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Making Transitions: A Multiple Case Study of Mathematics Classroom Teaching Reform in China

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MAKING TRANSITIONS: A MULTIPLE CASE STUDY OF MATHEMATICS CLASSROOM TEACHING REFORM IN CHINA

A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Doctor of Philosophy

In

The Department of Educational Theory, Policy and Practice

by

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PREFACE

Did You Know?

……

According to US Department of Labor…
1/4 of the workers today are working at a company for whom they have been employed less than a year.
1/2 of the workers are working for a company for whom they have been worked less than 5 years …
We are currently preparing the students for the jobs that do not yet exist…
Using technologies that haven’t be invented…
In order to solve the problems we do not even know are problems yet.
……

Did you know?
We live in exponential times.
There are over 2.7 billion searches performed on Google each month.
……

The amount of text messages sent and received every day exceeds the population on the planet.
Today, there are about 540,000 words in the English language…
About 5 times as many as during Shakespeare’s time.
There are 3,000 new books are published…
Daily!
It is estimated that a week’s worth of New York Times…
contains more information than a person was likely to come across in a lifetime in the 18th century.
It is estimated that 1.5 exabyte (1.5x10^{18}) of unique new information will be generated worldwide this year.
That’s estimated to be more than in the previous 5000 years.
The amount of new technical information is doubling every two years.
For students starting a four-year technical or college degree, this means that…
Half of what they learn in their first year of study will be outdated by their third year of study.
……

What do all these mean?
Shift happens.

From video (updated 2009) Did you know 1.0: Size Does Matter (http://www.youtube.com/watch?v=xj9Wt9G--JY), [Chinese vision ] (http://www.youtube.com/watch?v=_4gcDu2ESPE), [English vision1.0]
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ABSTRACT

The main purpose of this study is to investigate how teachers implemented teaching reform in secondary mathematics classrooms in China, and to understand the context of teaching transformation in general. Two groups of mathematics teachers were included in this study. One group was involved in a leadership project led by university-based mathematics teacher educators, and the other was engaged in a school teaching experiment led by teacher educators from local education department.

In this study, classrooms are viewed as social systems in which the teacher and students are interrelated through communication networks. This study examines the structures, patterns of social relationships, and socio-autonomy of the communication networks. The social relationships are focused on learning goals, social and socio-mathematical norms, and mutual relationships. Socio-autonomy refers to the abilities of the communication networks to adapt and to evolve. A combination of social network analysis and qualitative research methods is employed to analyze the dynamics of the structures and relationship patterns. In addition, the teachers’ perspectives of mathematics, instruction, and experiences, along with associated teacher communities are all examined to understand their impact on teaching practices.

The findings have indicated different types of classroom teaching with different learning goals, social norms, and mutual relationships. Overall, the teaching was centered upon students’ problem solving and presentations of solutions. The communication structures in most classroom teaching, however, are lack of dynamics. Constraints placed on communications controlled by the teacher result in linear patterns of communications and dampened emergent and dynamic communications in classroom teaching. Reform
efforts seemed not to fully support the development of mathematical insights and creativity. These issues suggest that classroom teaching reform need to address the power relationships between individuals’ development of understanding and socio-autonomy of the class. This research also indicates that teaching approaches are related to what the teachers have learned in the associated teacher learning communities. It suggests teacher education need to help teachers change expectations for learning from efficiency and skills to processes and communication dynamics. By embracing uncertainty and better understanding of individual autonomy and letting go of control, innovative transformation happens in classroom teaching.
CHAPTER 1: INTRODUCTION

Background of the Research

On May 22\textsuperscript{nd}, 2009, I attended a teacher enhancement activity at a secondary school in Beijing, China. During the activity, a Chinese mathematics high school teacher Mr. W presented a model lesson, and then talked about his teaching philosophy and experience with the participants. In his presentation, the teacher played the video \textit{Did you know?}, originally made by an American high school teacher and published on YouTube in 2007. Based on a series of statistical data, the video showed how our globalized society is undergoing profound transformation with rapid development and broad applications of information technology. The video intimated: “We are currently preparing students for jobs that do not yet exist, using technologies that have not been invented, in order to solve the problems we do not even know are problems yet.” Social changes call for educational changes. As mathematics educators, what can we do to address the challenges of a society in transition?

Indeed, early in the 1980’s, the \textit{A Nation at Risk} (1983) report warned that American education was lagging behind the rapid development of technology. This report, along with other research findings and documents, initiated a new wave of educational reform that continues today in the United States (US). In particular, various reform documents in mathematics education such as \textit{The Principles and Standards for School Mathematics} (2001) advocated that mathematics is the science of patterns and relationships. The reform calls the development of students’ mathematical power central to any school mathematics curriculum. A student’s mathematical power is indicated through understanding, communicating, reasoning, problem solving, and dispositions (NCTM, 1989, 1991, 2000). The National Research Council (NRC, 2001) suggested that mathematics learning consists of five intertwined strands: “conceptual
understanding,” “procedural fluency,” “strategic competence,” “adaptive reasoning,” and “productive disposition.” Reform-oriented school mathematics in the US strives for this integrated development of knowledge, abilities, and dispositions for their students.

The US educational reform wave has rippled out to affect the globe. In China, increasing numbers of educators have realized the weak creative and practical abilities among Chinese students in traditional education programs, despite their high performance on numerous international contests such as the International Math Olympiad. Zhong (2006) called these weaknesses a “threat” to the fast and sustainable development of economy or society in China. To ensure the economic and social development, an unprecedented large-scale curriculum reform has been advocated for all K-12 schools since 2001 in China. The reform attempts to transition from a “drill-and-kill” education to a more qualitatively oriented education. The new Chinese national standards for mathematics curriculum call for the integration of multiple dimensions for mathematics students: knowledge and skills, mathematical thinking and problem solving, and affect and attitudes (MEPRC, 2001). The re-conceptualization of school mathematics to focus on the meaningful mathematical development of every student has become an endeavor of mathematics educators worldwide.

Of particular interest is how educational reform ideas, suggestions, and research are increasingly being shared across the boundaries of countries. For example, the Did You Know video mentioned earlier - originally made and posted on the Internet by a US high school teacher to stimulate faculty discussions about improving local education efforts – was eventually downloaded and shown by Mr. W to Chinese colleagues in the aforementioned teacher meeting. Most likely, the US teacher will never know that his/her video had been used by an expert mathematics teacher in a key teacher meeting in Beijing; however, networks have enabled the
sharing of such ideas. With networks (the Internet, and even the Chinese teacher meeting), US and Chinese educators have been connected in new and exciting ways.

No one knows how many people will further be affected by such networks, or what impact the nested-networks will provide for individuals embedded in such activity. Argentinean author Jorge Luis Borges said, “Everything touches everything” (quoted in Barabasi, 2000, p. 5). In our increasingly connected world, Eastern and Western education, students, teachers, educators, curriculum, and policies interact and are interrelated through interconnected networks of communication.

Although reforms have been advocated and implemented for many decades in the US, research has indicated how difficult it is for essential changes to occur in the teaching and learning of mathematics on a large scale (Franke, Kazemi, & Battey, 2007). This is the real test of reforms. For example, despite the reform efforts, the demands for remedial mathematics education for incoming college students keeps growing. The National Mathematics Advisory Panel (NMAP, 2008) report indicated the percentage of students who were at or above the basic mathematics proficiency level in secondary schools was still unacceptably low.

Mathematics educators are dedicated to making essential changes in school mathematics globally. The current curriculum reform in China involves the largest population of students, teachers, and schools in the world. This large scale reform provides tremendous opportunities but also challenges for Chinese classrooms and educators (Carson, 2009). Over the past 10 years, this reform has emphasized the transformation of classroom teaching. How did Chinese mathematics educators implement classroom reform? What perspectives and strategies did Chinese mathematics educators have in implementing classroom teaching reform? What would be ideal conditions for mathematics classroom transformation? Motivated by these questions, I
visited six Chinese schools in three cities in the summer of 2009. I observed 24 lessons conducted by 12 teachers, and interviewed teachers, students, teacher educators, and school administrators. The classroom observations and videotapes, and interviews provided me with an opportunity to study these questions systematically and to examine mathematics classroom reform from multiple levels. In this study, I explore the reform implementation across different classrooms and teacher professional development communities in different cities where different approaches were used. I expect to bring forth insights about the conditions of mathematics classroom teaching transformation in mathematics education fields across the boundaries of China and the US.

Statement of the Research

The reform in China, as with those in the US, requires new perspectives in mathematics education to affect transformative change. Monge and Contractor (2003) pointed out, “If the phenomenon… is itself undergoing radical transformation, then we too, must change our ways of studying it. And to be effective, the ways in which we change must reflect the transformations that we seek to understand” (p. 7). In the past, researchers applied analytical methods focusing on examining the components of classroom teaching and learning separately, rather than examining the dynamic relationships occurring in and across classrooms. Studying classroom teaching transformation requires a holistic perspective respecting the integrity of system components while simultaneously allowing for their integration and support of change across the system. In other words, to understand systemic change, the various dimensions of classroom teaching and learning need to be examined for their relational dynamics and interactions.

From social systems theories perspectives, classrooms are networks of communications (Fleener, 2002; Lumman, 1990; Maturana & Veralá, 1987). The communication networks can
self-generate, self-maintain, and adapt. Despite the unpredictability of their evolution, the interactions among members of systems are meaning-based (Doll, 1993). Meaning relationships such as goals or values are produced and adapted through communication networks (Fleener 2002; Lumman, 1990). The communication networks, along with the meaning relationships, evolve as a unity. Because classrooms are social systems, individual experiences and understandings are intertwined with collective goals, knowledge, commitments, and norms. The transformation of classroom teaching and learning depends on supportive social relationships and collaborative actions among the members in the communication networks. Classrooms are also themselves embedded in larger educational systems; these larger education systems (such as schools and local educational systems) help to shape the overall classroom adoption of reform. Understanding the transformation of classrooms involves examining the communication structures and relationship patterns in classroom systems, as well as the impact from and relationship to larger educational systems.

This study investigates the implementation of teaching reform in secondary mathematics classrooms in the context of nation-wide curriculum reform in China. More specifically, it is an analysis of dynamic communication structures and meaning relationship patterns (such as learning goals, norms, mutual relationships, and classroom organizations) that underlie classroom teaching reform in two school sites selected for their different reform approaches: Site A, where teachers participating in a reform leadership project conducted by university-based teacher educators and the local educational government, was contrasted with Site B, where a school-wide teaching experiment was led by mathematics educators from the local district educational department. The structures and meaning relationships in the classrooms and the associated teacher professional development communities were analyzed to explore the different
approaches to systematic educational reform, and to examine what contributed to any perceived change. Through the examinations of the dynamics of communication structures and relationships, this research describes the features, the dynamics and driving forces underlying the different approaches of implementing classroom teaching reform. In addition, this study examines the relationships between classroom teaching and teacher professional development programs, and teachers’ beliefs and experiences of teaching mathematics. It provides insights about the context of classroom teaching transformation from three levels including the teacher, the classrooms and the teacher communities. This study thus contributes to the research that investigates mathematics classroom transformation from a systematic and progressive perspective.

Research Questions

The central question of this study is: What communication structures and meaning relationship patterns underlie the Chinese mathematics classroom teaching reform at these two sites? The sub-questions are:

1. What communication structures are demonstrated during classroom teaching? What relationship patterns are embedded in the structures? How do these network structures change in teaching?
2. What communication structures and relationship patterns are demonstrated in the associated teacher professional development communities at both sites?
3. What similarities and/or differences are demonstrated in the networks and relationship patterns across classroom practices and teacher professional development communities? What similarities and/or differences are demonstrated in the networks and relationship patterns across different sites?
4. What perspectives of mathematics and mathematics teaching do those teachers have?

Organization of the Study

This study is organized into seven chapters. The next chapter details the theoretical perspectives that guided the research. Chapter Three reviews relevant literature on mathematics teaching and reform. Chapter Four describes the research methods. Chapters Five and Six describe the analysis of data and reporting of results for the two sites. Chapter Seven concludes the study with a discussion of implications and limitations of this study, along with recommendations for further study.
CHAPTER 2: THEORETICAL FRAMEWORK

The purpose of this study is primarily to investigate mathematics classroom teaching reform in China and to understand the context of classroom teaching transformation in general. In this study, mathematics classrooms are viewed as social systems where teachers and students interrelate with one another in fundamental and significant ways. Since social systems can only be described in terms of their dynamics (Fleener, 2002), I apply social systems theories with a focus on understanding the dynamics of the classroom teaching context. According to social systems theories, classroom systems and their members constantly undergo a co-evolution through communication networks (Fleener, 2002; Luhmann, 1999; Maturana & Varela, 1987). The dynamic relationships between individuals and the emergence of collective properties in classrooms are not visible from the perspectives focusing on individuals. Instead, we need a “level-jump” (Davis & Sumara, 2006) view to consider emergent properties of the systems and the growth of individuals all at once. Social systems theories offer us alternatives to examine the changes not only of the structures of the systems, but also the dynamic relationships within the systems simultaneously.

In this chapter, I first introduce autopietic systems and social systems theories. Built on those and other research on social systems, particularly in educational systems that most pertain to my study of mathematics classroom teaching, I then construct my theoretical framework used in this study.

A Shift to Systems Perspectives

In Crosby’s (1997) book The Measurement of Reality, Western Europeans’ splendid accomplishment in science and technology are attributed to their quantification and measurement of reality. Galileo’s abandonment of the qualification of objects in science, Descartes’ analytical
methods, along with Newtonian universal laws contributed to the rooting of a clockwork worldview in Western European’s perception of reality. This mechanistic view sees the universe as a certain and controllable collection of dissociated parts. We strived to be “precise, punctual, calculable, standard, bureaucratic, rigid, invariant, finely coordinated, and routine” (p.230). In the last few decades, this long-held mechanical model of reality has been shaken by studies across the disciplines such as physics, biology, economics, and sociology that indicate numerous phenomena cannot be explained in static, reductionist, and predetermined ways (for example, properties that arise from the interactions among the components of a living system, which do not belong to any part of the organism). The appearance and disappearance of a city, the cooperation in an ant colony, and even political upheavals, like the overthrow of the thirty-year-ruled Egyptian government, are examples of social systems behaving in ways that exhibit coordinated and interactive dynamics which can best be understood from systems perspectives.

As researchers across all disciplines investigate these phenomena, theories such as systems theories, complexity theories, and social systems theories that focus on relationships, patterns, and dynamics have been developed and offer alternative perspectives to the mechanistic view. Social systems theories especially suggest that a social system is composed of interacting and interrelated components which exhibit coordinated and synergistic dynamics. From social systems theories, the components and the system as a whole constantly change in response to changes in their environment through interaction networks.

The Properties of Autopoietic Systems

Literally, Autopoiesis consists of two roots: auto and poiesis. In Greek they originally mean “self” and “produce” respectively. The term autopoiesis was introduced by the Chilean theoretical biologists Humberto Maturana and Francisco Varela in the 1970’s to define and
explain the nature of living systems. The term now has been used broadly to denote systems that have abilities to self-generate, to adapt and to maintain identities.

Maturana and Varela distinguished “organization” from “structure” of a living system. They pointed out, “Organization denotes those relations that must exist among the components of a system for it to be a member of a specific class. Structure denotes the components and relations that actually constitute a particular unity and make its organization real” (p.47). In other words, organization refers to configurations of the relationships of the components in a system, which defines the identities of the system. Structure, including the actual components and the relationships they hold, is an embodiment of the organization between components. Based on this differentiation, Maturana and Varela suggested the changes in a living system are a result of structural coupling, the recursive processes involving “recurrent interactions, each of which triggers structural changes in the system” (Capra, 1996, p. 219). Over time the structure of both system and environment changes through mutual perturbations while maintaining the identities of each system.

“Structural coupling” implies that an autopoietic system is self-referenced and autonomous system. As Maturana and Varela said, “The structure of the system determines its interactions by specifying which configurations of the environment can trigger structural changes in it” (p.135). When the system interacts with the environment, the environment provides sources or stimulation to the system; yet, internal correlations of the system as a whole determine how to instantaneously respond to the environment.

Moreover, a living system experiences co-evolution of the components and the system over time. According to Maturana and Varela, the components of a living system are dynamically related through a network of interactions. On the one hand, the components of the
network reproduce themselves in the process of interacting with each other. On the other hand, the entire interaction network transforms as a whole. “The network continually makes itself. It is produced by its components and in turn produces those components” (Capra, 1996, p. 162). In short, in living systems the components and the interaction network are inseparable as a unity, and they are constantly self-generating as a whole.

The characteristics of autopoiesis in living systems described by Maturana and Varela can be summarized as (1) structurally-closed (self-referential and autonomous); (2) self-generating (co-evolving of the system and its components); and (3) adapting (maintaining identities in changes) (Fleener & Pourdavood, 1997; Fleischaker, 1990). Those properties highlight recursive interaction and dynamic relationships between living systems and their components. Likewise, there are also dynamic relationships between the systems and their environment.

Social Systems as Autopoietic Systems

Living systems have the characteristics of being of self-generating, adapting, and structurally-closed. According to some social systems theories, social systems similarly possess these characteristics of living systems (Fleener, 2002; Luhmann, 1990; Maturana & Varela, 1987). While controversial, the question of whether social systems are living systems is addressed by understanding social systems as communication networks. From this perspective, “structural coupling” in a social system is communication of coordinated behaviors among the members in a social system. Maturana and Varela said, “We call communication the coordinated behaviors mutually trigged among the members of a social unity” (p. 193). They highlighted communication not as the “transmission of information,” but as “recurrent coordination” among the members, and through which both the members and the communication networks adapt and
evolve. Maturana and Varela regarded that the members in social systems have autonomy to adapt themselves, and meanwhile they build coordinated relationships through communication. Individual’s autonomy is impacted by the dynamic structures of the whole social systems, thus, to understand individuals, we need to understand relationships between individual structural changes and dynamics of the structure of the whole system in which the individuals are embedded.

Inspired by the work of Maturana and Varela, German socialist Luhmann (1990) extended social systems as autopoietic systems where communications are the elements of the social systems. Luhmann stated,

Social systems use communications as their particular mode of autopoietic reproduction. Their elements are communications that are recursively produced and reproduced by a network of communications that cannot exist outside of such a network. (p. 3)

Luhmann suggested communications as elements of social systems instead of introducing social systems as communication networks in which communications come out of communications. In other words, social systems, for Luhmann, are autopoietic. Therefore, social systems as communication networks are structurally-closed, self-generating and adapting. Moreover, Luhmann regarded communications as syntheses of selections of utterance, information, and understanding. When there are utterances in the system, communication participants select the utterance of interest, interpret it into information, and create understandings with corresponding actions within a social context (Kadirov & Varey, 2008; Luhmann, 1990; Rasmussen, 2005). For example, in Figure 2.1, the utterance “Do not touch the stove. It is hot” by a mother is not a communication until and unless the child hears the utterance, interprets the utterance as information (in this case a warning), and understands the warning as referring to the hot stove. So the communication is a complex unity of “utterance,” “information” and “understanding” shared by the mother and child. The communication occurs within the larger communication
network that may include that the mother has provided other warnings, for example, “Do not cross the street – it is dangerous.” So, it is the communication participants who make decisions on meaningful information and reproduce communications comprising the interaction network. Utterances in the communication network stimulate individuals’ construction of meaning. The communication process is a meaning-based continuous evaluation of understanding the relationships among instances of utterance, information and meaning in ever evolving social contexts, and the coordinated actions that occur among the participants in the communication network. Communication networks are recursive structures that change over time with individual participants as constituents or organizational components of the systems. The social systems are meaning-based networks of communications with properties of self-produing, adapting, and self-referentiality.
Figure 2.1 An Understanding of Luhmann’s Communication Networks
Influenced by the studies of Maturana and Varela on living systems, Sfard (2008) regarded mathematics as “the objects of talk along with the talk itself and that grows incessantly ‘from inside’ when new objects are added one after another” (p.128). Learning is a recursive process in which mathematics objects are realized in mathematical discourse. The realization is a personal construct; however, it is generated from and guided by the discourse. The discourse itself grows with the contributions from individuals’ participation in the discourse. Thus mathematical discourse has properties of autopoiesis. Sfard further defined “Mathematics is a multilayered recursive discourse structure of discourse-about-discourse” (p. 161). In other words, mathematics is a network of communications about communications. Mathematics understandings and communications comprise the system of mathematics.

Derived from Wittgenstein’s philosophy of meaning as embedded in actions, Sfard pointed out mathematics is a meaning-based system, and the meaning of mathematics is embedded in the use of mathematical language by persons according to context. The generation of meaning is thus not located inside the mind of individuals, but in the discourse itself. Mathematical understandings are co-adaptations of the discourse and the participants in the discourse. Sfard highlighted,

Discourse rules of the mathematics classroom, rather than being implicitly dictated by the teacher through her own discursive actions, are an evolving product of the teacher’s and students’ collaboration efforts. (p. 202)

Discourse and the participants are engaged in recursive modification as communications unfold. Sfard pointed out the richness and depth of understandings emerged in classroom discourse (the network of communications) reflect the quality of the discourse. Specifically, the richness of communications comprising the network of communications constantly creates and re-creates meanings and undergoes substantial transformation over time.
Davis and Simmt (2003) viewed mathematics classrooms as self-producing and adaptive systems where learning is understood as constant adaption arising from interactions. Davis and Simmt paid attention to the emergence of learning possibilities. They suggested “internal diversity”, “redundancy,” “neighbor interactions,” “decentralization of control,” and “organized randomness” as a set of conditions for the manifestation of learning opportunities.

Davis and Simmt described “internal diversity” as “a source of possible responses to emergent circumstances” (p. 148). Meanwhile, they also acknowledged the necessity of “redundancy” such as common goals, languages, and compatible experiences for the emergence of collective knowledge. Davis and Simmt highlighted the dynamic relationship between redundancy and internal diversity. In addition, they called for “decentralization of control” to encourage individual autonomy in discussion. They regarded students’ decisions about what ideas to pursue rather than the teacher as the final arbiter as important to the learning process and the development of student autonomy. Decentralization of control, however, does not mean there is not any constraint among students. On one hand, individuals have the ability to adapt independently; on the other hand, individuals adapt their behaviors in a way fitting in the adaption of the whole system through interactions. “Organized randomness” highlights the dynamic relationship between “centralization” and “decentralization” of control in mathematics discourse. Davis and Simmt used “neighbor interactions” to note the relationship of interactions between local interest and the appearance of larger level complexity. They pointed out that the richer local-level connections, the more chances of the emergence of up-level complexity. The interplay of various thoughts of individuals promotes broad and deep understanding.

believed they all emphasized interaction, especially the emerging mutual relationships among the participants through interaction; in particular, interaction should not only focus on final results but also on their intentions. The interplay of intentions creates opportunities for expanding knowledge and understanding. Doll highlighted the significance of students’ intentions as below, Curriculum designs and instructional strategies focusing on a learner’s intentions and bringing those intentions into dynamic interplay with the results produced by the actions would not only make learning far more efficient, it would also open space for the creative emergence of new ideas and procedures. (Personal conversation)

Intentions and misunderstandings, from Luhmann’s perspective, are all the participants’ selections of interpretations in communications. The elimination of misunderstanding and intentions constrains the possibilities of meaning-construction, and further the dynamics and complexity of communication networks. Doll (2005) also regarded interaction as cultural and argued for a democratic classroom environment in “bringing forth ideas into the light of cultural criticism and development” (p. 53). He said, “Public discourse and verification is key if a personal idea, belief, knowledge is to have any value in a community arena” (p.53). Classroom teaching is an ongoing conversation in which the teacher and students bring their own different ideas and perspectives into an inquiry matrix for deeper and richer understanding.

Fleener and Rodgers (1997) emphasized the dynamics of social learning systems, in particular the relationships between the individual’s autonomy and the socio-autonomy of social learning systems. Fleener and Rodgers regarded individuals as autopoietic systems in which autonomy refers to the “internal process of self-regulation” determined by the structures of individuals. Individuals’ decision-making is coordination with understanding of the relationships between individuals and the others. “Autonomy does not imply ‘freedom’ as the term is often used; rather it is acting in accordance to one’s core system of understandings, values and beliefs” (p. 14). In other words, autonomy is a process of meaning-making determined by individuals
interacting within the context of the system. In social learning systems, individuals are engaged in a variety of social relationships with the members within the systems and other systems that impact their autonomous actions.

From Luhmann’s theory of social systems, Piaget’s theory of equilibration, and Haken’s theory of synergetics, Fleener and Rodgers developed the concept of “socio-autonomy” to describe the dynamics of social learning systems and its influence on individual autonomy. In analogy to the definition of individual’s autonomy, socio-autonomy is “the potential for self-creation and self-production of social systems” (p. 15). The socio-autonomy of systems implies the process of coordination among the individuals within the systems, and with the environment to lead to the emergence of new collective abilities. Fleener and Rodgers examined the development of socio-autonomy and its relationships with individual autonomy with a group of early childhood majors. They found “the tension between conflicts and cooperation” stimulate the negotiation of meaning among individuals in the local level of the learning system, while mutual respect and a sense of belonging and commitment play the role that triggered the development of the growth of the group as a learning system. The dynamics of the social learning system is established on the openness to diversity of ideas, and negotiation about the difference of individuals, which further contributes to the growth of a learning system as a whole. Meanwhile, the dynamics of the social systems promotes the development of autonomy of individuals indicated by their tendency to spend more time and energy to adapt them.

Fleener and Pourdavood (1997) further argued educational reform has to be built on the understanding of “the process of the change and the structure of schools” (p. 3). In particular, with the difficulties teachers have experienced in the classroom teaching reforms, a dialogic community is important for promoting teachers’ own autonomy and the emergence of new
perspectives of teaching. Fleener and Pourdavood advocated an approach to school reform, based on the understanding of the relationships between the socio-autonomous environment of school social systems and autonomy of individual teachers and educators. They stated this approach to school reform is to view

That schools, as social institutions, are consensual, that changes cannot be forced or controlled, that visions of reform must change as individuals within the system continually change, and that change itself is fluid through time. (p. 20)

The structural changes of school systems are results of dialogical communications in which the individual teachers can develop their autonomy within understanding of the reform vision of school systems as a whole.

Schlechty (2009) studied the transformation of school systems and flagged the roles of social relationships in the transformation. Schlechty distinguished two types of systems: operating systems and social systems. Schlechty explained,

Operating systems define how work is done; social systems define the meaning of the work, the values that are attached to the work and its outcomes, the ends toward which the work is aimed, the manner in which authority is assigned, the knowledge that is honored, and so on. (p.26)

Operating systems are concerned with the procedures of actions, while social systems answer questions such as why we need to practice these procedures that determine the meaning of the work. According to Schlechty, “Social systems define social and cultural relationships within which operation systems must carry out their task” (p.26). Schlechty noticed that those social contexts such as beliefs, perceptions, shared norms, and roles are crucial for understanding educational systems and implementing reform.

Schlechty especially highlighted directional goals, shared norms, and mutual relationships as important aspects for classroom and school systems transformation. Schlechty regarded directional goals as determining the priorities and actions to achieve the goals of the
systems. For example, if a goal of a school is to ensure knowledge gaining, the priority will be placed on learning skills. However, if the goal is to develop comprehensive abilities, students’ interests will be the focus. In addition, values, social norms, and mutual relationships are also essential for the transformation of the systems. In particular, the self-discovered meaning of individuals, rather than the material stimulus, is the key to learning.

Schlechty discussed the conditions for the transformation of social systems, and pointed out that collective goals, social norms, and mutual relationships collaboratively promote the changes of social systems. In contrast, evaluation, boundary, power and authority limit the flexibility of social systems. Moreover, when changes threaten existing power and authority, discomfort arises among those who have power. Then, they give up their efforts to make the changes possible. The flexibility of power arrangements, evaluation and boundaries are preconditions for social systems changes.

Based on systems and social theories, Bielaczyc and Collins (1999) suggested a framework to study classroom communities involving the aspects of learning goals, roles of the teacher and students, mutual relationships, centrality of power and identity, and discourse. Learning goals are collective-benefit based, emphasizing learning from diverse perspectives. Individual learning is obtained through the advancement of collective knowledge and skills. The collective learning goals evolve along with the needs of individuals, and further promote the evolution of the individuals. Classroom learning communities need to foster a context where members are able to articulate learning goals and share the criteria for success. Through sharing and articulation of goals, members are able to better understand the collective goals, and to adjust their actions in alignment with the development of the collective.
Bielaczyc and Collins also noted that in classroom learning communities, the students are responsible for their own learning processes. Students decide their own needs and ways to evaluate their learning while the teacher helps create environments for learning to happen. Roles, identities and mutual relationships are important for learning communities as each member plays a different role and contributes to the growth of the collective in different ways. Bielaczyc and Collins suggested discourse as the “medium for formulating and exchanging ideas” for the members instead of the “medium for conveying knowledge asking students questions to test their knowledge” (p.276) in classrooms.

In summary, Maturana and Varela suggested social systems are autopoietic systems with the properties of structurally-closed, self-producing, and adapting. The components of social systems are interrelated through communication networks. Moreover, communications are not a transmission of information but coordinated actions among the participants in communications (Maturana & Varela, 1987). Communication networks and participants in the communications are engaged in on-going, recursive changes over time (Fleener, 2002; Luhmann, 1999, Maturana & Varela, 1987). Luhmann especially suggested social systems are meaning-based communication networks in which communications are the units of the networks. Applying the properties of autopoietic systems and Wittgenstein’s meaning as action-associative language, i.e., language as a “form of life,” Sfrad (2008) suggested mathematics is a discourse about discourse in which mathematics meaning is generated, and the discourse and participants co-evolve with their coordinated efforts. The quality of mathematics discourse is evaluated by the richness of connections in the discourse. Davis and Simmt discussed the conditions such as internal diversity, neighbor interactions, redundancy, decentralization of control, and organized randomness that promote the emergence of collective knowledge in mathematics classroom learning systems.
Doll (2008) noted the significance of students’ intentions in the communication of classroom teaching, and democratic environment for the emergence of classroom teaching and learning transformation. Fleener (2002) emphasized the dynamics of social systems. Fleener and Rodgers (1999) and Fleener and Pourdavood (1997) suggested studying the relationships between the autonomy of participants, and socio-autonomy of communication networks in the reform of school systems. They also identified the significance of context such as beliefs, values, and power relationships for the negotiation of meanings. Schlechty pointed out social norms, mutual relationships, and autonomy as essential aspects for the transition of social systems. Additionally, Bielaczyc and Collins proposed discourse, mutual relationships, and collective goals as main components for studying classroom communities.

The above studies suggest that communication networks, socio-autonomy, and social meaning relationships such as goals, norms and mutual relationships are central aspects to understand changes of classroom and school systems. Specifically, classroom social systems can be viewed as communication networks with meaning relationships embedded in and constituent of the networks. On the one hand, patterns of meaning relationships generate and create social systems through communications; on the other hand, the meaning relationships guide the actions of participants and the development of individual autonomy in the communication networks. The participants of the communication networks have autonomy to adapt themselves in accordance with their understanding of the relationships with others within the system. In turn, the evolvement of communication networks arises from the adaptations of the participants’ coordinated actions in the communications. The evolvement of the communication networks and individual advancement are integrated together.
A Framework for This Study

The main purpose of this research is to use empirical data to examine the implementation of teaching reform in secondary mathematics classrooms in China, and to understand the context for the transformation of classroom teaching. The above studies suggested a framework for this study.

In this study, classroom social systems are understood as communication networks in which communications are the units. Meaning relationships are connections in the communication networks. This study examines the structures of communication networks of classroom teaching, patterns of social relationships attached in the networks, and socio-autonomy of the networks. Structures refer to the actual communications between the teacher and students around mathematics and the connections to the communications. This study is also focused on social and cultural meaning relationships such as norms (including socio-mathematical norms), mutual relationships, and learning goals. Socio-autonomy refers to the abilities of the communication networks to adapt and to evolve. Thus, the purpose of this study is not only to examine the changes of classroom teaching but also to examine the context for the changes of classroom teaching. Network analysis is best suited to the requirement of studying the structure of communication networks and the evolvement of the networks. In Chapter 4, I describe the network-based qualitative research method used in this study.

From social systems theories, the teachers, the classrooms and the associated teacher professional development communities are nested systems. This study further examines the structures and meaning relationships in the associated teacher professional development programs, the cultural beliefs of mathematics and instruction, experiences of the teachers to see the relationships between the changes of classroom teaching and the teacher professional
development programs, and the beliefs and teaching experiences of teachers. Thus, my framework is built on social systems as communication networks and the above other research on social systems, which is illustrated by Figure 2.2 below.

This framework from social systems perspectives offers a dynamic and relational view to look at the evolvement of the structures of classrooms and relationships patterns in the classrooms from multiple levels. Using this framework, the researcher is able to incorporate essential dimensions of classroom teaching (such as communications and social relationships) and to examine the evolvement of the whole teaching process simultaneously. In this framework, the understanding of classroom teaching is also placed in the nested systems of the teacher, teacher professional development communities and nation-wide curriculum reform implementation.
Figure 2.2 Theoretical Framework for This Study
CHAPTER 3: LITERATURE REVIEW ON MATHEMATICS CLASSROOM REFORMS

Given the social system framework, classrooms are viewed as evolving communication networks with meaning relationships. Various dimensions of classroom teaching interplay together from which dynamics and complexity of classroom teaching arise.

The international mathematics education community has impacted Chinese mathematics education reforms over the recent decades. The current round of mathematics reform is especially impacted by the reforms in United States. In this chapter, I first describe reform trends of mathematics education in both China and the US. I then examine existing research regarding the implementation of mathematics classroom reform focusing on classroom discourse, social norms, and instructional organizations. Moreover, from social systems theories, teachers, classroom teaching, and teacher communities are nested systems. In the last section, I review the literature on the relationships between teacher’s beliefs, their teaching experiences in mathematics and the implementation of reform teaching, and efforts that teacher professional development communities have made in supporting the reform teaching.

A Brief History of Mathematics Education Reforms in the US

Mathematics education has undergone several transition periods in the US, often tied to the economy and other social trends. With each transition, however, mathematics reform agendas have oscillated between focusing on learners and focusing on mathematics content.

Mathematics education in the US is traditionally viewed from a subject content perspective. When the country was primarily rural, students were expected to master basic arithmetic facts. In the early twentieth century the progressive education movement became the theme of American education. John Dewey was one of the leading persons of the movement, who advocated children as the center of education and called for the integration of learners,
curriculum and society, thus shifting the pendulum to a child-focused curriculum. In the 1940’s progressive education was criticized for its deficit in academic content, especially when military recruits were found lacking basic skills among soldiers. Within the US the power of and need for mathematics as a discipline during post-World War II influenced mathematics education back toward scientific rationality and a content focus.

With the launch of the Soviet Union’s space satellite Sputnik in 1957, the New Math movement emerged. It was aimed at fostering engineers with high levels of mathematics and science to increase the competency of the nation in the technology fields internationally. The New Math curriculum focused on the development of logical thinking and systematic structures of mathematics-like set theory. Abstract mathematical concepts and theorems were introduced early in school (Klein, 2003). Unfortunately, because this approach alienated many students to mathematics and did not improve overall mathematical proficiency, another reform was enforced to change the focus back to the basics. Thus, during the 1960’s and the 1970’s, the mathematics curriculum pendulum shifted from the abstract study of the foundations of mathematics back to a more disciplinary focus. The curriculum was arranged and separated by disciplinary content and the historical development of mathematics, the hallmark of the Back-to-Basics movement in mathematics education. In both New Math and Back-to-Basics movements, however, the focus was on the curriculum and not on the learners.

During the 1980’s, with increasing integration of psychological and mathematics education research, the focus shifted again from what should be learned to how learning occurs. Moreover, the transitions of industry-based to information-based society challenged mathematics education. The A Nation at Risk (1983) report warned that America was losing competitive capabilities in the rapid changing of society and growing global competitions. A new round of
educational reform incorporating learning psychology and content focus in the US was initiated and the reform continues today.

In response to the *Nation at Risk* (1983) and consistent with a combined psychological/student and content focus, the NCTM *Curriculum Standards* (1989) set the agenda for mathematics curriculum and mathematics teacher education reform. In the new NCTM standards, the goals of school mathematics were stated from the perspective of learners (1) to value mathematics, (2) to reason mathematically, (3) to communicate mathematically, (4) to become confident of mathematical abilities, and (5) to become mathematical problem solvers. The NCTM (1989) standards indicated learners are part of the curriculum focus. In addition to the *Standards*, the NRC Report *Everybody Counts* (1989) addressed the needs of diverse learners. Mathematics education was shifting to the integration of mathematical knowledge and learners; process over product; and away from the pendulum swing of either content or learner.

In *Adding It UP* (NRC, 2002), mathematics learning is specifically defined as the development of mathematical proficiency as consisting of five intertwined strands: “conceptual understanding”, “procedural fluency”, “strategic competence”, “adaptive reasoning”, and “productive disposition”. This re-conceptualization of mathematics learning highlights the relationships between learners and mathematics. Connecting knowledge across different areas and applying it to solve problems in their life situations, developing critical and flexible thinking skills, improving the depth and breadth of mathematical thinking, and building intimate relationships with mathematics are further emphasized.

In alignment with the new vision of mathematics learning, the new vision of teaching focuses on classrooms as learning communities where students construct mathematical knowledge and understanding through negotiation in classroom social activities. In the
**Professional Standards for Teaching Mathematics** (NCTM, 1991), the following five movements of the reform of mathematics classroom teaching were particularly advocated:

- Toward classrooms as mathematical communities and away from classrooms as simply a collection of individuals;
- Toward logic and mathematical evidence as verification and away from the teacher as the sole authority for right answers;
- Towards mathematical reasoning and away from merely memorizing procedures;
- Towards conjecturing, inventing, and problem solving and away from an emphasis on mechanistic answer-finding;
- Towards connecting mathematics, its ideas, and its applications and away from treating mathematics as a body of isolated concepts and procedures. (NCTM, 1991, p. 3, slight modification of quote)

The document indicates teaching is to create opportunities to engage students in social practices to explore mathematical ideas, share understanding, present mathematical thought, and build confidence and identity in learning. In 2001, NCTM published the **Principle and Standards for School Mathematics**, a revised comprehensive version for guiding school mathematics teaching. **Principle and Standards** advanced the tenets that mathematics is the science of seeking patterns and relationships; while teaching should empower every student to explore, propose hypotheses, communicate, and verify the hypotheses. Mathematics teaching should provide opportunities for students to apply mathematical knowledge to solve the problems in real life situations.

Over the past few decades, the reform efforts in mathematics education in the US have moved away from the pendulum swing between content-focused and learners focused curriculum to the comprehensive development of individual learners and the relationships among the learners, mathematics and society. The trend has been echoed in international mathematics education reform efforts.

**Chinese Mathematics Education Reforms**

Similarly, the development of Chinese mathematics education has gone through several periods which reflect the changes of the policies, economy, and society in China. Ancient
Chinese mathematics is very different from that in Western countries. As noted by Martzloff (1987), Chinese ancient mathematics was concerned with numbers, computation and description of methods or process. Early mathematics education was focused on training people computation skills and its applications in real life situations. However, since the beginning of the 17th century, the development of mathematics in China was influenced by Western mathematics.

Contemporary Chinese mathematics education is greatly impacted by the development of mathematics education in western countries (Siu, 2004). The new round of reform is especially impacted by the recent reforms in the US.

