Experimental and numerical investigation of fluid flow and heat transfer in microchannels

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EXPERIMENTAL AND NUMERICAL INVESTIGATION OF FLUID FLOW AND HEAT TRANSFER IN MICROCHANNELS

A Thesis

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering

in

The Department of Mechanical Engineering

by

Wynn Allen Phillips, Jr.
B.S., Louisiana State University, 2007
August 2008
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## NOMENCLATURE

**General Terms**

- \( r \) - Radius
- \( D \) - Diameter
- \( D_h \) - Hydraulic diameter
- \( x \) - Axial/local
- \( A \) - Area
- \( A_c \) - Cross-sectional area
- \( \dot{m} \) - Mass flow rate
- \( c_v \) - Constant volume specific heat
- \( T \) - Temperature
- \( p \) - Pressure
- \( u/v/w/V \) - Velocity
- \( q \) - Heat
- \( h \) - Heat transfer coefficient
- \( \bar{h} \) - Average heat transfer coefficient
- \( P \) - Perimeter
- \( L \) - Length/channel length
- \( \text{Nu} \) - Nusselt Number
- \( k \) - Thermal conductivity
- \( \Delta T_{lm} \) - Log mean temperature difference
- \( \text{Nu} \) - Average Nusselt number
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Thermal diffusivity</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic Viscosity</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Nondimensional temperature</td>
</tr>
<tr>
<td>$f$</td>
<td>Friction factor</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Surface roughness</td>
</tr>
<tr>
<td>Kn</td>
<td>Knudsen number</td>
</tr>
<tr>
<td>$\Delta P$</td>
<td>Pressure drop</td>
</tr>
<tr>
<td>K</td>
<td>Loss coefficient</td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>Channel aspect ratio</td>
</tr>
<tr>
<td>$W$</td>
<td>Channel width</td>
</tr>
<tr>
<td>$H$</td>
<td>Channel depth</td>
</tr>
</tbody>
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**Superscripts**
- $^+$ - Nondimensional

**Subscripts**
- $m$ - Mean/average
- $conv$ - Convective
- $o$ - At the out position
\( i \) - At the in position
\( s \) - Surface
\( \text{avg} \) - Average
\( \text{hyd} \) - Hydrodynamic/hydrodynamically
\( \text{lam} \) - Laminar
\( \text{turb} \) - Turbulent
\( \text{fd} \) - Fully developed
\( \text{th} \) - Thermal/thermally
\( c \) - Characteristic
\( \text{app} \) - Apparent
\( \text{un} \) - Uncorrected
\( \text{dev} \) - Developing
\( \infty \) - Fully developed
\( \text{tot} \) - Total
ABSTRACT

Microchannels are of current interest for use in heat exchangers where very high heat transfer performance is desired. Microchannels provide very high heat transfer coefficients because of their small hydraulic diameters. In this study, an investigation of fluid flow and heat transfer in microchannels is conducted.

A review of the literature published regarding fluid flow and heat transfer in microchannels is provided in this study. A thorough background on the theory of internal convective heat transfer is provided as well. A critical analysis of some of the published heat transfer experiments on microchannels is also given. It was found that some of the experimental methods published recently employ experimental and data reduction techniques which may result in errors for reported Nusselt numbers. A brief computational fluid dynamics (CFD) investigation into the heat transfer and fluid flow performance of some channels with designed bumps is also presented. It was found that designed channels offer enhanced Nusselt numbers in the turbulent regime at the cost of a higher friction factor when compared to a plain channel.

Fluid flow and heat transfer experiments were conducted on a copper microchannel heat exchanger (MHE). An experimental method of imposing a constant surface temperature to the MHE was used. A description of the experimental apparatus and procedure is provided. The friction factor results from the experiments agree fairly well with theoretical correlations. The experimental Nusselt number results agree with theory very well in the transition/turbulent regime, but the results show a higher Nusselt number in the laminar regime than predicted by theoretical correlations. A full description of the data reduction and analysis of the experimental data is given. A CFD model was created to simulate the fluid in the inlet plenum and the
microchannels. The results from these simulations are presented, and they show good agreement with the experimental data in the transition/turbulent regime as well as with theoretical correlations for laminar and turbulent flow.
CHAPTER 1: INTRODUCTION

1.1 Microchannel Heat Exchangers

Microchannels have become very popular in applications where very high heat transfer rates are necessary. The study of microchannels for use in heat exchanger applications is generally agreed to have begun in 1981 when Tuckerman and Pease [1] produced a publication that outlined the benefits of using small diameter channels for cooling of very-large-scale integrated circuits. They noted that as the hydraulic diameter of a channel decreases, the heat transfer coefficient increases. They showed a forty-fold increase in heat transfer capabilities over previous heat exchanger designs. Since this publication, a good deal of research has been conducted in the fields of both microchannel heat exchanger (MHE) manufacturing technology and MHE performance improvements. While many of the first MHE’s constructed and tested were made from Si, metal-based MHE’s are currently being developed to meet even higher performance demands. The high heat conductivity and high strength of metal-based MHE’s make them a very promising prospect for high performance cooling applications.

Among the research that has been conducted within the academic and industrial communities are investigations of the validity of the macroscale equations of friction factor and Nusselt number on the microscale. Over the course of the past couple of decades, many conflicting accounts of results on the validity of classical macroscale equations for microchannel fluid flow and heat transfer have been given. Among this research are the investigations of the validity of the macroscale equations for friction factor, transition Reynolds number, and Nusselt number on the microscale. The following is a literature review of the research that has been completed on microchannels over the past two decades.
1.2 Literature Review

1.2.1 Friction Factor and Turbulent Transition

A study of the history of the discrepancies between microchannel friction factor measurements in experiments and macroscale laminar equations shows that it is primarily the early studies which present these contradictions [2-7]. Some of the first experimental results obtained for fluid flow in microchannels came from Wu and Little [8] for gas flow. Their measured friction factors in the laminar regime were higher than expected, and they found that the transition Reynolds number ranged from 350 to 900. They attributed the early transition to the roughness of the walls of the microchannels [7]. Peng et al [5, 9, 10] conducted experiments on microchannels, and they found that the laminar-to-turbulent transition period occurred at Reynolds numbers which were lower than expected from conventional theory [7]. Peng and Peterson [6, 11] also found disparities between conventional flow theory and experimental results for microchannels. They tested microchannels with hydraulic diameters ranging from 133 μm to 367 μm, and they showed a friction factor dependence on hydraulic diameter and channel aspect ratio. Pfund et al [12] found through flow visualization that turbulent transition occurred at lower Reynolds numbers for microchannels than for macrochannels [7].

It is likely that these contradictions between experimental results and classical laminar theory are due to factors involving experimental error and the lack of accounting for entrance losses [13-15]. More recent studies have shown good agreement with laminar theory. Xu et al. [14] presented results of experiments that showed good agreement with laminar theory. The largest microchannel they tested had a hydraulic diameter of 344 μm, and they tested Reynolds numbers from very low ranges (~ 20) to the turbulent regime (4000). Judy et al. [15] performed pressure drop experiments on both round and square microchannels with hydraulic diameters
ranging from 15 to 150 μm. They tested distilled water, methanol, and isopropanol over a Reynolds number range of 8 to 2300. Their results showed no distinguishable deviation from laminar flow theory for each case. Liu and Garimella [16] conducted flow visualization and pressure drop studies on microchannels with hydraulic diameters ranging from 244 to 974 μm over a Reynolds number range of 230 to 6500. They were able to measure the onset of turbulence through their flow visualization, and they compared their pressure drop measurements with numerical calculations. Their results showed that both conventional turbulent transition and pressure drop correlations are valid on the microscale. Qu and Mudawar [17] found that friction factor data from their experiments with microchannels of 349 μm hydraulic diameter agreed well with classical theory. Wu and Cheng [18, 19] also found that friction factor results for flow through trapezoidal microchannels agreed with theory for smooth channels. However, for rough channels, some deviations in friction factor from theory were found by Wu and Cheng. Their measured turbulence transition region occurred around Reynolds numbers from 1500 to 2000. Kohl et al [20] measured pressure drop in microchannels through internal pressure measurements. They constructed microchannel tap lines of 7 μm width and 10 μm depth using microfabrication techniques. Their results showed that both friction factor and turbulent transition Reynolds numbers agreed well with theoretical results. It should be noted, however, that their tests which used water as a working fluid were conducted in relatively smooth channels.

The study of transition to turbulence in microchannels has been studied by a number of researchers through the use of microscopic particle image velocimetry (microPIV) [13, 21-25]. This method of analyzing flow patterns and behavior in microchannels is beneficial since it is noninvasive. Many of the problems involved in past experiments conducted on microchannels
lie with using macroscale measurement techniques, and thus microPIV provides a more sophisticated solution to the problem of analyzing microscale fluid flow [13]. However, even with such a measurement device, it is important to understand how to quantify the onset of turbulence. Zeighami et al. [21] measured transition using the repeatability of velocity data and particle motion. Their results showed a transition region in the range of Reynolds numbers from 1200-1600 [13]. Lee et al [22] defined the onset of turbulence through deviations in velocity profiles, and they found the critical Reynolds number to be 2900 [13]. Sharp and Adrian [23] performed microPIV experiments on glass tubes, and they identified transition through the presence of unsteady changes in centerline velocity. Their findings more closely relate to conventional theory, with critical Reynolds numbers ranging from 1800 to 2200. Li et al. [24] performed similar studies on microchannels with transition criteria defined also with deviations in centerline velocity, and they found the transition Reynolds number to be 1535. They also found that the fully turbulent region began at Reynolds numbers of 2630 to 2853. Li and Olsen [13, 25] performed microPIV visualizations on microchannels and determined that no early transition to turbulence was present in their studies. Some of their investigations included microchannels with hydraulic diameters of 320 μm and aspect ratios of 1 to 5.7 being tested over a Reynolds number range of 200 to 3267. They quantified the onset of turbulence by an increase in centerline velocity fluctuations, and they found that the transition region occurred for Reynolds numbers from 1765 to 2315. They also found that fully developed turbulent flow began to occur at Reynolds numbers ranging from 2600 to 3200.

While many of the studies mentioned above involve only smooth microchannels, there has also been a good deal of research on the effects of roughness on friction factor and transition to turbulence. Mala and Li [26] tested circular microtubes with high relative roughnesses, and
they found that as the relative roughness increased, the friction factor in the laminar regime increased. Pfund et al [12] showed early transition in rough channels. Guo and Li [27] indicated from their experiments on microchannels with high relative roughnesses that friction factors increased with relative roughness, and they also proposed that wall roughness effects may incur early turbulence transition and higher than expected Nusselt numbers. Tu and Hrnjak [28] performed friction factor experiments on smooth microchannels, and they found that their results were well predicted by laminar theory. However, one of the channels that they tested had a significant surface roughness, and a friction factor increase and transition Reynolds number decrease to 1570 was found for this case [7].

As a summary of the information gathered on friction factor experiments conducted on microchannels, it can be said that conventional theory for macroscale conditions can be readily applied to smooth-wall microchannels. However, there is a great deal of investigation left to be done to make conclusions about the effects of surface roughness on both the friction factor and turbulent transition Reynolds number in microchannels.

1.2.2 Nusselt Number and Heat Transfer

Some well developed summaries of experimental results for heat transfer in microchannels found in literature are available in a number of publications [17, 29-31]. Some of the works mentioned here are commonly cited in these publications. Wu and Little [32] tested rectangular microchannels and found that the Nusselt number varied with Reynolds number in the laminar regime. This was one of the first studies that predicted a higher Nusselt number for microchannels when compared to macroscale equations. Choi et al. [33] also suggested from their experiments with microchannels that the Nusselt number did in fact depend on the Reynolds number in laminar microchannel flow. They also found that the turbulent regime
Nusselt numbers were higher than expected from the Dittus-Boelter equation. Rahman and Gui [2, 3] found Nusselt numbers to be high in the laminar regime and low in the turbulent regime as compared to theory [29, 31]. Similar to Choi et al., Yu et al. [34] found that their measured Nusselt numbers in the turbulent regime were higher than the Dittus-Boelter equation [31]. Adams et al. [35] tested microchannels in the turbulent regime and found their results to be higher than predicted by theoretical turbulent equations. Nusselt numbers in excess of theoretical predictions were also found by Celata et al. [36] and Bucci et al. [37] through experimental work with microchannels [31]. Recently, Jung and Kwak [29] tested microchannels of 100 μm hydraulic diameter. They found the Nusselt number to be a function of both the Reynolds number and the aspect ratio in the laminar regime.

Not all researchers have found disparities between experimental results and theoretical predictions with regard to heat transfer. Harms et al. [30] performed experiments on an array of microchannels and determined that local Nusselt numbers can be accurately predicted in microchannels by conventional correlations. They also determined that proper plenum design and consideration are necessary to be able to apply the theoretical Nusselt number and friction factor equations to microchannel experiments. Qu and Mudawar [17] performed both experimental and numerical studies on microchannels with 231 μm widths and 713 μm depths. They tested only in the laminar regime, and they indicated that the Navier Stokes and energy equations do in fact properly predict fluid flow and heat transfer in microchannels. Owhaib and Palm [38] tested channels with diameters ranging from 800 μm to 1700 μm, and found good agreement with theory for Nusselt number in the turbulent regime. Lee et al. [31] investigated the heat transfer characteristics of copper microchannels with widths ranging from 194 to 534 μm over a range of Reynolds numbers from 300 to 3500. They also completed a numerical
analysis to validate their test results. They found that a classical macroscale analysis can be applied to microchannels, although care must be taken to use the proper theoretical or empirical correlation. Many of the empirical correlations available did not match with their experimental data. However, their numerical analysis showed good agreement with their experimental results in the laminar regime. They indicated that considerations of entrance regions and turbulent transitions must be accounted for.

As a summary of the experimental and numerical results for heat transfer in microchannels, it can be said that there are no concrete conclusions regarding the validity of classic empirical or theoretical correlations for the prediction of microscale heat transfer effects. Many of the disparities may arise from experimental error, improper analysis, surface roughness effects, or channel entrance effects. Further investigation is required to definitively characterize the heat transfer performance of microchannels.

1.3 Setup for the Present Study of Microchannel Fluid Flow and Heat Transfer

From this literature review, it can be inferred that one of the fundamental difficulties with performing experiments on microchannel heat exchangers is proper instrumentation. Because of the size of microchannels, certain common measurement techniques are not available for use in experiments. Direct temperature measurement inside the microchannels does not seem feasible as thermocouples are too large to fit in the channels without disturbing the flow pattern significantly. Direct pressure measurement is also a problem. Given the small size of microchannels, constructing even smaller pressure taps is quite a challenge. For some experimental setups, this feat is easier than others. As mentioned earlier, Kohl et al. [20] used micromachining techniques to construct 8 μm diameter pressure tap lines along a microchannel. This size of pressure tap hole is necessary to get an accurate indication of the pressure inside the
microchannels. Too big of a pressure tap hole would create flow effects around the pressure tap that significantly influence the pressure reading and the flow field inside the microchannel.

