A Finite Element Analysis of High Temperature Fatigue and Creep at a Stress Concentration.

Faysal Abdou Kolkailah

Louisiana State University and Agricultural & Mechanical College

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Kolkailah, Faysal Abdou

A FINITE ELEMENT ANALYSIS OF HIGH TEMPERATURE FATIGUE AND
CREEP AT A STRESS CONCENTRATION

The Louisiana State University and Agricultural and Mechanical Col. Ph.D. 1982

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A FINITE ELEMENT ANALYSIS OF
HIGH TEMPERATURE FATIGUE AND CREEP
AT A STRESS CONCENTRATION

A Dissertation
Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy
in
The Department of Mechanical Engineering

by

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B.S., Cairo University, 1970
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>i</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>v</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>xii</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. BACKGROUND</td>
<td></td>
</tr>
<tr>
<td>2.1 Overview of Fatigue</td>
<td>4</td>
</tr>
<tr>
<td>2.2 Overview of Creep</td>
<td>6</td>
</tr>
<tr>
<td>2.3 Overview of Constitutive Models</td>
<td>9</td>
</tr>
<tr>
<td>2.4 Overview of Finite Element Technique</td>
<td>11</td>
</tr>
<tr>
<td>2.5 Literature Survey</td>
<td></td>
</tr>
<tr>
<td>A. Material Constitutive Models</td>
<td>14</td>
</tr>
<tr>
<td>B. The Finite Element Study</td>
<td>16</td>
</tr>
<tr>
<td>2.6 Objectives</td>
<td>18</td>
</tr>
<tr>
<td>III. NUMERICAL PRESENTATION OF TIME DEPENDENT MATERIAL MODEL</td>
<td></td>
</tr>
<tr>
<td>3.1 The Constitutive Theory of Bodner and Parton</td>
<td>21</td>
</tr>
<tr>
<td>3.2 Numerical Evaluation of the Material Parameters</td>
<td>25</td>
</tr>
<tr>
<td>3.3 Comparison of the Numerical Model Versus Published Data</td>
<td>28</td>
</tr>
<tr>
<td>3.4 Numerical Representation of Inconel 718 By the Bodner Constitutive Model</td>
<td></td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------</td>
</tr>
<tr>
<td>3.4.1</td>
<td>The Experimental Data</td>
</tr>
<tr>
<td>3.4.2</td>
<td>Numerical Evaluation Procedures</td>
</tr>
<tr>
<td>3.5</td>
<td>Discussion of the Results</td>
</tr>
<tr>
<td>3.6</td>
<td>Conclusions</td>
</tr>
<tr>
<td>IV</td>
<td>ELASTIC AND ELASTIC-PLASTIC ANALYSIS BY THE FINITE ELEMENT METHOD</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>4.2</td>
<td>Finite Element Formulation for the Elastic Analysis</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Displacement Functions</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Element Strain (total)</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Initial Strain (thermal strain)</td>
</tr>
<tr>
<td>4.2.4</td>
<td>Elasticity Matrix</td>
</tr>
<tr>
<td>4.2.5</td>
<td>The Governing Equation</td>
</tr>
<tr>
<td>4.3</td>
<td>Finite Element Formulation for the Elastic-Plastic Analysis</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Plasticity</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Yield Criteria</td>
</tr>
<tr>
<td>4.3.3</td>
<td>Finite Element Technique</td>
</tr>
<tr>
<td>V</td>
<td>COMPARISON OF THE FINITE ELEMENT RESULTS VERSUS ANALYTICAL AND EXPERIMENTAL RESULTS</td>
</tr>
<tr>
<td>5.1</td>
<td>Theoretical Elastic Stress Concentration Factor $(K_t)$</td>
</tr>
<tr>
<td>5.1.1</td>
<td>Discussion of the Results and Conclusion</td>
</tr>
<tr>
<td>5.2</td>
<td>Nonlinear Time-Dependent Cyclic Loading with Various Load Spectrum</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS (continued)

5.2.1 Discussion of the Results and Conclusion .................. 124

VI. CONCLUSIONS AND RECOMMENDATIONS .......................

6.1 Summary and Conclusions .................................. 136
6.2 Recommendations ........................................ 137

REFERENCES .................................................. 139

SELECTED BIBLIOGRAPHY ......................................... 142

APPENDIXES

A. The Experimental Program ....................................
   A.1 The Interferometric Strain/Displacement Gage (ISDG) .... 144
   A.2 Test Facilities and Methodology ....................... 147
   A.3 Experimental Results .................................. 150

B. Computer Program Listing for Numerical Evaluation of the Material Parameters .... 158

C. Computer Program Listing for the Elastic-Plastic Finite Element Code ................ 162

VITA ........................................................... 201
**LIST OF TABLES**

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Analytical and Numerical Variables for</td>
<td></td>
</tr>
<tr>
<td>IN100 at 1350°F</td>
<td>29</td>
</tr>
<tr>
<td>3.2 Analytical and Numerical Variables for Inconel 718 at 1200°F</td>
<td>35</td>
</tr>
<tr>
<td>3.3 Starting and Final Numerical Variables for Inconel 718 at 1200°F</td>
<td>45</td>
</tr>
<tr>
<td>(Individual Runs - First Case)</td>
<td></td>
</tr>
<tr>
<td>3.4 Starting and Final Numerical Variables for Inconel 718 at 1200°F</td>
<td>46</td>
</tr>
<tr>
<td>(Individual Runs - Second Case)</td>
<td></td>
</tr>
<tr>
<td>3.5 Starting and Final Numerical Variables for Inconel 718 at 1200°F</td>
<td>47</td>
</tr>
<tr>
<td>(Group Runs)</td>
<td></td>
</tr>
<tr>
<td>5.1 Theoretical Elastic Stress Concentration Factor</td>
<td>122</td>
</tr>
<tr>
<td>5.2 Material Variables for Inconel 718 at 1200°F used in the Finite Element Analysis</td>
<td>127</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Notched Sample with Nominal Stress</td>
<td>7</td>
</tr>
<tr>
<td>2.2</td>
<td>Idealized Creep Curve</td>
<td>8</td>
</tr>
<tr>
<td>3.1</td>
<td>Tensile Response for IN100 at 1350°F - Strain Rate $1.42 \times 10^{-3}$ sec</td>
<td>30</td>
</tr>
<tr>
<td>3.2</td>
<td>Tensile Response for IN100 at 1350°F - Strain Rate $6.6 \times 10^{-5}$ sec</td>
<td>31</td>
</tr>
<tr>
<td>3.3</td>
<td>Creep Response for IN100 at 1350°F - Stress Level 120 ksi</td>
<td>33</td>
</tr>
<tr>
<td>3.4</td>
<td>Creep Response for IN100 at 1350°F - Stress Level 72 ksi</td>
<td>34</td>
</tr>
<tr>
<td>3.5</td>
<td>Tensile Response for Inconel 718 at 1200°F - Strain Rate $10^{-3}$ sec</td>
<td>36</td>
</tr>
<tr>
<td>3.6</td>
<td>Tensile Response for Inconel 718 at 1200°F - Strain Rate $8.3 \times 10^{-5}$ sec</td>
<td>37</td>
</tr>
<tr>
<td>3.7</td>
<td>Tensile Response for Inconel 718 at 1200°F - Strain Rate $8.3 \times 10^{-7}$ sec</td>
<td>38</td>
</tr>
<tr>
<td>3.8</td>
<td>Creep Response for Inconel 718 at 1200°F - Stress Level 125 ksi</td>
<td>40</td>
</tr>
<tr>
<td>3.9</td>
<td>Creep Response for Inconel 718 at 1200°F - Stress Level 110 ksi</td>
<td>41</td>
</tr>
<tr>
<td>3.10</td>
<td>Button Head Specimen</td>
<td>42</td>
</tr>
<tr>
<td>3.11</td>
<td>Creep Response for Inconel 718 at 1200°F - Stress Level 125 ksi (Individual Run - First Case)</td>
<td>50</td>
</tr>
<tr>
<td>3.12</td>
<td>Creep Response for Inconel 718 at 1200°F - Stress Level 110 ksi (Individual Run - First Case)</td>
<td>51</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>3.13</td>
<td>Tensile Response for Inconel 718 at 1200°F - Strain Rate $1.6\times10^{-5}$ sec$^{-1}$ (Individual Run - First Case)</td>
<td>52</td>
</tr>
<tr>
<td>3.14</td>
<td>Tensile Response for Inconel 718 at 1200°F - Strain Rate $6.75\times10^{-5}$ sec$^{-1}$ (Individual Run - First Case)</td>
<td>53</td>
</tr>
<tr>
<td>3.15</td>
<td>Tensile Response for Inconel 718 at 1200°F - Strain Rate $10^{-6}$ sec$^{-1}$ (Individual Run - First Case)</td>
<td>54</td>
</tr>
<tr>
<td>3.16</td>
<td>Tensile Response for Inconel 718 at 1200°F - Strain Rate $1.1\times10^{-5}$ sec$^{-1}$ (Individual Run - First Case)</td>
<td>55</td>
</tr>
<tr>
<td>3.17</td>
<td>Tensile Response for Inconel 718 at 1200°F - Strain Rate $3.3\times10^{-3}$ sec$^{-1}$ (Individual Run - First Case)</td>
<td>56</td>
</tr>
<tr>
<td>3.18</td>
<td>Creep Response for Inconel 718 at 1200°F - Stress Level 110 ksi (Individual Run - Second Case)</td>
<td>57</td>
</tr>
<tr>
<td>3.19</td>
<td>Creep Response for Inconel 718 at 1200°F - Stress Level 110 ksi (Individual Run - Second Case)</td>
<td>58</td>
</tr>
<tr>
<td>3.20</td>
<td>Tensile Response for Inconel 718 at 1200°F - Strain Rate $1.6\times10^{-5}$ sec$^{-1}$ (Individual Run - Second Case)</td>
<td>59</td>
</tr>
<tr>
<td>3.21</td>
<td>Tensile Response for Inconel 718 at 1200°F - Strain Rate $6.75\times10^{-5}$ sec$^{-1}$ (Individual Run - Second Case)</td>
<td>60</td>
</tr>
<tr>
<td>3.22</td>
<td>Tensile Response for Inconel 718 at 1200°F - Strain Rate $10^{-6}$ sec$^{-1}$ (Individual Run - Second Case)</td>
<td>61</td>
</tr>
<tr>
<td>3.21</td>
<td>Tensile Response for Inconel 718 at 1200°F - Strain Rate $1.1\times10^{-5}$ sec$^{-1}$ (Individual Run - Second Case)</td>
<td>62</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>3.24 Tensile Response for Inconel 718 at 1200°F - Strain Rate $3.3 \times 10^7$ sec$^{-1}$ (Individual Run - Second Case)</td>
<td>63</td>
<td></td>
</tr>
<tr>
<td>3.25 Creep Response for Inconel 718 at 1200°F - Stress Level 125 ksi (7 Data - First Case)</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>3.26 Creep Response for Inconel 718 at 1200°F - Stress Level 110 ksi (7 Data - First Case)</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>3.27 Tensile Response for Inconel 718 at 1200°F - Strain Rate $1.6 \times 10^5$ sec$^{-1}$ (7 Data - First Case)</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>3.28 Tensile Response for Inconel 718 at 1200°F - Strain Rate $6.75 \times 10^4$ sec$^{-1}$ (7 Data - First Case)</td>
<td>67</td>
<td></td>
</tr>
<tr>
<td>3.29 Tensile Response for Inconel 718 at 1200°F - Strain Rate $10$ sec$^{-1}$ (7 Data - First Case)</td>
<td>68</td>
<td></td>
</tr>
<tr>
<td>3.30 Tensile Response for Inconel 718 at 1200°F - Strain Rate $1.1 \times 10^7$ sec$^{-1}$ (7 Data - First Case)</td>
<td>69</td>
<td></td>
</tr>
<tr>
<td>3.31 Tensile Response for Inconel 718 at 1200°F - Strain Rate $3.3 \times 10^7$ sec$^{-1}$ (7 Data - First Case)</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>3.32 Creep Response for Inconel 718 at 1200°F - Stress Level 125 ksi (7 Data - Second Case)</td>
<td>71</td>
<td></td>
</tr>
<tr>
<td>3.33 Creep Response for Inconel 718 at 1200°F - Stress Level 110 ksi (7 Data - Second Case)</td>
<td>72</td>
<td></td>
</tr>
<tr>
<td>3.34 Tensile Response for Inconel 718 at 1200°F - Strain Rate $1.6 \times 10^5$ sec$^{-1}$ (7 Data - Second Case)</td>
<td>73</td>
<td></td>
</tr>
<tr>
<td>3.35 Tensile Response for Inconel 718 at 1200°F - Strain Rate $6.75 \times 10^4$ sec$^{-1}$ (7 Data - Second Case)</td>
<td>74</td>
<td></td>
</tr>
<tr>
<td>3.36 Tensile Response for Inconel 718 at 1200°F - Strain Rate $10$ sec$^{-1}$ (7 Data - Second Case)</td>
<td>75</td>
<td></td>
</tr>
</tbody>
</table>
## LIST OF FIGURES (continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.37 Tensile Response for Inconel 718 at 1200°F - Strain Rate $1.1 \times 10^{-6}$ sec$^{-1}$ (7 Data - Second Case).</td>
<td>76</td>
</tr>
<tr>
<td>3.38 Tensile Response for Inconel 718 at 1200°F - Strain Rate $3.3 \times 10^{-7}$ sec$^{-1}$ (7 Data - Second Case).</td>
<td>77</td>
</tr>
<tr>
<td>3.39 Creep Response for Inconel 718 at 1200°F - Stress Level 125 ksi (5 Data - First Case).</td>
<td>78</td>
</tr>
<tr>
<td>3.40 Creep Response for Inconel 718 at 1200°F - Stress Level 110 ksi (5 Data - First Case).</td>
<td>79</td>
</tr>
<tr>
<td>3.41 Tensile Response for Inconel 718 at 1200°F - Strain Rate $10^{-5}$ sec$^{-1}$ (5 Data - First Case).</td>
<td>80</td>
</tr>
<tr>
<td>3.42 Tensile Response for Inconel 718 at 1200°F - Strain Rate $1.1 \times 10^{-6}$ sec$^{-1}$ (5 Data - First Case).</td>
<td>81</td>
</tr>
<tr>
<td>3.43 Tensile Response for Inconel 718 at 1200°F - Strain Rate $3.3 \times 10^{-7}$ sec$^{-1}$ (5 Data - First Case).</td>
<td>82</td>
</tr>
<tr>
<td>3.44 Creep Response for Inconel 718 at 1200°F - Stress Level 125 ksi (5 Data - Second Case).</td>
<td>83</td>
</tr>
<tr>
<td>3.45 Creep Response for Inconel 718 at 1200°F - Stress Level 110 ksi (5 Data - Second Case).</td>
<td>84</td>
</tr>
<tr>
<td>3.46 Tensile Response for Inconel 718 at 1200°F - Strain Rate $10^{-5}$ sec$^{-1}$ (5 Data - Second Case).</td>
<td>85</td>
</tr>
<tr>
<td>3.47 Tensile Response for Inconel 718 at 1200°F - Strain Rate $1.1 \times 10^{-6}$ sec$^{-1}$ (5 Data - Second Case).</td>
<td>86</td>
</tr>
<tr>
<td>3.48 Tensile Response for Inconel 718 at 1200°F - Strain Rate $3.3 \times 10^{-7}$ sec$^{-1}$ (5 Data - Second Case).</td>
<td>87</td>
</tr>
<tr>
<td>4.1 An Element of a Continuum in Plane Stress or Plane Strain</td>
<td>93</td>
</tr>
<tr>
<td>4.2 Example of Finite Element Model of 2-D.</td>
<td>102</td>
</tr>
<tr>
<td>4.3 Elastic and Plastic Components of the Strain.</td>
<td>105</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>4.4 Bauschinger Effect</td>
<td>110</td>
</tr>
<tr>
<td>5.1 Benchmark Double-Notched Specimen</td>
<td>119</td>
</tr>
<tr>
<td>5.2 Grid #5 -</td>
<td>121</td>
</tr>
<tr>
<td>5.3 Grid #1 -</td>
<td>125</td>
</tr>
<tr>
<td>5.4 Grid #4 -</td>
<td>126</td>
</tr>
<tr>
<td>5.5 Experimental and Calculated (F.E.M.) Cyclic Response for Continuous Fatigue (Grids #1, 4 and 5 - $E = 25\text{ksi} \ z_o = 9.46 + 10^5\text{psi}$)</td>
<td>129</td>
</tr>
<tr>
<td>5.6 Experimental and Calculated (F.E.M.) Cyclic Response for Tension Hold (Grids #1, 4 and 5 - $E = 25\text{ksi} \ z_o = 9.46 + 10^5\text{psi}$)</td>
<td>130</td>
</tr>
<tr>
<td>5.7 Experimental and Calculated (F.E.M.) Cyclic Response for Tension Hold (Grid #5 - $E = 22.5\text{ksi} \ z_o = 9.46 + 10^5\text{psi}$)</td>
<td>132</td>
</tr>
<tr>
<td>5.8 Experimental and Calculated (F.E.M.) Cyclic Response for Continuous Fatigue (Grid #5 - $E = 22.5\text{ksi} \ z_o = 9.46 + 10^5\text{psi}$)</td>
<td>133</td>
</tr>
<tr>
<td>5.9 Experimental and Calculated (F.E.M.) Cyclic Response for Continuous Fatigue (Grid #4 - $E = 22.8\text{ksi} \ z_o = 8.46 + 10^5\text{psi}$)</td>
<td>134</td>
</tr>
<tr>
<td>A.1 Schematic of the ISDG</td>
<td>145</td>
</tr>
<tr>
<td>A.2 Schematic of the Minicomputer-Controlled ISDG System</td>
<td>149</td>
</tr>
<tr>
<td>A.3 Load Spectra</td>
<td>151</td>
</tr>
<tr>
<td>A.4 Load vs. Strain - Cycle 3 - Continuous Cycle</td>
<td>152</td>
</tr>
<tr>
<td>A.5 Load vs. Strain - Cycle 252 - Continuous Cycle</td>
<td>153</td>
</tr>
<tr>
<td>A.6 Load vs. Strain - Cycle 502 - Continuous Cycle</td>
<td>154</td>
</tr>
<tr>
<td>A.7 Load vs. Strain - Cycle 3 - Tension Hold</td>
<td>155</td>
</tr>
<tr>
<td>A.8 Load vs. Strain - Cycle 35 - Tension Hold</td>
<td>156</td>
</tr>
<tr>
<td>A.9 Load vs. Strain - Cycle 70 - Tension Hold</td>
<td>157</td>
</tr>
</tbody>
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ABSTRACT

The Bodner-Partom model of nonlinear time-dependent material behavior was incorporated into a finite element code to predict the strain response at the notch root of an Inconel 718 specimen subjected to cyclic tension-compression at 649°C.

A numerical minimization technique was developed to establish the parameters in the Bodner-Partom model. This technique was applied to creep and stress strain data obtained from button-head test specimens tested at 649°C. These parameters were found to be in general close agreement with those obtained by other investigators.

The test specimen for the experimental phase of this research was double notched with an elastic stress concentration factor of 1.94. It was subjected to cyclic plastic loads at 649°C which were large enough to generate plastic deformation at the notch root on the tensile portion of the cycle. This behavior was then predicted by incorporating the Bodner-Partom model into a finite element code. The predicted strain response and the experimentally measured (using a laser-based technique) strain response agreed reasonably well.

xii
CHAPTER I
INTRODUCTION

In the design, fabrication and service operation of structures, the engineering purpose is to produce a structure that will perform the operating function efficiently, economically and safely. To achieve these objectives, engineers make predictions of service loads, the resulting stresses, and the loads that will cause failure of the structure. Potential modes of structure failure include: general yielding, buckling or structural instability (either elastic or plastic) and finally fracture (including fatigue).

In low-cycle-fatigue limited jet engine disk components, which operate under high temperatures, severe stresses and hostile environmental conditions, local stress raisers such as notches, cracks and fillet radii, are life-limiting areas. The material at these areas does not need to contain a pre-existing defect to experience a fatigue cracking problem. Because of the high stress concentration of the life-limiting areas, the material will undergo plastic deformations where the stress-strain response is nonlinear. When the level of cyclic plastic deformation is consistently high, a crack will initiate and subsequently grow under the influence of this deformation. Also, due to the high temperature environment for the engine disk, the material under load will undergo creep deformations, where the stress-strain response is time dependent. Changing the ductility of the material, the temperature
and the environment may reduce the ability of the material to strain prior to fracture.

The primary objectives in the present research are to develop a numerical simulation technique for determining the Bodner model parameters using tensile and creep data and to use the Bodner model in a two-dimensional elastic-plastic finite element model of a double-notched specimen. The predicted elastic-plastic cyclic behavior of the specimen is compared with experimental cyclic data generated for that purpose.

A high temperature super alloy which is developed for such application as a turbine disk in a jet engine (nickel base alloy Inconel 718) is employed in this study.

Chapter II provides an overview of fatigue, creep, constitutive model and the finite element technique. This is followed by a literature survey of the recent studies on the nonlinear time-dependent constitutive theories and the use of the finite element method for the stress analysis of cracked and notched specimens.

In Chapter III, the Bodner-Partom flow law is used as the visco-plastic time dependent material model for this study. A minimization technique, coupled with the numerical integration of the constitutive equation through time using the Runge-Kutta (fourth order) method was employed to determine the material constants for this model to the best curve fitting with experimental stress-strain and creep data. The validity of this numerical determination of the material constants was verified through a comparison with published results; then it was employed to determine the material constants in Inconel 718 at 1200°F.

In Chapter IV, the formulation of the developed finite element code is given. The equilibrium equation for the elastic code was derived.
from the minimization of the potential energy. As to the elastic-plastic code, the residual force method in a form equivalent to the initial strain method was used to solve the displacement elastic-plastic problem.

In Chapter V, the validity of the linear elastic finite element code was verified through the determination of the theoretical elastic stress concentration factor ($K_t$). The comparison with the published results shows good agreement. As to the elastic-plastic finite element code, it was employed to study the local stress and strain field in a double-notched specimen under cyclic loads at 1200°F (649°C). The finite element results compared with the measured stress-strain behavior, using the laser-based technique (ISDG) developed by W. N. Sharpe[1], for two load spectra at the same temperature shows good correlation.

Based upon the numerical results in this study and the comparison with the experimental and published ones, the conclusions and recommendations are given in Chapter VI.
2.1. Overview of Fatigue

When a metal is subjected to an alternating or fluctuating stress, it is liable to develop cracks that gradually propagate through the material. The resulting fracture is called "Fatigue Failure". Because of the effect of geometry changes, which act as stress and strain concentrations, fatigue failure nearly always initiates at a geometric discontinuity. Associated with every notch is a theoretical elastic stress concentration factor ($K_t$) which is dependent on only geometry and loading mode.

$$K_t = \frac{\sigma_{\text{max}}}{S}$$  \hspace{1cm} (2-1)

where, $\sigma_{\text{max}}$ = maximum actual or local stress at the stress concentration.