Since the establishment of the People’s Republic of China in 1949, mathematics education has gone through different periods of reforms. From 1949 to 1957, the entire Chinese education system followed the format of the Soviet Union. *The Syllabus for Secondary School Mathematics* was initially copied from that of the Soviet Union. The first series of textbooks were also translated from those of the Soviet Union, in which mathematics content was organized systematically. Since then, the two basics including basic knowledge and skills have been emphasized in Chinese education. The teaching method during this time period focused on the aspects of instruction organization - introduction to a new lesson, teaching the lesson, consolidation practice, and homework assignments (Xie, 2009). In short, mathematics education in this period concentrated on content and teaching, and was lack of consideration about students (Zhong, 2006).

From 1958 to 1961 China pursued their own curriculum reform efforts in order to separate from the Soviet Union education system. China was impacted by the national political movement called the “Great Leap Forward”, and was influenced by the New Math movement in the US as well. Modern mathematics content was added to new textbooks and systematic
structure of mathematics was emphasized. However, the curriculum reform was criticized for its lack of consideration of the reality in China (Xie, 2009; Yan, 1998). During this period of time, most of the population in China was not prepared for the abstractions of the new curriculum. As the reform in 1958-1961 failed to consider the needs of students to learn basic mathematics, Chinese mathematics education went through the processes of revising and making adjustments to the reforms starting in 1963. First, the national curriculum and guidelines were generated in this period, in which the mathematics curriculum was focused on scientific and systematical structures and combination of theories and practice. In particular, the Syllabus for School Mathematics, which was published in 1963, introduced the cultivation of students’ three capabilities including calculation, logical thinking, and spatial imagination in addition to basic knowledge and skills for the first time. Meanwhile, a series of mathematics textbooks for K-12 pertaining to Chinese conditions were developed and published. Chinese mathematics education gradually obtained their own textbooks, syllabi, and teaching pedagogy to meet Chinese circumstances. However, mathematics education still was focused on knowledge and teaching. The characteristics of learners and their learning experiences as a product were ignored.

From 1966 to 1975, the Cultural Revolution disrupted the entire education system in China. Students were “sent down” to the farm or factories to learn from peasants and to work. “The previous textbooks were no more in use and there was no unified syllabus” (ICME-11, 2008, p. 22). Mathematics education emphasized practical work through simple skills like calculation and measurement found in manufacturing and farming (Xie, 2009; ICME-11, 2008).

After the Cultural Revolution and Mao’s death in 1976, Chinese education rapidly developed. In 1977, the national college entrance examination system was restored. Educating people with knowledge and skills became a national priority. Students showed enthusiasm in
learning and parents paid great attention to their children’s education. Along with the publications of *Guidelines for Ten-Year Secondary School Mathematics* in 1978, and *Guidelines for Key Secondary School Mathematics* in 1980, mathematics education in China went back to the focus on basic knowledge and skills. Especially, beginning in the 1980’s, a variety of educational ideas from abroad, such as “standardized tests”, “Bloom’s taxonomy of education objectives”, and “problem solving” were embraced by the Chinese mathematics education community as China opened its economic doors to the western world (ICME-11, 2008). Although the national curriculum was still dominant in mathematics education, China has begun curriculum reform and teaching experiments in some places.

Starting in 1986, the *Compulsory Education Law* initiated China’s nine years compulsory education for every child. More important, the *Guidelines for Mathematics Education for Nine Year Compulsory Education* (1988) emphasized more than basic knowledge and skills to include real life applications and the cultivation of students’ good learning habits and dispositions. Those documents initiated the shift in Chinese mathematics education from knowledge to include learners’ experience as a part of curriculum. The trend was consistent with the international trend of mathematics education reform initialized in the US in the 1980’s. Yet, Chinese mathematics education was still mainly test-based and elitist education-based until the 1990’s. With test-oriented education, knowledge and skills were overemphasized and separated from students’ real life experiences. Although Chinese students often performed well on international comparison tests, the development of mathematics education was very unequal across the country in many cities and areas. Moreover, Chinese students showed weaknesses in creativity and practical ability in general, which was considered a constraint for the rapid and sustainable development of China in the new century.
Mathematics Teaching Reform in the Context of Nationwide Curriculum Reform

As China entered the 21st century with the world’s fastest developing economy and relying more on technology, issues concerning education became more prominent. A new round of mathematics curriculum reform was launched in 2001 with the publication of *The Guidelines on Curriculum Reform of Basic Education (for Experiment)* by the Chinese Ministry of Education (MEPRC, 2001a). Different from the previous mathematics reforms, this new round of mathematic reform does not just modify the existing syllabi or standardized textbooks; rather, it is accompanied by an innovative student-centered curriculum which is greatly influenced by Western philosophers and scholars such as John Dewey and Jean Piaget (Xu & Lerman, 2004; Zhong, 2006). The new round of curriculum reform sets up a new vision for Chinese education. As Zhong (2006) pointed out, “Under the new curriculum context, education is for the purpose of the sound development of each student, not for ‘instilling and training’” (p.373). The new mathematics curriculum standards emphasize learners’ experiences, interests, and attitudes toward mathematics.

The new standards (MEPRC, 2001) defined mathematics as descriptions of relationships of the real world via qualitative and quantitative processes. Mathematics is to generate rules and methods to describe the relationships, and to solve problems in real world. For learners, doing mathematics means exploring, communicating, evaluating, and applying mathematics in real life situations. In the new standards, learning is an integration of students’ experiences, dispositions, and cognitive development to meet the needs of a changing society. As stated in the standards, the new mathematics curriculum promotes comprehensive, sustainable, and harmonious development of learners.
The goals of the new mathematics curriculum standards are addressed in four essential aspects: (1) knowledge and skills, (2) mathematical thinking, (3) problem solving, and (4) emotion and attitudes. For example, the objectives for grades 1-9 mathematics learning include:

- To acquire fundamental mathematical knowledge (including experiences), thinking methods and application skills, to meet the needs in life and future advancement.
- To learn how to apply mathematical knowledge and methods of mathematical thinking to observe, analyze and solve problems in real world situations and in the studies in other subject matters;
- To realize the intimate relationships between mathematics, humanity and society; to appreciate the value of mathematics and develop mathematical understanding; and to build confidence in learning mathematics.
- To have innovational spirit and practical ability, and seek full development of abilities, emotion and attitudes.

More specifically, the first aspect (Knowledge and skills) highlights the mastery of fundamental knowledge and basic skills pertaining to numbers, algebra, space and figures, and statistics and probability. Students should gain the knowledge and skills through problem solving, and experience with real world applications.

The second aspect (Mathematical thinking) includes the establishment of number and symbol sense, the development of abstract, spatial, and statistical thinking, and inductive and deductive reasoning abilities. The third aspect (Problem-solving) emphasizes mathematical applications, strategies for problem solving, and the development of practical and creative abilities, and communication abilities. The fourth aspect (Emotion and attitudes) addresses the cultivation of enthusiasm, perseverance and confidence in mathematics learning, the establishment of intimate relationships between mathematics and human lives, and the development of independent and critical thinking abilities. In short, this latest reform advocates individual comprehensive development grounded on learning experiences, process, and connections to daily life.
In China, mathematics education has historically been elite-oriented, discipline-oriented, and teaching-oriented; In the US, mathematics education reforms have swung back and forth between discipline-oriented and students-centered education. However, the latest round of mathematics education reforms in both countries has shown a common trend: bi-focusing on learners and rigorous mathematics content. The goals of both reform efforts emphasize the development for every student with attentions to his/her experiences, attitudes, and connection between mathematics and applications in real life situations. Both reforms indicate an integrated perspective between knowledge and learners, process and product, and theories and applications. In particular, both countries have advocated inquiry-based teaching in which students learn through a process of exploring, conjecturing, testing, and applications. Despite their similarities, there are subtle differences between the two countries’ latest rounds of mathematics education reforms. It seems that Chinese reform emphasizes more fostering students’ innovation and practical abilities, while the US reform focuses on the depth and flexibility of knowledge and skills and the abilities of doing mathematics. Mathematics education in both countries has called for the development of intimate relationships between the learners and mathematics, which can be viewed as the goals guiding the direction of classroom mathematics education. The new round of reforms requires a restructuring of mathematics education, which has placed big challenges for classroom teachers, and professional development of classroom teachers in implementing reforms in both countries.

**Essential Aspects of Mathematics Classroom Teaching Reform**

Classroom communications, social norms, and organization of instruction have been regarded as essential dimensions in restructuring mathematics classroom teaching and
implementation of mathematics curriculum reforms (Franke, Kazemi, & Battey, 2007; Peressini & Knuth, 1998; Spillane & Zeuli, 1999).

**Communication.** Traditional mathematics classroom communication is often characterized by the IRE model in which the teacher *initializes* a series of questions; students *respond* and the teacher *evaluates* them according to his/her expectations (Franke, Kazemi, & Battey, 2007). During IRE, students have few opportunities to develop their own mathematical thinking. On the contrast, the reform mathematics calls for the sharing of ideas, developing mathematical thinking, and further building relationships with mathematics as they communicate.

New conceptions of classroom discourse focus on how conversations foster mathematical arguments so students can come to understand the different forms of mathematical explanation, create a public knowledge base that can be used by the class as a resource, align students with one another and the content, develop students’ mathematical identities, and generally foster a higher level of thinking. (Franke, Kazemi, & Battey, 2007, p. 231)

Despite the appeal of incorporating classroom discourse and encouraging student interactions in the mathematics classroom, it is not easy for teachers to break out of the traditional IRE approach to mathematics instruction. Spillane and Zeuli studied classroom communications to examine to what extent communications supported the reform ideas in twenty-five classrooms. They found only four out of twenty-five classroom practices used communications to help students to develop conjectures, elaborate their reasoning, and justify their solutions. In the rest of the classroom practices, communications were either lack of opportunities for building-up ideas or IRE-driven. In about forty percent of the classroom practices, students were exposed to a completed product or prompted with “a series of questions to elicit the answer” (p.15). Even in small groups, student communications still followed the “expert telling novice” (p. 14) model. Their study highlights the difficulties which teachers faced to foster the reform-oriented teaching.
To help teachers develop the reform-based teaching, mathematics educators have made great efforts to provide various strategies and approaches. For instance, Franke, Kazemi, and Battey (2007) identified the strategies commonly used for supporting reform ideas. For instance, getting students to elaborate on their understandings through questioning and explanation is one of “the most powerful pedagogical moves a teacher can make” (p.232) to support classroom discourse aligned with the reform agenda. Franke, Kazemi, and Battey also identified “revoicing”, providing tasks involving multiple strategies and connections as ways to support productive discourse.

Scaffolding and collaborative interactions are regarded as important in discourse (Franke, Kazemi, & Battey, 2007, Goos, 2004). Goos especially discussed the conditions for scaffolding and collaborative interactions. Goos pointed out the teacher appeared as a more capable knower and orchestrated the process of interaction in scaffolding interactions. Yet, essential to scaffolding interactions is that students needed to play an active role to create the Zone of Proximal Development (ZPD) through negotiation and reorganization of actions and goals rather than simple imitation of the teacher’s demonstration. In collaborative interaction, the comparable expertise among participants is important for the creation of collaborative ZPD, where the equal status of participants in conversations provides opportunities for learners to experience and to build ownership of ideas (Goos 2004). Goos’ research offered insights for shaping interactions between students and teacher, between students and students. Scaffolding and collaborative interactions are often intertwined each other. From a social systems perspective, both the teacher and students are participants in communication networks. More important, networks of communications that are “alive” with meaningful interactions are self-generating, adapting and maintaining, which suggests creating meaning networks in the classroom more naturally allows
information to flow throughout the classroom discourse rather than requiring the teacher to orchestra the communications toward expected learning.

From a social and cognitive psychology perspective, Peressini and Knuth (1998) studied the nature of discourse in a high school mathematics classroom and in a teacher professional development classroom. Peressini and Knuth distinguished between two types of communications based on their functions. One is dialogic communication in which new meaning arises from communication. In such classroom communication, a teacher listens to students; make sense of their intentions and solutions; and students and the teacher share the authority of mathematics. In contrast, in univocal communications, the teacher conveys his/her ways of thinking and solutions to students and students adapt their own thinking to match with the teacher’s expectations. Instead of generating new meaning for all participants, univocal communication is to reach conformity of meaning. Based on the framework of different types of communications, Peressini and Knuth found that discourse in the high school classroom was dominated by univocal communication despite the teacher being previously exposed to dialogical instructional strategies in the associated teacher professional development. Peressini and Knuth believed the status of the participants, in particular the role of the teacher, in the communication contributed to the nature of communication. When the teacher took the role of facilitators, he tended to invoke dialogical communication.

Brendenfur and Frykholm (2000) further extended the communication patterns into univocal, contributive, reflective, and instructive communications. In both univocal and contributive communication, the objectives are to help students internalize the predetermined information and knowledge rather than expand their understanding (Brendenfur & Frykholm, 2000; Lloyd, 2008; Wertsch & Toma, 1995). Different from univocal communication, students
have opportunities to share ideas with each other in contributive communication, but the primary focus is still on telling, showing, and explaining. Students are expected to internalize the curricular knowledge. In reflective communication, students do not just explain and share their reasoning processed but also make adjustments and generate new understandings from interactions. In instructive communication, new meanings are generated from students’ utterances and they also lead the modifications of instruction. Instructive communication thus highlights the significance of improvising and emerging natures of classroom communication (Brendenfur & Frykholm, 2000).

Applying the concepts of different types of communications, Lloyd (2008) studied a high school teacher’s experience in implementing a reform-based mathematics curriculum across two years. In the first year of implementing the new curriculum, the teacher often interacted with small groups, and both univocal and contributive communications were manifested in the first year. In the second year, as the teacher became more familiar with the new curriculum, he adapted small group interaction into whole class discussion with significantly lengthier teacher-student interactions. The univocal communication began to dominate the reminder of the second year. Lloyd (2008) found the teacher experienced a “pedagogical discomfort” when implementing new reform approaches, in which the teacher tended to rely on his or her past experience to overcome the discomfort.

The different types of communication offer us a framework to analyze the functions the communication in classrooms and the supports teachers need in order to cultivate meaningful communications. From a social systems perspective, meaningful classroom teaching is an evolving network of communications in which the teacher and students are interactive and co-emergent participants. Thus, the communication network includes the participants as a whole and
evolves with new and adaptive meanings. The evolvement of communication networks in the classroom is not visible from the examination of the interactions between individuals alone. It requires an “up-level” examination of dynamics in the whole classrooms (Davis & Sumara, 2006).

**Classroom Norms.** The participants’ behaviors in interactions are governed by social relationships in classrooms. Social relationships such as classroom social norms are considered as an important aspect in the reform of mathematics education. As with studies of classroom discourse, classroom norms studies focusing on individual interactions within classrooms rather than “up-level” dynamics fail to capture whole-class values with regard to classroom and sociomathematical norms. Furthermore, teachers implementing reform mathematics instruction techniques are found in many studies to implement the techniques but not the spirit of developing classroom norms that support mathematic discourse and inquiry.

NPR (2001) recommended norms for developing robust mathematics classroom communities, including (1) “value placed on ideas and methods, (b) student autonomy in choosing and sharing their problem-solving methods, (c) appreciation of values of mistakes as sites for learning for everyone, and (d) renegotiation of authority for whether something is correct or sensible” (p. 239, in Franke, Kazemi, & Battey 2007). Those norms indicate an emphasis on mathematical thinking, autonomy, experience, and mathematical evidence as authority that pervade the interactions within the classroom.

Depaepe, Corte, and Verschaffel (2007) investigated reform classroom culture in terms of classroom norms, in particular, the nature of problem solving and heuristics and metacognitive skills. The categories of heuristic and metacognitive skills in their research were drawn from the suggestions of the reform-based textbook. The researchers counted the frequencies of those
normative aspects addressed explicitly by teachers in their lessons. Findings from their study indicate that teachers had strong emphasis on heuristics, while the metacognitive strategies hardly were used in problem solving. This study shows to what degree mathematics teachers emphasized the class norms recommended by the reform education in their teaching. The norms appeared as statements or rules predetermined by the reform textbook and interpreted by the teachers as individual responses to students rather than social relationships that were developed in classroom teaching and learning processes.

Wood (1998) defined social norms as

Interlocking networks of obligations and expectations that exist for both the teacher and students [that] influence the regularities by which classrooms interact and create opportunities for communication to occur between the participants (p. 170).

Wood’s definition of social norms indicates that classroom norms are a network of social relationships among the participants in communication. Social norms connect participants through regulations, responsibilities, and expectations in the communication and support the development of communication. This notion of social norms highlights the dynamic social relationships in classroom communications.

Lampert (2000) developed a framework for teachers to create norms that fostered students’ good habits in problem solving. She emphasized sharing meaning publicly in solving problems. The norms of solving problem she suggested included articulating the conditions of the problem, making conjectures, and making adjustments based on mathematical evidence (Franke, Kazemi, & Battey, 2007). Students built up those social norms through their actions in solving problems in her study. In other words, the social norms of problem solving and the research techniques used to assess classroom norms are interactive actions rather than statements or rules.
Cobb, Wood, and Yackel (1993) focused on the processes of establishing social norms to support learning. For example, they noticed that the expectations of the teacher and students in mathematical activities did not match in their research. The teacher expected students to articulate their thinking; however, students thought about the answers. The teacher expected students to guess, justify, and test their hypothesis, while students expected the teacher to evaluate the correctness of their answers. By talking about and doing mathematics (as well as talking about talking about mathematics), and contributing in small group interactions and whole-class discussions, students and the teacher developed classroom norms that guided mathematical activities. Cobb and colleagues (Cobb, Wood, & Yackel, 1993; Cobb, 1990; Wood, 1998; Yackel & Cobb, 1996) explored how various normative aspects of the mathematics learning environment regulated teachers’ and students’ behaviors in mathematics activities in classrooms. Cobb and his colleagues’ studies highlighted the norms such as “working together”, “thinking through ideas for themselves”, “deciding on acceptable explanation” (Franke, Kazemi, & Battey, 2007). These social norms were also developed especially through ongoing negotiation among participants in the communication. They helped students understand what learning means and shaped their learning behaviors. In addition, Yackel and Cobb (1996) suggested sociomathematical norms to highlight the social norms that especially pertain to mathematical attributes; for example, the criteria for different solutions in mathematics problem solving. “Knowing that one is expected to explain one’s thinking is a social norm; knowing what counts as an acceptable mathematical explanation is a sociomathematical norm” (Franke, Kazemi, & Battey, 2007, p.239). Yackel and Cobb (1996) indicated that sociomathematical norms that focused on inquiry-based learning contribute in particular to the development of students’ mathematical thinking and social autonomy in participating in community learning activities.
The relationships between sociocultural norms and classroom teaching and learning practices provides a basis for framing a socio-constructivist understanding of the reflective and dialectical relationship between individuals’ constructions and the evolution of collective norms and understanding.

As Davis and Simmt (2003) argued, social norms are typically understood at the level of individual students rather than from the level of the whole learning system. Although emergence and negotiation are considered in social norms, they are also understood in terms of the benefit of regulating individual constructions of prescribed knowledge. Based on social systems theories, social norms should be considered beyond a concern of the individual level. The emergence of complex understandings and learning possibilities rather than conformity are strived for by the advancement of the whole system. Classroom teaching and learning activities should be organized and operated for the collective growth. In correspondence with this, expectations and obligations for individuals to learn must be considered from the larger perspective requiring an examination of collaborative and shared understandings as crucial to understanding the social norms of learning communities. Study of classroom norms therefore requires examination of interactions and whole-system behaviors at the social systems level.

**Instructional Organization.** The organizational forms of instructional activities often refer to whole class, small group, individual work, or a combination of them. In traditional mathematics classrooms, whole-class teaching is very common, while NCTM reforms recommended individual/ small-group interaction followed by whole-class discussion of student solutions and strategies (NCTM, 2000). Yackel, Cobb and Woods (1991) studied elementary classrooms where small-group problem solving was used and followed by whole class discussion. They found small-group problem solving provided opportunities for students to explain and
justify their solutions, which increased learning opportunities not typically seen in traditional classrooms. However, Lloyd (2008) found that despite small-group instruction was called for in reform-envisioned teaching, if the teacher felt discomfort about the reform curriculum, he or she tended to go back to the whole-class teaching which he or she was familiar with. Moreover, Spillane and Zeuli noticed it was easier for teachers to change instructional organization of their activities while maintaining a teacher-focused approach. In other words, changing instructional organization, for example, to include small-group work, did not necessarily reflect a reform mathematics perspective. These studies show that it is meaningless to study instructional organization without considering other dimensions of classroom practice. From a social network frame, different types of instructional organizations such as whole-class, small-group discussion, and individual learning are complementary and depend on each other. Instructional organization is a part of a larger classroom meaning systems.

Studies on international mathematics classrooms also indicate that the same type of instructional organization may even have different meanings. For example, Stevenson and Lee (1995) studied the perspectives of whole-class teaching in Eastern countries. Stevenson and Lee noticed that the perspective of whole-class teaching in Eastern countries was different from those in Western countries. In Eastern Asia whole-class teaching, individual’s expertise, and diverse learning strategies and approaches were used to complement each other and were treated holistically. For example, “inappropriate or inefficient approaches” served as informative signs of “cogent, powerful approaches” (p.159). “Less effective” learning styles and approaches were compared, contrasted, and adjusted with “relative effective” learning styles and approaches. Diverse interests, approaches, and ability levels were used to create the dynamics of classroom teaching. In whole-class teaching, “the contributions of all children are valued and the teacher
seeks to be their knowledgeable, experienced guide” (p.162) in the classrooms in Eastern countries.

Moreover, there were student groups across different subject matters in the classrooms. Stevenson and Lee noticed small han groups in Japanese classrooms were study groups across subject areas. The han groups consisted of students with different interests, abilities, and genders. The han groups engaged in diverse activities together not only in-class, but also after-class in a variety of school related activities including sports, work, and extracurricular engagements. The students developed their identities and healthy friendships through their han group activities. Stevenson and Lee said, “Because of this identification with a group, the motivation of slow learners to work hard and perform well may be enhanced and the eagerness of the fast learners to help their slower classmates may be increased” (p. 160). The research indicated the han group connected and motivated members to learn together. The student built intimate relationships across subjects through the han group organization. Similarly, from a social network perspective, the social relationships between the teacher and students and relationships among students contribute to classroom teaching and learning and become part of the communication system of mathematics learning.

Teachers’ Beliefs and Teaching Experiences

Studying teachers’ belief of mathematics and mathematics teaching and learning is a way to understand teachers’ actions in the classroom (Thompson, 1984). Thompson stated,

If teachers’ characteristic patterns of behavior are indeed a function of their views, beliefs, and preference about the subject matter and its teaching, then any attempt to improve the quality of mathematics teaching must begin with an understanding of the conceptions held by the teachers and how these are related to their instructional practice (p. 106).

Thompson (1984) highlighted the significance of studying teachers’ beliefs of mathematics, of teaching and learning mathematics and the relationship with their practice. In particular,
existing research has indicated teachers’ beliefs of mathematics, of mathematics teaching and learning, and their teaching experience have close relationships with the implementation of classroom reform (Chazan, 2000).

Research has indicated a consistency between the teachers’ beliefs and their teaching practices in general and it is hard for teachers to change their long-held beliefs (McGalliard, 1983; Thompson, 1985; Flores, Sowder & Schapelle, 1994). In the study conducted by Spillane and Zeuli, they found that despite teachers’ claims that they were familiar with reform practices, the degree to which their classroom practices were aligned with the reform agenda was quite different. Some of them used reform rhetoric language to describe their teaching; however, the instruction still remained the same as traditional ways. Similarly, Chan (2002) pointed out that in Chinese schools the teachers said they were doing qualitative-oriented education, but indeed, they were still using test-based education (Kappa Delta Pi, 2002). The inconsistency is often due to the introduction of reform agendas that are inconsistent or incompatible with their beliefs and experiences (Handal, 2003).

The inconsistency of beliefs and practices lead to uncomfortable feelings and frustration (Anderson & Piazza, 1996; Orton, 1991). For instance, the study conducted by Lloyd (2008) revealed that teachers experienced a “pedagogical discomfort” in implementing new approaches of teaching. Chazn (2001) pointed out those teachers who were familiar with the conventional models of teaching tended to resist changes, while those with less teaching experience were more likely to adapt to new teaching approaches. Carson (2009) further pointed out that teachers had to go through two processes in implementing new curriculum: to unlearn what they have learned in their experiences while learning what they do not know. This process, Carson
found, could be very difficult for teachers as they began to question old beliefs. Teachers’ resistance to changes of beliefs prevents the successful implementation of new curriculum.

Gu and Tan (2004) surveyed about 1,000 practicing teachers in K-12 in the implementing of the new curriculum in China. Their study indicated that the teachers wanted help with connecting new curriculum ideas with their teaching practices. In many cases, however, the mechanics of incorporating reform teaching methodologies without addressing teacher beliefs makes implementation of reform efforts futile. Spillane and Zeuli suggested that teachers’ understanding of reform ideas should go beyond getting teachers to know the reform agenda superficially, in particular, actions to connect the reform ideas and teachers’ existing practices and beliefs were needed for teachers to implement change (Spillane & Zeuli, 1999).

More importantly, the essential changes of classroom practice, besides the changes of “behavioral regularities”, require teachers to “fundamentally transform the epistemological regularities of instruction by recognizing and supporting new conceptions of knowledge and knowing in their classrooms” (p. 19, emphasis is original). This implies it is important to study the teachers’ beliefs of mathematics and instruction and their experiences in the reform context.

The understanding of teachers’ beliefs should not ignore cultural factors either. In TIMSS mathematics classroom videotape analysis, Stigler and Hiebert (1999) found significant differences between the beliefs of Japanese teachers and those of the U.S. mathematics teachers. Japanese teachers had a relational view of mathematics teaching and learning, in which they focused on the relationships between concepts and procedures, constructing connections between methods and problems, and between different ideas. In addition, they had a holistic view of the individual and the collective. Individuals’ differences were used as resources for collective discussion and reflections. Each part of their lesson was connected coherently as a
whole to reach their lesson objectives. In contrast, school mathematics teachers in the US tended to view mathematics as a collection of separated and isolated concepts and procedures. They focused on step-by-step procedures rather than on mathematical ideas, and on answers rather than procedures. Individual difference were treated in isolation, thus individual difference became obstacles to rather than advantages for teaching. In addition, they found American teachers and Japanese teachers had different views of students’ interest. Japanese teachers viewed students’ mathematical interest inherent in mathematics learning, while American teachers viewed students’ interest separated from mathematics learning.

Correa (2008) et al. studied American and Chinese teachers’ beliefs of mathematics learning with twenty-one American elementary teachers and twenty-nine Chinese elementary mathematics teachers. The findings indicated there was a significant difference in beliefs in terms of best ways for students to learn mathematics from teachers in the two countries and across grade levels. American teachers for the lower level grades believed students’ discoveries, divergent thinking, and hands-on activities were particularly important; while teachers in higher-level grades (4th or 5th grade) believed practice and repetition were more important. American teachers were concerned with different types of leaning styles of individual students. In contrast, Chinese teachers in lower level grades valued the development of students’ interest in mathematics learning and students’ own experience of using mathematics. The teachers teaching higher level elementary grades believed the importance of the connections between knowledge and motivation of learning mathematics. In general, Chinese teachers believed the development of students’ interest and the student-teacher relationship as a pedagogical component of their mathematics teaching were important in learning.
**Teacher Professional Development**

Mathematics education reform places tremendous pressure on classroom teachers. Teacher professional development is an inseparable aspect of helping teachers in implementing mathematics education reforms.

US attempts at mathematics reform are typically in the form of teacher professional development program where teachers learn new materials, understand new standards, and learn instructional techniques. The one-day workshop is the most common form of teacher professional development in the US. Some classroom teachers were fortunate to participate with university grant-funded programs that allow for follow-up classroom observations and meetings (Edwards, 1994; Kitchen, 2003; Lloyd, 2008; Peressini & Knuth, 1998). Other forms include development of reform-based curriculum. For example, the Math Learning Center at Portland State University developed teaching encouraging exploration, small group collaboration, and whole-class discussion. They provided workshops to engage teachers with activities that allowed them to better understand these materials and underlying strategies for application (Edwards, 1994).

Peressini and Knuth (1998) conducted a two-week summer course where mathematics teachers learned discrete mathematics and dialogical pedagogy. After that, teachers implemented the discrete mathematics lessons they designed in their own class, accompanied with follow-up group meetings and classroom observations. Likewise, Kitchen (2003) conducted three-week summer program focused on problem-based curricula and compatible instruction. A main finding from these studies indicates that despite the researchers’ efforts, teachers’ practice did not change. The study conducted by Peressini and Knuth concluded that teachers’ reflection and collaboration over personal teaching experience needs to be incorporated into any
meaningful professional development. Kitchen highlighted the lack of support from colleagues and administrators as a roadblock to reform. Many of Kitchen's teachers were the only representative of their school that participated in the professional development. A number of US reform efforts focus on individual teachers’ gaining predetermined reform-oriented knowledge, while neglecting teachers’ personal experience, classroom dynamics, and systematic classroom-based reform efforts in the US.

Recent research suggested that practice-situated and community-based approach provides more lasting teacher transition. For instance, Matos, Powell, and Sztajn (2009) suggested a community practice-based model of professional development in which communities of practice are fostered by teachers that work together to discuss and to understand their own teaching experiences and to develop their teaching through lesson studies, development of materials, and reflection on classroom discourse. In particular, lesson study (originally Eastern Asian forms of professional development) is receiving emphasis in the US. Two salient characteristics of lesson study are classroom practice and collaboration among a group of teachers (often teachers in a school or in a district). The community-based approach reflects an attempt to understand teachers’ practices through collaboration among teachers on daily practice. Moreover, research highlights that this form of teacher professional development deals not only with augmenting mathematics knowledge and teaching approaches, but also makes teachers’ beliefs more flexible. The community-based approach involves not only an individual teacher, but also his or her students, colleagues, and schools. All of these aspects interrelate and interact with each other to form a complex network of relationships.

It deals not only with beliefs, knowledge and practices of teachers but also students’ beliefs and knowledge, as well as with interaction between teachers and students, and the interaction between teacher educators and teachers… (Adler, Ball, Krainer, Lin, & Novotna, 2005, p. 369)
Matos, Powell, and Sztajn also suggested the community needs to build up cohesion: a sense of belonging among members to ensure commitment of members to the improvement of the community. The goal of understanding the nature and the experience of teachers’ learning needed to be imbued in community activities. Darling-Hammond and Richardson (2009) summarized that the current US trend in teacher education is to develop professional learning communities with goals centered on curriculum development and problem solving that improve both teacher and student learning; members in the communities should value mutual support rather than privacy, share responsibilities to improve instruction, and build up community identity. In addition, the communications should be inquiry-based where teachers examine their teaching practice, identify problems, discuss and reflect on their experience, and develop solutions.

The studies on teacher professional development indicate a shift to teachers as active learners in learning communities. The community approach offers opportunities for teachers to interact and connect with each other. The goals, social norms, communications, and instructional patterns are keys in both classroom and teacher learning communities. From a social systems perspective, teacher learning communities and classrooms are embedded nest-systems. These dimensions in the two communities are inseparable and push each other for the greatest outcome for both.

In summary, the literature review has indicated that the trend of school mathematics reforms has been toward a comprehensive development for every student. In China, innovative spirit and practical abilities are highly valued in the new round of reform. The literature has also indicated how those dimensions have impacted classroom reform, what challenges teachers face, and how various efforts have been made in helping teachers implement classroom reform.
Missing is a picture of the whole that includes how each aspect interacts and constitutes the context of classroom mathematics instruction reform. The understanding of teaching reform also needs to go beyond classrooms to include associated teacher professional organizations and teachers as nested systems. Overall, this literature review demonstrates that examination of the dynamics structure and meaning relationships such as goals, social norms, instructional organizations in classroom discourses, and teachers’ beliefs of mathematics and teaching, and the associated teacher professional development organizations is needed for deep understanding of the context of classroom reform.
CHAPTER 4: METHODOLOGY

The purpose of this study is to investigate how teaching reform was implemented in Chinese mathematics classrooms, in particular, to examine the communication structures and meaning relationships underlying classroom teaching, and to understand the context that impacts classroom teaching transformation. Social systems theories are used as the theoretical foundation to capture the dynamic relationships of various components of the reform implementation. The central question of this study is: what communication structures and relationship patterns underlie Chinese mathematics classroom teaching reform at the two different sites? The sub-questions are:

1. What communication structures are demonstrated during classroom teaching? What meaning relationships are embedded in the structures? How do these structures change in teaching?

2. What communication structures and meaning relationships are demonstrated in the associated teacher development communities at both sites?

3. What similarities and differences are demonstrated in the structures and meaning relationships across the classrooms and the teacher professional development communities? What similarities or differences are demonstrated in the structures and meaning relationships across different sites?

4. What perspectives of mathematics and mathematics teaching do those teachers have?

Based on the research purpose and questions, social network analysis provides an appropriate way to describe the structure of a system and meaning relationships within the system, and to reveal the dynamic relationships between the participants and the system in communications (Wasserman & Faust, 1994).
The existing research using social network analysis is often used in computer-based, collaborative learning settings where data is collected by computers, and is processed with computer network software packages. An example is the study conducted by Haythornthwaite (2005), who studied clusters and centered agents in interactions that emerged in an e-learning classroom setting. As pointed out by Martinez, Dimitriadis, Gómez, and Fuente (2003), social network analysis “by itself is not enough for achieving a full understanding of the problems, and needs to be complemented with other methods, like qualitative data analysis” (p. 354). Martinez et al. (2003) suggested a mixed methods approach to combine qualitative methods with network analysis to explore the interactions in a computer-based learning environment. From a social systems perspective, classrooms are communication networks with meaning relationships that guide the movement of the networks. Thus, a combination of qualitative data analysis and social network analysis is particularly appropriate for this study. Yet, there is no research to combine qualitative methods and social network analysis in normal mathematics classroom settings. This research is an attempt to use the combination of network analysis and qualitative analysis to understand the dynamics of the communication structures and meaning relationships in regular mathematics classroom teaching.

Among the methods of qualitative research, the principles of case study analysis can be used as an appropriate framework for the combination of qualitative and social network methods (Huberman & Miles, 1994; Martinez et al., 2003). Multiple types of data sources such as observations, questionnaires and interviews can be used to capture the views of the participants in case studies, and qualitative categories generated in qualitative analysis can be integrated with network theory to understand the occurrence of actions or events (Martinez et al., 2003). This study uses both multiple case and network analyses to elaborate the structures and meaning
relationships of classroom teaching in two sites where different approaches to implement mathematics classroom teaching reform were carried out.

**Multiple Case Study**

Based on the purpose of the research, a variety of dimensions of sample selection and multiple methods of data collection are necessary.

**Site Selection.** The researcher knows a Chinese mathematics education professor in a normal university in China, who has built good relationships with school mathematics educators in both universities and local teacher education departments. Through him, the researcher contacted one mathematics teacher educator from a university in the city of Beijing and three mathematics teacher educators from local mathematics education departments in the cities of Tianjing and Chengdu. Those educators are familiar with the local schools and surrounding mathematics teachers. Through them, the researcher initiated the sample selection of the teachers and classrooms. Fourteen mathematics teachers in six secondary schools in the three cities were initially included in this study. In the summer of 2009, the researcher visited the three cities, observed 24 lessons in 14 classes, and interviewed 11 teachers in the six schools in the three cities (See Appendix VII).

Among the six schools in the three cities, two schools in two cities (Beijing and Chengdu) were engaged in different approaches to educational reform. Based on the purpose of the research, the two schools were selected as the final two case sites for this study to examine how classroom teaching reform was implemented differently and the impact of the implementation efforts on classroom communications. Site A involved a rural school, although in Beijing, a developed city, while Site B involved a rural school in Chengdu, a developing city. Both schools are public schools located in disadvantaged districts in the cities (See Table 1).
Six teachers and three teacher educators in the two cities were included in this study. At Site A, the three teachers were involved in a teacher leadership project led by a team of four university-based teacher educators (professors in mathematics education) and local education government. At Site B, the three teachers were engaged in a district-wide teaching experiment led by two mathematics teacher educators from education department in the local district. The different teacher professional activities associated with classroom teaching practices in the two sites allowed the researcher to examine the relationships between classroom teaching and teacher professional development programs, while comparing the similarities and differences between the two sites.

Table 1: Characteristics of the Two Sites

<table>
<thead>
<tr>
<th>Site</th>
<th>School</th>
<th>District</th>
<th>City</th>
<th>Types of Reform Projects Involved</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site A</td>
<td>Secondary</td>
<td>Rural</td>
<td>Beijing</td>
<td>Key Teacher Professional Development Project</td>
</tr>
<tr>
<td>Site B</td>
<td>Secondary</td>
<td>Rural</td>
<td>Chengdu</td>
<td>School-wide teaching experiment</td>
</tr>
</tbody>
</table>

In addition, both sites include one exemplary lesson. For example, at Site A, one teacher presented an exemplary lesson and also shared his view of mathematics, the reform of mathematics education, and his mathematics teaching experiences. At Site B, one teacher presented an exemplary lesson to the principals from the district. Following the lesson, there was a seminar about the teaching experiment, primarily to assist teacher professional development among the school teachers, the district principals, and the district education department teacher educators. The exemplary lessons and the follow-up conversations among the teachers, administrators, and teacher educators embodied the reform efforts across schools, university, local teacher education departments and district governments in the two sites. The different teaching approaches between the two sites, and the associated different teacher professional
development communities create opportunities to see the classroom teaching reform efforts in Chinese schools from multiple angles.

It is worth noticing two other dimensions in the data selection. The three lessons included at Site A were from three teachers, which offer a general view of the reform classroom teaching at Site A. However, at Site B, where the teaching experiment was underway in the school, 13 consecutive lessons with three teachers were included. Each teacher’s teaching was observed and video-taped at least four times. The data at Site B offers an opportunity for a deeper inspection of current classroom teaching reform. Other characteristics of the schools related to the sample selection are listed in Tables 2.

Table 2: Data Sources and Size for This Study

<table>
<thead>
<tr>
<th>Site</th>
<th>Lessons video-taped</th>
<th>Classes Directly Observed</th>
<th>Teachers Interviewed</th>
<th>Teacher Educators Interviewed</th>
<th>Students Surveyed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site A</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>74</td>
</tr>
<tr>
<td>Site B</td>
<td>13</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>152</td>
</tr>
</tbody>
</table>

**Data Sources.** Multiple sources of data such as video-recordings, audio and survey data were employed to achieve the research purpose. Specifically, the following data were collected:

**Classroom Observations and Video-Taping.** The purpose of the classroom observations was to get first-hand experience of Chinese secondary classroom teaching practices. The video-taping allowed for further fine-grained analyses once the researcher returned to the US. The observations and video-taping were focused on classroom interactions, instructional organization and social and cultural relationships. Three video cameras were used to catch classroom teaching and learning from different angles and attempted to catch as much of the learning processes as possible, including initialization of interactions, problem-solving processes, emergent events, and overall conversations in the classrooms. One camera was fixed at the front of each classroom.
to catch the general context of the whole classroom. Another camera was fixed at the back of each classroom to catch the activities near the blackboards. The third camera was carried by the researcher to capture classroom discussion during teaching and learning activities. The primary focal point was the teacher, although student conversations were also captured.

During the school visits at Site B, there was a seminar regarding the implementation of the new model of teaching among the school teachers, middle school principals, and teacher educators from the district. The researcher observed and video-taped the seminar, and the data was analyzed to deepen the understanding of teacher professional development community associated with classroom teaching reform.

