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Elastohydrodynamic Analysis of Spur Gears Using Load-Sharing Concept: Running-In and Steady-State

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ELASTOHYDRODYNAMIC ANALYSIS OF SPUR GEARS USING LOAD-SHARING CONCEPT: RUNNING-IN AND STEADY-STATE

A Dissertation
Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
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in
The Department of Mechanical Engineering

by
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to

Hazrate Vallie Asr (AJ)
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Abstract

Gears are widely used in industry and hence their performance is of vital importance. Under the typical operating conditions of gears, the lubricant layer formed between the teeth of the pinion and the gear cannot completely separate the surfaces and contact of asperities of the pinion and gear occurs. This case is usually referred to as mixed lubrication problem.

In this research the load-sharing concept has been employed to predict the performance of the pinion-gear system. The load-sharing concept is an efficient method to solve the mixed lubrication problem and is capable to predict the thickness of the lubricant film, contribution of the fluid film and asperities in carrying the load, friction coefficient, lubricant temperature, and wear with fairly good accuracy.

During the initial stage of contact, a considerable number of plastic contact occurs between asperities resulting in permanent change of surface roughness profile. This period which is called running-in has a significant effect on the steady-state performance of the pinion-gear system. The developed model has the capability to predict the variation of surface roughness and contribution of fluid film as well as asperities in carrying the load during running-in. The steady-state wear of gears is predicted using the thermal desorption model.

A test rig is designed and built which is capable to mimic the operating condition of any point on the involute profile of gear. Two motors are used to rotate the rollers to generate the same rolling and sliding speed as the corresponding point of the involute profile of pinion-gear system. A hydraulic system is used to exert the desired load on the rollers and keep them in contact under the applied load. The sensors that are mounted on this test rig monitor the speed of each shaft, applied load, surface temperature, and wear depth. The results of the experiments that are conducted on the fresh rollers as well as broken-in roller are shown to be in good agreement with the predicted running-in behavior and steady-state behavior, respectively.


Introduction

1.1 Problem Statement

Gears are widely used in transmission systems and hence their performance is of great importance for the end users. During meshing, gear teeth engage in what is classified as non-conformal contact where the applied load must necessarily be carried over a small area. Conformal contact, on the other hand, occurs in cases such as journal bearings where the surfaces have a high degree of conformity and the load is carried over a relatively large area. Figure 1 shows examples of conformal and non-conformal contacts. As shown in Figure I.1, applications such as journal bearing are the examples of conformal contact.

![Figure I.1: Conformal and non-conformal contact](1)

There are different lubrication regimes in regard to each type of contact. Hydrodynamic lubrication is usually associated with conformal surfaces in which a positive pressure gradient is generated. This positive pressure gradient along with convergence surfaces enables the lubricant to carry normal load. The elastohydrodynamic lubrication (EHL) on the other hand occurs in non-conformal surfaces. EHL is a form of hydrodynamic lubrication in which the elastic deformation of surfaces is significant [1]. Contact of spur gear is an example of a EHL problem. It is a non-conformal contact in which the contact pressure is extremely high resulting in elastic deformation which is of the same order of magnitude of the lubricant film thickness. Now, let us turn our attention to the details of contact of pinion and gear.
The contact point of pinion and gear is always located on the line of action which is tangent to the base circles of the pinion and the gear. Figure I.2 illustrates the schematic of contact of pinion and gear.

![Figure I.2: Schematic of contact of pinion and gear](image)

The transmitted load varies during the contact since the number of teeth in contact changes. As the contact point moves along the line of action, for some duration of time there is only one pair of tooth in contact and for some duration of time there are two pairs of teeth in contact. Figure I.3 shows schematic of variation of transmitted force by a tooth of pinion.

![Figure I.3: Transmitted load](image)

Point A is the first point of contact of pinion. As the contact starts from point A, two pairs of teeth carry the load until it reaches the point B. Point B is the lowest point of single tooth contact. From point B to point C, there is only one pair of tooth in contact and therefore the entire load (F) is carried by one tooth. Point C is the highest point of single tooth contact. From point C to point D, there are again two pairs of teeth in contact. In other words, as point A in one tooth of pinion comes to contact, point C in the next tooth of pinion also comes to contact. As the contact in one tooth travels from point A to point B, the contact in the adjacent tooth travels from...
point C to point D. The F/3 and 2F/3 are calculated based on the constant stiffness of pinion and gear teeth and corresponds to pinion and gears without tooth modification. If the teeth are modified, the load distribution would be different.

Another important point in the contact of gears is surface roughness. Under high load, the asperities of the contacting surfaces undergo elastic and plastic deformation and cause an increase in friction, result in wear, and cause the surface temperature to rise.

Based on the thickness of the lubricant film that is formed and the roughness of the contacting surfaces, a non-dimensional parameter called film parameter $\Lambda$, is introduced which is the ratio of film thickness $h$ to the standard deviation of asperity heights of the two surfaces [1]:

$$\Lambda = \frac{h}{\sqrt{R_{q1}^2 + R_{q2}^2}}$$  \hspace{1cm} (I-1)

In Equation (1), $R_{q1}$ and $R_{q2}$ are the standard deviation of asperity heights of the contacting surfaces. Based on the value of the film parameter, three different lubrication regimes are defined in non-conformal contacts. Boundary lubrication occurs when $\Lambda < 1$, where the contacting surfaces are not separated and lubricant properties are of minor importance. Partial lubrication occurs when $1 < \Lambda < 3$ where the fluid film and the surface properties both have contributions in carrying the load. Full EHL $\Lambda > 3$ corresponds to the case where both surfaces are completely separated by the fluid film [1].

Friction is an important issue in contact of gears and it reflects the power loss in the transmission systems. There are two sources for friction in gears: a portion is due to the shearing of lubricant film and the rest of friction is due to the contact of asperities on the surfaces of pinion and gear. Friction coefficient between teeth of a typical pinion and gear versus film parameter is illustrated in Figure I.4.

Since contact of gears is usually in partial or mixed regime, it is necessary to include the role of asperities in carrying the load. As a result of contact of asperities, friction coefficient
increases, surfaces’ temperature increase and wear occurs. Wear of flank of pinion and gear is a major concern in gears. It results in deviations from the designed center distance, affects the load distributions, and increases noise and vibration of the gear-pinion system {reference?}. The tribological properties, however, are not constant during time. The initial transient regime is called running-in period during which a large number of asperities undergo plastic deformation and as a result the surface properties vary. The change in contact properties such as friction coefficient, arithmetic average of asperity heights, $R_a$, and the portion of load carried by fluid film and asperities continues until the running-in stage is complete and steady-state regime starts.

\[ \text{Friction Coefficient} \]

![Figure I.4: Stribeck curve for a spur gear](image)

### I.2 Overview of Dissertation

The involute profile of the pinion and gear ensures a non-conformal contact. The high transmitted load by gear teeth results in significant deformation of the bodies, making the EHL formulation the viable method to address the spur gear contact problem. The radii of curvature of pinion and gear continuously vary as the contact point travels along the line of action. The applied load as was shown in Figure 3 varies as the number of teeth in contact change. These
contact conditions make the solution of the gear contact complex. The provision of asperity contact results in more complexity. Solution of the governing equations for each contact point takes hours. For a full analysis of gears, these equations need to be solved for several points along the line of action which is a tedious job.

The traditional way to solve the contact of heavily-loaded gears includes simultaneous solution of several equations. Reynolds equation governs the pressure of the film that is formed between contacting surfaces:

\[
\frac{\partial}{\partial x} \left( \rho h^2 \frac{\partial P_f}{12 \mu} \right) = u \frac{\partial (\rho h)}{\partial x} \tag{I-2}
\]

where \(\rho\) is the density of lubricant, \(\mu\) is the viscosity of lubricant, \(u\) is the rolling speed of two surfaces, \(P_f\) is the fluid pressure and \(h\) is the thickness of the film that is formed between surfaces. A main feature in non-conformal contacts is the large increase in effective viscosity of the fluid due to increase in the film pressure [1].

\[
log \mu + 1.2 = (log \mu_0 + 1.2) \left( 1 + \frac{P}{C} \right)^Z \tag{I-3}
\]

where \(\mu\) is the effective viscosity of the lubricant in the contact zone, \(\mu_0\) is the viscosity at the ambient pressure, \(C\) is a constant and equal to 196.1 MPa and \(Z\) is the value of viscosity-pressure index. Equation (I-3), however, is valid for moderate contact pressures. Due to the significant contact pressure that develops in contact of gears, the elastic deformation of surfaces become significant and \(h\) can be written as:

\[
h(x) = h_0 + \frac{x^2}{2R_{eq}} - \frac{4}{\pi E'} \int_{-\infty}^{x} P(s) \ln(|x - s|) \, ds \tag{I-4}
\]

where \(x\) is the location inside the contact zone, \(R_{eq}\) is the equivalent radius of curvature, \(E_{eq}\) is the equivalent modulus of elasticity and \(P\) is the total pressure. The integral of pressure distribution inside the contact zone should be equal to the applied load:

\[
F = l \int_{-\infty}^{\infty} P(x) \, dx \tag{I-5}
\]
where $F$ is the applied load and $l$ is the width of rollers. The asperity contact pressure $P_c$ is calculated from work of Greenwood-Tripp [4]:

$$P_C(x) = \frac{8\sqrt{\pi}}{15} D_{sum}\beta^{1.5} R_q^{2.5} \int_0^\infty \left( h \left( \frac{R_q}{R_q} \right) \right)^{2.5} e^{-s^2/2} ds$$

(I-6)

where $D_{sum}$ is the density of asperities, $\beta$ is the radius of tip of asperities, and $R_q$ is the standard deviation of asperity height. This equation was developed for the contact of two rough surfaces.

The roughness properties of the surfaces which are required to calculate the contact pressure are obtained using the spectral moments of the roughness profile.

The contact of gears is replaced with contact of cylinders. As the contact point moves along the line of action, the transmitted load as well as the curvature of gear tooth changes. Therefore, the radii of these cylinders and the applied load vary. Equations (I-6) should be solved simultaneously to find the variation of film thickness $h$ and pressure inside the contact zone. The numerical procedure for solving this set of equations for a single operating condition is a time consuming task. In contact of gears, these equations need to be solved for hundreds of points along the line of action which is extremely a tedious job.

In this research, an algorithm based on the Johnson’s load sharing concept [3] is developed to predict the performance of spur gears. Based on the load sharing concept, it is assumed that a portion of total load $F_T$ is carried by fluid film $F_H$ and the rest of the load is carried by asperities $F_C$ [3]:

$$F_T = F_H + F_C$$

(I-7)

By introducing scaling factors as $\gamma_1$ and $\gamma_2$, Equation (I-7) is written as:

$$F_T = \frac{F_T}{\gamma_1} + \frac{F_T}{\gamma_2}$$

(I-8)

In other words, $1/\gamma_1$ is the portion of load carried by fluid film and $1/\gamma_2$ is the portion of load carried by asperities. By simplifying Equation (I-8):

$$1 = \frac{1}{\gamma_1} + \frac{1}{\gamma_2}$$

(I-9)
The central film thickness between the two rollers was developed by Moes [5]:

\[
H_c = \left[ (H_{RI}^{-7/3} + H_{EI}^{-7/3})^{3s/7} + (H_{RP}^{-7/2} + H_{EP}^{-7/2})^{2s/5} \right]^{1/p}
\]  
(I-10)

where

\[
s = \frac{1}{5} \left( 7 + 8e^{\left( \frac{-2H_{RI}}{H_{RI}} \right)} \right)
\]  
(I-11)

The dimensionless parameters used are defined as below [5]:

\[
H_{RI} = 3(WU_\Sigma^{-0.5})^{-1}
\]

\[
H_{EI} = 2.621(WU_\Sigma^{-0.5})^{-0.2}
\]

\[
H_{RP} = 1.287(GU_\Sigma^{-0.25})^{2/3}
\]

\[
H_{EP} = 1.311(WU_\Sigma^{-0.5})^{-1/8} \left( GU_\Sigma^{-0.25} \right)^{3/4}
\]  
(I-12)

\[
W = \frac{F_T}{E_p R'B}
\]

\[
G = \alpha E_p
\]

\[
U_\Sigma = \frac{\eta_0 u}{E_p R'}
\]

\[
H_C = \frac{h}{R'} U_\Sigma^{-0.5}
\]

Equation (I-10) was developed for perfectly smooth surfaces. In order to include the surface roughness effect, the scaling factors are introduced in the film thickness equation. According to Johnston’s load sharing method, a portion of the load equal to \((1/\gamma_1)\) is taken by fluid film. For calculating film thickness, the problem will be of two rollers with known geometry and modulus of elasticity of \(E'/\gamma_1\) being pressed together with a force equal to \(F_T/\gamma_1\). Rest of the load \((1/\gamma_2)\) is taken by asperities. Therefore for calculating asperity contact pressure \(P_C\), the problem will be two rollers with known geometry and modulus of elasticity of \(E'/\gamma_2\) being pressed together by a force equal to \(F_T/\gamma_2\). Applying the Johnson’s concept of scaling factor and substituting \(E'/\gamma_1\) for \(E'\) and \(F_T/\gamma_1\) for \(F_T\), the Moes’ equation takes the following form:

\[
H_c = \left[ \gamma_1^{s/2} \left( H_{RI}^{7/3} + \gamma_1^{14/15} H_{EI}^{7/3} \right)^{3s/7} + \gamma_1^{-s/2} \left( H_{RP}^{-7/2} + H_{EP}^{-7/2} \right)^{-2s/7} \right]^{1/s} (\gamma_1)^{1/2}
\]  
(I-13)
where

$$s = \frac{1}{5} \left( 7 + 8e^{\left( \frac{2(y_1)^{-2/5}H_0}{\eta R_1} \right)} \right)$$

Equation (I-13) has two unknowns: $y_1$ and $h_C$. In order to determine both of the unknowns, another equation is needed. This equation comes from making the asperity contact pressure obtained from Greenwood-Tripp model (Equation (I-6)) equal to Gelinck’s curve-fit (Equation (I-15)) [6]. Gelinck and Schipper [6] have shown that the central pressure is a good quantity to characterize the pressure distribution of a rough line contact:

$$P_C = \frac{1}{\gamma_2} \sqrt{\frac{F_T E_t}{2\pi B R T}} \left[ 1 + \left( 1.558 \left( D_{sum} R \sqrt{\beta R} \right)^{0.0337} \left( \frac{a_c}{R} \right)^{-0.442} \left( \frac{W}{W'} \right)^{0.4757} \right)^{-0.5882} \right]$$

The solution scheme is to choose an initial value for $y_1$ and find the film thickness from Equation (I-13), plug the film thickness in Equation (I-6) and check if equations (I-6) and (I-15) are close enough. If not, then a new value is chosen and the loop continues until the convergence criterion is satisfied. Satisfaction of convergence criterion means that the value chosen for $y_1$ and consequently $y_1$ are such that equations (I-6) and (I-15) are close enough.

Once the scaling factors are known, the film thickness and friction coefficient can be calculated. The friction coefficient has two components: shearing of fluid film ($F_{f,H}$) and the contact of asperity ($F_{f,C}$):

$$F_f = F_{f,H} + F_{f,C}$$

The asperity friction force is determined as:

$$F_{f,C} = \sum_{i=1}^{N} f_{ci} P_{ci} dA_{ci} = f_C \sum_{i=1}^{N} P_{ci} dA_{ci} = f_C F_C$$

In Equation (I-17) it has been assumed that all the asperities have the same coefficient of friction.

Hence, the coefficient of friction is:

$$f = \frac{F_f}{F_T} = \frac{F_{f,H} + f_C F_C}{F_T}$$

The hydrodynamic friction force for Newtonian lubricant is calculated as:
\[ F_{\nu, H} = 2aB\mu \frac{u_{\text{sliding}}}{h_c} \]  \hspace{1cm} (1-19)

In this equation \( a \) is the Hertzian half-width of contact, \( B \) is the roller width, \( u \) is the sliding velocity, \( h_c \) is the film thickness, and \( \mu \) is the lubricant viscosity at the contact pressure calculated from Equation (I-3).

Another important issue in lubricated contact of rough surfaces such as gears is the wear. Wear causes the load distribution to change and increases vibration and noise. One of the aims of this research is to employ the load sharing concept to predict wear. The wear behavior during the running-in stage and the steady-state regime has different nature and needs to be addressed differently.

The load sharing concept and the algorithm to evaluate the scaling factors \( \gamma_1 \) and \( \gamma_2 \) is explained in more details in Chapter 1. Chapter 1 also covers the isothermal analysis of gears for lubricants with shear thinning behavior. In Chapter 2, thermal analysis has been incorporated with the load sharing concept. The predicted friction coefficient for contact of two rollers is compared to published experimental data. A numerical algorithm to generate the surface roughness is developed in Chapter 3 which enables us to analyze the effect of different surface patterns on the performance of lubricated contact of rough surfaces. This method enables us to study the effect of different finishing procedures. Chapter 4 discusses the formulation to predict the steady state sliding wear based on thermal desorption model. Running-in problem is addressed based on the plastic failure of the asperities in Chapter 5 and the proposed model is validated by comparing the predicted variation of arithmetic average of asperity heights with time to the experimentally measured values. Using this method, the running-in behavior of contacting surfaces can be determined. The Gear Test Machine (GTM) that was built in Center for Rotating Machinery (CeRoM) is described in Chapter 6. The conclusions and future steps are discussed in Chapter 7.
I.3 References


Chapter 1: Performance of Spur Gears Considering Surface Roughness and Lubricants with Shear Thinning Behavior*

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1.1 Introduction

A gear is used to transmit force between two parts of the same machine or between two devices, often with a mechanical advantage that allows increasing or decreasing the rotational speed or torque from one shaft to another. As is commonly the case in the analysis of most tribological components, the key parameters of interest in gears are the lubricant film thickness, the dimensionless film parameter $\Lambda$, and the coefficient of friction. The first two parameters are important in terms of reliability and damage, while the third parameter is a measure of the efficiency of the gear set in terms of the required power.

It is well known that lubrication regime in gears is governed by mixed or partial elastohydrodynamic (EHL) regimes—a subject that has captured the attention of many researchers over the last four decades. Gear surfaces are typically much rougher than those of shaft/bushing surfaces, making it necessary to take surface roughness into consideration. Johnson et al. [1] proposed a theory of asperity contact in elastohydrodynamic lubrication. They combined the Greenwood-Williamson model of rough surfaces with EHL theory and introduced the load sharing concept. Later Patir and Cheng [2] solved the average Reynolds equation for rough surfaces. They studied both isotropic surfaces and surfaces with directional patterns. Lee [3] proposed an analytical model and conducted some experiments on the scuffing of rollers under elastohydrodynamic lubrication. Hua and Khonsari [4] solved the transient EHL equation in spur gears assuming the surfaces are smooth. Chang [5] proposed a model for partial EHL and considered the asperities to be frictionless. Later, Flodin and Andersson [6] studied wear in spur and helical gears assuming dry contact. Chapkov et al. [7] have recently proposed a model to predict roughness amplitude reduction in both Newtonian and non-Newtonian EHL contacts.

Their model includes the influence of surface roughness wavelength and contact operating conditions and predicts the deformed shape. They showed that surfaces with short wavelength will slightly deform and therefore there is a high probability of asperity contact, while surfaces
with longer wavelength will be deformed more and therefore less asperity contact will occur. The majority of pure mineral oils of similar molecular size exhibit Newtonian behavior, where their viscosities are independent of shear rate. Nevertheless, there are practical applications where the lubricant’s viscosity varies with the rate of shear. For example, the so-called shear thinning lubricants experience a drop with increasing shear rate \[8\]. An excellent experimental application of shear fluids for lubrication of two cylindrical rollers is reported by Dyson and Wilson \[9\]. They showed that their reported measurement of film thickness is significantly lower than what the well-established EHL film thickness formula predicts. Bair and Khonsari \[10\] showed that some gear oils show shear thinning behavior and compared the analytical and experimental flow curve. Since shear thinning lubricants are becoming more popular in some industrial applications, their behavior is studied in this research. To account for shear thinning effect, Bair \[11\] recently proposed a correction factor for predicting the film thickness in an EHL line contact in the form of \(\phi = \frac{h_N}{h_{NN}}\), where \(h_N\) is the Newtonian film thickness and \(h_{NN}\) is the actual film thickness for the lubricant. The expression for \(\phi\) is a function of the slide to roll ratio, lubricant properties, velocity and power-law exponent.

In this paper we apply the Johnson’s load-sharing concept to predict the performance of a spur gear. The surface properties are amongst the inputs to the model, which is capable of taking shear thinning effect into consideration. Once the values of film thickness along the line of action are obtained, the fluid friction force and hence the friction coefficient can be easily predicted.

1.2 Model

One of the most important issues in gear sets is the friction coefficient. The aim of this research is to present a model that takes into consideration the surface properties (i.e., spectral moments of profile) and type of lubricant as well as gear geometry and the loading condition as the input and predicts the film thickness and the friction coefficient along the line of action.
Surfaces profiles are described either by spectral moments \((m_0, m_2 \text{ and } m_4)\) or by determining the asperity density \(D_{sum}\), the radius of asperities \(\beta\) and the standard deviation of asperity height \(\sigma_s\). These parameters are related to the spectral moments of the surface through the following equations [12]. If \(z(x)\) is the profile in an arbitrary direction \(x\) and \(E[\ ]\) denotes the statistical expectations, then the zeroth, second and fourth profile moments are:

\[
m_0 = E(z^2) = \sigma^2
\]

\[
m_2 = E \left( \left( \frac{dz}{dx} \right)^2 \right)
\]

\[
m_4 = E \left( \left( \frac{d^2z}{dx^2} \right)^2 \right)
\]

For an isotropic surface, the asperity density, the average radius of the spherical caps of the asperities, and the standard deviation of asperity heights can be calculated from [12]:

\[
D_{sum} = \frac{m_4}{6\pi\sqrt{3}m_2}
\]

\[
\beta = \frac{3}{8} \sqrt{\frac{\pi}{m_4}}
\]

\[
\sigma_s = \sqrt{m_0}
\]

If the surface is anisotropic, the values of \(m_2\) and \(m_4\) will vary with the direction in which the profile is taken on the surface. The maximum and minimum values for \(m_2\) and \(m_4\) occur in two orthogonal principal directions. The use of an equivalent isotropic surface is recommended for which \(m_2\) and \(m_4\) are computed as harmonic mean of \(m_2\) and \(m_4\) along the principal directions.

In this model, the contact of gear teeth at each point along the line of action is represented by the contact of two cylinders having radii \(R_p\) and \(R_g\). The radii of these cylinders vary along the line of action. Figure 1.1 shows how two cylinders replace the contact of pinion and gear. In this figure, \(\psi\) represents the pressure angle and \(CD\) is the line tangent to the base circles of the pinion and gear. The points of contact in spur gears are always along this line, hence commonly referred to as the line of action (LoA). The positions of contact points are determined by their coordinates.
and variation of different parameters such as the Hertzian pressure, equivalent curvature, film thickness and the friction coefficient are evaluated along LoA.

Referring to Figure 1.1, in order to analyze the pinion and gear contact, LoA is divided to several segments and for each point on this line the radii of curvature of pinion and gear are calculated using following relations.

At the pitch line, the radius of curvature for pinion is:

$$\rho = \frac{d_{wp} \sin \alpha}{2}$$  \hspace{1cm} (1.7)

At any other diameter, such as \(d_1\), the radius of curvature of pinion is:

$$\rho_1 = \frac{d_1 sin \phi_1}{2}$$  \hspace{1cm} (1.8)

The angle \(\phi_1\) can be found from the relation:

$$\cos \phi_1 = \frac{d \cos \alpha}{d_1}$$  \hspace{1cm} (1.9)

The radius of curvature for gear is:

$$\rho_2 = \frac{d_{wp} + d_{wg}}{2} \sin \alpha - \rho_1$$  \hspace{1cm} (1.10)

In spur gears the load carried by each tooth varies along LoA since the number of tooth in contact changes along this line. This variation of load is shown in Figure 1.2. In derivation of this plot, it was assumed that the gear tooth stiffness changes along the tooth profile. Under this condition, it cannot be assumed that the load is equally shared among the pairs of teeth in contact since it is a statically indeterminate case [13]. This type of variation, however, does not include the effect of vibration nor does it have the effect of tooth modification. The effect of tooth modification on load distribution in gear tooth has been shown in [14]. In Figure 1.2, from point A —i.e. the beginning of mesh—until point B, there are two pairs of teeth in contact so the amount of carried load changes from one third to two thirds of the total load \(F\). From point B to point C, however, there is only one pair of tooth in contact and the transmitted force is equal to
From point C to point D, i.e. the end of the contact, the load changes from two thirds of $F$ to one third of $F$.

**Figure 1.1: Representation of pinion and gear with rollers.**

The variation of the Hertzian stress along LoA is given by the following equation:

$$P_{\text{Hertzian}} = \sqrt{\frac{F_T}{\pi B}} \left( \frac{\frac{1}{\rho_1} + \frac{1}{\rho_2}}{\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}} \right)$$  \hspace{1cm} (1.11)

**Figure 1.2: Variation of load along LoA.**

Having calculated the geometry of replacing cylinders and contact stresses, the problem now is to find the film thickness and coefficient of friction between two rollers with the known geometry and the loading condition. The analysis starts from the first point of contact and proceeds along LoA. At each point, the appropriate cylinders radii and the load are used.

The calculation of film thickness and friction coefficient for each point along LoA is based on Johnson’s concept of scaling factors for hydrodynamic part $\gamma_1$ and asperity contact part $\gamma_2$. That
is, the transmitted force $F_T$ is the sum of the load carried by the asperity, $F_C$, and the hydrodynamic load, $F_H$:

$$F_T = F_H + F_C$$  \hspace{1cm} (1.12)

By introducing the scaling factors, Equation (1.12) is written as:

$$F_T = rac{F_T}{y_1} + rac{F_T}{y_2}$$  \hspace{1cm} (1.13)

In a similar fashion the total friction force is composed of two components: the hydrodynamic friction force ($F_{f,H}$) and asperity friction force ($F_{f,C}$).

$$F_f = F_{f,H} + F_{f,C}$$  \hspace{1cm} (1.14)

The asperity friction force is determined as:

$$F_{f,C} = \sum_{i=1}^{N} f_{ci} P_{ci} dA_{ci} = f_c \sum_{i=1}^{N} P_{ci} dA_{ci} = f_c F_C$$  \hspace{1cm} (1.15)

In Equation (1.15) it has been assumed that all the asperities have the same coefficient of friction. Hence, the coefficient of friction is:

$$f = \frac{F_f}{F_T} = \frac{F_{f,H} + f_c F_C}{F_T}$$  \hspace{1cm} (1.16)

The hydrodynamic friction force for Newtonian lubricant is calculated as:

$$F_{f,H} = 2 a B \mu \frac{u_{sliding}}{h_c}$$  \hspace{1cm} (1.17)

In this equation $a$ is the Hertzian half-width of contact, $B$ is the roller width, $u$ is the sliding velocity, $h_c$ is the film thickness, and $\mu$ is the lubricant viscosity at the contact pressure. In this model, viscosity of lubricant changes with pressure according to Roeland’s equation [15]:

$$\log \mu + 1.2 = (\log \mu_0 + 1.2) \left(1 + \frac{P}{C}\right)^Z$$  \hspace{1cm} (1.18)

In this equation $\mu_0$ and $\mu$ are the viscosities of lubricant at the ambient pressure and at pressure $P$, both in mPa.s. The parameter $Z$ is the viscosity pressure index and for mineral oils is assumed to be 0.6 and value of $C$ is taken equal to 196.1 MPa.

The film thickness calculation is based on Moes’ equation [16] for central film thickness:
According to Johnston’s load sharing method, a portion of the load equal to \(\frac{1}{\gamma_1}\) is taken by fluid film. For calculating film thickness, the problem will be of two rollers with known geometry and modulus of elasticity of \(E'/\gamma_1\) being pressed together with a force equal to \(F_T/\gamma_1\). Rest of the load \((1/\gamma_2)\) is taken by asperities.

Therefore for calculating asperity contact pressure \(P_C\), the problem will be two rollers with known geometry and modulus of elasticity of \(E'/\gamma_2\) being pressed together by a force equal to \(F_T/\gamma_2\). Applying the Johnson’s concept of scaling factor and substituting \(E'/\gamma_1\) for \(E'\) and \(F_T/\gamma_1\) for \(F_T\), the Moes’ equation takes the following form:

\[
H_c = \left[ \left( H_{RI}^{7/3} + H_{EI}^{7/3} \right)^{3s/7} + \left( H_{RP}^{7/2} + H_{EP}^{7/2} \right)^{-2s/7} \right]^{1/s} (\gamma_1)^{1/2} \tag{1.22}
\]

where
\[ s = \frac{1}{5} \left( 7 + 8e^{-\frac{2(y_1)^2/5}{H_Rl}} \right) \] (1.23)

Equation (1.22) has two unknowns: \( y_1 \) and \( h_C \). In order to determine both of the unknowns, another equation is needed. This equation comes from making the asperity contact pressure obtained from Greenwood-Tripp model (Equation (1.25)) equal to Gelinck’s curve-fit (Equation (1.24)) [17]. Gelinck and Schipper [17] have shown that the central pressure is a good quantity to characterize the pressure distribution of a rough line contact.

In order to fit functions to this pressure distribution, they introduced velocity-independent parameters \( n' = D_{sum}R \sqrt{\beta R} \) and \( \sigma_s' = \sigma_s/R \) where \( D_{sum} \) represents the number of asperities per unit area, \( \beta \) denotes the radius of asperities, \( \sigma_s \) is the standard deviation of asperity heights.

After applying the Johnson’s concept of load sharing which leads to substituting \( E'/\gamma_2 \) for \( E' \), \( F_T/\gamma_2 \) for \( F_T \) and \( n\gamma_2 \) for \( \gamma_2 \), the following equation was fitted to the central contact pressure:

\[ P_C = \frac{1}{\gamma_2} \sqrt{\frac{F_T E'}{2\pi \beta R}} \left[ 1 + \left( 1.558 \left( D_{sum} R \sqrt{\beta R} \right)^{0.0337} \left( \frac{\sigma_s}{R} \right)^{-0.442} W^{0.4757} \right)^{-1.7} \right]^{-0.5882} \] (1.24)

The Greenwood-Tripp equation for contact pressure is:

\[ P_C = \frac{8\sqrt{2}}{15} \pi D_{sum}^2 \beta^{1.5} \sigma_s^{2.5} E' F_{5/2} \left( \frac{h_c - d_d}{\sigma_s} \right) \] (1.25)

The function \( F_{5/2} \) is basically defined as:

\[ F_{5/2}(H) = \frac{1}{\sqrt{2\pi}} \int_{H}^{\infty} (s - H)^{5/2} e^{-s^2/2} ds \] (1.26)

In EHL formulation, instead of \( H \), the difference between film thickness \( h_c \) and the distance between the mean plane through the summits and the mean plane through the heights of the surface \( d_d \) is used. That is:

\[ F_{5/2}(H) = \frac{1}{\sqrt{2\pi}} \int_{H}^{\infty} \left( s - \frac{h_c - d_d}{\sigma_s} \right)^{5/2} e^{-s^2/2} ds \] (1.27)

In Equation (1.27) \( d_d \) is approximately \( 1.15 \sigma_s \). To simplify the integration in Equation (1.27), we will use the polynomial of Equation (1.28) which is curve fitted to the \( F_{5/2} \) [18].
\[ F_{5/2}(H) = \begin{cases} 4.4086 \times 10^{-5} (4 - H)^{6.804} & \text{for } H < 4 \\ 0 & \text{for } H \geq 4 \end{cases} \quad (1.28) \]

Therefore, making Equation (1.24) equal to Equation (1.25):

\[
\frac{1}{\gamma_1} \sqrt{\frac{F_F E_f}{2 \pi R}} \left[ 1 + \left( 1.558 \left( D_{sum} R \sqrt{\beta R} \right)^{0.0337} \left( \frac{\sigma_s}{R} \right)^{-0.442} W^{0.4757} \right)^{-1.7} \right]^{-0.5882} =
\]

\[
\frac{8\sqrt{2}}{15} \pi D_{sum}^2 \beta^{1.5} \sigma_s^{2.5} E' F_{5/2} \left( \frac{h_c - d_a}{\sigma_s} \right) \quad (1.29)
\]

Equation (1.24) is a curvefit of Equation (1.25) but latter is a function of film thickness also.

