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Formative assessment in Algebra II

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FORMATIVE ASSESSMENT IN ALGEBRA II

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

This work reviews the assessment design process in depth, as relevant to classroom practice. The "assessment triangle'' model of assessment and other design components are described. We discuss the content standards and the assigned classroom curriculum of the unit of the East Baton Rouge Algebra II course, which deals with quadratics. We offer an assessment that meets standards of the models that were reviewed.

INTRODUCTION

Designing effective classroom assessment is important for many reasons. Good assessments challenge learners to think critically and apply their knowledge. Teachers have a duty to prepare students for higher-level mathematics, higher education, and the workforce through valid assessments.

I have taught Algebra II for several years but have lacked assessments that I felt adequate to the level of instruction that I try to uphold. This has given me motivation to design better Algebra II tests. I would like to give other teachers the opportunity to use high-quality formative assessments.

My plan is to use an accepted theory of assessment to examine some existing tests. I used the book *Knowing What Students Know* as a basis for my work on assessment design. The theory of the assessment triangle guides this evaluation and helps me in designing a higher quality test.

Test design must also take into account the distinction between formative and summative assessment as well as the issues of validity, reliability and fairness. Most important, good tests have take into account the relevant learning goals. I took a close look at the Core Curriculum Standards as a basis for my test questions. I examined the state and district curriculum and used them to design a valid test.

My work concerns the unit on quadratics. I examined the entire content of Algebra II to determine where the vocabulary and ideas of the quadratics unit were used elsewhere in the course in order to assure that I would have a useful formative assessment.

The test that I present in chapter 5 of this thesis incorporates all my research. I provide a solution key along with a list of correlations between the test questions and the state, district, and common core standards. I believe that I have demonstrated that the test I offer here is of a high-quality. I hope this test will be considered a contribution to the curriculum by the other teachers in my school and district.

CHAPTER 1: THEORY OF ASSESSMENT

This chapter lays out the basic theory of assessment. We review the conceptual framework of the assessment triangle and its connection to the classroom and teacher professional development.

1.1 The Assessment Triangle

Assessment is a process of reasoning from evidence. It involves making assumptions about how students learn, finding a means of generating evidence of student learning and creating a way of interpreting that evidence. The book, *Knowing What Students Know*, presents a survey and synthesis of thinking on educational assessment that clarifies the manner in which this reasoning is performed. The authors introduce the idea of the 'assessment triangle' as a means of summarizing the key features that every assessment possesses. These features refer to the manner in which the author of an assessment thinks about and makes assumptions about student learning using the categories of cognition, observation, and interpretation.

When teachers develop assessments, they need to answer questions such as following: "How does a teacher know if a student has mastered the concepts being tested?" "What would we accept as evidence of a desired proficiency level?" "What assignments or tasks can teachers assign that would let the teacher know if the students have grasped the concept and have mastered it?" "How can the results of assessments be interpreted in a meaningful way?" Quality assessments cannot be created and implemented without a lot of consideration of such questions and how they relate to one another.

The main points of concern can be represented in the form of a triangle; see Figure 1.

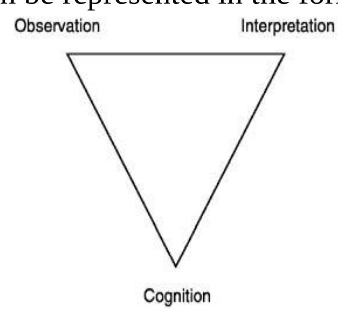

Figure 1. The Assessment Triangle

The vertices of the triangle are:

- cognition, the cognitive state or level of knowledge and skill of students who are to be assessed;
- observation, the tasks that will be used to generate evidence about student learning;
- interpretation, the methods used to draw conclusions about the cognitive state of the students from the evidence obtained by observing their performance (p. 19 kwsk).

These components of assessment are inter-connected and inter-reliant. The edges of the triangle are meant to indicate this.

An example is one of testing vocabulary words. The cognitive aspect would be the student internalizing the definition of the word. The observation aspect would be the teacher writing a phrase with missing information and the student recognizes what word fits into the missing blank. The interpretation aspect would be the teacher grading the student's test that includes a question using that word and based on the student's response, make a determination if the student understood that word.

1.2 Cognition

Cognition is the first vertex of the assessment triangle. Cognition is a model of how students think and learn in a subject. It can be productive for a teacher who wants to design an assessment to begin with a model of learning. Cognitive models can be used to classify the difficulties that students may encounter, the steps they take in the learning process, and the goals that the teacher expects. The design of the assessment begins with a description of knowledge and skills that are to be learned as well as the way in which that knowledge develops.

Bloom's Taxonomy is an example of a cognitive theory. It is a good example for illustrative purposes because it is used by many educators. In 1956, Benjamin Bloom led a group of educational psychologists who classified levels of intellectual behavior. During the 1990's, Lorin Anderson, a former student of Bloom, led a group of cognitive psychologists as they updated the taxonomy (Bloom, 1956) (Milton, Pollio, and Eison, 1986). In Bloom's Taxonomy, there are six levels of knowledge. In the following list, I will illustrate each with an example from mathematics:

- remembering: having students define vocabulary words or state the axis of symmetry of a quadratic function, given its vertex.
- understanding: having students classify functions by their attributes or describe the nature of the roots of a quadratic given the discriminant.
- applying: having students solve a quadratic equation or sketch the graph of a quadratic function after finding points on the graph.
- analyzing: having students compare and contrast the graphs of a linear function and a quadratic function, or examine a graph and discuss intercepts, extreme values, symmetry and end behavior.
- evaluating: having students solve a mathematics word problem and explain more than one way to get the same solution, or to solve a quadratic equation and explain how they could have solved it using different methods.
- creating: having students develop a new function by combining two more functions and define the new function's properties, or design a picture using many functions in a coordinate plane and write the equations of the functions used, with their restrictions.

It may be difficult to create a test that incorporates all levels of Bloom's taxonomy, but cognitive-based tests include some questions that incorporate higher levels of thinking rather than just recall and comprehension. Bloom's taxonomy can be condensed into three general levels: recall or recognition of specific information, comprehension and application, and problem solving. Problem solving is one of the best ways of assessing students' knowledge (Crooks, 1988).

1.3 Observation

The next vertex of the triangle is observation. This refers to the tasks or activities that students are asked to perform to provide evidence about learning. Observational tasks are designed for collecting evidence to support assessments. These tasks must be designed with the relevant cognitive model in mind. Designing tasks using a scientific and content-driven approach will result in evidence that can lead to valid and reliable inferences. This approach starts with identifying the knowledge and skills that are assessed which then guide the selection of relevant tasks and ways of scoring assessments (Messick, 1994) (p. 194, kwsk).

Observational tasks are what learners say or do such as their words, actions, and the work they produce. Student work includes observation and discussion in the classroom, written work done in the classroom or at home, and written assessments or projects. Examples include the learner working out a problem on the board, explaining a definition to another learner, or modeling a problem on their paper.

1.4 Interpretation

The third vertex of the triangle is interpretation, the process of drawing conclusions from the evidence that is collected from the students as they perform the assessment tasks (p. 19 kwsk). The interpretation vertex of the triangle pulls information from the cognition and observation vertices in order to draw conclusions about the student's mastery of the material being taught.

Interpretation is sometimes understood in terms of statistical analysis. While statistical analysis may provide a summation of what students have learned, only knowing the rates at which questions are answered correctly within some population of students is not as helpful as knowing why some students answered a particular item incorrectly.

The interpretation vertex of the assessment triangle is relevant to high-stakes testing in schools. The state reports standardized test results to individual schools and to the school authorities who then compile the results for their core content areas. Based on the test results, the school authorities then make recommendations of adjustments of teaching strategies to the core content teachers. Then the teachers adjust their teaching strategies with the goal of increasing student learning.

1.5 Assessment Triangle Connection to Classroom Assessment

As we have said, every assessment is based on ideas about how knowledge and understanding increase over time (p. 20 kwsk). Cognition, observation, and interpretation are based on knowledge of student learning and how learning is measured. The three vertices of the triangle are linked by teaching students, students showing their learning on an assessment, and the teacher interpreting the assessment results to be used for future teaching. Thus, the process of designing an assessment takes into consideration each of these three elements.

In the classroom setting, teachers teach their students using lecturing, questioning, and modeling, which increases students' understanding. Students perform observational tasks to demonstrate their content knowledge. Students may show evidence of their understanding by answering teachers' questions, writing or producing projects, or by interacting with peers. Teachers create assessments to assess student content knowledge, but this also provides students with feedback about their learning and how they can improve. Teachers may get a better understanding of student thinking patterns and use this knowledge to modify their instruction (p.292, kwsk). If teachers understand and implement the concepts of the assessment triangle, then students, teachers and schools will reap the benefits.

CHAPTER 2: PRACTICE OF ASSESSMENT

As we have seen in chapter one, assessment is a process of reasoning from evidence. It involves making assumptions about how students learn, finding a means of measuring student learning and a creating a way of interpreting that evidence. This chapter presents a discussion on the purposes of assessment, formative and summative assessments, and validity, reliability, and fairness of assessments. It also presents a discussion on the state standards that what students need to learn. It presents general facts and considerations that influenced my work on test design in a substantive way.

2.1 Purposes of Assessment

Assessments are a necessary education tool for teachers. An assessment has many functions. They help teachers evaluate and interpret student learning. They motivate students to study. Interpreting assessments aids teachers in knowing how successful they were in presenting the material. Also, assessments reinforce learning by showing what concepts students have not mastered and are advised to relearn.

When students know they have an assessment on certain concepts, they are motivated to pay attention and learn the material. A study by Crooks, McKeachie, and Wergin finds that students will change their study habits to reflect their expectations about the assessment. If the assessment expectations are memorization of details, that is all students will do. If students know they are expected to apply their knowledge and solve problems, they will study and work toward that goal (Crooks, 1988, McKeachie, 1986, and Wergin 1988).