Interviews with Teachers and Teacher Educators. The interviews with the teachers involved semi-structured questions specific to classroom observations such as learning goals, lesson plan preparation, instructional organizations, reflections on classroom teaching; and questions pertaining to their perceptions of mathematics and mathematics instruction, teaching experiences (see Appendix V). In addition, interviews with the university-based teacher educators and teaching researchers from the local education department were conducted to understand how the associated teacher professional communities influenced the classroom teaching reform implementation efforts. The interviews with the university-based teacher educators and the local district teacher educators were primarily focused on goals, processes and activities in the school mathematics teaching reform projects. In addition, documents about the projects were collected and used during the analyses. Rather than originally planned, the interviews with the teacher educators were emerged from sites visits; therefore, there were no structured questions to guide the interviews.
**Students Survey and Interviews about Mathematics Classroom Learning.** All the students in the observed classrooms were given a survey (see Appendix IV) and responses were collected the following day. The survey included questions about their attitudes toward mathematics learning, their perceptions of mathematics, mathematics learning, classroom learning environment, and instructional organization. Besides the survey, a few students from Site B were interviewed to learn more about their perceptions regarding the on-going experiment of the new model of teaching. Student interview at site B was not originally planned. In addition, copies of students’ written work during the classroom observations at Site B were also collected for analysis.

In summary, the data collection included: (1) classroom observations and video-taping focusing on the instruction process, classroom interactions and social relationships norms; (2) interviews with classroom teachers about their planning and classroom teaching to support the classroom observations, and about their views of mathematics, and mathematics teaching and learning to learn about the hidden aspects that drove their teaching practices; (3) survey of students’ perceptions of mathematics classroom teaching practices to learn more about how these students viewed mathematics, mathematics teaching and learning, and their mathematics classroom teaching and learning; and (4) interviews with the university-based teacher educators and the local district teacher educators to understand the goals and activities in the teacher professional communities that were designed to support classroom reform. Additionally, observation and video-taping of a seminar among the school teachers, middle school principals, and teacher educators from the district was conducted as emerging data collection.
Network Analysis

This section includes an introduction of network analysis theories in general and an explanation of how these network theories are employed in classroom teaching analysis in this study.

**Description of Network Analysis.** Monge and Contractor (2003) noted, “A network analysis consists of applying a set of relations to a set of identified entities” (p. 30). Entities, the components or agents of system, are often called nodes in network analysis. In educational organizations, the nodes can be people who interact with each other through communications in the organization (for example: students, teachers, administrators, or teacher educators). Nodes can also be groups of people or organizations, such as classrooms and teacher professional communities. Moreover, nodes can also be “nonhuman agents” (Monge & Contractor, 2003). In their research on understanding mathematics knowledge from a network perspective, Mowat and Davis (2010) defined mathematics conceptual domains as nodes. For example, the conceptual domain of “circle,” was considered a node in the network of conceptual mathematics knowledge. In their study, a node like “circle” was viewed as a network of a learner’s knowledge and experience related to the concept of “circle”. Each node should be understood as a sub-network consisting of the larger network (Barabási, 2002; Mowat & Davis, 2010).

In Luhmann’s social systems theory, communications are nodes of communication networks. When there is an “utterance,” the system recognizes “information” associated with it and takes action according to its “understanding.” Through the interaction among utterance, information and understanding, communication occurs. Thus, communication is a synthesis of selections among utterance, information, and understanding represented in the communications network by a communications node (Luhmann, 1999; Rasmussen, 2005).
Network analysis focuses on patterns of relationships among nodes. The links of a network constitute the relationships between nodes. In Haythornthwaite’s study, teachers, students, administrators, and institutions were possible nodes, and they interacted and were interrelated to each other in the network. Relations or links during communications were messages of information, knowledge, mathematical problems, or symbols; they could also be social and cultural relationships such as attitudes, and norms. (Haythornthwaite, 2005).

In the study conducted by Mowat and Davis, they used “conceptual metaphors” as links in the growing network of shared mathematics knowledge in classrooms. As Mowat and Davis stated, “Conceptual metaphors are referential systems of thought; metaphorlic expressions in language and gesture are simply surface manifestations of an underlying conceptual metaphor” (p. 11). With the definition of links, a mathematical conceptual domain (node) can be mapped onto a network of related experience and knowledge of a concept. More specifically, Mowat and Davis noted the establishment of links was a function of the structures of individuals and their participation in the interaction within the network.

In Luhmann’s communication networks, communications are nodes (units) of the communication networks. The links between communications are social and cultural meaning relationships, which can be expressed such as goals, values, norms, attitudes, and responsibilities. Meaning relationships are referential systems that guide the synthesis of the selections of “utterance,” “information” and “understanding.” The process of communication is a meaning-based continuous modification of understandings with reference to the coordination in the communication. The networks of communication go through recursive and constant reproducing processes. Thus, communication nodes are sub-networks of communication networks.
Nodes can be described from varying perspectives including content, direction and strength of connection. For example, in classroom learning systems, questions about what is exchanged during interactions between the teacher and students become the content of the node. Whether the teachers or students initiate a communication indicates the direction of the communication. And the strength of connections between nodes is determined by the frequency of the same communications between the two nodes (Barabási, 2002; Haythornthwaite, 2005).

Networks evolve as new nodes are added or modified, and new links emerge (Mowat & Davis, 2010). The nodes that attract multiple connections in a communication network are also called hubs. Hubs indicate central and shared interests of a particular community. Hubs can be further connected into clusters of sub-networks. The earlier a node forms in a network, the greater tendency for it to establish more links, and to become a hub (Barabási, 2002; Jeong & Bianconi, 2000). Mowat and Davis suggested early experiences of a child associated with mathematics concepts may provide for richer networks and conceptual hubs in personal mathematical learning networks. In addition, nodes that have a greater degree of ‘fitness’ tend to attract more connections, which is called “preferential attachment” (Barabási, 2002, p. 86). Links with more connections have more opportunities to further connect; this is called “richer-get-richer phenomenon” (Barabási, 2002, p. 80).

By contrast, weak links are nodes with fewer connections. Despite having lower frequency of connections, weak links are important for developing new ideas because they provide additional pathways for new resources to access the networks. Moreover, weak links play important roles in reducing the distance of connections among nodes (Barabási, 2002; Mowat & Davis, 2010). Often small changes may lead to dramatic effects on the overall structure of the network over time.
Haythornthwaite also notified latent relations between two nodes, where relations are not yet activated. Latent links provide potential connections. Latent links are often set up by authorities from a larger social network. In Haythornthwaite’s (2005) study of the e-learning class, the group email list given by the teacher at the beginning laid a foundation of latent links as the students did not yet know each other.

Three types of network structures are often highlighted in social network research (see Figure 3) (a) centralized, (b) decentralized or scale-free, and (c) distributed (Barabási, 2002; Davis & Sumara, 2006).

![Network Structures Diagram](image)

Centralized Decentralized Distributed

Figure 4.1. Three Types of Network Structures (Brent & Sumara, 2008)

In a centralized network, all components are connected to a central node, which maximizes the exchange of information; however, the network can easily break down if the center does not function well. In classroom teaching, the emphasis on just one solution may run the risk of collapsing knowledge in dealing with new situations.

In a distributed network, there are many alternatives to connections, ultimately making the search for novel solution paths to new situations more efficient and robust. Communication structures that are represented by distributed networks may possess richness of meanings including irony and pun, for example, that aren’t found in less rich connections.
A distributed network has more connections that offer richer alternative connections when central nodes are missing which is why some neurophysiologists examining brain trauma have begun to look at synaptic connections in the brain from the perspective of networks. A distributed network is more robust than a centralized or decentralized network because the failure of a few nodes does not cripple the overall network. However, distributed networks have more restricted information flow. Decentralized (scale-free) networks are formed through “preferential attachment.” (Barabási, 2002) A decentralized network is clustered around a few hubs. Decentralized networks demonstrate a happy medium between efficiency in exchanging information, flexibility and robustness (Barabási, 2002; Davis & Sumara, 2006) especially in systems where information flow may be slow.

In summary, network analysis offers a way to examine the structures and patterns of relationships in a social system. By combining qualitative analysis with social network analysis, the researcher is able to examine how participants communicate and what meaning relationships are developed during communication within a social system; therefore, we can understand the dynamic evolvement of the relationships and the participants in the community as a whole.

**Network Analysis of Classroom Teaching in This Study.** The researcher first transcribes the video recordings line by line. Following the processes described by Huberman & Miles (1994), the researcher reads the transcripts and writes summaries for each meaningful segment focusing on mathematical ideas, participants, or social norms embedded in the interactions. Multiple sources of data such as students’ worksheets and interviews with students and teachers are used as complementary material to ensure the accuracy of interpretation along with cross-checking particular episodes.
The researcher then examines the communication networks of mathematics classroom teaching. Using Luhmann’s theory, communications are nodes of communication networks. Communication units occur as resulting from a synthesis of selections of understandings as the dynamics among utterance, information, and meanings evolve. In mathematics classrooms, communication involves a process in which the teacher and students build understandings about mathematics and meaning relationships. The understanding is embodied by correlated actions taken by the teacher and students in the process of realizing the meaning of mathematics, and social and cultural relationships. In the analysis, the nodes are coded as correlated actions of understandings about mathematics objects or meaning relationships between the teacher and the students (e.g., as assigning problems by the teacher), or as communications among students (e.g., as students working together to solve problems), or as individual to whole-class and teacher communications (e.g., as a student describing the solution to the teacher and class). The links or connections between two communications are represented as social and cultural meaning relationships (e.g., students as problem-solvers and interpreters, and efficiency as a socio-mathematical norm in those communications). Those meaning relationships (learning goals, socio-mathematical norms, attitudes and responsibilities) guide the communications in the classrooms. Meaning relationships are shared and adapt through the process of understanding mathematics objects between the teacher and students over time and space. Thus, the movements of communications among the participants are composed of actions of constructing the understanding about mathematics objects and social meanings such as socio-mathematical norms and learning goals. The network of the entire classroom teaching is patterns of meaning relationships developed in the process of teaching and learning. Through the analysis of the actions of constructing understandings, the direction of communication and frequencies of the
nodes, a picture of what communications are exchanged between the teacher and students, and how and to what extent mathematical ideas, knowledge, and understandings are shared and developed among different participants are obtained. Applying the social network analysis, the researcher synthesizes the patterns and characteristics of the structures of communications and the patterns of relationships such as norms, roles, learning goals, and instructional organizations in each site and then compares the similarities and differences of them in the two sites.

**Analysis of Teacher Learning Communities and Teachers’ Beliefs and Experiences**

Besides network analysis of classroom teaching, the researcher also analyzes the structures and social relationships including the goals, values, and norms in the associated teacher learning communities. However, the analysis is primarily based on the interviews with the teachers, teacher educators, administrators, and the documents received from the two sites. Thus, in comparison with the classroom interaction network analysis, the evolution of the structures and relationship patterns of teacher professional development communities is not able to be as fully addressed. The researcher compares the similarities of the structures and relationship patterns of teacher learning communities in the two sites, and also discusses the similarities of structures and patterns of relationships across the teacher communities and the classrooms.

In addition, the teachers’ beliefs of mathematics, mathematics teaching and learning, and their experiences are analyzed based on teacher interviews and data from the seminars conducted by the teachers. The researcher discusses the impact of the teachers’ beliefs on their teaching practices. Thus, the analysis of the classroom teaching reform is across three nested systems – the teachers, the classrooms and the teacher learning communities. Employing the social network analysis and combining it with qualitative research methods, the researcher investigates the
context of mathematics classroom reform implementation consisting of interactions, social norms, goals, and organizations in the two sites. Additionally, the comparison and contrast between exemplary lessons and normal lessons, between different approaches guided by the institution and by the local education department in the two sites increase dynamics of views on the reform. Finally, the general study at Site A, although with limited numbers of classes, and the concentrated study at Site B offer complementary perspectives to view the reform implementation in different cities in the nation-wide reform.
CHAPTER 5: DATA ANALYSIS AT SITE A

This chapter is an analysis of three dimensions of classroom teaching at Site A including (1) classroom teaching, (2) teachers’ perspectives of mathematics and mathematics teaching and learning, and (3) the Key Teacher Professional Development Project. Three mathematics teachers and one university-based mathematics teacher educator are involved in this site. Among them, Ms. N and Ms. B are outstanding school teachers and were selected to participate in the district-level key teacher development project. Mr. W was in a city-level key teacher professional development project and was invited to present an exemplary lesson to the participating teachers in the district-level key teacher development project. Ms. K is a university-based teacher educator and was one of the main actors in the development and execution of the district-level key teacher professional development project.

I first analyze the communication structures of the three lessons and examine the meaning relationships embedded in the communication networks such as learning goals, values, norms, roles, and instructional organizations. I then describe and compare the three teachers’ perspectives of mathematics, mathematics instruction, and teaching experiences to understand the relationships among their perspectives, teaching experiences, and teaching practices. I also examine the structures of the teacher learning community based on the district leadership project, and the meaning relationships such as learning goals, values, norms, and activities. Finally, I discuss the relationships between the classroom and the project-based learning communities.

Classroom Teaching at Site A

The analysis of classroom teaching is primarily focused on the evolvement of communication networks and meaning relationships embedded in the networks.
**Ms. N's Lesson.** This section describes Ms. N’s lesson including a brief introduction of the teacher, classroom setting and students, and the analysis of classroom teaching communication network.

**The Teacher, the Students and the Setting.** Ms. N has taught middle school students for about 23 years since she graduated from high school in 1986. She taught nine years in countryside schools before she moved in this school. Ms. N obtained her college degree and continued to work toward her master degree while teaching mathematics in the school. Her class was a regular 8th grade class with thirty-four students sitting by rows. A big evaluation chart was on the wall in the class. The topic of the lesson was about quadratic formula.

**Phase 1: The Beginning of the Lesson.** Ms. N started her lesson with a review of the “completing the square” method of determining roots of quadratic equations, which was taught in the previous lessons. She reviewed it by asking students to solve a quadratic equation $2x^2+8x-1 = 0$. One student was called to the board to show the process to solve the equation. Others worked at their own seats. When students finished, they showed Ms. N their work. Ms. N gave each of them points for their work and praised the students who seemed to make good progress in the class. Then she went over the solution shown on the blackboard and highlighted two common errors and main points in solving the problem. Using network analysis, the communication in this period can be represented by the following diagram (Figure 5.1).

During this period of communication, the primary actions performed by the teacher included assigning the problem and evaluating students’ work, while the main action performed by students was to solve the problem. The role of the teacher was task assigner, monitor, and interpreter of solutions, and students were the problem solvers. The communication was focused
on procedures of solving an equation. In addition, the teacher encouraged students’ participation in the conversation through positive comments.

Figure 5.1 Communication during the Review Exercise in Ms. N’s Lesson

**Phase 2: The Main Part of the Lesson.** After the review, the lesson then moved to the main topic of the day: “The Quadratic Formula.” Ms. N did not lecture the lesson directly; instead, students worked on generating the solutions to the general form of quadratic equations $ax^2+bx+c = 0 \ (a\neq0)$ in an analogy with what they did in solving $2x^2+8x-1 = 0$. Ms. N said to the class,

You are to compare to the method we used in this problem (refers to $2x^2+8x-1 = 0$) to solve the equation $ax^2+bx+c = 0 \ (a\neq0)$. Try it by yourself first. If you do not know how to do it, turn to page 114 in your textbook to check up with the book and solve it. If it doesn’t work, ask your partner. If your still have difficulties, ask our elite group members. Now, you start to work on the problem. I’ll give you 8 minutes.

Ms. N called one student to go to the board to show the solution process, and she circulated the room to help students individually. When students finished the problem, they submitted their work to Ms. N. She checked their work, offered comments, and praised them for their work. After about five minutes, seeing both the student at the blackboard and other students finished the problem, Ms. N began to elaborate on the solution on the board.

T:  Now, let us look at the blackboard. Some of you do not know how to solve it. We will use an analogy method to show you the process step-by-step. When we learn new knowledge, we often use an analogy. First step, where should the constant be?

Ss:  On the right side.
T: It is not on the right side, here. Let us first divide the coefficient of the quadratic term, ok?

Ss: Yes.
T: Ok, divide $a$. In this problem, we divided 2 on each term (points to the step in the solution of $2x^2+8x-1 = 0$). But here, there is a problem. If you divide a symbol, under what condition you can do it?

Ss: (the symbol) Cannot be zero.
T: So, here, did he miss the “$a\neq 0$”?
Li: It is a given condition (so we do not need to mark it).
T: Yes. It is given, but you need to write it up here, so you know it. Good. Any questions? If you have any questions, you should ask the teacher. Li is doing well. He asked questions and participated actively in learning. All right, we should add $a\neq 0$ here. Now, looking at the steps of completing square, what do we need to add? Compare to here (pointing to the steps in solving $2x^2+8x-1 = 0$).

Ss: The square of half of the coefficient of x.
T: That is $(b/2a)^2$. Add them on the sides, both the right side and the left side. I do not think we have much trouble in this step from what I observed during my circulation. Did we make the analogy from the example (refers to $2x^2+8x-1 = 0$) to this problem (refers to $ax^2+bx+c = 0$)? What is the difficult part? Combining $(c/a) + (b/2a)^2$, right? Zhe (the student who did the problem on the board) did a very good job at here. What operation is there? Kids?
S: Addition.
T: What kind of addition? Addition with unlike denominators? Then, what do I need to do?
S: Find a common denominator.

The episode above illustrates how Ms. N explained the solving process of the general quadratic equation by comparing it with the steps in solving $2x^2+8x-1 = 0$. It is a typical IRE model of interaction between the teacher and students, except that the student first did the problem. The interaction pattern between the teacher and students can be illustrated in the following diagram (Figure 5.2).

![Figure 5.2 Communication during the Deduction of Quadratic Formula](image-url)

Figure 5.2 Communication during the Deduction of Quadratic Formula
During this period, the learning goal was to help students understand the procedure of obtaining the formula. Ms. N went through each step and asked a series of short-answer questions to ensure students’ understanding of the procedures. In going through the process of deducting the formula, Ms. N emphasized these steps: (1) dividing the coefficient of the quadratic term $a$, (2) adding the square of half of $a$ to make a perfect square on the left side, (3) taking a square root on both sides, and (4) simplifying the solution. She especially highlighted the errors and difficulties that students had based on her observations. For example, she highlighted why $b^2-4ac \geq 0$ and $a \neq 0$.

Despite the teacher pointing out that the method of generating the formula was analogical, the instruction was focused on explaining the procedures of solving the equation $ax^2+bx+c = 0$. In addition, the conversation was limited to the teacher’s promoting questions and the students’ brief responses.

After her explanation and clarification of each step, she asked students to get in groups and talk with each other to make sure they understood the solution process. The talking between partners also followed an “explaining” pattern. Then, Ms. N pointed out the result could be used as a formula. She then wrote down the topic of the day: “Quadratic equation formula and the conditions for the formula”, and asked students to recite the formula together for five times based on their understanding of the deduction of the formula.

The second part of the lesson concerned the application of the formula. Students were assigned two problems in order to practice using the formula to solve equations. Two more students were called to the board to demonstrate the processes, and similarly, Ms. N highlighted the main steps and errors students made. The following episode showed a similar pattern of interaction when the teacher led the class to go through the process of solving $2x^2-8x+3=0$ demonstrated by one student on the blackboard.
T: First, what do we need to do?
Ss: Identify a, b, c.
T: In order to identify a, b, c, what form of the equation should it be?
Ss: Standard form.
T: If it is not in standard form?
Ss: Modify it into standard form.
T: Now is it standard form?
Ss: Yes.
T: Can you identify a, b, c? (she pointed one student to answer)
S: a = 2, b = -8, c = 3.
T: What do we calculate next?
Ss: $b^2-4ac$.
T: Why do not we plug into the formula directly?
Ss: $b^2-4ac$ has to be greater than zero.
T: So, we need to calculate $b^2-4ac$ first: $b^2-4ac = (-8)^2 - 4\times2\times3$. Calculate them mentally. What is the answer?
Ss: 40.
T: 40, what does it mean?
Ss: Greater than zero.
T: Write down that it is greater than zero. Can we apply the formula now?
S: Yes.

The episode confirms that the conversation was mainly an explanation of the procedures of using the formula. The teacher promoted a series of questions to elaborate the procedures step by step. The objective of the conversation was to ensure students’ understanding of the steps and be able to use it correctly and efficiently. The class ended with the teachers’ explanation and evaluation of students’ work on another two exercises using the quadratic formula.

The Network of the Communications in the Lesson. Based on the moment-by-moment transcription, conversations between the teacher and students are chosen as nodes. According to Luhmann’s social systems theory, nodes embody a synthesis of the complex of understanding, utterance and information. In particular, the conversation nodes can be indicated by main coordinated actions taken by the participants in the communications. The arrow indicates the movement of conversation which was guided by meaning relationships in the conversation, such as learning goals, mutual understanding of the roles, and social norms/socio-mathematical norms.
The network of communications was generated along with the unfolding of the interaction in the lesson, as shown below (Figure 5.3).

![Communication Network Diagram](image)

**Figure 5.3 Communication Network in Ms. N’s Lesson**

From the network we can see that the conversation in the lesson was clustered around four problems. The first problem was a review of prior knowledge. Students solved the problem and then used the similar method to generate the quadratic formula. After that, students applied the formula to solve two more quadratic equations. The instruction followed the process that students solved the problem and demonstrated their solutions on the blackboard; and the teacher elaborated on key aspects of the procedure. The interaction pattern was the teacher initiated short answer questions and students brief responses from the. The communication structure appears consistent and predictable.

From the above communication network, we can see actions taken by the teacher included assigning the problems, elaborating and evaluating of the solutions. The actions taken by students were primarily the demonstrations of solving process and short answers to the teacher’s questions. The teacher was the person who determined the direction of communication even though students first solved the problems and presented solutions to the class. The roles of the teacher included task assigner, monitor, and evaluator. The conversations were focused on solving procedures as there were no discussions either on the formula of solving quadratic
equations or on different methods, but explanation of the steps in each method, and skills in calculation. The emphasis of accuracy and efficiency guided the movements of the communication.

The conversations in small group followed the same pattern of the whole classroom interactions, in which students checked on understanding and ensured solving steps were correct and efficient. The purpose of the lesson was to understand the formula and to apply it to solve equations efficiently. Thus, the nature of the conversation was primarily contributive. Additionally, Ms. N paid special attentions to motivate students’ participation in learning. The strategies included feedback on students’ work, in particular the praises on students’ progress, one-to-one learning partners, formative evaluation and rewards to those who made progress in learning, and close contact with parents regarding their children’s progress (Interview with Ms. N).

In summary, in Ms. N’s lesson, students were given opportunities to solve the problems; however, there were lack of discussions on dynamic ideas and methods. The lesson focused on understanding the quadratic formula and the procedures of using it to solve equations. Accuracy and efficiency were emphasized in the lesson. The typical interaction pattern involved the teacher’s elaboration on students’ work, difficulties and key points in the solving process.

**Ms. B’s Lesson.** The analysis of Ms. B’s lesson also includes a brief introduction of the teacher, classroom setting and students, and then the analysis of the classroom teaching communication network.

**The Teacher, the Students, and the Setting.** Ms. B is the head-teacher of the class, and is also the director of the school’s academic affairs office. She was recognized as an outstanding teacher. The students in Ms. B’s class were selected based on two criteria from the district:
outstanding academics, and from a lower-level income family. The class was titled “Hongzhi”, which literally means “great ambitions”. As Ms. B pointed out, the local district government and school offered special caring including financial help to the students throughout the whole middle school period. Students were given high expectations. The class had 37 students sitting in rows. There were a few encouraging quotes on the blackboard in front, such as, “Giving-up is worse than failure.”

Phrase 1: The Beginning of the Lesson. The class started with a review on the definition of similarity of two triangles.

T: What does similarity of two triangles mean?
Ss: The lengths of sides are proportional and angles are congruent correspondently.
T: This is how we use mathematical words to describe the relationship of geometrical figures, right? How do we describe the relationship in images?
S: They have the same shape but different sizes.
T: Good. We can use the definition to see if two triangles are similar and also the properties of similar triangles, right? In other words, the definition can be used to determine if two triangles are similar or not. Now, under what conditions two triangles are similar by this definition?
Ss: All angles and sides are correspondently equal.
T: How many conditions do we need in total?
Ss: Six.
T: We need six conditions. Six conditions are a lot, right?

The teacher then led students to review a preliminary postulate of similar triangles they learned in the previous lesson: If DE//BC, ΔABC ∼ ΔAED (Figure 5.4). Ms. B continued to ask students about the theorems they learned to judge the congruence of two triangles such as SSS, SAS, AAS, HL, ASA. Ms. B pointed out there were only three conditions in each congruence theorem. She then introduced that the day’s topic was to explore the theorems about similarities between two triangles in an analogy to the theorems of triangle congruence. In the review, the teacher promoted a series of questions to review the related concepts and theorems in the lesson.
Phrase 2: The Main Part of the Lesson./Exploring the Theorems about Similarities

between Triangles. The teacher asked students to think of a possible situation in which two triangles are possible to be similar. The students seemed had no difficulty to make the conjecture: If two angles are correspondently equal, then the two triangles are similar.

The communication quickly moved to how to prove the conjecture.

T: Here are ΔABC and ΔA’B’C’. What do we have?
S: \( \angle B = \angle B', \angle A = \angle A' \).
T: What do you need to do?
S: Prove \( \Delta ABC \sim \Delta A'B'C' \).
T: This is what you can use (refers to the preliminary postulate). Now, get into your groups to work on it.

Ms. B led students to identify the known and unknown in the problem first, and then she pointed out what they needed do was to bridge the unknown and known using the preliminary theorem.

While students worked in groups, Ms. B circulated around the class checking students’ work, answering questions, and offering comments.

After a few minutes, she seemed concerned about students’ difficulties in solving the problem. She said to the class,

“I want to ask you if you are trying to move the two triangles together first? … What do you do after that? What theorem will you use next? … What is the premise of the preliminary theorem?”

She neither told students how to put the two triangles together nor where to draw the parallel line. Instead, she emphasized why they needed to put the two triangles together and why a parallel
line was needed to solve the problem. After another five minutes, most of students almost finished; and Ms. B called one student to articulate her solution.

S: Let AM = A’B’, M on AB, and then make MN∥BC, let MN intersect with AC at point N (Figure 5.5).

T: (to the student) Slow down. (to the class) Do you feel her approach is different from yours? (pause) She first obtained AM = A’B’ and in order to avoid the lack of parallel relationship in the given conditions, she directly added the parallel lines.

![Figure 5.5 A Student’s Solution to Problem 1 in Ms. B’s Lesson](image)

As the student elaborated her solution, the teacher wrote down the key steps of the solution on the blackboard, and highlighted the key in solving the problem: the parallel line. She then summarized,

T: She is smart because she noticed the characteristics of the preliminary theorem. That is, the small triangle is overlapped with the big one (in the preliminary postulate). In the problem, the two triangles are separated, right? So, what is your first thought?

Ss: Put them together.

T: To put the triangles together means to move one triangle (to be with the other triangle), to construct a congruent relationship. But there are many ways to get a congruent triangle. What else do we need to consider about?

Ss: Parallel relationship.

Through reflective summary on the strategic steps in the solving process, Ms. B structured students’ thinking to the consideration of parallel and congruent relationships.

Following this solution, the teacher asked if there were other methods to solve the problem. Another student offered a different approach in which he constructed a congruent triangle AMN of triangle A’B’C’ by taking a point M on AB so that AM=A’B’, and a point N on
AC so that AN=A’C’, and then he proved MN//BC (Figure 5.5). After his presentation, the
teacher highlighted why constructing a congruent triangle works.

Figure 5.6 Communication during Problem 1 in Ms. B’s Lesson

A third student offered another method, which was to extend the segment of BA and CA to
construct a congruent triangle on the other side of triangle ABC. The teacher noticed it as a
similar method to the previous one. By the end of the presentations of solutions to the first
problem, the teacher asked the students to generalize the conclusion using accurate mathematical
language. The interaction in this period can be illustrated in the diagram above (Figure 5.6).

The emphasis of the conversation was the relationships between prior knowledge and the
new situation, and connections between different ideas; yet, the teacher provided hints and
guided the ways to solve the problem.

In analogy to the congruence theorem SSS, students proposed that if \( \frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'} \), then \( \triangle ABC \sim \triangle A'B'C' \), and then worked on how to prove it. Ms. B circulated in
the room to check students’ work. After a couple minutes, she seemed to notice the common
fractions among the students, so she said to the class,
T: Let me give you a hint. First you definitely need to put the two triangles together. Then you have two things to do: one is to get a congruent relationship, and the other is to get a parallel relationship. The problem now is that if you construct two congruent triangles, it seems hard to get parallel relationship; if you construct a parallel line, it seems hard to get congruent relationship too. I had conversation with Yan (a student in the class). If you construct a congruent triangle, you cannot prove there is a parallel relationship. But you can just go back and choose to construct a parallel line first and prove there is a congruent relationship. Ok, keep working on that.

The statement above indicates Ms. B realized the challenges the students had. Rather than initializing a discussion on those challenges, she guided students to the way that she thought appropriate for solving the problem. When Ms. B saw one student was still trying a different approach, she said,

“You want to construct a congruent triangle first and then to prove a parallel relationship. It is impossible. Switch to another approach.”

She appeared increasingly worried as she saw more students working on the “dead way” as she walked around the room. She said to the whole class,

T: We only have two things to do. One is to get congruent triangles, and the other is to get a parallel relationship. We cannot prove the parallel relationship if we construct congruent triangles first. So, we construct a parallel line and then prove congruent relationship. If you constructed a parallel line but you cannot prove congruent relationships, raise your hand.

Up to this point, the teacher showed a univocal way to communicate with students. Seeing more students were still working on the “dead way”, she even more so pushed the students to solve the problem in her way (constructing a parallel line first). She completely dismissed other possible methods, and paid attention only to the students who worked with her method and helped those students further diagnose the obstacles they had.

T: You first constructed a parallel line, right? Now tell us what obstacles you have.
S: I took a point M on AB to get AM = A’B’ (see Figure 5.5), and then made a parallel line.
T: Ok, you made MN//BC. Good. What makes you feel difficulty?
S: I cannot find conditions (for a congruence relationship).
T: What have you found?
S: AM=A'B'.
T: You found a pair of equal sides. But no more equal sides, right? Do you find any angles are equal?
S: No.
T: No. If you have, you would have gotten a parallel relationship already. Let us look at the parallel line (Figure 5.5). If we have parallel relationship, we have a proportional relationship, right? Let me give you another hint: we can get equal segments from proportional relationships among segments. For example, if \( \frac{\triangle}{\square} = \frac{\square}{\angle} \), then \( \triangle = \angle \). Now see what you may go from there.

The above episode shows how Ms. B asked students to speak out their struggles, and she diagnosed the difficulties and provided hints step by step. She also encouraged social interactions among students.

Although Ms. B emphasized that it was impossible to solve the problem by first constructing a congruent triangle, and she explained there were no theorem introduced in the new curriculum supporting another approach. Some students were still trying different ways.

While the teacher walked around the room, one student showed her work to Ms. B,

S: I constructed two pairs of equal segments first.
T: Is it possible to solve the problem in that way?
S: Yes. Just a little bothersome.
T: (checked the solution) No. I do not think it is possible. I tried this approach before too.
S: (inaudible, seemed to explain her method)
T: No such theorem.

Up to this stage, we can see although the teacher let students experience the challenges, and listened to students’ articulation of the obstacles in solving the problem, when students had approaches that were not compatible with hers, she directed them to her way. The teacher appeared as the authority who judged the feasibility of a solution.

About 10 minutes later, Ms. B asked one student to articulate the whole solving process, whose approach was what Ms. B expected. Ms. B corrected some errors and summarized the strategic steps.
T: Let’s go back to look at the steps. We needed to prove similarity between the two triangles using the preliminary theorem. Two things we need to do: moving triangle A′B′C′ to here (refers triangle AMN). But not only move it. What else do we need to get?

S: Parallel lines.

T: Because parallel relationship is a key, so we first create parallel lines. But what is the purpose of obtaining the parallel lines?

S: To get similarity.

The reflection on the solving process shown in the above episode continued with respect to the strategies in solving the problem. Overall, the reflection was focused on mathematical thinking and relationships of mathematical ideas. The interaction is illustrated as below, which indicates a univocal nature of communication between the teacher and students during this period (Figure 5.7).

Figure 5.7 Communication during Problem 2 in Ms. B’s Lesson

Phrase 3: The Ending of the Lesson. Ms. B asked students to re-write the proving process on their notebook, mark the difficulties they had, and also reflect on what they learned in the lesson. Students summarized what they learned including (1) two theorems that could be used to judge if two triangles are similar; (2) connections between new knowledge and prior knowledge; and (3) congruence as a special case of similarity. After that, Ms. B added on,
Besides those aspects you have mentioned, I also want you to learn how to learn... Think about how to make connections with prior knowledge. What you did very well today is to connect two separate triangles together. There are two purposes: to gain congruent and parallel relationships. Of course, similarity is one of them too. If you get this, the unknown can be transferred into what you have known. Sometimes it is difficult, though. When you encounter difficulties, if one method does not work, try another one. This is the most important thing I want to tell you today.

In the summary, the teacher reviewed the main points of the lesson and highlighted the learning goal of this lesson. It seems the emphasis of the lesson was on ways of thinking more than on specific knowledge and skills.

The Communication Network in the Lesson. The interaction network in the lesson can be illustrated as below (Figure 5.8).

![Communication Network in Ms. B’s Lesson](image)

Figure 5.8 Communication Network in Ms. B’s Lesson
From the interaction network, the actions taken by students included presenting solutions and obstacles and reflecting on and summarizing what they learned. The actions taken by the teacher included providing hints to help students deal with the difficulties, directing “the proper” way to solve the problems, highlighting main points, reflecting and summarizing, and pointing out the learning goals. The different actions taken by both the teacher and students in the conversation imply their different roles in the learning process. Students were active learners but the teacher was the authority of knowledge. For instance, the teacher listened to students’ articulation of their struggles; however, as the authority of knowledge, the teacher’s listening was to diagnose the difficulties and to provide treatments. Students listened to her in a univocal way. In particular, in solving the second problem, the direction of solving the problem was determined by the teacher rather than as a result of negotiation among members in the classroom community.

The power of control from the teacher seemed to limit the possibilities of students’ ways of making sense of the ideas in the communications. There was lack of negotiations about students’ ideas in the lesson. As a result, students were directed to the solution that the teacher expected. Thus, the whole network appears to lack of abilities of self-generating and adapting. Instead, the network revolved around the preference of the teacher’s selection. The nature of the interactions between the teacher and students in this period was mainly univocal and contributive.

Summarizing and reflecting have higher frequencies in the interaction network. Both the teacher and students reflected on methods of dealing with the obstacles in solving the problems, and on key points of the lesson. The higher frequencies of the nodes regarding summarizing and reflecting indicates a central interest in this lesson and imply the social norms promoted in the class.
The whole conversation was clustered around the hubs of conjecture-proving about the similarity between two triangles. The network shows that the sub-hubs were centered on students with focus on reasoning strategies, such as how to make connections between known and unknown, how to identify obstacles, and how to conquer the obstacles. Reasoning strategies, such as learning goals in Ms. B’s lesson, permeated in the network of interaction.

**Mr. W’s Lesson.** The analysis of Ms. W’s lesson also includes a brief introduction of the teacher, classroom setting and students, and then the analysis of classroom teaching communication network.

**The Setting of the Lesson, the Students and the Teacher.** This lesson was held in a big conference room in the school at Site A. The students were 9th graders. A movable blackboard was temporally set in the front of the room. Students sat in rows. There were also sixteen teachers presented who were the participants of the district-level key teacher development project. Other mathematics teachers in this the school also attended in this lesson.

The teacher, Mr. W, has 13 years of teaching experience; he taught gifted and talented students in Beihai High School at Site A, and he is also the associate director of the academic affairs of that school. Mr. W was recognized as an outstanding mathematics teacher in the city and was involved in a city-level key teacher enhancement project. In addition, he was well-known in new curriculum development, K-12 school educational research, and services in teacher education associations. Mr. W was invited to present an exemplary lesson to the teachers in the district-level key teacher development project. After this lesson, he also shared with the attendants his perspectives on mathematics, mathematics teaching and learning, and his teaching experience. It was worth noticing that Mr. W and the students did not know each other before and that no textbook was used in the lesson.
The Introduction: What Does Mathematical Inquiry Mean? Holding a piece of paper in his hand, Mr. W began the lesson with a question about the paper. He said to the class,

T: When I am looking at this piece of paper, there are a lot of questions I want to ask you. My first question is, have you ever paid attention to the length and width of it? What is the ratio between the length and width approximately? What do you think about it?

One student immediately said 3:2. Mr. W continually asked others, “What do you think?” Seeing no more different answers, he posed an argument, “Why not 5:3?” He further prompted students to think a way to test the answers. Some students said they could measure the paper. Mr. W then asked students for a ruler and measured the length and the width of the paper. They determined the ratio is close to 1.5. Mr. W did not stop his questioning.

T: But there is always some error (in measuring). Like what I said before, a little less than 1.5. It was 1.4 something. So what do you think about the ratio is? 1.4 something?

S: Square root of two?

T: What? Square root of two? Is it really square root of two? Who can calculate it for me? But how do we calculate it?

With his questioning, another conjecture of the ratio (the square root of 2) was generated. The lesson was then switched to seek a way to prove the conjecture. Mr. W folded the paper in half and asked the students to observe what he was doing.

T: This is a paper. I fold it in half. Did you discover anything? What did you find?

S: It is folded.

T: Folded. Of course, it is folded (the class was laughing). What else did you discover?

S: Did not change.

T: Did not change? What did not change? I did not hear you.

S: The ratio did not change.

T: The ratio between the length and the width did not change. How do we say this? What is this called in mathematical language? What is it called? Louder.

S: Similar.

Mr. W demonstrated the changes of the width and the length by folding the papers. He highlighted that, “it is not that nothing has changed, but the ratio of width and length did not
change”. He extended students’ knowledge of the standards of the printing paper sizes in general (In China, the length and width dimensions are based on geometrical series of 1, 2, 4, 8, 16, 32 openings, and these quadrilaterals with different sizes are similar). And then he drew a geometrical figure on the blackboard. Students worked with the teacher using the properties of similarities of two quadrilaterals to calculate the ratio.

Based on the network analysis described in Chapter 4, the network of communications between the teacher and the students in this period can be illustrated as below (Figure 5.9).

Figure 5.9 Communication during Mr. W’s Lesson Introduction

The communications between the teacher and students in the period of introduction can be summarized: (1) the teacher initiated the question about the ratio between length and width of the printing paper to invite students to participate in exploration; (2) students proposed their conjecture of 3:2; (3) the teacher provided an argument to stimulate students to make sense of the conjecture; and (4) students and the teacher measured the paper and proposed the second conjecture that the ratio is $\sqrt{2}$; (5) the teacher further led the class to explore the properties of the printing paper regarding its length and width, and to prove the conjecture of $\sqrt{2}$. In the discourse students’ ideas were a “generator of meaning” (Peressini & Knuth, 1998). Rather than posing himself as the authority of the class, Mr. W engaged students in the exploration of the problem to establish a learning community.
Typical questions asked by Mr. W in this period include “what do we do?”, “what do you think?”, and “how do we do this?”, which showed a tone of respect to students’ ideas and invitation for students to join into the conversation. Actions such as folding the paper and measuring imply that experiments are original sources of knowledge. Moreover, Mr. W did not stop at the result of experiment. He challenged students for more accurate results through mathematical proof. Logical reasoning, observation, and measurement were all involved in the inquiry. Students experienced mathematics learning by doing. This process was recursive and involved asking questions, making conjectures, and testing and modifying conjectures. The teaching approach indicated that Mr. W attempted to build “a community of mathematical inquiry” where “students learned to speak and act mathematically by participating in mathematical discussion and solving new or unfamiliar problems” rather than listening to the teacher, or an authorized reference (Goos, 2004, p.259). In other words, the socio-mathematics norm of inquiry guided the movement of conversation in this period.

