Performance modeling of explosively actuated devices

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PERFORMANCE MODELING OF EXPLOSIVELY ACTUATED DEVICES

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
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by

Adam Braud
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Abstract

Explosively actuated devices (pin pullers, cable cutters, valves, etc) are used extensively to perform critical functions for aerospace, industrial, and defense related applications. The failure of these devices have led to a greater effort to quantify device design and performance. This thesis describes the actuation process of an explosively actuated valve, including: 1) the burning of the solid explosive HMX (C\(_4\)H\(_8\)N\(_8\)O\(_8\)) and production of its high pressure gas products, 2) the mass transfer of gas products through an actuator to an expansion volume including choked flow effects, 3) the resulting piston motion due to high pressure gas products, and 4) the effects of device deformation on valve performance. Although the model presented is validated with a valve, it is kept general such that it can be applied to other explosively actuated devices. A key model objective is to qualify the effect of design modification (geometry, propellant mass, etc.) on device performance. A focus of this paper is to describe the leading order effects component deformation has on device performance, including the effects of material strain hardening and internal gas pressure. Model results for the axial resistive force exerted on the piston during actuation are compared to nonreactive quasistatic compression tests and a finite element study. Results from the compression tests and FEA indicate there is significant piston bending induced by the housing corner during skirt insertion. Results reasonably predict both the compression tests and finite element results if two friction coefficients are used as a simple way to describe piston bending. To characterize reactive valve performance, data from a reduced number of experiments was used to determine model parameters which are difficult to measure (propellant linear regression rate, friction coefficient, etc.) and characterize baseline valve performance. Results from a sensitivity study suggest that the piston is being overdriven by its current propellant load (150 mg HMX). As such, valve performance is insensitive to slight modifications around the baseline case. Valve performance does show sensitivity to propellant mass and friction coefficient. Valve failure is predicted with a propellant mass between 30 and 40 mg, and with elevated friction coefficients (≈ 1.0).
Chapter 1
Introduction

1.1 Background

Pyrotechnically and explosively actuated devices, such as pin pullers, cable cutters, thrusters, and valves, are routinely used to perform critical functions in numerous industrial, aerospace, and defense related applications because they can reliably and safely deliver large power in remote environments by the combustion of a self-contained energy source. To demonstrate the power producing capability of these devices, first consider a battery. While a typical 1.25 V, 500 mA nickel-cadmium battery is able to deliver approximately 0.625 W of power, 150 mg of HMX ($C_4H_8N_8O_8$) having a typical energy density of $5 \frac{MJ}{kg}$ can react in approximately 100 $\mu$s, delivering 7.5 MW of power. Historically, the design of these devices has been largely empirical and considered by some to be an art [12]. There has been greater effort to quantify device design and performance in response to a number of spacecraft failures (Landsat 6, Telstar 4, and the Mars Observer) that occurred when explosively actuated hardware did not properly function [1, 2, 4, 9]. Initially, devices were qualified based on simple go no-go tests in which the devices were fired with an 85% charge for functioning. If the devices actuated with an 85% propellant mass load, the lot of devices was considered suitable for use. Subsequently, more quantitative tests were utilized such as the weight drop test. A falling mass was used in the weight drop tests to actuate the device and quantify energy requirements. An inert pressure source has been used to quantify device performance in more recent experiments [24]. In recent years there has been much interest in scaling down these devices. For example, there has been extensive work done in the field of microthrusters for the application of new micro-rocket devices [15, 16, 18, 23]. While experiments are always needed to assess actual device performance, it is desirable to develop modeling tools that can assess the performance of current designs and facilitate the design and development of new generation devices.
1.2 Problem Statement

A focus of this study is describing the operation and performance of a nitrogen cartridge valve, but emphasize that our model can be easily adapted and applied to other devices such as pin pullers and cable cutters. The term valve performance includes typical quantities used to characterize valve operation, such as pressure history, piston motion history, operational timescales, etc. The axisymmetric valve shown in Fig. 1.1 in both its pre-fired and post-fired configurations; representative dimensions are indicated in the figure. The valve cross section in the post-fired configuration is shown in Fig. 1.2. The purpose of this valve is to enable the flow of stored nitrogen gas through a transfer conduit at a desired time. The valve is driven by the combustion of 150 mg of the commonly used solid high explosive HMX (C₄H₈N₈O₈). The explosive is contained in a small actuator cartridge that is threaded into the device directly above the product gas expansion chamber. Combustion of the explosive, which is initiated by an embedded hot wire, produces a mixture of high temperature gases that rapidly pressurize the actuator volume. The explosive and combustion products are initially sealed within the actuator by a metal burst disc that facilitates ignition and subsequent combustion of HMX by allowing for pressure build-up prior to rupturing. The burst disc ruptures when the pressure within the actuator exceeds a critical value (≈ 55 MPa), enabling the flow of product gases, and possibly some unreacted explosive, into the expansion chamber where it exerts a net axial force on the hollow piston pushing it into the valve bore. This movement induces plastic deformation of the piston and housing, and significant frictional resistance between these components due to mechanical interference. The cutter, attached to the closed bottom end of the piston, penetrates the diaphragm as the moving piston is brought to rest by contact with the stops. Valve operation is then complete approximately 90 μs following ignition as stored nitrogen gas flows through the newly created opening into the gas transfer conduit. The original design and subsequent modifications of the nitrogen cartridge valve were largely based on limited empirical data. The model formulated in this thesis is intended to give
designers a predictive capability that enables them to make informed decisions regarding design changes.

1.3 Review

There is a limited amount of modeling work reported in the archival literature on pyrotechnically and explosively driven devices. Previously developed models that are cited in the open literature give only partial descriptions of their operation. The model of Jones, et al [5, 7], uses analytical and computational results based on pressure vessel theory to estimate the work required to operate an explosively actuated valve. Their model, while useful, does not describe the combustion process and its influence on valve operation, nor does it describe the effects of internal gas pressure or material strain hardening on valve deformation. Similarly, the model of Gonthier, et al [3, 6] describes the coupled, time-dependent combustion and work processes required to operate a pyrotechnically actuated pin puller, but does not describe the effect of component material strength and deformation on device performance. The model of Ng and Kwon [10] describes the burning of an
explosive with an adiabatic gas expansion model, and subsequent piston deformation. It does not consider the thermochemistry of a burning explosive and its effects on valve performance. The deformation model used by Ng and Kwon is similar to that of Jones and does not account for the effects of internal gas pressure or material strain hardening on device deformation.

The emergence of micro-technology has led to the development of a new class of miniature devices (microvalves, micropumps, microthrusters, etc) for the purpose of reducing costs. The work of Rossi, et al [15, 16, 23] describes flow characteristics in an array of silicon mounted microthrusters for the use of attitude control on satellites. A common practice in analyzing explosively actuated devices is to assume a quasi-equilibrium process in which the thermodynamic quantities are spatially homogeneous at any time interval [3, 14, 17]. This assumption allows the model to consist of a system of ordinary
differential equations with respect to time. The work of Lee [21] explored the effects of unsteady gas dynamics within a closed bomb and determined that the quasi-equilibrium assumption becomes invalid as the device operation time approaches the gas dynamic wave propagation timescale. Though it is questionable whether the quasi-equilibrium assumption is appropriate for the present analysis, it does enable leading order estimates for valve operation to be obtained.

1.4 Goals of Current Study

The model presented in this thesis combines and extends key features of these existing models to account for coupled 1) time-dependent, multiphase combustion of HMX within the actuator, 2) flow of product gases from the actuator into the expansion chamber, 3) work performed by the expansion of high pressure combustion gases within the expansion chamber on the moving the piston, 3) elasto-plastic deformation of the piston and the outer housing that confines its motion, and 4) frictional resistance due to relative motion between the piston and housing.

Features included in this model that have not been previously addressed include consideration of the coupled combustion process with device deformation, consideration of combustion gas pressure on component deformation, and consideration of material strain hardening behavior on component deformation. Rigorous constitutive theories are sacrificed for tractability when feasible. The model tracks the time-dependent evolution of mass and energy for HMX and gas phase combustion products based on principles of mixture theory. Here, it is assumed that combustion products are formed in fixed ratios determined by chemical equilibrium calculations, as assumed by Gonthier, et al [3, 6], for the pin puller model. Forcing terms in the product energy equations account for heat and work interactions with the surrounding valve structure. The work interactions are coupled to an equation of motion for the piston that accounts for frictional resistance with the valve bore. This resistance is modeled based on an interface stress induced by the elasto-plastic deformation of the piston and housing. As in the work of Jones, et al [5, 7], we assume that
the spatial stress field within the piston and housing locally equilibrates during the device operation time; as such, the deformation is considered quasistatic. Other key assumptions are given in appropriate sections of this thesis. The inclusion of structural plastic deformation can be used to determine the likelihood of device failure due to uncontained plastic flow, for a given explosive composition and mass, and its dependence on valve geometry and material properties. Importantly, the comprehensive model is constructed so that the total system mass and energy are conserved.

The goal of the present work is to formulate a comprehensive, but simple, mathematical model that can be used to quickly explore and assess the effects of geometric, structural, and energetic material design modifications on the power producing capability and performance of explosively driven actuators in different environments. The focus of the work is on describing device deformation with specific objectives: 1) to formulate a device deformation model that accounts for leading order effects of piston and housing material properties, including the effects of material strain hardening and internal gas pressure, 2) to characterize the inert operation of the nitrogen cartridge valve using both compression tests and finite element study, and 3) to characterize the baseline performance of the nitrogen cartridge valve and explore the effects of design modifications on performance. It is anticipated that this model will prove useful in identifying optimal configurations for explosively actuated devices based on suitably defined performance measures; these configurations can then be further explored by performing a reduced set of detailed finite-element simulations and experiments.

An outline of this thesis is as follows. First summarized in Chapter 2 is the mathematical model for the evolution of explosive and combustion product mass and thermal energy within the actuator and gas expansion chamber, and the model used to describe valve deformation. Next, Chapter 3 gives comparisons between inert model predictions and quasistatic compression tests along with FEA results for the force and work needed to push the piston into the bore and puncture the diaphragm. In Chapter 4, the model
is used to characterize the baseline operation of the nitrogen cartridge valve including a parametric study to investigate the effect of modifications on valve performance. Last, some conclusions and recommendations for future work are given.
Chapter 2
Mathematical Model

As a preliminary step in model development, the valve was partitioned into three interacting subsystems: the actuator, gas expansion chamber, and surroundings. As indicated in Fig. 2.1, the actuator and gas expansion chamber collectively contain all explosive and gas phase combustion product mass, whereas the surroundings contain the mass of all structural members including the piston and housing. The actuator interacts with the expansion chamber by mass and energy transport, and both interact thermally with the surroundings by convective and radiative heat transfer. It is assumed that all unreacted explosive mass is confined to the actuator, and gas product mass blowby past the piston is ignored. The expansion chamber also undergoes a work interaction with the surroundings due to volume changes associated with piston motion. Strain work interactions between both the actuator and expansion chamber and their surrounding walls is ignored.

First outlined in Section 2.1 are the model equations for the actuator and expansion chamber subsystems. Because these equations are similar to those originally formulated by Gonthier, et al [3, 6], for a pyrotechnically actuated pin puller model, the discussion here is intentionally kept brief; the interested reader is referred to this cited work for a
Table 2.1: Reaction equation for HMX combustion.
\[ \text{C}_4\text{H}_8\text{N}_8\text{O}_8(s) \rightarrow 3.979 \text{N}_2(g) + 2.971 \text{CO}(g) + 2.908 \text{H}_2\text{O}(g) + 1.005 \text{CO}_2(g) + 0.999 \text{H}_2(g) + 0.062 \text{OH}(g) + 0.035 \text{H}(g) + 0.002 \text{NO}(g) \]

more comprehensive discussion. Given in Section 2.2 is a technique to estimate the axial resistive force induced by piston-housing deformation, followed by a brief analysis of its solution behavior.

2.1 Actuator and Expansion Chamber Model

Using principles of mixture theory, a set of mass and thermal energy evolution equations can be written for the solid explosive and gaseous combustion products contained within the actuator and expansion chamber. To this end, the reactant and gas products are assumed to acoustically equilibrate much faster than the valve operation time resulting in a time-dependent, well-stirred reactor. Though the modeling framework of Gonthier, et al.,\cite{3, 6} accounts for the combustion of generic fuel-oxidizer mixtures and the existence of both condensed and gas phase product species, the combustion of only a single solid reactant (HMX \( \text{C}_4\text{H}_8\text{N}_8\text{O}_8 \)) is considered. The solid reactant is assumed to form strictly gaseous product species (\( \text{N}_2, \text{NO}, \text{CO}_2, \text{CO}, \text{H}_2\text{O}, \text{OH}, \text{H}_2, \text{H} \)) as indicated by equilibrium chemistry that imposed constant internal energy and actuator volume; the equilibrium calculations were performed using the commercial software package CHEMKIN. The product species specified by CHEMKIN are similar to those reported by Tarver \cite{11} for finite rate thermal decomposition of HMX. The stoichiometric equation for this combustion process is given in Table 2.1. Product species having mole concentrations less than 0.01 are ignored.