Assessments are called by several different names: quizzes, tests, and exams. An examination is the most comprehensive form of testing (Gross, 1993). A test is more limited than an examination, focusing on certain parts of the course material. A quiz is even more limited in length than a test with both of them being formative in nature. The term “test” will be used throughout the paper (Jacobs and Chase, 1992).

2.2 Formative Assessment

Formative assessments include all activities that teachers and students do to obtain information that can be used to diagnose teaching and learning. Some examples of formative assessments are classroom discussion, teacher observation, and analysis of student work. Formative assessments involve evaluating the information gained from testing and using it to alter the teaching methods to increase student learning. Examples of adjustments are: trying alternative ideas to teaching, re-teaching concepts, and offering extra opportunities for practice. A variety of these activities can lead to improved student success (Black and Wiliam, 1998b).

Students receive feedback after a formative assessment which helps them become aware of any misconceptions they have about the concepts they are trying to master (Ramaprasad, 1983; Sadler, 1989). Feedback on suggestions for improvement encourages students to focus their attention on grasping the concept rather than on simply getting the correct answer (Bangert-

Drowns, Kulick, & Morgan, 1991; Elawar & Corno, 1985). Feedback from formative assessments can motivate students to have the desire to learn (Ames, 1992; Vispoel & Austin, 1995).

2.3 Summative Assessment

Summative assessment is an assessment of learning after a length of time of instruction. Students take summative assessments for the purposes of grading or standardized testing.

Midterm or final exams are summative assessments are a culmination of the coursework that the student has done. The results of a summative test are interpreted to see if students have understood and mastered the material taught. Teachers use the results of summative tests to evaluate how they have taught the material tested.

Summative assessments based on the state standards are in the form of standardized, high-stakes, and end-of-year testing which determine if students progress to the next level. Interpretation of high-stakes testing is used as a snapshot to show how a school is performing.

2.4 Validity, Reliability, and Fairness in Assessments

It is essential for tests to be valid, reliable and fair. Validity in testing refers to how much a test’s content represents the actual skills learned and whether accurate conclusions can be attained from the test. A test is considered valid if it tests what it is meant to test. In a valid test, results are appropriately interpreted, and the validity of a test refers to the results, not to the test itself (Gronlund and Linn, 1990). The content of a test should be proportional to the amount of time spent learning the content in the classroom. If this proportionality occurs, then interpretation of the test scores is likely to have greater validity.

The reliability of a test means it is accurate and consistent in evaluating a student's performance. A test is said to be reliable if a student takes a test many times under the same conditions and gets the same results (this is assuming the student did not remember the test questions from taking it the previous times). A reliable test has clear and concise questions and directions. A particularly short or lengthy test may not be particularly reliable.

Issues of fairness can arise in classroom assessment. A fair test can be defined as one that allows valid inferences to be compared from person to person and group to group and is not biased. A biased assessment is one in which certain characteristics of the assessment produce different meanings for different subgroups. An assessment of mathematical reasoning must include words and expressions that are generally used, and not associated with particular cultures or regions (p. 215, kwsk). Differences between the cultural backgrounds of the teacher and students can lead to teacher bias. Teachers of a different culture or socioeconomic background than their students may ask questions in class that may be substantially different from what the students may experience at home, therefore, putting them at a disadvantage.

2.5 How Do Standards Affect Teacher Practice and Assessments?

State and national standards are developed so that teachers will treat the same concepts at the same performance levels in the same grade levels. This way, students will get an equal level of education, no matter what school they attend. Classroom teachers have to follow the guidelines imposed upon them by the regulatory authorities that uphold the standards. They are required to cover certain material within the subject they are teaching; a textbook or curriculum may be mandated. Assessments must be consistent with curriculum standards and with the mandated materials.

2.6 Standards: The National Scene

In 1981, the National Commission on Excellence in Education was created by then Secretary of Education T.H. Bell (http://www2.ed.gov/pubs/NatAtRisk/index.htmlIts). Its mission was to examine the quality of education and make recommendations for educational improvements. Their findings were reported in 1983 in a report called *A Nation at Risk* (http://www2.ed.gov/pubs/NatAtRisk/index.html). *A Nation at Risk* gave a detailed report of recommendations on educational reforms that were to be implemented over the next few years. The first organizational response to A Nation at Risk's call for standards in math was NCTM (www.nctm.org).

In 1989, the National Council of Teachers of Mathematics (NCTM) released "The *Curriculum and Evaluation Standards for School Mathematics* (www.nctm.org). This document is a set of standards for the content to be included in the mathematics curriculum and was used to establish a framework to guide school reform. In 1991, the NCTM published *Professional Standards for Teaching Mathematics* that described the elements of effective mathematics teaching (www.nctm.org). In 1995, *Assessment Standards for School Mathematics* was published and established objectives for assessment practices (www.standards.nctm.org).

NCTM updated its existing standards documents; resulting in the Standards 2000 project which began in 1997 (www.standards.nctm.org). Many different sources were used, including curriculum materials, state curriculum documents, research publications, policy documents, and international curriculum materials. This resulted in the book *Principles and Standards for School Mathematics* (www.standards.nctm.org).

2.7 Standards in Louisiana

Louisiana's educational reform was developed on the basis of rigorous and challenging content standards. The Louisiana State Board of Elementary and Secondary Education, BESE, was established during the Louisiana Constitutional Convention in 1974 and became the policy-making body for elementary-secondary schools (www.louisianaschools.net). The board did not get involved with setting standards until much later.

Louisiana began a process of raising academic standards in the early 1990's (www.doe.state.la.us). In 1997, the first attempt was made by the state to establish standards for classroom curriculum. The resulting document was called the "Frameworks" (http://www.doe.state.la.us/lde/uploads/2910.pdf).

Standardized high-stakes testing in Louisiana started in 1999, placing focused demands on schools (www.doe.state.la.us). Schools were required to administer the LEAP exam beginning in 1999 and the GEE in 2002 (www.doe.state.la.us). The IOWA test of basic skills was administered from 1998 to 2005 and then was replaced by the ILEAP exam that is currently given (www.doe.state.la.us). In 2004, the Grade Level Expectations (GLEs) were created to provide a better description of what was necessary to teach, and also to respond to demands of the "No Child Left Behind Act," enacted in 2001 (www.doe.state.la.us).

In 2005, BESE ordered the creation of the Louisiana Comprehensive Curriculum to guide the implementation of the grade-level expectations (www.doe.state.la.us). The LACC was revised in 2008 (www.doe.state.la.us). This curriculum was based on the Grade-Level Expectations, which are statements of what students should know or could do by the end of each grade and for each core content area (www.doe.state.la.us).

The Louisiana Comprehensive Curriculum is given to every parish in the state. It is aligned with state content standards and organized into units with a description of the curriculum for each unit, sample activities, black-line masters, and some classroom assessments. This is the curriculum all parishes in Louisiana are expected to teach although they may revise it for their own use (www.doe.state.la.us).

2.8 Parish Level East Baton Rouge Parish Comprehensive Curriculum (EBRCC)

The state mandates the use of the LACC or a locally-modified version. The East Baton Rouge Parish school system created a revision of the Louisiana Comprehensive Curriculum in the summer of 2006 and adopted it as the official curriculum for the district (Dauzat, 2010). This is called the EBRPSS Comprehensive Curriculum. It has gone through several revisions since 2006, based on feedback from teachers and input from curriculum trainers. All the suggested tests included in the East Baton Rouge Parish Comprehensive Curriculum were created by East Baton Rouge Parish teachers.

Revisions of the EBRCC are based on teacher feedback and are supposed to preserve the continuity of the curriculum. The district's math content trainers ask teachers to fill out feedback forms to indicate where they feel the curriculum should be revised. The math supervisor for East Baton Rouge Parish says that "revisions are based on a consensus of teachers and what is best for the students of EBRPSS" (Dauzat, 2010).

The state does not exercise quality control over local versions. When the Louisiana Algebra II comprehensive curriculum was revised for East Baton Rouge Parish, the order of the units was changed. For example, quadratic functions appeared in Unit 5 of the LACC, but they were moved to Unit 2 in the EBRCC. The presentation of quadratics in the LACC presupposed familiarity with topics such as factoring, complex numbers, and simplifying radical expressions. After the rearrangement, these things were not taught prior to the quadratics unit. For example, factoring quadratic equations such as $x^2 - 5x - 14$ is taught in unit 3, but solving the quadratic equation $x^4 - 5x^2 - 14 = 0$ is taught in Unit 2, before students have even learned how to factor quadratics at all. Another example is solving the quadratic equation $x^2 - 2x + 8 = 0$. This problem has solutions of $x = 1 \pm i\sqrt{7}$, and finding them involves simplifying radical expressions and using complex numbers, which are not taught until unit 4.

2.9 Common Core Standards

The Common Core Standards is a state-led effort by the National Governors' Association and the Council of Chief State School Officers (www.corestandards.org). "The goal of these standards is to define the knowledge and skills students should have within their K-12 education careers so that they will graduate high school and be able to succeed in entry-level, credit-bearing academic college courses and workforce training programs" (www.corestandards.org). The intent of the standards is to build upon the standards that individual states are using. The standards were released in June of 2010 (www.corestandards.org). The intent of the authors was to use the highest and most effective models from many states and countries and to provide parents, teachers, and school authorities with common goals of student learning. The common core standards define a framework for mathematics curriculum for all students regardless of where they live (http://www.corestandards.org).

Louisiana will adopt these Common Core Standards as the foundation for the curriculum. The Louisiana Comprehensive Curriculum will have to be revised to reflect the new standards and the East Baton Rouge Parish Comprehensive Curriculum will also have to be revised. Then teachers will have to adjust their teaching strategies and student expectations to conform to the new standards. Hopefully, the new standards, curriculum and expectations will increase student achievement.

CHAPTER 3: ALGEBRA II

This chapter examines the Algebra II course that is mandated by East Baton Rouge Parish School System, outlining the concepts in each unit of the comprehensive curriculum. My goal is to describe what students are expected to learn in order to be successful. Throughout this chapter, "Algebra II" refers to the EBRPSS course.