The Main Part: Exploring the Conditions for Two Quadrilaterals to Be Congruent. This section describes the development of teaching in the main phrase.

Period One: Under What Conditions Are Two Quadrilaterals Congruent? In analogy to the similarities of quadrilaterals, Mr. W started the topic for the lesson, which was to explore the conditions in which two quadrilaterals are congruent. This topic was unusual for students as it had never been discussed in any textbooks. Most of textbooks were only focused on similarity and congruence of triangles. Mr. W said that he wanted students to learn the meaning of mathematical inquiry through exploring untouched topics in the textbooks.

Mr. W initiated the question, “How do you determine if two polygons are congruent?” When students offered a definition: “All the corresponding sides are equal, and also the angles;”
instead of simply evaluating student’s answers, Mr. W challenged students with the following arguments.

T:    Do you think the angles have to be the same? Isn’t equal sides enough? Two quadrilaterals, if all the corresponding sides are equal, then they are congruent to each other. (Some students are asking that, “teacher, you are making this up.”) But why not? I think it’s perfect. If all the sides are equal, the two quadrilaterals are the same. What do you think?

S:    Suppose there is a quadrilateral, whose sides are fixed, but the quadrilateral is movable.

T:    It is movable. Unstable. Can you give an example?

S:    A square and a rhombus.

T:    Give an example, like a square and a rhombus have the same side length. That’s what you meant, right? If a square and a rhombus have the same side length, their corresponding sides are equal, but they would not overlap completely like what he said before. So how can you correct that statement?

This episode illustrates how the teacher made arguments to promote students to make conjecture, test and revise their conjectures. At this point, the teacher intended to help students develop conjectures through exploring the conditions for two quadrilaterals to be congruent. Mr. W expected students to expand the discussion to include various situations within four conditions. For example, are two quadrilaterals congruent with one angle and three sides correspondently equal? Through that, Mr. W wanted to introduce grouping, which is one of the basic methods of mathematical inquiry. Furthermore, Mr. W asked students to revise the conjecture, “If all the sides are equal, then the two quadrilaterals are the same.” He expected that students were able to specify the conditions for the congruence of two quadrilaterals. However, students answered “when the corresponding sides are equal, the corresponding angles also need to be equal to each other.” Students’ answer was out of the teacher’s expectation. Encountering the conflict between his expectation and the student’s response, the Mr. W did not direct students to what he expected (the case of four-condition). Instead, he modified his question to engage students in discussion as shown below.
But if we use it as the way to determine if two quadrilaterals are congruent, would that be a good idea? What do you think? You think it’s good?

Too much work.

What do you mean by “too much work”?

Too many conditions.

There are too many conditions, and we do not need that many conditions. If there are some conditions that are not needed? We said we needed all the sides to be the same and all the angles to be the same, is there an extra condition that is not necessary? Right? Looks like that we do not need that many. So I have a question. Do you know what I am going to ask? What do you think I am going to ask?

Reduce the conditions.

Some of the things we say are obvious. Some of the things we say aren’t obvious. One condition is not possible; two conditions are also not possible. Three, probably not; four, also not possible. Why can’t four be possible?

Mr. W captured students’ utterance of “too much,” and helped students mathematize it as “too many conditions,” and further promoted the exploration of the possible conditions for two congruent quadrilaterals. He guided students to think about different cases of four-conditions. However, students were not able to think about that at that moment. Students thought they had already finished with the example they offered. Seeing no responses, Mr. W modified his question again.

Do you think four (conditions) is enough?

No.

What do we need to do to about this?

Add one more condition.

Once again, students did not realize there were different situations within four-conditions, so they responded that they should “add one more condition.” Mr. W tried to draw back the conversation to the case with four conditions of two quadrilaterals, so he negotiated with students,

No. I meant like, why not four conditions? Because if you say three conditions are not enough, I can understand. I think three is too few, but are four conditions really not enough to determine if two quadrilaterals are congruent? You can just point a direction for us, what should we do now?
S: The properties of quadrilaterals?  
T: I am thinking about four conditions to determine congruence.

Up to this stage, students seemed to go even further away from his expectation of listing all the possible four conditions. Instead of sticking with his original plan, Mr. W switched to a new plan to meet students’ needs. He modified his instructions again.

T: What about this? Let’s put this aside for now. Because everyone is like “teacher, you are so stubborn, We all think four is enough, but you have to say it’s not,” so what about this? Does everybody think that five conditions are enough? Let’s use 5 conditions. What are the 5 conditions?

The network of interactions emerged in this period is shown below (Figure 5.10).

![Diagram showing communication during Period 1 of Main Part in Mr. W’s Lesson](image)

**Figure 5.10 Communication during Period 1 of Main Part in Mr. W’s Lesson**

From the network we can see that the communication was unfolded around making, testing, and modifying conjectures concerning the conditions necessary for two quadrilaterals to be congruent in this period. The teacher initiated the question. Both students and the teacher made conjectures to the solution. Students rejected the teacher’s conjecture using a counter-example. However, there was a gap between students’ understanding and the teacher’s expectation. The teacher expected students to discuss all the possible four conditions in which
two congruent quadrilaterals are not able to be congruent, while students thought that they were
done because they had given one counter-example already. Instead of guiding students to what
he was expecting, the teacher modified his question according to the students’ ongoing responses.
The teacher chose to focus on five conditions since that was what students felt comfortable with
for next discussion. It appeared as though the teacher “surrendered” to students; however, that
indicated the instructive feature of the teacher’s teaching. The teacher was sensible and flexible
to adjust the teaching based on the dynamics of classroom interactions. The adjustment was a
comprehensive consideration about if the students were ready for the direction in which he
wanted them to go. In the later conversations, we see Mr. W made other attempts to go back to
his direction throughout the rest of lesson. In his reflection of the lesson, Mr. W wished he had
more time for the lesson so that students could generate various possibilities by themselves.

Period 2: Under Which Five Conditions Are Two Quadrilaterals Congruent? The
discussion then unfolded around five conditions necessary for two quadrilaterals to be congruent.
The teacher initiated the question, “Under which five conditions would it be possible two
quadrilaterals to be congruent?” One student offered a conjecture: three sides and two angles are
correspondently equal. The teacher drew two geometrical figures on the blackboard and marked
the conditions on the figures according to what the student said.

![Figure 5.11 Student’s Proof of SSSAA in Mr. W’s Lesson](image-url)
If \( AD = A'D' \), \( BC = B'C' \), \( DC = D'C' \), \( \angle D = \angle D' \), \( \angle C = \angle C' \), then \( ABCD \) was congruent to \( A'B'C'D' \) (Figure 5.11).

T: (to the class) She gave us this assumption, who can help? I think you can prove it. Is there anyone who disagrees? Do you agree with her? Do you think her assumption is good? Then can you try it? If you need to prove it, how would you do so?

S: Connect \( AC \), \( A'C' \).

T: Connect \( A \) to \( C \)? Okay, connect those, and \( A', C' \). Right? And then?

S: Using side-angle-side we can conclude that triangle \( ADC \) and triangle \( A'D'C' \) are congruent.

T: So the two triangles (\( ADC \) and \( A'D'C' \)) are congruent. These two are congruent.

S: From the two congruent triangles, we can know that \( \angle ACD \) is equal to \( \angle A'C'D' \). Because \( \angle BCD \) is equal to \( \angle B'C'D' \), after subtracting the same angle from them, the resulting angles are still the same, which are \( \angle ACB \) and \( \angle A'C'B' \).

T: Right. The two angles are equal at first, after subtracting the same thing. It’s like equal things minus equal things, the differences are still equal. So this angle and this angle are equal. Now, this angle equals this angle.

S: Also, because of the congruent triangles, \( AC \) is equal to \( A'C' \).

T: \( AC \) is equal to \( A'C' \), that’s correct.

S: And then using side angle side, triangle \( ACB \) and triangle \( A'C'B' \) are congruent

T: Can you repeat that? Using SAS, the triangles are congruent, what next?

S: And then the quadrilaterals are congruent.

At this stage, the teacher provided opportunities for the students to articulate and to prove their conjecture. The conversation was led by students, and the teacher made sense of their ideas, rather than guiding students’ ideas to match with what was in his mind or evaluate student work.

In addition, the teacher showed his appreciation to student work in a way different from what traditional Chinese teachers may do.

T: Ok, this is pretty good, this proves there can be another theorem, right? If we want to name this theorem, what should we name it? What do you think it should be called? Use her name? Sure, it’s just a name after all. What’s the name?

S: I do not know.

T: What’s your name? You do not know you name? (class laughing)

S: Huang Biyang.

T: Huang Biyang. Right? Okay, this theorem will just be called as Huang Biyang theorem (class laughing).
Up to this point, students proved their conjecture regarding the congruent conditions for two quadrilaterals, which was that if three sides and angles between two quadrilaterals were correspondingly congruent, then the two quadrilaterals were congruent. In the period, students went through a process of doing mathematics: making conjectures, testing and proving them. The teacher invited students to express their ideas and honored students’ efforts. Students’ ideas drove the conversations. The students had the ownership of mathematical knowledge. In addition, the teacher also extended student thinking toward the relationship among sides and angles in five conditions.

T: But how do we use math symbols to express this?
S: SASAS.
T: SA what?
S: SASAS.
T: Okay, why use this one? Do you think it’s a good name?
Ss: Yes.
T: It clearly expressed the conditions we need and their relationship. The three sides and the angles are?
Ss: Between them.

In pointing out the positional relationship between the sides and angles, the teacher attempted to help students understand methods of grouping. After students gave one conjecture of five conditions and proved it, the teacher asked about other conjectures. Students then generated another one and proved it as well. The conversation between the teacher and students generally followed the same pattern in which students explained their conjecture and solution, and the teacher made sense of them. Again the teacher extended student thinking to consider the propositional relationship among angles and sides with five conditions. The conversation about the relationships of angles and sides exemplified another effort that the teacher made to prepare students to categorize different situations for five conditions.

The network below (Figure 5.12) is constructed based on the ongoing conversation in this period. It is clear that students dominated the conversation. Two conjectures were generated and
proved by students independently. The teacher further extended students’ thinking on the relationships among angles and sides within five conditions. The network indicates how the teachers and students did mathematic inquiry: making conjectures, testing and proving them.

Figure 5.12 Communication during Period 2 of Main Part in Mr. W’s Lesson

**Period 3: How Many Cases of Five Conditions Are There?** So far, students made two conjectures about the cases of five conditions and proved them (SASAS and SSAAA). Mr. W wanted students to discuss all the cases of five conditions including situations in which two quadrilaterals were not congruent. To prove a conjecture wrong, students need to construct counter-examples. The method of using counter-examples to prove a proposition is not common in most of school-based geometry courses. Likewise, students at this stage did not recognize providing counter-examples as a method of proving.

**T:** Can anyone think of a counter-example? Can anyone think of five conditions that wouldn’t work?

**S:** If the two sides are not next to each other.
Mr. W expected students to offer a geometrical figure as a counter-example; yet, students naturally switched the verbal to the opposite in the statement SSAAA, “the two sides are not next each other.” Mr. W negotiated his meaning with students in the following way,

T: Yes, continue with this hypothesis. If the two sides are not next to each other, it wouldn’t have worked, right? Then we could not have proved that the two quadrilaterals are congruent. But how do we say it? How do we show that it would not prove the two quadrilaterals are congruent?

Mr. W expected students to use geometrical representations to illustrate the meaning of verbal language “sides not next to each other.” Yet, again students showed different understandings of the meaning of counter-example.

S: If we cannot prove the quadrilaterals congruent, we can prove that we cannot prove quadrilaterals congruent with five conditions.

T: (repeat student’s words) But what if I cannot prove it, but another person came and proved it. What do I do then? Huh? I am worried that would happen.

Instead of injecting his thought, Mr. W used “what if” questions to argue with students, and encouraged students to think critically. Students appeared to not be familiar with negotiation. Mr. W continued to use students’ thought as a device to promote their thinking as shown in the following conversation.

T: But let’s look at what he said, The two corresponding sides are equal but the two sides are- let’s look at his hypothesis- The two corresponding sides are equal; also there are three pairs of congruent angles, right? There are five conditions too. Three pairs of equal corresponding angles and two pairs of equal corresponding sides, but the two sides are not next to each other. They are on the opposite sides, right? What do we do then? What do you think? He said it’s not possible to use those five conditions to prove two quadrilaterals congruent, do you agree?

T: (walking to a student in the back) Do you think what he said is reasonable? Do you agree with what he said?

S: Yes.

T: Yes? Then what do we do? Even if you agree, I might not agree. What do you do? How can you convince me? What if I say ‘I just do not agree”? Some students might say “teacher, if you do not agree, when you cannot prove it, you would have to agree, right?” But even if I cannot prove it, some other genius might be able to prove it, right? Then we are in trouble, right? What do you think, what should we do?
Mr. W walked from the front to the back of the room, and from the left to the right to ask students what to do. He rephrased the question differently, but all questions were centered at the same meaning. He kept “disturbing” students to try different ways to tackle the problem. More important, he tried to establish the legitimacy based on agreements of the class community. Finally, one student came up with an idea of measuring it.

S: Use a ruler.
T: Use a ruler? Find some figures and use a ruler to measure and prove it, right? This sounds good, but we need to have some figures right? Can anyone draw some figures for us that meet these conditions but still are not congruent? Can anyone draw some?
S: A rectangle and a square, with two sides….
T: A rectangle and a square. What do you mean? A rectangle and a square… What about you come up here and draw it for us? Come, come, come. Ok, you draw it for us.

At this moment, students finally came out of a geometrical counter-example. The teacher and the class then made sense of the counter-example together. Mr. W summarized the method of using counter-examples, saying, “If you think it’s correct, you need to be able to give proofs, … and if you think it’s incorrect, if you want to prove it wrong, one counter-example would be enough.” The interaction in this period can be shown as the below (Figure 5.13).

The network indicates there were clusters around three questions, especially negotiations between the teacher and students regarding the meaning of a counter-example. The communication network shows ability to self-adapt as negotiation unfolded between the teacher and students. New understandings emerged from the conversation. For example, the class investigated an emerging conjecture regarding the number pattern of conditions necessary for a polygon to be the congruent. Despite students not being able to finish the proof of their conjecture, those “latent” links promoted their thinking. The teacher showed an ability to adjust his instruction according to “teachable moments.” In fact, if we put the networks of the three periods together, we can see that the teacher kept modifying his questions based on students’
ongoing understanding. Students' understanding also went through a recursive process of making and modifying conjectures. The interaction patterns were reflective and instructive.

The Ending: Grouping Method and Mathematical Inquiry. Up to this stage, students had investigated situations under which two quadrilaterals are congruent with five conditions including four sides and one angle (SSSSA), three sides and two angles (SASAS), two sides and three angles (SSAAA), and one side and four angles (SAAAA). But considering all the possibilities of five conditions appeared complicated to students. Despite the teacher making attempts to help students generate all the categories, students still could not reach the teachers' expectation. Seeing there was not much time left for the lesson, Mr. W gave a short-time talk on how to categorize cases. He said, “To analyze a problem thoroughly, we need to pick a standard first. The standard can be anything, for example, here I used the number of sides, from more sides to less.” Following this, he led the class in a discussion about the relationships among angles and sides in five conditions and figured out what cases they had not studied. Students
then conjectured about three more cases in which two quadrilaterals are not congruent within five conditions. In particular, students successfully generated counter-examples to prove some conjectures wrong, for example, the conjecture “if two quadrilaterals have three sides with an angle between, and the neighbor angle of that angle are corresponding equal, the two quadrilaterals will not be congruent.”

Below is the network of interactions of the ending part of the lesson (Figure 5.14). The interaction pattern between the teacher and students was contributive and in which the teacher generated different cases for the rest situations within five conditions about sides and angles. The students came up with solutions, and the teacher made sense of them. The main focus of the stage was on categorizing method and constructing counter-examples.

![Communication during Mr. W’s Lesson Ending](network-diagram)

Figure 5.14 Communication during Mr. W’s Lesson Ending

**The Network of Mr. W’s Lesson.** Now let’s look at the communication networks in the three periods during the main part together to examine the interaction between the teacher and students in the whole 50-minute lesson.
In the introduction stage, students investigated the ratio of length and width of a piece of printing paper. The teacher initiated questions and arguments while students made, revised, and proved conjectures. Methods of investigation such as observation, guessing, measuring, and proving were used during the whole process of the investigation in the introduction stage (Figure 5.9).

During the main part of the lesson, the teacher led students to investigate the topic of the lesson: Under what conditions are two quadrilaterals congruent? In the first period, interaction patterns still followed that the teacher initiated questions and arguments, and students made conjectures and revised conjectures. In the second period, the class explored cases within five conditions where two quadrilaterals are congruent. Students made conjectures and proved them correct. In the third period, the class discussed some more possible cases of five conditions. Besides making conjectures and proving them correct, students began to investigate cases in which two quadrilaterals are not congruent with five conditions. The meaning of constructing counter-examples to prove conjectures was negotiated between the teacher and students (Figure 5.13).

In the ending stage, the class discussed all possible cases of five conditions of two quadrilaterals. Students made conjectures and used both regular methods and the method of counter-examples to prove conjectures. The teacher highlighted what mathematical inquiry means.

When we look at the network in the main part of the lesson, we can notice that the initiative questions asked by the teacher in each period were derived from the question: “Under what conditions are two quadrilaterals congruent?” The introduction, main part, and ending part are further linked by the question: “what is mathematical inquiry?” (Figure 5.16).
Figure 5.15 Communication Network of Main Part in Mr. W’s Lesson
Figure 5.16 Coherence in Mr. W’s Lesson

The above diagram implies that the lesson had a cohesive topic, which was centered around “what mathematical inquiry is” (ability level) and the question: “Under what are conditions two quadrilaterals congruent?” (knowledge level). The lesson focused on the development of the ability of mathematical inquiry through exploring necessary conditions for two quadrilaterals to be congruent. The purpose of the lesson was more on developing students’ ability of doing mathematics rather than gaining pre-existing knowledge (Of course, this is not to deny the importance of the knowledge of congruent triangles in this lesson). The emphasis of thinking abilities was highlighted by Mr. W below:
First, go home and think about the two counterexamples. Go home and look at the problem and think “what’s with this problem, why talk about this problem so much?” I just wanted to test your thinking ability. I think it’s really good. Your thinking abilities are really good. And the theorems you learned before, you all understood them well. That’s really good. But what I want you to understand is that when we face a problem, when we solve it, we need to think.

These statements indicate that Mr. W’s intention of the lesson was to help students learn how to think mathematically rather than to gain the knowledge written in a textbook.

In addition, if we look closer at the development of students’ mathematical thinking, it was developed through recursive processes during the entire lesson. In each recursive process, the discussions of inquiry methods were similar (e.g., the teacher initiated a question, students made conjectures, revised, and proved the conjectures) but went to a higher level. In the introduction stage, it was a general process of inquiry including making conjectures, revising, and proving. In the main part stage, students learned about the method of using counter-examples to prove a conjecture wrong; and in the ending part, students learned how to categorize cases and further construct relatively complicated counter-examples. The recursive growing communication can be shown in the following diagram (Figure 5.17):

![Figure 5.17 The Development of Understanding of Mathematical Inquiry](image)
In the communication network (Figure 5.15), the actions taken by the teacher involved asking the initial question, generating a series of on-going sub-questions, arguing, conjecturing, summarizing, reflecting, and making comments on students’ solutions. The actions from students also involved conjecturing, revising conjectures, and proving. From the network, we can see clusters of interactions were formed around the teacher’s initiative questions. In other words, teacher’s initiative questions are the hubs in the network. Moreover, for each hub, clusters around students’ discussion were formed. This implies that the teacher played the roles of organizer and facilitator. He initiated questions, offered arguments, made sense of students’ thought, and led dynamic contextual conversation. The teacher orchestrated the whole conversation to build on students’ dynamic thinking; meanwhile, it was centered on the topic of mathematical inquiry. It also reveals that students’ utterances drove the development of conversation. On a closer view, the strongest links were making conjectures, testing and proving conjectures, and using counter-examples, which reflect the social norms – the legitimacy of mathematics is based on the agreement of the community.

As we can see from the communication networks of the lesson, there were some weak links too. One of the weakest links, for example, is about using the probability theory to investigate the numbers of possible conditions for two quadrilaterals to be congruent (Figure 5.15). Even though it appears as one of the weakest links in the network, however, it has opened doors for new understandings.

Overall, understanding mathematical inquiry was the learning goal in Mr. W’s lesson. The classroom norm was focused on the establishment of the legitimacy of mathematical truths. Typical interaction pattern was recursive processes of making conjectures, testing and revising them. The nature of interaction was reflective and instructive as the teacher keeps revising his
teaching based on the new meaning generated from ongoing conversations with students. A variety of mathematical thinking such as analogical thinking, grouping, guessing, measuring, logical reasoning, and giving counter-examples was involved in this lesson. The knowledge of congruence and similarity of triangles was comprehensively applied in this lesson. The following table summarizes the features of the three lessons at Site A (Table 3).

Table 3: Main Features of the Three Lessons at Site A

<table>
<thead>
<tr>
<th></th>
<th>Ms. N</th>
<th>Ms. B</th>
<th>Mr. W</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Topic</strong></td>
<td>Quadratic formula</td>
<td>Theorems about similarity</td>
<td>Congruence of quadrilaterals</td>
</tr>
<tr>
<td><strong>Learning Goals</strong></td>
<td>How to solve equations</td>
<td>How to make connections between the new</td>
<td>How to do mathematical</td>
</tr>
<tr>
<td></td>
<td>effectively using formula?</td>
<td>knowledge and prior knowledge?</td>
<td>inquiry?</td>
</tr>
<tr>
<td><strong>Class Norms</strong></td>
<td>Active participation; mastery</td>
<td>Identification of obstacles; reflections</td>
<td>Conjecture, justification,</td>
</tr>
<tr>
<td></td>
<td>of procedures</td>
<td></td>
<td>negotiation, collective</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>agreement</td>
</tr>
<tr>
<td><strong>Nature of Interaction</strong></td>
<td>Univocal and contributive</td>
<td>Univocal and contributive</td>
<td>Reflective and instructive</td>
</tr>
</tbody>
</table>

**Classroom Organizations**

Ms. N also mentioned there were student learning organizations in her class, which were the basis of her teaching approach. In her class, one advanced student in mathematics was paired with a lower level student. They established a one-on-one learning partner relationship and were evaluated as a team. For instance, when the lower level person solved a problem successfully, he/she got four points, and the advanced one got five points. Each week, those who had highest points were rewarded with learning supplies or certifications on achievement in the class. Besides the one-on-one learning groups, there was also a group consisting of students, called elites, who were at the top in mathematics in the class. Each of them was in charge of a certain
numbers of one-on-one groups. The organization in Ms. N’s class showed a hierarchal structure, which can be illustrated as the following diagram (Figure 5.18):

![Figure 5.18 The Learning Organization in Ms. N’ Class](image)

Likely, Ms. B also organized students into six teams. Each team had a team leader. All class activities or after school activities were executed in groups. In addition, they had different interest groups including groups such as mathematics, literatures, arts and sports. For example, the literature interest group was in charge of the activities that encourage students to learn, such as offering daily quotes on the blackboard. Each week, students in the mathematics interest group worked together to find some mathematical problems to share with other students. When students had problems in learning mathematics in the class, the team leader was the first responsible person to provide help, and the mathematics interest group members were the next responsible persons. The organization of the class can be illustrated as the following diagram (Figure 5.19).

![Figure 5.19 The Learning Organization in Ms. B’ Class](image)
Both Ms. N and Ms. B promoted students’ autonomous learning via those organizations in their class. Ms. N and Ms. B regarded the ideas of developing autonomous learning and teaching for students’ needs as adapted from what they learned in the Key Teacher Professional Development Project. In fact, Ms. K, the university-based teacher educator in the project, commented that the strategy of using one-on-one partnerships to improve autonomous learning was adapted from one of the strategies used in a well-known reform school. The participating teachers in the project had a trip to this school, where they observed and learned that this strategy worked very well in encouraging students to learn from each other. This can be considered as an example of how the project-based teacher learning community impacted the participating teachers’ teaching practices. There are more discussions in the section of key teacher development project at Site A.

Despite the classroom learning organizations playing roles in Ms. N’s and Ms. B’s classes, Mr. W’s lesson showed that classroom learning organizations alone did not play an essential role in classroom teaching. It seems that learning organizations need to work with other dimensions such as learning goals, classroom norms, and interaction patterns.

In the study of comparing different patterns of international teaching, Stigler and Hiebert (1999) pointed out that the reasons behind the different teaching could relate to the teacher-training programs and different cultures. In their study, they focused on teaching as a cultural activity and concluded:

The scripts for teaching in each country appear to rest on a relatively small and tacit set of core beliefs about the nature of the subject, about how students learn, and about the role that a teacher should play in the classroom. These beliefs, often implicit, serve to maintain the stability of cultural system over time (p. 87-88)

Stigler and Hiebert suggested that the understanding of teaching needs the understanding of cultural beliefs of mathematics and mathematics teaching and learning.
Teachers’ Perspectives of Mathematics and Mathematics Teaching and Learning

This section describes three teachers’ perspectives of mathematics and mathematics teaching and learning at Site A, and in particular, Mr. W’s philosophy of mathematics teaching and learning, and teacher education.

Ms. N’s and Ms. B’s Perspectives Ms. N’s practices reflected her view of mathematics and mathematics teaching and learning. Ms. N viewed mathematical knowledge and skills as fundamental. She regarded mathematics learning as a process of construction, so students’ participation in learning is very important for her teaching. In particular, she regarded that a good relationship between the teacher and students is the best way to motivate students to learn mathematics. Ms. N used a Chinese idiom to describe her view of mathematics and mathematics teaching, “love/respect the teacher, follow his/her doctrines.” She explained, “When students feel close to me, they more likely learn my doctrines, and achieve better.” Her view of mathematics as knowledge and skills was especially reflected in her emphasis of the mastery of the formulas and fluency in the procedures of using the formula. Her regard to a good relationship as essential to mathematics teaching and learning was reflected in her strategies used to encourage students’ participation in the learning activities. The strategies that Ms. N used to encourage students’ participation in learning included one-to-one learning groups, frequent evaluation of and rewards to those who made good progress in learning to motivate students’ participation in learning, and sharing with the parents their children’s progress.

Ms. B’s teaching practices were closely related to her view of mathematics and mathematics teaching and learning as well. Ms. B regarded mathematical thinking as the essential part of mathematics. Mathematics learning is not just about mathematical knowledge but more about developing abilities of thinking. She believed that it is often that students learned
school mathematics but do not apply mathematical thinking in real life. Her view of mathematics as methods of thinking was reflected in her lesson. For example, the learning goal of the lesson was to help students learn how to learn (to make connections between the unknown and known knowledge, and to take alternatives when one approach doesn’t work). She believed in the process of learning how to learn as students developed their thinking abilities. In addition, realizing the characteristics of students in her class (coming from lower level families, but having strong desires in learning and having been excellent in academics), Ms. B focused on developing independence of the students. Helping students to understand how to learn was a way that Ms. B used to develop students’ independence. As a head teacher of the special group of students, she was also concerned with the development of their well-being as human beings. She said, “I want them to know how to learn in class and how to live [a life]. I want them to have good attitudes, and to be confident with themselves. We care about them not only their financial needs but also emotional needs.” She further explained “how to learn” was also a way to encourage autonomous learning suggested in the key teacher development program in which she participated.

**Mr. W’s Presentation.** After the lesson, Mr. W gave a 1.5 hour-long talk regarding his views of mathematics, mathematics teaching and learning, and teacher education combined with his reflection on this lesson to the participating teachers in the district development project. Two main points regarding his view of mathematics and mathematics education are described below.

**What Is Mathematics?** Mr. W viewed mathematics as a combination of mathematical knowledge, thinking, and applications. In particular, he emphasized that mathematics is alive, and that the real world is the source for the life of mathematics. He used the metaphor of a fish to describe what mathematics means to him as the following:
I have even thought that if we compare math to a fish, what would be the head of the fish? I think the origin of math would be the fish head. What’s the origin of math? It’s life. The solving process of math, which is the formulas and calculations, is the body of the fish. Of course it’s important. They are the methods and the knowledge. The application of math is the tail. Just because I call it the tail doesn’t mean it’s not important. I think it’s supposed to be a whole. So I think the kind of math we are teaching the students should be a fish that’s complete and alive. There is life in mathematics. The math we show the students need to be alive too. It cannot miss any part or be a cold corpse missing a head or a tail.

In the presentation Mr. W displayed the episodes of his problem-centered teaching experiment to illustrate his view of mathematics as an entity of mathematical thinking, knowledge and skills, and applications of them. For him, mathematical ideas are especially important as they come from and serve for daily life.

Mr. W regarded the motivation of students’ interest and passion to learn as an essential for mathematics teaching and learning. He believed that the interest comes from students’ curiosity and the beauty of mathematics itself. He explained, at the beginning of the lesson, that he could choose to directly tell the students that they would study the congruence of quadrilaterals. However, he thought initiating a question related to students’ life would stimulate students’ interest in learning. In the reflection of the lesson, he said the hardest part of the lesson for him was how to start the lesson. The introduction part was not only to make connections between prior knowledge and new knowledge, but also to evoke students’ interest to learn. Once students had an interest in learning, the development of the lesson could just flow with ongoing talking and thinking, and with guidance of the goals of the lesson. In the analysis of introduction part, we have seen that the introduction not only played the role of connecting new knowledge with prior knowledge, but also introduced the process of mathematical inquiry to students.

**What Is Mathematics Teaching and Learning?** From Mr. W’s view, students should be given opportunities to experience the whole process of exploration, including the happiness of
success and painful moments of struggling throughout the process of seeking solutions. Mr. W stated his view of mathematics teaching and learning in this way,

    I have always thought that studying is the students’ business; do not take what’s theirs away from them. This includes the opportunities to explore and discover, the happiness they get when they succeed and the pain they get if they fail. We need to give the class back to the students.

In the lesson, we see that Mr. W attempted to withdraw himself from scaffolding the communication. For instance, in helping students learn how to use counter-examples to prove a conjecture, rather than providing hints, he created an environment for students’ negotiation in class. Later on, when students made the conjectures about five conditions necessary for the congruence of two quadrilaterals, he let students construct counter-examples independently without his help. Mr. W was good at adjusting the instructional time so that students had more time to struggle with their own ideas. For example, seeing that students had difficulties in constructing counter-examples, Mr. W took a great deal of the class time for students to struggle with that. When students proved their conjectures or construct successful counter-examples, he named them after the students’ name. When students were not able to come up with the counter-examples for two cases because of the limited class time, Mr. W did not offer answers but let students take home the problems to continue working on them.

    Moreover, Mr. W tried various strategies to engage students in learning activities. Mr. W liked to use variations of the same question to engage students into conversations, for instance, he asked “How do you determine if two polygons are congruent? If I let you to think of some theorems for that, how would you do it? What do you think?” These variations of a question had a tone of invitation and expressed his eagerness to students to be the active actors in the stage of the class to cast in their own thoughts, to take the responsibility for solving the problems.
He also tried to make students feel at ease in speaking out by positioning himself as a co-seeker for solutions rather than the one who already knew the answer. For example, he said, 

It’s ok (to be wrong). We need to dare to express ourselves. Doesn’t matter how you say it. What do you think? What do you think we should do to solve this problem? How can we solve this problem? I am worried that I just find one theorem, there may be another, then another one. And I won’t be able to tell if I have found all of them. What do you think?

Those expressions indicate that the teachers valued students’ thinking. Instead of evaluating students’ responses or providing help to the students, Mr. W appeared to be someone who needed help from them. Through those efforts, the teacher engaged students in active participation although there were some disadvantages in this teaching; for instance, the teacher and students were not familiar with each other, and there were more than 20 teachers and school administrators sitting in the room and observing the lesson. Below (Table 4) is a summary of the teachers’ perspective of mathematics, and mathematics instruction.

Table 4: Teacher’s Perspectives of Mathematics and Mathematics teaching and Teaching and Learning at Site A

<table>
<thead>
<tr>
<th>Perspectives of Mathematics</th>
<th>Ms. N</th>
<th>Ms. B</th>
<th>Mr. W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics</td>
<td>Mathematics as knowledge and skills based</td>
<td>Mathematics as ways of thinking</td>
<td>Mathematics as a combination of knowledge, thinking and application</td>
</tr>
<tr>
<td>Mathematics Teaching and Learning</td>
<td>Building good relationship between the teacher and students</td>
<td>Developing abilities to learn</td>
<td>Doing experiments</td>
</tr>
</tbody>
</table>

Looking together at the descriptions of the three teachers’ perspectives of mathematics and mathematics teaching and learning (Table 4), there is a consistency between the teacher’s perspectives and his/her teaching practice. In a closer look at the teaching practice, there were many similarities between Ms. N’s and Ms. B’s approaches. In fact, both Ms. B and Ms. N
claimed their teaching approaches were related to what they learned in the Key Teacher Development Project. In the following section, I examine the district-level key teacher development project in which the teachers were involved at Site A to provide an understanding of their teaching practices from the angle of teacher education.

The Key Teacher Development Project at Site A

Ms. K is a university professor in mathematics education in the Teacher Institution in Beijing, whose job is primarily to help practicing mathematics teachers to improve classroom teaching. She was one of the main implementers of the Key Teacher Professional Development Project. The analysis of the project is based on the interview data with Ms. K, Ms. N, and Ms. B, and the documentations they provided that were related to this project. The similarities and differences between the project-based teacher learning community and the classroom learning communities of the three teachers are also discussed as well.

**Background of the Project.** The project was at the request of the local district educational department at Site A. The district is located in a suburban area of the city where the economy is less developed and the opportunities for innovation are fewer. More specifically, students had lower scores in mathematics on high school entrance tests in comparison with those in other districts. In order to change the situation, the local district educational government requested mathematics education department in the Teacher Education Institute to help foster a team of key secondary mathematics practicing teachers to improve mathematics education in the whole district.

The Teacher Education Institute first initiated an investigation through surveys and classroom observations to identify the main issues in mathematics education in this district. The findings revealed that the teachers appeared to have overall weaknesses in both pedagogy and
mathematics content knowledge and a lack of a broad exposure to new educational perspectives.
The teachers in this district, however, demonstrated openness to new perspectives. The project was thus specifically designed for the improvement of teachers’ understanding of educational theories, reform ideas, and abilities to apply them in their practice.

**Goals of the Project.** Based on the issues of mathematics education in this district, three goals emerged for this project to help the teacher 1) be able to implement reform-based mathematics; 2) be able to conduct systematic analysis about and reflections on their own and other teachers’ teaching practices; and 3) be willing to share their experience and views of mathematics education to colleagues to help the improvement of mathematics education in the district. The goals of the project reflect an integrated perspective of teaching, research, and leadership. In particular, the third goal reflected an expectation for the participants to influence the mathematics education in the whole district. For instance, upon the completion of the project at least, half of the participants were expected to present to their school about their learning experiences and view in implementing reform ideas in classrooms, and to lead research projects in their schools.

**Participants of the Project.** Sixteen teachers were selected who were identified to have the potential to promote the mathematics education in the schools in the district. The admission to the program was based on a screening of teaching practices and school recommendations. The participants were all outstanding teachers recognized by the schools or the district. They demonstrated high level competence in mathematics problem solving, communication, and instructional organization. Moreover, those teachers had strong desires to further improve their teaching. The background of applicants was also taken into account. For example, they had to
have at least five years of teaching experience, with an undergraduate degree in mathematics, and less than 40 years old. The project took three semesters with 400 hours length in total.

**Vision of Mathematics Teaching and Learning Advocated by This Project.** Based on the investigation, the university-based project team first identified that the main problem of mathematics education in the district was teachers’ lack of faith in students’ abilities. Teachers lectured too much without giving students opportunities to develop their own thought. The university-based teacher educators (professors in mathematics education) advocated a vision of teaching and learning as promoting students’ autonomous learning and teaching based on students’ needs in the project. Ms. K explained the vision relating to the curriculum reform: “Students’ needs are understood from three integrated dimensions including appreciation the values of mathematics, mathematical thinking, and knowledge and skills” (Interview with Ms. K). She believed teachers need to give up the full control of classrooms and let students deal with new challenges independently. Ms. K regarded mathematics teaching as recursive attempts, in which teachers continue to reflect and modify their teaching practice.

The following diagram (Figure 5.20) illustrates the vision of mathematics teaching and learning in the project focusing on bridging the gaps between the ideas of reform teaching and teachers’ understanding of them, and the gaps between teachers’ understanding of students’ needs and the reality of students’ mathematical needs.

![Figure 5.20 Bringing the Gaps between Teachers’ Understanding and the Reform Ideas](image)
Learning Activities. In the project, teachers were involved in a series of learning activities to understand the reform ideas and to learn how to integrate the new vision in teaching practices. In stage one (across the first semester), the activities were arranged around understanding of the new vision of teaching – teaching for students’ needs. The activities primarily involved diagnosis of the teachers’ teaching behaviors and studies on theoretical foundations which were conducted by the university professors such as curriculum reform ideas, and mathematics educational ideas. Through classroom observation, analysis of teaching, and reflection, the teachers were motivated to study new teaching perspectives and underlying educational theories, which promoted the changes of classroom teaching.

Diagnosis of teaching (lesson study) was one of the learning activities in which each of the participating teachers designed and conducted a lesson while the university-based teacher educators and all other participating teachers observed the lesson. After that, the teacher reflected on her/his teaching, sharing his/her thought on the designing of the lesson, the issues emerging in implementing the lesson, and understanding of students’ needs. The teacher educators led an analysis and discussion about the issues in the lesson to generate suggestions for improvement. Right after the discussion, the teacher conducted the lesson again with another class. All the participating teachers compared and reflected on the lessons again.

The activities on the basis of the diagnosis of teaching not only provided the teachers opportunities to understand the gaps between their teaching and the reform ideas of teaching, but also offered empirical guidance from the educational authorities and through discussions with other teachers. Ms. K reflected on the teaching diagnosis activity and noted that the participating teachers especially liked this kind of activity because they saw the different effects of different
teaching approaches. According to Ms. K, the pattern of communications in the activities can be illustrated in the following diagram (Figure 5.21).

**Figure 5.21 Diagnosis of Teaching**

In the second stage (across the second semester), activities were focused on developing effective teaching approaches. Teachers’ learning in this period involved conducting research projects on a systematical understanding of middle school mathematical knowledge and understanding of students’ learning, in particular, understanding of students’ difficulties in learning. The participating teachers were placed into two groups and worked on the research projects in algebra or geometry. In the two research projects, the participating teachers investigated 1) the core mathematics knowledge across all the grades of middle school, the connections among the knowledge, and ways to understand them; and 2) students’ understanding of the knowledge, in particular, typical difficulties students had in their learning and reasons for the difficulties. The participating teachers collected numerous cases on the difficulties that their students had, analyzed the reasons behind them, and further built up effective teaching strategies. In the process of conducting their research projects, the teachers also studied educational theories for the underlying research perspectives. Moreover, they conducted classroom teaching observations to learn from others, to obtain more instructional strategies, and to broaden their perspectives. For example, Mr. W was invited to give the participating teachers an exemplary
lesson and also a reflection report on the lesson, his perspectives of mathematics, and his understanding of mathematics teaching. The teachers were also involved in the reflection on and sharing of what they had learned with other teachers and the administrators in the stage two.