So, the solution scheme is to choose an initial value for \( \gamma_1 \) and find the film thickness from Equation (1.22), plug the film thickness in Equation (1.24) and check if equations (1.24) and (1.25) are close enough. If not, then a new value is chosen and the loop continues until the convergence criterion is satisfied. Satisfaction of convergence criterion means that the value chosen for \( \gamma_1 \) and consequently \( \gamma_2 \) are such that equations (1.24) and (1.25) are close enough.

This procedure is a general method and is independent of type of lubricant. Hence, if the lubricant has shear thinning behavior, a similar procedure is used and the calculated film thickness is corrected using the Bair correction factor [11]. Since this correction factor \( \phi \) is a function of the slide to roll ratio and velocity in addition to lubricant properties, its variation along LoA should be considered. In results section, the behavior of a shear thinning lubricant is investigated.

### 1.3 Numerical Simulations Procedure

The numerical simulation procedure starts with calculating the surface properties. In other words, asperity radius, asperity density and the rms of the surface should be known. If these parameters are not provided and instead the spectral moments of the surface are given, then equations (1.4B)-(1.6B) will be used to calculate the surface properties from spectral moments. For each point along LoA, the radii of curvature are calculated using equations (1.7B)-(1.10B). These radii are the radii of the replacing cylinders. Equation (1.11) is used to calculate the Hertzian stress between
contacting cylinders along the line of action. So far, for all the points along LoA the radii of cylinders and the contact stress are known. Then a value for $\gamma_1$ is chosen and film thickness is calculated using Equation (1.22) and non-dimensional parameters of Equation (1.21). Then both sides of Equation (1.29) are calculated. If the difference between right and left side of Equation (1.29) divided by right hand side is larger than a specified tolerance error $\varepsilon$ (e.g., $1 \times 10^{-5}$), then a new value for $\gamma_1$ is assumed and the calculations are repeated until the convergence criteria is satisfied. Once the film thickness is known, Equation (1.17) is used to calculate the hydrodynamic friction force. Finally, Equation (1.16) is employed to get the value for friction coefficient.

As presented in the discussion section, the model has the ability to predict the gear performance when shear thinning lubricant is used. The corrections to the film thickness and hydrodynamic friction force are given in equations (1.31) and (1.32).

In our simulation, the line of action is divided to 4000 points and typical execution time for this number of points is around one hour on a Pentium 4 computer with CPU of 1.8 GHz. The convergence criterion was chosen to be $\varepsilon = 1 \times 10^{-5}$ meaning that when the right hand side of Equation (1.29) minus the left hand side divided by right hand side gets smaller than this number the iteration will stop.

1.4 Results and Discussions

1.4.1 Verification Tests

The model was applied to a set of gear data and loading conditions which were obtained from literature [4] for verification purposes. Table 1.1 shows the geometrical properties and loading conditions of pinion and gear. In [4] the lubricant was Newtonian and the surfaces were assumed to be smooth. For comparison of current simulation and the original paper [4], the following surface properties were selected to represent a very smooth surface [17]. It should be noted that surface properties are obtained from spectral moments of the surface and they are not
independent of each other. In other words, the roughness profile and its first and second
derivatives are used to evaluate these parameters.

The active length of LoA was divided to 4000 points and the aforementioned calculations
were done for each point. The variation of equivalent radius of curvature calculated from
equations (1.7) to (1.10) is shown in Figure 1.3.

**Table 1.1: Pinion and gear data [4]**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of pinion teeth</td>
<td>(z_p=28)</td>
</tr>
<tr>
<td>Number of gear teeth</td>
<td>(z_g=84)</td>
</tr>
<tr>
<td>Module</td>
<td>(m=0.003175) m</td>
</tr>
<tr>
<td>Pinion pitch diameter</td>
<td>(d_{wp}=0.0889) m</td>
</tr>
<tr>
<td>Pinion rotational speed</td>
<td>(\omega=1637) rpm</td>
</tr>
<tr>
<td>Gears width</td>
<td>(B=0.1) m</td>
</tr>
<tr>
<td>Load per unit width</td>
<td>(F_T=376.5) KN</td>
</tr>
<tr>
<td>Pressure angle</td>
<td>(\alpha=20) deg</td>
</tr>
<tr>
<td>Oil viscosity</td>
<td>(\mu=0.065) Pa.s</td>
</tr>
<tr>
<td>Viscosity-pressure index</td>
<td>(Z=0.6)</td>
</tr>
</tbody>
</table>

**Table 1.2: Surface properties [17]**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS of the surface</td>
<td>(\sigma_s=0.05) (\mu m)</td>
</tr>
<tr>
<td>Density of asperity</td>
<td>(D_{sum}=1.0 \times 10^{11}) (m^{-2})</td>
</tr>
<tr>
<td>Radius of asperity</td>
<td>(\beta=10.0) (\mu m)</td>
</tr>
</tbody>
</table>

In the following figures, the abscissa is the coordinate along the line of action i.e. the
coordinate of the point on pinion tooth that comes into contact. The negative part of the axis
refers to pinion dedendum and the positive part refers to pinion addendum. Hence, the largest
negative coordinate shows the beginning of mesh and largest positive point refers to end of mesh
for pinion tooth. Figure 1.4 shows the variation of transmitted load along LoA. At the first point
of contact (A) the transmitted load is shared by two pair of teeth in contact until the contact point
reaches point (C). Thereafter, until the contact point reaches (D), there is only one pair of tooth
in contact. From point (D) to point (B), there will, again, be two pair of teeth in contact.
Figure 1.3: Variation of equivalent radius of curvature along LoA.

The variation of the Hertzian stress along LoA is shown in Figure 1.5. The lowest point of single tooth contact experiences the largest contact stress. The film thickness from the present simulation and the film thickness as calculated in [4] are shown in Figure 1.6.

Figure 1.4: Variation of load along LoA.

Figure 1.5: Variation of contact stress along LoA.
The film thickness comparison shows a good agreement between this model and the work of Hua and Khonsari [4] who solved the EHL problem by direct solution of the Reynolds equation. The differences can be related to the fact that in [4] the surfaces were assumed to be perfectly smooth but in our simulation the surface roughness is considered. Probably that is why in pinion addendum, where the film thickness is larger and the effect of surface roughness becomes less pronounced, the results are closer. However, as shown in Table 1.2, the surface properties used for this comparison are selected such that the surface roughness is very small.

Next, we present simulation results corresponding to an experimentally measured friction coefficient. For this purpose, the experimental results of Lee [3] were selected. Lee’s experiment was done with a twin disk machine. The test rig had a continually variable speed motor and a set of gears which were used to provide different sets of roll to slide ratio. The experiment was designed for scuffing in heavily loaded EHL contacts and as part of his experiment, Lee measured the friction coefficient. The test conditions of Lee’s experiment are shown in Table 1.3.

**Table 1.3: Test conditions [3]**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smaller roller radius</td>
<td>$R_p = 13.97$ mm</td>
</tr>
<tr>
<td>Larger roller radius</td>
<td>$R_g = 55.88$ mm</td>
</tr>
<tr>
<td>Crown radius of large roller</td>
<td>$R_{crown} = 88.9$ mm</td>
</tr>
<tr>
<td>Rolling velocity</td>
<td>$v_{rol} = 3.83$ m/s</td>
</tr>
<tr>
<td>Sliding velocity</td>
<td>$v_{slide} = 3$ m/s</td>
</tr>
<tr>
<td>------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>Sliding to rolling ratio</td>
<td>$\Sigma = 0.7826$</td>
</tr>
<tr>
<td>Oil viscosity at operating temperature</td>
<td>$\mu = 0.063$ Pa.s</td>
</tr>
<tr>
<td>Rms of surface roughness</td>
<td>$\sigma_s = 0.374 \times 10^{-6}$ m</td>
</tr>
</tbody>
</table>

The surface properties that are used in our model are shown in Table 1.4. The parameter $\sigma_s$ is assumed to be exactly the same as standard deviation of asperity heights of the disks used in the experiment. Density of asperities and radius of asperity tip, however, are obtained from [17]. Coefficient of friction between asperities is a function of material and surface properties and is usually determined by experiments being conducted in boundary lubrication regime between two rollers. For a wide range of surface properties from pretty smooth to fairly rough surfaces, the literature give values from 0.10 to 0.13 for asperity friction coefficient. For this case, the variation from 0.1 to 0.13 for asperity friction coefficient will not influence the friction coefficient considerably and almost the same accuracy will be maintained. The reason for that is the large contribution of fluid in taking the load. Figure 1.7 shows the comparison between the present model and experiment for a range of loads.

Table 1.4: Surface properties [17]

<table>
<thead>
<tr>
<th>Density of asperities</th>
<th>$D_{sum} = 4 \times 10^{10}$ (1/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of asperity tip</td>
<td>$\beta = 10 \times 10^{-6}$ m</td>
</tr>
<tr>
<td>RMS of the surface</td>
<td>$\sigma_s = 0.374 \times 10^{-6}$ m</td>
</tr>
<tr>
<td>Coefficient of friction between asperities</td>
<td>$f_c = 0.1$</td>
</tr>
</tbody>
</table>

The differences seen in the plot can be attributed to the manner in which the experiment was conducted. Initially, the rollers were pressed together with a force equal to 425 N. The test was run for 5 minutes, the friction coefficient and surface temperatures of the disks were collected and then the normal load was increased and test continued for another 5 minutes. Our simulation is not a function of time and does not consider the effect of running-in process.
1.4.2 Shear Thinning Simulations

Having verified the model for Newtonian lubricant, next step is to check its validity for non-Newtonian lubricants such as shear thinning. These lubricants which are sometimes used by some industries exhibit shear thinning behavior as illustrated in Figure 1.8. At very low and very high shear rates, the lubricant behavior is the same fashion as a Newtonian fluid and in intermediate region it drops linearly with increase in shear rates. In order to be able to predict the gear performance with shear thinning lubricant, some corrections have to be made. First, the film thickness needs to be modified. A correction factor called $\phi$ was proposed by Bair [11] which is the ratio of Newtonian film thickness to shear thinning film thickness.

![Figure 1.7: Comparison model results with experimental data [3].](image)

![Figure 1.8: Characteristics of shear thinning lubricant](image)

This factor is a function of sliding to rolling ratio ($\Sigma$) and Weissenberg number ($\Gamma = \frac{\mu_0 u}{h_C G_C}$) where $u$ represents the rolling velocity, $G_C$ denotes the critical stress, $h_N$ is the...
central film thickness based on the Newtonian lubricant assumption calculation and \( \mu_0 \) is the low-shear viscosity at ambient pressure:

\[
\phi = \frac{h_N}{h_{NN}} = \left\{ 1 + 0.79 \left[ (1 + \Sigma) \Gamma \right]^{1+6.25} \right\}^{3.6(1-n)^{1.7}} \tag{1.30}
\]

For a shear thinning lubricant, the hydrodynamic friction force \( F_{f,H} \) is calculated using the Carreau’s equation [20]:

\[
F_{f,H} = 2aB \dot{\gamma} \left[ \mu_2 + (\mu_1 - \mu_2) \left( 1 + \left( \frac{\mu_1 \dot{\gamma}}{\mu_2} \right)^2 \right)^{\frac{m-1}{2}} \right] \tag{1.31}
\]

In this equation, \( \mu_1 \) and \( \mu_2 \) are the first and second Newtonian viscosities, \( B \) is the width of the rollers, \( a \) is the half width of contact, \( m \) is the power-law exponent and \( \dot{\gamma} \) shear rate is defined as the ratio of the difference in speeds of the two rollers to the film thickness \( h_c \):

\[
\dot{\gamma} = \frac{u_{sliding}}{h_c} \tag{1.32}
\]

Figure 1.9 shows the film thickness for shear thinning lubricant. For comparison, Newtonian film thickness for the same case is plotted in the same figure. The geometry and loading conditions are the same as Table 1.1. For this lubricant \( m=0.4 \) and \( G_c=1 \) MPa and the surface properties are derived from Table 1.5.

<table>
<thead>
<tr>
<th>Density of asperities</th>
<th>( D_{sum}=3.31\times10^2 ) (1/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of asperity tip</td>
<td>( \beta=6.02\times10^{-6} ) m</td>
</tr>
<tr>
<td>RMS of the surface</td>
<td>( \sigma_s=0.7\times10^{-6} ) m</td>
</tr>
<tr>
<td>Coefficient of friction between asperities</td>
<td>( f_c=0.12 )</td>
</tr>
</tbody>
</table>

The results predict that the film thickness tends to increase along the line of action. In the region of single tooth contact, there is a higher contact stress which results in a decrease in the film thickness. Around the pitch point when the effect of sliding becomes minor, the Newtonian and shear thinning film thickness become closer. Also, at the pitch point there is a change in the
The slope of film thickness. This change is due to the behavior of the correction factor which is a function of slide/rolling ratio and Weissenberg number. Figure 1.10 shows the variation of Bair correction factor along the line of action.

![Figure 1.9: Variation of film thickness along LoA.](image)

![Figure 1.10: Variation of φ along LoA](image)

As shown in Figure 1.10, as the contact point moves toward the pitch point, sliding decreases and the value of Bair correction factor $\phi$ decreases. As the contact point moves away from pitch point, the correction factor increases.

### 1.4.3 Variation of Film Parameter and Surface Roughness

The dimensionless film parameter which is the ratio of film thickness to the surface roughness ($\Lambda = h_c/\sigma_s$) is of great importance in tribology. The variation of this parameter along the line of action is plotted in Figure 1.11. The film parameter is a useful parameter for determining the
severity of load and the lubrication regime. It is generally believed that $\Lambda < 1$ corresponds to boundary lubrication, whereas partial or mixed-film lubrication occurs when $1 < \Lambda < 3$ and for full elastohydrodynamic lubrication $\Lambda > 3$ [15]. According to Figure 1.11, the lubrication regime for contact points in the pinion dedendum is boundary while in pinion addendum it is mixed.

![Figure 1.11: Variation of film parameter ($\Lambda$) along LoA.](image)

**1.4.4 Friction Coefficient**

Friction coefficient is a function of geometry, loading and the lubricant and determines the required torque (and power). Hence, this variable is a key parameter in performance of a gear set. Figure 1.12 plots the variation of friction coefficient along the line of action for a Newtonian lubricant and a shear thinning lubricant. The input data such as lubricant properties, geometry and loading are taken from Table 1.1. For comparison, it was assumed that the first Newtonian viscosity for shear thinning lubricant is the same as the viscosity of the other lubricant and the second Newtonian viscosity is zero. For Newtonian lubricant, the friction coefficient is highest at first point of contact and drops as contact point moves toward pitch point and sliding decreases. At the lowest point of single tooth contact, there is a sudden increase in friction coefficient which is due to shift from two pairs of teeth to one pair of tooth in contact. At pitch point, where there is pure rolling, the hydrodynamic friction force is zero and there is only asperity friction force. A
sudden decrease is seen in the point of shift from one pair of tooth to two pairs of teeth in contact and after that the friction coefficient remains almost constant. Shear thinning lubricant shows the similar behavior at points when number of tooth in contact changes and also at pitch point. This lubricant shows a smaller friction coefficient in comparison to Newtonian lubricant.

![Figure 1.12: Comparison of friction coefficient between Newtonian and shear thinning for $\sigma_s = 0.1 \times 10^{-6} m$.](image)

To illustrate the effect of surface roughness on performance of gear, the model was simulated for two other surface roughness parameters of $\sigma_s = 0.4 \times 10^{-6} m$ and $\sigma_s = 0.7 \times 10^{-6} m$. The results are shown in figures 1.13 and 1.14. As shown in Figure 13, the friction coefficient for both lubricants has a similar trend as in smoother case (Figure 1.12), except that there is a slight increase in value of friction coefficient. This increase is due to use of rougher surfaces, in this case more asperities come into contact and they carry a larger portion of load. Figure 1.14 compares the friction coefficient for even a rougher surface where $\sigma = 0.7 \times 10^{-6} m$. The value of the friction coefficient comparing to the other two smoother cases is higher. For smoother cases, asperity friction force is negligible and most of the load is carried by the fluid film. When the surface becomes rough, the scaling factor for the asperity part becomes smaller which means that more asperity contact will occur. Therefore the asperity friction force will become the dominant part and the friction coefficient increases.
Figure 1.13: Comparison of friction coefficient between Newtonian and shear thinning for $\sigma_s = 0.4 \times 10^{-6} m$

Figure 1.14: Variation of friction coefficient along the line of action and comparison with Newtonian lubricant for $\sigma_s = 0.7 \times 10^{-6} m$

One of the outputs of the model is the scaling factors. That is what percentage of load is taken by fluid film and what percentage is taken by asperities. For the case in Figure 1.14, the percentage of load taken by fluid film and asperities is shown in Figure 1.15. In the dedendum,
the asperities take a larger portion of load. As the contact point moves along the line of action, the film thickness increases and larger portion of load is carried by fluid film. The points of shift from two pair of teeth to one pair of tooth can also be seen in Figure 1.15.

Figure 1.15: Scaling factors along LoA for $\sigma_s = 0.7 \times 10^{-6}m$

1.5 Conclusions

In this chapter, a useful approach for predicting the film thickness and friction coefficient of spur gears with consideration of surface roughness and provision for lubricant with shear thinning characteristic is reported. The asperity density, asperity radius and rms of the surface are either among the inputs to the model or the model calculates those using spectral moments of the surface.

The model employs the Johnson’s concept of the load sharing and applies the Moes’ equation for calculating the Newtonian central film thickness. The model can also be applied for non-Newtonian lubricants such as shear thinning lubricants. In this case, the film thickness is corrected by using Bair’s correction factor for shear thinning lubricants. Having calculated the film thickness, the model then uses Carreau’s equation for calculating hydrodynamic friction force. The main advantages of this model is that i) the model does not require solving the full EHL equations ;ii) it generates reasonable results for rough surfaces; and iii) the results are in
acceptable agreement with experimental data. This model can be used as a rapid prediction for the gear performance since the code without need to solve the full EHL equations generates acceptable results.

Using the shear thinning lubricant instead of Newtonian will result in lower film thickness. At the beginning of contact, due to large sliding the film thickness is much smaller than Newtonian film thickness. Around the pitch line where there is small sliding the Newtonian and the shear thinning film thickness become closer.

The surface roughness obviously affects the friction coefficient. As the surface roughness increases, more asperities come into contact and larger portion of load is taken by asperities. Therefore, the friction coefficient will increase.

Comparing Newtonian and shear thinning lubricant indicates that shear thinning lubricant shows smaller friction coefficient. However, as the surface roughness increases, the difference between the friction coefficients of the two lubricants decreases. This decrease is due to the fact that for rougher surfaces, more asperities come into contact and carry larger portion of load. Hence, the asperity friction force becomes dominant term.

In the next chapter, the effect of thermal analysis on the current formulation is studied. A simplified form of energy equation along with the current model is solved to predict the temperature of the contacting surfaces and the lubricant. Including the simplified energy equation, affects the predicted friction coefficient and film thickness especially at high values of sliding speed.

1.6 Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>half width of Hertzian contact, m</td>
</tr>
<tr>
<td>$B$</td>
<td>gear width, m</td>
</tr>
<tr>
<td>$d_d$</td>
<td>distance between mean line of asperities and mean line of surface, m</td>
</tr>
<tr>
<td>$d_{wp}$</td>
<td>pinion pitch diameter, m</td>
</tr>
<tr>
<td>$R_p$</td>
<td>radius of roller p, m</td>
</tr>
<tr>
<td>$R_g$</td>
<td>radius of roller g, m</td>
</tr>
<tr>
<td>$u_{roll}$</td>
<td>rolling velocity $(u_1 + u_2)/2$, m/s</td>
</tr>
<tr>
<td>$u_{sliding}$</td>
<td>Sliding velocity, $u_2 - u_1$, m/s</td>
</tr>
<tr>
<td>$v$</td>
<td>Poisson ratio</td>
</tr>
<tr>
<td>$D_{sum}$</td>
<td>Density of asperities ($1/m^2$)</td>
</tr>
<tr>
<td>----------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td>$E'$</td>
<td>Equivalent Young modulus, N/m²</td>
</tr>
<tr>
<td>$F_T$</td>
<td>transmitted force, N</td>
</tr>
<tr>
<td>$F_H$</td>
<td>load carried by fluid, N</td>
</tr>
<tr>
<td>$F_C$</td>
<td>load carried by asperity, N</td>
</tr>
<tr>
<td>$F_{f,H}$</td>
<td>hydrodynamic friction force, N</td>
</tr>
<tr>
<td>$F_{f,C}$</td>
<td>friction force from asperity interaction, N</td>
</tr>
<tr>
<td>$f$</td>
<td>coefficient of friction</td>
</tr>
<tr>
<td>$f_c$</td>
<td>friction coefficient between asperities</td>
</tr>
<tr>
<td>$G_c$</td>
<td>Critical stress, Pa</td>
</tr>
<tr>
<td>$h_c$</td>
<td>central film thickness</td>
</tr>
<tr>
<td>$h_N$</td>
<td>Newtonian film thickness, m</td>
</tr>
<tr>
<td>$h_{NN}$</td>
<td>non-Newtonian film thickness, m</td>
</tr>
<tr>
<td>$m$</td>
<td>Power-law exponent</td>
</tr>
<tr>
<td>$\omega$</td>
<td>rotational speed, rpm</td>
</tr>
</tbody>
</table>

1.7 References


Chapter 2: Thermoelastohydrodynamic Analysis of Spur Gears*

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2.1 Introduction

In the previous chapter, the isothermal analysis of spur gears was presented. In gears as the contact point moves along the line of action, the rolling and sliding speed vary. Consequently, the temperature of surfaces and the film temperature change which affects the viscosity of the lubricant.

In what follows, some of the pertinent papers of interest to this study are reviewed. Sadeghi and Sui [1] solved thermal EHL in rolling/sliding line contacts by simultaneously solving the thermal Reynolds and energy equation. Later [2] they added the effect of surface roughness to their simulations. They calculated temperature distribution and friction coefficient for different values of surface roughness. Of interest in these papers was the lubrication of rolling element bearings. In applications that involve gears, surfaces are typically about two orders of magnitude rougher than in ball bearings.

Hua and Khonsari [3] solved the thermal EHL for steady state line contact. They showed that their prediction for friction coefficient for a range of slide-to-roll ratios is in very good agreement with experimental data. Later [4] they solved the transient EHL problem in spur gears. Larsson [5] solved the transient EHL problem for non-Newtonian lubricant. He showed that the transient effect is most pronounced in the points where the shift in number of tooth in engagement occurs. In all three of these references, simulations pertained to perfectly smooth surfaces only.

Hsu and Lee [6] also solved the thermal EHL for rolling/sliding line contacts for smooth surfaces. They developed a correlation formula of thermal reduction factor for film thickness. This factor gives the ratio of thermal film thickness to isothermal film thickness as a function of slip ratio, load, thermal loading parameter and materials parameter.

A common feature of most of the existing papers is treatment of TEHL problem by solving a simultaneous set of equations involving the fluid pressure, surface elasticity, and energy
equation. The solution algorithms that can accomplish this objective are often time consuming and tedious to implement. For characterization of the performance of gears, TEHL solutions for rough surfaces are needed at hundreds of points along the active profile of the line of action (LoA). This fact was the motivation for the authors to develop an efficient and powerful method to treat the problem of TEHL in gears.

The approach chosen utilizes the load sharing concept presented by Johnson et al. [7]. Gelinck and Schipper [8] extended the load-sharing concept to solve a EHL line-contact problem. Based on their approach, the contact of two surfaces is decomposed into two problems. A portion of load is shared by fluid film and the rest is taken by asperities. Scaling factors $\gamma_1$ and $\gamma_2$ represent inverse of portion of load carried by fluid film and asperities accordingly. Lu et al. [9] applied this method to journal bearings and the predicted value for friction coefficient was in excellent agreement with their experiments. Akbarzadeh and Khonsari [10] applied this method to steady state isothermal analysis of spur gears. The predicted value for film thickness and friction coefficient was in good agreement with previous published works. The aim of the present paper is to extend the analysis to predict the performance of spur gears with rough surfaces with considering thermal effect.

2.2 Model

The objective of this work is to develop a model for predicting the film thickness, temperature of surfaces of pinion and gear, lubricant temperature and friction coefficient. Surface roughness effects are to be taken into consideration by incorporating the load sharing concept proposed by Johnston et al. [7].

As shown in Figure 2.1, the contact of pinion and gear along LoA is replaced by the contact of a pair of rollers with varying radii of curvature under varying load to mimic the behavior of the gear-pinion system. Change in radii of cylinders as the contact point moves along LoA has been shown in schematics (a) and (b) in Figure 2.1.
Figure 2.1: Replacing contact of pinion and gear with rollers

The central film thickness between two rollers is evaluated using Moes’ equation [11]:

\[ H_c = \left[ \left( H_{RI}^7 + H_{EI}^7 \right)^{\frac{38}{7}} + \left( H_{RP}^{\frac{7}{2}} + H_{EP}^{\frac{7}{2}} \right)^{\frac{1}{2}} \right] \]

(2.1)

where

\[ s = \frac{1}{5} \left( 7 + 8e^{\frac{2H_{EI}}{H_{RI}}} \right) \]

(2.2)

The dimensionless parameters used are defined as below [8]:

\[ H_{RI} = 3(WU_\Sigma^{-0.5})^{-1} \]
\[ H_{EI} = 2.621(WU_\Sigma^{-0.5})^{-0.2} \]
\[ H_{RP} = 1.287(GU_\Sigma^{-0.25})^{2/3} \]
\[ H_{EP} = 1.311(WU_\Sigma^{-0.5})^{-1/8}(GU_\Sigma^{-0.25})^{3/4} \]

(2.3)

\[ W = \frac{F_T}{E_p R'B} \]
\[ G = \alpha E_p \]
\[ U_\Sigma = \frac{\eta_0 u}{E_p R'} \]
\[ h_c = \frac{h}{R'} U_\Sigma^{-0.5} \]

2.2.1 Load-Sharing Concept

The resulted film thickness relationship provided in the previous section is intended for “perfectly smooth” surfaces and does not include the effect of surface roughness. To incorporate
roughness, Johnson’s load-sharing concept is utilized. According to this method, prediction of the film thickness between two rollers with known geometry involves replacing the equivalent Young modulus with $E'/\gamma_1$ and the load with $F_T/\gamma_1$. Similarly, for calculating asperity contact pressure $P_c$, the Young modulus of $E'/\gamma_2$ and a force equal to $F_T/\gamma_2$ is used. Substituting $E'/\gamma_1$ for $E$ and $F_T/\gamma_1$ for $F_T$, the Moes’ equation takes the following form:

$$H_c = \left[ \gamma_1^{3/2} \left( H_{RI}^{7/3} + \gamma_1 \left( H_{EI}^{7/3} \right) \right)^{3s/7} + \gamma_1^{-5/2} \left( H_{RP}^{-7/2} + H_{EP}^{-7/2} \right)^{-2s/7} \right]^{1/5} (\gamma_1)^{1/2} \quad (2.4)$$

where

$$s = \frac{1}{5} \left( 7 + 8e \left( \frac{-2(\gamma_1)^{-2/s}H_{EI}}{H_{RI}} \right) \right) \quad (2.5)$$

Equation (2.4) has two unknowns: $\gamma_1$ and $h_c$. In order to determine both of the unknowns, another equation is needed. In order to provide the second equation, Gelinck and Schipper [12] developed a curvefit based on Greenwood-Williamson formula that related the contact pressure between asperities to surface properties and load. Since in the case of gears both surfaces are rough, in this study the formulation proposed by Greenwood-Tripp [13] was used and a new curvefit relationship was developed. For this purpose, the Greenwood-Tripp formula for contact of two rough surfaces along with the separation equation and load balance equation were solved simultaneously and a curvefit relationship relating the contact pressure to load and surface properties was developed. The solution procedure is explained in the next section.