3.1 What Is Algebra II?

The central content strands in Algebra II are summarized in the following quotation taken from the website of another school district (www.sbcusd.com). "This discipline complements and expands the mathematical content and concepts of Algebra I and Geometry. Students who master Algebra II will gain experience with algebraic solutions of problems in various content areas, including the solution of systems of quadratic equations, logarithmic and exponential functions, the binomial theorem, and the complex number system." This is an accurate description of the EBRPSS course, except that it omits mathematical modeling, which is also a focus of Algebra II. "Mathematical modeling consists of recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem" (www.achieve.org/ADP).

3.2 Topics in Algebra II

In Algebra II, many functions are investigated by looking at significant points on the graph, sketching the curve, and examining other relevant features. Specifically, the functions studied in the Algebra II curriculum include linear, quadratic, higher-order polynomial, radical, rational expression, logarithmic and exponential functions. The graphing calculator is used extensively in higher-level mathematics courses and the student should feel comfortable with its use after completing the Algebra II course.

Some topics in Algebra II have been introduced in Algebra I and are reviewed in Algebra II. Much of Algebra I concerns linear functions. Algebra I topics include:

- reasoning quantitatively and using units(i.e. feet, seconds, pounds) to solve problems with linear functions;
- finding and interpreting intercepts and the slope of linear functions;
- function notation and evaluating linear functions;
- writing and interpreting expressions;
- creating linear equations;
- reasoning with linear equations and inequalities;
- reasoning with absolute value inequalities in one variable;
- interpreting and building linear functions;
- comparing linear and exponential models and solving problems;

- using the modeling cycle with linear functions which includes identifying variables, formulating a model, analyzing and performing operations, interpreting the results, validating the conclusions and reporting on the conclusions and the reasoning behind them (www.corestandards.org).

Algebra II extends many ideas of Algebra I to polynomials functions. Topics include:

- extending the properties of exponents to rational exponents;
- using properties of rational and irrational numbers;
- reasoning quantitatively and using units to solve problems with polynomial functions;
- understanding the complex number system;
- arithmetic with polynomial and rational functions;
- creating polynomial equations;
- reasoning with polynomial equations and inequalities;
- interpreting and building polynomial functions;
- constructing and comparing linear, exponential and quadratic models and solving problems;
- using the modeling cycle with polynomial functions which is identifying variables, formulating a model, analyzing and performing operations, interpreting the results, validating the conclusions and reporting on the conclusions and the reasoning behind them;
- graphing, finding key points, and solving equations of the radical, polynomial, quadratic, rational, logarithmic, and exponential types;
- graphing conic sections (www.corestandards.org).

3.3 The Units of the Algebra II Comprehensive Curriculum

Unit 0 involves a review of basic skills mastered in Algebra I. These skills include simplifying and evaluating expressions, solving linear equations with variables on both sides of the equation. The unit also includes problems involving modeling real-world problems with linear functions. Students use calculators to assist in computations. Typical problems students should be able to solve at the end of the unit include:

1. Simplify $x^2 + 3x^2 + 4 – 2x + 3x^2$ and evaluate this expression at $x = 2$.
2. Solve $2x – 6 + 3(x – 4) = x + 2$.
3. Solve word problems such as: John has x number of baseballs, Jill has four more than twice as many as John has, and Jack has three less than half of the amount of John. If there are 29 baseballs in all, how many baseballs does each person have? Write and solve an algebraic equation to model this situation.

Unit 1 reviews facts about linear functions and explores absolute value, step, and piecewise functions by looking at them both symbolically and graphically. Unit 1 introduces functions that are not linear. It also introduces the concepts of composite and inverse functions. Concepts treated in the unit include: the difference between a function and a relation, slope, x-intercepts, y-intercept, domain and range, set notation, set-builder notation, interval notation, graphing inequalities and absolute value inequalities in one variable, translation, inverse, and composition of functions. Typical problems students should be able to solve at the end of the unit include:

1. Solve and graph: $|x - 3| > 2$.
2. Find the equation in slope-intercept form of the line passing through point (-1, -4) and perpendicular to the line $y = -3x + 1$.
3. Write an equation of an absolute value function shifted six units to the right and three units down from the basic graph.
4. Find $f(g(x))$ if $f(x) = 4x - 3$ and $g(x) = -\frac{1}{2}x - 4$.

Unit 2 covers solving quadratic equations and inequalities by graphing, factoring, and using the quadratic formula and modeling real-world situations. Graphs of quadratic functions are explored with and without technology. The unit develops an understanding of the relevance of the zeros and maximum or minimum values of the function as they relate to real-world situations. Typical problems students should be able to solve at the end of the unit include:

1. Find the vertex and axis of symmetry for $f(x) = 5x^2 - x - 3$.
2. Find the zeros of the function $f(x) = 2x^2 - 8x + 3$ by both completing the square and using the quadratic formula.
3. Graph $y > x^2 + 5x + 3$ using the graphing calculator and label all key points.

Unit 3 develops the procedures for factoring polynomial expressions in order to solve polynomial equations and inequalities and introduces the graphs of polynomial functions using technology. Typical problems students should be able to solve at the end of the unit include:

1. Solve the inequality $x^4 > x$, and graph the solution set.
2. Decide if $x + 3$ is a factor of the polynomial function $f(x) = x^3 - 2x^2 - 11x + 12$. If so, find all the factors of the polynomial. Then find the zeros, y-intercept and sketch a graph. Write the polynomial in factored form.
3. Use the graphing calculator to find and label the x-intercepts, maximum and minimum values, and sketch the graph of the following function. Round the answers to three decimal places. $f(x) = -2x^3 - 7x^2 + 3x + 3$.

Unit 4 expands on the 9th and 10th grade GLEs regarding simplification of radicals with numerical radicands to include adding, subtracting, multiplying, dividing, and simplifying radical expressions with variables in the radicand and solving equations containing radicals. The unit also includes the development of the complex number system in order to solve equations with imaginary roots. Typical problems students should be able to solve at the end of the unit include:

1. Compare the graphs of these radical functions using translations: $f(x) = \sqrt{x}$, $g(x) = \sqrt{x-1} + 2$, and $h(x) = -\sqrt{x+1} - 2$.
2. Simplify $(3 + 2i)(8 - 5i)$.
3. Solve $-3 + 4\sqrt{2x-5} = 4x + 1$.

Unit 5 is the study of rational equations and reinforces the students' factoring and expanding skills. This unit develops the process for simplifying rational expressions, adding, multiplying and dividing rational expressions, and solving rational equations and inequalities. Typical problems students should be able to solve at the end of the unit include:

1. Solve the equation. $\frac{-2}{x-1} = \frac{x-8}{x+1}$. Check each solution.
2. Simplify the rational expression, $\frac{x^2+6x-7}{x^2+11x+28}$.
3. Find the intercepts and asymptotes of the function $f(x) = \frac{x}{x-2}$, then sketch the graph.

Unit 6 explores exponential and logarithmic functions, their graphs, and applications. Typical problems students should be able to solve at the end of the unit include:

1. Solve $\log 5x + \log (x - 1) = 2$. (Check for extraneous solutions).
2. Graph the function $f(x) = 2ex^2$.
3. You deposit \$2000 in an account that pays 2% annual interest compounded quarterly. How long will it take for the balance to reach \$2400?

Unit 7 ties together all the functions studied throughout the year. It categorizes graphs and models data with them. Students are expected to be able to graph basic functions quickly and make connections between the graphical representation of a function and the mathematical description. They are expected to be able to translate among the equation of a function, its graph, its verbal representation, and its numerical representation. Typical problems students should be able to solve at the end of the unit include:

1. Use a graphing calculator to graph the function $f(x) = x^4 - 4x^3 - x^2 + 12x - 2$. Find all the x-intercepts and maximum and minimum points. Round the answers to two decimal places.
2. Use $f(x) = 6x^4 + 7x^3 - 33x^2 - 35x + 15$ to answer parts a-e.
    a. How many roots does the Fundamental Theorem of Algebra guarantee this equation has?
    b. How many roots does the Number of Roots Theorem say this equation has?
    c. List all the possible rational roots.
    d. Find all of the roots of the polynomial. Write the polynomial in factored form.
    e. List the end behavior of the polynomial and y-intercept. Then sketch the graph of $f(x)$ using the roots, y-intercept, and end-behavior without a graphing calculator.

Unit 8 focuses on the analysis of graphs and equations of conic sections and their real-world applications. The study of conics helps students relate the cross curriculum concepts of art and architecture to math. They define parabolas, circles, ellipses, and hyperbolas in terms of the distance of points from the foci and describe the relationship of the plane and the double-napped cone that forms each conic. Students identify various conic sections in real-life examples and in symbolic equations. Students solve systems of conic and linear equations with and without technology. Typical problems students should be able to solve at the end of the unit include:

1. How are the graphs $y = (x - 3)^2 - 5$ and $x = (y - 3)^2 - 5$ the same and how are they different?

2. Find the center and radius of the circle with the formula $(x - 1)^2 + (y + 4)^2 = 25$. Then draw the graph.
3. Classify the conic given by $2x^2 + y^2 - 4x - 4 = 0$. Then graph the equation.

CHAPTER 4: UNIT 2 OF EBRCC: QUADRATIC FUNCTIONS

The main ideas in Unit 2 concern quadratic expressions, equations, functions and models, with graphs of these functions being explored with and without technology. The present chapter is with an exposition of the mathematical ideas about quadratics that are important in Unit 2.

4.1 Polynomials and the Factor Theorem

An expression built from constants and the variable x by using addition and multiplication is called a polynomial. Two polynomials are said to be equivalent if one can be transformed into the other using the laws of arithmetic (commutative, associative, and distributive).

Every polynomial is equivalent to a polynomial of the form

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0,$$

where $a_n,\ldots,a_0$ are constants and $a_n \neq 0$. This is called the standard form, and n is the degree of $p(x)$.