In the stage three (across the third semester), the teacher educators continued classroom observations and discussions with the teachers to help the implementation of new teaching perspectives. The focus of this period was teachers’ reflection on learning and sharing with others about their learning in schools in the district. Many of the participating teachers published articles on city or nation-wide educational journals. By this stage, the participating teachers became a growing influential group on mathematics education in the district through research activities and presentations.

The learning process for the teachers was an integration of learning, practice, and research. Doing and reflecting were the norms guided the learning process. The process of learning in the three stages can be illustrated in the following diagram (Figure 5.22):

![Diagram of Three Stages of Learning in the Key Teacher Project](image)

Figure 5.22 Three Stages of Learning in the Key Teacher Project
Communication Structure in the Key Teacher Project. In further examining the communication between the teacher educators and participating teachers in the learning activities, there were community learning approaches underlying the teachers’ professional development. For instance, the lesson study can be summarized as follows: the teachers presented their lesson and reflected on the designing of the lesson and on the understanding of the content and students; then, the teacher educators led the participating teachers to discuss the issues generating in the implementation of the lesson, providing suggestions for improvement.

In the theoretical learning seminars conducted by the university-based teacher educators, the communication pattern was primarily the teacher educators’ lectures and the participating teachers’ reflection. In the activities such as observing exemplary lessons, attending follow-up seminars, and visits to schools that demonstrated different approaches of teaching, the project participating teachers focused on reflecting their own learning.

In the research activities, the participating teachers worked together. By the end of the second stage of the project, the teachers shared what they learned in the project to the educational administrators from the schools and the district. Ms. K reflected on this activity saying it promoted a comprehensive reflection of the participating teachers on what they had learned. In addition, the participating teachers were also involved in various presentations to share their learning experience with other teachers. The interaction pattern between the participating teachers in the key teacher development projects and other educators and administers was mainly sharing-reflecting.

Overall, through the activities carried out in the Key Teacher Professional Development Project, the participating teachers were connected to a variety of educators. The communication network can be illustrated as the below (Figure 5.23). Taken together, the university-based
teacher educators, the participating teachers, and school and local government administrators formed a teacher learning community.

Figure 5.23 Communication Structure in the Key Teacher Project

**A Project-Based Teacher Learning Community.** Considering all the participants, the university-based teacher educators, the participating teachers, the administrators from schools and local educational department, and the outstanding teachers out of the district were connected through the Key Teacher Professional Development Project and they formed a teacher learning community. From the network diagram, as we can see in the community, the participating teachers had the largest cluster of links in the communication network, which indicates that they were in the center of the community.

Schlechty (2009) suggested that collective goals, social norms, and mutual relationship collaboratively determine the adaptation of the community. The goals serve as a directional
guide for the community, which should rise from the bottom-up (Jones & Bozeman, 2009). In this project-based teacher learning community, the learning goal was focused on the improvement of teaching practices as a whole consisting of three sub-goals: to be able to implement reform-based teaching, to be able to conduct research on teaching and learning, and to influence mathematics education in the district. The learning goals were based on the investigation of mathematics education conducted by the university-based teacher educators with the teachers in the district. For each individual participating teacher, the overall goal was that they would be able to form different approaches of teaching based on their own situations and interest. From the interaction network, the learning goals permeated into all the learning activities. The activities such as lesson studies (diagnosis of teaching), studies on reform ideas, seminars, visits to different exemplary classrooms, sharing teaching and learning experiences and perspectives, and research projects ensured the alignment of the work of the community members. The direction of the community moved toward the advancement of mathematics teaching in the district.

Centered at the learning goals, the community members built up norms that valued collaboration and collective advancement. For instance, in diagnosis activity, the teachers gathered together to observe teaching, share opinions, and reflect on the lessons. There was a close correlation between the teachers. They respected each other and put learning and advancement over privacy and individuality. For example, Ms. N and Ms. B are from the same school and often planned lessons together and exchanged their teaching experiences. There was a feeling of belonging, and a sense of obligation arose in the project-based community (Schlechty, 2009). In the learning activities, they looked for a new understanding of teaching and learning, became learners, and worked toward new teaching approaches. The teachers demonstrated a
commitment for the advancement of the group as a whole. They communicated actively with their colleagues about their learning and understanding, and took the role of leaders to transform classroom teaching in their schools and in the district.

The administrators of the schools and local educational department also provided support for the community. For example, they allowed the participating teachers a flexible teaching schedule based on their learning activities in the project. They arranged the seminars for the teachers to communicate with other teachers in the schools and the district, and supported the teachers to conduct research projects in the schools. They also provided financial assistance for the teachers’ traveling to well-known classes out of the city or state to broaden their teaching perspectives.

The teacher learning community advocated autonomous learning, and teaching based on students’ needs. The approaches that helped the teachers build the understanding of the new perspective reflected what it advocated. In the projects, the teacher educators played the role of designers rather than instructors. They arranged various activities to engage the teachers in doing, sharing and reflecting. In these activities, the teachers constructed their own learning. For example, in teaching diagnosis activity (lesson study), one teacher presented his/her daily teaching in a classroom, while the teacher educators and other teachers worked together to diagnose and to provide suggestions for improvement. The teacher then reflected on and made modifications to his/her teaching. All of the teachers took turns to conduct the lesson study and to reflect on their own teaching, while at the same time to participating in other teachers’ teaching analysis. The teachers learned through doing-and-reflecting. As Ms. K indicated, her main job as a teacher educator is to see what help teachers need in their classroom teaching. In this key teacher project, she and the Institute team designed and arranged activities such as
inviting outstanding teachers to give seminars or present exemplary lessons. Through those activities, the teachers were exposed to alternative teaching approaches and were expected to form their own effective teaching approaches.

One of the salient features in this teacher learning community was the teachers, university-based teacher educators, teacher colleagues across schools, the district and the city, and administrators in local educational department were all closely connected together. For example, Mr. W, as a city-level key teacher who was invited to share his view and experience with other teachers regarding the reform ideas of teaching and learning. He gave his email address with password for logging onto an account where teachers could email him, shared the teaching materials there, and participated in the conversations among a group of teachers who worked hard to improve mathematics education in the city. Mr. W appeared as a “hub” in the network. In fact, the participating teachers in this project were all master teachers and expected to be “hubs” in the network of improving mathematics education in the city. Using master teachers to spread reform ideas and experience is a popular method in Chinese classroom reform.

The learning goals, commitments, and social norms permeated through the network of interactions in the project-based teacher learning community. All of the aspects worked together and constituted the context for the changes of classroom teaching.

The Similarities and Differences across the Classroom Learning and the Teacher Learning Communities

Autonomous learning and teaching on students’ needs were advocated in the Key Teacher Professional Development Project. Ms. K interpreted that students’ needs, in relation to the reform ideas, were an integration of three dimensions: knowledge and skills, processes and methods, and values and affects.
Both Ms. N and Ms. B took an approach in which students worked on problems individually or with peers first and then shared solutions with the class. The purpose was to give time and space for students to struggle with the problems and develop autonomous learning. The autonomous learning approach taken by Ms. N and Ms. B involved the steps: (1) students working on the problems independently; (2) studying the textbook when they had obstacles to solve the problems; (3) talking with partner(s) if they still had difficulties; and (4) talking with expert group members if the students still could not solve the problems.

Ms. N reflected on her lesson saying, “I used to deduct the formula, but I felt it was not effective. Now I let students try it first. When they encounter difficulties, I help them. Thus, they can think more about the problems.” Ms. N’s class was a regular class in the school, and she believed especially she needed to develop her students’ abilities of autonomous learning. When students worked on the problems, Ms. N studied students’ needs through observation and talking with students. Ms. N believed it was better to offer more detailed or direct help to her students because their learning abilities were relatively lower in her class.

Ms. B believed that students would benefit from autonomous learning as it helps students connect with real life situations. Ms. B also believed that the essential aspects of autonomous learning include students’ ability to identify the obstacles and the teacher is to the students find methods to overcome the obstacles. In her teaching, she paid special attention to studying students’ needs. She studied students’ needs before a lesson, during a lesson and after a lesson. Ms. B suggested a strategy for offering students hints step by step. She called it as “3-2-1”. She explained that during the first three minutes of students’ thinking about the problem, the teacher observed students, communicated with them to understand their difficulties, and offered first a hint to promote their work but then allowing students to continue to think about the problem;
after another two minutes of students’ thinking, the teacher offered a second hint according to students’ ongoing needs. Ms. B’s strategy indicated her understanding of the dynamics of students’ needs. In addition, Ms. B emphasized the role of reflection on learning. Both Ms. N and Ms. B studied student’s needs and offered help based on their understanding of students’ needs. Yet, their strategies in offering either direct treatments or providing hints were based on the diagnosis of the teachers rather than on collective agreement through negotiation, which implies that the teachers were the authorities of knowledge.

If we look at the teaching approach in the project-based teacher learning community, Ms. K emphasized a recursive process of doing and reflecting to understand students’ needs. In diagnosis teaching activity, Ms. K asked the participating teachers to present lessons and diagnosed the problems of their teaching with respect to students’ needs in classrooms. The teachers reflected on their teaching and on students’ learning and then modified their teaching strategies. In both Ms. N’s and Ms. B’s classrooms and in the project-based teacher learning community, learners experienced challenges and demonstrated their process of learning. Diagnosis and reflection were common in both the classroom and the teacher learning communities. Thus, it is evident that Ms. N’s and Ms. B’s teaching were influenced by the project learning. Their teaching approaches demonstrated similarities to the approaches employed in the project-based teacher learning community.

Yet, there were some differences regarding the teaching approaches between classrooms and the key teacher development community as well. Although the diagnosis method was used to identify teachers’ needs in the professional development activities, the teachers had choices to modify their teaching in their own ways. The university-based teacher educators did not make specific decision for the teachers on how to implement a lesson. As one of the goals of the
project, the teachers had to create their own teaching approach. In fact, in order to help teachers form their own teaching approaches, Ms. K exposed the participating teachers to a variety of different teaching approaches. She invited outstanding educators to give seminars to the teachers and took the teacher to visit different classrooms where their teaching approaches were well-recognized. Ms. K attempted to help the teachers with “intraperspective exploration” and “interperspective exploration” (Richardson, Gilliers & Lissack, 2007). In the “intraperspective exploration” such as teaching diagnosis activity, the critical examination of differences was encouraged; while in “interperspective exploration,” any possible approaches were encouraged to try. From the consideration of the both aspects, Ms. K hoped the teachers would come to their own teaching styles. Thus, rather than Ms. K’s approach as the only resource, any approaches the teacher learned could be resource of his/her decision of teaching approaches. This was the main difference between Ms. N’s and Ms. B’s teaching approaches in classrooms and Ms. K approaches in the teacher learning community.

Mr. W, a city-level key teacher, was invited to present an exemplary lesson and a seminar on his teaching perspectives. He demonstrated an understanding of students’ needs from another perspective. He especially related students’ needs to the changes of the society. He explained that mathematics education needs to foster students’ abilities of autonomous learning and creativities because, as Mr. W puts it, “we take the scores on test too seriously. We forgot to help the students to develop fully in every area. We do not really help the students develop their autonomous learning abilities and creativities.” Mr. W meant that students have the power to decide which to learn. In Mr. W’s lesson, his main goal was to let students know how to perform mathematical inquiry—a recursive process of making, testing, and modifying conjectures. In this process, he attempted to build mathematics legitimacy in the class. Mr. W delivered a clear
message that mathematical truth came from doing experiments and through agreement of the community rather than any authority. The topic and learning task Mr. W chose for the lesson were innovative to the students. Mr. W expected students to create knowledge rather than just to understand and to take the existing knowledge.

In all, this chapter described the communication structures and meaning relationships in classroom teaching and the associated teacher professional development project. Additionally, those teachers’ beliefs and experiences were investigated as well. The understanding of classroom teaching transformation in this site was gained from the following dimensions across teachers, classrooms and teacher professional community: (1) classroom community and project-based key teacher development community, (2) regular and exemplary lessons, and (3) outstanding school teachers, city-level key teachers, and university-based teacher educators.
CHAPTER 6: DATA ANALYSIS AT SITE B

Chapter 5 indicates how Chinese mathematics teachers implemented the classroom reform at Site A with support from university-based mathematics educators in general. In this Chapter, the researcher focuses on a teaching experiment in one school intensively. The researcher observed 13 continuous lessons across 7th and 8th grades with three mathematics teachers over one week. The 13 lessons included 12 normal lessons, and one model lesson which was presented to the middle school principals in this district.

Similar to the analysis at Site A, the researcher analyzes the communication networks of classroom teaching at Site B and reveals the general patterns of communications and social meaning relationships embedded in the 13 lessons in the teaching experiment. The researcher also examines the associated teacher professional development community with focuses on the structures of communications and meaning relationships, and discusses the similarity and difference of them between classrooms and teacher learning communities.

Additionally, perceptions of the teachers, teacher educators, administrators, and students regarding this experiment are described to further understand the impact of the teaching experiment on teaching and learning. Furthermore, two teachers’ beliefs of mathematics, and mathematics teaching and learning, and their teaching experiences are examined to further understand their classroom teaching. Students’ perceptions of mathematics, and mathematics teaching and learning are examined as well to enrich the understanding of classroom teaching and learning.

The School-wide Teaching Experiment

The school is a rural school located in a suburban area of a city in southwest of China. The main industry of this area is agriculture and it is known for growing various fruits especially
peaches. Many young parents went to the advanced cities in east of China to make more money and left their children with elderly grandparents in the villages. About 20% students in this study were left-behind children staying with their grandparents. The school ranked in the last place in the local district before implementing the new teaching experiment in March 2007. As the principal H indicated that the school changed greatly since the enactment of the teaching experiment: More and more students liked to continue their study in this school; The rank of the school in academic testing climbed up to the fourth in this district; Students also won the first place in the city-level PE contest in 2009. “It is not easy for our students to compete with those in the city”, said principal H, “This indicates our school is changing. In fact, the changes are not only in academics but also in their mentality”. According to the principal, the school ranked in top three in the district school overall evaluation in 2009.

The teaching experiment is an educational research project, funded by the city education department. The teaching experiment is called “DJP” teaching model: D (导), guided self-learning; J (讲), sharing solutions; and P (评), evaluation of learning and reflection. In the DJP teaching model, students first learned the content knowledge independently with guide of the study plan (designed by the teachers in local schools), then talked with classmates and the teacher to further develop understanding. The local teacher education department led implementation of the new model of teaching. According to Mr. F, a teacher educator from the local educational department, the purposes of this model of teaching were to encourage students’ autonomy in learning and to provide opportunities for them to communicate and to share understanding. The ultimate goal was to help students learn how to learn including how to reason, communicate, inquire and evaluate their learning (Wang, Wang, & Tan, 2009). The DJP model of teaching was originated in teaching mathematics subject and was gradually spreading out to
other subjects teaching such as English, physics etc. This study is focused on mathematics classroom teaching.

**Ms. Y’s Lesson.** The analysis of Ms. Y’s lesson includes a brief introduction of the teacher, classroom setting and students, and the analysis of classroom teaching communication network.

**The Teacher, Students and Class Setting.** Ms. Y has a bachelor degree in mathematics. She taught mathematics in middle schools more than five years. She was involved in the research project of the DJP teaching model; and in particular, she was in charge of one of the sub-projects. She was also the head teacher of the class described before. There were 53 students in the class. Among them, more than 20 students were left-behind with their grandparents.

Different from traditional Chinese classroom settings, there were six blackboards on the wall around Ms. Y’s class. Two bigger ones were on the front and the back walls. Four small portable blackboards were on left and right side walls. This lesson was a model lesson presented to the middle school principals in the district to study the teaching experiment. The coming visitors sat in the back and aisles of the classroom. This lesson was a typical lesson in Ms. Y’s teaching except the classroom appeared more crowded that it did usually.

**The Beginning of the Lesson.** The lesson started with small group working on the problems on the study plan (a local school-based curriculum complementary to the textbook, more information is available in the discussion of the DJP teaching model). The teacher Ms. Y said to the class, “Give your questions to the group to discuss”. The students in the front row turned back and talked with their group members in the back. The class then became noisy due to the discussion on the problems from the study plan. Students worked on them individually at home before the lesson. The teacher assigned group presentations of those problems and other
tasks on the blackboard to seven groups including: Review exercise, Problem 1, Problem 2, Problem 3, Problem 4, Problem 5, Problem 6, Retrospection, and Evaluation. It is worth to notice that two groups were particularly assigned to do “retrospection” and “evaluation”, which indicated students were expected to monitor their own learning processes each other.

**Working on the Problems in Groups.** Students in groups worked on the given tasks and prepared for sharing their solutions with the class. Ms. Y explicated her expectation of small group discussion, “Discuss how to solve the problem, and reach a consensus to the solution. Then pick up someone from your group to talk about the solution to the class. Others pay attention to their use of mathematical language.” Ms. Y seemed to concern about the accurate use of mathematical language.

About five minutes later, students began to move around in the classroom. Some students worked at blackboard drawing graphs and writing down their solutions; some still sat talking about how to solve the problems. Ms. Y circulated in the classroom to provide help. After another 15 minutes, Ms. Y said to the class, “Now, go back to your seat.” Seeing some students were still around the class and talking, she said, “Do not forget when I am speaking or your classmates are presenting, you should stop talking and to listen to them. If you did not get enough time finish all the steps of solving the problem, you can finish them while you present the solution. You do not need to write every step, though. Just be sure to explain your thought clearly”. Those indicated Ms. Y’s expectations of students and their presentations.

**Group Presentations and the Whole Class Interactions.** Ms. Y then highlighted the topic of the lesson, “Today we will apply the properties of similar triangles to solve problems. First, let’s see the review exercise presented by the 8th group.” The review exercise was to ask students
to write proportional expressions of the length of sides in the given similar triangles (Figures 6.1-6.4).

(1). \( \triangle ADE \sim \triangle ABC \), AD and AB are corresponding sides.

![Figure 6.1. A–shaped Similar Triangles](image1) ![Figure 6.2. X–shaped Similar Triangles](image2)

(2). \( \triangle ABC \sim \triangle AED \), \( \angle B = \angle AED \)

![Figure 6.3. Skewed A–shaped](image3) ![Figure 6.4 Skewed X–shaped](image4)

The study plan listed the above figures and categorized them into the two types of similar triangle figures, namely, A-shaped (Figure 6.1 & 6.3) and X-shaped (Figure 6.2 & 6.4). Four students from 8th group participated in the presentation. Each played a different role in the presentation. The first two students articulated the group solutions to the two questions in the exercise,

**S1:** (pointed to Figure 6.1) This is an “A”–shaped figure, so the corresponding sides are proportional, that is, \( \frac{AE}{AC} = \frac{AD}{AB} = \frac{DE}{BC} \). If we rotate \( \triangle ADE \) around point A for 180 degree, we get “X”–shaped figure (Figure 6.2), and we have \( \frac{EA}{AC} = \frac{DA}{AB} = \frac{DE}{BC} \).

**S2:** In the skewed cases of A- and X–shaped figures 6.3 and 6.4, we know \( \triangle ABC \sim \triangle AED \), \( \angle ADE \) is congruent with \( \angle ACB \)

The teacher offered students some technique help for them to explain their thought clearly. The third student of the group went to the board and added another case of similarity of triangles, which was not listed on the study plan.
S3: We want to add a special case of similarity for a right triangle with an altitude from the vertex of the right angle. For example, in Figure 6.5, in Rt $\triangle ABC$, AD is perpendicular with BC, we can get $\triangle ABD \sim \triangle ABC$, $\triangle ADC \sim \triangle ABD$, and also $\triangle ADC$ is similar to the big $\triangle ABC$.

![Figure 6.5 Similar Triangles in a Right Triangle](image)

T: If I were you, I would use I, II, III to mark the three triangles. Thus, it appears more clear and simple (e.g. $\triangle ABD$ as I; $\triangle ADC$ as II; $\triangle ABC$ as III). How many pairs of similar triangles in this figure (Figure 6.5) ?

S4: Three pairs.

Another member in the group pointed out some small errors that his teammates made, such as the vertexes of the triangles were not in corresponding orders. At the end of their presentation, the team concluded that the exercise was to a review of the basic figures of similar triangles, and the properties of similar triangles. The teacher also highted,

This exercise is a review of three typical figures of similar triangles: A-shaped, X-shaped and similar triangles embeded in a right triangle with an altitude. We have reviewed the properties of similar triangles. Now, we are going to use these typical figures and the properties of similar triangles to solve the problems.

In short, the exercise was not only to review the proportions of similar triangles but also to highlight typical figures of similar triangles. The following diagram (Figure 6.6) illustrates the conversation during the mathematical exercise in the class. The actions students took primarily included explaining their solutions, and monitoring and reflecting on learning processes. The teacher was a monitor to make sure students’ explanations were clear and effective.
Following the review exercise, the class applied the properties of similar triangles and the typical figures of similar triangles identified in the review exercise to solve other five given problems (Problems 2-6) from the study plan. These problems were marked as examples on the study plan, which involved calculating length of sides, and proving congruent of angles in geometrical graphs.

Problem 2: Given $DE//BC$, $D$ is the midpoint of $AB$, $DC$ and $BE$ intersect at $G$. Find (1) ratio of $DE$ and $BC$; (2) ratio of the perimeter of $\triangle GDE$ and $\triangle GBC$ (Figure 6.7).

Similar to the first presentation, one student presented his group’s solution on the board.

S: I am going to talk about the solution to example 1 (Figure 6.7). Because given $DE//BC$, and this is a right “A” type of triangle similarity. So, $\triangle ADE$ is similar to $\triangle ABC$, so $AD/AB = AE/AC$. Because $D$ is the midpoint of $AB$, so $AD/AB = AE/AC = \frac{1}{2}$. And because $AD/AB=DE/BC$, so $DE/BC = \frac{1}{2}$…

Figure 6.7 Problem 2 in Y’s Lesson
In solving this problem, the student first identified the A-shaped figure containing similar triangles \( \triangle ADE \) and \( \triangle ABC \). He then applied the properties of similar triangles and the theorem regarding the midpoint of a segment to solve the first question. He asked his group to read the theorem about the property of similar triangles to support his presentation. In the presentation of the solution to the second question, he proved that \( \triangle DEG \) is similar to \( \triangle BCG \) via two pairs of congruent angles generated by two parallel lines DE and BC.

After his presentation, the teacher asked comments from his group and the class. One student pointed out there is an X-shaped figure between parallel lines DE and BC, so \( \triangle DEG \) is similar to \( \triangle BCG \) directly. The teacher added,

T: From DE//BC, we can get \( \triangle DEG \) is similar to \( \triangle BGC \). Do we need to prove the angles are equal (in order to prove \( \triangle DEG \) is similar to \( \triangle BGC \))?  
Ss: No.

Meanwhile, the teacher pointed out that the explanation to the first question given by the first student was not concise enough. She rephrased it in the following way:

T: Because DE//BC, \( \triangle ADE \) is similar to \( \triangle ABC \), so \( \frac{AD}{AB} = \frac{DE}{BC} \). Because D is the midpoint of AB, so \( \frac{AD}{AB} = \frac{1}{2} \). And because \( \frac{AD}{AB} = \frac{DE}{BC} \), so \( \frac{DE}{BC} = \frac{1}{2} \).

Ms. Y further explained why there was redundant language in the student’s explanation:

T: Parallel, similar; and similar, and proportions. So, \( \frac{DE}{BC} = \frac{AD}{AB} \). Do we need to consider AE/AC?  
S: No.  
T: AE/AC has nothing with the question, so we do not need it. Now because D is the midpoint, \( \frac{AD}{AB} = \frac{1}{2} \), so \( \frac{DE}{BC} = \frac{1}{2} \). What I want to emphasize here you do not need to write those down that are not needed for the solution.

The movement of interactions during Problem 2 can be illustrated in the following diagram (Figure 6.8). During this period of time, the teacher assigned the problem, and students in group took the responsibility to solve and to articulate their solution to class, and to reflect on their
solving-process. The typical figures were used as a way to make prove efficiency. The Teacher’s expectation of conciseness, and efficiency drove the conversation.

Figure 6.8 Communication during Problem 2 in Y’s Lesson

The third problem was another application of the properties of similar triangles and the typical geometric figures.

Problem 3: Given D, E, F as midpoints of each side of equilateral triangle \( \triangle ABC \), respectively. If the length of each side of \( \triangle ABC \) is \( a \), what is the area of \( \triangle DEF \) and \( \triangle ABC \) respectively.

Figure 6.9 Problem 3 in Y’s Lesson

Students used Pythagorean theorem to calculate the length of AE first, and then calculate the area of \( \triangle ABC \). After that, one student noted that in \( \triangle ABE \) the length of AE can be obtained through the relationship \( BE:AE=1:2: \sqrt{3} \), among the three sides.
S: Because $\triangle ABC$ is an equilateral triangle, $\angle C$ is 60 degree, $\angle EAC$ is 30 degree. So, we can get $AE = \frac{\sqrt{3}}{2}a$.

Ms. Y summarized, “He takes out $\triangle ABE$, and uses what we learned about the relationship of the three sides: $BE: AB: AE = 1: 2: \frac{\sqrt{3}}{2}$ to get $AE = \frac{\sqrt{3}}{2}a$”. She commented, “When we have a bicycle, why do we want to walk”, a common Chinese saying that means to take the easiest route. The identifying of the typical figures was served as a way to make proof efficient.

The forth problem was to prove the congruence of angles using the properties of similar triangles.

Problem 4: Given $AB / AD = BC / DE = AC / AE$. Prove $\angle BAD = \angle CAE$ (see Figure 6.10).

After one student presented his group’s solution, another student from the “Retrospection” group summarized the key points in solving the problem:

S: I am going to pick up the main points in solving this problem.
T: Finally someone is doing this [reflection] for us.
S: This example is to prove the congruence of angles via similar triangles. We cannot prove the two angles $\angle BAD = \angle CAE$ are congruent directly because they are not in two similar triangles. But from $AB / AD = BC / DE = AC / AE$, we can get $\angle BAC = \angle DAE$. Thus, to prove $\angle BAD = \angle CAE$, we can prove $\angle BAC = \angle DAE$.
T: Very nice. I want to say a little more. We used to get congruent angles from similarity of triangles directly. Now, we do not have similar triangles from the given conditions, and the angles we need (to prove congruence) are not in two similar triangles, either. But we have three pairs of sides which are proportional; we can check if they are in two triangles. So, in this situation, we need to switch from the relationship of proportion of sides to the relationship of congruence of angles.
The above episode indicates the responsibility for “Retrospection” group was to summarize key points and reasoning strategies in solving the problems provided by each group. The teacher emphasized the reasoning strategy in solving the problem: using the properties of similar triangles to transfer the relationship of sides in the given conditions to the relationships of angles. The episode implies the teacher’s expectation to students’ reflection on their learning, and the emphasis on logical reasoning. After that, one student offered the comments on the solution given by the group. She said,

S: In this problem, we did not use the segment of CE, so we can ignore it. Thus, we easily focus on the two triangles $\triangle BAC$ and $\triangle DAE$.

T: [erases the segment of EC] Very good. When we encounter complex figures, we need to learn to how to extract typical figures.

The student’s suggestion of erasing segment CE can be viewed as an indication that how socio-mathematical norms of efficiency in the class regulated students’ mathematical thinking. The teacher further emphasized that complicated figures can be reduced into simpler figures. The movement of the communication during the fourth problem can be illustrated in Figure 6.11.

Figure 6.11 Communication during Problem 4
The Problem 5 was to calculate the length of a square: A piece of wood is represented by right $\triangle$ ABC, with $AB = 1.5$ m, $BC=2$ m, and area $1.5$ m$^2$. To make a square table top from the piece of wood, what is the length of the side of the square BFED (Figure 6.12)?

![Figure 6.12 Problem 5 in Ms. Y’s lesson](image)

One student group presented their solution using the properties of similar triangles. After that, a student offered an unexpected solution. This student did not apply the properties of similar triangles that were used predominately in this lesson; instead, her solution was based on the combination of areas. At the beginning, no one in the class, including the teacher, understood her solution:

S: Because it (quadrilateral BFED) is a square, so the quadrilateral DEAB is a trapezoid. Its area plus the area of $\triangle$ ADE together is $1.5$ m$^2$.

T: I did not get you. [to the class] Did you all understand her? [to the student] Can you repeat it again?

S: Because $\triangle$ ADE is a right triangle. Its area is $\frac{1}{2}x(2-x)$. And the trapezoid [referring to DEAB] is a right trapezoid. The area is half of the sum of the bases, times the altitude, which is $x \cdot (x + 1.5) / 2$.

She then wrote down $x \cdot (x +1.5) / 2 + \frac{1}{2}x \cdot (2-x) = 1.5$ on the board.

T: [to the class] Do you accept this method?

Ss: No. It is troublesome.

T: Do you think this approach is correct? Is it practical?

Ss: Yes.

Some students thought the method was correct but the equation was too hard to solve; some realized the equation was not difficult to solve. A small turbulence appeared in the class as
different opinions and confusion were emerged. Students talked each other, and Ms. Y simplified the equation as: \( \frac{1}{2} x^2 + \frac{3}{4} x + x - \frac{1}{2} x^2 = 1.5 \) on the board. Students were surprised to see that the second-degree terms were canceled. The method brought fresh air to class as most of students had never thought it could be solved using other mathematical ideas besides the properties of similarity, which was what had been emphasized in the lesson. Students looked excited about this solution. The teacher gave special praise to the student and her group.

T: This method is very subtle. I’ll add one extra point to each of you in the group. Chung [the student] will be given 2 extra points, and the group will get one extra point. Let’s give a round of applause to them.

After that, one student summarized the key points of solving Problem 5, and another student pointed out some mistakes in the presentation too.

The interaction during the problem Five can be shown in Figure 6.13. The interaction during this period of time exhibited flexibility as an unexpected solution appeared and the class made sense of it. The class finally accepted the solution based on mathematical evidence.

![Figure 6.13 Communication during Problem 5 in Ms. Y’s Lesson](image)

In the last problem (Problem 6), the interaction followed the same pattern. Of interest was that one student posed a completely incorrect solution after the first student presenter gave a correct solution. This was the only time that a student gave a totally wrong solution during the
whole period of whole-class interaction. However, there was no discussion about the wrong solution at all; instead, it was replaced immediately by another solution. The whole presentation and problem-solving explanations were done in haste before the bell rang. The interaction chart during Problem 6 is shown in Figure 6.14.

![Interaction Chart]

Figure 6.14 Communication during Problem 6 in Mr. Y’s lesson

Discussion of Ms. Y’s Lessons. The whole-class interactions between the teacher and students are shown in Figure 6.15. In the network, the actions from the teacher mainly included: (1) offering techniques to better articulate the solving process, (2) summarizing and highlighting key points and strategies, (3) comparing different approaches, and (4) correcting errors (except for the last one). The actions from students primarily included: (1) articulating group’s solutions, (2) offering different approaches, (3) summarizing and reflecting on key points of solving the problems, (4) reciting of key mathematical theorems, and (5) correcting errors. The whole-class communication was centered at students’ presentations.
The clusters in the communication network indicate that learning activities were unfolding around students’ presentations of solutions to the problems. Students took the responsibilities of solving the problems, and explaining their solving process accurately, and efficiently, and reflecting on the key points of mathematical knowledge and skills. The teacher played the roles of organizer, monitor, and evaluator.

The dynamics of the whole-class interaction was primarily indicated by students’ different solutions, especially when there were unexpected thoughts or solutions, the class appeared generated new understanding. For example, in solving Problem 5 when the student presented an unexpected method, Ms. Y and students listened to the student and made sense of the method. The teacher and others students generated new understanding out of this conversation. However, errors were treated more as weakness rather than a learning opportunity.
in the lesson. The teacher and students showed less interest in studying why certain methods did not work and how wrong answers came from.

All the problems were centered at the applications of properties of similar triangles and development of the skill to identify typical figures (as reviewed in the beginning of the lesson) to make the proof more efficient. These tasks and instructional activities were consistent with the main goal of the lesson. The utilization of typical figures of similar triangles and avoiding of “redundant language” or “figures” can be viewed a way to simplify the process. Additionally, expectations of group presentations and small group discussions, and reflections on the core knowledge and skills were typical social norms that regulated the conversations in the lesson.

The main interaction pattern was students presenting different solutions and summarizing the solving process and strategies. There was little argumentation or debate about the solutions and strategies. The nature of the whole-class interaction was contributive where students demonstrated their understanding of predetermined knowledge and skills (Peressini & Knuth, 1998). The main features of the lesson are summarized below (Table 5).

Table 5: Main Features of Ms. Y’s Lesson

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Learning Goals</th>
<th>Social norms</th>
<th>Interaction Patterns</th>
<th>Nature of Interaction</th>
<th>Instructional Organizations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms. Y’s Lesson</td>
<td>Applying knowledge and skills to solve problems</td>
<td>Reflecting on solving process; emphasizing on efficiency; and communicating mathematics accurately.</td>
<td>The teacher assigning the problems; students sharing of different solutions and reflecting on key knowledge and skills</td>
<td>Contributive</td>
<td>Combination of individual work; small group discussion and presentations</td>
</tr>
</tbody>
</table>
Overall, the features described in this lesson were consistent across all the five observed lessons in Ms. Y’s class. In some cases, Ms. Y also extended students’ understanding by asking “what if” questions. For example, in one lesson students were given two sides of an isosceles triangle with lengths 3 and 5, and asked to find the perimeter of the triangle. After students presented their solutions, Ms. Y asked, “[What if] the lengths of two sides are 3 and 6 of an isosceles triangle? Are there still two situations?” Different opinions and meaningful discussions were arisen. In general, the most popular communication was contributive in which students had opportunities to contribute their solutions to the class; however, the focus of communication was on gaining predetermined knowledge rather than generating new knowledge.

**Ms. R’s Lesson.** The analysis of Ms. R’s lesson also includes a brief introduction of the teacher, classroom setting and students, and then the analysis of the classroom teaching communication network.

**The Teacher, the Students and the Classroom Setting.** Ms. R has a bachelor’s degree in mathematics and has taught mathematics in middle schools for more than ten years. She participated in the seminar on the Revision of National Curriculum Standards in 2008, and was a district-level key teacher training project in 2009. She was involved in school-level research project entitled *Autonomous Learning: Promoting the Comprehensive Development of Students*, and a district-level research project entitled *Inquiry-based Mathematical Learning*. She was recognized as district-level key teacher. Meanwhile, she is also the director of academic affairs at her school. Ms. R was the head teacher of the class described in this study.

There were 59 students in her class; however, more than 20 of her students were living with their grandparents. Ms. R said some of her students did not even want to go home after school because they felt lonely at home. Those students told her that they felt no common
language with their grandparents. Before the school implemented the new teaching model, many of students also showed no interest in learning. They often skipped classes to play games at a local internet bar nearby the school.

Similar to Ms. Y’s class, six blackboards were hung on the classroom’s walls. Two big blackboards were on the front and back walls, and four other small movable blackboards were hung on the left and right walls of the classroom.

**The Beginning of the Lesson.** The topic of this lesson was about the angles formed by a traversal across two lines. Ms. R asked students to take out the study plan, read the objectives of the lesson on the lesson plan. The teacher then began with a review of the definitions and relationships of the angles formed by a traversal across two lines. The review was based on an exercise from the study plan asking students to identify different types of paired-angles with a traversal across two lines. She called on students for answers, and gave explanations why the correct answers were indeed correct. After reviewing key points regarding the relationships of angles formed by a traversal across two lines, she asked students to work on other four problems from the study plan. Ms. R said, “First work on them alone, then talk with your group members; after that, you will demonstrate to us your solutions”.

**Working on the Problems Individually.** While the students worked on the problems individually, Ms. R circulated around the room and reminded students to use the theorems they had just reviewed to prove the problems. Ms. R told students that if they got stuck they could talk to each other. After about five minutes, Ms. R said to the students, “Now work in your groups. You will need to explain why you can solve the problem in that way.” This indicates Ms. R’s expectation to her students in small group discussion.
Working on the Problems in Small Groups. Students who sat in front row turn back to those in the back, where every six of them formed a group. There were about nine groups in the classroom. Students began to talk about the solutions and helped to make sense of them each other. Ms. R circulated in the room, asking students to think about alternatives if they already found one solution. When most of groups found at least one solution to the questions, she began to assign groups to present their solutions to the problems respectively. Ms. R said to students “Now prepare to present your solutions to the class”. Students then focused on practicing presentation skills in groups, e.g. how to express mathematical terms more accurately. After about five minutes, students began to share their solutions to the class.

Group Presentations and the Whole-Class Interactions. The first problem was an open-ended question shown below. One boy went to the blackboard to present a solution as below.

Problem 1: Given AB//CD as in the following figure. What is the relationship among \( \angle B \), \( \angle C \), \( \angle E \) in each situation (Figure 6.16 and 6.17)? Prove your conjecture.

\[
\begin{align*}
\text{Figure 6.16 Problem 1 in Ms. R’s Lesson} \\
\end{align*}
\]

\[
\begin{align*}
\text{Figure 6.17 Problem 2 (A Variation of Problem 1)} \\
\end{align*}
\]

S: To prove the relationship of \( \angle B \), \( \angle C \), \( \angle E \)...

R: First give your conjecture.

S: The conjecture is \( \angle C = \angle B + \angle E \).
R: Write it down.
S: [Writing \( \angle C = \angle B + \angle E \) on the board, marking the interception point of AB, CE as O] Since AB // DC, so \( \angle AOC + \angle C = 180^\circ \); and the sum of \( \angle E, \angle B, \angle EOB \) is \( 180^\circ \). The sum of interior angles of triangle BOE is \( 180^\circ \). Because \( \angle EOB \) and \( \angle AOC \) are vertical angles, they are congruent. So \( \angle C = \angle B + \angle E \).

R: According to?
S: Substitution.
R: OK. What he said here is AB // DC, so \( \angle AOC + \angle C = 180^\circ \). Because \( \angle AOC = \angle EOB \), and the sum of \( \angle E, \angle B, \angle EOB \) is \( 180^\circ \), so \( \angle C = \angle B + \angle E \). Anything comments? Or different approaches?

As the student explained his solving process, Ms. R questioned him, and to make sure his reasoning was supported. After the student’s presentation, Ms. R summarized the approach and asked for comments from the class. Some students said they had different solutions; Ms. R invited one student to the board to explain his way of solving the problem:

S: Because AB // DC, the corresponding angles are congruent, so \( \angle C = \angle AOE \). \( \angle AOE = \angle B + \angle E \), and the measure of an exterior angle equals to the sum of the measure of its two remote interior angles, so \( \angle C = \angle B + \angle E \).
R: By substitution, right? Good. He gave us another approach. Any others?

Ms. R highlighted the main mathematical idea in the second method, which was the measurement of an exterior angle equals to the sum of the measure of its two remote interior angles. Some students also suggested using alternate interior angles as well. Ms. R explained this third method. She reminded students about some techniques for presenting their ideas to the class, such as facing to the audience when they presented their solutions.

In short, as indicated by the above episodes, in the presentations students shared their different solutions and explained the solution to the class. Ms. R paid attention to logical thinking of students’ solutions, offered help with their presentation skills, and compared these different approaches in terms of their effectiveness. It seems that everyone understood the solutions and there was no discussion of the solutions and negotiations of the methods.