As an alternative to direct pressure tapping inside the microchannels, many researchers have used pressure taps inside the supply regions at the channel inlet and the channel outlet. A few of the papers published suggest that if this procedure is adopted, accounting for the inlet and exit losses is essential to accurately predicting the friction factor inside of the microchannel [12-14]. However, it can often be fundamentally difficult to accurately predict the entrance losses. This is due to the effects that entrance geometries have on the losses. For example, a perfectly sharp cornered entrance has approximately 400% higher head loss than a slightly rounded entrance (r_{corner}/D_h = 0.1) [41]. The exit losses can be readily estimated since loss coefficients are well defined and unchanging with exit geometry. Also important for consideration in correlating the friction factor is the hydrodynamic entrance length. The hydrodynamic entrance region has a higher friction factor than the developed region, and therefore the presence of a hydrodynamic development region in a microchannel will give a high apparent friction factor.

In this study, some background and theory is presented for insight on the internal flow heat transfer problem at hand. Some secondary investigations in microchannel fluid flow and heat transfer are presented as further background on the subject and for insight into the design of microchannel heat exchanger experiments. The main topics of this study are the experiments conducted on a Cu MHE and the Computational Fluid Dynamics (CFD) analysis conducted to interpret the experimental results. In the experiments conducted, some of the aforementioned issues with experimentation on microchannels were investigated. Effects of entrance lengths and entrance and exit losses are discussed, and a new method of heat application was used in the experiments. All of the studies mentioned above used a heater as a heat source. This implies
that the surface boundary condition for the microchannels in each of these studies was that of a constant surface heat flux. In the experiments presented here, a constant surface temperature condition was imposed on the microchannel walls, thus eliminating the need for multiple wall temperature measurements. The benefits and shortcomings of this design are discussed. Correlations found in literature are used for comparison with the experimental results. In the CFD analysis, Fluent was used to compute the flow and heat transfer fields in the microchannels.
CHAPTER 2: BACKGROUND AND THEORY FOR INTERNAL CONVECTIVE HEAT TRANSFER

When performing an experiment or analysis, proper care must be taken to measure the correct quantities, accurately measure those quantities, and perhaps most importantly, perform the proper calculations to interpret the data. Knowing which quantities need to be measured prior to performing an experimental analysis is essential. Therefore, before designing an experiment, a thorough review of the physical phenomena involved in the system in question must be completed.

Microchannels involve internal flow and internal heat transfer. There are many analytical solutions to the laminar flow situations involved in the heat transfer and fluid flow of internal flow. This chapter includes a derivation of the heat transfer involved in practical internal tube flows. Although many microchannels in practical use are in fact not round, it has been shown by many studies that the circular tube theory of internal flows is applicable to rectangular channel flows if the hydraulic diameter $D_h$ is substituted for the circular diameter $D$ in the results of circular flow analysis. An analysis of a circular tube is more simplified than a rectangular analysis, and the study provides more readily apparent insight into the phenomena involved because of its simplicity. Also included in this chapter are correlations for developing flow Nusselt numbers and turbulent flow Nusselt numbers.

### 2.1 Tube Flow Energy Balance

Consider a circular tube of length $L$, radius $R$ (diameter $D$), and coordinates $(r, \varphi, x)$ as shown in Figure 2.1. Now consider the control volume (shaded region) shown in Figure 2.1 through which fluid flows and heat is convected through the pipe walls. The energy balance of this control volume is given by
\[
\dot{m}(c_v T_m + p u_x) + dq_{conv} = \left[ \dot{m}(c_v T_m + p u_x) + \dot{m} \frac{d}{dx} (c_v T_m + p u_x) dx \right]
\]

(1)

where \(dq_{conv} = \dot{m}c_p dT_m\) for both ideal gases and incompressible liquids.

\[\begin{align*}
\frac{d}{dx} (q_{conv}) &= \dot{m} \frac{d}{dx} (c_v T_m) \\
(2)
\end{align*}\]

Through integration over the length of the tube, the heat applied to the control volume through convection is found to be

\[q_{conv} = \dot{m}c_p (T_{out} - T_{in})\]

(3)

Note also that through the use of Newton’s law of cooling, \(q_s'' = h(T_s - T_m)\), and the fact that \(dq_{conv} = q_s''Pdx\), Eq. (3) can be converted to

\[\frac{dT_m}{dx} = \frac{P}{\dot{m}c_p} h(T_s - T_m)\]

(4)
2.2 Constant Surface Heat Flux

For the constant heat flux condition, Eq. (4) can be integrated to an arbitrary distance \( x \) using \( q_{\text{conv}} = q_s^"PL \) to obtain

\[
T_m(x) = T_{mi} + \frac{q_s^"P}{mc_p}x
\]

(5)

This implies that the mean fluid temperature varies linearly with axial distance in a tube. Also, the heat transfer coefficient for the constant surface heat flux condition is given by Newton’s law of cooling:

\[
q_s^" = h(T_s - T_m)
\]

(6)

This can be further adapted to the tube situation by using the linearity of the mean fluid temperature along the tube length to obtain

\[
q_s^" = h \left( T_s - \frac{T_{mi} + T_{mo}}{2} \right)
\]

(7)

It is important to note that the surface heat flux in a channel with a constant heat flux on the wall does not have a constant surface temperature [39]. From Eq. (7), the varying Nusselt number can be substituted to obtain [40]

\[
T_s(x) - T_m(x) = \frac{q_s^"D}{\text{Nu}_k k}
\]

(8)

If \( \text{Nu}_k \) is constant (i.e. fully developed conditions), then the following can be shown:
\[ \frac{dT_m}{dx} = \frac{dT_s}{dx} \]  

(9)

Thus, the temperature of the wall varies linearly and parallel to the mean fluid temperature in the fully developed region, and in the developing region the wall temperature varies according to Eq. (8) (which means that it is nonlinear in this region).

2.3 Constant Surface Temperature

The constant surface temperature condition shows a much different behavior than the constant surface heat flux condition. Defining $\Delta T$ as $(T_s - T_m)$ and integrating Eq. (5) over the length of the tube $L$, the following expression is obtained:

\[ \ln \frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{mc_p} \bar{h} \]

(10)

where $\Delta T_o = (T_s - T_{mo})$, $\Delta T_i = (T_s - T_{mi})$, and $\bar{h} = \frac{1}{L} \int_0^L h \cdot dx$. This then provides

\[ \frac{\Delta T_o}{\Delta T_i} = e^{-\frac{PL}{mc_p} \bar{h}} \]

(11)

This result highlights the fact that the difference in surface temperature and mean fluid temperature decays exponentially along the tube length. This is in direct contrast to the constant surface heat flux condition, where the difference in surface temperature and mean fluid temperature is constant in the fully developed region [39]. Furthermore, from Eqs. (3) and (11),

\[ q_{conv} = \bar{h}A_s \frac{\Delta T_o - \Delta T_i}{\ln \left( \frac{\Delta T_o}{\Delta T_i} \right)} = \bar{h}A_s \frac{T_{mi} - T_{mo}}{\ln \left( \frac{T_{wall} - T_{mo}}{T_{wall} - T_{mi}} \right)} = \bar{h}A_s \Delta T_{lm} \]

(12)
where \( \bar{h} \) is the average heat transfer coefficient over the length of the tube and \( \Delta T_{lm} \) is the log mean temperature difference across the tube. The inclusion of the average heat transfer coefficient is an important point to stress since it means that Eq. (12) may be readily applied to flow situations that involve both developing and fully developed flows. The average heat transfer coefficient does not assume a fully developed condition. However, when comparing the average heat transfer coefficient from one set of experimental results with another, it is important to include effects of the thermally developing region since this region provides higher local heat transfer coefficients. It is noted here that the log mean temperature difference is considered to be an appropriate average temperature difference for use in constant surface temperature conditions. This is in contrast to the constant heat flux condition, where the average temperature difference includes an arithmetic mean [39].

2.4 Nusselt Numbers for Fully Developed Laminar Flow

The energy equation for the circular tube of Figure 2.1 with constant properties is given by

\[
u \frac{\partial T}{\partial x} + v_r \frac{\partial T}{\partial r} = \alpha \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial x^2} \right]
\]

(13)

Two different solutions may be found for Eq. (13). The constant surface heat flux \( q_s \) is investigated first. An analytical solution to Eq. (13) is more readily found for the constant surface heat flux condition if the assumptions of symmetric heat transfer \( (\partial^2 T/\partial \phi^2 = 0) \), hydrodynamically fully developed flow \( (v_r = 0) \), and no axial conduction \( (\partial^2 T/\partial x^2 = 0) \) are made. For a thermally developed condition for constant surface heat flux, the axial temperature gradient simplifies to \( (\partial T/\partial x) = (\partial T/m/\partial x) \) [40]. These assumptions simplify the energy equation to
\[
\frac{\partial T_m}{\partial x} = \alpha \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \right]
\]  

(14)

The goal of solving the energy equation here is to define a function or expression for the local Nusselt number and average Nusselt number, which are defined as

a) \( \text{Nu}_x = \frac{h_x D}{k} \)

b) \( \overline{\text{Nu}} = \frac{\overline{h} D}{k} \)

(15)

Applying the parabolic velocity profile \( u/V_{\text{avg}} = 2(1 - r^2/R^2) \) to Eq. (14) and integrating gives

\[
T_s(x) - T_m(x) = \frac{11 q_s^* D}{48 k}
\]

(16)

which indicates that the surface temperature differs from the mean fluid temperature by a constant [40]. Applying Newton’s law of cooling (Eq. (6)), the Nusselt number for the constant surface heat flux condition can be shown to be

\[
\text{Nu}_x = \overline{\text{Nu}} = 4.364
\]

(17)

which indicates that the average Nusselt number is equal to each local Nusselt number [40]. This solution of a constant Nusselt number for fully developed conditions implies that in the laminar regime, the Nusselt number is independent of the Reynolds number and the Prandtl number.

The solution of the constant surface temperature \( T_s \) condition is slightly more involved than the constant heat flux condition. Application of the same assumptions of symmetric heat transfer, a fully hydrodynamically developed condition, and no axial conduction is still valid. However, the axial temperature gradient is different for the constant surface temperature
condition. It is defined as \( \frac{\partial T}{\partial x} = \frac{(T_s - T(x,r))(T_s - T_m)(\partial T_m/\partial x)}{T_s - T_m} \) [40]. Applying these conditions to Eq. (13) gives the energy equation for the constant surface temperature condition as

\[
\nu \left( \frac{T_s - T(x,r)}{T_s - T_m} \right) \frac{\partial T_m}{\partial x} = \alpha \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \right]
\]

(18)

Applying the parabolic velocity profile \( u/V_{avg} = 2(1 - r^2/R^2) \) to Eq. (18) and integrating gives the infinite series solution [40] of the form

\[
\frac{T_s - T(x,r)}{T_s - T_{centerline}} = \sum_{n=0}^{\infty} C_{2n} \left( \frac{L}{R} \right)^{2n}
\]

(19)

The solution to this equation is given by Kays and Crawford [40], and it can be shown to yield the Nusselt number for the constant surface temperature condition as

\[
\text{Nu}_x = \bar{\text{Nu}} = 3.657
\]

(20)

Therefore, the Nusselt number for the constant surface temperature condition is also a constant for fully developed conditions.

2.5 Entrance Lengths

Almost every study cited in literature involving fluid flow and heat transfer in microchannels involves situations where significant portions of the microchannels are in either a hydrodynamically developing region, a thermally developing region, or a combination of the two (referred to as simultaneously developing flow). Having a large portion of the microchannels in these developing regions can give measured values of friction factor and heat transfer coefficient that are higher than expected. Therefore, when performing an experiment involving microchannels, it is important that the entrance lengths be considered.
The hydrodynamic entrance length is defined as the distance from the channel entrance where the shear stress at the walls becomes constant with increasing distance along the channel. In common engineering practice, this is practically defined as the distance from the entrance where the wall shear stress reaches within 2% of the fully developed value [41]. In the developing region, the velocity profile is changing along the length of the channel. Once the velocity profile is constant, the fully developed region has been reached.

The hydrodynamic entrance length in the laminar regime is defined as [41]

\[
L_{hyd,lam} \approx 0.05ReD
\]

(21)

where \( Re = \frac{\rho VD}{\mu} \) is the Reynolds number. This can be developed from the Graetz problem (see Section 2.6) which solves the thermally developing region of internal flows. In the turbulent regime, the hydraulic entrance length is much shorter because of the mixing involved in turbulence. The turbulent entrance length can be approximated by [41]

\[
L_{hyd,turb} \approx 1.359Re^{\frac{1}{4}}D
\]

(22)

It should be noted that the above correlations for entrance lengths assume a uniform velocity inlet. In many practical cases, including the situations where flow is entering from a large manifold, the entrance does not have a uniform velocity. It is proposed by some engineers that an abrupt entrance from a manifold to a microchannel significantly decreases the hydrodynamic entrance length [31, 42]. It was shown by Rosenhow et al. [42] that for a microchannel array similar to the one in consideration here, the hydrodynamic entrance length for \( Re = 300 \) was approximately 79% shorter than what was predicted by Eq. (21).
Thermal entrance length is defined as the distance it takes the flow along the channel to reach where the relative shape of the temperature profile becomes constant. This can be mathematically defined as

\[
\frac{d}{dx} \left[ \frac{T_s(x) - T(x,y,z)}{T_s(x) - T_m(x)} \right]_{fd,th} = 0
\]  

(23)

where \(T_s(x)\) is the surface temperature and \(T_m(x)\) is the mean fluid temperature [40]. Another way to define the thermal entrance length is the length along the channel at which the local heat transfer coefficient, \(h_x\), becomes constant. This is an important point to note since when calculating or measuring the heat transfer coefficient in a channel, care must be taken to account for the thermal entrance length when comparing measured heat transfer coefficients with established correlations.

