2013

Time-variant performance assessment and improvement of existing bridges

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DEDICATION

To my parents, my wife and son
ACKNOWLEDGEMENTS

At this moment of accomplishment, I would like to express my sincere appreciation to my advisor Prof. Steve C. S. Cai for his continuous support of my Ph.D. study and research. His guidance helped me in all the time of research and writing of this dissertation. It is my honor to work with Dr. Cai and in his research group. All the knowledge accumulated during my learning and living experience here at LSU is valuable and will lead me to face any challenges in the future.

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ABSTRACT

The serviceability and safety of buildings and bridges are expected to be maintained within a reasonable safety level throughout their lifetimes. However, the increase of the applied loads and degradation of structural performances reduce the safety of these structures over time. Therefore, the performance assessment of existing bridges with reliability theories is a worldwide problem in civil infrastructure systems. Theoretically, the bridge reliability, usually expressed by a reliability index, is quantified by comparing the structural capacity (R) with the load effects (Q), using the predefined limit state functions. A limit state function is a mathematical description of a boundary between the desired and undesired performance of a structure. The resistances of structures and live loads on the bridge are none stationary processes, where their statistic parameters, e.g., mean values and deviations, are time variant. Thus, traditional reliability analysis methods cannot be applied to the entire service life of bridges.

In this research, the entire life cycle of bridges is treated as the sum of time series. During each time segment, both the load effect Q and the structural capacity R are assumed to be a stationary random process, and are expressed with a certain type of distribution. Thus, after obtaining the reliability probabilities for each time segments, the reliability probability for any length of mean recurrent intervals is obtained by the continued multiplication of the yearly reliability.

The extreme structure response which reflects the extreme live load distribution for mean recurrence intervals is derived based on a short-term monitoring of a field bridge. The flexural capacity of bridge girders considering variation of concrete strength, corrosion of steel reinforcements in the concrete and steel components is discussed in details.
The flexural capacity of bridge beams can be retrofitted with fiber reinforced polymers (FRP) materials. Finally, the flexural capacity of concrete bridge girders and steel girders strengthened with prestressed carbon fiber reinforced polymers (CFRP) are introduced. The time-variant reliability after the rehabilitation is calculated.

The reliability of a bridge keeps decreasing all the time. There is a jump in the reliability when the bridge is strengthened. Rehabilitation of a bridge also slows down the rate of the performance degradation of the bridge.
CHAPTER 1. INTRODUCTION

This dissertation is composed of six chapters based on papers that are under review or to be submitted for publications in peer-reviewed journals. The technical paper format is intended to facilitate and encourage publications of research results by graduate degree candidates. Thus, each chapter is independent, though some information of the reviews and references may be repeated for the completeness of these chapters. All chapters document the research work of the Ph. D. candidate under the guidance of the major advisor and committee members. This introductory chapter presents the general motivation of the study and the review of previous studies related to this research topic. More detailed information can be found in the subsequent chapters.

1.1 Purpose of Reliability Evaluation

Buildings and bridges are expected to maintain their serviceability and safety within a reasonable safety level throughout their lifetimes (Nowak and Collins 2000; Nowak and Zhou 1990; Saydam and Frangopol in press; Sharifi and Paik 2011; Stewart 2001). Factors determining the performance of a structure throughout its lifecycle are inherently uncertain. Furthermore, many sources of uncertainties are constantly time-variant throughout the entire service life of structures. At present, the safety of structures is usually measured in terms of reliability. The structure reliability, usually expressed with a reliability index, is quantified by comparing the structural capacity R with the load effects Q, using the predefined limit state functions. The reliability of a structure illustrates its ability to fulfill its design purpose for its life cycle. It is often understood as the probability that a structure will not fail to perform its intended function (Ge et al. 2000; Law and Li 2010; Liu 2002; Micic et al. 1995; Nowak and Collins 2000; Saydam and Frangopol 2013; Stewart et al. 2001; Trautner and Frangopol 1990). Structure
reliability evaluation can be applied to both the new structures and the existing ones. Reliability estimation will allow more efficient maintenance, repair, and rehabilitation strategies of existing bridges and will also help design more safer and economical new bridges. The reason of using reliability or safety/failure probability to express a structure’s safety level is due to the uncertainties of factors related to the R and Q. The uncertainties come from two sources as discussed below.

1.2 Event Inherent Uncertainties

The constructed bridge is always, to some degree, different from that on the construction drawings. For example, the strength of the materials, such as the concrete and steel, are different from that marked on the drawings.

Concrete is a mixture of cement, water, fine and coarse aggregates, and other admixtures. Any slight difference of mixing proportions will lead to a strength variety for each batch. Furthermore, the final strength of the concrete greatly depends on the conditions of moisture and temperature during the curing period. It is reported that thirty percent or more of the strength are lost by the premature drying out of the concrete (Nilson et al. 2004). Compared with those factory-made materials, such as steel, the compression strength of concrete has a larger variance.

Unlike the concrete material, steel is a homogeneous alloy, and their principal components consist of iron and carbon. The various properties of the structural steel, including the strength and ductility, are determined by its chemical composition. The most common type of reinforcing steel applied in reinforced concrete structures is in the form of round bars. These bars are furnished with surface deformations with the purpose to increase the resistance of slipping behaviors between the steel and concrete (Nilson et al. 2004). For steel structures, various shaped
steel members with standard cross-section are available. For example, *S-shape*, *C-shape*, *W-shape* and *Structure Tee* shape steel members are commonly used in structures, and they are produced by the hot-rolling method. Among other factory-made members, the strength of steel components has a smaller variance; however, the slight error of the cross-section dimensions may lead to a significant difference of the load carrying capacity. In addition, the mechanical properties of concrete and steel are time-variant and will be discussed in details later.

Another important factor for reliability evaluation is the live load. Live load acting on a bridge is a random process. Statistics of load and its effect is based on a predefined mean recurrent interval, i.e., 75 years, according to the *AASHTO LRFD Bridge Design Specifications* (AASHTO 2007). The structure response due to the live load is determined by the weight of the vehicle, the roughness of the pavement, and the velocity of vehicles while running on the bridge. Models developed by Nowak (1993) and used in the calibration of *AASHTO LRFD Bridge Design Specifications* (AASHTO 1994) were derived based on the available statistical data on 9,250 selected truck surveys, and weigh-in-motion measurements. The derived load effects, applied in the design code are uniformly used in new bridge designs around the country. For an existing bridge, the live load acting on the bridge is more specific than that defined by the design code. Its statistic characteristics may not fit the values defined for new bridge design. The expected load effects for an existing bridge should be estimated based on the actual load information.

### 1.3 Uncertainties due to Simplifications and Assumptions

In order to calculate the capacity of a structure, some assumptions are made to simplify the calculation. For instance, to calculate the flexural capacity of beams, a plane cross section before loadings is assumed to remain plane under loadings. In truss structure analysis, the
secondary moment due to the rotation stiffness of truss member connections is neglected. Other than inherent uncertainties (random error) of events, the uncertainties or errors due to assumptions or simplifications are always unidirectional. The capacity is either overestimated or underestimated only due to the simplifications and assumptions. Among these uncertainties or errors, the system error can be eliminated only by improving the comprehensive understanding of mechanisms of structure performance (Taylor 1997). Thus, in present research, the system error is not taken into consideration to evaluate the bridge performance in its life cycle.

1.4 Deterministic Parameters and Random Variables

Theoretically, all the factors related to the structural capacity and its subjected loads are random variables. They are affected by many sources including the inherent uncertainties due to the construction, materials, and the environment that the structure is exposed to. To evaluate the bridge performance, it is not necessary and impossible to take every variable into account. For the aim of simplification, some of the variables can be treated as deterministic parameters. Two of the following principles are applied to distinguish the deterministic parameters and random variables:

1. If it is easy to be measured
   Factors can be measured easily are usually treated as deterministic parameters.

2. If it is sensitive for the structure performance

3. Factors with a large variance and sensitive for the structure performance are always treated as random variables. The properties of variables are obtained by acquisition and analysis of numerous relative data. The quantification of variables is described in terms of statistic items.
1.4.1 Deterministic Parameters

Due to the limitation of fabrication and construction techniques, the dimensions of structures are different from the drawings. For the visible dimensions, it is easy to obtain the real values by measuring. Compared to the large size of the structure, it is reasonable to believe that the small dimension difference is limited to a small range. Thus, in practice, the visible structure dimensions, such as the bridge span and width or depth of a girder, can be reasonably treated as deterministic parameters.

1.4.2 Time Independent Variables

Unlike the visible structure dimensions, some invisible structure dimensions usually have a large variance and are sensitive for the bridge performance. For example, the position of the reinforcement in the reinforced concrete structures is sensitive for its flexural capacity. During the concrete casting in bridge constructions, the longitudinal reinforcements may be easily moved away from the designed position. The final position of the reinforcement will change the length of the arm of force. Moreover, for the reinforcement, the thinner of the concrete cover, the more serious corrosion it will experience. Therefore, the relative positions of the reinforcement are usually treated as variables.

1.4.3 Time-Variant Variables

Time-variant variables are mostly related to the properties of materials. The compressive strength of concrete keeps varying from its initial value since being casted. It increases dramatically in the first 120 day. After reaching the peak at the age of about 1 year, then it decreases gradually as the age increases. Although compared to the concrete compressive strength, the steel tension strength is steadier, yet the cross section area of the steel reinforcement decreases due to the material corrosion. The decrease of intersection area of steel reinforcement
is a key issue of capacity when a structure is exposed to an aggressive environment. Besides properties related to materials, live loads, especially the live load on bridges are time-dependent because of the increasing traffic every year.

1.4.4 Summary of Deterministic Parameters and Random Variables

Since the variable identification is a very complicated process, researchers have devoted a lot of efforts on this subject. Akgul and Frangopol (2005) defined deterministic and random parameters involved in the structure capacity as shown in Tables 1-1 to 1-4. The variables listed below include the deterministic parameters, time independent variables and time-variant variables.

**Table 1-1 Random Variables for Reinforced Concrete Slabs**

<table>
<thead>
<tr>
<th>Random Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{sr}$</td>
<td>Area of top transverse steel reinforcing</td>
</tr>
<tr>
<td>$f_{cs}'$</td>
<td>Compressive strength of concrete slab</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Yield strength of reinforcing steel in slab</td>
</tr>
<tr>
<td>$y_{mf}$</td>
<td>Modeling uncertainty for flexure in slab</td>
</tr>
<tr>
<td>$\lambda_a$</td>
<td>Asphalt weight uncertainty factor</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>Concrete weight uncertainty factor</td>
</tr>
<tr>
<td>$\lambda_{def}$</td>
<td>Effective depth of top reinforcing uncertainty factor</td>
</tr>
<tr>
<td>$\lambda_{trk}$</td>
<td>Uncertainty factor for an HS20 truck load</td>
</tr>
</tbody>
</table>

**Table 1-2 Deterministic Parameters for Reinforced Concrete Slabs**

<table>
<thead>
<tr>
<th>Deterministic Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_f$</td>
<td>Continuity factor</td>
</tr>
<tr>
<td>$d_{eff}$</td>
<td>Effective depth of top slab reinforcement</td>
</tr>
<tr>
<td>$L_s$</td>
<td>Span length of the slab between two girders</td>
</tr>
<tr>
<td>$P_{HS20}$</td>
<td>Load on one middle or rear wheel of an HS20 truck</td>
</tr>
<tr>
<td>$t_a$</td>
<td>Thickness of asphalt pavement</td>
</tr>
<tr>
<td>$t_s$</td>
<td>Thickness of concrete slab</td>
</tr>
<tr>
<td>$\omega_{ps}$</td>
<td>Uniform weight of utility piping for slab</td>
</tr>
<tr>
<td>$\gamma_a$</td>
<td>Asphalt unit weight</td>
</tr>
<tr>
<td>$\gamma_c$</td>
<td>Concrete unit weight</td>
</tr>
</tbody>
</table>
### Table 1-3 Random Variables for Reinforced Concrete Girders

<table>
<thead>
<tr>
<th>Random variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_s$</td>
<td>Area of flexural steel reinforcement in concrete girder</td>
</tr>
<tr>
<td>$A_g$</td>
<td>Area of shear steel reinforcement in concrete girder</td>
</tr>
<tr>
<td>$D_{fc, cg}$</td>
<td>Distribution factor for concrete girder</td>
</tr>
<tr>
<td>$f'_{c, cg}$</td>
<td>Compressive strength of concrete girder</td>
</tr>
<tr>
<td>$\sqrt{f'_{c, cg}}$</td>
<td>Derived random variable for $f'_{c, cg}$</td>
</tr>
<tr>
<td>$f_y, cg$</td>
<td>Yield strength of reinforcing steel of concrete girder</td>
</tr>
<tr>
<td>$l_{f, cg}$</td>
<td>Impact factor for concrete girder</td>
</tr>
<tr>
<td>$M_{trk, cg}$</td>
<td>Moment due to truck load for concrete girder</td>
</tr>
<tr>
<td>$V_{trk, cg}$</td>
<td>Shear due to truck load for concrete girder</td>
</tr>
<tr>
<td>$\lambda_{mfc}$</td>
<td>Modeling uncertainty for flexure in concrete</td>
</tr>
<tr>
<td>$\lambda_{misc}$</td>
<td>Modeling uncertainty for shear in concrete</td>
</tr>
<tr>
<td>$\lambda_a$</td>
<td>Asphalt weight uncertainty factor</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>Concrete weight uncertainty factor</td>
</tr>
<tr>
<td>$\lambda_d$</td>
<td>Depth of reinforcing uncertainty factor</td>
</tr>
</tbody>
</table>

### Table 1-4 Deterministic Parameters for Reinforced Concrete Girders

<table>
<thead>
<tr>
<th>Deterministic parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_a$</td>
<td>Width of asphalt pavement</td>
</tr>
<tr>
<td>$b_c$</td>
<td>Width of concrete curb</td>
</tr>
<tr>
<td>$b_{eff, cg}$</td>
<td>Effective slab width</td>
</tr>
<tr>
<td>$b_{w, cg}$</td>
<td>Width of the girder</td>
</tr>
<tr>
<td>$d_{cg}^s$</td>
<td>Depth of steel reinforcement at shear section</td>
</tr>
<tr>
<td>$d_{cg}^f$</td>
<td>Depth of steel reinforcement at flexural section</td>
</tr>
<tr>
<td>$h_{cg}$</td>
<td>Height from slab top to girder bottom</td>
</tr>
<tr>
<td>$l_{cg}$</td>
<td>Span length of the girder</td>
</tr>
<tr>
<td>$N_G$</td>
<td>Number of girders</td>
</tr>
<tr>
<td>$P_{diaph}$</td>
<td>Concentrated diaphragm weight</td>
</tr>
<tr>
<td>$S$</td>
<td>Spacing of shear reinforcement</td>
</tr>
<tr>
<td>$t_a$</td>
<td>Thickness of asphalt pavement</td>
</tr>
<tr>
<td>$t_c$</td>
<td>Thickness of concrete curb</td>
</tr>
<tr>
<td>$t_s$</td>
<td>Thickness of concrete slab</td>
</tr>
<tr>
<td>$\omega_{pg}$</td>
<td>Uniform weight of piping per girder</td>
</tr>
<tr>
<td>$\omega_r$</td>
<td>Uniform weight of railing</td>
</tr>
<tr>
<td>$\lambda_a$</td>
<td>Asphalt unit weight</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>Concrete unit weight</td>
</tr>
</tbody>
</table>
1.5 Description of Variables

1.5.1 Description of Parameters Involved in Capacity

The random uncertainties are the primary and unavoidable uncertainties in analysis and design of structures. The entire or long-term information of an existing bridge will be helpful for its reliability analysis. For the purpose of reliability analysis, variables must be described quantitatively based on this information. Several methodologies have been developed and applied to deal with the randomness of structures and load effects. The methodologies include probability theory, statistic theory, and stochastic processes theory. For different aims of research, different methodologies are selected.

To deal with the randomness of variables, based on the observed data, the variables are usually expressed with probability distributions. In probability theory, the probability density or probability distribution is a function that describes the probability of a random variable taking certain values. Akgul and Frangopol (2005) assigned these variables with lognormal distributions. The parameters of the distribution factors for the variables are listed in Tables 1-5 and 1-6.

Table 1-5 Random Variables for Reinforced Concrete Slab of Bridges E-17-HS

<table>
<thead>
<tr>
<th>Random variable</th>
<th>Design Value</th>
<th>$\mu^b$</th>
<th>$\sigma^b$</th>
<th>$\lambda$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_y$ (MPa)</td>
<td>275.8</td>
<td>308.90</td>
<td>33.98</td>
<td>5.7272</td>
<td>5.7272</td>
</tr>
<tr>
<td>$f_{cs}'$ (MPa)</td>
<td>20.70</td>
<td>19.03</td>
<td>3.43</td>
<td>2.9300</td>
<td>0.1786</td>
</tr>
<tr>
<td>$\lambda_a$</td>
<td>1.00</td>
<td>1.00</td>
<td>0.25</td>
<td>-0.0303</td>
<td>0.2462</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>1.00</td>
<td>1.05</td>
<td>0.11</td>
<td>0.0438</td>
<td>0.0998</td>
</tr>
<tr>
<td>$A_{sr}$ (cm$^2$)</td>
<td>5.33</td>
<td>5.33</td>
<td>0.16</td>
<td>1.6736</td>
<td>0.0300</td>
</tr>
<tr>
<td>$\lambda_{def}$</td>
<td>1.00</td>
<td>1.00</td>
<td>0.02</td>
<td>-0.0002</td>
<td>0.0200</td>
</tr>
<tr>
<td>$\gamma_{mf}$</td>
<td>1.00</td>
<td>1.02</td>
<td>0.06</td>
<td>0.0180</td>
<td>0.0599</td>
</tr>
<tr>
<td>$\lambda_{trk}$</td>
<td>1.00</td>
<td>0.60</td>
<td>0.20</td>
<td>-0.5626</td>
<td>0.3201</td>
</tr>
</tbody>
</table>
Table 1-6 Random Variables for Reinforced Concrete Girders of Bridge E-17-HS

<table>
<thead>
<tr>
<th>Random variable</th>
<th>Design Value</th>
<th>$\mu^b$</th>
<th>$\sigma^b$</th>
<th>$\lambda$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{y, cg}$ (MPa)</td>
<td>275.80</td>
<td>308.90</td>
<td>33.98</td>
<td>5.7270</td>
<td>0.1097</td>
</tr>
<tr>
<td>$f'_{c, cg}$ (MPa)</td>
<td>20.70</td>
<td>19.03</td>
<td>3.43</td>
<td>2.9300</td>
<td>0.1786</td>
</tr>
<tr>
<td>$\sqrt{f'_{c, cg}}$ (MPa)</td>
<td>4.55</td>
<td>4.36</td>
<td>0.79</td>
<td>1.4571</td>
<td>0.1786</td>
</tr>
<tr>
<td>$A_s$ (cm$^2$)</td>
<td>110.84</td>
<td>110.84</td>
<td>3.33</td>
<td>4.7076</td>
<td>0.0300</td>
</tr>
<tr>
<td>$A_p$ (cm$^2$)</td>
<td>2.58</td>
<td>2.58</td>
<td>0.08</td>
<td>0.9476</td>
<td>0.0300</td>
</tr>
<tr>
<td>$\lambda_d$</td>
<td>1.00</td>
<td>1.00</td>
<td>0.02</td>
<td>-0.0002</td>
<td>0.0200</td>
</tr>
<tr>
<td>$\gamma_{mfc}$</td>
<td>1.00</td>
<td>1.02</td>
<td>0.06</td>
<td>0.0180</td>
<td>0.0599</td>
</tr>
<tr>
<td>$\gamma_{msc}$</td>
<td>1.00</td>
<td>1.02</td>
<td>0.06</td>
<td>0.0180</td>
<td>0.0599</td>
</tr>
<tr>
<td>$M_{trk, cg}$ (kN m)</td>
<td>268.94</td>
<td>187.18</td>
<td>50.80</td>
<td>5.1965</td>
<td>0.2666</td>
</tr>
<tr>
<td>$V_{trk, cg}$ (kN)</td>
<td>119.77</td>
<td>74.86</td>
<td>19.14</td>
<td>4.2839</td>
<td>0.2517</td>
</tr>
<tr>
<td>$f_{cg}$</td>
<td>1.31</td>
<td>1.15</td>
<td>0.12</td>
<td>0.1361</td>
<td>0.0998</td>
</tr>
<tr>
<td>$D_{f, cg}$</td>
<td>1.44</td>
<td>1.44</td>
<td>0.18</td>
<td>0.3598</td>
<td>0.1235</td>
</tr>
</tbody>
</table>

1.5.2 Description of Live Load Parameters

During the service life cycle of a bridge, it withstands dead load, traffic live load, wind load, ice load, seismic load, etc. Traffic live load, wind load, ice load, and seismic load are classified as live loads. Dead load does not change a lot during the entire service life cycle of the bridge. On the contrary, for the live load, it changes in a large range. A bridge is expected to withstand extreme live loads in a service life cycle, thus, one concerns the extreme value of every live load in the given interval. The expected extreme load effect is seriously dependent on the length of the intervals. The longer the interval is, the larger the extreme load effects is expected. With the help of the recently developed weigh-in-motion system (WIM) and structural health monitoring (SHM) system, information of load effects and structure responses due to the traffic live load can be easily obtained. Based on these monitored data, the instant distribution of live load effects or the structure response due to the live load can be derived with distribution fitting techniques. Since the service life cycle is typically long, it is impossible to estimate an
extreme live load only by a monitoring system. One of the purposes of this research is to fit distributions of extreme live load values in a long-term interval based on a short-term structure health monitoring.

### 1.6 Time-Variant Reliability

Though it is a gradual process, one of the typical properties of structural reliability is time-variant. Two facts determine the time dependent structure reliability. As discussed above, the reliability is related to the structural capacity $R$ and load effects $Q$. Any time-dependent sources leading to the changes and uncertainties of $R$ and $Q$ may result in the structure reliability being time-variant. In practice, the resistance variation with time cannot be ignored especially for structures exposed to aggressive environments, such as industrial buildings and structures near the seaside. As the structure age increases, the capacity of the structure decreases because of the degradation of material strength and loss of cross section area due to the steel corrosion. On the other hand, due to the constant increase of the traffic demand, for an existing bridge, the probability of experiencing larger traffic live load increases constantly.

At the present, most bridges are constructed with timber, concrete and steel. In the last few decades, more and more new type materials, especially fiber reinforced polymer (FRP), are introduced to civil engineering field to replace the traditional materials. Theoretically, mechanical properties of all the materials are time-variant due to various reasons. Concrete strength and durability characteristic of reinforced concrete structures are seriously affected by its age and the actions of environmental factors such as acidic rain water, alternate wetting and drying, temperature variations and ground moisture according to the experiment conducted by Ismail et al. (2010). Based on their research, the concrete strength increases rapidly till the age of 120 days, and then decreases gradually because of the aggressive environment it is exposed to.
Relatively, the steel strength is more stable than the concrete strength. Whereas, the corrosion due to the aggressive environment will decrease the cross section area of steel reinforcements in concrete and various shape steel components in steel structures. The decrease of cross section area may lead to the decrease of component capacities. More seriously, the reduction of the thickness of web and flange due to the corrosion may lead to failures of buckling that occurs without significant precautions.

The expected length of a new bridge service life is 75 years defined by AASHTO. In such a long period, it is reasonable to expect that many new designed and heavier trucks are put into operation which will lead to larger live loads on the bridges.

These two random processes (variations of structure capacity and load) determine the decrease of structure reliability as its age increases. When the safety of a bridge does not meet the traffic requirement, rehabilitation and strengthening are needed. After that, the bridge will be restored to their original or better condition and the reliability will be increased instantly.

1.7 Methodology of Reliability Index Calculation

Structures and infrastructures are supposed to maintain adequate levels of serviceability and safety throughout their lifetime (Saydam and Frangopol in press). Bridge performance is often expressed in a reliability format. The aim of reliability calculation is to assess the safety level using a probabilistic approach, typically as the probability of failure (unsatisfactory performance) (Stewart 2001). This will allow optimum maintenance strategies and will help in designing more crucial repair and retrofit applications (Catbas et al. 2008).

Reliability analysis methodology applied to new bridges is not suitable for existing bridges. Compared with the new bridge, the load and the environment that an existing bridge
withstands and is exposed to are more specific. Reliability analysis for an existing bridge in service will take into the consideration of the present condition of the bridge, the expected service life cycle in the future, and the load test and load history.

For an existing bridge having served for decades, the expected subsequent service life cycle depends on the traffic demand, bridge condition, and maintenance cost. Therefore, the definition of reliability needs to be characterized using the same length of service cycle and the same failure principles.