Mass and thermal energy evolution equations for the explosive and gas products, coupled with an equation for piston motion, are given by the following ordinary differential equations (ODE’s):

\[
\frac{d}{dt} (\rho_s V_s) = -\rho_s A_b r_b, \quad (2.1)
\]

\[
\frac{d}{dt} (\rho_{g_1} V_{g_1}) = \rho_s A_b r_b - \dot{m}_g, \quad (2.2)
\]
\[
\frac{d}{dt} (\rho_s V_s e_s) = -\rho_s e_s A_b r_b, \quad (2.3)
\]
\[
\frac{d}{dt} (\rho_{g1} V_{g1} e_{g1}) = \rho_s e_s A_b r_b - h_{g1} \dot{m}_g - \dot{Q}_{g1}, \quad (2.4)
\]
\[
\frac{d}{dt} (\rho_{g2} V_{g2} e_{g2}) = \dot{m}_g, \quad (2.5)
\]
\[
\frac{d}{dt} (\rho_{g2} V_{g2} e_{g2}) = h_{g1} \dot{m}_g - \dot{Q}_{g2} - \dot{W}_{out}, \quad (2.6)
\]
\[
m_p \frac{d^2}{dt^2} (z_p) = F_p - F_R. \quad (2.7)
\]

In these equations, subscripts "1" and "2" indicate quantities associated with the actuator and expansion chamber, respectively, and subscripts "s" and "g" indicate quantities associated with the solid explosive and gas phase products, respectively. The independent variable is time \( t \). Dependent variables include the density \( \rho_{gi} \) \((i = 1, 2)\); the volumes \( V_s \) and \( V_{gi} \); the specific internal energies \( e_s \) and \( e_{gi} \); the specific enthalpy \( h_{g1} \); the piston position measured relative to the top of the expansion chamber \( z_p \) as indicated in Fig. 2.2; the explosive linear regression burn rate \( r_b \); the area of the burn surface \( A_b \); the product gas mass flow rate from the actuator to the expansion chamber \( \dot{m}_g \); the heat transfer rates from the gas phase products to the surroundings \( \dot{Q}_{gi} \); the work rate done by product gases contained within the expansion chamber in moving the piston \( \dot{W}_{out} \); and the net gas pressure and resistive force acting on the piston, \( F_p \) and \( F_R \), respectively. Constant parameters contained in Eqs. (2.1)-(2.7) are the piston mass \( m_p \) and the unreacted solid explosive density \( \rho_s \). Equations (2.1) and (2.2), and Eqs. (2.3) and (2.4), govern the evolution of mass and internal energy for the solid explosive and gas phase products contained within the actuator, respectively, while Eqs. (2.5) and (2.6) govern the evolution of gas product mass and internal energy within the expansion chamber. Kinetic and potential energy are ignored. Summing Eqs. (2.1), (2.2), and (2.5) gives

\[
\frac{d}{dt} (\rho_s V_s + \rho_{g1} V_{g1} + \rho_{g2} V_{g2}) = 0;
\]
consequently, the total explosive mass is conserved by this model. Likewise, summing Eqs. (2.3), (2.4), and (2.6) gives

\[
\frac{d}{dt} (\rho_s V_s e_s + \rho_{g1} V_{g1} e_{g1} + \rho_{g2} V_{g2} e_{g2}) = - \left( \dot{Q}_{g1} + \dot{Q}_{g2} \right) - \dot{W}_{\text{out}}
\]

indicating that the total explosive and gas product energy changes only due to heat and work interactions with the surroundings. Further, it can be shown by multiplying Eq. (2.1) by \( e_s \), and subtracting the result from Eq. (2.3), that the internal energy of the solid explosive remains constant for this analysis (i.e., \( e_s = e_{s0} \)) because heat transfer from the hot product gases to the explosive is ignored. Equation (2.7) is Newton’s Second Law which governs the motion of the piston.

Constitutive relations needed to mathematically close Eqs. (2.1)-(2.7) include the following equations. Geometrical constraints require that

\[
V_1 = V_s + V_{g1}, \quad V_2 = V_{g2}, \quad z_p = z_{p0} + \frac{V_2 - V_{20}}{A_p}, \quad (2.8)
\]

\[
A_{w1}(z_p) = 2\sqrt{\frac{\pi}{A_1}} V_1 + 2A_1 - A_e, \quad A_{w2}(z_p) = 2\sqrt{\frac{\pi}{A_p}} V_2 + 2A_p - A_e, \quad (2.9)
\]

\[
A_p(z_p) = \pi b(0, t)^2 \quad (2.10)
\]

where \( A_p \) is the cross-sectional area of the top of the piston skirt which is initially located at axial position \( z_{p0} \), \( A_1 \) is the constant cross-sectional area of the cylindrical actuator, \( A_e \) is the cross-sectional area of the port separating the two subsystems, \( A_{w1} \) and \( A_{w2} \) are the surface areas of the actuator and expansion chamber through which heat transfer can occur with the surroundings, and \( b(0, t) \) is the instantaneous piston outer radius at the top of the skirt, as discussed in the next section. The skirt is conical and the cross-sectional area of the top of the piston skirt is changing as it is pushed in the bore. It is assumed that the solid explosive will fragment into \( N \) individually burning spherical grains immediately following ignition by the embedded hot wire. Expressions for the radius of each resulting
spherical grain, their regression rate, and their total burn surface area, are respectively given by
\[
r = \left( \frac{3V_s}{4\pi N} \right)^{1/3}, \quad r_b = -\frac{dr}{dt} = bP_{g1}^{n}, \quad A_b = (4\pi r^2 N) \tag{2.11}
\]
As commonly done in solid propellant combustion modeling, the burn rate is taken to be dependent on the gas pressure within the actuator \(P_{g1}\). Here, \(b\) and \(n\) are burn rate constants whose values are chosen based on constant volume HMX combustion data, as discussed later. The behavior of the product gases is taken as ideal. Their thermal and caloric equations of state, and constant volume specific heats, are given by
\[
P_{g1} = \rho_{g1}RT_{g1}, \quad e_{g1} = \sum_{j=1}^{N_g} Y_{gj}^j e_{gj}, \quad c_{vg1} = \sum_{j=1}^{N_g} Y_{gj}^j \frac{d}{dT_{g1}} \left( e_{gj} \right). \tag{2.12}
\]
Because internal energy is a function of temperature only for ideal gases, the constant volume specific heat is obtained by differentiating the caloric equation of state with respect to temperature. Occurring in these expressions are the gas temperature \(T_{g1}\), the ideal gas constant for the gas phase products \(R\) (the ratio of the universal gas constant and the mean molecular weight of the product gases), and the constant mass fractions \(Y_{gj}^j\) of the \(N_g\) product species. The notation superscript \(^{\prime}\) is used to label quantities associated with individual chemical species. The thermodynamic properties of each species are calculated using the CHEMKIN subroutine library and database. Expressions for the specific enthalpy and constant pressure specific heat for the product gases contained within the actuator are given by
\[
h_{g1} = \sum_{j=1}^{N_g} Y_{gj}^j h_{gj}^j, \quad c_{pg1} = \sum_{j=1}^{N_g} Y_{gj}^j \frac{d}{dT_{g1}} \left( h_{gj}^j \right). \tag{2.13}
\]
Again, for ideal gases, the constant pressure specific heat is obtained by differentiating the enthalpy with respect to temperature. Heat loss from the high temperature product gases to the cooler surroundings is assumed to occur by both convective and radiative heat
transfer. The heat loss rates are given by

\[ \dot{Q}_{g1} = hA_w (T_{g1} - T_w) + \sigma A_w (\epsilon T_{g1}^4 - \alpha T_w^4), \quad \dot{Q}_{g2} = hA_w (T_{g2} - T_w) + \sigma A_w (\epsilon T_{g2}^4 - \alpha T_w^4), \]

(2.14)

where \( h \) is a constant convective heat transfer coefficient, \( T_w \) is the temperature of the surrounding walls, \( \sigma \) is the Stefan-Boltzmann constant, \( \alpha \) is the absorptivity of the walls, and \( \epsilon \) is the net emissivity of the product gases. The pressure-volume work done by the gas contained within the expansion chamber in moving the piston is given by

\[ \dot{W}_{out} = P_{g2} \frac{dV_2}{dt}, \]

(2.15)

and the net axial pressure force acting on the piston is given by

\[ F_p = P_{g2} A_p. \]

(2.16)

All product mass is contained within the actuator until its pressure exceeds a critical value \( P_{crit} \) that causes the burst disc to rupture. Once the disc is ruptured, the flow rate of gas product mass from the actuator to the expansion chamber is governed by

\[ \dot{m}_g = \begin{cases} \rho_{g1} A_e \sqrt{\gamma RT_{g1}} \left( \frac{2}{\gamma - 1} \left( \frac{P_{g1}}{P_{g2}} \right)^{\gamma+1} - \left( \frac{P_{g1}}{P_{g2}} \right)^{\gamma-1} \right) & \text{if } \left( \frac{P_{g1}}{P_{g2}} \right) < \left( \frac{\gamma+1}{2} \right)^{\frac{\gamma}{\gamma-1}} \vphantom{\left( \frac{P_{g1}}{P_{g2}} \right)^{\frac{\gamma+1}{\gamma-1}}} \right) \\
\rho_{g1} A_e \sqrt{\gamma RT_{g1}} \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} & \text{if } \left( \frac{P_{g1}}{P_{g2}} \right) \geq \left( \frac{\gamma+1}{2} \right)^{\frac{\gamma}{\gamma-1}} \end{cases}, \]

(2.17)

Occurring in this expression is the specific heat ratio for the product gases contained within the actuator \( \gamma (= c_{pg1}/c_{vg1}) \). This expression accounts for mass choking at elevated actuator/expansion chamber pressure ratios.

An important, yet undefined, variable in Eq. (2.7) is the force \( F_R \) that resists piston motion due to geometrical interference between the piston and housing. Properly estimating the magnitude of this force and its dependence on both piston and housing material
properties and geometry, and on gas pressure within the expansion chamber, is key to obtaining a predictive model. Fundamental design decisions often focus on component material selection and geometry that can significantly affect the resistive force. The technique used to estimate this force is outlined in the following section.

Equations (2.1)-(2.17) can be reduced by mathematical operations to a final autonomous system of first order ODE’s that can be numerically solved to predict valve performance. To this end, it is necessary to define a new variable \( \dot{V}_2 \) representing the time derivative of the gas expansion chamber volume:

\[
\dot{V}_2 \equiv \frac{dV_2}{dt}.
\]  

(2.18)

The final system consists of six first order ODE’s of the form

\[
\frac{d\mathbf{u}}{dt} = \mathbf{f}(\mathbf{u}),
\]

(2.19)

where \( \mathbf{u} = (V_2, V_s, \rho_{g1}, T_{g1}, T_{g2}, \dot{V}_2)^T \) is a vector of dependent primary variables and \( \mathbf{f} \) is a non-linear vector function. All remaining variables can be expressed in terms of these six primary variables. The operations used to express the model equations in this reduced form are omitted for brevity as they are discussed in detail by Gonthier, et al.[3, 6] Initial conditions for these equations are

\[
V_2(0) = V_{20}, \quad V_s(0) = V_{s0}, \quad \rho_{g1}(0) = \rho_{g10}, \quad T_{g1}(0) = T_0, \quad T_{g2}(0) = T_0, \quad \dot{V}_2(0) = 0.
\]

(2.20)

### 2.2 Piston-Housing Deformation Model

A technique is described in this section for predicting time-dependent resistance to piston motion due to interference between the piston and housing as the piston in pushed into the bore by the high pressure gas contained within the expansion chamber. A similar technique was used by Jones, et al [5, 7], to model piston motion within an explosively...
actuated valve, though differences exist between our descriptions which are highlighted later. A premium is placed on tractability; thus, simple analytical solutions for material mechanics that do not require the application of elaborate finite-element modeling techniques are utilized. The use of analytical solutions facilitates extensive parametric studies with minimal computational times due to their relative simplicity. Because thermal energy evolution within the surrounding structure is not accounted for, emphasis is placed on estimating mechanical stresses induced within the piston and housing by geometrical interference and internal gas pressure only. Thermal stresses within this structure, while potentially significant at elevated temperature, would require a more detailed thermomechanics model. The formulation of the deformation model is addressed in Section 2.2.1. Section 2.2.2 discusses the uniqueness of the solutions obtained in Section 2.2.1. Section 3.1 reviews inert, quasistatic compression tests that have been performed to determine the work requirements of the valve and compare with results from the deformation model.

### 2.2.1 Formulation

The deformation model presented in this section shall be referred to as the simple deformation model in that it is analytically based. It is assumed that the piston initially lies entirely within the tapered region of the expansion chamber, as indicated in Fig. 1.1(a), referred to in this thesis as the skirt region. Immediately following actuation, part of the
piston is pushed into the bore, while part remains within the skirt region, as indicated in Fig. 2.2. Thus, time-dependent resistance to piston motion is generally due to interference experienced within both the skirt and bore regions of the valve:

\[ F_R(t) = F_{\text{skirt}}(t) + F_{\text{bore}}(t). \] (2.21)

In the discussion that follows, parentheses ( ) are used to denote a functional dependence on the enclosed variables so that the coupling between spatial and temporal quantities is apparent. Initially, \( F_{\text{skirt}} = F_{\text{bore}} = 0 \), and \( F_{\text{skirt}} \rightarrow 0 \) as the entire piston is pushed into the bore. The geometrical interference and gas pressure within the hollow piston increase the magnitude of the compressive radial stress at the axisymmetric piston-housing interface, i.e., \( \sigma_r = -\tilde{P} \). This radial interface stress is assumed to locally induce a tangential frictional stress, \( \tilde{\tau} = \mu \tilde{P} \), where \( \mu \) is a constant friction coefficient. Because the interference varies with both position along the piston axis and time, \( \tilde{P} = \tilde{P}(\xi, t) \) and \( \tilde{\tau} = \tilde{\tau}(\xi, t) \), where \( \xi \) is the position measured relative to the upper piston surface as indicated in Fig. 2.2. Due to geometric variations in the piston and housing, the inner, interface, and outer radii for a cross-section of the piston-housing structure are locally given by \( a(\xi, t) \), \( b(\xi, t) \), and \( c(\xi, t) \), respectively; these radii are indicated in the figure. Using the coordinates defined in the figure and the geometry illustrated in Fig. 2.3, where the valve attached axial coordinate \( z \) is measured relative to the top of the expansion chamber, the following expressions for the time-dependent resistive force components in the direction of piston motion result:

\[ F_{\text{skirt}}(t) = (\mu_s \cos \theta + \sin \theta) \int_{A_{\text{skirt}}} \tilde{P}(\xi, t) \, dA, \quad F_{\text{bore}}(t) = \mu_b \int_{A_{\text{bore}}} \tilde{P}(\xi, t) \, dA, \] (2.22)

where \( \theta \) is the inclination of the skirt with respect to the vertical. The integration indicated here is performed over the interfacial surface area of the skirt and bore regions of the piston and can be numerically integrated using the trapezoidal rule. In this work, the skirt region has the shape of a partial cone and the bore region has the shape of a cylinder; thus, the
area integrals in Eq. (2.22) can be reduced to 1-D integrals in terms of the axial piston coordinate ζ:

\[ F_{\text{skirt}}(t) = (\mu \cos \theta + \sin \theta) \int_0^{L-z_p(t)} \tilde{P}(\xi, t) g(\xi, t) d\xi, \quad F_{\text{bore}}(t) = 2\pi \mu R_b \int_{L-z_p(t)}^{L} \tilde{P}(\xi, t) d\xi, \]

(2.23)

where

\[ g(\xi, t) = \pi \sqrt{[b(0, t) - b(\xi, t)]^2 + \xi^2} \left\{ \frac{\partial b}{\partial \xi} + \frac{[b(0, t) + b(\xi, t)] \left( \xi - [b(0, t) - b(\xi, t)] \frac{\partial b}{\partial \xi} \right)}{[b(0, t) - b(\xi, t)]^2 + \xi^2} \right\}, \]

and

\[ b(\xi, t) = \left[ \frac{R_b - b(0, t)}{L - z_p(t)} \right] \xi + b(0, t). \]

Here, \( b(0, t) \) is the interface radius at the top of the piston, \( R_b \) is the constant bore radius, and \( b(\xi, t) \) is the variable interface radius between these limits. Both the piston and housing are assumed to experience only small deformations; thus, variations in piston length associated with its deformation are ignored. This assumption is reasonable for the nitrogen cartridge valves studied in this work because \( \theta \ll 1 \).
To complete the model, it is necessary to estimate $\bar{P}(\xi, t)$ so that the total resistive force can be computed from Eq. (2.23) and the result coupled to the equation of motion for the piston given by Eq. (2.7). The local interface stress is estimated based on a 2-D plane strain analysis which assumes that the stress state within the piston and housing rapidly equilibrates during piston motion. To determine the validity of this assumption, it is necessary to look at the time required for an acoustic wave to travel through the housing thickness. The characteristic time for an elastic wave to traverse the housing of thickness $d_c = 14$ mm is $t_c = 2d_c\sqrt{\rho/E} \approx 3\mu s$ (for $E = 214$ GPa and $\rho = 7800$ kg/m$^3$ which are representative of steel). Because $t_c$ is not significantly less than $t_{op}$, where $t_{op} \approx 90\mu s$ is the characteristic valve operation time, transient effects may be important in accurately describing the mechanical behavior of the valve. However, the equilibrium assumption is used for its simplicity and to capture leading order effects.