$S$ = Nominal net stress on a notched member.

In fatigue, notches may be less damaging than predicted by $K_t$, therefore, a fatigue notch factor ($K_f$) is often employed and is determined by taking the ratio of unnotched fatigue strength to notched fatigue strength at a given life level.

$$K_f = \frac{\sigma_{\text{unnotched}}}{\sigma_{\text{notched}}}$$  \hspace{1cm} (2-2)

Often, a notch sensitivity index is defined as:
\[ q = \frac{(K_f - 1)}{(K_t - 1)} \] (2-3)

where the value of "q" varies from zero (no notch effect) to one (full theoretical effect), and is dependent on the material and the radius of the notch root.

In the low and intermediate life region, when yielding can occur at a notch, it must be looked upon as a strain concentration as well as a stress concentration. By definition we can write:

\[ K_\sigma = \frac{\Delta \sigma}{\Delta S} \] (2-4)

and \[ K_\varepsilon = \frac{\Delta \varepsilon}{\Delta e} \] (2-5)

where \( \sigma = \) Actual or local stress at the stress concentration
\( \varepsilon = \) Actual or local strain at the stress concentration
\( e = \) Nominal strain
\( \Delta S, \Delta \sigma, \Delta e, \Delta \varepsilon = \) Peak to peak change in the above quantities during one half cycle.

Before yielding, \( K_\sigma \) and \( K_\varepsilon \) are equal, but after yielding, \( K_\varepsilon \) increases while \( K_\sigma \) decreases, which means that the material near the notch went into the plastic region. Thus, to handle this plasticity problem, Neuber's [2] rule is employed, where the theoretical stress concentration factor, \( K_t \), is equated to the geometric mean of the stress concentration factor, \( K_\sigma \), and strain concentration factor, \( K_\varepsilon \), or:

\[ K_t = (K_\sigma K_\varepsilon)^{\frac{1}{2}} \] (2-6)

combining equations (2-4); (2-5) and (2-6):
\[ K_t = \left( \frac{\Delta \sigma \Delta e}{\Delta S \Delta e} \right)^{\frac{1}{2}} \]  
(2-7)

In case of fatigue stress, \( K_f \) is often substituted for \( K_t \), so that:

\[ K_f = \left( \frac{\Delta \sigma \Delta e}{\Delta S \Delta e} \right)^{\frac{1}{2}} \]  
(2-8)

or

\[ K_f (\Delta S \Delta e)^{\frac{1}{2}} = (\Delta \sigma \Delta e)^{\frac{1}{2}} \]  
(2-9)

Illustrated schematically, the quantities of interest are shown in Figure 2-1.

2.2. Overview of Creep

Some materials flow plastically with time under a sustained load, a phenomenon called creep. In creep, strain increases while stress remains constant. With a lapse of time, this can cause large deformations that cannot be tolerated. Figure 2-2 is an idealized creep curve that shows the transient phase (from \( t = 0 \) to \( t = t_1 \)) in which the strain rate \( \dot{\varepsilon} \) decreases with time, the steady-state phase (from \( t = t_1 \) to \( t = t_2 \)), and tertiary phase (from \( t = t_2 \) to \( t = t_3 \)) which is terminated at \( t = t_3 \) by failure. In general, the transient creep period is short, the duration of the steady-state phase is comparatively long but dependent on the magnitude of the applied load; and the tertiary phase is very brief. It has been found that the creep curve for a number of materials can be expressed by a relationship of the form [3],

\[ \varepsilon = A + Bt + C(t) \]  
(2-10)

where, \( A \) is the elastic strain at \( t = 0 \), \( Bt \) is the steady-state creep, and \( C(t) \) is the transient or primary creep. A term for tertiary creep
Figure 2-1.
Figure 2-2. Idealized Creep Curve
is not included because laboratory studies indicate its duration is usually so short that once it is initiated, failure cannot be arrested. The creep rate, \( \dot{\varepsilon} \), is dependent on the magnitude of the applied stress. It has been found experimentally that at a given time, \( t \), usually in the steady state phase, the creep rate \( \dot{\varepsilon} \) may be expressed by [3],

\[
\dot{\varepsilon} = D\sigma^n
\]

or \( \varepsilon = D\sigma^n \)

where, \( \dot{\varepsilon} \) is the creep strain rate, \( \sigma \) is the uniaxial stress, and \( D \) and \( n \) are empirical constants. However, the dependence of the creep rate on stress is complex and equation 2-11 may not be valid for all ranges of strain or for creep occurring in the steady-state phase. Also, the creep rate strongly depends on the temperature to the extent that in high temperature cases it will be a factor in design considerations.

2.3. Overview of Constitutive Models

Because of the high stress and the high temperature environment for the low-cycle fatigue limited jet engine disk component, there is an essential need for accurate constitutive models capable of predicting the material response under complicated load histories where rate-dependent "creep" and rate-independent "plasticity" effects are united. Three of these constitutive models are: the Bodner-Partom flow law; the Malvern flow law, and Norton's creep law. Each of these models consists of a set of coupled, first order, nonlinear differential equations. One of these differential equations relates the plastic strain rate \( \dot{\varepsilon}^P \) to the stress and a number of internal state variables, while the others relate the plastic strain history to the growth of the internal state
variables. In general, assuming small strain, the total strain rate $\dot{\varepsilon}_t$ is a superposition of the elastic strain rate $\dot{\varepsilon}^e$ and the plastic strain rate $\dot{\varepsilon}^p$ components.

$$
\dot{\varepsilon}_t = \dot{\varepsilon}^e + \dot{\varepsilon}^p
$$

where

$$
\dot{\varepsilon}^e = \frac{\sigma}{E}
$$

where, $E$ = Young's modulus of elasticity.

The $\dot{\varepsilon}^p$, assuming incompressibility and isotropy, is assumed to follow the Prandtl-Reuss flow law of classical plasticity:

$$
\dot{\varepsilon}^p_{ij} = \lambda S_{ij}
$$

where, $\dot{\varepsilon}^p_{ij}$ = the plastic strain rate. 

$\lambda$ = a scalar that reflects the viscosity of the material 

$S_{ij}$ = the deviatoric stress, which is left over after the hydrostatic average stress has been subtracted from the actual stress at a point.

In Malvern's [4] flow law, $\lambda$ has the form of:

$$
\lambda = \frac{3\nu_p}{2\sigma_e} \left[ \frac{\sigma_e}{\bar{\sigma}(\varepsilon^p_e)} - 1 \right] 
$$

If $\sigma_e > \bar{\sigma}(\varepsilon^p_e)$

or

$$
\lambda = 0 
$$

If $\sigma_e \leq \bar{\sigma}(\varepsilon^p_e)$

where $\nu_p$ is a fluidity constant, $\sigma_e$ is the effective stress, $\bar{\sigma}(\varepsilon^p_e)$ is
the strain hardening yield stress and \( \varepsilon^p_{e} \) is the effective plastic strain, defined incrementally as \( d\varepsilon^p_{e} = \frac{2}{3} d\varepsilon^p_{ij} \).

In Norton's [4] creep law, \( \lambda \) has the form:

\[
\lambda = \frac{3v_c}{20\varepsilon_c} [\sigma_0]^{\beta}
\]

(2-17)

where, \( v_c \) and \( \beta \) are constants determined from uniaxial creep test results.

Finally in Bodner's [5] flow law, squaring equation (2-15) gives

\[
\frac{1}{2} \varepsilon^p \varepsilon^p_{ij} = \frac{\mu^p_{ij}}{2} = \frac{1}{2} \lambda^2 S_{ij} S_{ij} = \lambda^2 J_2
\]

(2-18)

where, \( \mu^p_{ij} \) is the second invariant of the plastic strain rate and \( J_2 \) is the second invariant of the stress deviator.

or

\[
\lambda = [\frac{\mu^p_{ij}}{J_2}]^{\frac{1}{2}}
\]

(2-19)

The Bodner model was selected to be employed in this study, and it will be discussed in detail in chapter III.

2.4. Overview of Finite Element Techniques

The finite element method has proved to be a very useful tool and the most widely used method in industry for solution of complex stress analysis problems. The finite element process is a method of approximation to continuous problems in which the continuum is divided to a finite number of parts (elements). The behavior of these elements is specified by a finite number of parameters, and the solution of the complete system as an assembly of its elements follows precisely the same rules as those applicable to standard discrete problems. In many phases of engineering where the solution of stress and strain distribu-
tions in elastic continua is required, the approximation [6] can be made in the following manner:

1. The continuum is separated by imaginary lines or a surface into a number of "finite elements". 2. The elements are assumed to be interconnected at a discrete number of nodal points which will be the basic unknown parameters of the problem. 3. A set of functions is chosen to define uniquely the state of displacement within each "finite element" in terms of its nodal displacements. 4. The displacement functions now define uniquely the state of strain within an element in terms of the nodal displacements. These strains together with any initial strains and the constitutive properties of the material will define the state of stress throughout the element and also on its boundaries. 5. A system of forces concentrated at the nodes and equilibrating the boundary stress and any distributed loads is determined, resulting in a stiffness relationship of the form of

\[ \{q\} = [K]\{U\} + \{f_b\} + \{f_{e_0}\} \]  

(2-20)

where, \{q\} are the forces acting on all the nodes within an element, \{U\} is the corresponding nodal displacements, \([K]\) is the element stiffness matrix which will always be a square matrix, \{f_b\} are the nodal forces required to balance any distributed loads acting on the element, and \{f_{e_0}\} are the nodal forces required to balance any initial strains such as may be caused by temperature change if the nodes are not subject to any displacement.

Difficulties with the finite element method are: (a) It is not always easy to ensure that the chosen displacement function will satisfy
the requirement of displacement continuity between adjacent elements. Thus, the compatibility condition on such a line may be violated (though within each element it is satisfied due to uniqueness of displacement implied in their continuous representation). (b) By concentrating the equivalent forces at the nodes, equilibrium conditions are satisfied in the overall sense only. Local violation of equilibrium conditions within each element and on its boundaries will usually arise. The choice of element shape and the form of the displacement function for specific cases will very much affect the degrees of approximation.

In solid mechanics there are many situations in which such phenomena as plasticity, creep, or other complex constitutive relations supersede the simple linear elasticity assumptions. However, while in linear problems the solution is always unique because the material properties are constant, this no longer is the case in many nonlinear problems since the coefficients in the stiffness matrix vary as a function of loading. Therefore, there are two methods to find the displacement in a non-linear problem:

(1) The residual force method, where one applies a small load increment followed by adjustment of the stiffness matrix coefficients.

(2) The plastic load vector method, in which one adds on a so-called plastic load vector to the force side of the equilibrium equation.

In chapter IV, the finite element analysis will be discussed in more details.
2.5. Literature Survey

In this section, the recent studies on the non-linear time-dependent material constitutive models will be briefly reviewed, as well as the recent studies on the use of the finite element method for the stress analysis of cracked and notched plates where nonlinear time-independent and non-linear time-dependent material constitutive models are employed.

A. Material Constitutive Models

Bodner [5] utilized the constitutive equations of Bodner and Partom to represent the inelastic behavior of Rene 95 over a wide range of uniaxial loading conditions. In that study, a physical interpretation of the parameters in the constitutive equations was given. The determination of the material constants was based on experimental data for Rene 95 at 1200°F. The comparison between the experimental and the calculated curves as a measure of the curve fitting capability of the equations showed reasonable agreement. As to the stress-strain curves, time effects become more pronounced at lower rates due to the influence of the hardness recovery. Concerning creep curves, since the initial starting points correspond to the knee region of the stress-strain curve (because of the less precise fitting), the largest difference between the experimental and the calculated results occurs at the stress level corresponding to the upper transition knee of the curve.

Stouffer, Papernik and Bernstein [7] experimentally evaluated the mechanical behavior of Rene 95. Through their experimental program they concluded that the effect of the changing of strain-rate on the engineering stress-strain response at rates above 5%/min is insignificant. Also the minimum creep-rate in tension is about one decade faster than the corresponding one in compression.
Stouffer [8], in his study titled "A Constitutive Representation for IN100", used the state variable constitutive equations of Bodner and Partom to calculate the mechanical response of IN100 at 1350°F. He demonstrated that the coefficients in constitutive equations for the state variable can be determined from a systematic analysis of tensile and creep data. In general, he concluded that the calculated response reasonably agreed with the experimental results. Also the strain-rate sensitivity in the tensile tests, low stress or short time creep response, and amount of stress relaxation up to 500 minutes were reasonably predicted. But the long time stress relaxation response and the tensile, tertiary creep response was not reasonably predicted.

Hinnerichs [4] estimated the material constants for IN100 from only three experiments: one stress-strain test data at a specific value of the strain rate and two creep test data at two different stress levels. Using Bodner-Partom constitutive equations, the viscoplastic material constants in these equations were broken into "short time response" and "creep" groupings for determination. Based on one stress-strain test data at a specific value of the strain rate, the short time response constants were determined. Also, based on data from at least two creep tests at two different stress levels, the creep constants were determined.

In her study, Milly [9], represented the experimental data for Inconel 718 at 1200°F, from which the material constants were determined by the method given by Stouffer [8]. The Bodner constitutive equations were then applied. Milly compared the theory and the experimental data. She concluded that the overall behavior is very good. Concerning Stouffer's method of determining the material constants, although it is

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consistent, the constants are not suitable to fit the experimental data. As to the Bodner theory, Milly reported that it does not model the softening behavior at large strains.

B. The Finite Element Study

For high temperature creep and fatigue, several researchers have recently considered the use of finite element methods to study the threshold fatigue and creep crack growth at high temperatures, and many reports have been published on this subject [10-14]. The review here will be restricted to the central cracked and double edge notched specimens in both elastic and plastic conditions.

Concerning the crack growth from the initial notch in 304 stainless steel, Yokobori, Sakata and Yokobori, Jr., [13] continuously observed the geometrical change of the notch shape from the instant of load application under high temperature. They also used the finite element method to calculate the effective crack length (taking into account the effect of the initial notch instead of the equivalent crack length) for the accurate estimation of local stress distribution near the tip of the crack initiated from the initial notch root. They found that the geometrical change of the notch shape is nearly completed by the time when the crack initiates at the notch root, and that the shape at the instant of the crack initiation is almost independent of the experimental conditions such as temperature, gross section stress and holding time.

Newman, [16] used an elastic-plastic finite element analysis in conjunction with a crack-growth criterion based on crack-tip strain (whenever the crack-tip strain is equal to or greater than the critical strain, the crack will grow) to study crack growth behavior under monotonic and cyclic loading. He found that the finite-element analysis
predicted three stages of crack growth behavior (no crack growth, stable
crack growth and instable crack growth). Also the growth was found to
be dependent upon the mesh size, the material strain hardening, the
critical strain and the specimen type. As to crack growth under cyclic
loading, the crack-closure effect was accounted for in the analysis.

Varanasi [17] developed a two-dimensional finite element analysis
to model the slow crack growth prior to instability under rising loads,
where the plastic region behind the advancing crack tip is unloaded
while the region ahead of the crack tip is being loaded. The crack-
growth criterion was based on local failure (whenever the maximum prin­
ciple stress at the crack tip node is equal to or greater than the
ultimate stress, the crack will grow). He concluded that the results of
his study compare reasonably well with test results.

Newman, [18] investigated crack extension and crack closure in a
center-crack panel under cyclic loading. In his study, he developed a
two-dimensional finite-element program, which accounts for both elastic-
plastic material behavior and changing boundary conditions associated
with crack extension and intermittent contact of the crack surfaces
under cyclic loading. To model the crack tip region, three different
mesh sizes were used to show how the element-mesh size in the crack-tip
region influences the calculated crack closure and crack opening stress-
es.

Ahmad [19] in his report, provides a user's guide for a special
purpose finite element code developed primarily for two-dimensional
linear elastic analysis of test specimens. It includes some supporting
programs, such as mesh generating and mesh plotting for commonly used
test specimens.
Domas, Sharpe, Ward and Yau [20] in the analytical task of their study for the "Benchmark Notch Test for Life Prediction", used the finite element method for good estimation of the elastic stress concentrating factor ($K_t$). One two-dimensional and two three-dimensional models were employed. The two-dimensional model was for the elastic analysis, but as to the two three-dimensional models, an eight-noded isoparametric brick element in elastic-plastic-creep code was used. From calculated stresses and strains at element centroids (computer output), an acceptable value of ($K_t$) was determined. Concerning the finite element solution using the cyclic stress strain curve, for a better correlation, they suggested that the material properties of the finite element model should be properly chosen for selected areas.

Hinnerichs [4] analyzed the creep crack growth in a nickel alloy at elevated temperatures through a simultaneous use of creep crack growth test displacement data from center cracked plate specimens of IN-100 at 1350°F and a theoretical two-dimensional (plane stress-plane strain) finite element program which accounts for both nonlinear viscoplastic material behavior and changing boundary conditions due to crack growth. Also, he investigated several crack growth rate criteria, one of which is the stress intensity factor that showed good agreement with the published results, but some other criteria such as $C^*$ integral and load point displacement rate which are closely related theoretically did not provide a unique solution for the crack growth rate.

2.6. Objectives

Even though during the last few years many reports have been published on the subject of constitutive theory of nonlinear materials, unfortunately, no numerical simulation results are available for com-
parison with the experimental and analytical results. Therefore, a numerical simulation consideration of nonlinear time-dependent material, where the material constants are numerically calculated using a curve fitting technique, is needed for complete comparison and to establish a good evaluation for the material variables of Inconel 718 during high temperature fatigue and creep. Also, for good correlation with experimental and published results on the Benchmark notched specimens, a theoretical model of the test specimen, which accounts for the cyclic behavior, is needed to give a good study for the local plastic stress and strain field.

Therefore, a stress analysis model that includes the significant geometrical variables was selected for this study. Other variables such as temperature, environment and number of repeated load cycles can be investigated in terms of their influence on the parameters that emerge from the stress analysis at the life limiting areas such as crack tip, notches, fillet radii, etc.

Two main factors have been considered during the course of this study:

(1) The nonlinear time-dependent material behavior of the laboratory specimens (because of the large stress concentration at the life limiting areas).

(2) The fact that the finite element method (FEM) is currently the most widely used method in the industry for solution of complex stress analysis problems.

The objectives of this research, in this light, are:

1. To develop a numerical simulation technique for determining the nonlinear time-dependent material variables using the
Bodner constitutive model coupled with tensile and creep data generated at one stage of the experimental task of this study for that purpose.

2. To develop a constant-strain-triangle, two-dimensional, elastic-plastic finite element model of the double-notched specimen in which the Bodner model is incorporated. This finite element code accounts for elastic and elastic-plastic cyclic behavior of the material to compare with experimental cyclic data (for two load spectra) generated at one stage of the experimental task of this study for that purpose.
CHAPTER III

NUMERICAL REPRESENTATION OF TIME DEPENDENT MATERIAL MODEL

The design and analysis of structural components which operate at elevated temperature levels and severe stresses, such as a low-cycle fatigue-limited jet engine disk, require an accurate prediction of the nonlinear stress-strain response encountered during the cyclic loading conditions. Nonlinear analysis of such components are normally carried out in a finite element code which makes use of constitutive theories in which the material response is separated into the two important groups of phenomena known as rate-dependent "creep" and rate-independent "plasticity". A number of viscoplastic constitutive theories in which "creep" and "plasticity" effects are combined into a unified plastic strain model have recently been proposed and are still undergoing active development. One of these theories is the constitutive theory of Bodner and Partom, or, simply, Bodner Model.

In this chapter, we will consider the constitutive equations of Bodner-Partom to represent the time-dependent inelastic properties of Inconel 718 at 1200°F (649°C) over a wide range of loading conditions.

3.1. The Constitutive Theory of Bodner and Partom

Assuming small strains, the constitutive theory of Bodner and Partom [5] is based on the assumption that the total strain rate $\dot{\varepsilon}_t(t)$
is separated into elastic (reversible), $\varepsilon^e(t)$, and plastic (irreversible), $\varepsilon^p(t)$, components, which are both non-zero for all loading/unloading conditions.

$$\dot{\varepsilon}^t(t) = \dot{\varepsilon}^e(t) + \dot{\varepsilon}^p(t) \quad (2-13)$$

$$\dot{\varepsilon}^e(t) = \frac{\sigma(t)}{E} \quad (2-14)$$

As to the plastic strain rate, $\dot{\varepsilon}^p(t)$, the specific representation used by Bodner and Partom is given by

$$\dot{\varepsilon}^p(t) = \frac{2}{\sqrt{3}} \cdot \frac{\sigma(t)}{\sigma(t)} \cdot D_0 \exp \left[ \frac{1}{2} \cdot \left( \frac{\varepsilon(t)}{\sigma(t)} \right)^{2n} \cdot \left( \frac{n+1}{n} \right) \right] \quad (3-1)$$

where; $\sigma(t) = \text{The current value of the stress.}$

$D_0 = \text{Constant, represents the limiting value of the plastic strain rate in shear. Generally it is taken at } 10^4 \text{ sec}^{-1}$

except for conditions of very high rates of straining.

$n = \text{Material Constant, it is related to the viscosity of the dislocation motion. It controls the strain rate sensitivity.}$

$Z(t) = \text{the plastic state variable measure of the overall resistance to plastic flow.}$

The evolution equation, i.e. history dependence, of the plastic state variable is generally sought in the form of a differential equation for the hardening rate, $\dot{Z}$, that depends on stress, temperature and hardness, $Z$. A more specific representation is based on the concept that only the plastic rate of working, $W^p$, and current hardness, $Z$, control the rate of hardening. The complete expression for, $Z$, can be written as [5],
\[ Z = Z_1 - (Z_1 - Z_0) \exp(-mW^p) \]

or

\[ (Z_1 - Z) = (Z_1 - Z_0) \exp(-mW^p) \]

where, \( m \) = material constant that controls the rate of work hardening.

\( Z_1 \) = the saturation value of \( Z \) for large \( W^p \), i.e., it is the maximum value of \( Z \) which is taken to be constant.

\( Z_0 \) = the initial value of \( Z \), corresponding to the reference slate from which \( W^p \) is measured with the limits \( (0 \leq Z_0 \leq Z_1) \).

and

\[ W^p = \int \sigma \dot{e}^P dt + \int \frac{\dot{Z}_{\text{recovery}}}{m(Z_1 - Z)} dt \quad (3-3) \]

or

\[ \dot{W}^p = \sigma \dot{e}^P + \frac{\dot{Z}_{\text{rec.}}}{m(Z_1 - Z)} \quad (3-4) \]

where,

\[ \dot{Z}_{\text{rec.}} = -AZ_1 \left( \frac{Z - Z_i}{Z_1} \right)^r \]

\[ (3-5) \]

Therefore, the complete expression for \( \dot{Z} \), can be written as

\[ \dot{Z} = m(Z_1 - Z) \dot{W}^p - AZ_1 \left( \frac{Z - Z_i}{Z_1} \right)^r \quad (3-6) \]
where, \( A \) = the coefficient controlling the rate of hardening recovery
\( r \) = the exponent controlling the rate of hardening recovery
\( Z_i \) = a state variable corresponding to the complete non-work hardened condition. It is a function of the temperature.