After that, the class moved to Problem 2, which was a variation of the Problem 1 (see Figure 6.17). A girl went to the board to explain her group’s solution to class.
S: Given AB // CD, it asks us what relationship exists among $\angle C$, $\angle B$, $\angle E$. It is hard to prove it directly using this diagram; we need to add an auxiliary line first.

R: Tell us what the relationship is. Still $\angle C = \angle B + \angle E$?

S: $\angle E + \angle C = \angle B$, the same as before.

R: [To the class] The same relationship?

Ss: No.

Ss: Yes.

R: Some agree and some do not. [To the student at the board] Write down your conclusion first. [To the class] Listen to what she says.

The girl wrote down $\angle E + \angle C = \angle ABE$. And she also added an auxiliary line (see Figure 6.18).

![Figure 6.18 A Student’s Solution to Problem 2](image)

R: She said $\angle E + \angle C = \angle ABE$. Let’s see if she can prove that.

S: Because it is given that AB // CD. Extending AB to F, so AF // CD. We can get corresponding angles $\angle C = \angle 1$. And then, $\angle E + \angle 1 = \angle EBA$, because $\angle EBA$ is an exterior angle of the two angles $\angle E$ and $\angle 1$. Use $\angle 1$ to substitute $\angle C$, we get $\angle E + \angle C = \angle EBA$.

R: Is it correct?

Ss: Yes.

R: Any question?

Ss: No.

R: Did some of you have questions before?

Ss: It [the conclusion] is different from the previous one.

Ss: There are other ways.

R: Ok. Did she explain well about the solution?

Ss: Yes.

R: Let’s give her a round of applause.

In this episode, the student made a mistake by saying the conclusion was the same as the one in the previous problem; although she actually meant a different conjecture. The class was confused at the beginning. The teacher could have stepped in and made a clarification. Instead, she asked the student to write the conjecture down and asked the class to listen to her explanation. The “confusion” stimulated students’ interest at what the presenter was going to say, and the student corrected the mistake when she explained her solution later. In this process, Ms. R emphasized
the social norms of listening and sense-making. There was still little negotiation or discussion about the solutions.

After her presentation, other groups were eager to offer different approaches to solve the problem. Ms. R called them to the board. Three more solutions were shared in the class (see Figures 6.19).

![Figure 6.19 More Solutions to Problem 2](image)

After students presented those different methods, Ms. R summarized and compared them in terms of efficiency, e.g. the first solution was the most efficient one. The class then moved to the next problem, which was another variation of the Problem 1:

Problem3: Given AB//CD, prove \( \angle B + \angle E + \angle C = 360^0 \) (Figure 6.20).

![Figure 6.20 Problem 3 in Ms. R’s lesson](image)
After one student presented the solution (Figure 6.21), the teacher and students corrected some errors the student made in the proving process. Then, the teacher led the class to recite the proving process in an accurate and concise way together:

T & Ss: From E, make EF // AB (see Figure 51). Because AB // FE, ∠B + ∠1 = 180° (because given two parallel lines, the interior angles on the same side of the transversal are supplementary).

T & Ss: And because AB // DC, AB // EF, so, EF // DC (because if a line parallel to one of two parallel lines, it also parallel another lines). ∠2 + ∠C = 180° (because if two parallel lines, the interior angles on the same side of the transversal are supplementary).

T & Ss: So, ∠B + ∠1 + ∠2 + ∠C = 360°.

T & Ss: So, ∠B + ∠BEC + ∠C = 360°.

Figure 6.21 Solution 1 to Problem 3

The repeating of the solving process was a typical strategy used to help students get familiar with the mathematical language and the proving process. After that, some students indicated they had different approaches to the problem. Ms. R let them present their different solutions as well. One boy did his presentation too quickly to be understood by the class. Ms. R helped him mark some angles on the graph and asked him to re-present the solution until it was understood by the class (see Figure 6.22). She then summarized the key points of his method.

Figure 6.22 Solution 2 to Problem 3
After his presentation, when Ms. R asked for more approaches, one student offered a third solution.

S: Make BF // EC and intersect with DC at F. ∠ABF and ∠BFC are a pair of alternate interior angles, so they are equal. There is a qualiteral BFCE, and the sum of the measures of all interior angles are $360^0$.

![Figure 6.23 Solution 3 to Problem 3](image-url)

His solution was simple and unexpected to the class as he used partition, not by triangles but qualiterials (see Figure 6.23). The students and the teacher became visibly excited. They were surprised at how simple and elegant the solution was. After the two problems, the class continued to the group presentation of Problem 4. Two students from two different groups articulated their solutions to the problem. The teacher summarized key points in their solutions and pointed out there were other ways to solve the problem as well. The class ended with an applause for the students’ presentations. The group presentations last about 30 minutes. In reflecting on the lesson, Ms. R said she did not expect that students would have so many different approaches to solving these problems. Some of the solutions she had never thought of or seen before. She was satisfied with what the students did in the class.

**Discussion of Ms. R’s Lessons.** Ms. R’s classroom demonstrated many similar features to Ms. Y’s classroom in terms of the organization of instructional activities, interaction patterns, nature of interactions, and class norms (both social and socio-mathematical norms). The interaction network of the whole-class discussion in this lesson can be illustrated in Figure 6.24:
Similar to Ms. Y’s lesson, the communication network of the whole class interaction, is a decentralized structure. The lesson started with a review of key definitions and theorems of angles formed by a traversal cutting across two lines, and was unfolded around four problems that required the application of the knowledge.

In the network of the whole-class discussion, the actions from the teacher mainly were assigning problems, making sense of students’ solutions, helping students articulate their solutions, and summarizing key points of solving the problems. The actions from students were primarily articulating their solutions to the class. The interaction pattern was students shared their solutions and the teacher helped students elaborate on their thinking and make sense of different solutions. The dynamics of communication network was primarily reflected by different solutions; however, still there were less argumentation or discussions about different approaches. The teacher especially encouraged presentations of multiple approaches, and she also encouraged students to listen to others. As a result, students came up with a variety of ways to solve the
problems by making different auxiliary lines. Table 6 is a summary of different solutions that students generated in this lesson.

Table 6: Different Methods in Solving Problems 1 and Its Variations

<table>
<thead>
<tr>
<th>Problems</th>
<th>Conjectures</th>
<th>Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1 Given AB/CD as in the following figure. Provide a conjecture about the relationship among ∠B, ∠C, ∠E? Prove your conjecture.</td>
<td>∠C = ∠B + ∠E</td>
<td>Using interior angles on the same side of the transversal</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Using corresponding angles</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Using alternate interior angles to prove the conjecture</td>
</tr>
<tr>
<td>Problem 2 (a variation of the problem 1)</td>
<td>∠B = ∠E + ∠C</td>
<td></td>
</tr>
<tr>
<td>Problem 3 (another variation of the Problem 1)</td>
<td>∠B + ∠E + ∠C = 360°</td>
<td></td>
</tr>
</tbody>
</table>

![Diagram for Problem 1](image1)

![Diagram for Problem 2](image2)

![Diagram for Problem 3](image3)
When Ms. R reflected on her lesson, she said the main learning goals were (1) to apply knowledge of the angles formed by a traversal cutting across two parallel lines and (2) to develop skills to solve problems. Ms. R especially stressed on adding additional lines, which was also a difficulty part of the topic. The learning goals were embedded in all the tasks given to the students. Those tasks demanded procedures and connections with mathematical ideas. The interaction patterns reflected the learning goals. For instance, in group presentations, the teacher encouraged students to share different ways of adding auxiliary lines and demonstrate various ways to apply the definitions and theorems. Some of the solutions went beyond what the teacher had expected. The sharing of different solutions to solve the problems indicates that students were able to apply those mathematical theorems involving a traversal cutting across parallel lines, and were skillful in adding additional lines to solve problems. It seemed, however, that there was lack of discussion about how they found those different solutions. In other words, the lesson still was focused more on products instead of processes.

In this lesson, the first two problems appeared as open-ended questions, which asked students to first make conjectures first. The conjectures, however, seemed so obvious that students had no any difficulties to make them. The interactions were mainly focused on proof rather than testing and revising conjectures. Despite that, it can be viewed as an attempt of going beyond the given conclusions to embrace making conjectures and testing conjectures about conclusions. Yet, in general, the purpose of interaction was to help students gain the preset knowledge and develop competence to use them to solve mathematical problems, thus the nature of interaction in this lesson is mainly — like Ms. Y’s lesson — contributive communication.

Ms. R also paid attentions to mathematical communication skills. When students presented their solution methods, she asked students to first articulate their goals in the problems,
and offer the mathematical theorems or definitions underlying their proofs. She led the class to recite the proving process orally together so that students would become familiar with the mathematical language, and become proficient in using them. The main features of Ms. R’s lesson are illustrated in Table 7.

Table 7: Main Features of Ms. R’s Lesson

<table>
<thead>
<tr>
<th>Learning goals</th>
<th>Social norms</th>
<th>Interaction patterns</th>
<th>Nature of interaction</th>
<th>Instructional Organizations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms. R</td>
<td>Applying knowledge and skills to solve problems</td>
<td>Listening, making sense of others’ ideas, and reach consensus; communicating mathematical ideas proficiently</td>
<td>Students’ sharing of different solutions and class makes sense of other’s solutions</td>
<td>Contributive</td>
</tr>
</tbody>
</table>

Overall, the classroom teaching practices in all the four lessons of Ms. R shared similar features as listed in the above table. Moreover, Ms. R showed tolerance to errors. When the student made mistakes, she left some time to for the students to correct them. She asked others to listen to the student and to make sense of the solution. For instance, in the lesson regarding the definitions of mathematical propositions, Ms. R gave students a mathematical statement: “The end points of a side of a triangle have the same distance to the median of this side” and asked them to rephrase it into an “if…then” question. It seemed easy; however, students struggled a lot to do it correctly. Ms. R encouraged group discussions. In addition, she did not stop at one answer, but continued to ask for all possible answers to explore students’ thought. When different answers were given one by one, she did not evaluate each answer immediately but highlighted key points for each solution, and asked the class if they were satisfied with the answers. After a variety of responses, she realized the difficulties were originating from the lack of understanding of the proposition itself. She then asked students to use geometrical figures and
symbols to represent the proposition in multiple ways. Still, Ms. R led students to articulate their thoughts and to modify them rather than through a discussion that engaging dialogical conversation among students in most of time.

Ms. Y’s and Ms. R’ teaching demonstrated many similarities, which is to be expected as their teaching is under the same DJP model of teaching. The following section describes the common features of the DJP classrooms by looking at all 13 lessons together.

The Features of the DJP Model of Teaching

The features of the DJP model of teaching are primarily inters of classroom communications, instructional organizations, and social and cultural norms advocated in the classrooms.

**Classroom Communications.** In the DJP teaching model, classroom communications often include both whole classroom and small group communications.

**Whole-class Communications.** From the above description of Ms. R’s and Ms. Y’s lesson, the primary pattern of whole-class communications in the DJP model of teaching was: (1) students presented their group’s solution, (2) explained their solution, and (3) the whole class reflected on the main points. Different solutions were provided and compared as well. The teachers (1) offered help for students’ elaboration of their solution, (2) highlighted the main points, (3) gave comment to their presentation, and (4) evaluated their presentation. Specifically, Both Ms. Y and Ms. R emphasized simplicity and efficiency of solutions, and reflection on key knowledge, and development of skills to communicate mathematically. In Ms. R’s class, she allowed errors and tended to give students time to ponder with the errors. In Ms. Y’s class, it seems that students tended to demonstrate error-free solutions. If there was a wrong solution, it
was often replaced by a correct solution or the teacher’s examination of the error, instead of a discussion of the errors.

In the whole 13 lessons observed in this school in 7th and 8th grades, the teaching followed similar pattern: students worked in small groups and presented their solutions to the class. In 7th grade, the teacher Mr. C tended to use more univocal conversations, such as asking students to repeat the correct solving process after him. The 8th graders seemed showing stronger capability to solve the problems and to articulate their solution methods than 7th graders. Overall, in all the classes, errors were treated as something to be overcome rather than as a start-point for discussion. Despite Ms. R provided time for students to rethink the mistakes they made, but there were lack of discussion or negotiation about the errors among students.

The Small-group Communications. The primary interaction patterns in small-group discussion were univocal and contributive. Group members shared their solutions, exchanged ideas and made sense of groups’ solutions. The students with advanced-level of mathematics learning abilities often came up with a solution and told the rest of the members about the solution. The advanced-level students played a role to get the answer and to explain the solving process to the lower-level students in their group. The small group discussion was primarily to make sure every member in the group understood the solution(s). The following episodes are examples of communication between advanced-level students and lower-level students, and between two advanced-level students.

Episode 1: Student (Qian) was the advanced-level student in the group, and Yong was a lower-level achiever. The problem was:

Given a square ABCD, with CE = MN and \( \angle MCE = 25^0 \), what is the measure of \( \angle ANM \) (see Figure 6.25 (1))?
In the conversation between Qian and Yong, Qian told Yong how to add an auxiliary line and to finish solving the problem, step by step. Yong followed Qian’s ideas to make sense of the solution. It was primarily an one-way univocal transmission of the solution from Qian to Yong.

When students were closer in ability-level, their conversations tended to be a dialogic type of communication. Here is an example of that between two students:

Given a right \( \triangle ABC \), \( \angle ACB = 90 \), \( CD \perp AB \) at \( D \), \( AC = 4 \), and \( BC = 3 \), find the following measures: \( CD \), \( AD \), and \( \frac{CD}{BC} = ? \) (see Figure 6.25 (2)).

Ming: Look, we can use \( AC^2 = AB \cdot AD \) (called Project Theorem). We know \( AB \), right? We can calculate \( AB = 5 \), then get \( AD \).
Rui: We need to find \( CD \) first. We should use the Pythagorean Theorem to calculate \( CD \).
Ming: It is the same.
Rui: Using the Pythagorean Theorem, it is 2.4, 12 / 5 [Rui tries to use the answer he got].
Ming: What?
Rui: \( CD \) is in \( \triangle CBD \). In \( \triangle CBD \) and \( \triangle CBA \), \( \frac{CD}{AB} = \ldots \ldots \)
Ming: [interrupting] Use the Project theorem. \( CD^2 = AD \cdot BD \). You know \( AD \) and \( BD \)?
Rui: We do not know \( BD \), but we know \( AB \) in the bigger triangle, \( \frac{CD}{AB} = \frac{CB}{AC} \) [writing on a piece of paper], \( \triangle ACD \) and \( \triangle CBA \).

In this conversation, Ming and Rui brought in different approaches to solve the problem. Rather than one telling the other how to solve the problem, they shared their ideas with each other to make sense of the problem. Their final solution was not the result of only one person’s ideas, but a synergistic combination of multiple people’s ideas. In general, it seems that univocal
conversation primarily occurred when higher-level achievement students told their personal solution method to their lower-level achievement group members, while contributive conversations were more likely happen among students shared similar ability-levels in mathematics problem solving.

The univocal communication seemed to dominate the 7th grade class, in which one advanced student often told the rest of members his or her way of solving the problem. In 8th grade classes, more students in a group were engaged in sharing their different thought. Overall, small group interactions were primarily to 1) get a solution to the given problem, 2) allow advanced students to help lower level students understand the solution. Similar to the whole classroom interaction, there was less negotiation among students, but more explanation of solutions from the advanced students to lower-level students.

Taken together, in the DJP model of teaching, students showed high motivation to participate in the learning activities, and they took responsibility for learning in their groups in all the observed classes. The nature of whole-class interactions was mainly contributive communication focusing on sharing of knowledge and skills rather than expanding of students’ thinking across all the classes. There was a sense of seeking “correct” answer and “efficient” solving process. There were fewer cases in which students’ utterances were used as “thinking devices” (Peressini & Knuth, 1998) for the generation of new ideas. In most of students’ group discussions, presentations, and whole-class discussions, they did not explain where the strategies to solve the problems came from, but focused on how to use them to solve the problems. These problems mostly were context-free, and the pathways to the solutions to some of the problems were suggested by the study plan. There was fewer cases where students were challenged to make conjectures, to test, and modify their conjectures recursively.
**Classroom Instructional Organizations.** In the DJP model of teaching, the class was typically organized in small groups with about six or seven students in each group. Each group had a leader, who was selected by the teacher based on their overall ranking (by computing their average of grades in tests) in the class. The leader then selected members to form groups, and the teacher offered some adjustments in genders, interests, and learning abilities, etc. Small groups were the basic units for all class activities including academics such as discussions, and non-academic activities such as after-school classroom cleaning (as students were responsible for cleaning the classroom each day). The small groups worked together across all subject matters.

The teachers and the teacher educators explained the underlying assumption of the grouping, which was that all students in the group were contributors to the growth of the group in different areas. For example, students who were good in mathematics helped those who had difficulties in mathematics learning; at the same time, questions and struggles from them help the others in the group to think more critically. In addition, those who had difficulties in mathematics learning sometimes might excel in other subjects, and so helped other group members to learn these subjects. For example, those who were talented in sports, arts, or drama might contribute to the group in school sports games, class activities, and events. Small groups thus gave each student responsibility to help each other in order to advance the group as a whole.

The instructional activities were organized in the sequences of individual study, small-group discussion and presentation, and whole-class discussion. In individual study, students read the textbook and figured out solutions to the problems on the study plan. Individual study was done mostly at home as homework in preparation for the next day’s class. In class, there were about five minutes for individual students to prepare the questions and solutions for small group discussion. Then students got into small groups to discuss the questions brought by individuals.
The small group discussion took about 15 minutes. After that, small groups presented their solutions to the given problems and the whole class discussed their presentations. The structure of instructional activities is shown in Figure 6.26. In particular, there was a reinforcement and enhancement of learning in the combination of individuals study, small-groups discuss and the whole-class discussion over the presented solutions.

![Figure 6.26 Instructional Organization in the DJP Teaching](image)

**The Reinforcement and Enhancement among Individual Study, Small Group Work, and Whole-class Interaction.** The DJP model of teaching showed the potential to expose students’ learning through the reinforcement and enhancement in individual study, small group discussion and whole-class discussion.

In individual study, students first read the textbook and worked on the problems from the study plan. During this period of time, students made the first attempt to learn the mathematical content by themselves. In small-group discussion, students brought their personal solutions, questions, and comments to small groups where students had the opportunity to explain, to make
sense of others’ thinking, to justify their own and others’ thinking, and to reach a group consensus.

In whole-class discussion, each small group as a unit presented their solutions to the whole class and demonstrated their understanding. When students articulated their thinking process, and reflected on the knowledge and skills they used to solve the problems to the class, students got another chance to re-examine the problems with the whole class. There were possibilities for them to extend or get new understanding in the whole-class discussion. In other words, small-group discussion could be a resource for whole-class discussion and meanwhile whole-class discussion enhanced and reinforced small group discussion. The following episode is an example of how small-group discussion and whole-class discussion complemented each other to reinforce and enhance students’ understanding. The episode was observed in one of the 8th grade classes. During small-group discussion, Ming and Rui shared their different approaches (using the properties of similar triangles and the project theorem) to figure out the length of CD and AD when given AC and BC (see Figure 6.25(2)).

While in whole-class discussion, one group showed another totally different approach drawn on the area of a right triangle:

Jian: It is known that \( \triangle ABC \) is a right triangle, and we already knew \( AC = 4 \), \( BC = 3 \), so \( AB = 5 \). Because \( \triangle ABC \) is a right triangle, \( (AC \cdot BC) / 2 = (AB \cdot CD) / 2 = \) the area of the triangle.

After the group finished their presentation, Rui raised his hand eagerly and shared his group’s solution with the class.

Rui: From the previous figure we reviewed, we know that the three triangles are similar to each other [marking \( \triangle BCD \) as I, \( \triangle ACD \) as II and \( \triangle ABC \) as II]. Because I is similar to III, so we get \( \frac{BC}{AB} = \frac{CD}{AC} \) and we get \( CD = 2.4 \). For the second question, we can use the Project theorem \( AC^2 = AD \cdot AB \). AB = 5, and \( AC = 4, 16 = 5 \cdot AD \), so we solved \( AD = 16 / 5 \).
This episode illustrated reinforcement and enhancement between small-group discussion and the whole-class discussion. Both Rui’s and Jian’s solution represented the solutions generated from small groups as entities. Through sharing different approaches with the whole class, learning was reinforced and enhanced each other.

**Classroom Social and Cultural Norms.** Throughout the analysis of the 13 consecutive classroom lessons, the following social and socio-mathematical norms were emphasized in the DJP model of teaching.

**Sharing Solutions.** In either small-group discussion or whole-class presentations, students were obligated to explain their reasoning, and to reflect on key mathematical knowledge and strategies used in solving problems. Knowledge was constructed as something that one could learn directly from the teacher or classmates. Mathematics learning appeared as a *sharing of logical thinking process*. In the survey, students indicated that teacher’s teaching became students’ teaching in the DJP teaching. Many students indicated they had more chances to think, and to learn different solutions from classmates. Especially, they could ask group members to explain to them until they understood.

**Communication Skills.** The ability and skills to express mathematical ideas and explain thinking processes were emphasized in all the observed classes. Mathematical language, geometrical figures and symbols were often combined together to articulate the meaning of mathematical concepts and theorems from different angles. Students were also required to remember mathematical concepts and theorems accurately. In class, it was common for students to recite mathematical concepts or theorems together. Moreover, some communication skills and techniques were emphasized, for instance, using special marks to help articulation be simpler. The teachers often asked students to first identify the goals that they needed to reach, and then
addressed the deductive reasoning process in their presentations. If one student did not explain well his or her solving processes, the teacher would ask the student to reorganize his or her thinking and them. The teacher also led the class to restate the solution in a well-organized and succinct way. One of the evaluation criteria regarding group presentations was related to the clarification of their presentations. In the survey, students, especially the students in advanced-level (self-evaluated) of mathematical learning indicated that the DJP model of teaching helped them to develop their communications skills; for instance, they learned how to organize their thoughts in speaking, and to explain their ideas clearly to the others.

**Appreciation of Different Approaches.** It was common for students to offer more than one solutions to a problem in the DJP teaching model. When one group offered a different solution, the group was rewarded points. In addition, if it was a novel solution, the group would be rewarded with extra points. A typical example was the presentation of the solutions to the last problem in Ms. Y’s class, in which one student provided an unexpected solution. After the presentation, the teacher said, “This method is very subtle. I’ll add one extra point for each of you in the group. Chung [the presenter] will get two extra points.” The on-going criteria of evaluation promoted students to think different ways to solve problems.

**Socio-mathematical Norms of Simplicity and Efficiency.** In addition to alternative solutions, students often compared different approaches in terms of their simplicity and efficiency after students presented their different solutions. For example, one of the strategies of helping students solve mathematical problems efficiently was to use typical figures. In Ms. Y’s exemplary lesson, the identification and application of typical similar triangle figures such as A-shaped and X-shaped similar triangles reflected the norm of efficiency. The comments by the
teacher, “When we have bicycle, why do want to walk?” in particular indicated the teacher valued simplicity and efficiency during problem-solving processes.

**Collaboration.** In the DJP classrooms, students in groups collaborated and took the responsibility of learning together. For example, the advanced students often helped lower-level students in solving problems. In the self-evaluated survey, the students in the average and lower-level abilities were likely to say they understood more through the help from small groups. One student wrote, “When I do not understand, the teammates taught me until I understand, which helps me learn.” In the survey, students indicated mathematics became easier to understand when they listened to their peers talking about mathematics. The comments were consistent with the analysis of classroom discussions which highlighted the learning goals of understanding. The nature of interactions was contributive conversation. Additionally, students said they built closer relationships in small groups. For example, in an interview with one 7th grader, she said, “the small group let us get closer. There are many ways we can improve ourselves. We can either ask the team leader, the teacher, or another group” An 8th grader commented in her interview, 

There is an affinity when students help students. Students easily understand students. We know what we are thinking like each other. The teacher’s teaching most of time is either for the best or for the worst in a whole class. That cannot meet every individual’s needs. But in small group, students teach students specifically to stick to individual’s needs.

**A Culture of Learning.** In the student survey, the answer that appeared with the highest frequencies regarding the impact of the DJP teaching was that they became more interested in learning mathematic for all the students at difference levels of mathematical abilities. Students indicated their classrooms had a culture of learning of mathematics knowledge and logical thinking skills; and mathematics classroom became alive and fun.

In comparison with traditional teaching, the teachers believed the new model of teaching encouraged students’ autonomy and engagement with the learning activities. Ms. R said,
“Despite the difference in levels of engagement, almost every student is engaged with learning through small group work.” Helping student motivation to learn in a poverty school is a great success of this new model of teaching.

**Evaluation and Reward Systems.** In the DJP classrooms, small groups were the basic units accountable for learning. The teacher rewarded the groups that made good progress each week based on the calculation of performance in the categories created by the class. Despite that, there were flexible and ongoing evaluations and reward systems across the group, the class, and the school. The evaluations had multiple dimensions including academic aspects such as presentations, homework, exams, and non-academic aspects such as class routines, duties participation in non-academic activities, etc. The criteria for class evaluations were flexible. For instance, group presentations could be evaluated based on the accuracy of mathematical language or the creativity of solutions. In addition, students in lower-level mathematics learning ability would receive extra points if they presented the solutions to the problem for the group. Moreover, evaluations were not done only by the teacher. Students in groups evaluated each other (both within groups, and groups evaluate other groups). There were rewards for progress made overall, as well as for in a specific area. Meanwhile, each small group rewarded their members who made great contribution to the group. At the end of each semester, the school also rewarded the students, the classes who had good performance in all areas such as academic improvement and performance in school or district educational activities. In summary, there were evaluations crossed different levels of school, class, small group, and individual work. Evaluations were also involved multiple dimensions to encourage every student to contribute to group learning.
The evaluation and reward systems on the one hand strengthened the relationship among the group members and motivated the participation of them in group activities. On the other hand, they might reduce dynamic changes of teaching and learning in the classrooms. For example, to get high points in presentations, the advanced students often came out with the solutions, and then told lower-level students in small group discussion. Lower-level students’ own thinking were ignored. Despite the encouragement for different approaches to solve a problem, the purpose of different solutions was to compare which one was more efficient, rather than to explore diverse thinking. The solutions appeared wrong or complicated were likely to be discarded. In short, evaluation and reward systems showed double-sides effects on learning. It increased members’ commitment; at the same time, it reduced diversity, and the possibilities for new ideas to emerge as the ignorance of ideas from those students in lower-levels of mathematics learning.

**Learning Goals.** The learning goals in this model of teaching were to help students learn core mathematical knowledge and skills, and to develop competence to apply them to solve problems. The learning goals focusing on knowledge and skills were consistent with the perspectives of mathematics teacher educators from the local education department. Mr. F, the teacher educator from local education department, commented on the initiative of this experiment,

The new curriculum advocates learning through experiment, exploration, communication, and autonomous learning. Yet, evaluations of teaching and learning are still based on standardized test scores. Classroom teachers have been struggling to find a way to implement the reform ideas and also to meet the college entrance requirement. The struggles are especially strong for those teachers who taught in “poverty” schools where students often lacked the motivation to learn. The DJP teaching model was to bridge the gap between implementing reform ideas and improving testing scores. [Interview with Mr. F]
Mr. F’s comments indicated the DJP teaching model were to seek a compromise between implementing reform ideas and meeting the requirement of testing systems.

The interview from the teachers further illustrated the learning goals focusing on knowledge gaining and application. For example, Ms. Y believed the DJP model of teaching provided students with adequate opportunities to learn knowledge and skills. She said, “In general, an average student is able to master the knowledge and skills through self-guided study by the study plan. If the student still has problem, he/she can always ask classmates or the teacher.” She further explained there were four opportunities to learn in the DJP model of teaching including (1) working individually, (2) working within the small group, (3) working with the teacher, and (4) working with other groups in the class. Ms. Y also pointed out that the pressure in meeting with high school entrance examinations drives teachers to emphasis knowledge and skills. She explained, “No matter how hard you work, without good scores on the examinations, nobody will acknowledge your success.”

Overall, the learning goals focusing on gaining knowledge and skills guided the direction of the DJP teaching. The class dimensions such as instructional organization (individual studying, small-group discussion and whole-class discussion), communication patterns, classroom social norms, and evaluation systems worked together to support the achievement of the learning goals.

Feedback on the DJP Model of Teaching from Ms. R and Ms. Y. In reflecting on her teaching, Ms. R believed that her students showed more interest in learning mathematics after implementing the new model of teaching for two years. Students especially developed their abilities to communicate mathematically, and to collaborate with others. In addition, students’ learning habits were improved as well. Ms. R was satisfied with her class, in particular with her students richness of thought. She felt that more students were actively engaged in learning than
before. Ms. R, however, also felt the tensions between class time and learning tasks. She wished students had more time and space for themselves.

Ms. Y thought both she and her students changed greatly since the implementation of the DJP teaching. Especially, students liked to learn mathematics more and more, and to spend more time working on mathematical problems after class. Seeing students made good progress, she had more and more interest in developing her teaching too. In addition, there was less homework load than before, and better relationships between her and the students as they both cared about each other more than before.

**Teachers’ Perspectives of Mathematics and Mathematics Teaching and Learning**

This section describes two teacher’s perspectives of mathematics, mathematics teaching and learning, and their teaching experiences.

**Ms. Y’s Perspectives.** Ms. Y viewed mathematics as a science with fundamental knowledge and ways of thinking. She believed mathematical knowledge and skills were foundational for learning mathematics. Mathematics teaching for her is a way to cultivate students’ abilities of computation and logical thinking. In her class, it was typical that students recited mathematical theorems and concepts together in the class. Ms. Y explained,

> I think memorization is a basic component of learning, either in algebra or geometry. If you cannot remember basic facts, you cannot apply them. Therefore, when learning a simple concept, the basic thing is to let students remember it.

Ms. Y believed that students can learn mathematics best when they set up learning goals and master effective learning methods. In her teaching, she often articulated the goals at the beginning of each lesson, and required students to state the goals clearly when they presented their solutions to the problems. For her, mathematical thinking and learning methods are the most important things in mathematics learning. Her ideal class is where students can learn easily
and enjoy learning. Those perspectives of mathematics, and mathematics teaching and learning are translated into her teaching. In her teaching, she emphasized mastery of fundamental knowledge and skills, articulations of learning goals, efficiency in solving mathematical problems, and reflections of core knowledge and skills.

**Ms. R’s Perspectives.** Ms. R believed mathematics is embedded in real life problems. For her, mathematics is to describe the patterns and relationships. Mathematical teaching and learning is to develop logical thinking, spatial thinking, and number sense, to help students develop the abilities of questioning and critically thinking. Ms. R regarded building interest in mathematics and applying mathematical knowledge to solve practical problems are the most valuable components in mathematics learning. She believed teaching mathematics needs to develop students’ interest in mathematics, higher-order thinking, and problem-solving ability. Ms. R’s ideal class is a class with harmonious relationships where students have a passion to learn mathematics, and are given plenty time to explore mathematics without pressure. She wished there are more ways to learn mathematics. Ms. R’s perspectives of mathematics, and mathematics teaching and learning were in agreement with her teaching. In her teaching, she attempted to give more time for students to talk about their different solutions despite some solutions appearing complicated. When students made mistakes, she did not step in to give the correct answer, but allowed students to describe their thinking and left time for them to make justifications. In her teaching, she valued the relationship between her and the students. As a head teacher, she was concerned not only about students’ academic study, but also their emotional needs. She said, “Middle school kids, especially for 8th graders, are experiencing physical and emotional changes. I pay more attentions to their emotional needs. When they were
in 7th grade, I paid more attention to their learning habits. Now, most of them have good learning habits.”

The Teacher Learning Community Focused on the DJP Model of Teaching

The DJP model of teaching was an attempt to make classroom teaching transition in the local district education department. Behind the model of teaching, there was a teacher learning community consisting of teachers, school administrators, and district teacher educators. In this section, I describe the communication structure of the teacher professional community underlying the DJP teaching model, and discuss the meaning relationships attached in the structure such as learning goals, and social relationships between the DJP classroom learning communities and the teacher learning community.

**Background of the DJP Model of Teaching.** Mr. F and Ms. Z were teacher educators from the local in-service teacher education department, who played the main roles in the DJP model of teaching. Being teacher educators, especially mathematics classroom teacher educators, their work include three main aspects: research, teaching and service. In other words, their job is to conduct teaching research and activities with the school teachers, to provide professional help for teachers to improve their teaching practices. Under the context of the national curriculum reform, their main responsibility was to help practicing teacher understand the reform ideas and integrate them into teaching practices to improve classroom teaching.

Ms. Z reflected on her observations of classroom teaching before the implementation of teaching experiment, “One typical phenomenon was that teacher often lectured diligently while the students fell into asleep. To change the situation, students have to take ownership of their learning; and teachers play the role of facilitators.” Mr. F and his colleagues identified autonomy as a key in changing classroom teaching. The teacher educators shared their perspectives with
school teachers and administrators in this district. However, it was not easy to integrate the reform ideas into classroom teaching, especially for high poverty schools where the teachers lacked faith in students; in particular test scores were still the main aspects to evaluate the quality of education.

While many school administrators and teachers hesitated to adopt the new ideas of teaching, Mr. H (the principal of Huaihai Junior High school) showed interest in reform. Huaihai Junior High school was ranked in the bottom for a long period of time in the district. Not only were students lack of learning interest, so did the teachers to teach as well. Mr. H hoped students love learning and teachers love teaching in his school. He said, “I have a vision of the school as a home for teachers and students where teachers are delighted to teach and students are delighted to learn.” Seeing fewer and fewer students enrolled in the school, and more and more teachers got tired of teaching, Mr. H felt a crisis looming. Mr. H decided to take the advantage offered by the local educational department. Both the district teacher educators and the school principal valued the autonomy of students’ learning and teachers’ teaching.

One of the keys to the initiative of the experiment was the willingness to change and support from school administrators. Ms. Z said, “It doesn’t matter in which way you want to make changes, it matters that you are willing to change and to open to new ideas. If you are willing to change, you can always find ways suitable for specific situations.” The teacher educator Ms. Z and Mr. F highly appreciated the willingness and support from the school administrators. “Principal H showed great interest and provided support for the new model of teaching” said Ms. Z.

When we asked for extra small blackboards for each class, Principal H equipped the classrooms with them in one week. When our teachers needed more experience with the new ways of teaching, he provided financial support for the teachers to visit the well-known reform schools outside the state. He sent teachers to reform workshops by the city
education department and invited teachers outside of the school to observe classroom teaching and discussed issues arose. Because of the support from the administrators, the teachers in this school were able to engage actively in the new model of teaching. [Interview with Ms. Z]

The teacher Ms. Y reflected her involvement in the teaching experiment and said she was an outsider at the beginning of the new model of teaching; and thought this was just a passing fad like they had had before. One of the main reasons for her to actively engage in the teaching experiment was her own desire for an alternative of teaching and learning. Seeing young kids had more and more on homework, she worried her own child would lose a joyful childhood. Promoted by the school and local educational department, she began to engage in the reform activity. Overall, the teacher educators from the district educational department, the school administrators, and teachers showed the same values, commitments, and desire for changes. Those constituted a precondition for the meaningful and long-lasting change in the school.

In the following section, I further describe how the teacher, the teacher educators, and the school worked together in making the changes in this school through a practice-based learning community around the DJP teaching experiment.

**Strategies of Implementing the Teaching Experiment.** The main strategies employed in the implementation of the new model of teaching included sharing the reform ideas, doing and reflecting, and developing school-based curriculum.

**Sharing Reform Perspectives and Spreading the Reform from Micro to Macro.** The unfolding of the experiment followed the strategy of sharing and using leading teachers to engage more and more teachers to the new model of teaching.

The DJP teaching started with three classes in 7th grade in the spring of 2007, and expended to the new 7th grade in the fall of 2007, and then more and more grades were engaged in the experiment. The local education department teacher educators held meetings every week
in the school with the teachers to share their perspectives of reform classroom teaching. The
district teacher educators presented seminars on autonomous learning to the teachers, discussed
ways to implement the new model of teaching with the teachers, such as classroom organizations,
roles of the teacher, and students. In addition, the teacher educators shared their perceptions and
expectations of the new model of teaching with students too. Ms. Z explained to students why
they needed to take the responsibilities of learning and guided students to play the role of
dedicated autonomous learners. She said, “I gave a metaphor to the students. I said, like a bottle
with a lid, nothing can get in unless you open the lid.” In reflecting on the implementation of the
DJP model of teaching, Ms. Z said,

   It was tremendously hard to let students open their mouth to speak in front of the class at
the beginning. Some students said they knew how to do it, but they did not know how to
express their thoughts to others. They were afraid they could not articulate their thoughts
well. Our strategy was to use group leaders as breakthrough points. The teachers and
teacher educators spent extra time working with group leaders on how to explain their
thoughts to others and set up examples for presentation. And then group leaders taught
the members to articulate their thoughts about the solving process in the same way.
[Interview with Ms. Z]

With gradual progress, not only could the group leaders articulate their solutions, but more and
more average students learned to communicate mathematically.

   Likewise, more and more teachers were involved in this teaching experiment through the
leading teachers in the school. Exemplary lessons conducted by leading teachers were popular
ways to spread the reform ideas and teaching practice in the school and the district. Teachers
gathered together to study the lesson and discuss issues around the new way of teaching. Along
with more experience gained in the teaching experiment, the local educational department began
to spread the teaching experiment in other subjects of learning. Moreover, the leading teachers
also conducted exemplary lessons and workshop to engage the teachers and administrators
outside this district to further spread the reformed teaching practice beyond the school and this district.

**A Process of Doing- Reflecting- Revising**. The DJP experiment itself was a continuously refining process as Ms. Z said, “The experiment has gone through a process of doing-reflecting-refining.” The strategy of doing-reflecting was applied in the every aspect of the experiment. During the implantation of the teaching experiment, the teachers and teacher educators encountered many difficulties such as how to engage students, and how to understand autonomous learning. The school invited educational experts to the school to talk about the reform visions, and sent teachers to various seminars, and visited well-known reform schools. Ms. Y reflected on her visiting to the well-known reform school located, saying, “In their class, I saw teachers do not have to tell students. Students could teach each other. They could articulate their thinking very well.” When she came back from the trip, she began to let students work on the problems and present their solutions without her direct teaching. The principal H noted that the students became actively engaged in learning in Ms. Y’s class after the trip. Principal H concluded, “I felt the trip was worthy spending money when I saw our teaching is changing and our classes come alive.”

The development of the study plan is also a typical example of doing-reflecting-revising strategy. The study plan was a complementary material of the textbook, used to guide students’ independent learning in the DJP model of teaching. The study plan was developed by mathematics teachers from this district and teacher educators from the district.

**The Development of Study Plan**. In the beginning of the teaching experiment, there was no study plan, but a list of questions to guide students’ self-study. During the experiment, the teachers found it was not effective in helping them if just throwing questions to students; instead,
students needed more specific guidelines. In particular, the teachers felt there were gaps between the reform curriculum and students’ understanding of them. The reform curriculum emphasized exploring mathematical concepts through activities; however, it was hard for students to master mathematical concepts through activities. Thus, the teachers and local teacher educators had the idea of creating a study plan to elicit core mathematical content in those exploring activities to bridge students’ understanding and the requirement of the reform curriculum. Ms. Z explained this, “The textbook offers an activity for students to explore the definition of a quadratic equation, but after the activity, students had no clear ideas about what is a quadratic equation. The study plan seeks to provide examples and exercises to bring forth the essential features of a quadratic equation.” To bridge the gap, the mathematics teachers and the teacher educators in the local education department gathered together to discuss questions such as: What were the primary objectives of the lesson? To what degree did students understand the topic? How to assess their understanding? What kind of prior knowledge was related to the lesson? What errors or difficulties did students have in learning this topic? Through discussion of those questions, the teachers and the teacher educators exchanged their ideas. They came up with short questions, examples, hints to address the core content, difficulties in each topic teaching experience, and understanding of students (See Appendix VI).