The thermal entrance length can be related to the hydrodynamic entrance length by the Prandtl number. If \(Pr > 1\), the hydrodynamic boundary layer will develop faster than the thermal boundary layer. If \(Pr < 1\), the thermal boundary layer will develop faster than the hydrodynamic boundary layer. This is evident from the definition of the Prandtl number:

\[
Pr = \frac{\nu}{\alpha} = \frac{\text{momentum diffusivity}}{\text{thermal diffusivity}}
\]

(24)

It is therefore unsurprising that the thermal entrance length for laminar flows is simply [39]

\[
L_{th, lam} \cong 0.05 \text{RePrD}
\]

(25)
For turbulent flow, the thermal entrance length is not well defined, but is generally considered to
be small and nearly independent of Prandtl number [39]. For transitional flow, no correlations
for hydrodynamic or thermal entrance lengths exist.

2.6 Nusselt Number for Laminar Thermally Developing Flow

With an understanding of the fully developed Nusselt number and an understanding that
entrance regions must be treated differently than fully developed regions, some solutions to the
entrance region problem are presented here. This discussion involves only circular tubes for
simplicity. The problem involving internal thermally developing flow is referred to as the Graetz
problem. The complete derivation is not presented here (for the derivation, see Kays and
Crawford [40] and Kakac et al. [43]), but the results are discussed.

The method of developing and solving the Graetz problem is as follows. Starting with
the energy equation for a circular tube (with axial conduction neglected),

\[ \frac{\partial T}{\partial x} = \alpha \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \right] \]

a nondimensionalized energy equation can be developed. The most important nondimensional
parameter involved is the nondimensional axial distance since \( \text{Nu}_x = f(x^+) \). The nondimensional
axial distance is given by

\[ x^+ = \frac{2(x/D)}{\text{RePr}} \]

(26)

The other nondimensional parameters are \( r^+ = r/R \), \( u^+ = u/V_{avg} \), and the nondimensional
temperature \( \theta = (T_s - T_m)/(T_s - T_{mi}) \). Solving the nondimensionalized energy equation for the
constant surface temperature and the constant surface heat flux condition each yield different solutions.

The solution to constant surface temperature problem is [40, 43]

\[ \vartheta(x^+, r^+) = \sum_{n=0}^{\infty} C_n R_n(r^+) e^{-\lambda_n^2 x^+} \]  

(28)

\[ \text{Nu}_{x,T} = \frac{\sum_{n=0}^{\infty} G_n e^{-\lambda_n^2 x^+}}{2 \sum_{n=0}^{\infty} \frac{G_n}{\lambda_n^2} e^{-\lambda_n^2 x^+}} \]  

(29)

\[ \overline{\text{Nu}}_T = \frac{1}{2x^+ ln} \left( \frac{1}{8 \sum_{n=0}^{\infty} \frac{G_n}{\lambda_n^2} e^{-\lambda_n^2 x^+}} \right) \]  

(30)

where \( G_n = -0.5C_n R'_n(1) \). The constants and eigenvalues can be found for the above equations, and the Nusselt number distribution can be found.

While the above equations provide a very good solution to the thermally developing constant surface temperature problem, simpler equations are necessary for use in practical engineering applications. A correlation by Hausen as presented by Incropera and Dewitt [39] is valid for constant surface temperature and thermally developing laminar flow in a circular tube. It is expressed as

\[ \overline{\text{Nu}}_{Hsn} = 3.657 + \frac{0.0668 \frac{D}{L} \text{RePr}}{1 + 0.04 \left[ \frac{D}{L} \text{RePr} \right]^{2/3}} \]  

(31)
2.7 Nusselt Number for Laminar Simultaneously Developing Flow

The solution to the simultaneously developing flow problem is slightly more complicated than the thermally developing solution. A correlation by Sieder and Tate gives a useful solution to the mean Nusselt number for simultaneously developing flow with a constant surface temperature. This correlation, as presented by Incropera and Dewitt [39], is

$$\bar{Nu}_{ST} = 1.86 \left( \frac{RePr}{x/D} \right)^{1/3} \left( \frac{\mu}{\mu_s} \right)^{0.14}$$  \hspace{1cm} (32)

2.8 Nusselt Number for Fully Developed Turbulent Flow

For turbulent flow, the Nusselt number is not constant in the fully developed regime. As the following correlations suggest, the Nusselt number in the fully developed turbulent regime is mainly a function of the Reynolds number and the Prandtl number. The friction factor is also used in some correlations, and this can give a good first estimate for surface roughness effects on Nusselt number.

The Dittus-Boelter equation for fully developed turbulent flow in smooth circular tubes is presented by Incropera and Dewitt [39] and Rosenhow et al. [42] as

$$\bar{Nu}_{DB} = 0.023 Re^{4/5} Pr^{0.4}$$  \hspace{1cm} (33)

Rosenhow et al. recommends the Dittus-Boelter equation for $2500 \leq Re \leq 1.24 \times 10^5$. This equation is extended down to $Re \geq 1000$ for comparison with the experimental results presented in Chapter 5 since there is a possibility that turbulence existed in the experimental cases in this Reynolds number range. The Dittus-Boelter equation is derived from an extension of the
Reynolds analogy, which relates nondimensional terms such as friction factor, Nusselt number, Reynolds number, and Prandtl number for turbulent flow [39].

The Petukhov equation for fully developed flow in circular tubes is presented by Rosenhow et al. [42] as

\[
\overline{Nu}_p = \frac{(f/8)RePr}{c + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}
\]

(34)

where \( c = 1.07 + 900/Re - 0.63/(1 + 10Pr) \) and \( f \) is the friction factor. This equation is recommended by Rosenhow et al. for \( 4000 \leq Re \leq 5 \times 10^6 \). Like the Dittus-Boelter equation, the Petukhov equation is extended down to \( Re \geq 1000 \) for comparison with the experimental results presented in Chapter 5 since there is a possibility that turbulence existed in the experimental cases in this Reynolds number range. Notice that the Petukhov equation accounts for the friction factor in the microchannels. The inclusion of the friction factor implies that Nusselt number enhancement from roughness effects is included in the Petukhov equation. Due to the possibility of experimental error in determining the friction factor for the microchannels, the friction factor for the Petukhov equation is calculated from the Haaland equation for friction factor, which is expressed as

\[
f_{Hind} = \left\{ -1.8 \log \left[ \frac{6.9}{Re} + \left( \frac{\varepsilon}{D} \right)^{1.11} \right] \right\}^2
\]

(35)

where \( \varepsilon/D \) is the relative surface roughness [41]. For the 4 \( \mu \)m roughness estimated for the microchannels tested in the experiments presented in Chapters 4 and 5, \( \varepsilon/D = 0.017 \). The procedure of using turbulent regime friction factor correlations to find the friction factor for use in the Petukhov equation is a generally recommended procedure [42].
The Gnielinski equation for transition and turbulent flow in circular tubes is presented by Incropera and Dewitt [39] and Rosenhow et al. [42], and it is expressed as

\[
\text{Nu}_{\text{Gn}} = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}
\]

(36)

This equation is recommended by Rosenhow et al. for \(2300 \leq Re \leq 5 \times 10^6\). The Gnielinski equation, like the Petukhov equation, includes the friction factor of the microchannels, and therefore the Nusselt number enhancement from surface roughness effects is included in the Gnielinski equation. The Haaland equation is also used for friction factors in calculating Nusselt numbers from the Gnielinski equation.

**2.9 Knudsen Number**

This experiments conducted in this involve the flow of water through microchannels. While this does not by definition create a problem with applying continuity approximations, situations do occur where continuity is not valid. Gas flow in microchannels imposes more restrictions on continuum approaches for certain conditions. In order to analyze a gas flow microchannel problem with the continuum assumption, one must evaluate the Knudsen number (Kn). Knudsen number is defined as

\[
Kn = \frac{l}{L_c} = \frac{k_B T}{\sqrt{2\pi p \sigma_c^2 L_c}}
\]

(37)

where \(l\) is the mean free path of molecules (m), \(L_c\) is the characteristic length, \(k_B\) is Boltzmann’s constant (1.38066x10^{-23} J/K), \(T\) is temperature (K), \(p\) is pressure (Pa), and \(\sigma_c\) is the collision diameter of molecules (m). For the continuum approach to be valid, the Knudsen number must be lower than 0.001. For \(0.001 \leq Kn \leq 0.1\), the slip-flow regime is in effect, and the no-slip
condition at the wall is not valid. However, as long as the slip-velocity boundary condition at the wall is enforced, the Navier-Stokes equations can be applied. For $0.1 < \text{Kn}$, the Navier-Stokes equations begin to become invalid, and particle based solutions become necessary. For $\text{Kn} \geq 10$, the continuum analysis is completely invalid. This condition can only be analyzed as free molecular flow [44].

Figure 2.2 and Figure 2.3 below show variations of Knudsen number for air (where $\sigma_c = 3.673x10^{-10}$ m) in a 200 $\mu$m diameter tube [42]. Notice that as the temperature of air rises at a constant pressure, the Knudsen number increases linearly. Also notice that as the pressure decreases, the Knudsen number increases quite swiftly. Figure 2.3 indicates that at pressures below atmospheric, the no-slip conditions are generally not applicable for air in microchannels. Also, as the system reaches much lower pressures, the continuum approach breaks down.

![Air Knudsen Number Variation with Temperature](image-url)

**Figure 2.2:** Air Knudsen number variation with temperature.
Figure 2.3: Air Knudsen number variation with pressure.
CHAPTER 3: SECONDARY INVESTIGATIONS IN MICROCHANNEL FLUID FLOW AND HEAT TRANSFER

In addition to the work completed on the main thesis topic of experimentation and analysis of fluid flow and heat transfer in a Cu MHE, two secondary investigations were carried out to gain some insight on the field of microchannel fluid flow and heat transfer. The first study involves a computational fluid dynamics (CFD) analysis of four different microchannels with bumps placed on their bottom walls. These four geometries were analyzed for friction factor and Nusselt number to determine the benefits of each design. Also, a study was conducted to analyze the validity of some of the experimental and data reduction techniques found in the literature review of Chapter 1. CFD was used in this analysis to determine the best methods of experimentation and data reduction for microchannel heat transfer investigations. The findings from this analysis are part of the basis of design for the main experiment presented in Chapters 4 and 5.

3.1 Designed Geometries

3.1.1 Setup of Analysis

With metal-based microchannel manufacturing technology advancing swiftly, channel geometry designs other than plain channels may begin to emerge soon. Enhancing heat transfer through designed bumps in microchannel walls is a topic that has begun to receive some notice in the research community. An investigation of the fluid flow and heat transfer performance of four different designs were tested through the use of CFD. Each design has a bump on the bottom wall of the microchannel with a height and width of 50 μm. The four designs differ from each other only in their spacing of bumps. The spacings tested were 2x, 5x, 10x, and 20x (spacings were measured with respect to the bump size of 50 μm). The nominal outer
dimensions of these microchannels were 150 μm wide, 400 μm tall, and 10 mm long. In addition to these microchannels with bumps, a plain channel with the same nominal outer dimensions was modeled for comparison purposes. To illustrate what one of these channels looks like, a model of the 5x case is shown in Figure 3.1.

![Figure 3.1: 5x geometry model.](image)

### 3.1.2 Governing Equations and Meshing

The details of the governing equations involved with these models are the same as those for the computational analysis performed on the primary experiments presented in Chapter 6. These governing equations are the continuity equation (Eq. (65)), the x-momentum equation (Eq. (66)), the y-momentum equation (Eq. (67)), the z-momentum equation (Eq. (68)), and the energy equation (Eq. (69)). For this analysis, a structured rectangular mesh was used with boundary layer-type mesh growth ratios near the walls. This method of mesh refinement near the walls allows for good resolution of the boundary layer, which is especially important in convective heat transfer analyses since at the walls the convection coefficient is given by the boundary condition
\[-k \frac{dT}{dy}\bigg|_{y=0} = h(T_s - T_m)\]

(38)

Thus, the slope of the temperature profile determines the heat transfer coefficient, and the mesh near the wall should be fine in order to resolve the slope. For each geometry, the mesh size nearest to the wall was 3.75 $\mu$m, and the mesh grew at a ratio of 1.1 from the walls. To ensure good grid refinement and reasonable computational expense, a symmetry plane was used along the width-wise channel centerline. This type of simplification is valid because the bumps on the bottoms of the channels are symmetric about the width of the channels. The grid sizes for the 2x, 5x, 10x, and 20x cases were all approximately 300,000 cells (differences arose due to spacings and voids left in the grids by the bumps themselves). Grid independence tests based on Nusselt number similar to those shown in Chapter 6 were conducted, and similar convergence trends were found with the grid sizes used for calculations being the intermediate grid size in the grid independence tests. These grid sizes were found to be sufficient to resolve the heat transfer of the channels. Another grid independence test for friction factor was conducted, and again these intermediate grid sizes were found to be sufficient. An example of this grid independence test is given in Figure 3.2 for $Re = 2500$, where the data point plotted in the middle represents the grid used for calculations (further discussion of grid independence is left for Chapter 6 since the main CFD analysis presentation is in Chapter 6). As an example of what the grids look like, Figure 3.3 shows the 5x grid. Notice in Figure 3.3 that the grid is refined near the walls, but it becomes coarser towards the center of the channel. The boundary layer mesh functions available in the meshing program Gambit made this type of mesh gradient possible.
Figure 3.2: Grid independence for the 10x case, Re=2500.

Figure 3.3: 5x geometry mesh: (a) side view; (b) raised view of a section.
3.1.3 Solving for Friction Factor and Nusselt Number

To solve the governing equations and obtain a flow field and temperature field, the commercial code Fluent was used. A constant temperature boundary condition was imposed on the walls of the microchannels in each of the four cases. Water with properties measured at 300K was used as the working fluid. For cases in the laminar regime, the laminar model was used. For cases in the transition/turbulent regime, the k-ω model was used. It should be noted that not all Reynolds numbers could be calculated by either model. In the range $800 < \text{Re} < 1750$, both the laminar model and k-ω could not converge upon a solution to a satisfactory degree for some of the geometries. This problem could occur for a number of reasons. One possibility is the complex flow patterns that occur with a relatively high speed flow passing over a relatively stagnant flow in between the bumps. Also, there is flow separation that occurs at the edge of the bumps. The laminar model in particular may break down for these Reynolds numbers because the flow patterns may be too complex to be resolved without turbulent equations. The k-ω model is based off of approximations for transition flow, so it is known not to work with every situation, especially in these low Reynolds numbers. The difficulties encountered in modeling the flow through these microchannels with bumps illustrates some of the shortcomings of CFD, and it especially illustrates the fact that low Reynolds number transition regions are not well handled by commercially available CFD codes. It is shown in Chapter 6 that for plain geometries, Fluent is more than adequate for obtaining a converged result in this Reynolds number range. However, the inclusion of sharp bumps in the channels have proven to present a computational challenge in the transition Reynolds number range that is beyond the scope of this study.
The friction factor was calculated from Eq. (46) and Nusselt number was calculated from Eq. (15) for each geometry for $250 < \text{Re} < 750$ and $2000 < \text{Re} < 3000$. The results for friction factor are shown in Figure 3.4, and the results for Nusselt number are shown in Figure 3.5. Also plotted in Figure 3.4 are the laminar equation for friction factor (Eq. (50)) and the Haaland equation for turbulent friction factor (Eq. (35)). These are placed in Figure 3.4 for a reference to some theoretical values. It should be noted that although each case has simultaneously developing flow, the friction factors and Nusselt numbers were calculated from the same points in the channels, thus making a comparison between geometries valid for this study. It can be seen in Figure 3.4 that the four designed geometries hardly differ in friction factor. This is an interesting result since the geometries differ so greatly. This type of result suggests the possibility that in the cases with tight spacing (2x and 5x), the flow tends to pass over the bumps without creating a great deal of forward flow in the areas between bumps. The cases with larger spacing (10x and 20x) most likely do not exhibit this behavior, and instead they most likely have forward moving fluid hitting every bump, thus creating a significant pressure drop at each bump. Without this direct impact of fluid at every bump, the fluid in the 2x and 5x cases most likely glides over the bumps. Note, however, that all of the designed geometries exhibit higher friction factors than the plain channel, as expected.