### 2.2.2 Rough Line Contact

According to Greenwood and Tripp [13], the total pressure carried by asperities in contact of two rough surfaces is:

$$p(x) = \frac{8\sqrt{2}}{15} \pi n^2 \beta \sigma^{2.5} F_5 \frac{h(x)}{\sigma} \quad (2.6)$$

The function $F_{5/2}(h(x)/\sigma)$ is defined as:
\[ F_{5} \left( \frac{h(x)}{\sigma} \right) = \frac{1}{\sqrt{2\pi}} \frac{1}{\frac{h}{\sigma}} \int_{-\infty}^{\infty} \left( s - \frac{h(x)}{\sigma} \right)^{2.5} e^{-s^{2}/2} \]  
(2.7)

where \( h(x) \) represents the separation between the two surfaces. It is defined as:

\[ h(x) = h_0 + \frac{x^2}{2R} + w(x) \]  
(2.8)

where \( R \) is the equivalent radius of the curvature and \( w(x) \) is the elastic deformation of the surfaces. According to Timoshenko and Goodier [14]:

\[ w(x) = \frac{-4}{\pi} \int_{-\infty}^{\infty} p(s) \ln(|x - s|^2) ds + w_0 \]  
(2.9)

After inserting Equation (2.9) in Equation (2.8), separation will be of the form:

\[ h(x) = h_{00} + \frac{x^2}{2R} - \frac{4}{\pi} \int_{-\infty}^{\infty} p(s) \ln(|x - s|^2) ds \]  
(2.10)

Also the total load is equal to:

\[ P = B \int_{-\infty}^{\infty} p(x) dx \]  
(2.11)

Next, the following non-dimensional parameters were introduced:

\[ \tilde{p} = \frac{p}{p_{Hertz}} \]

\[ b = \frac{4P}{\sqrt{\pi B R_1 + R_2}} \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right) \]

\[ p_{Hertz} = \frac{4}{\pi} \frac{P}{2bB} \]  
(2.12)

\[ X = x/b \]

\[ H = hR/b^2 \]

\[ \tilde{\sigma}_s = \sigma_s R/b^2 \]

\[ \tilde{n} = 4nb^2\sqrt{\beta/R} \]
In Equation (2.12), $\bar{n}$ is proportional to load $P$ and $\bar{\sigma}_s$ is inversely proportional to load. Therefore, their product is independent of load:

$$\bar{n}\bar{\sigma}_s = 4D_{sum}\sigma_s\sqrt{\bar{\beta}R}$$  \hspace{1cm} (2.13)

The dimensionless form of Equations (2.6), (2.10) and (2.11) will take on the following form:

$$\tilde{p} = \frac{8\sqrt{2}}{15}\pi D_{sum}^2\beta^{1.5}\sigma_s^{2.5}\left(\frac{W}{2\pi}\right)^{-0.5}F_{5/2}(H)$$  \hspace{1cm} (2.14)

$$H(X) = H_{00} + \frac{X^2}{2} - \frac{1}{\pi}\int_{-\infty}^{\infty}\tilde{p}(S)\ln(|X - S|^2) dS$$  \hspace{1cm} (2.15)

$$\frac{\pi}{2} = \int_{-\infty}^{\infty}\tilde{p}(X) dX$$  \hspace{1cm} (2.16)

The function $F_{5/2}$ is defined as:

$$F_{5/2}(H) = \frac{1}{\sqrt{2\pi}}\int_H^{\infty}(s - H)^{5/2}e^{-s^2/2} ds$$  \hspace{1cm} (2.17)

To simplify the integration in Equation (2.17), we will use the polynomial curve fit shown in the following equation [15].

$$F_{5/2}(H) = \begin{cases} 
4.4086 \times 10^{-5}(4 - H)^{6.804} & \text{for } H < 4 \\
0 & \text{for } H \geq 4
\end{cases}$$  \hspace{1cm} (2.18)

The objective is to determine a pressure distribution and an initial separation $H_{00}$ that satisfies Equations (2.14) to (2.16). The solution scheme starts with assuming an initial pressure distribution such as the Hertzian pressure. Then $H_{00}$ is chosen such that after the resulted separation in Equation (2.15) is inserted in Equation (2.14), the pressure distribution satisfies the load balance in Equation (2.16). This algorithm is repeated until the difference between two consecutive pressure distributions become smaller than $10^{-4}$. The error tolerance of $10^{-4}$ gives a fairly accurate curvefit. The above procedure was applied to a series of different loads and surface properties to determine the asperity pressure. The results are plotted in Figure 2.2. At low loads all the curves have a similar slope and at high loads they approach each other. Therefore, a
function can be fitted to relate the central pressure to load and surface properties. This curvefit is shown in Equation (2.19).

![Figure 2.2: Asperity pressure versus dimensionless load and surface properties.](image)

Following the work of Gelinck and Schipper [12], the same form of the equation shown below was used to curve fit the results:

\[
\frac{P_c}{P_{Hertz}} = [1 + (a_1 n^{a_2} \sigma^{a_3} W^{a_2-a_3})^{a_4}]^{1/a_4}
\]  
(2.19)

After fitting the results, the following equation was derived:

\[
\frac{P_C}{P_{Hertzian}} = \left[1 + \left(1.30795 D_{sum} R \sqrt{\beta R} \right)^{-0.02834} \left(\frac{\sigma_s}{R_1}\right)^{-0.4189} W^{0.39} \right]^{-0.5915}
\]  
(2.20)

After applying the Johnson’s concept of load sharing which leads to substitute $E'/\gamma_2$ for $E'$, $F_T/\gamma_2$ for $F_T$ and $D_{sum} \gamma_2$ for $D_{sum}$, Equation (2.20) is written in the following form:

\[
P_C = \frac{1}{\gamma_2} \sqrt{\frac{F_T E'}{2 \pi B R'}} \left[1 + \left(1.30795 D_{sum} \gamma_2 R \sqrt{\beta R} \right)^{-0.02834} \left(\frac{\sigma_s}{R_1}\right)^{-0.4189} W^{0.39} \right]^{-0.5915}
\]  
(2.21)

### 2.2.3 Solution Procedure

Setting Equation (2.21) equal to equation (2.14) yields:

\[
\frac{1}{\gamma_2} \sqrt{\frac{F_T E'}{2 \pi B R'}} \left[1 + \left(1.30795 D_{sum} \gamma_2 R \sqrt{\beta R} \right)^{-0.02834} \left(\frac{\sigma_s}{R_1}\right)^{-0.4189} W^{0.39} \right]^{-0.5915} = \frac{8^{1/2}}{15} \pi D_{sum}^2 \beta^{1.5} \sigma_s^{2.5} E' F_{5/2} \left(\frac{h_c-d_d}{\sigma_s}\right)
\]  
(2.22)
The film thickness predicted in Equation (2.4) is based on temperature of inlet fluid. In order to correct this film thickness, the thermal reduction factor developed by Hsu and Lee [6] is employed.

They solved the coupled Reynolds and energy equation and developed a correlation formula for the ratio of thermal film thickness to the isothermal film thickness as a function of dimensionless load, dimensionless materials parameter and slip ratio.

\[
C_t = \frac{h_{\text{thermal}}}{h_{\text{isothermal}}} = \frac{1}{1 + 0.0766 G^{0.687} W^{0.447} L^{0.527} \mu^{0.875} sr} \tag{2.23}
\]

In this equation, \( W \) is the dimensionless load, \( sr \) is the slide to roll ratio, \( G \) is the dimensionless material parameter and \( L \) is the thermal loading parameter:

\[
L = \frac{\mu_0 \gamma u_{\text{rolling}}}{K_f} \tag{2.24}
\]

In Equation (2.24), \( \mu_0 \) is the viscosity at ambient pressure and temperature, \( \gamma \) is the temperature-viscosity coefficient of lubricant, \( u_{\text{rolling}} \) is the rolling velocity and \( K_f \) is the thermal conductivity of the lubricant.

To take thermal effects into account, the thermal film thickness is calculated by multiplying film thickness obtained from Equation (2.4) by the correction factor of Equation (2.23) and the new film thickness is plugged in Equation (2.22). The film thickness calculated in this way includes the thermal effect.

The solution to this part starts from an initial guess for scaling factors \( \gamma_1 \) and \( \gamma_2 \). Then, the non-dimensional parameters of Equation (2.3) are evaluated and film thickness is calculated from Equation (2.4). The thermal film thickness is obtained by multiplying film thickness in Equation (2.4) by the thermal correction factor of Equation (2.23). The calculated film thickness is inserted in Equation (2.21). If the difference between the two sides of Equation (2.22) is larger than a specific tolerance (0.001 in this simulation), then new values for scaling factors are chosen and the calculation continues until the convergence criterion is satisfied.
2.2.4 Temperature Analysis

Once the scaling factors and film thickness are known, then the surface temperature and film temperature can be calculated. The average film temperature is needed to find the viscosity of the lubricant and calculate the friction coefficient. The energy equation for line contact is:

\[ \rho C_p \left( \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = K_f \frac{\partial^2 T}{\partial y^2} - u \frac{T}{\rho} \frac{\partial}{\partial x} \frac{dP}{dx} + \tau \dot{\gamma} \]  \hspace{1cm} (2.25)

In this equation, \( x \) represents the coordinate along the direction of motion and \( y \) is the coordinate across the film. Neglecting pressure gradient in the \( x \) direction \( dP/dx \) velocity in the \( y \) direction \( v \) and temperature gradient in the \( x \) direction \( \partial T/\partial x \), Equation (2.25) can be written in the simple form of:

\[ \frac{d^2 T}{dy^2} = -\frac{\mu \dot{\gamma}}{h_c k_f} \]  \hspace{1cm} (2.26)

In this equation, \( \mu \) is the lubricant viscosity at operating pressure and temperature, \( \dot{\gamma} \) denotes the sliding velocity, \( K_f \) is the thermal conductivity of the fluid and \( h_c \) represents the film thickness. In Equation (2.26) viscosity is a function of pressure and temperature and is evaluated using the following equation attributed to Roeland’s equation [16]:

\[ \mu = \mu_0 \exp \left\{ (\ln \mu_0 + 9.67) \left[ -1 + (1 + 5.1 \times 10^{-9} p) \frac{T-138}{T_0-138} \right] \right\} \]  \hspace{1cm} (2.27)

In this equation, \( \mu_0 \) is the viscosity of the lubricant at ambient pressure and temperature, \( T_0 \) is the inlet temperature of oil, \( T \) is the temperature of oil, \( P \) is pressure and \( Z \) is the viscosity-pressure index.

The boundary conditions for this equation are:

\[ \begin{align*}
  y &= 0, T = T_1(x) \\
  y &= h, T = T_2(x)
\end{align*} \hspace{1cm} (2.28)\]

Integrating Equation (2.26) twice and applying the boundary conditions, yields:

\[ T(x, y) = T_1(x) + (T_2(x) - T_1(x)) \frac{y}{h_c} + \frac{\mu \dot{\gamma}}{2K_f} \left( \frac{y}{h_c} \right) - \frac{\mu \dot{\gamma}}{2K_f} \left( \frac{y}{h_c} \right)^2 \]  \hspace{1cm} (2.29)
Distribution of film temperature inside the contact zone will be obtained by averaging film temperature along film thickness, or:

\[
T_{film}(x) = \frac{1}{h_c} \int_0^{h_c} T(x,y) dy = \frac{T_1(x) + T_2(x)}{2} + \frac{\mu u_s^{sliding}}{12K_f}
\]  

(2.30)

where \( T_1(x) \) and \( T_2(x) \) represent the surfaces temperature distributions within the contact. Appropriate expressions for surface temperature of fast moving rollers are [17, 18]:

\[
T_1(x) = T_0 + \frac{1}{\sqrt{\pi \rho_1 C_{p1} K_1 U_1}} \int_{-\infty}^{x} \left( \frac{K_f}{h_c} \left[ T_2(\xi) - T_1(\xi) \right] + \frac{q(\xi)}{2} \right) \frac{d\xi}{\sqrt{x-\xi}}
\]  

(2.31)

\[
T_2(x) = T_0 + \frac{1}{\sqrt{\pi \rho_2 C_{p2} K_2 U_2}} \int_{-\infty}^{x} \left( \frac{K_f}{h_c} \left[ T_1(\xi) - T_2(\xi) \right] + \frac{q(\xi)}{2} \right) \frac{d\xi}{\sqrt{x-\xi}}
\]  

(2.32)

where \( \rho_1 \) and \( \rho_2 \) represent the density of each of the rollers; \( C_{p1} \) and \( C_{p2} \) are the specific heat of the rollers; \( K_1 \) and \( K_2 \) are the thermal conductivity of the rollers; and \( U_1 \) and \( U_2 \) are the rolling speed for two rollers. The heat generated by hydrodynamic friction and asperity friction is shown by \( q(\xi) \) and is defined as:

\[
q(x) = \left( \tau(x) + f_c P_c \sqrt{1 - \left( \frac{x}{a} \right)^2} \right) u_{sliding}
\]  

(2.33)

In this equation, \( f_c \) is the friction coefficient between asperities and \( P_c \) is the asperity pressure which is calculated from Equation (2.20). The parameter \( a \) represents the half-width of Hertzian contact, \( x \) is the coordinate of points inside the contact zone and \( \tau(x) \) is the fluid shear stress.

Surface temperatures are calculated using Gaussian quadrature with 32 terms. Therefore, the contact zone is divided to 32 points and initially a temperature distribution is assumed for each surface. Then using the Gaussian quadrature, the integrals in Equations (2.31) and (2.32) are calculated and the film temperature is evaluated using Equation (2.30). The average of film temperature inside the contact zone is input to the Equation (2.27) to calculate the viscosity. Finally, the modified film thickness and viscosity are used in Equation (2.34) to evaluate the hydrodynamic friction force:
\[ F_{f,H} = 2aB \mu \frac{u_{sliding}}{h_c} \]  

(2.34)

Assuming that all the asperities have the same coefficient of friction, the asperity friction force is determined as:

\[ F_{f,c} = \sum_{i=1}^{N} f_{Ci} p_{Ci} dA_{Ci} = f_c \sum_{i=1}^{N} P_{Ci} dA_{Ci} = f_c F_c \]  

(2.35)

Hence, the coefficient of friction is:

\[ f = \frac{F_f}{F_T} = \frac{F_{f,H} + f_c F_c}{F_T} \]  

(2.36)

### 2.3 Results and Discussions

#### 2.3.1 Verification Tests

The simulation results for friction coefficient were compared to the EHL results of rolling/sliding of rollers with rough surfaces published in [1]. The simulations reported in that paper was based on solving the thermal Reynolds, modified elasticity, and energy equation. The rollers and lubricant properties as well as surface roughness data are shown in Table 2.1.

<table>
<thead>
<tr>
<th>Young’s modulus of disks</th>
<th>( E = 200 \times 10^9 ) Pa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of larger roller</td>
<td>( R_1 = 0.0225 ) m</td>
</tr>
<tr>
<td>Radius of smaller roller</td>
<td>( R_2 = 0.0254 ) m</td>
</tr>
<tr>
<td>Viscosity</td>
<td>( \mu_0 = 0.04 ) Pa.s</td>
</tr>
<tr>
<td>Lubricant density</td>
<td>( \rho_f = 840 ) kg/m³</td>
</tr>
<tr>
<td>Lubricant thermal conductivity</td>
<td>( K_f = 0.14 ) N/s.K</td>
</tr>
<tr>
<td>Rollers density</td>
<td>( \rho = 7850 ) kg/m³</td>
</tr>
<tr>
<td>Rollers thermal conductivity</td>
<td>( K = 47 ) N/s.K</td>
</tr>
<tr>
<td>Inlet temperature</td>
<td>( T_0 = 313 ) K</td>
</tr>
<tr>
<td>Dimensionless load</td>
<td>( W = 2.3 \times 10^{-4} )</td>
</tr>
<tr>
<td>Dimensionless material property</td>
<td>( G = 3500 )</td>
</tr>
<tr>
<td>Dimensionless speed</td>
<td>( U = 9.2 \times 10^{-11} )</td>
</tr>
<tr>
<td>Radius of curvature of the asperities</td>
<td>( B = 3.61 \times 10^{-6} ) m</td>
</tr>
<tr>
<td>Amplitude of asperity</td>
<td>( A = 1.4 \times 10^{-7} ) m</td>
</tr>
</tbody>
</table>

Table 2.1: Operating conditions for simulation

Variation of friction coefficient with slip is shown in Figure 2.3. Slip is defined as:

\[ \text{slip} = \frac{sr}{2} \]  

(2.37)
As slip increases, the sliding velocity increases which results in increase of film thickness. On the other hand, an increase in the slide-to-roll ratio increases the surfaces and film temperature which results in a decrease of viscosity. Therefore, as the slide-to-roll ratio increases, initially the sliding speed effect dominates the thermal effect and viscosity drop. Gradually, the viscosity drop dominates the increase in film thickness and sliding speed. The presented approach predicts results very close to the method in [1].

The second comparison was made to experimental data reported in [19]. A set of experiments were conducted on two rollers being pressed together and friction coefficient was measured for different values of slide to roll ratio.

![Figure 2.3: Comparing friction coefficient with results from [1]](image)

**Figure 2.3: Comparing friction coefficient with results from [1]**

Table 2.2 shows the operating conditions of the experiment. In original paper the surfaces were assumed to be perfectly smooth but in our simulations we have considered the effect of surface roughness. Therefore, a set of surface properties which represents a very smooth surface were obtained from [8] and are shown in Table 2.3.

<table>
<thead>
<tr>
<th>Table 2.2: Operating conditions of the experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
</tr>
<tr>
<td>Equivalent radius of curvature</td>
</tr>
</tbody>
</table>
Table 2.2: Continued

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity</td>
<td>$\mu_0 = 0.068$ Pa.s</td>
</tr>
<tr>
<td>Lubricant density</td>
<td>$\rho_f = 878$ kg/m$^3$</td>
</tr>
<tr>
<td>Lubricant thermal conductivity</td>
<td>$K_f = 0.14$ N/s.K</td>
</tr>
<tr>
<td>Rollers density</td>
<td>$\rho = 7876$ kg/m$^3$</td>
</tr>
<tr>
<td>Rollers thermal conductivity</td>
<td>$K = 38$ N/s.K</td>
</tr>
<tr>
<td>Inlet temperature</td>
<td>$T_0 = 318$ K</td>
</tr>
<tr>
<td>Dimensionless load</td>
<td>$W = 5.5185 \times 10^{-5}$</td>
</tr>
<tr>
<td>Dimensionless material property</td>
<td>$G = 5152$</td>
</tr>
<tr>
<td>Velocity of roller 1</td>
<td>$U_1 = 2.8$ m/s</td>
</tr>
</tbody>
</table>

Table 2.3: Surface properties [8]

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of asperities</td>
<td>$D_{sum} = 1 \times 10^{11}$ 1/m$^2$</td>
</tr>
<tr>
<td>Radius of asperity tip</td>
<td>$\beta = 10 \times 10^{-6}$ m</td>
</tr>
<tr>
<td>RMS of the asperity heights</td>
<td>$\sigma_s = 0.05 \times 10^{-6}$ m</td>
</tr>
<tr>
<td>Coefficient of friction between asperities</td>
<td>$f_c = 0.1$</td>
</tr>
</tbody>
</table>

Figure 2.4: Comparing friction coefficient with experimental data of [19]

Figure 2.4 shows the friction coefficient from this simulation and those obtained from the experiment. An increase in sliding speed initially results in increase in friction coefficient. Further increase in slide to roll ratio will generate more heat. As a result viscosity drops and consequently the friction coefficient gradually decreases.
2.3.2 Results for Spur Gears

Having verified the simulation for rollers with rough surfaces under rolling and sliding, we now turn our attention to prediction of spur gears performance. Table 2.4 shows the geometry and loading conditions for the pinion and gear under study in this paper.

Table 2.4: Gear and pinion properties [4]

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of pinion teeth</td>
<td>( z_p = 28 )</td>
</tr>
<tr>
<td>Number of gear teeth</td>
<td>( z_g = 84 )</td>
</tr>
<tr>
<td>Module</td>
<td>( m = 0.003175 ) m</td>
</tr>
<tr>
<td>Pinion pitch diameter</td>
<td>( d_{wp} = 0.0889 ) m</td>
</tr>
<tr>
<td>Pinion rotational speed</td>
<td>( \omega = 300 ) rpm</td>
</tr>
<tr>
<td>Gears width</td>
<td>( B = 0.1 ) m</td>
</tr>
<tr>
<td>Load per unit width</td>
<td>( F = 0.3765 ) MN/m</td>
</tr>
<tr>
<td>Pressure angle</td>
<td>( \alpha = 20 ) deg</td>
</tr>
<tr>
<td>Oil viscosity</td>
<td>( \mu_0 = 0.065 ) Pa.s</td>
</tr>
<tr>
<td>Viscosity-pressure index</td>
<td>( Z = 0.6 )</td>
</tr>
<tr>
<td>RMS of the surface</td>
<td>( \sigma_s = 0.4 \times 10^{-6} ) m</td>
</tr>
<tr>
<td>Density of asperity</td>
<td>( D_{sum} = 3.31 \times 10^{10} ) 1/m²</td>
</tr>
<tr>
<td>Radius of asperity</td>
<td>( \beta = 6.02 \times 10^{-6} ) m</td>
</tr>
</tbody>
</table>

- **Geometry Analysis**

The analysis started with dividing the length of LoA into 100 points, where computations are to be performed. Line of action is the active profile of pinion and gear and for this simulation has a length of 16.26 mm. Figure 2.5 shows the variation of radii of curvature of pinion and gear and also equivalent radius of curvature along the line of action.

- **Load Cycle and Slip-to-Roll Ratio**

In spur gears, the load carried changes along LoA as the number of teeth in contact changes. This variation is illustrated in Figure 2.6. At the beginning of contact (point A), there are two pairs of teeth in contact until point B where the number of engaged teeth shifts from two to one. In this paper, point B is referred to as Lowest Point of Single Tooth Contact (LPSTC). In this region, the load linearly changes from one-third to two-third of total load.
Then, there is only one pair of tooth in contact and the carried load suddenly jumps to the total load and remains constant until it reaches point C which is known as Highest Point of Single Tooth Contact (HPSTC). From this point to point D which is the end of contact, the carried load decreases from two-third to one-third at the end of mesh [10].

The Hertzian pressure was calculated for each point along LoA based on Equation (2.12) and with considering the variation of transmitted load along LoA. Figure 2.7 shows variation of Hertzian pressure. As can be seen, the maximum pressure occurs at the lowest point of single
tooth contact and has the value of $P_{\text{max}}$=1.13 GPa. The rolling and sliding velocity of pinion and gear for each point along LoA are illustrated in Figure 2.8.

![Figure 2.7: Variation of Hertzian pressure along LoA](image)

![Figure 2.8: Variation of rolling and sliding speeds along LoA](image)

Another important parameter in gear analysis is the ratio of sliding speed to rolling speed which is known as slide-to-roll ratio or $sr$. The variation of this parameter is shown in Figure 2.9.

The maximum value of $sr$ occurs at the first point of contact and this parameter decreases along the line of action until pitch point where there is pure rolling or $sr=0$, then $sr$ increases again.
Figure 2.9: Variation of slide to roll ratio along LoA.

• EHL Film Thickness

Having calculated all the required parameters for analysis, next the thermal film thickness was evaluated and plotted in Figure 2.10. Also shown in Figure 2.10 is the isothermal film thickness.

Figure 2.10: Comparison of isothermal and thermal film thickness along LoA

Consideration of thermal effect results in lower film thickness for all the points along the line of action. The trend for both cases is the same and film thickness increases as the contact point moves along LoA. The thermal film thickness has a lower value because of the thermal
correction factor introduced in Equation (2.23). The thermal correction factor varies along LoA as the sliding speed, rolling velocity, transmitted load and the equivalent radii of curvature changes. Comparing to the isothermal case, thermal analysis predicts a 20% to 50% drop for film thickness along LoA.

- **Surfaces and Film Temperature**

Figure 2.11 shows the variation of surfaces and film temperature along LoA. Surface temperature and lubricant exhibit a similar trend along LoA. The maximum temperature occurs at the first point of contact where the sliding speed has its largest value and, according to Equation (2.33), more heat is generated. As the contact point moves toward the pitch point, the temperature decreases reaching its minimum where only pure rolling takes place. At this point, pinion, gear and the lubricant all have the temperature equal to the oil inlet temperature. As the contact point moves toward the end of the mesh, the temperatures rise. Referring to Equation (2.30), the film temperature is equal to the average of temperature of both surfaces plus a term proportional to the square of the sliding velocity that arises from the viscous dissipation. Therefore, at the beginning of the mesh, where the slide-to-roll ratio ($sr$) is the highest, film temperature is much higher than the temperature of pinion and gear. As the contact point moves along the LoA toward pitch point, $sr$ decreases and the difference between film temperature and surfaces temperature drops. As the contact point moves away from pitch point, sliding speed and therefore $sr$ increases and the difference between surfaces and film temperature rises.

- **Effect of Speed on Temperature**

Figure 2.12 shows the film temperature for three different rotational speeds. The trend for all three speeds is the same. The film temperature according to Equation (2.30) is directly proportional to the square of sliding velocity. Therefore, as the rotating speed of the pinion increases, the sliding speed increases and as a result more heat is generated, leading to a rise in the film temperature.
Figure 2.11: Variation of surface and film temperature along LoA

Figure 2.12: Effect of speed on film temperature

- Load

Figure 2.13 shows the portion of load taken by asperity and by fluid film. Asperities take a considerable portion of load at the first point of contact. As the contact point moves along the line of action the hydrodynamic portion increases and the asperity portion decreases until the lowest point of single tooth contact where there is a drop in asperity part and a sudden jump in
hydrodynamic part. When the contact point passes the highest point of single tooth contact, the increase in hydrodynamic portion and decrease in asperity portion continues.

![Figure 2.13: Portion of load taken by film and by asperities](image)

- **Friction Coefficient**

The variation of friction coefficient along LoA is shown in Figure 2.14. Starting from the first point of contact (point A in Figure 2.14), the friction coefficient due to small viscosity has a large value. As the contact point moves toward the pitch point (point C in Figure 2.14), the friction coefficient decreases until LPSTC (point B in Figure 2.14). At that point due to sudden increase in the load, a sudden change in friction coefficient is observed. After that, toward the pitch point, the load remains constant but due to decrease in slide-to-roll ratio, the film temperature drops, and this results in an increase in fluid viscosity. In this region, decrease in sliding velocity dominates the effect of increase in viscosity and, therefore, friction coefficient drops. At the pitch point where the motion is pure rolling, the hydrodynamic friction force is nil, but the simulation predicts existence of a small friction force due to interaction between
asperities. As the contact point moves from pitch point toward HPSTC (point D in Figure 2.14), the slide-to-roll ratio increases which results in an increase in fluid temperature and a decrease in fluid viscosity. The combined effect of these factors results in an increase in the friction coefficient until HPSTC. At this point, due to the shift from one pair of tooth in contact to two pair (see Figure 2.6), the load and the friction coefficient suddenly drops. As the contact point moves from HPSTC to the end of mesh (point E in Figure 2.14), the carried load reduces, fluid viscosity decreases and sliding speed increases. As a result of all of these factors, initially there is a slight increase in the friction coefficient which gradually becomes constant. Figure 2.15 shows the comparison for the friction coefficient of a rough surface (Figure 2.14) and the friction coefficient for a fairly smooth surface.

![Figure 2.14: Variation of friction coefficient along LoA](image)

The difference between these two graphs is due to the asperity friction force. For smooth surfaces, there is no interaction between asperities and friction force is merely hydrodynamic friction force. Therefore, at pitch point where there is pure rolling, friction coefficient is zero. For rough surfaces, however, the interaction between asperities has a major effect in friction
force and this effect can be seen along the line of action. For the first point of contact (point A in Figure 2.14), rougher surface has a friction coefficient twice the smoother surface.

**Figure 2.14:** Comparison of friction coefficient between smooth and rough surfaces

- **Effect of Speed on Friction Coefficient**

Figure 2.16 shows the variation of friction coefficient as speed changes. Friction coefficient is a function of film thickness, pressure and temperature. Variation of rotational speed affects film thickness and film temperature. However, this variation is not the same for all points along the line of action. The hydrodynamic Friction force is proportional to viscosity and inversely proportional to film thickness. As the speed increases, the film temperature increases which results in a decrease in viscosity. An increase in the speed, on the other hand, decreases the film thickness. The combined effect of both factors is shown in Figure 2.16. In the regions where there are two pairs of tooth in contact, viscosity effect dominates the film thickness effect. Therefore, in these regions as rotational speed increases, friction coefficient decreases. In the region of single tooth in contact, the viscosity effect and film thickness effect for these rotational
speeds compensate each other. Therefore, there is no obvious distinction between friction coefficients for different speeds.

- **Effect of Roughness on Friction Coefficient**

Figure 2.17 shows the effect of surface roughness on friction coefficient. In this comparison, density of asperities $D_{sum}$ and radius of tip of asperities $\beta$ are assumed to be constant for all three surfaces (as given in Table 2.4) and $R_q$ or $\sigma_s$ is the only variable.

**Figure 2.16: Variation of friction coefficient as speed changes**

Variation of friction coefficient along LoA shows a similar trend for all three surfaces. As surface roughness increases, more asperities come into contact and therefore friction coefficient increases.

**Figure 2.17: Effect of Surface Roughness on Friction Coefficient**
2.4 Conclusions

In this paper the thermal EHL in spur gears with rough surfaces was studied using Johnson’s load sharing concept. Contact of pinion and gear along the line of action was replaced by contact of cylinders that are under a varying load and their radii change to mimic the performance of a gear-pinion pair during the engagement. The scaling factors and film thickness based on isothermal analysis were determined and these two parameters were modified using the thermal correction factor developed by Hsu and Lee [5]. The corrected values of film thickness and scaling factors were employed to find the gear and pinion surface as well as fluid film temperature. The film temperature was used to calculate the viscosity and friction coefficient. The computational algorithm used in this paper is very efficient in handling gear problems.

A new curvefit relationship based on Greenwood-Tripp model was developed for the contact of rough surfaces as a function of surface properties, geometry and load. Since in gears both surfaces are rough, using Greenwood-Tripp formula is more pertinent for the analysis of gears to that of Greenwood-Williamson formula.

The validity of this model was verified by comparing this simulation to the results of two other papers. One paper [1], gives theoretical results for friction coefficient between two rough cylinders. The other comparison was made with experiment [19] conducted between two rollers with varying slide to roll ratio. The comparisons show a very good agreement between results predicted by this method and theoretical and experimental work of others. Considering the effect of surface roughness, it was shown that for rough surfaces the friction coefficient for pure rolling is not zero.

Effect of speed on performance of gears has also been studied. As the speed increases, more heat is generated and film temperature will increase which results in a decrease in film thickness. Also the temperature increase will lead to a decrease in viscosity. Variation of friction coefficient with speed for the first point of contact is of interest. The combined effect of decrease in film
thickness and decrease in fluid viscosity for the beginning of mesh results in a drop in friction coefficient. Effect of roughness on friction coefficient has also been studied. As surface roughness increases, more asperity contact will occur and therefore friction coefficient will increase.

The film thickness is lowest at the beginning of contact. Also, the largest portion of load carried by asperities occurs at the first point of contact. In spur gears, the largest wear depth is typically seen to occur in the pinion dedendum and gear addendum where according to our simulation a considerable portion of load is carried by asperities.

The model that has been developed so far is capable to predict the temperature of contacting gears for all the points along the line-of-action. Another important concern in gears is the issue of wear. Two wear mechanisms that might occur during the steady-state regime of spur gears are discussed in the next chapter. Based on the surface temperature, one of the mechanisms will be dominant.