Failure principles are expressed with limit states, such as ultimate limit states and service limit states. Ultimate limit states are mostly related to the loss of load-carrying capacity, while serviceability limit states are related to gradual deteriorations, user’s comfort or maintenance costs (Nowak and Collins 2000). Deflection or permanent deformations beyond a reasonable limit and vibration, such as reaching human acceptable limits, belong to serviceability limit states. Each limit state is associated with a particular limit function. Different limit states have different limit functions; the general limit function can be defined as

\[ g(R - Q) = R - Q \]  

Probabilities of failures or reliability index are calculated based on these limit functions. Since many variables are involved in these functions, two methodologies are optional to calculate the reliability index, analytical and numerical or simulation method.

First-order second-moment method deals with the means and standard deviation of the random variables only. This simple method is the most commonly used analytical method in the engineering field. It is applied for linear limit functions and variables following normal distributions. Hasofer and Lind proposed to evaluate the reliability index at a point known as the
“design point” of the limit state function. Once the distributions of the random variables are known, Rackwitz-Fiesssler procedure can be applied to calculate reliability indexes (Nowak and Collins 2000). This iterative method guarantees a sufficient accuracy even for nonlinear limit functions and variables following non-normal distributions.

Monte Carlo methods are a class of computational algorithms that rely on repeated random sampling to compute the reliability index. They are often used for simulating system with many coupled degrees of freedoms. For complex nonlinear state limit functions with which analytical methods are difficult to apply, Monte Carlo simulation method is a relatively straightforward method. It should be noted that the procedure can become computationally intensive.

1.8 Rehabilitation or Strengthening with FRP Materials

For bridges classified as structurally and/or functionally deficient, rehabilitation and strengthening are needed to restore their capacities to their original conditions. One of the most effective ways to solve the problem is to use composite materials to strengthen existing bridges. As rapidly developed over the past several decades, different kinds of composite fiber reinforced polymers (FRP) have been regarded as one of the best solutions to several problems associated with transportation and civil engineering infrastructures. In the present study, a demonstration bridge was used to illustrate the design process to strengthen the flexural capacity with FRP laminates and rods.

1.9 Purpose of the Research

A structure’s entire service life cycle can be divided into three periods, namely construction period, service and degradation period. There are no clear boundaries between these
periods. In most cases, service period and degradation period are overlapped. In each service life period, the natures of the structure change constantly and deteriorate most of the time. Due to the uncertainties of load effects, environmental and man-made effects, structures’ safety, serviceability and durability change constantly. Rehabilitation and strengthening of existing bridges lead to an instant increase of reliability, and then the bridges deteriorate according to the natures of the new composite component section.

The aim of the present research is to describe the variation of the bridge in the service period. The second chapter describes the frame work of deriving the extreme live load distributions due to traffic for any length of time intervals based on monitoring data. The bridge reliability variation versus time before rehabilitation is quantified in the third chapter. The degradation of the structure due to steel corrosion and variation of concrete strength are taken into account. The fourth chapter presents the flexural capacity estimation of bridge girders rehabilitated with post tensioned CFRP materials. The following chapter, chapter five describes the reliability variation of bridges after rehabilitation. Finally, the conclusions are given in chapter 6.

1.10 References


CHAPTER 2. ESTIMATION OF EXTREME STRUCTURE RESPONSE DISTRIBUTION BASED ON SHORT-TERM MONITORING

2.1 Introduction

Performance assessment of existing bridges with reliability theories is a worldwide problem in civil infrastructure systems. Bridge reliability, usually expressed by a reliability index, is quantified by comparing the structural capacity $R$ with the load effects $Q$, using the predefined limit state functions. A limit state function is a mathematical description of a boundary between the desired and undesired performance of a structure. Both ultimate limit states, related to the load-carrying capacity, and serviceability limit states, related to the gradual deterioration of structures, user’s comfort, and maintenance costs, are described by limit state functions (Nowak and Collins 2000). To precisely calculate the reliability index of a bridge in its lifetime, the key step is to convert the two random processes, i.e., the structural capacity $R$ and the load effects $Q$, into variables following certain distribution types. Furthermore, both $R$ and $Q$ are non-stationary because of the deterioration of the materials and the potential increasing of the traffic demand during the life-cycle of a bridge; then, the bridge reliability is time-dependent. Since the degradation of the load carrying capacity and increasing of traffic demands are a long-term gradual process, they do not change dramatically. Therefore, it is reasonable to treat $R$ and $Q$ as stationary processes in a relatively short time interval. With this assumption, it is possible to convert these two random processes into variables following certain distribution types.

The previous studies on the performance assessment of a bridge’s lifetime are focused on the development of the capacity degradation models. For example, random variables and deterministic parameters were respectively identified for concrete, prestressed concrete, and steel girder bridge superstructure (Akgul and Frangopol 2004a; Akgul and Frangopol 2004b; Akgul and Frangopol 2005a; Akgul and Frangopol 2005b). According to Akgul and Frangopol, the
random variables related to the capacity of a girder can be assumed to have a lognormal distribution, and each random variable is characterized by its mean value $\mu$, and standard deviation $\sigma$. The corresponding lognormal distribution parameters ($\lambda$, $\nu$) are accordingly proposed in their research. The limit state functions are derived strictly following the load and capacity formulas and the requirements in AASHTO specifications (AASHTO 2007). In Akgul and Frangopol’s research, for the reinforced concrete girder and prestressed concrete girder bridges, a model simulating the propagation of chloride within the cross section of reinforced concrete girders and prestressed concrete girders were applied to estimate the degradation of capacity (Kong et al. 2002). For the steel girder bridges, the capacity degradation caused by the deterioration of steel girders was estimated using a model based on the salt water exposure and atmospheric corrosion of structural metals (McCuen and Albrecht 1994).

In the studies discussed above, the time-variant live load, that is applied to calculate the reliability index, was computed based on the live load models developed by Nowak (1993) and used in the calibration of *AASHTO LRFD Bridge Design Specifications* (AASHTO 1994). The models were derived based on the available statistical data on 9,250 selected truck surveys, and weigh-in-motion (WIM) measurements. For a structure with a given capacity without considering the damage accumulation, its reliability index is only related to the maximum load effect distribution corresponding to the structure’s service life. Assuming a normal distribution for the individual truck load, the maximum live load effects (moment or shear) for various time periods are determined by extrapolation as described below.

Let the live load effects following a certain distribution $\Omega$, and the number of trucks in the surveying interval $T_{\text{sur}}$ is $N_{\text{sur}}$. Then, the total number of trucks $N_{\text{exp}}$ passing through the bridge in an expected service life $T_{\text{exp}}$, will be
\[ N_{\text{exp}} = N_{\text{sur}} T_{\text{exp}} / T_{\text{sur}} \]  

(2-1)

The maximum live load effects \( Q_{l,\text{max}} \) corresponding to any expected bridge service life is

\[ Q_{l,\text{max}} = \Phi^{-1} \left( 1 - \frac{1}{N_{\text{exp}}} \right) \]  

(2-2)

where \( \Phi^{-1} \) is the inverse of the standard distribution function. According to AASHTO specifications (AASHTO 1994), the expected service life for a new bridge \( T_{\text{exp}} \) is 75 years. Therefore, the maximum live load effect \( Q_{l,\text{max}} \) calculated based on 75 years is applied to calculate the reliability of a bridge. Thus, a non-stationary random process was reduced to a constant value \( Q_{l,\text{max}} \). The live-load factor in the AASHTO LRFD specifications has been calibrated for use along with the HL93 design load such that bridge members designed with AASHTO LRFD specifications would achieve a uniform target reliability index \( \beta = 3.5 \). However, the reliability calculation of existing bridges using the live load model defined by AASHTO specifications has the following drawbacks.

1. The mean recurrence interval of live load effect defined by AASHTO is 75 years, and it is usually longer than the expected remaining service life of an existing bridge that has been in service for several decades. Therefore, a reliability index calculated with AASHTO live load effects may be too conservative for existing bridges.

2. Live load acting on a bridge is very site-dependent. A structural reliability index calculated based on load effects defined by design specifications is different from that derived from an actual routine service traffic load. Thus, the AASHTO load model cannot precisely describe the actual live load effect for a given bridge.
To evaluate the performance of existing bridges, more accurate live-load models are needed. Recently developed structural health monitoring (SHM) systems of bridges can provide useful information, and with which the bridge performance can be assessed and predicted more precisely. Instead of measuring the weight of every vehicle passing through bridges, SHM techniques can conveniently record structural response such as strains under routine service traffic load. The procedure of reliability assessment using the monitoring data includes collecting the survey data, identifying the distribution type of live load effects, and estimating its distribution parameters by curve fitting. The extreme load effects are derived from the response of a bridge directly including each possible combination of the number of loaded lanes multiplied by a corresponding multiple presence factor to account for the probability of simultaneous lane occupation (Orcesi and Frangopol 2010). For reliability calculations, with the help of the monitored data, one can minimize the uncertainties and apply fewer assumptions in structural analysis, thus make the reliability calculation more rational. Catbas et al. (2008) investigated the reliability of a longest cantilever truss bridge in the United States with the consideration of dead load, wind pressure, traffic loads, temperature effects, and their combinations. Furthermore, one can calculate the reliability of a component simply by comparing the material’s limit strain with the total strain (the measured strain plus the strain induced by the dead load) without calculating the capacity and load effects.

Liu et al. (2009b) presented an efficient approach to assessing the bridge system performance based on the long-term monitored strain data induced by the heavy vehicle traffic on an existing bridge. Orcesi and Frangopol (2010) developed a methodology for lifetime serviceability analysis of existing steel girder bridges including crawl tests and long-term
monitoring information. In their studies, the extreme value distribution of monitored data is assumed to approach a Gumbel probability distribution (Gumbel 1958).

\[
F_l(x; \mu, \sigma) = \exp \left( -\exp \left( -\frac{x-\mu}{\sigma} \right) \right) \quad -\infty < x < +\infty
\]  \tag{2-3}

where \( F(x) \) is the cumulative distribution function of Gumbel probability distribution, a particular case of generalized extreme value distribution; \( x \) is the extreme value of a random variable; \( \mu \) is the location parameter, referred to the mode (The mode is the value that occurs the most frequently in a data set or probability distribution) of the distribution and \( \sigma \) is the scale parameter. Both \( \mu \) and \( \sigma \) are constants to be determined from the measured data by either theory of order statistics or a graphical method. Thus, the extreme values of the monitored data in a mean recurrence interval, \( T \), \( (T \) is longer than the monitoring period), can be predicted as

\[
x(T) = \mu - \sigma \cdot \ln \left( -\ln \left( 1 - \frac{1}{N_{exp}} \right) \right)
\]  \tag{2-4}

where \( N_{exp} \) is defined by Eq. (2-1). In addition, trigger levels were set in their monitoring program. Therefore, only truncated probability density distributions of the maximum stress under heavy vehicles were obtained from these monitored data. It should be noted that the effects of the trigger levels on the histograms of the maximum stress on different components vary significantly (Liu et al. 2009a). To estimate the extreme value of load effects in a mean recurrence interval, the information of the number of the trucks running through the bridge must be available. However, it is not always easy to identify the number of trucks only by dealing with the recorded strain data. Even the number of the trucks is known, some cases whose maximum structural response is induced by multiple presences of vehicles side by side or one after another in a same span are still excluded in the reliability calculation.
The live load model developed for AASHTO LRFD Bridge Design Specifications (1998) is used by Akgul and Frangopol (2004c) to calculate the time-variant reliability index. The live load model was derived from data collected using weigh-in-motion (WIM) studies. WIM system records weights of trucks by means of sensors attached to bridge deck and girders. The results are used to quantify the actual load effects on any girder of the bridge. The extrapolation is used for the load model to predict the maximum moment and shear in a certain length of a mean recurrent interval. For a mean recurrent interval, the maximum moments and shear forces due to the live load in bridge components is described by the Type I extreme value distribution (Gumbel distribution). The mean value and standard deviation are given as

\[ \mu_{Y_n} = \mu_X + u_n \sigma_X + \frac{\gamma}{\alpha_n} \sigma_X \]  
\[ \sigma_{Y_n} = \frac{\pi}{\sqrt{6} \alpha_n} \sigma_X \]

where \( u_n \) and \( \alpha_n \) are the location and scale parameters of Type I extreme value distribution, \( \gamma \) is the Euler number (0.577216), and \( \mu_X \) and \( \sigma_X \) are the mean value and standard deviation of the maximum moment or shear at initial time \( t = 0 \), i.e. due to a single truck.

NCHRP Report 683 (2011) proposed three methods, i.e., convolution or numerical integrations, Monte Carlo simulations, and simplified statistical projections, to estimate the maximum loading over a longer period based on a short-term WIM data. In their studies, the upper 5% of the values were assumed to follow a normal distribution. A linear fitting on the normal probability plot gives a slope, \( m \), and an intercept, \( n \), which will give the mean of the equivalent normal distribution that best fit the tail end as \( \mu_{\text{event}} = -n/m \). The standard deviation of the best-fit normal distribution is \( \sigma_{\text{event}} = (1 - n)/m - \mu_{\text{event}} \). According to Ang
and Tang (2007), if the parent distribution of the initial variable S has a general normal distribution with mean \( \mu_{\text{event}} \) and \( \sigma_{\text{event}} \), then the maximum value after N repetitions approaches asymptotically an Gumbel distribution. The most probable value, \( u \), for the Gumbel distribution that models the maximum value in a specific mean recurrent interval is given as

\[
U_N = \mu_{\text{event}} + \sigma_{\text{e}} \times n_t \times \left[ \sqrt{2\ln(N)} - \frac{\ln(\ln(N)) + \ln(4\pi)}{2\sqrt{2\ln(N)}} \right] \tag{2-7}
\]

The dispersion coefficient for the Gumbel distribution that models the maximum load effect is given as

\[
\alpha_N = \frac{\sqrt{2\ln(N)}}{\sigma_{\text{event}}} \tag{2-8}
\]

where N is the total number of events for the return period of interest.

Among all the current studies, to estimate the maximum live load effect in a specific mean recurrent interval, the number of the truck passage is needed. In most cases the number of the truck passage is difficult to be obtained by analyzing the monitored data only. The roughness of the deck surface, the number of the axis of the vehicle, and the vibration of the vehicles create numerous multi peaks in the monitored data. For cases that the bridge span is shorter than the axle intervals of vehicles, there are multi peaks in the monitored data for even one vehicle passing. In addition, the efforts are needed to count the number of the vehicles passing through the bridges in a given monitoring interval, the method of considering the upper 5% of the values is based on experiments and there is limited data to evaluate its accuracy. The aim of this study is to develop a methodology to establish the maximum live load effect distribution for a mean recurrence interval with extreme value theories based on short-term monitored data of structural
response without counting the number of the vehicles. The accuracy of the distribution is evaluated by the convergence of the distribution parameters.

2.2 Extreme Value Theory

Since only the maximum structure response is considered in the present study, it is reasonable to use extreme theories to estimate the long-term maximum response from the short-term records of structure response. In probability theory and statistics, the generalized extreme value (GEV) distribution is used to model the extreme values of long (finite) sequences of independent, identically distributed random variables.

The GEV distribution is a family of continuous probability distributions that focus on the behavior of the extreme values (maximum or minimum) of a data set. There are essentially three types of extreme value distributions, Gumbel, Fréchet, and Weibull distributions, also known as Type I, II and III extreme value distributions developed within the extreme value theory.

Let the variable \( X \) be the maximum of \( n \) independent random variables \( Y_1, Y_2, Y_3 \). Since the inequality \( X \leq x \) implies \( Y_i \leq x \) for all \( i \ (i = 1, 2, ..., n) \), it follows that

\[
F(X \leq x) = \text{Prob} (Y_1 \leq x, Y_2 \leq x, ..., Y_n \leq x)
\]

\[
= F_{Y_1}(x)F_{Y_2}(x) ... F_{Y_n}(x)
\]  

(2-9)

The distributions \( F_{Y_i}(x) \) are referred to as the initial distributions of the variables \( Y_i \). The latter constitutes the parent population from which the largest values \( X \) have been extracted. In the particular case in which all the variables \( Y_i \) have the same probability distribution \( F_Y(x) \), the probability distribution of \( X \) becomes

\[
F_X(x) = [F_Y(x)]^n
\]  

(2-10)
If the number $n$ becomes large enough, the cumulative distribution $F_X(x)$ of the largest values approach limits known as Type I or Type II extreme value distributions if the initial distributions are of the exponential or of the Cauchy type, respectively (Simiu and Scanlan 1986). The extreme value Type I distribution has two forms. One is based on the smallest extreme, i.e. the minimum case, and the other is based on the largest extreme, i.e. the maximum cases. As mentioned above, the extreme value Type I distribution is also referred to as the Gumbel distribution. Eq. (2-3) presents the maximum case cumulative distribution function (CDF) of the Gumbel distribution. The cumulative distribution function for the Type II distribution also known as Frechet distribution is

$$F_H(x; \mu, \sigma, k) = \begin{cases} 0 & x \leq \mu \\ \exp\left(-\left((x - \mu)/\sigma\right)^{-k}\right) & x > \mu \end{cases} \quad (2-11)$$

Both Type I and Type II extreme distributions have an unlimited tail length.

2.3 Modeling of Maximum Live Load Effects for a Mean Recurrence Interval

To establish the probability model of maximum live load effects, the length of the mean recurrence interval must be determined first since a maximum live load effect corresponds to a certain mean recurrence interval. According to AASHTO (2007), for a new bridge, the mean recurrence interval is 75 years; for an existing bridge, the mean recurrence interval is the expected remaining service life-cycle of the bridge. For simplification, one can use the yearly maximum live load effects as a demonstration. The yearly maximum live load effect is the extreme value of live load effects the structure is subjected to in a year that can be divided into $n$ time segments. The maximum live load effects in each time segment, $Q_{seg,t}$, is a variable and can be obtained with a bridge health monitoring system. The extreme value in each segment is assumed to be independent from each other and have the same cumulative distribution function.
\( F_{\text{seg}}(Q) \) that is referred to as the initial distribution. The distribution of yearly maximum live load effect can be derived according to Eqs. (2-9) and (2-10) as:

\[
F_y(Q) = [F_{\text{seg}}(Q)]^n
\]  

(2-12)

The accuracy of the estimation of the yearly maximum live load effects probability distribution depends on the accuracy of the distribution of the initial population and the number of the intervals. For different number of time segments \( n_1 \) and \( n_2 \) (or different length of time segments, \( \text{seg}_1 \) and \( \text{seg}_2 \)), the initial distributions \( F_{\text{seg,1}}(Q) \) and \( F_{\text{seg,2}}(Q) \) can be obtained. The distribution of the extreme live load effects in a mean recurrence interval can be derived in terms of \( F_{\text{seg,1}}(Q) \) and \( F_{\text{seg,2}}(Q) \) as follow:

\[
F(Q) = [F_{\text{seg,1}}(Q)]^{n_1} = [F_{\text{seg,2}}(Q)]^{n_2}
\]  

(2-13)

For example, the yearly extreme structure response distribution can be derived from initial distributions based on time segments of an hour or a minute as:

\[
F_y(Q) = [F_{\text{hr}}(Q)]^{365 \times 24} = [F_{\text{min}}(Q)]^{365 \times 24 \times 60}
\]  

(2-14)

where \( F_{\text{hr}}(Q) \) and \( F_{\text{min}}(Q) \) are the initial distributions and they represent the cumulative distribution function of the maximum structural response for 1 hour and 1 minute, respectively.

The initial distribution can be derived by curve fitting of the monitored data. It shows in Eq. (2-12) and Eq. (2-14) that the yearly maximum live load distribution is unique; the initial distributions corresponding to different lengths of time segments are not unique.

To estimate a reasonable number of intervals or to determine the length of the time segment requires that the following two principles be satisfied (Duan et al. 2002):
1. The time segment is long enough so that the maximum live load in every interval satisfies independence requirement.

2. The length of the time segment is reasonable so that the maximum live loads in every interval follow the same distribution.

These two conditions require the length of the time segments is long enough so that the structural response recorded in every time segment is a stationary and ergodic process. The property of stationary of a stochastic process always refers to the process being unchanged when shifting along the time axis (Lutes and Sarkani 2004). A strongly stationary process is a stochastic process whose joint probability distribution does not change when a shift in time or space satisfies the two principles discussed above for any length of intervals. In practice, the live load on a bridge is not a strongly stationary process but an interval-dependent quasi stationary process. With a sufficient length of the time segment, the structural response due to routine traffic in every time segment is a weak stationary random process, meaning that the 1st and 2nd moments do not vary with respect to time. A continuous time-weak stationary random process \( x(t) \) has the following restriction on its mean function

\[
E\{x(t)\} = m_x(t) = m_x(t + \tau) \quad \forall \tau \in R \tag{2-15}
\]

and autocorrelation function

\[
E\{x(t_1)x(t_2)\} = R_x(t_1, t_2) = R_x(t_1 + \tau, t_2 + \tau) = R_x(t_1 - t_2, 0) \quad \forall \tau \in R \tag{2-16}
\]

For an ergodic process, its statistical properties (such as its mean and variance) can be deduced from a single, sufficiently long sample (realization) of the process. In other words, statistical properties obtained from a single time-series will approach definite limits independent
of the particular series as the length of the series increases. As a stationary ergodic process, it is possible to estimate the process statistics from the observed values of a single time series. Mathematically, various ergodicity of various properties of a stochastic process can be discussed. For example, a weak stationary process \( x(t) \) has a mean \( \mu = E[x(t)] \) and autocovariance \( R_x(\tau) = E[(x(t + \tau) - \mu)(x(t) - \mu)] \) that do not change with time. One way to estimate the mean is to perform a time averaging. For a given sufficient time, they include or impinge on all points in a given space and can be represented statistically by a reasonably large selection of points as

\[
\hat{\mu}_T = \frac{1}{2T} \int_{-T}^{T} x(t) \, dt \tag{2-17}
\]

If the time averaged mean \( \hat{\mu}_T \) converges in squared mean to \( \mu \) as \( T \to \infty \), then the process \( x(t) \) is said to be mean-ergodic or mean-square ergodic in the first moment. Similarly, autocovariance \( R(\tau) \) can be calculated by performing a time averaging:

\[
\hat{R}_x(\tau) = \frac{1}{2T} \int_{-T}^{T} [x(t + \tau) - \mu][x(t) - \mu] \, dt \tag{2-18}
\]

If this expression converges in squared mean to the true autocovariance, i.e., \( \hat{R}_x(\tau) = E[(x(t) - \mu)(x(t + \tau) - \mu)] \), then the process is said to be autocovariance-ergodic or mean-square ergodic in the second moment. A process that is ergodic in the first and second moments is sometimes called ergodic in the broad sense.

For a stationary and ergodic process, if the sufficient length of time segments is used, the structural response distribution for a time segment can be identified using any set of response record in a time segment because the distributions in any time segments are the same. Thus, the maximum response in every time segment are independent and following the same distribution.
Once the initial distribution of the parent population is determined, the maximum distribution in any mean recurrent intervals can be derived from Eq. (2-12) and Eq. (2-13).

2.4 Case Study I

2.4.1 Bridge Description

The bridge selected for this study is the CORIBM Bridge on route LA 70 in District 61, Assumption Parish, Louisiana. The bridge was built in 1988, with a design load of HS20-44 and ADTT about 6000. The bridge, with a total length of 44.2m and a roadway width of 14m, consists of six 6.1m spans and a 7.6m span. The 6.1m spans are concrete structures and the 7.6m span consists of a steel grid deck supported on steel girders. The 7.6m steel span is designed for being lifted for river navigation when needed. Figure 2-1 shows the damaged grid deck that needs to be replaced in the 7.6 m span. The requirement of being movable and the span length of 7.6 m make this steel span a good candidate to be replaced with a FRP slab system.

The span to be replaced has eight 7600×1800 mm deck panels across the traffic direction, as shown in Figure 2-2. The FRP deck panels that have been bonded on the I-girders have the same dimensions as the steel grid deck panels. Labels A through J in Figure 2-2 stand for the girder positions, and 2 through 4 are the reference lines where some sensors are located. In this project, the bridge performance monitoring is concerned with: (1) integrity of the FRP wrapped Balsa wood bridge deck system; (2) the strains in the transverse direction of the deck and the longitudinal direction of the individual girders, and (3) bridge deck–girder interface bond integrity. Potentially, the measured strains can be used to identify truck weight and axle configuration, i.e., serve as a weigh-in-motion system. The present study focuses on the reliability of the steel beam’s flexural capacity.
The instrumentation plan was designed to measure the live load response behavior of the superstructure. The central four composite panels and supporting girders were instrumented with sensors. Externally attached fiber optic FBG sensors were used in this project. FBG sensors were attached at the bottom of all eight I-girders named 9-1 to 16-4 as shown in Figures. 2-3 and 2-4.
Three positions of the I-girders are chosen for monitoring, which are the mid-span and the other two positions about 1.00 m away from either end of the girders.

