<td>13.3</td>
<td>14.31</td>
<td>&lt; 2x10^{-10}</td>
<td>0.3-3</td>
<td>1,13,14,15,16,17</td>
<td></td>
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<tr>
<td>H2215-046</td>
<td>241.5, 40</td>
<td>13.5</td>
<td>22.8</td>
<td>&lt; 2x10^{-10}</td>
<td>0.1-3</td>
<td>1,18,19</td>
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<tr>
<td>DU Herculis</td>
<td>278.8, 21</td>
<td>14.6</td>
<td>20.9</td>
<td>&lt; 2x10^{-10}</td>
<td>0.6</td>
<td>1,20,21,22,23</td>
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</tr>
<tr>
<td>TV Columbae (3A 0526-328)</td>
<td>329.2, 6</td>
<td>13.5</td>
<td>5795</td>
<td>&lt; 5x10^{-7}</td>
<td>0.3-4</td>
<td>1,24,25,26,27,28</td>
<td></td>
</tr>
<tr>
<td>V533 Herculis*</td>
<td>403.2, 15.7</td>
<td>13.3</td>
<td>5795</td>
<td>&lt; 5x10^{-7}</td>
<td>0.3-4</td>
<td>1,24,25,26,27,28</td>
<td></td>
</tr>
<tr>
<td>AE Aquarii</td>
<td>592.8, 11.5</td>
<td>11.5</td>
<td>5.45</td>
<td>&lt; 5x10^{-14}</td>
<td>0.06</td>
<td>1,30,31</td>
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<tr>
<td>GK Perseus</td>
<td>2875.7, 13.5</td>
<td>13.5</td>
<td>5.85</td>
<td>&lt; 5x10^{-14}</td>
<td>0.06</td>
<td>1,30,31</td>
<td></td>
</tr>
</tbody>
</table>

Comment:
[V533 Herculis may be a pulsating white dwarf (Robinson and Nathan 1983).]

References:
which are the prototypes of each classification will be used.

An historical perspective of CVs is given by Payne-Gaposchkin (1977). The X-ray characteristics of CVs are discussed by Bradt and McClintock (1983), and Cordova and Mason (1983). Other reviews of CVs are by Robinson (1976), Gallagher and Starrfield (1978), and Livio and Shaviv (1982).
II. PHYSICS OF ACCRETION IN MAGNETIC CVS

In this section, the accretion process will be discussed using simple arguments and assumptions about CVs. First, an estimate of the luminosity of CVs will be found. Next, changes in the geometry of the system will be discussed as a result of the disruption of the accretion disk by a strong magnetic field. Finally, the physical conditions near the surface of the white dwarf due to the formation of a "stand off" shock will be investigated. This brief discussion of accretion in CVs will provide a basis for the more detailed discussions in the succeeding chapters.

The major luminosity component in CVs is due to the accretion process. Specifically, this luminosity is derived from the increase in kinetic energy of the accreted matter as it accelerates in the strong gravitational field of the white dwarf. The existence of other luminosity components, such as (1) thermonuclear reactions - either steady nuclear burning which may be likely in some CVs (see e.g. Imamura 1981) or a thermonuclear runaway which is the most likely mechanism for outbursts of classical novae (see e.g. Schatzman 1949; Kraft 1964), or (2) instability in the ionization structure of the disk which may explain the outbursts of dwarf novae (see e.g. Meyer and Meyer-Hofmeister 1981), will not be discussed in this thesis. Only the accretion process and those modifications of the accretion process which are produced in the presence of a strong magnetic field will be discussed.

The maximum radiant energy released from matter falling onto a compact object (white dwarf, neutron star, or black hole) cannot be
greater than the available gravitational energy:

$$h_a = \frac{GMm}{R},$$

(1.1)

where $G$ is the gravitational constant, $M$ and $R$ are the mass and radius of the compact object, and $m$ is the mass of the accreted material.

Likewise, the total luminosity $L$ cannot be greater than the accretion rate onto the compact object:

$$L = \frac{GM\dot{m}}{R},$$

(1.2)

where $\dot{m}$ is the accretion rate. Inserting values characteristic of an accreting white dwarf, the luminosity

$$L_{WD} = 1.34 \times 10^{33} \left(\frac{M}{M_\odot}\right) \left(\frac{\dot{m}}{10^{16} \text{ yr}^{-1}}\right) \left(\frac{R}{10^9 \text{ cm}}\right)^{-1} \text{ ergs s}^{-1}.$$  

(1.3)

Similarly, the luminosity of an accreting neutron star is approximately $10^3$ greater than a white dwarf, because the neutron star has a much smaller radius ($R_{NS} \sim 10^6 \text{ cm}$).

The radiation pressure may become sufficiently strong to slow and eventually to halt the infall of the accreted matter, thereby setting an upper limit to the accretion luminosity of a compact object. The limit is found by equating the radiation force and the gravitational force on the infalling matter:

$$\frac{\alpha_{\text{r}} L}{4\pi k^2 c} = \frac{GM\dot{m}}{R^2},$$

(1.4)
where $\sigma_T$ is the Thomson scattering cross-section, $c$ the speed of light, and $m_p$ the mass of the proton. The parameter $f = A/4\pi k^2$ is the ratio of the surface area of the emitting region $A$ to the surface of the compact object. This parameter is called the polar cap fraction and is important in the discussion of accretion onto magnetic CVs which will be discussed later. For the present discussion, the emitting region is the entire surface of the compact object, so $f = 1$. This luminosity is called the "Eddington limit" (Eddington 1926):

$$L_{\text{Edd}} = \frac{4\pi c m_p G M f}{\sigma_T},$$

$$= 1.25 \times 10^{38} \left(\frac{M}{M_\odot}\right) (f) \text{ erg s}^{-1}. \quad (1.5)$$

The substitution of equation (1.2) for the luminosity in equation (1.5) gives a maximum accretion rate for CVs:

$$\dot{m} = \frac{4\pi c m_p R f}{\sigma_T},$$

$$= 10^{21} \left(\frac{R}{10^9 \text{ cm}}\right) (f) \text{ g s}^{-1}. \quad (1.6)$$

Thus, an accretion rate of $10^{16} \text{ g s}^{-1}$ (for $f = 1$), found in CVs, is much less than the accretion rate set by the Eddington limit.

All CVs probably have a magnetic field of some strength. Therefore the magnetic CV is defined, not by the presence of a magnetic field in the white dwarf, but by the strength of its magnetic field. When the energy density of the magnetic field is greater than the kinetic energy density of the accreted matter (which is assumed to be fully ionized),
\[
\frac{B^2(r)}{8\pi} > \frac{\rho(r)v^2(r)}{2}, \tag{1.7}
\]

the magnetic field channels the accreting matter along the field lines. If the magnetic field is assumed dipolar \(B(r) = B_0(R_W/r)^3\) and the mass density \(\rho\) and velocity \(v\) take their free fall values \(\rho_{ff} = \frac{L_{WD}}{\pi GM v_{ff}^2}, v_{ff} = (2GM/r)^{1/2}\), then the radius at which the energy densities are equal is called the Alfven radius \(r_A\) or the magnetospheric radius and is given approximately by (Lamb, Pethick, and Pines 1973; Chanmugam and Wagner 1977):

\[
r_A = \left(\frac{G}{32}\right)^{1/7} B_0^{4/7} L^{-2/7} M^{1/7} M_{\odot} R^{10/7},
\]

\[
= 4 \times 10^{11} \left(\frac{10^5 \text{ gauss}}{10^3 \text{ ergs s}^{-1}}\right)^{4/7} \left(\frac{L}{10^{33} \text{ ergs s}^{-1}}\right)^{-2/7}
\]

\[
\times \left(\frac{M}{M_{\odot}}\right)^{1/7} \left(\frac{R}{10^9 \text{ cm}}\right)^{10/7} \text{ cm.} \tag{1.8}
\]

In this thesis, the condition \(r_A > R_{WD}\) defines a magnetic CV. For a white dwarf, the minimum magnetic field strength \(B_{\text{min}}\) necessary to satisfy this condition is found by setting \(r_A = R_{WD}\),

\[
B_{\text{min}} = \left(\frac{G}{32}\right)^{-1/4} L^{1/2} M^{1/4} R^{-3/4},
\]

\[
= 3.4 \times 10^3 \left(\frac{L}{10^{33} \text{ ergs s}^{-1}}\right)^{1/2} \left(\frac{M}{M_{\odot}}\right)^{1/4} \left(\frac{R}{10^9 \text{ cm}}\right)^{-3/4} \text{ gauss.} \tag{1.9}
\]

Because the matter must flow along the magnetic lines of force, the
accreting matter reaches the surface of the white dwarf at the magnetic poles. If \( r_A > > R_{\text{WD}} \), then the direction of the magnetic field at the poles is almost perpendicular to the surface of the star. Therefore, the mass flow near the surface (\( r < 2 R_{\text{WD}} \)) is to a good approximation radial and forms an accretion column. The ratio of the cross-sectional area of the accretion column at the stellar surface to the area of the stellar surface is the polar cap fraction and is approximately (Kiny and Lasota 1980):

\[
f = \frac{R_{\text{WD}}}{r_A} = \left(\frac{G}{32}\right)^{-1/7} B^{-4/7} L^{2/7} M^{-1/7} R^{-3/7},
\]

\[
= 2.5 \times 10^{-3} \left(\frac{B}{10^6 \text{ gauss}}\right)^{-4/7} \left(\frac{L}{10^{33} \text{ ergs s}^{-1}}\right)^{2/7}
\times \left(\frac{M}{M_\odot}\right)^{-1/7} \left(\frac{R}{10^9 \text{ cm}}\right)^{-3/7}.
\]

If all the radiation is emitted at the polar cap, then the Eddington luminosity for magnetic CVs is

\[
L_{\text{Edd}} = 3.13 \times 10^{35} \left(\frac{M}{M_\odot}\right) \left(\frac{f}{2.5 \times 10^{-3}}\right) \text{ ergs s}^{-1}.
\]

This luminosity is much less than the Eddington luminosity for nonmagnetic CVs, but the luminosity is still greater than the observed luminosities.

For the condition of radial accretion, the formation of a "stand off" shock (Aizu 1973; Hoshi 1973; see Fig. 1.9) above the magnetic pole is expected with the postshock conditions given by the Rankine-Hugoniot strong shock jump conditions (Chu and Gross 1969):
Fig. 1.9.--Illustration of the postshock region at the magnetic pole of the white dwarf. A standing shock forms above the surface of the white dwarf heating the accreted matter. The hot magnetized plasma emits cyclotron radiation in the optical spectrum and bremsstrahlung in the hard X-ray spectrum.
\[ kT_1 \rightarrow kT_2 = (3/16)\mu m_p v_f^2, \quad (1.12a) \]

\[ \rho_1 \rightarrow \rho_2 = 4\rho_1 = \frac{\dot{m}}{\pi v_{ff} R^2f}, \quad (1.12b) \]

\[ v_1 \rightarrow v_2 = \frac{v_f}{4}, \quad (1.12c) \]

where subscripts 1 and 2 indicate preshock and postshock parameters, respectively, and \( \mu \) is the mean molecular particle number (\( \mu = 1/2 \), for an electron-proton plasma). Therefore, the expected temperature and number density \( N_2 \) (\( = \rho_2/m_p f \)) of the postshock for magnetic CVs using equations (1.12a) and (1.12b) are:

\[ kT_2 = 26.1 \left( \frac{M}{M_\odot} \right) \left( \frac{R}{10^9 \text{ cm}} \right)^{-1} \text{ keV}, \quad (1.13) \]

and

\[ N_2 = \frac{1}{\pi} (2G)^{-1/2} \dot{m}^{-1/2} M^{-1/2} R^{-3/2} f^{-1}, \]

\[ = 4 \times 10^{15} \left( \frac{\dot{m}}{10^{15} \text{ g s}^{-1}} \right) \left( \frac{M}{M_\odot} \right)^{-1/2} \left( \frac{R}{10^9 \text{ cm}} \right)^{-3/2} \left( \frac{f}{2.5 \times 10^{-3}} \right)^{-1} \text{ cm}^{-3}. \quad (1.14) \]

The temperature of the postshock is much greater than the ionization energy of hydrogen (\( kT_{\text{ion}} = 13.6 \text{ eV} \)). Hence, the plasma which is composed of electrons and protons is considered completely ionized.

The steady-state height of the postshock \( h \) (see Langer, Chamuyam, and Shaviv 1982 for time dependent oscillations of the postshock height)
is found by taking the product of the postshock velocity and the cooling time \( t_{\text{cool}} \) of the plasma \( h = v_{\text{ff}} t_{\text{cool}} / 4 \). This means that the kinetic energy acquired during accretion must be radiated away by the time the matter reaches the white dwarf surface. If bremsstrahlung emission with emissivity

\[
J(T) = 2.4 \times 10^{-27} \, T^{1/2} N^2 \, \text{ergs cm}^{-3} \, \text{s}^{-1}, \tag{1.15}
\]

is the dominant radiative process in the postshock region, where the bremsstrahlung cooling time (Tucker 1975)

\[
t_{\text{brem}} = \frac{3NkT}{J(T)},
\]

\[
= 0.195 \left( \frac{kT}{10 \, \text{keV}} \right)^{1/2} \left( \frac{N}{10^{16} \, \text{cm}^{-3}} \right)^{-1} \, \text{s}, \tag{1.16}
\]

then the height of the postshock is:

\[
h_{\text{brem}} = 2.5 \times 10^7 \left( \frac{kT}{10 \, \text{keV}} \right)^{1/2} \left( \frac{N}{10^{16} \, \text{cm}^{-3}} \right)^{-1} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{R}{10^9 \, \text{cm}} \right)^{-1/2} \, \text{cm}. \tag{1.17}
\]

On the other hand, if cyclotron radiation is the dominant radiative process, then the height of the postshock is less than the the bremsstrahlung postshock height because the cyclotron cooling time is much less than the bremsstrahlung cooling time.

To estimate the conditions at which cyclotron and bremsstrahlung emission are equal, \( L_{\text{Cyc}} = L_{\text{brem}} = GM_{\text{WD}} m / 2 R_{\text{WD}} \), the cyclotron luminosity is approximated by a Rayleigh-Jeans spectrum of frequency \( \omega^* \):
where $\omega^*$ is the frequency at which the optical depth $\tau \sim 1$. The spectrum is then truncated above $\omega^*$ because the plasma becomes optically thin. The results of detailed numerical calculations by Lamb and Masters (1979) are presented in Figure 1.10 for various scaled luminosities ($L/f$) and magnetic fields.

Bremsstrahlung usually dominates in the UV and X-ray parts of the spectrum (see Masters 1978; Imamura 1981; see Fig. 1.11), while cyclotron emission is dominant in the infrared, optical, and, possibly, near-UV parts of the spectrum. The concern of this thesis is a comparison of theoretical models and observations of cyclotron radiation in the optical spectrum of magnetic CVs.
Fig. 1.10-- This diagram is relevant to spherically symmetric accretion at rate $\dot{m}$ g s$^{-1}$ onto a magnetic white dwarf of mass 1 M$_{\odot}$, radius $5 \times 10^8$ cm and surface field $B$ gauss. The line on which bremsstrahlung and cyclotron emission are comparable is drawn for various values of $\omega^*/\omega_B$, the cyclotron harmonic above which the emitting region is optically thin. It is expected $120 > \omega^*/\omega_B > 25$. The dashed line derived (erroneously) by Fabian et al. 1976, is also shown. In practice, $\omega^*/\omega_B$ will be a slowly decreasing function of $B$ (for a given value of $\dot{m}$). It is therefore expected the true curve to be (like that in Fabian et al. 1976) slightly less steep than the lines drawn here for constant $\omega^*/\omega_B$. From Masters et al. 1977.
Fig. 1.11.—Theoretical spectrum of an AM Her binary of $1 \, M_\odot$ and $B = 5 \times 10^7$ gauss at a normalized luminosities of $L/f = 10^{37}$ and $10^{35}$. The abbreviations CYC, BB, and BREMS represent cyclotron, blackbody, and bremsstrahlung radiation in the optical, UV, and X-ray band, respectively. From Lamb and Masters 1979.
CHAPTER 2

RADIATION FROM HOT MAGNETIZED PLASMAS
I. CHARACTERISTICS OF A HOT MAGNETIZED PLASMA

a) Introduction

The propagation of radiation in a fully ionized homogeneous plasma without an external magnetic field is isotropic (i.e. the radiation does not have a preferred direction of propagation). By introducing a magnetic field into the plasma, the isotropy of the plasma is destroyed (Ginzburg 1970). The anisotropic plasma interacts with the radiation propagating perpendicular to the magnetic field differently than with the radiation propagating parallel to the magnetic field, and is therefore called a magnetoactive plasma. The magnetic field also causes the charged particles to gyrate around the magnetic field (see Fig. 2.1). Since the charged particles are being accelerated, they must emit radiation which is called cyclotron, gyrosynchrotron, or synchrotron radiation. This results in the emitted radiation from the plasma being frequency dependent (cyclotron lines), angle dependent (beaming with respect to the magnetic field), and, possibly, polarized (linear and circular polarization). These properties of the emitted radiation as they apply to magnetic CVs will be discussed in Chapters 3 and 4.

In this section, the properties and particle behavior of hot magnetized plasmas are discussed. A $T - B$ (temperature - magnetic field) plane is used to define and delineate the classical and quantum mechanical domains of cyclotron, gyrosynchrotron, and synchrotron radiation. This section ends with a discussion of the effects of collisions on radiative processes (absorption, emission, scattering) in hot magnetoactive plasmas.
Fig. 2.1--An electron with velocity $v$ gyrating in a constant magnetic field $B$ which is parallel to the $z$-axis. The position of the electron makes an angle $\phi$ with the $x$-axis.
A fundamental parameter which is used to describe plasmas is the Debye length $\lambda_D \equiv (kT/4\pi Ne^2)^{1/2}$ (e.g. see Ginzburg 1970; Ichimaru 1973). This parameter is the effective screening length for a singly charged particle in the plasma. For distances nearer than $\lambda_D$ the effective potential of the charged particle is approximately that of a bare charge. If the distance is farther than $\lambda_D$, then the effective potential decreases exponentially (i.e. the charged particle is affected by other charged particles only within a few $\lambda_D$). Consequently, a disturbance to the equilibrium state of the plasma will cause the charged particles to react either collectively or individually depending on the characteristic wavelength of the disturbance. More specifically, an electromagnetic wave will interact individually with each particle when the wavelength $\lambda < \lambda_D$. If $\lambda \gtrsim \lambda_D$, the particles will interact collectively with the wave. Therefore, a plasma may exhibit either collective or individual particle behavior.

In the postshock of magnetic CVs, the electron number density $N_e \sim 10^{16}$ cm$^{-3}$ and $kT \sim 10$ keV (p. 24), thus:

$$\lambda_D \approx 7 \frac{(kT/10 \text{ keV})^{1/2}(N_e/10^{16} \text{ cm}^{-3})^{-1/2}}{\text{\mu m}}. \tag{2.1}$$

This length is much larger than the wavelength of optical radiation $\lambda = 0.4$-$0.8$ microns. Therefore, the plasma will exhibit individual particle behavior at these wavelengths. If the wavelength of the radiation is increased to $\lambda \sim 7$ microns (the far infrared), then $\lambda = \lambda_D$ and the charged particles will behave collectively. Collective particle behavior will also become important if $\lambda = 0.7$ microns and the conditions in the postshock are such that $\lambda_D = 0.7$ microns ($kT < 0.1$
keV, or \( N_2 > 10^{18} \, \text{cm}^{-3} \)). When the wave interacts collectively with the charged particles in a magnetized plasma, the different types of waves that may propagate are acoustic waves, low-frequency waves, and hydromagnetic waves (e.g. see Ginzburg 1970; Ichimaru 1973). If a magnetized plasma exhibits individual particle behavior with an electromagnetic wave \( (\omega > 2\pi c/\lambda_D) \), and has a sufficiently weak magnetic field so that \( \omega >> \Omega_B \), where \( \Omega_B = (eB/m_p c) \) is the ion cyclotron frequency, then only one type of wave, a high-frequency wave (Ginzbury 1970), will propagate in the plasma.