### 2.5 Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>half width of contact (m$^2$)</td>
</tr>
<tr>
<td>$B$</td>
<td>Roller’s width (m)</td>
</tr>
<tr>
<td>$C_{pi}$</td>
<td>Specific heat of roller i (J/Kg/K)</td>
</tr>
<tr>
<td>$d_{wp}$</td>
<td>pitch radius of pinion (m)</td>
</tr>
<tr>
<td>$d_{wg}$</td>
<td>pitch radius of gear (m)</td>
</tr>
<tr>
<td>$D_{sum}$</td>
<td>density of asperities (1/m$^2$)</td>
</tr>
<tr>
<td>$E_i$</td>
<td>Modulus of elasticity of roller i (N/m$^2$)</td>
</tr>
<tr>
<td>$E'$</td>
<td></td>
</tr>
<tr>
<td>$F_R$</td>
<td>Applied force (N)</td>
</tr>
<tr>
<td>$F_C$</td>
<td>Load carried by asperities (N)</td>
</tr>
<tr>
<td>$F_{Hf}$</td>
<td>Load carried by fluid film (N)</td>
</tr>
<tr>
<td>$F_{HC}$</td>
<td>Asperity friction force (N)</td>
</tr>
<tr>
<td>$F_{HF}$</td>
<td>Hydrodynamic friction force (N)</td>
</tr>
<tr>
<td>$f_c$</td>
<td>friction coefficient between asperities</td>
</tr>
<tr>
<td>$f_f$</td>
<td>Asperity friction force (N)</td>
</tr>
<tr>
<td>$f_H$</td>
<td>Hydrodynamic friction force (N)</td>
</tr>
<tr>
<td>$f_i$</td>
<td>friction coefficient between asperities</td>
</tr>
<tr>
<td>$R_p$</td>
<td>Radius of curvature for pinion (m)</td>
</tr>
<tr>
<td>$R_g$</td>
<td>Radius of curvature for gear (m)</td>
</tr>
<tr>
<td>$R$</td>
<td>equivalent radius (m)</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Oil inlet temperature (K)</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature (K)</td>
</tr>
<tr>
<td>$U$</td>
<td>non-dimensional speed $\mu_0(U_1 + U_2)/E'R$</td>
</tr>
<tr>
<td>$v_i$</td>
<td>rolling speed of roller i (m/s)</td>
</tr>
<tr>
<td>$w(x)$</td>
<td>elastic deformation</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Pinion pressure angle</td>
</tr>
<tr>
<td>$\beta$</td>
<td>radius of tip of asperities (m)</td>
</tr>
</tbody>
</table>
$G$: material number ($\alpha E'$)  
$\gamma_1$: scaling factor for hydrodynamic part

$h_C$: central film thickness (m)  
$\gamma_2$: scaling factor for asperity part

$H_C$: non-dimensional film thickness  
$\gamma$: temperature viscosity coefficient (1/K)

$K_f$: thermal conductivity of fluid (W/m/K)  
$\nu_i$: Poisson ratio for roller $i$

$K_i$: thermal conductivity of roller $i$ (W/m/K)  
$\sigma$: standard deviation of asperities (m)

$L$: thermal loading parameter  
$\mu_0$: viscosity at ambient condition (Pa.s)

$m_i$: $i^{th}$ moment of the surface  
$\mu$: viscosity (Pa.s)

### 2.6 References


Chapter 3: Prediction of Steady State Adhesive Wear in Spur Gears Using the EHL Load-Sharing Concept*

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3.1 Introduction

Wear is the removal of material from solid surfaces as a result of mechanical action and can be categorized into four different forms: adhesive, abrasive, corrosive, and surface fatigue [1]. Adhesive wear, the subject of this paper, occurs when two bodies are slid over each other and fragments that are pulled off from one surface tend to adhere to the other surface. Later these fragments may come off the surface on which they are formed and be transferred back to the original surface or else form loose wear particles.

In elastohydrodynamic lubrication (EHL), the fluid film separates the two surfaces from contact. In applications such as gears, the lubrication regime is typically mixed or partial EHL, implying the lubricant film alone cannot completely separate the two surfaces and therefore asperities tend to interact and, in conjunction with the fluid film, provide load-carrying capacity. The main objective of this paper is to present a method that applies the thermoelastohydrodynamic EHL (TEHL) analysis to gears based on load sharing concept [2] and follow the work of Wu and Cheng [3 and 4] to predict the adhesive wear in spur gears.

3.2 Model

In this model, the contact of pinion and gear along the line of action is replaced by contact of equivalent rollers with varying radii of curvature under varying load to mimic the behavior of the gear-pinion system. The radii of cylinders replacing pinion and gear and the applied load for each point along LoA are determined based on the formulation that was presented in [5].

Briefly, the interaction between the fluid film and asperities is based on the load-sharing concept proposed by Johnson et al. [6] for mixed lubrication. According to this method, a part of load \( F_T/y_1 \) is carried by fluid film and the rest of the load \( F_T/y_2 \) is carried by asperities:

\[
F_T = F_H + F_C \quad \text{(3.1)}
\]

\[
F_T = \frac{F_T}{y_1} + \frac{F_T}{y_2} \quad \text{(3.2)}
\]
The parameters \( \gamma_1 \) and \( \gamma_2 \) are called the scaling factors, \( F_T \) is the total transmitted load, \( F_H \) is the load carried by hydrodynamic film and \( F_C \) is the load carried by asperities. The wear formulation proposed by [4] was used in this paper. The wear rate based on surface temperature of two rollers with sliding speed of \( U \) can be written in the following form:

\[
\dot{W} = k_m A_n \left\{ 1 - \exp \left[ - \frac{X}{U t_0} \exp \left( - \frac{E}{RT_s} \right) \right] \right\} \left( \frac{A_c}{A_n} \right) \quad T_s < 200 \, ^\circ C
\]

\[
\dot{W} = \frac{A_0 A_n}{C_{3/4} U} \exp \left( - \frac{Q}{RT_s} \right) \left( \frac{A_c}{A_n} \right) \quad 200 \, ^\circ C < T_s < 350 \, ^\circ C
\]

\[
\dot{W} = \frac{A_0 A_n}{C_{2/3} U} \exp \left( - \frac{Q}{RT_s} \right) \left( \frac{A_c}{A_n} \right) \quad 350 \, ^\circ C < T_s < 570 \, ^\circ C
\]

\[
\dot{W} = \frac{A_0 A_n}{C_{1/2} U} \exp \left( - \frac{Q}{RT_s} \right) \left( \frac{A_c}{A_n} \right) \quad T_s > 570 \, ^\circ C
\]

(3.3)

In this equation \( k_m \) is the non-dimensional wear coefficient, \( U \) is the sliding velocity, \( X \) is the area associated with an adsorbed molecule, \( t_0 \) is the fundamental time of vibration of the molecule in the adsorbed state, \( E \) is the heat of adsorption, \( R \) is the gas constant, \( T_s \) is the absolute temperature of the surface, \( A_n \) is the nominal area of contact, \( A_c \) is the real area of contact, \( A_0 \) is the Arrhenius constant for linear oxidation, \( Q \) is the activation energy for linear oxidation and \( C_x \) is the oxide constant.

The wear rate in Equation (3.3) has the dimension of \( m^3/m \). Here, we define a new term called volumetric wear rate which is the worn volume divided by time and is calculated by multiplying the wear rate in Equation (3.3) by sliding velocity:

\[
\dot{V} = \dot{W} u_{sliding}
\]

(3.4)

Therefore volumetric wear rate can be written as:

\[
\dot{V} = k_m A_n U \left\{ 1 - \exp \left[ - \frac{X}{U t_0} \exp \left( - \frac{E}{RT_s} \right) \right] \right\} \left( \frac{A_c}{A_n} \right) \quad T_s < 200^\circ C
\]

(3.5)

\[
\dot{V} = \frac{A_0 A_n}{C_{3/4}} \exp \left( - \frac{Q}{RT_s} \right) \left( \frac{A_c}{A_n} \right) \quad 200^\circ C < T_s < 350^\circ C
\]
3.3 Simulation Procedure

The solution algorithm starts from finding the radii of rollers which replace the pinion and gear. The radii of these rollers as well as the transmitted force between the two rollers vary along the line of action (LoA). Then, the load sharing scaling factors $\gamma_1$ and $\gamma_2$, film thickness $h$, and contact pressure between surfaces, $P_c$, are calculated for each point along the LoA. The film temperature is calculated based on the simplified energy equation for Newtonian lubricants. The boundary conditions for this simplified equation are the surfaces temperature which are calculated based on [7, 8]. The details of this analysis are given in [2]. The predicted surface temperature is used to determine the dominating wear mechanism. Based on the surface temperature, volumetric wear rate for each point along LoA is evaluated using Equation (3.5).

3.4 Results and Discussions

3.4.1 Verification Test: Two Rollers

In this section, the simulation results are compared to the experimental data obtained from measuring the wear rate between two rollers [3]. The experiment was conducted between two rollers with variable slide-to-roll ratio. The rollers specifications are presented in Table 3.1.

<table>
<thead>
<tr>
<th>Table 3.1: Experiment conditions [3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of roller 1</td>
</tr>
<tr>
<td>Diameter of roller 2</td>
</tr>
<tr>
<td>Rolling speed</td>
</tr>
<tr>
<td>Contact pressure</td>
</tr>
<tr>
<td>Oil inlet temperature</td>
</tr>
<tr>
<td>Oil viscosity</td>
</tr>
<tr>
<td>Initial Standard deviation of asperities height</td>
</tr>
</tbody>
</table>

The surface roughness shown in Table 3.1 pertains to initial surface roughness before the experiment. Based on the slide-to-roll ratio of the rollers, the standard deviation of asperity heights of rollers after the test varied between 0.4 µm for slide-to-roll ratio below 0.1 to 0.34 µm for higher values of slide-to-roll ratio. Therefore, these values were used in order to predict the
steady state wear rate. The surface properties shown in Table 3.2 are used for this simulation. The values for wear parameters that are used in Equation (3.3) are given in Table 3.3. These values are originally derived from experiments conducted in [9, 10].

**Table 3.2: Surface properties of the rollers**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation of asperities height $\sigma_s$</td>
<td>$0.34-0.4 \mu m$</td>
</tr>
<tr>
<td>Density of asperities $D_{sum}$</td>
<td>$2.77 \times 10^{10} , 1/m^2$</td>
</tr>
<tr>
<td>Radius of asperity tip $\beta$</td>
<td>$5 \times 10^{-6} , m$</td>
</tr>
</tbody>
</table>

Simulations were performed for a range of slide-to-roll ratio, $sr$. Shown in Figure 3.1 are experimentally measured steady state wear rate and predicted values based on Equation (3.3). The predicted values are in agreement with experimentally measured wear rate. As the slide-to-roll ratio increases, the adsorbed molecules will have less time to detach from the surface and the wear rate decreases. Further increase in slide-to-roll ratio will cause an increase in the real area of contact to dominate the effect of decrease in contact time of adsorbed molecules. Therefore, at high slide-to-roll ratios the wear rate increases.

**Table 3.3: Values for wear parameters [3]**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_m$</td>
<td>$5 \times 10^{-4}$</td>
</tr>
<tr>
<td>$A_0$</td>
<td>$4 \times 10^{-10} , kg/m^2.s$</td>
</tr>
<tr>
<td>$E$</td>
<td>49 , kJ/mole</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>193 , kJ/mole</td>
</tr>
<tr>
<td>$R$</td>
<td>8.31 , J/mole.K</td>
</tr>
<tr>
<td>$X$</td>
<td>$3 \times 10^{-10} , m$</td>
</tr>
<tr>
<td>$t_0$</td>
<td>$3 \times 10^{-12} , sec$</td>
</tr>
<tr>
<td>$\rho_{Fe}$</td>
<td>7800 , kg/m$^3$</td>
</tr>
<tr>
<td>$M_{O_2}$</td>
<td>32 , kg/mole</td>
</tr>
<tr>
<td>$M_{Fe}$</td>
<td>56 , kg/mole</td>
</tr>
</tbody>
</table>

### 3.4.2 Results for Gears

Having validated the simulation results by comparing to experimental data with two rollers in contact, we now proceed to present the results for gears. The specifications of the pinion and gear used in the simulation are given in Table 3.4.
Load sharing among teeth has been considered in this model as detailed in references [2] and [5]. Figure 3.2 shows the variation of transmitted load along the line of action. In spur gears, the load carried changes along LoA as the number of teeth in contact changes. At the beginning of contact (point A), there are two pairs of teeth in contact until point B (known as lowest point of single tooth contact) where the number of engaged teeth shifts from two to one. In this region, the load linearly changes from one-third to two-third of total load. Then, there is only one pair of tooth in contact and the carried load suddenly jumps to the total load and remains constant until it reaches point C (known as highest point of single tooth contact). From this point to point D
which is the end of contact, the carried load decreases from two-third to one-third at the end of mesh. The contact ratio for the pinion-gear system of this paper is about 1.6.

\[\Lambda = \frac{h_c}{R_q}\]  

\(\text{(3.6)}\)

**Figure 3.2: Variation of transmitted load along LoA**

One of the important parameters in tribology is the film parameter (\(\Lambda\)). The film parameter is the film thickness (\(h\)) divided by standard deviation of asperity heights (\(R_q\)):

This parameter is useful for determining the severity of load and the lubrication condition. It is generally believed [12] that \(\Lambda < 1\) corresponds to boundary lubrication whereas partial or mixed lubrication occurs when \(1 \leq \Lambda < 3\) and for full EHL \(\Lambda \geq 3\). According to Figure 3.3, all the points along the line of action are in the mixed lubrication regime. The scaling factor for asperity part \(1/\gamma_2\), which represents the percentage of load carried by asperities, is always of interest in the lubricated contact of rough surfaces. Figure 3.4 shows the variation of this parameter along the LoA.

Among all the points along LoA, it is in the first point of contact (point A) where the largest amount of load is carried by asperities. As the contact point moves along LoA, the asperity portion decreases and, hence, the fluid-film portion increases. At the lowest point of single tooth contact (point B), there is a drop in asperity portion. On the other hand, at the highest point of
single tooth contact (point C), there is an increase in asperity portion. From there until the last point of engagement (point D) there is no significant change in load carried by asperities and fluid film.

![Figure 3.3: variation of film parameter along LoA](image)

![Figure 3.4: Variation of scaling factors along LoA](image)

The variation of surface temperature which is an output of the thermal analysis is plotted in Figure 3.5. The surface temperature has its highest value at the first point of contact where the sliding is greatest and this translates to more heat generation there. As the contact point moves toward the pitch point and the sliding decreases, the surface temperature drops until it reaches the minimum value at the pitch point. After the pitch point, the sliding increases and the surface temperature again rises.
The volumetric wear rate along LoA is shown in Figure 3.6. This parameter shows the worn volume as a function of time. The volumetric wear rate has the highest value at the first point of contact. At the pitch point where there is no sliding, the volumetric wear rate is nil.

Variation of steady state wear depth along LoA after one million cycles is illustrated in Figure 3.7. The wear depth has its highest value at the first point of contact and as the contact point moves toward the pitch point, the wear depth decreases. The wear depth is nil at the pitch point. As illustrated in Figure 3.6, for the points near pitch point, the volumetric wear rate has small values leading to minute wear depth in this region. This simulation also predicts that the wear in pinion dedendum is more significant than wear in pinion addendum. Experimental observations [13-18] attest to the validity of this prediction. The wear seen in pinion dedendum and gear addendum is usually more noticeable than the wear observed in pinion addendum and gear dedendum.
3.4.3 Speed Effect

As the speed increases, a thicker fluid film is formed and larger portion of load is taken by fluid film. Therefore, the load carried by asperity decreases. Figure 3.8 compares the volumetric wear rate for different speeds. An increase in speed results in the formation of a thicker lubricant layer. Therefore, less asperity contacts will occur and the portion of load carried by asperities decrease. A decrease in asperity portion results in decrease in volumetric wear rate. As shown in Figure
3.8, for similar points on the LoA as speed increases, the volumetric wear rate decreases. Also, due to small sliding speed, volumetric wear rate has small values around the pitch point.

![Figure 3.8: Comparison of volumetric wear rate for different speeds](image)

3.4.4 Surface Roughness Effect

In this section, effect of surface roughness on wear behavior is studied. The surface roughness mentioned here refers to the standard deviation of asperity heights after running in. Based on the running in speed, running in load and other operating conditions, the running in time and the resulted surface properties corresponding to the steady state regime will be different. To consider that effect, the standard deviation of asperity heights ($R_q$) is varied and the rest of the parameters are the same as Tables 3.2, 3.3 and 3.4. An increase in $R_q$ means that standard deviation of asperity heights is larger implying that asperities have larger heights. Therefore, increasing $R_q$, results in more asperity-to-asperity interaction, and so the contribution of asperities in carrying load increases. Figure 3.9 shows variation of wear depth with increase in surface roughness. As was shown in Figure 3.9, increase in $R_q$ results in increase in the asperity-asperity contacts. As a result of increase in asperity interactions, wear rate and therefore wear depth increases.
3.5 Conclusions

The adhesive wear prediction model proposed by Wu and Cheng [3 and 4] is employed in conjunction with TEHL analysis of rough surfaces based on the load sharing concept to predict the steady state adhesive wear in gears and the predicted wear rate for a specific case is compared to the experimental data. The comparison shows good agreement between predicted values and experimentally determined wear rate for contact of two rollers representing the gears and the pinion. Also presented are the results of series of wear calculations for a set of pinion and gear and variation of scaling factors and volumetric wear rate. The effect of speed on scaling factors and volumetric wear rate is also shown. As speed increases, the fluid film takes a larger portion of load and smaller portion of load is carried by asperities. Therefore, less asperity-to-asperity contact occurs, which will decrease volumetric wear rate. Consequently, volumetric wear rate will decrease with increasing speed.

An advantage of this algorithm beside its reasonable accuracy in predicting wear rate is its efficiency in terms of execution time. The execution time for simulation which consists of
dividing the line of action to 300 points, performing the rough TEHL analysis for all the points
and calculating the wear depth takes around 20 seconds on a Pentium 4 computer with a CPU of
2.4 GHz.

The effect of surface roughness on volumetric wear rate was also investigated. As surface
roughness increases, asperities carry a larger portion of the load and the wear rate increases.
Lubricant properties have an enormous effect on wear. These properties are viscosity, the area
associated with an adsorbed molecule $X$, fundamental time of vibration of the lubricant
molecules in the adsorbed state $t_0$ and heat of adsorption $E$.

3.6 Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$</td>
<td>Arrhenius constant for oxidation (kg/m$^2$.s)</td>
</tr>
<tr>
<td>$A_{ci}$</td>
<td>Local asperity contact area for asperity i (m$^2$)</td>
</tr>
<tr>
<td>$A_l$</td>
<td>Arrhenius constant for linear oxidation (kg/m$^2$.s)</td>
</tr>
<tr>
<td>$A_n$</td>
<td>Nominal contact area (m$^2$)</td>
</tr>
<tr>
<td>$B$</td>
<td>Gear tooth width (m)</td>
</tr>
<tr>
<td>$C_X$</td>
<td>Oxide constants</td>
</tr>
<tr>
<td>$D_{sum}$</td>
<td>Density of asperity (1/m$^2$)</td>
</tr>
<tr>
<td>$E$</td>
<td>Heat of adsorption of mineral oil molecules on steel surface (J/mole)</td>
</tr>
<tr>
<td>$F_C$</td>
<td>Load carried by asperity contacts (N)</td>
</tr>
<tr>
<td>$F_H$</td>
<td>Load carried by hydrodynamic film (N)</td>
</tr>
<tr>
<td>$F_T$</td>
<td>Total load (N)</td>
</tr>
<tr>
<td>$h$</td>
<td>Film thickness (m)</td>
</tr>
<tr>
<td>$k_m$</td>
<td>Wear coefficient parameter specific to contacting asperities</td>
</tr>
<tr>
<td>$L$</td>
<td>Sliding distance (m)</td>
</tr>
<tr>
<td>$m$</td>
<td>Gear’s module (m)</td>
</tr>
<tr>
<td>$M_{Fe}$</td>
<td>Molecular weight of iron (kg/kmole)</td>
</tr>
<tr>
<td>$M_{O2}$</td>
<td>Molecular weight of oxygen (kg/kmole)</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>Activation energy for oxidation (J/mole)</td>
</tr>
<tr>
<td>$R$</td>
<td>Molar gas constant (J/mole.K)</td>
</tr>
<tr>
<td>$R_1$</td>
<td>Radius of roller replacing pinion (m)</td>
</tr>
<tr>
<td>$R_2$</td>
<td>Radius of roller replacing gear (m)</td>
</tr>
<tr>
<td>$R_q$</td>
<td>Rms of asperity heights (m)</td>
</tr>
<tr>
<td>$sr$</td>
<td>Slide-to-roll ratio</td>
</tr>
<tr>
<td>$t_0$</td>
<td>Fundamental time of vibration of molecule in adsorbed state (s)</td>
</tr>
<tr>
<td>$t_r$</td>
<td>Average lifetime of a molecule on a given surface site (s)</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Contacting surface temperature (K)</td>
</tr>
<tr>
<td>$U$</td>
<td>Sliding speed (m/s)</td>
</tr>
<tr>
<td>$V_i$</td>
<td>Local wear volume for asperity i (m$^3$)</td>
</tr>
<tr>
<td>$V$</td>
<td>Volumetric wear rate (m$^3$/sec)</td>
</tr>
<tr>
<td>$W$</td>
<td>Total wear rate (m$^3$/m)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>( \chi )</td>
<td>Diameter of area associated with an adsorbed lubricant molecule (m)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Fractional film defect</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Radius of tip of asperities (m)</td>
</tr>
<tr>
<td>( \xi )</td>
<td>Critical thickness of oxide layer (m)</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>Scaling factor for hydrodynamic part</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>Scaling factor for asperity part</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>Film parameter</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density (kg/m³)</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>Scaling factor for asperity part</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Standard deviation of asperity heights (m)</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Rotational speed</td>
</tr>
<tr>
<td>( \Psi )</td>
<td>Pressure angle</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Viscosity (Pa.s)</td>
</tr>
</tbody>
</table>

### 3.7 References


Chapter 4: Effect of Surface Pattern on Strubeck Curve*

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4.1 Introduction

The characterization of friction in tribological components in the form of a single curve is attributed to Strubeck [1]. The so-called Stribeck curve is often used to classify different lubrication regimes: boundary regime in which the film parameter $\Lambda$ which is the ratio of lubricant film formed between surfaces $h$ to the standard deviation of asperity heights $R_q$ ($\Lambda = h/R_q$) is less than one, mixed regime where $1 < \Lambda < 3$ and hydrodynamic regime in which $\Lambda > 3$ [2]. The study of mixed lubrication regime—which covers a wide range of operating conditions in lubricated contact of rough surfaces—is the major goal of this research.

Important parameters in mixed lubrication are the surface roughness and asperities distribution pattern. These properties pertain to the procedure used in surface finishing such as grinding, lapping, etc. To characterize the asperities orientation, a parameter called $\Gamma$ is employed to represent the surface pattern. For surfaces with transverse roughness orientation $\Gamma < 1$ and for longitudinal orientation $\Gamma > 1$. When $\Gamma = 1$, the surface pattern is said to be isotropic implying that there is no preferred direction. Most engineering surfaces are often assumed to be isotropic.

Figure 1 shows the different surface patterns with dashed lines representing the fluid path.

![Figure 4.1: Different surface patterns: (a) transverse ($\Gamma < 1$), (b) isotropic ($\Gamma = 1$) and (c) longitudinal ($\Gamma > 1$) [3].](image)

The surface roughness properties can be measured using a stylus profilometer. Alternatively, surface roughness can be numerically generated for specific values of $\Gamma$. Patir [4] developed a numerical algorithm for generating three-dimensional surface roughness for different values of $\Gamma$. 

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This algorithm was based on randomly generating surface roughness for surfaces with Gaussian and non-Gaussian height distributions.

In this research, using the load-sharing concept [5], the Strubeck curve is predicted for the non-conformal contact of rollers and conformal contact of a relatively heavily-loaded pin-bushing assembly. The load-sharing is a powerful tool for solving the lubricated contact of rough surfaces, which separately addresses the hydrodynamic film formation between surfaces and the contact of asperities. In order to use the load sharing concept, the surface roughness properties of the contacting surfaces are needed.

In this paper, for different values of surface pattern parameter Γ and for given values of standard deviation of asperity heights \(R_q\), the surfaces are numerically generated. The properties of the generated surfaces are calculated and used as input to predict the friction between the contacting surfaces (Strubeck curve). Effect of surface pattern on Stribeck surface which presents the simultaneous effect of load and speed on friction coefficient is also studied.

4.2 Model

4.2.1 Load-Sharing Concept

In this section, the load-sharing concept is briefly described. Moes [6] developed appropriate expressions for predicting the central film thickness for EHL problems assuming smooth surfaces. The non-dimensional film thickness \(H_c\) is:

\[
H_c = \left[ \left( H_{RI}^{7/3} + H_{EI}^{7/3} \right)^{3s/7} + \left( H_{RP}^{-7/2} + H_{EP}^{-7/2} \right)^{-2s/7} \right]^{1/s} \tag{4.1}
\]

where

\[
s = \frac{1}{5} \left( 7 + 8e^{\left( \frac{-2H_{RI}}{H_{RI}} \right)} \right) \tag{4.2}
\]

The non-dimensional parameters used are defined as [6]:

\[
H_{RI} = 3 \left( W U^{-0.5} \right)^{-1}
\]
Based on the load-sharing concept, the hydrodynamic film and asperities both contribute in carrying the total load $F_T$:

$$F_T = F_H + F_C$$  \hspace{1cm} (4.4)$$

In other words, a part of the total load $F_T / \gamma_1$ is carried by the fluid film and the rest of load, $F_T / \gamma_2$, is carried by asperities:

$$F_T = \frac{F_T}{\gamma_1} + \frac{F_T}{\gamma_2}$$  \hspace{1cm} (4.5)$$

The parameters $\gamma_1$ and $\gamma_2$ are called scaling factors and have the following relation:

$$1 = \frac{1}{\gamma_1} + \frac{1}{\gamma_2}$$  \hspace{1cm} (4.6)$$

The application of load sharing to Moes equation was first formulated in [7], experimentally verified in [8], and recently applied to gears in [9, 10]. Referring to [9, 10], the resulted equation for film thickness with consideration of surface roughness is:

$$H_c = \left[ \gamma_1^{s/2} \left( H_{RL}^{7/3} + \gamma_1^{14/15} H_{EI}^{7/3} \right)^{3s/7} + \gamma_1^{-s/2} \left( H_{RP}^{-7/2} + H_{EP}^{-7/2} \right)^{-2s/7} \right]^{1/s} \left( \gamma_1 \right)^{1/2}$$  \hspace{1cm} (4.7)$$

where
Friction force between contacting surfaces is a result of hydrodynamic film and the contact of asperities:

\[ F_f = F_{f,H} + F_{f,C} \]  

(4.9)

Friction coefficient \( f \) is calculated as:

\[ f = \frac{F_{f,H}}{F_T} + \frac{F_{f,C}}{F_T} \]  

(4.10)

Assuming all the asperities have the same friction coefficient \( f_c \), the friction coefficient \( f \) can be written as:

\[ f = \frac{F_{f,H}}{F_T} + \frac{f_c}{\gamma_2} \]  

(4.11)

The algorithm used in this paper for determining the scaling factors is similar to reference [8]. The operating conditions along with the surface roughness properties are used as input to the model. In reference [8], Greenwood-Williamson [11] formulation which corresponds to contact of one rough and one smooth surface was used whereas the Greenwood-Tripp [12] development—which enables formulating the contact problem of two rough surfaces—is employed in the present model.

According to Greenwood-Tripp formulation, when two rough surfaces contact, the asperity contact pressure \( P_C \) is calculated as [12]:

\[ P_C = \frac{8\sqrt{2}}{15 \pi D_{sum}} \beta^{1.5} R_q^{1.5} E_p F_{S/2} \left( \frac{h}{R_q} \right) \]  

(4.12)

where \( D_{sum} \) is the density of asperities, \( \beta \) is the radius of tip of asperities, \( R_q \) is the standard deviation of asperity heights, \( E_p \) is the equivalent modulus of elasticity and \( h \) is the thickness of the film that is formed between two surfaces. The function \( F_{S/2} \) is defined as:

\[ F_{S/2} \left( \frac{h}{R_q} \right) = \frac{1}{\sqrt{2\pi}} \int_{\frac{h}{R_q}}^{\infty} \left( S - \frac{h}{R_q} \right)^{2.5} e^{-0.5s^2} \, ds \]  

(4.13)
To simplify the integration in Equation (4.13), we will use the polynomial in Equation (4.14) which is curve fitted to the $F_{5/2}$ [13],

$$F_{5/2} = \begin{cases} 
4.4086 \times 10^{-5} (4 - \frac{h}{R_q})^{6.804} & \text{for } \frac{h}{R_q} < 4 \\
0 & \text{for } \frac{h}{R_q} \geq 4 
\end{cases}$$  \tag{4.14}

On the other hand, the central contact pressure $P_C$ in the contact of two rough surfaces was shown to be [9]:

$$P_C = \frac{1}{\gamma_2} \sqrt{\frac{F_T E_p}{2\pi B R}} \left[ 1 + \left( 1.30759 \left( D_{sum} Y_2 B \sqrt{\beta D_{sum}} \right)^{-0.0283} \left( \frac{R_q}{\beta} \right)^{-0.418} \left( \frac{F_T}{E_p R_B} \right)^{0.39} \right)^{-1.69} \right]^{-0.591}$$ \tag{4.15}

where $B$ is the width of the roller. To determine the values for scaling factors $\gamma_1$ and $\gamma_2$, an initial value for $\gamma_1$ is chosen and using Equation (4.6), the corresponding value for $\gamma_2$ is computed. Then, the film thickness is calculated from Equation (4.7) and used in Equation (4.12) to find the contact pressure $P_c$. The calculated contact pressure from Equation (4.12) is compared to the contact pressure calculated from Equation (4.15). The iterative for determining the scaling factors continues until the contact pressure calculated from Equations (4.15) and (4.12) become identical. Once the film thickness corresponding to this set of scaling factors is determined, the friction coefficient is computed, and the process is repeated for different operating conditions to plot the Stribeck curve.

In order to predict the friction coefficient, the Bair-Winer [14] model to calculate the shear stress of the lubricant is used. This model is based on typical pressures observed in EHL problems. Based on this model, the limiting shear stress in the lubricant film is:

$$\tau_L = \tau_0 + aP$$ \tag{4.16}

where $\tau_0$ is the shear stress at the ambient pressure, $a$ is a constant, and $P$ is the pressure. The friction coefficient can be written in the following form:

$$f = \frac{2b B \tau_L (1-\exp\left(\frac{\mu L}{kT}\right))}{F_T} + \frac{f_C}{\gamma_2}$$ \tag{4.17}
In Equation (4.17), $b$ is the half-width of Hertzian contact, $B$ is the width of the roller, $U$ is the speed, $h$ is the central film thickness, $\mu$ is the viscosity, $\tau_L$ is the limiting shear stress and $f_c$ is the friction coefficient between asperities. Alternatively, Gelinck and Schipper [7] applied the so-called Ree-Erying model to calculate the friction coefficient:

$$f = \frac{2bB\tau_0 \arcsinh\left(\frac{\mu U}{\tau_0 h}\right)}{F_T} + \frac{f_c}{\gamma_2}$$  (4.18)

In Greenwood-Tripp formulation [12] the contact of two rough surfaces requires values for density of asperities $D_{sum}$, radius of tip of asperities $\beta$ and standard deviation of asperity heights $R_q$. In this paper, a numerical algorithm based on the work of Patir [4] is used to generate the surfaces and to determine the asperity contact parameters. This algorithm is presented in the next section.