![Figure 2-3 Plan View of All Installed FBG Sensors at the Bottom of I Girder](image)

![Figure 2-4 Elevation of Arrangement of Typical FBG Sensor Array along Girder 5](image)

The strain values induced by traffic load were obtained by converting the wavelength shift of light traveling in the optical fiber sensors continuously. The rate of data acquisition is 62.5Hz. Figure 2-5 shows a time history record of strains of a steel girder G8. In this example, the three hours monitoring data is used to estimate extreme strain distribution for mean recurrence intervals of 1 day, 10 days, 30 days, 180 days and one year.
2.4.2 Parameter Estimation of Initial Distributions

The initial distributions with time segment lengths from 2s to 300s were determined using distribution fitting techniques. For example, if the time segment is 10s, then the maximum response in every 10s were identified from the recorded data. The identified maximum values constitute a new set of data sequence and its distribution (initial distribution) was simulated with a type of extreme value distributions. The parameters of the initial distribution were estimated using distribution fitting techniques. Under the independence assumption discussed earlier, it is straightforward to compute the estimator of unknown model parameters. A classical method of approaching the problem of estimations is the method of moments. In this method it is assumed that the distribution parameters can be obtained by replacing the expectation and the mean square value of the random variable by the corresponding statistics of the sample.

In some aspects, when estimating parameters of a known family of probability distributions, the maximum likelihood estimation method is a better choice, because the
maximum likelihood estimators have a higher probability of being close to the quantities to be estimated. The concept behind the maximum likelihood parameter estimation is to determine the parameters that maximize the probability (likelihood) of a set of fixed data. If \( x \) is a continuous random variable with a PDF \( f(x; \theta_1, \theta_2, ..., \theta_k) \), where \( \theta_1, \theta_2, ..., \theta_k \) are \( k \) unknown constant parameters that need to be estimated by obtaining \( N \) independent observations, \( x_1, x_2, ... x_N \) from an experiment. Then the likelihood function is given by the following product:

\[
L(x_1, x_2, ... x_N | \theta_1, \theta_2, ... \theta_k) = L = \prod_{i=1}^{N} f(x_i; \theta_1, \theta_2, ..., \theta_k) \quad i = 1, 2, ..., N \tag{2-19}
\]

The logarithmic likelihood function is given by:

\[
\Lambda = \ln L = \sum_{i=1}^{N} \ln f (x_i; \theta_1, \theta_2, ..., \theta_k) \tag{2-20}
\]

The maximum likelihood estimators of \( \theta_1, \theta_2, ..., \theta_k \) are obtained by maximizing \( L \) or \( \Lambda \). By maximizing \( \Lambda \), which is much easier to work with than \( L \), the maximum likelihood estimators of \( \theta_1, \theta_2, ..., \theta_k \) are the solutions of \( k \) simultaneous equations such that:

\[
\frac{\partial \Lambda}{\partial \theta_j} = 0, \quad j = 1, 2, ..., k \tag{2-21}
\]

In this study, the initial distribution was fitted with Gumbel distribution function (maximum cases) shown in Eq. (2-3). The distribution parameters were estimated by the maximum likelihood estimation method. The parameters corresponding to various time segments are summarized in Table 2-1. For example, the distribution of structural response induced by extreme live load effects with mean recurrences of 90s and 300s are

\[
F_{90s}(Q) = \exp \left( -\exp \left( -\frac{Q - \mu}{\sigma} \right) \right)
\]
respectively. The initial distribution fitting using *Gumbel* distribution functions with time segments of 8s, 50s, 90s and 300s are shown in Figure 2-6.

<table>
<thead>
<tr>
<th>Time segments (Seconds)</th>
<th>Gumbel Distribution</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>µ</td>
<td>σ</td>
</tr>
<tr>
<td>2</td>
<td>2.605</td>
<td>2.613</td>
</tr>
<tr>
<td>4</td>
<td>3.419</td>
<td>4.076</td>
</tr>
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<td>5.379</td>
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<td>8</td>
<td>5.119</td>
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</tr>
<tr>
<td>10</td>
<td>5.973</td>
<td>7.365</td>
</tr>
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<td>20</td>
<td>9.823</td>
<td>11.153</td>
</tr>
<tr>
<td>30</td>
<td>13.190</td>
<td>14.004</td>
</tr>
<tr>
<td>40</td>
<td>16.724</td>
<td>16.317</td>
</tr>
<tr>
<td>50</td>
<td>19.042</td>
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<td>20.095</td>
</tr>
<tr>
<td>70</td>
<td>24.604</td>
<td>21.674</td>
</tr>
<tr>
<td>80</td>
<td>27.166</td>
<td>23.171</td>
</tr>
<tr>
<td>90</td>
<td>29.768</td>
<td>24.367</td>
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<tr>
<td>100</td>
<td>31.921</td>
<td>26.044</td>
</tr>
<tr>
<td>120</td>
<td>35.260</td>
<td>27.678</td>
</tr>
<tr>
<td>140</td>
<td>39.251</td>
<td>28.749</td>
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<td>160</td>
<td>43.817</td>
<td>29.289</td>
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<td>180</td>
<td>44.904</td>
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<td>200</td>
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<td>31.328</td>
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<td>220</td>
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<tr>
<td>240</td>
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<td>280</td>
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</tr>
<tr>
<td>300</td>
<td>61.584</td>
<td>32.682</td>
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</tbody>
</table>
2.4.3 Extreme Live Load Effects Prediction and Verification

*Gumbel* distribution (maximum cases) was also applied to describe the extreme strain distribution in mean recurrence intervals of 1 day, 10 days, 30 days and one year. These distributions were derived from Eq. (2-13). For instance, the daily or yearly extreme strain distribution can be derived from the initial distributions with time segments of 90s or 300s as

**Figure 2-6** Initial Distribution Fitting Using Gumbel Distribution Function with Time Segments of: (a) 8s; (b) 50s; (c) 90s; (d) 300s
\[
F_d(Q) = \left[ F_{90s}(Q) \right]^{24 \times 3600 / 90} = \left[ F_{300s}(Q) \right]^{24 \times 3600 / 300}
\] (2-24)

\[
F_y(Q) = \left[ F_{90s}(Q) \right]^{365 \times 24 \times 3600 / 90} = \left[ F_{300s}(Q) \right]^{365 \times 24 \times 3600 / 300}
\] (2-25)

where the initial distributions \( F_{90s}(Q) \) and \( F_{300s}(Q) \) have been obtained from distribution fitting previously.

It is difficult to prove directly, for the monitored data, that Eq. (2-24) and Eq. (2-25) are tenable and it is difficult to derive the parameters of daily or yearly extreme response distribution through an analytical method. An alternative method of verification is to generate samples using Monte Carlo simulation following the distribution functions on the right side of Eq. (2-24) and Eq. (2-25), and then, fit the generated samples with the selected distribution function, Gumbel distribution (maximum cases). If the parameters obtained by the distribution fitting procedure, based on different lengths of time segments, are the same, then Eq. (2-24) and Eq. (2-25) are tenable and verified. Take yearly extreme response for example. According to Eq. (2-25), for a given \( F_y(Q) \) we have,

\[
Q = F_{300s}^{-1}\left\{ F_y(Q) \right\}^{300 \times 24 \times 3600 / 300}
\] (2-26)

\[
Q = F_{90s}^{-1}\left\{ F_y(Q) \right\}^{90 \times 24 \times 3600 / 90}
\] (2-27)

The yearly extreme response probabilities \( [F_y(Q)] \) are generated randomly between 0 and 1. The corresponding extreme responses Q can be derived from Eq. (2-26) and Eq. (2-27) and modeled with Gumbel distribution. The distribution parameters of Q are determined with the maximum likelihood estimation method as discussed earlier. If the parameters of Q from both Eq. (2-26) and (2-27) are close, then Eq. (2-25) are verified. The extreme distribution parameters, \( \mu \)
and \( \sigma \) with mean recurrence intervals of 1 day, 10 days, 30 days, 180 days and one year are derived from the initial distribution corresponding to different lengths of time segments. The obtained \( \mu \) and \( \sigma \) are listed in Table 2-2 and Table 2-3 and are shown in Figure 2-7 and Figure 2-8, respectively.

**Table 2-2** Extreme Strain Distributions Parameter, \( \mu \), Derived from Different Initial Distributions with Various Lengths of Time Segments

<table>
<thead>
<tr>
<th>Time segments (seconds)</th>
<th>( \mu ) (1.0E-6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>65.065</td>
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<tr>
<td>50</td>
<td>151.859</td>
</tr>
<tr>
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<tr>
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<td>178.898</td>
</tr>
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Table 2-3 Extreme Strain Distributions Parameter, \( \sigma \), Derived from Different Initial Distributions with Various Lengths of Time Segments

<table>
<thead>
<tr>
<th>Time segments (seconds)</th>
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<td>33.235</td>
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<td>32.857</td>
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From Figure 2-7 and Figure 2-8, it is seen that both the location factor \( \mu \) and the scale factor \( \sigma \) converge as the length of time segment increases. They become constant when the length of time segment is longer than 200s, which verifies Eqs. (2-12), (2-13), (2-24) and (2-25). These results also show that structural response for a time segment is a stationary and ergodic process when the time segment is longer than 200s. For example for a mean recurrence interval of 10 days, the distribution based on 300s time segment is
\[ F_{10d}(Q) = \exp\left(-\exp\left(-\frac{Q-321.988}{32.523}\right)\right) \quad (2-28) \]

and for a mean recurrence interval of 1 year

\[ F_y(Q) = \exp\left(-\exp\left(-\frac{Q-439.830}{32.857}\right)\right) \quad (2-29) \]

It should be noted that the extreme structural response distributions for mean recurrence intervals can be expressed based on initial distributions with time segments other than 300s, and the distribution parameters are a little different from that derived from time segment of 300s. For instance, the extreme structural response distributions for mean recurrence intervals derived based on initial distributions with time segment of 200s were determined as follows:

for a mean recurrence interval of 10 days,

\[ F_{10d}(Q) = \exp\left(-\exp\left(-\frac{Q-311.706}{31.173}\right)\right) \quad (2-30) \]

for a mean recurrence interval of 1 year

\[ F_y(Q) = \exp\left(-\exp\left(-\frac{Q-424.656}{31.493}\right)\right) \quad (2-31) \]

The small differences between Eqs. (2-28) and (2-30) and between Eqs. (2-29) and (2-31) are due to the distribution fitting error of recorded data.

For different mean recurrence intervals, the \( \mu \) converges to different values, but the \( \sigma \) converges to a fixed value. The PDF of extreme strain distribution for mean recurrence intervals of 1 day, 10 days, 30 days, and one year are shown in Figure 2-9. It indicates that the extreme structural response distribution for different mean recurrence intervals have the same shape but with different locations. The location factor \( \mu \) determines the mode value of the distribution.
while the shape factor $\sigma$ determines the variance or the standard deviation of the distribution. Its location shifts to the right direction (larger value) as the mean recurrence interval increases. The distributions have different mode values but same variance for different mean recurrence intervals. Figure 2-10 shows that the mode values of the extreme response distribution increase gradually as the length of the mean recurrence increases.

**Figure 2-7** Extreme Strain Distributions Parameter, $\mu$, Derived from Different Initial Distributions with Various Lengths of Time Segments

**Figure 2-8** Extreme Strain Distributions Parameter, $\sigma$, Derived from Different Initial Distributions with Various Lengths of Time Segments
2.4.4 Discussion of Predicted Extreme Values

As shown above, both $\mu$ and $\sigma$ of the extreme values converge to constant values when the time segment is longer than 200s. Therefore, the extreme strains due to live load for different mean recurrence intervals were predicted only using time segments from 200s to 300s based on $\mu$ and $\sigma$ in Table 2-1 and Eq. (2-4), and are listed in Table 2-4, along with the average (mean) value based on different time segments.
Table 2-4 Extreme Strains for Different Mean Recurrence Intervals Calculated Using Eq. (2-4)

<table>
<thead>
<tr>
<th>Mean Recurrence Intervals (day)</th>
<th>Extreme Strain Derived from Eq. (2-4) (1.0E-6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>200 (s)</td>
</tr>
<tr>
<td>1</td>
<td>239.49</td>
</tr>
<tr>
<td>30</td>
<td>346.078</td>
</tr>
<tr>
<td>180</td>
<td>402.211</td>
</tr>
<tr>
<td>365</td>
<td>424.359</td>
</tr>
</tbody>
</table>

The extreme strain for each mean recurrence interval derived based on different time segments are not exactly the same, but is close. The difference is induced because the monitored data does not fit the Gumbel distribution perfectly as shown in Figure 2-6. The error can be measured by coefficient of variation, $V_x$, defined as the standard deviation divided by the mean:

$$V_x = \frac{\sigma_x}{\mu_x}$$  \hspace{1cm} (2-32)

The coefficient of variation of extreme strain calculated based on different time segments (from 200s to 300s) for different mean recurrence intervals 1 day, 10 days, 30 days, 180 days and one year is between 2.06% to 2.20%. It demonstrates that though the extreme responses calculated based on different time segments are not exactly the same, its accuracy is sufficient. Therefore, it is reasonable to use mean value of extreme strain derived based on different time segments (from 200s to 300s) as the extreme response, as shown in the last column of Table 2-4 and plotted in Figure 2-11.

Meanwhile, the distribution of the extreme strain has been predicted using the proposed methodology and the Gumbel distribution parameters have already been shown in Tables 2-2 and 2-3. Comparing between Table 2-4 and Table 2-2, it is found that the extreme response calculated using Eq. (2-4) is close to the mode value (location parameter) of the extreme
response distribution (Gumbel distribution) derived from the present methodology. Table 2-5 shows the ratio of the mode value of extreme strain distribution predicted using the proposed methodology to that calculated using Eq. (2-4). It is concluded that Eq. (2-4) only predicts a constant extreme value that is the same as the mode value of the Gumbel distribution as observed in Table 2-5. It does not consider the type and variation of the distribution of the extreme values. The present methodology predicts a distribution for the extreme response which can be used in the reliability calculation.

![Figure 2-11](image)

**Figure 2-11** Mean Values of Extreme Strain Distribution Mode for Mean Recurrence Intervals of 1 Day, 10 Days, 30 Days, 180 Days and One Year

<table>
<thead>
<tr>
<th>Mean Recurrence Intervals (day)</th>
<th>200 (s)</th>
<th>220 (s)</th>
<th>240 (s)</th>
<th>260 (s)</th>
<th>280 (s)</th>
<th>300 (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00024</td>
<td>1.00025</td>
<td>1.00028</td>
<td>1.00028</td>
<td>1.00030</td>
<td>1.00032</td>
</tr>
<tr>
<td>10</td>
<td>1.00015</td>
<td>1.00024</td>
<td>1.00016</td>
<td>1.00024</td>
<td>1.00026</td>
<td>1.00025</td>
</tr>
<tr>
<td>30</td>
<td>0.99961</td>
<td>0.99970</td>
<td>0.99960</td>
<td>0.99969</td>
<td>0.99971</td>
<td>0.99969</td>
</tr>
<tr>
<td>180</td>
<td>0.99985</td>
<td>0.99994</td>
<td>0.99985</td>
<td>0.99994</td>
<td>0.99996</td>
<td>0.99994</td>
</tr>
<tr>
<td>365</td>
<td>1.00070</td>
<td>1.00079</td>
<td>1.00070</td>
<td>1.00079</td>
<td>1.00080</td>
<td>1.00079</td>
</tr>
</tbody>
</table>

**Table 2-5** Ratio of the Mode Value of Extreme Response Distribution to That Calculated Using Eq. (2-4)
2.4.5 Reliability Calculation

Reliability index of a steel girder under the routine traffic live load is calculated. The limit state is defined based on the normal stress due to the applied loads at the mid-span exceeding the yield strength. Figure 2-12 shows a composite cross-section with a FRP deck and steel girders used in the calculation of stress. Following the structural reliability theory, the limit state function, \( g(R, Q_d, Q_l) \), is defined as follows:

\[
g(R, Q_d, Q_l) = R - Q_d - Q_l
\]  

where \( R \) represents the normal stress capacity, and \( Q_d \) and \( Q_l \) represent the dead and live load effects, respectively.

Akgul and Frangopol (2004b) identified the random variables individually. In their research, the yield strength and the dead load effects are assumed following a lognormal distribution and are characterized by their mean value \( \mu \), standard deviation \( \sigma \), and the corresponding lognormal distribution parameters (\( \lambda \), \( \nu \)). Table 2-6 lists the lognormal distribution parameters of \( g_{1832} \), \( g_{1827} \), and \( g_{1831} \). The elastic modulus of steel and FRP wrapped Balsa wood deck are assumed to be constant as \( E_s = 2 \times 10^5 \text{Mpa} \), and \( E_{deck} = 9.239 \text{Mpa} \). The extreme structural response induced by the live load is derived with the method developed in the present methodology based on monitored data.

The reliability of the bridge can be measured with the first-order second-moment mean value reliability index which is calculated from the following formula:

\[
\beta = \frac{\mu_R - \mu_{Q_d} - \mu_{Q_l}}{\sqrt{\sigma_R^2 + \sigma_{Q_d}^2 + \sigma_{Q_l}^2}}
\]  

(2-34)
where $\mu_R$ and $\sigma_R^2$, $\mu_{Qd}$ and $\sigma_{Qd}^2$,and $\mu_{Ql}$ and $\sigma_{Ql}^2$ represent mean values and variance of resistance, dead load effects, and live load effects, respectively.

The calculation of the *first-order second-moment mean (FOSM) value reliability index* only concerns about the mean value and variance of the variables that are assumed to follow normal distribution. In fact, detailed information on the type of distribution for each random variable can improve the accuracy of the reliability index. The Rackwitz-Fiessler procedure, an iteration procedure provides a way to calculate reliability index with variables following non-normal distribution by calculating “equivalent normal” values of the mean and standard deviation for each non-normal random variable (Nowak and Collins 2000). The reliability indices calculated using this modified method are presented in Figure 2-13 and a significant difference is observed.
2.5 Case Study II

To further confirm the developed strategy, a second example is used below.

A cable-stayed bridge over Haihe River located in Tianjin, China, was monitored after the cables were replaced. The bridge, built in 1987, consists of two 25.15m approach spans and three spans in the main crossing section, namely two 99.85m spans and one 260m span. Forty four pairs of cables were fan-designed on two towers. Some parts of the bridge were damaged due to the increasing overweight trucks recent years. Cables in the longest span were replaced with smart cables in 2006. The smart cables are made with combining FRP bars and optic fiber grating (OFBG) sensors. A picture and elevation of the bridge are shown in Figure 2-14. The stress in the cables was monitored using the smart cables for 120 hours (not continued). The monitored stress of cable 10 is shown in Figure 2-15 (Lan 2009).
The initial extreme distributions with time segment lengths of 20s to 6000s were determined using distribution fitting techniques, similarly to Case Study I. The initial extreme distribution fitting using the Gumbel distribution function with segments of 600s, 1200s, 1800s,
Tables 2-7 and 2-8 list the Gumbel distribution parameters $\mu$ and $\sigma$ for various mean recurrent intervals from 1 year to 75 years based on different initial extreme distributions; and the parameters $\mu$ and $\sigma$ are also shown in Figures 2-17 and 2-18.

As in Case Study I, for different mean recurrence intervals, $\mu$ converges to different values, but $\sigma$ converges to a fixed value. The PDF of extreme strain distributions for mean recurrence intervals of 1 year, 10 years, 30 years, 50 years and 75 year are shown in Figure 2-19.

Figure 2-20 shows the extreme response, the mode value of extreme strain derived based on different time segments (from 1500s to 6000s) for different mean recurrent intervals.

2.6 Conclusions

This study developed a framework to estimate the extreme strain distribution for mean recurrence intervals due to live load effects based on short-term monitoring. Two example bridges were studied to demonstrate the application of the developed methodology in reliability calculations. The following conclusions are drawn based on the developed methodology and example applications:

1. The structure’s strain response due to live loads is a weak stationary random process. The duration of the monitoring can be divided into a series of time segments. If the length of the time segment is long enough, the following two principles are satisfied, otherwise the distribution will not be convergent:

   a. The maximum live load effect in each time segment is independent.

   b. The maximum live load effect in each time segment follows the same distribution.

   The appropriate length of the time segments may be different for different bridges.
Figure 2-16 Initial Distribution Fitting Using Gumbel Distribution Function with Time Segments of: (a) 600s; (b) 1200s; (c) 1800s; (d) 2400s; (e) 3600s; (f) 6000s
Table 2-7 Extreme Strain Distributions Parameter, $\mu$, Derived from Different Initial Distributions with Various Lengths of Time Segments

<table>
<thead>
<tr>
<th>Time Segment (seconds)</th>
<th>Mean Recurrent Intervals (year)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>36.878</td>
</tr>
<tr>
<td>30</td>
<td>43.824</td>
</tr>
<tr>
<td>60</td>
<td>60.923</td>
</tr>
<tr>
<td>120</td>
<td>84.235</td>
</tr>
<tr>
<td>240</td>
<td>109.961</td>
</tr>
<tr>
<td>300</td>
<td>116.325</td>
</tr>
<tr>
<td>360</td>
<td>119.819</td>
</tr>
<tr>
<td>420</td>
<td>124.906</td>
</tr>
<tr>
<td>510</td>
<td>128.851</td>
</tr>
<tr>
<td>600</td>
<td>130.271</td>
</tr>
<tr>
<td>750</td>
<td>131.413</td>
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<tr>
<td>900</td>
<td>133.105</td>
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<td>1800</td>
<td>130.050</td>
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<tr>
<td>2100</td>
<td>130.441</td>
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<tr>
<td>2700</td>
<td>126.494</td>
</tr>
<tr>
<td>3000</td>
<td>124.166</td>
</tr>
<tr>
<td>3300</td>
<td>125.511</td>
</tr>
<tr>
<td>3900</td>
<td>128.452</td>
</tr>
<tr>
<td>4200</td>
<td>127.699</td>
</tr>
<tr>
<td>4500</td>
<td>129.551</td>
</tr>
<tr>
<td>4800</td>
<td>117.967</td>
</tr>
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<td>5400</td>
<td>121.542</td>
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<tr>
<td>5700</td>
<td>120.268</td>
</tr>
<tr>
<td>6000</td>
<td>122.520</td>
</tr>
<tr>
<td>Time Segment (seconds)</td>
<td>1</td>
</tr>
<tr>
<td>-----------------------</td>
<td>------</td>
</tr>
<tr>
<td>20</td>
<td>2.420</td>
</tr>
<tr>
<td>30</td>
<td>2.966</td>
</tr>
<tr>
<td>180</td>
<td>7.574</td>
</tr>
</tbody>
</table>
2. The extreme strains due to live loads for each time segment was identified from the monitored data. Its initial distribution was modeled with the Gumbel distribution function (maximum cases). The distribution parameters were determined using the maximum likelihood estimation method.
3. The distribution of extreme strains due to live loads in a mean recurrence interval was determined. The distributions were derived based on extreme values in every time segment instead of using the upper 5% values during the monitoring.
4. The distributions of extreme strains for different mean recurrence intervals have different mode values (position value) but the same scale parameter. The mode values increase smoothly as the length of the mean recurrence increases.

5. The reliability index was presented using two research methods, FOSM method and Rackwitz-Fiessler procedure. In the first one, the variables R and Q are assumed to follow normal distributions; while in the second one, the variables are assumed to follow non-normal distributions. The difference of reliability index is significant due to different distributions.

2.7 References


CHAPTER 3. TIME-VARIANT RELIABILITY ANALYSIS OF EXISTING BRIDGES CONSIDERING CORROSION OF STEEL

3.1 Introduction

During the entire life cycle, infrastructures often experience three major periods, including constructions, services, and deteriorations. In each period, structure performance may change due to various natural and manmade factors. Among all, the variation of concrete strength, the corrosion of steel, and the increasing traffic load are the main factors that affect the reliability of existing bridges. Traditional, the resistances are always modeled with random variables during the reliability analysis of structures in design. These variables are assumed to be independent of time, and the randomness is only due to the uncertainty of materials used in constructions and dimensions of components. In addition, they are also assumed following the lognormal distributions (Akgul and Frangopol 2004; Akgul and Frangopol 2004; Akgul and Frangopol 2005; Akgul and Frangopol 2005). This assumption provides an easy way to estimate the reliability with probability functions only, but it does not take into the consideration of the effect of the aggressive environment on the material properties.