In equation 3-6, the second term corresponding to hardening recovery is negligible during rapid load histories. Therefore, for long time response, such as creep, the second term in equation 3-6 is necessary, but during a tensile test that is fast compared to a creep test, equation 3-6 reduces to the first term only, i.e., for a tensile test equation 3-6 becomes

\[
\dot{Z} = m(Z_i - Z)^r W^P \tag{3-7}
\]

where, in this case, \( \dot{W}^P \) is determined by

\[
\dot{W}^P = \sigma \dot{\varepsilon}^P \tag{3-8}
\]

Since test data can be resolved into the forms, \( \sigma \) and \( \dot{\varepsilon}^P \), equation 3-1 is solved for \( Z \) which then is a function of \( \sigma \) and \( \dot{\varepsilon}^P \) as follows

\[
Z = \sigma \left[ \frac{2n}{n + 1} \ln \left( \frac{2}{\sqrt{3}} \frac{D_0}{\varepsilon^P} \right) \right]^{\frac{1}{2n}} \tag{3-9}
\]

In order to determine the viscoplastic material constants in these constitutive equations, the constants can be broken into two groups, "Creep response" and "short time response". The short time response constants are \( D_0, n, m, Z_0, Z_1 \) and they are determined based on stress-strain test data. The creep response constants are \( Z_i, r, A \), and they are determined based on data from at least two creep tests at two dif-
ferent stress levels. Step-by-step theoretical evaluation of the mate-
rial parameters was developed by Bodner [5], for Rene 95, and by 
Stouffer [8], for In 100. Also, Stouffer and Bodner [21], studied the 
relationship between theory and experiment for the state variable con-
stitutive equation. Therefore, for a complete study of the constitutive 
equations, a numerical evaluation of the material parameters is needed.

3.2 Numerical Evaluation of the Material Parameters

In this section, a numerical study of nonlinear time dependent 
material will be considered, where the material variables are 
numerically calculated using a curve fitting technique.

In general the Bodner material parameters are dependent on tempera-
ture, but by performing the material characterization tests (stress-
strain and creep) at the same temperature that the Bodner model will be 
applied, the temperature dependence is suppressed.

To determine the Bodner variables \((n, Z_0, Z_1, m, A, r, Z_J \text{ and } D_0)\) 
numerically, we will consider the actual and the theoretical evaluation 
of the plastic strain rate. For the actual evaluation of \(\dot{\varepsilon}^p(t)\), the 
total strain rate is the sum of elastic and plastic strain rates

\[
\dot{\varepsilon}^t(t) = \dot{\varepsilon}^e(t) + \dot{\varepsilon}^p(t) \quad (2-13)
\]

where

\[
\dot{\varepsilon}^e(t) = \frac{\dot{\sigma}(t)}{E} \quad (2-14)
\]

Therefore, we can rewrite equation 2-13 in the form

\[
\dot{\varepsilon}^p(t) = \dot{\varepsilon}^t(t) - \frac{\dot{\sigma}(t)}{E} \quad (3-10)
\]
where
\[ \dot{\varepsilon}^t(t) = \frac{d\varepsilon^t}{dt} = \text{the slope of (\varepsilon - t) curve} \]

and
\[ \dot{\sigma}(t) = \frac{d\sigma}{dt} = \text{the slope of (\sigma - t) curve} \]

then, equation 3-10 becomes
\[ \varepsilon^P(t) = (d\varepsilon^t(t)/dt) - (d\sigma(t)/dt)/E \] (3-11)

Having the experimental data for the stresses, strain and time, the actual value of \( \varepsilon^t \) is evaluated:
\[ \varepsilon^t(t) = \varepsilon^t(0) + \int_0^t \varepsilon^P(t)dt + \frac{\sigma(t)}{E} = f_{\text{Actual}}(t) \] (3-12)

As to the theoretical evaluation of \( \varepsilon^P \), we have
\[ \varepsilon^P(t) = \lambda S \] (2-15)

where, \( S \) is the deviatoric stresses, which are the stresses left over after the hydrostatic average stresses has been subtracted from the actual stresses at a point, and depends only on time.

\[ \lambda = \lambda(X) \]

where
\[ <X> = <n, Z_0, Z_1, m, A, r, Z_i, D> \] (3-13)
therefore, equation 2-15 becomes
\[ \varepsilon^p(t) = \lambda(X)S(t) \]

and
\[ \varepsilon^t(t) = \varepsilon^t(o) + \varepsilon^p(t)dt + \frac{\sigma(t)}{E} \]

where,
\[ = f_{\text{Theoretical}}(t,X) \] \hspace{2cm} \text{(3-15)}

Now, let
\[ Q = \sum_{\text{data}} w(t)(f_{\text{th}} - f_{\text{act}})^2 \]

or
\[ Q = \int_{\text{data}} w(t)(f_{\text{th}} - f_{\text{act}})^2 dt \] \hspace{2cm} \text{(3-17)}

where, \( w(t) \) is a positive weight function with the value of one for equal importance, \( f_{\text{th}} - f_{\text{act}} \) is the error. \( Q \) was minimized in a computerized numerical scheme by varying values of the material coefficients. In this computer program, the Runge Kutta (fourth order) algorithm was employed for the numerical time integration of the Bodner equations in the following order:

\[ \dot{\varepsilon}^p = \lambda S \]

\[ Z = Z_1 - (Z_1 - Z_0) \exp[-MW^p] \]

\[ D_2^p = D_0^2 \exp[-\left(\frac{Z_2^2}{3J_2}\right)^n \left(\frac{n + 1}{n}\right)] \]

\[ \varepsilon^p = [D_2^p J_2]^{\frac{3}{2}} S \] \hspace{2cm} \text{(3-18)}
\[
\dot{Z}_{\text{rec}} = -A\left(\frac{Z - Z_i}{Z_1}\right)^{rZ_1}
\]

\[
W^p = \sigma \cdot d\epsilon^p + \int \left[\frac{\dot{Z}_{\text{rec}}}{m(Z_1 Z)}\right] dt
\]

Using Powell's [22] iteration algorithm coupled with the least square method, the specific material variables for the Bodner model were then determined to best fit the tensile and creep data.

3.3. Comparison of the numerical model versus published data

The numerical results for this study are given in computer printout and graphs. To evaluate these numerical results, a comparison of the model versus full set of data for IN100 from Stouffer's study [8], with his analytical results and limited set of data for Inconel 718 from Millys study [9], with her analytical results, has been made. Then, Coupling the model with the data for Inconel 718 at 1200°F from the experimental task of this study, the material variables for the model were numerically determined to best fit to the experimental data.

Table 3-1 shows the value of the material variables of IN100 at 1350°F determined from the full data set which were the assumed values for the numerical evaluation. The table also shows the numerical results and the value of Q for the same material.

The numerical calculations were done for two sets of tensile data at steady strain rates $1.42 \times 10^{-3}$ and $6.67 \times 10^{-6}$ sec$^{-1}$, and two sets of creep data at constant stress levels 120 and 72 Ksi.

The numerical, analytical and experimental tensile response is shown in figures 3-1 and 3-2. It can be seen that the agreement is very well with very good curve fitting at the higher rate $1.42 \times 10^{-3}$ sec$^{-1}$
<table>
<thead>
<tr>
<th>MATERIAL VARIABLE</th>
<th>UNITS</th>
<th>Analytical Values From Stouffer's Study (the starting values for the model)</th>
<th>Numerical Values from the Model with $Q = 1.0373 \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>-</td>
<td>0.7</td>
<td>0.707</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>psi</td>
<td>$9.15 \times 10^5$</td>
<td>$9.14 \times 10^5$</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>psi</td>
<td>$10.15 \times 10^5$</td>
<td>$10.03 \times 10^5$</td>
</tr>
<tr>
<td>$m$</td>
<td>psi$^{-1}$</td>
<td>$2.57 \times 10^{-3}$</td>
<td>$1.21 \times 10^{-3}$</td>
</tr>
<tr>
<td>$A$</td>
<td>sec$^{-1}$</td>
<td>$1.90 \times 10^{-3}$</td>
<td>$1.86 \times 10^{-3}$</td>
</tr>
<tr>
<td>$R$</td>
<td>-</td>
<td>2.66</td>
<td>2.60</td>
</tr>
<tr>
<td>$Z_i$</td>
<td>psi</td>
<td>$6 \times 10^5$</td>
<td>$5.83 \times 10^5$</td>
</tr>
<tr>
<td>$D_0$</td>
<td>sec$^{-1}$</td>
<td>$10^4$</td>
<td>$8.48 \times 10^3$</td>
</tr>
<tr>
<td>$E$</td>
<td>psi</td>
<td>$21.3 \times 10^6$</td>
<td>$24.5 \times 10^6$</td>
</tr>
</tbody>
</table>
Figure 3-1. Tensile Response of IN100 at 1350°F, strain rate = 1.42*10^{-3} \text{ sec}^{-1}

Triangles are the Data, Diamonds are the Model and Circles are Stouffer's.
Figure 3-2. Tensile Response of IN100 at 1350°F, strain rate = 6.6×10⁻⁶ sec⁻¹.
Triangles are the Data, Diamonds are the Model and Circles are Stouffer's.
but the analytical and the model (up to the knee of the curve) are slightly high for the lower rate $6.67 \times 10^{-6} \text{ sec}^{-1}$. This is due to the fact that, at high rates, the steady state values of $Z$ are equal to the saturation value of $Z$, $Z_1$, since thermal recovery effects are small, but these effects become noticeable at the lower rates for which the steady state $Z$ is less than $Z_1$.

The numerical, analytical and experimental creep response is shown in figures 3-3 and 3-4 from which it can be seen that the numerical response compares very well and the curve fitting at both stress levels is very good. But since the model formulation does not include effects that would lead to tertiary creep, therefore, the tertiary creep is not indicated in the numerical results. As to the analytical results, at the lower value of the stress 72 ksi the agreement is good, however, at the higher value of the stress 120 ksi the shape of the experimental and analytical curves do not agree.

Table 3-2 shows the value of the material coefficients for Inconel 718 at 1200°F determined from limited data set which were again the values for the numerical calculations. It is also shown in table 3-2 the numerical results and the value of $Q$ for the same material.

The numerical calculations were done for three tensile data set at steady strain rate $10^{-3}$, $8.3 \times 10^{-5}$ and $8.3 \times 10^{-7} \text{ sec}^{-1}$ and two creep data at constant stress levels 125 and 110 ksi.

The numerical, analytical and experimental tensile response is shown in figures 3-5 through 3-7. At the rate $10^{-3} \text{ sec}^{-1}$, the analytical agreement is better than the numerical agreement which is higher than the experimental data, while at the rate $8.3 \times 10^{-5} \text{ sec}^{-1}$, the numer-
Figure 3-3. Creep Response of IN100 at 1350°F, stress = 120 ksi
Triangles are the Data, Diamonds are the Model and
Circles are Stouffer's.
Figure 3-4. Creep Response of IN100 at 1350°F, stress = 72 ksi. Triangles are the Data, Diamonds are the Model, and Circles are Stouffer's.
### TABLE (3.2) ANALYTICAL AND NUMERICAL VARIABLES FOR INCONEL 718
(AT 1200°F)

<table>
<thead>
<tr>
<th>MATERIAL VARIABLE</th>
<th>UNITS</th>
<th>Analytical Values From Milly's Study (the starting values for the model)</th>
<th>Numerical Values from the Model with ( Q = 4.3113 \times 10^{-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td></td>
<td>1.167</td>
<td>1.215</td>
</tr>
<tr>
<td>( Z_0 )</td>
<td>psi</td>
<td>4.537 \times 10^5</td>
<td>4.836 \times 10^5</td>
</tr>
<tr>
<td>( Z_1 )</td>
<td>psi</td>
<td>6 \times 10^5</td>
<td>5.87 \times 10^5</td>
</tr>
<tr>
<td>( m )</td>
<td>psi (^{-1})</td>
<td>1.674 \times 10^{-4}</td>
<td>2.57 \times 10^{-4}</td>
</tr>
<tr>
<td>( A )</td>
<td>sec (^{-1})</td>
<td>1.1 \times 10^{-4}</td>
<td>2.848 \times 10^{-4}</td>
</tr>
<tr>
<td>( R )</td>
<td></td>
<td>2.857</td>
<td>4.73</td>
</tr>
<tr>
<td>( Z_i )</td>
<td>psi</td>
<td>4 \times 10^5</td>
<td>4.83 \times 10^5</td>
</tr>
<tr>
<td>( D_0 )</td>
<td>sec (^{-1})</td>
<td>10^4</td>
<td>5.49 \times 10^4</td>
</tr>
<tr>
<td>( E )</td>
<td>psi</td>
<td>24 \times 10^6</td>
<td>21 \times 10^6</td>
</tr>
</tbody>
</table>

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Figure 3-5. Tensile Response of Inconel 718 at 1200°F, strain rate = $10^{-3}$ sec$^{-1}$
Triangles are the Data, Diamonds are the Model and Circles are Milly's.
Figure 3-6. Tensile Response of Inconel 718 at 1200°F, strain rate = $8.3 \times 10^{-5}$ sec$^{-1}$. Triangles are the Data, Diamonds are the Model and Circles are Milly's.
Figure 3-7. Tensile Response of Inconel 718 at 1200°F, strain rate = 8.3 x 10^{-7} sec^{-1}
Triangles are the Data, Diamonds are the Model and Circles are Milly's.
ical agreement is better than the analytical agreement which is lower than the experimental data. At the rate $8.3 \times 10^{-7} \text{ sec}^{-1}$, both agreements in the plastic region are not as good as the numerical agreement in the elastic region.

The numerical, analytical and experimental creep response is shown in figure 3-8 and 3-9. At the higher stress level 125 ksi the numerical agreement is good, but the shape of the analytical and experimental curves do not agree. However, at the lower stress level 110 ksi, both numerical and analytical responses do not agree with the experimental response.

This inconsistent behavior of both numerical and analytical response could be attributed to the sensitivity of the Bodner constitutive model to the data and the starting values for the variables coupled with using a limited set of data.

3.4 Numerical Representation of Inconel 718 by the Bodner Constitutive Model

3.4.1 The Experimental Data

At one stage in the experimental task of this study, a group of tensile and creep tests were performed for Inconel 718 at 1200°F (650°C). The Bodner model, coupled with this set of data, was employed to determine the material variables of Inconel 718 to the best fitting of the data.

To help eliminate inconsistencies in data, the same specimen geometry was used for all tests. A drawing of the button-head specimen, used in this study, is presented in figure 3-10.
Figure 3-8. Creep Response of Inconel 718 at 1200°F, stress = 125 ksi.
Triangles are the Data, Diamonds are the Model and Circles are Milly's.
Figure 3-9. Creep Response of Inconel 718 at 1200°F, stress = 110 ksi
Triangulars are the Data, Diamonds are the Model and Circles are Milly's.
All experiments were performed in an electrohydraulic testing machine equipped with a special high-temperature furnace. Special attention was given to the alignment of the specimen to minimize the eccentricity of the load and to obtain a uniform temperature profile in the test section. The machine was run under strain control. The data (stress, strain, time) was obtained by using the Interferometric Strain/Displacement Gage (ISDG) technique discussed in chapter V.

3.4.2 Numerical Evaluation Procedures

The numerical evaluations were done for five tensile tests at steady strain rates: \( \dot{\varepsilon} = 1.6 \times 10^{-3}, 6.75 \times 10^{-5}, 10^{-5}, 1.1 \times 10^{-6} \) and \( 3.3 \times 10^{-7} \) sec\(^{-1} \) and two creep data at constant stress levels: \( \sigma = 125 \) and 110 ksi.

At the outset of this study we had to smooth the experimental data, and the numerical evaluations were done individually by employing the computer code for each set of data. Then, the numerical evaluations of the material variables were redone by running all the sets of data in unison. The result of this procedure revealed the best fit data and consequently the final numerical evaluations depended exclusively on the data with the best fitting.

To study the validity of Bodner model parameters, the above explained procedure was employed twice with two different starting (assumed) values for the material parameters.

In the numerical evaluations, Young's modulus of elasticity, \( E \), was included in the material variables. The idea behind setting, \( E \), as one of the variables was based on the following points:

(i) In Stouffer's study [8] for IN100 at 1350°F, two different values for "\( E \)" were observed for the limited data and the full
data, hence it can be seen that the value of "E" is somewhat variable.

(ii) from equations 2-14 and 3-10, it can be recognized that the strain rate, $\dot{\varepsilon}$, depends on the value of "E". However, the disadvantage of setting "E" as a variable in the numerical evaluations is increasing the number of search variables by one.

A computer program listing for the numerical evaluation of the Bodner parameters is presented in Appendix B. The program has been run on the IBM 3033 at Louisiana State University.

3.5 Discussion of the Results

The individual runs show that the agreement between the experimental data and the numerical model is quite accurate with a precise curve fitting of each and every set of data with $Q$ in the order of $10^{-6}$. As to the seven sets of data run in unison, two tensile data sets ($1.6 \times 10^{-3}$ and $6.75 \times 10^{-5}$ sec$^{-1}$) were not as good as the rest of the data and $Q$ was in the order of $6 \times 10^{-4}$. This result is due to the fact that in these two tensile data sets the value of the modulus of elasticity, $E$, is too high ($E = 31.18 \times 10^6$ and $29.91 \times 10^6$ psi respectively) as compared to the other data where $23 \times 10^6 \leq E \leq 25 \times 10^6$. In other words, these two tensile data sets were then excluded and the numerical evaluations were applied only to the other five sets of data (2 creep tests at $\sigma = 125$ and 100 ksi and 3 tensile tests at $\dot{\varepsilon} = 10^{-5}$, $1.1 \times 10^{-6}$ and $3.3 \times 10^{-7}$ sec$^{-1}$) as a group.

Tables 3-3, 3-4 and 3-5 show the first and the second case of the assumed values with the numerically calculated (from the model) values.
<table>
<thead>
<tr>
<th>Material Variable</th>
<th>Starting Values</th>
<th>Final Numerical Values for Individual Runs</th>
<th>$\alpha=125$ ksi</th>
<th>$\alpha=110$ ksi</th>
<th>$\dot{\varepsilon}=1.6\times10^{-3}$ sec$^{-1}$</th>
<th>$\dot{\varepsilon}=6.75\times10^{-5}$ sec$^{-1}$</th>
<th>$\dot{\varepsilon}=10^{-5}$ sec$^{-1}$</th>
<th>$\dot{\varepsilon}=1.1\times10^{-6}$ sec$^{-1}$</th>
<th>$\dot{\varepsilon}=3.3\times10^{-7}$ sec$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>0.7112</td>
<td>0.7144</td>
<td>0.6839</td>
<td>0.6115</td>
<td>0.7005</td>
<td>0.6805</td>
<td>0.7246</td>
<td>0.7166</td>
<td></td>
</tr>
<tr>
<td>$Z_0$</td>
<td>9.162\times10^5</td>
<td>9.711\times10^5</td>
<td>9.158\times10^5</td>
<td>9.161\times10^5</td>
<td>9.288\times10^5</td>
<td>9.851\times10^5</td>
<td>9.659\times10^5</td>
<td>9.475\times10^5</td>
<td></td>
</tr>
<tr>
<td>$Z_1$</td>
<td>11.39\times10^5</td>
<td>11.42\times10^5</td>
<td>10.71\times10^5</td>
<td>9.228\times10^5</td>
<td>10.700\times10^5</td>
<td>11.059\times10^5</td>
<td>11.328\times10^5</td>
<td>11.328\times10^5</td>
<td></td>
</tr>
<tr>
<td>$m$</td>
<td>2.86\times10^{-4}</td>
<td>2.581\times10^{-4}</td>
<td>4.119\times10^{-3}</td>
<td>8.716\times10^{-4}</td>
<td>2.757\times10^{-4}</td>
<td>2.720\times10^{-3}</td>
<td>1.036\times10^{-3}</td>
<td>5.358\times10^{-4}</td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>5.419\times10^{-1}</td>
<td>1.911\times10^{-4}</td>
<td>6.120\times10^{-1}</td>
<td>18.689\times10^{-1}</td>
<td>99.774\times10^{-1}</td>
<td>1.739\times10^{-2}</td>
<td>15.197\times10^{-1}</td>
<td>5.489\times10^{-1}</td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>3.96</td>
<td>1.28</td>
<td>4.45</td>
<td>4.73</td>
<td>4.73</td>
<td>3.58</td>
<td>4.73</td>
<td>4.73</td>
<td></td>
</tr>
<tr>
<td>$Z_1$</td>
<td>9.139\times10^5</td>
<td>9.168\times10^5</td>
<td>9.142\times10^5</td>
<td>9.151\times10^5</td>
<td>9.215\times10^5</td>
<td>8.671\times10^5</td>
<td>8.756\times10^5</td>
<td>8.820\times10^5</td>
<td></td>
</tr>
<tr>
<td>$D_0$</td>
<td>1.108\times10^5</td>
<td>1.118\times10^5</td>
<td>1.102\times10^5</td>
<td>1.299\times10^5</td>
<td>1.102\times10^5</td>
<td>1.657\times10^3</td>
<td>1.106\times10^3</td>
<td>1.100\times10^3</td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>2.5\times10^7</td>
<td>2.499\times10^7</td>
<td>2.499\times10^7</td>
<td>2.887\times10^7</td>
<td>2.938\times10^7</td>
<td>2.5\times10^7</td>
<td>2.5\times10^7</td>
<td>2.366\times10^7</td>
<td></td>
</tr>
<tr>
<td>$Q$</td>
<td>-</td>
<td>6.437\times10^{-6}</td>
<td>8.368\times10^{-7}</td>
<td>4.537\times10^{-6}</td>
<td>8.778\times10^{-6}</td>
<td>4.100\times10^{-6}</td>
<td>3.582\times10^{-6}</td>
<td>2.066\times10^{-6}</td>
<td></td>
</tr>
<tr>
<td>Number of Iterations</td>
<td>-</td>
<td>3004</td>
<td>873</td>
<td>1007</td>
<td>650</td>
<td>2033</td>
<td>719</td>
<td>564</td>
<td></td>
</tr>
<tr>
<td>CPU Time (sec)</td>
<td>-</td>
<td>35.4</td>
<td>10.55</td>
<td>12.34</td>
<td>8.51</td>
<td>23.30</td>
<td>10.12</td>
<td>10.93</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 3-5. STARTING AND FINAL NUMERICAL VARIABLES FOR INCONEL 718 AT 1200°F (GROUP RUNS)

<table>
<thead>
<tr>
<th>Material Variable</th>
<th>Starting Values</th>
<th>Final Numerical Values (First Case)</th>
<th>Final Numerical Values (Second Case)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First Case</td>
<td>Second Case</td>
<td>Set of Seven Data</td>
</tr>
<tr>
<td>n</td>
<td>0.7112</td>
<td>0.7</td>
<td>0.7224</td>
</tr>
<tr>
<td>Z_1</td>
<td>11.39*10^5</td>
<td>10.15*10^5</td>
<td>12.563*10^5</td>
</tr>
<tr>
<td>m</td>
<td>3.286*10^-4</td>
<td>2.57*10^-3</td>
<td>7.661*10^-4</td>
</tr>
<tr>
<td>A</td>
<td>5.419*10^-1</td>
<td>2.701*10^-4</td>
<td>31.682*10^-1</td>
</tr>
<tr>
<td>R</td>
<td>3.98</td>
<td>4.704</td>
<td>3.87</td>
</tr>
<tr>
<td>Z_i</td>
<td>9.139*10^5</td>
<td>5.084*10^5</td>
<td>8.889*10^5</td>
</tr>
<tr>
<td>D_0</td>
<td>1.108*10^3</td>
<td>10^4</td>
<td>1.102*10^3</td>
</tr>
<tr>
<td>E</td>
<td>2.5*10^7</td>
<td>2.5*10^7</td>
<td>2.337*10^7</td>
</tr>
<tr>
<td>Q</td>
<td>-</td>
<td>-</td>
<td>5.609*10^-4</td>
</tr>
</tbody>
</table>
TABLE 3-5 (continued)

<table>
<thead>
<tr>
<th>Material Variable</th>
<th>Assumed Values</th>
<th>Numerical Values (First Case)</th>
<th>Numerical Values (Second Case)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First Case</td>
<td>Second Case</td>
<td>Set of Seven Data</td>
</tr>
<tr>
<td>Number of Iterations</td>
<td>-</td>
<td>-</td>
<td>739</td>
</tr>
<tr>
<td>CPU Time (sec)</td>
<td>-</td>
<td>-</td>
<td>62.05</td>
</tr>
</tbody>
</table>
of the material variables for Inconel 718 at 1200°F determined to the best fitting of the data, individually, as a group of seven and as a group of five sets of data for each case.