The following analysis is largely based on cylindrical pressure vessel theory. Shear stresses ($\tau_{r\theta} = \tau_{r\bar{z}} = \tau_{\theta z} = 0$) are ignored and it is assumed that $\hat{\sigma} = \hat{\sigma}(r; \xi, t)$, where there exists a parametric dependence on $\xi$ and $t$; consequently, the stress state is locally defined by the radial stress $\sigma_r$, the hoop stress $\sigma_\theta$, and the axial stress $\sigma_z$, which are the principal values of $\hat{\sigma}$. Based on these assumptions, the equilibrium form of the angular and axial momentum field equations for the composite piston and housing structure are identically satisfied for fixed $\xi$ and $t$, and the radial momentum equation reduces to

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0. \quad (2.24)$$

It is desired to obtain analytical elasto-plastic solutions to the Boundary Value Problem (BVP) defined by Eq. (2.24) for the generic cross-sectional member illustrated in Fig. 2.2 subject to the surface stresses $\sigma_r(a; \xi, t) = -P_{g_a}(t)$ and $\sigma_r(c; \xi, t) = 0$ and a geometric interference between the piston and housing given by $\delta(\xi, t)$. It is essential to account for plastic deformation as it significantly affects interface stress and enables hardening to
be described. It is later shown in Section 3.2 that plastic deformation is dominant in the piston. The strategy is to first obtain a general solution to Eq. (2.24) for a single annular disc undergoing elasto-plastic deformation that can be used to separately describe the displacement and stress fields within the piston and housing. A particular solution for the composite member is then obtained by imposing $\sigma_r(a; \xi, t) = -P_g(t)$ at the internal piston surface, $\sigma_r(c; \xi, t) = 0$ at the external housing surface, and $\sigma_r(b; \xi, t) = -\bar{P}$ at the piston-housing interface. The geometrical interference between the piston and housing requires that $u_h(b; \xi, t) - u_p(b; \xi, t) = \delta(\xi, t)$, where $u_p$ and $u_h$ are radial displacements for the piston and housing, respectively. In the following paragraphs, general solutions of this problem are summarized; detailed derivations of these solutions are not provided as they are published elsewhere [22].

Equation (2.24) can be combined with the stress-strain relations for a Hookean elastic solid, and the strain-displacement relations $\epsilon_r = du/dr$ and $\epsilon_\theta = u/r$, where $u$ is the radial displacement field, to obtain the following linear, homogeneous ODE:

$$r^2 \frac{d^2 u}{dr^2} + r \frac{du}{dr} - u = 0.$$  

The general solution of this equation is given by

$$u(r; \xi, t) = \frac{A(\xi, t)}{r} + B(\xi, t) r,$$  

(2.25)

where $A$ and $B$ are integration constants that are parameterized by $\xi$ and $t$. The following stress fields result from this displacement field:

$$\sigma_r(r; \xi, t) = \frac{E}{1 + \nu} \left[ \frac{A(\xi, t)}{r^2} + \frac{B(\xi, t)}{1 - 2\nu} \right],$$  

(2.26)

$$\sigma_\theta(r; \xi, t) = \frac{E}{1 + \nu} \left[ \frac{A(\xi, t)}{r^2} + \frac{B(\xi, t)}{1 - 2\nu} \right].$$  

(2.27)
\[ \sigma_z(\xi, t) = \frac{2\nu EB(\xi, t)}{(1 + \nu)(1 - 2\nu)}, \]  

(2.28)

where \( E \) and \( \nu \) are Young’s modulus and Poisson’s ratio, respectively. It is noted that \( \sigma_z \) is constant for fixed \( \xi \) and \( t \) in this case.

A general plastic solution to Eq. (2.24) is obtained based on Tresca’s yield criterion with linear strain hardening. To this end, total strain is partitioned into elastic and plastic components. The yield criterion is given by \( \sigma_1 - \sigma_{III} = \sigma_y(|\varepsilon^p_r|) \), where

\[ \sigma_y(|\varepsilon^p_r|) = \sigma_0 (1 + \eta |\varepsilon^p_r|). \]

Here, \( \sigma_1 \) and \( \sigma_{III} \) are the maximum and minimum principal stresses, and \( \sigma_y \) is the material yield strength in simple tension that increases with the magnitude of the plastic radial strain \( \varepsilon^p_r \). The linear hardening parameter is \( \eta \), and the reference yield stress corresponding to \( \eta = 0 \) is \( \sigma_0 \). The magnitude of the plastic radial strain is directly proportional to the equivalent plastic strain, i.e. \( \varepsilon^p_r \propto \varepsilon_{EQ} \), since compatibility of the plastic flow relations associated with the yield criterion requires that \( \varepsilon^p_r = -\varepsilon^p_\theta \). It is important to note that the ordering of principal stresses is important for properly establishing yielding and subsequent plastic flow. Two cases are considered: \( \sigma_r > \sigma_z > \sigma_\theta \) and \( \sigma_\theta > \sigma_z > \sigma_r \).

The first yield criterion considered is Case I): \( \sigma_r > \sigma_z > \sigma_\theta \). For this case, the equilibrium condition of Eq. (2.24), together with the yield criterion, the stress-total strain relations, and the strain-displacement relations, results in the following linear, inhomogeneous ODE for the radial displacement field:

\[ \frac{r^2}{d^2 u}{dr^2} + \frac{r}{dr} \frac{du}{dr} - u = -\frac{2\sigma_0 (1 + \nu)(1 - 2\nu)}{E [1 + H (1 - \nu^2)]} r, \]

where \( H \) is the dimensionless hardening parameter given as \( H \equiv \eta \sigma_0/E \). Its general
solution is given by
\[ u(r; \xi, t) = \frac{A(\xi, t)}{r} + B(\xi, t)r - \frac{\sigma_0 (1 + \nu) (1 - 2\nu) (2 \ln r - 1) r}{2 [1 + H (1 - \nu^2)]}, \] (2.29)

where, again, \( A \) and \( B \) are integration constants. The corresponding stress fields are given by

\[ \sigma_r(r; \xi, t) = -\frac{EH A(\xi, t)}{[2 + H (1 + \nu)] r^2} + \frac{EB(\xi, t)}{(1 + \nu) (1 - 2\nu)} - \frac{\sigma_0 (2 \ln r - 1)}{2 [1 + H (1 - \nu^2)]}, \] (2.30)

\[ \sigma_\theta(r; \xi, t) = \frac{EH A(\xi, t)}{[2 + H (1 + \nu)] r^2} + \frac{EB(\xi, t)}{(1 + \nu) (1 - 2\nu)} - \frac{\sigma_0 (2 \ln r + 1)}{2 [1 + H (1 - \nu^2)]}, \] (2.31)

\[ \sigma_z(r; \xi, t) = \frac{2E\nu B(\xi, t)}{(1 + \nu) (1 - 2\nu)} + \frac{2\sigma_0 \nu \ln r}{1 + H (1 - \nu^2)}. \] (2.32)

In this instance, the axial stress varies with radial position unlike that for purely elastic deformation.

The second yield criterion considered is Case II): \( \sigma_\theta > \sigma_z > \sigma_r \). For this case, the radial displacement field is governed by the ODE

\[ r^2 \frac{d^2 u}{dr^2} + r \frac{du}{dr} - u = \frac{2\sigma_0 (1 + \nu) (1 - 2\nu)}{E [1 + H (1 - \nu^2)]} r, \]

whose general solution is given by
\[ u(r; \xi, t) = \frac{A(\xi, t)}{r} + B(\xi, t)r + \frac{\sigma_0 (1 + \nu) (1 - 2\nu) (2 \ln r - 1) r}{2 [1 + H (1 - \nu^2)]}. \] (2.33)

The corresponding stress fields are

\[ \sigma_r(r; \xi, t) = -\frac{EH A(\xi, t)}{[2 + H (1 + \nu)] r^2} + \frac{EB(\xi, t)}{(1 + \nu) (1 - 2\nu)} + \frac{\sigma_0 (2 \ln r - 1)}{2 [1 + H (1 - \nu^2)]}, \] (2.34)

\[ \sigma_\theta(r; \xi, t) = \frac{EH A(\xi, t)}{[2 + H (1 + \nu)] r^2} + \frac{EB(\xi, t)}{(1 + \nu) (1 - 2\nu)} + \frac{\sigma_0 (2 \ln r + 1)}{2 [1 + H (1 - \nu^2)]}, \] (2.35)
\[
\sigma_z(r; \xi, t) = \frac{2E\nu B(\xi, t)}{(1 + \nu)(1 - 2\nu)} + \frac{2\sigma_0 \mu \ln r}{1 + H (1 - \nu^2)}.
\] (2.36)

Depending on the material properties of the piston and housing, a given loading scenario (i.e., geometric interference \( \delta \) and gas pressure \( P_{g_2} \)) will involve an initially elastic response followed by plastic deformation with strain hardening within the piston and/or housing. It can be easily shown that plastic deformation of the piston and housing initiates at their respective inner surfaces and subsequently flows radially outward until their annuli are fully plastic. Due to the intense gas pressure generated by HMX combustion, and the short valve operation time, it is anticipated that transition to full plasticity will rapidly occur for reasonably thin members such as the piston skirt. To simplify the analysis, the plastic flow rate in the piston is assumed fast compared to to the valve operation time and contained plasticity is ignored; as such, the piston instantaneously becomes fully plastic at a cross-section following the onset of yielding. The housing is generally much thicker than the piston, therefore it is more appropriate to model it as undergoing contained rather than uncontained plasticity. Figure 2.4 shows the cross-section of the piston/housing apparatus where the piston has yielded through its thickness and the housing is undergoing contained plasticity. The plastic radius \( d(\xi, t) \) has formed at the housing inner radius and propagated to a location between its inner and outer radii \( (b < d < c) \).

Figure 2.4: Schematic illustrating contained plasticity in the valve housing.
Equations (2.25)-(2.36) are separately applicable to both the piston and housing. Assuming uncontained plasticity following the onset of yielding in the piston and contained plasticity in the housing, values for the integration constants $A_p\bar{\beta}$, $B_p\beta$, $A_{h\kappa}$, and $B_{h\kappa}$, $A_h\bar{\kappa}$, and $B_h\bar{\kappa}$, and plastic radius $d$, are determined, for fixed $\xi$ and $t$, by simultaneously solving the following coupled nonlinear equations that result from imposing the boundary conditions:

$$
\sigma_{r,p\beta}(a; \xi, t) = -P_{g2}(t), \quad \sigma_{r,p\beta}(b; \xi, t) = \sigma_{r,h\beta}(b; \xi, t),
$$

$$
\sigma_{r,h\kappa}(d; \xi, t) = \sigma_{r,h\beta}(d; \xi, t), \quad \sigma_{r,h\kappa}(c; \xi, t) = 0,
$$

(2.37)

$$
\frac{du_{h\beta}(b; \xi, t) - u_{p\beta}(b; \xi, t)}{\delta(\xi, t)} = \frac{u_{h\kappa}(d; \xi, t) - u_{h\beta}(d; \xi, t)}{0},
$$

$$
\sigma_{\theta,h\kappa}(d; \xi, t) = \sigma_{\theta,h\beta}(d; \xi, t).
$$

This reduced set of nonlinear equations is iteratively solved using a Newton-Raphson numerical technique. Here, subscripts “$p$” and “$h$” denote quantities associated with the piston and housing, and subscripts $\kappa$ and $\beta$ refer to the plastic and elastic regions of the housing, respectively. It is important to note that appropriate expressions for $u$ [Eq. (2.25), (2.29), or (2.33)] and $\sigma_r$ [Eq. (2.26), (2.30), or (2.34)] used with these boundary conditions depends on whether the piston and housing are elastic or plastic and, if plastic, whether $\sigma_r$ is less or greater than $\sigma_{\theta}$. The solution behavior of the coupled nonlinear equations is addressed in Section 2.2.2

Lastly, the assumption of plane strain maintains consistency between the elastic and plastic solutions, unlike the work of Jones, et al.[5, 7] which assumes plane stress for the elastic response and plane strain for the plastic response. It is shown in Section 3.2 that axial stresses are important in describing the stress field within the piston, so the plain strain assumption would seem more representative. Nonetheless, predictions indicate that little difference exists between the plane strain and plane stress response of the system.
2.2.2 Solution Behavior

As discussed in Section 2.2.1, the plastic solution for the stress and displacement fields within both the piston and housing depends on the ordering of the principal stresses. The ordering depends on the sign of the hoop stress in each member; i.e., whether the member yields due to an internal (positive hoop stress) or external (negative hoop stress) pressure. This section will first discuss the logic used for determining this ordering, address the uniqueness of the solution for \( \tilde{P} \), and characterize the special case of \( H_p = H_h = 0 \).

![Diagram of piston and housing with stress fields](image)

Figure 2.5: Illustration of a general loading condition.

Consider a loading state induced only by geometric interference which produces a non-zero stress field within both the piston and housing, as illustrated in Fig. 2.5. The arrows indicate the direction the interface stress is acting in each member. For the purpose of this example, it is assumed that both the piston and housing are plastic. Because the piston has deformed plastically due to an external pressure (negative hoop stress), the appropriate ordering of principal stresses is \( \sigma_r > \sigma_z > \sigma_\theta \). The housing, however, has deformed due to an internal pressure (positive hoop stress), and the appropriate ordering of principal stresses is \( \sigma_\theta > \sigma_z > \sigma_r \). It should be noted that because the housing will always yield due to an internal pressure (positive hoop stress), the ordering of the principal stresses will always be \( \sigma_\theta > \sigma_z > \sigma_r \). In this illustration, the ordering of principal stresses in the piston is easily determined in the absence of any internal gas pressure because it will always yield
due to an *external* pressure (negative hoop stress) due to geometric interference. However, it is more difficult to predict the proper ordering when there is combined internal gas pressure and geometric interference. For example, consider the piston loading condition in Fig. 2.5 with $P_{g2} \neq 0$. Because the sign of the hoop stress cannot be predicted a priori, it is necessary to make an assumption regarding the principal stress ordering.