In practice, the time-varying resistance cannot be ignored especially for structures exposed to aggressive environments such as industrial buildings or structures near the seaside. In these cases, random processes should be adopted to simulate the structural resistance rather than random variables. There is a significant difference between random processes and random variables. When one describes random variables or random process using distributions, the parameters for random variables do not change with time, while random processes do. The resistance is represented with random variables following certain distributions. The distributions are usually derived from samples collected at the starting of a service cycle, so that the resistance distributions, characterized with mean value, variation, etc., describe the state of a structure when
the samples are collected. The information of the resistance provided by the distribution is not sufficient to describe its variation during the entire service cycle. It describes resistance at a certain time moment only, and does not consider the resistance variation over time, i.e., the resistances are assumed constant. On the contrary, the resistances change all the time during the structure’s life cycle. The resistance at one moment $R(t)$, to a great extent, determines the resistance of the structure in the future, $R(t + \Delta t)$; and theoretically it is an auto-correlated process. If the resistance is high at one moment, there is a great chance that the resistance is still high thereafter. This means the resistances at different time moment are positively correlated. Though it is a highly auto-correlated process, its variation cannot be ignored. In general, the resistance may increase at the beginning of the structural service cycle because of the concrete hardening as the age increases; and then, it decreases because of the environment and the aging of the materials especially for those exposed to aggressive environments. Furthermore, it should be noted that the resistance varying with time is an irreversible process. The random process resistance model follows three conditions:

1. The mean values of resistance are monotonically decreasing with time. The mean value functions reflect the general trend of the resistance. Although to some extent some factors changing with time are beneficial to increase the resistance. For example, the strength of concrete at early age, the resistance has a trend of decrease especially for components exposed to aggressive environments in most cases.

2. The variations of resistance are monotonically increasing functions with time. The variation functions reflect the uncertainty of resistance. The more factors involved in the resistances and the longer the structure exposed to an aggressive environment, the greater uncertainty the resistances have. Factors such as environment temperature, humidity and various
aggressive medium do not make significant effects on resistance at the early age of the components, while as the time increases the effects become more and more serious and cannot be neglected. Because of the diversity and complexity the environment acting on the structures, the variation of resistance increases while the mean value of the resistance decreases due to the aggressive environment.

3. Autocorrelations are monotonically decreasing functions of time intervals. Autocorrelation functions represent the relationship of resistance at two given time moments. The relationship of resistances at two different time moments becomes weak as the time intervals increase, thus the autocorrelation decreases correspondingly.

Apparently, the resistance is a non-stationary random process and the statistic properties change with time. These three conditions describe the characters of the resistance random processes.

Contrary to the structural resistance, the traffic load on the bridge is a monotonically increasing process. This is due to the dramatic increasing traffic recent years. In addition, a property of traffic loads is site specific. A key bridge or a bridge near a factory may have large opportunity to accommodate heavy trucks. Because of the large variation, extreme live load defined by the AASHTO specifications (American Association of State Highway and Transportation Officials. 2007) may not represent the actual live load acting on the bridge precisely. Fortunately, recently developed structural health monitoring (SHM) technique can record structural response such as strains, and deflections at critical locations of bridge components and dynamic response of structures. These acquisitions reflect the bridge performance under routine traffic. The internal force of structural components, and the live load
acting on the bridge; can be derived from these acquisitions. The methodology to estimate structural extreme response distributions for mean recurrence intervals based on short-term monitoring has been presented in Chapter 2.

Time-variant reliability considering corrosion of steel has received increasing attentions recently. Previous researches were concentrated to develop steel corrosion models in concrete and many corrosion models have been proposed. The previous methods of reliability calculation are based on the assumption that both the resistance and load effects are stationary processes, however, both of them are time-variant. Thus, it is impossible to describe the time-variant reliability of bridges using previous methods. In the present research, the entire life cycle is assumed to be the sum of a time series, during each interval (a year), both the resistance and load effects are assumed to be stationary processes. The safe probabilities for any length of mean recurrent intervals are obtained by continued multiplication of the yearly safe probability.

3.2 Time-variant Properties of Materials

3.2.1 Corrosion of Concrete Reinforcement

Corrosion of reinforcement is an electrochemical process. CO$_2$ (carbon dioxide) and Cl$^-$ (chloride ion) are the most common causes that damage the reinforcement’s passive film which preserves the reinforcement from corrosion.

Carbonization of concrete is a precondition to induce reinforcement corrosion. The stability of the reinforcement’s passive film relies on the pH value of the concrete around the reinforcement. There are two critical pH values for reinforcements. One is pH=9.88 at which the passive films begin to emerge. The other one is pH=11.5 at which the passive film is fully established, in other words, the passive film is unstable when the pH value is under this value.
(Niu 2003). Exposed to CO$_2$ for a long time, chemical reactions occur between Ca(OH)$_2$ and NaOH in the concrete structural components. As a consequence of carbonization, the decreasing of pH value in the concrete destroys the passive film on the surface of the reinforcement, and the reinforcement loses the protection from the concrete. Type of concrete, type of cement, and the water cement ratio determine the rate of the concrete carbonization.

A more common and aggressive type of corrosion that bridges are experiencing is chloride-induced corrosion. Thoft-Christensen (1998) estimated the starting time of corrosion considering the thickness of concrete coverage, factor of chloride ions diffusion, density of chloride ions on the surface of concrete, and the critical chloride density at which the corrosion begins. Vu and Stewart (2000) established formulas to calculate the rate of corrosion based on survey on concrete bridges. The distribution type and statistic parameters of the factors in the formula were proposed.

The diffusion process which represents chloride ions penetration through concrete is assumed following the Fick’s second law of diffusion (Stewart and Rosowsky 1998), mathematically,

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial^2 x^2}$$  \hspace{1cm} (3-1)

where $C$ is the chloride ion concentration at a distance $x$ from the surface at $t$ years and $D$ is the apparent diffusion coefficient. By solving the equation above, the chloride content $C(x, t)$ at distance $x$ from the concrete surface and at time $t$ for bridge located in areas where de-icing salts are used is

$$C(x, t) = C_0 \left[1 - erf\left(\frac{x}{2\sqrt{tD}}\right)\right]$$  \hspace{1cm} (3-2)
where \( C_0 \) is the surface chloride content, \( D \) is the apparent diffusion coefficient and \( \text{erf} \) is the error function. Given the threshold concentration \( C_{th} \) triggering chloride corrosion and the location of every reinforcement, the time for the initiation of chloride corrosion, \( T_{corr} \), can be obtained by rearranging Eq.(3-2) as:

\[
T_{corr} = \frac{x^2}{4D[\text{erf}^{-1}(C_{th}/C_0)]} 
\]  

(3-3)

where \( T_{corr} \) is the time of corrosion initiation (years), \( x \) the depth of cover (mm), \( D \) the diffusion coefficient for chloride in concrete (mm\(^2\) /year), \( \text{erf}^{-1} \) denotes the inverse complimentary standard error function, and \( C_{th} \) and \( C_0 \) the threshold and surface chloride concentrations (gm/mm\(^3\)), respectively.

Two types of corrosion models, a general corrosion model and a pitting corrosion model are proposed to estimate the loss of cross-section due to corrosion. The general corrosion model assumes that the rate of steel loss is constant over the entire surface area of the reinforcement. The cross-section area of reinforcement after corrosion is (Marsh and Frangopol 2008)

\[
A_{st}(t) = \frac{n\pi(D_0 - r_{corr}(t-T_{corr}))^2}{4} 
\]  

(3-4)

where \( n \) is the number of bars experiencing active corrosion, \( D_0 \) the initial bar diameter (mm), \( r_{corr} \) the instantaneous corrosion rate (mm/year), and \( T_{corr} \) the time to corrosion initiation (years). The pitting corrosion model describes the local corrosion where chloride ions break down the passive film in localized areas along the length of the reinforcement. Stewart (2004) established a pitting corrosion model shown in Figure 3-1.
Given an average corrosion rate, $r_{corr}$, the maximum pit depth along a given length of reinforcement will be

$$p(t) = r_{corr} R t$$

(3-5)

where $r_{corr}$ is the average instantaneous corrosion rate (mm/year), $R$ the ratio of the maximum pit depth over the average pit depth ($P_{max}/P_{AV}$) along a given length of reinforcement, and $t$ the time since the corrosion initiation in years. The cross-sectional area of pit $A_{pit}(t)$ (mm$^2$) can be expressed as

$$a = 2p(t) \sqrt{1 - \left(\frac{p(t)}{D_0}\right)^2}$$

(3-6)

$$A_{pit}(t) = \begin{cases} A_1 + A_2, & \text{for } p(t) \leq \frac{D_0}{\sqrt{2}} \\ \frac{\pi D_0^2}{4} - A_1 - A_2, & \text{for } \frac{D_0}{\sqrt{2}} < p(t) \leq D_0 \\ \frac{\pi D_0^2}{4}, & \text{for } p(t) > D_0 \end{cases}$$

(3-7)

where

$$A_1 = 0.5 \left[ \left( \frac{D_0}{2} \right)^2 - a \left| \frac{D_0}{2} - \frac{p(t)^2}{D_0} \right| \right]$$

(3-8)
and

\[ A_2 = 0.5 \left[ \theta_2 p(t)^2 - a \frac{p(t)^2}{D_0} \right] \quad (3-9) \]

\[ \theta_1 = 2\arcsin\left( \frac{a}{D_0} \right) \quad (3-10) \]

and

\[ \theta_2 = 2\arcsin\left( \frac{a}{2p(t)} \right) \quad (3-11) \]

The remaining cross-sectional area of the reinforcing steel at \( t \) years since the corrosion initiation due to the pitting corrosion of \( n \) bars is

\[ A_{st}(t) = A_{stnorm} - \sum_{m=1}^{n} A_{pit_m}(t) \quad (3-12) \]

where

\[ A_{stnorm} = \frac{n\pi D_0^2}{4} \quad (3-13) \]

3.2.2 Corrosion of I Section Steel Girder

"I" section steel girders of bridges exposed to salt water and atmosphere is inevitable to experience corrosion too. The corrosion decreases the thickness of the web and flange of the steel girders; and it thus decreases the stiffness of the girders. More seriously, it may lead to structure failures without significant signs. Figure 3-2. shows a corrosion model (Akgul and Frangopol 2004) due to the heavy exposure to leaking salt water. Corrosion is assumed to occur throughout the web height at the supports while it is assumed to occur only at the bottom quarter of the web height along the rest of the girder length including the mid-span location. Townsend and Zoccola (1982) and McCuen and Albrecht (1995) proposed a power function for the corrosion model

\[ p = b_0 t^{b_1} \quad (3-14) \]
where $b_0$ and $p$ = corrosion losses after one and $t$ years respectively, and $b_1$ is the slope of the logarithmic transformation of Eq. (3-14).

Figure 3-2 Corrosion Propagation Model for Steel Girders (adapted from Akgul, F., and Frangopol 2005)

3.2.3 Concrete Time-Variant Compressive Strength

The time-variant strength of concrete is a non-stationary random process. Few publications in the literature are available to describe the variation of concrete strength corresponding to time. Variation of concrete strength is related to several factors such as the 28 day concrete strength, the age of concrete, sustained load the component subjected to and the aggressive environment the component exposed to which may induce the deterioration of concrete. Ismail et al. (2011) investigated degradations due to long-term weathering actions on a
reinforced concrete structure. They monitored concrete compressive strength and reinforcement corrosion developments of a prototype reinforced concrete structure for 6 years using destructive and nondestructive tests. The result is shown in Figure 3-3.

Figure 3-3 shows that the compressive strength of concrete increases dramatically in the first 120 day; it reaches a peak at the age of 1 year, and then decreases smoothly over time. A strength loss is as much as 27.6% of the maximum strength at age of 6 years. Al-Khaiat and Fattuhi (Al-Khaiat and Fattuhi 2001) investigated the long-term development of the compressive strength of various concrete subjected to the Kuwait hot and arid environmental conditions. Their research indicated that, after 5 years, the difference in strength was marginal. Niu (1995) proposed a time-variant model of compressive strength based on the information of the exposing experiment of concrete all over the world. The mean value and standard deviation of concrete are expressed as:

\[
\mu(t) = 1.4529 \exp[-0.0246(\ln(t) - 1.7154)^2]
\]

\[
\sigma(t) = 0.0305t + 1.2368
\]
where $t$ is the age (year) of the concrete.

Figure 3-4 shows the variation of mean value and standard deviation of concrete compressive strength as the age of the concrete increases (based on Eqs. (3-15) and (3-16)). The standard deviation increases linearly as the age increases.

**Figure 3-4** Variation of Mean Value and Standard Deviation of Concrete Compressive Strength as the Age of the Concrete Increases (adapted from Niu 1995)
3.3 Estimation of Extreme Live Load for a Mean Recurrent Interval

A reliability index is meaningful only when it corresponds to a determined mean recurrent interval, the expected service life cycle in the future. Once the length of the interval is determined, the extreme live load model for this determined interval is to be established. As discussed above, the live load acting on a bridge is site specific; it varies from bridge to bridge. To precisely estimate the reliability, a particular live load model for the bridge should be developed for a determined mean recurrent interval.

In Chapter 2, a methodology to estimate the extreme live load for any length of mean recurrent interval based on shorter monitored data is proposed. In that study, the monitoring duration was divided into numbers of time segments, and in each segment, the extreme response induced by the live load was selected. The selected extreme response composed a new set of data which was assumed to approach a *Gumbel* probability distribution (Gumbel 1958).

\[
F_{seg}(x; \mu, \sigma) = \exp \left( -\exp \left( -\frac{x-\mu}{\sigma} \right) \right) \quad -\infty < x < +\infty
\]  
(3-17)

where \( F(x) \) is the cumulative distribution function of a *Gumbel* probability distribution, a particular case of generalized extreme value theory; \( x \) is the extreme value of a random variable; \( \mu \) is the location parameter, referred to the mode (The mode is the value that occurs the most frequently in a data set or probability distribution) of the distribution and \( \sigma \) is the scale parameter.

The extreme live load expressed in terms of structure response in any length of mean recurrent interval were predicted using

\[
F_{exp}(Q) = [F_{seg}(Q)]^{N_{exp}}
\]  
(3-18)

where \( N_{exp} \) is the ratio of the mean recurrence interval to the time segment
\[ N_{exp} = \frac{T_{exp}}{T_{seg}} \] (3-19)

To determine the length of the time segment two conditions must be satisfied,

1. The time segment is long enough so that the maximum live load in every interval satisfies independence requirement.

2. The length of the time segment is reasonable so that the maximum live loads in every interval follow the same distribution.

It is difficult to derive the extreme response distribution parameters for the expected mean recurrent interval through an analytical method. An alternative method is to generate samples using Monte Carlo simulations following the distribution functions on the right side of Eq. (3-18), and then, fit the generated samples with the selected distribution function, the Gumbel distribution (maximum cases). The distribution parameters of Q are determined with the maximum likelihood estimation method. If the distribution parameters of Q obtained from different lengths of time segments are close, then Eq. (3-18) are verified. Thus, the structure extreme response due to live load for any length of mean recurrence intervals based on monitoring is obtained in terms of a Gumbel distribution. According to the research present in Chapter 2, it indicates that the extreme structural response distribution for different mean recurrence intervals have the same shape but with different locations. Unlike previous methods, the extreme strain due to live load in a mean recurrence interval is modeled with a Gumbel distribution instead of a constant value. Compared with other method, the location parameter in the Gumbel distribution \( \mu \) is close to that derived from the method used in developing the AASHO specifications. These two properties are useful in deriving equivalent extreme live load
distribution for any length of mean recurrent intervals based on the live load defined by the AASHTO specification.

For bridges lack of the monitoring information, the extreme live load can be derived using HL-93 truck load defined by AASHTO (2007). The AASHTO has defined different load combinations and load factors for different load combination limit states, strength states and service states, respectively. These load factors guarantee a higher reliability of ultimate limit states than service limit states with a mean recurrent interval of 75 years. To calculate the reliability, only service state load combination and load factors need to be considered. Let $\mu_{AA}$ be the structure response due to the live load, the AASHTO live load predicts the extreme structure response in a mean recurrent interval of 75 years. To calculate the time-dependent reliability, it is necessary to transfer it into a *Gumbel* distribution with a mean recurrent interval shorter than 75 years, for example one year. As discussed above, the location parameter in the *Gumbel* distribution $\mu$ is close to that derived from method used in developing the AASHTO specifications and the extreme structural response distribution for different mean recurrence intervals have the same shape but with different locations, i.e., they have the same variance. It is assumed that the shape factor is 31.55 (based on Chapter 2). It should be noted that for different bridges, the shape factors may be different. Substituted to Eq. (3-17), the extreme structure response is expressed in a *Gumbel* distribution

$$F_{75_y}(Q) = \exp \left( -\exp \left( -\frac{Q - \mu}{\sigma} \right) \right)$$

$$= \exp \left( -\exp \left( -\frac{Q - \mu_{AA}}{31.55} \right) \right) \quad (3-20)$$
To estimate the yearly extreme structure response due to the live load, after rearranging Eq.(3-20), we have

\[ F_y(Q) = \left[ F_{75y}(Q) \right]^{\frac{1}{75}} \quad (3-21) \]

where \( F_y(Q) \) follows a Gumbel distribution too.

Monte Carlo Simulation was used to estimate the distribution factors of \( F_y(Q) \) as discussed in Chapter 2. According to Eq. (3-21), for a given \( F_y(Q) \) we have,

\[ Q = F_{75y}^{-1}\left\{ \left[ F_y(Q) \right]^{75} \right\} \quad (3-22) \]

To obtain the distribution factors, samples were generated using Monte Carlo Simulation following the right side of Eq. (3-22), and then, fit the generated samples of Q with the selected distribution function, i.e., the Gumbel distribution (maximum cases).

### 3.4 Reliability Estimation

Bridge reliability, usually expressed with a reliability index, is quantified by comparing the structural capacity R with the load effects Q, using predefined limit state functions. For a reliability calculation, using traditional methods (first-order second-moment and other iteration methods) cannot provide a precise prediction for long mean recurrent intervals as long as a bridge’s life cycle of several decades. This is because the assumption used in the reliability calculation where both the structure capacity R and load effect Q are assumed as stationary random processes that do not reflect their variations with time. For stationary processes, their statistical properties do not change with time. In reality, both the structure capacity R and load effect Q are non-stationary processes because of the deterioration of the materials and the potential increasing traffic demand during the life-cycle of a bridge. Fortunately, since these
variations are long gradual processes, it is reasonable to divide a mean recurrent interval into short intervals in series, and then calculate the reliability in each interval using stationary process.

By combining the time series reliability, the total reliability can be evaluated through a series system’s reliability analysis. Assuming a structure life-cycle $t_L$ is uniformly divided into $n$ time segments $(0, t_1), (t_1, t_2), \ldots, (t_{n-1}, t_n)$ with a length of one year. The variable load capacity $R(t)$ and load effects $Q(t)$ are expressed with $n$ random processes, respectively. Though it is known that $R(t)$ is autocorrelated, it is difficult to describe $R(t)$ precisely. Since the variations of R and Q are gradual processes, it is reasonable to treat these random processes as stationary random processes in each segment. Thus, both $R(t)$ and $Q(t)$ are described using statistic properties and the structure reliabilities for each time segment are calculated using first-order second-moment and other iteration methods. Assuming that the failure of the structure is independent in different time segments, the reliability in each time segment is expressed as $P[Z(t_i) > 0]$. The reliability for the total life cycle is

$$P_s(t) = \prod_{i=1}^{n} P[Z(t_i) > 0]$$ (3-23)

3.5 Case Study I

The superstructure of the LA 415/Missouri Pacific Railroad overpass on US 190 is located at West Baton Rouge Parish and was constructed in 1940. It is a grade-crossing structure of the Federal Highway system and a National Bridge Inventory (NBI) structure. The purpose of this study is to examine whether the flexural capability satisfies the recent increasing traffic requirement or not. The structure consists of twenty 38’-0”cast-in-place concrete tee beam approach spans and five steel I-beam spans in the main crossing section, namely one 64’-6”, two
38’-0”, and two 47’-0” steel spans. All the beams are simply supported between the piers. The other bridge information is listed below:

- **Bridge Length**: 995’-0”
- **Number of Spans**: 25
- **Roadway Width**: 2@23’-9”
- **Number of Traffic lanes**: 2
- **Shoulder Widths**: None
- **Sidewalks**: 1’-2”
- **Design Load**: H15

According to the latest La DOTD Bridge Inspection Report (dated 05/14/98), no significant section loss which warrants a reduction in the capability of the primary load carrying members was indicated. Also included in the inspection report was documentation of cracks and spalls in the concrete decks resulting in exposure of the reinforcing steel throughout the structure. Additionally the inspection report indicated the presence of corrosion in some areas of the steel bridge members. This structure was built before 1950. Hence, the weight of the concrete rail was assumed to be distributed equally to each beam.

In this study, the flexural capability of the longest span, the 64’-6” span is calculated. The span consists of ten girders simply supported between the steel floor beams, with spacing of 7 ft (between exterior and interior girders) and 5 ft (between interior girders). The girders are classified as interior (In), exterior (Ex), and interior-exterior (I-E). The steel girders are stiffened with diaphragms located at the end and intermediate of the span, respectively. The cross-section of the steel span is shown in Figure 3-5.
Two possible critical girders, exterior girders and interior-exterior girders were examined with respect to the flexural limit states in this study. The cross-sections of these two girders are shown in Figure 3-6 and their cross-section properties are listed in Table 3-1.

### Figure 3-6 Cross-section of Exterior Girder, Interior-exterior Girder and I-Beam

#### 3.5.1 Dead Load

The statistical dead load used in the development of the AASHTO LRFD Code and OHBDC are listed in Table 3-1. All variables are treated as normal random variables (Nowak and Szerszen 1998).

<table>
<thead>
<tr>
<th>Component</th>
<th>Bias factor</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factory-made members</td>
<td>1.03</td>
<td>0.08</td>
</tr>
<tr>
<td>Cast-in-place members</td>
<td>1.05</td>
<td>0.10</td>
</tr>
<tr>
<td>Asphalt</td>
<td>75mm</td>
<td>0.25</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>1.03-1.05</td>
<td>0.08-0.10</td>
</tr>
</tbody>
</table>
According to Akgul and Frangopol, (Akgul and Frangopol 2004; Akgul and Frangopol 2004; Akgul and Frangopol 2005; Akgul and Frangopol 2005) the random variables related to girder capacity are assumed following a lognormal distribution, and each random variable is characterized by its mean value $\mu$, and standard deviation $\sigma$. In this study, the dead load effect due to steel girders is assumed to be time independent variable following a lognormal distribution.

3.5.2 Live Load Effect

Because of lack of direct monitoring data, the live load effect was derived using the live load effect defined with AASHTO (2007). The AASHTO codes provide a possible extreme heavy truck a bridge may experience in its service life cycle, 75 years, as specified. As discussed above, the structure response due to the live load is described with a Gumbel distribution. The location parameter $\mu$ is obtained by calculating the structure response due to live load proposed by AASHTO. The flexural moments were obtained with a line girder model of a bridge. The cross section for this span is type (a). The distribution factor for moment in interior beams shall be taken as follows,

$$gM = 0.075 + \left( \frac{S}{9.5} \right)^{0.6} \left( \frac{\delta}{L} \right)^{0.2} \left( \frac{K_g}{12.0L\ell_d^2} \right)^{0.1}$$

(3-24)

in which: $K_g$ is $n\left( l + Ae_g^2 \right)$, where $n = \frac{E_B}{E_D} E_B$ = modulus of elasticity of beam material (ksi), $E_D$ = modulus of elasticity of deck material (ksi), $l = \text{moment of initial of beam (in}^4\text{), and } e_g = \text{distance between the centers of gravity of the basic beam and deck. The flexural moment at the critical position for the in-exterior girder induced with HL-93 is 1295kip \cdot ft$. The maximum strain is
\[ f_l = \frac{M_{\text{truck}} l_m DF}{S_{bn} E_s} = 425 \text{ microstrain} \] (3-25)

where \( l_m \) is the impact factor, \( DF \) is the distribution factor, and \( S_{bn} \) is the short-term composite elastic section modulus. The shape parameter, is induced in Chapter 2 from a monitoring test of CORIBM Bridge on route LA 70 in District 61, Assumption Parish, Louisiana, is \( \sigma = 31.584 \text{ microstrain} \). Thus, the extreme truck is described using a Gumbel distribution as follow:

\[ F_{75, \gamma}(Q) = \exp \left( -\exp \left( -\frac{Q-425}{31.584} \right) \right) \] (3-26)

The yearly maximum structural response due to the live load was generated using Monte Carlo simulation according to Eq.(21). Then, the generated data were fitted with a Gumbel distribution and the distribution factors were obtained. Then, the yearly maximum structural response was derived as:

\[ F_y(Q) = \exp \left( -\exp \left( -\frac{Q-288.6}{31.584} \right) \right) \] (3-27)

and shown in Figure 3-7.

\[ \text{Figure 3-7 PDF of Extreme Live Load with Mean Recurrence Intervals of One Year and 75 Years (steel span)} \]
Figure 3-7 shows the extreme structural response with mean recurrence intervals of 1 year and 75 years, respectively.