The experimental and numerical response for the individual tests is shown in figures 3-11 through 3-17 for the first case and in figures 3-18 through 3-24 for the second case. The response for the seven tests in one group is shown in figures 3-25 through 3-31 for the first case and in figures 3-32 through 3-38 for the second case, where figures 3-39 through 3-43 and figures 3-44 through 3-48 show the response of the five tests as one group in the first and second cases.

From tables 3-3, 3-4 and 3-5 and figures 3-11 through 3-48 it can be seen that:

(1) For the individual runs, the curve fitting is extremely good with \( Q \) on the order of \( 10^{-6} \).

(2) In some of these individual runs such as: \( \sigma = 125 \) ksi and \( \dot{\varepsilon} = 10^{-5} \text{ sec}^{-1} \) in the first case and \( \sigma = 125 \) ksi and \( \dot{\varepsilon} = 6.75 \times 10^{-5} \text{ sec}^{-1} \) in the second case, the values of "\( Z_0 \)" and "\( Z_1 \)" are equal and this pushes the definitions of these two parameters in which \( 0 \leq Z_0 \leq Z_1 \).

(3) For the group of the one set of seven data the value of \( Q \) was on the order of \( 6 \times 10^{-4} \) with good data agreement for the two creep tests (\( \sigma = 125 \) and 110 ksi) and the three tensile tests (\( \dot{\varepsilon} = 10^{-5}; 1.1 \times 10^{-6} \) and \( 3.3 \times 10^{-7} \text{ sec}^{-1} \)), but the agreement in the other two tensile tests (\( \dot{\varepsilon} = 1.6 \times 10^{-3} \) and \( 6.75 \times 10^{-5} \text{ sec}^{-1} \)) was poor because of the inconsistency of "\( E \)".

(4) Excluding the two tensile data with the poor agreement, slightly improved the value of \( Q \) which became in the order
Figure 3-11. Creep Response of Inconel 718 at 1200°F, stress = 125 ksi, (Individual Run, 1st Case)

Triangles are the Data and Diamonds are the Model.
Figure 3-12. Creep Response of Inconel 718 at 1200°F, stress = 110 ksi, (Individual Run, 1st Case)
Triangles are the Data and Diamonds are the Model.
Figure 3-13. Tensile Response of Inconel 718 at 1200°F, strain rate = 1.6×10⁻³ sec⁻¹ (Individual Run, 1st Case)
Triangles are the Data and Diamonds are the Model.

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Figure 3-14. Tensile Response of Inconel 718 at 1200°F, strain rate = $6.75 \times 10^{-5}$ sec$^{-1}$ (Individual Run, 1st Case). Triangles are the Data and Diamonds are the Model.
Figure 3-15. Tensile Response of Inconel 718 at 1200°F, strain rate = 10^{-5} \text{ sec}^{-1} (Individual Run, 1st Case) 
Triangles are the Data and Diamonds are the Model.
Figure 3-16. Tensile Response of Inconel 718 at 1200°F, strain rate = 1.1*10^{-6} sec^{-1} (Individual Run, 1st Case). Triangles are the Data and Diamonds are the Model.
Figure 3-17. Tensile Response of Inconel 718 at 1200°F, 
strain rate = 3.3*10^-7 sec^-1 (Individual 
Run, 1st Case) 
Triangles are the Data and Diamonds 
are the Model.
Figure 3-18. Creep Response for Inconel 718 at 1200°F, stress = 125 ksi, (Individual Run, 2nd Case). Triangles are the Data and Diamonds are the Model.
Figure 3-19. Creep Response of Inconel 718 at 1200°F, stress = 110 ksi, (Individual Run, 2nd Case) Triangles are the Data and Diamonds are the Model.
Figure 3-20. Tensile Response of Inconel 718 at 1200°F, strain rate = 1.6 x 10^-3 sec^-1 (Individual Run, 2nd Case) Triangles are the Data and Diamonds are the Model.
Figure 3-21. Tensile Response of Inconel 718 at 1200°F, strain rate = 6.75*10^{-5} sec^{-1} (Individual Run, 2nd Case)
Triangles are the Data and Diamonds are the Model.
Figure 3-22. Tensile Response of Inconel 718 at 1200°F, strain rate = $10^{-5}$ sec$^{-1}$ (Individual Run, 2nd Case)
Triangles are the Data and Diamonds are the Model.

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Figure 3-23. Tensile Response of Inconel 718 at 1200°F, strain rate = 1.1*10^-6 sec^-1 (Individual Run, 2nd Case).
Triangles are the Data and Diamonds are the Model.
Figure 3-24. Tensile Response of Inconel 718 at 1200°F, strain rate = 3.3*10^-7 sec^-1 (Individual Run, 2nd Case) Triangles are the Data and Diamonds are the Model.
Figure 3-25. Creep Response of Inconel 718 at 1200°F, stress = 125, (7 sets of Data, 1st Case)
Triangles are the Data and Diamonds are the Model.
Figure 3-26. Creep Response of Inconel 718 at $1200^\circ$F, (7 Sets of Data, 1st Case)
Triangles are the Data and Diamonds are the Model.
Figure 3-27. Tensile Response of Inconel 718 at 1200°F, strain rate = $1.6 \times 10^{-3}$ sec$^{-1}$ (7 Sets of Data, 1st Case). Triangles are the Data and Diamonds are the Model.
Figure 3-28. Tensile Response of Inconel 718 at 1200°F, strain rate = 6.75*10\(^{-5}\) sec\(^{-1}\) (7 Sets of Data, 1st Case)
Triangles are the Data and Diamonds are the Model.
Figure 3-29. Tensile Response of Inconel 718 at 1200°F, strain rate = $10^{-5}$ sec$^{-1}$ (7 Sets of Data, 1st Case)
Triangles are the Data and Diamonds are the Model.
Figure 3-30. Tensile Response of Inconel 718 at 1200°F, strain rate = $1.1 \times 10^{-6}$ sec$^{-1}$ (7 Sets of Data, 1st Case)
Triangles are the Data and Diamonds are the Model.
Figure 3-31. Tensile Response of Inconel 718 at 1200°F, strain rate = $3.3 \times 10^{-7}$ sec$^{-1}$ (7 Sets of Data, 1st Case).
Triangles are the Data and Diamonds are the Model.
Figure 3-32. Creep Response of Inconel 718 at 1200°F, stress = 125 ksi, (7 Sets of Data, 2nd Case)
Triangles are the Data and Diamonds are the Model.
Figure 3-33. Creep Response of Inconel 718 at 1200°F, stress = 110 ksi, (7 Sets of Data, 2nd Case)
Triangles are the Data and Diamonds are the Model.
Figure 3-34. Tensile Response of Inconel 718 at 1200°F, strain rate = 1.6\times10^{-3} \text{ sec}^{-1} (7 Sets of Data, 2nd Case) Triangles are the Data and Diamonds are the Model.
Figure 3-35. Tensile Response of Inconel 718 at 1200°F, strain rate = 6.75\times10^{-5} \text{ sec}^{-1} (7 Sets of Data, 2nd Case)
Triangles are the Data and Diamonds are the Model.
Figure 3-36. Tensile Response of Inconel 718 at 1200°F,
strain rate = $10^{-5}$ sec$^{-1}$ (7 Sets of Data,
2nd Case)
Triangles are the Data and Diamonds
are the Model.
Figure 3-37. Tensile Response of Inconel 718 at 1200°F, strain rate = $1.1 \times 10^{-6}$ sec$^{-1}$ (7 Sets of Data, 2nd Case)
Triangles are the Data and Diamonds are the Model.
Figure 3-38. Tensile Response of Inconel 718 at 1200°F, strain rate = $3.3 \times 10^{-7}$ sec$^{-1}$ (7 Sets of Data, 2nd Case). Triangles are the Data and Diamonds are the Model.

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Figure 3-39. Creep Response of Inconel 718 at 1200°F, stress = 125 ksi, (5 Sets of Data, 1st Case)
Triangles are the Data and Diamonds are the Model.
Figure 3-40. Creep Response of Inconel 718 at 1200°F, stress = 110 ksi, (5 Sets of Data, 1st Case)
Triangles are the Data and Diamonds are the Model.
Figure 3-41. Tensile Response of Inconel 718 at 1200°F, strain rate = $10^{-5}$ sec$^{-1}$ (5 Sets of Data, 1st Case)
Triangles are the Data and Diamonds are the Model.
Figure 3-42. Tensile Response of Inconel 718 at 1200°F, strain rate = $1.1 \times 10^{-6}$ sec$^{-1}$ (5 Sets of Data, 1st Case) Triangles are the Data and Diamonds are the Model.
Figure 3-43. Tensile Response of Inconel 718 at 1200°F, strain rate = $3.3 \times 10^{-7}$ sec$^{-1}$ (5 Sets of Data, 1st Case).

Triangles are the Data and Diamonds are the Model.
Figure 3-44. Creep Response of Inconel 718 at 1200°F, stress = 125 ksi, (5 Sets of Data, 2nd Case). Triangles are the Data and Diamonds are the Model.
Figure 3-45. Creep Response of Inconel 718 at 1200°F, stress = 110 ksi, (5 Sets of Data, 2nd Case). Triangles are the Data and Diamonds are the Model.
Figure 3-46. Tensile Response of Inconel 718 at 1200°F, strain rate = $10^{-5}$ sec$^{-1}$ (5 Sets of Data, 2nd Case) Triangles are the Data and Diamonds are the Model.
Figure 3-47. Tensile Response of Inconel 718 at 1200°F, strain rate = $1.1 \times 10^{-6}$ sec$^{-1}$ (5 Sets of Data, 2nd Case) Triangles are the Data and Diamonds are the Model.
Figure 3-48. Tensile Response of Inconel 718 at 1200°F, strain rate = 3.3x10^-7 sec^-1 (5 Sets of Data, 2nd Case)
Triangles are the Data and Diamonds are the Model.

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of $2 \times 10^{-4}$ with slightly better curve fitting, meaning that these two tensile data sets have only a slight influence on the others.

(5) In both group runs concerning the seven and five sets of data, inspite of the fact that $Q$ has the same value of $6 \times 10^{-4}$ for both the first and second cases, the calculated values of the parameters $m$, $A$, $Z_i$ and $D_0$ are not the same. This difference causes some distinction between response agreement in the first and second cases specially in the creep response since $Z_i$ is a major creep parameter. However, this was not expected since the value of $Q$ was almost the same.

(6) In the first and second cases of the individual runs, the difference between the value of the number of iterations as well as the central processing (CPU) time is pronounced. However, that difference is small in the first and second cases of group runs.

3.6 Conclusions

From the general view of this chapter, we can make the following conclusions:

(1) A numerical evaluation of the material variables can be made by using Bodner constitutive theory through a numerical prediction of the tensile and creep response with reasonable curve fitting to the experimental response.

(2) Bodner Constitutive theory is very sensitive to the variability of the experimental data. Since the stress, $\sigma$, is the driving force in the constitutive model, a special attention should be given to the time data for a smooth $(\sigma-t)$ curve.
(3) As in Reference [8], Bodner constitutive theory may need further work to decide on improvements which can be made to include effects that would lead to tertiary creep in the representations.

(4) From the fact that the calculated values for some of the variables are different for the same value of Q (and they should not be) and from the fact that for some runs "Z₀" and "Z₁" have the same value, (and they should not), it can be recognized that the material variables in Bodner's Constitutive model are too many and not well defined. More specifically, they are not universal.
CHAPTER IV

ELASTIC AND ELASTIC-PLASTIC ANALYSIS
BY THE FINITE ELEMENT METHOD

4.1. Introduction

Not only is the finite element method a very useful tool, but it is also the most widely used in industry to obtain quantitative solutions of practical problems with complicated geometries and to verify approximate continuum analysis results numerically.

There are large number of finite elements in use today for both plane stress and plane strain analysis such as constant strain triangular elements, hybrid elements and higher order isoparametric elements. For linear elasticity, the mathematical development of these elements is simple. However, for nonlinear materials, both the higher order isoparametric and hybrid elements are no longer simple because the continuous variation in stress and strain within each element must conform to the properties of the nonlinear material. As to the constant strain triangular elements, they have the advantages of being simple and economical, and since they give a single point representation of stress and strain, they can conform to the constitutive behavior of the nonlinear material. Therefore, constant strain triangular elements were selected to be employed in this study.

In the modeling of an elastic-plastic continuum which contains a crack (or notch), i.e. problems with singularities, the finite element approaches fall into two general classes. The first approach ignores the singular nature of the solution and uses the same element type.
incorporated in the model remote from the singularity source. This
approach ensures that the finite element convergence criteria are
satisfied but requires a very fine finite element grid near the source
of singularity (e.g. the crack tip) to obtain good results. A disad­
advantage of this approach is that it is costly. The second approach is
to use a special element that has appropriate functions to include the
stress or strain singularity. This approach requires knowledge of the
stress or strain singularity at the singular point. Further, this
method has no provision for including material history dependence. For
this reason and for the benefit of simplicity, the first approach was
chosen to be applied in this study.

This chapter describes the development of a two dimensional plane
stress/plane strain finite element model which is composed of constant
strain triangular elements. The linear elastic code was used in the
numerical determination of the elastic \( K_c \) for benchmark notched speci­
men under constant tensile load. The nonlinear elastic-plastic-creep
code was employed to study the local plastic stress and strain field in
a benchmark notch specimen under various load spectrum.

4.2. Finite Element Formulation for
the Linear Elastic Analysis

The application of the finite element method to problems involving
linearly elastic materials is straightforward because the material
properties are constant and only one solution is required to obtain dis­
placements for the elastic structure. The numerical solution should
converge to the exact solution as the size of the elements become small.
To assure the convergence of the complete solution, the two conditions
of displacement compatibility and equilibrium have to be satisfied. As to the displacement function, it must be chosen so that structure displacements do not cause straining of the elements and a constant state of strain is obtained as the element size approaches zero. The simplest representation of the displacement which assures this convergence is the linear polynomial function. The polynomial function is simple for differentiation and integration and as the size of the element becomes small, the approximation to the exact solution is simple.

4.2.1 Displacement Functions

A typical finite element, \( e \), is defined by nodes \( i, j, m, \) etc., and straight line boundaries. For the constant strain triangular elements (plane stress/plane strain), figure 4-1 shows the typical triangle element considered, \( e \), with nodes \( i, j, m, \) numbered in a counterclockwise order. The displacements of a node have two components, \([6]\),

\[
a_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix}
\]

and the six components of element displacements are listed as a vector

\[
a_e = \begin{bmatrix} a_i^1 \\ a_i^2 \\ a_i^3 \end{bmatrix}
\]

The displacements within an element have to be uniquely defined by these six values. The simplest representation is clearly given by two linear polynomials

\[
\begin{align*}
u &= \alpha_1 + \alpha_2 x + \alpha_3 y \\
v &= \alpha_4 + \alpha_5 x + \alpha_6 y
\end{align*}
\]
Figure 4-1. An Element of a Continuum in Plane Stress or Plane Strain.
The six constants \( \alpha \) can be evaluated easily by solving the two sets of three simultaneous equations which will arise if the nodal co-ordinates are inserted and the displacements equated to the appropriate nodal displacements. Writing for example,

\[
\begin{align*}
\mathbf{u}_i &= \mathbf{a}_i + \mathbf{a}_x i + \mathbf{a}_y j \\
\mathbf{u}_j &= \mathbf{a}_j + \mathbf{a}_x j + \mathbf{a}_y j \\
\mathbf{u}_m &= \mathbf{a}_m + \mathbf{a}_x m + \mathbf{a}_y m
\end{align*}
\]

or

\[
\begin{bmatrix}
\mathbf{u}_i \\
\mathbf{u}_j \\
\mathbf{u}_m
\end{bmatrix} = 
\begin{bmatrix}
1 & x_i & y_i \\
1 & x_j & y_j \\
1 & x_m & y_m
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3
\end{bmatrix}
\]

we can easily solve for \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) in terms of the nodal displacements \( u_i, u_j \) and \( u_m \) and obtain

\[
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3
\end{bmatrix} = \frac{1}{2\Delta}
\begin{bmatrix}
a_i & a_j & a_m \\
b_i & b_j & b_m \\
c_i & c_j & c_m
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_i \\
\mathbf{u}_j \\
\mathbf{u}_m
\end{bmatrix}
\]

Similarly

\[
\begin{bmatrix}
\alpha_4 \\
\alpha_5 \\
\alpha_6
\end{bmatrix} = \frac{1}{2\Delta}
\begin{bmatrix}
a_i & a_j & a_m \\
b_i & b_j & b_m \\
c_i & c_j & c_m
\end{bmatrix}
\begin{bmatrix}
\mathbf{v}_i \\
\mathbf{v}_j \\
\mathbf{v}_m
\end{bmatrix}
\]

Therefore;

\[
\mathbf{u} = \frac{1}{2\Delta}[(a_i + b_i x + c_i y)\mathbf{u}_i + (a_j + b_j x + c_j y)\mathbf{u}_j + (a_m + b_m x + c_m y)\mathbf{u}_m]
\]
and

$$v = \frac{1}{2\Delta} (a_i + b_i x + c_i y)v_i + (a_j + b_j x + c_j y)v_j + (a_m + b_m x + c_m y)v_m$$

(4-9)

in which

$$a_i = x_j y_m - x_m y_j$$

$$b_i = y_j - y_m = y_{jm}$$

(4-10)

$$c_i = x_m - x_j = x_{mj}$$

with the other coefficients obtained by a cyclic permutation of subscripts in the order, i, j, m, and where

$$2\Delta = 2(\text{area of triangle } ijm)$$

$$= \begin{vmatrix}
1 & x_i & y_i \\
1 & x_j & y_j \\
1 & x_m & y_m
\end{vmatrix}$$

(4-11)

Equations 4-8 and 4-9 can be represent in the form

$$U = \{u\} = Na^e = [IN_i, IN_j, IN_m]a^e$$

(4-12)

with I a two by two identity matrix, and

$$N_i = \text{shape function} = (a_i + b_i x + c_i y)/2\Delta \text{ etc.}$$

(4-13)

The chosen displacement function automatically guarantees continuity of displacements with adjacent elements because the displacements vary linearly along any side of the triangle and, with identical displacement
imposed at the nodes, the same displacement will clearly exist all along an interface.

4.2.2 Element Strain (total)

The total strain at any point within the element can be defined by its three components which contribute to internal work. Thus

\[
\varepsilon = \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix}
\partial/\partial x, & 0 \\
0, & \partial/\partial y \\
\partial/\partial y, & \partial/\partial x
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
\]

(substituting equation 4-12 we have)

\[
\varepsilon = \begin{bmatrix}
\partial/\partial x, & 0 \\
0, & \partial/\partial y \\
\partial/\partial y, & \partial/\partial x
\end{bmatrix}
\begin{bmatrix}
\{a_i \}
\{a_j \}
\{a_m \}
\end{bmatrix}
\]
with a typical matrix \( B \) given by

\[
B_i = \begin{bmatrix}
\frac{\partial Ni}{\partial x} & 0 \\
0 & \frac{\partial Ni}{\partial y}
\end{bmatrix} = \frac{1}{2\Delta} \begin{bmatrix}
b_i, 0 \\
0, C_i
\end{bmatrix}
\]

(4-17)

therefore

\[
[B] = \frac{1}{2\Delta} \begin{bmatrix}
b_i & 0 & b_j & 0 & b_m & 0 \\
0 & C_i & 0 & C_j & 0 & C_m \\
C_i & b_i & C_j & b_j & C_m & b_m
\end{bmatrix}
\]

(4-18)

It can be seen that the \( B \) matrix is independent of the position within the element, and hence the strains are constant throughout it. Therefore, the criterion of constant strain is satisfied by the shape functions.

4.2.3 Initial Strain (thermal strain)

Initial strains, that is strains which are independent of stress, may be due to many causes. Shrinkage, crystal growth or, most frequently, temperature changes will, in general, result in an initial strain vector.
\[ \varepsilon_o = \{ \varepsilon_{xo} \} \]  

(4-19)

To be consistent with the constant strain conditions imposed by the prescribed displacement function, initial strains will be defined by average, constant, values.

4.2.4 Elasticity Matrix

Assuming general linear elastic behavior and isotropic materials, the relationship between stresses and strains will be linear and of the form

\[ \sigma = D(\varepsilon - \varepsilon_o) + \sigma_o \]  

(4-20)

Where \([D]\) is an elasticity matrix containing the appropriate material properties and \(\sigma_o\) is the initial stresses. Excluding \(\sigma_o\), which is simply additive, we can rewrite 4-20 in the form

\[ \sigma = D \begin{bmatrix} \varepsilon_x \\
\varepsilon_y \\
\gamma_{xy} \end{bmatrix} = D \left( \begin{bmatrix} \varepsilon_x \\
\varepsilon_y \\
\gamma_{xy} \end{bmatrix} - \varepsilon_o \right) \]  

(4-21)

For plane stress (\(\sigma_z = \tau_{xz} = \tau_{yz} = 0\)), the elasticity matrix \([D]\) is given by

\[ D = \frac{E}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & (1-\nu)/2 \end{bmatrix} \]  

(4-22)

In case of plane strain (\(\varepsilon_z = 0, \sigma_z = \nu(\sigma_x + \sigma_y)\)), the elasticity matrix \([D]\) is given by

\[ D = \frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix} 1 & \frac{\nu}{(1-\nu)} & 0 \\
\frac{\nu}{(1-\nu)} & 1 & 0 \\
0 & 0 & (1-2\nu)/2(1-\nu) \end{bmatrix} \]  

(4-23)
where $E$ is Young's modulus of elasticity and $v$ is Poission's ratio.