Figure 3.5 shows interesting behavior for the Nusselt number. Each geometry behaves similarly in the laminar regime, but in the turbulent regime, the 2x case shows a marked performance gain over the other geometries. This is most likely due to a heat fin effect that comes from having so many bumps present in the channel in the 2x case. This type of spacing is effectively like a large surface roughness. All of the other geometries behave similarly in the turbulent regime, and all of the designed geometries show higher Nusselt numbers than the plain
channel case, as expected. Therefore, it can be said in general that the designed bumps provide higher Nusselt numbers in the turbulent regime at a cost of a higher friction factor.

Figure 3.4: Friction factors for designed geometries.

Figure 3.5: Nusselt numbers for designed geometries.
3.2 Critical Analysis of Experiments from Literature

3.2.1 Setup of Analysis

The methods of experimentation and data reduction of two of the papers mentioned in Chapter 1 are analyzed here to determine the strengths and weaknesses of their arguments. One paper is by Lee et al. [31], and the other is by Jung and Kwak [29].

Lee et al. [31] conducted an investigation of the validity of classical macroscale correlations for predicting the heat transfer of microchannels. They used a heater block on one side of a copper MHE for a heat source. This indicates that there was a constant heat flux over the length of the microchannel (and this is stated by Lee et al.). However, they calculated heat transfer coefficients under the assumptions that the surface temperature over the length of the channels was constant and that the surface temperature could be measured from a thermocouple half-way down the length of the channels. It is shown in Chapter 2 that the surface temperature is not constant along a constant heat flux wall, and it is not even linearly varying in the developing flow region. The microchannels in Lee et al.’s study were only 25.4 mm long, so the flow certainly had a significant portion (if not all of it) in the thermally developing regime. They did mention that there is some effect from the entrance region, and showed in a numerical simulation that this was the case. The validity of their measurement and calculation techniques is questioned here. Also, while they made the assumption of a constant wall temperature, they calculated the temperature difference from the wall to the water using a mean water temperature from the arithmetic average from the inlet and outlet temperatures as opposed to using the log mean temperature difference.

Jung and Kwak [29] recently tested silicon microchannels with 15 mm lengths. They also applied heat to their microchannels through a heater on one side, but instead of measuring
just one temperature on the wall, they measured the temperature at approximately 7 locations along the wall. They showed that the temperature of the walls did in fact vary along the length of the channels. However, when calculating the heat transfer coefficients, they assumed a constant surface temperature based on the average of the surface temperatures measured. In this way, they assumed a constant surface temperature similar to the way that Lee et al. did. However, instead of using the arithmetic average of the inlet and outlet water temperatures for use in heat transfer coefficient calculations, Jung and Kwak used the log mean temperature difference.

### 3.2.2 Problem Definition, Governing Equations, and Meshing

For this analysis, a 200 μm diameter, 25 mm long tube was considered. The working fluid was chosen to be water since water was used in the studies in question. The choice of water also helped to ensure that the continuum approach was valid for the study (see examination of Knudsen number in Chapter 2). The surface condition of a constant wall heat flux was employed to allow comparison to the papers in question (heat flux was specified in this model to give a total of 7 watts of heat addition to the tube). The inlet condition was a uniform velocity and temperature, so the flow was simultaneously developing, much like in the experiments in question. A representation of the tube along with the coordinate system and velocity directions to be used in the analysis is shown in Figure 3.6.

The first step in analyzing any model is to define the governing equations for the problem. In this case, cylindrical coordinates were used. The governing equations consist of the continuity equation, the momentum equation, and the energy equation. In this model, the flow is hydrodynamically developing, so many of the assumptions for the fully developed model cannot be applied.
Figure 3.6: Circular microtube (note: not to scale).

The assumptions used here are axisymmetric flow, constant properties, no body forces, pressure variation only in the $x$-direction, and no axial conduction. Therefore, the continuity equation, the $x$-momentum equation, the $r$-momentum equation, and the energy equation for this model are, respectively,

\[
\frac{1}{r} \frac{\partial (rv_r)}{\partial r} + \frac{\partial (u_x)}{\partial x} = 0
\]

(39)

\[
\rho \left( v_r \frac{\partial u}{\partial r} + u \frac{\partial u}{\partial x} \right) = - \frac{dp}{dx} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{\partial^2 u}{\partial x^2} \right]
\]

(40)
\[
\rho \left( \frac{\partial v_r}{\partial r} + u \frac{\partial v_r}{\partial x} \right) = \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_r}{\partial r} \right) - \frac{v_r}{r^2} + \frac{\partial^2 v_r}{\partial x^2} \right]
\]

(41)

\[
u \frac{\partial T}{\partial x} + v_r \frac{\partial T}{\partial r} = \alpha \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \right]
\]

(42)

Note that the terms \( \frac{\partial^2 u}{\partial x^2} \) and \( \frac{\partial^2 v_r}{\partial x^2} \) can be neglected to more easily obtain a solution to the governing equations, but they are included here because a numerical solution from Fluent was used anyway. Properties of water were taken at 350K.

### 3.2.3 Solution and Analysis

To solve this problem, Fluent was used. A mesh of the tube shown in Figure 3.6 was created with a triangular mesh on the cross-section extruded into wedges along axial lengths. The density of the mesh near the wall and near the tube entrance was enhanced to ensure that the areas with the highest gradients were properly resolved. A short grid convergence study showed that further mesh refinement did not yield any more useful results than the mesh used for the results presented. The outlet condition was simply assumed to be atmospheric pressure. The continuity, momentum, and energy equations were all solved using a second-order upwind scheme.

Once the solution converged to a satisfactory level (residuals < \( 10^{-6} \)), data was extracted from the model. The local Nusselt number was calculated from

\[
\text{Nu}_x = \frac{q_s D}{[T_s(x) - T_m(x)]k}
\]

(43)

The average Nusselt number for the whole channel was then calculated as
\[
\overline{\text{Nu}} = \frac{1}{L} \sum_{x=0}^{L} \text{Nu}_x \Delta x
\]

(44)

Figure 3.7 below shows the solution for the Nusselt number over \(100 \leq \text{Re} \leq 1000\). Also plotted in Figure 3.7 is the fully developed value of Nusselt number for the constant heat flux boundary condition, \(\text{Nu} = 4.364\). It can be seen that the solution for simultaneously developing flow is higher than the fully developed solution. It can also be seen that as the Reynolds number increases, the measured average Nusselt number also increases. As Re goes to zero, it can be seen that the simultaneously developing flow average Nusselt number approaches the fully developed value. This is due to the fact that as Re gets smaller, so does the hydrodynamic and thermal entrance lengths. The smaller these entrance lengths are, the less of a deviation there is from the fully developed solution. As Re gets bigger, the simultaneously developing average Nusselt number continues to get larger because of the increasing length of the thermal entrance length.

![Average Nu vs. Re](image)

Figure 3.7: Critical analysis simulation results.
The effects of the entrance lengths on the local Nusselt number can be seen in Figure 3.8, where the distribution of local Nusselt number with x is given for various Reynolds numbers. Notice that each case starts at a large Nusselt number but converges to approximately the fully developed value at the exit of the tube. The higher Reynolds number cases show a higher local Nusselt number in the beginning part of the tube. Taking the integral of these curves easily shows that the average Nusselt number gets larger as Re increases because of the longer entrance length that comes from a higher Re. Both Lee et al. and Jung and Kwak most likely had similar entrance length effects. Lee et al. did note that the entrance effects have an impact on Nusselt number, and they used correlations for thermally developing and simultaneously developing flow to compare against their data in the laminar regime. However, in the turbulent regime, they did not correct the correlations for fully developed turbulent flow for comparison with their data. Even in the turbulent regime, the thermal entrance length can be significant. Jung and Kwak did not compare their data to any known correlations.

To compare the results from this simulation to those from the experiments in question (Lee et al. and Jung and Kwak), the methods of calculating average Nusselt number from these experiments was used to construct curves for Nu vs. Re. Figure 3.9 shows the comparison of the results of the present model with integrated average Nusselt numbers plotted against the Nusselt numbers calculated from the model using the same measurement points and data reduction as Lee et al. (labeled as “Matching Lee and Garimella.”). Recall that Lee et al. used the temperature at the center of the channels as the average, constant wall temperature. They also used the arithmetic mean of the inlet and outlet water temperatures in determining the heat transfer coefficients.
Figure 3.8: Critical analysis local Nusselt numbers.

Figure 3.9: Comparison of simulation results with calculation techniques of Lee et al.
Figure 3.9 shows that Lee et al.’s measurement and data reduction techniques do indeed create errors in the measured average Nusselt number. The main source of this error comes from assuming a constant surface temperature. Their assumption that the temperature in the center of the channel length is the average wall temperature is also somewhat flawed. Figure 3.9 shows this with a plot of the Nusselt number curve for a properly calculated average wall temperature (labeled as “Matching L. & G. (proper average)”). This kind of average can be obtained in an experiment by measuring the wall temperature at multiple locations along the length of the tube. This is one of the benefits of Jung and Kwak’s experimental method.

Even though Jung and Kwak used the right method to find the average surface temperature, the method of using the log mean temperature is questionable. A curve of the Nusselt number that shows how their results would compare to the above graphs cannot be completed for the given boundary conditions. This is the case because the average wall temperature is lower than the water outlet temperature in some cases. This creates an undefined log mean temperature difference (see Eq. (12)). For Jung and Kwak’s results, the heat flux may have been such that this undefined log mean temperature difference never occurred. It should be noted the log mean temperature difference often does not result in Nusselt numbers much different than from using arithmetic means. However, this is a dangerous technique to use because some of the experimental data could create an undefined log mean temperature difference while others do not. Therefore, it can be said that with a constant surface heat flux condition, the log mean temperature difference should not be used (especially when the experiment also proves that the surface temperature is not constant). For a constant surface temperature condition, the log mean temperature difference certainly should be used, as stated in Chapter 2.
This exercise in analyzing the methods employed in experiments published recently outlines the need for new experimental techniques to be tested. When employing these new techniques, it is imperative that the proper methods of data reduction are used. The current study provides a new method of experimentation on MHE’s, and the methods of data reduction are followed exactly from theory based on the prescribed boundary conditions.
CHAPTER 4: EXPERIMENTAL SETUP

4.1 Experimental Theory

To test the fluid flow and heat transfer characteristics of an MHE, an experiment was created that allowed water to flow through a Cu MHE while heat was applied. From the literature review in Chapter 1, it can be said that almost every paper published with reports on experimental testing of MHE’s employed a resistance heater to create a heat transfer effect in the microchannels. This type of heat application is most closely approximated by the constant heat flux boundary condition. Some researchers [29, 31] used resistance heaters but assumed a constant wall temperature based on an average wall temperature. However, this can incur some errors in overall heat transfer coefficient calculations because of the inherent temperature variation that occurs along a microchannel with a constant heat flux applied to it. There are especially large errors involved with this assumption when the flow is in a thermally developing regime since the beginning portion of the channels would then have a nonlinear wall temperature variation. To properly use the resistance heating method of heat application to an MHE, wall temperature measurements are required along the length of the microchannels. With this information and the mean fluid temperature from Eq. (5), the local heat transfer coefficients can be calculated at a few points along the channel from Eq. (8). These local heat transfer coefficients would account for the developing region of the channels, and therefore they allow for the true mean Nusselt number of the channels to be calculated.

The fundamental difficulty in constructing an experiment with a constant heat flux boundary condition is then to adequately instrument the channel walls. However, it is very difficult to get accurate readings of wall temperatures along the microchannels in an MHE such as the one investigated here. An infrared camera cannot be used here because the channels are
enclosed. Thermocouples along the channel walls would provide some level of resolution of the wall temperature variation, but the placement of a large number of thermocouples along the channel lengths would affect the heat flow path from a heater to the channels. The holes drilled for the thermocouples would create a complex heat flow path geometry, and therefore some sections of the channels having a higher heat flux than others would be a problem. Also, since thermocouples have a finite size, it would be very difficult to get good temperature resolution in the entrance region of the channels, which is where the largest gradients occur.

It is for these reasons that a new approach to heat input to the MHE was employed in this study. Instead of a resistance heater, and therefore a constant heat flux, a constant temperature hot swirling water bath was used to impose a constant surface temperature boundary condition on the walls of the microchannels. This allowed for only one wall temperature measurement to be required, and the average heat transfer coefficient (and therefore mean Nusselt number) could be easily calculated from Eq. (12). While this method does not allow for local heat transfer coefficients to be calculated, the average heat transfer coefficient is more easily and more accurately calculated from this method than from the constant heat flux method.

4.2 MHE Construction

The Cu MHE used in the experiments was constructed by Dr. Wen Jin Meng’s group at Louisiana State University. To construct the microchannels, molding replication was employed. The base metal into which the microchannels were molded was a 42 mm x 42 mm x 6.4 mm Cu 110 (99.9+ wt.% Cu) plate. This process involved high temperature molding of this copper plate with a Si:N coated Inconel insert. A total of 26 microchannels with lengths of 17.32 mm were molded into the Cu plate. The average cross-section dimensions of the microchannels were 178 μm wide and 341 μm tall (thus giving hydraulic diameters of 234 μm). The microchannels were
spaced about 750 \( \mu m \) apart from each other (on centers). As an inherent part of the molding replication process, a surface roughness was created. The surface roughness was measured by Dr. Meng’s group to be on average about 4 \( \mu m \). Details of the molding replication process are reported by Mei et al. [45].