3.5.3 Elastic Section Modulus

Different from dead load effect, the section modules are time dependent. They decrease with time because of the decrease of the cross section area due to steel corrosion. Thanks to the good manufacture quality and stability of I-beam products the initial section modules are treated as constant functions. The depth of the corrosion is described using Eq.(3-19) and the section modules at any age are determined. Figures 3-8, to 3-10 show the elastic section modulus of steel girders, short-term and long-term composite section varying over time. Three pairs of \( b_0 \) and \( b_1 \) were considered. The first and second pairs (\( b_0 = 31.9 \) \( b_1 = 0.697 \), and \( b_0 = 36.1 \) \( b_1 = 0.602 \)) are based on carbon steel samples located in rural environments in Pennsylvania and Germany for eight years exposure, respectively. Morcillo et al. (1995) and Albrechr and Naeemi (1984) proposed these two factors for 15 cities with rural-urban environment. The mean value of \( b_0 \) and \( b_1 \) are assumed for the third pair.

![Figure 3-8 Elastic Section Modulus of Steel I-beam](image)
3.5.4 Reliability Analysis

The reliability of a component, usually expressed with a reliability index, is quantified by comparing the structural capacity $R$ with the load effects $Q$, using predefined limit state functions. The reliability index of a steel girder under routine traffic live load is calculated. The limit state is defined based on the normal stress due to the applied loads at the mid-span exceeding yield strength. The reliability can be calculated using either first-order second-moment
or other iteration methods. Using these methods, the variables are assumed to be stationary random processes; they follow the constant distribution during the service life cycle. In reality, the structural capacity decreases due to the corrosion while the live load increases due to the increasing traffic demand during the life-cycle of a bridge. Both the variation of capacity and live load are very slow and gradual processes, thus, in short intervals they can be treated as stationary random processes. In this study, as discussed earlier, the entire service life cycle was divided into 75 segments, one year long each, and the reliabilities for each one year mean recurrent interval were calculated. The dead load is assumed following a lognormal distribution, live load following a Gumbel distribution, and the capacity is assumed to be a time independent in each segment and the value is taken at the middle of the year.

Reliability index of a steel girder under routine traffic live load is calculated. The limit state is defined based on the normal stress due to the applied loads at the mid-span exceeding the yield strength. The stress under service load at the bottom flange can be derived from the following equation

\[ f_s = \frac{M_{beam}}{S_b} + \frac{M_{deck}}{S_{b3n}} + \frac{M_{bar}}{S_{b3n}} + \frac{M_{FW}}{S_{b3n}} + \frac{M_{truck}l_nDF}{S_{bn}} + \frac{M_{lane}l_m}{S_{bn}} \]  

(3-28)

where \( M_{beam} \), \( M_{deck} \), \( M_{bar} \), \( M_{FW} \) and \( M_{lane} \) are moments at the mid-span due to the steel girder, concrete deck, barrier, future wearing and lane traffic load, respectively. \( M_{truck} \) is the moment due to the extreme truck load in a year, derived from HL-93 truck load defined by AASHTO. It is expressed using Eq. (3-27).

Figure 3-11 shows the yearly reliability index of the steel girder corresponding to the age of the bridge. The yearly reliability decreases as the age of the bridge increases. The reliability indexes were converted into failure probability using standard normal distribution function. The
failure probability is 1.9461E-8 at the first year and 2.9184E-8, 2.6234E-8 and 2.292E-8 at the 75th year corresponding to three pairs of \( b_0 \) and \( b_1 \), respectively. The failure probability at the 75th year is 1.2 to 1.5 times of that at the first year. The decrease is due to the corrosion of the steel. Once the safe probabilities for each year are obtained, the safe probabilities for any length of mean recurrent intervals are obtained by continued multiplication of the yearly safe probability using Eq. (3-23). The corresponding reliability indexes for 1 to 75 years intervals are calculated using an inverse of the standard normal distribution function

\[
\beta = \Phi^{-1}[F(x)]
\] (3-29)

and are shown in Figure3-12.

Similarly, the failure probabilities for any mean recurrent intervals were calculated. For a mean recurrent interval of 75 years, as specified in AASHTO, the failure probabilities are 1.8528E-6, 1.7549E-6, 1.6225E-6 and 1.4596E-6 corresponding to the three pairs of \( b_0 \) and \( b_1 \) and none corrosion cases, respectively. The failure probability increases 12% to 25% because of the corrosion of the steel girder.

![Figure 3-11 Yearly Reliability Index](image-url)
In this section, the flexural capacity of the concrete span is calculated. The span consists of ten girders simply supported between bents. The spacing between the girders are 6 ft (between exterior and interior girders) and 5 ft (between interior girders). According to their locations, the girders are named as interior (In), exterior (E), and interior-exterior (I-E). The concrete girders are also stiffened by the end and intermediate diaphragms. The cross-section of the concrete span is shown in Figure 3-13.

3.6.1 Load Effects

Table 1 lists the statistical dead load parameters applied in the development of the AASHTO LRFD Code. In this study, all the random variables related to the dead load are
assumed to be time independent and following a lognormal distribution characterized with its mean value $\mu$ and standard deviation $\sigma$.

The live load effect, the maximum moment due to live load, was derived using AASHTO as discussed in Case Study I. The flexural moment at the critical position for the in-exterior girders due to HL-93 load is $385.575 \text{kip} \cdot \text{ft}$. The maximum flexural moment due to live load is assumed following an extreme distribution (Gumbel distribution). The calculated value is the mode of the distribution with the assumed coefficient of variation of 0.12. This distribution represents the maximum flexural moment due to live load in a mean recurrent interval of 75 years. It was transferred to the yearly maximum flexural moment following steps as for steel span illustrated in Case Study II. Figure 3-14 shows the distribution of maximum flexural moment with mean recurrent intervals of one year and 75 years.

![Figure 3-14 PDF of Extreme Live Load with Mean recurrence Intervals of One Year and 75 years (concrete span)]
3.6.2 Description of Steel Corrosion

As discussed above, for steel reinforcements in the concrete girders or decks, corrosions take place when the chloride ions meet the trigger level. The time for the initiation of the chloride corrosion is related to the thickness of the concrete cover, the surface chloride, and the diffusion coefficient and the trigger level is estimated using Eq. (3-3). Since the chloride ions diffusion in concrete is such a complicated chemical process and so many parameters are involved, the parameters in Eq. (3-3) are treated as variables.

a) Surface Chloride Concentration

Surface chloride concentration $C_0$ is determined by the environment. In coastal areas and northern areas where salts are commonly used to get rid of snows on the bridge in the winter, chloride ions concentrations are higher than other locations. Bamforth (1996) investigated bridges located in coastal areas of United Kingdom, Japan, Norway, Denmark, Austria and Singapore, and proposed the values ranged between 0.3 and 0.7% by the weight of concrete. Based on a survey of four bridge decks in the United States that were 13 years old, Funahashi (1990) reported that the $C_0$ was between 0.56 and 0.65% by the weight of concrete, corresponding to a mean value and standard deviation of 0.61% and 0.05% by the weight of concrete. Thoft-Christensen (1998) classified deterioration into low, medium and high levels, the corresponding values of $C_0$ are reported as 0.575, 0.650 and 0.725% by the weight of cement, respectively. Based on the experimental values of $C_0$ obtained from bridges at different ages, chloride concentration of 15 years were calibrated as 0.10 by Enright (1998). The chloride in concentration in the deck would be higher than in the girders, because salts are directly applied to the surface of the bridge deck. Akgul and Frangopol (2005b) proposed the mean value and standard deviation of chloride surface concentration on reinforced concrete girder as 0.13% and
0.0195\% by the weight of concrete and is adapted in this research to determine corrosion start time of reinforcement.

\textit{b) Diffusion Coefficient}

Diffusion property of concrete mainly depends on the ingredient of concrete. Water-cement ratio, composite action between aggregates and cement paste, temperature, and free chloride concentration affect the chloride diffusion coefficient in concrete. Table 3-2 list the diffusion coefficient of Colorado bridges that depend on mixture proportion of concrete (Bentz et al. 1996).

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Concrete Strength (MPa) & Water-cement ratio (w/c) & Slump (cm) & Air Content (\%) & Dc (cm\(^2\)/year) \\
\hline
31.03 & 0.44 & 10.16 & 6 & 0.265 \\
0.265 & 0.5 & 10.16 & 6 & 1.097 \\
\hline
\end{tabular}
\caption{Calculated Mix-design Proportion and Chloride ion Diffusion Coefficients for Reinforced Concrete Slab and Girder Strengths of Colorado Bridges}
\end{table}

\textit{c) Critical (Threshold) Chloride Concentration}

Zemajtis (1998) claimed that reinforcement inside concrete components is covered with an oxide layer on the surface. The corrosion process starts when the chloride concentration reaches the trigger level (critical chloride concentration) and the oxide layer coat is destroyed. The chloride concentration trigger level is obtained either by substantial amount of published data (ACI 1985; Coggins and French 1990; Manning and Ip 1996; Thoft-Christensen 1998; Zemajtis 1998; Thompson et al. 2000) or experiment/field testing directly. By aggregating research achievements of Zemajtis (1998) and Thoft-Christensen et al. (1998), the mean and standard deviation of values are assumed to be 0.055 and 0.046\% by weight of concrete,
respectively (Akgul et al. 2005). If only the high quality concretes with a low water to cement (w/c) ratio were taken into account, the critical chloride concentration were treated following a lognormal distribution with the mean value and standard deviation becoming 0.037 and 0.018% by weight of concrete, respectively. Enright (1998) proposed the coefficient of variation of critical chloride concentration as 0.15, and it is adapted in this research.

\( d) \) Distribution of Corrosion Starting Time

Three variables involved in determining the corrosion starting time discussed above are assumed to follow lognormal distributions, and their mean values and standard variations have been determined.

Another factor involved in determining the start time of corrosion is the thickness of concrete cover that is treated as constants in the present study. Figure 3-15 shows the position of reinforcements in the interior-exterior girder (with respect to the flexural capacity, exterior girders and interior-exterior girders are two possible critical girders). The four bottom reinforcements and two side reinforcements in the top row are exterior reinforcements (6 reinforcement), and the central reinforcement in the top row are interior reinforcements. The starting time of corrosion of both exterior and interior reinforcements are discussed below.

The time for initiation of chloride corrosion, \( T_{corr} \), is obtained with Eq. (3-3). All the variables involved in Eq. (3-3) have been defined with their distributions. Monte Carlo simulations were applied to determine the starting time of corrosion where 10,000 simples for each variable were generated according to their distributions, and the calculated starting time of corrosion was then fitted with Gumbel distributions. Figure 3-16 shows the starting time distributions of corrosion for both exterior and interior reinforcements. For the exterior
reinforcement, the corrosion is most likely to occur at age of 20 years, and for the interior reinforcement, the corrosion usually happens after age of 60 years. In this study, the corrosion of the exterior reinforcement is considered only.

**Figure 3-15** Position of Reinforcements in Girders

**Figure 3-16** PDF of Starting Time of Reinforcement Corrosion
Two assumptions are adopted to estimate the reduction of volume of steel due to the penetration of chloride (Akgul et al. 2005). First, steel is assumed to be corroding at a constant rate after the corrosion occurs. Second, corrosion is assumed to be uniform along the reinforcing bar, reducing the cross-sectional diameter uniformly along the bar perimeter. Thoft-Christensen (1998) proposed a rate in terms of physical corrosion thickness per unit time as

\[ r_{corr} = C_{corr}i_{corr} \quad (30) \]

where \( i_{corr} \) is the corrosion current density generally expressed in terms of microamperes per unite time, corrosion coefficient, \( C_{corr} \), acts as a proportionality constant that is used to directly convert current density in terms of microamperes per unit area to physical thickness reduction in terms of distance per unit time. Akgul et al. (2005) proposed the corrosion rate following a log-normal distribution with the mean and standard deviation of 0.0762 and 0.0023 cm/year, respectively which was also applied in this present. The reduced diameter of reinforcing steel can be modeled using the following simple formula (Thoft-Christensen 1998)

\[
D(t) = \begin{cases} 
D_0, & \text{for } t < T_1 \\
D_0 - r_{corr} \cdot (t - T_1), & \text{for } t > T_1 \\
0, & \text{for } t \geq t^* 
\end{cases} \quad (31)
\]

where \( D_0 \) is the initial bar diameter, \( T_1 \) is the time of initiation of corrosion described with the Gumbel distribution and \( t^* \) is the time at which the reinforcement bar completely disappears due to corrosion activity. According to the discussion above, \( t^* \) is assumed to be 15 years.

Thus, the reduced section area of reinforcing steel is derived as

\[
A_s(t) = \begin{cases} 
\frac{\pi}{4} \sum_{j=1}^{n} D_{0j}^2, & \text{for } t \leq T_1 \\
\frac{\pi}{4} \sum_{j=1}^{n} D_j^2, & \text{for } T_1 < t \leq t^* \\
0, & \text{for } t \geq t^* 
\end{cases} \quad (3-32)
\]
3.6.3 Time-Variant Reliability Analysis

The nominal moment $M_n$ is then obtained as follows:

$$M_n = \rho f_y b d^2 \left( 1 - \frac{\beta f_y \rho}{\alpha f_c} \right) \quad (3-33)$$

With the specific values for $\alpha$ and $\beta$ being obtained experimentally, Eq. (3-33) becomes

$$M_n = \frac{A_s}{b d} f_y b d^2 \left( 1 - 0.59 \frac{f_y A_s}{b d f_c} \right) \quad (3-34)$$

The limit state function becomes

$$g = M_n - M_l - M_d$$

$$= \frac{A_s}{b d} f_y b d^2 \left( 1 - 0.59 \frac{f_y A_s}{b d f_c} \right) - M_l - M_d \quad (3-35)$$

where $M_l$ and $M_d$ are moments due to the live load and dead load. In this equation, $A_s$ and $f_c$ are time-variant variables, $M_l$ and $M_d$ are time-independent variables, and the geometry dimension $b$ and $d$, and the yield strength of steel reinforcement $f_y$ are treated as constants.

Similar to the steel spans, the structural capacity decreases due to the corrosion while the live load increases due to the increasing traffic demand during the life-cycle of a bridge. Both the variation of capacity and live load are very slow and gradual processes, thus, in short intervals they are treated as stationary random processes.

The limit state function is changed to

$$g = \frac{A_s(t)}{b d} f_y b d^2 \left( 1 - 0.59 \frac{f_y A_s(t)}{b d f_c(t)} \right) - M_l - M_d \quad (3-36)$$
In this study, the entire service life cycle was divided into 75 segments, one year long each, and the reliabilities for a year of the mean recurrent intervals were calculated. The dead load is assumed following a lognormal distribution, live load following a Gumbel distribution. The yearly reliabilities were calculated using the Rackwitz-Fiessler procedure.

Figure 3-17 shows the yearly reliability index of concrete girder corresponding to the age of the bridge. The yearly reliability increases slightly at the early age of the bridge, and then decreases dramatically. The early increase of reliability is due to the strength increase of concrete; and then, the reliability decreases as the age of the bridge increases. The degradation of concrete and deduction of steel due to corrosion result in the decreasing of reliability. Compared with Figure 3-4, it should be noted that while the strength of concrete increase dramatically at its early age, while the corresponding reliability increases smoothly. It is concluded that, for a tension failure governed mode, the increase of concrete strength does not enhance the flexural capacity significantly; on the other hand, the reduction of volume of steel reinforcement decreases the flexural capacity dramatically. The reliability indexes were converted into failure probability using a standard normal distribution function. The failure probability is 4.4088E-9 at the first year and 1.3701E-4 at the 75th year. The failure probability at the 75th year is 31,076 times of that at the first year. The decrease is due to the corrosion of the steel. The safe probabilities for any length of mean recurrent intervals are obtained by continued multiplications of the yearly safe probability using Eq. (3-23). The corresponding reliability indexes are calculated using an inverse of the standard normal distribution function and are shown in Figure 3-18. For a mean recurrent interval of 75 years, as specified in AASHTO, the reliability index is 2.9825 for this specific bridge.
Conclusions

1. Both the resistance and the live load of a bridge are non-stationary auto-correlated random process during its service life cycle. In a reliability calculation, they cannot simply be treated as time independent variables. The mean values of resistance are monotonically
decreasing functions with time; the deviations of resistance are monotonically increasing functions with time while the live load effects are monotonically increasing function of time.

2. Since the decrease of resistance and increase of live load are gradual processes, it is recommended that in a reasonable time segment, a year in this study, they can be treated as stationary processes and expressed with variables following a certain type of distributions. Thus, the reliability for the time segment is obtained.

3. The entire life cycle is the sum of a time series. Thus the reliability for the entire service cycle of the bridge is calculated through a reliability analysis of a series system.

4. In this study, the corrosion of steel material for steel bridges does not affect the structure reliability significantly. For the steel span, it increases the failure probability by 12% and 25% for the entire service cycle of the case bridge. It is expected that it may play a more important role in small size steel components. For the concrete span, the corrosion of steel reinforcements is much more sensitive to the reliability of structures.

3.8 References


CHAPTER 4. STRENGTHENING OF BRIDGES WITH POST-TENSIONED FRP LAMINATES AND FINITE ELEMENT ANALYSIS

4.1 Introduction

One of the challenges that the transportation agencies are facing is to keep the bridges in good condition during their service life. Numerous of bridges are classified as structurally and/or functionally deficient in the country. In the State of Louisiana, 4,591 bridges or 34% of the total 13,426 bridges are classified as substandard. Load capacity degradation, increased gross vehicle weight, and increasing traffic demand lead to the deficiencies.

One of the most effective ways to solve the problem is to use composite materials to strengthen existing bridges. As rapidly developed over the past several decades, different kinds of composite fiber reinforced polymers (FRP) have been regarded as one of the best solutions to several problems associated with transportation and civil engineering infrastructures. Some of the major benefits of FRP include its high strength to weight ratio, high fatigue endurance, excellent corrosion resistance, low thermal expansion, and the ease of fabrication, manufacturing, handling and installation.

The main objective of this research is to develop a flexural resistance designing process using post-tensioning prestressed carbon reinforced polymers (CFRP) laminates adhering on bridge girders to avoid various possible flexural failure modes. It is noted that in the original plan, a steel bridge and a concrete bridge will be rehabilitated with prestressed FRP laminates or rods and the bridge performance will be monitored. However, the sponsor has decided not to pursue the field implementation due to the cost issue and this report summarizes the up-to-date work by the research team.
This chapter presents a review of the up-to-date work on bridges strengthened with FRP materials. Mechanical properties of FRP fibers and composite are presented in detail. The investigators presented previous research findings on experiments of FRP composite materials used as various prestressed tendons, and the analyses for different failure modes are introduced. To investigate the effects of rehabilitation with prestressed CFRP laminates, two 3-D finite element analyses are conducted to examine the deflection and bottom fiber stress at the mid-span. A detailed designing process of rehabilitation with prestressed CFRP laminates was presented in this report. A feasible plan to enhance the flexural capability of an existing bridge with externally prestressed Carbon Fiber Reinforced Polymer (CFRP) laminates according to AASHTO and ACI code specifications are also proposed in this report.

4.2 Mechanical Properties

Among the three categories of FRP materials, namely aramid, carbon and glass fiber reinforced polymers, carbon fiber reinforced polymers (CFRP) is the most popular one in civil engineering field. Two types of commercial products of FRP are widely used in civil engineering field, laminates and bars. FRP materials are composite of fibers and resins system. The mechanical properties of fibers and resins system and their ratio determine the properties of FRP materials.

4.2.1 Fibers

Fibers provide the FRP system strength and stiffness, while the resin transfers stress among fibers and protects them. Fibers used for manufacturing composite materials usually have high strength and stiffness, toughness, and durability. The most commonly used fibers for FRPs are carbon, glass, and aramid. On the contrary, to the conventional steel that behaves in an elasto-plastic manner, the FRP product in general behaves in linear elastic manner and fails at
large strains. There is no yielding point before it fails. The mechanical properties are shown in Figure 4-1 compared with reinforcing steel and resins. Typical mechanical properties of these fibers can also be found in Table 4-1.

**Figure 4-1** Tensile Stress-strain Behavior of Reinforcing Fibers as Compared with Steel

4.2.2 Resins System

The resins are other important constituents in composites. They not only coat the fibers and protect them from mechanical abrasion but also transfer stresses between the fibers. The matrixes transfer inter-laminar and in-plane shear in the composite and provide lateral support to fibers against buckling while subjected to compressive loads. Epoxy and polyester are most commonly used resins. Resins in manufacture of composites have relatively low strain to failure, resulting in low impact strength. Mechanical properties of some thermo set resins are provided in Table 4-2.

To resist the aggressive service condition the FRP system selected should include a resin matrix resistant to alkaline, acidic or other special environments.
### Table 4-1 Typical Mechanical Properties of Fibers

<table>
<thead>
<tr>
<th>FIBER TYPE</th>
<th>Tensile Strength (MPa)</th>
<th>Modulus Elasticity (GPa)</th>
<th>Elongation (%)</th>
<th>Coefficient of Thermal Expansion (10E-6)</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>CARBON</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PAN</td>
<td>High Strength</td>
<td>3500</td>
<td>200-240</td>
<td>1.3-1.8</td>
<td>(-1.2) to (-0.1) ((\alpha_{frpL})), 7 to 12 ((\alpha_{frpT}))</td>
</tr>
<tr>
<td></td>
<td>High Modulus</td>
<td>2500-4000</td>
<td>350-650</td>
<td>0.4-0.8</td>
<td>-0.2</td>
</tr>
<tr>
<td></td>
<td>Pitch</td>
<td>Ordinary</td>
<td>780-1000</td>
<td>38-40</td>
<td>2.1-2.5</td>
</tr>
<tr>
<td></td>
<td>High Modulus</td>
<td>3000-3500</td>
<td>400-800</td>
<td>0.4-1.5</td>
<td>(-1.6) to (-0.9) ((\alpha_{frpL}))</td>
</tr>
<tr>
<td>ARAMID</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Kevlar 29</td>
<td>3620</td>
<td>82.7</td>
<td>4.4</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>Kevlar 49</td>
<td>2800</td>
<td>130</td>
<td>2.3</td>
<td>2.0 ((\alpha_{frpL})), 59 ((\alpha_{frpT}))</td>
</tr>
<tr>
<td></td>
<td>Kevlar 129</td>
<td>4210 (est.)</td>
<td>110 (est.)</td>
<td>--</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>Kevlar 149</td>
<td>3450</td>
<td>172-179</td>
<td>1.9</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>Twaron</td>
<td>2800</td>
<td>130</td>
<td>2.3</td>
<td>2.0 ((\alpha_{frpL})), 59 ((\alpha_{frpT}))</td>
</tr>
<tr>
<td></td>
<td>Technara</td>
<td>3500</td>
<td>74</td>
<td>4.6</td>
<td>N/A</td>
</tr>
<tr>
<td>GLASS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>E-Glass</td>
<td>3500-3600</td>
<td>74-75</td>
<td>4.8</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>S-Glass</td>
<td>4900</td>
<td>87</td>
<td>5.6</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>Alkali Resistant Glass</td>
<td>1800-3500</td>
<td>70-76</td>
<td>2.0-3.0</td>
<td>N/A</td>
</tr>
</tbody>
</table>

(adapted from Design Manual No. 3 Sep. 2001, Reinforcing Concrete Structures with Fiber Reinforced Polymers ISIS CANADA)

### Table 4-2 Typical Properties of Thermosetting Resins

<table>
<thead>
<tr>
<th>Resin</th>
<th>Specific Gravity (MPa)</th>
<th>Tensile Strength (MPa)</th>
<th>Tensile Modulus (GPa)</th>
<th>Cure Shrinkage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epoxy</td>
<td>1.20-1.30</td>
<td>55.00-130.00</td>
<td>2.75-4.10</td>
<td>1.00-5.00</td>
</tr>
<tr>
<td>Polyester</td>
<td>1.10-1.40</td>
<td>34.50-103.50</td>
<td>2.10-3.45</td>
<td>5.00-12.00</td>
</tr>
<tr>
<td>Vinyl Ester</td>
<td>1.12-1.32</td>
<td>73.00-81.00</td>
<td>3.00-3.35</td>
<td>5.40-10.30</td>
</tr>
</tbody>
</table>

(adapted from Design Manual No. 3 Sep. 2001, Reinforcing Concrete Structures with Fiber Reinforced Polymers ISIS CANADA)
4.2.3 FRP Reinforcing Products and Material Properties

FRP materials are composed of a number of continuous fibers, bundled in a resin matrix. FRP tendons are available in the form of rods or cables, rectangular strips, braided rods and multi-wire strands. Normally, the volume fraction of fibers in FRP strips is about 50-70% and that in FRP fabrics is about 25-35%. The mechanical properties of the final FRP product depend on the types and quality of fibers, fiber to resin volumetric ratio, orientation, shape, fiber adhesion to the matrix, and on the manufacturing process. The tensile behaviors of FRP bars are similar to FRP fibers, when loaded in tension. They are characterized by a linearly elastic stress-strain relationship until failure without exhibiting any plastic behaviors. The kind of fiber and the fiber to overall volumetric ratio affect the mechanical properties of FRP materials most because fibers are the main load-carrying constituents, while the resin transfers stresses among fibers and protects them. The tensile properties of some commonly used FRP bars are shown in Table 4-3 compared with steels. Figure 2 demonstrates the tensile strain stress behaviors of construction materials (FRP, steel, and concrete). Compared with Figure 1, the Young’s modulus of FRP composite materials is always smaller than that of steels; even the Young’s modulus of fibers is usually larger than that of steels.