The study plan includes learning objectives, analysis of core knowledge and skills, analysis of learning difficulties, and guidelines of learning process. For instance, the learning process addresses the aspects such as preparation, understanding the core knowledge, and reflecting on, and extending of that knowledge. Specifically, preparation is focused on the connections between prior knowledge and new knowledge. The section of “understanding the textbook” breaks essential mathematical concepts or theorems into small parts and are
accompanied with questions and connections with other mathematical ideas. In addition, there is a section called “digging into the textbook”, in which typical examples, geometrical figures are collected to deepen students’ understanding and thinking skills. Some examples do not have a full solution, rather some hints to promote students to solve the problems by themselves. Following those are some self-assessment items for students to check out their understanding of the content and skills and reflection on their learning experience.

**Learning Activities and Communication Structures.** During the DJP teaching experiment, the teacher educators offered ongoing professional support in classroom teaching through a variety of activities besides the development of the study plan. The learning activities included classroom observations and discussion, mathematics teachers and teacher educators, visiting well-known reform schools and classrooms, attending seminars regarding the reform ideas conducted by educational experts outside this district, and inviting university professors from mathematics education to their classrooms to provide advice for improvement. The activities provided multiple avenues of communications among the classroom teachers, district teacher educators, and educational experts across schools, districts, and cities. Through exchanging ideas and collaborative work among teachers, researchers, educational experts, the teachers re-conceptualized their teaching perspectives, reflected on, and revised their teaching practices. Along with the development of the teaching experiment, the teachers began to present their teaching and to share their teaching perspectives and experience to the teachers in other schools and districts. In addition, more and more teachers came to the school to observe the classrooms and discuss educational issues together. Through the recursive process of doing, reflecting and revising, teachers kept learning and improving their teaching.
Ms. Y highlighted the help she got in the teaching experiment including attending reform seminars from the educational experts, visiting to other schools, and discussing with teacher educators in this district and researchers from a university. In particular, Ms. Y noticed that the teacher educators in the local distinct education department helped her reflect on her teaching practice, and conduct research on her teaching and students’ learning. She has published some journal articles with the support from the local teacher educators, and felt more confident in teaching and research.

Below is another example of the activities around the DJP teaching experiment among the teachers, school administrators, and teacher educators in this district that promoting teachers’ professional development. Such activities seem not common in the US.

The Principals’ Meeting. This meeting was held right after the exemplary lesson presented by Ms. Y in the school. Sixteen principals from the middle schools in this district, the local teacher educator (Mr. F), and the school administrators attended this meeting. The main purpose of activity was to discuss the implementation of the DJP teaching experiment in those schools, in particular, to discuss the concerns about its influence on teaching and learning, and teachers’ professional development. In this meeting, Ms. Y shared her experience with the DJP teaching experiment from a teacher’s angle. After that, the school administrators including the principal of the school (Principal H) shared their thoughts about the new approaches of teaching, and the changes in the school since the implementation of the new teaching. Generally speaking, the teacher and the administrators indicated many positive changes in the school, especially in the following aspects since the implementation of the new model of teaching:

1) Students’ and teachers’ attitude toward teaching and learning changed positively. More and more teachers showed passions to teach, and more and more students developed interest in learning.
2) The relationship between the teacher and students improved. Teachers tended to more appreciate students rather than to criticize them.

3) Teachers improved their professional levels in content knowledge and pedagogical knowledge. For instance, through working on the study plan, teachers were engaged in studying the curriculum, and gaining comprehensive understanding of the curriculum. In addition, teachers built understanding of their students and developed abilities to organize, to guide, and to adjust the teaching.

4) The role of teachers changed from just teacher to teacher-researcher. In the process of the teaching experiment, teachers were involved in research on engaging students and reflecting on teaching. Teachers were encouraged to conduct lessons from a researcher’s perspective to identify problems, to explore solutions to the problems, to reflect on own teaching, and to use research results to guide teaching. There were about 41 articles written by the school teachers that were published in difference levels of journals and magazines.

Principal H highlighted two important aspects during the implementation of the reform process from the perspectives of school administrators: (1) periodic reflection and summary on what they did and what problems they faced, which gave a chance to make sure they went in the right direction, and (2) offering teachers chances to visit the reform schools outside the city so that teachers could gain insights to further guide their modification of their own teaching. Principal H said, “The reform ideas of collaboration, communication, and exploration could not make sense to the teachers until they actually saw how those ideas were integrated into teaching.”

In the second part of the meeting, the attending principals discussed their concerns and issues arose regarding the teaching experiment in their own school. For example, an attending principal (Mr. C) shared the unfolding of the DJP teaching experiment in his school. He said,

In general, our teachers are not resistant to the DJP model of teaching. The content knowledge is not an issue in our school. The main issue is related to pedagogical knowledge related to implement this model of teaching, for example, how to organize classroom instructional activities? How to promote discussion? How to help students conduct evaluation? We now ask each key teacher [about 40] in our school to present a lesson to study those questions together. Overall it has impacted greatly on teachers’ understanding of the reform ideas.
Another principal pointed out that although his school tried the DJP model of teaching; there were few essential changes in his school. One of the reasons was the teachers and school administrators appeared to lack of understanding about the new model of teaching in his school. Other principals talked about the obstacles that inhibited the implementation of new teaching in their schools. The main issue brought up by the principals was how to engage teachers in the reform teaching. For instance, some teachers were comfortable with their current teaching, and they neither want to change nor like the pressure brought by the new model of teaching. In addition, there were inconsistencies among the administrators and among the teachers regarding the values and purposes of the new model of teaching. The attending principals hoped more successful examples to show them ways to implement the new model of teaching; meanwhile they needed time to make adjustment between the new model of teaching and traditional ways of teaching based on different situations in each school. Some others suggested flexible forms of integrating reform ideas, and more collaboration with the local education department.

Following that, the local teacher educator Mr. F also shared his perspective of the DJP teaching model with those participating administrators from the angle of teacher educators. In particular, Mr. F highlighted three aspects of teacher professional development including

1) Mathematical content knowledge,

2) Pedagogical knowledge such as knowledge of educational theories, of curriculum, abilities to organize classroom teaching and make adjustments according to on-going classroom situations,

3) Passion in teaching, and love of students.

Mr. F talked about how the DJP teaching experiment could contribute to the development of those aspects. He especially used some examples to illustrate various ways and possibilities to
develop teachers’ knowledge and passion during the DJP teaching experiment. For example, he believed the process of creating the study plan was a process of studying curriculum, students, and reflecting on teaching. Response to the issues that the principals brought up in the conversation, Mr. F pointed out that the confusion around the DJP model of teaching was normal. For example, teachers had questions about if the study plan was necessary, how to organize instruction, and how to make justification in teaching. Mr. F highlighted that the transformation itself is an evolving process—it occurs in the process of doing, communicating, and reflection. Without practicing, communicating, and reflecting, transformation could not happen. Mr. F emphasized the attitude of transformation of educational perspectives would be the most important for classroom changes to happen.

Moreover, he agreed that the implementation of the new model of teaching should gradually spread out from leading teachers and their classrooms, from Micro to Macro rather than trying to do it all at once. The vital thing for teacher educators and school administrators, however, from his perspective, was to help teachers reflect on their practice and to offer support and opportunities for teachers to learn from each other. He also suggested that the principals invite educational experts to their schools to talk with teachers, to encourage their teachers to share their teaching with other schools to promote teaching reform. He acknowledged that understanding teachers’ needs is important as well.

Overall, in this meeting, the teachers and the school administrators of the school at Site B shared their successful experiences in implementing the DJP model of teaching, while other principals spoke out about their confusion and needs. For example, they wanted to know how to motivate teachers to participate in reform teaching, especially those who did not have much pressure to reform. The principals hoped to get more understanding about the new model of
teaching. The local teacher educator exchanged his ideas with those principals from teacher educator’s perspective and offered suggestions. This meeting reflected how administrators from different schools and teacher educators from local educational professional department came together to exchange ideas and discussed ways to promote classroom teaching reform. The principals’ meeting can be viewed as an example of using the target school as a starting point for large scale reform.

**A Teaching Experiment-based Learning Community.** The description of the implementation of the DJP model of teaching experiment revealed a teacher professional development learning community around the DJP model of teaching in the school and the local district. Based on the activities described above, the structure of the teacher learning community is illustrated in Figure 6.27.

Figure 6.27 Communication Structure of the School-based Teacher Community
The teacher professional learning community involved three levels: (1) school level (including teachers and administrators in this school), (2) district level (including the teacher educators in the local educational department, teachers and administrators in the local district), and (3) schools beyond the local district (including the teachers, teacher educators, and administrators outside the district). The three-level community indicated that the DJP classroom teaching reform was embedded in nested systems – school, district, and outside-district educational systems.

From this diagram, the teachers in this school and the teacher educators in this district had larger clusters of connections, which indicates that they were the two hubs in the network of the teacher learning community. Specifically, at the school level, the local teacher educators played the leadership role. They helped the school identify the problems and set up directional goals for this community: to improve students’ autonomous learning and promote teachers’ professional development. The goals were generated from the needs of the students, the teachers, and the school, and communicated across the school administrators, teachers, and students. The teacher educators, school administrators, and teachers all shared the same value, and took responsibilities respectively to make change happen. For example, the teacher educators in the local teaching educational department played the role of “facilitators”, school teachers were the “actors”, and the school administrators provided material, financial, and emotional support. The coordinated actions between the classroom teachers and the teacher educators included research lessons, classroom observations, research projects, and creating the local curriculum, which were all teaching-based activities. In those activities, the teachers and the teacher educators committed to the improvement of learning and teaching. They built good relationships with each other and worked cooperatively.
There were multiple ways to communicate among members in the community regarding the new model of teaching. For instance, research lessons, research projects, and regular meetings (once a week for the teacher educators with the school teachers, once a week for collective preparation of lesson plans between mathematics teachers in this school, and once a month for the district mathematics teaching and research activities). Both the teacher educators and the teachers were engaged in the collective learning process of doing-reflecting-revising. In the study conducted by Kitchen (2003), one of the barriers that prevented teachers from carrying out the reform ideas in their own teaching was the lack of support from the school administrators and from colleagues. This study indicates that the classroom teachers, the teacher educators in the local educational department, and the administrators in this school formed a school-based learning community where they continuously practiced, reflected on, and learned through multiple activities on the basis of their daily teaching practices.

Additionally, the teacher educators in the local education department were responsible for the improvement of classroom teaching and learning in all the schools in this district. One of the strategies that was used to spread the DJP model of classroom teaching reform to the district was sharing and using leading teachers and their classrooms as starting point to spread the teaching experiment from Micro to Macro scale. Along with more and more successful experience from the DJP teaching experiment, teachers in this school began to share their teaching in district schools and beyond. The sharing included either the teachers going to other schools to demonstrate their teaching, or teachers from other schools being invited to the school to observe the teaching practice. In addition to teachers, the administrators were involved in the reform enactment as well; for instance, the district principals’ meeting showed how the principals of middle schools in this district got together to discuss the issues and concerns that were related to
the implementation of the DJP teaching in their schools. Through those ways, the new model of teaching spread its impact to the level of district and beyond. In other words, the unfolding of the new model of teaching showed a gradual process from small area to larger areas.

The Similarities and Differences across the Classroom and the Teacher Learning Communities

From the previous analysis, the DJP classrooms and the teacher professional community in this school showed similarities across different scales.

**Different Levels of Learning Communities.** At the level of teacher learning community in this school, the teacher educators played the role of organizers and guiders of learning activities. They offered seminars and discussion on the reform ideas and the teaching experiment. The teachers were the practitioners of the teaching experiment, who enacted and refined their teaching practice. The administrators were financial and administration support behind the teaching experiment. Teachers, teacher educators, and school administrators shared the desire for change and the same vision of classroom teaching reform, which was to motivate students’ autonomy to promote learning, to motivate teachers’ autonomy, and to promote teachers’ professional development. The communication was manifest in multiple ways among the teacher educators, teachers, and the administrators, such as sharing of the lessons, research projects, and creation of study plan. The relationships among the teacher educators, teachers and the school administrators were supportive of each other. The communication demonstrated dynamic characteristics as it was based on the idea of “doing experiments” —doing, reflecting and revising. It was contributive and dialogic.

At the level of the classroom, the teacher distributed the learning responsibilities to the student learning groups. The teacher mainly became the organizer of learning activities, and students worked on in groups and came up with solutions to the given problems. The teacher and
the small groups were committed to the achievement of learning goals in the class together. They had common learning goals and valued the mastery of mathematical knowledge and skills, which were consistent with those of the teacher professional community. The learning goals and values were embedded in the tasks in the classroom learning activities and were also reflected by the nature of interactions in the whole classroom. The social and socio-mathematical norms such as reflection on knowledge and solving processes, and simplicity and efficiency were all in the alignment with the learning goals and values of the whole class.

In the level of small groups, the members of each group took the responsibility of learning together. They also shared the common goals and values for the achievement of the small group as a whole. The learning goals and values in small groups were in alignment with the whole class’s learning goals and values. The members built collaboration in small group work. The communication patterns within small groups were also sharing understanding and solutions to problems.

The three levels of communities demonstrated a nested structure, which can be shown in Figure 6.28. Each level of community had similar learning goals and values, social norms, interaction patterns and organizations of activities.

The nested network indicated the success of the teaching experiment in this school was based on the context that had sharing learning goals and values, coherences, shared responsibilities, and supportive communication patterns for the members across levels of the communities in this school.
In contrast, in the level of district, as it was indicated in the principals’ meeting, the schools in this district had not yet reached shared values and goals for the new model of teaching experiment, and therefore, the implement of the reform teaching model encountered obstacles. Despite the obstacles the principals had in implementing the new model of teaching, the communication structures across classrooms, schools, and districts have set up a foundation for the teachers, school administrators, and local education department educators to exchange ideas, share teaching reform experiences. As we have seen, the reform teaching started from the leading teachers and classrooms and spread out through sharing of teaching practices and ideas to the whole schools. With the establishment of communication network among schools in the district, the new model of teaching was unfolding from the leading school to the schools in the district.
The different levels of learning communities also demonstrated a decentralized structure as it is shown in Figure 6.29:

According to Davis & Sumara (2006), a decentralized structure “is a more viable structure for a system that relies on the efficient exchanges of information” (p. 88). The structure underlying the DJP model of teaching has a decentralized structure which shows robustness and efficiency in sharing of problem-solving process and understanding across different communities. Thus, when the learning goal is to gain predetermined knowledge and skills and logical thinking process, this structure promotes achievement effectively. This can be used to interpret why students could improve their testing score effectively in this school.

**Dynamics of Interactions.** As pointed out by Davis and Sumara (2006) “a decentralized network will decay into a more vulnerable (but informatively efficient) centralized network if stressed” (p. 88). As described in the previous part, the interaction patterns in the DJP model of
teaching were primarily sharing of different solutions to a problem and reflections on the core knowledge and skills including the techniques of presentation skills. The interaction pattern was consistent with the learning goals of mastery of pre-existing knowledge and skills to pass the examinations. In other words, the interactions in the classes were a sharing of solving processes rather than a process of developing new understanding through interaction. Unexpected and emerging mathematical thoughts or ideas were few, and errors or mistakes were treated as something less value than correct answers. In small-group discussion those advanced-level students often played the role of “tellers” who explained their solutions to the lower-level students. Lower-level students appeared as listeners, whose ideas were often ignored. Many lower-level students indicated they improved their learning more easily as the advanced-students explained to them than the teacher did. In addition, the advanced-students would teach them until they understood the problem. However, this learning was more on understanding the preset knowledge rather than developing their own mathematical ideas.

As Davis & Sumara noted (2006), “For complexivists, the emergence of new interpretive possibility is framed more in terms of expansiveness and outward movement” (p. 57). In other words, learning is measured as knowledge expanding through interactions of students and the teacher rather than as accumulations of knowledge. Individuals are active agents who bring different experience and backgrounds to the learning. Through interaction, students explore possible solutions and make conjectures and justifications. In this process, learning should go beyond reflection to embrace spontaneously emerging ideas. In looking at the network of the DJP model of classroom interactions, small group presentations to each problem were the hubs of the interaction network. Around the hubs were different solutions to the problems and summaries of main points of mathematical core knowledge. Such structure reflects a
convergence of knowledge. What we need is a more divergent structure of communication where students’ talk becomes the driving force of the interaction.

One possible reason for the tendency of “centralized” structure may be related to the stress from evaluations of small groups. Despite the evaluations were aimed to encourage students’ engagement in learning activities, as we have seen, the pressure on gaining higher points for the group pushed small groups to get correct solutions quickly, and then to make sure everyone understood the solutions. Thus, it promoted the advanced students helping less-advanced students to make sense of the solution given by the advanced students, rather than the advanced students to make sense of the lower-level students’ thinking.

Additionally, limited time for discussion in class might promote the occurrence of centralization as well. As it was shown, there were a variety of knowledge and skills to master but limited time available for discussion and develop ideas in each lesson. While network theorists suggested weak links to play a role in increasing ways to solve a problem, it also reduces the distance between links (Barabási, 2002; Mowat & Davis, 2010). The lower-level students’ thinking can be viewed as weak links. Ignoring their ideas loses these weak links in the class communication network, which reduces learning opportunities that arise from mutual interactions between students, and possibilities for the expanding of new understanding. Moreover, on a broader scale, the requirement of examinations for college entrance secures the emphasis of gaining knowledge and skill.

Cognitive Demands of the Tasks. Stein et al., (2000) highlighted different tasks that engage students in developing different levels of mathematical thinking. They defined tasks based on the cognitive demands as involving low-level mathematics thinking (such as
memorization or procedures-without-connections tasks) and high-level mathematics thinking (such as procedures-with-connections and “doing mathematics” tasks).

In the DJP model of teaching, each lesson consisted of a review of the main mathematical knowledge and skills and a series of problems that addressed different aspects of those knowledge and skills. Through solving those problems, students learned the expected mathematical knowledge and skills, and developed competence in mathematics.

Most of the geometrical tasks given to students were from the study plan, which were almost context-free and close-ended. The problems were either with the results were out there, and what students needed to do was to bridge the conclusion and integrate the known conditions together through logical proof; or the pathways to solutions were explicitly (or implicitly) suggested by the tasks. Students had no need to explore possible situations and justify their approaches. Despite those tasks demanding understanding of mathematical concepts, the cognitive demands of the tasks were primarily procedure-with-connection among mathematical ideas rather than “doing mathematics”. Those tasks reflected the learning goals of mastery of predetermined knowledge and skills. Mathematics tasks of “doing mathematics”, however, offer no “predicted” ways to solutions. They demand explorations of situations and monitoring and justification of thinking process (Stein et al., 2000).

The over-reliance on procedure-with-connection tasks and lack of “doing mathematics” tasks provided rare opportunity for students to develop their deep level thinking, especially adopting reasoning and creativity. The research conducted by Langer and her colleagues (1989) indicated that it is uncertainty rather than certainty that encourages creative thinking. Langer advocated a conditional view of teaching to help students become mindful. To have a conditional
view, we can simply ask “what if” questions. What if the situation is changed? Is conclusion still the same? For example, In Ms. R’s lesson, she gave students the following problem:

Given $AB // CD$ as in the (Figure 6.30). Provide a conjecture about the relationship among $\angle B, \angle C, \angle E$ in each situation? Prove your conjecture.

Ms. R asked students to make conjectures and then prove them. Since conclusions were implicitly suggested by the task, students could easily guess the conclusion and prove it. The focus of the task was still on “procedure-with-connection” rather than “doing mathematics”, even though it appears to ask students to make conjectures. Thus, the learning was still on logical proving rather than exploration of situations. But we could increase learning opportunities by making a little modification to the task to increase the thinking level. For example, can we ask students to think all those situations together? for instance: Suppose $BC$ is a flexible rubber band in the Figure 6.31, which can be stretched at point $E$ to create different situations of the angles at $B, E, C$. Now, can we make a conjecture of the relationship among those angles formed at $B, C, E$?

When we change the given conditions, we open the geometrical questions to more possibilities. In the above problem, we could start from the last question— not even give students
any figures, let students to explore all possibilities. As Langer pointed out to “encourage a conditional view, a sense of possibility” (p.123), the sensitivity to learning possibilities in teaching makes differences in classroom teaching.

In summary, the analysis of site B offers understanding of classroom teaching reform from the following dimensions: (1) classroom learning community and the associated teachers’ professional development community, (2) students and the classroom teachers, classroom teachers and teacher educators in the local educational department, and the school administrators in this district. Summary and discussions of research findings at Sites A and Site B will be described in Chapter 7.
CHAPTER 7: SUMMARY, DISCUSSION, AND CONCLUSIONS

This multiple case study provides empirical data about teaching reform in secondary mathematics classrooms in China. In this study, mathematics classrooms are viewed as social systems with networks of communications between teachers and students engaging in mathematics. This study focuses on the dynamic structures of classroom teaching and the meaning relationships embedded in the structures such as goals, norms, and mutual relationships. Furthermore, it examines the teachers’ perspectives of mathematics, of teaching, and their experiences along with the associated teacher communities involved in professional development programs in order to understand their influence on classroom teaching practices.

In this chapter, I first summarize the research findings, discuss the strategies and issues about teaching reform in the two sites, and finally provide implications for reform implementation for classroom teaching and teacher education. The limitations of this study and future research recommendations are described as well. Overall, the understanding of the transformation of classroom teaching involves an examination of the context of classroom teaching as well as implementation efforts by teachers in order to examine and interpret classroom communications and the evolution of the structures of communications in classrooms. Teachers’ beliefs and experiences and teacher professional development communities impact the changes of the context of classroom and thus serve as an important lens for understanding what occurs in classrooms.

Summary of Findings

The central question of this study is, “What communication structures and meaning relationship patterns underlie the Chinese mathematics classroom teaching reform at these two sites?” This question is explored through the following sub-questions:
1. What communication structures are demonstrated during classroom teaching? What relationship patterns are embedded in the structures? How do these network structures change in teaching?

2. What communication structures and relationship patterns are demonstrated in the associated teacher professional development communities at both sites?

3. What similarities and/or differences are demonstrated in the networks and relationship patterns across the classroom practices and teacher professional development communities? What similarities and/or differences are demonstrated in the networks and relationship patterns across different sites?

4. What perspectives of mathematics and mathematics teaching do those teachers have?

**The Communication Structures and Meaning Relationships in Classroom Teaching.**

In this section, the researcher summarizes the characteristics of communication structures, describes the meaning relationships embedded in the networks in five classroom teaching from both Site A and Site B. Furthermore, the research discusses possible factors behind the different teaching.

![Figure 5.3 Communication Network in Ms. N’s Lesson](image)

Figure 5.3 Communication Network in Ms. N’s Lesson
At Site A. The network diagram from the analysis in Chapter Five (Figure 5.3) describes the communication flow of Ms. N’s lesson. The communication nodes (coded by the main actions in the conversation) from the teacher primarily involve assigning problems, highlighting main points, and offering comments on students’ solutions in the conversation network. The nodes from students mainly include solving the problems and demonstrating solutions.

The communication network reflects the cluster around four given problems from the teacher. The network starts with reviewing previous knowledge of the method of “completing squares to solve a quadratic equation”, and then moves to deducing the quadratic formula in a way analogous to the preceding exercise, and finally ends with applications of the formula. The flow is driven by the goal of understanding the predetermined knowledge and corresponding practical skills. There were similar interacting patterns around each problem: students’ demonstration of solutions, and the teacher’s elaboration of key aspects in solving the problems.

The structure indicates the teacher played multiple roles of organizer, monitor and evaluator in the lesson. Meanwhile, students learned through solving the given problems. The focus of the lesson was on knowledge, understanding, and application. It was consistent with and demonstrated in interview data and information about the implementation efforts to reform mathematics instruction in this school. It seems that Ms. N’s beliefs supported her teaching. The communication structure in general is linear and consistent.

In the network structure (Figure 5.8) of Ms. B’s lesson, the communication nodes from students primarily include articulating problem-solving processes, offering alternative approaches, explaining difficulties in solving the problems, and reflecting on their learning. The nodes from the teachers vary from assigning tasks, offering comments and hints, guiding approaches, and articulating learning goals. Those nodes indicate that the teacher took the roles
of organizing the learning activities, monitoring the learning process, and evaluating the appropriateness of approaches.

Figure 5.8 Communication Network in Ms. B’s Lesson

The structure is a clustering around two conjectures regarding the proof of similarity of two triangles. For example, the communication cluster around the first task involved different solutions from students and reflections from both the teacher and students about the strategies they used in solving the problem. Concerning the second problem, despite students’ different attempts at the beginning, the teacher directed students to the approach to which she preferred, and students’ attempts became focused on the only solution. The structure indicates that the movement of the communications was guided by the goal of seeking the “acceptable” solutions approved by the teacher to the given problems. It was the teacher, rather than the conversation between the teacher and student, that determined the direction of communication. This unequal
power relationship declined the opportunities to generate new understandings and decreased the dynamics of communication network.

The nodes related to the summaries and reflections have a higher frequency rate in the interacting network, which suggests them as social norms established in the lesson. In addition, connecting the known and unknown knowledge, identifying obstacles, and solving the obstacles were the main socio-mathematical norms emphasized in the lesson. The structure of communication network was consistent with Ms. B’s perspective of mathematics and mathematics teaching – reasoning strategies and learning strategies are essential for students to learn by themselves.

The communication structure in Mr. W’s lesson (Figure 5.15) is built on the exploration of congruent quadrilaterals. In the network, the communication nodes are primarily questioning, conjecturing, revising, proving, and reflecting on the problem-solving process. Those similar actions between the teacher and students indicate that both the teacher and students played similar roles in the conversations. There are also weak links such as the investigation of the numbers of possible conditions for two quadrilaterals to be congruent. The network structure shows rich and dynamic discussion, which starts with the central question of the lesson: under what conditions are two quadrilaterals to be congruent? The central question was adapted into several sub-questions, evolving into nested-clusters along the unfolding of the conversations between the teacher and students in their recursive modifications of their questions or conjectures. The evolvement of the network is unpatterned in a short time as communications were driven by students’ comments and conversations, but it is also patterned in a long period of time as it was guided by the learning task and the goals of learning what mathematical inquiry means.
Figure 5.15 The Communication Network during Main Part of Mr. W’s Lesson
Barabasi (2002) identified “preference attachment,” or “rich-get-richer,” as one of the rules that evolve network communications, and includes how nodes are connected, with the nodes with more links tending to represent richer communications. Findings from the analysis of the evolvement of structures of the three lessons at Site A are in agreement with that. Specifically, as indicated in the structure of Mr. W’s lesson, when students’ talking drove the directions of communications, the numbers of links increase; and as indicated in Ms. N’s and Ms. B’s networks, when the teacher had the authority, the numbers of links in the conversation were stable or decreased. This implies the evolvement of conversation network in classroom teaching is affected by the power relationships in the classroom. The structures of the lesson indicates that when the teacher had the control over the collective socio-autonomy in the conversation, the network tended to evolve based on the teacher’s preference and thus limited the generation of new connections. When the class had the power of control, the network tended to evolve based on collective discussion, where more connections were generated. In other words, the findings confirmed that the generation of understanding, ideas, meanings is not located in individuals, but in the conversation itself (Sfard, 2008). It is the communication that determines the evolvement of the communications.

At Site B. The DJP teaching experiment (DJP refers to three components of teaching: $D$, (导), self-learning with study plan; $J$, (讲), sharing solutions; and $P$, (评), evaluating learning processes) was implemented in the whole school at site B. It is not surprising that the instructional approaches in all the lessons observed at Site B showed a very similar pattern. The instruction followed the following key points: (1) students’ learning independently with the guide of study-plan; (2) small-group discussions on the problems from the study-plan; and (3) group presentations on the solutions to the class and the whole-class discussion on the
presentation including evaluations of the presentations. The main goals were to master the core mathematical knowledge and skills. Instead of the teacher lectured the lessons, students worked on the problems and presented their solutions to the class. The following diagrams (Figure 6.15 & 6.24) demonstrate the typical structures underlying the DJP model of teaching.

Figure 6.15 Communication Network in Ms. Y’s Lesson

The communication structures were built on a review of the related knowledge and skills, and followed by students’ presentations of their solutions to a few of the problems from the study plan. The clusters of the networks are around students’ presentations. The nodes from students in the communication network primarily include articulating solutions, summarizing, and reflecting on the knowledge, skills, and strategies applied in solving the problems. The nodes also include reciting of key mathematical theorems and correcting errors. The nodes from the teacher mainly include helping with articulation, monitoring, evaluating their performance, and highlighting the main points of the lesson.
The structures indicate students assumed responsibility for solving the problems, explaining their solving process, providing different solutions, and reflecting on the key mathematical knowledge and skills. The teacher was the organizer, monitor, evaluator, and helper in guiding students to articulate their solving process and to use mathematical language correctly. Overall, the presentations focused on the completeness of explanation rather than “unpacking every response in ways that support learning” (Franse, Kazemi, & Battey, 2007, p. 236). There were few argumentations and discussions regarding the solutions in the whole-class communications. Small group discussions also followed similar patterns; the advanced-level students in mathematics learning played the role of explaining the solutions and strategies to the lower-level students. In general, the movement of communications was guided by the learning goals of understanding and by applying knowledge and the socio-mathematical norms of
conciseness and efficiency. The network structures are consistent across the 13 consecutive lessons at Site B.

There were few fine differences between Ms. R’s and Ms. Y’s teaching at Site B. It seemed that Ms. R showed more tolerance for wrong answers. She gave more time to students to adjust their solutions when they had wrong answers. Meanwhile, she encouraged alternatives whether they were more or less efficient than the previous solutions; Ms. R also often included fewer problems in one lesson with a slower pace. In contrast, Ms. Y tended to emphasize more on accuracy and efficiency of solutions along with reflections on what and how the knowledge and skills were used in solving the given problems. The focus of communication for both classes, however, was on seeking solutions to the given problems, instead of exploring possibilities to increase understanding. In other words, the nature of communications in both of their lessons was contributive. In addition, most tasks given to students were almost context-free and close-ended. The lack of “doing mathematics” tasks provided rare opportunity for students to develop their higher level thinking, especially practical reasoning and creativity. The school-based curriculum of study plan in this site is more like a collection of problems reflecting the core knowledge and strategies of using that knowledge to solve various related problems.

The Similarity and Differences of Classroom Teaching across the Two Sites. In the lessons implemented by Ms. N. and Ms. B at Site A, and the lessons by Ms. Y and Ms. R at Site B, the communication networks have similar structures: both started with an exercise that reviewed previous knowledge or main points of the lesson and moved to more problems to learn new knowledge and skills and to apply them. The networks are unfolded following the pattern of students’ presentations, evaluation of the solutions, and reflection on key knowledge and skills. The learning goals were focused on knowledge, understanding and application, and logical
thinking skills. Social norms mainly included reflection and collaboration. Socio-mathematical norms primarily included accuracy, simplicity and efficiency. The nature of communications was typically contributive in those lessons (see Table 8).

Table 8: Main Features of the Five Lessons at Both Sites

<table>
<thead>
<tr>
<th></th>
<th>Learning Goals</th>
<th>Social/socio-mathematical Norms</th>
<th>Interaction Patterns</th>
<th>Nature of Interaction</th>
<th>Instructional Organizations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms. N</td>
<td>How to solve equations proficiently using the quadratic formula</td>
<td>Procedure proficiency</td>
<td>Sharing of solutions, and teacher’s monitoring of the solving process.</td>
<td>Contributive</td>
<td>Combination of individual, peer-study and whole-class teaching</td>
</tr>
<tr>
<td>Ms. B</td>
<td>How to learn (make connections between prior and new knowledge)</td>
<td>Identification of obstacles, and reflection</td>
<td>Sharing of different solutions, obstacles, strategies of overcoming the difficulties.</td>
<td>Contributive</td>
<td>Combination of individual, small-group and whole-class teaching</td>
</tr>
<tr>
<td>Mr. W</td>
<td>How to conduct mathematical inquiry</td>
<td>Conjecture, justification, negotiation, and collective agreement</td>
<td>Driving by students’ utterances, emerging and unexpected patterns</td>
<td>Reflective and instructive</td>
<td>Whole-class teaching</td>
</tr>
<tr>
<td>Ms. R</td>
<td>Mastering and applying knowledge and skills to solve problems</td>
<td>Communication efficiency, multiple approaches, collaboration, and reflection</td>
<td>Sharing of different solutions.</td>
<td>Contributive</td>
<td>Individual work, small-group discussion and presentations</td>
</tr>
<tr>
<td>Ms. Y</td>
<td>Mastering and applying knowledge and skills to solve problems</td>
<td>Communication, reflection, efficiency simplicity, collaboration, and multiple approaches</td>
<td>Sharing of different solutions, reflection.</td>
<td>Contributive</td>
<td>Individual work, small-group discussion and presentations</td>
</tr>
</tbody>
</table>
Discussion on the structures of communication network and the meaning relationships in those lessons. The findings of this study indicate that mathematics classroom teaching at the two sites have common characteristics that are different from those in traditional Chinese mathematics classroom teaching. For example, the roles of the teacher are changed from a lecturer to an organizer of learning activities and a monitor of the learning process. The teacher assigned the problems, and students worked on the problems and presented their solutions to the class; the teacher diagnosed the difficulties, highlighted and reflected on key points of the knowledge and skills, and discussed strategies in the problem-solving process. Moreover, social communications were encouraged and students helped each other in groups. Instead of working alone, students were grouped together to learn from each other.

The essential properties of discourse, however, remained unchanged. The communication structures in most of the lessons were not self-organized. The communication structures offered little opportunities for sustained self-generation and adaptation. It seems that learning goals, socio-mathematical norms, and power relationships deeply influenced the dynamics of communications.

Learning goals determine the priorities of classroom teaching (Schlechty, 2009). In most of the observed lessons, the learning goals were focused on understanding certain knowledge and reasoning strategies of applying them to solve mathematics problems rather than supporting emerging understandings and expanding of knowledge (Doll, 2005; Davis & Sumara, 2006). Although students had opportunities to present their different solutions and to explain their problem-solving processes, the presentations were focused on final products in which errors and mistakes were treated as something to be avoided. There was little argumentation regarding the solutions and discussions of students’ ideas and intentions. The teachers gave variations of the
problems to help students make connections in knowledge, and students were encouraged to present different solutions, to reflect on the main points, and to diagnose misunderstanding and obstacles. Yet, those instructional activities and communications were still focused on “a particular content objective” rather than the richness and depth of mathematics understandings (Reeder, et al, 2006). The erasing or wiping-out of misunderstanding, especially, intentions from the conversations limits the possibilities of syntheses of selections of individuals and further constrains the dynamic and the development of communications (Lumann, 1990; Rasmussen, 2005). Despite the fact that mathematical problems were carefully arranged around core mathematical ideas, the lack of questioning and argumentation limited the dynamics of the conversation fundamentally, thereby limiting the value of the lessons for supporting the emergence of new understandings and creativity (Doll, 2005). The communication networks are linear and reflect a lack of dynamics. Besides logical thinking, however, there were rarely other ways of mathematical thinking such as inquiry and exploration in most of the observed lessons (except in Mr. W’s lesson).

In contrast, the learning goal in Mr. W’s lesson was to learn mathematical inquiry. The knowledge and skills were tools for students to develop their mathematical thinking abilities. The communications were focused on various attempts at tackling the problem and evaluations and modifications of solutions. Students explored the relationships among mathematical ideas and regulated their thinking process through solving given problems. In communications, students developed multiple ways of thinking such as guessing, measuring, and grouping. Beyond deductive thinking, there was also evidence of inductive and abductive reasoning. Students were engaged in learning and applying knowledge in both algebra and geometry. The communication
structure became evolved and dynamic rather than convergent and linearly patterned – this implies richness and depth of mathematical understandings of classroom communications.

The power relationships between the individuals’ autonomy and the socio-autonomy of the classroom also impacted the dynamics and growth of communication networks, suggesting self-similar relationships between individual’s autonomy and the unfolding classroom socio-autonomy (Fleener & Rodgers, 1997). In most of the lessons in the other classrooms, although students solved the problems, shared their problem-solving strategies, and presented their struggles in solving the problems, the teacher was the one who monitored the learning process, evaluated the learning process, and most importantly, determined the direction of the communications. Students’ autonomy was conditioned, especially when their approaches conflicted with what the teacher expected. Similarly, in the DJP model of teaching at Site B, students worked in groups to come up with solutions to the given problems and presented their solutions to the class. It seemed that students and teacher were co-participants of learning. However, the conversations were guided by seeking correct answers and efficiency of problem solving. Specifically, in small-group discussions in DJP teaching, it was often those advanced-level students who came up with the solutions and explained to the lower-level students. Lower-level students were the listeners, whose ideas were ignored. Davis & Sumara (2006) argued that it is the conversation system itself that determines the directions rather than an authority: “To impose a singular or centralized authority would extinguish the potential of the collective as a knowledge-producer” (p. 144). The adaption and evolvement of communication networks are coordination between individual’s structural changes and the dynamics of the communication networks.
In Mr. W’ lesson, students and Mr. W demonstrated an equal power relationship. Instead of assigning problems to students, monitoring students’ learning process or diagnosing students’ difficulties in problem solving, Mr. W was a participant of the learning process. Both his students and he made conjectures and tested and modified conjectures. When the teacher and students had different opinions, Mr. W revised his questions to fit in the emerging classroom conversations rather than directing students to his way of thinking. Both the teacher and students modified their thinking through negotiation. Mr. W was especially sensitive to students’ ideas and thoughts for their potentials and helped students develop those ideas. He tried to create an environment to stimulate negotiation of meaning-making rather than seek a correct answer. The discourse was a process of dynamic exchange of ideas through negotiations among the participants. When students developed their own ideas, Mr. W paid close attention to honor their ideas. In Mr. W’s lesson, legitimacy of mathematics reality was built on mathematical evidence, rather than the teacher’s authority. There was a dynamic and balanced relationship between the teacher and students. Communication networks of teaching were recursive structural changes of the participants in the communication network over time.

International comparative studies show that Chinese students often outperform their US counterparts in knowledge-based, constrained-process problem solving and abstract thinking, but do not have advantages in other ways of thinking such as guessing and grouping, or in open-process problem solving. The features of most of those classroom teaching dynamics may offer insights into the reasons behind the successes. However, in considering the development of students’ creativity and practical ability advocated by the newest round of Chinese curriculum reform, the gap between teaching in most classrooms and the expectations of reforms is critical.
Mr. W’s lesson is a good example of how to develop students’ practical ability and creativity that
are aligned with the reform visions.

The findings in this study indicate the reform of teaching is not only if students have
opportunities to present their solutions, but about what is valued in communication and who
determines the direction of discourse, i.e. the power relationship between individuals and the
collectives. This implies that the professional development needs to address students’ autonomy
and the relationships with the socio-autonomy of the collective. In particular, considering the
Chinese culture where teachers have been historically respected as authority of knowledge in
Chinese society, students tend to accept teachers as the authority of knowledge naturally in
classrooms. The reform of classroom teaching needs to address the fundamental issues of power
relationships between individuals’ autonomy and the autonomy of the classroom as a whole in
classrooms, and the dynamic and emergent transformative properties of curriculum,

**The Communication Structures and Meaning Relationships in the Associated
Teacher Communities.** The study revealed a project-based teacher learning community led by
the university-based teacher educators at Site A and a school-based teaching experiment learning
community led by the mathematics teacher educators in local education department at Site B.
Both the teacher professional communities demonstrate some unique features of Chinese
mathematics practicing teacher education with their influences upon the transformation of
classroom teaching at both sites.