Figure 2.6 demonstrates the logic for choosing the correct principal stress ordering for a given piston cross-section. The first step in the logic is to assume elastic deformation and determine $\bar{P}$. It is then determined if the piston has yielded at its inner surface. If it has not, the elastic solution is taken as correct and the resistive force contribution from the cross-section is calculated according to Eq. 2.22 before the logic advances to the next cross-section. If it has yielded, it is assumed that $\bar{P} < P_{g2}$, i.e. $\sigma_\theta > \sigma_z > \sigma_r$, and the solution for a fully plastic piston and elastic housing is calculated and stored. Then, the inner surface of the housing is checked for yielding. If it has yielded, it is again assumed that $\bar{P} < P_{g2}$, and the solution for a fully plastic piston and plastic housing is calculated and stored. If it has not yielded, the solution for a plastic piston and elastic housing is stored and the logic continues. The next step is to determine if the assumption regarding the ordering of principal stresses is correct. If the assumption is correct, the resistive force contribution from the cross-section is calculated and the logic advances to the next cross-section. If the assumption is incorrect, it is assumed that $\bar{P} > P_{g2}$, i.e. $\sigma_r > \sigma_z > \sigma_\theta$, and the solution for a plastic piston and elastic housing is determined and stored. Yielding at the inner surface of the housing is then determined. If it has yielded, it is assumed that $\bar{P} > P_{g2}$, and the solution for a plastic piston and plastic housing is calculated and stored. The resistive force contribution is then calculated before the logic continues to the next cross-section. If it has not yielded, the solution for a plastic piston and elastic housing is used to determine the resistive force contribution and the logic advances to the next cross-section.

Implicit in this logic is the assumption that the piston must always yield before the housing. This assumption is reasonable in that the housing is generally much thicker than
Given a piston displacement and local piston coordinate

Assume elastic deformation—determine interface pressure

Does piston yield?

Assume internal gas pressure is greater than interface pressure—determine interface pressure

Does housing yield?

Assume internal gas pressure is greater than interface pressure—determine interface pressure

Is internal gas pressure greater than interface pressure?

Assume internal gas pressure is less than interface pressure—determine interface pressure

Continue
the piston. However, it is possible that a much thicker housing yields before the piston according to the steady solutions presented in Section 2.2. Consider the case of a single piston-housing cross-section subject to a large internal gas pressure and interference such that the internal gas pressure and interface pressure are comparable in magnitude. In this situation, there is nothing to drive the stresses in the piston to the yield point. Even though the internal and interface pressures may be very large in magnitude, it is the difference in the two that causes yielding. Because the external pressure is always zero in the housing, the large interface pressure produces a large stress gradient through its thickness causing the housing to yield. This case is a limitation of the steady solutions used to determine the interface pressure. In reality, the applied internal gas pressure cannot be instantaneously applied to the inner surface of the piston. To illustrate this, consider a piston cross-section subject to a constant interference and an internal gas pressure, and the gas pressure is increased from zero. For small pressures, the piston will yield due to the interference loading according to the steady solution. As the pressure is increased, the magnitude of the interface and internal gas pressure will approach each other reaching a gas pressure where the piston no longer yields according to the steady solution. Even though the piston plastically deformed at lower gas pressures, the steady solutions give an elastic solution for the piston at larger gas pressures. The steady solutions do not account for strain history inherent in plasticity. It can be shown that this behavior occurs only for small piston displacements and as such is tolerated in the implementation of the deformation model.

Figure 2.7 graphically summarizes the different solutions for $P$ discussed in the flowchart. Solutions 2 and 3 correspond to $\hat{P} < P_{g2}$, where solution 2 is for an elastic housing and plastic piston, and solution 3 is for a plastic piston and plastic housing. Solutions 4 and 5 correspond to $\hat{P} > P_{g2}$, where solution 4 is for an elastic housing and plastic piston, and solution 5 is for a plastic piston and plastic housing. The solutions track a piston cross-section as it is axially displaced into the bore. and are for a constant $P_{g2}$ (100 MPa). Notice there is a small region where the solution for $\hat{P}$ is not unique, indicated by the
shaded region in Fig. 2.7. In this region, solutions 2,3,4, and 5 all satisfy their assumptions regarding the ordering of principal stresses, i.e. $\tilde{P} > P_{g2}$ or $\tilde{P} < P_{g2}$. Because solutions 2 and 3 are only satisfied in the shaded region, whereas solutions 3 and 4 are not, it is assumed that solutions 3 and 4 are the proper solutions within the shaded region. To avoid choosing the wrong solution in the implementation of the deformation model, an algorithm is used to maintain continuity in $\tilde{P}$ as the cross-section moves into the bore.

- **Special Case of $H_p = H_h = 0.0$: Uncontained Plasticity**

  Consider the special case where both the piston and housing have deformed plastically with no hardening such that each is fully plastic. This section will show there is no mathematical solution for such a case. First, fully plastic deformation of a single cross-section with no hardening is analyzed, followed by a coupled piston/housing cross-section.

  Figure 2.8 shows a cross-section subject to an internal pressure $P_i$ at $r = r_i$ and external pressure $P_o = 0$ at $r = r_o$. The cross-section is assumed fully plastic with zero hardening. The equilibrium equation in the radial direction is given by Eq. 2.24. The Tresca yield criterion for the cross-section is given as $\sigma_\theta - \sigma_r = \sigma_y$, where $\sigma_y$ is the material yield
strength. The yield criterion can be substituted into the equilibrium equation and directly integrated to give the solution for the radial stress as:

$$\sigma_r(r) = \sigma_y \ln(r) + c,$$  \hspace{1cm} (2.38)

where $c$ is a constant of integration. This solution is different from a cross-section with nonzero hardening in that there is only one integration constant (See for example Eq. 2.34). Mathematically, this means because the radial stress distribution in the cross-section is fixed by imposing only one boundary condition, i.e. $\sigma_r = -P_i$ at $r = r_i$ or $\sigma_r = -P_o$ at $r = r_o$, both boundary conditions cannot be satisfied and thus there is no solution. Physically, this means once the cross-section has yielded plastically through its thickness without hardening, it has no strength to resist further strain and will continue to deform until rupture.

Consider now a piston/housing cross-section such that both are fully plastic with zero hardening subject to $\sigma_r = 0$ at $r = a(\zeta, t)$, $\sigma_r = 0$ at $r = c(\zeta, t)$, and $\sigma_{r,p} = \sigma_{r,h}$ at $r = b(\zeta, t)$, as illustrated in Fig. 2.5. The radial stress distribution in each cross-section is given by Eq. 2.38 and fixed by imposing only two boundary conditions (one for each cross-section). As such, the three boundary conditions listed above cannot be satisfied so there is no solution. In the event of this special case, it is suggested that a small hardening parameter be imposed to obtain a steady solution.
Chapter 3
Inert Quasistatic Analysis

Section 2.2.1 outlined a method to predict the piston axial resistive force, $F_R$. Certain assumptions were made in the derivation, such as contained or uncontained plasticity in both the piston and housing. This chapter first describes quasistatic compression tests performed to quantify the energy requirements of the nitrogen cartridge valve. Next, results from a finite element study are given to validate assumptions made in the simple deformation model and to gain insight into valve deformation and stress fields. Results are given that compare simple model predictions for $F_R$ with those given by quasistatic compression tests and finite element analyses.

3.1 Quasistatic Compression Tests

An important question to designers of explosively actuated devices is this: How much energy is required for successful device operation? To partially answer this question, inert quasistatic compression tests were performed using an MTS machine having a 88,960 N (20,000 lbf) load cell. Figure 3.1 shows the experimental apparatus where the actuator has been removed from the valve. The valve was firmly attached to a support frame and placed onto the lower fixed platen of the MTS machine. A specially designed cylindrical ram made of hardened tool steel was vertically inserted into the valve through the actuator port until it firmly rested on top of the piston. The upper platen of the MTS machine was carefully lowered until it was flush with the top surface of the ram; it was then lowered at a constant extension rate of 2.54 mm/min, until the piston contacted the stops, while the applied axial force and displacement were simultaneously recorded. Six tests were performed, although two of these tests were prematurely terminated due to a support frame malfunction.

It must be made clear that these inert, quasistatic tests are much different than an explosive actuation. While the timescale for explosive actuation is on the order of microseconds, the MTS tests actuated the valve in minutes. Any strain rate effects present in
an explosive actuation are not reproducible in the MTS tests. It has been shown in some typical steels that ultimate strength can increase as much as 10% at high strain rates [20]. Differences in loading conditions and resulting stress states also separate explosive from quasistatic actuation. During an explosive actuation, the piston skirt is loaded with hydrostatic pressure, as illustrated in Fig. 3.2 (a). In the MTS tests, the valve is loaded with a concentrated force where the ram contacts the piston, shown in Fig. 3.2 (b). Although the MTS tests do not replicate explosive actuation, they were performed for their simplicity and their ability to give quantitative data on the energy requirements of the valve.

Figure 3.3 summarizes results of the inert quasistatic compression tests performed on six, new Nitrogen Cartridge Valves. As seen in the figure, there is initially no resistive force due to zero interference between the piston and housing. As the ram pushes the piston into the bore, interference develops which increases the resistive force. The resistive force continues to rise and reaches a peak ($\approx 12$ kN) before the piston is entirely displaced into the bore. After this peak force is reached, the force decreases as the piston is further
displaced into the bore. The small kink that is visible near the approximate location where
the skirt is first completely displaced into the bore (2.54 mm), and during the initial loading
(5 kN), is attributable to the support frame, as discussed below. As the piston comes in
contact with the diaphragm, there is a gradual increase in resistive force as the diaphragm
starts to deform. Once the diaphragm is punctured, the resistive force decreases. For
the quasistatic tests, there is a bulging out of the diaphragm before it is ruptured as the
puncturing process takes place over a distance of approximately 0.75 mm. Otherwise, the
resistive force is relatively constant after the piston is displaced into the bore. The resistive
force decreases rapidly leaving little residual force after the piston is fully displaced into
the bore. This phenomena is consistent with extrusion type processes in which there are
large deformations. The housing corner appears to induce large bending on the piston such
that there is little resistance once the piston reaches the bore. This behavior suggests that
using a smaller friction coefficient in the bore region may be appropriate. These qualitative
trends were observed for all valves tested, but there were quantitative variations in the
measured force-displacement profiles. It is not apparent what caused these differences,
however, slight differences in valve geometry or eccentric valve loading are possible causes.

Table 3.1 shows the contributions of work required to insert the piston into the bore,
push the piston down the bore, and puncture the diaphragm, as a percentage of the total
work required to function the valve. Only valves 3-6 are shown due to incomplete data sets taken from valves 1 and 2. The work requirements were calculated by numerically integrating the experimental force-displacement data.

The work required to insert the piston was computed by integrating the data from the initial piston location to the location where it was first entirely in the bore. The work required for bore travel was computed by integrating the data from the location where the piston was entirely displaced in the bore to the location where the piston hit the stops. The work required to puncture the diaphragm was computed by integrating the data from the location where the resistive force starts to rise due to piston-diaphragm contact to the location where the force levels off after puncture. It is apparent that the work needed to insert the piston into the bore is the largest fraction of total valve work. Diaphragm puncture plays a minimal role in the valve work requirements, accounting for less than 16 percent of the work requirements in any of the valves. The average work required to
puncture the diaphragm was approximately 2.1 J, corresponding to an equivalent piston velocity of 23 m/s based on kinetic energy.

<table>
<thead>
<tr>
<th>Valve</th>
<th>Total Work (J)</th>
<th>Skirt Insertion (Percentage)</th>
<th>Bore Travel (Percentage)</th>
<th>Puncture (Percentage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>34.56</td>
<td>54.07</td>
<td>45.93</td>
<td>7.79</td>
</tr>
<tr>
<td>4</td>
<td>28.76</td>
<td>51.00</td>
<td>49.00</td>
<td>10.50</td>
</tr>
<tr>
<td>5</td>
<td>9.82</td>
<td>61.57</td>
<td>38.43</td>
<td>15.81</td>
</tr>
<tr>
<td>6</td>
<td>23.45</td>
<td>79.76</td>
<td>20.24</td>
<td>4.33</td>
</tr>
</tbody>
</table>

Figure 3.4: Experimental apparatus with push disc used in compliance tests.

A second MTS experiment was performed to quantify the effect of support plate compliance on the experimental data shown in Fig. 3.3. Figure 3.4 shows the experimental apparatus used for the compliance tests. The apparatus is very similar to the previous test, except that there is a push disc rather than a ram. The apparatus was clamped onto the MTS lower platen, and the upper platen was slowly lowered until it was flush with the push disc. The upper platen was then lowered at a constant extension rate of 2.54 mm/min while the applied axial force and displacement were recorded. The experiment was terminated when the applied force reached approximately 20 kN. This force value was chosen to
be slightly larger than the maximum force observed in Fig. 3.3. Figure 3.5 (a) shows the raw displacement-force data taken for the seven tests. The curves are quasilinear except for a small kink around 5 kN which is consistent with the kinks seen in Fig. 3.3. This kink, therefore, results from reversible loading of the valve support hardware, possibly the screws holding the valve to the valve plates. All tests displayed similar behavior. Figure 3.5 (b) shows a linear curve fit to the compliance experimental data. The curve fit was constrained to have a zero y-intercept and the slope of the fit is 0.046 mm/kN. The experimental data presented in Fig. 3.3 was subsequently adjusted to account for the compliance of the support plates.

Figure 3.5: (a) Compliance test raw data. (b) Linear fit.

Figure 3.6 compares an experimental result with a prediction for the axial resistive force given by the simple model. To this end, the force-displacement profile for Valve 3 was chosen as representative for all valves tested. The predicted net axial resistive force is shown, along with the contributions from the skirt and bore regions. A coefficient of friction value $\mu_s = 0.9$ was used for the skirt region, and a value of $\mu_b = 0.2$ was used for the bore region. This value of $\mu_s$ was chosen to approximately match the peak experimental resistive force, whereas the value of $\mu_b$ was chosen to approximately match the residual resistive force once the piston was fully displaced into the bore. The raw data presented in Fig. 3.3 suggests that there is little residual force in the bore, thus a smaller friction coefficient in this region is appropriate. The use of different values for $\mu_s$ and $\mu_b$
implicitly accounts for complex stress fields not described by the simple model. Again, this is consistent with extrusion processes which involve sharp decreases in the resistive force across corners. The predicted resistive force is initially stiffer than the experimental results, even when the experimental data is adjusted for compliance in the valve support plates. There is a qualitative difference between the experimental and predicted result in that the experimental result is more curved while the predicted result is very sharp. This difference may be the result of a combination of factors, namely the deviation of actual plastic material behavior from the linear strain hardening model and the existence of large bending stresses not accounted for in the deformation model.