When FRP materials are subjected to a constant stress, they can fail suddenly. This phenomenon is referred to as creep rupture that exists for all structural materials including steel. In general, carbon fibers are the least susceptible to creep rupture; aramid fibers are moderately susceptible, and glass fibers are most susceptible. The creep rupture happens due to resins not fibers; therefore, the orientation and volume of fibers have a significant influence on the creep performance of tendons. Studies on GFRP composites indicate that stress rupture diminishes if
the sustained loads are limited to 60% of the short-term strength while that of prestressing steel is 75%. Figure 3 shows the variation of strength of FRP subjected to a long term load.

<table>
<thead>
<tr>
<th>Nominal yield stress, ksi (Mpa)</th>
<th>Steel</th>
<th>GFRP</th>
<th>CFRP</th>
<th>AFRP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>40-75 (276-517)</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Tensile strength, ksi (Mpa)</td>
<td>70-100 (483-690)</td>
<td>70-230 (483-1600)</td>
<td>87-535 (600-3690)</td>
<td>250-368 (1720-2540)</td>
</tr>
<tr>
<td>Elastic modulus, x10E3 ksi (Gpa)</td>
<td>29 (200.0)</td>
<td>5.1-7.4 (35.0 to 51.0)</td>
<td>15.9-84.0 (120.0-580)</td>
<td>6.0-18.2 (41.0-125.0)</td>
</tr>
<tr>
<td>Yield strain, %</td>
<td>1.4-2.5</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Rupture strain, %</td>
<td>6.0-12.0</td>
<td>1.2-3.1</td>
<td>0.5-1.7</td>
<td>1.9-4.4</td>
</tr>
</tbody>
</table>

Typical values for fiber volume fractions ranging from 0.5 to 0.7

**Figure 4-2** Tensile Stress-Strain Behaviors of Construction Materials. (adapted from Ambrose Inc.)

CFRP and GFRP bars exhibit good fatigue resistance. Research on FRP Composites made of high-performance fibers for aerospace applications shows that carbon-epoxy composites have better fatigue strength than steel; while the fatigue strength of glass composites is lower than steel.
Figure 4.3 Comparison of Creep-rupture Curve for Aramid and Carbon FRP Rods under Environmental Exposure (adapted from Prestressing Concrete Structures with FRP Tendons, reported by ACI Committee 440.4R-04)

4.3 Mechanical Performance of Girders Strengthened with Prestressed FRP Materials

4.3.1 Concrete Flexural Components Prestressed with FRP Materials

The structural systems strengthened with externally bonded FRP laminates combine the benefits of mechanical properties of FRP composites, the compressive characteristics of concrete, and the ductility and deformation capacity of steel. This improves the load capacity of the structure definitely. The main advantages are shown by FIB Bulletin 14 as follows.

a. Control the deflection at the early stage and provides stiffer behavior.

b. Delay crack formation in the shear span.

c. Close pre-existing cracks.

d. Improve serviceability and durability due to reduced cracking.

e. Improve the shear resistance of member.

f. The same strengthening is achieved with smaller areas of FRP reinforcement.
g. Greater structural efficiency as the neutral axis remains at a lower level in the
prestressed case.

h. The internal steel begins to yield at a higher applied force compared to non-prestressed
member.

Besides these, there are other two advantages of being used as prestressed reinforcement.
One is the unloading of the steel reinforcement which is beneficial for fatigue resistance of the
structure because the stress in the steel can be maintained in a relatively low stress level. The
other one is that, due to the excellent corrosion resistance of FRP, it can be easily used as
externally prestressed reinforcements with minor protection.

Numerous studies have been carried out on flexural components strengthened with FRP
materials. Experiments studies revealed the behavior of beams strengthened with FRP
composites by means of different methods (Badawi and Soudki 2009; Ceroni 2010; Czaderski
and Motavalli 2007; Maalej and Leong 2005; Mukherjee and Rai 2009; Rosenboom et al. 2007;
Saqan and Rasheed 2011; Stoll et al. 2000; Woo et al. 2008; Xue et al. 2010). Failure modes
were identified based on these experiments (Pham and Al-Mahaidi 2004; Yang et al. 2009).
Calculation formulas were established and load capacity estimation was developed based on the
mechanical models simplified from failure modes (Almusallam and Al-Salloum 2001; Gunes et
al. 2009; Woo et al. 2008; Wu and Davies 2003). Special failure mode, the debonding of FRP
composite off the surface of the concrete was investigated in detail (Chen and Pan 2006; Smith
and Teng 2002; Smith and Teng 2002). Long-term and time dependent performances were also
evaluated (Arockiasamy et al. 2000; Youakim and Karbhari 2007; Zou and Shang 2007).
Badawi and Soudki (2009) investigated effectiveness of strengthening reinforced concrete (RC) beams with prestressed near-surface mounted (NSM) carbon fiber reinforced polymer (CFRP) rod. In their study, four RC beams (254 mm deep by 152 mm wide by 3500 mm long) were tested under monotonic loading including an un-strengthened control one and one with non-prestressed NSM CFRP rod. The setup of the experiments is shown in Figure 4-4. Strain gages were placed on the concrete, the FRP rod and reinforcing bars. Strain profile versus beam depth using strain readings show that, similar to ordinary RC beams, beams strengthened with prestressed NSM CFRP rods satisfy the plane-section assumption, i.e., a cross section that was plane before loading remains plane under load as shown in Figure 4-5.

![Figure 4-4 Specimen Design (adapted from Badawi and Soudki 2009)](image)

The first one is characterized with concrete crushing at the top fiber of the cross-section after yielding of tension steel reinforcement. With respect to capacity, it shows that compared with the control beam, the RC beams strengthened with prestressed (40% and 60%) NSM CFRP rods increased their yield and ultimate capacity up to 90% and 79%, respectively. The failure mode of prestressed CFRP rods is characterized with rupture in the CFRP rod after yielding of the tension steel reinforcement.
Mukherjee and Rai (2009) made an experimental study on the flexural behavior of RC beams that have reached their ultimate bearing capacities and then retrofitted with externally prestressed carbon fiber reinforced composite (CFRC) laminates. The RC beams were firstly damaged with a four point bending test. It was observed that the failure mode of the beams was due to yielding of tension steel prior to the application of any CFRC. And then, the CFRP laminate were pulled to the desired tensile force and bonded to the tension face of the beam with specially designed machine thereafter. To avoid peeling off of CFRC laminates, the ends of laminates are secured by means of a wrap of CFRC sheet. Therefore, due to the rehabilitation of the bending capacity the failure mode shifts to crushing of concrete in the compression zone and the beams were fully utilized. The load-mid-span deflection curves of the beam at all the different phases of the test are shown in Figure 4-6. It is noted that the failure did not lead to a sudden loss of stiffness as commonly expected due to the compression failure of the concrete.
Stoll et al. (2000) carried out research involved the design, fabrication, and testing to failure of bridge beams strengthened with FRP products for prestressing and shear reinforcement. They noted that for different manufacture-supplied CFRP products ratio of guaranteed-strength to ultimate-strength are different. Thus, there is not a consistent methodology in use by different tendon manufactures to establish characteristic strength value. Two 12.19 m long AASHTO Type 2 beams were built using different high-strength concrete formulations, and the twenty-eight day compressive strength of cylinder were 86.3 MPa and 71.1 MPa, respectively. The Leadline cables were used as prestressing cables. The standard cross-section of an AASHTO Type 4-2 beam is shown in Figure 4-7. These two beams were tested to ultimate failure in four-point bending. Both beams failed due to tension failure of the CFRP tendons in the bending zone between the load points and exhibited extensive cracking and large deflections before the failure of the tendons, as shown in Figure 4-8.
Externally prestressed tendons can improve load carrying capacity of composite beams too. Chen and Gu (2005) carried out study on the ultimate moment and incremental tendon stress of steel-concrete composite beams prestressed with external tendons under positive moment. Two beams, prestressed and non-prestressed, were tested for comparison. The non-prestressed
beam was loaded to the yielding of the bottom flange and was unloaded. The beam was prestressed thereafter, and then loaded to the ultimate failure. The ultimate stress increment in tendons is a substantial factor in the design of composite beams prestressed with external tendons. In their research, the ultimate stress increment in tendons was expressed in terms of ratio of prestress–span to deflection and is shown in Figure 4-9. The experimental investigation shows that adding prestressed tendons to composite beams significantly increases both the yield and ultimate flexural capacity and lead to less deflection.

![Incremental Prestress–span/deflection Curves with Different Eccentricities](adapted from Chen and Gu 2000)

**Figure 4-9** Incremental Prestress–span/deflection Curves with Different Eccentricities (adapted from Chen and Gu 2000)

4.3.2 Steel Flexural Components Prestressed with FRP Materials

Park et al. investigated (2010) studied the improvement of flexural capacity and the effect of deviator when a steel I-beem member is strengthened with externally unbounded prestressing tendons. Four point loading tests were conducted for steel I-beem member strengthened with external steel bars and strands. The setups of the experiments are shown in Figure 4-10 and 4-11.
As expected, the flexural capacity was improved significantly when the external post-tensioning technique was applied when the draped tendon was utilized.

**Figure 4-10** Steel I-beam Prestressed with Straight Tendons (adapted from Park et al. 2010)

**Figure 4-11** Steel I-beam Prestressed with Drape Tendons (adapted from Park et al. 2010)
4.4 Mechanical Flexural Capacity Analysis of Girders Strengthened with FRP Materials

To estimate the flexural capacity of reinforced concrete girders strengthened with prestressed CFRP laminates three type of failure modes, tension failure (i.e. rupture of CFRP plate prior the crushing of concrete in compression), debonding failure (i.e. force in the prestressed CFRP plate could not be sustained by the concrete substrate, which results in the CFRP plate debonding prior to the concrete crushing), and compression failure (i.e. crushing of concrete in compression prior to the rupture or debonding of CFRP plate) must be identified. These three types of failure modes control the ultimate capacity in RC beams. The boundary to distinguish tension, debonding failure and compression failure is at balance state, as the tensile strain in the prestressed CFRP plate equals to the tensile strain limitation \( \varepsilon_{p,fu} \), simultaneously with the crushing of concrete in compression.

4.4.1 Strengthened with Bonded Prestressed FRP Laminates

Bonded non-prestressed beam strengthened with one layer of FRP laminate tend to fail due to brittle intermediate crack-induced debonding from the mid-to end-span when the strain of the laminates reach about 6500-7000\( \mu \), while beams strengthened with more laminates tend to plate-end debonding when the CFRP plate strain reached about 5200\( \mu \). It is concluded that the strengthening efficiency of the member strengthened with one laminate is better than that of the member strengthened with two or more laminates with FRP anchored at the two end of the member (Yang et al. 2009).

Badawi and Soudki (2009) and Xue et al. (2010) proposed analytical model and flexural capacity prediction formulas for reinforced concrete beams strengthened with prestressed NSM CFRP rods and bonding CFRP plates, respectively. They both induced fundamental assumptions relating to flexure used in calculating the nominal flexural for reinforced concrete girders. It
seems that these assumptions are still applicable in flexural capacity estimation for reinforced concrete girders strengthened with prestressed CFRP materials:

1. A cross section that was plane before loading remains plane under load. The strain in the reinforcement and concrete are directly proportional to the distance from the neutral axis.

2. The bending stress at any point depends on the strain at the point in a manner given by the stress-strain diagram of the material.

3. The tensile strength of concrete is ignored.

The analysis models are based on force equilibrium and strain compatibility. Xue et al. (2010) induced compressive stress concrete corresponding to a given strain, $f_c$, are given by (Park and Paulay 1975)

$$f_c = \begin{cases} f_c' \left[ \frac{2\varepsilon_c}{\varepsilon_0} - \left( \frac{\varepsilon_c}{\varepsilon_0} \right)^2 \right] & \text{if } 0 \leq \varepsilon_c \leq \varepsilon_0 \\ f_c' \left[ 1 - \frac{0.15}{0.004-\varepsilon_0} (\varepsilon_c - \varepsilon_0) \right] & \text{if } \varepsilon_0 \leq \varepsilon_c \leq 0.003 \end{cases} \quad (4-1)$$

where $f_c'$ is the cylinder compressive strength of concrete; $\varepsilon_c$ the compressive concrete strain and $\varepsilon_0$ the compressive strain in concrete at the peak stress. In this calculation, the concrete is about to crush when the ultimate compressive strain reaches 0.003 for normal-density concretes.

Reinforcing steel is assumed to behave elastic-perfectly plastic response, and the FRP plate has a linear elastic stress–strain relationship up to failure. The shear deformation within the adhesive layer is neglected since the adhesive layer is very thin with slight variations in its thickness.

Figures 4-12 to 4-14 show the diagram of tension, debonding, compression failure modes and balanced state.
After the decompression state, the extreme precompressed fiber to reach zero strain due to the additional strain in the prestressed CFRP laminate, $\varepsilon_d$, the prestressed concrete beam is treated as the corresponding nonprestressed beam in the capacity analysis. The tensile strain limitation, $\varepsilon_{pfu}$, the ultimate strain increase in the CFRP laminate after decompression, is
proposed for predicting the maximum tensile strain level in the prestressed CFRP laminate under the debonding failure or tension failure.

\[
\varepsilon_{prefu} = \begin{cases} 
\varepsilon_{pe} + \varepsilon_d + \kappa_m \varepsilon_{pfu} < \varepsilon_{pfu} & \text{(debonding failure)} \\
\varepsilon_{pfu} & \text{(tension failure)} 
\end{cases}
\]  

(4-2)

In Eq.(4-2), the item of \(\kappa_m \varepsilon_{pfu}\) refer to the strain increase limitation for the prestressed CFRP laminate, which can be determined by following equation suggested by ACI 440.2R-02 to prevent the debonding failure of nonprestressed CFRP laminate

\[
\kappa_m \varepsilon_{pfu} = \begin{cases} 
\frac{1}{60} \left(1 - \frac{nE_t t_f}{360000}\right) \leq 0.9 \varepsilon_{pfu} & \text{for} \quad nE_t t_f \leq 180000 \\
\frac{1}{60} \left(\frac{90000}{nE_t t_f}\right) \leq 0.9 \varepsilon_{pfu} & \text{for} \quad nE_t t_f > 180000 
\end{cases}
\]  

(4-3)

where \(\kappa_m\) is the reduction factor, \(n\) the number of plies of CFRP laminate at the location along the length of the member where the moment is being calculated; \(E_t\) the tension modulus of elasticity of CFRP laminate (MPa) and \(t_f\) the thickness of CFRP laminate (mm). The identification of failure mode based on strain compatibility and plane strain assumption:

\[
\frac{\varepsilon_{cu}}{\Delta \varepsilon_{pfu}} = \frac{c_b}{h-c_b} = \frac{a_b/\beta_z}{h-a_b/\beta_z} 
\]  

(4-4)

from which the \(a_b\) is determined. The balanced CFRP reinforcement ratio of strengthened section is implied from Eq. (4-5)

\[
\rho_{fb} = \frac{A_{fb}}{bd} = \frac{0.85 f'_c b a_b - f_y A_s + f'_y A_s}{b d E_f \varepsilon_{pfu}} 
\]  

(4-5)
The concrete crushing failure of compression zone occurs when the CFRP reinforcement ratio \( \rho_f = A_f / bd \) exceeds \( \rho_{fb} \) or the depth of equivalent rectangular concrete stress block \( a \) exceeds \( a_b \), the strengthened beams will fail by concrete crushing in compression zone, otherwise, the debonding failure or tension failure occur in the strengthened beam.

For compression failure, based on the assumption of linear strain distribution, the following equation can be obtained:

\[
\frac{\varepsilon_{cu}}{\Delta \varepsilon_{pf}} = \frac{c}{h-c} = \frac{a/\beta_1}{h-a/\beta_1}
\] (4-6)

where \( c \) is the depth of neutral axis; \( a \) the depth of the equivalent rectangular concrete stress block and \( \Delta \varepsilon_{pf} \) the ultimate strain increment in the prestressed CFRP materials for the strengthened beam. The equilibrium of internal forces leads to the following equation:

\[
0.85f'c' b\beta_1 c + f_y' A_s' = f_y A_s + E_f A_f (\varepsilon_{pe} + \varepsilon_d + \Delta \varepsilon_{pf})
\] (4-7)

and the corresponding nominal flexural strength under compression failure can be given by summing the moments about the centroid of the concrete compressive force:

\[
M_n = f_y A_s \left( d - \frac{a}{2} \right) + f_y' A_s' \left( \frac{a}{2} - d_s' \right) + E_f A_f (\varepsilon_{pe} + \varepsilon_d + \Delta \varepsilon_{pf}) \left( h - \frac{a}{2} \right)
\] (4-8)

When the tension of debonding failure occurs, the compression strain in the extreme fiber of concrete, \( \varepsilon_c^t \), is derived from following equation obtained based on the plane strain assumption:

\[
\varepsilon_c^t = \frac{c}{h-c} \Delta \varepsilon_{pf}
\] (4-9)

The concrete compression force is solved by integration of the concrete stress within the range of compression zone.
\[
C_c = \int_0^c f'_c \; bc \left[ \frac{2 \varepsilon_{fy}}{\varepsilon_{0c}} - \left( \frac{\varepsilon_{fy}/c}{\varepsilon_{0}} \right)^2 \right] dy = f'_c \; bc \; \varepsilon'_c \left( 1 - \frac{\varepsilon'_c}{3\varepsilon_{0}} \right)
\]  \hspace{1cm} (4-10)

The equilibrium of internal forces leads to the following equation:

\[
C_c + f'_y A_s' = f_y A_s + E_f A_f \left( \varepsilon_{pe} + \varepsilon_d + \Delta \varepsilon_{pf} \right)
\]  \hspace{1cm} (4-11)

The length of the range of the compression zone is solved using Eq. 4-12. The distance from top concrete fiber to the centroid of concrete compressive force $y_c$

\[
y_c = \frac{f'_c \; f'_c \; bc \left[ \frac{2 \varepsilon_{fy}}{\varepsilon_{0c}} - \left( \frac{\varepsilon_{fy}/c}{\varepsilon_{0}} \right)^2 \right] \left(c-y\right) dy}{c_c} = \frac{c(\varepsilon_c - 4\varepsilon_{0})}{4(\varepsilon_c - 3\varepsilon_{0})}
\]  \hspace{1cm} (4-12)

The corresponding nominal flexural strength is computed by summing moments about the centroid of the concrete compressive force:

\[
M_n = f_y A_s (d - y_c) + f'_y A_s' (y_c - d_s) + E_f \left[ \varepsilon_{pu} \right] A_f (h - y_c)
\]  \hspace{1cm} (4-13)

4.4.2 Strengthened with External Unbonded Prestressed FRP Materials

ACI 440.4R-04 proposed method to calculate ultimate nominal flexural capability of prestressing concrete structures with FRP tendons. For unbounded prestressed members, the stress in the prestressing tendons at failure of the beam must be determined using the following relation

\[
f_p = f_{pe} + \Delta f_p
\]  \hspace{1cm} (4-14)

where $f_{pe}$ is the effective prestress in the tendon when the beam carriers only the dead load after the prestress losses have occurred, and $\Delta f_p$ is the stress increase above $f_{pe}$ due to any additional
applied load. $\Delta f_P$ can be derived using strain compatibility as if the tendon were bonded and applies a strain reduction factor $\Omega$ to account for the fact that the tendons were unbonded. Assuming linear elastic behavior of the tendon, the change in stress $\Delta f_P$ in the unbounded tendon is given by

$$\Delta f_P = \Omega u E_p \varepsilon_{cu} \left( \frac{d_p}{c_u} - 1 \right) \quad (4-15)$$

where $\varepsilon_{cu}$ is the strain in the extreme compression fiber at ultimate, and $c_u$ is the depth of the neutral axis at ultimate. According to Alkhairi and Naaman (1993), the strain reduction coefficient at ultimate, $\Omega_u$ can be determined by

$$\Omega_u = \frac{2.6}{(L/d_p)} \quad (for \ one \ point \ loading) \quad (4-16)$$

$$\Omega_u = \frac{5.4}{(L/d_p)} \quad (for \ two - point \ or \ uniform \ loading) \quad (4-17)$$

For design purposes, the above formulas were emended as

$$\Omega_u = \frac{1.5}{(L/d_p)} \quad (for \ one \ point \ loading) \quad (4-18)$$

$$\Omega_u = \frac{3.0}{(L/d_p)} \quad (for \ two - point \ or \ uniform \ loading) \quad (4-19)$$

ACI 440.4R-04 proposed a method to estimate stress in external unbonded prestressed at ultimate state. According to Aravinthan et al. (1997), equations for the strain reduction coefficient $\Omega_u$ used to predict the behavior at ultimate of beams with external prestressing or a combination of internal and external prestressing, are as follows

$$\Omega_u = \frac{0.21}{(L/d_p)} + 0.04 \left( \frac{A_p \text{int}}{A_p \text{tot}} \right) + 0.04 \quad (4-20)$$
for one-point loading; and

\[ \Omega_u = \frac{0.231}{(L/d_p)} + 0.21 \left( \frac{A_{p \text{ int}}}{A_{p \text{ tot}}} \right) + 0.06 \] (4-21)

for three-point loading where \( A_{p \text{ int}} \) is the area of the internal prestressed reinforcement, and \( A_{p \text{ tot}} \) is the total area of internal and external prestressed reinforcement.

### 4.5 Rehabilitation with External Bonded Prestressed CFRP Materials

#### 4.5.1 CFRP Material Mechanical Properties and Anchorage System

To rehabilitate the girders with external post-tensioning materials is an effective way to enhance girders flexural capability. In the tentative design, the CFRP laminates were selected to serve as prestressed reinforcements and the description of the bridge was demonstrated in last chapter. The CFRP laminates were prestressed before they are bonded to the bottom surfaces of the girders. All the construction can be conducted with special designed machines. As discovered above, several characters, such as high strength, relative high modulus of elasticity, excellent corrosion and fatigue resistance make CFRP material one of the best choices of external post-tensioning tendons. Sika CarboDur is a pultruded carbon fiber reinforced polymer (CFRP) laminate designed for strengthening concrete, timber and masonry structures and its mechanical properties is shown in Table 4-4.

Sika Carbodur, carbon fiber laminate for structural strengthening is widely used in civil engineering field. Commercial CFRP products are available in forms laminates and bars. Table 4-5 presents mechanical characters of commercial products of Sika CarboDur laminates.
Table 4-4 Properties of Sika CarboDur Laminate

<table>
<thead>
<tr>
<th>Property</th>
<th>Mean value</th>
<th>Design value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tensile Strength</strong></td>
<td>4.49E5 psi</td>
<td>4.06E5 psi</td>
</tr>
<tr>
<td><strong>Modulus of elasticity</strong></td>
<td>23.9E6 psi</td>
<td>23.2E6 psi</td>
</tr>
<tr>
<td>Elongation at break</td>
<td>1.69%</td>
<td></td>
</tr>
<tr>
<td>Design Strain</td>
<td>0.85%</td>
<td></td>
</tr>
<tr>
<td><strong>Thickness</strong></td>
<td>0.047 in</td>
<td>1.2 mm</td>
</tr>
<tr>
<td><strong>Temperature resistance</strong></td>
<td>&gt;300 °F</td>
<td>&gt;150 °C</td>
</tr>
<tr>
<td><strong>Fiber volumetric content</strong></td>
<td>&gt;68%</td>
<td></td>
</tr>
<tr>
<td><strong>Density</strong></td>
<td>0.058 lbs/c.in</td>
<td>1.60 g/c.cm</td>
</tr>
</tbody>
</table>

Because long-term exposure to various type of environments can reduce the tensile properties and creep-rupture and fatigue endurance of FRP laminates, the material properties used in design equations should be reduced based on the environmental exposure condition. The environmental-reduction factor, 0.85, is induced from ACI 440.2R-2 Table 8.1. Thus, the design value of Sika CarboDur laminate, $f_{pu}$, is reduced to $3.451 \times 10^5$ psi.