In this thesis, the discussion of waves propagating in a magnetically active plasma will henceforth be limited to the frequency range: \( \frac{1}{2} \omega_B < \omega < 25\omega_B \) (Fig. 2.2), and thus to high frequency waves because \( \omega \sim \omega_B >> \Omega_B \) where \( \omega_B = (eB/m_e c) = (m_p/m_e)\Omega_B \) is the electron cyclotron frequency. These frequencies are practical limits. The lower frequency \( ((1/2)\omega_B) \) is chosen so that the cyclotron spectrum includes the entire absorption profile of the fundamental cyclotron harmonic. The upper frequency \( (25\omega_B) \) is chosen because that frequency is roughly the harmonic at which the cyclotron absorption is approximately equal to the absorption from collisions (i.e. inverse bremsstrahlung and Thomson scattering, which dominates bremsstrahlung at temperatures and densities found in the postshock). An actual lower frequency limit (Pawsey and Bracewell 1955; Ginzburg 1970; Ramaty 1969) using the cold plasma approximation (see p. 62) is \( (\omega_{uh})_0 = \omega_p \) for the ordinary mode and \( (\omega_{uh})_x = (1/2)\omega_B + ((1/4)\omega_B^2)^{1/2} \) for the extraordinary mode of wave propagation (Fig. 2.3), where \( \omega_p^2 = 4\pi Ne^2/m \) is the plasma frequency. These frequencies are called the upper-hybrid frequencies and are the upper limits to the "stop band", a frequency band where wave propagation...
Fig. 2.2.—Spectrum of cyclotron absorption coefficient (solid line) at 50 keV and $\theta = 60^\circ$. The dashed line represents the bremsstrahlung absorption coefficient at 50 keV. The steeper slope of the cyclotron absorption coefficient as compared to the bremsstrahlung absorption coefficient is evident. The absorption coefficients are in units of $\omega_p^2/\omega_B$. 
Fig. 2.3 -- $\omega$ - $k$ diagrams of a cold magnetized plasma indicating the "stop bands" in both (left and right circularly polarized) modes of wave propagation. The functional relationship of $\omega$ and $k$ in a cold magnetized plasma is indicated by the solid lines. The dashed lines indicate this relationship in free-space.
is not allowed. At these frequencies, the index of refraction (i.e. the real part of the complex index of refraction) is zero. In an inhomogeneous plasma, the extraordinary and ordinary modes with frequency \( \omega \) will be reflected at depths where \( \omega = \omega_{\text{nh}} x, 0 \), respectively.

Finally, the absorption of radiation by protons is of order of \( (m_e/m_p)^2 \approx 2.5 \times 10^{-7} \) times the absorption by electrons and is therefore negligible.

b) The T-B Plane

The calculation of radiative processes in a magnetized plasma can be divided into four cases (see Fig. 2.4) depending on the importance of quantum and relativistic effects.

In a quantum mechanical description of radiation, one assumes discrete radiative processes and considers the recoil of the electron during the radiative process. If the energy of the radiation is much less than the energy of the electron \( (h\omega < kT) \), then the recoil of the electron is usually neglected during the radiative process, and the radiative process is assumed to occur continuously. Under these conditions, classical electrodynamics is used. But this criterion \( (h\omega < kT) \) may be invalid, depending on the strength of the magnetic field, when the energy of the electron becomes relativistic (see Fig. 2.4). This occurs when the energy of peak emission of the emitted radiation during one cycle approaches the kinetic energy of the electron. Because radiation is emitted discretely, not continuously, quantum effects in the form of fluctuation excitations of the amplitude of radial
Fig. 2.4.—*T*–*B* diagram indicating the classes of cyclotron radiation. The diagonal line delineates the quantum and classical domains in the nonrelativistic approximation. The conditions of \( kT, h\omega_B = mc^2 \) are indicated by the upper horizontal and rightmost vertical lines, respectively. Above the curved line, a quantum mechanical treatment must be used. The shaded areas locate the many fields of physics and astrophysics where cyclotron radiation is an important radiative process.
oscillations become important (i.e. the recoil of the electron is nonnegligible). The temperature at which this occurs is \( kT > (kT)_\text{th} = mc^2 \left( B_c / B \right)^{1/5} \) (Sokolov and Ternov 1968). The constant \( B_c = (e^2 / c m)^2 \) = 4.4 x 10^{13} \text{ gauss} and is found by equating \( h\omega_B \) and \( mc^2 \). Magnetic fields approaching this magnitude have only been found in neutron stars. For example, Her X-1 has a field \( = 5 \times 10^{12} \text{ gauss} \) (Trumper et al. 1977; Voges et al. 1983).

A second criterion determines whether relativistic effects can be neglected or not. If \( kT \ll mc^2 \) and \( h\omega_B \ll mc^2 \), then a nonrelativistic classical or quantum treatment is acceptable.

i) Quantum Domain

A quantum treatment of radiation must be used when the discrete nature of the interaction of radiation and matter is important (\( h\omega \geq kT \)). In this case, the calculations are most generally made using quantum electrodynamics (QED). When the magnetic field \( B \sim B_c \), new quantum effects become important. The vacuum begins to behave like a medium, or becomes polarized (vacuum polarization). Another quantum effect is the creation of an electron-positron pair from the decay of a photon with energy \( h\omega > 2mc^2 \). Calculations of synchrotron radiation with vacuum polarization and photon decay were made by Daugherty and Ventura (1978) and Bussard (1980) for the neutron star Her X-1.

The conditions for a nonrelativistic quantum treatment are found in metals and semiconductors in a magnetic field \( B \sim 10^4 \text{ gauss}, T \lesssim 10^2 \text{ K} \), e.g. Callaway 1974; Ashcroft and Mermin 1976; Ryu and Choi 1984 for electron-phonon systems), in the photosphere of magnetic white dwarfs (\( B \sim 10^{12} \text{ gauss} \)) for example, Her X-1 has a field \( = 5 \times 10^{12} \text{ gauss} \) (Trumper et al. 1977; Voges et al. 1983).
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\[ \sim 10^8 \text{ gauss}, T \sim 10^4 \text{ K}, \text{ e.g. Kemp 1970; Lamb and Sutherland 1974; Angel 1977; Pavlov, Mitrofanov, and Shibanov 1980; Mitrofanov and Pavlov 1981}, \text{ and possibly, in the photosphere of neutron stars (B} \sim 10^{13} \text{ gauss, T} \sim 10^7 \text{ K), if relativistic effects are negligible (e.g. Mitrofanov and Pavlov 1981)}. \]

ii) Classical Domain

In the classical domain (\( \omega_B \ll kT \)), the radiative process of gyrating electrons in a magnetized plasma is divided into three classes according to the value of the Lorentz factor \( \gamma = (1 - \beta^2)^{1/2} \). The first class which is called cyclotron radiation (or gyroresonance or magnetobremsstrahlung) has \( \gamma \ll 1 \) (\( kT < 0.5 \text{ keV} \)). In this case, relativistic effects can be neglected resulting in a simplification of the calculations without losing the important behavior displayed by the plasma at higher temperatures (e.g. see Stix 1962; Bekefi 1966; Ginzburg 1970; Montgomery and Tidman 1966; Ichimaru 1973). The cyclotron spectrum has sharp, narrow lines, equally spaced in frequency (Fig. 2.5). The strength of these lines decreases very rapidly with increasing harmonic number.

The second class which is called gyrosynchrotron radiation has \( 1 \leq \gamma \leq 2 \) (i.e. the electrons are mildly relativistic, \( 5 \text{ keV} \leq kT \leq 1.5 \text{ MeV}, \text{ Dulk 1985} \)). The calculations in this class are usually very difficult because relativistic effects are important. The harmonic line profiles in the spectrum are broader and the line peak has shifted to lower frequency. The spectrum progresses from a series of broad peaks into a continuum (Fig. 2.5).
Fig. 2.5.--Spectrum of Cyclotron Absorption Coefficients at 0.1 keV, 10.0 keV, and 1.0 MeV (solid lines). The absorption coefficient of Thomson scattering (dashed line) is also shown. It is assumed here to be a true absorption process. Notice the strong dependence of the line width on temperature.
Calculations of gyrosynchrotron radiation were initially made for studies of magnetically confined fusion reactors (Trubnikov 1958; Hirshfield, Baldwin, and Brown 1961; Drummond and Rosenbluth 1963; Bekefi 1966; Tamor 1978) and have found possible applications in solar flare emission (Ramaty 1969; Dulk 1985) and in magnetic CVs (Chamuyam and Dulk 1981; Meggitt and Wickramasinghe 1982).

The third class which is synchrotron radiation has $\gamma \gtrsim 2$ (i.e., the electrons are relativistic, $kT \gtrsim 1.5$ MeV, and ultrarelativistic, $kT \gtrsim 50$ MeV). In this class, approximations can be made for $\gamma \gg 1$, and the calculations become easier and simpler than the gyrosynchrotron class. The spectrum is now completely continuous (see Fig. 2.5). Synchrotron radiation which has derived its name from a class of electron accelerators is suggested to be the radiation mechanism in nonthermal radio sources such as the radio sources around some galaxies and the Crab pulsar (see e.g. Ginzburg and Syrovatskii 1969; Pacholczyk 1977).

In this thesis, two methods are used to calculate radiative processes in magnetized plasmas. In the first method, the emission for a single electron gyrating in a magnetic field (Fig. 2.1) is calculated using the Lienard-Wiechert potentials (see e.g. Jackson 1975). The total emission for the plasma is then found by summing (integrating) the emission from many electrons. We shall follow the lead of Bekefi (1966) and call this the single particle method. The second method uses the Boltzmann equation in conjunction with the Maxwell equations to determine the dielectric tensor $\varepsilon_{ij} i; j = x,y,z$ of the plasma. The rate of absorption of radiation passing through the plasma may then be found from the dielectric tensor. We shall call this second method the dielectric formulation.
Calculations of gyrosynchrotron radiation using both methods are difficult, specifically at low harmonics when the line profile is important. The calculations are normally performed numerically, because a summation of several harmonics is necessary to obtain the total radiation at a frequency. If the line profile is not important, then an integration over harmonic number can be performed instead as suggested by Petrosian (1981). This method leads to good results and the accuracy improves with increasing harmonic number (e.g. Robinson 1984; Robinson and Melrose 1984; Dulk 1985). Another numerical technique using the method of steepest descents has been suggested by Drummond and Rosenbluth (1965) and implemented by De Barbieri (1977).

c) Effects of Collisions

The previous section briefly discussed the important characteristics of emission, absorption, and propagation of radiation in a magnetized plasma. But little mention was given to the importance of electron-ion collisions. The primary importance of collisions is to maintain an equilibrium (or Maxwellian) particle distribution. Hence, the collision frequency will always be assumed sufficiently high to maintain a Maxwellian distribution, whether bremsstrahlung radiation is or is not the dominant radiative process in the postshock (see Chapter 1). If collisions are important, then they can significantly change the shape of the spectrum and the amount of fractional polarization emitted by the postshock. The importance of collisions will be shown in the following paragraph.

The absorption coefficient \( \alpha_{\text{col}} \) due to inverse bremsstrahlung is
given by the simple equation (Pavlov, Mitrofanov, and Shibanov 1980):

\[
\alpha_{\text{col}} = \frac{2^{2}(\nu_{c} + \nu_{r})}{\omega^{2}C},
\]  

(2.2)

where

\[
\nu_{c} = \frac{4}{3} \frac{2\pi^{1/2}}{m_{e}} \frac{N e^{2}}{(kT)^{3/2}} \ln \Gamma, \quad \nu_{r} = \frac{2}{3} \omega^{2}e^{2},
\]

(2.3)

are the effective collision and damping frequencies. For postshock values, the magnitude of the absorption coefficient becomes:

\[
\alpha_{\text{col}} = 6.68 \times 10^{-9} \left(\frac{N}{10^{16} \text{ cm}^{-3}}\right)^{2} \left(\frac{kT}{1 \text{ keV}}\right)^{3/2} \left(\frac{\lambda}{0.5 \text{ \mu m}}\right)^{2} \text{ cm}^{-1},
\]

(2.4)

By comparison, an approximate value of the cyclotron absorption coefficient is (Dulk and Marsh 1982)

\[
\alpha_{\text{cyc}} = 4.8 \times 10^{9} \ T^{7} B^{9} N \omega^{-10} \sin^{6} \theta.
\]

(2.5)

For \( \theta = 60^\circ \), \( kT = 10 \text{ keV} \), \( B = 3 \times 10^{7} \text{ gauss} \), \( N = 10^{16} \text{ cm}^{-3} \), \( \omega = 10^{15} \text{ s}^{-1} \), one finds

\[
\alpha_{\text{cyc}} = 3.8 \times 10^{-9} \left(\frac{kT}{1 \text{ keV}}\right)^{7} \left(\frac{B}{3 \times 10^{7} \text{ gauss}}\right)^{9}
\]

\[
x \left(\frac{N}{10^{16} \text{ cm}^{-3}}\right) \left(\frac{\omega}{10^{15} \text{ s}^{-1}}\right)^{-10} \text{ cm}^{-1}.
\]

(2.6)

Thus the two values are of similar magnitude and hence the effects of collisions can be important.
The expected conditions for collisions to be important are at high harmonics, small $\theta$, and between cyclotron harmonics at low temperatures ($kT \lesssim 5$ keV), where the cyclotron absorption becomes very small (Fig. 2.6).

Another important radiative process is Thomson scattering. The angular, temperature and frequency dependence of Thomson scattering is different from those of bremsstrahlung. Thomson scattering affects the incident radiation in three important ways: (1) It does not change the radiation spectrum, because the frequency of the scattered radiation is considered to have the same frequency as the incident radiation at low radiative energies. (2) The incident radiation is scattered anisotropically, thereby producing an angular distribution which is called the Law of Darkening. (3) The angular distribution of the incident polarized light is changed. Such effects in a very strong magnetic field ($B \sim 10^{13}$ gauss) have been discussed by Ventura (1978). If the effective scattering frequency $\nu_r$ is treated as an effective collision frequency, then a comparison with bremsstrahlung can be made. In Figure 2.6 are shown, those conditions in which Thomson scattering is more important than bremsstrahlung.

In this thesis, the effects of collisions and Thomson scattering are included in the calculation of the nonrelativistic cyclotron absorption coefficient as effective collision and damping frequencies, respectively. For inverse bremsstrahlung, this approximation is satisfactory, but the approximation also assumes that Thomson scattering is a "true" absorption process, which is not strictly correct (Pavlov, Mitrofanov, and Shibanov 1980; Barrett and Channugam 1984; Meyyitt and Wickramasinghe 1985).
Fig. 2.6.—Ne - $\omega/\omega_B$ diagram delineating bremsstrahlung and Thomson scattering dominated domains. The condition where $v_e$ and $v_T$ are equal at 0.1, 1.0, 10, and 30 keV is indicated by the solid lines. Other parameters are the magnetic field $B = 10^7$ gauss, the ionic charge $Z = 1$ (hydrogen), and the Gaunt factor $\Gamma = 10$. 
II. CYCLOTRON ABSORPTION COEFFICIENTS

The cyclotron absorption coefficients play a crucial role in determining the emission from hot magnetized plasmas. In this section, the numerical methods used to evaluate cyclotron absorption coefficients are discussed in detail and the relative accuracy of each method is discussed.

In the single particle model (Liemohn 1965; Ramaty 1969; Trulsen and Fejer 1970; and Channuyam 1980), the emissivity from a distribution of electrons is obtained by integrating the emission from a single electron over the appropriate momentum distribution \( y(p) \). In this thesis, the electron distribution is an isotropic Maxwellian (for discussions of the emissivity from anisotropically, non-Maxwellian electron distributions, see e.g. Ramaty 1969; Petrosian 1981; Dulk and Marsh 1982; Dulk 1985). The two modes of propagation in the plasma are not implicit to the single particle method, but must be included in the calculation by using the polarization coefficients \( a_{x,0} \) and refractive indices \( n_{x,0} \). Therefore, one limitation of the single particle method is in the determination of the polarization coefficients and refractive indices. A second limitation is the implicit assumption that the plasma is tenuous and that therefore plasma dispersion effects are negligible. The polarization coefficients and refractive indices are actually calculated using the dielectric formulation and the cold plasma approximation, but because the plasma is tenuous \( (\omega_p^2/\omega^2 \ll 1) \), these parameters may also be reliably applied at higher temperatures.

The advantage of the dielectric formulation (Bernstein 1958; Stix 1962; Montgomery and Tidman 1964; Ginzburg 1970; Tamor 1978; Pavlov,
Mitrofanov, and Shibanov 1980 to name a few) is the wider range of plasma conditions to which it can be applied. This method is not limited to tenuous plasmas. It can also be applied to dense plasmas ($w \gtrsim 1$) where plasma dispersion effects become important.

The dielectric formulation is used to calculate the absorption coefficient $\alpha_{x,0}$ of each mode by solving the determinant of the dispersion tensor which contains the dielectric tensor $\varepsilon_{ij}$. In principle, the dielectric tensor can completely describe all macroscopic electromagnetic properties of a plasma (Ichimaru 1973) which respond linearly to the electromagnetic field (i.e. $j_i = \sigma_{ij} E_j$). The dielectric tensor is calculated to first order in both the electric field and the electron distribution function using a linearized Boltzmann equation for relativistic electrons.

This section begins with a discussion of the dielectric formulation. One such solution (Pavlov, Mitrofanov, and Shibanov 1980) includes the effects of collisions and Thomson scattering for nonrelativistic electrons by means of an effective collision and damping frequency. The breakdown of this nonrelativistic approximation is also discussed. Finally, this section concludes with the solution of the radiative transfer equations for the Stokes parameters, for the case of large Faraday rotation, in a magnetoactive plasma.

a) Dielectric Formulation

i) The Dielectric Tensor

The propagation of electromagnetic waves through an
anisotropically, dispersive medium must be solved on the basis of the Maxwell equations:

\[ \nabla \times H = \frac{4\pi}{c} J + \frac{i\omega}{c} \mathcal{D}, \quad (2.7a) \]

\[ \nabla \cdot D = 4\pi \rho, \quad (2.7b) \]

\[ \nabla \times E = -\frac{i\omega}{c} H, \quad (2.7c) \]

\[ \nabla \cdot H = 0, \quad (2.7d) \]

where \( E \) and \( H \) are the electric and magnetic fields, \( D \) is the electric induction or displacement current, and \( J \) and \( \rho \), the current and space densities of free charge. This form of the Maxwell equations implicitly assumes that all parameters are harmonic functions \( e^{-i\omega t} \) of time \( t \) and frequency \( \omega \).