Inlet and outlet plenums were cut into the microchannel Cu plate at the ends of the channels using \( \mu EDM \) and traditional milling. An identical plenum spaced exactly like the one on the microchannel plate was also machined onto another Cu plate to create a symmetric plenum. The total plenum dimensions were 3 mm tall, 25 mm wide, and 7 mm long (measured longitudinally from the microchannels). Two holes were drilled into the inlet plenum as well as into the outlet plenum. These 11/64” holes served as the inlet and outlet ports for the water to flow through. Also, a 3/32” hole was drilled into the inlet port and the outlet port to serve as pressure tap holes. Finally, a 0.94 mm hole was drilled into the side of both Cu plates all the way to the center of the plates. These holes were positioned (on center) 1.75mm from the bonding surface. These holes served as thermocouple holes for wall temperature measurements. The two completed Cu plates are shown in Figure 4.1.

The two Cu plates shown in Figure 4.2 were bonded together through eutectic bonding. This type of bonding created a nearly seamless connection between the two Cu plates. The details of the eutectic bonding process implemented here are detailed by Mei at al. [45]. The completed MHE with plastic inlet and outlet adapters attached is shown in Figure 4.2.

4.3 Experimental Apparatus

The experimental apparatus can be split into three main systems: the pressurizing system, the heat exchanger system, and the data acquisition system. The experimental setup is shown in Figure 4.3.
Figure 4.1: Cu MHE with channels molded, plenums cut, and holes drilled.

Figure 4.2: Cu MHE after bonding.
The compressed nitrogen tank pressurized the water holding tank. A metal tube inserted down to the bottom of the water holding tank was connected to a valve for flow adjustment. Water flowed through this valve, through plastic tubing, and into the MHE. The water flowed through the MHE and into a catch bucket. The MHE was suspended in a water bath at approximately 80°C. A digital manometer measured the pressure in the inlet and outlet plenums, and a total of 8 thermocouples monitored the fluid and wall temperatures. These thermocouples were connected to a data acquisition system where the data were collected.

4.3.1 Pressurizing System

To create pressure to drive the flow through the MHE, a water holding tank and a compressed nitrogen tank were used. This setup provided a smoother and more stable flow of water than a pump would have provided. Figure 4.4 shows the pressurizing system.
The compressed nitrogen tank (rated for 2000 psi) was fitted with a control valve and pressure regulator. This allowed for the pressure of the nitrogen leaving the tank to be monitored and adjusted while also monitoring the pressure of the nitrogen still in the tank. The compressed nitrogen tank and the pressure regulator can be seen in Figure 4.5. The pressure regulator has two readouts. The gage on the left shows the pressure of the nitrogen leaving the tank while the gage on the right shows the pressure of the nitrogen remaining in the tank.

The 15 gallon water holding tank, shown in Figure 4.6, was connected to the compressed nitrogen tank with a metal tube and a needle valve. The compressed nitrogen entered the water holding tank from the side, and this kept the water holding tank pressurized. With this tank pressurized, the water could be forced through another metal tube coming out of the top of the water holding tank. This metal tube extended down to the bottom of the water holding tank. Also shown in Figure 4.6 is a pressure relief valve which was used to release the pressure inside the water holding tank should the pressure have exceeded 100 psi. Figure 4.6 also shows the valve on the top of the tank that allowed for the flow of water to be adjusted.
Figure 4.5: Nitrogen tank: (a) compressed nitrogen tank; (b) pressure regulator.

Figure 4.6: Water holding tank.
4.3.2 Heat Exchanger System

The MHE was instrumented with thermocouples and pressure taps to monitor the fluid flow and heat transfer through the microchannels. Figure 4.7 shows a cross-section side view of the MHE. Notice that fluid thermocouples were placed in the plenums in two locations: the inlet and outlet tubes as well as the pressure tap holes. Also notice the locations of the wall temperature measurement thermocouples. These thermocouples were placed at a distance of 1.75 mm from the channels to ensure that in the drilling process the microchannels were not disturbed. Figure 4.8 shows a top view of the instrumented MHE. From this view, the positions of the thermocouples placed inside the plenum can be seen. The inlet and outlet plenums, as well as the instrumentation inside each of them, are identical to each other.

![Figure 4.7: Microchannel instrumentation cross-section side view.](image)

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The MHE was fitted with plastic barbed tube fittings (as shown in Figure 4.2) to allow for plastic tubes to connect to the inlet and outlet ports. The plastic tubing on the inlet side connected directly to the valve on top of the water holding tank shown in Figure 4.6, while the plastic tubing on the outlet side drained to a water bucket for measurement of water pumped during a given period of time. Also fitted to the MHE were 316 stainless steel tubes with an outside diameter of 3/32” and an inside diameter of 1/32” for use as pressure tap tubes. These tubes were marked to a depth equal to the depth of the copper into which they needed to penetrate prior to insertion into the MHE to ensure that the tubes remained fairly close to being level with the bottom floor of the plenums. Plastic tubing was fitted over the stainless steel tubes for connection to a digital manometer. Thermocouples were also fitted into the pressure tap tubes to allow for measurement of water temperature as close as possible to the channel inlets.

Figure 4.8: MHE top view (note: drawing not to scale).
and outlets. Figure 4.9 shows the MHE fitted with the inlet tubes, the outlet tubes, and the pressure tap tubes.

![Figure 4.9: MHE fitted with tubes and sealed with epoxy.](image)

Figure 4.9 also shows that epoxy was used to seal the area surrounding the inlet ports, the outlet ports, and the outer area of bonded region of the MHE. In the process of eutectic bonding, a small leak path was created. The use of epoxy proved adequate for sealing the bonded area. Epoxy was also used to seal the area surrounding the thermocouples in the wall thermocouple holes to ensure that water from the hot water bath did not touch the thermocouples. The thermal resistance between the walls and the thermocouple was assumed to be small, despite the existence of the epoxy, because the layer of epoxy was so thin (approximately 75 μm from the thermocouple to the Cu walls). The epoxy used in the application of sealing the MHE and filling the gaps in the wall thermocouple holes was Loctite Hysol two-part epoxy, as shown in Figure 4.10.
The hot water bath was created by placing a large beaker filled with water onto a heater plate. The heater plate had the capability to stir the water through the use of a magnetic stirrer. This method of stirring was employed to ensure that the water temperature surrounding the MHE was uniform and that natural convection did not create any pressure gradients in the region surrounding the MHE. The heater plate also had different heat settings to allow for the temperature to be varied in the water in the beaker. The heater plate and the beaker are shown together in Figure 4.11.
4.3.3 Data Acquisition System

The thermocouples used in the acquisition of temperature measurements inside the walls and inside the plenums were 36 gauge type-K Omega thermocouples (5SC-TT-K-36-36). Figure 4.12 shows one of these thermocouples. These thermocouples have PFA insulation with a maximum allowable temperature of 260°C. A subminiature plastic connector was attached to the thermocouple for easy connection to the data acquisition system. A total of 8 of these thermocouples were used in the instrumentation of the MHE. Three thermocouples were placed in the inlet plenum, three were placed in the outlet plenum, and two were placed in the walls.

![Figure 4.12: Omega brand type-K thermocouple.](image)

To record all of the temperature data, an InstruNet model 100 analog/digital input/output system was used to acquire the thermocouple data and send the data to a PC. The two thermocouple leads from each thermocouple were connected to a pair of Vin+ and Vin- input terminals on the InstruNet model 100. The InstruNet model 100 reported all temperatures to the PC in °C with an accuracy of ±0.6°C. The InstruNet model 100 is shown in Figure 4.13.
Figure 4.13: Instrunet model 100 data acquisition system.

To measure the pressure differential between the inlet and outlet plenums, a wet/wet Dwyer Series 490 digital manometer was used. This battery-operated hand-held device has a pressure range of 0-50 psi with an accuracy of ±0.5% of its full scale. This pressure measurement device was used to set a flow rate at a given differential pressure as read by the digital readout on the digital manometer. Figure 4.14 below shows the Dwyer Series 490 digital manometer. Also shown in Figure 4.14 are the 316 stainless steel 1/8” FNPT fittings which were constructed to fit to the tubing coming from the pressure taps on the MHE.

Figure 4.14: Dwyer digital manometer.
To measure the water which drained from the MHE, a bucket was used to collect the water that drained from the MHE over a measured period of time. After the measured period of time had passed, the bucket was emptied into a volumetric beaker, shown in Figure 4.15, in sections until the bucket was emptied and the total volume of water collected was measured. With the total volume of water measured during the given amount of time, the flow rate could be calculated. From the volumetric flow rate, the mass flow rate could be calculated, and therefore the Reynolds number was calculated. The volumetric beaker used has a resolution of 2.5 ml, and the stopwatch used has a resolution of 0.01 s.

![Figure 4.15: Volumetric beaker used for flow rate measurements.](image)

### 4.4 Experimental Procedure and Uncertainty

The first step in conducting the experiments was to degas the water in the water holding tank. This was accomplished by letting some nitrogen flow into the tank while the cap of the tank was uncovered. Once the water was degassed, the water holding tank was sealed. The large beaker was filled with water and placed on top of the heater plate. The heater plate was turned on, and the temperature of the water was allowed to reach about 80°C. This temperature was
monitored to ensure that it remained constant throughout the data taking process. Once the water in the large beaker reached this temperature, the valve on top of the water holding tank was opened. This allowed water to flow through the MHE. A set flow rate was established by monitoring the digital manometer and setting the valve at a position where a predetermined pressure was measured on the digital manometer. Once the desired differential pressure was read, a stopwatch was started and the water collection bucket was placed under the fluid outlet tubes. The temperatures measured from the InstruNet model 100 were monitored on the PC, and once they reached a steady state value, the temperatures of the inlet, the outlet, and the walls were recorded. It should be noted that during the temperature measurement that all temperatures remained constant with time. Once the temperature measurements were completed and the set time for flow measurement had elapsed, the water in the bucket was measured using the volumetric beaker. This procedure was repeated several times for differential pressure readings of 1 to 30 psi. Temperature differences from inlet to outlet ranged from 6.4°C to 15°C, temperature differences from wall to inlet ranged from 12.2°C to 21.5°C, and temperature differences from wall to outlet ranged from 4.3°C to 8.6°C. Volumes of water collected were always above 300 ml. Reynolds numbers from 236 to 2946 were tested.

A standard uncertainty analysis was conducted for each measurement [46]. The uncertainty of pressure measurements was found to be approximately 0.27 psi, the uncertainty of temperature measurements was found to be approximately 0.61°C, and the uncertainty of flow rate measurements was found to be less than 0.8%. Mei et al. [45] found that the uncertainty of the channel dimensions were 6 μm for height, 5 μm for width, and 50 μm for length for Cu microchannels constructed in the same way as the microchannels considered here.
Using these uncertainties, an uncertainty analysis for uncorrected friction factor and Nusselt number based on propagation of errors was conducted. These uncertainties were calculated through

\[ u_R = \pm \left( \sum_{i=1}^{n} \left( \frac{\partial R}{\partial x_i} u_{x_i} \right)^2 \right)^{1/2} \]  

(45)

where \( u_R \) is the uncertainty of value \( R \) and \( u_{x_i} \) is the uncertainty of a given quantity \( x_i \) upon which \( R \) is based [46]. For friction factor, the \( x_i \)'s included \( \Delta P \), \( W \), \( H \), \( L \), and \( V_{avg} \). For Nusselt number, the \( x_i \)'s included \( Q \), \( W \), \( H \), \( L \), \( (T_{out} - T_{in}) \), \( (T_{wall} - T_{out}) \), and \( (T_{wall} - T_{in}) \). This method of determination of uncertainties allowed for the errors in each measurement to be compounded together to estimate the total error for both friction factor and Nusselt number.

For low Reynolds numbers (approximately 300), the uncertainty in friction factor was found to be as high as 55%. This is primarily due to the accuracy of the manometer at low pressures. This level of uncertainty is not uncommon in microchannel experiments for friction factor [14, 15]. Above \( Re = 1000 \), the uncertainty of friction factors was found to be below 11%. For Nusselt number, the uncertainty was found to have a maximum of 26% at the higher Reynolds numbers. The uncertainty of Nusselt numbers could be lowered in future experiments by increasing the difference between inlet temperatures and wall temperatures.
CHAPTER 5: EXPERIMENTAL DATA REDUCTION, RESULTS, AND ANALYSIS

5.1 Friction Factor

5.1.1 Friction Factor Data Reduction

By measuring the pressure drop from the inlet plenum to the outlet plenum, the apparent Darcy Friction Factor ($f_{app}$) could be determined. The uncorrected apparent Darcy Friction Factor is

$$f_{app, un} = \Delta P_{in-out} \frac{2D}{L \rho V^2_{avg}}$$

(46)

where $\Delta P_{in-out}$ is the pressure drop that was measured in the experiments [41]. The average velocity was calculated from $Q = \rho V_{avg} (26A_c)$. Since the pressure was measured from the inlet and outlet plenums, it is important to note that the apparent friction factor should not be confused with the fully developed Darcy Friction Factor, $f$ (commonly referred to as simply the friction factor), which is

$$f = \frac{8\tau_w}{\rho V^2_{avg}}$$

(47)

where $\tau_w$ is the wall shear stress [41, 43]. Note that the fully developed friction factor is equal to the apparent friction factor if the pressure change is measured in a region where the flow is fully hydrodynamically developed. A corrected apparent friction factor was calculated which accounted for the losses incurred at the channel inlet and exit, and it is given by
where $\Delta P_{adj}$ is the adjusted pressure drop. This adjusted pressure drop takes into account the losses from the inlet condition and the exit condition. The adjusted pressure drop was calculated from

$$f_{app} = \Delta P_{adj} \frac{2D}{\rho V_{avg}^2}$$

(48)

where $\Delta P_{adj}$ is the adjusted pressure drop. This adjusted pressure drop takes into account the losses from the inlet condition and the exit condition. The adjusted pressure drop was calculated from

$$\Delta P_{adj} = \Delta P_{in-out} - K_{in} \frac{\rho V_{avg}^2}{2} - K_{out} \frac{\rho V_{avg}^2}{2}$$

(49)

where $K_{in}$ is the loss coefficient for the abrupt entrance and $K_{out}$ is the loss coefficient for the abrupt exit [41]. These loss coefficients were determined based off of the experimental values widely available for circular tubes. Of the two loss coefficients, $K_{out}$ is the most readily identified and is equal to 1. For any abrupt exit to a large reservoir (i.e. an exit from a microchannel to a large plenum), the geometry of the exit has no effect on the loss coefficient. The loss coefficient for the inlet, $K_{in}$, is highly dependent on the geometry of the inlet area. For a sharp-edged inlet (i.e. 90° corners), $K_{in} = 0.5$. For a well rounded inlet (defined as $r_{corner}/D_h > 0.2$), $K_{in} = 0.03$. For a slightly rounded inlet (defined as $r_{corner}/D_h > 0.1$), $K_{in} = 0.12$ [41]. The ratio of $r_{corner}/D_h$ is unknown for the microchannels tested, and the ratio certainly varies for each individual channel. Therefore, an intermediate loss coefficient of $K_{in} = 0.25$ is used as an estimate.