Table 4-5 Mechanical properties of Sika CarboDur Commercial Products

<table>
<thead>
<tr>
<th>Product</th>
<th>Thickness</th>
<th>Width</th>
<th>Cross Section Area</th>
<th>Tensile Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type S 512</td>
<td>47.2 (1.2 mm)</td>
<td>1.97 (50 mm)</td>
<td>0.093 sq.in. (60 mm²)</td>
<td>37.8E3 lbs. (168kN)</td>
</tr>
<tr>
<td>Type S 812</td>
<td>47.2 (1.2 mm)</td>
<td>3.15 (80 mm)</td>
<td>0.149 sq.in. (96 mm²)</td>
<td>60.4E3 lbs. (269kN)</td>
</tr>
<tr>
<td>Type S 1012</td>
<td>47.2 (1.2 mm)</td>
<td>3.94 (100 mm)</td>
<td>0.186 sq.in. (120 mm²)</td>
<td>75.5E3 lbs. (336kN)</td>
</tr>
</tbody>
</table>

Laminates and anchorages are usually provided together by manufacturers. The shape of stressing anchorage Type Es and fix anchorage type Ef are shown in Figure 4-15 and 4-16.
4.5.2 Concrete Span

In the tentative design, two S 1012 CFRP laminates (3.94 in x 0.047in, 100 mm x 1.2mm) were applied to restore the flexural capacity of both the exterior girders with a total section area of $0.372 \text{ in}^2$. The initial prestress applied to the CFRP laminates is $0.50 \times 0.85 f_{pu}$. Because the information of stress loss is limited, in this calculation, the stress loss is assumed to be 15%. The effective stress in the CFRP laminates after all losses is

$$f_{pe} = 0.50 f_{pu}(1 - 0.15) = 146.667 ksi$$

and correspondingly, the effective strain is

$$\varepsilon_{pe} = f_{pe}/E_f = 6.322 \times 10^{-3}$$
where \( f_{pu} \) is the nominal tensile strength of prestressed CFRP laminate; and \( E_f \) the tension modulus of elasticity of CFRP laminate. In this calculation, for the external girder, the additional strain in the prestressed CFRP laminate, \( \varepsilon_d \), leading to the state of decompression is

\[
\varepsilon_d = \frac{h - c}{E_c c} \left( \frac{f_{pe} A_f}{A_e} - \frac{f_{pe} A_f (h - c)}{l_e} \right) = 7.210 \times 10^{-5}
\]

The strain increase limitation for the prestressed CFRP laminates is

\[
\kappa_m \varepsilon_{pfu} = \frac{1}{60} \left( \frac{90000}{nE_f t_f} \right) = 7.8125 \times 10^{-3}
\]

thus, strain increase for the prestressed CFRP laminates is equal to \( \kappa_m \varepsilon_{pfu} \).

Translate Eq. (4-4),

\[
c_b = \frac{\varepsilon_{cu} h}{\varepsilon_{cu} + \Delta \varepsilon_{pfb}} = 8.879 \text{ in}
\]

The depth of the corresponding concrete compressing block is

\[
a_b = 0.85 c_b = 7.547 \text{ in}
\]

The balanced CFRP reinforcement ratio of strengthened section is implied from Eq. (4-5)

\[
\rho_{fb} = \frac{A_{fb}}{bd} = \frac{0.85 f' c}{bd E_f \varepsilon_{pfu}} = 1.384 \times 10^{-3}
\]

The failure mode is identified using Eq. (4-5). Since \( \rho_f < \rho_{fb} \), it is confirmed that the exterior girders strengthened with prestressed CFRP laminates will experience tension or debonding failure.
The compressive strain in concrete at the peak stress is

\[ \varepsilon_0 = 2 \frac{f_c}{E_c} = 1.922 \times 10^{-3} \]

The compressive concrete strain is derived using

\[ \varepsilon_c = \frac{c}{h - c} \Delta \varepsilon_{pfu} = 1.803 \times 10^{-3} \]

The total strain of CFRP laminates

\[ \varepsilon_{pfu} = \varepsilon_{pe} + \varepsilon_d + \kappa_m \varepsilon_{pfu} = 0.0143 \]

By solving equilibrium equations, the depth of the concrete compressing zone is 4.186 in.

The nominal flexural capacity of the exterior concrete girders rehabilitated with prestressed CFRP laminates is

\[ M_{n,e} = f_y A_s (h_s - y_c) + \varepsilon_{pfu} E_f A_f (h - y_c) = 1140.74 \text{ kip ft} \]

Multiplying the factor 0.9,

\[ 0.9 M_{n,e} = 1026.2 \text{ kip ft} > M_{T,e} = 1048.734 \text{ kip ft} \]

The flexural capacity after rehabilitation satisfies the requirement.

Check service stress in the CFRP laminates

The CFRP laminates share the load effects of future wearing, truck and lane load. For the service state, all the load combination factors are 1.0.

\[ M_{s,e} = F_w + M_{truk} + M_{lane} = 360.78 \text{ kip ft} \]
Stress in the CFRP laminates is

\[ f_s = \frac{M_{s,e}}{S_{b,e}} + f_e = 0.522f_u < 0.55f_u \]

The stress in the CFRP laminates under service state satisfies the requirement.

Same to the exterior girders, two S 1012 CFRP laminates (3.94 in x 0.047in, 100mm x1.2mm) were applied to restore the flexural capacity of both the interior-exterior girders, with a total section area of 0.372 in². The initial prestress applied to the CFRP laminates is \(0.20 \times 0.85f_{pu}\). Because the information of stress loss is limited, in this calculation, the stress loss is assumed to be 15%.

The effective stress in the CFRP laminates after all losses is

\[ f_{pe} = 0.50 f_{pu}(1 - 0.15) = 146.667 ksi \]

and correspondingly, the effective strain is

\[ \varepsilon_{pe} = f_{pe}/E_f = 6.322 \times 10^{-3} \]

where \(f_{pu}\) is the nominal tensile strength of prestressed CFRP laminate; and \(E_f\) the tension modulus of elasticity of CFRP laminate. In this calculation, for the interior-external girder, the additional strain in the prestressed CFRP laminate, \(\varepsilon_d\), leading to the state of decompression is

\[ \varepsilon_d = \frac{h - c}{E_c c} \left( \frac{f_{pe} A_f}{A_e} - \frac{f_{pe} A_f (h - c)}{I_e / c} \right) = 7.813 \times 10^{-5} \]

The strain increase limitation for the prestressed CFRP laminates is

\[ \kappa_m \varepsilon_{pfu} = \frac{1}{60} \left( \frac{900000}{nE_f t_f} \right) = 7.8125 \times 10^{-3} \]
thus, strain increase for the prestressed CFRP laminates is equal to $\kappa_m \varepsilon_{pf}$.

Translate Eq. (4-4),

$$c_b = \frac{\varepsilon_{cu}h}{\varepsilon_{cu} + \Delta \varepsilon_{pf_b}} = 8.879 \text{ in}$$

The depth of the corresponding concrete compressing block is

$$a_b = 0.85c_b = 7.547 \text{ in}$$

The balanced CFRP reinforcement ratio of strengthened section is implied from Eq. (4-5)

$$\rho_{fb} = \frac{A_{fb}}{bd} = \frac{0.85 f'_c b a_b - f_y A_s + f'_y A'_s}{bd E_f \varepsilon_{pfu}} = 1.35 \times 10^{-3}$$

The failure mode is identified using Eq. (4-5). Since $\rho_f < \rho_{fb}$, it is confirmed that the exterior girders strengthened with prestressed CFRP laminates will experience tension or debonding failure.

The compressive strain in concrete at the peak stress is

$$\varepsilon_0 = 2 \frac{f_c}{E_c} = 1.922 \times 10^{-3}$$

The compressive concrete strain is derived using

$$\varepsilon_c = \frac{c}{h - c} \Delta \varepsilon_{pf_b} = 1.264 \times 10^{-3}$$

The total strain of CFRP laminates

$$\varepsilon_{pfu} = \varepsilon_{pe} + \varepsilon_d + \kappa_m \varepsilon_{pfu} = 0.0154$$
By solving equilibrium equations, the depth of the concrete compressing zone is 4.456 in.

The nominal flexural capacity of the interior-exterior concrete girders rehabilitated with prestressed CFRP laminates is

\[ M_{n,i} = f_y A_s (h_s - y_c) + \varepsilon_{p_{fu}} E_f A_f (h - y_c) = 1101.74 \text{ kip ft} \]

Multiplying the factor 0.9,

\[ 0.9M_{n,i} = 991.566 \text{ kip ft} > M_{T,e} = 962.114 \text{ kip ft} \]

The flexural capacity after rehabilitation satisfies the requirement.

Check service stress in the CFRP laminates

The CFRP laminates share the load effects of future wearing, truck and lane load. For the service state, all the load combination factors are 1.0.

\[ M_{s,i} = F_w + M_{truk} + M_{lane} = 360.78 \text{ kip ft} \]

Stress in the CFRP laminates is

\[ f_s = \frac{M_{s,i}}{S_{b,i}} + f_e = 0.523 f_u < 0.55 f_u \]

The stress in the CFRP laminates under service state satisfies the requirement.

4.5.3 Steel Span

In order to reduce the steel girder stress under service load, the steel I-beam girder can also be rehabilitated with externally prestressed CFRP laminates. The stress under service load can be reduced to 55% of the steel yield strength \( f_y \). A S1024 CFRP laminate were installed to
each girder, and they are located at the bottom of the steel girder. The initial prestress applied to the CFRP laminates are assumed to be $0.45 f_{pu}$ and the stress loss is assumed to be 15%. The effective stress in the CFRP laminates after all losses is

$$f_{pe} = 0.45 f_{pu} (1 - 0.15) = 155.25 ksi$$

The steel girder stress under service is obtained from following equation

$$f_s = \frac{M_{beam}}{S_b} + \frac{M_{deck}}{S_{b3n}} + \frac{M_{bar}}{S_{b3n}} + \frac{M_{FW}}{S_{b3n}} + \frac{M_{trucklmDF}}{S_{bn}} + \frac{M_{lanelmDF}}{S_{bn}} + \frac{T_{ten}}{A_{c3n}} - \frac{T_{ten}y_{b3n}}{S_{b3n}} \quad (4-22)$$

For exterior girders

$$f_{s,e} = 22.271 ksi = 0.543 f_y$$

and for interior-exterior girders

$$f_{s,i} = 21.138 ksi = 0.516 f_y$$

Both of them are smaller than 0.55$f_y$. The tension stress in the CFRP laminates under service traffic load is obtained from following equation

$$f_{pf} = \frac{M_{FW}}{S_{b3n}} + \frac{M_{trucklmDF}}{S_{bn}} + \frac{M_{lanelmDF}}{S_{bn}} + f_{pe} \quad (4-23)$$

For exterior girders

$$f_{pf,e} = 171.235 ksi = 0.496 f_y$$

and for interior-exterior girders

$$f_{pf,i} = 169.314 ksi = 0.491 f_y$$
Both of them are smaller than $0.55f_{p\text{f}u}$.

4.6 3-D Finite Element Analysis

Two 3-D finite element analysis models were developed for both the concrete approach span and main crossing steel span with ANSYS (Release 13.0), respectively.

4.6.1 Finite Element Type

For the concrete span, both the concrete deck and the concrete girder were simulated with SOLID 45 elements. SOLID45 is used for the 3-D modeling of solid structures. The element is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. For the steel girder span, the concrete deck is simulated with SOLID 73 and the steel girder flanges and web were simulated with SHELL63 elements. Unlike SOLID 45, besides three translation freedom at each node, each node of SOLID 73 has additional three degrees of rotation freedoms. SHELL 63 has both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedoms at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. Stress stiffening and large deflection capabilities are included. A consistent tangent stiffness matrix option is available for use in large deflection (finite rotation) analyses. The top flange and bottom deck surface were connected with stiff arms which were simulated with BEAM 4, a uniaxial element with tension, compression, torsion, and bending capabilities. BEAM 4 elements has six degrees of freedoms at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z axes. With the connection of the stiff arms between the concrete deck and steel girders, the main cross section was considered as a full composite section, without relative displacement between these two materials. To ensure the side stability of the slender web, the contribution of the diaphragms were realized by coupling the transverse
deformation of the web at the diaphragms position, one at the mid-span, two at the location one foot away from the two ends.

4.6.2 Load Combination

The dead load and live load were included in the models. For the purpose of simplicity, the dead load of wearing surface, diaphragms and barrier were ignored in the preliminary analysis. Two HL-93 trucks were put in the worst position side by side along the longitudinal direction. The location of the two trucks is shown in Figures 4-17 and 4-18. Two load combination cases, considering strength limit state and service limit state, were calculated using different combination factors. For the service limit state, combination factors for dead load and live load are 1.0, for the strength limit state, the factors are 1.25 and 1.75, respectively. For both load combination cases, the live load impact factor, 1.33, was included.

![Figure 4-17 Truck Position in Concrete Span](image1)

![Figure 4-18 Truck Position in Steel Span](image2)
4.6.3 3-D Finite Element Analysis Result

The external prestressed CFRP laminates were simulated with external force for exterior and interior-exterior girders applied at the anchorage positions. The prestressed force is equal to the effective prestress force when the CFRP materials were applied on the girders, which was used in the tentative design mentioned above. This simplification does not take the consideration of the increments in the prestress force when the live load is applied on the girders, thus the improvement of the performance of the girders rehabilitated with prestressed CFRP laminates is conservatively underestimated in the finite element model.

Tables 4-6 and 4-7 list the mid-span deflection and bottom fiber stress of concrete girders and steel girders, respectively. The deformation of the entire bridge, and the longitudinal stress among the bridge under live load only in both service limit state and strength limit state are shown in Figure 4-19 to 4-30.

The deflection due to the truck load is 0.207 in. for the concrete span and 1.347 in. for the steel span. This deflection in steel span exceeds the requirement of L/800. After the rehabilitation, the deflection reduces to 0.157 in. and 1.009 in., respectively. It is shown that the rehabilitation with prestressed CFRP reduces the bottom stress by 5% to 10%. One should notice that, the stress calculated with 3-D finite element model is much smaller than that calculated from AASHTO (2005). For the steel span, the result is sensitive to the connections between the shell elements and solid elements. It is recommended a field test is needed to improve the accuracy of the finite element models. In addition, since the stress increments in the CFRP are ignored, the realistic contribution of the CFRP laminate is greater than the calculation result.
Table 4-6 Concrete Girder Mid-span Stress and Deflection

<table>
<thead>
<tr>
<th></th>
<th>before rehabilitation</th>
<th>after rehabilitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>live load</td>
<td>service limit state</td>
<td>Strength limit state</td>
</tr>
<tr>
<td>stress (psi)</td>
<td>611.8</td>
<td>1235.9</td>
</tr>
<tr>
<td></td>
<td>1850.8</td>
<td>402</td>
</tr>
<tr>
<td>deflection (in.)</td>
<td>0.2066</td>
<td>0.4177</td>
</tr>
<tr>
<td></td>
<td>0.6242</td>
<td>0.1571</td>
</tr>
</tbody>
</table>

Table 4-7 Steel Girder Mid-span Stress and Deflection

<table>
<thead>
<tr>
<th></th>
<th>before rehabilitation</th>
<th>after rehabilitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>live load</td>
<td>service limit state</td>
<td>Strength limit state</td>
</tr>
<tr>
<td>stress (psi)</td>
<td>2672.1</td>
<td>5522.5</td>
</tr>
<tr>
<td></td>
<td>8239.2</td>
<td>2233.4</td>
</tr>
<tr>
<td>deflection (in.)</td>
<td>1.347</td>
<td>3.1016</td>
</tr>
<tr>
<td></td>
<td>4.5475</td>
<td>1.009</td>
</tr>
</tbody>
</table>

Figure 4-19 Concrete Span Deformation under Service Limit State before Rehabilitation
**Figure 4-20** Concrete Span Deformation under Service Limit State after Rehabilitation

**Figure 4-21** Concrete Girders Longitudinal Stress under Service Limit State before Rehabilitation
**Figure 4-22** Concrete Girders Longitudinal Stress under Service Limit State after Rehabilitation

**Figure 4-23** Concrete Girders Longitudinal Stress under Strength Limit State before Rehabilitation
Figure 4-24 Concrete Girders Longitudinal Stress under Strength Limit State after Rehabilitation

Figure 4-25 Steel Span Deformation under Service Limit State before Rehabilitation
**Figure 4-26** Steel Span Deformation under Service Limit State after Rehabilitation

**Figure 4-27** Steel Girders Longitudinal Stress under Service Limit State before Rehabilitation
Figure 4-28 Steel Girders Longitudinal Stress under Service Limit State after Rehabilitation

Figure 4-29 Steel Girders Longitudinal Stress under Strength Limit State before Rehabilitation
4.7 Conclusion

1 Some of the major benefits of FRP include its high strength to weight ratio, high fatigue endurance, excellent corrosion resistance, low thermal expansion, and the ease of fabrication, manufacturing, handling and installation which make it one of the best materials for bridge rehabilitations.

2 For the design of the structures strengthened with FRP, failure modes should be identified first. Debonding failures should be avoid in the design.

3 Structures strengthened with FRP can significantly improve their performance, including higher capacity, smaller deflection, and excellent durability due to reducing cracking.

4.8 References


CHAPTER 5. TIME-VARIANT RELIABILITY ANALYSIS OF EXISTING BRIDGES STRENGTHENED WITH PRESTRESSED CFRP LAMINATES

5.1 Introduction

Performance of existing bridges and other infrastructure facilities degrades due to the degradation of materials and increase in applied loads. The continuous degradation of structures leads to a sustained reduction of reliability of structures. The structure reliability, in terms of reliability index, expressed with a function of time, can be described precisely using time-varying variables reflecting the characters of material degradation. The methodology of calculation for time-varying reliability of existing bridges has been presented in Chapter 3. The decreasing reliability according to the increasing age for both the steel and concrete girders is demonstrated separately. Once the bridge’s time-varying reliability is calculated, firstly, one can determine whether an aged bridge needs rehabilitation or not, and secondly, one can estimate the time when a bridge needs rehabilitation.

The potential danger of highly risky infrastructures may damage people’s life or properties. Retrofitting is needed when structure’s performance or reliability does not meet their current requirement. Rehabilitation of structures includes adding additional components or increase cross section of components with the same or different materials.

Two aims of rehabilitation of structures are, firstly, to increase the reliability instantly to meet current requirement, and secondly, to slow down the rate of the degradation of structure performance. To realize these two aims, a relatively new material in the area of civil infrastructure, carbon fiber reinforced polymer (CFRP) strengthening systems have been applied for various structures ranging from beams to slabs to resist load effects leading to flexural failure.
The methodology to restore flexural capacity of concrete and steel girders with prestressed CFRP laminates has been proposed in Chapter 4.

Previous research was concentrated to gain an understanding of behavior of structures strengthened with CFRP materials (Adom-Asamoah and Kankam 2009; Atadero et al. 2005; Dolan and Swanson 2002; Karbhari 2004; Karbhari et al. 2001; Lekou and Philippidis 2008; Nanni 2003; Rosenboom et al. 2007; Sen et al. 2001; Smith and Teng 2002; Smith and Teng 2002; Teng et al. 2012; Woo et al. 2008). Research related to environmental degradation of CFRP and long term durability of strengthening measures has been conducted in recent years (Abanilla et al. 2005; Karbhari and Abanilla 2007; Karbhari and Ghosh 2009; Walker and Karbhari 2007; Youakim and Karbhari 2007; Zhang et al. 2003), but few of them describe their long term degradation properties quantitatively. Apparently, if properly rehabilitated, the reliability of the bridge improves instantly with an instant jump of structure reliability. It should be noted that the rehabilitation with external prestressed CFRP laminates is to add additional reinforcement to the beams; but it does not improve the properties of original materials, concrete and steel rebars. From bad to worse, the deterioration of existing structural materials accelerates as the age increases. Furthermore, the characteristics of CFRP materials themselves are also time dependent. It cannot avoid the degradation either. This is a new factor affecting the reliability of structures in the future. Consequently, the degradation process continues even the beams are rehabilitated. The reliability decreases from the new level, and the decreasing rate is determined by more factors.

The aim of this chapter is to describe the long term flexural performance of bridge girders strengthened with prestressed CFRP materials in term of reliability index. In this Chapter, the durability characters of CFRP materials are introduced. The time variant reliability after
rehabilitation is described. Compared with the reliability of bridge beams without rehabilitation, the rehabilitation effect is evaluated.

5.2 Time-Variant Properties of Original Materials

To describe the time-variant reliability after rehabilitation, the variation of strength and other mechanical properties of materials according to time need to be considered. The original materials may have experienced decades of deterioration before rehabilitation. At this moment, their actual mechanical properties may be far away from their original values. If their actual values are not available, they can be estimated based on experience or experiments. When using distributions to describe the time-variant material properties, it should be noted that the variation increases as the age accumulates.

In chapter 3, the variation of concrete strength, corrosion of steel reinforcement in concrete, and corrosion of wide flange steel girder are described. The mean value and standard deviation of concrete are expressed using Eq. (3-15) and (3-16), respectively.

Two key parameters describe the corrosion of steel reinforcement, one describes the initial time when the corrosion begins, and the other determines the rate of corrosion in terms of physical corrosion thickness per unit time. The distance of the reinforcement from the concrete surface, the threshold of chloride concentration of corrosion, and chloride concentrations at surface of concrete determine the initial time of steel corrosion. Monte Carlo simulation and distribution fitting are carried using Eq. (3-3) to derive the distribution of the initial time of corrosion, showing that the initial time of corrosion fits the Gumbel distribution. The rate in terms of the physical corrosion thickness per unit time is in proportion to the corrosion current
density generally expressed in terms of microamperes per unit time. As mentioned above, the rate can be described with a lognormal distribution.

Corrosion in “I” section steel girders is assumed to occur throughout the web height at the supports while it is assumed to occur only at the bottom quarter of the web height along the rest of the girder length including the mid-span location. The power function for corrosion is expressed using Eq. (3-14).

5.3 Time-Variant Properties of CFRP Materials

As one of the most promising new development for civil structures, fiber reinforced polymers (FRP) are increasingly being used to rehabilitate and renew the existing infrastructure as reinforcing elements for strengthening deteriorating and under-strength concrete and steel components. FRP reinforced polymers provide lighter, easier to assemble and more durable structures. The general advantages of FRP reinforcement are:

(a) High ratio of strength to mass density.
(b) Excellent fatigue characteristics.
(c) Excellent corrosion resistance and electromagnetic neutrality.
(d) Low axial coefficient of the thermal expansion.

Compared with steel, CFRP materials have lower elastic modulus and much higher tension strength. Based on these particular characteristics, it is more economical to use CFRP materials as internally embedded or externally bonded prestressed elements. Embedded elements are usually applied in new structures, while externally bonded elements are usually applied in existing structures. Numerous studies have been carried out on investigating structure
performance after rehabilitation with CFRP materials (Arockiasamy et al. 2000; Ascione et al.
2011; Pisani 2000; Zou and Shang 2007).

CFRP material itself is subject to variability in its own properties. Geometry uncertainty,
typically referring to the thickness of the CFRP laminates or the diameters of CFRP rods, are
treated as time-independent variables. Atadero and Karbhari (2008) developed a methodology
for the calibration of preliminary resistance factors for the design of externally-bonded FRP
composite renewal strategies for reinforced concrete structures using the load and resistance
factor design (LRFD) approach. In their research, the strength, modulus, and thickness of FRP
were treated as uncertainties. The FRP strength was assumed following a Weibull distribution
and the modulus and thickness of FRP following lognormal distributions. The properties and
distribution factors of FRP composite are listed in the following Tables 5-1 and 5-2.