![Figure 3.6: Comparison of MTS results with simple model predictions.](image)

### 3.2 Finite Element Analysis

In Section 2.2.1, a method was presented for determining the resistive force the piston is subjected to as it travels down the bore. The method consisted of developing 2-D equilibrium solutions for a piston/housing cross-section to determine the interface pressure \( \tilde{P} \). In the derivation, certain assumptions were made, including the ordering of the principal stresses during plastic deformation, a 2-D state of stress (plane strain), uncontained plas-
ticity in the piston, and contained plasticity in the housing. To determine the validity of these assumptions and gain insight into the deformation behavior of the valve, a finite element study was done using the commercial FEA package ANSYS. First, simulation details are discussed including geometry and boundary conditions. Next, results are given that compare the ANSYS simulations to the simple model predictions and experimental MTS results. Last are some brief conclusions.

- **ANSYS Model Details**

  A quasistatic finite element study was done to investigate deformation within the Nitrogen Cartridge Valve during piston insertion. Three cases were investigated, one with a friction coefficient between the piston and housing of 0.0, a second with a coefficient of 0.1, and the last with a friction coefficient of 0.3. Computer memory constraints prevented cases with higher friction coefficients from being investigated. Figure 3.7 shows the axisymmetric geometry and boundary conditions for the simulations. To replicate the MTS experiments as closely as possible, the displacement boundary condition was applied on...
the same surface that the ram was applied, and the top of the housing was modeled as fixed, simulating the screws holding the valve in place during the experiments. The valve centerline is the axis of symmetry. All other surfaces are stress free initially. The piston was given a displacement, and the resulting steady solution determined. Data was then extracted using ANSYS postprocessing. This process was repeated until the piston was displaced into the bore. Figure 3.8 illustrates the piston being displaced into the valve bore. For each piston displacement, the image is zoomed in to the upper valve housing to illustrate the piston moving into the bore. Ten simulations were used to replicate piston insertion although only six are shown in Fig. 3.8. All pertinent data (interface stress, net axial force, etc.) was then extracted using ANSYS postprocessing. This was repeated until the piston was completely inserted into the bore. Elastic-plastic material models, including linear strain hardening, were used in describing the mechanical behavior of both the piston and housing. Table 3.2 gives the material data used in the simulations. This material data was chosen to approximately match tensile test data for both the piston and housing taken by the Los Alamos National Laboratory (LANL) ESA-GTS/WR Material Properties Testing Laboratory. A penalty based method was used to model contact between the piston and housing.

Figure 3.8: Deformed simulation geometries for varying piston displacements.
Table 3.2: Material data used in ANSYS simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Piston (SS 17-4PH)</th>
<th>Housing (304L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus</td>
<td>200.0 GPa</td>
<td>214.0 GPa</td>
</tr>
<tr>
<td>Yield Stress</td>
<td>758.0 MPa</td>
<td>275.0 MPa</td>
</tr>
<tr>
<td>Plastic Modulus</td>
<td>0.0 GPa</td>
<td>74.9 GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Figure 3.9: Comparison of predicted resistive force with ANSYS simulations.

Figure 3.9 compares the predicted axial resistive force with the ANSYS and experimental results. The ANSYS results are qualitatively similar in that the force initially rises and reaches a maximum before approaching a constant value as the piston is completely displaced into the bore. The stiffness of the initial force-displacement response increases with friction coefficient. There is good agreement between the predicted and ANSYS results for both qualitative trend, and approximate location of the maximum force. It is expected that if the coefficient of friction used in the ANSYS study was increased, then the maximum force would more closely approximate the experimental maximum force. The ANSYS results display similar qualitative trends as the experimental results, in that the
force increases, reaches a maximum, then decreases to a constant value as the piston is displaced into the bore. However, both the predicted and ANSYS force profiles are not as broad as the experimentally determined profile. One of the possible reasons for this is that the linear strain hardening model may not be representing the true material behaviors of the piston and housing. Another reason is that there may be some compliance in the MTS machine not considered. In the remainder of this section, results are presented from the ANSYS study for a coefficient of friction of 0.3 since it more closely represents the experimental data.

![Von Mises stress contours](image)

Figure 3.10: Von Mises stress contours for a piston displacement of 25 % into the bore.

Figure 3.10 shows the Von Mises stress for the piston/housing geometry for a 25% piston insertion to illustrate plasticity in the piston and housing. For this piston displacement, the Von Mises stress is equal to the yield stress everywhere in the piston skirt, indicating the entire skirt has yielded. This Von Mises stress in the piston is not allowed to increase due to the perfectly plastic material model. Plastic deformation has propagated through the piston thickness for even this small insertion distance. This suggests that the uncontained plasticity model for the piston presented in Section 2.2 is appropriate. Plastic deformation in the housing, however, is localized near the housing corner. This suggests that the contained plasticity model for the housing presented in Section 2.2 is appropriate.
Figure 3.11: Comparison of predicted interface stress distribution with ANSYS results for 25% piston insertion.

Figure 3.12: Radial stress contours for 25% insertion.

One way to determine the validity of the deformation model presented in Section 2.2.1 is to compare the predicted interface stress with the results from the finite element study. Figure 3.11 shows the predicted and finite element interface stress along the piston skirt for a
piston insertion of 25%. Figure 3.12 illustrates the radial stress contours for the insertion and gives the deformed geometry. The radial and axial directions shown in Fig. 3.12 correspond to the direction of the radial and axial stresses shown in Fig. 3.11. Notice that the normal stress at the piston-housing interface is not necessarily in the radial direction depicted in Fig. 3.12 due to inclination of the housing. The ordering of the predicted piston stresses in Fig. 3.11 is consistent with the model in Section 2.2.1, \( \sigma_r > \sigma_z > \sigma_\theta \), for no internal gas pressure. There is little change in the predicted stresses along the interface, due to both a perfectly plastic piston and small inclination angle. Looking at the ANSYS results, the stress magnitudes significantly increase in the vicinity of the valve corner. This suggests that the housing corner is inducing significant bending stresses within the piston. The simple model does not predict bending stresses, but the integrated effect of the corner stresses may not be large. Thus, the simple model gives reasonable net axial resistive force predictions, even though complex bending stresses are not described.

To determine contact between the piston and housing, it is helpful to look more closely at the radial stress at the piston-housing interface. Figure 3.13 shows the deformed geometry and radial stress distribution for different piston insertion depths. One trend common for all insertion depths is that the radial stress is concentrated at a position corresponding to the housing corner. In fact, the piston is in contact only in this concentrated region around the housing corner. The piston loses contact as the radial stresses shown in Fig. 3.13 go to zero. The radial stresses do not go exactly to zero because of the orientation problem previously discussed. It was verified that stresses normal to the interface vanish where contact is lost.

Figure 3.14 shows the results of a convergence study done for the ANSYS simulations. The figure shows the resistive force vs. nondimensional element length scale for a piston insertion depth of 12.5%. The nondimensional element length scale is defined as \( l_c \), where \( l_c \) is the element length scale of the coarse mesh. Because of computational constraints, larger nondimensional lengthscales could not be analyzed. However, it does appear from
Fig. 3.14 that the solution is converging. All of the simulation results presented in this section were based on a grid size corresponding to a nondimensional element lengthscale of 4. Although there is some error associated with using this grid size, the solution appears to be close to the actual solution. The convergence rate is approximately quadratic if it is assumed that the solution corresponding to an element nondimensional lengthscale of 5 is the true solution.
Figure 3.13: Deformed geometry and radial interfacial stress.
Figure 3.14: Convergence results from ANSYS simulations.
Chapter 4
Results

Results are given in this chapter that describe explosive actuation of the Nitrogen Cartridge Valve. Three model parameters must be first fitted against experiments before a comprehensive parametric study can be performed. The burn rate must be correlated with closed bomb experiments to predict the combustion timescale. The friction coefficient in the skirt and bore regions must also be correlated with reactive valve experiments to predict the valve operation timescale. These coefficients are difficult to directly measure due to the complex state of stress in the piston during actuation. After a small set of experiments are conducted to validate these three parameters, an extensive parametric study can then be performed. In Section 4.1, results from closed bomb tests used to determine HMX burn rate are presented. Next, Section 4.2 characterizes the baseline operation of the Nitrogen Cartridge Valve. Results from a parametric study are then presented in Section 4.3 first from model parameters difficult to measure, then from model parameters important from a design perspective. All reactive experiments were conducted at LANL [19].

4.1 Explosive Burn Rate Characterization

The explosive burn rate must be correlated with closed bomb data to reasonably estimate combustion time. For the closed bomb experiments, an actuator containing 150 mg of HMX was fired into a 1 cc volume, and pressure-time data recorded. Figure 4.1 shows the representative geometry of the closed bomb experiments. The experimental apparatus consists of the actuator where the solid explosive is housed, the 1cc closed bomb volume, and the pressure transducers. When the explosive starts burning, it produces high pressure combustion gases which then move into the 1cc volume, while pressure-time data is simultaneously recorded.

Figure 4.2 shows the pressure-time history of both the closed bomb experiment and prediction. The burn rate coefficients \((b, n)\) used in the simulation replicating the closed
bomb test were taken from the literature [13], and the number of burning grains, \( N \), was adjusted to match the experimental timescale of combustion. This is a reasonable approach in that the number of burning grains of HMX is difficult to predict. The number of grains used in the simulation was 3000, corresponding to a burning grain size of approximately 400 \( \mu m \) diameter, compared to a typical HMX grain size of approximately 50 \( \mu m \). There exist oscillations in the closed bomb data, possibly due to either burn instabilities or reflected pressure waves within the closed bomb. The simulation prediction for peak pressure (\( \approx 200 \) MPa) overshoots the experimental peak pressure within the closed bomb (\( \approx 150 \) MPa). However, matching the magnitude of the peak pressure in the closed bomb tests was not as critical as matching the combustion timescale, because there is some doubt regarding the pressure measurement in the closed bomb tests.

![Figure 4.1: Representative geometry of closed bomb experiments.](image)

For the data shown in Fig. 4.2, the pressure was recorded using a transducer having a side wall orientation, where the transducer is normal to the axis of the closed bomb. Further experiments have shown that the measured pressure can be as much as 30% higher if the pressure transducers are oriented in the end wall position, with the transducer aligned with the flow of combustion gases into the 1 cc volume. In addition to the uncertainty regarding the pressure measurement, it is also not clear whether the entire mass of propellant was
Figure 4.2: Determination of burn rate from 1 cc closed bomb experiments.

burned in the closed bomb simulations. For these reasons, it was not as critical that the predictions for peak pressure in the closed bomb tests agree with the experimental result.

4.2 Baseline Valve Operation

This section characterizes the operation of the nitrogen valve for the baseline model parameters. A key goal of the baseline study is to establish that key experimental quantities can be predicted with the adjustment of a minimal number of parameters (rb, µs, and µb). The baseline case is matched to experimental data by the adjustment of these three parameters. After the baseline valve performance is characterized, a parametric study is performed to determine sensitivity to model parameters. Baseline parameters used in the simulations are listed in Table 4.1, where r_ho is the outer radius of the housing, r_hi is the inner radius of the housing in the constant radius bore region, tk is the piston skirt thickness. The parameters \( \hat{L}_p, t_k, r_{ho}, r_{hi}, \) and \( \theta \) define the valve geometry. The coefficients of friction were chosen to approximately predict valve operation timescales. Whereas larger coefficients of friction were used to replicate the MTS tests, it has been shown by Jones [7], et al, that smaller friction coefficients are more representative of the more dynamic valve actuation. A stiff ODE solver contained in the package LSODE is used to numerically integrate Eqs. (2.19). The integration domain given by Eq. (2.23) was numerically divided
into 20 equally spaced intervals, and were numerically integrated using the trapezoidal rule. It can be shown the results are insensitive to increasing the number of intervals beyond 20. The domain in the skirt region is decreasing as the piston moves into the bore region, while the domain in the bore region is increasing as the entire piston moves into the bore. There could exist numerical errors due to the possibility of the domain size becoming the same order of magnitude as the interval size. As the piston is fully displaced into the bore, the domain size of the skirt region approaches the interval size and could possibly become smaller than the interval size. However, the numerical error associated with this is likely small because the resistive force from the bore region is dominating the net resistive force at large piston displacements. The nonlinear algebraic system defined by Eq. (2.37) was solved using a standard Newton-Raphson method, utilizing the LAPACK linear equation solver DGESVX. The numerical implementation elastic-plastic model was validated against the predictions given in Ref. [22].

All simulations were performed on a Linux workstation having an INTEL Pentium IV, 1.3 GHz processor with 528 Mb RAM. The average computational run time for a simulation was approximately 45 minutes.