To compare against the friction factor data obtained from the experiments, three equations are used. One involves fully developed laminar flow, one involves flow in the laminar, transition, and turbulent regimes, and the last involves developing flow in the laminar regime.
For the fully developed friction factor for laminar flow in rectangular channels, an analytically derived equation is [43]

\[
f_{lam,fd} = \frac{96}{Re} \left[ 1 + \frac{1}{\alpha^*} \right]^2 \left[ 1 - \frac{192}{\pi^5 \alpha^*} \sum_{n=1,3,5} \left( \tanh \left( \frac{n\pi\alpha^*}{2} \right) \right) \right] = \frac{61.615}{Re}
\]

(50)

where \(0 < \alpha^* < 1\) is the aspect ratio of the channels. For the microchannels tested, \(\alpha^* = 0.522\), so \(fRe = 61.615\).

No complete analytical solutions for the friction factor for transitional flow exist for channel or tube flow. However, a few equations have been developed for flow in circular tubes that range from the laminar regime, through the transition regime, and into the turbulent regime. Such an equation from Churchill gives a continuous function for the friction factor for the laminar, transition, and turbulent regimes. The Churchill equation for fully developed friction factor is [43]

\[
f_{Ch} = 8 \left\{ \frac{1}{\left[ \left( \frac{8}{Re} \right)^{10} + \left( \frac{Re}{36500} \right)^{20} \right]^{1/2}} + \left[ 2.21 \ln \left( \frac{Re}{7} \right) \right]^{10} \right\}^{-1/5}
\]

(51)

While Eq. (51) was developed for circular tubes, it should still be applicable here since the aspect ratio of the microchannels gives such a close correlation to the laminar flow conditions in circular tubes:

\[
\left( \frac{f_{lam,fd} Re}{f_{lam,fd} Re}_{rect} \right) = 0.963
\]

(52)
It should be noted that the Churchill equation has been shown to be in almost exact agreement with the Colebrook equation for turbulent flow, and for the transition region of $2100 < \text{Re} < 4000$, the Churchill equation is generally in good agreement with experimental results [43].

Since the microchannels tested here each contain some portion of their length that is in the hydrodynamically developing region, a correlation for hydrodynamically developing flow is included. Some correlations for the hydrodynamically developing region friction factor are available from Shah and London [47]. All of the correlations available assume both fully laminar flow and a uniform inlet condition. Some correlations assume that the channel in question is relatively short in comparison with the hydrodynamic length. If the assumption is made that the flow entering the tubes is uniform and the flow in the channels is completely laminar, the effects of the hydrodynamically developing region may be overestimated for the experimental cases. Both the assumptions of a uniform inlet and of fully laminar flow in the channels are questionable for the cases seen in the experiments.

Due to the ambiguity of the Reynolds number at which transition to turbulence occurs (see Chapter 1), it is very difficult to say which test cases should compare well with correlations for developing laminar flow. Several factors could induce early transition, among which are surface roughness and the severity of the inlet from the plenum to the microchannels. The hydrodynamic entrance length is much shorter for the case of turbulent flow than for the case of laminar flow. With a shorter entrance length, there is less of an effect on apparent friction factor from the developing region. With the laminar hydrodynamic entrance length given by Eq. (21) and the turbulent hydrodynamic entrance length given by Eq. (22), the hydrodynamic entrance lengths involved in the experiments can be approximated if the assumption of a uniform inlet condition is made. The variation of estimated hydraulic entrance length with Reynolds number
for the microchannels tested is given in Figure 5.1. Note that the hydraulic entrance length is in fact significantly long in the laminar regime, while it is less prominent in the turbulent regime. This indicates that the increased losses due to the entrance length are more pronounced for laminar flow than for turbulent flow with a uniform inlet. As noted earlier, there are no correlations available for the entrance length of transitional flows, but it can be reasonably inferred that the hydrodynamic entrance length lies somewhere between the laminar and turbulent predictions shown in Figure 5.1.

![Estimated Hydrodynamic Entrance Lengths](image)

Figure 5.1: Hydrodynamic entrance lengths for the entire range of Reynolds numbers tested.

The assumption of uniform flow at the inlet is especially questionable for the experimental cases since there are complex flow patterns that arise from the inlet from a plenum into a microchannel. It was found by both Lee et al. [31] and Rosenhow et al. [42] that in cases where flow enters from a large plenum into a microchannel, the flow can often be assumed to be almost fully hydrodynamically developed at the inlet. While this assumption is not made here, their findings are cause for concern in choosing to evaluate the experimental results under the assumption of a uniform inlet condition.
With these reservations about the available developing flow apparent friction factor
correlations, the following equation was chosen from Shah and London [47] for comparison with
the experimental data:

\[
f_{app,dev} = \frac{K(\infty)}{4x^+} + f_{lam,fd} \text{Re}
\]

(53)

where \(K(\infty)\) is the constant value Hagenbach factor (effectively a loss coefficient),
\(x^+ = L/(D_h \text{Re})\) is the dimensionless axial distance, and \(f_{lam,fd} \text{Re}\) is given by Eq. (50). The
constant value Hagenbach factor is a function of the aspect ratio of a rectangular channel given
by

\[
K(\infty) = 0.6796 + 1.2197a^* + 3.3089a^{*2} - 9.5921a^{*3} + 8.9089a^{*4} - 2.9959a^{*5}
\]

(54)

In the case here where \(a^* = 0.522\), \(K(\infty) = 1.28\). Eq. (53) was chosen because it assumes a fairly
long channel in comparison to the hydrodynamic entrance length. This seems to be a better
approximation of the actual experimental conditions than the assumption of a hydrodynamic
entrance length that is, in comparison with the length of the channel, fairly long due to a uniform
inlet.

5.1.2 Friction Factor Results and Analysis

The uncorrected apparent friction factor from the results of the experiments, the corrected
apparent friction factor with uncertainties, Eq. (50), Eq. (51), and Eq. (53) are plotted against
Reynolds number in Figure 5.2. The error bars which are plotted on the corrected apparent
friction factor include experimental uncertainties (see Chapter 4) in addition to errors which arise
from the range of apparent friction factor values that could occur for the entire range of \(K_{in}\), \(0 \leq
K_{in} \leq 0.5\).
It can be seen in Figure 5.2 that there is indeed a significant decrease in friction factor when the inlet and exit losses are considered. Notice from Figure 5.2 that the corrected apparent friction factor matches reasonably well with both the trend and magnitude of the Churchill equation throughout the range of Reynolds numbers. At \( \text{Re} \approx 3000 \), the data seems to begin to follow the Churchill equation more exactly. This may be the point at which the flow begins to reach a fully turbulent condition. It is not surprising that the apparent friction factor does not exactly follow the Churchill equation in the transition region of \( 1500 < \text{Re} < 3000 \) since there are no definite solutions available for the friction factor in the transition region. Also note that the hydrodynamically developing region does not greatly affect the friction factor. The plot of Eq. (53) for hydrodynamically developing laminar flow does not deviate greatly from the fully developed laminar flow prediction. This is to be expected given the assumption of a somewhat
shorter hydrodynamic entrance length due to the abrupt entrance. It can then be assumed that measured friction factors inside the channel are very close to the actual fully developed friction factors. Therefore, the corrected apparent friction factor plotted in Figure 5.2 can be considered, to a good approximation, to be equal to the fully developed friction factor.

It is inferred from the data in Figure 5.2 that the transition to turbulence begins at $Re \approx 1500$. The corrected friction factor begins to deviate from the trend of the fully developed laminar equation, the Churchill equation, and the developing laminar equation at this value. The transition Reynolds number $Re \approx 1500$ seen here agrees well with other studies done with microchannels with a similar roughness and inlet condition. The relative roughness of the microchannels tested here was $\varepsilon/D_h = 0.017$. Gui and Scaringe [4] found that transition in channels with $\varepsilon/D_h \approx 0.015$ occurred at $Re \approx 1400$ [30]. Harms et al. [30] found the transition Reynolds number to be $Re \approx 1500$ for channels with a relative roughness of $\varepsilon/D_h \approx 0.02$. However, Harms et al. attributed the early transition to the abrupt inlet condition present in their MHE. It is very likely that the early transition to turbulence seen in the microchannels tested here is in fact also due to the severe inlet condition at the entrance from the plenum. This is a commonly cited reason for early transition in microchannel studies [30].

It is interesting to note that were the uncorrected apparent friction factor taken to be the true friction factor, the transition to turbulence would have been estimated to occur at $Re \approx 1000$. This would be a misinterpretation of the actual phenomena occurring in the microchannels. Many researchers have stated low turbulent transition Reynolds numbers from friction factor data that was not corrected for inlet losses and exit losses [2-6, 8-11]. It is imperative in analyzing the friction factor data from microchannel flow experiments to include all factors affecting the pressure drop.
It should be noted that while the microchannels tested had a relative roughness of \( \varepsilon/D_h = 0.017 \), the apparent friction factor may not show an increase in friction factor from smooth wall correlations in the range of Reynolds numbers given since surface roughness is only supposed to affect turbulent flows. No highly accurate correlations for rough wall transition region flow are available, so it is difficult to tell whether or not the apparent friction factor data is appropriately high in the transition region. The Churchill equation’s transition region curve can be best used as an approximation of the transition flow.

5.2 Heat Transfer

5.2.1 Heat Transfer Data Reduction

The heat applied to the working fluid in the channels is given by

\[
q_{\text{tot}} = \dot{m}c_p(T_{\text{out}} - T_{\text{in}})
\]

where the mass flow rate, \( \dot{m} \), was measured from the volume of water collected from the outlet of the channels over a specified period of time [39]. The inlet temperature and outlet temperature were taken from the thermocouples inserted into the pressure tap tubes. It is noted again that the experimental condition was that of a constant surface temperature for the walls of the microchannels. Under this assumption, the average heat transfer coefficient is given by Eq. (12), and written in terms of the measured quantities for the experiment, the average heat transfer coefficient for a single microchannel is

\[
\overline{h} = \frac{q_{\text{tot}}/26}{A_s} \ln \left( \frac{\Delta T_o}{\Delta T_i} \right) = \frac{q_{\text{tot}}/26}{A_s \Delta T_{\text{lm}}}
\]

(56)
where $A_s = PL = 2(W+H)L$ is the surface area available for convective heat transfer in the microchannels. The average Nusselt number for a single microchannel is then [45]

$$\overline{Nu} = \frac{hD_h}{k}$$

(57)

5.2.2 Heat Transfer Results and Discussion

Using the definition of the average Nusselt number from Eq. (57) for the microchannels of the tested MHE, the Nusselt number was calculated for each case and is shown in Figure 5.3. The error bars plotted in Figure 5.3 show the uncertainties which were presented in Chapter 4. The errors in the high Reynolds number range are significantly higher than those in the low Reynolds number range. The experimental results for average Nusselt number shown in Figure 5.3 seem to be higher than expected at first glance. However, there are many factors at play in the current situation which give rise to high average Nusselt number results.

![Experiment Average Nu vs Re](image)

Figure 5.3: Experiment results for average Nusselt number.
A very important factor in the current analysis is the presence of a thermally developing flow field. As stated in Eq. (25), the thermal entrance length is approximately equal to the hydrodynamic entrance length multiplied by the Prandtl number. Since the working fluid in the experiments was water in a temperature range of approximately 300K to 315K, the Prandtl number for the experiments was approximately $Pr = 5$. This means that the thermal entrance lengths of the microchannels tested were approximately 5 times longer than the hydrodynamic entrance lengths. It is shown in Chapter 6 that the flow is indeed thermally developing throughout the lengths of the entire microchannels in every case tested. Therefore, it is reasonable to assume for the entire range of data collected that the flow field is at least thermally developing. It is not known what portion of the channels was in a simultaneous development region (where both the hydrodynamic and thermal boundary layers were developing simultaneously) for reasons stated in Section 5.1.2. However, the influence of the entrance region should be markedly more significant in the laminar regime than in the turbulent regime since the thermal entrance length in the turbulent regime is so much smaller than in the laminar regime.

Another very important factor at play in the current investigation is the existence of transition and/or turbulence. The Nusselt numbers for turbulent flow are not only much higher than for laminar flow, but turbulent flow Nusselt numbers also depend on the Reynolds number and Prandtl number. As stated in the Section 5.1.2, the transition Reynolds number is unknown for this investigation, but it can be estimated to occur at $Re \approx 1500$.

As derived in Chapter 2, the fully developed laminar Nusselt number is $Nu_x = \overline{Nu} = 3.657$. However, since the flow in the microchannels was thermally developing, this Nusselt number would be an incorrect choice for comparison. The entrance region provides for much
higher Nusselt numbers than in the fully developed region [39, 40, 42, 43]. Therefore, correlations for developing laminar flow are necessary for comparison with the experimental data in Figure 5.3. Each correlation assumes either a constant surface temperature or a constant surface heat flux, as well as either a simultaneously developing flow or an exclusively thermally developing flow. While it is known that the case in the experiments was that of the constant surface temperature, the development regime is less known. It is believed that the development regime falls somewhere between the simultaneously developing situation and the exclusively thermally developing situation.

The Hausen equation for thermally developing flow laminar flow (Eq. (31)) and the Sieder and Tate equation for simultaneously developing laminar flow (Eq. (32)) are plotted against the experimental data for average Nusselt number in Figure 5.4 below. It can be seen in Figure 5.4 that besides the very low Reynolds number cases, the developing laminar flow correlations do not match well with the experimental data. This could be a result of a number of causes. Beside experimental error, the biggest unknown in the present analysis is the type of flow regime that is present at each Reynolds number. Certainly if the flow is either transitional or turbulent at any given Reynolds number, it will not correlate with laminar developing flow Nusselt number correlations. While the possibility is not explored here, it should be noted that there may be some effects from either the surface roughness or inlet condition at play here in the laminar regime. A stated in Chapter 1, there is still much debate as to what effect surface roughness has on both the friction factor and the Nusselt number in microchannel flows. While not many researchers have found a surface roughness or inlet condition effect on the Nusselt number in the laminar regime, a few have. This alone makes the possibility worth mentioning.
Since the laminar developing flow correlations do not fit the data well, turbulent Nusselt number correlations must then be used for comparison as well. There is a wide range of equations available for correlation. However, none of the correlations available are applicable to thermally developing or simultaneously developing flow. This is likely due to the fact that entrance lengths are so much shorter in the turbulent regime than in the laminar regime. Therefore, as a first comparison, fully developed turbulent flow correlations are used.