One obtains a wave equation for the electric field by first, taking the curl of equation (2.7c), and then making a substitution for the right hand side using equation (2.7a):

\[ \nabla \times (\nabla \times E) - \frac{\omega^2}{c^2} (D - \frac{4\pi i}{\omega} J) = 0. \quad (2.8) \]

This can be rewritten as

\[ \nabla^2 E - \nabla(\nabla \cdot E) + \frac{\omega^2}{c^2} (D - \frac{4\pi i}{\omega} J) = 0, \quad (2.9) \]
using a standard vector identity. If \( \mathbf{D} \) and \( \mathbf{J} \) depend linearly on \( \mathbf{E} \) such that

\[
\mathbf{D}_i = (\varepsilon_0)_{ij} \mathbf{E}_j, \quad \mathbf{J}_i = \sigma_{ij} \mathbf{E}_j; \quad i, j = x, y, z
\]  

(2.10)

where \( \varepsilon_0 \) is the dielectric permittivity tensor and \( \sigma_{ij} \) is the conductivity tensor, then a substitution for \( \mathbf{D}_i \) and \( \mathbf{J}_i \) gives the equation:

\[
\nabla^2 \mathbf{E}_i - \nabla_i (\nabla \times \mathbf{E}) + \frac{\omega^2}{c^2} [(\varepsilon_0)_{ij} - \frac{4\pi i}{\omega} \sigma_{ij}] \mathbf{E}_j = 0.
\]  

(2.11)

The dielectric permittivity tensor and conductivity tensor are usually combined to form the complex dielectric tensor:

\[
\varepsilon_{ij} = (\varepsilon_0)_{ij} - \frac{4\pi i}{\omega} \sigma_{ij} \quad i, j = x, y, z.
\]  

(2.12)

If the monochromatic wave with wave vector \( \mathbf{k} \) is such that \( \mathbf{E} = E_0 \mathbf{e}^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} \) (no spatial dependence), then the wave equation becomes

\[
-k^2 E_i + k_i (k \times \mathbf{E}) + \frac{\omega^2}{c^2} \varepsilon_{ij} E_j = 0.
\]  

(2.13)

A solution to equation (2.13) is easily obtained by making the wave propagate along the z-axis so that \( k_x, k_y = 0 \) (Ginzburg 1970; Akhiezer et al. 1967). Equation (2.13) written in matrix form is:
where \( n = k_z c/\omega \) is the complex index of refraction. By eliminating \( E_z \), the system reduces to a 2x2 matrix in \( E_x, E_y \):

\[
\begin{bmatrix}
-n^2 + \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\
\varepsilon_{yx} & -n^2 + \varepsilon_{yy} & \varepsilon_{yz} \\
\varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz}
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix}
= 0, \quad (2.14)
\]

The requirement for a nontrivial solution (\( E_x, E_y = 0 \)) is that the determinant of the 2x2 matrix must vanish. This gives a solution in \( n^2 \):

\[
n^2 = \frac{1}{2} \left[ \varepsilon_{yy} + \varepsilon_{xx} - (\varepsilon_{xz} \varepsilon_{zx} + \varepsilon_{yz} \varepsilon_{zy})/\varepsilon_{zz} \right]
+ \frac{1}{2} \left[ (\varepsilon_{yy} - \varepsilon_{xx} - (\varepsilon_{yz} \varepsilon_{zy} - \varepsilon_{xz} \varepsilon_{zx})/\varepsilon_{zz})^2
+ 4(\varepsilon_{xy} - \varepsilon_{xz} \varepsilon_{zy}/\varepsilon_{zz})(\varepsilon_{yx} - \varepsilon_{yz} \varepsilon_{zx}/\varepsilon_{zz}) \right]^{1/2}. \quad (2.16)
\]

The absorption coefficient \( \alpha \) in each mode is found from the imaginary part \( n_1 \) of the complex index of refraction:

\[
\alpha_{x,0} = \frac{\omega}{c} (n_1)_{x,0}. \quad (2.17)
\]
In the tenuous plasma approximation (\( w << 1 \)), the complex
dielectric tensor \( \varepsilon_{ij} \) is rewritten as

\[
\varepsilon_{ij} = \delta_{ij} - \frac{4\pi i}{\omega} \sigma_{ij}.
\]  

(2.18)

The term (4\( \pi i/\omega \))\( \sigma_{ij} = O(\omega) \). Therefore, the solution of the determinant
of equation (2.14) is, to first order in \( \omega \),

\[
(1 - \frac{4\pi i}{\omega} \sigma_{zz})[(1 - n^2 - \frac{4\pi i}{\omega} \sigma_{yy})(1 - n^2 - \frac{4\pi i}{\omega} \sigma_{xx}) - (\frac{4\pi i}{\omega})^2 \sigma_{xy} \sigma_{yx}] = 0.
\]  

(2.19)

The term in square brackets is associated with the transverse wave and
the other term is associated with the longitudinal wave. In this
approximation, the longitudinal wave is separated from the transverse
wave, but does not propagate. In general, the longitudinal wave and
transverse wave cannot be separated as was shown in the derivation of
equation (2.16).

A second approximation is called the transverse wave approximation.
This approximation assumes from the outset and without rigorous
justification that the longitudinal component of the electric field \( E_z = 0 \). This assumption reduces the 3x3 matrix (eq. 2.14) to a 2x2 matrix in
\( E_x, E_y \). A solution to the determinant of the 2x2 matrix is:

\[
n^2 = \frac{1}{2}(\varepsilon_{yy} + \varepsilon_{xx}) + \frac{1}{2}[(\varepsilon_{yy} - \varepsilon_{xx})^2 - 4 \varepsilon_{xy} \varepsilon_{yx}]^{1/2}.
\]  

(2.20)

If the first term of equation (2.20) is rewritten as
1 + \frac{1}{2}(\varepsilon_{yy} + \varepsilon_{xx} - 2), \quad (2.21)

then using the binomial expansion when \((\varepsilon_{i,j} - \delta_{i,j}) \ll 1\) (see Pavlov, Mitrofanov, and Shibanov 1980; Trubnikov 1958; Shafranov 1967), one obtains

\[
n_{x,0} = 1 + \frac{1}{4}(\varepsilon_{yy} + \varepsilon_{xx} - 2) + \left[\frac{1}{16}(\varepsilon_{yy} - \varepsilon_{xx})^2 - \frac{1}{4}\varepsilon_{yy}\varepsilon_{xx}\right]^{1/2}
\]  

(2.22)

which is identical to the transverse wave of the tenuous plasma approximation.

ii) The General Formulation

In the dielectric formulation, once the conductivity tensor of the plasma is obtained, the absorption coefficients can also be obtained. If the particle distribution function \(y = y^{(0)} + y^{(1)} + \ldots\), where \(y^{(0)}\) is the equilibrium distribution and \(y^{(1)} \ll y^{(0)}\) is the first order perturbation, then the Boltzmann equation may be made linear to first order. The conductivity tensor is determined by making the assumption that the perturbation of the electromagnetic wave is given by the first order current density:

\[
J^{(1)}_i = \sigma_{ij}E_j = eN\int v_i y^{(1)}(v) d^3v,
\]  

(2.23)

where the superscript \((1)\) means the first order current density and particle distribution function. The first order distribution function is found by solving the linearized Boltzmann equation to first order (see
Stix 1962; Ginzburg 1970; Ichimaru 1973; Montgomery and Tidman 1964). Substituting for \( g^{(1)} \), it may be shown that the conductivity tensor \( \sigma_{ij} \) is of the form (Bekefi 1966; Bernstein 1958; Bers 1964)

\[
\sigma_{ij} = 2\pi w J_{\perp} d\rho_{\perp} \int d\rho_{11} \frac{1}{s \pi^2} \frac{\gamma^{-1} S_{ij}}{e^{\gamma (\omega - k_{11} v_{11} - s\omega_B)}} , \tag{2.24}
\]

where

\[
S_{ij} = \begin{bmatrix}
\frac{sJ_s(x)}{x} & \frac{sJ_s(x)J'_s(x)}{x} & \frac{sJ^2_s(x)}{x} \\
iv_1 \Phi \frac{sJ_s(x)J'_s(x)}{x} & iv_1 \Phi \frac{sJ^2_s(x)}{x} & iv_1 \Psi J_s(x)J'_s(x) \\
v_{11} \Phi \frac{sJ^2_s(x)}{x} & iv_{11} \Phi J_s(x)J'_s(x) & iv_{11} \Psi J^2_s(x)
\end{bmatrix} . \tag{2.25}
\]

The integration is over the perpendicular \( \rho_{\perp} \) and parallel \( \rho_{11} \) components of momentum. The variable \( \chi = \gamma k_{11} v_{11} / \omega_B \) and the functions

\[
\Phi = \gamma m_e \omega \frac{\partial y^{(0)}}{\partial \rho_{\perp}} + \kappa_{11} \frac{\partial y^{(0)}}{\partial \rho_{11}} \rho_{\perp} - \frac{\partial y^{(0)}}{\partial \rho_{\perp}} \rho_{11} , \tag{2.26a}
\]

\[
\Psi = \gamma m_e \omega \frac{\partial y^{(0)}}{\partial \rho_{11}} - m_e \omega_B \left( \frac{\partial y^{(0)}}{\partial \rho_{11}} \rho_{\perp} - \frac{\partial y^{(0)}}{\partial \rho_{\perp}} \rho_{11} \right) , \tag{2.26b}
\]

where \( y^{(0)} = g^{(0)}(\rho_{\perp}, \rho_{11}) \) is the zero order distribution function. In this thesis, \( y^{(0)} \) will always be a relativistic or nonrelativistic Maxwellian distribution. The functions \( J_s(x) \) and \( J'_s(x) \) are the Bessel function and its first derivative.
iii) Nonrelativistic Cyclotron Absorption Coefficient

If a nonrelativistic Maxwellian distribution function $y^{(0)} = y_0 e^{-\gamma^2/\text{2} (\beta_2^2 + \beta_1^2)}$ is used, where $y_0$ is the normalization factor, then an evaluation of the integrals gives for the dielectric tensor (Sitenko and Stepanov 1957; Pavlov, Mitrofanov, and Shibanov 1980):

$$
\varepsilon_{xx} = 1 + \frac{i \sqrt{\pi}}{\beta |\cos \theta|} \exp - \chi \sum_{s=-\infty}^{+\infty} s^2 I_s W_s(z_s), \quad (2.27a)
$$

$$
\varepsilon_{yy} = \varepsilon_{xx} - \frac{2i \sqrt{\pi} \chi}{\beta |\cos \theta|} \exp - \chi \sum_{s=-\infty}^{+\infty} (I_s - I_s) W_s(z_s), \quad (2.27b)
$$

$$
\varepsilon_{xy} = - \varepsilon_{yx} = \frac{2i \sqrt{\pi} \chi}{\beta |\cos \theta|} \exp - \chi \sum_{s=-\infty}^{+\infty} s (I_s - I_s) W_s(z_s), \quad (2.27c)
$$

$$
\varepsilon_{xz} = \varepsilon_{zx} = \frac{\tan \theta}{\chi} \exp - \chi \sum_{s=-\infty}^{+\infty} s I_s [1 + i \sqrt{\pi} z_s W_s(z_s)], \quad (2.27d)
$$

$$
\varepsilon_{yz} = - \varepsilon_{zy} = i \tan \theta \exp - \chi \sum_{s=-\infty}^{+\infty} (I_s - I_s) [1 + i \sqrt{\pi} z_s W_s(z_s)], \quad (2.27e)
$$

$$
\varepsilon_{zz} = 1 + \frac{2}{\beta |\cos \theta|} \exp - \chi \sum_{s=-\infty}^{+\infty} I_s z_s [1 + i \sqrt{\pi} z_s W_s(z_s)], \quad (2.27f)
$$

where $I_s = I_s(\chi)$ is the modified Bessel function of integer order $s$, and $I_s = dI_s/d\chi$. The argument of the modified Bessel function is

$$
\chi = \beta^2 u^2 \sin^2 \theta, \quad \text{where} \quad \beta = (2\eta)^{1/2} \text{ and } \eta = kT/mc^2. \quad \text{The complex plasma dispersion function is}
$$

$$
W_s(z_s) = \frac{i}{\pi} \int_{-\infty}^{+\infty} e^{-t^2} \frac{e^{-t^2}}{(z_s - t)}, \quad (2.28)
$$

where
Thus the effects of collisions appear in the plasma dispersion function as an imaginary term in the argument $z_s$. In this approximation, Thomson scattering is treated as pure absorption.

A further approximation can be made at low temperatures where only the first term of the series of the modified Bessel function is important. The following results for each mode of wave propagation (Mitrofanov and Pavlov 1981) are obtained using this approximation (simple approximations have also been found by Ronnmark 1977 using Padé approximants):

\[
\alpha_{x,0} = \frac{1}{\delta} \frac{\nu_c + \nu_r}{(\omega + \omega_B)^2} \cdot \left(1 + \cos^2 \theta - a_{1}^{x,0} \sin^2 \theta - 2a_{c}^{x,0} \cos \theta\right) \\
+ 2 \frac{(\nu_c + \nu_r)}{\omega} (1 + a_{1}^{x,0}) \sin^2 \theta \frac{(\nu_c + \nu_r)}{[(\omega + \omega_B)^2 + (\nu_c + \nu_r)^2]} \\
x (1 + \cos^2 \theta - a_{1}^{x,0} \sin^2 \theta + 2a_{c}^{x,0} \cos \theta), \tag{2.30}
\]

where

\[
a_{1}^{x,0} = \mp \frac{C}{(1 + \xi)^{1/2}} : \quad a_{c}^{x,0} = \mp \frac{\text{sign}(\cos \theta)}{(1 + \xi^2)^{1/2}}, \tag{2.31a,b}
\]

and $\xi = \sin^2 \theta/2u \cos \theta$. The terms $a_{1,c}^{x,0}$ are the degrees of linear (1) and circular (c) polarization in the extraordinary (upper sign) and ordinary (lower sign) modes. The third term has been simplified by using the condition for a "cold plasma" ($\nu_D \ll \nu_c + \nu_r$), and now only represents
the cyclotron radiation from the fundamental harmonic.

If \( \nu_d \gg \nu_c + \nu_r \), then the "not plasma approximation" should be used. The third term now becomes

\[
\frac{\sqrt{\pi}}{\beta \cos \Theta} \sum_{s=1}^{\infty} (\beta \sin \Theta / 2)^{2s-2} \frac{sH_s}{(s-1)!} \]

\[\times (1 + \cos^2 \Theta - a^{x,0}_1 \sin^2 \Theta + a^{x,0}_c \cos \Theta), \tag{2.32}\]

where \( H_s \) is the real part of the plasma dispersion function:

\[
W_s(z_s) = H_s(z_s) + iF_s(z_s) = \frac{i}{\pi} \int_{-\infty}^{+\infty} dt \frac{e^{-t^2}}{(z_s - t)}, \tag{2.33}\]

and

\[
z_s = x_s + iy = \frac{(\omega - s\omega_b)}{\nu_d} + \frac{(\nu_c + \nu_r)}{\nu_d} : s = 1, 2, \ldots \tag{2.34}\]

The difference between the two approximations is that in the cold plasma approximation the fundamental harmonic is the only harmonic which absorbs a significant amount of radiation compared with the collisional absorption. So, the absorption from the higher harmonics does not appear in the equation. The higher harmonics contribute significantly to the absorption process in the "hot collisionless plasma approximation".

Collisional effects in the derivation of the cyclotron absorption coefficients are made by including a characteristic collisional damping term in the collision term of the Boltzmann equation. The characteristic collision frequency is assumed to be independent of velocity and to act isotropically. This simplification of the collisional process could
\[
\sigma_{ij} = \frac{\eta^2 u}{8k_2(\eta)^2} \sum_{s=0}^{\infty} e^{-\eta \gamma_0} \frac{s^2}{\kappa_1^2} \left( \frac{x}{\lambda} \right) \sum_{q=0}^{\infty} C_q^i(x)q^j S_{ij} \quad i,j = x,y,z, \quad (2.3b)
\]

where

\[
S_{xx} = A_s + q(\xi), \quad (2.36a)
\]

\[
S_{xy} = -S_{yx} = i(1+q/s)A_{s+q}(\xi), \quad (2.36b)
\]

\[
S_{yy} = \kappa_1^4 s_{s+q}(\xi), \quad (2.36c)
\]

\[
S_{xz} = S_{zx} = \frac{\kappa_{11}}{\kappa_1^4} [A_{s+q}(\xi) - u \frac{\sqrt{1-\kappa_1^2/\kappa_{11}}}{|\kappa_{11}|} A_{s+q}^i(\xi)], \quad (2.36d)
\]

\[
S_{yz} = -S_{zy} = -i(1+q/s)S_{xz}, \quad (2.36e)
\]

\[
S_{zz} = (\frac{\kappa_{11}}{\kappa_1^4})^2 [A_{s+q}(\xi) - 2u \frac{\sqrt{1-\kappa_1^2/\kappa_{11}}}{|\kappa_{11}|} A_{s+q}^i(\xi)] + (u \frac{\sqrt{1-\kappa_1^2/\kappa_{11}}}{|\kappa_{11}|})^2 A_{s+q}^i(\xi), \quad (2.36f)
\]

The parameters are \( \eta = mc^2/kT, \gamma_0 = su/(u^2 - \kappa_1^2), \gamma_1 = [s^2 - (u^2 - \kappa_1^2)]^{1/2}/(u^2 - \kappa_1^2), \chi = \kappa_1^2 [s^2 - (u^2 - \kappa_1^2)]/(u^2 - \kappa_1^2), \) and \( \xi = \eta \gamma_1 |\kappa_{11}|, \) where \( u = \omega/\omega_b, \) and \( \kappa = \kappa c/\omega. \) The constants \( C_q^i \) and \( \kappa_q^i \) are

\[
C_q^i = (-1)^q(2s+2q)! \frac{[s+q]!^2(2s+q)!q!}{[s+q]!^2(2s+q)!q!}, \quad (2.37a)
\]

and

\[
\kappa_q^i = \frac{[2(s+q)(2s+q-1)(2s+q)]}{(2s+2q-1) s^2} - \frac{2q}{s} - 3). \quad (2.37b)
\]
possibly have a large effect on the angular flux and polarization of the emitted radiation.

In this thesis, collisional effects due to both inverse bremsstrahlung and Thomson scattering are included. These effects were shown in section I of this chapter to be of comparable importance at high harmonics and small $\theta$. However, the absorption due to inverse bremsstrahlung is a "true" absorption process. The absorption due to Thomson scattering is not.

At low temperatures and high electron densities, inverse bremsstrahlung dominates over Thomson scattering as the most important collisional effect. But Thomson scattering becomes more important as the frequency increases.

Between the cyclotron line centers, the cyclotron absorption becomes negligible for $kT \lesssim 5$ keV. This is the region of the cyclotron spectrum where collisional absorption becomes important. The polarization in this region decreases significantly compared to the polarization in the region of the cyclotron line centers for optically thin plasmas because the difference between the two absorption coefficients is much less.

The nonrelativistic calculations are satisfactory for temperatures below $kT \lesssim 1$ keV, but as the temperature increases the absorption coefficient becomes larger than the absorption coefficient calculated using the relativistic theory. At $kT = 5$ keV and harmonic number $s = 6$, the absorption coefficient from the nonrelativistic theory is a factor of three greater than the absorption coefficient from the relativistic theory. This discrepancy is due to the nonrelativistic approximation used by Pavlov, Mitrofanov, and Shibanov (1980). The approximation also
results in a disagreement as $\theta \to \pi/2$, since the relativistic mass effect is neglected, then $V_D \to U$. For calculations of the polarization from homogeneous plasma slabs, this error is probably not significant, but it will be important for determining the thickness of optically thin plasmas (see Chapters 3 and 4).

iv) Relativistic Cyclotron Absorption Coefficient

The conductivity tensor given in equations (2.24) and (2.25) is fully relativistic. The calculations in the nonrelativistic case have been made assuming the distribution function for a nonrelativistic Maxwell-Boltzmann distribution. Also the square of the Bessel function has been transformed into the modified Bessel function of the first kind because the argument is $\ll 1$.

If no approximations are made to the Bessel functions and the distribution function is taken to be that of a relativistic Maxwellian, then the full solution will be completely relativistic. Normally, the integration over the velocity is evaluated analytically for only the perpendicular component, while the parallel component of the velocity is integrated numerically. Tamor (1978) has derived an analytic solution for the conductivity tensor by first changing variables $v_{\parallel} \gamma$, and $\phi$ and then evaluating the half residue for $v_{\parallel}$ and the $\phi$ integration. Next, another change of variables is made so that the Bessel function can be expanded in a power series and each of the terms can be integrated analytically. The resulting expression for the six independent components of the conductivity tensor are:
The prime indicates the derivative of the function with respect to the argument, where

\[ A_{s+q}(\xi) = 2\left(\frac{2}{\xi}\right)^{s+q+1} \sqrt{\frac{\pi}{2}} I_{s+q+1/2}(\xi), \quad (2.38a) \]

\[ A'_{s+q}(\xi) = \frac{dA_{s+q}(\xi)}{d\xi} = -\frac{\xi}{2(s+q+1)} A_{s+q+1}(\xi), \quad (2.38b) \]

and

\[ A''_{s+q}(\xi) = \frac{d^2A_{s+q}(\xi)}{d\xi^2} = A_{s+q}(\xi) - A_{s+q+1}(\xi). \quad (2.38c) \]

b) Single Particle Model

Consider a magnetic field \( \mathbf{B} \) parallel to the z-axis and a wave vector \( \mathbf{k} \) in the x-z plane making an angle \( \theta \) to the magnetic field. By choosing \( \mathbf{B} \) along the z-axis, the problem is simplified without a loss of generality (Fig. 2.1). In a cold collisionless plasma (see Ginzburg 1970 for addition of collisions), the real component of the index of refraction is found by substituting the dielectric tensor:

\[ \varepsilon_{xx} = 1 - \frac{w}{1-1/u} = \varepsilon_{yy}, \quad (2.39a) \]

\[ \varepsilon_{xy} = -\varepsilon_{yx} = -i \frac{w}{u(1-1/u)}, \quad (2.39b) \]

\[ \varepsilon_{zz} = 1 - w, \quad (2.39c) \]

\[ \varepsilon_{xz} = \varepsilon_{zx} = \varepsilon_{yz} = \varepsilon_{zy} = 0, \quad (2.39d) \]
into equation (2.20) to obtain (Ginzburg 1970; Kamaty 1969):

\[
\eta_{0,0}^2 = 1 - \frac{2w/u^2}{2(1-w) - (\sin\theta/u)^2 \pm [(\sin\theta/u)^4 + 4(\cos\theta/u)^2(1-w)^2]^{1/2}},
\]

(2.40)

where the (+) and (-) correspond to the ordinary (0) and extraordinary (X) modes of wave propagation. The two modes are clockwise (X-mode) and counterclockwise (0-mode) circular polarization. If the polarization coefficients are defined by

\[
a_\theta = \frac{iE_\theta}{E_y} = \frac{iE_{xy}}{E_{yy} - n^2}, \quad a_k = \frac{iE_k}{E_y} = \frac{iE_{zy}}{E_{yy}},
\]

(2.41a,b)

then for the cold plasma approximation, they become

\[
(a_\theta)_{x,0}(\omega,\theta) = \frac{2(1-w)\cos\theta}{-(\sin\theta/u)\pm[(\sin\theta/u)^4 + 4(1-w)^2\cos^2\theta]},
\]

(2.42a)

and

\[
(a_k)_{x,0}(\omega,\theta) = \frac{w\sin\theta/u - (a_\theta)_{x,0} \cos\theta \sin\theta/u^2}{[w(1-\cos^2\theta/u^2) - (1-w)]},
\]

(2.42b)

using equations (2.39a-d). From equation (2.42b), \((a_k)_{x,0} = 0(w)\), and hence \(E_k \ll E_y\) for a tenuous plasma \((w \ll 1)\). Therefore, the longitudinal component \(E_k\) is negligible and is set to zero. Thus only the transverse components \(E_\theta\) and \(E_y\) are considered.