The rest of this section is not used until section 4.4 so the reader may go immediately to section 4.2 now, if desired. The Remainder Theorem, which we do not prove but only recall, concerns long division of polynomials. It says that if you divide a polynomial $p(x)$ by another polynomial $s(x)$, then the remainder has degree strictly less than the degree of $s(x)$. Thus, we can write $p(x) = q(x)\, s(x) + r(x)$, where the degree of $r(x)$ is strictly less than the degree of $s(x)$. Moreover, $q(x)$ and $r(x)$, are unique.

We will use the Remainder Theorem to prove the Factor Theorem**. Factor Theorem:** Suppose $p(x)$ is a polynomial. Then the number a is a root of the equation $p(x) = 0$ if and only if $x - a$ is a factor of $p(x)$. *Proof*. If $x - a$ is a factor of $p(x)$ then obviously $p(a) = 0$. On the other hand, suppose $p(a) = 0$. Then by the Remainder Theorem, $p(x) = q(x)(x - a) + r(x)$, where the degree $r(x) <$ degree of $(x - a)$. So $r(x)$ = a constant = r. To find r, let $x = 0$. Then $p(0) = q(0)\,(0 - a) + r$, so $0 = 0 + r$. So, $r = 0$. Thus, $p(x) = q(x)(x - a)$.

4.2 Quadratic Polynomials: A Symbolic Approach

A quadratic polynomial is a polynomial of degree two. The standard form of a quadratic is

$$p(x) = ax^2 + bx + c,$$

where x is a variable, and a, b, and c are constants, with $a \neq 0$. The constant a is called the quadratic coefficient, the constant b is called the linear coefficient and the constant c is called the constant term.

A quadratic polynomial in factored form is $a(x - \gamma)(x - \delta)$, where a, γ and δ are constants. You can transform factored form into standard form by using the distributive law repeatedly. If $a = 1$, this goes as follows:

$$(x - \gamma)(x - \delta) \;=\; (x - \gamma)\, x - (x - \gamma)\, \delta \;=\; x\, x - \gamma\, x - x\, \delta + \gamma\delta \;=\; x^2 - (\gamma + \delta)\, x + \gamma\delta.$$

An important case of this is the perfect-square identity: $x^2 + 2kx + k^2 = (x + k)^2$. Transforming an expression from standard form to factored form is called factoring. Factoring is harder to accomplish and is not always possible (unless complex numbers are allowed).

Suppose $q(x) = x^2 + bx + c$ is a quadratic polynomial in standard form. Then $q(x)$ can be rewritten in vertex form by the method of completing the square. This is done by making the part $x^2 + bx$ of $q(x)$ into a perfect square by adding $(b/2)^2$. To avoid changing $q(x)$, we also subtract $(b/2)^2$:

$$x^2 + bx + c = x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

Using the perfect square identity, we get:

$$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2. \qquad \text{(Identity 1)}$$

When the quadratic coefficient is not 1, we can factor it out of all terms, apply (1) and then simplify.

$$ax^2 + bx + c = a(x^2 + \frac{b}{a}x + \frac{c}{a}) = a((x + \frac{b}{2a})^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2).$$

Thus,

$$ax^2 + bx + c = a(x + \frac{b}{2a})^2 + \frac{4ac - b^2}{4a} \qquad \text{(Identity 2)}$$

The quantities $h = h(a, b) = \frac{-b}{2a}$ and $k = k(a, b, c) = \frac{4ac - b^2}{4a}$ have a special significance which we will see later. In terms of these

$$ax^2 + bx + c = a(x - h)^2 + k \qquad \text{(Identity 2.1)}$$

So far, everything we have derived has come from manipulating expressions using the rules of arithmetic. The equals sign has been used only to show equivalence of expressions. Obviously this is not the way to present the material in a classroom, but this shows that many things about quadratic polynomials that we see in the curriculum are actually direct consequences of the rules of arithmetic.

4.3 Quadratic Functions and their Graphs

A quadratic function is a function of the form

$$f(x) = ax^2 + bx + c,$$

where a, b, and c are constants with $a \neq 0$. In this section we will begin by looking at graphs of quadratic functions.

The graph of $y = x^2$ has a familiar shape that students sketch and study. If $a = 1$, then by (Identity1) above, $f(x) = x^2 + bx + c = (x - h)^2 + k$ where $h = \frac{-b}{2}$ and $k = c - h^2$. This shows that the graph of f is a translation of the graph of $y = x^2$ to the right by h units and up by k units. When $a \neq 1$, (Identity 2) shows that $f(x) = a(x - h)^2 + k$, for constants $h = \frac{-b}{2a}$ and $k = \frac{4ac - b^2}{4a^2}$. Thus, the graph of f is a translation of the graph of $y = ax^2$.

The shape of the graph of a quadratic is called a parabola. Parabolas may open upward or downward and vary in size, but they all have the same basic "*U*" shape. All parabolas are symmetric with respect to a line called the axis of symmetry, which for $y = ax^2$ is the line $x = 0$. A parabola intersects its axis of symmetry at a point called the vertex of the parabola. The points where the quadratic function intersects the x-axis are called the x-intercepts or zeros and where the function intersects the y-axis is called the y-intercept **(www.uncwil.edu).**

Given a quadratic function $f(x) = ax^2 + bx + c$, as the coefficient of each term changes, the graph of the function shifts. If the coefficient of the quadratic term a changes, the graph appears to widen or narrow. If the coefficient of the linear term b changes, then the y-intercept stays the same but the vertex changes. The vertex of the parabola is always on the opposite side of the y-axis as the sign of the linear term. As the coefficient b goes from negative infinity to positive infinity, the vertex travels on a parabola of the form $y = c - ax^2$ from right to left on the graph, since $k = c - ah^2$. As the coefficient c changes, the y-intercept of the parabola changes and the vertex shifts up or down the same number of units as the change of the y-intercept.

The zeros of a quadratic function are the values of x where $f(x) = 0$. The points on the x-axis corresponding to the zeros are called the x-intercepts. They are the points where the graph crosses the x-axis. Let m and n be the x-intercepts of a quadratic function and suppose the vertex is the point (h, k). Because h is the x-coordinate of the vertex, h is halfway between m and n. If the function is tangent to the x-axis, then the vertex (h, k) becomes the same as the only x-intercept and therefore $m = n = h$.

4.4 Quadratic Equations

A quadratic equation is an equation of the form,

$$ax^2 + bx + c = 0,$$

where a, b, and c represent constants, with $a \neq 0$. Quadratic equations can be solved by the methods of factoring, using the quadratic formula or by the method of completing the square. Completing the square is a method for finding the solutions of a quadratic equation. This method involves rewriting the equation from the standard form of $ax^2 + bx + c = 0$ to the vertex form of $a(x - h)^2 = -k$ and then solving for x.

$$a(x - h)^2 = -k$$

$$(x - h)^2 = \frac{-k}{a}$$

$$x - h = \pm\sqrt{\frac{-k}{a}}$$

$$x = h \pm \sqrt{\frac{-k}{a}}$$

The quadratic formula arises by evaluating h and k explicitly where $h = \frac{-\boldsymbol{b}}{\boldsymbol{2a}}$ and $k = c - \frac{b^2}{4a}$. From (Identity 2), the equation $ax^2 + bx + c = 0$ is true of x if and only if:

$$a(x+\frac{b}{2a})^2=\frac{b^2-4ac}{4a}$$

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

Given a quadratic equation in standard form, the quadratic formula can be used to find the solutions by substituting the values of *a*, *b*, and *c* with constants from the given equation and simplify the resulting equation.

We refer to the Factor Theorem. It says that if γ is a root of *f*, then x- γ is a factor and similarly, if you know a factor of *f* is x – γ, then γ is a root. If the quadratic equation can be factored, it can be written in factored form $f(x) = (x - \frac{-b+\sqrt{b^2-4ac}}{2a})\,(x - \frac{-b-\sqrt{b^2-4ac}}{2a})$, if $b^2 - 4ac \geq 0$ and $a \neq 0$.

<u>Quadratic inequalities in one variable</u> are inequalities which can be written in one of the following forms: $ax^2 + bx + c > 0$, $ax^2 + bx + c < 0$, $ax^2 + bx + c \geq 0$, or $ax^2 + bx + c \leq 0$ where *a*, *b* and *c* are real numbers and $a \neq 0$. The steps for solving quadratic inequalities are to move all terms to one side, simplify, find the roots of the corresponding quadratic equation, use the roots to divide the number line into regions and then test each region using the inequality. Then the solution can be written based on the graph.

<u>A quadratic inequality in two variables</u> can be written in one of the forms $y < ax^2 + bx + c$, $y \geq ax^2 + bx + c$, $y \leq ax^2 + bx + c$, or $y > ax^2 + bx + c$. Such inequalities can be graphed by shading the portion of the plane above or below the graph, as appropriate.

4.5 Quadratic Models

One of the goals of the unit on quadratic functions is to apply the knowledge of quadratic functions to application problems. Let us look at some specific examples of some real-world applications that may be included in the Algebra II curriculum.

Example 1: For each corner of a square piece of sheet metal, remove a square of side 9 centimeters. Turn up the edges to form an open box. If the box is to hold 144 cubic centimeters (cm^3), what should be the dimensions of the piece of sheet metal? The function that models this situation is $V(x) = 9(x - 18)^2$, and we want to solve $V(x) = 144$.

Example 2: The table shows how wind speed affects a runner's performance in the 200 meter dash. Positive wind speeds correspond to tailwinds, and negative wind speeds correspond to headwinds. Positive changes in finishing time mean worsened performance, and negative changes mean improved performance. Use a system of equations to write a quadratic model for the change *t* in finishing time as a function of the wind speed *s*. The function that models this situation is $f(x) = .012x^2 - .309x - .0004$.

Wind speed (m/sec),*s*	-6	-4	-2	0	2	4	6
Change in finishing time (sec), *t*	2.28	1.42	0.67	0	-.57	-1.05	-1.42

Figure 2. Wind Speed of a Runner's Performance

The following problem is an example from calculus that demonstrates the need for some of the main ideas of quadratics.