Figure 5.5 shows a plot of the experimental data, the Dittus-Boelter equation (Eq. (33)), the Petukhov equation (Eq. (34)), and the Gnielinski equation (Eq. (36)). It can be seen in Figure 5.5 that the Petukhov equation follows the trend of the experimental data in the transition and turbulent range of Reynolds number. The Dittus-Boelter equation is also in the range of comparability with the experimental data. Reasonable agreement between experimental results for microchannels in the turbulent regime with both the Petukhov and the Dittus-Boelter
equations was also found by Lee et al. [31]. The Gnielinski equation matches the slope of the experimental data, but the magnitude of the Gnielinski equation is far removed from the data. This is likely due to the nature of the Gnielinski equation. Notice from the Gnielinski equation that instead of having simply the Reynolds number in the numerator (like in the Petukhov equation), the Reynolds number is subtracted by 1000. This implies that the Gnielinski equation is strictly defined for a certain Reynolds number range assuming that transition to turbulence occurs at the typical value of $Re = 2300$. Extending the Gnielinski equation into a situation where transition can occur at a lower Reynolds number is therefore believed to yield an invalid comparison. Therefore, the Gnielinski equation is not recommended for use as a measure of comparison with the experimental data presented here.

![Turbulent Correlations: Mean Nu vs. Re](image)

Figure 5.5: Turbulent correlations for Nusselt number plotted with the experimental data.

While the experimental data correlates reasonably well with the Dittus-Boelter equation and the Petukhov equation, these equations assume fully developed flow, and therefore they should be adjusted for entrance effects to give a proper comparison to the experimental data. As
noted earlier, the thermal entrance length is roughly 5 times larger than the hydrodynamic entrance length for flow in tubes with water. Since the hydrodynamic entrance lengths for turbulent flows can be assumed to be on the order of 2 mm, it is reasonable to treat the turbulent flow situations as thermally developing situations.

The most commonly cited method of correcting Nusselt numbers for the thermally developing region is the Al-Arabi correlation [40, 42, 43]. The correlation is

\[
\frac{\bar{Nu}}{Nu_\infty} = 1 + \frac{C}{\left(\frac{L}{D_h}\right)}
\]

(58)

where \(\bar{Nu}\) is the Nusselt number in the fully developed region and the constant \(C\) is given by

\[
C = \left(\frac{L}{D_h}\right)^{0.1} \frac{Pr^{1/6}}{Pr^{1/6}} \left(0.68 + \frac{3000}{Re}\right)
\]

(59)

This correlation is valid for both the constant surface temperature condition and the constant heat flux condition. The correlation allows for a mean Nusselt number to be calculated for a given length of channel if the fully developed Nusselt number is known. This is beneficial when using given correlations for the fully developed Nusselt number because those correlations can then be adjusted for thermally developing flow in a certain length of channel.

In addition to adjusting for the effects of thermally developing flow, the Dittus-Boelter equation can be adjusted to include roughness effects. The Petukhov equation already accounts for these effects, but since the Dittus-Boelter equation was derived for smooth channels [39, 42], roughness adjustments are applicable. An empirical correlation by Norris is presented by [40] and [43]. The correlation is
Kays and Crawford [40] note that when \( \frac{f_{\text{rough}}}{f_{\text{smooth}}} > 4 \), the Nusselt number no longer increases with more surface roughness. In the experimental conditions, however, this ratio is nowhere close to 4. This ratio is calculated from the Haaland equation (Eq. (35)) for both smooth walls \( (\varepsilon = 0) \) and for walls with the roughness of the microchannels \( (\varepsilon = 4 \mu \text{m}) \).

New equations were developed from these adjustments to the Petukhov equation and the Dittus-Boelter equation. To summarize the adjustment for thermally developing flow applied to the Petukhov equation and the adjustments made to the Dittus-Boelter equation for thermally developing flow and surface roughness, the following correlations are given:

\[
\text{Nu}_{\text{Pt, th. devloping}} = \left[ \frac{(f/8)\text{RePr}}{c + 12.7(f/8)^{1/2}((\text{Pr}^{2/3} - 1))} \right] \left\{ 1 + \frac{\left[ \frac{L}{D_h} \right]^{0.1} \left( \frac{0.68 + 3000}{\text{Re}} \right)}{\left[ \frac{L}{D_h} \right]} \right\}
\]

(61)

\[
\text{Nu}_{\text{DB, th. dev & rough}} = 0.023\text{Re}^{4/5}\text{Pr}^{0.4} \left( \frac{f_{\text{rough}}}{f_{\text{smooth}}} \right)^{0.68\text{Pr}^{0.215}} \left\{ 1 + \frac{\left[ \frac{L}{D_h} \right]^{0.1} \left( \frac{0.68 + 3000}{\text{Re}} \right)}{\left[ \frac{L}{D_h} \right]} \right\}
\]

(62)
Figure 5.6 shows the experimental data plotted against the Petukhov equation, the adjusted Petukhov equation, the Dittus-Boelter equation, and the adjusted Dittus-Boelter equation. The plot of the adjusted Petukhov equation fits the experimental data even closer than the unadjusted Petukhov equation. The adjusted Petukhov correlation lies within the error bar of each experimental data point. The adjusted Dittus-Boelter equation fits the experimental data closer than any of the other correlations. Since the Dittus-Boelter equation is recommended by Rosenhow et al. [42] for Reynolds numbers down to 2500, it is reasonable that the adjusted Dittus-Boelter equation fits the data more closely than the adjusted Petukhov equation. The good agreement between the experimental data and the adjusted Dittus-Boelter and Petukhov equations indicates that it is in fact worthwhile and beneficial to account for both surface roughness and thermal entry length effects when comparing experimental data with Nusselt number correlations in the transition/turbulent regime.

Figure 5.6: Turbulent correlations for Nusselt number with corrections for entrance effects and surface roughness plotted with the experimental data.
As a summary of the analysis of the experimental results for heat transfer, it can be said that while the experimental data does not correlate well with developing laminar flow theory, the data shows very good agreement with turbulent correlations with adjustments for thermal entry length and surface roughness. Further experimentation in the fully laminar regime is required to analyze the discrepancies observed between the current experimental data in the fully laminar regime with developing laminar flow theory.

It is possible that additional experimental error is to blame for the discrepancies shown in laminar regime Nusselt numbers. There were temperature differences between the water inlets and the pressure tap tubes measured inside the inlet plenum on the order of 1-3°C, and this level of underestimation of inlet temperature would certainly cause an abnormally high Nusselt number to be measured (the outlet plenum, however, was nearly isothermal due to the small difference in water and wall temperature at the outlet). However, while this measurement error may have increased the Nusselt number, there was also most likely a temperature difference on the order of 1-2°C between the channel walls and the thermocouples monitoring them (with the measurement being higher than the actual wall temperature). This possible experimental error would then decrease the measured Nusselt number. These two errors act such that they tend to cancel each other out, but the degree to which they are canceled is unknown. Therefore, for further analysis of this Cu MHE in the area of heat transfer, more experiments with revised measurement techniques must be employed in order to resolve the issues presented here. Using a thermally insulating material for the plenums would reduce the underestimation of the temperatures measured in the inlet plenum. Also, installing a series of thermocouples vertically spaced apart from each other above the channel walls would allow for an extrapolation of the actual wall temperature to be made.
CHAPTER 6: COMPUTATIONAL FLUID DYNAMICS ANALYSIS

6.1 Modeling Setup and Governing Equations

In order to shed some light on the experimental results presented, a CFD analysis of the heat transfer in the microchannels was completed. The first step in completing a CFD analysis of a system is to set up the governing equations. The equations involved in incompressible flow through microchannels include the continuity equation, the momentum equation, and the energy equation [40, 43]. The momentum and energy equations are, respectively

\[
\frac{\rho D\bar{u}}{Dt} = -\nabla p + \rho \bar{g} + \mu \nabla^2 \bar{u}
\]

(63)

\[
\left(\rho u \frac{\partial \bar{i}}{\partial x} + \rho v \frac{\partial \bar{i}}{\partial y} + \rho w \frac{\partial \bar{i}}{\partial z} + \rho \frac{\partial \bar{i}}{\partial t}\right)
\]

\[
= \left[\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right)\right] + \left[ u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + w \frac{\partial p}{\partial z}\right] + \mu \varphi
\]

(64)

where \( \bar{g} \) is the body force vector, \( \bar{i} = c_p \partial T + (\partial p/p) \) is the specific enthalpy, and \( p \) is pressure. For the specific case of heated flow through microchannels, the above equations can be further simplified. Assuming steady flow, constant properties, no body forces, only axial pressure variation, no viscous dissipation, and no axial heat conduction (valid for RePr > 100), the governing equations for heated flow through a rectangular microchannel are

Continuity:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

(65)
x-Momentum:  
\[ \rho \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{dp}{dx} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \]  
(66)

y-Momentum:  
\[ \rho \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \]  
(67)

z-Momentum:  
\[ \rho \left( u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \]  
(68)

Energy:  
\[ \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = \frac{1}{\alpha} \left( \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \]  
(69)

Given the complexity of these equations, computational methods of solving them are required. All of these equations are coupled, so a computational code must be used to solve Eqs. (65)-(69) simultaneously. For this analysis, the commercial code Fluent was used to solve these equations.

In order to accurately model the fluid flow and heat transfer effects in the microchannels, direct modeling of the existing surface roughness in the microchannels would be necessary. Also, a model of both the inlet and outlet plenum would be required to have the inlet and exit conditions of the channels properly modeled. This method of modeling the microchannels has proven to be an unfeasible task given the available computational resources. However, a good approximation of the fluid flow and heat transfer in the microchannels can be made by breaking the problem up into a model of the inlet plenum and a model of a channel. The outlet plenum is not necessary to model since the channel merely outlets directly to it.
6.2 Basic Overview of the k-ω Turbulence Model

Because many of the flow conditions in the experiments involve transition and/or turbulent flow, a model for solving turbulent flow is also needed. Each turbulence model available in Fluent was created for different flow conditions. The model most suitable for the present analysis is the k-ω model. The k-ω model is useful for low-Reynolds number turbulent flows and transition flows [48]. While not all details of the model are presented here, a basic overview of the fundamentals behind the k-ω model are presented to provide some insight into the structure of the model.

The k-ω model is a Reynolds-averaged Navier Stokes (RANS) type of turbulence model [48]. This means that the k-ω model uses the RANS equation for momentum in order to determine the transport of averaged flow quantities. In this method, the entire range of scales of turbulence in the problem is simultaneously solved. In this modeling method, the velocity components are replaced by

\[ u_i = \bar{u}_i + u'_i \]

(70)

where \( \bar{u}_i \) is the mean velocity component and \( u'_i \) is the fluctuating velocity component (fluctuations are inherent to turbulence). All other scalar quantities involved in the momentum equation are treated in this manner as well. Therefore, the steady incompressible momentum equation becomes [40, 41, 48]

\[ \rho u_j \frac{\partial}{\partial x_j} (u_i) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_l}{\partial x_l} \right) + \rho \frac{\partial}{\partial x_j} \left( -\rho u'_i u'_j \right) \]

(71)
where \(-\overline{u_iu_j}\) are the Reynolds stresses and \(\delta_{ij}\) is the Kronecker delta. In the k-ω model, the Boussinesq hypothesis is employed to model the Reynolds stress tensor. The Boussinesq hypothesis is stated as [48]

\[
-\overline{u_iu_j} = \frac{\mu_t}{\rho} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \left( \rho k + \mu_t \frac{\partial u_k}{\partial x_k} \right) \delta_{ij}
\]

(72)

where \(\mu_t\) is the turbulent viscosity. This definition of the Reynolds stresses is used in the transport equations of the k-ω model.

In the k-ω model, two transport equations are solved in addition to the momentum equations to resolve the turbulent fluctuations, the Reynolds stresses, and the turbulent viscosity. These two equations solve for the turbulence kinetic energy, \(k_t\), and the specific dissipation rate, \(\omega\) (thus the model is known as the k-ω model). The steady-state, incompressible forms of these transport equations are given by [48] as

\[
\rho \frac{\partial}{\partial x_i} (k_t u_i) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{Pr_{t,k_t}} \right) \frac{\partial k_t}{\partial x_j} - \rho \overline{u_iu_j} \frac{\partial u_j}{\partial x_i} - \left[ \rho \beta^* f \omega k_t \omega \right] \right]
\]

(73)

\[
\rho \frac{\partial}{\partial x_i} (\omega u_i) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{Pr_{t,\omega}} \right) \frac{\partial k_t}{\partial x_j} - \frac{\omega}{k_t} \rho \overline{u_iu_j} \frac{\partial u_j}{\partial x_i} - \left[ \rho \beta^* \omega^2 \right] \right]
\]

(74)

where \(Pr_{t,k_t} = Pr_{t,\omega} = 2\) and the turbulent viscosity combines \(k_t\) and \(\omega\) by

\[
\mu_t = \alpha^* \left( \frac{\rho k_t}{\omega} \right)
\]

(75)
The coefficient $\alpha^{**}$ is very important in the k-$\omega$ model since it provides a low Reynolds number correction because it effectively damps the turbulent viscosity [48]. The coefficient $\alpha^{**}$ is defined by

$$\alpha^{**} = \left( \frac{0.072 + \frac{\rho k_i}{3 \mu \omega}}{1 + \frac{\rho k_i}{6 \mu \omega}} \right)$$

(76)

The coefficient $\alpha$ in Eq. (76) is related to $\alpha^{**}$ by

$$\alpha = \frac{0.52}{\alpha^{**}} \left( \frac{1}{\frac{9}{2} + \frac{2.95 \mu \omega}{\rho k_i}} \right)$$

(77)

The dissipation terms $[\rho \beta^* f^* \cdot k_i \omega]$ and $[\rho \beta f^* \omega^2]$ from Eqs. (73) and (74), respectively, are given in detail in [48].

An important factor to consider when creating a mesh for use with the k-$\omega$ model is the resolution of the grid near the walls. The k-$\omega$ model follows the law of the wall, and therefore it attempts to resolve the viscous sublayer. In order to resolve the viscous sublayer, the mesh size must be fine enough at the wall to have $y^+ \leq 5$ [48]. The dimensionless term $y^+$ is defined as

$$y^+ = \frac{y u}{v}$$

(78)

with $u$ being the velocity. For the model created, the highest average value of $y^+$ was 1.22 for the case where Re = 3000. Thus, the model used was sufficiently meshed for the k-$\omega$ model.