The volume emissivity from a single electron is (Liemohn 1965; Chanmugam and Dulk 1981; Ramaty 1969 for an apposite discussion):
\[ J_{x, o}(\omega, \theta, \beta) = \frac{e^2 \omega^2}{2 \pi c} \sum_{s=-\infty}^{+\infty} \frac{n_{x, o}}{1 + (q_{\theta})^2 / x_{s, o}} \{- \beta_{1}^1 J_s'(\chi) \}
\]

\[ + [(a_{\theta})_{x, o} (\cot \theta / n_{x, o} - \beta_{11} / \sin \theta) J_s(\chi)]^2 \delta[\omega_{p} / \gamma - \omega(1 - n_{x, o} \beta_{11} \cos \theta)], \quad (2.43) \]

where \( \gamma = (1 - \beta^2)^{1/2} \) is the Lorentz factor, \( \beta = v/c \), \( v \) is the velocity of the electron and \( \beta_{11} \) and \( \beta_{1} \) are the components parallel and perpendicular to the magnetic field direction, respectively. \( J_s(\chi) \) is the Bessel function of the first kind of order \( s \), and \( J_s'(\chi) = dJ_s(\chi)/d\chi \) is its first derivative with respect to the argument \( \chi = \gamma n_{x, o} \beta_{1} \sin \theta \).

The emissivity \( J_{x, o} \) for a distribution of electrons is:

\[ J_{x, o}(\omega, \theta) = \int J_{x, o}(\omega, \theta, \beta') g(0)(\beta') d^3 \beta'. \quad (2.44) \]

The distribution function is assumed Maxwellian:

\[ g(0)(\beta) = N_y e^{-\eta}, \quad (2.45) \]

where \( \eta = mc^2 / kT \), \( 1/y = -2 \pi^2 i m^2 k T H^2_2(i \eta) \), and \( H^2_2 \) is the Hankel function of the first kind of second order (Bekefi 1966; Ramaty 1969; Chanmugam and Dulk 1981). Thus substituting equations (2.43) and (2.45) into equation (2.44) and integrating over solid angle gives (Chanmugam and Dulk 1981):

\[ J_{x, o}(\omega, \theta) = 2 \pi e^2 N_y m \gamma c^2 \int_{-\infty}^{+\infty} \frac{d\beta_{11}}{1 + (a_{\theta})^2} \sum_{s=1}^{+\infty} \frac{n_{x, o}}{1 + (q_{\theta})^2 / x_{s, o}} \{- \beta_{1} J_s'(\chi) \}
\]

\[ \times \{- \beta_{1} J_s'(\chi) \} + [(a_{\theta})_{x, o} (\cot \theta / n_{x, o} - \beta_{11} / \sin \theta) J_s(\chi)]^2 \]
\[
\exp[-s \frac{\eta}{u} \frac{\omega u^{-3}}{1 - n_{x,0}^2 \beta_{11} \cos \theta}] \quad (2.4b)
\]

with \( s_1 \) the minimum value of \( s \) such that

\[
\beta_1^2 = 1 - \beta_{11}^2 - (u/s)^2 (1 - n_{x,0} \beta_{11} \cos \theta)^2 > 0. \quad (2.47)
\]

The absorption coefficients \( x_{x,0} \) are obtained from the emissivity by using Kirchoff's Law:

\[
\alpha_{x,0}(\omega, \theta) = J_{x,0}(\omega, \theta)/\left( n_{x,0}^4 I_{RJ} \right) \quad (2.48)
\]

where \( I_{RJ} = kT \omega^2/8 \pi^3 c^2 \) is the Rayleigh-Jeans intensity (for \( kT \gg \hbar \omega \)) per polarization and \( n_{x,0}^r \) is the ray refractive index which is \( n_{x,0}^r = 1 \). Equation (2.48) is a special case for an isotropic Maxwellian distribution (see Ramaty 1969 for the general equation).

III. Radiative Transfer

Consider a homogeneous plasma slab with a uniform magnetic field \( B \). Let the frequency of the emitted radiation be \( \omega \) and assume that \( \omega > \omega_p \gg \omega_0 \), so that the component of the electric field parallel to \( k \) may be neglected. If \( E_y, E_\theta \) are the transverse components of the electric field with \( E_\theta \) in the plane containing \( B \) and \( k \) (see Fig. 2.7), the polarization
Fig. 2.7.---Coordinate system with the magnetic field \( B \) parallel to the \( z \)-axis. The wave vector \( \mathbf{k} \) is at an angle \( \theta \) to \( B \). The transverse components of the wave \( E_\theta, E_y \) are also shown. Note that there is no loss of generality in the solution of the equations of radiative transfer when \( \mathbf{k} \) is in the \( x-z \) plane.
coefficients $a_{x,0}(\theta)$ for the two modes (with $i a_{x,0}(\theta) = \frac{E_x}{E_y}x,0$) are then given by (Ginzburg 1970; Chanygum and Dulk 1981)

$$a_{x,0} = \frac{\zeta}{\pm(1+\xi^2)^{1/2} - 1}, \quad (2.49)$$

where

$$\zeta = -\frac{2ucos\theta}{sin^2\theta}. \quad (2.50)$$

The observable properties of the radiation are described by the Stokes parameters (see the lucid discussion by Jackson 1975):

$$I = \langle E_x^* \rangle_x + \langle E_y^* \rangle_y, \quad (2.51a)$$

$$Q = \langle E_x^* \rangle_x - \langle E_y^* \rangle_y, \quad (2.51b)$$

$$U = \langle E_x^* \rangle_x + \langle E_y^* \rangle_y, \quad (2.51c)$$

$$V = -i(\langle E_x E_y^* \rangle - \langle E_y E_x^* \rangle). \quad (2.51d)$$

The Stokes parameters (e.g. Born and Wolf 1964; Ramaty 1969) can be written in terms of $I_s$, $I_x$, $I_0$, $I_c$ and $(a_\theta)_{x,0}$:

$$I = I_x + I_0, \quad (2.52a)$$

$$Q = I_x \left[ \frac{1-(a_\theta)^2}{1+(a_\theta)^2} \right] + I_o \left[ \frac{1-(a_\theta)^2}{1+(a_\theta)^2} \right] + \frac{4}{c \left[ 1+(a_\theta)^2 \right]^{1/2} \left[ 1+(a_\theta')^2 \right]^{1/2}}, \quad (2.52b)$$
\[ U = I_s \frac{2[(a_\theta)_x - (a_\theta)_0]}{[1+(a_\theta)_x^2]^{1/2}[1+(a_\theta)_0^2]^{1/2}}, \quad (2.52c) \]

\[ V = 2[I_x 1+(a_\theta)_x^2 + I_0 1+(a_\theta)_0^2 + I_c \frac{(a_\theta)_x + (a_\theta)_0}{[1+(a_\theta)_x^2]^{1/2}[1+(a_\theta)_0^2]^{1/2}}, \quad (2.52d) \]

where \( I_x \) and \( I_0 \) are the normal mode intensities of the radiation, \( I_s = (I_x I_0)^{1/2} \sin \delta \), and \( I_c = (I_x I_0)^{1/2} \cos \delta \). The angle \( \delta \) is the phase difference between the two modes (see Fig. 2.8).

The equations for the Stokes parameters may be simplified by limiting the treatment of the radiation to the case of large Faraday rotation in the source. This assumption means the phase relations are randomized and \( I_s = I_0 = 0 \), or that the anisotropy (Faraday rotation) of the medium is more important than the inhomogeneities of the medium.

The rotation angle \( \Delta \psi = (1/\lambda)(\omega_p^2 / \omega^3) \) (Kamaty 1969), where \( \lambda \) is the characteristic length for the inhomogeneities in the plasma and \( \lambda \) is the wavelength of the radiation. If \( \omega \sim 100 \omega_p \) and \( \omega = 2\omega_p \) with \( \lambda \sim 10^5 \) cm and \( \lambda = 5 \times 10^{-5} \) cm, then \( \Delta \psi \sim 10^5 \ll 2\pi \). Thus the condition of large Faraday rotation is expected to be satisfied under most conditions (i.e. when \( \lambda \gg 1 \) cm).

The simplified relations for the Stokes parameters are

\[ I = I_x + I_0, \quad (2.53a) \]

\[ Q = I_x \left[ \frac{1-(a_\theta)_x^2}{1+(a_\theta)_x^2} \right] + I_0 \left[ \frac{1-(a_\theta)_0^2}{1+(a_\theta)_0^2} \right], \quad (2.53b) \]

\[ U = 0, \quad (2.53c) \]
Fig. 2.8.--Coordinate system of an elliptically polarized wave. The amplitude of the electric field $E$ draws an ellipse when one sits in the rest frame of the wave. In one coordinate system, the semimajor axis $a = E_0 \cos \beta$ at an angle $\chi$ to the $y$-axis and the semiminor axis $b = E_0 \sin \beta$ can be used to define the ellipse where $E_0$ is the amplitude of the electric field and $\beta$ is the ratio of semimajor and semiminor axes. In another coordinate system, a clockwise $E_x$ and a counterclockwise $E_0$ rotating electric fields are used to define the ellipse. The angle $\delta$ is the phase angle between the two electric fields.
For the extreme conditions of $\theta = \pi/2$ and $\theta = 0$, the radiation is completely linearly polarized and circularly polarized, respectively. Since $I_s = I_c = 0$, the equations of radiative transfer decouple and become

$$V = 2[I_x \frac{(a_\theta)_x}{1+(a_\theta)_x^2} + I_o \frac{(a_\theta)_o}{1+(a_\theta)_o^2}]$$ \hspace{1cm} (2.53d)

For a homogeneous source, one finds

$$\frac{dI_{x,0}(\omega,\theta)}{dz} + a_{x,0}(\omega,\theta)I_{x,0}(\omega,\theta) = J_{x,0}(\omega,\theta),$$ \hspace{1cm} (2.54)

which are easily solved for the case of a homogeneous source region.

For a homogeneous source, one finds

$$I_{x,0}(\omega,\theta) = I_{RJ}[1 - \exp(-\tau_{x,0})],$$ \hspace{1cm} (2.55)

where $I_{RJ}$ is the Rayleigh-Jeans intensity for each mode and $\tau_{x,0}$ is the optical depth in the two modes. The Stokes equations which were derived by Ramaty (1969) may be considerably simplified by noting from equation (2.49) (Barrett and Chanmugam 1984) that

$$a \equiv a_x = -1/a_0.$$ \hspace{1cm} (2.56)

If one substitutes equation (2.56) into equations (2.53b) and (2.53d), it follows that

$$Q = \frac{(1-a^2)}{1+a^2}(I_x - I_o),$$

$$= \frac{(1-a^2)}{1+a^2}I_{RJ}[\exp(-\tau_x) - \exp(-\tau_o)].$$ \hspace{1cm} (2.57a)
From equations (2.55), (2.57a), and (2.57b) we note that if \( \tau_x, \tau_0 >> 1 \), then \( Q = V = 0 \) and the emitted radiation is unpolarized. Analytic expressions for \( Q \) and \( V \) have also been obtained for the homogeneous case (Pacholczyk 1977; Meygitt and Wickramasinghe 1982). However, their derivation is obtained in terms of the three emissivities \( J_I, J_Q, J_V \) or their corresponding absorption coefficients. We believe our results are simpler because \( I, Q, \) and \( V \) are obtained in terms of the two absorption coefficients in the ordinary and extraordinary modes and is thus more physically appealing.

It follows if \( I_x \neq I_0 \), then

\[
Q/V = \frac{1}{2}(\frac{1}{a} - a) = -\frac{1}{\zeta} = \frac{\sin^2 \theta}{2u \cos \theta}.
\]  

This result has also been obtained by Meygitt and Wickramasinghe (1982) using a modification of the single particle method. They calculate each Stokes parameter and find equation (2.58) in the limit of large Faraday rotation. The result is remarkable in that the expression for \( Q/V \) is independent of the optical depths in the plasma and hence can provide a simple and valuable probe \((\omega/\omega_B \) and \( \theta \) of the plasma.

\[
V = (\frac{2a}{1+a^2})(I_x - I_0),
\]

\[
\text{= } -\frac{2a}{1+a^2}I_{Qy}[\exp(-\tau_x) - \exp(-\tau_0)].
\]
CHAPTER 3

POLARIZED RADIATION FROM MAGNETIC CVS
I. AM HERCULIS BINARIES

This section contains the results and conclusions of a study of the polarized optical radiation from AM Her binaries. The first two subsections are an orientation to the subject. Subsection (a) is a discussion of the important observational properties of these systems such as the linear polarization pulse and the periodically modulated circular polarization. In subsection (b), the successes and failures of the early calculations on AM Her binaries are discussed. This section ends with a presentation of new results (subsections c and d) and conclusions (subsection e) of work which has been published in a paper by Barrett and Chanmuyam (1984a). These new results are more quantitative than the early calculations so that the shortcomings of the basic model which was used in both studies are more pronounced.

a) Early Observations

The first two AM Her binaries (AM Her, AN UMa) were discovered in 1977 (see Chapter 1, section 1). By 1980, the number had risen to four: AM Her, AN UMa, VV Pup, EF Eri (EF Eridani). A striking feature of all observations was the presence of a linear polarization pulse once per orbital period. Interestingly, though the circular polarization curves of these binaries were different for each one (see Figs. 3.1-3.4), it was possible to separate the circular polarization curves into three cases: (1) the polarization never changes sign (AN UMa, Fig. 3.2; EF Eri, Fig. 3.4), (2) the polarization does change sign (AM Her, Fig. 3.1), and (3) the polarization is zero for large fractions of
Fig. 3.1.--Sketch of visual light curve (Priedhorsky et al. 1973), polarization curves (Tapia 1977a), and phases of the soft X-ray maximum and minimum (Hearn and Richardson 1977) for AM Her. The position of the active magnetic pole of the white dwarf at selected phases is shown projected onto the stellar disk in the above illustrations. The short term (~5 minute) optical variations (flickering) are smoothed over in these light curves. From Chanmuyam and Dulk (1981).
Fig. 3.2.--Same as Figure 3.1 but for VV Pup. The optical light and polarization (solid lines) curves are from Tapia (1977b and private communication). The circular polarization (dashed line) is from Liebert and Stockman (1979) during a more active observing period. The X-ray light curve is from Patterson et al. (1984). From Chanmuyam and Dulk 1981.
Fig. 3.3.--Same as Figure 3.1 but for AN UMa. The light and polarization curves in the B band are from Krzeminski and Serkowski (1977). The X-ray observations (Hearn and Marshall 1979) and optical observations were made about a year apart. From Chanmuyam and Vulk 1981.
Fig. 3.4.--Same as Figure 3.1 but for EF Eri. The optical light curve is from Bond, Chanmuyam, and Grauer (1979). The polarization light curves are from Tapia (1979 and private communication). From Chanmuyam and Dulk 1981.
the orbital period (VV Pup, Fig. 3.3). In addition to the polarization curves, two binaries (AM Her, AN UMa) were observed in X-rays. The maximum X-ray intensity corresponds with the maximum circular polarization in AN UMa (Fig. 3.2), but the opposite situation occurs in AM Her where the maximum X-ray intensity corresponds to minimum negative circular polarization (Fig. 3.1). Also in AM Her, the minimum X-ray intensity corresponds to positive circular polarization; a change of sign from the X-ray maximum.

In their model of AM Her binaries (see Chapter 1, section 1) Chanmuyam and Wagner (1977) described a magnetic white dwarf with a hot postshock region which is the result of accreting matter from its companion star at one or both magnetic poles. The postshock is located on the surface of the white dwarf at colatitude $\delta$ (the angle between the rotation axis and the magnetic dipole axis; see Fig. 3.5). Therefore, the postshock circles the star in one orbital period. Since the inclination $i$ (the angle between the rotation axis and the line of sight) of the white dwarf can be at any angle, this postshock region can (see Fig. 3.6): (1) always remain in view (AN UMa, EF Eri), (2) remain in view but approach the limb of the white dwarf (AM Her), and (3) pass behind the limb and be eclipsed by the white dwarf (VV Pup). These three cases would then appear to correspond to the three cases of circular polarization. Of course, a fourth case can exist where the postshock (assuming one pole accretion) is always behind the limb of the white dwarf, but none of these binaries would be identified as AM Her binaries because no polarization or X-rays would be detected. The highly polarized radiation is indicative of cyclotron emission as noted by Tapia (1977) and suggested by Ingham, Brecher, and Wasserman
Fig. 3.5.--Sketch of coordinate system fixed at center of magnetic white dwarf. The line of sight is along the positive y-axis. The angle between the magnetic field $\mathbf{B}$, which is assumed to be along the magnetic dipole axis, and the line of sight is $\theta$. An equation for $\theta$ can be written in terms of the inclination $i$ (the angle between the line of sight and the rotation axis) and the colatitude $\delta$ of the magnetic pole (the angle between the magnetic dipole axis and the rotation axis). The phase $\phi = 0$ is defined by the maximum linear polarization pulse (if two pulses are observed). The magnetic dipole axis is perpendicular to the line of sight at this phase.
Fig. 3.6.--A sketch of views of the accretion column at selected orbital phases and magnetic colatitudes. The accretion column: is never eclipsed by the white dwarf at colatitude (1), is on the limb of the white dwarf at colatitude (2), and is eclipsed at colatitude (3).
If Faraday rotation is important in the radiative transfer, which it should be when a strong magnetic field is present, then a linear polarization pulse should appear when the circular polarization changes sign as is observed in AM Her (Fig. 3.1).

b) Past Calculations

The first calculations of polarized radiation from hot plasmas in AM Her binaries were made by Chanmugam and Dulk (1981) and Megyitt and Wickramasinghe (1982). The results of these calculations confirmed the speculation that the linear polarization pulse observed in all AM Her binaries (presently ten known) occurs at the orbital phase when the magnetic field in the accretion column is nearly perpendicular to the line of sight and when the circular polarization changes sign. From the results of these calculations, it was also deduced that the polarized optical radiation is from moderately high harmonics (5 < s < 15) at a magnetic field \( \lesssim 4 \times 10^7 \) gauss and is not from the fundamental cyclotron harmonic as was initially believed. It is also a prediction of the calculations that the low harmonics (s < 4) would have \( \tau_{x,0} \gg 1 \) (optically thick; see Chapter 2, section III), because the absorption coefficients are large (\( \alpha_{x,0} \gtrsim 10^{-5} \)) at these harmonics and the path lengths are long (\( l \gtrsim 10^6 \)). Thus the intensity I would approach a (Rayleigh-Jeans) blackbody intensity \( I_{\text{BJ}} \) (see Fig. 3.7). Because the optical depth decreases with increasing harmonic number, eventually, at some harmonic \( s^* \) where \( 5 \lesssim s^* \lesssim 15 \), \( \tau_{x,0} \approx 1 \) and the spectrum reaches its maximum intensity. Above \( s^* \), \( \tau_{x,0} \ll 1 \) (optically thin) and the spectrum declines because of the small absorption coefficients.
Fig. 3.7.--Comparison of the Rayleigh-Jeans spectrum and cyclotron spectra at selected values of the dimensionless plasma parameter $\Delta = \omega_p^2/\omega_c$. The maximum cyclotron flux defines the cyclotron harmonic $\omega^*/\omega_b$ which has also an optical depth $\tau \sim 1$. 
Qualitative comparisons of observations and theory were made in the papers of Chanmugam and Dulk (1981) and Wickramasinghe and Meggitt (1982). This latter paper was an application of calculations by Meggitt and Wickramasinghe (1982). Quantitative comparisons were made in the paper by Barrett and Chanmugam (1984) (see the next subsection).