### 4.2.2 Surface Roughness Pattern

To numerically generate the surface roughness, the frequency density function and the autocorrelation function of the surface are required. The method introduced here has the capability of handling surfaces with different frequency density functions. The autocorrelation function $R$ for a homogenous surface — a surface whose statistical properties do not change with translation along the surface — is described as:

$$R(\lambda_x, \lambda_y) = E\{z(x, y)z(x + \lambda_x, y + \lambda_y)\}$$  (4.19)

In Equation (4.19), $E$ represents the expectancy, $z$ denotes the asperities heights, $\lambda_x$ and $\lambda_y$ are the auto-correlation lengths in the $x$ and $y$ direction, respectively. $R(0,0) = R_q^2$ where $R_q$ is the standard deviation of surface heights. Let $z_{ij}$ denote the roughness amplitude at $x = i\Delta x$ and $y = j\Delta y$ where $\Delta x$ and $\Delta y$ are the sampling intervals in $x$ and $y$ directions, respectively. Then at each location, $R_{mn}$ is defined as:

$$R_{mn} = R(m\Delta x, n\Delta y) = E(z_{ij}z_{i+m,j+n})$$  (4.20)
In this paper, an auto-correlation matrix of order \( r \times s \) is formed such that \( R_{mn} \) is zero if \( m \geq r \) or \( n \geq s \), where \( m \) and \( n \) are the indices for the auto-correlation function. The expression for the auto-correlation function for surface generation is [4]:

\[
R_{mn} = R_q^2 \left(1 - \frac{m}{r}\right) \left(1 - \frac{n}{s}\right) \quad m \leq r, \quad n \leq s \tag{4.21}
\]

where

\[
R_{mn} = 0 \text{ if } m \geq r \text{ or } n \geq s \tag{4.22}
\]

As the arguments of the auto-correlation function increases, \( R_{mn} \) gradually decreases and approaches to zero. The correlation length of a profile in the x and y direction is denoted by \( \lambda^*_x \) and \( \lambda^*_y \) respectively. The correlation length is defined as the length at which the auto-correlation function becomes zero:

\[
\lambda^*_x = r \Delta x \tag{4.23}
\]

\[
\lambda^*_y = s \Delta y
\]

The degree of non-isotropy of the surface is shown by \( \Gamma \) and is the ratio of correlation lengths in the x and y direction:

\[
\Gamma = \frac{\lambda^*_x}{\lambda^*_y} \tag{4.24}
\]

Most engineering surfaces have a Gaussian asperity height distribution. The generation of Gaussian surface which has an auto-correlation function as Equation (4.21) is accomplished by using the formulation introduced in Equation (4.25) [3]:

\[
z_{ij} = \frac{R_q}{\sqrt{rs}} \sum_{k=1}^{r} \sum_{l=1}^{s} \eta_{i+k,j+l} \quad i = 1, 2, \ldots, N \quad j = 1, 2, \ldots, M \tag{4.25}
\]

where \( \eta_{ij} \) are mutually-independent, normally-distributed random numbers with zero mean and unit variance. To generate a surface with Gaussian height distribution, these random numbers should also have a Gaussian distribution. The surface pattern parameter \( \Gamma \) as well as the size of auto-correlation matrix and the standard deviation of asperity heights \( R_q \) are input variables to the
algorithm in order to generate the asperity heights using Equations (4.21)-(4.25). Once the
surface roughness is generated, the next step is to evaluate the surface roughness properties
which are required to predict the Stribeck curve. These parameters are standard deviation of
asperity heights $R_q$, density of asperities $D_{sum}$ and radius of tip of asperities $\beta$ which can be
calculated from spectral moments of the surface shown in Equation (4.26) [15]:

$$m_0 = E(Z^2) = R_q^2$$
$$m_2 = E \left[ \left( \frac{dz}{dx} \right)^2 \right]$$
$$m_4 = E \left[ \left( \frac{d^2z}{dx^2} \right)^2 \right] \quad (4.26)$$

Equation (4.25) shows the spectral moments for an isotropic surface. If the surface is
anisotropic, there exist two principal directions along which the profile value of $m_2$ is a minimum
and maximum. The equivalent value of $m_{2e}$ for an equivalent isotropic surface can be calculated
as [16]:

$$m_{2e} = \sqrt{m_{2max}m_{2min}} \quad (4.27)$$

The $m_4$ values in the same two directions are calculated in the same way as Equation (4.27) to
give $m_{4e}$. Once the equivalent moments for each surface are calculated, the total moments are
calculated as [16]:

$$m_{0total} = m_{0surface_1} + m_{0surface_2}$$
$$m_{2total} = m_{2esurface_1} + m_{2esurface_2} \quad (4.28)$$
$$m_{4total} = m_{4esurface_1} + m_{4esurface_2}$$

The density of asperities, radius of asperity tips and standard deviation of asperity heights are
calculated from the following equations [16]:

$$D_{sum} = \frac{m_{4total}}{6\pi \sqrt{3m_{2total}}}$$
$$\beta = \frac{3}{8} \sqrt{\frac{\pi}{m_{4total}}} \quad (4.29)$$
\[ R_q = \sqrt{m_0} \epsilon_{\text{total}} \]

These properties are used in Equations (4.12) and (4.15) to calculate the contact pressure. Then using the algorithm discussed in the previous section, the scaling factors \( \gamma_1 \) and \( \gamma_2 \), film thickness \( h \), and friction coefficient \( f \) are predicted.

4.3 Results and Discussion

4.3.1 Surface Generation

In this section, the results of a series of simulations for predicting the Stribeck curve are presented. The parameter \( r \) and \( s \) which represent the size of the auto-correlation function matrix as well as surface pattern parameter \( \Gamma \) are input to the algorithm. Assuming \( r=60 \) and \( s=60 \), representative surface patterns were generated for transverse \( \Gamma = 0.2 \), isotropic \( \Gamma = 1 \) and longitudinal \( \Gamma = 3 \) with \( R_q = 0.1 \mu m \) and the surfaces are shown in Figures 4.2, 4.3 and 4.4.

Figures 4.2, 4.3 and 4.4 present surface roughness for transverse, isotropic and longitudinal orientations, respectively. In each figure, the sliding direction is shown. The difference in asperity orientation in longitudinal and transverse case can be easily observed. No preferred direction can be recognized for the isotropic surface.

4.3.2 Comparison to Other Published Works

- Non-Conformal Contact

In this part of the paper, a comparison of the predicted friction coefficient for the case of non-conformal contact of rollers to simulation results of Gelinck and Schipper [7] is made. In [7] a “generalized Stribeck curve” is presented in the form of the friction coefficient versus a non-dimensional lubrication parameter \( L \) defined as:

\[ L = \frac{\mu(U_1 + U_2)}{P \sqrt{R_{a1}^2 + R_{a2}^2}} \]  
(4.30)

where \( U_1 \) and \( U_2 \) are the velocities of the rollers and \( R_{a1} \) and \( R_{a2} \) are the arithmetic average of asperity heights for surfaces 1 and 2, respectively.
The operating conditions for this problem are reported in Table 4.1.

Figure 4.2: Generated surface roughness for transverse surface pattern ($\Gamma = 0.2$, $R_q = 0.1\mu m$).

Figure 4.3: Generated surface roughness for isotropic surface pattern ($\Gamma = 1$, $R_q = 0.1\mu m$).
Figure 4.4: Generated surface roughness for longitudinal surface pattern ($\Gamma = 3$, $R_q = 0.1 \mu m$).

<table>
<thead>
<tr>
<th>Operating condition [7]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent radii of curvature of rollers $R = 0.02$ m</td>
</tr>
<tr>
<td>Width of the roller $B = 0.01$ m</td>
</tr>
<tr>
<td>Equivalent Young’s modulus $E’ = 231$ GPa</td>
</tr>
<tr>
<td>Load $F_{T} = 500$ N</td>
</tr>
<tr>
<td>Friction coefficient between asperities $f_c = 0.13$</td>
</tr>
<tr>
<td>Viscosity of the oil $\mu = 0.02$ Pa.s</td>
</tr>
<tr>
<td>Non-dimensional load number $W = 1.08 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

In order to observe the effect of surface pattern, three surfaces are generated using the presented algorithm. Figure 4.5 shows the comparison between the predicted results of [7] and the model used here for three different generated surfaces $\Gamma = 0.2$, $\Gamma = 1$, and $\Gamma = 3$ assuming $R_q = 0.05 \mu m$. For comparison purpose, the fluid is assumed to obey Equation (4.18) and the contact pressure was calculated based on Greenwood-Williamson formulation [11].

As the value of lubrication number increases, the friction coefficient decreases. This decrease is due to increase in the thickness of the fluid film and the associated decrease in the contribution of asperities in carrying the load. The decrease in friction coefficient continues until the lubrication regime shifts from mixed to full EHL where the entire load is carried by fluid film.
According to Figure 4.5, transverse surface shows a higher friction coefficient compared to isotropic and longitudinal. This difference can be explained by the difference in the asperity orientation for each surface. The asperity orientation in transverse surfaces impedes the flow of fluid film, resulting in an increase in asperity-asperity contact and, hence, an increase in friction coefficient. According to Figure 4.5, the generalized Striebeck curve for the generated surfaces is fairly close to the prediction of Gelinck and Schipper [16].

- **Conformal Contact**

In this section, the predicted Striebeck curve for a pin-bushing assembly is compared to the experimental data of Lu et al. [8]. The pin-bushing studied in this paper usually run under relatively high pressure (on the order of 100 MPa) and small speeds (0.05 m/s). Under these operating conditions, pressure is much greater than typical hydrodynamic bearings and the elastic deformation of the surfaces becomes significant and, at the same time, asperity contact is expected to occur.

![Figure 4.5: Effect of surface pattern on generalized Striebeck curve.](image)

Experiment in [8] was conducted by maintaining the constant load and changing the speed of shaft which is rotating uni-directionally. At each speed, the system was allowed to reach the steady state and then the friction coefficient was recorded. The geometry of the shaft and bushing are tabulated in Table 4.2.
Table 4.2: Shaft and Bushing properties [8]

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft diameter</td>
<td>$R_s = 24.54$ mm</td>
</tr>
<tr>
<td>Bushing inner diameter</td>
<td>$R_b = 24.71$ mm</td>
</tr>
<tr>
<td>Bearing length</td>
<td>$L = 25.4$ mm</td>
</tr>
<tr>
<td>Shaft Young’s modulus</td>
<td>$E_s = 209$ GPa</td>
</tr>
<tr>
<td>Bushing Young’s modulus</td>
<td>$E_b = 100$ GPa</td>
</tr>
<tr>
<td>Shaft’s Poisson’s ratio</td>
<td>$\nu_s = 0.29$</td>
</tr>
<tr>
<td>Bushing’s Poisson’s ratio</td>
<td>$\nu_b = 0.33$</td>
</tr>
<tr>
<td>Asperity friction coefficient</td>
<td>$f_c = 0.2$</td>
</tr>
<tr>
<td>Viscosity at 40$^\circ$ C</td>
<td>$\mu_{40} = 93$ cSt</td>
</tr>
<tr>
<td>Viscosity at 100$^\circ$ C</td>
<td>$\mu_{100} = 10.8$ cSt</td>
</tr>
<tr>
<td>Specific gravity at 15$^\circ$C</td>
<td></td>
</tr>
<tr>
<td>Viscosity at ambient conditions</td>
<td>$\eta_0 = 0.0815$ Pa.s</td>
</tr>
<tr>
<td>Limiting shear stress at ambient pressure</td>
<td>$\tau_{l,0} = 2.5 \times 10^6$ Pa</td>
</tr>
<tr>
<td>Slope of limiting shear stress and pressure</td>
<td>$\beta_0 = 0.047$</td>
</tr>
</tbody>
</table>

For the surface roughness properties, an isotropic surface with $R_q$ equal to the $R_q$ of the pin-bushing assembly used in the experiment was generated based on the numerical method discussed in this paper. The resulted roughness properties for these surfaces are:

Table 4.3: Surface roughness data for isotropic surface pattern ($I = 1$)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of asperities</td>
<td>$D_{sum} = 1.559 \times 10^{12}$ m$^2$</td>
</tr>
<tr>
<td>Radius of tip of asperities</td>
<td>$\beta = 5.89 \times 10^{-7}$ m</td>
</tr>
<tr>
<td>Standard deviation of asperity heights</td>
<td>$R_q = 0.35$ $\mu$m</td>
</tr>
</tbody>
</table>

The comparisons which correspond to different inlet oil temperature are shown in Figures 4.6 and 4.7. In pin-bushings, characterization of friction is usually represented in form of Strubeck curve [1], which presents the friction coefficient versus the non-dimensional Sommerfeld number:

$$ S = \frac{P}{\omega \mu} \left( \frac{c}{R_s} \right)^2 $$ 

where $P$ is the pressure, $\omega$ is the rotational speed of the shaft, $\mu$ is the viscosity, $c$ is the clearance between shaft and bearing and $R_s$ is the radius of the shaft. At low values of Sommerfeld number, due to small velocities, the lubricant film that is formed is not thick enough
to protect the surfaces from contacting. Therefore, contact of asperities occurs and high value of friction coefficient is observed. As speed increases, a better film is formed resulting in lower friction coefficient.

**Figure 4.6: Comparison of predicted and measured friction coefficient for oil inlet temperature of 40°C for an isotropic surface.**

As shown in Figures 4.6 and 4.7, the predicted Strubeck curve with isotropic generated surface is fairly close to the experimental results of Lu et al [8].

**Figure 4.7: Comparison of predicted and measured friction coefficient for oil inlet temperature of 60°C for an isotropic surface.**

The film parameter, $\Lambda$ often used in the tribology literature to distinguish different lubrication regimes is defined as:
\[ \Lambda = \frac{h_C}{\sqrt{R_{q1}^2 + R_{q2}^2}} \]  

(4.32)

where \( h_C \) is the film thickness and \( R_{q1} \) and \( R_{q2} \) are the standard deviation of asperity heights for surfaces 1 and 2, respectively. Figure 4.8 shows variation of film parameter with rotational speed for different oil inlet temperatures. The applied load was assumed to be \( F_T = 667 \) N for all cases and surfaces were assumed to be isotropic with surface properties reported in Table 4.3. For each inlet oil temperature, as rotational speed increases, a thicker film is formed which results in an increase in film parameter. At a constant rotational speed, as the inlet oil temperature increases, the viscosity drops which results in a smaller film thickness and hence smaller film parameter. It is interesting that for this specific operating conditions, for oil inlet temperature of 40°C, the hydrodynamic regime is achieved when the shaft speed reaches to \( \omega = 15 \) rpm while for oil temperature of 60°C, a shaft speed of \( \omega = 50 \) rpm is required for obtaining hydrodynamic lubrication regime. As the oil temperature rises, its viscosity decreases and a smaller film is formed. This decrease can be compensated by increasing rotational speed.

![Figure 4.8: Variation of film parameter for isotropic generated surface.](image)

- **Effect of Surface Pattern**

Having verified the validity of simulations, the next step is to investigate the effect of surface pattern on the Strubeck curve for the pin-bushing assembly. For this purpose, surfaces with
different non-isotropy values \( \Gamma \) were generated. The film thickness that is formed for each surface is shown in Figure 4.9.

**Figure 4.9: Effect of surface pattern on film thickness**

For a transverse surface, the asperities orientation impedes the lubricant and causes a higher film to form. On the other hand, in longitudinal surfaces, the asperity orientation is parallel to the lubricant flow and results in formation of a thinner lubricant film. The friction coefficient was calculated for each value of \( \Gamma \). Figure 4.10 shows variation of friction coefficient for surfaces with different surface patterns.

**Figure 4.10: Effect of surface pattern on friction coefficient.**

The results of Figure 4.10 are consistent with Figure 4.5 which investigates the effect of surface pattern for non-conformal contacts. Surfaces with transverse patterns show higher friction coefficient than surfaces with longitudinal pattern.
- **Stribek Surface**

Another way of looking at variation of friction in pin-bushing is to simultaneously change the load and rotational speed of shaft and calculate the friction coefficient. This will lead to the Stribeck surface shown in Figure 4.11 for surfaces with different surface patterns. The pattern of the Stribeck surface shown here is consistent with the results of Wang et al. [17].

At very low speeds for all the values of applied load, friction coefficient is almost constant. The reason is that at low speeds a very large portion of load is carried by asperities and hydrodynamic part plays an insignificant role leading to constant friction coefficient with variable load. Also the plot indicates that at higher loads the transition from mixed lubrication to full film lubrication regime occurs at higher speeds.

![Figure 4.11: Stribeck surfaces for different surface patterns.](image)

For equal applied load, the transition from mixed regime to full film occurs at lower speeds for longitudinal surface patterns compared to isotropic and transverse surfaces. For equal rotational speeds, the friction coefficient increases as the applied load increases. This increase is more noticeable in the speeds close to the lift-off speed.
4.4 Conclusions

A model based on the load-sharing concept was developed to predict the Strieber curve for a pin-bushing system. A numerical algorithm was developed to generate isotropic, transverse and longitudinal surfaces. The generated surfaces are used to calculate the surface roughness properties which are required for predicting the Strieber curve. The comparisons of non-conformal contact between two-cylindrical rollers with other published works and the predicted friction coefficient of a pin-bushing assembly with experimental values of friction coefficient confirm the validity of the approach. The study on the effect of surface pattern on Strieber curve reveals that transverse surfaces show a higher friction coefficient than isotropic and longitudinal surfaces because in the surfaces with transverse pattern the asperities orientation is such that it impedes the fluid film and causes more asperity-asperity contact. Comparison of film thickness for different surface patterns shows that transverse surfaces have a higher film-forming capacity than isotropic and longitudinal surfaces. Comparing the effect of rotational speed on film parameter for two different oil inlet temperatures show that as rotational speed increases a better film is formed and therefore the film parameter increases. For the pin-bushing assembly, the Strieber surfaces which have the simultaneous effect of load and speed have been shown for different surface patterns. In the generalized Strieber curve for the non-conformal contact, the lift-off phenomenon can be identified. In transverse surface patterns, lift-off occurs at larger lubrication number compared to isotropic and longitudinal surfaces.

4.5 Nomenclatures

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>Hertzian half-width of contact (m)</td>
</tr>
<tr>
<td>B</td>
<td>Pin-bushing length (m)</td>
</tr>
<tr>
<td>c</td>
<td>Clearance (m)</td>
</tr>
<tr>
<td>D_{sum}</td>
<td>Density of asperities (1/m^2)</td>
</tr>
<tr>
<td>E(x)</td>
<td>Expectation of x</td>
</tr>
<tr>
<td>E_b</td>
<td>Young’s modulus of bushing (N/m^2)</td>
</tr>
<tr>
<td>E_s</td>
<td>Young’s modulus of shaft (N/m^2)</td>
</tr>
<tr>
<td>E_p</td>
<td>Equivalent Young’s modulus</td>
</tr>
<tr>
<td>f</td>
<td>Friction coefficient</td>
</tr>
<tr>
<td>f_c</td>
<td>Friction coefficient between asperities</td>
</tr>
<tr>
<td>F_C</td>
<td>Load carried by asperities (N)</td>
</tr>
<tr>
<td>F_f</td>
<td>Friction force</td>
</tr>
<tr>
<td>F_{f,H}</td>
<td>Friction force due to hydrodynamic film (N)</td>
</tr>
<tr>
<td>F_{f,c}</td>
<td>Friction force due to asperity contact (N)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$F_H$</td>
<td>Load carried by hydrodynamic film (N)</td>
</tr>
<tr>
<td>$F_T$</td>
<td>Total load (N)</td>
</tr>
<tr>
<td>$h(x)$</td>
<td>Film thickness (m)</td>
</tr>
<tr>
<td>$H_c$</td>
<td>Non-dimensional film thickness</td>
</tr>
<tr>
<td>$L$</td>
<td>Lubrication number</td>
</tr>
<tr>
<td>$m_0$</td>
<td>Zeroth moment of the surface</td>
</tr>
<tr>
<td>$m_2$</td>
<td>second moment of the surface</td>
</tr>
<tr>
<td>$m_4$</td>
<td>fourth moment of the surface</td>
</tr>
<tr>
<td>$p$</td>
<td>Index for auto-correlation function</td>
</tr>
<tr>
<td>$q$</td>
<td>Index for auto-correlation function</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure (Pa)</td>
</tr>
<tr>
<td>$P_c$</td>
<td>Contact pressure (Pa)</td>
</tr>
<tr>
<td>$P_{Hertz}$</td>
<td>Hertzian pressure (Pa)</td>
</tr>
<tr>
<td>$R$</td>
<td>Auto-correlation function</td>
</tr>
<tr>
<td>$R_{as}$</td>
<td>Arithmetic average of asperity heights of shaft (m)</td>
</tr>
<tr>
<td>$R_{ab}$</td>
<td>Arithmetic average of asperity heights of bushing (m)</td>
</tr>
<tr>
<td>$R_{mn}$</td>
<td>Value of auto-correlation function at point p, q</td>
</tr>
<tr>
<td>$R_q$</td>
<td>Standard deviation of asperity heights (m)</td>
</tr>
<tr>
<td>$R_s$</td>
<td>Shaft radius (m)</td>
</tr>
<tr>
<td>$R_b$</td>
<td>Bushing radius (m)</td>
</tr>
<tr>
<td>$R'$</td>
<td>Equivalent radius of curvature (m)</td>
</tr>
<tr>
<td>$S$</td>
<td>Sommerfeld number</td>
</tr>
<tr>
<td>$z_{ij}$</td>
<td>Asperity heights (m)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Pressure-viscosity constant (1/Pa)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Radius of tip of asperities</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>Slope of limiting shear stress and pressure</td>
</tr>
<tr>
<td>$\mu_{40}$</td>
<td>Viscosity at 40°C (cSt)</td>
</tr>
<tr>
<td>$\mu_{100}$</td>
<td>Viscosity at 100°C (cSt)</td>
</tr>
<tr>
<td>$\eta_{ij}$</td>
<td>random numbers with zero mean and unit variance</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>Scaling factor for asperity parts</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>Scaling factor for hydrodynamic part</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Surface pattern parameter</td>
</tr>
<tr>
<td>$\lambda_x$</td>
<td>Delay length in x direction</td>
</tr>
<tr>
<td>$\lambda_y$</td>
<td>Delay length in y direction</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Film parameter</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Rotational speed (rpm)</td>
</tr>
<tr>
<td>$\nu_s$</td>
<td>Poisson ratio of shaft</td>
</tr>
<tr>
<td>$\nu_b$</td>
<td>Poisson ratio of bushing</td>
</tr>
<tr>
<td>$\tau_L$</td>
<td>Limiting shear stress at pressure $p$ (Pa)</td>
</tr>
<tr>
<td>$\tau_0$</td>
<td>Limiting shear stress at ambient pressure (Pa)</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>Sampling interval in x direction</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>Sampling interval in y direction</td>
</tr>
</tbody>
</table>
4.6 References


Chapter 5: On the Prediction of Running-In Behavior in Mixed-Lubrication Line Contact
5.1 Introduction

Running-in is a transient process occurring between contacting fresh surfaces that are exposed to relative rolling/sliding motion. It is often accompanied by transitions in tribological parameters such as coefficient of friction, wear rate, etc. During the running-in process, the surface characteristics of the mating surfaces continuously change as they undergo relative motion and the process continues until the surfaces are fully broken in. Noticeably, the surface asperities with high slopes and short wavelengths plastically deform and wear away under physical contact [1].

The ability to predict the characteristics of the running-in process, its duration, the effect of surface finish, the wear volume generated, and friction coefficient are all of great interest in the design process of tribological components. It is however a difficult process to model because of the transient effects involved.

To begin with, let us briefly review the parameters involved in the steady-state wear process. One of the earliest works in predicting wear was conducted by Holm [2]. He suggested that the worn volume $V$ is proportional to the applied load $F_T$, sliding distance $s$, and inversely proportional to the hardness of the softer material $H$:

$$V = k \frac{F_T s}{H}$$

(5.1)

where $k$ is the proportionality constant. Burwell and Strang [3] modified Equation (1) in terms of relating wear depth $d$ to the applied pressure $P$:

$$d = k \frac{Ps}{H}$$

(5.2)

According to Archard [4], the wear rate $\dot{W}$ is proportional to the load, i.e., $\dot{W} \propto P$, and thus the worn volume is proportional to the cubic of the radius of contact area, $V \propto a^3$. Archard also claimed that the wear coefficient, $k$, is inversely proportional to the number of cycles for a wear fragment to be formed. Equation (1) —commonly referred to as Archard’s equation— is widely
used in prediction of wear. Predictions of wear using Equation (1) match experimental results with reasonable accuracy provided that the value for the wear coefficient $k$ is appropriately selected [5-10].

As surfaces undergo rolling/sliding contact, asperities undergo elastic and plastic deformations. Stolarski [11] decomposed the wear coefficient $k$ into an elastic $k_e$ and plastic $k_p$ components. Rowe [12] introduced a so-called fractional-film defect parameter, $\alpha$, into Archard’s equation to correlate wear to the effectiveness of lubricant. He rewrote Equation (5.1) as:

$$V = k_m \alpha s^p_H$$

(5.3)

where $k_m$ is a dimensionless constant. It is specific to the rubbing materials but independent of the lubricant used. The fractional film defect, $\alpha$, is defined as the ratio of the number of sites on the friction surface unoccupied by molecules, $N_m$, divided by the total number of sites on the friction surface, $N_r$, viz.

$$\alpha = \frac{N_m}{N_r}$$

(5.4)

The wear prediction approach based on including the fractional-film defect in wear coefficient was later used by many researchers for predicting “steady state” wear in rolling/sliding systems [13-15]. During the running-in period, however, the surface properties such as arithmetic average of asperity heights, $R_a$, and asperity height distribution vary with time. Therefore, to model the running-in process in rolling/sliding contacts, appropriate modifications must be made to incorporate the variation of the asperity height distribution.

Several models are available for predicting the behavior of the rolling/sliding systems during “transient wear.” Lin and Cheng [16] postulated that the value of wear rate during running-in is proportional to a forcing term, $I$, and is inversely proportional to anti-wear strength of material, $S_m$. As running-in progresses, the asperity height distribution changes. Hence, anti-wear strength
and shear force vary. Lin and Cheng compared the results of this dynamic wear model with the experimental work of Stout et al. [17] as well as analytical work of Sugimura et al. [18], and reported reasonably close agreement. Sugimura’s work was based on the assumption that the surfaces have stationary random roughness and that the micro-topographical changes are only due to wear with resulting wear particles having a rectangular shape. Hu et al. [19] investigated the effect of surface roughness on the dynamic behavior of a lubricated sliding wear system. Blau [20] proposed a model assuming that friction coefficient during running-in is a function of lubrication effect, initial material’s deformation, and variability in friction coefficient. Kumar and Sethuramian [21] developed a model based on the experimentally determined parameters such as wear rate as a function of time to predict the running-in duration and steady-state wear rate. Wang and Wong [22] developed a curve-fit relationship to relate the worn volume to the change in arithmetic average of surface roughness $R_a$. They numerically generated several surfaces with different properties and simulated wear by removing successive layers of the contacting surfaces. They showed that the variation of wear volume and the change of average roughness can be described by a second order polynomial. They also verified their model with comparing the predicted values of wear volume to experimental results [23]. Later, they extended their work to propose a dynamic model for running-in phenomenon [24]. By incorporating the lubrication analysis in partial EHL algorithm, their model had the capability to predict the arithmetic average of surface roughness, $R_a$, as a function of time.

Many of the existing running-in models are limited to specific pair of sliding materials. In this paper, we present a model for predicting the behavior of running-in process in mixed-lubrication line contact problems which uses the plastic deformation of asperities in conjunction with the mixed-lubrication load-sharing concept. The predicted results for variation of arithmetic average of asperity heights are compared to the experimental results of [24] and the results are shown to be reasonably close to the experimental data. We present the results of an extensive set of
simulations that provide insight into the behavior of surfaces with different initial roughness pattern as they undergo the running-in process.

5.2 Running-In Model

5.2.1 Problem Statement

Consider the lubricated contact of two rollers with radii $R_1$ and $R_2$ and moduli of elasticity of $E_1$ and $E_2$ and Poisson’s ratio of $\nu_1$ and $\nu_2$ under the load $F_T$. The initial roughness profiles for these rollers are $z_1(x)$ and $z_2(x)$, respectively. Figure 5.1 shows the schematic of the contact.

![Schematic of the contact problem](image)

**Figure 5.1: schematic of the contact problem**

The equivalent radius of curvature $R'$, modulus of elasticity $E_p$, and surface roughness profile $z$ are defined as follows:

\[
\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} \\
\frac{2}{E_p} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \\
z(x) = z_1(x) + z_2(x)
\]  

(5.5)

The approach for studying the running-in behavior is based on the plastic failure of asperities. As the rough surfaces slide over each other, the asperities undergo elastic and plastic deformation. Over time, the height of the asperities, the percentage of the load carried by the fluid film and the asperities, as well as the friction coefficient vary as the surfaces tend to break-
in gradually until they reach a “steady-state.” At this stage, the performance parameters of the system—friction coefficient, wear-rate, etc.—stabilize. The duration of running-in depends on the operating conditions such as load and speed, as well as the lubricant properties, film thickness, the initial surface roughness, and their orientation.

5.2.2 Asperity Contact Model

In this section, we present how the contact of asperities during running-in is modeled. Figure 5.2 shows a schematic of the contact of an asperity of tip radius $\beta$ with a rigid, smooth flat. The interference $w$ between an asperity and a rigid smooth plane is defined as:

$$w = z - h_c + y_s$$  (5.6)

where $z$ denotes the height of asperity, $h_c$ is the central film thickness and $y_s$ is the distance between the mean of asperity heights and mean of surface heights.