Fluent simultaneously solved for the original governing equations (Eqs. (65)-(69)) in Reynolds averaged form along with the k-$\omega$ transport equations, Eqs. (73) and (74), when the k-
model was employed. This model proved to be a powerful tool in predicting heat transfer behavior in low-Reynolds number transitional flows since it creates some of the effects present in transition flows that laminar models do not incorporate.

6.3 The Plenum Model

The plenum was modeled to obtain some insight on the flow patterns inside the plenum and to obtain velocity profiles at the channel inlets for use in the model of the channel. A model of the plenum was created given the dimensions of the MHE which were measured. A small section of the microchannels was included in order to establish the proper flow out of the plenum and into the channels. Also, a symmetry plane was used in the middle of the plenum in order to reduce computational expense. The mesh was created using a triangular unstructured mesh on the bottom surface of the plenum and channels which was then extruded upwards to the top walls. This method of meshing allowed for the different shapes involved (circles and rectangles) to be meshed without high skewness of mesh. The density of the mesh was magnified in the region near the channel inlets in order to provide for good mesh quality in the region of the highest pressure and velocity gradients. Figure 6.1 shows three views of this model.

A velocity inlet was imposed on the circular inlet hole in order to set a channel Reynolds number for each case which was run. For the cases in which laminar flow was surely the dominant flow regime (i.e. Re ≤ 1000), the laminar model was used to solve the governing equations for the system. For cases in which Re ≥ 1000, the k-ω was used for two main reasons. First, the experimental data shows that there is a possibility of transition/turbulent phenomena happening in this region. Second, the model would not converge upon a constant exit Reynolds number when the laminar model was used in cases where Re ≥ 1000. This could have happened for any number of reasons, but it is hypothesized that because the plenum model involves
different length scales and sharp inlet corners, the laminar model does not have the proper type of modeling capabilities to resolve all of the complex flow patterns and recirculation which can occur near the channel inlets.

Figure 6.1: Inlet plenum model: (a) side view; (b) top view; (c) model view.
Each case was run using second order upwind schemes for each governing equation. It was ensured that residuals dropped to at least $10^{-6}$ for each case. Channel exit velocities were monitored, and the models were considered to be sufficiently converged when the maximum deviations in channel exit velocity over a significant range of iterations (approximately 100) was less than 0.5% of the average value.

To analyze the directionality of the flow velocity near the channels, a plane was created in the plenum which was 200 μm away from the channels and was normal to the flow direction. The area-weighted average magnitude of velocity on this plane was found, and it was compared to the area weighted average x-velocity, y-velocity, and z-velocity. For the case tested with Re = 2054, the ratio of x-velocity, y-velocity, and z-velocity to the total velocity magnitude was found to be $u/V = 0.87$, $v/V = 0.08$, and $w/V = 0.02$, respectively. Each case tested (from 300 < Re < 3000) showed ratios of similar magnitudes. This indicates that the direction of the flow near the channel entrances was primarily in the direction parallel to the flow in the microchannels. This indicates that the plenum was sufficiently big to allow the flow to become nearly unidirectional in the region near the channel inlets.

Were the flow more three-dimensional, the channel inlet velocity profiles could be unsteady or highly uneven. The differences in x-velocity in the plenum on a plane 200 μm from the microchannel inlets for the Re = 2054 case are shown in Figure 6.2. From Figure 6.2, the channels toward the outside of the plenum and in the middle of the plenum seem to get slightly less flow than the channels very near to the inlets of the plenum (the channels in the middle of Figure 6.2 are the channels very near to the inlets of the plenum). However, the differences in actual flow rates entering the channels were very small. Indeed, for the case shown in Figure 6.2, the difference between the flow rates in the outermost channels and the channel nearest to
the plenum inlet was ≈ 1%. All of the other cases (from 300 < Re < 3000) exhibited similar behavior for flow entering the channels. Therefore, it is concluded from the plenum model that the plenum used in the experiments was properly designed to ensure even flow rates in each microchannel.

![Figure 6.2: Velocity variations along a plane 200 μm away from the channel inlets at Re = 2054.](image)

In order to create a realistic inlet condition for the channel model, the velocity profile from one of the channels was copied into a raw data file for use in defining an inlet condition in the microchannel model. The channel used for this velocity profile was somewhat arbitrarily chosen to be the channel nearest to the middle of the MHE. While other channels may have slightly different inlet flow profiles, one of the channels had to be chosen, and it is difficult to determine which channel is most representative of the actual inlet profiles from the experiment. Since the actual profiles may not be accurately represented in the channel model, it is assumed to be a valid assumption that one of the channels can be arbitrarily chosen as the channel with a representative inlet profile.
6.4 The Microchannel Model

6.4.1 Microchannel Model Mesh

With the plenum modeled and inlet velocity profile conditions obtained, the microchannels themselves could be modeled. Because of the severe inlet condition that exists in the MHE, a uniform velocity inlet in the microchannels cannot be assumed when attempting to make a direct comparison with the experiments. This would misrepresent the heat transfer characteristics of the entrance region of the microchannels, and since this is the region with the largest gradients in heat transfer coefficient, it is important to attempt to model the inlet condition as closely as possible to the actual situation. It is for this reason that the velocity profiles from the plenum model were used in the microchannel model.

The microchannel model is a conjugate heat transfer/fluid flow analysis that includes a section of copper surrounding the microchannel. No symmetry planes were assumed in the microchannel because the velocity inlet from the plenum was not assumed to be symmetric about the cross-section of the channel inlet. The microchannel model consists of a structured mesh with boundary layer-type mesh growth ratios near the walls. For the mesh of the channel used for full calculations, the mesh size on the vertical wall employed 39 elements with a starting size of 2.5 \( \mu \text{m} \) at the wall. The horizontal walls employed 23 elements with a starting size of 2.5 \( \mu \text{m} \) at the wall. The longitudinal direction employed 150 elements with a growth ratio of 1.015 beginning from the inlet. This finer meshing at the inlet ensured that the gradients in the beginning of the channel could be resolved more completely. The total mesh count for this model was 134,550 rectangular elements. The mesh for the copper surrounding the channel grew from the mesh size at the channel walls at a growth ratio of 2 until a maximum element size
of 200 μm was reached. Such a coarse mesh is sufficient in modeling heat transfer through such a conductive metal as copper. Figure 6.3 shows this microchannel model.

Figure 6.3: Microchannel model: (a) side view; (b) channel side view close-up; (c) inlet plane view (triangular mesh is the solid); (d) model view.
6.4.2 Grid Independence

A grid independence test was conducted for the microchannel model. Grid independence was seen at every Reynolds number when measuring Nusselt number. The intermediate grid size was chosen for use because further refinement did not yield more useful data, and a smaller grid expedited the computational process. Some examples of the grid independence found from the microchannel model are shown in Figure 6.4.

![Grid Independence - Re=600](image1)

![Grid Independence - Re=1450](image2)

![Grid Independence - Re=2050](image3)

Figure 6.4: Grid independence: (a) Re = 600; (b) Re = 1450; (c) Re = 2050.
6.5 CFD Results and Analysis

Each case was run using second order upwind schemes for each governing equation. It was ensured that residuals dropped to at least $10^{-6}$ for each case. Nusselt numbers were calculated for the microchannel model for $300 < \text{Re} < 3000$. Figure 6.5 shows the laminar results ($300 < \text{Re} < 2000$) plotted with the experimental data, the Hausen correlation (Eq. (31)), and the Sieder and Tate correlation (Eq. (32)).

![Laminar CFD: Nu vs Re](image)

- **Figure 6.5:** CFD results for the laminar model plotted with the experimental data and developing laminar flow correlations.

It can be seen in Figure 6.5 that the microchannel model matches the Hausen correlation for thermally developing laminar flow. It is interesting that the results do not match the Sieder Tate correlation for simultaneously developing flow as closely. This indicates that the velocity profile at the inlet which was taken from the plenum model does in fact make the hydrodynamic entry length shorter than a uniform inlet condition. It is hypothesized that this was the case for the experimental conditions. Therefore, it can be said that the plenum model velocity inlet
profile performs its expected function of approximating the condition at the inlet of the microchannel more accurately than a uniform velocity inlet would. The results for the laminar microchannel model do not match the experimental data any closer than the laminar correlations for developing flow. Therefore, the laminar microchannel model confirms the conclusion presented in Section 5.2.2 that the Nusselt numbers measured in the experiments do not follow the trend of laminar Nusselt number theory. The results for the k-ω microchannel model are shown in Figure 6.6.

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**Figure 6.6: CFD results for Nusselt number with the k-ω model plotted with the experimental data and turbulent Nusselt number correlations.**

The k-ω microchannel model matches the Dittus-Boelter equation reasonably well. The computational data is not expected to match perfectly with any turbulent correlation since the assumptions behind the k-ω model are unique and may not follow the same assumptions as the
turbulent correlations. The same can be said for why the Petukhov correlation and the Dittus-Boelter correlation do not perfectly match each other.

As an illustration of the fact that thermal entrance lengths are of great importance in the analysis of Nusselt numbers for these experiments, Figure 6.7 shows plots of local Nusselt numbers for Re = 600 (laminar model), Re = 1060 (laminar model), and Re = 2450 (k-ω model). It can be seen in Figure 6.7 that throughout the range of Reynolds numbers, the Nusselt number never reaches its steady state value. Thus, it is inferred that for each case tested in the experiments, the thermal entrance length was longer than the channel length itself.

![Figure 6.7: Local Nusselt numbers for various Reynolds numbers as calculated from the simulations.](image)

Just as the turbulent correlations were corrected for surface roughness in Section 5.2.2, the k-ω microchannel model can be corrected for surface roughness. Again employing Eq. (60) to correct the calculated Nusselt number for surface roughness, adjusted results from the k-ω
microchannel model were created and are plotted in Figure 6.8 along with the adjusted Petukhov equation and the adjusted Dittus-Boelter equation.

The adjusted k-ω microchannel model Nusselt number matches the experimental data more closely, and it also matches the adjusted Petukhov correlation fairly well. Just as with the roughness and entrance length corrections performed in Section 5.2.2, the roughness correction of the k-ω microchannel model is a necessary step to legitimately compare the computational results to the experimental data. While the computational model does not perfectly match the experimental data, this is to be expected to some degree because the k-ω model is merely an approximation of transitional and turbulent flow. The computational results from the k-ω microchannel model follow a trend similar to the experimental data, which indicates that the use
of the k-ω model is more appropriate for modeling the experimental results than the laminar model.

As with any computational model, the results are only as good as the assumptions made in creating the model. Without complete knowledge of all of the conditions and phenomena involved in the flow inside the microchannels, a computational model which completely matches an experiment cannot be made (other than by chance). In this case, the inlet condition is still a bit of an unknown (although the inlet condition taken from the plenum model is a good approximation), and the true effect that the surface roughness plays is also an unknown. As a summary of the findings from the computational modeling completed and the correlations found in literature, Figure 6.9 shows the most applicable results and correlations with the experimental data. It can be seen in Figure 6.9 that the area of greatest uncertainty is in the low Reynolds number range. Further experimentation is necessary to resolve these discrepancies.

Figure 6.9: Final graph for Nusselt number with the experimental data plotted with the most appropriate theoretical correlations and the CFD results.
CHAPTER 7: CONCLUSIONS AND RECOMMENDATIONS

An investigation of the fluid flow and heat transfer phenomena in microchannels and microchannel heat exchangers was conducted. A review of the literature published on research conducted in microchannel fluid flow and heat transfer over the two decades was completed. An analysis of some of the methods of experimentation and data reduction found in the literature was performed, and it was found that some of the methods presented in the literature create errors in the calculated Nusselt number. An investigation of the performance capabilities of designed microchannels with bumps on the bottoms of the channels was conducted, and it was found that the designed channels provided higher Nusselt numbers than a plain channel in the turbulent regime at the cost of a higher friction factor.

Experiments were conducted on a Cu MHE that was constructed through molding replication and eutectic bonding. The microchannel dimensions in the MHE tested were 178 μm wide, 341 μm tall, and 17.32 mm long. A new method of heat application was employed in the heat transfer experiments where instead of a resistance heater being used, a hot water bath was used to provide a constant surface temperature boundary condition rather than a constant heat flux boundary condition. Friction factor and Nusselt number were calculated for a Reynolds number range of 236 to 2946. Friction factor data was adjusted for entrance and exit losses, and the corrected data matched reasonably well with the theoretical equations. The Nusselt number results showed higher than expected values in the low Reynolds number range while the Nusselt numbers in the high Reynolds number range showed excellent agreement with a modified Dittus-Boelter equation which was adjusted for entrance region and roughness effects. A CFD model was created in Fluent to model the inlet plenum and the microchannels. The inlet condition from the microchannels was taken from the inlet plenum model in order to better simulate the
experimental conditions. The laminar Nusselt numbers from the microchannel model showed excellent agreement with the Hausen equation for thermally developing flow, which suggests that the inlet condition did in fact reduce the hydrodynamic entrance length. The transition/turbulent Nusselt numbers from the microchannel model were corrected for surface roughness effects and showed reasonable agreement with the magnitude and trend of the experimental turbulent Nusselt numbers.

For future work in the experimentation of MHE’s, it is suggested that temperature measurements be made closer to the microchannel inlets and that temperatures be measured at more locations in the solid surrounding the microchannels and the plenums in order to obtain a better sense of how the heat flows through the entire MHE. Also, a thermally insulating material for the plenums would reduce the underestimation of the temperatures measured in the inlet plenum. Installing a series of thermocouples vertically spaced apart from each other above the channel walls would allow for an extrapolation of the actual wall temperature to be made. Also, providing a higher wall temperature and/or lower inlet temperature would decrease the experimental errors that are incurred from the temperature measurements. There is still a considerable amount of work to be done with the experimentation of microchannels and MHE’s, and the prospect of the use of MHE’s in high performance heat transfer applications looks more promising than ever.
REFERENCES


VITA

W. Allen Phillips was born in Lafayette, Louisiana, in 1985. He received his Bachelor of Science in Mechanical Engineering degree from Louisiana State University in May of 2007. He participated in the 2007 Louisiana State University Formula SAE project as a senior design member, and he helped the team compete at the 2007 Formula SAE competition in Michigan. Allen conducted research during his graduate studies on both turbine blade film cooling and microchannel heat exchangers. Allen plans to begin his career with ExxonMobil as a machinery engineer at the ExxonMobil Baton Rouge Chemical Plant.