Chanmugam and Dulk (1981) applied their model to the four AM Her binaries (c. 1980; see Figs. 3.1-3.4). They reached the following conclusions about AM Her and VV Pup. In AM Her, the polarized light is from the cooler \((kT < 1 \text{ keV})\) accretion column overlying the shock because the circular polarization is observed when the X-ray emission beneath the shock is eclipsed (Fig. 3.1) and the hot \((kT > 20 \text{ keV})\) shock or postshock does not produce linear polarization except at optically thin high harmonics (Fig. 3.9a). Chanmugam and Dulk (1981) also noted that their simple slab model cannot explain all features of the observations. First, no unpolarized component was included in their calculations to reduce the polarization which is much larger than observed. Second, a second linear polarization pulse is expected when the circular polarization reverses sign the second time, but is not observed. Third, Chanmugam and Dulk (1981) suggested that a better model would be a cylinder with a boundary layer instead of a homogeneous plasma slab. In the analysis of the circular polarization (Fig. 3.9a), Chanmugam and Dulk (1981) have concluded that the low polarization and visual flux in the V spectral band at phase 0.6 (the "standstill"), where the X-ray flux is a maximum (Fig. 3.1), probably results from viewing the accretion column at a small angle \((\theta \lesssim 30^\circ)\). The small angle gives a correspondingly low optical depth, small
Fig. 3.8.--The cyclotron absorption coefficients in the extraordinary (X) and ordinary (O) modes at the sixth harmonic and 1 keV as a function of \( \cos \Theta \). The line center is \( \omega/\omega_B = 5.93 \) at this temperature. The dashed curves correspond to the nonrelativistic approximation of Engelmann and Curatolo (1973). From Chanmugam and Dulk (1981).
Fig. 3.9.--(a) The circular (solid line) and linear (dashed line) polarization from a plasma slab for cyclotron harmonics \( s = 4, 5, 6, \) and 7 as a function of \( \cos \theta \). The line centers occur at \( \omega / \omega_0 = 3.97, 4.95, 5.93, \) and 6.90, respectively. The temperature of the plasma \( kT = 1 \) keV, the dimensionless parameter \( \Lambda = 10^8 \), and \( l = 10^8 \) cm. (b) The log of the intensity (in units of ergs cm\(^{-2}\)) as a function of \( \cos \theta \) for the same values as in (a). From Chammugam and Dulk 1981.
apparent area, and low intensity of the cyclotron emission (Fig. 3.9b). They have also accounted for the relative lack of variation in the blue and UV spectral bands as a result of optically thin high harmonic cyclotron emission.

In Chanmugam and Dulk's analysis of VV Pup, all of the remarks about AM Her also apply. Their only comment on VV Pup has been about the circular polarization observed during bright active periods. They have suggested that the polarization is due to an extended accretion column which is always in view; contrary to the interpretation by Liebert and Stockman (1977) for the second magnetic pole becoming active.

Wickramasinghe and Meggitt (1982) only applied their model to VV Pup. Their results were in qualitative agreement with the observations. However, as Chanmugam and Dulk found, the optical polarization from their model calculations were ~ 3-5 times larger than the observed polarization (Figs. 3.10 and 3.11). They interpreted this result as evidence for the presence of an unpolarized or weakly polarized component of radiation which dominates in the blue and UV spectral band. As for the geometry of the accretion column, no definite conclusions were reached using the model calculations of optical polarization.

In addition to the important papers by Chanmugam and Dulk (1981) and Meggitt and Wickramasinghe (1982) on the polarization properties of magnetic CVs, a third paper by Brainerd and Lamb (1984) should also be mentioned. Their analysis did not include the polarization properties of cyclotron radiation. Instead, they determined the inclination $i$ and colatitude $\delta$ of five AM Her binaries using data of the
Fig. 3.10.--The intensity and polarization spectra for optically thin emission at 10 keV at various angles $\theta$ to the magnetic field. The solid, dash, and dash-dot lines correspond to intensity, linear polarization, and circular polarization, respectively. From Meggitt and Wickramasinghe 1982.
Fig. 3.11.--Broadband linear and circular polarization curves as a function of rotational (orbital) phase $\phi$ for the model with $i = 30^\circ$ ($\delta = 100^\circ$), $kT = 10$ keV, $B = 3.18 \times 10^7$ gauss, $A = 10^5$. The dotted sections correspond to the phases when the emission region (assumed to be a point source) will be eclipsed by the body of the white dwarf. The observations are from Liebert et al. (1978) obtained when the system was in a state with $m_{\text{min}} \sim 17.6$, $m_{\text{max}} \sim 17.4$. From Meggitt and Wickramasinghe 1982.
change of the linear polarization position angle with phase (Figs. 3.12, 3.13). Their results are included in Table 1 at the end of Chapter 1. From their analysis, Brainerd and Lamb (1984) concluded that the discovery of AM Her stars is strongly affected by observational selection effects and that three times the present number (1U) of AM Herculis binaries exist.

The calculations of Chanmuyam and Dulk (1981) and Meggitt and Wickramasinghe (1982) were made in the collisionless approximation. On the other hand, Pavlov, Mitrofanov, and Shibanov (1980) have shown that absorption due to collisional effects (meaning inverse bremsstrahlung and Thomson scattering) would be important in plasmas at relatively low temperatures and high frequencies. However they did not explain the important polarization observations such as the linear polarization pulse. In subsection (c), results of comparisons between the cyclotron calculations of Pavlov, Mitrofanov, and Shibanov (1980) and the flux and polarization observations by Tapia (1977) are presented. Results of similar comparisons using a two component model (a polarized and an unpolarized component) and the single particle method to calculate the absorption coefficients are presented in subsection (d). Conclusions about these two studies are given in subsection (e).
Fig. 3.12.—Comparison of the fitted theoretical curves and the observational data for the change in the linear polarization position angle with phase for AM Her. The eclipse length $\Delta \phi$ is fixed at 0.15 in all three fits. From Brainerd and Lamb 1984.

Fig. 3.13.—Same as Fig. 3.12, but for AN UMa. The $i + \delta$ is fixed at $85^\circ$ in all three fits.
c) The Nonrelativistic Approximation

(From the paper by Barrett and Chanmugam 1984a)

i) Results

1) Absorption Coefficients

The absorption coefficients $\alpha_{X,0}(\omega, \theta)$ were calculated using the nonrelativistic dielectric formulation described in Chapter 2, section II for plasmas with temperatures $kT \lesssim 1$ keV. For the collisionless case ($\nu = 0$), excellent agreement was obtained with the results of Chanmugam and Dulk (1981) for temperatures $kT \lesssim 1$ keV. Hence we infer from the agreement with the collisionless calculations that the collisional calculations ($\nu \neq 0$) are also valid in this region. As the temperature is increased above 1 keV, the results begin to disagree increasingly with temperature. For example with $kT = 5$ keV and harmonic number $s = 6$ the results differ by about a factor of two from those of Chanmugam and Dulk (1981). This discrepancy is probably due to the nonrelativistic nature of the method of Pavlov, Mitrofanov, and Shibanov (1980). The nonrelativistic approximation also results in a disagreement when $\theta \to \pi/2$, because the relativistic mass effect is neglected and $\nu_d \to 0$ as is discussed in Chapter 2.

The absorption coefficients in the ordinary and extraordinary modes are plotted as a function of angle $\theta$ for a plasma at a temperature $kT = 1$ keV and 0.2 keV in Figures 3.14 and 3.15, respectively. Results are presented for both the collisional (meaning inverse bremsstrahlung and
Fig. 3.14.--The absorption coefficient $\alpha$ is plotted against angle $\theta$ for a plasma at a temperature $kT = 1$ keV and magnetic field $B = 3 \times 10^7$ gauss. The absorption coefficients are plotted in units of $\omega_p^2/\omega_B c$. The chain-dash and chain-dot curves correspond to the extraordinary and ordinary modes, respectively, when the effects of collisions and Thomson scattering are included. The dash and dot curves correspond to the extraordinary and ordinary modes, respectively, for the collisionless case. Plots for harmonic numbers $\omega/\omega_B = 5, 7, \text{ and } 9$ are in Figs. 3.14a, 3.14b, and 3.14c, respectively.
Fig. 3.15.--Same as Fig. 3.14, except that the temperature is $kT = 0.2$ keV. Harmonics $\omega/\omega_B = 4, 6, \text{ and } 8$ are plotted in Figs. 3.15a, 3.15b, and 3.15c.
Thomson scattering as is in the previous subsection) and collisionless cases for cyclotron harmonics $s = 5, 7, \text{and } 9$ (Fig. 3.14) and $s = 4, 6, \text{and } 8$ (Fig. 3.15). We note from Figures 3.15a,b that for $s = 4$, the collisional effects (in this case, those of Thomson scattering, not inverse bremsstrahlung) become important only for $\theta \leq 45^\circ$. Whereas for $s = 6$, they are important for $\theta \leq 60^\circ$. In figures 3.14 and 3.15, even though the effects of collisions are dominant at small $\theta$, there are differences between $\alpha_x$ and $\alpha_0$ due to the anisotropy imposed on the plasma by the magnetic field. Hence polarized radiation may be emitted even when collisions are dominant (see (2) below) since $I_x \neq I_0$ (eqs. 2.57a,b, provided $\tau_x, \tau_0$ are not much greater than 1).

In Figure 3.16, the behavior of the absorption coefficient as a function of harmonic number at a fixed angle $\theta = 60^\circ$ and $kT = 1$ keV and collisional effects become important with increasing harmonic number. Another important point is that the harmonic features are not discernible for $s \geq 5$ at $\theta = 60^\circ$ (see also Pavlov, Mitrofanov, and Shibanov 1980). Furthermore, collisional effects become important for frequencies between the line centers.

To summarize, several important points can be made about the collisional effects on the absorption coefficient: (1) As is well known (Trubnikov 1958; Chanmugam and Dulk 1981; and references therein), the collisionless cyclotron absorption coefficient including the effects of collisions decreases strongly with the angle $\theta$ between the line of sight and the magnetic field (see Figs. 3.14 and 3.15). The contribution to the absorption coefficients from collisions, on the other hand, depend only weakly on the angle $\theta$. As $\theta \to 0$, the collisional effects become more important and eventually are the dominant absorption mechanism.
Barrett and Channugam 1983). (2) The collisional effects also become important with increasing harmonic number. This is a result of the rapid decrease in the cyclotron absorption coefficient with increasing harmonic number in the collisionless case, whereas the absorption from collisions depends only weakly on harmonic number (see Figs. 3.16). (3) Collisional effects become more important at lower temperatures (Pavlov, Mitrofanov, and Shibanov 1980). The cyclotron absorption coefficient decreases with decreasing temperature while the bremsstrahlung absorption coefficient actually increases moderately with decreasing temperature. Note in particular that at $kT = 20$ keV and $\theta = 60^\circ$, collisional effects are important only for $s \gtrsim 16$ whereas at $kT = 1$ keV they are important for $s \gtrsim 6$.

(b) Polarization

The polarization of the radiation emitted by a homogeneous plasma slab was calculated by using the method described in Channugam and Dulk (1981) for the case of large Faraday rotation. The results were checked against equation (2.57a,b) which were derived only after most of the calculations were completed. For a plasma slab of thickness $l$ which is parallel to the magnetic field (see Fig. 3.17) the optical depths are given by

$$\tau_{x,0} = \sigma_{x,0} l / \sin \theta.$$

The fractional polarization versus angle is plotted for temperatures $kT = 1.0$ and 0.2 keV in Figures 3.18 and 3.19. We have included the polarization for both the collisional and collisionless cases as in Figures 3.14 and 3.15 for the absorption coefficients. Figures 3.18a and 3.19a show the fractional linear polarization at
Fig. 3.16.—Plots of the absorption coefficients versus $\omega/\omega_B$ for a magnetic field of $3 \times 10^7$ gauss, temperatures $kT = 1$ keV, and $\vartheta = 60^\circ$. The units and notation are the same as in Fig. 3.14.
Fig. 3.17.---Sketch of an infinite plasma slab of thickness \( l \) and magnetic field parallel to its surface. The wave vector \( \mathbf{k} \) is at an angle \( \theta \) to the magnetic field \( \mathbf{B} \). The optical depth along \( \mathbf{k} \) is 
\[ \tau_{x,0} = \alpha_{x,0} l / \sin \theta. \]
Fig. 3.18.--Plots of (a) the fractional linear polarization, (b) the fractional circular polarization, and (c) the flux per unit frequency versus angle $\theta$ at $\omega/\omega_B = 5, 7, \text{ and } 9$ from a homogeneous plasma slab with $kT = 1 \text{ keV}$. The plasma slab of thickness $l = 10^8 \text{ cm}$ and dimensionless parameter $\Lambda = 10^9$ is oriented parallel to the magnetic field $B = 3 \times 10^7 \text{ gauss}$. The solid and dashed lines correspond to the collisional and collisionless cases, respectively.
Fig. 3.19.--The same as Fig. 3.18, except $kT = 0.2$ keV and $\omega/\Omega_B = 4, 6, \text{and} 8$. 
magnetic field $B = 3 \times 10^7$ gauss, dimensionless parameter $\Lambda = 10^8$ and a slab thickness $l = 10^8$ cm for $kT = 1.0$ and 0.2 keV, respectively. Figures 3.18b and 3.19b show the corresponding circular polarization. The harmonics plotted in Figures 3.18 and 3.19 are those harmonics we believe would be seen in the visible and infrared portion of the spectrum of AM Herculis stars. A couple of general points can be made about the fractional polarization: (1) for cyclotron harmonics above some value $s^*$, the fractional polarization decreases with increasing harmonic number, and (2) the fractional polarization in the collisional case is lower than in the collisionless case.

For the fractional linear polarization specifically, the collisional effects are large. The linear polarization pulse at 1 keV (Fig. 3.18a) drops dramatically to just a few percent until at harmonic $s = 9$, the pulse completely disappears. The linear polarization at 0.2 keV (Fig. 3.19a) shows behavior similar to that shown in Figure 3.18a except that the pulse disappears at $s = 8$.

The effects of collisions on the fractional circular polarization are even more pronounced. We note firstly the significant decrease in the fractional circular polarization. Another interesting effect is that the fractional circular polarization approaches 0% when $\theta \rightarrow 0^0$ for a plasma slab parallel to the magnetic field.

This behavior of the circular polarization gives a natural explanation of the "standstill" observed, for example, in AM Herculis and PG 1550+191, if we assume the polarization is emitted from a long column. Part of the explanation for this decrease in the circular polarization is that the optical depths $\tau_{x,0} \rightarrow \infty$ as $\theta \rightarrow 0^0$ with an infinite plasma slab. But even for the case of a plasma slab
perpendicular to the magnetic field (Fig. 3.20) where the optical depth \( \tau_{x,0} + \tau_0 \) (some constant value < \( \infty \)) as \( \theta \rightarrow 0^0 \), there is still a decrease in the polarization to a non-zero value < 100\%. In both cases, as \( \theta \rightarrow 0^0 \), the polarization decreases from a maximum value.

(c) Emitted Flux

The emitted flux per unit surface area from the slab, parallel to the magnetic field,

\[
F = \pi I \sin \theta
\]

(3.1)

is presented in Figures 3.18c and 3.19c for the temperatures \( kT = 1 \) and 0.2 keV, respectively. There are two points to be noticed in the emitted flux: (1) The flux in the collisionless approximation decreases by \( \sim 10 \) for each increase in harmonic number. This behavior does not occur in the collisional approximation. The flux in such a case varies by only a factor of 2 or 3 for each increase in harmonic number. (2) The flux in the collisional case has a weak dependence on angle in contrast to the flux in the collisionless case which has a strong dependence on angle.

(d) The Shock-Heated Region

Calculations of the radiation emitted by the shock-heated region were made for the collisionless case using the single particle method of Chanmugam and Dulk (1981). The temperature of this region was taken to
Fig. 3.20—Same as Fig. 3.17, but magnetic field is perpendicular to the surfaces of the slab and the optical depth $\tau_{x,0} = \alpha_{x,0} l / \cos \theta$. 
be \( kT = 30 \) keV consistent with the hard X-ray temperature observed for AM Herculis (Rothschild et al. 1981). The collisionless approximation is satisfactory at temperatures above 20 keV and \( s \lesssim 16 \) because the effects of collisions and Thomson scattering are less important. The shock-heated region was represented by an infinite plasma slab perpendicular to the magnetic field (see Fig. 3.20).

The absorption coefficients for the harmonics \( s = 16 \) and 18 are shown in Figures 3.21a,b. From the calculations of King and Lasota (1979), and Lamb and Masters (1979), the shock height is expected to lie in the range \( 10^{7} \) cm \( \lesssim h \lesssim 10^{9} \) cm. The optical depth \( \tau_{x,0} = \alpha_{x,0} h > 1 \) for all angles. Therefore the polarization from this region should be negligible. Figures 3.22a,b show the fractional linear and circular polarization, respectively, from a plasma slab of height \( h = 10^{6} \) cm. Despite the low value of \( h \), we see that it is difficult to obtain a linear polarization pulse unless \( s \gtrsim 18 \). Such a high value for \( s \) shows that the polarized radiation cannot arise in the shock-heated region as a result of thermal cyclotron emission. The emitted flux per unit area is given in Figure 3.22.

ii) Application to AM Herculis

In this section we briefly compare our result with optical observations of AM Herculis. The theoretical calculations of the emitted radiation, with collisions included, are carried out for a homogenous plasma slab parallel to the magnetic field \( B \). These results are compared to the observed linear polarization (\( Q/I \)), circular polarization (\( V/I \)), the ratio \( Q/V \), and the visual flux (Tapia 1977).
Fig. 3.21—(a) Absorption coefficient versus angle $\theta$ at $\omega/\omega_B = 16$ for $B = 3 \times 10^7$ gauss and $kT = 30$ keV. The absorption coefficient was calculated in the collisionless approximation. The dash and dot curves correspond to the extraordinary and ordinary modes, respectively, in units of $\omega_p^2/\omega_B c$. (b) Same as above for $\omega/\omega_B c = 18$. 
Fig. 3.22--Plots of (a) linear and (b) circular polarization versus angle $\theta$ for $\omega/\omega_B = 16$ and 18 at $kT = 30$ keV. The dimensionless parameter $\Lambda = 10^6$ and $l = 10^6$ for a plasma oriented perpendicular to the magnetic field $B = 3 \times 10^7$ gauss. (c) Plots of flux against $\theta$ for the same parameters.
The viewing angle $\theta$ is related to the orbital phase $\phi$, the inclination $i$ and the colatitude $\delta$ of the magnetic pole of the white dwarf by the equation:

$$\cos \theta = \cos i \cos \delta - \sin i \sin \delta \cos [2\pi(\phi - \phi_0)]. \quad (3.2)$$

Thus when the accretion column (or homogeneous plasma slab) is perpendicular to the line of sight $\cos \theta = 0$ and $\cot i \cot \delta = \cos [2\pi(\phi - \phi_0)]$. The phase $\phi = 0$ is chosen to correspond to the primary linear polarization pulse. Hence, when the field is perpendicular to the line of sight for the second time, the circular polarization changes sign at phase $\phi_2 = 2\phi_0$. However a second linear polarization pulse is seen infrequently at this phase.

In Figure 3.23, the smoothed version of the data of Tapia (1977) in the V band (5400 Å) is compared to the "best fit" of the theoretical calculations where $B$, $T$, $i$, $\delta$ and the slab thickness $l$ were varied. The parameter values that give the best fit are $B = 2.7 \times 10^7$ gauss, $kT = 0.2$ keV, $l = 2.6 \times 10^8$ cm, $i = 46^\circ$, and $\delta = 55^\circ$. From equation (2.58), it follows that $\phi_0 = 0.13$ while the circular polarization changes sign for the second time in each cycle at phase $\phi_2 = 0.26$ in agreement with Tapia (1977). By fitting the change in the position angle of the linear polarization, Brainerd and Lamb (1984) find $i = 35^\circ$ and $\delta = 60^\circ$ so that $\phi_2 = 0.19$. This disagreement is probably due to there being few observational data points near phase $\phi_2$ in the data we have used.