![Figure 5.2: Schematic of plastic contact of asperities.](image)

The interference is a measurement of the extent of the asperity deformation. As $w$ increases, the asperity experiences three different deformation regimes: elastic, elasto-plastic and plastic [25]. The critical indentation of an asperity at the point of initial yielding is defined as [26]:

$$w_e = \left(0.94 \frac{H}{E_p}\right)^2 \beta$$  (5.7)

where $H$ is the hardness of the softer material and $E_p$ is the equivalent modulus of elasticity. The contact interference at the beginning of fully plastic deformation is [27]:

$$w_p = 54w_e$$  (5.8)
If the asperity interference is between $w_e$ and $w_p$, then the asperity is in elasto-plastic deformation regime. The asperity contact area and the load carried by each asperity are dependent upon their respective deformation regime. If the interference for an asperity is negative, then the asperity will not contribute to carrying the load. If the interference for an asperity $i$ is in the elastic range, then the contact area and associated load support are given by the following expressions [28]:

$$A_{ie} = \pi \beta_i w_i$$  \hspace{1cm} (5.9)

$$F_{ie} = \frac{4}{3} E \beta_i^{0.5} w_i^{0.5}$$  \hspace{1cm} (5.10)

For an asperity in elasto-plastic regime, the contact area and the load support expressions are [28]:

$$A_{iep} = \pi \beta_i \omega_i \left( 1 - 2 \left( \frac{w_i - w_e}{w_p - w_e} \right)^3 + 3 \left( \frac{w_i - w_e}{w_p - w_e} \right)^2 \right)$$  \hspace{1cm} (5.11)

$$F_{iep} = \left( H - 0.6H \ln \frac{w_p - w_i}{w_p - w_e} \right) A_{iep}$$  \hspace{1cm} (5.12)

Finally, for an asperity which is plastically deformed, we have [28]:

$$A_{ip} = 2\pi \beta_i w_i$$  \hspace{1cm} (5.13)

$$F_{ip} = HA_{ip}$$  \hspace{1cm} (5.14)

Therefore, the total load carried by asperities and the real area of contact are:

$$F_c = \sum_{i=1}^{n} F_{ei} + F_{epl} + F_{pi}$$  \hspace{1cm} (5.15)

$$A_c = \sum_{i=1}^{n} A_{ei} + A_{epl} + A_{pl}$$  \hspace{1cm} (5.16)

### 5.2.3 Film Thickness

Let us now turn our attention to the lubricant film thickness and its relationship with surface roughness. As was shown in the previous section, the film thickness has a significant effect on the interference of asperities. Moes [29] developed expressions for the EHL film thickness in non-conformal line contact problem without considering the effect of surface roughness. According to his derivations, the central film thickness in dimensionless form can be expressed as follows.
\( H_c = \left[ \left( H_{RI}^{7/3} + H_{EI}^{7/3} \right)^{3s/7} + \left( H_{RP}^{-7/2} + H_{EP}^{-7/2} \right)^{-2s/7} \right]^{1/s} \) \hspace{1cm} (5.17)

where

\( s = \frac{1}{5} \left( 7 + 8e \left( -\frac{2H_{RI}}{H_{RI}} \right) \right) \) \hspace{1cm} (5.18)

The non-dimensional parameters used are defined as [29]:

\[
H_{RI} = 3(WU_{\Sigma}^{-0.5})^{-1}; \quad H_{EI} = 2.621(WU_{\Sigma}^{-0.5})^{-0.2}; \quad H_{RP} = 1.287(GU_{\Sigma}^{-0.25})^{\frac{2}{3}};
\]

\[
H_{EP} = 1.311(WU_{\Sigma}^{-0.5})^{-\frac{1}{2}}(GU_{\Sigma}^{-0.25})^{\frac{3}{2}}
\]

\( W = \frac{F_T}{E_{PR} \text{R}^2}; \quad G = \alpha E_p; \quad U_{\Sigma} = \frac{\eta_0 u}{E_{PR} \text{R}^2}; \quad H_C = \frac{h_c}{R} U_{\Sigma}^{-0.5} \)

To take surface roughness into account, Gelinck and Schipper [30] applied the load-sharing concept pioneered by Johnson [31] to film thickness equation derived by Moes. An experimental validation of this approach is reported by Lu et al. [32] and later applied to analyze the lubrication of gears [33,34] and to study the general characteristics of Stribeck curve [35].

The crux of the load-sharing concept is that the total load \( F_T \) is carried partly by hydrodynamic film \( F_H \) and partly by asperities \( F_C \):

\( F_T = F_H + F_C \) \hspace{1cm} (5.20)

The contribution of each part is represented by scaling factors \( \gamma_1 \) and \( \gamma_2 \). In other words it is assumed that:

\( F_T = \frac{F_T}{\gamma_1} \frac{F_T}{\gamma_2} \) \hspace{1cm} (5.21)

Equation (5.21) can be simplified as:

\( 1 = \frac{1}{\gamma_1} + \frac{1}{\gamma_2} \) \hspace{1cm} (5.22)

The problem is then divided into two sections. The contact of two rollers with equivalent modulus of elasticity of \( E_p / \gamma_1 \) under the applied load of \( F_T / \gamma_1 \) is solved to find the film thickness \( h_c \). The second part of the problem is the contact of two rollers with modulus of elasticity of \( E_p / \gamma_2 \) under the applied load equal to \( F_T / \gamma_2 \). Replacing the applied load by \( F_T / \gamma_1 \) and the modulus of elasticity by \( E_p / \gamma_1 \) in Equation (5.17), the modified form for film thickness will be:
\[ H_c = \left[ \gamma_1^{s/2} \left( H_{RI}^{7/3} + \gamma_1^{14/15} H_{EI}^{7/3} \right)^3 s/7 + \gamma_1^{-s/2} \left( H_{RP}^{-7/2} + H_{EP}^{-7/2} \right)^{-2s/7} \right]^{1/s} (\gamma_1)^{1/2} \]  

(5.23)

where

\[ s = \frac{1}{5} \left( 7 + 8e^{-2(y_1)^{-2/5}H_{RI}} \right) \]  

(5.24)

Once \( H_c \) is determined, the dimensional film thickness \( h_c \) is calculated using Equation (5.19). The film thickness predicted in Equation (5.19) is based on the fluid temperature at the inlet. In order to correct this film thickness, the thermal reduction factor developed by Hsu and Lee [36] is employed. They solved the coupled Reynolds and energy equation and developed a correlation formula for the ratio of thermal film thickness to the isothermal film thickness as a function of dimensionless load, dimensionless materials parameter and slip ratio.

\[ C_t = \frac{h_{thermal}}{h_{isothermal}} = \frac{1}{1 + 0.0766G^{0.683}W^{0.447}T^{0.527} e^{0.578fr}} \]  

(5.25)

In this equation, \( W \) is the dimensionless load, \( sr \) is the slide to roll ratio, \( G \) is the dimensionless material parameter and \( TL \) is the thermal loading parameter:

\[ TL = \frac{\mu_0 \gamma u_{rolling}^2}{K_f} \]  

(5.26)

In Equation (5.26), \( \mu_0 \) is the viscosity at ambient pressure and temperature, \( \gamma \) is the temperature-viscosity coefficient of lubricant, \( u_{rolling} \) is the rolling velocity and \( K_f \) is the thermal conductivity of the lubricant. The interference for each asperity can then be calculated using Equation (5.6).

5.2.4 Surface Roughness Profile

One of the requirements of the method described in Section 5.2.3 is specification of the surface roughness. One can measure the surface profile of the sample using a profilometer and evaluate the roughness properties. Alternatively, the surface profile can be numerically generated. In this paper a numerical algorithm based on the work of Patir [37] is used to generate the surface roughness. This algorithm was recently used to characterize the behavior of Stribeck curve in
different types of contacts with experimental verification [35]. A brief description of the method is as follows:

The surface roughness generation requires the height distribution, the auto-correlation function, standard deviation of asperity heights, and the surface pattern parameter $\Gamma$ as input. The height distribution in this analysis is assumed to be Gaussian. The roughness profile is generated using an auto-correlation matrix of size $r \times s$ with the following form for each location $m$ and $n$:

$$R_{mn} = \begin{cases} R_q z \left(1 - \frac{m}{r}\right) \left(1 - \frac{n}{s}\right), & m < r \text{ and } n < s \\ 0, & m \geq r \text{ or } n \geq s \end{cases}$$  \hspace{1cm} (5.27)

The correlation length of a profile in the $x$ and $y$ directions are denoted by $\lambda_x^*$ and $\lambda_y^*$, respectively. They are defined as the lengths at which the auto-correlation function becomes zero, or:

$$\lambda_x^* = r \Delta x$$  \hspace{1cm} (5.28)
$$\lambda_y^* = s \Delta y$$

where $\Delta x$ and $\Delta y$ are the sampling intervals in $x$ and $y$ directions, respectively. The degree of non-isotropy of the surface or surface pattern parameter is measured by $\Gamma$ which is defined as the ratio of correlation length in the $x$ and $y$ directions:

$$\Gamma = \frac{\lambda_x^*}{\lambda_y^*}$$  \hspace{1cm} (5.29)

The surface pattern parameter, $\Gamma$, represents orientation of the surface asperities which varies depending upon the manufacturing processes used in the surface finishing process, e.g., grinding, lapping, honing, etc. For modeling purposes, surfaces are categorized into transverse, isotropic, or longitudinal pattern. For surfaces with transverse orientation $\Gamma < 1$, for isotropic surfaces $\Gamma = 1$, and for longitudinal surfaces $\Gamma > 1$. An illustrative schematic of these surface patterns adapted from [38] is shown in Figure 5.3.

The generation of Gaussian surface which has an auto-correlation function as Equation (5.27) is accomplished by using the following linear transformation [37]:

$$R_{mn} = \begin{cases} R_q z \left(1 - \frac{m}{r}\right) \left(1 - \frac{n}{s}\right), & m < r \text{ and } n < s \\ 0, & m \geq r \text{ or } n \geq s \end{cases}$$  \hspace{1cm} (5.27)
Figure 5.3: Different surface patterns: (a) transverse (Γ<1), (b) isotropic (Γ=1) and (c) longitudinal (Γ>1) [38]

\[ z_{ij} = \frac{R_q}{\sqrt{v_{fs}}} \sum_{k=1}^{N} \sum_{l=1}^{M} \eta_{i+k,j+l} \]

where \( \eta_{ij} \) are mutually-independent, normally distributed random numbers with zero mean and unit variance and \( z_{ij} \) is the generated asperity height matrix. Once the surface roughness is generated, the next step is to evaluate the surface roughness properties such as standard deviation of asperity heights \( R_q \), density of asperities \( D_{sum} \), and radius of tip of asperities \( \beta \). These parameters are calculated from spectral moments of the surface using following expressions [39]:

\[ m_0 = E(z^2) = R_q^2 \]
\[ m_2 = E \left[ \left( \frac{dz}{dx} \right)^2 \right] \]
\[ m_4 = E \left[ \left( \frac{d^2z}{dx^2} \right)^2 \right] \] (5.31)

Equation (5.31) shows the spectral moments for an isotropic surface. If the surface is anisotropic, there exist two principal directions along which the value of \( m_2 \) is minimum and maximum. The equivalent value of \( m_{2e} \) for an equivalent isotropic surface can be calculated as [39]:

\[ m_{2e} = \sqrt{m_{2max} m_{2min}} \] (5.32)

The \( m_4 \) values in the same two directions are calculated in the same way as Equation (5.32) to give \( m_{4e} \). The density of asperities, radius of asperity tips and standard deviation of asperity heights are calculated from the following equations [40]:

\[ D_{sum} = \frac{m_{4e}}{6\pi\sqrt{3m_{2e}}} ; \quad \beta = \frac{3}{8} \sqrt{\frac{\pi}{m_{4e}}} ; \quad R_q = \sqrt{m_0} \] (5.33)
5.2.5 Wear Volume Calculations

As running-in progresses, the heights of asperities with plastic and elasto-plastic interferences vary while the rest of asperities will remain unchanged. As the asperity heights change, the non-dimensional wear volume is calculated using the following equation. It should be noted that Equation (5.34) only addresses the wear due to the plastic deformation of asperities which occur during running-in.

\[ \bar{V} = \frac{1}{L_{Ra_1}} \int_{0}^{L} |z_b(x) - z_a(x)| \, dx \]  

(5.34)

where \( L \) represents the length of the roughness profile, \( R_{a1} \) is the initial arithmetic average of the asperity heights, and \( z_b \) and \( z_a \) are the profile before and after considering plastic deformation of asperities, respectively. The arithmetic average of asperity heights, \( R_a \), is calculated as:

\[ R_a = \frac{1}{N} \sum_{i=0}^{N} |z(x)| \]  

(5.35)

The wear volume is calculated using the following equation:

\[ V = 2\pi R'B R_{ai} \bar{V} \]  

(5.36)

where \( R' \) is the equivalent radius of curvature of the rollers, \( B \) is the width of the rollers, and \( R_{ai} \) is the arithmetic average of the surface roughness of the specimen at the simulation time step \( i \), respectively.

Another pertinent parameter of interest is the “transient wear coefficient” during the running-in process. Rearranging Equation (5.1), transient wear coefficient as a function of time can be written as:

\[ k(t) = \frac{V_H}{F_T U t} \]  

(5.37)

where \( V \) is the wear volume during each step and is calculated using Equation (5.36). \( H \) is the hardness of the softer material, \( F_T \) is the force, \( U \) is the sliding velocity, and \( t \) represents time. The time, \( t \), required for this amount of volume to wear is calculated based on the assumption that the number of cycles required for the material to wear is the inverse of the dry wear
The dry wear coefficient, \( k_m \), is a parameter that can be easily found in the literature for many different combinations of sliding pairs of materials.

### 5.2.6 Friction Coefficient and Striebeck Curve

The variation of friction coefficient during the running-in process is of particular interest. The friction force \( F_f \) has two components: friction force due to the shearing of lubricant \( F_{f,H} \) and the friction force due to the contact of asperities \( F_{f,C} \):

\[
F_f = F_{f,H} + F_{f,C}
\]

(5.38)

For a lubricant with Newtonian behavior between the rollers shown in Figure 5.1, the hydrodynamic friction force is written as:

\[
F_{f,H} = \int \tau dA = \int_{-a}^{a} \mu \frac{U_{\text{dif}}}{h_c} B dx = 2aB\mu \frac{U_{\text{dif}}}{h_c}
\]

(5.39)

where \( a \) is the Hertzian half width of contact, \( B \) is the width of the roller, \( \mu \) is the effective viscosity of the lubricant, \( U_{\text{dif}} \) is the sliding distance of the rollers, and \( h_c \) is the thickness of the lubricant film. The effective viscosity of the lubricant under the applied load is calculated from Roeland’s equation \([42]\):

\[
log_{10}\mu + 1.2 = (log_{10}\mu_0 + 1.2) \left(1 + \frac{P_H}{196.1 \times 10^6}\right)^{Z_{\text{lub}}}
\]

(5.40)

where \( \mu \) and \( \mu_0 \) are the viscosities of the lubricant at the pressure \( P_H \) and ambient pressure in mPa.s, respectively. The parameter \( Z_{\text{lub}} \) is the viscosity-pressure index. This relationship is valid for moderate pressures. The friction force between asperities \( F_{f,C} \) is:

\[
F_{f,C} = f_c F_C = f_c \frac{F_T}{y_2}
\]

(5.41)

where \( f_c \) is the friction coefficient between asperities. Friction coefficient \( f \) is written as:

\[
f = \frac{f}{F_T} = \frac{F_{f,H} + F_{f,C}}{F_T} = \frac{2aB\mu U_{\text{dif}}}{F_T h_c} + f_c \frac{F_C}{y_2}
\]

(5.42)

During running-in as the asperities deform, the thickness of the lubricant film as well as the contribution of asperities in carrying the load changes which result in the change in friction coefficient.
5.2.7 Numerical Solution

In this section, the numerical procedure for the running-in model is described. The roughness properties of the surfaces are calculated from the numerically generated surface profile. Using the bisection method, the scaling factor for asperity part $\gamma_2$ is determined so that the load carried by asperities from Equation (5.15) is equal to the total load divided by asperities scaling factor, $F_c = F_T / \gamma_2$. The initial range for asperity scaling factor, $\gamma_2$, is assumed to be $1.00001 < \gamma_2 < 10000$. It typically takes less than 15 iterations to converge on the asperity scaling factor, $\gamma_2$. The values of scaling factors are then used to determine the film thickness $h_c$. Then, the local interference for each asperity is evaluated using Equation (5.6). The asperities whose interference values are in the plastic or the elasto-plastic regime will deform and the rest of asperities are left unchanged. The change in surface profile is calculated, and using Equations (5.36) and (5.37), the wear volume and the wear coefficient are determined. As a result of elasto-plastic and plastic deformation of asperities, the roughness properties change. Hence, the surface roughness profile is updated and new values for scaling factors and film thickness are evaluated. Calculations are repeated until a steady state condition is reached. In the present algorithm, steady state is assumed to occur when the difference in portion of load carried by asperities at two consecutive iterations fall below a specified tolerance value, i.e.,

\[
\left| \frac{1}{\gamma_2^{\text{new}}} - \frac{1}{\gamma_2^{\text{old}}} \right| < \varepsilon
\]

(5.43)

where $\varepsilon$ is the error tolerance and it is assumed to be $\varepsilon = 0.00005$. The results with more stringent tolerance values were found to be negligibly different.

5.3. Results and Discussions

5.3.1 Validation

In order to verify the validity of this approach, a set of comparisons was made to the results of two published experiments. Table 1 shows the operating conditions for the experiments. The
experiment was conducted between two rollers with fresh surfaces brought in contact under a specified load and speed in the presence of lubricant. As the contact of the rollers continues, the rollers run-in and the asperities deform, thus resulting in a change in the surface roughness properties as a function of time. According to Wang et al. [24], the arithmetic average of asperity heights were measured every two minutes using optical means. The geometry, applied load, and the lubricant are the same in both experiments. However, the sliding speed and the surface roughness are different in Case 1 and Case 2.

Table 5.1: Operating condition and lubricant properties of the experiment [24]

<table>
<thead>
<tr>
<th>Dimensions and Operating condition</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of larger roller</td>
<td>43 mm</td>
<td>43 mm</td>
</tr>
<tr>
<td>Radius of smaller roller</td>
<td>17 mm</td>
<td>17 mm</td>
</tr>
<tr>
<td>Speed of the larger roller</td>
<td>0.8 m/s</td>
<td>0.7 m/s</td>
</tr>
<tr>
<td>Speed of the smaller roller</td>
<td>1.2 m/s</td>
<td>1.3 m/s</td>
</tr>
<tr>
<td>Applied force</td>
<td>40 N</td>
<td>40 N</td>
</tr>
<tr>
<td>Lubricant viscosity</td>
<td>0.0283 Pa.s</td>
<td>0.0283 Pa.s</td>
</tr>
<tr>
<td>Average roughness of larger roller</td>
<td>0.5825 µm</td>
<td>1.2678 µm</td>
</tr>
<tr>
<td>Pressure-viscosity index</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Average roughness of smaller roller</td>
<td>0.1 µm</td>
<td>0.1 µm</td>
</tr>
<tr>
<td>Hertzian pressure</td>
<td>0.22 GPa</td>
<td>0.22 GPa</td>
</tr>
<tr>
<td>Modulus of elasticity of each roller</td>
<td>230 GPa</td>
<td>230 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio of each roller</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Dry wear coefficient</td>
<td>0.0003</td>
<td>0.0003</td>
</tr>
<tr>
<td>asperity-asperity friction coeff.</td>
<td>0.12</td>
<td>0.12</td>
</tr>
</tbody>
</table>

The hardness and the dry wear coefficient of the rollers in the experiment are not reported in [24]. The dry wear coefficient for two steels have been reported in several literature to be somewhere from $10^{-4}$ to $5 \times 10^{-4}$. The time required for the rollers to reach steady-state depends on the dry wear coefficient. For Case 1, it was found that if the dry wear coefficient, $k_m$, is assumed to be $3 \times 10^{-4}$, then the predicted time for the rollers to reach steady-state is close to the duration measured by the experiment for this case. Therefore, the same value of $k_m$ was used for Case 2. It is worthwhile to mention that the variation of surface roughness during running-in is not a function of dry wear coefficient and that it depends on the operating conditions of the
experiment. Therefore, varying the dry wear coefficient would only change the time required for
the rollers to reach steady-state.

The dependency of the running-in behavior is highly influenced by the operating conditions
such as normal load, rolling speed, and sliding speed during the running-in process and less so on
the material properties such as hardness. The hardness value used in the simulations was 2 GPa.
Another input to the running-in model developed in this paper is the surface roughness. Since
this information is not available in [24], it was decided to numerically generate the profile using
the procedure outlined in section 2.4. Hence, a pair of isotropic surfaces which has the same \( R_a \)
as reported in Table 1 is numerically generated.

Figure 5.4 shows the comparison between the predicted and the experimentally measured
values of surface roughness. This comparison shows that the proposed method is capable of
predicting the variation of surface roughness. Comparison between Case 1 and Case 2 reveals
that in Case 1, where the surfaces are smoother, steady state is reached faster and the drop in
arithmetic average of asperity heights is smaller. Case 2, however, corresponds to contact of
rougther surfaces and larger changes in variation of \( R_a \) is observed.

Figure 5.4: Comparison of experimental with predicted values.

The variation of scaling factors during the running-in process for both cases is shown in
Figure 5.5. The model predicts that in Case 1, which has a relatively smoother surface, the
asperities carry a smaller portion of the load. As the surfaces undergo running-in, the asperity heights decrease and therefore less asperity-asperity contact occurs resulting in gradual increase in contribution of hydrodynamic film in carrying the load.

According to this model, when the variation of the portion of load carried by the fluid film reaches a steady-state value, the running-in stage is complete. It is worthwhile to note that after running-in, still some asperity-to-asperity contact occurs but the load carried by the asperities during steady-state is small. Note that in Case 2, after the running-in process is complete, even though the surface roughness remains relatively high ($R_a = 0.8 \, \mu m$), under the conditions simulated only a minute number of asperity-to-asperity contact remains. Nevertheless, if at the end of running-in is operated under different load or speed, considerably more asperity contacts may occur.

![Figure 5.5: Variation of scaling factors during running-in.](image)

Having validated the predictions of the model by comparison to two experiments, we now proceed to present a series of simulations for transient wear that occurs in the contact of two rollers.

### 5.3.2 Parametric Simulations

- **Effect of Surface Topography**

In this section, the effect of surface topography on the running-in behavior is studied. Three surfaces with different values for surface pattern are generated and used in the running-in model.
The formulas introduced in the section 5.2.4 are employed to generate surfaces with three values of $\Gamma$. The surface properties for these surfaces are reported in Table 5.2.

**Table 5.2: Properties of the generated surfaces**

<table>
<thead>
<tr>
<th>Surface pattern number</th>
<th>$R_a$ (m)</th>
<th>$\beta$ (m)</th>
<th>$D_{sum}$ ($1/m^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transverse ($\Gamma = 0.2$)</td>
<td>$0.72\times10^{-6}$</td>
<td>$2.35\times10^{-6}$</td>
<td>$6.1\times10^{10}$</td>
</tr>
<tr>
<td>Isotropic ($\Gamma = 1$)</td>
<td>$0.72\times10^{-6}$</td>
<td>$2.787\times10^{-6}$</td>
<td>$7.217\times10^{10}$</td>
</tr>
<tr>
<td>Longitudinal ($\Gamma = 3$)</td>
<td>$0.72\times10^{-6}$</td>
<td>$3.4\times10^{-6}$</td>
<td>$6.297\times10^{10}$</td>
</tr>
</tbody>
</table>

Having generated the surface roughness, we now proceed to investigate how surface topography affects the running-in behavior. Figure 5.6 shows how a particular surface pattern affects the asperity scaling-factor and wear volume. The asperity orientation in transverse surfaces, as illustrated in Figure 5.3, impedes the flow of lubricant and hence more asperity-to-asperity contact occurs resulting in larger contribution of asperities in carrying the load. The results also indicate that with longitudinal ($\Gamma = 3$) and isotropic surfaces ($\Gamma = 1$), the running-in process comes to a completion sooner than that of the transverse surface ($\Gamma = 0.2$).

![Figure 5.6: Effect of surface pattern on asperity scaling factor.](image)

The wear volume initially increases with a high slope. As the running-in time proceeds, the slope decreases until its variation with time becomes nil, and the system attains steady-state. The slope of the wear volume depicting the wear rate versus time is plotted in Figure 5.7.
The wear rate gradually decreases as the running-in time progresses. Based on Figure 5.6, the wear volume increases until it reaches the steady-state during which the wear volume increases linearly. This point pertains to a constant wear rate which occurs at the end of running-in period. The transverse surface pattern has a higher wear rate. The wear rate values corresponding to the end of the running-in period are referred to as the steady-state wear rate.

![Wear Rate vs Time for Different Surface Patterns](image)

**Figure 5.7: Variation of wear rate with time for different surface patterns.**

Another way to verify the validity of the proposed running-in model is to compare the steady-state values for wear rate from this analysis with the values calculated from the steady-state wear model based on the concept of fractional film defect, \( a \), introduced by Rowe [12], which was discussed in the introduction of this paper. The authors in an earlier publication [15] extended the thermal desorption model developed by Wu and Cheng [16] using the load-sharing concept to study the steady-state wear in EHL line contact problem. By rearranging equations (5.3) and (5.4) and substituting the fractional film defect with corresponding terms from thermal desorption theory, the wear rate is calculated as:

\[
\dot{V} = k_m A_n U_d \left\{ 1 - \exp \left[ -\frac{x}{U_d t_0} \exp \left( \frac{-E}{R_g T_s} \right) \right] \right\} \left( \frac{A_C}{A_n} \right)
\]  

(5.44)
where $A_n$ is the nominal area of contact, $U_d$ represents the sliding speed, $X$ is the diameter of area associated with an adsorbed lubricant molecule, $t_0$ is the time of vibration of a molecule in the adsorbed state, $E$ is the heat of adsorption of lubricant molecules on the steel surface, $R_g$ is the molar gas constant, $T_s$ is the surface temperature, and $A_c$ is the real area of contact. The surface roughness properties at the end of the running-in period were used as input to this model.

The properties of the oil which are required in this analysis are taken from [16] are:

- $X = 3 \times 10^{-10}$ m,
- $t_0 = 3 \times 10^{-12}$ sec,
- $E = 49$ KJ/mole.

The sliding speed, $U_d$, and nominal area of contact, $A_n$, were calculated based on the operating conditions of Table 5.1. The surface temperature was calculated using the thermal EHL analysis which was developed earlier by the authors for line contact problem [34]. The ratio of real area of contact to the nominal area of contact $A_c/A_n$ is calculated from the running-in model proposed in this paper.

The predicted values for steady-state wear rate are illustrated in Figure 5.8. The results reveal that the steady-state wear values obtained using the running-in model are consistent with the predictions of the thermal desorption model both in trend and magnitude. It is worthwhile to note that as the running-in stage finishes and the steady-state regime starts, the portion of asperities that plastically deforms decreases and the asperity contacts are elastic. Therefore, there is a change in the dominant wear mechanism from plastic deformation of asperities to thermal desorption which is based on elastic deformation of asperities.

Another interesting parameter is the variation of asperity height distribution during running-in. The asperity height distribution for isotropic surface pattern is plotted in Figure 5.9 for different time steps during running-in stage for Case 1. The asperities of the fresh surface initially have a Gaussian type of distribution. As running-in continues, the asperities plastically deform and their height decreases. As a result, the asperity height distribution changes during time. In fact, the asperities with high height deform more since they are not fully protected by the lubricant film while the asperities with lower height remain unchanged. As a result, the
roughness distribution shows a drastic change in the end right of the roughness histogram while the rest of the histogram is almost unchanged.

![Roughness Distribution](image)

**Figure 5.8: Comparison of wear rate from the current model and the thermal desorption model.**

![Histograms of Asperity Heights](image)

**Figure 5.9: Evolution of surface profile during running-in for case 1.**

The evolution of asperity height distribution for this case shows that the concentration of asperity heights moves to the region where $0 < z < 0.2 \mu m$. Variation of friction coefficient during running-in period is shown in Figure 5.10.
The asperities in transverse surfaces impede the lubricant flow and therefore compared to isotropic and longitudinal surfaces, a larger friction force is generated. As the asperities polish, the contribution of asperities in carrying the load as well as the real area of contact decreases resulting in gradual decrease in friction coefficient until it reaches the steady-state friction coefficient.

- **Effect of Speed**

Effect of rolling speed on the variation of arithmetic average of the asperity heights, $R_a$, during running-in is shown in Figure 5.11. In these simulations, the isotropic surface generated in the previous section is used. As the rolling speed increases, a thicker film is formed and the arithmetic average of asperities height, $R_a$, at the end of the running-in period is higher. Although, the final values for $R_a$ for these speeds are close to each other, increasing the rolling speed decreases the time required for the surfaces to reach the steady state. The effect of speed on the wear volume and friction coefficient is shown in Figure 5.12. As the speed decreases, a
smaller protecting film is formed and, therefore, more asperity-to-asperity contact occurs resulting in more wear.

Figure 5.11: Roughness variation during running-in with different speeds (isotropic).
A higher speed, on the other hand, translates into a thicker lubricating film. Hence, more asperities are protected by the film and less asperity-to-asperity contacts occur resulting in the decrease of friction coefficient. Effect of applied load on the running-in behavior of the rollers is studied in the next section.

Figure 5.12: Variation of friction coefficient and wear volume during running-in (isotropic).
• **Effect of Load**

In this section, the effect of load on the running-in behavior is studied. For this part of the analysis the applied load was changed while all other parameters reported in Table 5.1 for Case 1 were maintained constant. The isotropic surface profile which was generated in the previous section is used. With increasing the applied load, a smaller protecting lubricant film is formed and therefore the number of asperity-asperity contacts increases. An increase in the asperity-asperity contact results in an increase in the load carried by asperities. Therefore, more asperities experience plastic and elasto-plastic deformation and thus the wear volume increases. Figure 5.13 illustrates variation of wear volume with time.

![Figure 5.13: Variation of wear volume during running-in period (isotropic).](image)

Increase in the wear volume occurs with increasing the applied load. The manifestation of increased wear volume is observed in the larger decrease in arithmetic average of asperity heights $R_a$ as load increases.

**5.3.3 Effect of Running-In on Strubeck Curve**

In this section the effect of running-in on the Strubeck-type curve is studied. There are several variations of Strubeck curve. While the ordinate is always the friction coefficient, the abscissa can be speed, Hersey number, Sommerfeld number, or film parameter [43]. For this study, the
rollers introduced in Table 5.1 are used with varying the rolling speed. This analysis was done with longitudinal, isotropic, and transverse surface patterns which were generated in Section 5.3.1. The Stribeck curve was generated for the fresh surface, the surface which was partially run-in and the surface which is fully run-in under the operating conditions of Table 5.1. The results are shown in Figure 5.14.

![Diagram](image)

**Figure 5.14: Effect of running-in on Stribeck curve a) transverse $\Gamma=0.2$, b) isotropic $\Gamma=1$, c) longitudinal $\Gamma=3$.**

As the surfaces run-in, due to the decrease in the surface roughness, the friction coefficient decreases. For run-in surfaces, the shift from boundary lubrication to the partial lubrication occurs at smaller rolling speeds compared to the fresh surface. The reason is that run-in surfaces are smoother than fresh surfaces and therefore can be protected by a smaller film thickness. In other words, the rolling speed that can protect the asperities is smaller for run-in surfaces compared to a fresh surface. Another interesting observation is that in transverse surfaces, the shift from boundary lubrication to mixed lubrication occurs at smaller speed compared to isotropic and longitudinal. This is due to the better film-forming capacity of surfaces with transverse surface pattern. It is worthwhile to note that this trend exists for both fresh surface and run-in surface.

As the running-in stage progresses, the Stribeck curve shifts to the left which means that the lift-off speed decreases. This can be explained by knowing that as the surfaces run-in, the asperities get polished and a smaller film thickness will completely separate the surfaces.
Therefore, lift-off will occur at smaller values of rolling speed. This trend is in agreement to the qualitative prediction reported in [44, 45].

5.4 Comments on Running-In Operating Conditions

The parametric study of the running-in shows that the operating condition during running-in has a tremendous effect on the steady-state performance of the contacting bodies. In other words, to improve the steady-state performance of the system, the operating conditions for the running-in period should be carefully selected. Usually the user cannot choose the surface finish and the surface pattern. The lubricant is also not a choice of the user. Among all the different factors that affect the running-in behavior, it is only the load and the speed which could be controlled by the operator of the machinery.