Example 3: If $s = f(t)$ is the position function of a particle that is moving in a straight line, then $\Delta s/\Delta t$ represents the average velocity over a time period Δt, and $v = \frac{ds}{dt}$ represents the instantaneous velocity (the rate of change of displacement with respect to time). The position of a particle is given by the equation $S = f(t) = t^3 - 6t^2 + 9t$.

a) Find the velocity at time t.
b) What is the velocity after 2 s? After 4 s?
c) When is the particle at rest?
d) When is the particle moving forward (that is, in the positive direction)?

This function can be modeled by $s(t) = f(t) = t^3 - 6t^2 + 9t$, and $v(t) = \frac{ds}{dt} = 3t^2 - 12t + 9$.

CHAPTER 5: ALGEBRA II ASSESSMENT IN EBRCC

This chapter includes a discussion of the current activities in Unit 2 of the comprehensive curriculum. The unit contains ten activities on the topic of quadratic functions and five additional activities on polynomials at the end, but Unit 3 covers polynomials in depth and these last five activities can be integrated into that unit, so I do not consider them. Each activity in Unit 2 begins by listing the GLEs it covers, the materials needed and by giving a general description of the curriculum that the activity is designed to cover (EBRCC).

Each of the following descriptions includes a discussion of the course content on quadratics from each activity. Then I give a listing of the Core Content Standards that correspond to the quadratic content in each unit. They also indicate the relation of the mathematics in this unit to other mathematics topics.

Following this, we discuss the current Unit 2 test on quadratics in the Algebra II curriculum. Finally, I present my test. I include a detailed explanation of how this test was created, based on my research.

5.1 Test Questions Correlated with Standards and Curriculum

This section considers the activities in Unit 2. For each activity, we give a description of the mathematical concepts, the Common Core Standards addressed, and the relationships to other mathematics. The Common Core Standards are indexed by heading and number. See Table 1 below, and see Appendix B for the complete listing of the Common Core Standards. They can also be found at www.corestandards.org.

Table 1. Sections of the Common Core Standards

N	**Number and Quantity**
N-RN	The real number system
N-Q	Quantities
N-CN	The complex number system
N-VM	Vector and matrix quantities
A	**Algebra**
A-SSE	Seeing Structure in Expressions
A-APR	Arithmetic with polynomials and rational expressions
A-CED	Creating equations
A-REI	Reasoning with equations and inequalities
F	**Functions**
F-IF	Interpreting Functions
F-BF	Building functions
F-LE	Linear, quadratic, and exponential models
F-TF	Trigonometric functions

Activity one gets the students familiar with quadratics. They start with a problem concerning the area of a rectangle whose sides vary linearly with a parameter. From this problem they write an equation to model it. They are asked to state connections between the situation, the equation, the symbols in it and the graph. They graph quadratic functions given in standard form $y = x^2 + bx$ and factored form $y = (x - a)(x - b)$ by plotting points. They look at the leading coefficient of the function to determine the general shape of the graph. This activity includes some work with real-world data and integrates the use of the graphing calculator. This activity gives a broad view of quadratics on a basic level.

Standards: The Common Core Standards call for general competencies in:

- interpreting expressions A-SSE 1a,
- identifying and using zeros to construct a graph A-APR 3,
- understanding graphing in two variables could form a curve A-REI 10,
- interpreting key features of graphs and tables using intercepts, intervals, extrema, symmetries, and end behavior F-IF 4,
- relating the domain of a function to its graph F-IF 5,
- graphing quadratic functions showing intercepts and maxima and minima F-IF 7a,
- using the process of factoring to show zeros F-IF 8a,
- graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior F-IF 7c.

Relate to other math: This activity involves finding the zeros of a quadratic function in factored form. It involves factoring and solving a quadratic equation, and finding the vertex, the equation of the axis of symmetry, the zeros and the y-intercept, and using these concepts to sketch a graph. Factoring is used throughout Algebra II and is taught more completely in Unit 3. Finding features of a graph are skills needed with other types of functions studied in Algebra II.

Activity two teaches students how to determine the value of the vertex and axis of symmetry of functions in both standard and vertex form. They graph various quadratics given in vertex form. Students are shown by examples that the x-coordinate of the vertex of the graph of a quadratic is the average of the zeros. Students learn to apply the vertex formula and sketch a graph. Students discover the changes that shift the graph horizontally, vertically, and obliquely.

Standards: The standards expect general competencies in

- interpreting graphs and tables using key features such as intercepts, intervals, extrema, symmetries, end behavior F-IF 4,
- graphing quadratic functions showing intercepts and maxima and minima F-IF 7a,
- identifying the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$ and $f(x + k)$ for specific values of k, find the value of k given the graphs, illustrate translations using technology F-BF 3,
- use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context F-IF 2.

Relate to other math: This activity teaches students to find the vertex of a quadratic function written in both vertex and standard form. Students learn how to find the equation of the axis of symmetry, find the vertex, make determinations using the leading coefficient, and use this information to sketch the graph. Finding features of a graph are skills needed with all other types of functions studied in Algebra II.

Activity three covers the concept of completing the square. This activity describes how to determine a value for c so the expressions $x^2 + bx + c$ and $x^2 - bx + c$ will be perfect square trinomials and find exact zeros of a quadratic function in standard form by completing the square. This activity includes applying knowledge of quadratic equations to area and perimeter of a rectangle.

Standards: The standards expect general competencies in

- solving quadratic equations with real coefficients that have complex solutions N-CN 7,
- producing an equivalent form of quadratic expression through completing the square A-SSE 3b,
- solving quadratic equations using the method completing the square to transform $f(x)=ax^2 + bx + c$ to $f(x) = a(x - p)^2 + q$ where the vertex is (p,q) and derive the quadratic formula from this form A-REI 4a,
- using the process of completing the square in a function to show zeros, symmetry, and extreme values of the graph and interpret their meaning in context of the graph F-IF 8a.

Relate to other math: This activity asks for the knowledge of the process of completing the square to find the solutions to a quadratic equation. This method is widely used in mathematics. One such example is for graphing conic sections.

Activity four derives and applies the quadratic formula through the method of completing the square. This method can be used to find the zeros of a quadratic function.

Standards: The standards expect general competencies in

- solving quadratic equations with real coefficients that have complex solutions N-CN 7,
- using the quadratic formula to solve quadratic equations and recognize when the quadratic formula gives complex solutions A-REI 4b.

Relate to other math: This activity asks students to find the solutions to a quadratic equation using the quadratic formula. The quadratic formula is widely used in mathematics.

Activity five discusses using the discriminant to determine the nature of the roots. Graphs are examined to determine the types of roots and zeros. This activity also relates quadratic functions to geometry by using the discriminant to find the number of possible solutions

Relate to other math: This activity asks for an interpretation of the discriminant by looking at its sign. This sign determines the nature of the roots and determines how many times the graph of a quadratic function crosses the x-axis.

Activity six compares linear functions to quadratic functions. In this activity, similarities and differences between linear and quadratic functions are observed. This activity compares equations of the form $y = mx + b$ and $y = x(mx + b)$. Data is examined to decide if it represents a pattern that is linear or quadratic by using the method of finite differences. Linear and quadratic equations that model data are found using the graphing calculator.

Standards: The standards expect general competencies in

- understanding graphing A-REI 10,

- comparing properties of two functions each represented is algebraically, graphically numerically in tables, or by verbal description F-IF 9,
- recognize situations in which one quantity changes at a constant rate per unit interval relative to another F-LE 1b,

Relate to other math: This activity uses the method of finite differences to determine the degree of a polynomial. This can be useful if the data can be represented by a model where every point in the data set is on the model.

Activity seven has students analyzing how varying the coefficients in $y = ax^2 + bx + c$ affects the graphs. The students use their prior knowledge of translating graphs to predict how changes in a, b, and c in the equation $y = ax^2 + bx + c$ will affect the graph. They use the graphing calculator.

Standards: The standards expect general competencies in

- interpreting parts of an expressions such as terms, factors and coefficients A-SSE 1a,
- identifying the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$ and $f(x + k)$ for specific values of k, find the value of k given the graphs, illustrate translations using technology F-BF 3.

Relate to other math: This activity asks how changing a particular coefficient changes the graph of a quadratic function. This can be used to help predict the shape of a graph of a polynomial. Experience here will help students understand translations of radical, rational, logarithmic and exponential functions.

Activity eight is an optional parabolic graph lab activity that can be used with an honors course if there is a motion detector available to the class. Students collect data with a motion detector to determine a quadratic equation for the position of a moving object and use the equation to answer questions. There are some real-world problems in this activity that are excellent for students to apply their knowledge of quadratic functions that do not require the use of the motion detector.

Standards: The standards expect general competencies in

- reasoning quantitatively and use units as a way to understand problems, choose and interpret units NQ 1,
- defining appropriate quantities for modeling functions NQ 2,
- choosing a level of accuracy appropriate to limitations on measurement NQ 3,
- understanding graphing in two variables could forma curve A-REI 10,
- interpreting graphs and tables using key features such as intercepts, intervals, extrema, symmetries, and end behavior F-IF 4,
- relating the domain of a function to its graph F-IF 5,
- graphing quadratic functions showing intercepts and maxima and minima F-IF 7a.

Relate to other math: This activity is representative of projectile motion problems that is an application of quadratic functions. It uses the concepts of finding the vertex, and zeros learned in prior activities in this unit. This question asks for the students' understanding of how to interpret points on a graph in the context of a real-world problem. This interpretation is essential for a complete understanding of quadratics in a real-world context and used in the rest of Algebra II, and higher-level mathematics classes.

Activity nine has students solving equations in quadratic form. They will solve quadratic equations of the form $a(x-k)^2 - b(x-k) + c = 0$, $x - b\sqrt{x} + c = 0$ and $x^4 + bx^2 + c = 0$ where a, b, c, and k are constants and $a \neq 0$.