4.2.5 The Governing Equation

The equation which governs the elastic response of a discretized structure can be derived from the minimization of the total potential energy.

Let (for one element),

$$U = \text{the total potential energy}$$

$$= U_s + W \quad (4-24)$$

where

$$U_s = \text{the strain energy of the system}$$

$$= \frac{1}{2} \int \varepsilon^T D \varepsilon \cdot dv - \int \varepsilon^T D \varepsilon_o \cdot dv - \int \varepsilon^T \sigma \cdot dv \quad (4-25)$$

and

$$W = \text{the work done by the external loads}$$

$$= - \int y^T f^B \cdot dv - \int y^T f^S \cdot dx - \int y^T f^N \quad (4-26)$$

where the superscript $T$ denotes the matrix transpose,

$f^B$ = body force = Force/volume

$f^S$ = surface force = Force/area

$f^N$ = concentrated force

$y$ = the displacement vector

Substituting 4-25 and 4-26 into 4-24 with ($\varepsilon = Bu$) gives,
\[ U = \left[ \frac{1}{2} \int_{v} (u^T B^T D B u) dv - \int_{v} (u^T B^T D \varepsilon_0) dv - \int_{v} (u^T B^T \sigma_0) dv \right] \]

\[- \left[ \int_{v} (u^T N^T f^B) dv + \int_{s} (u^T N^T f^S) ds + \sum_{n} (u^T N^T f^n) \right] \]

(4-27)

For equilibrium to be ensured, the total potential energy must be stationary for variation of admissible displacements, therefore

\[ \frac{\partial U}{\partial u} = \text{Zero} \] (4-28)

or

\[ \int B^T D B dv \cdot u = \left[ \int B^T D \varepsilon_0 dv + \int u^T B^T \sigma_0 dv \right] \]

\[ + \left[ \int N^T f^B \cdot dv + \int N^T f^S \cdot ds + \sum_{n} N^T f^n \right] \] (4-29)

or

\[ [K]{u} = \{Q\} + \{P\} \] (4-30)

where

\[ \{Q\} = \text{the force vector due to the presence of initial stress and/or initial strain} \]

\[ = \left[ \int ([B^T][D]\{\varepsilon_0\} + [B]^T\{\sigma_0\}) \text{dvol.} \right] \] (4-31)

\[ \{P\} = \text{the external applied load vector due to the body, surface and concentrated forces} \]

\[ = \left[ \int N^T f^B dv + \int N^T f^S ds + \sum_{n} N^T f^n \right] \] (4-32)

\[ \{u\} = \text{the generalized displacement vector (unknown)} \]
\[ [K] = \text{the elastic stiffness matrix} \]

\[ = \int [B]^T[D][B] \text{dvol} \quad \nu \]

\[ = \int [B]^T[D][B] t \text{dxdy} \quad \nu \]

(4-33)

where \( t \) is the thickness and the integration is taken over the area of the triangle element.

Equation 4-30 is the governing equation of the elastic response of each element. By assuming expressions similar to equation 4-30 overall elements, the assembled governing equation for the structure is obtained. Let us consider, for example, the system shown in figure 4-2, also let us consider, for simplicity, that \{Q\} is negligible. Therefore, equation 4-30 becomes,

\[ [K]\{u\} = \{P\} \quad (4-34) \]

now, let us write the stiffness matrix for each element, \( e \), as follows,

(i) \( e = 1 \): \( i = 1, j = 2, m = 3 \), therefore,

\[ [K]^1 = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \quad (4-35) \]

(ii) \( e = 2 \): \( i = 2, j = 4, m = 3 \), therefore,

\[ [K]^2 = \begin{bmatrix} k_{22} & k_{24} & k_{23} \\ k_{42} & k_{44} & k_{43} \\ k_{32} & k_{34} & k_{33} \end{bmatrix} \quad (4-36) \]
Figure 4-2 Example of Finite Element Model of 2-D. Circled Numbers Denote Element Numbers. Uncircled Numbers Denote Node Numbers.
(iii) \( e = 3 \): \( i = 4, j = 5, m = 3 \), therefore,

\[
[K]^3 = \begin{bmatrix}
k_{44} & k_{45} & k_{43} \\
k_{54} & k_{55} & k_{53} \\
k_{34} & k_{35} & k_{33}
\end{bmatrix}
\] (4-37)

(iv) \( e = 4 \): \( i = 5, j = 1, m = 3 \), therefore,

\[
[K]^4 = \begin{bmatrix}
k_{55} & k_{51} & k_{53} \\
k_{15} & k_{11} & k_{13} \\
k_{35} & k_{31} & k_{33}
\end{bmatrix}
\] (4-38)

with reference to figure 4-2 and equations 4-35 through 4-38, the assembled stiffness matrix (GLOBEL), displacements vector, and assembled forces vector are:

\[
\begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
1 & (k^{1} + k^{e}) & k^{1} & (k^{1} + k^{e}) & 0 & k^{4} \\
2 & k & (k^{1} + k^{e}) & (k^{1} + k^{e}) & k^{2} & 0 \\
3 & (k^{1} + k^{e}) & (k^{1} + k^{e}) & (k^{1} + k^{e}) & (k^{2} + k^{e}) & (k^{2} + k^{e}) \\
4 & 0 & k^{2} & (k^{2} + k^{e}) & (k^{2} + k^{3}) & k^{3} \\
5 & k^{4} & 0 & (k^{3} + k^{e}) & k^{3} & (k^{3} + k^{e})
\end{bmatrix}
\begin{bmatrix}
u_{1} \\
u_{2} \\
u_{3} \\
u_{4} \\
u_{5}
\end{bmatrix} = \begin{bmatrix}
p^{0} \\
p^{2} \\
p^{0} + p^{0} + p^{0} \\
p^{0} + p^{0} + p^{0} \\
p^{0} + p^{0} + p^{0}
\end{bmatrix}
\] (4-39)

The numbers, in \( \Box \)'s are simply for bookkeeping purposes (denote row and column numbers of stiffness matrix). The assembly process essentially consists of taking elements from the expressions derived for individual elements and storing them in the appropriate locations in the overall system equations. The "appropriate" locations are determined by the connectivity of the elements assembled to represent the system. The number of potentially non-zero elements in any row is determined by the
number of elements connected to the node for which the corresponding equation applies.

4.3. Finite Element Formulation For Elastic-Plastic Analysis

4.3.1 Plasticity

One definition of plasticity is the presence of irrecoverable strains on load removal. Plastic behavior of solids is characterized by a non-unique stress-strain relationship. One of the basic assumptions in the theory of plasticity is that the total strain, $\varepsilon^t$, can be decomposed into elastic strain, $\varepsilon^e$, and plastic strain, $\varepsilon^p$, components as shown in figure 4-3.

$$\varepsilon^t = \varepsilon^e + \varepsilon^p$$  (4-40)

where,

$$\varepsilon^e = \sigma/E$$  (4-41)

where, $\sigma$ is the current state of stress and $E$ is the young's modulus of elasticity. As to the plastic strain component, $\varepsilon^p$, assuming incompressibility and isotropy and by following the Prandtl-Reuss flow law of classical plasticity we can write,

$$\{\varepsilon^p\} = \lambda[S]$$  (4-42)

where,

$$\{\varepsilon^p\} = \text{the plastic strain rate} \quad \lambda = \text{a scalar that has the significance of material viscosity}$$
Figure 4-3. Elastic and Plastic Strain.
\([S] = \) the deviatoric stress, which is left over after the hydrostatic average stress (mean normal stress), \(s^*\), has been subtracted from the actual stress, \(\sigma\).

i.e.

\[
[S] = \left[ \begin{array}{ccc}
\sigma_x & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_y & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_z \\
\end{array} \right] - \left[ \begin{array}{ccc}
s^* & 0 & 0 \\
0 & s^* & 0 \\
0 & 0 & s^* \\
\end{array} \right]
\]

\[
= \left[ \begin{array}{ccc}
(\sigma - s^*) & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & (\sigma - s^*) & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & (\sigma - s^*) \\
\end{array} \right]
\]

(4-43)

where,

\[
s^* = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)
\]

(4-44)

\[
S_x = \sigma_x - s^*
\]

(4-44)

\[
S_y = \sigma_y - s^*
\]

\[
S_z = \sigma_z - s^*
\]

In Bodner's flow law, \(\lambda\) takes the form,

\[
\lambda = \left[ D^P_2/J_2 \right]^{1/2}
\]

(4-45)

where,

\[
D^P_2 = \text{the second invariant of the plastic strain rate}
\]

\[
= D_0^2 \exp\left[\left(-Z^2/3J_2\right)^n(n+1/n)\right]
\]

(3-17)
and

\[ J_2 = \text{the second invariant of the deviatoric stress, } [s] \]

The principal deviatoric stresses, \( s \), can be found just as the ordinary principle stresses as follows:

\[
\begin{vmatrix}
(S -s) & \tau_{xy} & \tau_{xz} \\
\tau_{xy} & (S_{xy}^2) & \tau_{yz} \\
\tau_{xz} & \tau_{yz} & (S_{yz}^2)
\end{vmatrix} \tag{4-46}
\]

The principal deviatoric stresses are in the same direction as the principal stresses. Therefore,

\[ s_1 = \sigma_1 - s^* \]
\[ s_2 = \sigma_2 - s^* \tag{4-47} \]
\[ s_3 = \sigma_3 - s^* \]

where,

\[ \sigma_1, \sigma_2 \text{ and } \sigma_3 = \text{the principal stresses} \]

and \( s_1, s_2 \text{ and } s_3 = \text{the principal deviatoric stresses} \)

when equation 4-46 is expanded, the following cubic equation is obtained

\[ s^3 - J_2 s - J_3 = 0 \tag{4-48} \]

where the scalar invariants of the deviatoric stress matrix, \( s \), are:

\[ J_1 = 0 \]
\[ J_2 = -(S_{xy} S_{yz} + S_{yz} S_{xz} + S_{xz} S_{xy}) + \tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2 \tag{4-49} \]
The invariants of the principal deviatoric stress matrix are:

\[ J_3 = \begin{vmatrix} S_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & S_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & S_z \end{vmatrix} \]

\[ J_3 = S_x S_y S_z \]

\[ = \frac{1}{3} (S_x^3 + S_y^3 + S_z^3) \quad (4-51) \]

Therefore, from equations 4-44 and 4-50, \( J_2 \) for two-dimensional problems \((\sigma_z = \tau_{xz} = \tau_{zx} = \tau_{yz} = \tau_{zy} = 0, \tau_{xy} = \tau_{yx} \neq 0)\), has the form:

\[ J_2 = \frac{1}{2} [(\sigma_x - s^*)^2 + (\sigma_y - s^*)^2 + (-s^*)^2] + \tau_{xy}^2 \quad (4-52) \]

or

\[ J_2 = \frac{1}{2} [\sigma_x^2 + \sigma_y^2 - 2s^*(\sigma_x + \sigma_y) + 3s^* + \tau_{xy}^2] \]

4.3.2 Yield Criteria

A yield criterion is a mathematical statement concerning the combinations of stresses which will cause yielding of the material. Many different yield criteria have been proposed. Each of these criteria use some physical reason for the occurrence of yielding in the material. In general, the criteria all are consistent for some special simple combination of stresses (uniaxial tension, for example), but disagree to a greater or lesser extent for the more complicated combination of
stresses. The better theories are substantiated by experimentation and the poorer theories are disproved by experimentation. The assumptions for the yield criteria are:

1. There are always exists a function, \( f(\sigma) \), such that the material is elastic for \( f < 0 \) and \( f \leq 0 \), and plastic for \( f = 0 \) and \( f > 0 \). The dot stands for the time derivative.

2. There is no Bauschinger effect. That is, tensile yield is the same magnitude as compressive yield, as shown in figure 4-4.

3. Yielding is independent of hydrostatic stresses and depends only on deviatoric stresses. This may be reasoned theoretically by noting that hydrostatic stress (the mean normal stress) simply tends to strain the bonds between the atoms, but after the stress is removed the atoms return to their original positions and no plastic strain is obtained. On the other hand, the deviatoric (distortional) stresses tends to deform the body and cause shearing of layers of atoms and movement of dislocations, both of which can be irreversible and result in some plastic strain. Since yielding is dependent on the deviatoric stresses. We can refer to the equation for the principle deviatoric stresses,

\[
s^3 - J_2s - J_3 = 0 \tag{4-48}
\]

From this we conclude that the yield criteria is

\[
f(J_2, J_3) = 0 \tag{4-53}
\]

4. The material is isotropic. Therefore, the yielding is independent of the directions of the principal stresses. This
Figure 4-4. Bauschinger Effect.
means that "f" is a function that is symmetric in terms of the stresses (the principal stresses may be interchanged and the function "f" is unchanged).

The Von Mises yield condition is one of the most widely used criterion. Von Mises made the assumption that equation 4-53 can be written more simply if it is assumed that yielding is independent of $J_3$. Then,

$$ f(J_2) = 0 $$  \hspace{1cm} (4-54)

The Von Mises yield criterion based on the uniaxial tension test has the form

$$ Y^2 = \frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] $$  \hspace{1cm} (4-55)

where

$$ \sigma_1 = Y \text{ when } \sigma_2 = \sigma_3 = 0. $$

Therefore, for the two dimensional problem, where

$$ s^* = (\sigma_x + \sigma_y)/3 $$  \hspace{1cm} (4-56)

and

$$ \sigma_{1,2} = \frac{(\sigma_x + \sigma_y)}{2} \pm \left[ \frac{(\sigma_x + \sigma_y)^2}{4} - (\sigma_x \sigma_y - T_{xy}) \right] $$  \hspace{1cm} (4-57)

and

$$ \sigma_3 = 0 $$  \hspace{1cm} (4-58)

it can be shown that equation 4-52 becomes,

$$ J_2 = \frac{1}{3}[\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3T_{xy}^2] $$  \hspace{1cm} (4-59)
and equation 4-55 becomes,

\[ Y^2 = [\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2] \]  \hspace{1cm} (4-60)

from which it can be seen that the yield criteria is accounted for in the Bodner Constitutive model by the "J_2" term where,

\[ \lambda = \left[ \frac{D^p}{J_2} \right]^{\frac{1}{2}} \]  \hspace{1cm} (4-45)

and

\[ \{\dot{\varepsilon}^p\} = \lambda[S] \]  \hspace{1cm} (4-42)

Therefore, using equations 3-17, equation 4-42 becomes:

\[ \{\dot{\varepsilon}^p\} = \left\{ \frac{D^p_0}{D^p_0} \exp \left[ \frac{-\gamma^2}{\left( \sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2 \right)} \right] \cdot \left( \frac{n + 1}{n} \right)^{\frac{1}{2}} \right\} \cdot \left( \frac{3}{\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2} \right)^{\frac{1}{2}} \cdot \left[ \begin{array}{c} (\sigma - s^*) \\ \tau_{xy} \\ (\sigma - s^*) \end{array} \right] \]  \hspace{1cm} (4-61)

4.3.3 Finite Element Technique

As to the finite element method, while its application to linear problems was straightforward since the solution was always unique because the material properties are constant, this no longer is the case in elastic-plastic problems since the coefficients in the stiffness matrix vary as a function of loading. However, there are two techniques to solve the small displacement elastic-plastic problems incrementally within a finite element code. The first technique is the "tangent modulus" method in which the effects of plasticity are accounted for directly in the stiffness matrix by updating the coefficients of the
stiffness matrix after each load increment. The second technique is
the "residual force" method in which the plastic behavior is accounted
for through the addition of an effective plastic load vector to the
force side of the governing equation of the discretized structure after
each load increment. The latter technique was chosen to be employed in
this study.

The general governing equation for the "residual force" method is:

\[ [K]\{u\} = \{P\} + \{Q\} \quad (4-30) \]

However, there are two forms of the "residual force" method. The first
is the initial stress form, in which only the initial stress, \( \sigma_o \), is
considered. Then

\[ \{Q\} = \int_v [B^T]\{\sigma_o\} \cdot \text{dvol} \quad (4-62) \]

The second form is the initial strain form, in which only the initial
strain, \( \varepsilon_o \), is considered. Then

\[ \{Q\} = \int_v [B^T][D]\{\varepsilon_o\} \cdot \text{dvol} \quad (4-63) \]

In this study, the "initial strain" form was used as follows:

\[ \varepsilon^t = \varepsilon^e + \varepsilon^p \quad (4-40) \]

or

\[ Bu = \varepsilon^e + \varepsilon^p \]

i.e

\[ (4-64) \]
\[ \varepsilon^e = B u - \varepsilon^p \]

therefore,

\[ U_s = \text{the strain energy of the system} \]
\[ = \frac{1}{2} \int_{v} \{ B u - \varepsilon^p \}^T D \{ B u - \varepsilon^p \} \cdot dv \quad (4-65) \]

or

\[ U_s = \frac{1}{2} \left[ U^T B^T D B u - \varepsilon^P D B u - U^T B^T D \varepsilon^p + \varepsilon^P D \varepsilon^p \right] \cdot dv \quad (4-66) \]

By substituting (4-66) and (4-26) into (4-24) equation (4-27) becomes:

\[ U = \frac{1}{6} \int v \left[ U^T B^T D B u - \varepsilon^P D B u - U^T B^T D \varepsilon^p + \varepsilon^P D \varepsilon^p \right] \cdot dv \]
\[ - \left[ \int v U^T N^B f^B . dv + \int S^T f^S . ds + \sum_{n} U^T N^T f^n \right] \quad (4-67) \]

Again, for \( \frac{\partial U}{\partial u} = 0 \), equation (4-30) becomes:

\[ [K]\{u\} = \{P\} + \{Q^P\} \quad (4-68) \]

where, \([K], \{u\} \text{ and } \{P\}\) are the same as in the elastic formulation and \(\{Q^P\}\) is the "effective" plastic load vector which accounts for elements in a plastic state.

\[ \{Q^P\} = \sum_{el=1}^{M} \int v \left[ B^T D \{ \varepsilon^P \} \right] \cdot dv \quad (4-69) \]

where, \(M\) is the total number of the elements. The integration is taken over the volume of each element and the summation is over all elements in the structure.
The residual force method in the initial strain form approaches the solution to the elastic-plastic problem through the application of the plastic load vector \( \{Q^P\} \). The governing equation for the structure at a specific plastic strain rate is:

\[
[K]\{u\} = \{P(t)\} + \{Q^P(t)\}
\]  

(4-70)

or

\[
\{u\} = [K]^{-1}\{P(t)\} + \{Q^P(t)\}
\]  

(4-71)

where,

\[
\varepsilon^P(t) = \varepsilon^P(0) + \int_0^t \dot{\varepsilon}^P(t)\,dt
\]  

(4-72)

For each total load the problem is elastic and the displacement \( \{u\} \), are employed to compute the total strain, \( \{\varepsilon\} \), with the corresponding elastic stress, \( \{\sigma^e\} \):

\[
\{\varepsilon\} = [B]\{u\}
\]  

(4-73)

and

\[
\{\sigma^e\} = [D]\{\varepsilon^e\}
\]  

(4-74)

However, since the material is nonlinear, the correct stress increment for the corresponding strain is \( \{\sigma^e\} \). Therefore, a set of effective plastic load vector \( \{Q^P\} \), caused by the initial plastic strain, \( \{\varepsilon^P\} \), is needed to maintain the stress components on the yield surface or compatible with the uniaxial stress-strain curve.

\[
\{\sigma\}_I = \{\sigma\} + \{\sigma^e\}
\]  

(4-75)
where $\bar{\sigma}$ is the uniaxial yield stress.
CHAPTER V

COMPARISON OF THE FINITE ELEMENT RESULTS VERSUS
ANALYTICAL AND EXPERIMENTAL RESULTS

5.1 Theoretical Elastic Stress Concentration Factor ($K_t$)

Associated with every notch is a theoretical elastic stress concentration factor, $K_t$, which is dependent only on geometry and loading mode and has the form:

$$K_t = \frac{\sigma_{\text{max}}}{S} \quad (2-1)$$

where, $\sigma_{\text{max}}$ is the maximum actual or local stress at the stress concentration, and $S$ is the nominal net stress on the notched member.

Chin-Bing-Ling [23] presented an analytical solution for the stress in an infinite strip under longitudinal tension, the strip containing a pair of semicircular notches of equal radii symmetrically located on the opposite edges. The solution involves an infinite set of linear equations. The set consists of two infinite sets. One set was solved by using the method of successive approximations and the other by using the method of elimination of unknowns. In a discussion that followed shortly after the publication of Ling's solution, Peterson [24] pointed out that the numerical values of $K_t$ given by Ling are higher than those obtained by Neuber from an approximate solution. However, Ling had grounds to believe that Neuber's values somewhat underestimate the true stress-concentration factor. In reference [23] it was stated that, based on values obtained from photoelastic experiment, Peterson's
contention was later supported by Frocht and his associates. Similar remarks, based on respective values computed from the perturbation method, were made by Isida [25]. By this method, the stress-concentration factor is expressed in a power series of the radius of the notches. Therefore, Ling performed his computations again to attain a higher accuracy. In these computations, the stress-concentration factor $K$ was given by

$$K = (1-\lambda) \frac{\sigma_{\text{max}}}{\sigma}$$  \hspace{1cm} (5-1)

where, $\lambda$ is the radius of the notch.

Domas, Sharpe, Ward and Yau [20], in the analytical task of their study for the "Benchmark Notch Test for Life Prediction", used the finite element method to estimate the theoretical elastic stress concentration factor ($K_t$) using the same specimen geometry employed in this study (see figure 5.1). Three finite element codes were employed. One code was a two dimensional plane stress model for the elastic analysis employed by Ward at Louisiana State University. The remaining two programs were conducted by General Electric. In these two codes, eight-noded isoparametric brick elements were employed in elastic-plastic-creep code.

To determine $K_t$ from the finite element results, Domas, Sharpe, Ward and Yau plotted the axial stresses of the element centroid versus the distance from the surface and with an extrapolation to the surface the value of $K_t$ was determined.

As to this research, the constant strain triangle, two dimensional, elastic-plastic finite element model of the double-notched specimen was employed to determine the value of $K_t$. 

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Figure 5.1 - Specimen dimensions.
The notched member for this study was the specimen shown in figure 5.1. Since the specimen is symmetrical about its two axis, only one fourth of it was used in this analysis.