It was shown that reducing the rolling speed will result in a smaller steady-state $R_a$. One, however, should not draw the wrong conclusion that the lower the rolling speed during running-in, the smoother the surfaces would be for the steady-state. By reducing the speed, a smaller lubricant film will be formed and therefore more asperity-asperity contacts will occur which may result in scuffing and severe damage to the surfaces.

Increasing the rolling speed was shown to reduce the duration of running-in period. Another advantage of increasing the rolling speed is that a thicker lubricant film is formed and more asperities are protected. Excessive increase in the rolling speed will generate a large heat and can have an adverse effect on the performance of the contacting bodies in terms of thickness of the lubricant film and its viscosity.

The same scenario holds for the applied load during running-in. The higher the applied load during running-in, the smaller is the $R_a$ of the surface. Increasing the load also results in an increase of the wear volume. Excessive increase in the load might result in other failures such as scuffing or even pitting. By applying a small load during running-in, however, there will be very few asperity-to-asperity contacts and the polishing of the surfaces will not take place.
5.5 Discussion

The model proposed in this paper has the advantage of being extremely time-efficient. The execution time for the simulations shown with the error tolerance $\epsilon = 0.00005$ on a Pentium IV computer with a CPU of 2.4 GHz is about 30 minutes. On the other hand, an important input to this model is the dry wear coefficient $k_m$ which affects the time required for each iteration. There are several published papers on the experimental data for dry wear coefficient and the value $k_m = 3 \times 10^{-4}$ has been used in this paper. The available data on this parameter are sometimes different by a factor of 2. The surface profile is another input to this model which could be either measured by a profilometer or numerically generated using the surface generation algorithm described in this paper. Alternatively, the surface profile measured using the profilometer can be used. The running-in model proposed in this paper is not constrained to the surfaces with Gaussian height distribution.

5.6 Conclusions

In this paper a detailed review of the previous studies on running-in is presented. During the running-in many tribological parameters of the contacting surfaces such as arithmetic average of asperity heights and asperity height distribution change. As the running-in period progresses, the portion of the load carried by the asperities decreases while the portion of load carried by fluid film increases. Hence, the friction coefficient decreases during the running-in process. An efficient deterministic model for running-in of two rollers based on the load-sharing concept is proposed which is capable of predicting the variation of arithmetic average of surface roughness as well as wear volume as a function of time. The predicted values for variation of asperity heights $R_a$ related to the contact of two rollers are compared to two published experimental works and the results are shown to be fairly close. A parametric study on the running-in behavior is presented. Effect of surface pattern on wear volume and real area of contact is studied. The asperity orientation in transverse surfaces impedes the fluid flow and comparing to longitudinal
and isotropic surfaces, causes more asperity-asperity contact and larger wear to occur. The steady-state wear rate are compared to the wear rate calculated from thermal desorption model and are shown to be in good agreement. An increase in rolling speeds results in a thicker film to form between surfaces and the wear volume decreases. Increasing the load causes more asperity-to-asperity contact and hence worn volume rises and the steady-state $R_a$ is smaller. Running-in is shown to have a profound effect on Stribeck curve. As the surfaces run-in, the lift-off speed decreases. The lift-off speed for transverse surface pattern is smaller than the isotropic and longitudinal surfaces.

5.7 Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_e$</td>
<td>Contact area pertinent to elastic deformation ($m^2$)</td>
</tr>
<tr>
<td>$A_p$</td>
<td>Contact area pertinent to plastic deformation ($m^2$)</td>
</tr>
<tr>
<td>$A_r$</td>
<td>Real area of contact ($m^2$)</td>
</tr>
<tr>
<td>$B$</td>
<td>Width of roller (m)</td>
</tr>
<tr>
<td>$C_t$</td>
<td>Thermal correction factor for film thickness</td>
</tr>
<tr>
<td>$d$</td>
<td>Wear depth (m)</td>
</tr>
<tr>
<td>$D_{as}$</td>
<td>Density of asperities ($1/m^3$)</td>
</tr>
<tr>
<td>$E$</td>
<td>Heat of adsorption (J)</td>
</tr>
<tr>
<td>$E_1$</td>
<td>Elasticity modulus of roller 1</td>
</tr>
<tr>
<td>$E_2$</td>
<td>Elasticity modulus of roller 2</td>
</tr>
<tr>
<td>$E_p$</td>
<td>Equivalent Young’s modulus ($N/m^3$)</td>
</tr>
<tr>
<td>$f_C$</td>
<td>Asperity-asperity friction coefficient</td>
</tr>
<tr>
<td>$F_C$</td>
<td>Load carried by asperity (N)</td>
</tr>
<tr>
<td>$F_H$</td>
<td>Load carried by fluid film (N)</td>
</tr>
<tr>
<td>$F_T$</td>
<td>Applied load (N)</td>
</tr>
<tr>
<td>$H$</td>
<td>Material hardness ($N/m^3$)</td>
</tr>
<tr>
<td>$h_c$</td>
<td>Central film thickness (m)</td>
</tr>
<tr>
<td>$k_e$</td>
<td>Elastic wear coefficient</td>
</tr>
<tr>
<td>$k_f$</td>
<td>Thermal conductivity of the lubricant ($W/m/K$)</td>
</tr>
<tr>
<td>$k_p$</td>
<td>Plastic wear coefficient</td>
</tr>
<tr>
<td>$k_m$</td>
<td>Wear coefficient for rubbing materials</td>
</tr>
<tr>
<td>$k$</td>
<td>Wear coefficient for lubricated contact</td>
</tr>
<tr>
<td>$L$</td>
<td>Sampling length (m)</td>
</tr>
<tr>
<td>$m$</td>
<td>Index for auto-correlation function</td>
</tr>
<tr>
<td>$n$</td>
<td>Index for auto-correlation function</td>
</tr>
<tr>
<td>$N_m$</td>
<td>Number of sites on the friction surface unoccupied by molecules</td>
</tr>
<tr>
<td>$N_n$</td>
<td>Total number of sites on the friction surface</td>
</tr>
<tr>
<td>$P_{H}$</td>
<td>Hydrodynamic pressure ($N/m^2$)</td>
</tr>
<tr>
<td>$R$</td>
<td>Auto-correlation function</td>
</tr>
<tr>
<td>$R_1$</td>
<td>Radius of roller 1 (m)</td>
</tr>
<tr>
<td>$R_2$</td>
<td>Radius of roller 2 (m)</td>
</tr>
<tr>
<td>$R_a$</td>
<td>Arithmetic average of asperity heights (m)</td>
</tr>
<tr>
<td>$R_g$</td>
<td>Gas molar constant ($J/mole/K$)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$R_{q1}$</td>
<td>Standard deviation of asperity heights for surface 1 (m)</td>
</tr>
<tr>
<td>$R_{q2}$</td>
<td>Standard deviation of asperity heights for surface 2 (m)</td>
</tr>
<tr>
<td>$R'$</td>
<td>Equivalent radii of curvature (m)</td>
</tr>
<tr>
<td>$s$</td>
<td>Sliding distance (m)</td>
</tr>
<tr>
<td>$t_0$</td>
<td>Time of vibration of a molecule in the adsorbed state (s)</td>
</tr>
<tr>
<td>$t_\bar{c}$</td>
<td>Average time that a molecule remains at a given surface site (sec)</td>
</tr>
<tr>
<td>$t_x$</td>
<td>Time an asperity travels the distance equivalent to the diameter of an adsorbed molecule (sec)</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Surface temperature (K)</td>
</tr>
<tr>
<td>$TL$</td>
<td>Thermal loading parameter</td>
</tr>
<tr>
<td>$U$</td>
<td>Sliding velocity (m/s)</td>
</tr>
<tr>
<td>$V$</td>
<td>Worn volume (m$^3$)</td>
</tr>
<tr>
<td>$\dot{V}$</td>
<td>Wear rate (mm$^3$/s)</td>
</tr>
<tr>
<td>$W$</td>
<td>Wear rate (m$^3$/m)</td>
</tr>
<tr>
<td>$w$</td>
<td>Interference (m)</td>
</tr>
<tr>
<td>$w_e$</td>
<td>Interference at the point of initial yielding (m)</td>
</tr>
<tr>
<td>$w_p$</td>
<td>Interference at the beginning of fully plastic deformation (m)</td>
</tr>
<tr>
<td>$X$</td>
<td>Diameter of the adsorbed area (m')</td>
</tr>
<tr>
<td>$y_s$</td>
<td>Distance between mean of asperity heights and mean of surface heights (m)</td>
</tr>
<tr>
<td>$z$</td>
<td>asperity height (m)</td>
</tr>
<tr>
<td>$Z_{hub}$</td>
<td>Viscosity-pressure index of lubricant</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Fractional film defect</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Radius of tip of asperities (m)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Temperature-viscosity coefficient of the lubricant (1/K)</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>Scaling factor for hydrodynamic part</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>Scaling factor for asperity interaction part</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>Sampling interval in the $x$ direction (m)</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>Sampling interval in the $y$ direction (m)</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Surface pattern parameter</td>
</tr>
<tr>
<td>$\lambda_x$</td>
<td>Correlation length of the profile in the $x$ direction (m)</td>
</tr>
<tr>
<td>$\lambda_y$</td>
<td>Correlation length of the profile in the $y$ direction (m)</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Film parameter</td>
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<tr>
<td>$\sigma_{eq}$</td>
<td>Equivalent standard deviation of asperity heights (m)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Plasticity index</td>
</tr>
</tbody>
</table>

### 5.8 References


Chapter 6: Experimental Verification of EHL Line Contact Model
6.1 Introduction

Gears are one of the most widely used machine elements in power transmission systems and hence their performance is of great importance to both the designer and the user. Typically, the lubricated contact area between gear teeth has a rectangular shape and is classified as a line contact problem. Gears are typically heavily-loaded non-conformal contact in which the load transmitted by the gear teeth causes considerable elastic deformation of the surfaces. This substantial elastic deformation and the non-conformal contact of the gears involute profile is categorized as line contact ElastoHydrodynamic Lubrication (EHL) problem.

In reality, the finished surfaces of gears are not perfectly smooth. The contact of asperities on the surface of the gears contributes in carrying the total load. Therefore, a gear commonly operates in the mixed lubrication regime. In this regime, the total load is supported in part by the fluid film and in part by the surface asperities. The contact pressure and associated load carrying capacity play a role in the friction coefficient and the associated power loss.

The goal of this research is to investigate the wear in gears analytically and experimentally. The analytical section of this research is based on the load-sharing concept [1] and is discussed in Section 2. The experimental phase of this research involved designing a test rig that can mimic the operating conditions of different points along the line of action of gears and examine friction and wear. This requires implementation of two motors to independently drive two rollers under desired load and speed to create exactly the same amount of load, rolling, and sliding speed specific to a point along the active profile of gear. Five sets of experiments are conducted to investigate the transient wear behavior of fresh rollers during running-in based on the rolling speed and the material hardness. Two steady-state experiments are also conducted on run-in rollers to measure the effect of rolling speed and sliding speed on the friction coefficient and wear rate.
6.2 Simulation

The details of the simulation of this research are discussed in this section. The numerical algorithm is based on the load-sharing concept [1] which is a powerful method in solving the lubricated contact of rough surfaces. In these types of applications, the applied load is carried by both the lubricant and the tip of the surface asperities. According to the load-sharing concept, the total load, \( F_T \), is partly carried by the asperities, \( F_C \), and partly by the fluid film, \( F_H \):

\[
F_T = F_H + F_C
\]

(6.1)

The contribution of each part is represented by scaling factors \( \gamma_1 \) and \( \gamma_2 \). In other words it is assumed that:

\[
F_T = \frac{F_T}{\gamma_1} + \frac{F_T}{\gamma_2}
\]

(6.2)

Equation (2) can be simplified as:

\[
1 = \frac{1}{\gamma_1} + \frac{1}{\gamma_2}
\]

(6.3)

The application of the load-sharing method to line contact problems was first formulated by Gelinck and Schipper [2]. Later Lu et al. [3] applied this method to heavily loaded pin-bushing assembly. This method was used by the authors for analysis of gears during running-in and steady-state [4-6].

When two fresh surfaces come into a contact in the presence of a lubricant, the tip of the asperities that protrude from the film thickness come into an intimate contact. Initially, the asperities plastically deform and thus the surfaces become smoother. This process is called running-in. The schematic of a hemi-spherical asperity with height \( z_{before} \) and radius \( \beta_{before} \) in the presence of lubricant with thickness of \( h_c \) before the contact and the same asperity after the contact with a rigid flat is shown in Figure 6.1(a) and 6.1(b), respectively. As a result of plastic deformation, the height and the radius of asperities change. In these figures \( y_s \) is the distance between the mean of surface height and the mean of asperity height. The interference \( w \) which
plays an important role in determining the deformation regime of asperities (elastic, plastic, and elasto-plastic) is calculated as:

\[ w = z - h_C + y_s \]  

(6.4)

**Figure 6.2: Schematic of asperity contact (a) before contact (b) after plastic deformation.**

where the central film thickness, \( h_C \), is calculated with the provisions for the effect of surface roughness and is discussed in the next section. The value of interference is calculated for all the asperities that experience contact. Depending on the operating conditions and material properties, the contacting asperities are in elastic regime, elasto-plastic regime, or plastic regime. Each contacting asperity carries some portion of total load based on its deformation regime. The summation of the carried load by asperities represents \( F_C \).

The scaling factors \( \gamma_1 \) and \( \gamma_2 \) are determined in an iterative process by equating \( F_T/\gamma_2 \) to the load that is carried by all the contacting asperities, \( F_C \). The equation for non-dimensional central film thickness, \( H_c \), for the lubricated line contact with provision of surface roughness is [7]:

\[ H_c = \left[ \gamma_1^{s/2} \left( H_{RI}^{7/3} + \gamma_2^{14/15} H_{EI}^{7/3} \right)^{3s/7} + \gamma_1^{-s/2} \left( H_{RP}^{-7/2} + H_{EP}^{-7/2} \right)^{-2s/7} \right]^{1/s} \gamma_1^{1/2} \]  

(6.5)

where

\[ H_{RI} = 3 \left( W U_\Sigma^{-0.5} \right)^{-1} \]

\[ H_{EI} = 2.621 \left( W U_\Sigma^{-0.5} \right)^{-0.2} \]

\[ H_{RP} = 1.287 \left( G U_\Sigma^{-0.25} \right)^{2/3} \]
\[ H_{EP} = 1.311 \left( WU_{\Sigma}^{-0.5} \right)^{-1/3} \left( GU_{\Sigma}^{-0.25} \right)^{3/4} \] (6.6)

\[ W = \frac{F_T}{E_p R B} \]

\[ G = \alpha_{EHL} E_p \]

\[ U_{\Sigma} = \frac{\mu_0 u_{\text{roll}}}{E_p R'} \] (6.7)

\[ H_C = \frac{h_C}{R'} U_{\Sigma}^{-0.5} \]

\[ s = \frac{1}{5} \left( 7 + 8e^{-2(h_{RI})^{-2/5} H_{RI}} \right) \]

In Equation (7), \( E_p \) and \( R' \) represent the equivalent modulus of elasticity and the equivalent radius of rollers, respectively. \( B \) denotes the width of the roller, \( u_{\text{roll}} \) is the rolling velocity, \( h_C \) is the film thickness, and \( \mu_0 \) is the viscosity at the ambient pressure and temperature.

The plastic deformation of the asperities results in a permanent change in the roughness profile of the contacting surfaces, thus affecting the friction coefficient, the load-sharing scaling factors, and the wear rate. Progressively, during the running-in process, the percentage of asperities that plastically deform decrease and the percentage of asperities that undergoes elastic deformation increases. The details of the prediction of the running-in behavior based on the plastic deformation of asperities are in [4]. The input parameters used in the simulations are the surface roughness profile, the geometry of the contacting rollers, material properties, the speed, the load, and the lubricant properties and the predicted parameters are the variation of arithmetic surface roughness, \( R_a \), and the variation of scaling factors \( \gamma_1 \) and \( \gamma_2 \). The steady-state regime starts when the running-in stage is complete and the asperities mainly experience elastic deformation. In this regime parameters such as the friction coefficient and the wear rate remains constant.

The wear rate and wear depth are predicted based on the thermal desorption model discussed in [8]. At moderate asperity temperatures, the thermal desorption is the major mechanism for
adhesive wear. The metal junctions that are not protected by the lubricant film form wear particles. The volumetric wear rate based on the thermal desorption model is calculated using [9]:

$$\dot{V} = k_mA_nU_{dif}\left\{1 - \exp\left[-\frac{X}{U_{dif}t_0}\exp\left(\frac{-E_{ads}}{RT_s}\right)\right]\right\}\left(\frac{A_c}{A_n}\right)$$  \hspace{1cm} (6.8)

where $k_m$ represents the dry wear coefficient, $A_n$ is the nominal area of contact, $U_{dif}$ is the sliding distance, $X$ is the diameter of adsorbed molecule, $t_0$ is the fundamental time of vibration of a molecule in the adsorbed state, $E_{ads}$ is the heat of adsorption, $R$ is the universal gas constant, $T_s$ is the surface temperature, and $A_c$ is the real area of contact.

The surface roughness properties that are of importance in this model are the standard deviation of asperity heights, $R_q$, the density of asperities, $D_{sum}$, and the radius of tip of asperities, $\beta$. It is worthwhile to note that during running-in, the plastic deformation of asperities polishes the surfaces and affects the roughness properties of the contacting surfaces and, hence, influences the wear behavior. The roughness properties are obtained from the spectral moments of the surface $m_0$, $m_2$, and $m_4$ which are calculated from the surface profile $z(x)$. For isotropic surfaces these parameters are obtained from the following expression [10]:

$$m_0 = E(z^2) = R_q^2$$

$$m_2 = E\left[\left(\frac{dz}{dx}\right)^2\right]$$

$$m_4 = E\left[\left(\frac{d^2z}{dx^2}\right)^2\right]$$  \hspace{1cm} (6.9)

where $E$ denotes the expectancy. The moments $m_0$, $m_2$, and $m_4$ are called zeroth, second, and fourth moment of the roughness profile, respectively. Equation (8) shows the spectral moments for an isotropic surface.

If the surface is anisotropic, there exist two principal directions along which the profile value of $m_2$ is a minimum and maximum. The equivalent value of $m_{2e}$ for an equivalent isotropic surface can be calculated as [10]:
\[ m_{2e} = \sqrt{m_{2\text{max}} m_{2\text{min}}} \]  \hspace{1cm} (6.10)

The \( m_4 \) values in the same two directions are calculated in the same way as Equation (9) to give \( m_{4e} \). Once the equivalent moments for each surface are calculated, the total moments are calculated as [10]:

\[ m_{0\text{total}} = m_{0\text{surface 1}} + m_{0\text{surface 2}} \]
\[ m_{2\text{total}} = m_{2\text{surface 1}} + m_{2\text{surface 2}} \]  \hspace{1cm} (6.11)
\[ m_{4\text{total}} = m_{4\text{surface 1}} + m_{4\text{surface 2}} \]

The required surface properties are calculated using the following expression [11]:

\[ D_{\text{sum}} = \frac{m_{4\text{total}}}{6\pi\sqrt{3m_{2\text{total}}}} \]
\[ \beta = \frac{3}{8} \left[ \frac{\pi}{m_{4\text{total}}} \right] \]
\[ R_q = \sqrt{m_{0\text{total}}} \]  \hspace{1cm} (6.12)

Another pertinent parameter in calculating the friction coefficient and predicting wear (Equation (6.7)) is the surface temperature. The temperature of the contacting surfaces and the lubricant are calculated based on the simplified form of energy equation:

\[ \frac{dT}{dy^2} = -\frac{\mu U_{\text{diff}}}{K_f h_c^2} \]  \hspace{1cm} (6.13)

where \( T \) is the lubricant temperature, \( \mu \) is the lubricant viscosity, \( U_{\text{diff}} \) is the sliding velocity, \( K_f \) is the thermal conductivity of the lubricant, and \( h_c \) is the central film thickness. The boundary conditions for this equation are:

\[ \begin{cases} y = 0 & T = T_1 \\ y = h_c & T = T_2 \end{cases} \]  \hspace{1cm} (6.14)

where \( T_1 \) and \( T_2 \) are the temperature of roller 1 and roller 2, respectively. The boundary conditions imply that the film temperature at the surfaces is equal to the temperature of the surfaces. The integration of Equation (6.13) and appropriately applying the boundary conditions
of Equation (6.14) result in the lubricant temperature inside the contact zone to be equal to the average of the surfaces in addition to a term which is proportional to the second power of sliding velocity:

\[ T(x) = \frac{T_1(x) + T_2(x)}{2} + \frac{\mu U_{df}^2}{12K_f} \]  

(6.15)

The average of lubricant temperature inside the contact zone, \( T_L \), is then used to evaluate the actual viscosity of the lubricant.

\[ \mu = \mu_0 \exp \left\{ (\ln \mu_0 + 9.67) \left[ -1 + (1 + 5.1 \times 10^{-9} P)Z_{lub} \left( \frac{T_L - T_0}{T_L - T_0} \right)^{-1.1} \right] \right\} \]  

(6.16)

In this equation, \( \mu_0 \) is the viscosity of the lubricant at ambient pressure and temperature, \( T_0 \) is the inlet temperature, \( T_L \) is the lubricant temperature, \( P \) is the pressure, and \( Z_{lub} \) is the viscosity-pressure index. This equation provides a reasonable prediction of viscosity only up to moderately high pressures.

The traction force is composed of two components hydrodynamic traction force and asperity traction force:

\[ F_f = F_{fH} + F_{fC} \]  

(6.17)

The hydrodynamic traction force is calculated using the famous Bair-Winer [12] formula for a non-Newtonian lubricant:

\[ F_{fH} = 2aB \tau_L \left( 1 - \exp \left( -\frac{\mu U_{df}}{h_c \tau_L} \right) \right) \]  

(6.18)

where the limiting shear stress, \( \tau_L \), is calculated from:

\[ \tau_L = \tau_0 + \beta_0 P \]  

(6.19)

where \( \tau_0 \) is the shear stress at ambient conditions, \( \beta_0 \) is the slope of the limiting shear stress and the pressure, and \( P \) is the pressure.

The asperity traction force is:

\[ F_{fC} = \sum_{i=1}^{N} f_{Ci} P_{Ci} dA_{Ci} = f_c \sum_{i=1}^{N} P_{Ci} dA_{Ci} = f_c F_C \]  

(6.20)
In this equation, \( f_C \) is the friction coefficient between asperities. The traction coefficient can be written as:

\[
f = \frac{F_f}{F_T} = \frac{F_{fH} + F_{fC}}{F_T} = \frac{2aBt_L}{F_T} \left(1 - \exp\left(-\frac{\mu f d f}{h_c^* t_L}\right)\right) + \frac{f_c}{\gamma_2}
\] (6.21)

6.3 Gear Test Rig Assembly

The test rig consists of motors, drives, lubrication system, hydraulic pressure system and different sensors to continuously monitor the variation of the designed parameters such as the applied load, the surface temperature, the oil reservoir temperature, the consumed power and the wear depth.

Figure 6.2 shows a schematic of the test rig. The speed of each motor is independently set by a Variable Frequency Driver (VFD). The load is applied by the hydraulic cylinder and its value is recorded by the pressure transducer. The hydraulic fluid is pumped from the reservoir and using the arm attached to the body valve, the rollers are set in contact under the desired load. The lubricant is heated in the reservoir and then pumped to the contact zone of the rollers. The output of the sensors are connected to a data acquisition board and transferred to the computer.

Figure 6.3 shows a picture of the test rig with emphasis on the motors and the drivers. VFD1 and VFD2 are used to adjust the speed of Motor 1 and Motor 2, respectively. The handle shown in Figure 6.3 is used to for adjusting the applied load by the hydraulic cylinder on the rollers. The applied load is continuously recorded using the pressure transducer. More details of the test rig are shown in Figure 6.4 and 6.5.

6.3.1 Hydraulic System

To apply the required contact force on the rollers, a hydraulic ram is used. The elements of the hydraulic system are shown in Figures 6.3 and 6.3. This system consists of a ram, a pump motor, a pressure transducer, and a valve body. The ram used has a maximum pressure of 3000 psi (20.7 MPa). A valve body inserted between the ram and the hydraulic pump regulates the
flow by either extending or retracting the ram to achieve the specified pressure. On the discharge side of the ram, connected to the valve body, is a block valve to prevent bypass leaking in the valve body and maintain pressure in the lines. A pressure transducer attached to the line measures the pressure using which the force applied to the roller is determined.

Figure 6.2: A schematic of the gear test rig.

Figure 6.3: Motors and drives in the gear test rig.

6.3.2 Lubrication System

The lubrication system contains heating unit (which is used to heat the lubricant to the working temperature of oil in real applications which is close to 85 °C), directional spout, catch basin, tubing, oil pump, filter (a filter is used in the lubrication system to remove the wear debris before
the oil enters the contact zone), and valves. The elements of the lubrication system are shown in Figure 6.4.

The lubricant is contained in a 5-gallon reservoir at the far end of the test rig. It is wrapped in fiberglass insulation and coated with a layer of foil duct tape to keep the heat in and ensure safety from accidental contact. To heat the lubricant, a 1.1 kW heater is used. It usually takes about 30 minutes to heat the lubricant from room temperature to the working temperature 85°C. The lubricant flowing toward the contacting rollers first goes through an oil filter with a 19 micron rating. This will keep debris from entering the contact between the rollers during the test. Once the lubricant comes into contact with the rollers it drips freely down into a catch basin which directs it back into the reservoir.

![Figure 6.4: Elements of the hydraulic and lubrication system](image)

### 6.3.3 Motors

Each point on the active profile of gears experiences a distinct rolling and sliding speed. Hence, in order to mimic the operating conditions of any point on the line of action, the drivers must be appropriately set. For this purpose, two single-phased 220V motors are employed to set the speed at the specified level which generates the desired rolling speed and slide-to-roll ratio. The motors and the drivers are shown in Figure 6.2. The rolling speed \( u_{\text{roll}} \) which is the average of the
speed of the rollers and the slide-to-roll (sr) ratio which is sliding speed divided by the rolling speed are defined as:

\[
\frac{u_{roll}}{u} = \frac{u_1 + u_2}{2} \quad (6.22)
\]

\[
sr = \frac{2(u_1 - u_2)}{u_1 + u_2} \quad (6.23)
\]

where \( U_1 \) and \( U_2 \) are the speed of roller 1 and 2, respectively.

6.4 Instrumentation

In order to measure the desired parameters during the test, several sensors are attached in different parts of the test rig. These sensors are used to record the speed of the shafts, the temperature of the contact zone, the displacement, and the applied load.

6.4.1 Tachometer

Two fixed, mounted tachometers are chosen to record the speeds of each of the rotating shafts. The tachometer can measure speeds up to 250,000 RPM and withstand temperatures up to 92°C. It can effectively measure speeds from up to three feet away and up to a 45° angle and the accuracy of the sensor is 0.1 rpm.

6.4.2 LVDT

A Linear Variable Displacement Transducer (LVDT) which has ±2.5mm of maximum travel, and is 68 mm long is used to continuously measure the displacement. An internally mounted spring ensures that the probe head will remain in contact with the measured surface at all times. The values continuously recorded by LVDT are translated to wear depth. The accuracy of the LVDT is 1.6 µm.

6.4.3 Pressure Transducer

A pressure transducer is used to measure the pressure being applied by the hydraulic ram. This sensor can measure pressures up to 5,000 psi (35 MPa) and can operate in high temperatures and the accuracy of the sensor is 1 psi(7 KPa).
6.4.4 Infrared Thermocouple

To measure the contact zone temperature, an infrared thermocouple is selected. These thermocouples rely on the incoming infrared radiation that they receive from the surface in order to produce the output signal to the data acquisition unit. The infrared thermocouple chosen is a Type – K thermocouple with a stainless steel body and it can measure temperatures up to 538°C and the accuracy is 1 °C.

The output of all of these sensors is sent via a data acquisition board to a Pentium IV computer with a CPU of 1.9 GHz. This board allows for 16 analog inputs and 2 analog outputs, and it also has 24 digital inputs and 24 digital outputs. The analog outputs on the board operate at 6000 samples per minute. This board is fully compatible with LabVIEW™.

6.4.5 Friction Coefficient Measurement

In order to measure friction coefficient, the current and the voltage consumed by each motor must be evaluated. The consumed power is then related to the friction force. The idea of using the consumed power to find the friction force has been previously used by other researchers such as [1].

Initially the rollers are set at the desired speed but under no load and the voltage and current of each motor are registered:

\[ P_i = \sqrt{3}\eta(V_{S1}I_{S1} + V_{L1}I_{L1}) \]  

(6.24)

In this equation, \( P_i \) is the power consumed by the rollers under no load while the rollers are rotating in the air, \( \eta \) is the efficiency of the motor, \( V_S \) and \( V_L \) are the voltage of the small and large rollers, respectively. The parameters \( I_S \) and \( I_L \) are the current drawn by the small and large roller, respectively. When the rollers are set under load, the voltage and current are again recorded and the power is:

\[ P_f = \sqrt{3}\eta(V_{S2}I_{S2} + V_{L2}I_{L2}) \]  

(6.25)
The power consumed due to the friction is the difference between the recorded power in the loaded and the unloaded case:

\[ P = P_f - P_l \]  

(6.26)

The friction force is calculated as the consumed power divided by the sliding speed of the rollers:

\[ F_f = \frac{P}{u_2 - u_1} \]  

(6.27)

The normal force is measured by the pressure transducer which records the pressure in the hydraulic cylinder:

\[ F_N = \pi r_c^2 P_{hyd} \]  

(6.28)

where \( r_c \) is the radius of the hydraulic cylinder and \( P_{hyd} \) is the pressure in the hydraulic cylinder which is measured by the pressure transducer. The friction coefficient is:

\[ \text{cof} = \frac{F_f}{F_N} \]  

(6.29)
6.5 Specimen Preparation and Experiment Procedure

The materials used for the rollers are AISI 4140. The rollers are cut from the raw material in the machine shop at Louisiana State University. The rollers are heat treated in different series to different values of Hardness outside the university. The small rollers are all hardened to 42~44 RC. The large rollers, however, are either without any heat treatment or hardened to 40~42 RC. Hence, the large rollers are always softer than the small rollers. Evaluating the wear weight for rollers with different values of hardness is one of the goals of this research.

The rollers are then taken to a different facility for surface machining. The surfaces are ground up to the typical value of arithmetic average of asperity heights for gears, i.e. $R_a=0.2 \, \mu m$. The roughness value for all the samples is fairly close and it is in the range of $R_a=0.15 \, \mu m$~$0.25 \, \mu m$.

There are some steps that need to be taken before starting an experiment with this test rig. Initially, using the stylus profilometer Surftest SV-2000 shown in Figure 6.5, the surface roughness of the rollers is measured. The stylus of the profilometer travels along a specified length on the surface and records the surface topography. These data are then used to evaluate roughness parameters of the surface such as arithmetic average $R_a$, standard deviation of the surface heights $R_q$, skewness $R_{sk}$, kurtosis $R_{ku}$ and etc. The values of $R_a$ and $R_q$ as well as the surface profiles before and after the experiment are evaluated and compared.