Standards: The standards expect general competencies in

- explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents N-RN 1,
- solving quadratic equations with real coefficients that have complex solutions N-CN 7,
- use the structure of an expression to identify ways to rewrite it A-SSE 2,
- factoring a quadratic expression to reveal the zeros of the function it defines A-SSE 3a,
- creating equations in one variable and use them to solve problems A-CED 1,
- explaining each stem in solving an equation constructing a viable argument to justify a solution method A-REI 1,
- solving quadratic equations by inspection, taking square roots, and factoring A-REI 4a,
- compose functions F-BF 1c.

Relate to other math: This activity demonstrates knowledge of substitution of variables to make a simpler problem out of a harder problem. It also asks students for complete factoring of quadratic equations and to find their solutions. This activity uses the composition of functions to solve an equation, this can be applied to integration techniques in calculus.

Activity ten involves solving quadratic inequalities. This involves finding the zeros and constructing a number sign chart. Solutions are placed on a number line and then test values in each interval; writing false or true above the interval if the value is a solution to the inequality. Solutions are written in interval notation or set notation using the words: and, or, intersection, and union. Then the quadratic inequalities are graphed on an appropriate graph.

Standards: The standards expect general competencies in

- creating inequalities in one variable and use them to solve problems A-CED 1,
- representing constraints by inequalities and interpret solutions as options in a modeling context A-CED 3.

Relate to other math: This activity asks the students to solve a quadratic equation, test the points and find the solution of the inequality. This is done with polynomials in the next unit.

5.2 Current Algebra II Assessment in EBRCC

We now discuss the Unit 2 test that is currently in the EBRCC; see appendix D. This test is intended to cover the ten activities from Unit 2 on quadratics. It consists of fourteen questions, of which eight are multiple-choice and six are matching. Six out of fourteen questions involve simple recall and most of the rest of the questions require basic mechanics only. None of the questions ask for sketching or reading a graph. Some questions are redundant. The test assesses only about half the activities on quadratics in the unit.

After looking at the format of this test, I cannot be sure this is a quality test teachers would want to use. This test is not representative of what students need to know from the unit.

5.3 New Test Analysis

The test I have created is in section 5.4 following this section. It is designed using the content from the Unit 2 activities in the EBRCC, the core curriculum standards, and the research I have done on developing good formative assessments.

Decisions about the cognitive vertex of the assessment triangle are reflected in the kind of questions included in my test. While parts of some questions are on the recall level, many of the questions use verbs such as compare, contrast, sketch and show. This kind of questioning is on the applying and analyzing levels of Bloom's Taxonomy. Decisions about the observation vertex are reflected in the requirement that students show work on all the questions. By requiring students to show their work, the teacher can make an accurate interpretation of the student's knowledge of the concepts on this exam, thus addressing the interpretation vertex of the assessment triangle.

Graphing quadratic functions is a central concept on this test. Students are expected to know:

- the shape of a parabola and that the graph of a quadratic is a parabola;
- that a parabola has a vertex and an axis of symmetry;
- that the graph of a quadratic may be a bigger or a smaller parabola and may open up or down depending on the sign of the leading coefficient;
- that the vertex may be anywhere in the plane;
- that the size and direction of opening of the quadratic function and position of the vertex determine the graph completely;
- that if the function is written in vertex form, these features of the graph can be read off the equation.

As a described in the exposition of quadratics in chapter 4, the following knowledge is assessed in this exam.

- given $f(x) = ax^2 + bx = c$, the students are expected to know and use the quadratic formula to find the zeros;
- they are expected to know that the y-intercept is the point (0,c);
- they are expected to look at the sign of a to find the direction of opening of the graph;
- they are expected to know that the vertex is at the point $(-b/2a, f(-b/2a))$;
- they are expected to know that the graph of $x = -b/2a$ is the axis of symmetry.

Questions one, two and seven were designed to assess these concepts. Question seven assesses these concepts on a higher-level than the other two by asking the student to apply knowledge in a real-world context.

Finding the zeros of a function is a concept used throughout Algebra II and higher-level mathematics classes. This test assesses several ways of finding the zeros. To find the zeros, students are expected to know these methods:

- factoring a trinomial or factoring out a common monomial;
- completing the square;
- using the quadratic formula;
- using the graphing calculator.

Students have to demonstrate knowledge of finding the zeros by completing the square on questions three and five, by using the quadratic formula on questions four and five, and by factoring on questions one, eight, nine, and ten.

Completing the square is used as a technique for solving quadratic equations or for rewriting the standard form of a quadratic function in the vertex form. Students are expected to

- know and write the steps for completing the square;
- be able to find the zeros of a quadratic equation using the method of completing the square;
- understand and explain the rationale behind the strategy that is involved;
- know that completing the square means to write the equation with a binomial squared term;.

The understanding of completing the square is addressed in questions three and five.

The quadratic formula is used as a technique for problem solving. Students are expected to:

- memorize the formula;
- know what values of *a*, *b*, and *c* are to be used to evaluate the function;
- simplify a radical expression that may involve complex solutions;
- solve a quadratic equation using the formula.

The understanding of the quadratic formula is addressed in problems four and five.

Four of the questions ask for an explanation of how processes or solutions compare and contrast. These questions assess students' knowledge on a higher-level of thinking than recall and understanding. Students are expected to know how to:

- understand when it is necessary to use the method of completing (questions 3).
- compare the solutions of the same equation solved two different ways. Students sometimes treat solution methods as disconnected processes and do not see solutions as entities that are independent to the process. Question five assesses the students' knowledge that regardless what method you use to find the zeros, you always get the same result (question 5).
- to recognize "quadratic equations in disguise," that is, quadratic expressions in which the variable has been replaced by a more complicated expression. In question 9, they are expected to see that both equations are of the same quadratic form $x^2 - 18x + 81 = 0$, but after the substitution for x is made, they result in different solutions.

- to investigate how the function changes if the coefficients a, b, and c of the function $f(x) = ax^2 + bx + c$ change one at a time. This question takes the concept of what the coefficient means from a recall level to an analyzing level (question 7).

In question five, students are expected to know the formula for the discriminant and how to interpret its sign.

From question six, students are expected to know:

- what the degree of a polynomial means;
- the method of finite differences and how to interpret its meaning in determining the degree of a polynomial;
- how to see the connection between the degree of the polynomial, the shape of the graph and the equation of best fit.
- how to plot points to graph a set of data;
- how to recognize linear trends in data;
- how to use the graphing calculator to find the regression equation; See question six.

In question ten, students are expected to know:

- how to finding the solutions to a quadratic inequality;
- how to graph the solutions of a linear inequality in one variable on the number line;
- how to write solutions in set builder and interval notation.

In questions two and seven, students are expected to know:

- how to translate functions, that is, how the graph of $y - y_0 = f(x - x_0)$ relates to the graph of $y = f(x)$;
- that if the function is in vertex form, $y = a(x - h)^2 + k$, then the value of h shifts the vertex h units horizontally and the value of k shifts the vertex k units vertically.

Being able to memorize formulas and perform calculations are skills that are necessary to be successful in Algebra II. However, a true understanding of the concepts goes beyond memorizing and calculating. Students are expected to make connections in processes. Students are expected to show an understanding of how the concepts relate to real world applications.

This test assesses all of the main concepts in the activities in Unit 2 on quadratics that students are expected to know. Based on my research on formative testing, I believe the test I have created is valid and fair. It tests students' understanding of quadratics and how to apply them to appropriate situations. It covers the standards that the students are expected to know on quadratic functions. The test integrates the use of the graphing calculator.

5.4 New Test for Unit 2 Quadratic Functions

Algebra II   Quadratic Functions Unit 2   Name ___________________________

Show all your work for the entire test.

For problems 1-2, find the zeros, y-intercept, direction of opening, vertex, axis of symmetry. Draw and label these features on the graph.

1. $f(x) = x^2 - 4x - 12$

2. $f(x) = (x - 3)^2 - 4$; How is this graph translated from the basic graph of $f(x) = x^2$?

3. Solve the quadratic equation by completing the square: $x^2 - 4x - 2 = 0$. Is the method of completing the square a good approach for finding the roots of $x^2 - 2 = 0$? Why or why not?

4. Solve the quadratic equation by using the quadratic formula. $-5x^2 - 4x - 9 = 0$

5. Let $\frac{1}{2}x^2 - 3x + 2 = 0$. Find the discriminant, determine the nature of the roots, and state how many times the graph of the function crosses the x-axis. Find the zeros of the function by using the quadratic formula. Another student used the method of completing the square to find the roots, and concluded that they were at $6 \pm \sqrt{3}$. Did they make any mistakes? How do you know?

6. Given the following table, plot the data points. Using the method of finite differences, find the degree of the polynomial to model the data. After knowing the degree of the polynomial, find the model to represent the data using the graphing calculator. The answers are to be rounded to three decimal places.

x	-4	-3	-2	-1	0	1	2	3	4
y	77	44	19	2	-7	-8	-1	14	37

Figure 3. Data Table for Regression Equation

7. Given the function $f(x) = x^2 + 3x + 2$, use a graphing calculator to answer parts $a - c$.

   a. Change the constant term to 1, 0, and -1. How do these graphs differ? How are they the same?
   b. Change the coefficient of the linear term to 2, 0, and -2. How do these graphs differ? How are they the same?
   c. Change the coefficient of the quadratic term to -3, 0, and 3. How do these graphs differ? How are they the same?

8. A rocket is shot vertically with an initial velocity of 40 feet per second. Its height above the ground after t seconds is given by the function $h(t) = 40t - 16t^2$. What is its maximum height? When will it hit the ground? Where is the rocket after 6 seconds?

9. Factor completely and then solve $x^4 - 18x^2 + 81 = 0$. Factor completely and solve $x - 18\sqrt{x} + 81 = 0$. Compare the methods used to factor each quadratic equation.

10. Solve and graph the inequality. Write the solution in set builder and interval notation: $x^2 - x - 12 < 0$.

5.5 Solution Key to New Test

The following is a solution key to the exam I created on quadratics for Unit 2. It includes each question from the test and a worked out solution to the question.