From Figure 3.24 we note that good agreement with the circular polarization is obtained everywhere except near phase $\phi = 0.5$. Since
Fig. 3.23.--Plots of fractional linear and circular polarization, $Q/V$, and visual magnitude versus phase for one cycle of AM Herculis. The symbol (+) corresponds to the observed data taken from Tapia (1977) and the solid line corresponds to the theoretical calculation. The plasma has a slab width $l = 2.6 \times 10^8$ cm, temperature $kT = 0.2$ keV, and is oriented parallel to a magnetic field of $2.7 \times 10^7$ gauss. The inclination $i = 46^\circ$ (the angle between the line of sight and the rotation axis) and $\delta = 55^\circ$ (the angle between the dipole axis and the magnetic field).
Fig. 3.24 -- Same as Fig. 3.23, except for key, \( R = 2 \times 10^7 \) gauss.

and \( I = 3.5 \times 10^8 \) cm.

\( \text{Fig. 3.24} -- \text{Same as Fig. 3.23, except for key, } R = 2 \times 10^7 \text{ gauss.} \)
the observed circular polarization curve is asymmetric, this is expected because there is no asymmetry in the model we have used. The ratio $Q/V$ derived theoretically in equation (2.58) is not in agreement with observations near phase $\phi \approx 0.5$. Since the theoretical expression is independent of $kT$, $N$ and $l$, if the criterion for large Faraday rotation is satisfied, this shows that a simple change of these values cannot overcome the discrepancy. The discrepancy must therefore arise because of other effects such as the breakdown of the assumption of homogeneity and symmetry of the accretion column. It is possible that a region of higher temperature, possibly the shock-heated region, emits an unpolarized optical flux which reduces the polarization (see e.g. Chanmugam and Dulk 1981). Such a flux may also explain the discrepancy with the visual light curve. The latter, except for the region near phase $\phi = 0.5$, may be understood to arise from a region of cross-sectional area $A \sim 10^{16}$ cm$^2$. The size of this area appears reasonable.

We must emphasize that the values we obtained to fit the data are not necessarily correct. First, the observed value of the magnetic field is probably closer to $2 \times 10^7$ gauss. (e.g. Schmidt, Stockman, and Maryon 1981). Secondly, the temperature of 0.2 keV appears to be too hot for the plasma above the shock and too cool for the shock region itself.

Nevertheless we are very surprised that such a simple model does fit most of the data so well. Whether or not this model is correct, such good fits provide encouragement for simple, but more realistic models in the future.

If the magnetic field is taken to be $2 \times 10^7$ gauss the best fits are obtained with $i = 46^\circ$, $\delta = 55^\circ$, $kT = 1$ keV and $l = 3.5 \times 10^6$ cm
Fig. 3.25.--Same as Fig. 3.24, except i = 35° and g = 60°.

AME HERCULIS
(Fig. 3.24). Figure 3.25 shows the theoretical calculations using Brainerd and Lamb's values for $\delta$ with $B = 2 \times 10^7$ gauss, and $kT = 1$ keV. The best fit was obtained for $l = 3.5 \times 10^8$ cm.

d) The Relativistic Approximation

In the application to AM Herculis using the collisional nonrelativistic approximation, all comparisons of the model calculations to the observations were made for a single spectral band (V band). More stringent requirements are obtained when the model calculations are applied simultaneously to the circular polarization and flux from two or more spectral bands. This type of comparison was made using the collisional nonrelativistic approximation. The data of AM Herculis used for this comparison was by Priedhorsky et al. (1978). They obtained circular polarization, color (B-V), and flux data (see Figs. 3.26-3.29) in the V and I spectral bands. Reasonable fits to the circular polarization observations were found from the results (not shown) of this analysis, but the fits to the fluxes in both bands were poor; specifically, the slope of the spectrum could not be fit properly. Therefore, comparisons to this data using the relativistic approximation and a two component model were attempted.

$Q$ and $V$ (see eqs. 2.57a,b) have been calculated using the two component model in the same manner as the single component model is used in section (b). The only difference is that the total flux is the sum of a polarized (cyclotron radiation) and an unpolarized component ($I_{tot} = I_{pol} + I_{unp}$). The origin of this unpolarized component is only
Fig. 3.26.—Plots of fractional circular polarization (lower) and V band flux (upper) versus phase for more than one cycle of AM Herculis. The symbol (+) corresponds to the observed data from Priedhorsky et al. (1978) and the solid line corresponds to the theoretical calculation. The plasma has a slab width $l = 3.5 \times 10^8$ cm, temperature $kT = 20$ keV, and is oriented parallel to a magnetic field of $2 \times 10^7$ gauss. The inclination $i = 35^\circ$ and $\delta = 60^\circ$. The unpolarized component contributes twice the flux as the polarized component. Notice that the level of polarization is nearly correct, but that the shape of the theoretical polarization curve varies too rapidly. The flux was normalized to that of the V band.
Fig. 3.27.—Same as Fig. 3.26, but for the I band. Notice that the circular polarization varies wildly and the I band flux is too low.
Fig. 3.28.—Same as Fig. 3.26, but $kT = 10$ keV.
Fig. 3.29.--Same as Fig. 3.26, but for the I band at $kT = 10$ keV. Notice that the flux level is nearly the same, but that the shape of the circular polarization curve is poor.
conjecture. The ratio of unpolarized flux to polarized flux was 2:1. All other model parameters remained the same as in the previous nonrelativistic approximation calculations in section (b).

The results of comparisons using the two component model are presented in Figures 3.26-3.29. The results for a 20 keV plasma temperature are poor for two reasons (see Figs. 3.26 and 3.27): (1) The slope of the spectrum between the I and V bands is fit poorly. The slope of the spectrum is too steep. (2) Even though the level of polarization appears correct in both spectral bands, the shape of the polarization is poor. It is too steep at some phases and too flat at other phases of the orbital period. Therefore, a lower temperature (kT = 10 keV) plasma was used. The results (see Figs. 3.28 and 3.29) at this temperature for the slope of the spectrum were much better. Comparisons of the circular polarization, though, were still poor (Figs. 3.28 and 3.29).

E) Conclusions

In section (b), we have corrected the collisionless calculations of the cyclotron absorption coefficients made in Chanmugam and Dulk's (1981) paper to include the effects of collisions and Thomson scattering. These effects increase the opacity and hence reduce the polarization of the emitted radiation. For example, in the calculations of Chanmugam and Dulk (1981) for plasmas with temperatures kT ~ 1 keV and harmonic number ~ 6, the circular polarization approached 100% for small $\theta$ while the linear polarization approached 100% for $\theta \approx \pi/2$. The effect of collisions and Thomson scattering is to reduce these values
substantially and bring them down to the observed values \( \sim 10\% \). In this regard it should be noted that the principal absorbing mechanism does not necessarily have to be due to cyclotron absorption. Any absorbing mechanism where \( \tau_x \neq \tau_o \) and with \( \tau_x, \tau_o \) not both much greater than 1 is sufficient to produce the observed levels of polarization.

In the previous papers of Chanmugam and Dulk (1981) and Meygitt and Wickramasinghe (1982), only qualitative comparisons between theory and observations were made. Quantitative comparisons are, however, extremely difficult because the observations are complex. Nevertheless, we have attempted to make such comparisons using the (still oversimplified) model of a homogeneous plasma slab in a parallel magnetic field. Surprisingly good agreement with the optical observations of AM Her (Tapia 1977) were obtained except near orbital phase \( \phi = 0.5 \). The disagreement between the predicted and observed values of \( Q/V \) near this phase shows that the simple model breaks down. The best fit value for the magnetic field of \( 2.7 \times 10^7 \) gauss is higher than the observed field of \( \sim 2 \times 10^7 \) gauss. This means that the optical emission must come from a higher harmonic and higher temperature than the best fit value of \( kT = 0.2 \) keV. This suggests that the shock-heated region, and hence the inhomogeneity in the plasma, must be taken into account (cf. Meygitt and Wickramasinghe 1984). Alternatively, the assumption that the plasma is Maxwellian may not be valid (Liebert and Stockman 1983). Finally, it should be emphasized that the treatment of Thomson scattering as an absorbing mechanism is not strictly correct.

In section (c), we have found that the slope of the spectrum is fit by a plasma with a temperature of 10 keV and that the polarization is reduced by the addition of an unpolarized component to the level seen in
the observations. Thus, we conclude that the polarized radiation comes from near the shock or postshock because the temperature of the polarized radiation is approximately the postshock temperature, and that the polarization is a mixture of polarized and unpolarized radiation. In reference to a previous comment on the transfer of polarized radiation in a hot magnetized plasma, the polarization must be emitted from a region where $\tau_{x,0} \sim 1$, because no polarization is emitted for $\tau_{x,0} \gg 1$ (blackbody radiation) or $\tau_{x,0} \ll 1$ (very weak intensity). Therefore, we conclude that the polarized radiation comes from an inhomogeneous temperature boundary layer around the postshock of the accretion column (inhomogeneities in electron number density in the boundary layer are probably not important because the plasma is already quite tenuous; cf. Meggitt and Wickramasinghe 1984).

We also conclude that the standstill seen in the V band is a result of the plasma becoming optically thin at small $\theta$ and high harmonics. Whereas in the I band, the plasma is not optically thin, because the emission is at lower harmonics and hence the opacity is larger.

Finally, if the postshock height is $\sim 10^8$ cm, then a proper treatment of Thomson scattering of the polarized radiation in the equations of radiative transfer should be made, because the value of the scattering (or in our case, absorption) coefficient is $\sim 10^8$ cm$^{-1}$. Otherwise, if the postshock height is much less $\sim 10^8$ cm, then Thomson scattering can probably be neglected (see Chapter 2, section I and Meggitt and Wickramasinghe 1984).
II. DQ HERCULIS BINARIES

(From the paper by Barrett and Chanmuyam 1984b)

a) Introduction

Cataclysmic variables are binary systems in which a white dwarf accretes matter from a red dwarf companion (Robinson 1976). The detection of strong optically polarized radiation (Tapia 1977) from a subclass of these systems, the AM Herculis binaries (Liebert and Stockman 1984) has been interpreted (Chanmuyam and Dulk 1981; Meggitt and Wickramasinghe 1982) as cyclotron radiation arising near a magnetic pole of the white dwarf of strength $B \sim \text{few } \times 10^7$ gauss. Conversely, the absence of optically polarized radiation from other cataclysmic variables, has been interpreted (e.g., Warner 1983; Jameson et al. 1982; Robinson and Nather 1983; Lamb and Patterson 1983) to imply that the white dwarf's magnetic field is weaker ($B \lesssim 10^7$ gauss) than in the AM Her binaries. This latter interpretation is, in general, incorrect and results in erroneous conclusions regarding the magnetic fields in these systems. In particular, we show that if $B \sim \text{few } \times 10^7$ gauss and the binary separation is sufficiently large to accommodate an accretion disk, then no significant optically polarized radiation would be expected. This is either because the unpolarized radiation from the accretion disk dilutes the polarized radiation from near the magnetic pole, or because the principal pulsed optical radiation observed is due to reprocessed radiation which is not significantly polarized.
b) Polarization

Consider accretion onto a magnetic white dwarf of mass $M_{\text{WD}} (= M_0)$ and radius $R_{\text{WD}} (= 5 \times 10^8 \text{ cm})$ from a companion star. The magnetic field controls the flow of the accreting matter within the magnetospheric (or Alfven) surface of radius $r_A$. If $r_A > > R_{\text{WD}}$, the accreting matter within the magnetosphere flows onto the magnetic pole(s) via an accretion column(s). If $r_A < r_{\text{orb}}$, where $r_{\text{orb}}$ is the orbital separation, an accretion disk forms outside the magnetosphere, as is observed in many CVs. This estimate is only a rough one. More accurately the transferred matter only has enough angular momentum to form a disk at radius $0.1r_{\text{orb}}$. Hence, for $r_A < 0.1r_{\text{orb}}$ a disk certainly forms. If $0.1r_{\text{orb}} < r_A < r_{\text{orb}}$, it appears plausible that a ring of disklike structure could form outside the magnetosphere. Such a ring of gas, like an accretion disk would also emit unpolarized radiation. If $r_A > r_{\text{orb}}$, as is believed to occur in the AM Herculis binaries, no accretion disk is formed and the white dwarf rotates synchronously (Chanmugam and Wayner 1977; Stockman et al. 1977). Whether a disk is formed or not, a standing shock is formed in the accretion column near the surface of the white dwarf. The temperature (Hoshi 1973) of the postshock region is given by $kT_s \sim 10 \text{ keV}$ (see Chapter 1) and is independent of the mass accretion rate $\dot{m}$. The polarized radiation is believed to arise in or just above the postshock region, at cyclotron harmonic number $\sim 5$ to 15 (Chanmugam and Dulk 1981; Megyitt and Wickramasinghe 1982). Only fields $\sim 5 \times 10^7 \text{ gauss}$ place these harmonics in the optical band. For weaker fields ($B \lesssim 10^7 \text{ gauss}$) the radiation is polarized at IR and longer wavelengths and for stronger
fields \((B \geq 5 \times 10^7 \text{ gauss})\) polarized at UV and shorter wavelengths. If \(A\) is the cross-sectional area at the base of the accretion column and the height of the postshock region is \(\lesssim A^{1/2}\) the luminosity of the optically polarized radiation, at a frequency \(\omega_{\text{opt}}\), is limited by the Rayleigh-Jeans luminosity from the postshock region and satisfies

\[
L_{\text{pol}} \lesssim A k T_s \omega_{\text{opt}}^3 / 12 \pi^2 c^2. \tag{3.3}
\]

The value of \(A\) (= \(4\pi R^2 f\)) is difficult to estimate. Different estimates for \(f\) have been made for the AM Herculis binaries and range widely: \(f \sim 10^{-4}\) to \(10^{-5}\) (Patterson et al. 1984; see also Imamura and Durisen 1983), \(f \sim 10^{-3}\) (Raymond et al. 1979) and up to possibly \(f \sim 0.1\) (King and Shaviv 1984).

Next consider CVs with roughly the same system parameters as in the AM Herculis binaries, but having a sufficiently large orbital separation, \(r_A \lesssim r_{\text{orb}}\), so that an accretion disk may then form in these systems. Since all of the AM Herculis binaries (Robinson 1976) have orbital periods \(P_{\text{orb}} < 3.7\) hr, these CVs are likely to have \(P_{\text{orb}} \gtrsim 4\) hr. If the disk is optically thick, the disk luminosity, at optical frequencies, is given by \(L_d = \alpha \pi R_d^2 k T_d \omega_{\text{opt}}^3 / 12 \pi^2 c^2\), where \(R_d \sim 5 \times 10^4 \) cm is the radius, and \(T_d \sim 10^4 \) K is the temperature of the disk. The quantity \(\alpha \sim 0.5\) corrects for the inaccuracy of the Rayleigh-Jeans approximation. The radiation from the disk would be unpolarized and would exceed the luminosity of the optically polarized radiation from the accretion column by a factor of at least \((R_d^2/4fR^2) (T_d/T_s) \approx 0.13 f^{-1}\) if \(f \ll 0.1\). Now, the fractional polarization of the radiation emitted by the accretion column is unlikely to exceed the maximum value
of \approx 40\% \text{ (Liebert and Stockman 1984)} observed in the AM Herculis binaries. Thus, if \( f \lesssim 10^{-3} \) as has been suggested by several authors (e.g. Patterson \textit{et al.} 1984; Raymond \textit{et al.} 1979) the fractional observable polarization would be \(< 1.0\%\). Such weak polarization is difficult to detect. If \( f \) is large (> 0.1), then it would appear that significant polarization may be expected. However, if the polar cap is large then the region emitting cyclotron radiation will contain an inhomogeneous magnetic field. If the polar cap, assumed to be circular, subtends an angle \( 2\beta \) at the center of the star, then \( \cos \beta = 1 - 2f \).

Thus, if \( f = 0.1 \) to 0.25, \( \beta = 37^\circ \) to \( 60^\circ \). Hence, for a dipolar field \( B_{r,\theta,\phi} = B_0(R/r)^3 \) \( (2 \cos \alpha, \sin \theta, 0) \), in obvious spherical polar coordinates, the strength and the direction of the field varies significantly across the polar cap. Also for a given accretion rate, the postshock height \( h = f \), if bremsstrahlung is the dominant cooling mechanism, and can become comparable to the radius of the star. In addition for accretion from a disk, the magnetic field is likely to be sheared so that the inhomogeneity of a dipole field is likely to be enhanced. Thus because of a combination of these factors, the emitted polarization would be lower than in the AM Herculis binaries, but a precise estimate is difficult to make.

These results are relevant for the DQ Herculis binaries (Robinson 1976; Lamb and Patterson 1983) which are another subclass of CVs. These binaries are believed to contain asynchronously rotating magnetic white dwarfs (see however Robinson and Nather 1983; where it is suggested that one of these, V533 Herculis, is instead a pulsating white dwarf). There is no direct evidence for magnetic fields in these binaries, since they neither emit detectable polarized light nor show spectroscopic evidence
of magnetic fields, as in the AM Herculis binaries. Estimates for the fields have been deduced from the spin-up properties of the white dwarf for disk accretion, and range from \( \sim 6 \times 10^4 \) gauss for AE Aquarii to \( 3 \times 10^5 - 1 \times 10^8 \) gauss for TV Columbae (Lamb and Patterson 1983). However, these values are model dependent (Anzer and Börner 1983). In the past, the absence of optically polarized radiation has given preference to the lower values \( (B \lesssim 10^7 \) gauss) ( Warner 1983; Jameson et al. 1982; Lamb and Patterson 1983).

The optical radiation from these systems may show variations with one or more periods corresponding to the rotational and orbital periods (e.g. Warner 1983). Radiation with the rotation period, presumably equal to the period of X-ray variations when observed, probably arises in the accretion columns of the white dwarf. It has been suggested that the optical variations with the beat period arises as a result of reprocessing of the X/EUV flux by a component of the system which remains stationary in the rotating frame (Hassall et al. 1981; Patterson and Price 1981), such as the companion star or the accretion disk. Such reprocessed radiation is unlikely to have significant polarization. It should be emphasized that there are energetic (Patterson and Price 1981) and geometrical (King and Shaviv 1984) difficulties in models invoking reprocessing.

c) Discussion

Any significant polarized radiation emitted by the DQ Herculis binaries should arise in the accretion column of the magnetic white
dwarf and hence will show variations with the rotation period of the white dwarf. If the amplitude of these variations is $\xi$ and if it is assumed that the unpulsed radiation is unpolarized, any polarized radiation emitted would be diluted by a factor $\sim 1/\xi$. Optical observations show that $\xi$ varies from $\sim 0.01$ for AE Aquarii (Patterson 1979) to $\sim 0.4$ for H 2215-086 (Shafter and Targon 1982). However, it is not always clear whether these amplitudes correspond to optical variations with the rotation period. For example, H 2252-035 exhibits three distinct periods corresponding to: the orbital period of 3.59 hr, the X-ray period of 805 seconds which is most likely the rotation period of the white dwarf, and one of 859 seconds which is the beat period between the other two. The amplitudes of the optical variations are 0.1, 0.02 (Warner et al. 1981) and 0.05, respectively. Thus if this system had a sufficiently strong magnetic field to emit polarized light, it would be diluted by a factor $\sim 50$ which would be difficult to detect. H 2215-086 shows large amplitude oscillations $\sim 0.4$ with a period of 21 minutes. However, no X-ray pulsations have been detected yet from this system so it is unknown whether or not these large amplitude oscillations arise from the accretion column. If they do, then the absence of optical polarization would be consistent with the system having a magnetic field which is not in the range $(1-5) \times 10^7$ gauss, or would imply that there is significant depolarization of the radiation emitted by a large polar cap. We note that large polar cap models also have difficulties. If the X-ray emission is due to bremsstrahlung radiation, a large postshock height $h \gtrsim R$ is required in order to account for the emission measure.

In conclusion, we emphasize that the absence of optical
polarization does not necessarily imply that the system has a weak magnetic field ($< 10^7$ gauss). This implies that there may exist several systems with orbital periods $> 4$ hr in which the white dwarf has a strong magnetic field. This also supports the hypothesis that the AM Her binaries were once DQ Her binaries in the past when their orbital separation was sufficiently large to allow formation of an accretion disk (Chanmuyam and Ray 1984).
I. INTRODUCTION

The ten known AM Herculis binaries form a subclass of cataclysmic variable stars. Their distinguishing characteristic is the emission of strongly polarized light (~10\%, Tapia 1977). Characteristics more in common with other classes of cataclysmic variables are rapid optical variability (flickering) and X-ray variability (see e.g. the reviews by Chiappetti, Tanzi, and Treves 1980; and Liebert and Stockman 1984). The standard model for these systems envisions a magnetic white dwarf accreting matter from a red dwarf companion. The strong magnetic field prevents formation of an accretion disk; instead the accreting matter flows along the magnetic field lines and is funneled into an accretion column above one or both the magnetic poles of the white dwarf (Chanmugam and Wagner 1977). A static shock above the magnetic poles slows and heats the infalling matter, so that the hot postshock plasma (with a temperature \( T \) where \( kT \sim 10 \) keV; Hoshi 1973) settles onto the white dwarf surface.