Then the weight of specimens is measured using a balance with 1 mg accuracy. The next step is to heat the lubricant using the heater immersed in the oil reservoir. It takes about 30 minutes to heat the oil from room temperature to 85 °C which is close to the working temperature of oil in the real applications.

The speed of each of the shafts is then set appropriately to provide the desired rolling speed and the slide-to-roll ratio. The required load to produce the desired contact pressure is provided by hydraulic cylinder. Then the data acquisition starts and the output of all the sensors are
registered into different files on the computer and are displayed real time in the LabView interface. After each experiment, the weights of the rollers are measured.

![Image of a stylus profilometer](image)

**Figure 6.6: Stylus profilometer**

Seven experiments are performed on the rollers. Five sets of experiments (Test#1-5) are done to investigate the running-in behavior of rollers and two sets of experiments (Test #6 and 7) to investigate the steady-state wear. The results of simulations (see Chapter 5) show that the time required for rollers subjected to a Hertzian pressure of $P_{\text{Hertz}}=0.7$ GPa (typical load for gears) and the $u_{\text{roll}}=0.1$ m/s–0.2 m/s is around 30 minutes. Therefore, the running-in experiments are conducted for more than 2 hours to clearly show the running-in stage, the transition to steady-state, and the beginning of the steady-state.

Two of the running-in (Test #1 and 2) experiments are conducted on two pairs of rollers which have clear distinction in terms of hardness value whereas the other two experiments (Test #3 and 4) are conducted on the rollers which have slight difference in the value for hardness. These running-in experiments will show the effect of material hardness and rolling speed, $u_{\text{roll}}$, on the running-in behavior of the contacting rollers. One of the running-in experiments (Test#5) is performed in an intermittent fashion. In this experiment, the test is stopped every 10 minutes, the softer roller is cleaned with acetone and the weight and surface roughness of the roller are recorded. Then the roller is again mounted and the test is continued for another 10 minutes.
The steady-state experiments (Test #6 and 7) are conducted on broken-in rollers to measure wear rate (worn volume divided by sliding distance), surface temperature, and friction coefficient as a function of sliding speed and rolling speed. In Test#6 the rolling speed is kept constant and the sliding speed is increased. For each value of sliding speed, the experiment is conducted for one hour and then the test rig is shut down for at least two hours to ensure that the system has completely cool down and reached the room temperature. Then the experiment is started for a new value of sliding speed. This procedure assures that two consecutive tests do not affect each other in terms of rollers and bearings temperature. The same procedure in terms of length of each test and the time between each consecutive test is used for Test#7 in which the sliding speed is constant and the rolling speed is varied.

6.6 Test Results

In this section the results of different experiments are presented. The lubricant properties used for the simulation are shown in Table 6.1 [9]. The required lubricant properties for SAE 30 to be used in the Bair-Winer model (Equation (6.19)) are chosen as \( \tau_0 = 2.5 \text{ MPa} \) and \( \beta_0 = 0.047 \) [6].

<table>
<thead>
<tr>
<th>Lubricant viscosity</th>
<th>( \mu_0 = 0.03017 \text{ Pas} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity pressure index</td>
<td>( Z_{\text{lub}} = 0.56 )</td>
</tr>
<tr>
<td>Heat of adsorption</td>
<td>( E_{\text{ads}} = 49 \text{ kJ/mol} )</td>
</tr>
<tr>
<td>Molar gas constant</td>
<td>( R = 8.31 \text{ J/mol.K} )</td>
</tr>
<tr>
<td>Diameter of the adsorbed area</td>
<td>( X = 3 \times 10^{-10} \text{ m} )</td>
</tr>
<tr>
<td>Time of vibration of the lubricant molecule</td>
<td>( t_0 = 3 \times 10^{-12} \text{ sec} )</td>
</tr>
</tbody>
</table>

6.6.1 Running-In Experiments

- **Hard/Soft Continuous Running-In Experiment (Tests 1 and 2)**

In this experiment, a pair of fresh rollers with one being considerably softer than the other is tested and the variation of wear depth, weight loss, and surface roughness are measured. This experiment is conducted for two different rolling speeds. The operating conditions for this test are shown in Table 6.2.
Table 6.2: Operating conditions for running-in experiment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolling speed</td>
<td>( u_{roll}=0.1 \text{m/s and 0.2 m/s} )</td>
</tr>
<tr>
<td>Range of slide-to-roll ratio</td>
<td>( sr= 0.15 )</td>
</tr>
<tr>
<td>Applied Load</td>
<td>( F= 5700 \text{ N} )</td>
</tr>
<tr>
<td>Radius of larger roller</td>
<td>( R_1=0.051 \text{ m} )</td>
</tr>
<tr>
<td>Radius of smaller roller</td>
<td>( R_2=0.038 \text{ m} )</td>
</tr>
<tr>
<td>Width of the rollers</td>
<td>( B=0.01 \text{ m} )</td>
</tr>
<tr>
<td>Initial roughness of the soft roller</td>
<td>( R_a=0.23 \mu\text{m} )</td>
</tr>
<tr>
<td>Density of asperities</td>
<td>( D_{sum}= 5\times10^9 \text{ 1/m}^2 )</td>
</tr>
<tr>
<td>Radius of tip of asperities</td>
<td>( \beta=5\times10^{-3} \text{ m} )</td>
</tr>
<tr>
<td>Arithmetic average of the asperity height</td>
<td>( R_a=0.22 \mu\text{m} )</td>
</tr>
<tr>
<td>Lubricant viscosity</td>
<td>( \mu_0=0.03017 \text{ Pas} )</td>
</tr>
<tr>
<td>Viscosity pressure index</td>
<td>( Z_{lub}=0.56 )</td>
</tr>
<tr>
<td>Oil inlet temperature</td>
<td>( T_{in}=315 \text{ K} )</td>
</tr>
<tr>
<td>Hardness of the softer roller</td>
<td>( H_1=22 \text{ RC} )</td>
</tr>
<tr>
<td>Hardness of the harder roller</td>
<td>( H_2= 42 \text{ RC} )</td>
</tr>
</tbody>
</table>

In these two experiments, the effect of rolling speed on the running-in behavior is studied. Figure 4 shows the comparison of variation of wear depth measured by LVDT during the test and the predicted wear depth.

The predicted wear depth is calculated based on the thermal desorption model with provision of plastic deformation of asperities. In other words, during the running-in process the asperities experience plastic deformation, which affects the roughness properties of the contacting surfaces. Once the running-in stage is complete, the equilibrium surface is obtained and asperities experience mainly elastic deformation.

The running-in model that is presented in [4] and the thermal desorption wear model [8] are used to predict the worn volume during running-in and the steady-state regime. At each instant, the wear depth is calculated from:

\[
\text{wear depth} = \frac{\text{wear rate} \times \text{time}}{2\pi RB} \tag{6.30}
\]

where wear rate is calculated from Equation (6.7), \( R \) is the radius of the softer roller, and \( B \) is the width of the roller. The initial rate of change of wear as a function of time is quite high while surfaces run in, a period which last 25 minutes for the 0.2 m/s rolling speed.
Figure 6.7: Effect of rolling speed on wear depth during running-in.

Then, as the steady-state regime starts, the slope of wear depth decreases. The polishing of the contacting surfaces which occurs as a result of plastic deformation of asperities results in a decrease in the slope of wear depth during steady-state regime. When the rolling speed increases, a better protecting film is formed and therefore wear is reduced. The steady-state wear slope, thus, decreases as the rolling speed increases. The weight loss of the hard and soft samples during the experiment is shown in Figure 6.8.

Clearly, most of the wear occurs in the soft roller. The harder roller, however, experiences some weight loss as well. Comparing $u_{roll}=0.1$ m/s and $u_{roll}=0.2$ m/s, at higher rolling speed a better lubricant film is formed and therefore the amount of weight loss decreases. The weight loss of harder roller in this case ($u_{roll}=0.2$ m/s) is considerably smaller than the case with $u_{roll}=0.1$ m/s. The increase in the lubricant film thickness that occurs as a result of increasing the rolling speed, inhibits the asperity-asperity contact and therefore wear depth and weight loss decreases. The comparison of surface roughness before and after the experiment is shown in Figure 6.9.
During the running-in process, as a result of plastic deformation, the asperities are polished and the roughness decreases. The surface roughness of the harder roller changes slightly when there is significant difference in the hardness of the contacting materials. In contrast, the soft roller experiences a more pronounced variation in surface roughness. In the case of higher rolling speed ($u_{\text{roll}} = 0.2 \text{ m/s}$), less wear occurs and therefore there is less change in the steady-state roughness of the surfaces. This is a fascinating finding regarding the running-in operating
conditions. As previously predicted by the authors [4], decreasing the rolling speed during running-in will result in a smaller value for steady-state roughness at the expense of higher wear.

- **Hard/Hard Running-In Experiment (Test#3 and 4)**

In order to study the effect of hardness on the wear behavior, the above experiment is repeated with rollers whose hardness values are fairly close. In this case, one of the rollers is only slightly harder than the other (44RC for harder roller and 40 RC for softer roller). The operating condition for this test is reported in Table 6.3.

**Table 6.3: Operating condition for hard/hard experiment**

<table>
<thead>
<tr>
<th>Rolling speed</th>
<th>$u_{\text{roll}}=0.1 \text{m/s and 0.2 m/s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range of slide-to-roll ratio</td>
<td>$s_r=0.15$</td>
</tr>
<tr>
<td>Applied Load</td>
<td>$F=5700$ N</td>
</tr>
<tr>
<td>Initial roughness of the soft roller</td>
<td>$R_a=0.23 \mu m$</td>
</tr>
<tr>
<td>Density of asperities</td>
<td>$D_{\text{sum}}=1.52 \times 10^9 1/m^2$</td>
</tr>
<tr>
<td>Radius of tip of asperities</td>
<td>$\beta=9.56 \times 10^{-5}$ m</td>
</tr>
<tr>
<td>Hardness of the softer roller</td>
<td>$H_1=40$ RC</td>
</tr>
<tr>
<td>Hardness of the harder roller</td>
<td>$H_2=44$ RC</td>
</tr>
</tbody>
</table>

The weight loss comparison is illustrated in Figure 6.10. Comparing to the Hard/Soft experiment, less wear occurs since the wear volume is inversely proportional to the hardness of the material. The values predicted by the simulations correspond to the total weight loss which should be compared to the summation of the experimentally determined weight loss of softer roller and the harder roller.

It is also noteworthy that comparing the Hard/Soft and the Hard/Hard experiment, a higher percentage of wear occurs on the harder roller in the Hard/Hard case. This difference in the wear can be attributed to the relatively small difference in the hardness values of the contacting materials. In the extreme cases, it is expected that if both of the materials have similar hardness and roughness, they experience equal weight loss during running-in test.
Figure 6.10: Comparison of measured weight loss and predicted weight loss

Figure 6.11 shows the comparison of roughness before and after the running-in test. Comparing $u_{\text{roll}}=0.1$ m/s and $u_{\text{roll}}=0.2$ m/s, in higher rolling velocity a thicker lubricant film is formed, therefore less asperity-asperity contact occurs. Hence, the final roughness in the case with higher rolling velocity ($u_{\text{roll}}=0.2$ m/s) is higher than the case with lower rolling velocity ($u_{\text{roll}}=0.1$ m/s).

Figure 6.11: Roughness variation for hard/hard experiment

- Hard/Soft Interrupted Running-In Experiment (Test#5)

In order to carefully observe the running-in phenomenon, an experiment under the operating conditions of Table 4 was conducted in which the experiment is stopped every 10 minutes.
Table 6.4: Operating conditions for interrupted experiment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolling speed $u_{roll}$</td>
<td>0.2 m/s</td>
</tr>
<tr>
<td>Range of slide-to-roll ratio $sr$</td>
<td>0.15</td>
</tr>
<tr>
<td>Applied Load $F$</td>
<td>5700 N</td>
</tr>
<tr>
<td>Initial roughness of the soft roller $R_s$</td>
<td>0.26 µm</td>
</tr>
<tr>
<td>Density of asperities $D_{sum}$</td>
<td>$3.67 \times 10^{9}$ 1/m²</td>
</tr>
<tr>
<td>Radius of tip of asperities $\beta$</td>
<td>$3.73 \times 10^{-5}$ m</td>
</tr>
<tr>
<td>Oil inlet temperature $T_{in}$</td>
<td>315 K</td>
</tr>
<tr>
<td>Hardness of the softer roller $H_1$</td>
<td>22 RC</td>
</tr>
<tr>
<td>Hardness of the harder roller $H_2$</td>
<td>42 RC</td>
</tr>
</tbody>
</table>

After each stopping period of the test, the soft roller is cleaned with acetone and dried. Then the weight and the roughness of the roller are recorded and the roller is again mounted on the shaft and the test continues for another 10 minutes. The variation of surface roughness during running-in is shown in Figure 6.12.

![Figure 6.12: Variation of surface roughness during running-in experiment.](image)

Initially, there is a substantial drop in the value of $R_a$ which is followed by smaller decrease of surface roughness until the running-in process is completed and the steady-state regime starts. The simulations predict the final value for surface roughness fairly accurate. The variation of worn weight during running-in is shown in Figure 6.13.
Figure 6.13: Comparison of worn weight during running-in experiment.

Similar to the variation of wear depth shown in Figure 4, the worn weight starts with a relatively high slope and then increases linearly. Also, examination of the changes in the slope reveals that the results of the simulations accurately capture the duration of running in (35 minutes in this case) with reasonable accuracy.

6.6.2 Steady-State Experiments (Test 6)

Two steady-state experiments are performed on the broken-in rollers to observe the steady-state wear and friction behavior. For this test, the rolling speed is constant \( u_{\text{roll}} = 0.15 \text{ m/s} \) and the slide-to-roll ratio \( (s_r) \) is varied from 0.2 to 1.4 in the increments of 0.2. Therefore 7 experiments \( (s_r=0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4) \) are conducted. The duration of each test is 1 hour and the time between each two consecutive test is 2 hours. After each test, the soft roller is dismounted, cleaned with acetone, dried and weighted. The operating condition of this experiment is reported in Table 6.5.

<table>
<thead>
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<th>Table 6.5: Operating conditions for steady-state experiment</th>
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<tr>
<td>Rolling speed ( u_{\text{roll}} ) &amp; 0.15 m/s</td>
</tr>
<tr>
<td>Range of slide-to-roll ratio &amp; ( s_r = 0.2\sim1.4 )</td>
</tr>
<tr>
<td>Applied Load &amp; F= 5700 N</td>
</tr>
<tr>
<td>Surface roughness of the roller &amp; ( R_a=0.1 \mu m )</td>
</tr>
<tr>
<td>Density of asperities &amp; ( D_{\text{sum}}=1.52\times 10^9 \text{ 1/m}^2 )</td>
</tr>
<tr>
<td>Radius of tip of asperities &amp; ( \beta=5\times 10^{-6} \text{ m} )</td>
</tr>
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</table>
One of the pertinent parameters in steady-state wear experiments is the wear rate. Wear rate shows the worn volume per unit of sliding distance and is measured using following equation:

\[
\text{Wear rate} = \frac{\text{worn weight}}{\text{density} \times \text{sliding distance}}
\]

In this experiment, the worn weight is measured using an accurate balance each time the experiment is stopped. The comparison of measured and predicted wear rate is illustrated in Figure 6.14. As was discussed in [8], at relatively low surface temperatures such as this case, the wear mechanism is thermal desorption wear mechanism. In analyzing the data of wear rate versus slide-to-roll ratio, there are many parameters involved which have different impacts on the wear behavior.

![Figure 6.14: Wear rate as a function of slide-to-roll ratio](image)

As the slide-to-roll ratio increases, the film thickness decreases and therefore more asperity-asperity contacts might occur which results in an increase in wear rate. The sliding distance, however, increases with increasing the slide-to-roll ratio resulting in decrease of wear rate. Based on thermal desorption model, as sliding speed increases the molecules have less time to detach from the surfaces and therefore the wear volume rate decreases. Further increase in the slide-to-roll ratio may result in excessive heat generation and substantial wear or even scuffing. The variation of temperature is shown in Figure 6.15.
For each value of slide-to-roll ratio, the temperature starts from the room temperature and rises until it reaches a steady-state value. The temperature is recorded using the infra-red thermocouple. The temperature reported in Figure 12 corresponds to the steady-state value for surface temperature.

As the slide-to-roll ratio increases, the sliding speed rises. The increase in the sliding speed decreases the film thickness. It was shown in [3] that the generated heat in the lubricant is proportional to the second power of sliding speed. Therefore, as the sliding speed increases, the temperature of the lubricant as well as the contacting surfaces rise. The traction coefficient behavior is shown in Figure 6.16.

As the sliding speed increases, the traction coefficient initially increases. Further increase in the sliding speed results in a decrease in friction coefficient. The drop in the traction coefficient at higher values of slide-to-roll ratio is attributed to the thermal effects involved in the problem. If one neglects the thermal effects in the simulation, the predicted traction coefficient for small values of slide-to-roll ratio would be similar to what is shown in Figure 6.16. At higher values of slide-to-roll ratio, however, the isothermal model predict an increasing trend for traction.
coefficient as the slide-to-roll ratio rises whereas the current model predicts that the thermal
effects become overriding and the traction coefficient drops.

![Graph of traction coefficient vs slide-to-roll ratio](image)

**Figure 6.16: Variation of traction coefficient with slide-to-roll ratio.**

An increase in the surfaces temperature as a result of the increase in slide-to-roll ratio results
in a lower lubricant viscosity and smaller film thickness. A decrease in viscosity brings about a
decrease in traction coefficient whereas the decrease in film thickness results in an increase in
traction coefficient. Initially, the decrease in film thickness has a more pronounced effect than
the decrease in viscosity. For higher values of slide-to-roll ratio (sr=0.6 in this case), the effect of
viscosity drop becomes more pronounced and traction coefficient decreases.

### 6.6.3 Effect of Rolling Speed (Test 7)

In this experiment, the slide-to-roll ratio is kept constant \( sr=0.1 \) and the rolling speed is changed.
The operating conditions for this experiment are shown in Table 6.6. In this experiment, for each
rolling speed, the rollers are set at specific rotational speeds such that for all points the \( sr=0.1 \)
whereas the rolling speed changes from 0.1 m/s to 0.22 m/s. For each rolling speed the test is
conducted for 60 minutes and the time between each two consecutive tests is about 2 hours.


<table>
<thead>
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<th>Table 6.6: Operating conditions for the experiment</th>
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<tr>
<td>Rolling speed ( u_{\text{roll}} ) = 0.11~0.22 m/s</td>
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<tr>
<td>Range of slide-to-roll ratio ( s_r = 0.1 )</td>
</tr>
<tr>
<td>Applied Load ( F = 5800 \text{ N} )</td>
</tr>
<tr>
<td>Surface roughness of the roller ( R_a = 0.2 \mu m )</td>
</tr>
<tr>
<td>Density of asperities ( D_{\text{sum}} = 4 \times 10^{9} \text{ } \text{1/m}^2 )</td>
</tr>
<tr>
<td>Radius of tip of asperities ( \beta = 5 \times 10^{-5} \text{ m} )</td>
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</table>

The variation of traction coefficient versus rolling speed is shown in Figure 6.17 (a). The surface roughness data used to calculate the friction coefficient are the surface roughness data after the running-in. A small range of the rolling speed in the mixed regime is chosen for this experiment and friction coefficient is measured for eight values of rolling speed. The effect of variation of rolling speed on traction coefficient is shown in Figure 6.17 (b). As the rolling speed increases, a better protecting film is formed and therefore less asperity-asperity contact occurs. Hence, friction coefficient decreases. Decreasing the rolling speed, on the other hand, reduces the thickness of the film that is formed. In this case, a large number of asperity-asperity contact occurs which results in high surface temperature and high friction coefficient. Further increase in the rolling speed will reduce the traction coefficient until the rollers reach the lift-off speed in which the lubrication regime shifts from mixed regime to full film regime. In other words, the lubrication regime in which the total load is carried by the fluid film is obtained at higher rolling speed. Further increase in rolling speed after the lift-off speed increases the traction coefficient. The plot shown in Figure 6.17 (a) is usually referred to as Stribeck curve.

6.7 Conclusions

In this chapter, the experimental results conducted with the Gear Test Rig are compared to the simulation results of the model based on the load-sharing concept which is developed in chapters 1-5. In this test rig, contact of points of interest on the involute profile of gears is replaced with contact of two rollers. The speed of the rollers and the applied load is selected such to ensure that the contact conditions between the rollers are the same as the contact properties on the
corresponding point of the gear. The sensors that are mounted on the contacting rollers enable the user to monitor the variation of wear depth, surface temperature, and friction coefficient.

![Figure 6.17: (a) Stribeck curve predicted by simulation (b) Comparison of predicted and measured friction coefficient.](image)

A set of running-in experiments are conducted on the rollers and evolution of wear depth, worn weight and surface roughness during the experiment are measured. The wear depth and worn weight has a similar trend during running-in stage, i.e. they both initially increase with a relatively sharp slope. As the asperities polish and the number of plastically deformed asperities reduce, the slope of wear depth and worn weight decreases.

Running-in experiments are performed on pairs of rollers with different values for hardness and different operating conditions. If there is substantial difference in the hardness of the rollers, most of the wear occurs in the softer material and the latter experiences more reduction in surface roughness. Whereas when the contacting materials have little difference in the value for hardness, the worn weight of the slightly harder roller is comparable to the worn weight of the slightly softer roller.

Running-in operating conditions affects the steady-state performance of the contacting rollers. In real world applications, among all the influential parameters in the running-in behavior such
As material, roughness, hardness, lubricant, applied load, rolling speed, and sliding speed, there are very few such as speed which can be selected by the user. In these experiments the effect of rolling speed was investigated. It is shown by experiment as well as by simulation that increasing the rolling speed will decrease the worn weight and wear depth during running-in. On the other hand, a lower rolling speed results in a smaller value for steady-state surface roughness which means less asperity-asperity contact and hence smaller value for friction coefficient during steady-state regime. Therefore, it is believed that there are a set of optimized operating conditions for running-in stage which can be determined based on the working conditions of the steady-state which is the main course of working life of the rollers.

After the rollers run-in, their surfaces polish and experiments pertaining to steady-state is conducted. Several sets of experiments are conducted on the run-in rollers to measure the steady-state friction, surface temperature, and wear rate. In lubricated contact of rollers, keeping the sliding speed constant and increasing the rolling speed in the mixed regime results in a better protecting film and therefore a decrease in friction coefficient. Formation of a thicker film reduces the wear depth. Maintaining the rolling speed constant and increasing the sliding speed results in an initial increase in the friction coefficient. Further increase in sliding results in excessive heat generation. Hence, viscosity as well as film thickness decreases resulting in decrease of the friction coefficient.

### 6.8 Nomenclature

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<th>Description</th>
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<tr>
<td>(a)</td>
<td>Hertzian half-width of contact (m)</td>
</tr>
<tr>
<td>(B)</td>
<td>Width of roller (m)</td>
</tr>
<tr>
<td>(D_{sum})</td>
<td>Density of asperities (1/m(^2))</td>
</tr>
<tr>
<td>(E_P)</td>
<td>Equivalent Young’s modulus (N/m(^2))</td>
</tr>
<tr>
<td>(f_C)</td>
<td>Asperity-asperity friction coefficient</td>
</tr>
<tr>
<td>(F_{C})</td>
<td>Load carried by asperity (N)</td>
</tr>
<tr>
<td>(F_T)</td>
<td>Friction force (N)</td>
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<tr>
<td>(F_H)</td>
<td>Load carried by fluid film (N)</td>
</tr>
<tr>
<td>(F_T)</td>
<td>Applied load (N)</td>
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<tr>
<td>(H)</td>
<td>Material hardness (N/m(^2))</td>
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<tr>
<td>Symbol</td>
<td>Description</td>
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<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>$h_c$</td>
<td>Central film thickness (m)</td>
</tr>
<tr>
<td>$I_{Sl}$</td>
<td>Current of motor s (Amp)</td>
</tr>
<tr>
<td>$I_{Ll}$</td>
<td>Current of motor l (Amp)</td>
</tr>
<tr>
<td>$P_f$</td>
<td>Consumed power of the rollers after contact (W)</td>
</tr>
<tr>
<td>$P_{hyd}$</td>
<td>Hydraulic pressure (N/m$^2$)</td>
</tr>
<tr>
<td>$P_i$</td>
<td>Consumed power of the rollers before contact (W)</td>
</tr>
<tr>
<td>$r_c$</td>
<td>Bore of hydraulic jack (m)</td>
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<tr>
<td>$R_1$</td>
<td>Radius of larger roller (m)</td>
</tr>
<tr>
<td>$R_2$</td>
<td>Radius of smaller roller (m)</td>
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<td>$R_a$</td>
<td>Arithmetic average of asperity heights (m)</td>
</tr>
<tr>
<td>$R_q$</td>
<td>Standard deviation of asperity heights for surface 2 (m)</td>
</tr>
<tr>
<td>$R'$</td>
<td>Equivalent radii of curvature (m)</td>
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<td>$U_{roll}$</td>
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<td>$U_{sliding}$</td>
<td>Sliding velocity (m/s)</td>
</tr>
<tr>
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</tr>
<tr>
<td>$U_2$</td>
<td>Velocity of larger roller (m/s)</td>
</tr>
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<td>$V_{Sl}$</td>
<td>Volts of the motor (V)</td>
</tr>
<tr>
<td>$V_{S2}$</td>
<td>Volts of the motor (V)</td>
</tr>
<tr>
<td>$Z$</td>
<td>Viscosity-pressure index of lubricant</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Radius of tip of asperities (m)</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>Scaling factor for hydrodynamic part</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>Scaling factor for asperity interaction part</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Viscosity of the lubricant (Pas)</td>
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</table>

### 6.9 References


Chapter 7: Conclusions and Future Steps
7.1 Conclusions

In this research, a method based on the load-sharing concept is presented to analyze the transient and steady-state performance of gears. The isothermal analysis of the gears is discussed in Chapter 1. In this chapter it is shown that the load-sharing concept can be applied to analysis of spur gears and the calculated film thickness is compared to the film thickness predicted by the numerical method. The thermal analysis of gears is addressed in Chapter 2. The simplified energy equation is added to the load-sharing analysis of Chapter 1. Including the thermal analysis resulted in a correction factor for film thickness and modified values for viscosity. The modified values for viscosity are used to predict friction coefficient. The predicted results for friction coefficient versus slide-to-roll ratio are shown to be in good agreement with experimental results. The steady-state sliding wear based on thermal desorption is studied in Chapter 3. In this formulation based on the surface temperature the thermal desorption or the oxidative wear mechanism is active. For the operating condition of typical gears, the surface temperature is not high enough for oxidative mechanism to become active and the dominant wear mechanism is thermal desorption. The predicted wear rate is shown to be close to the measured wear rate between two rollers. Gears usually operate in the mixed lubrication regime in which the surface roughness plays an important role. Based on the procedure used to finish the surface, the asperities on the surface have different orientations. A numerical algorithm is developed which takes the standard deviation of asperity heights and the surface pattern parameter as input and generates the roughness profile. The surface pattern parameter determines the asperities orientation. The effect of surface pattern on the friction behavior is studied in Chapter 4. The transverse, longitudinal, and isotropic surface patterns are generated and it is shown that the friction coefficient in longitudinal surface pattern is lower compared to isotropic and transverse surface patterns. The running-in behavior of gears is modeled based on the plastic deformation of asperities and is discussed in Chapter 5. This model is a powerful method in
predicting the variation of surface roughness, friction coefficient, wear volume, scaling factors, etc. during running-in. The predicted variation of surface roughness is shown to be fairly close to the experimentally measured values. It is also shown that the wear volume is higher in the transverse surface pattern compared to isotropic and longitudinal surface patterns. The wear rate gradually decreases until it reaches the steady-state. The steady-state value for wear rate in contact of two rollers is shown to be close to the wear rate calculated from thermal desorption model. A gear test rig is built in Center for Rotating Machinery (CeRoM). The structure of the test rig as well as the experimental data is shown in Chapter 6. The running-in experiments are performed on five pair of rollers under different operating conditions and the wear depth and the worn weight are recorded during the test. Two sets of steady-state experiments are also conducted on run-in rollers. The friction coefficient, the surface temperature, and the wear rate are compared to the results predicted by simulation.

7.2 Future Steps

In this dissertation the load-sharing concept is used to predict the performance of spur gears during running-in stage as well as steady-state. The current method can be extended to other problems which are listed here:

- **Developing Curvefit Relation for Running-In Behavior**

The model that is developed in this research for running-in is shown to be capable of predicting the running-in behavior in close agreement with experimental data. A group of dimensionless parameters can be used to develop curvefit relation to predict the running-in time, final value for arithmetic average of surface roughness $R_a$, and final values for friction coefficient.

- **Visualization of the Model**

The model in its present form is capable to be used to take the geometry and loading conditions of a pair of spur gears, as well as the surface properties as input and predicts the transient and the steady-state performance. The current algorithm can be visualized to make it easier for the users
to observe the effect of different parameters such as load, speed, geometry, surface pattern parameter, surface roughness, etc on the transient and steady-state behavior of gears.

- **Extending the Model to Helical Gears**

Besides spur gears which have been studied in this research, helical gears are also widely used in industry. In helical gears at each instant, there are several pairs of teeth in contact. Therefore, helical gears have the capability to transmit large amount of loads in a smoother fashion. The present model can be extended to predict the performance of helical gears.

- **Extending the Model to Cam-Follower**

The proposed model is developed for line contact problem such as gears. Some of the real world application of lubricated contact problems such as cam-follower and ball bearings are point contact. Extending the current algorithm to point contact problem, will facilitate handling the cam-follower problem.

- **Conducting Experiments on Point Contact Problem**

The current test rig (GTR) is capable to be used for point contact problem. By making a crown on the rollers outer diameter, the contact area will be changed from line contact to point contact and the simulation predictions for cam-follower can be experimentally verified.
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Vita

Saleh Akbarzadeh was born in September 1981 in Neishabour, northeast of Iran. His family and he then traveled to the USA when his father was continuing his studies toward a doctorate in physics at Purdue University. The family returned back to Iran in 1984 and went to Isfahan. He went to the primary and high schools which were located inside the campus of Isfahan University of Technology. He finished high school in 1999.

He was accepted in mechanical engineering at Sharif University of Technology in Tehran in 1999. After one semester he transferred to Isfahan University of Technology. He finished his bachelor’s degree in summer 2003. That year he was accepted in Sharif University of Technology as a graduate student. He finished his master’s degree in June 2005. His master’s thesis was on the analysis of a new generation of power transmission systems known as CVT (Continuously Variable Transmission). During his master’s studies, he worked in the Vehicle Fuel and Environment Research Institute in College of Engineering at University of Tehran.

He moved to USA in January 2006 after he was accepted in the doctoral program in Louisiana State University. He finished his studies under the supervision of Prof. M. M. Khonsari who is the director of Center for Rotating Machinery (CeRoM). The main focus of his doctoral research has been on the performance of the gears. He was nominated as the Best Research Assistant in the Department of Mechanical Engineering in April 2008.