Problem 1. Find the zeros, y-intercept, direction of opening, vertex, axis of symmetry. Draw and label these features on the graph for $f(x) = x^2 - 4x - 12$.

Solution.

zeros: $f(x) = (x - 6)(x + 2)$, then $x = 6$, $x = -2$.

y-intercept: (0, -12)

vertex: $h = -(-4)/(2(1)) = 2$; $k = f(2) = 2^2 - 4(2) - 12 = -16$; so the vertex = (2, -16).

axis of symmetry: $x = 2$

direction of opening: upward

sketch the graph:

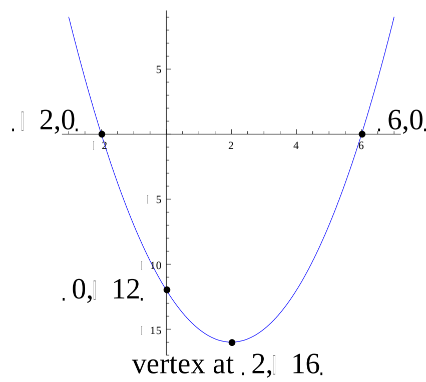

Figure 4. $f(x) = x^2 - 4x - 12$

Problem 2. Find the zeros, y-intercept, direction of opening, vertex, axis of symmetry. Draw and label these features on the graph for $f(x) = (x - 3)^2 - 4$. How is this graph translated from the basic graph of $f(x) = x^2$?

Solution.

zeros: $0 = (x - 3)^2 - 4 \rightarrow \pm 2 = x - 3 \rightarrow x = 5$ or $x = 1$

y-intercept: $(0 - 3)^2 - 4 = 5$

vertex: (3, -4)

axis of symmetry : $x = 3$

direction of opening: upward

sketch the graph:

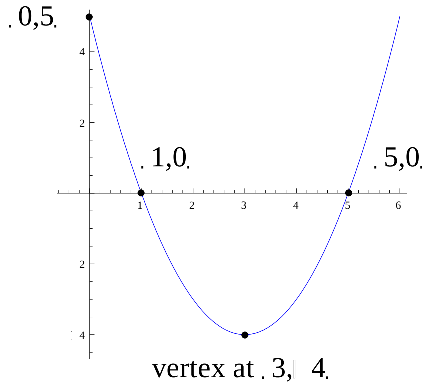

Figure 5. $f(x) = (x - 3)^2 - 4$

The graph of this function is translated from the basic graph by shifting three units to the right and four units down.

Problem 3. Solve the quadratic equation by completing the square: $x^2 - 4x - 2 = 0$. Is the method of completing the square a good approach for finding the roots of $x^2 - 2 = 0$? Why or why not?

Solution.

$$x^2 - 4x - 2 = 0, \text{ given to solve;}$$
$$x^2 - 4x = 2, \text{ put constant terms on right-hand side;}$$
$$x^2 - 4x + 4 = 2 + 4, \text{ complete the square;}$$
$$(x - 2)^2 = 6, \text{ factor the square;}$$
$$x - 2 = \pm\sqrt{6}, \text{ take roots;}$$
$$x = 2 \pm \sqrt{6} \text{ isolate } x.$$

For the second part of the questions,

$$x^2 - 2 = 0, \text{ given to solve;}$$
$$x^2 = 2, \text{ put constant term on right-hand side;}$$
$$x^2 = 2, \text{ square is complete;}$$
$$x = \pm\sqrt{2}, \text{ take roots; } x \text{ is already isolated.}$$

Problem 4. Solve the quadratic equation by using the quadratic formula, $-5x^2 - 4x - 9 = 0$.

Solution.

$$-5x^2 - 4x - 9 = 0, \text{ given to solve;}$$
$$5x^2 + 4x + 9 = 0, \text{ multiply by -1;}$$
$$= \frac{-4 \pm \sqrt{16 - 180}}{10}$$
$$= \frac{-2 \pm i\sqrt{41}}{5}$$

Problem 5. Let ½ $x^2 - 3x + 2 = 0$. Find the discriminant, determine the nature of the roots, and state how many times the graph of the function crosses the x-axis. Find the zeros of the function by using the quadratic formula. Another student used the method of completing the square to find the roots, and concluded that they were at $6 \pm \sqrt{3}$. Did they make any mistakes? How do you know?

Solution. Discriminant = $(-3)^2 - 4(½)(2) = 9 - 4 = 5$. Since this value is positive and 5 is not a perfect square number, there are two real irrational roots. Since there are two real roots, the graph of the function will cross the *x*-axis twice.
Quadratic formula:

$$\tfrac{1}{2}x^2 - 3x + 2 = 0, \text{ given to solve;}$$

$$\frac{-(-3) \pm \sqrt{(-3)^2 - 4\left(\frac{1}{2}\right)(2)}}{2\left(\frac{1}{2}\right)}, \text{ substitute values}$$

$$3 \pm \sqrt{9-4}, \text{ simplify}$$

$$3 \pm \sqrt{5}, \text{ simplify}$$

Yes, the other student made mistakes. If you solve the same problem different ways, you must get the same solution regardless of the method you choose. Clearly the solution of $6 \pm \sqrt{3}$ is not the same as the solution when the quadratic formula was used.

Problem 6. Given the following table, plot the data points and find and graph an appropriate regression equation. Round answers to three decimal places.

x	-4	-3	-2	-1	0	1	2	3	4
y	77	44	19	2	-7	-8	-1	14	37

Figure 6. Data Table Used to Find Regression Equation

Solution. The method of finite differences requires finding the differences of successive entries in the y-row to make a new row, and then repeating the process until a row in which all entries are equal is obtained. If we plot the points in a coordinate plane, it looks quadratic. By using the graphing calculator, entering the coordinates in the lists, and calculating a quadratic equation to model the data, the resulting function is $f(x) = 4x^2 - 5x - 7$.
A sketch of the graph is:

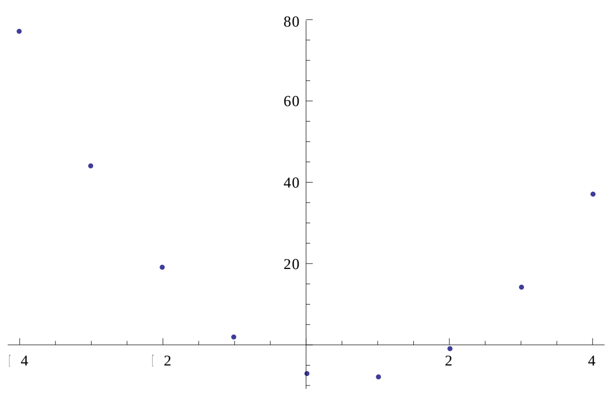

Figure 7. $f(x) = 4x^2 - 5x - 7$

Problem 7. Given the function $f(x) = x^2 + 3x + 2$,

a. Change the constant term to 1, 0, and -1. How do these graphs differ? How are they the same?

b. Change the coefficient of the linear term to 2, 0, and -2. How do these graphs differ? How are they the same?
c. Change the coefficient of the quadratic term to -3, 0, and 3. How do these graphs differ? How are they the same?

Solution.

a. The size, shape, x-coordinate of the vertex, axis of symmetry, and direction of opening stays the same. The y-intercept changes, the vertex of the other graphs shifts up or down the same number of units that the constant c changed.

b. The size, shape, and direction of opening of the graph (which is a parabola) stay the same but the parabola is shifted to a new position. If the linear term b stays the same sign, then the vertex stays on the same side of the y-axis as the original graph, if the linear term b changes signs, then the vertex moves to the opposite side of the y-axis. In the case that $b = 0$, the y-intercept and the vertex become the same point.

c. If $a \neq 0$, the shape, and y-intercept stay the same. The width of the graph changes according to how a changes, if $a > 0$, the parabola opens upward, if $a < 0$, the parabola opens downward. If the quadratic term has $a = 0$, then the function becomes linear. The vertex will change also.

Problem 8. A rocket is shot vertically with an initial velocity of 40 feet per second. Its height above the ground after t seconds is given by $h(t) = 40t - 16t^2$. What is its maximum height? When will it hit the ground? Where is the rocket after 6 seconds?

Solution. The height at time t is:

$$h(t) = 40t - 16t^2,$$

Because the function has a leading coefficient that is negative, its graph (a parabola) opens downward with a maximum at the vertex. The x-coordinate of the vertex is $\frac{-40}{2(-16)} = 1.25$, and the y-coordinate of the vertex is $f(1.25) = 25$. This can be interpreted in the context of the problem. At time $t = 1.25$ seconds, the rocket reaches a maximum height of 25 feet. It is at the ground at two times since its height is a quadratic function with a discriminant of 1600, which is a positive rational number. To find when the height = 0, solve

$$0 = 40t - 16t^2$$
$$t(40 - 16t) = 0$$
$$t = 0 \text{ or } t = \frac{40}{16} = 2.5.$$

This means the height is 0 feet at time $t = 0$ and at time t = 2.5. The time $t = 0$ represents the initial time, so the time $t = 2.5$ seconds is the time when the rocket will hit the ground.

Problem 9. Factor completely: $x^4 - 18x^2 + 81 = 0$, then factor: $x - 18\sqrt{x} + 81 = 0$. Compare the methods used to factor each quadratic equation.

Solution.

$$x^4 - 18x^2 + 81 = 0$$
$$= (x^2 - 9)(x^2 - 9)$$
$$= (x - 3)^2(x + 3)^2$$
$$= \pm 3$$

$$x - 18\sqrt{x} + 81 = 0$$
$$= (\sqrt{x} - 9)(\sqrt{x} - 9)$$
$$= \sqrt{x} - 9 = 0$$
$$= \sqrt{x} = 9$$
$$= 81$$

Both problems are solved using the basic quadratic form $f(x) = x^2 - 18x + 81$, the first one uses the substitution $u = x^2$, the second uses the substitution $u = \sqrt{x}$.