Several methods have been used to determine the magnetic fields of AM Herculis binaries. The first method explains the large fractional optical polarization, as being due to cyclotron emission from the hot postshock region or its vicinity at cyclotron harmonics \( n \sim 5-20 \). In order to place these harmonics in the optical band, one requires magnetic fields \( \sim (1-4) \times 10^7 \) gauss (see Chanmugam and Uulk 1981; Meggitt and Wickramasinghe 1982; and Barrett and Chanmugam 1984). The second method uses Zeeman spectroscopy and has led to the identification of magnetic fields of \( \sim 2 \times 10^7 \) gauss in AM Her (Schmidt, Stockman, and Margon 1981; Latham, Liebert and Steiner 1981; Hutchings, Crampton and Cowley 1981), CW11U3 + 2b4 (Schmidt, Stockman and Grandi 1983), and
HU139-268 (Wickramasinghe, Visvanathan, and Tuohy 1984). This method leads to the determination of the magnetic field at the surface of the white dwarf. However, it must be used when the system is in a "low" state (a state when the system is optically faint, implying low accretion rates) or a faint phase (an orbital phase when the white dwarf eclipses the shock and postshock regions). It gives no direct information about the accretion column. The third method uses observations of cyclotron lines and is potentially the most accurate one for determining the magnetic field strength in the accretion column. However, it has been the least fruitful since only one AM Herculis binary, VV Puppis, during one time span, February 1979 (Visvanathan and Wickramasinghe 1979; Stockman, Liebert and Bond 1979) has shown these spectral features corresponding to a field of $3.08 \times 10^7$ gauss. The advantage of this method is that it can be used to probe the structure of the accretion column. This method was also used to obtain the first direct measurement of a magnetic field of a neutron star ($B \sim 5 \times 10^{12}$ gauss, Trümper et al. 1977; Voges et al. 1983) and a solar flare ($B \sim 10^4$ gauss, Willson 1984).

There are several papers on the subject of cyclotron lines in accreting magnetic white dwarfs. In particular, Mitrofanov (1980) analyzed the cyclotron line profile at the fundamental and second harmonics for different optical depths. The calculations were made using a non-relativistic theory which took into account absorption in the ordinary and extraordinary modes. Mitrofanov made the assumption that the magnetic fields of AM Herculis stars were of the order of $10^5$ gauss, so that the fundamental and second cyclotron harmonics were in the visible spectrum. However, the determination of $B = 3.08 \times 10^7$
gauss in VV Puppis by VW and of $B = 2.9 \times 10^9$ gauss by Wickramasinyhe and Visvanathan (1980) was based on the assumption that the broad spectral features, equally spaced in frequency, were absorption lines at cyclotron harmonics $n = 6, 7$ and $8$. A more recent analysis by Wickramasinghe and Meggitt (1982) however, concludes that the spectral features are actually cyclotron emission lines. The broad peaks correspond to cyclotron harmonics $n = 6, 7, 8$; with the radiation emitted perpendicular to the magnetic field of strength $3.18 \times 10^7$ gauss in a plasma of characteristic temperature $kT \sim 10$ keV, and dimensionless parameter $\Lambda = \omega_p^2 l/\omega_B^2 c \sim 10^5$ (Wickramasinghe and Meggitt 1982). Here, $\omega_p$ is the plasma frequency, $l$ is the thickness of the plasma slab, and $\omega_B = eB/mc$ is the cyclotron frequency.

The purpose of this chapter is to study in further detail the formation of cyclotron lines in accreting magnetic white dwarfs. In section II, the absorption coefficients for cyclotron radiation are discussed and a table comparing the methods of Chanmuyam and Uulk (1981) to Meggitt and Wickramasinghe (1982) is given. Next, a general analysis of the cyclotron spectrum is presented for a homogeneous plasma slab parallel to a constant magnetic field. Several figures show how the spectrum changes with various plasma temperatures, viewing angles, optical depths and magnetic field strengths. Finally in this section, a simple equation relating the frequency of the cyclotron line in terms of the cyclotron frequency, harmonic number, plasma temperature, and viewing angle is presented. The homogeneous model is then applied in section III to the observations of cyclotron lines and circular polarization in VV Puppis (Stockman, Liebert, and Bond 1979; Visvanathan and Wickramasinghe 1979). The results obtained from numerical fits to
the data are compared to the results of Wickramasinghe and Meggitt (1982). Finally, in section IV, we calculate the size of the postshock region and the distance to VV Puppis using the results of section III. The orientation of the accretion column of VV Puppis is also briefly discussed.

II. ABSORPTION AND EMISSION OF CYCLOTRON LINES FROM A PLASMA SLAB

To calculate the absorption or emission of radiation from a source, it is necessary to solve the equations of radiative transfer. In general, the equations are a set of four linear integro-differential equations for the Stokes intensities I, Q, U, and V in three spatial variables (Chandrasekhar 1960). If the effects of scattering are small, the equations simplify to a set of four linear partial differential equations. Normally, the polarizable effects of the medium are neglected and the set of four linear partial differential equations are reduced to a single linear partial differential equation for the total radiation intensity. For a magnetoactive plasma, the effects of polarization can be quite large and therefore should be considered (Ginzburg 1970). Two methods are frequently used to solve the equations. The first step of these methods involves reducing the equations to a set of linear differential equations in one spatial variable by using the geometry of a plane parallel atmosphere or an infinite slab. One method presented by Pacholczyk (1977) then solves a system of four coupled equations in I, Q, U, V in the limit of large Faraday rotation (Meggitt and Wickramasinghe 1982). The other method makes the assumption of large Faraday rotation at the outset (Kamaty
1969). This allows the set of four coupled equations to be reduced to two uncoupled equations in $I_0$, $I_x$, one for each mode (ordinary and extraordinary) of wave propagation. We have used this method in the present paper (see also Chanmugam and Dulk 1981; and Barrett and Chanmugam 1984). The result derived by Ramaty (1969) is

$$\frac{dI_{x,0}}{dz} + \alpha_{x,0}(\omega,\theta) I_{x,0}(\omega,\theta) = J_{x,0}(\omega,\theta),$$

where $I_{x,0}$ is the specific intensity, $\alpha_{x,0}$ the absorption coefficient, and $J_{x,0}$ the emissivity, for the extraordinary ($X$) mode and the ordinary ($O$) mode. For a Maxwellian distribution of electrons, Kirchhoff's Law is applied to obtain the absorption coefficient $\alpha_{x,0}$ in terms of the blackbody intensity and the emissivity:

$$\alpha_{x,0}(\omega,\theta) = J_{x,0}(\omega,\theta)/(n_{x,0} I_{RJ}),$$

where $n_{x,0}$ is the index of refraction for both modes, and $I_{RJ} = kT\omega^2/8\pi^3c^2$ is the Rayleigh-Jeans intensity for each mode. Therefore, a solution for $I_{x,0}$ is found in terms of just the absorption coefficient $\alpha_{x,0}$. The absorption coefficient for cyclotron radiation depends on four independent variables: the frequency of the radiation $\omega$, the plasma temperature $kT$, the angle $\theta$ between the magnetic field direction $B$ and the wave vector $k$, and the dimensionless constant $\Lambda$. Calculations of the total absorption coefficient ($\alpha = 1/2 (\alpha_x + \alpha_O)$, in units of $\omega_p^2/\omega_Bc$), were made for various temperatures, angles $\theta$, and $\omega/\omega_B$ for both the first method employed by Meggitt and Wickramasinghe (1982) and the second method given by Chanmugam and Dulk (1981) (Table
Visvanathan and Wickramasinghe (1979) and Stockman, Liebert, and Bond (1979) initially assumed that the broad features in the spectrum of VV Puppis were caused by absorption in a cool magnetized plasma surrounding the postshock region of the accretion column and determined the polar magnetic field to be $3.08 \times 10^7$ gauss. We thought it useful to include calculations of absorption in a magnetized plasma slab. Our simple model (Figure 4.1) has a magnetized plasma slab with $B = 3 \times 10^7$ gauss aligned parallel to the slab's surface. A blackbody source with temperature $kT = 30$ keV is behind the plasma slab. This background source is meant to represent the postshock region shining on the cooler foreground plasma slab of temperature $kT = 10$ keV. Assuming $J_{x,0} = 0$ for the plasma slab, we obtain a solution to equation (4.1):

$$I(\omega, \theta) = I_{RJ}(kT_S) \left( \exp(-\tau_x) + \exp(-\tau_0) \right),$$

(4.3)

where $kT_S$ is the temperature of the background source and $\tau_{x,0} = \alpha_{x,0}^1$ the optical depth of the plasma. Theoretical spectra (Figure 4.2) for this absorption model are then calculated using equation (3) and the collisionless cyclotron absorption coefficients (Chanmugam and Dulk 1981). We note the major features of these spectra (Wickramasinghe and Meygitt 1982; Masters 1978): (1) the troughs are wide and rounded, (2) the crests are narrow and sharply peaked, (3) the troughs have an extended blue wing, and (4) the spectrum rises steeply toward the blue. The troughs correspond to the cyclotron lines in absorption with the profile of the lines being asymmetric. The asymmetry exists because a cyclotron line is the sum of the absorption
### TABLE 4.1

Values of Total Cyclotron Absorption Coefficient

<table>
<thead>
<tr>
<th>$kT$ (keV)</th>
<th>$\theta$</th>
<th>$\omega/\omega_B$</th>
<th>$\log(\alpha_{MW})$</th>
<th>$\log(\alpha_{CD})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>90.0</td>
<td>5.93</td>
<td>-7.64</td>
<td>-7.64</td>
</tr>
<tr>
<td>1.0</td>
<td>78.5</td>
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<td>-8.01</td>
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<td>1.0</td>
<td>66.4</td>
<td>5.93</td>
<td>-8.51</td>
<td>-8.50</td>
</tr>
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<td>53.1</td>
<td>5.93</td>
<td>-9.12</td>
<td>-9.18</td>
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<td>-10.31</td>
<td>-10.46</td>
</tr>
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<td>-5.09</td>
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<td>-5.25</td>
</tr>
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<td>-5.65</td>
</tr>
<tr>
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<td>-6.25</td>
</tr>
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</tr>
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<td>-5.00</td>
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<td>-4.98</td>
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<td>-5.96</td>
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<td>10.0</td>
<td>90.0</td>
<td>10.00</td>
<td>-7.12</td>
<td>-7.16</td>
</tr>
<tr>
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<td>4.00</td>
<td>-2.99</td>
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<td>-7.47</td>
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<tr>
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<td>60.0</td>
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<td>-7.36</td>
<td>-8.35</td>
</tr>
<tr>
<td>30.0</td>
<td>30.0</td>
<td>18.00</td>
<td>-9.14</td>
<td>-9.21</td>
</tr>
</tbody>
</table>
Fig. 4.1.—Diagram of a hot \((kT = 30\, \text{keV})\) blackbody source shining on a cooler \((kT = 10\, \text{keV})\) infinite plasma slab of thickness \(l\) with a magnetic field \((B = 3 \times 10^7\, \text{gauss})\) parallel to the surface. The radiation, with wave vector \(\mathbf{k}\), propagates at an angle \(\theta\) to the magnetic field. This diagram describes the geometry of the absorption model. The emission model is identical to the absorption model except for the omission of the background source.
Fig. 4.2--Theoretical absorption spectra for $\theta$ (the angle between the magnetic field $B$ and the wave vector $k$) = $90^\circ$, $75^\circ$, $60^\circ$ and $\Lambda = \omega_p^2 / \omega_B c = 10^6, 10^5, 10^4$ at 10 keV and $B = 3 \times 10^7$ gauss.
from that harmonic and all higher cyclotron harmonics. At finite
temperatures, each cyclotron harmonic is shifted in frequency by
different amounts, resulting in greater absorption on the blue side of
the line center than the red side (see discussion of eq. 4.6). The
steep rise in the blue region of the spectrum results from our
assumption that the background source is an optically thick plasma at a
high temperature of 30 keV. The emission can then be approximated
throughout the entire optical spectrum by a Rayleigh-Jeans spectrum.

Next, a simple model for the emission from a plasma slab was
considered. In this model, the plasma slab represents the postshock
region of the accretion column. We solve equation (4.1) by including
the emissivity of the plasma slab, assuming no incident flux on it.
This is done by substituting equation (4.2) into equation (4.1) to
obtain:

\[ I(\omega,\theta) = I_{\text{RJ}}(kT) \left[ 2 - \exp(-\tau_x) - \exp(-\tau_0) \right] . \quad (4.4) \]

The theoretical spectra (Figure 4.3-4.6) for this model show features
which are the inverse of those features in the absorption model. The
major spectral features present in the emission model spectra are: (1)
deep, narrow troughs, (2) broad, rounded crests, (3) the troughs have an
extended red wing, and (4) the flux decreases toward the blue
for \( \lambda < 10^6 \). Now, the broad peaks correspond to the cyclotron lines and
the asymmetry in the troughs is reversed. A notable feature of this
model is the decrease of the flux toward the blue as the plasma slab
becomes optically thin.

The four independent variables affect the appearance of cyclotron
Fig. 4.3.--Theoretical emission spectra for $\theta = 90^\circ$, $75^\circ$, $60^\circ$, and $\Lambda = 10^8, 10^7, 10^6$ at $kT = 5$ keV and $B = 3 \times 10^7$ gauss.
Fig. 4.4.—Same as Fig. 4.3, except $\Lambda = 10^6, 10^5, 10^4$, and $kT = 10$ keV.
Fig. 4.5.—Same as Fig. 4.3, except $\Lambda = 10^6$, $10^5$, $10^4$, and $kT = 15$ keV.
Fig. 4.6.--Same as Fig. 4.3, except $B = 2 \times 10^7$ gauss and $kT = 10$ keV.
lines in several ways. The most important of these is the plasma
temperature. As the plasma temperature increases, the cyclotron lines
broaden and become less distinctive until they completely disappear.
Wickramasinghe and Meggitt (1982) restricted the temperature for the
production of cyclotron lines to $kT < 20 \text{ keV}$. However, Figure 4.6 shows
that these features are barely visible at 15 keV. We believe $\approx 15 \text{ keV}$
is probably a better upper limit on the plasma temperature for $A > 10^5$,
if distinct cyclotron lines are to be seen from a hot plasma. Many
other contributing factors; such as background noise, the shift in the
frequency of the lines with $\theta$, and the inhomogeneous temperature
distribution, will degrade the spectral quality even further. Another
important limiting factor to the visibility of cyclotron lines is the
strength of the magnetic field. For $B < 1.5 \times 10^7 \text{ gauss}$, one requires $s$
$> 10$ in the visible spectrum. At these high harmonics, the lines become
broader and closer to one another. Eventually, the individual harmonics
become indistinguishable and the spectrum forms a continuum. The
harmonic at which the spectrum becomes a continuum, decreases with
increasing temperature. Finally, the cyclotron lines become smeared
together because of Doppler broadening when $\theta < 60^\circ$ and $kT > 10 \text{ keV}$. For $kT < 10 \text{ keV}$, the cyclotron lines become smeared together at smaller
angles because Doppler broadening is less important. Most AM Her
binaries have $kT > 10 \text{ keV}$, so Doppler broadening is significant.
Therefore, for cyclotron lines to be observable in these systems, the
accretion column probably needs to be approximately perpendicular to the
line of sight ($\theta \sim 90^\circ$) for large fractions of the orbital period with
the postshock temperature $\lesssim 15 \text{ keV}$.

Hirshfield et al. (1961) presented a temperature dependent formula
for the frequency of cyclotron lines:

\[ \frac{\omega}{\omega_B} = n(1 - s/\eta) \]  

(4.5)

where \( s \) is the harmonic number and \( \eta = mc^2/kT \). They obtained this formula by considering a two dimensional Maxwellian distribution of electrons and propagation of radiation perpendicular to the magnetic field. We have generalized the equation of Hirshfield et al. (1961) by considering a three dimensional Maxwellian distribution for the electrons and propagation at an angle \( \theta \) to the magnetic field (see Appendix for derivation) to obtain:

\[ \frac{\omega}{\omega_B} = \left[ -1 + (1 + 8s \sin^2\theta/\eta)^{1/2} \right] / (4s \sin^2\theta/\eta) \]  

(4.6)

Our equation could be very useful in determining the strength of the magnetic field, the temperature, and the geometry of the cyclotron emitting region. We will later apply equation (4.6) in section III.

III. FIT TO OBSERVATIONS OF VV PUPPIS

(a) The Cyclotron Lines

The brightness of AM Herculis binaries alternate between "high" and "low" states. The high state is normally 2-3 magnitudes brighter than the low state and may last from several months to many years. The high state is the result of high accretion rates onto one or both magnetic poles, whereas the low state corresponds to low accretion rates onto
either pole. VV Puppis has been observed in both states. Normally, the high state has $m_{\text{max}} \sim m_{\text{min}} \sim 15$ (Wickramasinghe and Megyitt 1982) (though $m_{\text{max}} = 13.9$ has been observed, Visvanathan and Wickramasinghe 1981), where $m_{\text{max}}$ corresponds to the maximum and $m_{\text{min}}$ to the minimum brightness of the system during an orbital period. Both poles were believed to be accreting at that time, because circular polarization measurements showed a constant negative polarization outside of the bright phase (Figure 4.1 of Liebert and Stockman 1979; Visvanathan and Wickramasinghe 1981). During the low state, the system's brightness drops to $m_{\text{max}} \sim m_{\text{min}} \sim 17.8$ with an observed lack of significant H and He emission and flickering (Liebert et al. 1978). These observations indicate little accretion during the low state. In February 1979, VV Puppis was between high and low states. The brightness of the system varied from $m_{\text{max}} \sim 15.7 - 16.5$ to $m_{\text{min}} \sim 16.9$ to $m_{\text{min}} \sim 17.8$ at the end of the month (Stockman, Liebert, and Bond 1979). Presumably, only one pole was accreting during this time span because of the large change in brightness between bright and faint phases and the lack of negative circular polarization during the faint phase (Wickramasinghe and Megyitt 1982). This period of time was when the cyclotron lines were observed by Visvanathan and Wickramasinghe (1979) and Stockman, Liebert, and Bond (1979).

We first study the spectrophotometric data of Stockman, Liebert, and Bond (1979), specifically their Figure 2, instead of Visvanathan and Wickramasinghe (1979) data, for two reasons: (1) the phase dependence of their spectra is better, and (2) the frequency range of their spectra is wider. Stockman, Liebert, and Bond's (1979) data covers an entire orbital period ($P \sim 100$ min.) broken up into nine spectral frames of
eight minutes each. Frames 6, 7, and 8 are the observations which contain the bright phase of the orbital period and exhibit the cyclotron lines. A comparison of the absorption model calculations (Figure 4.2), the emission model calculations (Figures 4.3-4.6), and the data of Stockman, Liebert, and Bond (1979) (Frames 6-8 of their Figure 2), show that the emission model represents the data much better than the absorption model. Thus, we confirm the conclusion of Wickramasinghe and Meygitt (1982) that the cyclotron lines are a result of cyclotron emission from a single accreting magnetic pole, and not the result of absorption through a cooler plasma boundary layer as proposed earlier by Visvanathan and Wickramasinghe (1979) and Stockman, Liebert, and Bond (1979).

By making a detailed examination of Frames 6-8, we find that the minimum of the trough, at a wavelength $\lambda = 0.57$ microns, is displaced by about 0.01 micron in Frame 7 from its value in Frames 6 and 8. This shift in wavelength (or frequency) of the cyclotron lines could be a result of changes in the magnetic field strength, the plasma temperature, or the orientation of the accretion column. We believe a change in the orientation of the accretion column with the magnetic field strength and plasma temperature remaining constant, the most likely explanation. In the succeeding few paragraphs, we will show using equation (4.6), how one may place limits on the orientation of the accretion column by observing the frequency shift of the cyclotron lines at various angles $\theta$.