The material parameters calculated in Chapter III from the Bodner model and the experimental tensile and creep data are used in these evaluations. In these parameters the value of Young's modulus of elasticity, E, was $25 \times 10^6$ psi. Assuming the Poisson's ratio, $v$, to be 0.3, the value of the modulus of rigidity, G, was calculated where:

$$\frac{E}{2G} (1 + v) \quad (5-2)$$

The numerical results are given in computer print out. Each node with its coordinates and each element (triangle elements) with its nodes are presented in the printout.

Based on the surface nodal point stresses (mid line positions) and the applied stress, the calculated stress concentration factor was an output at different time intervals (steps).

To study the influence of the finite element mesh (size, the ratio between the elements at the boundary of the notch) on the value of $K_t$, five different mesh sizes were employed in this study. Figure 5.2 shows grid number 5 with 176 nodes and 300 elements.

5.1.1 Discussion of the Results and Conclusion

The calculated values of $(K_t)$ from the different studies discussed above are presented in table 5.1. From figure 5.2 and from table 5.1 it can be seen that:

1. The general agreement between all the studies is very good where $1.87 \leq K_t \leq 2.18$. 

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Figure 5.2 Grid # 5.
<table>
<thead>
<tr>
<th>The Study</th>
<th>Calculated $K_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ling's Analytical Solution</td>
<td>1.87</td>
</tr>
<tr>
<td>Peterson's Analytical Solution</td>
<td>1.90</td>
</tr>
<tr>
<td>L.S.U.'s 2-D Finite Element Code</td>
<td>1.90</td>
</tr>
<tr>
<td>G.E.'s 3-D Finite Element Code</td>
<td>1.94</td>
</tr>
<tr>
<td>Present Finite Element Study:</td>
<td></td>
</tr>
<tr>
<td>Grid #1 = 70 Nodes and 108 Elements</td>
<td>2.18</td>
</tr>
<tr>
<td>Grid #2 = 80 Nodes and 126 Elements</td>
<td>2.04</td>
</tr>
<tr>
<td>Grid #3 = 120 Nodes and 198 Elements</td>
<td>1.97</td>
</tr>
<tr>
<td>Grid #4 = 140 Nodes and 234 Elements</td>
<td>1.94</td>
</tr>
<tr>
<td>Grid #5 = 176 Nodes and 300 Elements</td>
<td>1.94</td>
</tr>
</tbody>
</table>
2. From the results of the present study, the size of the mesh (number of nodes and elements) coupled with the uniform distribution of the elements at the boundary of the notch influences the value of $K_t$. The finer the mesh with good uniform distribution for the nodes and elements, the more consistent the results with the other studies.

3. For the same specimen geometry, while the calculated value of $K_t$ in the finest grids (#4 and #5) agreed exactly with G.E's 3-D model, it nearly, but not exactly, agreed with L.S.U.'s 2-D model.

From the results discussed above, it can be concluded that the finite element model developed in this study was employed successfully to determine the theoretical elastic stress concentration for the double-notched specimen shown in figure 5.1, the value of which is, conclusively, 1.94.

5.2 Nonlinear Time-Dependent Cyclic Loading with Various Load Spectrum

Because of the fact that many materials change their deformation behavior (softening, hardening, or remaining neutral) after repeated loading, the need for cyclic stress-strain response is essential. Therefore, the 2-D finite element code developed in this study was used to develop a numerical cyclic stress-strain response of the double-notched specimen (figure 5.1) with the two load patterns shown in figure A-3. In the load pattern, the time for each cycle is 6 seconds and the tension hold is for 120 seconds. The Bodner constitutive model was incorporated in the finite element code. The second order Runge-Kutta
algorithm was employed to integrate, numerically, the four components 
\( \varepsilon'^x, \varepsilon'^y, \varepsilon'^{xy}, \) and \( z \) in the Bodner constitutive equation.

There are two major factors influencing the cyclic response. The 
first factor is the material factor presented by the modulus of elasticity, \( E \), and the value of, \( Z_0 \), which determines the yielding point. The 
second factor is the geometry factor presented by the finite element 
mesh (size, the number and the ratio of the elements at the boundary of 
the notch).

To study the geometry influence on the calculated response, the numeri­
cal evaluations were done using 3 different mesh sizes: one coarse 
mesh, #1, (see table 5.1 and figure 5.3) and two fine meshes, #4 and #5, 
(see table 5.1 and figures 5.2 and 5.4). As to the material influence 
on the numerical results, 3 different values of, \( E \), and 2 different 
values of, \( Z_0 \), were used in the model: \( E = 25\times10^6, 22.5\times10^6 \) and 
\( 22.8\times10^6 \) psi and \( Z_0 = 9.46\times10^5 \) and \( 8.46\times10^5 \) psi.

Using equation 5.2 with \( v = 0.3 \), the corresponding values of the 
modulus of rigidity, \( G \), came to be \( 9.6\times10^6, 8.7\times10^6 \) and \( 8.8\times10^6 \) psi respectively.

Table 5.2 shows the material variables used in the numerical devel­
opment of the cyclic response of the double-notched specimen. The 
values of these material variables are the same as the ones given in 
table 3.5 for the 7 sets of data in the second case.

5.2.1 Discussion of the Results and Conclusion

At the outset of this study, the numerical evaluations were done 
using meshes 1, 4 and 5 with \( E = 25\times10^6 \) psi and \( Z_0 = 9.46\times10^5 \) psi, see 
figures 5.5 and 5.6. The calculated results revealed the influence of 
the geometry factor (finite element mesh) on the cyclic response. Then,
Figure 5.3 Grid #1.
Figure 5.4 Grid #4.
TABLE 5.2. The Values of the Material Variables for Inconel 718 at 1200°F used in the Finite Element Analysis.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>-</td>
<td>0.737</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>psi</td>
<td>(9.46 and 8.46)×10^5</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>psi</td>
<td>10.197×10^5</td>
</tr>
<tr>
<td>m</td>
<td>psi$^{-1}$</td>
<td>4.729×10^{-3}</td>
</tr>
<tr>
<td>A</td>
<td>sec$^{-1}$</td>
<td>6.82×10^{-4}</td>
</tr>
<tr>
<td>R</td>
<td>-</td>
<td>4.72</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>psi</td>
<td>5.35×10^{-5}</td>
</tr>
<tr>
<td>$D_0$</td>
<td>sec$^{-1}$</td>
<td>1.033×10^{4}</td>
</tr>
<tr>
<td>E</td>
<td>psi</td>
<td>(25, 22.5 and 22.8)×10^6</td>
</tr>
<tr>
<td>G</td>
<td>psi</td>
<td>(9.6, 8.7 and 8.8)×10^6</td>
</tr>
</tbody>
</table>
for better correlation with the experimental results, the finest mesh (#5) was employed in a second evaluation with a value of, $E$, equals to $22.5 \times 10^6$ psi, where the experimental observed value of $E$ was $22.8 \times 10^6$ psi, and $Z_0$ was the same as in the first run ($Z_0 = 9.46 \times 10^5$ psi). See figures 5.7 and 5.8. The calculated results revealed the influence of $E$, on the cyclic response. As to the influence of $Z_0$ on the location of the yielding point, mesh number 4 (for better correlation with the experimental response, and for the optimum cost of the computer time) was employed in a third evaluation with the exact observed value of, $E$, by the experimental data, $22.8 \times 10^6$ psi, and a reduced value of, $Z_0$, ($Z_0 = 8.46 \times 10^5$ psi), only for the continuous fatigue load pattern as a check on the influence of reducing the value of $Z_0$. See figure 5.9.

The numerical results are given in computer print out. The value of Von Mises stress, hardness $(Z)$, $(\varepsilon^t, \varepsilon^p)_x$, $(\varepsilon^t, \varepsilon^p)_y$ and $(\varepsilon^t, \varepsilon^p)_{xy}$ are an output for the value of applied loads at some selected elements on the boundary of the notch at different time steps.

From figures 5.5 through 5.9, it can be seen that:

1. In figures 5.5 and 5.6, the calculated results from grids number 4 and 5 (fine mesh) with $E = 25 \times 10^6$ psi and $Z_0 = 9.46 \times 10^5$ psi, agreed with each other all over the cycle with a slight deviation in the last linear portion of the unloading cycle. However, both results have a good agreement with the experimental response in the first linear portion of the loading cycle, but then they do not yield as much as the experimental results. This could be attributed the high value of $E$ and $Z_0$. 

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Figure 5.5 Tension Hold

LOAD-1.85.

- Experimental
- Grid #1
- Grid #4
- Grid #5

E = 25 x 10^6 psi

Zo = 9.46 x 10^5 psi

[Graph showing tension hold with specific data points and annotations]
Figure 5.6 Continuous Cyclic
2. In figures 5.5 and 5.6, the calculated response from mesh number 1 (coarse) with $E = 25 \times 10^6$ psi and $Z_0 = 9.46 \times 10^5$ psi, shows better agreement with the experimental results. Actually, this should not be the case since the mesh is coarse and the values of $E$ and $Z_0$ are the same as above. The cause of this apparently good agreement is that the geometry factor (the finite element mesh) is masking the material factors ($E$ and $Z_0$). Therefore, the good agreement in this case is false agreement.

3. In figure 5.7, the calculated response for the tension hold from mesh number 5 (very fine) with $E = 22.5 \times 10^6$ psi and $Z_0 = 9.46 \times 10^5$ psi, shows very good correlation with the experimental results. However, there was slight deviation in the yielding rate at the end of the loading cycle.

4. For the continuous fatigue, it can be seen (from figures 5.7 and 5.8) that the calculated response from mesh number 5 with $E = 22.5$ ksi and $Z_0 = 9.46 \times 10^5$ psi, has the same attitude as observed in the calculated tension hold response. The portion of loading cycle in both responses (continuous fatigue and tension hold) is exactly the same, and both have very good agreement with the experimental results. As to the unloading cycle, the agreement is not as close as in the loading cycle. However, the last linear portion of the unloading is not presented for the experimental results because of test failure at this stage of this specific experiment.

5. From figure 5.9, the calculated response from mesh number 4 (for optimum cost of the computer time with the same accuracy
Figure 5.7  Tension Hold
Figure 5.9 Continuous Cyclic
as mesh number 5) with $E = 22.8 \times 10^6$ psi (the exact experimental value) and $Z_0 = 8.46 \times 10^5$ psi (the reduced value) shows an earlier yielding than the case with $Z_0 = 9.46 \times 10^5$ psi. This earlier yielding agrees with the thought of the influence of $Z_0$ on the level of the yielding point. However, the general agreement for the reduced value of $Z_0 (8.46 \times 10^5$ psi) is not as good as the agreement for the higher value of $Z_0 (9.46 \times 10^5$ psi). From this result it can be recognized that not only $Z_0$ should be adjusted but also the rest of the bodner parameters influencing the tensile response. Again, this comes to be consistent with the fourth conclusion in Chapter III.

6. Generally, in the linear portion of the loading and unloading cycle, the value of $E$ is slightly decreasing at the beginning of the cycle and is rapidly decreasing at the end of the cycle. This could be due to an initiating crack or propagation of existing crack at the root of the notch.

7. From the response of the tension hold, as can be seen, the creep effect is existing but, on a small scale.

From the results discussed above, it can be concluded that:

By using a reasonably fine mesh (geometry parameter) coupled with the adjusted material parameters, to avoid any masking of one factor on the other, the elastic-plastic finite element model developed in this study successfully evaluate the cyclic stress-strain response for the continuous fatigue and the tension hold load pattern.
CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Summary and Conclusions

1. A numerical simulation technique for determining the nonlinear time dependent material variables for Inconel 718 at 1200°F (650°C) using the Bodner constitutive model coupled with tensile and creep data was developed in this study. The Runge-Kutta algorithm was employed for the numerical integration of the constitutive mode first order differential equation w.r.t. time. Also, the least square method coupled with Powell's iteration algorithm were used for the curve fitting analysis.

The numerically predicted tensile and creep response show reasonable curve fitting to the experimental response.

As to the calculated values of the material variables, it was observed that they are:

(i) very sensitive to the accuracy of the input data.
(ii) different for the same value of the error function Q.
(iii) inconsistent with their definition.

This reveals that the material variables in the Bodner constitutive model are too many and not well defined.

The experimental data in this study (for the button-head specimen) was obtained from the experimental program conducted by W. N. Sharpe, in which, the Interferometric Strain/Displacement Gage (ISDG) technique was employed.

136
2. A constant strain triangle, two dimensional, elastic-plastic finite element model of the double-notched specimen, in which the Bodner model is incorporated, was developed in this study. This finite element model accounts for elastic and elastic-plastic cyclic behavior of the material.

The finite element code was employed successfully to determine the theoretical elastic stress concentration, $K_t$, for the double-notched specimen. The influence of the geometry factor presented by the finite element mesh size was studied and a perfect agreement with the published results was observed.

3. The developed elastic-plastic finite element code was used to predict the cyclic behavior of the material to compare with experimental data (for two load patterns) generated at one stage of the experimental task of this study for that purpose. The experimental cyclic data was obtained by the ISDG technique developed by W. N. Sharpe, Jr.

The influence of the material factor ($E$ and $Z_0$) and the geometry factor (the mesh size) were studied and it was found that the geometry factor in a very coarse mesh conceals the material factor.

However, a reasonably fine mesh coupled with well adjusted material variables provided a good correlation with the experimental results.

In the response of the tension hold load pattern, the effect of the creep is present but it is not very large.

6.2 Recommendations

1. A study to develop a new nonlinear time-dependent model is needed. In the new model the effects that would lead to tertiary creep and microstructural damage must be included in the representation.
Also, the new model should consist of a minimum number of well defined variables.

2. A finite element study with different type of elements (hybrid elements for example) for different specimen geometry (such as the compact tension specimen), will be a suitable continuation for this study.

3. As to the creep effect in the tension hold load pattern, a longer hold time (more than 120 seconds) may be considered.
REFERENCES


11. Yamada, Y., Okumura, H., "Finite Element Analysis of Stress and Strain Singularity Eigen State in Inhomogeneous Media or Composite Materials", Institute of Industrial Science, University of Tokyo, Tokyo, Japan.


SELECTED BIBLIOGRAPHY


142

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APPENDIX A

THE EXPERIMENTAL PROGRAM

The experimental data used for comparison in Chapter V was provided by W. N. Sharpe, Jr. The specimens and load patterns were a continuation of the work of reference [20], but at higher loads. In this experimental program, the data was obtained by using the Interferometric Strain/Displacement Gage (ISDG) technique. The principles of (ISDG) technique are briefly discussed in this section followed by a description of the tests used in this study. Also, the experimental results are displayed in this section.

A.1 The Interferometric Strain/Displacement Gage (ISDG)

W. N. Sharpe and D. R. Martin [26], described in detail the laser-based interferometric system (controlled by a minicomputer) for measuring displacement/strain over short gage-lengths at high temperature. Only a basic review of the technique follows here.

When two closely spaced diamond indentation are illuminated with a laser light, two interference patterns are formed at approximately 45 degrees to the specimen. These diffraction patterns overlap, creating interference fringe patterns on either side of the laser beam. The motion of these formed patterns is related to the relative displacement between the indentations. The arrangement of the (ISDG) is illustrated in figure A.1. The equation which governs the position of bright interference fringes is
\[ d \sin \alpha = m\lambda \quad (A-1) \]

where, "d" is the spacing between indentations, "\( \lambda \)" is the wave length of the laser and "m" is an integer (±1, 2, 3,...). In order to measure the relative displacement between the two indentations, \( \delta d \), the fringes are counted as they move past a fixed observation angle, \( \alpha_o \), then equation A-1 becomes:

\[ \delta d \sin \alpha_o = \delta m\lambda \quad (A-2) \]

or

\[ \delta d = \frac{\delta m\lambda}{\sin \alpha_o} \quad (A-3) \]

where, \( \delta m \), is the number of fringes (or fraction thereof) passing the observation position.

The strain, \( \varepsilon \), is given by

\[ \varepsilon = \frac{\delta d}{d_o} \quad (A-4) \]

or

\[ \varepsilon = \frac{\lambda \delta m}{d_o \sin \alpha_o} \quad (A-5) \]

where, \( d_o \) is the original spacing between indentations.

Also, the rigid-body motion of the specimen causes the fringes to move. In a carefully aligned testing machine, the rigid body motion can be sufficiently eliminated except for motion in the direction of the applied load. A rigid-body motion in one direction would cause each pattern to move an equal amount in the same direction. However, if fringe motion toward the incident laser beam is defined as positive,
then averaging the two pattern motions eliminates the rigid-body effect. So, in practice, relative displacement is measured by:

$$\delta d = \frac{\delta m_u + \delta m_l}{2} \cdot \frac{\lambda}{\sin \alpha_0} \quad (A-6)$$

and the strain is measured by:

$$\varepsilon = \frac{\delta m_u + \delta m_l}{2} \cdot \frac{\lambda}{d_0 \sin \alpha_0} \quad (A-7)$$

where the subscripts $u$ and $l$ refer to the upper and lower patterns respectively. More details of the optics associated with the ISDG and various applications are given in References [27, 28].

A.2 Test Facilities and Methodology

The material used in this work is, again, Inconel 718; the material variables were numerically determined in Chapter III. The notched low-cycle fatigue specimens, as well as the corresponding grips were supplied by General Electric. To help eliminate inconsistencies in data, the same specimen geometry (the double-notched specimen, see figure 5.1) was used for all tests.

The specimens were cyclically loaded in an electrohydraulic testing machine equipped with a special high-temperature furnace designed and constructed at Louisiana State University specifically for the specimens and grips utilized in this research.

Strain was measured with a minicomputer-controlled scanner that measured the amount and direction of fringe motion. The minicomputer is a Digital Equipment Corporation, MlNC System. It has an analog/digital-digital/analog converter, a flexible disk memory, solid disk
control unit, graphics terminal, linear printer, digital plotter and upgrade package which extends the capabilities of the system to include FORTRAN IV. A schematic of the system is shown in figure A-2.

The methodology of the (ISDG) technique is as follows [29]; as the fringe patterns move in correlation with the relative displacement of the indentations, a predetermined minimum on each channel is tracked by the computer. A computer-generated sixty step ramp is fed to the servocontroller, causing each mirror to rotate about its axis. The total angular rotation of the mirror is adjusted via the gain control on the servocontroller to scan only the "trough" region surrounding the predetermined minimum of each fringe pattern. The fringe pattern is swept over a narrow slit over the photomultiplier tube face. At each of the sixty increments of the mirrors constituting a sweep, the photomultiplier tubes sample the fringe intensities and relay their electric analogues to the minicomputer via an inverting amplifier. Typically the minicomputer averages the sixty intensities recorded per channel with a six point sliding average routine to mask any noise present and then locates the new minimum intensity locations for each channel using a simple comparison loop. The next mirror sweep will now be centered about this new minimum. The total fringe displacement is determined by subtracting this new minimum location from the original minimum location prior to any load on the specimen. The total fringe displacement is then multiplied by the appropriate constant to determine strain. This entire process takes 100 milliseconds to generate one data point. Normally sixty data points are gathered per cycle. At the end of each cycle, the sixty strain and the corresponding load values are stored on a flexible disk to be inspected after the test is completed.
Figure A.2 - Schematic of the minicomputer-controlled ISDG system.
A.3 Experimental Results

The experimental program consists of two different load patterns: continuous cycle and tension hold. These two load patterns are shown in figure A-3. Each test was conducted with the notch region of the specimen maintained at 1200°F (650°C). The duration of each test was determined by either flaws developing on the specimen surface, or by the tension strain limitation of the measuring system, approximately 2%.

The load range for the "continuous cycle" load pattern is (8000, -4800) lbs. Cycles number 1, 252 and 502 are shown in figures A-4 through A-6. As to the "tension hold" load pattern, the load range was again (8000, -4800) lbs. The hold at the tensile load (8000) lbs. was two minutes. During this hold period thirty data points were measured at equal intervals by the ISDG. Cycles number 1, 35 and 70 are shown in figures A-7 through A-9.
Figure A.3. Load Patterns

(a) Continuous Cyclic

(b) Tension Hold

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Figure A.4. Continuous Cyclic Load-Strain Diagram

LOAD-LBS.

STRAIN = 1.0E-03

SPECIMEN NO. 10

CYCLE 1

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Figure A.6. Continuous Cyclic
Figure A.7. Tension Hold
Figure A.8. Tension Hold
Figure A.9. Tension Hold
APPENDIX C

Computer Program Listing for the Elastic-Plastic Finite Element Code
MAIN PROGRAM FOR GE SPECIMEN, STAGE 1.