Problem 10. Solve and graph the quadratic inequality. Write the solution in set builder and interval notation: $x^2 - x - 12 < 0$

Solution. $x^2 - x - 12 < 0$

$\Leftrightarrow (x - 4)(x + 3) < 0$

$\Leftrightarrow \{(x - 4) < 0$ and $(x + 3) > 0$ or $(x - 4) > 0$ and $(x + 3) < 0$

$\Leftrightarrow ((x - 4) < 0$ and $(x + 3) > 0$, since $(x - 4) > 0$ and $(x + 3) < 0$ is impossible;

$\Leftrightarrow -3 < x$ and $x < 4$;

therefore, the solution to the inequality is $-3 < x < 4$ in set notation or $(-3, 4)$ in interval notation.

(=================)

-4 -3 -2 -1 0 1 2 3 4 5 6

Figure 8. $x^2 - x - 12 < 0$, Graph Inequality

CONCLUSION

Quality test design is important in assessing students' knowledge, for motivating students and for other reasons as well. Teachers ought to have quality assessments that truly and usefully measure the learning of the concepts they are teaching. Quality assessments challenge students to think critically and apply their knowledge.

After teaching Algebra II in East Baton Rouge for several years, I found a need for quality assessments. Based on my study of test design and a careful analysis of the stated learning goals, I created an assessment to be used with the unit on quadratics that addresses the standards of the East Baton Rouge Comprehensive Curriculum.

As background and support for my test design, I employed the assessment theory described in the book *Knowing What Student Know*. I used the assessment triangle as a model to guide me in designing my test. I also took into account the requirements of formative (as opposed to summative) assessment, as well as the issues of validity and fairness. Issues of test reliability were beyond the scope of this work.

I examined the Core Curriculum Standards as a basis for my test questions. I reviewed the topics in Algebra II in the East Baton Rouge Curriculum and in the unit on quadratics that I focused on. I reviewed the district curriculum and used this to design my test. I argued for the validity of my test based on the apparent intentions of the curriculum and the structure of the math.

I looked at the main ideas, vocabulary and concepts of quadratics required by the curriculum. I showed relationships between the core mathematical ideas I viewed as central and the Core Curriculum Standards, to assure I would have a valid formative assessment. I showed how each question on my test was created by showing the content students are expected to know and how each question assesses that knowledge.

The test I designed is a culmination of my research on test design, the standards, and the curriculum for the unit on quadratics. I created a quality test that is consistent with the learning goals it was designed to assess. The test I offer here can be used by other teachers in my school district. I believe the test I designed it of high-quality and hope it can be considered a quality contribution to East Baton Rouge Parish.

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APPENDIX A: BLOOM'S TAXONOMY

Remembering: can the student recall or remember the information?	define, duplicate, list, memorize, recall, repeat, reproduce state
Understanding: can the student explain ideas or concepts?	classify, describe, discuss, explain, identify, locate, recognize, report, select, translate, paraphrase
Applying: can the student use the information in a new way?	choose, demonstrate, dramatize, employ, illustrate, interpret, operate, schedule, sketch, solve, use, write.
Analyzing: can the student distinguish between the different parts?	appraise, compare, contrast, criticize, differentiate, discriminate, distinguish, examine, experiment, question, test.
Evaluating: can the student justify a stand or decision?	appraise, argue, defend, judge, select, support, value, evaluate
Creating: can the student create new product or point of view?	assemble, construct, create, design, develop, formulate, write.

APPENDIX B: COMMON CORE STANDARDS FOR MATHEMATICS FOR HIGH SCHOOL

The Real Number System N – RN

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want $(5^{\frac{1}{3}})^3 =$ $5^{\wedge}((1/3))^3$ to hold, so $(5^{\frac{1}{3}})^3$ must equal 5.

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities N - Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

2. Define appropriate quantities for the purpose of descriptive modeling.

3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

The Complex Number System N - CN

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.

6. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.

8. Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.

9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities N -VM

Represent and model with vector quantities.

1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, v).

2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

3. Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

4. Add and subtract vectors.

a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

5. Multiply a vector by a scalar.

a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v\mathrm{x}, v\mathrm{y}) = (cv\mathrm{x}, cv\mathrm{y})$.

b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Perform operations on matrices and use matrices in applications.

6. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

7. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

8. Add, subtract, and multiply matrices of appropriate dimensions.

9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

12. Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Seeing Structure in Expressions A-SSE

Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)n$ as the product of P and a factor not depending on P.

2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ *as* $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15*t* can be rewritten as (1.151/12)12*t* ≈1.01212*t* to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

Arithmetic with Polynomials and Rational Expressions A -APR

Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems

4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

5. Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by

Pascal's Triangle. The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations A -CED

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.

Reasoning with Equations and Inequalities A -RE I

Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

4. Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Solve systems of equations

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

8. Represent a system of linear equations as a single matrix equation in a vector variable.

9. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Interpreting Functions F-IF

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.

Estimate the rate of change from a graph.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)t$, $y = (0.97)t$, $y = (1.01)12t$, $y = (1.2)t/10$, and classify them as representing exponential growth or decay.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, explain which has the larger maximum.

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

c. Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Include recognizing even and odd functions from their graphs and algebraic expressions for them.

4. Find inverse functions.

a. Solve an equation of the form f(x) = c for a simple function *f* that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x–1)$ for $x \neq 1$.

b. Verify by composition that one function is the inverse of another.

c. Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. Produce an invertible function from a non-invertible function by restricting the domain.

5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models F -LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

APPENDIX C: INTERVIEW WITH MICHAEL DAUZAT

This interview was done via email on June 1, 2010. Michael Dauzat is the K-12 math supervisor for East Baton Rouge Parish. He was involved in writing and revising the East Baton Rouge Parish Comprehensive Curriculum.

1. When was the Louisiana Comprehensive Curriculum revised for East Baton Rouge Parish to make the EBRP Comprehensive Curriculum? It has gone through several revisions. It was first revised summer 2006.

2. Why was the Louisiana Comprehensive Curriculum revised? The revision was based on teacher feedback and is continually based on teacher feedback. At the end of each unit, there is a feedback form that teachers can submit after each unit.

3. Who was involved in creating the revisions? Teachers who taught the course and C&I (curriculum & instruction) content trainers.

4. Who created the tests that are in the EBR Comprehensive Curriculum, specifically Algebra II, specifically the unit 2 on quadratic and polynomial functions? The tests were created by EBR teachers.

5. What tools and inspirations were used to create the quadratic and polynomial functions Algebra II test? I do not know how these were created.

6. Why was the quadratic and polynomial function unit that was unit 5 in the Louisiana Comprehensive Curriculum moved to unit 2 of the East Baton Rouge Comprehensive Curriculum? This was based on teacher feedback in May 2006, based on continuity. Since then, no teacher has requested the unit be moved back. Movement back is based on consensus and what is best for the students of EBRPSS.

APPENDIX D: CURRENT UNIT 2 TEST IN EBRCC

The following is from the East Baton Rouge Parish Comprehensive Curriculum for Algebra II

Name:__________________________ Teacher:__________________

School:_________________________ Period:___________________

Algebra II Unit 2 Test

For each of the following questions, choose the BEST answer:

For numbers 1 and 2, consider the graph of the equation $y=(x+3)^2-4$.

1. The vertex is:

A. $(3,4)$ **B.** $(3,-4)$ **C.** $(-3,4)$ **D.** $(-3,-4)$

2. The y-intercept is:

A. $(0,4)$ **B.** $(0,-4)$ **C.** $(0,5)$ **D.** $(0,-5)$

For numbers 3-6, consider the graph of the equation of $y=-x^2-4x+32$.

3. The vertex is:

A. $(0,32)$ **B.** $(32,0)$ **C.** $(-2,44)$ **D.** $(-2,36)$

4. The y-intercept is:

A. $(0,-32)$ **B.** $(0,32)$ **C.** both of these **D.** none of these

5. The x-intercept is:

A. $(4,0)$ **B.** $(-8,0)$ **C.** both of these **D.** none of these

6. The graph opens:

A. up **B.** down **C.** right **D.** left

7. Solve $2x^2-8x+3=0$:

A. $x = \pm\sqrt{\frac{1}{2}}$ **B.** $x = \pm\sqrt{40}$ **C.** $x = 2 \pm 2\sqrt{10}$ **D.** $x = \frac{4 \pm \sqrt{10}}{2}$

For numbers 9-11 match the quadratic equations on the left with their proper number of solutions on the right

9. $x^2 + 5x - 3 = 0$ **A.** one real solution

10. $-2x^2 + 4x - 2$ **B.** two real solutions

11. $x^2 + 2x + 5$ **C.** two complex solutions

For numbers 12-14 match the quadratic equations on the left with the number of times their graphs cross the x-axis on the right.

12. $x^2 + 5x - 3 = 0$ **A.** crosses the x-axis in two places

13. $-2x^2 + 4x - 2$ **B.** crosses the x-axis in one place

14. $x^2 + 2x + 5$ **C.** never crosses the x-axis

13. $\left(x^2 - 5x - 14\right) \div \left(x + 2\right) =$

A. x **B.** $-x$ **C.** $x + 7$ **D.** $x - 7$

VITA

Angie Ann Byrd was born 1968 in Milwaukee, Wisconsin, to Vladimir and Ruth Aleksic. She is the second of three siblings. She graduated from the University of Wisconsin at Green Bay with a Bachelor of Science in mathematics, a secondary education certification, and a minor in human development in June 1991. She taught Algebra I, Algebra II, and Geometry at the Learning Skills Center in Pensacola Florida for three years. She taught Algebra I, middle school math and science, and physical education at Starkey Academy in East Baton Rouge Parish for one year. She taught Applied Math I and II, Algebra II, Advanced Math and Calculus at Capitol High School in East Baton Rouge Parish for thirteen years. She taught Algebra II, and is currently teaching Advanced Math, ACT Math, Advanced Placement Calculus, and Advanced Placement Statistics at Belaire High School in East Baton Rouge Parish for the third year.