**At Site A**, Ms. N and Ms. B participated in the two-year long district-level key teacher
leadership enhancement project. This project was to foster a group of teachers to lead the
improvement of teaching practices in the district. Three specific sub-goals to help the
participating teachers were (1) to implement reform ideas-based teaching based on their own
ways, (2) to conduct research on teaching and learning reform, and (3) to influence mathematics education in the district.

All participants of the leadership teacher development project were recognized as outstanding teachers in the local district. Besides the university-based teacher educators, the project included educational experts in curriculum reform, the well-known teachers of reform teaching in other districts, and the administrators from both schools and local government.

The learning activities primarily included diagnosis of teaching, seminars on reform ideas, group research projects on students’ learning and teaching approaches, observation of exemplary lessons, workshops on teaching perspectives from the well-known teachers, and conversing with colleagues and with the local administrators. Those activities were specifically tailored to the learning goals and reflected a comprehensive view of teaching, learning and research. The learning activities helped the teachers not only with their understanding of the new vision of teaching, but also ways to implement the reform ideas with the assistance of experienced colleagues and educational experts. The teachers were also required to present lessons and to initiate seminars with other colleagues in their own schools. In this sense, the participating teachers took the role of facilitators to impact the transformation of classroom teaching in their schools and in the district. Moreover, the participating teachers built the norms that valued the collaboration and collective advancement, forming a collaborative and practice-based learning community. The following chart (Figure 5.23) shows the underlying structure of the teacher learning community at Site A.
Figure 5.23 The Communication Structure of the Key Teacher Project

**At Site B.** The teachers in the school were involved in a school-based DJP teaching experiment led by mathematics teaching researchers in the local district teaching research department. Based on the DJP teaching experiment, the teachers, school administrators, and local teacher educators in this district constituted a practice-based teaching professional community. The learning goals were focused on improving students’ autonomous learning. The goals were tailored with the needs of the students and were communicated to classroom students, teachers and administrators. The members in this community committed to carrying out the teaching experiment collaboratively. The learning activities included classroom observations and discussion of the issues in teaching with district mathematics teachers and teacher educators, visiting well-known classrooms, attending seminars regarding the reform ideas embedded in the new curriculum conducted by educational experts outside this district, and inviting university professors in mathematics education to classrooms for professional advice. One of the main
approaches the community used in the teaching experiment was similar to that at Site A: doing-and-reflecting. The teaching experiment started with fewer key teachers, and gradually spread out to a larger scale in the school. The structure of the learning community can be illustrated as below (Figure 6.27).

![Communication Structure of the School-based Teacher Community](image)

**Figure 6.27 Communication Structure of the School-based Teacher Community**

The Similarity and Differences of the Teacher Learning Communities in the Two Sites.

The DJP teacher learning community at Site B exhibited a similar structure as the project-based learning community at Site A with focus on a practice-and-reflective approach. The commonalities in both sites included:

- Learning activities based on sharing, practicing, and reflecting, such as teachers’ presentations of teaching, sharing of experience, discussions on the teaching, and reflecting on teaching.
• Focusing on teachers’ comprehensive understanding of the content knowledge, pedagogical knowledge and research abilities. At Site A, teachers were required to work together on the diagnosis of the obstacles students have in learning and strategies to help them overcome; at Site B, teachers worked together to create study-plans to address the difficulties, typical examples, and connection between knowledge. Both communities encouraged teachers to conduct researches on their own teaching and students’ learning. Teachers in both sites were involved in research projects and publications.

• Encouraging students’ autonomous learning. At Site A, the reform teaching was more focused on understanding students’ needs; at Site B, it was more focused on mathematical communications and students’ collaboration.

Findings from the study on the teacher professional communities indicate that mathematics teachers were not alone in complementing classroom reform. There were cohorts of mathematics educators consisting of classroom teachers, district teaching researchers, university-based mathematics educators, and administrators from schools and local government. The members took the responsibilities of promoting learning respectively, and showed commitment and mutual support. The university-based teachers and local education department teacher educators were the facilitators of the learning of the teachers. The teachers were not only learners, but also leaders of the classroom teaching reform in schools. The activities showed that teachers received on-going classroom-based practical support to implement new ideas. The learning goals, norms, mutual relationships were intertwined to promote teachers’ learning in the two teacher learning communities.

The teachers were involved in a variety of learning activities, but those activities were more focused on how to help students learn the knowledge and skills, rather than how to create
learning opportunities in class teaching, i.e. those opportunities that helped them develop argumentation and further develop students’ ideas in communications. The teachers’ diagnosis of students’ needs appeared more often than discussions on inquiry into students’ ways of thinking in the documents.

Considering the lack of literature on the education of teacher educators, especially practicing mathematics teacher educators, the study on the teacher professional community provided empirical information about mathematics teacher education in China, especially the structures and meaning relationships underlying teacher learning communities.

Implications, Limitations and Recommendations for Future Research

**Teacher Professional Development Programs.** The findings of classroom teaching indicate the relationships between the autonomy of the individuals and the socio-autonomy in classrooms, and emergent dynamic views of teaching played essential roles in understanding the transformations of classroom teaching. This research especially demonstrates that teachers’ classroom teaching approaches were directly related to what they learned in the associated teaching communities. For example, at Site A, both Ms. N and Ms. B participated in the Key Teacher Professional Development Project at district level, and their teaching showed consistency with what was advocated in the project. Additional research is needed to explore how to help teachers change their expectations for learning from efficiency and skills to processes and class dynamics, and especially how to construct professional development to build on Mr. W’s approaches, which includes changing teacher expectations and goals.

**Teachers’ Beliefs and Teaching Experiences.** Overall, the teachers agreed that mathematics is essential in terms of its knowledge and skills, ways of thinking, and abilities of solving problems with slight differences among them. Students’ personal interests, ways of
learning, and relationship with the teacher were highly valued in teaching and learning for all of them. Below is the summary of the beliefs of mathematics, and mathematics teaching and learning (Table 9).

Table 9: Teachers Perspectives of Mathematics and Mathematics Teaching and Learning at Both Sites

<table>
<thead>
<tr>
<th>Perspectives of Mathematics</th>
<th>Perspectives of Mathematics Teaching and Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms. N</td>
<td>Knowledge and skills</td>
</tr>
<tr>
<td></td>
<td>Building students’ learning interest, good relationship between the teacher and students</td>
</tr>
<tr>
<td>Ms. B</td>
<td>Ways of thinking</td>
</tr>
<tr>
<td></td>
<td>Developing abilities to learn and to be independent in life</td>
</tr>
<tr>
<td>Mr. W</td>
<td>A combination of knowledge, ways of thinking and applications in real life</td>
</tr>
<tr>
<td></td>
<td>Doing experiments</td>
</tr>
<tr>
<td>Ms. R</td>
<td>Abilities of thinking, and problem solving</td>
</tr>
<tr>
<td></td>
<td>Developing interest, and abilities to think, to ask questions, and to solve problems</td>
</tr>
<tr>
<td>Ms. Y</td>
<td>Knowledge and skills, abilities of thinking and problem solving</td>
</tr>
<tr>
<td></td>
<td>Learning basic knowledge, how to think, and develop interest in mathematics learning</td>
</tr>
</tbody>
</table>

This study indicates that those teachers’ beliefs of mathematics and mathematics teaching and learning impact their teaching. For example, Mr. W had very different perspectives of mathematics and teaching mathematics. His learning goals in this lesson were also different from most teachers in this study. Mr. W regarded mathematics learning as doing experiments, while other teachers valued more of knowledge, skills, and logical reasoning. For Mr. W, mathematical knowledge and skills were tools for students to develop their mathematical thinking, and there were different ways of mathematical thinking besides logical thinking, such as guessing, measuring, and intuition. Particularly, this study indicates that even though the teacher might
apply a different approach from traditional teaching, for example, the DJP teaching model at site B, her/his teaching would be focused on knowledge achieving rather than knowledge generation if the teacher valued fundamental knowledge and skills of mathematics. This result is in agreement with the existing research indicating that innovative transformation of classroom teaching is not merely about the teaching method, but also about values, beliefs of certainty, and domination (Fleener & Fry, 1998).

Besides teachers’ beliefs of mathematics, teaching and learning, it seems that teachers’ teaching experience and relationships with students impacted their teaching too. Mr. W taught gifted-students for about ten years. In his presentation, he talked about how he was challenged and inspired by the students. Facing those students, Mr. W did not feel he needed to exert his “authority”; rather, he felt “thrilled” that he could be a learner from his students. His teaching experience seemed to help him build faith in students’ potentials and his attitude to appreciate students’ creativity. Ms. B taught a special group of students, who were from lower-lever income families and received special attention from the school and society. Ms. B believed that independence for her students was essential, and mathematics teaching was to develop students’ abilities and skills to solve the problems independently. In the lesson, Ms. B emphasized learning how to find appropriate ways to overcome the obstacles they encountered. In the DJP school, where about 20% of the students were left behind by their parents to be raised by their grandparents and had no interest in studying. One of the teachers’ responsibilities was to motivate students’ interest and to build their confidence in learning. The DJP teaching model is an effective way for the teachers to help students work together, to take responsibility for their learning, and to demonstrate their abilities in mathematics learning.
For future study, it would be valuable to examine how teachers’ teaching experience and relationships with students affect their teaching practice over time, especially for students from different backgrounds. It would also be important to explore further the relationship between control and emergence in classrooms as learning goals focus on creativity and emergence of ideas.

**Analysis of Classroom Teaching.** The communication structures have indicated different types of teaching with different learning goals and social norms, and mutual relationships in the classrooms at the two Sites. Overall, all the classroom teaching was centered upon students’ problem-solving and presentations of solutions. The communication structures in most classroom teaching, however, are lack of dynamics. Reform efforts seemed not to support the development of mathematical insights and creativity. These issues suggest classroom teaching reform needs to address the power relationships between individuals’ development of mathematical understandings and the socio-autonomy of the class as revealed in communications patterns. Constraints placed on communications controlled by the teacher resulted in linear patterns of communications and dampened emergent and dynamic communications in classroom teaching.

It is noticeable that the analysis of the structures of communication networks is limited within the period of time in each lesson; in particular, the data includes only one lesson for each teacher at Site A, and thus the structures only indicate the teaching in the specific lesson during that class period. The analysis of teaching is based on a synthesis of classroom video-taping, observation, and immediate interviews with the teachers after the lesson, but there is lack of sufficient data to visualize the development of the communication structures over time. Also, At site B, in the DJP model of teaching, small-group discussions were important components. This
study showed that small-group discussion had similar patterns of communications, but there were variations of small-group discussion between advanced students and lower-level students.

Future research is needed to examine the continuous evolution of communication structures in each class, and how the meaning relationships develop and guide the communications across lessons. It is also necessary to explore the relationships between individual autonomy and socio-autonomy in small-groups discussions and in the whole-classroom discussions.

**Roles of the Teacher Educators.** This study indicates two teacher learning communities: the key teacher project-based community at district-level at Site A and the teaching experiment-based learning community at Site B. In the two teacher learning communities, the teacher educators act like hubs to help the classroom teachers understand and implement the reform versions; the leading teachers in the professional learning communities act like sub-hubs to connect to more classroom teachers and form a decentralized structure for the reform implementation in a large scale. Infusing and implementing the reform teaching, those pilot teachers also became teacher educators. For example, at Site A, the teachers have double responsibilities to improve their own teaching practices and to influence mathematics education as a whole in the district. Mr. W’s presentations of the exemplary lesson and his sharing with participating teachers of his views of mathematics and mathematics teaching are an example of how a city-level key teacher acted as teacher educator. From network theory, the decentralized structure is an effective way to infuse the reform ideas (Davis & Sumara, 2006); however, “a decentralized network will decay into a more vulnerable (but informationally efficient) centralized network if stressed” (p. 88). This suggests it is necessary for the classroom teaching reform to consider the relationships between the autonomy and socio-autonomy, between the
teacher educators and the associated communities across different levels. Future research is needed to understand the beliefs and perspectives of those teacher educators regarding mathematics curriculum, reform ideas, and experience with the implementation of teaching. It is also necessary to investigate the impact of the teacher educators as hubs of the networks of communication in the transformation of teaching across different levels (university, local education department, city-level key teachers, and district-level key teacher).

A Final Thought

At the end of this dissertation, I want to share a story about my teaching experience involving my (at that time) five-year-old son. One day I read him a bedtime story about bear mom and dad going on a second honeymoon trip (Berenstain, 1986). Before I read the book, I was curious whether he knew the meaning of the word “honeymoon.” So, I asked him and he replied, “The moon is made out of honey.” I had never thought of such an explanation -- it was refreshing. After I read the book, I asked him the question again.

Mom: Do know what “honeymoon” means now?
Son: Yes, that is a trip.
Mom: What kind of trip?
Son: A bears’ trip.
Mom: Why is it a bears’ trip?
Son: Because bears like honey.
Mom: Will you be going to have a honeymoon when you grow up?
Son: No. I am not a bear and I do not like honey at all.

What another surprise for me! The book does not explain much what honeymoon means, instead, it tells the story of how the bear mom and dad had fun in the trip. What do you think that a child will know about the vital meaning of honeymoon to a couple? Children construct their own meaning based on their experiences and understandings of the conversation in context. Because of that, every child brings different ideas and inspiration into classrooms. As the inspiration song, “The power of the dream” from the Olympic game in Atlanta in 1996 expressed,
Deep within each heart
There lies a magic spark
That lights the fire of our imagination
...
There’s nothing ordinary
In the living of each day
There’s a special part
Every one of us will play

.......
Your mind will take you far
The rest is just pure heart
You’ll find your fate is all your own creation
Every boy and girl
As they come into this world
They bring the gift of hope and inspiration

When teaching is focused on accomplishment of tasks, we often pay attention to how much knowledge they can gain, but forget the meaning students can make, and the meaning of teaching (Fleener & Rogers, 1999). Mathematics education reform advocates a comprehensive development of every student, especially creativity and practical ability in China. The implementation of teaching reform needs a soul that values the dynamics of curriculum, and appreciates and believes in individual potentials.

Curriculum dynamics as a journey, a dance, a song, a dangerous across, is the soul of teaching. It cannot be understood as a prescription, method, or disciplined approach (Fleener, 2004, p.41).

As suggested by Fleener in the above quotation, teaching and learning is process of dialogue in which meaning emerges and develops recursively. Only with an embrace of dynamics, better understanding of individual autonomy and socio-autonomy, and letting go of the control, can an innovative transformation happen in classroom teaching.
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APPENDIX I: TEACHER CONSENT FORM  
(ENGLISH VERSION)

Study Title: An investigation into Chinese mathematics reform classroom teaching.

Location: Interviews and classroom observations including video-taping are conducted in mathematics classrooms in secondary schools in three cities including Beijing, Tianjin and Chengdu in China.

Investigators: The following investigator is available for questions pertaining to this project: Lianfang Lu, 221-M Peabody Hall, LSU (225) 931-2115, llu1@lsu.edu.

Purpose: To investigate Chinese classroom teaching including instructional communication patterns and meaning relationships attached including learning goals, classroom norms, and instructional organizations. In addition, teachers’ and students’ perceptions of mathematics and classroom teaching and learning are examined as well. In doing so, this research is attempted to provide understanding of the holistic and dynamic relationships of the aspects of classroom culture to inform classroom teaching reform and teacher professional development.

Participants: Approximately 10-15 Chinese high school mathematics teachers are selected to participate in this study. Participation is based on recommendations of local educational institutions/departments and teachers’ and students’ willingness.

Procedures: The researcher will observe and video-taping selected teachers’ classroom teaching and lessons. The selected teachers will be interviewed about their perceptions of classroom culture and practices for 30 -45 minute. Interviews will be audio-recorded.

Benefits: This research is to document characteristics of classroom culture and practices that support learning and perspectives of mathematics teaching and learning to seek potential alternatives to support mathematics classroom teaching reform.

Risks: There are no known risks.

Right to Refuse: Participants will not receive any compensation from the researcher for their participation. At any time the participants have the right to withdraw from the project without penalty.
**Privacy:**

All information obtained during this project will be reported using pseudonyms. Some subject identity may be disclosed in the video tapes or pictures for scientific and instructional research activities. Data will be kept indefinitely in a locked file cabinet.

**Signatures:**

The project has been discussed with me and all my questions have been answered. I may direct additional questions regarding study specifics to the investigators. If I have questions about subjects' rights or other concerns, I can contact Robert C. Mathews, Chairman, LSU Institutional Review Board, (225)578-8692.

I agree to participate in the study described above and allow the researchers to use these videotapes and all other information obtained during this project used for research, and for scientific and instructional purposes. I acknowledge the researchers' obligation to provide me with a copy of this consent form if signed by me.

______________________________  __________
Signature                      Date
TEACHER CONSENT FORM
(CHINESE VERSION)

尊敬的老师，

我们想邀请您参加一项关于数学课堂教学的研究。这项研究的目的是要了解在不同的文化背景下数学课堂教学是怎样进行的, 教师和学生对数学教学有什么观点及看法。这项研究结果将用于改善数学课堂教学以及教师培训。我们要录制您的三到五节数学课，然后问您一些关于教学方面的问题。参与这项研究将不会影响您的正常教学。录像带以及问卷只是用于科学研究及教学研究活动及报道。我们将会妥善保管。所得到的数据将经过整理分析后，我们将以匿名的形式书面报道结果。你是否参加这项研究是自愿的。如果有什么问题请与我们联系。我们的电子邮件是 llu1@lsu.edu。电话是 225-931-2115 或 225-578-0091。对参与的权利有什么疑问请与路易斯安娜州立大学研究中心 Robert C. Mathews 联系。联系电话是 225-578-8692。

如果你同意参加这项研究，请在下面签上您的姓名、日期及联系方式。感谢你的支持并附上美好的祝愿！

我读了上面的信并同意参加这项研究。

姓名_________________________  日期_________________  

联系电话及邮件地址 ________________________________
APPENDIX II: PARENT CONSENT FORM
(ENGLISH VERSION)

Study Title: An investigation into Chinese mathematics reform classroom teaching.

Location: Classroom observations including video-taping, survey on students’ perceptions of mathematics classroom culture and practices, and interviews with students are conducted in secondary mathematics classrooms in schools in three cities including Beijing, Tianjin and Chengdu in China.

Investigators: The following investigator is available for questions pertaining to this project: Lianfang Lu, 221-M Peabody Hall, LSU (225) 931-2115, llul1@lsu.edu.

Purpose: To investigate Chinese classroom teaching including instructional communication patterns and meaning relationships attached including learning goals, classroom norms, and instructional organizations. In addition, teachers’ and students’ perceptions of mathematics and classroom teaching and learning are examined as well. In doing so, this research is attempted to provide understanding of the holistic and dynamic relationships of the aspects of classroom culture to inform classroom teaching reform and teacher professional development.

Participants: Approximately 10-15 Chinese high school mathematics teachers are selected to participate in this study. Students 12 -17 years of age who are taking course with the selected Chinese mathematics teachers. Participation is based on recommendations of local educational institutions/departments and teachers’ and students’ willingness.

Procedures: The researcher will observe lessons and video-taping each participating teachers’ classroom teaching. Students’ work in these lessons may be collected or copied as well. Also, a questionnaire will be used to assess your child’s perceptions of mathematics learning. It will take about 20-30 minutes. A short interviews regarding classroom learning culture will be conducted based on willingness.

Benefits: This research is to document characteristics of classroom culture and practices that support learning and perspectives of mathematics teaching and learning to seek potential alternatives to support mathematics classroom teaching reform.

Risks: There are no known risks.
Right to Refuse: Participants will not receive any compensation from the researcher for their participation. At any time the participants have the right to withdraw from the project without penalty.

Privacy: All information obtained during this project will be reported using pseudonyms. Some subject identity may be disclosed in the video tapes or pictures for scientific and instructional research activities. Data will be kept indefinitely in a locked file cabinet.

Signatures: The project has been discussed with me and all my questions have been answered. I may direct additional questions regarding study specifics to the investigators. If I have questions about subjects' rights or other concerns, I can contact Robert C. Mathews, Chairman, LSU Institutional Review Board, (225)578-8692.

I agree my child to participate in the study described above and allow the researchers to use these videotapes and all other information obtained during this project used for research, and for scientific and instructional purposes. I acknowledge the researchers' obligation to provide me with a copy of this consent form if signed by me.

________________________________________________________________________
Student’s Name                                      Classroom Teacher

________________________________________________________________________
Parent/ Guardian Signature                           School

Date

Reader: The parent/guardian has indicated to me that he/she is unable to read. I certify that I have read this consent form to the parent/guardian and explained that by completing the signature line above he/she has given permission for the child to participate in the study.

________________________________________________________________________
Reader Signature                                      Date
尊敬的家长，

我们想邀请您的孩子参加一项关于数学课堂教学的研究。这项研究的目的是要看在不同的文化背景下数学课堂教学是怎样的, 教师和学生对数学教育有什么观点及看法。这个研究课题的结果将用于改善数学课堂教学以及教师培训。我们要对您孩子的数学课堂教学进行录像, 您孩子也会参加一个有关数学学习的问题调查。问卷调查大概需要 20-30 分钟。参加这项研究将不会影响您的孩子的正常学习。录像带以及问卷只是用于科学研究及教学研究活动及报道。我们会对它们进行妥善保管。所得到的数据经过整理、分析后，我们将以匿名的形式书面报道结果。

您的孩子是否参加这项研究是自愿的。如果有什么问题请与我联系。我们的电子邮件是 llu1@lsu.edu。电话是 225-931-2115。对参与的权利有什么疑问请与路易斯安娜洲立大学研究服务中心 Robert C. Mathews 联系。联系电话是 225-578-8692。

如果您同意您的孩子参加这项研究, 请填好下面的表并让您的孩子把它交给老师。

感谢你的支持并附上美好的祝愿！

我读了上面的信并同意我的小孩参加这项研究。

学生姓名__________________________ 数学老师姓名__________________________

家长签字__________________________ 学校__________________________

联系电话或邮件地址__________________________ 日期__________________________
APPENDIX III: CHILD ASSENT FORM
(ENGLISH VERSION)

I, _____________________________, agree to be in a study to find ways to improve mathematics teaching and learning. I can decide to stop being in the study at any time without getting in trouble.

__________________________________________
Student’s Signature

Age

__________________________________________
Witness

Date
学生同意书

我，________________________同意参加这项有助于改善数学课堂教学的研究。我知道我可以随时退出这项研究。

学生签名________________________  年纪________________________

日期________________________
APPENDIX IV: SURVEY ON STUDENTS’ PERCEPTIONS OF MATHEMATICS, OF MATHEMATICS TEACHING AND LEARNING
(ENGLISH VERSION)

Section A: The questions in this section are related to your background information. Please circle the following items that most accurately describe you or write your response in the space provided.

1. Your gender:
   a. Female       b. Male

2. Your grade level:
   a. 7th  b. 8th  c. 9th  d. 10th  e. 11th  f. 12th

3. The name of your school: ________________________________

4. If you have received any award:
   a. Yes       b. No

   If yes, what kinds of awards have you received:
   ________________________________

5. How would you rate your level of mathematics?
   a. Excellent  b. Above average  c. Average  d. Below average

Section B: The items in this section are about your perceptions of mathematics, mathematics learning, and classroom culture. Please circle a number in correspondence to each of statements which you think best expresses the extent you agree or disagree, according to the following rating scale:

1 = Strongly Disagree; 2 = Disagree; 3 = Not Sure; 4= Agree; 5 = strongly Agree.

I: About Your Attitude toward mathematics learning

1. I am confident with my ability of learning mathematics.
   a. Strongly agree       b. Agree       c. Not sure       d. Disagree       e. Strongly disagree

2. To do well in mathematics, you need to work hard.
   a. Strongly agree       b. Agree       c. Not sure       d. Disagree       e. Strongly disagree

3. To do well in mathematics, you need to be smart.
   a. Strongly agree       b. Agree       c. Not sure       d. Disagree       e. Strongly disagree

4. Mathematics is boring.
   a. Strongly agree       b. Agree       c. Not sure       d. Disagree       e. Strongly disagree

5. I feel happy in learning mathematics.
   a. Strongly agree       b. Agree       c. Not sure       d. Disagree       e. Strongly disagree

6. There is no need to learn mathematics.
a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
7. I like to do mathematical problem solving in real life
   a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
8. I have a strong desire to seek solutions when I encounter challenging mathematical problems.
   a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
9. I only like to solve simple mathematics problems.
   a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree

II. About what is mathematics
10. Mathematic is a set of abstract concepts, rules and procedures
    a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
11. Mathematics is a symbolic system to describe the patterns of relationships in real life.
    a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
12. Mathematics is fixed and predetermined.
    a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
13. Mathematics concepts, rules and procedures are consensus. It is changeable.
    a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
14. Mathematical solutions are accurate and free of contradictions and ambiguities.
    a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
15. It is possible that mathematical problems not only have one correct solution.
    a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
16. List the characteristics of mathematics you think.
    __________________________________________________________

17. The motivations for you to learn mathematics are:
    a. It is a required course   b. interested
    c. to get teachers and students’ acceptance   d. to please parents
    e. future advanced education   f. useful tool in daily life
    g. others

III: About mathematics learning processes

18. Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.
    a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
19. The teacher is the primary source of information and resources in the student learning process.
    a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree

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20. Students should be encouraged to justify their solutions, thinking, and conjectures.
   a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
21. The abilities of inquiry are more important than gaining rich mathematics knowledge in mathematics learning.
   a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
22. The process of solving mathematical problem is not important as long as you find a correct answer.
   a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
23. Students should exchange different thinking and approaches of solving a problem each other.
   a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
24. Mathematics problem solving is to develop mathematics thinking abilities.
   a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
25. I learn mathematics better when I learn with partners.
   a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
26. The most valuable things in mathematics learning are

VI. About your recent mathematics learning activities
27. Listen to teachers’ lectures.
   a. often   b. some times   c. very few   d. never
28. Do a lot of drilling exercises.
   a. often   b. some times   c. very few   d. never
29. Propose my own thinking, assumptions or solutions.
   a. often   b. occasionally   c. very few   d. never
30. Discuss with partners about different solutions and mathematical thinking.
   a. often   b. some times   c. very few   d. never
31. Discuss with the teacher about needs and expectation of my mathematics learning
   a. often   b. some times   c. very few   d. never

V. About mathematics classroom learning environment
32. Good student-teacher relationship promotes learning.
   a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
33. Good student-student relationship promotes learning
   a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
34. Mathematics classroom climate affects learning.
   a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
35. Mathematics classroom is an organic learning community. Individual and the collective have close relationship.
   a. Strongly agree   b. Agree   c. Not sure   d. Disagree   e. Strongly disagree
36. Mathematics classroom teacher is:
   a. A deliver of knowledge  b. an organizer of learning  c. a problem-solver  d. a participant of knowledge creation
   e. others

37. My role in the classroom is:
   a. A listener  b. an acceptant of knowledge  c. a question-generator  d. a participant of knowledge creation
   e. others

38. Does your classroom have any learning organization? What learning activities involved in it?
   a. Yes  b. No
   If yes, please describe the organization, activities involved, and its impact on your learning.

39. Please describe the characteristics of a typical mathematics classroom you have experienced. How do they impact your mathematics learning, and what differences and commonalities do your mathematics classroom have from other classrooms?

40. Describe your ideal mathematics classroom.

41. Describe approaches you communicate with your teacher.

42. In what situation, you learn mathematics best?

43. In what situation you learn mathematics worst?
44. Describe characteristics of your favorite (or dislike) mathematics teacher.

_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________

Thank you!
SURVEY ON STUDENTS’ PERCEPTIONS OF MATHEMATICS,
OF MATHEMATICS TEACHING AND LEARNING
(CHINESE VERSION)

学生数学学习及数学课堂教学观问卷调查

第一部分：这部分是有关您的个人基本信息。请于适当处勾选或填写。

1. 您的性别：
   a. 女  b. 男
2. 您就读的年级：
   a. 初中一年级  b. 初中二年级  c. 初中三年级  d. 高中一年级  e. 高中二年级  f. 高中三年级
3. 您所在的学校的名称是：______________________________
4. 您是否受过奖励：
   a. 是  b. 否
   如果是，何种奖励：________________________________
5. 您的数学水平在班级：
   a. 好  b. 平均水平以上  c. 平均水平  d. 平均水平以下

第二部分：这部分是有关您对数学学习及数学课堂教学的看法。请于适当处勾选。

I. 关于数学学习的态度

1. 我自信有能力学好数学。
   a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意
2. 数学需要努力才能学好。
   a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意
3. 数学需要聪明才能学好。
   a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意
4. 我觉得数学枯燥无味。
   a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意
5. 数学学习使我感到充实和快乐。
   a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意
6. 我认为没有必要学习数学。
   a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意
7. 我喜欢提出生活中的数学问题，并努力解决问题。
   a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意
8. 遇到难题时，我有强烈欲望去寻求解题方案。
   a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意
9. 我只愿意解决简单的数学问题。
   a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意
II. 关于什么是数学

10. 数学是一套抽象的概念、定理、方法。
    a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意

11. 数学是描述自然界各种关系模式的符号语言系统。
    a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意

12. 数学结论是绝对的，是永恒不变的。
    a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意

13. 数学概念、定理、方法是人们在一定程度上达成的共识，它是可以变化的。
    a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意

14. 数学答案要求准确无误，不能模棱两可。
    a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意

15. 数学问题可能没有唯一正确的答案。
    a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意

16. 您认为什么是数学？有什么特点？
    ________________________________________________________________

17. 我学习数学的动机是:
    a. 它是学校必修课  b. 浓厚的兴趣
    c. 得到老师同学的认可  d. 让父母高兴
    e. 将来学习的需要  f. 数学日常生活中是有用的工具
    g. 其他__________________________________________________________

III. 关于数学学习过程

18. 数学学习的过程就是学生吸收信息，通过反复练习，强化所获得的信息，再以零散的形式储存起来，以便以后提取信息。
    a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意

19. 教师是学生学习过程中主要的知识和信息来源。
    a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意

20. 教师应该鼓励学生提出自己单独的想法、猜想，或解答。
    a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意

21. 在数学学习中，探究新事物的能力比获得丰富的数学知识更重要。
    a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意

22. 在解答数学问题时只要找到正确答案，过程并不重要。
    a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意

23. 学生应当彼此交流不同的解决问题的方法和思想。
    a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意

24. 解数学题的目的是为了锻炼数学思维能力。
    a. 非常同意  b. 同意  c. 不清楚  d. 不同意  e. 非常不同意

25. 在与同学的共同学习中我学得更好。
26. 您认为什么是数学学习中最重要的，有价值的？

IV. 近期数学学习过程包括
27. 聆听老师的讲解。
   a. 经常       b. 偶尔       c. 很少       d. 从不
28. 做大量的练习题。
   a. 经常       b. 偶尔       c. 很少       d. 从不
29. 提出自己的想法、猜想、或解答。
   a. 经常       b. 偶尔       c. 很少       d. 从不
30. 同学之间交流解决问题的不同方法和思想。
   a. 经常       b. 偶尔       c. 很少       d. 从不
31. 与老师讨论我的学习需要和期望。
   a. 经常       b. 偶尔       c. 很少       d. 从不

V. 关于数学课堂及管理方式
32. 良好的师生关系促进我的学习。
   a. 非常同意       b. 同意       c. 不清楚       d. 不同意       e. 非常不同意
33. 良好的同学关系促进我的学习。
   a. 非常同意       b. 同意       c. 不清楚       d. 不同意       e. 非常不同意
34. 良好的数学课堂氛围促进我的学习。
   a. 非常同意       b. 同意       c. 不清楚       d. 不同意       e. 非常不同意
35. 数学课堂是一个有机整体，我与这个集体息息相关。
   a. 非常同意       b. 同意       c. 不清楚       d. 不同意       e. 非常不同意
36. 数学老师是数学课堂上的:
   a. 知识的传道者       b. 学习的组织者       c. 疑难问题解答者       d. 知识生成的参与者       e. 其他

37. 您所在的班级里是否有与数学学习有关的组织形式，或活动？
   a. 是       b. 否
   如果是，请简述，比如组织形式、管理方式、任何小组活动，对你学习的影响 …………

_________________________________________________________________________

_________________________________________________________________________

_________________________________________________________________________
38. 描述您所在的数学课堂最典型的特征。它们怎样影响您的数学学习？您所在的数学课堂与别的课堂有什么相同和不同之处？

____________________________________________________________________________________
____________________________________________________________________________________
____________________________________________________________________________________

39. 描述您最理想的数学课堂。

____________________________________________________________________________________
____________________________________________________________________________________
____________________________________________________________________________________

40. 您与老师交流的渠道和方式有：

____________________________________________________________________________________
____________________________________________________________________________________
____________________________________________________________________________________

41. 在什么情况下您的数学学得最好？

____________________________________________________________________________________
____________________________________________________________________________________
____________________________________________________________________________________

42. 在什么情况下您的数学学得不好？

____________________________________________________________________________________
____________________________________________________________________________________
____________________________________________________________________________________

43. 描述您最喜欢（或不喜欢）的数学老师的特征？

____________________________________________________________________________________
____________________________________________________________________________________
____________________________________________________________________________________

谢谢您！
APPENDIX V: INTERVIEW PROTOCOL WITH TEACHERS (ENGLISH VERSION)

I. Biographical information and teaching and teacher professional experience

- How many years have you been teaching mathematics?
- What experience do you have in teaching?
- What education/profession development do you have?

II. Questions related to classroom observation (goals, class norms, communication, and instructional organization in general)

- What are your goals, objectives, representations used, type of tasks, and concepts of the lesson?
- What are the main factors affecting your preparation? How?
- What components are usually included in your lesson plan?
- What criteria to decide which questions to ask in classroom?
- What impact you adjustment in implementing your lesson plan?
- Comment about/reflection of this lesson?

III. Questions regarding teacher’s beliefs of mathematics, mathematics teaching and learning

- What is mathematics? Why do we need mathematics?
- What is mathematics teaching?
- What are the most valuable things in learning mathematics?
- How can students learning best?
- What is your ideal classroom teaching?
INTERVIEW PROTOCOL WITH TEACHERS
(CHINESE VERSION)

教师访谈提纲

I: 个人教育及教学信息:

- 简述过去教学经验
- 简述教师培训经验

II. 与课堂教学相关的问题

- 您本节课的教学目的及教学任务是什么？
- 影响您备课的主要因素是什么？
- 您教案的主要组成部分有什么？
- 您课堂提问有什么标准吗？
- 您根据什么来调整课堂教学？
- 您自我感觉这堂课怎样？

III. 教师数学，以及数学教育观

- 您认为什么是数学？
- 什么是数学教育？为什么数学学习重要？
- 学生怎样学习数学最好？
- 您认为什么是数学学习中最重要、最有价值的东西？
- 您最希望的课堂是怎样的？
APPENDIX VI: AN EXAMPLE OF STUDY PLAN

第 7 课时 三角形的中位线

【学习目标】1. 探究并掌握三角形中位线的概念及特性；
2. 理解三角形中位线定理的推导过程；
3. 能利用三角形中位线定理解决有关的计算及证明。

【学习重点】理解三角形中位线定理，并运用它求解问题。

【学习难点】灵活运用三角形中位线定理解决有关问题。

【学习过程】

一. 学习准备：
1. 三角形、中点、中位线；
2. 如图 1，在△ABC 中，D 是 BC 中点，那么 AD 是△ABC 的中位线。
3. 一个三角形的中位线有 3 条，如图 1 中标出它的中位线。
4. △ABC 的中位线 AD 与△ABD 和△ACD 的中位线有什么关系？为什么？

二. 探究破解——三角形的中位线的定义及性质
5. 如图 2，若，D，E 是△ABC 中 AB，AC 的中点，那么 DE 是△ABC 的中位线。
三角形的中位线定义，连接三角形两边中点的，叫做三角形的中位线。

三. 你认为三角形的中位线与三角形的中线有什么不同？
（1）如下图 3，在△ABC 中，D，E 是 AB，BC 的中点，DE 是△ABC 的中位线；
（2）如图 4，在△ABC 中，D 是 BC 的中点，AD 是△ABC 的中位线。

7. 在△ABC 中，D，E 是 AB，AC 的中点，猜想 DE 与 BC 有什么数量关系与位置关系？（用刻度尺量一量，验证你的猜想）

已知：△ABC 中，D，E 是 AB，AC 的中点，试估算 DE // BC 且 DE = \frac{1}{2} BC

证明：延长 DE 到 F，使 EF = DE，连接 CF。
在△ABD 与△BDE 中，D 是 AB 的中点
\angle AED = \frac{1}{2} \angle ABC
\angle D = \frac{1}{2} \angle ACB
\angle ADE = \angle BDE
\therefore \triangle ADE \cong \triangle BDE (SAS)
\therefore AD = BD, CF = BF = \frac{1}{2} BC
\therefore 四边形 BCFD 是平行四边形
8. 如图，△ABC中，AC=BC，M、N分别为AB、BC的中点，MN=3cm，则 AB=_____cm。

9. △ABC中，AB=6cm，AC=4cm，BC=10cm，D、E、F分别作AB、BC、AC的中点，则△DEF的周长=_____cm，△ABC的周长=_____cm。

三、典型例题

10. 如图1，已知四边形ABCD中，对角线AC、BD交于点O，ED=EF，E、F分别是OC、OD、AB的中点。

求证：（1）BE⊥AC，（2）EG=EF

证：

（1）∵E、F是OC、OD的中点，
∴ED是△OCD的中位线，
∴ED∥BC，
∴BE⊥AC

（2）∵E、F是OC、OD的中点，
∴EF是△OCD的中位线，
∴EF=1/2CD

同时练习

已知四边形ABCD中，AB=CD，EG=EF，求证：EG∥AB

证：

（A）∵EG=EF
∴EF是三角形OCD的中位线
∴AB=CD

四、拓展思维——典型思维陷阱

11. 如图2，若△ABC中，D、E、F分别为AB、AC、BC的中点，求证：EF=1/2BC

证：

（A）∵D、E、F分别为AB、AC、BC的中点，
∴EF是△ABC的中位线，
∴EF=1/2BC
# APPENDIX VII: CLASS OBSERVATION SCHEDULE

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<th>Date</th>
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VITA

Lianfang Lu received her Bachelor of Science degree in mathematics from Sichuan Normal University in China in 1990. During 1990-2000 she was a high school mathematics teacher where she obtained teaching experience with high school students and built passion in mathematics education. She began her graduate study in mathematics education at the University of Oklahoma in 2002 and got her master’s degree in mathematics education in 2005. After that, she worked as a curriculum developer intern at the Advanced Academics Inc. in OKC, where she evaluated the teaching and curriculum materials submitted by teachers for on-line teaching and learning, and provided interactive teaching materials to assist student learning. In 2006, she followed her advisor M. Jayne Fleener at the University of Oklahoma to Louisiana State University to continue her doctoral study in the Department of Curriculum and Instruction. Meanwhile, she taught elementary math methods course and worked with the Assessment and Accountability project as a graduate assistant in the College of Education.

In the years of pursuing her degrees and working in mathematics education in the USA and China, she taught different levels of students across elementary schools to colleges and worked in the areas of curriculum and instruction, teaching, research, assessment and educational services. Her experiences have kept her growing as a mathematics educator. She believes mathematics teaching is to invoke students’ thoughts and to assist them in constructing rich and robust understanding of mathematical ideas and to build an intimate life-long relationship with mathematics. Her research interests are particularly in classroom teaching and learning, teacher education, curriculum development, and international mathematics education.