![Graphs](image)

Figure 4.3: (a) Pressure histories of actuator and expansion chamber. (b) Predicted and experimental expansion chamber pressure.
Table 4.1: Baseline parameter values used for the valve simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
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<td>$A_1$</td>
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<td>cm$^2$</td>
<td>$E_h$</td>
<td>214</td>
<td>GPa</td>
</tr>
<tr>
<td>$A_e$</td>
<td>0.112</td>
<td>cm$^2$</td>
<td>$E_p$</td>
<td>200</td>
<td>GPa</td>
</tr>
<tr>
<td>$b$</td>
<td>$4.638 \times 10^{-6}$</td>
<td>(dyne/cm$^2$)$^{-0.8}$cm/s</td>
<td>$H_h$</td>
<td>0.35</td>
<td>—</td>
</tr>
<tr>
<td>$h$</td>
<td>$1.25 \times 10^6$</td>
<td>g/s$^3$/K</td>
<td>$H_p$</td>
<td>0.0</td>
<td>—</td>
</tr>
<tr>
<td>$m_p$</td>
<td>8.0</td>
<td>g</td>
<td>$\sigma_{h0}$</td>
<td>344.7</td>
<td>MPa</td>
</tr>
<tr>
<td>$n$</td>
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<td>$\sigma_{p0}$</td>
<td>1089</td>
<td>MPa</td>
</tr>
<tr>
<td>$N$</td>
<td>3000</td>
<td>grains</td>
<td>$\nu_h$</td>
<td>0.3</td>
<td>—</td>
</tr>
<tr>
<td>$m_{s0}$</td>
<td>150.0</td>
<td>mg</td>
<td>$\nu_p$</td>
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<td>—</td>
</tr>
<tr>
<td>$P_{crit}$</td>
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<td>MPa</td>
<td>$\mu_s$</td>
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<td>—</td>
</tr>
<tr>
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<td>K</td>
<td>$\mu_b$</td>
<td>0.088</td>
<td>—</td>
</tr>
<tr>
<td>$T_w$</td>
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<td>K</td>
<td>$\tilde{L}_p$</td>
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<td>mm</td>
</tr>
<tr>
<td>$V_1$</td>
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<td>mm</td>
</tr>
<tr>
<td>$V_{20}$</td>
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<td>cm$^3$</td>
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<td>12.7</td>
<td>mm</td>
</tr>
<tr>
<td>$V_{s0}$</td>
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<td>cm$^3$</td>
<td>$r_{hi}$</td>
<td>5.969</td>
<td>mm</td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>$\theta$</td>
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<td></td>
</tr>
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<td>$\epsilon$</td>
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<td>—</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{g1o}$</td>
<td>$6.202 \times 10^{-6}$</td>
<td>g/cm$^3$</td>
<td>$\rho_s$</td>
<td>1.91</td>
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<td>1.91</td>
<td>g/cm$^3$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.3 (a) illustrates the pressure in both the actuator and expansion chamber for the baseline case. The pressure in the actuator builds up rapidly and reaches a maximum of approximately 1430 MPa. Meanwhile, the expansion chamber pressure increases more slowly due to choked flow until it equilibrates with the actuator pressure at approximately 35 $\mu$s. The pressures decrease due to the combined effect of volume expansion and heat transfer to the surroundings. Fig. 4.3 (b) shows the predicted and experimental expansion chamber pressure. There is no experimental data available for the actuator pressure history. The predicted expansion chamber pressure increases as propellant burns and explosive gases move into the expansion chamber and reaches a maximum of approximately 235 MPa near 25 $\mu$s. Then the pressure decreases due to volume expansion and heat transfer to the surroundings. Both the prediction and experimental data exhibit the same trends, however, the predicted pressure is larger than the experimental pressure. For the same argument
presented in Section 4.1, predicting the experimental trend and timescale is more critical than the *magnitude*. Of course if the difference in the predicted and experimental pressure magnitudes is large there will exist errors.

![Graph showing piston velocity over time](image)

**Figure 4.4: Comparison of VISAR velocity measurements with predicted values.**

Figure 4.4 shows the predicted and experimental piston velocity profiles. A VISAR (Velocity Interferometer System for Any Reflector) system was used to record the piston velocity. The experimental valve used for the velocity measurements was similar to an actual valve, except a bore tube was used to simulate the housing. The difference in the bore tube and the actual valve housing is there are no piston stops or nitrogen reservoir in the bore tube, and the volume below the piston is open to atmospheric conditions. This volume must be open to allow the VISAR laser optical access to the piston bottom. The predicted velocity is both quantitatively and qualitatively similar to the experimental results. Both reach a velocity of approximately 200 m/s as the piston exits the bore tube. The time to reach stroke distance if the piston stops were contained in the bore tube is

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approximately 90µs, with a velocity near 150 m/s. Recall from the MTS experiments in Section 3.1 that the diaphragm requires approximately 23 m/s for puncture, suggesting that the piston is significantly overdriven by the 150 mg of HMX.

Figure 4.5 illustrates the forces acting on the piston for the baseline simulation. The pressure force from the combustion gas combined with the deformation resistive force produces a net force acting on the piston. The pressure force is directly proportional to the magnitude of the pressure within the expansion chamber and follows the same trend. The resistive force follows a similar trend to those shown in Section 3.2. It increases and reaches a maximum of approximately 17.5 kN before decreasing as the piston is fully inserted into the bore. After this point, the resistive force is relatively constant because there is no further interference, however, it does slightly decrease due to a decreasing expansion chamber pressure. For this case the pressure force is always larger than the resistive force, producing a positive net force everywhere. After 60µs, the net force is decreasing until the stroke is reached. This can be interpreted as a decrease in piston acceleration, or a decrease in the slope of the velocity-time data. Looking back at Fig. 4.4, the slope of the predicted velocity-time data is in fact decreasing after 60µs.

It is necessary to analyze the forces acting on the piston when the diaphragm is punctured to determine if the piston will be held in place. It is undesirable for the piston to become dislodged and rebound after having punctured the diaphragm. When the diaphragm is punctured, the piston has a force exerted on it due to the combustion gases driving it into the bore, a nitrogen gas pressure force on the bottom side of the piston, and a deformation resistive force opposing piston motion. The nitrogen reservoir is at approximately 6.9 MPa, and the piston area it is applied to is approximately 6.43x10⁻⁶ m², producing a force of approximately 44 N. It is clear from Fig. 4.3 (b) that the expansion chamber pressure at piston stroke (≈ 120 MPa, exerting a force of ≈ 13 kN on the piston) is much larger than the pressure of the nitrogen gas in the reservoir (≈ 6.9 MPa), and thus the piston should be held in place. This equilibrium analysis of the forces acting on the
piston does not consider any transient vibration of the piston after it contacts the piston stops. Although there may be some vibration, the piston will not become dislodged due to the forces holding it in place. The worst case scenario would be the case of no combustion gas pressure acting on the piston. In this situation, only the deformation resistive force holds the piston in place, opposing the nitrogen reservoir pressure. Without any combustion gas pressure, the residual piston deformation force holding the piston in place at stroke is approximately 1 kN, more than enough to oppose the force from the nitrogen reservoir (≈ 44 N).

Figure 4.5: Piston motion dynamics for baseline case.

It is convenient to look at the deformation model uncoupled from the combustion model to determine at what internal pressure the piston and housing yield. The following discussion uses results from the uncoupled deformation model imposing a constant internal gas pressure for the entire motion history of the piston to determine what pressure the piston and housing yield, and the extent of plasticity within the housing. Results are shown for the first discretized cross-section in the skirt region, corresponding to the top of
the piston skirt. The Tresca stresses are computed at the inner radius of the piston and the inner radius of the housing, since that is the location of the maximum Tresca stress in each member.

Figure 4.6 illustrates the nondimensional Tresca stress in both the piston and housing for zero internal gas pressure as a function of piston displacement. The Tresca stress is scaled by the material initial yield stress such that the material begins to yield as the nondimensional Tresca stress goes to one. Both the piston and housing are initially elastic. The interference is increased as the piston is displaced causing the Tresca stress to increase in both the piston and housing. The piston yields near 0.25 mm, and remains on the yield surface. Once the piston yields, the Tresca stress in the housing remains relatively constant with small variation due to an increasing interference and geometry change. The piston does not harden and thus the interface pressure is fixed, causing the Tresca stress in the housing to be relatively constant.

![Nondimensional Tresca Stress](image)

Figure 4.6: Tresca stresses in both the piston and housing for zero internal pressure.

Figure 4.7 shows the Tresca stress in the housing for different internal pressures to determine what pressure the housing first yields. The trends are the same in that the Tresca stresses increase and reach a maximum, then remain relatively constant. The Tresca
stress in the housing is relatively constant because the piston does not harden. For internal pressures of 0 and 5 MPa, the housing remains elastic for the entire piston motion history. The housing begins to yield at approximately 15 MPa.

Figure 4.8 shows the nondimensional housing plastic radius as a function of piston displacement for various internal pressures in order to determine the extent of plastic deformation. The nondimensional plastic radius goes from zero at the inner radius of the housing to one at the outer radius. The plastic radius is initially zero signifying zero plasticity, instantaneously increases near 0.25 mm, then slightly decreases due to geometry changes. The housing thickness increases as the piston moves into the bore due to the piston taper and thus strengthens, causing the plastic radius to decrease with piston displacement. While this strengthening effect may be physical, the model does not account for plastic history important in describing actual material behavior for this unloading event. Plasticity in the housing appears to go uncontained near 200 MPa, signifying the plastic radius has propagated through the housing thickness. For the baseline case, the max-
imum pressure in the expansion chamber is approximately 235 MPa, suggesting a fully plastic housing. However, the model predictions for maximum expansion chamber pressure overshoots the experimental peak pressures (≈ 175 MPa). If the experimental expansion chamber pressures are representative, the housing would not yield through its thickness according to the deformation model, as shown in Figure 4.8.

### 4.3 Parametric Analysis

One of the key capabilities of the comprehensive model presented in this work is the ability to perform quick parametric analyses. The goal of this section is to analyze the sensitivity of the model to various valve parameters. This section is comprised of two main parts. The first part contains a parametric study on the model parameters with the most uncertainty associated with them. In this section sensitivity to the friction coefficient, number of burning grains, and propellant burn rate is investigated. Following will be the results of a parametric study for parameters important from a design perspective, including propellant mass, piston and housing thickness, and skirt angle.
4.3.1 Parameters with Significant Uncertainty

- Friction Coefficient

The friction coefficient is a value difficult to measure, particularly when such a complex state of stress exists in the piston. To determine the model sensitivity on the friction coefficient, a parametric study was performed. Results are given illustrating valve performance for varying friction coefficients.

Figure 4.9: (a) Piston velocity profiles. (b) Piston energy.

Figure 4.9 (a) shows piston velocity profile for different skirt friction coefficients, ranging from 0.0 to 1.0. The corresponding coefficients of the bore are scaled such that the ratio between the skirt and bore coefficient is the same as the baseline case ($\frac{\mu_s}{\mu_b} = 4.5$). The case of a zero friction coefficient yields the largest piston stroke velocity ($\approx 170$ m/s) and the fastest operation time ($\approx 73 \mu$s). The absence of friction enables the mechanical energy that would be dissipated by friction to go into piston kinetic energy, yielding a larger stroke velocity. Likewise, there is no frictional force retarding the motion of the piston, yielding faster stroke times. Notice that for $\mu_s = 0.8$ and 1.0, the velocity trend is different in that there is a local maximum near 30$\mu$s. This maximum is directly related to the forces acting on the piston, both the gas pressure force and the resistive deformation force. With a friction coefficient of 0.8 and 1.0, the magnitude of the resistive force is large enough to balance the pressure force at approximately 30$\mu$s, producing the local maximum in the velocity profile.
For the case of $\mu_s = 1.0$, the resistive force is large enough in magnitude to overcome the pressure force and decelerate the piston until motion is stopped at approximately 50 $\mu$s.

Figure 4.9 (b) shows the piston stroke kinetic energy scaled by the reaction energy for different friction coefficients. The case of $\mu_s = 1.0$ is ignored because the piston does not reach stroke distance. For the range of friction coefficients, there appears to be a linear relationship between the piston stroke kinetic energy and the friction coefficient. Notice that even for $\mu_s = 0.0$, the system is only about 15% efficient in converting reaction energy into piston kinetic energy at stroke. The remaining energy can be accounted for by dissipation through heat transfer, volume expansion, and stored residual mechanical energy in the form of pressure when the piston reaches its stroke.

![Figure 4.10: Variation in expansion chamber pressure.](image)

Fig. 4.10 shows the expansion chamber pressure history for different friction coefficients. All cases regardless of friction coefficient have the same initial response. Piston motion cannot occur instantaneously, there is a finite timescale required for the piston to move, regardless of friction coefficient. The cases with a larger friction coefficient have a fuller pressure profile. The rate of volume increase is slowed by friction, allowing the pressures for larger friction coefficients to reach higher values.
Figure 4.11 shows the resistive force profiles for varying friction coefficients as a function of piston displacement. The qualitative trends in the force-displacement are the same, but the peak force is larger for larger friction coefficients. The difference in the curves is essentially due to multiplying by a different constant ($\mu_s$). Notice the case of $\mu_s = 0$ has a nonzero force profile due to piston taper. Even though there is no tangential frictional stress opposing motion, there is still a normal component in the direction of motion of the piston which damps motion. The resistive force does go to zero as the piston is fully displaced into the bore when the normal stress no longer has a component in the axial direction.

- **Number of Burning Grains**

  In general when the HMX is ignited, it will burst into smaller particles and then continue burning. An embedded hotwire facilitates sudden energy release causing this breakup. The larger the number of these particles, or grains, the larger the burn surface and the faster the propellant will burn. It is very difficult to predict or measure the number of burning grains as it is a function of many parameters, such as propellant amount and composition,
etc. The burn parameters of this study were formulated by choosing a burnrate similar to experimental data available, then adjusting the number of burning grains in order to match the timescale of experimental closed bomb data. Because the number of burning grains is difficult to measure, a parametric study is presented to determine model sensitivity.

Figure 4.12: Variation in expansion chamber pressure to number of burning grains.

Figure 4.12 shows the expansion chamber pressure for 100, 1000, 100,000, and 1,000,000 grains. The qualitative trends for all cases are the same in that the pressure rises, reaches a maximum due to the combined effect of gas pressure and volume expansion, then decreases as the volume expands until piston stroke is reached. However, what is different about each case is its associated timescale. The timescale to stroke is much larger (≈ 300 µs) for the case of 100 grains than the case with 1,000,000 grains (≈ 85 µs). The larger amount of surface area associated with larger number of burning grains causes the propellant to burn more quickly, and device operation timescales to be shorter. In addition to the timescales being different, the magnitude of peak pressure varies from case to case. This effect can be explained by the interaction between the rate of gas transfer to the expansion chamber and volume expansion. Consider the case of 100 grains. As the piston starts to move,
the expansion chamber volume increases. In order for the expansion chamber pressure to increase, combustion gases must fill the newly increased volume at a rate which overcomes the volume expansion effect. For the 100 grain case, the propellant is burning very slowly and the pressure in the expansion chamber does not increase as rapidly as in the other cases. For the cases with larger numbers of grains, the propellant is burning fast enough to overcome the volume expansion effect and reaches a higher peak pressure. In addition, the expansion chamber pressure history is virtually unaffected by increasing the number of grains beyond 10,000 grains, with the 100,000 and 1,000,000 cases being indistinguishable. This effect is partly due to mass choking through the actuator port and is looked at in more detail in the next section. The number of burning grains used in the baseline case (10,000) is much less than the number of grains present in 150 mg of HMX based on a grain size of 50µm diameter (≈ 1,200,000).

- **Propellant Burn Rate**

The propellant burn rate is a quantity difficult to measure. In practice, a pressure dependent burn rate model is commonly used to describe combustion. Closed bomb experiments are often performed to determine the burn rate prefactor \( b \) and exponent \( n \), as outlined in Section 4.1. A sensitivity analysis is presented to investigate model dependence on the propellant burn rate.