REAL*4 FN(400), DN(400), XN(400), EGT(3600)
INTEGER*2 NUE(1800)
LOGICAL*1 BN(400)

BYTE J0BNM(6), FILNM(14), CB, CD, CE, CF, CL, CN, CS, CU, CV

COMMON/FLUNSD/NSND, NSED, NSEBD, NSESV, NSUGS, NSFGS
+ , NNUED, NNUED, NNUED, NNUED, NNUED, NNUED, NNUED

COMMON/URKPTN/NXS, NELS, NUUP, NUR, NRUP, NRBUP
+ , NHRUP, NNBUP, NHBUP, IPANIC

EQUIVALENCE (MB, NSND, NWR), (NBUP, NRBUP), (CD, FILNM(1)), (CL, FILNM(2))

DATA NSND, NSED, NSEBD, NSESV, NSUGS, NSFGS
+ , 00, 00, 000, 000, 000, 000
DATA NWED/16/, NSED/16/, NWED/64/, NSED/64/
DATA NWIE/256/
DATA MSZ/3600/
DATA MXND, MXELS, NELS/200, 300, 00/
DATA FILNM(1), FILNM(2), FILNM(3), FILNM(10), FILNM(14)
+ , / 'D', 'L', '1', 'I', 'I', /
DATA CB, CE, CF, CG, CH, CS, CU, CV
+ , / 'B', 'E', 'F', 'G', 'N', 'S', 'U', 'W'/

1000 READ(5,1) J0BNM, IRUN

IF IRUN.EQ.00 RETURN

DO 1050 J=1, 6
1050 FILNM(J+63) = J0BNM(J)

READ(5,*) NSND, NSED
WRITE(6,1) J0BNM, IRUN, NSND, NSED

IF IRUN.LT.00 READ(5,1) FILNM(J), J=4,9

IF NSND.LE.00 GOTO 1060

FILNM(11) = CN
FILNM(12) = CD
FILNM(13) = CD

OPEN
+ (UNIT=NSND, NAME=FILNM, ACCESS='DIRECT', ASSOCIATE=NSND, RECORDSIZE=NSND, TYPE='NEW', DISP='KEEP')

1060 IF NSED.LE.00 GOTO 1070

FILNM(11) = CE
FILNM(12) = CL
FILNM(13) = CD

OPEN
+ (UNIT=NSED, NAME=FILNM, ACCESS='DIRECT', ASSOCIATE=NSED, RECORDSIZE=NSED, TYPE='NEW', DISP='KEEP')
CALL AUTOG(MNDS,XN,BN,DM,FN,NELS,NUE,EGT,MXND,MXELS)

CALL BWFP(NUE,NELS,NWR,EGT,M31,EGT(M31+NELS/2+3),XN,EGT)

IF(NNDS.LE.00) GO TO 1000

NXS = NNDS + NNDS
NRWP = MSZ/NWR
IF(NRWP.LT.NWR) GO TO 2100

NRWB=NWB/NWR
IF(NRWP.LE.(NBWF-01)*NRWB+01) GO TO 2200

NRWB=NRWB-01
IF(NRWB.GT.00) GO TO 2000

WRITE(6,2)MSZ,MBAND,NRWP
GO TO 1000

NRWP=NRWB*NBW
NRWB=NRWB*NWR
NWLB = (( NWLB+127 )/128 )*128
NLBDS = ( NXS+NRWB-01 )/NRWB
NPBDS = ( NWLB/128 )*NLBDS
WRITE(6,1)JOBNM,IRUN,NXS,NELS,NWR,NWB,NWLB,NLBDS,NPBDS

WRITE(7,'(CLOSING DATA SETS JOBNAME,NND & ELD)')
IF( NSND.GT.00 ) CLOSE(UNIT=NSND)
IF( NSED.GT.00 ) CLOSE(UNIT=NSED)
GOTO 1000

FORMAT(1X,6A1,12I5/1X,15I5)
2 FORMAT(' SIZE OF WORKING ARRAY INSUFFICIENT:MSZ,MBAND,NWFP',315)
3 FORMAT(12X, 15I5)

END

BAND WIDTH AND PROCESSING VECTOR PROGRAM

SUBROUTINE BWFP(NU,NOEL,MP,NLP,MX,XN,EGT)

REAL*4 EGT(3,1)
REAL*4XN(2,1)
INTEGER*2 NU(3,1),NLP(1),MX(1)

COMMON/FLUNSD/MNDS,NSED,NSED,NSED,NSENG,NSEV,NSUGS,NSFGS
+ ,NVSD,NVMD,NVMS,NVNS,NVUGS,NVFGS

THIS ROUTINE DETERMINES THE BAND WIDTH IN TERMS OF
NODE INDEX DISTANCE. IT ALSO SAVES THE ELEMENT DATA
ON UNIT NSED. THIS INCLUDES THE MAX NODE NUMBER REFERENCED
BY THIS ELEMENT

MP=3
DO 2000 I=1,NOEL

MNI = 10000

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MXI = 00
C
DO 1000 J=1,3
C
IF(MNI.GT.NU(J,I))MNI=NU(J,I)
1000 IF(MXI.LT.NU(J,I))MXI=NU(J,I)
C
IF(MP.LT.MXI-MNI)MP=MXI-MNI
C
MX(I)=MXI
NLP(I)=I
2000 CONTINUE
C
COMPUTE BAND WIDTH. ( INCLUDES DIAGONAL )
C
MP=(MP+1)*2
NM1 = NOEL-01
IF( NM1.LE.00 ) RETURN
C
ORDER THE ELEMENTS FOR PROCESSING OF THE GLOBAL STIFFNESS MATRIX.
C
THE ELEMENTAL DATA WILL BE WRITTEN TO DISK IN ORDER OF PROCESSING.
C
THE NUMBER OF THE ELEMENT IS THE FIRST PIECE OF DATA IN THE RECORD
C
THE ORDERING IS BASED ON THE MIN-MAX NODE REFERENCED BY THE
C
ELEMENT BEING PROCESSED, WITH THE AUTOGEN THE ORDER OF PROCESSING
C
SHOULD BE THE SAME AS THE ELEMENT NUMBERING.
C
DO 4000 I=1,NM1
C
JMN=I
IP1 = I+01
C
DO 3000 J=IP1,NOEL
IF(MX(J).LT.MX(JMN))JMN=J
3000 CONTINUE
C
MXI=NLP(JMN)
NLP(JMN)=NLP(I)
NLP(I)=MXI
MXI=MX(I)
MX(I)=MX(JMN)
MX(JMN)=MXI
4000 CONTINUE
C
WRITE THE ELEMENT DAT TO NSED IN THE ORDER OF PROCESSING
C
DO 5000 I=1,NOEL
C
J = NLP(I)
5000 WRITE(NSED,'(I),(NU(K,J),K=1,3),(XN(L,NU(K,J))+L=1,2),K=1,3),
C
RETURN
C
END
SUBROUTINE DIST(X1,X2,A),#7»
REAL XI(2),X2(2)
A2=X1(1)+X2(1)
A(2)=X1(1)+X2(1)
RETURN
END

SUBROUTINE ETXEL(NEL,NUE,EST,EGL,MXEL;11,12,J1,J2,XN,H2)
REAL44 XN(2),XEL)»EST(J),MIXL)»EGL(J)
INTEGER NUE(MXEL)
IF (PLT.GT.0) 3 CALL PENUM
J = J1

451 IF (J1+101)
J(J1)=1.E20
R(J1)=1.E20
IF (J1.LE.J2) GO TO 452
IF (J1.GT.J2) GO TO 456
R(J1) = 0.0
GO TO 454

452 IF (J1.LE.J2) GO TO 453
R(J1) = 0.0
GO TO 454

453 J(IP) = HOM(E1(1),E1(2),E1(3),E1(4),E1(5),E1(6),E1(7),E1(8))
J(IP) = HOM(E1(1),E1(2),E1(3),E1(4),E1(5),E1(6),E1(7),E1(8))
J(IP) = HOM(E1(1),E1(2),E1(3),E1(4),E1(5),E1(6),E1(7),E1(8))
IF (J1.I.E-20) D1=1.E-20
CALL DIST(E1(I),E1(2),E2(S1))
IF (J2.I.E-20) D2=1.E-20
CALL DIST(E1(I),E1(2),E2(S2))
IF (J3.I.E-20) D3=1.E-20
CALL DIST(E1(I),E1(2),E2(S3))
IF (J4.I.E-20) D4=1.E-20
CALL DIST(E1(I),E1(2),E2(S4))
IF (J5.I.E-20) D5=1.E-20
CALL DIST(E1(I),E1(2),E2(S5))
IF (J6.I.E-20) D6=1.E-20
CALL DIST(E1(I),E1(2),E2(S6))
J(IP) = HOM(E1(I),E2(I),E3(I),E4(I),E5(I),E6(I),E7(I),E8(I))
J(IP) = HOM(E1(I),E2(I),E3(I),E4(I),E5(I),E6(I),E7(I),E8(I))
J(IP) = HOM(E1(I),E2(I),E3(I),E4(I),E5(I),E6(I),E7(I),E8(I))
IF (PLT.LE.0.0) 00 TO 451
NUE(J1) = NUE(J1)+1
CALL PENUM
GO TO 456

457 NUE(J2) = 1.P

458 IF (PLT.LE.0.0) 00 TO 451
NUE(3) = NUE(3)+1
CALL PENUM
GO TO 456

500 CALL PENUM(E1(I),E2(I),E3(I),E4(I),E5(I),E6(I),E7(I),E8(I))

GOTO 451
IF (NUE(J1).LE.NUE(J2) 00 TO 505
WRITE(6,6) X(NUE(J1),J1,J2,3,I=NUE(J1).NUE(J2))
GOTO 505
IF (PLT.GT.0.0) 00 TO 451
RETURN

FUNCTION RA0T(A2,C2,A6,C6)
RA0T=A6SC(C6))/(A6SC(A2)+C6)
RA0T=A6SC(A6SC(C6)+A2)
RA0T=A6SC(A6SC(A2)+C6)
RETURN
END

FUNCTION GBRD(I2,I2,I2)
REAL X2(I2),X2(I2),X2(I2)
RETURN
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SUBROUTINE ELMTX( A, E, K, B, C, B1C, J )
C THIS PROGRAM CALCULATES THE DISPLACEMENT FUNCTION MATRIX.
REAL*8 (A,E,K,B,C,B1C)
REAL*8 211,231
C DO
  100  J = 1,4
  100  A(J,1) = X(J(2))
  100  A(J,4) = 2*X(J)
  100  A(J,2) = 2*X(J)
C ALIAS LOADED
C COMPUTE DE(144) AND COMPLETE AZ
UTA = 0.00
  200  J = 1,4
  200  UT(J,0) = 1.0
  200  A(J,3) = UT(J,0)
C ALIAS LOADED AND DE(44) COMPUTED
IF (DABS(XA(J)) < 1.0D-0) WRITE (213)
WRITE (6,1) N,DX
17 (DX,2,2.0) DX = 1.0-DX
210  DO
  210  J = 1,4
  210  J1 = J+J
  210  J2 = J+J
  210  A(J1,1) = DMIN(X(J1,1),J1)/UTA
  210  B(J1,2) = 0.0
  210  B(J1,3) = 0.0
  210  B(J1,4) = DMIN(X(J1,4),J1)/UTA
  210  B(J1,0) = B2(D2)
C RETURN
C FOR 10**** CLAVES 10 UT(AJ) FOR ELE(J),I,J,JUP.Y=1)
C LAST OF UTAW
END
SUBROUTINE CTX( N,J,J1,J2,EK,ER,E,C,BIC)
C THIS PROGRAM DEFINES THE ELASITCITY AND ELEMENTAL STIFFNESS
REAL*8 (A,E,J1,J2,EK,ER,E,C,BIC)
REAL*8 3.0
C C(J,J) = 0.00
C C(1,2) = (C(1,1))/231(4)**0.5 - 1.0
C C(1,1) = (C(1,1))/(1.0-C(1,2)**2*C(1,2))
C(1,2) = (C(1,1))
C C(J,2) = (C(J,1))
C C(J,1) = (C(J,2))
C C(J,2) = 0.00
C C(J,1) = 0.00
C C(J,2) = 0.00
C C(J,1) = 0.00
C C(J,2) = 0.00
C C(J,1) = 0.00
C C(J,2) = 0.00
C C IS CALLED FROM EKL(IJ) YOU MUST MODULN
C EK2) = MODULN OF EUPICITY
C MUST BE MULTIPLIED BY 1.0 TO PROPERLY SCALE
C ELEMENTAL STIFF FROM NODE LOCATION MATRIX CALCULATED BY LDST.
C CALL LDSTX( N,J,J1,J2,J1,J2
C J IS NOU LOADED
C J = THICKNESS OF ELEMENT
be ACCOUNTS FOR FACT THAT ELASIC PARAMETERS NEED TO BE MULTIPLIED
C 1000  DO
  1000  J = 1,4
  1000  C(J,J) = C(J,J)*1.006
  1000  CC(J,J) = VOL*C(J,J)
C CALL RXS(BIC,4,-J,J,J,-J,J,J)
C CALL RXS(BIC,4,-J,J,J,J,J,J)
C RX = BIC**2
BIC = RX**C
C REASS
END
SUBROUTINE RPPL(A,NWB,NBR)
REAL A(NWB)
C
C A=WORKING PARTITION OF NBA BLOCKS
C NWB=NO. OF WORDS IN EACH BLOCK
C NB=SIZE AND DIRECTION OF RIPPLE
C = + CLEARS TOP OF A(HIGH)
C = - CLEARS BOTTOM OF A(LOW)
C
C NWB=NBA-IABS(NBR)
INC=ISIGN(1,NBR)
C
J1 = 01
IF( NBR.LT.00 ) J1 = NBA
C
DO 200 J=1,NBM
C
DO 100 K=1,NWB
100 A(K,J1)=A(K,J1+NBR)
C
200 J=J1+INC
C
RETURN
END
SUBROUTINE LOAD (NB,NWB,I1,NDS,NBP,A,J1)
REAL A(NWB)
C
C NB=NUMBER OF BLOCKS TO BE LOADED + OR -
C NWB=NUMBER OF WORDS IN A READ BLOCK
C I1=FIRST BLOCK TO BE LOADED FROM DATA SET
C NDS=FLUN OF DATA SET
C NBP=PRESENT DATA SET BLOCK POINTER
C A=WORKING ARRAY
C J1=LOCAL BLOCK POSITION IN A
C
NBP = I1
FIND(NDS,NBP)
C
IN=ISIGN(1,NB)
NBA=IABS(NB)
C
DO 1000 IR =1,NBA
C
READ(NDS,NBP)(A(J,J1),J=1,NWB)
C
J1=J1+IN
NBP = NBP + IN
C
IF( IN.GT.00 ) GOTO 1000
IF( NBP.LE.00 ) GOTO 1000
C
FIND(NDS,NBP)
C
1000 CONTINUE
C
RETURN
END
SUBROUTINE UNLOAD(NB,NWB,I1,NDS,NBP,A,J1)
REAL A(NWB)
C
C NB=NUMBER OF BLOCKS TO BE WRITTEN
C POSITIVE BLOCK NUMBERS INCREASE
C NEGATIVE BLOCK NUMBERS DECREASE
C NWB=NUMBER OF WORDS IN WRITE BLOCK
C I1=DATA SET BLOCK NUMBER BEGINNING OF WRITE
C NDS=FLUN OF DATA SET
C NBP=DATA POINTER
C A=WORKING ARRAY
C J1=BLOCK NUMBER LOCALLY OF WRITE
C
NBP = I1
FIND(NDS'NBP)
C
IN=ISIGN(1,NB)
NB=IABS(NB)
C
DO 1000 IW=1,NBA
C
WRITE(NDS'NBP)(A(J,J1),J=1,NWWB)
C
J1=J1+IN
NB=NB+IN
C
IF( IN.GT.00 ) GOTO 1000
IF( NBP.LE.00 ) GOTO 1000
C
FIND( NDS'NBP )
C
1000 CONTINUE
C
RETURN
END
C
SUBROUTINE RDWRT(A, NLR, NHR, NR, NC)
C
REAL A(I)
C
GLOBAL STIFFNESS MATRIX DATA HANDLING
C
NR=NUMBER OF A ROW FOR TASK
C
MC=CONTROL FLAG
C
-1=INITIALIZE A FOR GSM LOADING
C
1=UPDATE FOR GSM LOADING. IF NR=0 FLUSH A.
C
-2=INITIALIZE A FOR REDUCTION OF GSM
C
2=UPDATE FOR REDUCTION OF GSM
C
-3=INITIALIZE A FOR REDUCTION OF RHS VECTOR
C
3=UPDATE FOR REDUCTION OF RHS VECTOR
C
-4=INITIALIZE A FOR BACK SUB OF SOLUTION VECTOR
C
4=UPDATE FOR BACK SUB OF SOLUTION VECTOR
C
-5=INITIALIZE FOR REDUCTION REPAIR OF GSM
C
5=UPDATE FOR REDUCTION REPAIR SAME AS 2
C
COMMON/FLUNSD/NSND, NSED, NSEBD, NSESV, NSUGS, NSF GS
+ * NVSD, NVSED, NVESD, NVESV, NVUGS, NVFGS, NVFFS
C
FLUNSD=FORTAN LOGICAL UNIT NUMBERS FOR SAVING DATA
C
NSND=NODE DATA
C
NSED=ELEMENT DATA
C
NSEBD=ELEMENT BASIC DATA
C
NSESV=ELEMENT STATE VECTOR
C
NSUGS=UNFACTORED GLOBAL STIFFNESS MATRIX
C
NSFGS=FACTORED (REDUCED) GLOBAL STIFFNESS MATRIX
C
NV'S ARE ASSOCIATED VARIABLES TO DATA SETS
C
COMMON/WRKPTN /NXS, NELS, NWP, NHR, NRB, NRRB, NRBP, NLRBP, NLRWP, NHRWP, NLEWP, NNEWP, IPANIC
C
WRKPTN=WORKING PARTITION OF UGSN OR FGSN
C
NWBP=NUMBER OF WORDS IN WORKING PARTITION(WP)
COMMON/DSCNTL, NBD5, LBUD, LBF D, NBD7, NBFD, NDTWP

DSCNTL = DATA SET CONTROL
NBD5 = NUMBER OF BLOCKS OF DATA STORED/STORED
LBUD = LAST BLOCK OF UNFACTORED DATA READ/WRITTEN
LBF D = LAST BLOCK OF FACTORED DATA READ/WRITTEN
NBD7 = NEXT BLOCK OF UNFACTORED DATA THAT WOULD BE READ/WRITTEN, RECORD POINTER
FOR UNFACTORED DATA, NSUDS
NBFD = NEXT BLOCK OF FACTORED DATA THAT WOULD BE READ/WRITTEN, RECORD POINTER, NSFGS
NDTWP = DATA TYPE IN WORKING PARTITION
0 = NO DATA IN WORKING PARTITION
-1 = ALL UNREDUCED DATA
-2 = ALL REDUCED (FACTORED) DATA
*NWR = MIXED, FACTORED DATA IN HIGH BLOCKS,
UNFACTORED DATA IN LOWER BLOCKS,
VALUE IS NUMBER OF FIRST UNFACTORED RCW
DATA IZERO, ZERC/00, 0, 0/

IF (NC.GT.00) GO TO 6000
NC = IABS(NC)
GO TO (1000, 2000, 3000, 4000, 5000), NCA

C INITIALIZE GLOBAL STIFFNESS MATRIX AND DATA SET
1000 DO 1010 J=1,NWRP
1010 A(J) = 0.0

LBUD = 00
NBUD = 01
NLRWP = 01
NHRWP = NWRP
NLBP = 01
NBHWP = NRBWP
NDTWP = -01
GO TO 9000

C INITIALIZE WORKING ARRY FOR START OF REDUCTION
C OF GLOBAL STIFFNESS MATRIX
2000 IF (NDTWP.NE.-01) GO TO 2020
IF (NHBWP.EQ.0,NBD5) GO TO 2060
2020 NB = -NRBWP
II = NBD5
JJ = NRBWP

C
CALL LOAD(NB, NRB, I1, NSFGS, NBUD, A, J1)
C
2060 NBFD = NBDS
NDTWP = -1
NLSWP = NBDS
NLWP = NHBWP - NRBWP + 01
NHRWP = NHBWP * NRRB
NLWP = NHRWP - NRBWP + 01
GO TO 9000
C
C INITIALIZE WORKING ARRAY FOR REDUCTION OF RHS VECTOR
C
3000 IF(NDTWP .NE. -02) GO TO 3010
C
3010 NB = -NRBWP
I1 = NBDS
J1 = NRBWP
C
CALL LOAD(NB, NRB, I1, NSFGS, NBUD, A, J1)
C
NDTWP = -02
C
3030 NHBWP = NBDS
NHRWP = NHBWP * NRRB
NLBP = NHBWP - NRBWP + 01
NLWP = NHRWP - NRBWP + 01
GO TO 9000
C
C INITIALIZE FOR EACH SUBSTITUTION OF SOLUTION VECTOR
C
4000 IF(NDTWP .NE. -02) GO TO 4010
C
4010 NB = NRBWP
I1 = 01
J1 = 01
C
CALL LOAD(NB, NRB, I1, NSFGS, NBUD, A, J1)
C
NDTWP = -02
C
4040 NLBP = 01
NLWP = 01
NHBWP = NRBWP
NHRWP = NRBWP
GO TO 9000
C
C INITIALIZE WORKING ARRAY FOR REPAIR REDUCTION
C
5000 NRB = (NR + NRRB - 01) / NRRB
C
5010 IF(NRB .NE. NHBWP) GO TO 5010
C
5020 IF(IABS(NDTWP) .LT. 02) GO TO 5010
C
5030 IF(NDTWP .LT. 00) GO TO 5020
IF(NDTWP .LE. NR) GO TO 5020
I1 = (N0TMP * NRRB - 0II) / NRRB
NB = NHBWP - I1
J1 = NRBWP - NB
NB = -(NB + 01)
GO TO 5015
C
5010 J1 = NRBWP
I1 = NHBWP
NB = -(NHBWP - NRB + 01)
C
5015 CALL LOAD(NB, NWRB, I1, NSFGS, NBFD, A, J1)
C
NHBWP = NHB
NMRP = NHBWP * NRRB
C
LOAD IN PARTIAL BLOCK
C
5020 NM = NRRB - 01
NLBW = NHBWP * NRBWP - 01
JZ = NWRB * (NMB - NLBW) + 01
NLRWP = (NLBW - 01) * NRRB + 01
JMAX = (NR - NLRWP - 01) * NWR - 01
NREST = NRB - JMAX
C
POSITION RECORD POINTER CN NSGSM
C
NBUD = NRB
FIND(NSUGS, NBUD)
C
READ(NSUGS, NBUD) (A(J), J = JZ, JMAX), (DUM, J = 1, NREST)
NBUD = NBUD + 01
C
LOAD REST OF WORKING PARTITION
C
NB = NRB - NLBW
IF (NB .LE. 00) GO TO 5030
C
CALL LOAD (-NB, NRB, NRB, NSFGS, NBUD, A, NB)
C
5030 NLRWP = (NLBW - 01) * NRRB + 01
C
GO TO 9000
C
END OF INITIALIZATION ROUTINES
C
START OF UPDATE ROUTINES
C
6000 GO TO (6100, 6200, 6300, 6400, 6200) * NC
C
UPDATE GSM LOADING, FLUSH IF NR .NE. 0
C
6100 I1 = NLBW
J1 = 01
NB = 01
IF (NR .NE. 0) NB = NRBWP
C
CALL UNLOAD(NB, NWRB, I1, NSUGS, NBUD, A, J1)

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C IF(NR.NE.0) GO TO 6105
ENDTNWP=01
GOTO 9000
C 6105 CALL RPPL(A,NRBWP,NWRB,01)
C JMX=NWRP
J1=JMX-NWRP+01
C DO 6110 J=J1,JMX
6110 A(J)=0.0
C NDTWP=01
C NLRWP=NLRPWP+NRBB
NHRWP=NHRWP+NRBB
NLBWP=NLBWP+01
NHBWP=NHBWP+01
C GO TO 9000
C UPDATE FGSM DURING REDUCTION FROM UGSM TO FGSM
C 6200 J1=NRBWP
II=NHBWP
NB=01
IF(NR.EQ.0) NB=NRBWP
C CALL UNLOAD(NB,NRBWP,II,NSFGS,NBFD,A,J1)
C IF(NR.GT.0) GO TO 6210
C NDTWP=02
GO TO 9000
C 6210 CALL RPPL(A,NRBWP,NWRB,-01)
C NDTWP=NR
NLWP=NLWP-01
NLBWP=NLBWP+01
NHRWP=NHRWP-01
NHBWP=NHBWP-01
C II=NLBWP
J1=01
CALL LOAD(NB,NRBWP,II,NSFGS,NBUD,A,J1)
C GO TO 9000
C UPDATE CURING REDUCTION OF RHS VECTOR
C 6300 CALL RPPL(A,NREWP,NWRB,-01)
C J1=01
II=NLBWP-01
NB=01
C
CALL LOAD(NB,NWBR,I1,NSFGS,NBFD,A,J1)
C
NLBWP = NLBWP+01
NHBWP = NHBWP+01
NLRRP = NLRRP - NRRB
NHRRP = NHRRP - NRRB
C
GO TO 9000
C
UPDATE DURING BACK SUBSTITUTION OF SOLUTION
C
6400 CALL RPPL(A,NRBMP,NRRB,01)
C
J1=NRRBP
I1=NHBWP+01
NB=01
C
CALL LOAD(NB,NWBR,I1,NSFGS,NBFD,A,J1)
C
NDTWP=02
C
NLRRP=NLRRP+NRRB
NHRRP=NHRRP+NRRB
NLBWP=NLBWP+01
NHBWP=NHBWP+01
C
9000 NLRR = NLRRP
NHR = NHRRP
WRITE(7,*),ROWR=NLRRP:NHRRP,NLRRP,NLBWP,NHBWP
C
IF(IPANIC=EQ.00) RETURN
C
WRITE(7,1)(A(J),J=1,NWBP)
1 FORMAT(IP 13E10.2)
C
RETURN
C
END
SUBROUTINE SVDSBR ( SVD, SV, D )