Table 5-1 Generalized FRP Properties Used for Calculation

<table>
<thead>
<tr>
<th>Material type</th>
<th>Ultimate strength MPa (ksi)</th>
<th>Modulus GPa (ksi)</th>
<th>1-Layer Thickness mm (in.)</th>
<th>Ultimate strain mm/mm (in./in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>620.5 (90)</td>
<td>51.7 (7500)</td>
<td>1.27 (0.05)</td>
<td>0.012</td>
</tr>
<tr>
<td>2</td>
<td>689.5 (100)</td>
<td>61.4 (8900)</td>
<td>1.27 (0.05)</td>
<td>0.011</td>
</tr>
<tr>
<td>3</td>
<td>758.4 (110)</td>
<td>58.6 (8500)</td>
<td>1.27 (0.05)</td>
<td>0.013</td>
</tr>
<tr>
<td>4</td>
<td>827.4 (120)</td>
<td>59.3 (8600)</td>
<td>1.27 (0.05)</td>
<td>0.014</td>
</tr>
<tr>
<td>5</td>
<td>896.3 (130)</td>
<td>68.9 (10000)</td>
<td>1.27 (0.05)</td>
<td>0.013</td>
</tr>
</tbody>
</table>

It should be noted that CFRPs are relatively new materials and are provided with various
forms, including laminates, bars and fabrics. The scanty manufactures may use different
technical processing on CFRP productions. This leads to a large quality difference between
CFRP products provided by different manufactures.
Table 5-2 Statistical Distributions of FRP Variables Used in Reliability Analysis

<table>
<thead>
<tr>
<th>Variable</th>
<th>Statistical Distribution</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRP Strength</td>
<td>Weibull</td>
<td>Allowed to vary from 0.05 to 0.3</td>
</tr>
<tr>
<td>FRP Modulus</td>
<td>Lognormal</td>
<td>0.2</td>
</tr>
<tr>
<td>FRP Thickness (1-layer)</td>
<td>Lognormal</td>
<td>0.05</td>
</tr>
</tbody>
</table>

To describe the time-varying reliability of bridges strengthened with CFRP materials, the degradation properties of CFRP are needed. A popularly used general function for estimating the long-term response of a given limit state against harsh, challenging environments is (Karbhari and Abanilla 2007):

\[
P(T) = \frac{P_0}{100} [A \ln(T) + B]
\]  
(5-1)

where \(P(T)\) = performance attribute at time \(T\), and \(P_0\) = performance attribute at the unexposed condition; \(A\) = constant that denotes degradation; and \(B\) = material constant accounting for posture effects (\(B > 100\)). Karbhari and Abanilla (2007) proposed the time-dependent functions for tensile modulus and tensile strength for CFRP and GFRP composites with following equations:

\[
E(T) = \frac{E_0}{100} \left[ -0.4182 \cdot \ln \left( T \cdot 365 \frac{days}{year} \right) + 100 \right]
\]  
(5-2)

\[
\sigma_{ult}(T) = \frac{\sigma_0}{100} \left[ -3.366 \cdot \ln \left( T \cdot 365 \frac{days}{year} \right) + 100 \right]
\]  
(5-3)

where subscript “0” indicates initial values or as-built properties and \(E(T)\) and \(\sigma_{ult}(T)\) = time-dependent FRP composite tensile modulus and tensile strength, respectively. Figures 5-1 and 5-2
show the changes of the elastic modulus and strength of Sika CarboDur laminates according to the length of service time, respectively.

![Figure 5-1 Elastic Modulus of Sika CarboDur Laminates](image1)

![Figure 5-2 Strength of Sika CarboDur Laminates](image2)

Some durability characteristics of the CFRP materials also affect the long term performance of bridges. Creep rupture is a time dependent phenomenon that exists in FRP materials as steel does. When FRP materials are subjected to a constant stress, they can fail
suddenly due to the creep rupture. The creep rupture happens due to resin not fibers; therefore, the orientation and volume of fibers have a significant influence on the creep performance of tendons. According to ACI 440.2R-04, carbon fibers are the least susceptible to the creep rupture. Stress relaxation is the decay in stress with time when the material is kept under a constant strain condition. The relaxation phenomenon is characterized by the time dependent decrease in load in a FRP tension element held at a given constant temperature with prescribed initial load applied and held at a given constant strain. It has been estimated that the relaxation rates by setting the service life of the structures to 50 years of CFRP materials are 2.0 to 3.1%. The relaxation rate is related to the environment temperature; the higher the temperature, the greater relaxation is obtained. Compared with other FRP materials such as aramid fiber reinforced polymer (AFRP) and glass fiber reinforced polymer (GFRP), CFRP exhibits the best structural characteristic and durability. It has the highest strength, the least relaxation and is least susceptible to creep rupture.

Furthermore, since most CFRP laminates are bonded to the surface of existing structures by epoxy, the degradation of interaction behavior between CFRP and the existing structures is also a significant factor in evaluating composite component performance. The bonding character is highly related to the type of the materials used, the construction method and the environment when it is constructed. According to ACI 440.2R-02, debonding failure can be avoided through limiting the amount and thickness of CFRP materials. In this research, only the degradation of the material is considered. In addition, it should be noticed that not all characteristics changes affect the ultimate capacity. Relaxations of CFRP prestressing tendons that cause the long-term deformation in concrete structures do not affect the ultimate capacity of a prestressed concrete member (Youakim and Karbhari 2007).
5.4 Case Study

The superstructure of the LA 415/Missouri Pacific Railroad overpass on US 190 is located at West Baton Rouge Parish and was constructed in 1940. The detailed information of the bridge is introduced in Chapter 3. Since the bridge was built more than 70 years ago following design load H15, and has been experiencing deterioration for several decades, it is necessary to evaluate its reliability according to the current increasing traffic loads. The extreme load distributions have been derived in Chapter 2 and Chapter 3. The time-variant reliabilities of a concrete girder in typical concrete approach spans and of a wide flange steel girder in the main crossing section were described in Chapter 3. Corresponding to the mean recurrent of 75 year, which is close to the bridge’s real age, the reliabilities are 2.9138 for the concrete girder and 4.0084 for the steel girder, respectively. Apparently, the reliability of the concrete girders does not meet the current requirement; thus, the steel girder is much more reliable due to overdesign. As demonstration, Chapter 4 has indicated the methodology of strengthening bridge girders’ flexural capacity with post-tensioned CFRP laminates. In this section, the time-variant flexural reliabilities of both concrete and steel girders after are evaluated.

5.4.1 Concrete Girders

Chapter 3 has proposed methodology of calculating time-variant reliabilities of existing bridges considering corrosion of steel. In that research, the time-variant flexural reliability index of girders of a concrete span was calculated. For mean recurrent intervals from one year to 75 years, the reliability index drops from 5.8 to 2.9.

For the reliability index calculation for concrete girders, the statistic model of concrete strength and corrosion of steel reinforcement in concrete were introduced. The mean value of concrete strength is time variant and assumed following a lognormal distribution, meanwhile, its
deviation increase linearly as the age increases. The starting time of corrosion of steel reinforcement in concrete was assumed following an extreme value distribution, a Gumbel distribution. The corrosion of reinforcement begins at different time because of their different locations in the concrete. The corrosion rate is assumed following a lognormal distribution. Because of these uncertainties, the reliability degrades nonlinearly. The yearly reliability index of concrete girder, as shown in Figure 3-17 in Chapter 3, indicates the yearly reliability index increases slightly and decreases smoothly in the first 15 years. This is because the strength of concrete increases in the first 10 years. After that, with a sharp turn, the reliability index drops dramatically. This is because the steel corrosion for the exterior rebars happens around 20 years. Figure 17 shows the variation of yearly reliability index up to 75 years. The decreasing tendency continues till the interior rebars begin to be corroded. It is expected that the decreasing rate increases after the corrosion for the interior rebars happens.

In Chapter 4 a calculation was carried on the flexural capacity of concrete girder after rehabilitation. Two prestressed strips of CFRP laminates were applied to restore the flexural capacity of both the interior-exterior girders. The CFRP laminates, Sika CarboDur (S 1012) were provided by Sika Corporation. Sika CarboDur is a pultruded carbon fiber reinforced polymer (CFRP) laminate designed for strengthening concrete, timber and masonry structures. Sika CarboDur is bonded onto the structure as external reinforcement using Sikadur 30 epoxy resin as the adhesive. The CFRP nominal total cross section area of CFRP is 0.372 in$^2$. Because of lack of information, it is assumed following a lognormal distribution with a coefficient of variation (COV) of 0.05. The nominal cross section area is regarded as mean value of the distribution. The design and mean value of tension strength of Sika CarboDur are 406 ksi and 449 ksi, respectively. It is assumed following a Weibull distribution with a COV of 0.3.
The original materials and the new added CFRP laminates are not in the same age. The concrete is at age of 75 years old at the time of rehabilitation. Its strength is described using a lognormal distribution. The mean value and standard deviation of the distribution are defined using Eq. (3-15) and (3-16). According to Figure 3-16, at age of 75 years old, the exterior steel reinforcement have been corroding for several decades, and the interior steel reinforcement may begin to corrode.

The flexural reliability index of the concrete girder under the routine traffic live load is calculated. The Chapter 4 has verified that the tension failure instead of the debonding failure will occur in this case. The nominal flexural capacity is

\[ M_n = f_y A_s (d - y_c) + f_y \frac{A_s}{A_s} (y_c - d_s) + E_f \left[ \varepsilon_{p_{fu}} \right] A_f (h - y_c) \]  (5-4)

where \( y_c \) is the distance from the top concrete fiber to the centroid of concrete compressive force. \( y_c \) is derived from Eq. (4-6), (4-9) and (4-12) based on the assumption that a plane cross section before loading is assumed to remain plane under loading.

Using equivalent rectangular stress distribution, a more simplified equation is proposed to estimate the nominal flexural capacity

\[ M_n = (A_s f_y + A_{frp} f_{frp}) \left( d - 0.59 \frac{A_s f_y + A_{frp} f_{frp}}{b f_c} \right) \]  (5-5)

The nominal moment is a function of time, and it is rewritten as

\[ M_n(t) = \left( A_s(t) f_y + A_{frp} f_{frp}(t) \right) \left( d - 0.59 \frac{A_s(t) f_y + A_{frp} f_{frp}(t)}{b f_c(t)} \right) \]  (5-6)

This equation is used to calculate the time-variant reliability of concrete girders.
The load effect consists of dead load and live (traffic) load. In Chapter 4, the dead load has been calculated, it is assumed following lognormal distribution. As mentioned in Chapter 3, the AASHTO live load predicts the extreme structure response in a mean recurrent interval of 75 years. The load factors guarantee a specific reliability for both ultimate limit states and service limit states. To calculate the time-dependent reliability, it is necessary to transfer it into Gumbel distribution (a type of extreme distribution) with a mean recurrent interval shorter than 75 years, for example one year. Chapter 3 presents the methodology of transforming live load defined by AASHTO to Gumbel distribution with mean recurrent interval of one year using Monte Carlo Simulation.

In short intervals, both the flexural capacity and load effects are regarded as stationary random process. The limit state function is expressed as

\[ g = (A_s(t)f_y + A_{frp}f_{frp}(t)) \left( d - 0.59 \frac{A_s(t)f_y + A_{frp}f_{frp}(t)}{b_f(t)} \right) - M_i - M_d \]  \hspace{1cm} (5-7)

All the variables have been defined with distributions. The yearly reliabilities were calculated using Rackwitz-Fiessler procedure. An iteration procedure provides a way to calculate the reliability index with variables following non-normal distributions by calculating “equivalent normal” values of the mean and standard deviation for each non-normal random variable. The calculation is realized using Matlab platform. Figure 5-3 shows the yearly reliability index of the concrete girder up to thirty years after rehabilitation. For the first year, the reliability index of the rehabilitated beam is 5.2415; and for the thirtieth year, it is 4.2710. The corresponding failure probabilities are 7.964E-08 and 9.73E-06, respectively. They are derived by converting reliability index using a standard normal distribution function. Once the safe probabilities for each year are obtained, the safe probabilities for any length of mean recurrent intervals are
obtained by continued multiplication of the yearly safe probability using Eq. (3-23). The corresponding reliability indexes are calculated using an inverse of the standard normal distribution function. The time-variant reliability of the concrete girder after rehabilitation is shown in Figure 5-4. As mentioned above, reliability is meaningful only when it corresponds to a determined mean recurrent interval. Figure 5-4 shows the reliabilities corresponding to mean recurrent intervals from one year to thirty years. As a comparison, the corresponding reliability of the un-strengthened bridge girder at age of 75 to 105 years (0 to 30 years after rehabilitation) is also shown in Figure 5-4.

It is a common sense that no matter what kind of rehabilitation method is applied, the performances of structural components after strengthening mainly rely on the remaining strength of original materials. The economical way of rehabilitation is to eliminate the weakness of the components. By making the strength of different materials in the component matched each other; the maximum potential capacity can be achieved. Since the properties of structural materials keep changing all the time, the same rehabilitation method at different ages leads to different subsequent structural performance after rehabilitation. The time-variant reliabilities of structural component after rehabilitation at different ages are connected to the reliability before rehabilitation and are shown in Figure 5-5. For girders without rehabilitation, with the mean current intervals from one year to 75 years, the reliability drops from 5.7546 to 2.9138. For the mean recurrent interval of 59 years, the reliability is 3.4788, which is less than 3.5. Figure 5-5 shows that the later the rehabilitation is applied, the lower subsequent reliability is achieved. Corresponding to rehabilitation at age of 50, 60, 75, 90, and 100 years old, the reliability for the first year after rehabilitation are 5.763, 5.551, 5.240, 4.883, and 4.620, respectively. Due to the excellent duration character, the deterioration rate of CFRP is slower than the corrosion rate of
steel in concrete. Thus, the performance degradation of the composite components strengthened with CFRP is more smoothly than the original components without strengthening. Similarly, the later the rehabilitation is applied, the faster the degradation takes place.

![Figure 5-3 Yearly Reliability Index of a Concrete Girder](image1.png)

**Figure 5-3** Yearly Reliability Index of a Concrete Girder

![Figure 5-4 Reliability Index of a Concrete Girder during Service Life after Rehabilitation](image2.png)

**Figure 5-4** Reliability Index of a Concrete Girder during Service Life after Rehabilitation
5.4.2 Steel Girders

Time-variant reliabilities of existing bridges considering corrosion of steel were calculated and presented in Chapter 3. For the steel girder span, the corrosion of wide flange steel girders significantly affects the performance of a bridge. The time-variant flexural reliability index of girders of the main cross span, the steel span, was calculated. For the mean recurrent intervals from one year to 75 years, the reliability index drops from 4.95 to 3.8.

For the reliability index calculation for steel girders, the statistics model of elastic section modulus considering corrosion of steel reinforcement was introduced. Thanks to the good quality and stability of I-beam products the initial section modules are treated as constant functions. The depth of the corrosion is described using Eq. (3-19) and the section modules at any age are determined. Chapter 3 presents the elastic section modulus of wide flange steel beams, and short-term and long-term composite elastic section modulus varying over time with three pair of factors. The parameters reflect the environments related to corrosion where the bridges are
located. They are assumed following lognormal distributions. The limit state is defined based on
the normal stress due to the applied loads at the mid-span exceeding the yield strength.

In Chapter 4, a calculation was carried on the extreme strain at the bottom of the steel
girder strengthened with prestressed CFRP strips. A S1024 CFRP laminate were installed to each
girder, and they are located at the bottom of the steel girder. The effective prestress applied to the
CFRP laminates are assumed to be 0.45 \( f_{pu} \) and the stress loss is assumed to be 15%. Since this
is not a service state analysis, stress relaxation of CFRP need to be considered. The relaxation
rate is expressed by dividing the load measured in the relaxation test by the initial load.
Relaxation for CFRP tendons after 50 years of loading can be estimated 2.0% to 10.0%,
depending on the initial tensile stress (10.0% is applied in this research). The steel girder stress
under service at any age is obtained from following equation

\[
f_s(t) = \frac{M_{beam}}{S_b(t)} + \frac{M_{deck}}{S_{b3n}(t)} + \frac{M_{bar}}{S_{b3n}(t)} + \frac{M_{FW}}{S_{b3n}(t)} + \frac{M_{truck}(t)l_mDF}{S_{bn}(t)} + \frac{M_{lane}(t)l_mDF}{S_{bn}(t)} + \frac{T_{cen}(t_1)}{A_{c3n}(t)} - \frac{T_{cen}(t_1)y_{b3n}}{S_{b3n}(t)}
\]

(5-8)

where \( t \) is the age of the bridge, \( t_1 \) is the age after rehabilitation, \( \tau \) is a time interval, during which
the area of section, \( A_{c3n} \), the short-term composite elastic section modulus, \( S_{bn}(t) \), and the long-
term composite elastic section modulus, \( S_{b3n}(t) \) are treated as stationary process. The moment
due to the traffic load is calculated based on the AASHTO live load. To calculate the time-
dependent reliability, it is transferred into Gumbel distribution with a mean recurrent interval of \( \tau \)
(one year for this case). Chapter 3 presents the methodology of transforming live load defined by
ASSHTO to Gumbel distribution with mean recurrent interval of one year using Monte Carlo
Simulation. The yearly reliabilities were calculated using Rackwitz-Fiessler procedure. The time-
variant reliabilities of structural component after rehabilitation at different ages are related to the
reliability before rehabilitation and are shown in Figure 5-6. For girders without rehabilitations, with the mean current intervals from one year to 75 years, the reliability drops from 4.9067 to 3.9429. Similar to the concrete girder the later the rehabilitation is applied, the lowersequent reliability is achieved. Corresponding to rehabilitation at age of 50, 60 and 75 years old, the reliability for the first year after rehabilitation are 5.2691, 5.2393 and 5.2246, respectively.

![Figure 5-6 Reliability Index of a Steel Girder during Entire Service Life](image)

5.5 Comments and Conclusions

1. The increasing applied loads and degradation of structural performance reduce the safety of existing bridges or other infrastructure facilities over time. No matter what kind of rehabilitation method is applied, the performances of structural components after strengthening mainly rely on the remaining strength of original materials.

2. The economic way of rehabilitation is to enhance the weakness of the components. FRPs are increasingly being used to restore the flexural capacity of existing bridges because their high strength, excellent corrosion resistance, and fatigue characteristics.
3. Two aims of rehabilitation of structures are, firstly, to increase the reliability instantly to meet current requirement, and secondly, to slow down the rate of the degradation of structure performance.

4. The time when the bridges are strengthened determines itssequent reliability, the later the rehabilitation is applied, the faster the degradation takes place.

5.6 References


CHAPTER 6. CONCLUSIONS AND RECOMMENDATIONS

In the present study, using the field monitoring study and statistical analysis methods, the entire life flexural performance of bridges is described in terms of the time-variant reliability index. The entire service life of a bridge is separated into two sections, i.e., before and after the rehabilitation with externally bonded prestressed FRP laminates. The uncertainty factors that determine the flexural reliability are discussed in Chapter 1. Chapter 2 describes the framework of deriving the extreme live load distributions due to the traffic for any length of time intervals based on the monitoring data. The bridge reliability variation versus time before rehabilitation is quantified in the Chapter 3, where the degradation of the structure due to the steel corrosion and variation of concrete strength are taken into account. Chapter 4 presents flexural capacity estimation of bridge girders rehabilitated with post tensioned CFRP materials. Chapter 5 describes the reliability variation of bridges after rehabilitation. Some of the conclusions can be drawn as follows.

6.1 System Error and Random Error

Assumptions or simplifications applied in the structural analysis lead to the system error in the calculation of reliability index. System errors are always unidirectional. In other words, they have one way effect on the structural performances. If the cause of the system error can be identified, then it can usually be eliminated. Random error is caused by inherently unpredictable fluctuation of construction, materials and the environment the structure is exposed. Though the randomness of variables are inherent, they have expected value. Namely, they are scattered about the true value.
6.2 Deterministic Parameters and Random Variables

Theoretically, all the factors related to the structural capacity and its subjected loads are random variables. Parameters that can be measured easily or not sensitive to the performance of structures are treated as deterministic parameters.

Random variables are classified into two categories, time independent variables and time-variant variables. For time independent variables, they are described by distributions. Their mathematical expectations or other statistic parameters do not change over time. For time-variant variables, however, their mathematical expectations or other parameters change constantly over time. Time-variant variables are always used to represent mechanical characteristics. Since this variation is very smooth, time-variant variables can be regarded as time independent variables with a reasonable interval.

6.3 Extreme Live Load Model for Mean Recurrence Intervals

Extreme live load for a mean recurrent interval is a weak stationary random process if the length of the interval is long enough, so that the extreme live load in it is independent and follows the same distribution.

The extreme live load for each interval can be identified from the monitoring data. Its initial distribution is modeled with the Gumbel distribution function (maximum cases). The distribution parameters can be determined using the maximum likelihood estimation method.

For the studied cases, the distributions of extreme strains for different mean recurrence intervals have different mode values (position value) but the same scale parameter. The mode values increase smoothly as the length of the mean recurrence increases.
6.4 Time Variant Reliability of Existing Bridges

Infrastructures experience three periods during their entire service life cycle, namely, construction period, service period and deterioration period. Service period and deterioration period overlap each other most of the time. In each period, structure performance may change due to various environmental and man-made factors.

Both the resistance and the live load of a bridge are non-stationary auto-correlated random process during its service life cycle. In a reliability calculation, they cannot simply be treated as time independent variables. For example, the mean values of resistance are typically monotonically decreasing functions with time; the variations of resistance are monotonically increasing functions with time; and the mean values and variations of live load effects are monotonically increasing functions of time.

Since the decrease of resistances and increase of live loads are gradual processes, it is recommended that in a reasonable time segment, e.g., one year considered in this study, they can be treated as stationary processes and expressed with variables following a certain type of distributions. Thus, the reliability for each time segment is obtained.

The entire life cycle is the sum of a time series, thus the reliability for the entire service cycle of the bridge is calculated through the reliability analysis of a series system.

6.5 Time Variant Reliability of Existing Bridges after Being Strengthened

Two aims of rehabilitation of structures are, firstly, to increase the reliability instantly to meet the current requirement, and secondly, to slow down the rate of the degradation of structure performance.
No matter what kind of rehabilitation method is applied, the performances of structural components after strengthening mainly rely on the remaining strength of original materials.

The economical way of rehabilitation is to eliminate the weakness of the components. FRP materials are increasingly being used to restore the flexural capacity of existing bridges because their high strength, excellent corrosion resistance, and fatigue characteristics.

The time when the bridges are strengthened determines its subsequent reliability. The later the rehabilitation is applied, the faster the degradation takes place.

6.6 **Recommendation for Future Research**

Time-variant reliability of existing bridges and performance evaluation of bridges strengthened with presstressed FRP materials are relatively new topics in bridge engineering. Many factors affect the performance of existing bridges, including the original condition, and the environment the bridges are exposed to. Some factors, such as the live load and environmental conditions, are site specific; thus, it is impossible to apply uniform parameters to calculate reliabilities for all the bridges around the country. In addition, constructions of the original bridges and the rehabilitation also play an important role in bridge performance. Therefore, many meaningful aspects exist for research in the future.

Long term reliability of a bridge is seriously determined by the traffic (live) loads acting on it. It is well known that the live load is very site specific. With the help of the recently developed weigh-in-motion system (WIM) and structural health monitoring (SHM) system, information of load effects and structure response due to traffic live load can be easily obtained. Based on these monitoring data, the live load distribution for any length of mean recurrent intervals can be derived with distribution fitting techniques. In this research, the methodology of
deriving extreme live load distributions for long term intervals based on short term monitoring has been developed. This distribution does not reflect the variation of live load during the bridges’ long term service life. It is suggested that monitoring should be carried out every couple of years. Then, based on these monitoring data, one can estimate the variation of the life load and determine how long each monitoring should be conducted for different bridges. The other useful information is the characters of live load identified for different type of bridges. This information may help researchers to build general models of live loads. For bridges lack of the live load information, a general live load model can be used to calculate its reliability.

The environment that a bridge is exposed to affects the material deterioration process of bridges. Critical environment may decrease the strength of concrete, corrode steel reinforcements in concrete or steel components, and damage cohesion between reinforcements and concrete. Fragile analysis may help to find the key factors that determine a bridge’s long term performance. It is also very useful to build the relationship between key factors and surrounding environment.

The flexural capacity of concrete and steel girders may be improved when strengthened with prestressed FRP materials due to their excellent characteristics. Strengthening can increase the reliability instantly to meet current requirement and slow down the rate of the degradation of structural performance. It should be noted that no matter what kind of rehabilitation method is applied, the performances of structural components after strengthening mainly rely on the remaining strength of the original materials. Since the mechanical characteristics of material change constantly, it is proposed here to develop a method to determine an optimized aim of rehabilitation. The optimized aim should include when a structure should be strengthened and how much its reliability should be increased.
VITA

Miao Xia was born in 1971 in Jinan, China. He received his Bachelor of Science degree from the Department of Civil Engineering at Shandong Institute of Architecture and Engineering, China, in June 1993, and Master of Science degree from the Department of Structure Engineering at Tongji University, China, in March 1996. Mr. Xia has worked as a Graduate Research Assistant at Louisiana State University since August 2007.

Mr. Xia has been involved in several research areas, such as structural health monitoring, and FRP material applications for bridge strengthening engineering. He has written several journal articles and conference papers with Prof. Steve C. S. Cai. These journal articles are being under review.