![Graphs](image.png)

Figure 4.13: (a) Variation in expansion chamber pressure with burn rate prefactor \( b \) and exponent \( n \).
Figure 4.13 illustrates the expansion chamber pressure sensitivity to burn rate prefactor (a) and exponent (b). Figure 4.13 (a) shows pressure sensitivity for burn rate prefactors ranging from $1 \times 10^{-6}$ (dyne/cm$^2$)$^{-0.8}$ cm/s to $1 \times 10^{-4}$ (dyne/cm$^2$)$^{-0.8}$cm/s. For the case of $b = 1 \times 10^{-6}$ (dyne/cm$^2$)$^{-0.8}$cm/s, the propellant begins to burn very slowly and reaches a maximum pressure of about 125 MPa before reaching stroke distance near 300 $\mu$s. The propellant is only 69% burned at stroke completion. This operation timescale is much greater than experimental timescale of approximately 90 $\mu$s. As the prefactor is increased, the maximum pressures increase and operation timescales decrease due to a faster burn rate. For the case of $b = 2 \times 10^{-6}$ (dyne/cm$^2$)$^{-0.8}$cm/s, the valve completes the stroke, however, only 90% of the propellant is burned. As the prefactor is further increased, there is not much change in the pressure profiles, even though the burn times are decreasing from 65 $\mu$s for $b = 4 \times 10^{-6}$ (dyne/cm$^2$)$^{-0.8}$cm/s to 0.58 $\mu$s for $b = 1 \times 10^{-4}$ (dyne/cm$^2$)$^{-0.8}$cm/s. There are similar qualitative trends in the pressure sensitivity to the burn rate exponent $n$. For the case of $n = 0.6$, the burn rate is very slow such that the piston velocity is stopped before stroke distance at approximately 2100 $\mu$s at which point only 10% of the HMX is burned. As the exponent is increased, the maximum pressures increase and operation timescales decrease. The pressure profiles become indistinguishable for exponents of 0.8 – 1.0, even though propellant burn times are decreasing.

To understand why the pressure profile in the expansion chamber is insensitive to further increases in the burn rate, it is necessary to look both at the propellant burn times and the flow of explosive gases through the actuator port. Figure 4.14 (a) shows the propellant burn times for different prefactors, while Fig. 4.14 (b) shows the variation of explosive gas velocity through the actuator port. Notice in Fig. 4.14 (a), the burn times decrease as the prefactor is increased, reaching a minimum near 0.58 $\mu$s for $a = 1 \times 10^{-4}$. There is little change, however, in the exhaust gas velocity profile as the prefactor is increased beyond $6 \times 10^{-6}$. The flow is clearly choked at this point and the rate of gas transfer to the expansion chamber is independent of propellant burn time. This choked
Figure 4.14: (a) Variation of propellant burn times (a) and explosive gas velocity through actuator port with burn rate prefactor $a$.

flow effect is why there is little change in the expansion chamber pressure history in Fig. 4.13 (a) as the prefactor is increased beyond $6 \times 10^{-6}$.

### 4.3.2 Parameters of Design Importance

- **Propellant Mass**

  It is apparent that the amount of HMX used in the Nitrogen Cartridge Valve will greatly affect its performance. If too little is used, the propellant may not be able to sustain a burn, or will not produce enough energy to function the valve. Too much HMX may result in excessive valve deformation. This section will first give results from a parametric study on the propellant mass around the baseline value of 150 mg. Then, results from a threshold study will be given in which a minimum mass of HMX to function the valve is identified.

  Figure 4.15 shows the expansion chamber pressure history for different propellant masses, ranging from 50 mg to 250 mg of HMX. For all cases, the initially zero pressure rises due to energy released from combustion until a maximum is reached, then decreases until the completion of the stroke. Operation timescales range from 135 $\mu$s for the 100 mg case, to 63 $\mu$s for the 250 mg case. Propellant burn timescales range from 116 $\mu$s for the 100 mg case, to 29 $\mu$s for the 250 mg case. Operation timescales are smaller for larger propellant masses because the propellant burn rate is pressure dependent. The more
Figure 4.15: Expansion chamber pressure for different propellant masses.

energy released through combustion allows the pressure, and thus the burn rate to fur-
ther increase. This is also why the combustion timescales are smaller for larger propellant
masses. In addition, larger propellant amounts produce higher pressures enabling operation
to occur more quickly. Notice that the peak pressures do not correspond to the combustion
timescales. There are two major physical processes competing against each other. Com-
bustion is releasing energy and increasing the pressure within the expansion chamber, and
volume expansion due to subsequent piston motion decreases the pressure. The rate at
which the volume can expand is governed by the relative difference in the forces acting
on the piston, a pressure force and a frictional force. The interaction between combustion
and volume expansion produces a peak pressure. As the propellant mass is increased, the
combustion timescales appear to asymptotically approach a minimum timescale. Beyond
this point the combustion timescale cannot be lowered. This is because the combustion
process must have a finite timescale, it cannot instantaneously occur.
Figure 4.16 illustrates the variations in resistive force for varying propellant masses, going from 50 mg of HMX to 250 mg in increments of 25 mg. The quantitative force profiles are very similar to those previously discussed in Section 3. The profiles with larger force values correspond to the cases with a larger propellant mass. The larger propellant masses produce higher pressures within the expansion chamber which provides additional interference increasing the interface pressure between the piston and housing.

Figure 4.16: Resistive force variations for different propellant masses.

Figure 4.17 shows the piston velocity profiles for different propellant masses. Piston stroke velocities vary from 95 m/s for the case with 50 mg of HMX to 211 m/s for the case with 250 mg.

As the propellant mass is lowered, it will eventually reach a critical mass where the piston will not receive enough energy to actuate the valve. A threshold study was performed to identify this value. This section will identify a threshold propellant based on the propellant burn model and piston deformation model presented in Sections 2.1 and 2.2. It should be noted that the pressure dependent burn model may be inadequate in describing combustion near explosive quenching conditions. A more detailed initiation model is needed to characterize the propellant burn in this regime.
Figure 4.17: Piston velocity variations for different propellant masses.

Figure 4.18: Piston velocity variations for propellant mass threshold study.

Figure 4.18 shows the velocity profiles for pistons loaded in a valve containing 30, 40, 50, and the baseline case of 150 mg of HMX. For the cases with 40 and 50 mg of HMX, the valve is successfully actuated. For both of these cases, the piston velocity initially increases,
starts to level off, then increases until the valve is actuated. Actuation timescales for these
two propellant masses are 367 $\mu$s for the 40 mg case and 202 $\mu$s for the 50 mg case, much
larger than the baseline actuation time of approximately 90 $\mu$s. Piston stroke velocities for
these two propellant masses are near 76 m/s and 63 m/s, much lower than the baseline
stroke velocity of 150 m/s. Because of lower pressures produced from the smaller propellant
masses, the HMX combustion times for 40 mg and 50 mg of HMX are 110 $\mu$s and 127 $\mu$s,
much greater than the baseline combustion timescale of approximately 47 $\mu$s. The smaller
HMX masses produce lower pressures in the actuator and are not able to accelerate the
burn as quickly as the larger propellant masses. For propellant masses of 30 mg, the valve
is not successfully actuated as piston motion is stopped before actuation is complete. The
velocity profile for the 30 mg case initially increases as the gas pressure starts to build and
force the piston into the bore. However, the gas pressure is not large enough to overcome
the deformation force and the piston motion is stopped. A more in depth analysis of both
the 30 mg and 40 mg case is performed to describe piston motion near the propellant
threshold.

Figures 4.19 and 4.20 illustrate the piston motion dynamics for the 30 mg and 40 mg
cases. The 30 mg case is first analyzed, followed by the 40 mg case. Figure 4.19 illustrates
the piston velocity, force acting on the piston due to gas pressure, resistive force acting
against the piston due to deformation, and the net force acting on the piston which is the
difference in the pressure and resistive forces. The simulation was terminated for the 30
mg case and identified as a failed actuation when the piston velocity went below 0.1 m/s.
There are two forces acting on the piston which govern its motion history, the pressure
force driving the piston into the bore and the resistive force due to piston deformation.
Initially, the pressure force is greater than the resistive force and the piston begins to
accelerate. Near 40 $\mu$s, the increasing resistive force is equal to the pressure force. The
net force of zero yields a local maximum in the velocity of 5.5 m/s. After this point
the resistive force is always greater than the pressure force and the piston continues to
*piston travel distance = 0.22 mm
*30 mg HMX

Figure 4.19: Piston dynamics for 30 mg of HMX.

*40 mg HMX

Figure 4.20: Piston dynamics for 40 mg of HMX.
decelerate. Piston motion is finally extinguished near 68 $\mu s$ when the resistive force has damped piston motion such that the piston velocity is effectively zero. For the 40 mg case, the valve is successfully actuated. Again, the forces acting on the piston must be studied to explain its motion history. Shown in Fig. 4.20, the pressure force is initially greater than the resistive force and begins to accelerate the piston. The pressure and resistive forces are equal at approximately 45 $\mu s$, at which there is a local maximum in the piston velocity ($\approx 10$ m/s). The resistive force then overcomes the pressure force and continues to decelerate the piston until the pressure force is again equilibrated with the resistive force near 110 $\mu s$. At this point, there is a local minimum in the velocity profile of 3 m/s. After this point, the pressure force overcomes the decreasing resistive force and accelerates the piston until it reaches its stroke.

It is clear the interaction between the gas pressure and deformation resistive force governs piston motion history, and thus valve actuation. Based on the results presented in this section, the model predicts that the minimum propellant mass that can successfully actuate the valve is between 30 mg and 40 mg. It should be repeated, however, that a more accurate ignition model may be needed to describe propellant quenching for small propellant masses.

- **Piston and Housing Thickness**

  This section discusses the effects of both piston and housing thickness on valve performance from a design perspective. It is important to know the extent of plastic deformation in key structural components in order to predict failure. The extent of plastic deformation in the piston and housing for various internal pressures was discussed in Section 4.2. The ANSYS simulations displayed contained plasticity of the housing, however, they were modeled after the MTS tests and had no internal gas pressure. The remainder of this section will give model results from a parametric study on the piston and housing thickness, but it must be made clear that the deformation of both the piston and housing in an explosive firing is not well characterized.
Figure 4.21: (a) Variation in resistive force for different piston (a) and housing (b) thicknesses.

Figure 4.22: (a) Variation in piston velocity profiles for different piston (a) and housing (b) thicknesses.

Figure 4.21 illustrates the effect of piston (a) and housing (b) thickness on the resistive force, where \( t^* \) is the piston/housing thickness scaled by the baseline thickness. The resistive force trends for both the piston and housing are similar to the baseline case. As the piston thickness is increased, the resistive force profile is also increased, reaching a peak of near 40 kN for \( t^* = 4.0 \). The thicker piston requires more energy to deform it into the bore than the thinner piston. There is little change in the resistive force profile as the housing thickness is increased from \( t^* = 0.5 \) to 1.0. The insensitivity of the model to housing thickness can in part be explained by the piston material model, namely perfect uncontained plasticity. Because the piston instantaneously yields through its thickness, the solution for \( \tilde{P} \) becomes uncoupled from the housing, as explained for the special case presented in Section 2.2.
Figure 4.22 shows the effect of piston (a) and housing (b) thickness on piston velocity. As the piston thickness is increased, the resistive force increases, and thus the velocity profile decreases. The resistive force increases with piston thickness and reaches a critical value of \( t^* = 4.0 \) where the resistive force overtakes the pressure force driving the piston and the valve fails. The velocity profile is virtually unaffected by housing thickness.

- **Skirt Angle**

This section addresses the effects skirt angle has on valve performance. The skirt angle is an important parameter for designers. If the skirt angle is too small, there may be problems with sealing the explosive gases and there may be excessive blowby. If the angle is too large, the energy required to deform the piston may be so large the valve does not function. It should be noted that the deformation model will start to break down as the skirt angle is increased due to the large deformations required for larger skirt angles. In addition, the state of stress becomes highly 3-d as the skirt angle is increased, departing from the 2-d assumption used in the development of the deformation model.

![Variation in resistive force for different skirt angles.](image)

Figure 4.23: Variation in resistive force for different skirt angles.
Figure 4.23 illustrates the effect skirt angle has on resistive force. The skirt angles shown range from zero to fifteen degrees in increments of three degrees. The qualitative trend for all cases is similar in that the force increases from zero and reaches a maximum before decreasing to a quasi-steady state value as the piston is displaced into the bore near 2.54mm. After this point the resistive force decreases slightly as the expansion chamber pressure decreases. The peak force increases with skirt angle as expected. The larger skirt angles require a larger interference to displace the piston, thus giving a larger resistive force. Notice that for the case of a zero skirt angle, the resistive force does not approach the same steady state value as the other cases as the piston is displaced into the bore. This can be explained by the origin of the resistive force. The resistive force is caused by both a geometrical interference and also internal pressure. The case of a zero skirt angle has a lower steady state value because there is no interference associated with it, unlike the cases with a nonzero skirt angle. Notice for the cases with a nonzero skirt angle, the resistive force profiles are very close to each other after the piston is displaced into the bore. This is attributed to the small friction coefficient ($\mu_b = 0.088$) used in the bore region.

In Section 4.2, the piston deformation force was shown to be more than adequate to hold the piston in place once it reached stroke for the worst case scenario in which only the resistive force opposes the nitrogen reservoir pressure force acting on the piston. As the piston skirt angle goes to zero, the resistive force at piston stroke also goes to zero without any internal gas pressure. There will exist a critical skirt angle which will be unable to hold the piston in place from the nitrogen reservoir pressure acting on the piston. In order to determine this, simulations were performed with zero combustion gas pressure for different skirt angles to determine when the resistive force is no longer adequate to hold the piston in place. It was determined that even for a skirt angle of 1 degree, the resistive force at stroke ($\approx 640$ N) is more than enough to hold the piston in place from the nitrogen reservoir pressure force acting on the piston ($\approx 44$ N).
Figure 4.24 shows the effect of skirt angle on expansion chamber pressure (a) and piston velocity (b). Shown are skirt angles ranging from zero to fifteen in increments of three degrees. The pressures in the expansion chamber are identical up to 25 $\mu$s, then decrease at different rates. The pressures for the larger skirt angles decrease at a slower rate than the smaller skirt angles due to larger resistive forces associated with them. Stroke times range from 80 $\mu$s for a skirt angle of 0 degrees to 92 $\mu$s for a skirt angle of 15 degrees. The stroke piston velocity is larger for smaller skirt angles due to decreased deformation dissipation and range from 164 m/s for a skirt angle of zero degrees to 150 m/s for a skirt angle of fifteen degrees. The valve certainly contains enough explosive mass to function the valve even for a skirt angle of fifteen degrees that produces a maximum resistive force of approximately 22 kN.
Chapter 5
Conclusions

This thesis has outlined a method to model the entire actuation process of an explosive valve, including: the burning of a solid explosive and production of high pressure gas products, the mass transfer of combustion products from the actuator to the expansion volume, the resulting piston motion due to high pressure gas phase products, and the effects of piston and housing deformation on piston motion.