If we assume that the orientation of the accretion column during Frames 6 and 8 was perpendicular to the line of sight, then $\theta = 90^\circ$. Equation (4.6) can then be used to determine the strength of the
magnetic field. The wavelengths for the maxima of the broad peaks are \( \lambda = 0.60, 0.525, \) and 0.47 microns for Frames 6 and 8 of Stockman, Liebert, and Bond (1979) and Frame 1,2,9-11 of Visvanathan and Wickramasinghe (1979). Since the cyclotron lines are nearly equally spaced in frequency, it is not obvious which set of cyclotron harmonics at a specific temperature fits the data correctly. The correct set of cyclotron lines can only be found by trial and error as shown in Table 4.2. We see that at \( kT = 0 \) keV, a magnetic field strength of \( 2.56 \times 10^7 \) gauss is consistent for cyclotron harmonics \( n = 7, 8, \) and 9, and at 10 keV, a magnetic field strength of \( 3.56 \times 10^7 \) gauss is consistent with \( n = 6, 7, \) and 8. We may determine which of the two solutions is more correct by also considering the width of the cyclotron lines. The \( kT = 0 \) keV solution is clearly inconsistent with the observed broad line width, unless this is due to other inhomogeneities. We are then left with the \( kT = 10 \) keV solution and a magnetic field strength of \( 3.56 \times 10^7 \) gauss.

Now equation (4.6) can be used to find the wavelength of the cyclotron harmonics for angles other than \( 90^\circ \). As an example, the wavelength for the center of the trough between cyclotron harmonics \( n = 7 \) and 8 is 0.575 microns at \( \theta = 90^\circ \). Table 4.3 shows the wavelengths of this trough for other angles. The shift of \( \approx 0.01 \) microns between Frame 7, and Frames 6 and 8 corresponds to a change in the angle \( \theta \) from \( 90^\circ \) to about \( 70^\circ \).

We also fit the emission model in section II of this chapter, using a chi squared minimization analysis, to the observations in Frames 6, 7, and 8. The free parameters used were the magnetic field, the plasma temperature, the angle \( \theta \), and the dimensionless parameter \( \Lambda \). As stated
### TABLE 4.2a

**Magnetic Field Strength for Cyclotron Lines**

at \( kT = 0 \text{ keV} \)

\[
\begin{array}{ccc}
\text{B/10}^6 \text{ (gauss)} & \text{group 1} & \text{group 2} & \text{group 3} \\
5 & 35.70 & \cdots & \cdots \\
6 & 34.00 & 29.75 & \cdots \\
7 & 32.55 & 29.14 & 25.50 \\
8 & \cdots & 28.48 & 25.50 \\
9 & \cdots & \cdots & 25.32 \\
\end{array}
\]

### TABLE 4.2b

**Magnetic Field Strength for Cyclotron Lines**

at \( kT = 5 \text{ keV} \)

\[
\begin{array}{ccc}
\text{B/10}^6 \text{ (gauss)} & \text{group 1} & \text{group 2} & \text{group 3} \\
5 & 38.85 & \cdots & \cdots \\
6 & 37.57 & 32.88 & \cdots \\
7 & 36.49 & 32.67 & 28.59 \\
8 & \cdots & 32.37 & 28.98 \\
9 & \cdots & \cdots & 29.16 \\
\end{array}
\]
TABLE 4.2c
Magnetic Field Strength for Cyclotron Lines
at kT = 10 keV
B/10^6 (gauss)

<table>
<thead>
<tr>
<th>s</th>
<th>group 1</th>
<th>group 2</th>
<th>group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>41.67</td>
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<td>...</td>
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<td>9</td>
<td>...</td>
<td>...</td>
<td>32.29</td>
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</table>

footnote to table 4.2.

The observational spectra give wavelengths of three distinct cyclotron lines which are to be associated with three cyclotron harmonics. If the lines are taken to be cyclotron harmonics n = 5, 6, and 7 (group 1) for example, then a magnetic field strength may be calculated using equation (4.6) for each harmonic. A probable solution is found when each cyclotron harmonic in the group has the same magnetic field strength.
### TABLE 4.3

**Wavelength of Trough at Various Values of θ.**

<table>
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<tr>
<th>θ</th>
<th>λ (microns)</th>
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<td>70°</td>
<td>0.565</td>
</tr>
<tr>
<td>60°</td>
<td>0.554</td>
</tr>
</tbody>
</table>

### TABLE 4.4

**Best Fit Parameters to Figure 4.7 of Stockman, Liebert, and Bond (1979)**

<table>
<thead>
<tr>
<th>Frame</th>
<th>B/10^6 (gauss)</th>
<th>kT (keV)</th>
<th>θ</th>
<th>Λ/10^5</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>31.8</td>
<td>8.8</td>
<td>90.0</td>
<td>7.7</td>
</tr>
<tr>
<td>7</td>
<td>31.4</td>
<td>7.1</td>
<td>85.0</td>
<td>50.1</td>
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<td>8</td>
<td>31.5</td>
<td>8.7</td>
<td>88.5</td>
<td>19.3</td>
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</table>
the absorption coefficient and are in excellent agreement with one another.

Wickramasinghe and Meygitt (1982), fit theoretical model intensities to data taken from Visvanathan and Wickramasinghe (1979) and Stockman, Liebert, and Bond (1979) of the mean bright phase intensity spectrum. Theoretical models fit to the mean bright phase intensity data will result in higher temperatures than theoretical models fit to phase dependent bright phase intensity data, such as that of Stockman, Liebert, and Bond (1979). This occurs because the frequency of the cyclotron line width is narrowest at $\theta = 90^\circ$. As $\theta$ decreases, the peak frequency of the line changes and the width of the line broadens. For example, the mean bright phase flux of Wickramasinghe and Meygitt (1982), has $\theta$ vary from $90^\circ$ to about $70^\circ$. Whereas, Frame 6 of Stockman, Liebert, and Bond (1979) is taken over a shorter interval of time and $\theta$ probably varies from $90^\circ$ to about $85^\circ$. Note however that the width of the cyclotron line becomes broader as the temperature of the plasma increases. If theoretical models are fit to mean bright phase data at a specific angle of $\theta$, then the required temperature will increase to compensate for the broader line width of these observations. The higher temperature then increases the amount of absorption in the plasma which results in a lower value for $\Lambda$.

Wickramasinghe and Meygitt (1982), fit the data of VV Puppis in this manner. This explains why our values for the temperature of the plasma are lower than Wickramasinghe and Meygitt (1982), and also why we obtain a significantly larger value for $\Lambda$.

Wickramasinghe and Meygitt (1982) showed that the orientation of VV Puppis did not vary by more than $\Delta\theta = 20^\circ$ during the bright phase. They
earlier in this section, we decided to fit frames 6, 7, and 8, though the noise in these spectra are larger than in Figure 3 (which is a composite of Frame 6, 7, and 8) of Stockman, Liebert, and Bond (1979), to obtain more information on the orientation of the accretion column.

The values which best fit the data are given in Table 4.4 and are presented in Figure 4.7. The value for the magnetic field strength, $B = 3.15 \times 10^7 \text{ gauss}$, is consistent with the value found by Wickramasinghe and Meggitt (1982), but significantly less than the value found by using equation (4.6). We note that the calculations are very sensitive to the magnetic field strength, and therefore three digit accuracy in the results is not unreasonable. The plasma temperature found in the analysis ($kT = 8.7 \text{ keV}$) is lower, but not significantly lower, than the plasma temperature indicated by the analysis of Wickramasinghe and Meggitt (1982), ($kT > 10 \text{ keV}$). However, our values for $\Lambda$ are approximately an order of magnitude greater than Wickramasinghe and Meggitt (1982).

The best fit values obtained from the analysis of Stockman, Liebert, and Bond (1979) data were then applied to Frames 2 and 9 of Visvanathan and Wickramasinghe (1979). The fit to this data is very good (see Fig. 4.8), when one considers that absolutely no adjustments were made to the best fit values except for scaling to the observations. In addition, we fit the data of Visvanathan and Wickramasinghe (1979) using the values of Wickramasinghe and Meggitt (1982), ($kT = 10 \text{ keV}, B = 3.18 \times 10^7 \text{ gauss}, \Lambda \sim 10^5$) with our model calculations. The fits to the data (Fig. 4.8) are not as good as the fits using our values. We note that our calculations were made using two different methods (Chamuyam and Dulk 1981; Tamor 1978) to calculate
Fig. 4.7.--Best fit to the bright phase data of Frames 6, 7, and 8 of Stockman, Liebert, and Bond (1979). The best fit values for these frames are given in Table 4.4.
Fig. 4.8.--Fit to the bright phase data of Visvanathan and Wickramasinghe (1979). The upper figure corresponds to their Frame 2 and the lower figure to their Frame 9. For upper figure our best fit values ($B = 3.15 \times 10^7$ gauss, $kT = 8.7$ keV, $\theta = 90^\circ$) for $\Lambda = 3 \times 10^6$ (thick solid line) and $\Lambda = 10^6$ (thin solid line) are compared to the best fit values of Wickramasinghe and Meggitt (1982), ($B = 3.18 \times 10^7$ gauss, $kT = 10$ keV, $\theta = 90^\circ$, $\Lambda = 10^5$, dashed line). For lower figure our best fit values (same as upper figure) for $\Lambda = 10^6$ (thick solid line) compared to Wickramasinghe and Meggitt (1982), (same as upper figure).
determined this range by presenting theoretical spectra of a plasma at \( kT = 10 \text{ keV}, \beta = 3.18 \times 10^7 \text{ gauss}, \) and \( \Lambda = 10^5 \) for various values of \( \theta \). Their estimate for \( \Delta \theta \) is in good agreement with our earlier analysis of the VV Puppis spectra using equation (4.6).

(b) The Circular Polarization

The circular polarization (Visvanathan and Wickramasingale 1979) was observed during two bright phases to be 4% and 6% for \( \lambda \lambda 4000-6000 \AA \). Our calculations of the circular polarization for a complete bright phase was found by averaging the polarization over \( \lambda \lambda 4000-6000 \AA \) and over angles 90°, 85°, 80°, and 75°. The resulting value of 9% is in good agreement with the observations.

IV. DISCUSSION

The results from section III may also be used to determine the distance to VV Puppis and the size of its postshock region. Aizu (1973) gives the temperature of the postshock plasma as

\[
kT_s = \left(\frac{3}{16}\right) \frac{G M m_p}{R}, \tag{4.7}
\]

where \( m_p \) is the proton mass. For a degenerate nonrelativistic electron gas, the mass-radius relation of a white dwarf (Kittel 1969) is

\[
R = 7.94 \times 10^8 (M/M_\odot)^{-1/3} \text{ cm}. \tag{4.8}
\]
Substitution of equation (4.8) in equation (7) gives

\[ kT_S = 32.7 \left( \frac{M}{M_\odot} \right)^{4/3} \text{keV}. \tag{4.9} \]

Using \( kT_S = 8.7 \text{ keV} \) (Table 4.2), it follows that \( M = 0.37 \, M_\odot \) and \( R = 1.1 \times 10^9 \, \text{cm} \).

If the cooling mechanism in the postshock region is optically-thin bremsstrahlung radiation, then the height \( h \) of the shock front above the surface of the white dwarf is given by (Aizu 1973),

\[ h = 3.77 \times 10^6 \left( \frac{M}{M_\odot} \right) (\frac{R}{R_\odot})^{-1} (\frac{N}{N_{16}})^{-1}. \tag{4.10} \]

Here \( N \) is the electron number density, \( N_{16} = N/10^{16} \, \text{cm}^{-3} \) and \( R_9 = R/10^9 \, \text{cm} \). The height of the shock above the stellar surface was found by Stockman, Liebert, and Bond (1979) to be \( < 0.04 \, \text{R} \) or \( = 4.4 \times 10^7 \, \text{cm} \) from simultaneous photometry. Hence, by using equations (4.9) and (4.10), the electron number density in the postshock region during these observations may be deduced to be \( N > 2.9 \times 10^{14} \, \text{cm}^{-3} \). Therefore an upper bound on the diameter of the accretion column \( l = \Delta B/4\pi eN \) is \( < 3.4 \times 10^7 \, \text{cm} \) for \( \Delta = 2 \times 10^6 \) and \( B = 3.16 \times 10^7 \, \text{gauss} \). Knowledge of \( h \) and \( l \) gives the apparent surface area, \( A = hl \) of the postshock region. Therefore an upper bound on the distance to V V Puppis may be found:

\[ D = (2kT_S A/\lambda^2 f_\lambda)^{1/2} < 350 \, \text{parsecs}. \tag{4.11} \]

If the surface temperature of the white dwarf is \( \approx 9000 \, \text{K} \) as determined by Liebert et al. (1978), when V V Puppis was very faint with an observed
flux $f_\lambda \sim 2 \times 10^{-27}$ ergs/sec/cm$^2$/Hz, then the distance $D \sim 340$ parsecs.

Widely varying values are given for the area of the polar cap $A = 4\pi R^2 f$. Here, $R \sim 10^9$ cm is the radius of the white dwarf and $f$ is the polar cap fraction. These values range from $f \sim 0.1$ (King and Shaviv 1984), to $10^{-3}$ (Raymond et al. 1979), and to $f \sim 10^{-4} - 10^{-5}$ (Patterson et al. 1984; Imamura 1984). Using our diameter of the postshock region, we find $f = 6 \times 10^{-5}$. Our results then favor the smaller polar cap size.

Wickramasinghe and Meggitt (1982), placed constraints on the orbital inclination $i$ and the angle $\delta$ between the rotation axis and the magnetic dipole field axis. Their conclusion listed the angle between the line of sight and the direction of the magnetic field $\theta > 70^\circ$, because the cyclotron features were present throughout the entire bright phase. This conclusion and the fact that the bright phase was present for about 40% of the orbital period, implied geometries of $0^\circ < i < 30^\circ$, $90^\circ < \delta < 100^\circ$ or $80^\circ < i < 90^\circ$, $150^\circ < \delta < 180^\circ$. Preference was given to the former set of values because of the absence of an eclipse of the accretion column by the secondary. Brainerd and Lamb (1984) obtained the geometry for VV Puppis by analyzing linear polarization position angle data. They found $i = 75^\circ \pm 5^\circ$ and $\delta = 148^\circ \pm 5^\circ$, which agrees with the second set of values due to Wickramasinghe and Meggitt (1982), and our results.

Finally, Wickramasinghe and Meggitt (1982), concluded from their analysis that another component of radiation either unpolarized or weakly polarized dominates in the blue and UV regions of the spectrum. They claim this is necessary in order to fit the cyclotron spectrum and reduce the calculated polarization. Our analysis does not support this
We see no need for the presence of a weakly polarized or unpolarized component of radiation in the blue and UV spectral regions.

V. CONCLUSIONS

We have made calculations of cyclotron line spectra for homogeneouse plasmas and applied them to VV Puppis. Our main conclusions are: (1) the visibility of cyclotron lines in the optical spectrum is limited to a small parameter space in magnetic field strength, plasma temperature, and system orientation. The magnetic field strength probably needs to be between (2-10) $\times$ $10^7$ gauss in order to observe optical cyclotron lines. Hence, cyclotron lines would be difficult to observe in most DQ Her binaries since their magnetic fields are on the average $\sim 10^6$ gauss. The plasma temperature needs to be $\leq 15$ keV and the accretion column perpendicular to the line of sight for extended periods of time ($\sim 10$ minutes, depending on the brightness). (2) equation (4.6) generalizes the equation by Hirshfield et al. (1961) for the frequency of the cyclotron line to a three dimensional Maxwellian electron distribution and propagation at an angle $\theta$ to the magnetic field. This equation is shown to be useful in determining the magnetic field strength of the postshock plasma and in placing limits on the system orientation from observational spectra. (3) VV Puppis has a polar magnetic field strength of $3.15 \times 10^7$ gauss, and a postshock plasma temperature of 8.7 keV. (4) the orientation of the accretion column varies between $\theta = 70^\circ$-90$^\circ$, during the bright phase, confirming the orientation determined by Brainerd and Lamb (1984). (5) the size of the polar cap ($f = 6 \times 10^{-5}$) for VV Puppis is consistent with estimates of
the polar cap fraction $f \lesssim 10^{-3}$ proposed by Raymond et al. (1979), Patterson et al. (1984), and Imamura (1984). (6) the suggestion by Wickramasinghe and Meggitt (1982), for a weakly polarized or unpolarized component of radiation in the blue and UV spectral region is not required by our analysis.
Previous calculations of cyclotron emission from hot magnetized plasmas, when applied to the AM Herculis binaries, were able to explain qualitatively many of the optical polarization observations of these systems (Chanmugam and Dulk 1981, Meggitt and Wickramasinghe 1982). However, the maximum value of the linear and circular polarization reached ≈ 100%, contrary to the observed values of < 40%. These authors therefore suggested that an unpolarized background flux could reduce the polarization to the observed levels. These calculations, however, did not include the effects of collisions and Thomson scattering. In this thesis, we include these effects. The principal conclusions reached are as follows: (1) In the calculations of this thesis, the addition of collisions to the calculations of the cyclotron absorption coefficients results in a significant reduction in the polarization. We find that an additional unpolarized flux is not necessary in order to obtain agreement between the calculations and the data. (2) The circular polarization does not approach 100% for small viewing angles with respect to the magnetic field as in the collisionless calculations. The circular polarization, instead, approaches 0% or some value << 100%, depending on the geometry of the system. This result provides a natural, qualitative explanation of the "standstill"; a behavior of the circular polarization seen at small viewing angles and at those cyclotron harmonics which are optically thin. (3) A best fit to data of AM Herculis (Tapia 1977) is obtained
for a magnetic field $B = 2.7 \times 10^7$ gauss, a plasma temperature $kT = 0.2$ keV, and a slab thickness $= 2.8 \times 10^8$ cm. (4) Fits of the circular polarization data of AM Herculis using the calculations of cyclotron absorption coefficients which neglect the effects of collisions are attempted. We find that the addition of an unpolarized flux reduces the amount of polarized radiation to observed levels, but the shape of the curves are fit poorly. (5) These fits of the data of AM Herculis match the slope of the spectral flux well, when the plasma temperature is $\approx 10$ keV. (6) From the theoretical cyclotron spectra, we find that the visibility of cyclotron lines in the optical spectrum is limited to a small parameter space in magnetic field strength ($(2-10) \times 10^7$ gauss), in plasma temperature ($< 15$ keV), and in viewing angle ($\approx 90^\circ$). (7) We also confirm the conclusion by Wickramasinghe and Meeghitt (1982) that the broad lines in the optical spectrum of VV Puppis are due to cyclotron emission. (8) The best fit to the data of VV Puppis yields a polar magnetic field strength $B = 3.15 \times 10^7$ gauss, a plasma temperature $kT = 8.7$ keV, and a dimensionless plasma parameter $\Lambda \approx 10^6$. (9) We do not require the addition of an unpolarized flux in the blue and UV spectral regions as suggested by Wickramasinghe and Meeghitt (1982).

It appears that the model proposed by Chanmuyam and Dulk (1981) and Meeghitt and Wickramasinghe (1982) is essentially correct. The problems which we encountered during the attempts to fit the circular polarization data of AM Herculis are probably a result of the model of the accretion column which is assumed to be a plasma slab with a constant temperature. Future work should model the transfer of polarized radiation through plasmas with a temperature gradient, as is probably found at the surface of the accretion column.
APPENDIX

Petrosian (1981) gives the equation

\[ \omega / \omega_B = n \gamma / (1 + t_0^2) \]  

(A1)

where \( t_0^2 = \gamma_0^2 \beta_0^2 \sin^2 \theta \). For the non-relativistic case \((\omega / \omega_B \mu << 1)\), \( \beta_0 \gamma_0 \sim 2 \omega / \omega_B \mu \) and \( \gamma_0^2 \sim 1 \). This gives

\[ \omega / \omega_B = n / (1 + 2 n \sin^2 \theta / \omega_B \mu). \]  

(A2)

Some additional algebra gives

\[ \omega / \omega_B = (-1 + (1 + 8 n \sin^2 \theta / \mu)^{1/2})/(4n \sin^2 \theta / \mu). \]  

(A3)

If \( 8 n \sin^2 \theta / \mu << 1 \), then we obtain

\[ \omega / \omega_B = n(1 - 2 n \sin^2 \theta / \mu). \]  

(A4)

For \( \theta = 90^\circ \), equation (A4) is similar to that obtained by Hirshfield et al. (1961) (see equation 4.5)).
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Major Field: Physics & Astronomy

Title of Dissertation: Cyclotron Radiation from Magnetic Cataclysmic Variables.

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