A device deformation model has been formulated that accounts for piston and housing material properties, including the effects of strain hardening and internal gas pressure. The model assumes linear strain hardening in device components, uncontained plasticity in the piston, contained plasticity in the housing, and a two dimensional state of stress for each piston-housing cross-section. The deformation model is easily correlated with a minimal set of experiments that give HMX combustion time and valve operation time (closed bomb test and reactive valve test).

Quasistatic compression tests were performed to characterize the integrated work requirements of the device and an FEA was performed to characterize both the quasistatic stress and deformation fields within the piston and housing. Results from the quasistatic compression tests indicate that skirt insertion dominates the work requirements of the valve, accounting for approximately 62% of the total work required to function the valve. Diaphragm puncture requires only approximately 10% of the entire work required for actuation. Results from the FEA indicate that the contained and uncontained plasticity assumptions in the housing and piston are appropriate. Results also agree with the compression tests indicating there is little residual resistive force left once the piston passes the housing corner due to a strong bending effect. To model this bending effect, the deformation model utilizes two friction coefficients ($\mu_s$ and $\mu_b$). Although it does not predict local piston bending stresses associated with the housing corner, the deformation model
was demonstrated to reasonably well describe finite element results for axial piston resistive force. The assumption of a 2-d state of stress in the deformation model disables it from predicting these largely 3-d bending stresses in the piston. The model agrees less so with the compression tests, likely due to deviation of actual plastic material behavior from the linear strain hardening approximation. The deformation model was then further extended to include internal pressure that is more representative of the loading conditions that exist in the explosive firing of the valve.

To determine the baseline operation of the valve, two experiments must be performed to set parameters that are difficult to measure ($r_b$, $\mu_s$, and $\mu_b$). The explosive burn rate must be correlated with closed bomb tests to estimate the combustion time. The friction coefficients ($\mu_s$ and $\mu_b$) must then be chosen such that valve operation time can be estimated. Baseline operation timescale was approximately 90 $\mu$s with a piston stroke velocity of approximately 150 m/s. Once the baseline operation of the valve was determined, an extensive parametric study was performed. Results from a parametric study suggest that the piston is being significantly overdriven with 150 mg of HMX. As such, there was little sensitivity in valve performance to slight modifications around the baseline propellant load of 150 mg. Quasistatic compression tests give a piston velocity of approximately 23 m/s required to puncture the diaphragm, while predictions give a piston stroke velocity of near 150 m/s. The overdriving of the piston is in part related to the skirt design. Because so much energy ($\approx 15$ J compared to the $\approx 2.1$ J required for diaphragm puncture) is required for skirt insertion initially, it is inevitable that piston stroke velocities become excessive. Conversely, the skirt plays an important role in sealing off the explosive gases from escaping past the piston.

The model presented in this thesis is a useful tool for design engineers. It can be used to better characterize current device designs, and to develop new generation devices. The model gives designers an additional tool for making design changes or developing new hardware, rather than basing them on previous designs.
There has not been extensive work done in comprehensively modeling valve actuation, as a result there is much additional work to be done. One possible extension to the work presented in this thesis could be the addition of an ignition model to more accurately describe explosive burn dynamics near quenching conditions. This addition would give more accurate predictions of a propellant threshold required to function the valve. In addition, a plasticity model that accounts for component deformation history would further strengthen the model. The geometry of skirt design is another important topic that should be investigated. Results from the ANSYS study show that valve deformation is very sensitive to the housing corner. The corner induces large bending stresses in a concentrated region around the corner. If the housing corner had a more gradual curvature it is expected that piston deformation be more predictable using the deformation model outlined in this thesis. Another area important to new generation devices is the effects of scaling. It is anticipated that the effects of heat transfer and friction will increase as the characteristic device lengthscale is decreased. As such, more detailed models describing them may be necessary.
References


Appendix: User’s Manual

NITVALVE: A Computer Program for the Simulation of Explosively Actuated Valves

Adam M. Braud, Keith A. Gonthier, and Michele Decroix

NITVALVE is a computer program written in FORTRAN 77 which simulates explosive valve actuation, including 1) the burning of a solid explosive and production of multiphase products, 2) the mass transfer of combustion products through an actuator port including possible mass choking, 3) the resulting piston motion due to high pressure gas products, and 4) the effects of device deformation on piston motion. The program was validated on an explosive valve, however, can be easily adapted to other axisymmetric explosively actuated devices.

The program is based upon a multi-phase combustion model formulated by Gonthier and Powers; the details of the model including model assumptions, model equations, and results are documented in Ref. [1]. A thorough description of the combustion methodology used by the authors for modeling explosively actuated systems is given in Ref. [2]. A description of the model used in describing device deformation is discussed in detail in Ref. [3]. This manual summarizes program input/output and other relevant program information.

The program uses subroutines and the database contained in the chemical kinetics package CHEMKIN 4.0.2 for the evaluation of temperature dependent properties of reactant and product species. The package LSODE is used for the numerical integration of the

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governing ordinary differential equations. The LAPACK subroutine DGESVX is used to solve systems of linear equations encountered in the deformation model, and the author written subroutine NEWTON uses a standard Newton method to solve the system of nonlinear equations appearing in the deformation model. Details of these packages will not be given in this manual.

The structure of NITVALVE is shown in Fig. A.1. Input is read from the file NITVALVE.INPUT and VALVE.INPUT. NITVALVE.INPUT contains input data relating to the combustion model, while VALVE.INPUT contains input data relevant to the deformation model. The file CHEM.ASC is a linking file containing chemical species information; this file is the primary output of the CHEMKIN 4.0.2 Interpreter (described below). The governing equations are numerically integrated from the time of chemical ignition \( t = 0 \) until completion of the piston’s stroke \( t_{\text{stroke}} \) using the subroutine LSODE. The code LSODE requires the user-supplied subroutine FCN for evaluation of the ordinary differential equations. Before updating the time, results are written to the output data file NITVALVE.DATA, and a check is performed to assure that both the total system mass and the total initial energy of the system are accounted for; if not, an error message is printed and the program is terminated. Upon completion of the pin’s stroke, pertinent information (i.e., the stroke time, the pin kinetic energy at completion of the stroke, etc.) is written to the output file NITVALVE.OUTPUT. Details of the input/output files, the subroutines, and the printed error messages are given below. Also, a brief description of the the CHEMKIN 4.0.2 package as it relates to the code NITVALVE is given.

**Input File: NITVALVE.INPUT**

This input file contains data pertinent to the combustion model, including: geometric data for the valve, values for physical parameters contained in the model, and the stoichiometric coefficients for the explosive reaction. As an example, the input file is attached to this manual.
The stoichiometric coefficients are determined using constant volume chemical equilibrium calculations performed by CHEMKIN 4.0.2. Stoichiometric coefficients for the reactant and product species are stored in the ordered array \( nu(i) \) where \( i = 1, N_s \) designates reactant species, \( i = N_s + 1, N_s + N_{cp} \) designates condensed phase product species, and \( i = N_s + N_{cp} + 1, N_s + N_{cp} + N_g \) designates gas phase product species. Here, \( N_s, N_{cp}, \) and \( N_g \) are the total number of reactant, condensed phase product, and gas phase product species, respectively. This ordering of reactant and product species is used for all arrays in the program which contain species information \([\text{e.g., the specific internal energy } e(i), \text{ etc.}]\)

Also, it is necessary to initially specify a small amount of product mass \((p_{mass0})\) to avoid singularities associated with zero product mass.

**Input File: VALVE.INPUT**

This input file contains inputs pertinent to the deformation model, including piston and housing geometry and material properties. In addition, the number of discrete points used to numerically integrate the integrals contained in the deformation model is chosen. There also exists the option to use a contained or uncontained plasticity model for the housing. It is suggested that the user use the uncontained plasticity model for a piston displaying near perfectly plastic material behavior to save computational time. As an example, the input file is attached to this manual.

**Output File: NITVALVE.OUTPUT**

This output file contains the following printed information: 1) the initial conditions used in the simulation, 2) the maximum predicted pressure, gas phase density, and temperature in the NSI assembly, 3) the maximum predicted pressure, gas phase density, and temperature in the expansion chamber, 4) the predicted stroke time, the velocity and kinetic energy of the pin at completion of the stroke, and the pressure in the NSI and expansion chamber at completion of the stroke, and 5) the chemical heat release. 6) the time required for the propellant to burn, if it has completely burned at the termination of
the code. 7) the maximum average stress for a given instant in time at the inner surface of
the piston and housing. 8) an error message if the solution from the linear solver DGESVX
is ill-conditioned. 9) an error message if the iterated solution in NEWTON is diverging. A
sample output file is attached to this manual.

Output File: NITVALVE.DATA

This output file contains the following formatted data: 1) the time, \( \mu s \) (column 1), 2)
the pressures within the actuator and expansion chamber, MPa (columns 2 and 3), 3) the
piston velocity, m/s (column 4), 4) the piston position, mm (column 5), 5) the deformation
force resisting piston motion, kN (column 6), 6) the gas phase density inside the actuator
and the expansion chamber, \( \frac{kg}{m^3} \) (columns 7 and 8), 7) the product temperatures within the
actuator and expansion chamber, \( K \) (columns 9 and 10), 8) the velocity of the explosive
gases through the actuator port, m/s (column 11), 9) the propellant burn rate, cm/s
(column 12), 10) the net force acting on the piston, kN (column 13), 11) the pressure force
acting on the piston, kN (column 14), and 12) the percentage of burned explosive, (column
15).

Since small time steps are initially needed to resolve the fast time scales associated
with the increase in temperature and pressure, values for the dependent variables are not
written to the output file after every time step to avoid unnecessarily large data files. To
this end, a counter \( (icount) \) is used in the program which is incremented after each new
time; when a specified maximum value for this counter is reached, values for the dependent
variables are written to the data file and the counter is re-initialized to zero. The maximum
value for the counter can be easily changed within the program.

Subroutines

Subroutines used in the program are listed in Table A.1.
Table A.1: Subroutines used in the code NITVALVE.

<table>
<thead>
<tr>
<th>Subroutine</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ckinit</td>
<td>Creates internal work arrays for CHEMKIN 4.0.2 (CHEMKIN 4.0.2 subroutine).</td>
</tr>
<tr>
<td>ckwt</td>
<td>Returns the molecular weights of the chemical species (CHEMKIN 4.0.2).</td>
</tr>
<tr>
<td>ckrp</td>
<td>Returns the value for the ideal gas constant (CHEMKIN 4.0.2).</td>
</tr>
<tr>
<td>ckums</td>
<td>Returns specific internal energy of the chemical species (CHEMKIN 4.0.2).</td>
</tr>
<tr>
<td>ckhms</td>
<td>Returns specific enthalpy of the chemical species (CHEMKIN 4.0.2).</td>
</tr>
<tr>
<td>ckcvms</td>
<td>Returns specific heat at constant volume for the chemical species (CHEMKIN 4.0.2).</td>
</tr>
<tr>
<td>ckcpms</td>
<td>Returns specific heat at constant pressure for the chemical species (CHEMKIN 4.0.2).</td>
</tr>
<tr>
<td>lsode</td>
<td>Returns values of dependent primary variables at new time (LSODE).</td>
</tr>
<tr>
<td>fcn</td>
<td>Subroutine used by LSODE to evaluate ordinary differential equations.</td>
</tr>
<tr>
<td>energy</td>
<td>Returns internal energy of system.</td>
</tr>
<tr>
<td>valve</td>
<td>Subroutine used by NITVALVE to determine the deformation force resisting piston motion.</td>
</tr>
<tr>
<td>newton</td>
<td>Subroutine used by VALVE to solve systems of nonlinear equations.</td>
</tr>
<tr>
<td>dgesvx</td>
<td>Subroutine used by VALVE and NEWTON to solve systems of linear equations.</td>
</tr>
</tbody>
</table>

The program prints an error message when either the total system mass or the total initial energy of the system changes by more than 0.001 percent during the computations. Such errors can possibly be eliminated by decreasing the time step specified in the input file (NITVALVE.INPUT). Additional error messages are included in the file NITVALVE.OUTPUT as previously explained.

**CHEMKIN 4.0.2 Interpreter**

The Interpreter is a program contained in the CHEMKIN 4.0.2 package that reads a symbolic description of the chemical species considered in the study as input and then extracts the needed thermodynamic data for each species from the CHEMKIN 4.0.2 Thermodynamic Database. The Interpreter outputs both a Linking File (CHEM.ASC) which contains the pertinent thermodynamic information and a printed output file which contains a listing of the species, diagnostic error messages (if needed), and work space requirements.
The code NITVALVE reads the file CHEM.ASC and calls subroutines contained in the CHEMKIN 4.0.2 Subroutine Library. Note that the order of the species in the input file described above must be consistent with the order of the stoichiometric coefficients contained in the file NITVALVE.INPUT.

Uncoupled Deformation Model

It is convenient to look at the deformation model contained in the subroutine VALVE uncoupled from the main combustion code NITVALVE to gain insight to both piston and housing deformation for various pressures. For example, the uncoupled deformation model can be used to estimate at what internal pressure and displacement the piston and housing yield, as discussed in Ref. [3]. Included is the subroutine VALVE written as a main code, along with its input file VALVE.INPUT. This input file is similar to the input file used in the coupled deformation code, except there is an option to choose the internal gas pressure for the simulation. There are two output files for the uncoupled deformation model, VALVE.DATA and STRESS.DATA. The output file VALVE.DATA contains the following information: 1) the piston displacement, mm (column 1), 2) the net force acting on the piston, kN (column 2), 3) the force acting on the piston from the skirt region, kN (column 3), and 4) the force acting on the piston from the bore region, kN (column 4). The output file STRESS.DATA contains stress data for the first ‘disc’ of the force integration corresponding to the top of the piston. The output file contains the following information: 1) the piston displacement, mm (column 1), 2) the maximum tresca stress in the housing scaled by the housing original yield strength, – (column 2), 3) the maximum tresca stress in the piston scaled by the piston original yield strength, – (column 3), 4) the piston/housing interface radius, mm (column 4), 5) the housing plastic radius, mm (column 5), and 6) the outer radius of the housing, mm (column 6).


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**Figure A.1:** Structure of the code NITVALVE.
Vita

Adam M. Braud was born October 27, 1981, at Lakeside Hospital in Metairie, Louisiana. He grew up in Covington, Louisiana, where he attended Covington High School and received his diploma in 2000. He next attended Louisiana State University where he received his bachelor’s degree in mechanical engineering December 2004. He was a member of one of the first classes of the 3-2 Accelerated Master’s Degree Program in mechanical engineering and is scheduled to graduate December 2006.