REAL SVD(5), SV(4), D(6), EE(3), SG(3)

COMMON/ELMDTA/ B(18), E(9), BTE(18), NU(3)

COMMON/TINE/T, DT, DTRM, DTRX, DTP, TPR, DTPR, TLOAD, F1LOAD,
+ T2LOAD, F2LOAD, FSLF, FSLP, JUMP, IR, IWR

COMMON/STASH/TST1, TST2, DDTR, PSRMX, SVMMX(10), WSTRS(10)

COMMON/CNSTTV/ZI, Z0, Z1, D0, EM, EN, A, R, YW, C1, C2, C3, C4, C5

CALL MXM( EE(3), 3, 1, EE, 3, 3, 3, 6, 6 )

DO 0100 I = 1, 3

0100 EE(I) = EE(I) - SV(I)

CALL MXM( SG(3), 3, 1, EE, 3, 3, 3, 3 )

IF( IWR .LE. 0.006 ) GOTO 0110

WRITE(7,*) 'EE* EE

0110 SGA = ( SG(1) + SG(2) ) * 0.3333333
SG(1) = SG(1) + SGA
SG(2) = SG(2) + SGA

SECONd INVARIANT OF DEVIATORIC STRESS TENSOR
SJ2 = 0.5 * ( SG(1)* SG(1) + SG(2)* SG(2) ) * SG(3)
+ SG(3)* SG(3)

VDN MISES STRESS
SVM = SQRT( 3.0 * SJ2 )

IF( IWR .GT. 0.02 ) WRITE(7,*) 'SVM, SGA, SVM, SGA
Z = SV(4)
D2 = SVM / Z

IF( D2 .GT. 0.005 ) GOTO 0150

SL = 0.0
D2 = 0.0
GOTO 0155

0150 D2 = C4 + C2*D2**C1
D2 = TEXP(D2) 
SL = D2/SVM 
IF(IMRT.LE.05) GOTO 0155
C WRITE(7,*) SL, D2, SVM, SL, D2, SVM 
C
0155 ZII = AMAX1(.001, (Z-ZII)/ZII) 
ZD1 = C5 + R*ALOG(ZII) 
ZD1 = TEXP(ZD1) 
ZD2 = AMAX1(0.0, C5*(ZII-Z)*SL*SJ2) 
0160 ZD = ZD2 - ZD1 
C IF(IMRT.GT.04) WRITE(7,*) ZD, ZD, ZD1, ZD2 
C
0200 I=1,3 
0200 SVD(1) = SL*SG(1) 
SVD(4) = ZD 
SVD(5) = SVM 
C RETURN 
C
END
C
******************************************************************************
C
SUBROUTINE PUTOUT( BN, DN, FN, FPN )
C
JUST A SIMPLE ROUTINE FOR DUMPING THE RESULTS, IF ANY. 
C
REAL*4 DN(2,1), FN(2,1), FPN(2,1), SV(10), FP(2,3) 
LOGICAL*1 BN(2,1) 
COMMON/FLUNSO/, NSND, NSED, NSEBV, NSES, NSUGS, NSFGS, 
+ NVND, NVED, NVE3D, NVESV, NVUGS, NVFGS, NSFFN
COMMON/ELMDTA/, BM(3,6), CM(3,3), BTDMN(18), NU(3) 
COMMON/SPCMN/, A, B, C, D, RA, RB, RC, E, G, TMK, FCLUD(10), FCSTR(20) 
+ YSTR(20), 
+ NA, NB, NC, LNDN(10), NND, NELS 
COMMON/TIME/, T0, DT, DTRMN, DTRMX, DTP, TPR, TILOAD, FLLOAD, 
+ TLOAD, FLOAD, FSLF, FSLP, JUMP, IRUN, IMRT 
COMMON/STASH/, TST1, TST2, DTR, PRMX, SVMX(10), PLSTRN(10) 
+ MSSTRS(10) 
C
INITIALIZE PLASTIC NODE FORCES 
C
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DO 0400 I=1,2
DO 0400 J=1,NNDS
0400 FPN(I,J) = 0.0
C
IF( IRUN.LE.02 ) GOTO 0900
C
DO 0600 I=1,3
J = N(I)
DO 0500 K=1,2
0500 FPN(K,J) = FPN(K,J) + FP(K,I)
C
0600 CONTINUE
C
SUM X & Y FORCES AS CHECK ON SOLUTION VERACITY.
C
0900 SUMX = 0.0
SUMY = 0.0
C
WRITE(6,1) T
DO 1000 J=1,NNDS
C
SUMX = SUMX + FPN(J,J)
FN(J,J) = FPN(J,J) - FPN(J,J)
SUMY = SUMY + F(J,J)
FN(J,J) = FN(J,J) - FN(J,J)
C
IF( IWRT.LE.01 ) GOTO 1000
WRITE(6,2) (I=1,2)
1000 CONTINUE
C
WRITE(7,*), SUMX, SUMY, SUMX, SUMY
C
COMPUTE THE STRESS PROFILE AT THE MID LINE
C
GET THE SUM OF FORCES AND STRESSES
C
NL = NA + 01
SUMY = 0.0
DO 2000 I=1,NL
SUMY = SUMY + FN(I,J)
FN(I,J) = FN(I,J)/FCSTR(I)
2000 CONTINUE
SUMY = -SUMY
STRAVG = SUMY/THK4A)
IF( STRAVG.EQ.0.0 ) STRAVG = 1.0
STCNTR = -FN(I,J)/STRAVG
C
SUMY = SUMY*2.0
WRITE(6,*), APPLIED LOAD*, AVG STRESS*, SUMY, STRAVG
WRITE(6,*), VON MISES/MID NFRC**, SVMMX(I), FN(I,J)
WRITE(6,**), STRESS CONCENTRATION**, STCNTR
C
IF( IWRT.LE.01 ) RETURN
C
WRITE(6,**), STRESS VS MIDLINE POSITION*
WRITE(6,**), "\{FN(I,J),YSTR(I)\},I=1,NL**
WRITE(6,**), MAX STRESS ELEMENTS*
WRITE(6,**), \{MXSTRS(I),SVMMX(I),PLSTRN(I)\},I=1,10**
C
RETURN
C
1 FORMAT(*0 PUTOUT PUTS OUT AT TIME *****F8.2)
2 FORMAT(*24IP 2L2**2E13.4)
C
END
C Stage 2 of Ill GE Program.

LREAL* F1(00),DF(40),AM(400),Z1(400)
INTEGER* 2 NDX(100)
LLOGICAL* CNS(400)
DATA JU00,(JU00)
C COMMON/MASK,DSEV,NSOS,NSUBD,NFSCS,
* AM10,DF10,AMF10,AMF20,AMF21,AMF30,NSUBD,NSOS,DFSCS
C COMMON/KNIGHT/MASK,NSUS,NSWP,NHNP,NSUB,
* NSWP,NSKIM,NSUBD,NSOS
C EQUIVALENCE(MASK,NDX),JU00,AM10,AMF10,(CD,FILN(1)),(CL,FILN(2))
C DATA NW3/10/,NW4/10/,NL20/40/
DATA NW3/10/
DATA NAU/10/
DATA MASK,NM3/10,00/
DATA FILN(1),FILN(2),FILN(8),FILN(10),FILN(14)
C IF ( I50 .LT. 00 ) READ(1) [FILN(1),J=1,9]
C IF ( I50 .LT. 00 ) GOTO 1060
FILN(1) = CH
FILN(12) = CH
FILN(13) = CD
C IF ( I50 .LT. 00 ) [FILN(1),J=1,9]
FILN(11) = CC
FILN(12) = CL
FILN(13) = CD
C IF ( 150 .LE. 10 ) GOTO 1060
FILN(11) = CC
FILN(12) = CL
FILN(13) = CD
C IF ( I50 .LE. 30 ) GOTO 1060
FILN(11) = CC
FILN(12) = CL
FILN(13) = CD
C CONTINUE
GET THE PARAMETERS FROM STAGE 1.

\begin{verbatim}
READ (b,j) NLS,NLB,NWB,NWb,NLB,NLBS,NFBS
WRITE(7,*)'SU4',NLS,NWB,NLB,NLBS,NFBS

CALCULATE MISSING PARAMETERS

NWB = NWL/NWB
NWb = NWB/NWB
NWDB = NWL/NWb
NWBP = NWL*NWB
NWMP = NWL*NWB

IF ( NSUB.LT.2.00 ) 2000

FILE(11) = CS
FILE(12) = CS
FILE(13) = CS

OPEN (UNIT=NSUB,MODE=FILE,ACCESS='DISC',ASSOCIATABLE=EVENTS,
       TITLE='EVENT',DISPO='KEEP')

2000  CALL CHRJRM (ZST,NAB Km NLS )

WRITE(7,*)'CLOSING DATA DIS.'

IF ( NSED.GT.0.0 ) CLOSE(UNIT=NSED)
IF ( NSED.GT.0.0 ) CLOSE(UNIT=NSED)
IF ( NSED.GT.0.0 ) CLOSE(UNIT=NSED)

GT0 1000

FO101 (12,3,1,1,1,1,1,1,1,1)
FO101 (12,3,5,1,1,1,1,1,1,1)
FO101 (12K,13,5,1,1,1,1,1,1,1)

END
\end{verbatim}
SUBROUTINE MXM(C, NC, M, N, A, NA, B, NB, NCTR)
REAL C(NC,1), A(NA,1), B(NB,1)
C C = A*B OR C = AT*B OR C = A*BT
C NCTR IS THE CONTRACTION INDEX RANGE
C M IS THE NUMBER OF ROWS IN THE RESULT. C = AT*B IF M<0
C N IS THE NUMBER OF COLUMNS IN RESULT. C = A*BT IF N<0
C A DOUBLE PREC INNER PRODUCT IS ACCUMULATED.
REAL SUM, AD, BD
C SET UP BRANCHES AND INDEX RANGES.
NROW = IABS(M)
NCOL = IABS(N)
ASSIGN 100 TO IGET
IF(N GT 00) GOTO 10
ASSIGN 200 TO IGET
10 ASSIGN 400 TO JGET
IF(N GT 00) GOTO 20
ASSIGN 500 TO JGET
20 DO 700 I=1,NROW
DO 700 J=1,NCOL
SUM = 0.00
DO 600 K=1,NCTR
GOTO IGET *(100,200)
100 AD = A(I,K)
GOTO 300
200 AD = A(K,I)
300 GOTO JGET *(400,500)
400 BD = B(K,J)
GOTO 600
500 BD = B(J,K)
600 SUM = SUM + AD*BD
700 C(I,J) = SUM
RETURN
END
SUBROUTINE PLSSTP( DN, FN )
REAL DN(2,1), FN(2,1), SV0(4), SV(4), DLP(2,3), DL0(2,3), FPI(2,3)
INTEGER NU(3)
REAL B(18), E(9), BTE(18), SVDI(5), SVD2(5)
REAL A(18), ATE(18), SVOCL(5), SVO2C5)
INTEGER NU(3)
COMMON/TIME/T, DT, TDRMN, TDRMX, DTP, TPR, DTPR, TLOAD, F1LOAD,
+ T2LOAD, F2LOAD, FSLF, FSLP, JUMP, IR, IWT
COMMON/STASH/TST1, TST2, DDTR, PSRMX, SVMXX(10), PLSTRN(10)
+ NSTRS(10)
COMMON/ELMDA/E, BTE, NU
COMMON/FLUUSD/NSND, NSED, NSEBD, NSESY, NSUMG, NSSF,
+ NVND, NVED, NVBD, NVSY, NVUG, NVFG, NVSTF
COMMON/WRKPTN/NX, NELS, IDM(11)
COMMON/CNSTTV/Z1, Z2, Z3, D0, EM, EN, R, YW, C1, C2, C3, C4, C5
COMMON/OALP/O, O, O, O, O, O, O, Z
DATA DLP/0,0,0,0,0,0,0,0,
WRITE(7,*) PL, T, DT, DTP, T, DT, DTP
IF( DTP, NE, 0 ) GOTO 0050
DTP = 0.0
GOTO 0050
0050 DTP = FSLF4DT/DTP
0060 DT2 = DT/2.0
PSRMX = 0.0
DO 0070 I = 1, 10
NSTRS(I) = 00
0070 SVMXX(I) = 0.0
DO 1000 NELS = 1, NELS
READ(NSES, NSEL) SV0, DLP, FP, SVD2
READ(NSEBD, NSEL) B, E, BTE, NU
DO 0100 I = 1, J
J = NU(I)
DO 0100 K = 1, 2
DL0(K, I) = DN(K, J)
DLP(K, I) = DLO(K, I) + DTDTP* ( DLO(K, I) - DLP(K, I) )
0100 CONTINUE
CALL SVDSBR(SVD1, SV0, DLO)
DO 0200 I = 1, 4
SV(I) = SV0(I) + DT* SVD1(I)
C
C COMPUTE STEP CHANGER PARAMETER
C
PSR = ABS( SVD(1)+SVD2(1) )
PSR = ABS( SVD(1)-SVD2(1) )/ ( TST2+PSR )
IF( PSR.LE.PSRMX ) GOTO 0200
PSRMX = PSR
0200 CONTINUE
C
C TRUNCATE SV(4), IF NEEDED
IF( SV(4).GT.Z1 ) SV(4) = Z1
C
CALL SVD58R(SVD2,SV,DL)
C
DO 0300 I=1,4
C
SV(I) = SVD(I) + DT2* ( SVD(1)+SVD2(1) )
0300 CONTINUE
C
C TRUNCATE SV(4) IF NEEDED
IF( SV(4).GT.Z1 ) SV(4) = Z1
C
LOCATE 10 HIGHEST VON MISES STRESSES
C
SV = 0.5* ( SVD1(5)+SVD2(5) )
C
DC 0350 I=1,10
J = I-1
IF( SVM.LE.SVMX(J) ) GOTO 0360
0350 CONTINUE
J = 00
0360 J = J + 01
IF( J.GT.10 ) GOTO 0370
SVMX(J) = SVM
MXSTRS(J) = NEL
PLSTRN(J) = SV(2)
0370 CONTINUE
C
C COMPUTE PLASTIC FORCES
C
CALL MXM( FP,6,6,1,BTE,6,SV0,3,3 )
C
DO 0400 I=1,3
J = NUC(I)
DO 0400 K=1,2
FN(K,J) = FN(K,J) + FP(K,I)
0400 CONTINUE
C
IF( IMRT.LT.5 ) GOTO 0410
WRITE(7,*) NEL,NEL
WRITE(7,*) SVM,SVD(I),I=1,4
WRITE(7,*) SVD1, SVD2
WRITE(7,*) PLSF,FP
0410 CONTINUE
C
WRITE(NSES,SV,NEL) SV0,DL0,FP,SVD2
C
1000 CONTINUE
C
T = T + DT
WRITE(7,*) PLSF,DT,PLSF,DT
DTP = DT
C
RETURN
C----------------------------------------------
END
SUBROUTINE UPDBOFI BN.DN.FN  
C 
C THIS IS THE LOADING PROGRAM 
C 
LOGICAL*! B N ! 2.1) 
REAL** DN(2.1,1,FN) .A.B.C.D*RA,RB.RC.E.G,T.FCLD,FCSTR,YSTR 
+ . FORC,SPCM(16,5),DUM(6),ZERC(3) 
INTEGER*2 NA, NB, NC, LDND, JUMP, KNTRL 
C 
COMMON/ S P C M N  / A .B.C.O.RA,RB>RC.E.G.THK 
+ *FCLO(!0).FCSTR(20)>YSTR(20) 
C 
COMMON/FLUNSD/ NSND,NSED,NSEBD,NSESV,NSUGS,NSFGS, 
+ NVND,NVED,NVEBD,NVESV,NVUGS,NVFGS,NFFN 
C 
COMMON/T I M E/T .DT.DTRMN,DTRMX,DTP,TPR,DTPR,T1,T2,F2 
+ *FSLF,FSLP,JUMP,IR,INRT 
C 
COMMON/E L M D TA/ B MX(18),CBMX(27),NU(3) 
C 
COMMON/STASH/TST1,TST2,DDTR,PSRMX,SVMX(10),PLSTRN(10) 
+ ,MXSSTRS(10) 
C 
EQUIVALENCE(ISPCM,1,1),A) 
C 
DATA ZERO/O..0..0./ 
C 
C GET SPECIMEN PARAMETERS FROM NSND DATA SET 
C 
IF( IMRT.GT.00 ) 
+ WRITE(7.*)'UPDATE*' JUMP,NSND,NSED,NSEBD,NSESV,NSUGS,NSFGS 
IF( JUMP.LT.00 ) RETURN 
IF( JUMP.EQ.00 ) GOTO 3000 
IF( JUMP.EQ.01 ) GOTO 4000 
C 
NNDS = JUMP 
C 
NNP = NNDS + 01 
FIND( NSND*NNP ) 
C 
DO 0400 J=1,5 
IF( IWRIT.GT.00 ) WRITE(7.*)'SPCM*',J 
NWP = NNDS + J 
0400 READ(NSND*NNP) (SPCM(1,J),J=1,16) 
C 
WRITE(6.*)'SPECIMEN PARAMETERS' 
WRITE(6.*) A,B,C,D*RA,RB.RC.E.G,T,FCLD,FSTR,YSTR 
WRITE(6.*) NA,NC,NDX,NELS 
WRITE(6.*) (FD(1,J),J=1,10) 
WRITE(6.*) ('YSTR(J),FCSTR(J),J=1,NL) 
NL = NA + 01 
C 
WRITE(6.*) IF( IR.LE.02 ) GOTO 460 
C 
READ5,8) ZI,Z0,Z1,DD,EM,EN,AMTL,AMTL,YN 
WRITE(6.*) 'EODNER MATL PARMS*',ZI,Z0,Z1,DO 
WRITE(6.*) 'EODNER MATL PARMS*',EN,EM,AMTL,AMTL,YN 
C 
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C1 = (-E0 + E1)
C2 = (E0 + E1)/C1
C3 = E0 + E1
C4 = ALOG(D0)
C5 = ALOG(Z1*ANL)
WRITE(6,*), DDE COFF', C1, C2, C3, C4, C5

C NULL ALL FORCES AND DISPLACEMENTS

0460 DO 0500 J=1, NNDs
  FN1(J) = 0.0
  FN2(J) = 0.0
  DN1(J) = 0.0
  DN2(J) = 0.0
  O500 CONTINUE

C ALL BN SHOULD BE SET BY THE AUTOGENERATE PRG, READ NSND

C IF( NSND > LE.00 ) RETURN

C FIND( NSND + I )

C DD 2000 I=1, NNDs
  READ( NSND + I ) DU4, (BK(J,I), J=1, 2)
  2000 CONTINUE
  CLOSE( UNIT = NSND )

C IF( NSBD > LE.00 ) GOTO 3000

C DD 2500 NEL = 1, NELS

C INITIALIZE ALL ELEMENTS TO START CONDITIONS

C WRITE( NESV + NEL ) ZERO, ZO, ZERC, ZERC, ZERG, ZERO

C IF( YM > LE.00 ) GOTO 2500

C UPDATE THE ELEMENT ELASTIC DATA BY SCALING THE MODULI

C READ( NSEBD + NEL ) BMX, CBTMX, NU

C DD 2400 J = 1, 27
  2400 CBTMX(J) = CBTMX(J)*YM

C WRITE( NSEBD + NEL ) BMX, CBTMX, NU

C 2500 CONTINUE

C 3000 IF( IR > GT. 02 ) GOTO 3100

C READ( 5, * ) FORCE

C DD 3050 I = 1, NXS
  FN(I) = 0.0
  3050 FN(I) = 0.0
  GOTO 4200

C 3100 T1 = -1.0
  T2 = 0.0
  F1 = 0.0
  F2 = 0.0
  FORCE = 0.0
  FSLP = 0.0
  FSLF = 0.0

C KICK AROUND ON SET UP CALL
  GOTO 4200

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C DETERMINE REFERENCE STEP SIZE IN T
C
4000 IF( PSRMX.GE.TST2 ) GOTO 4010
IF( PSRMX.LE.TST1 ) GOTO 4020
DTR = DTRMX-( PSRMX-TST1 )*DDTR
GOTO 4025
4010 OTR = OTRMN
GOTO 4025
4020 OTR = OTRMX
C CHECK TIME TO NEXT SEGMENT OF FORCE PROFILE
C
4025 T2T = T2 - T
DT = DTR
IF( T2T.GT.0.03125*DTR ) GOTO 4100
T1 = T2
F1 = F2
IF( NSFFN.LT.00 ) GOTO 9000
READ(5,*), T2,F2
WRITE(6,*), T,F,T2,F2
WRITE(7,*), T,F,T2,F2
IF( T2.LE.T1 ) GOTO 9000
FSL = (F2-F1)/(T2-T1)
IF( ABS(FSL).GE.ABS(FSLF)*0.5 ) GOTO 4030
FSLF = 0.0
GOTO 4060
4030 IF( FSL.FSLP.GE.0.0 ) GOTO 4040
FSLF = 0.0
GOTO 4060
4040 FSLF = 1.0
4060 FSLP = FSL
DT = DTR
GOTO 4150
4100 FSLF = 1.0
DT = DTR
IF( T2T.GT.1.25*DTR ) GOTO 4150
DT = T2T
4150 FORCE = (T-T1)*FSLP + F1
IF( IWRITE.GT.00 ) WRITE(7,*),FORCE,*,FCRCE
C
4200 FORCE = 0.5*FORCE
ML = NC + 01
DO 4300 J=1,ML
LJ = LOND(J)
IF( LJ.LE.00 ) GOTO 4300
FN(2,LJ) = FN(2,LJ) + FORCE*FCLD(J)
4300 CONTINUE
JUMP = 01
WRITE(7,*),DT,DTR,FSLF,DT,DTR,FSLF
C
RETURN
C
9000 JUMP = -01
C
RETURN
C-------------------------------------------------------------------------------

END
C STAGE 3 PLASTIC/ELASTIC DRIVER PROGRAM

REAL A F N C 4 0 0 ) .DN(* 0 0 1 .F P N (400 I.EGTC3600)
INTEGER*2 NUE(1800)
LOGICAL*1 BN(400)
BYTE J03NM(6).FILNM(14).CB.CD.CE.CF.CG.CL.CN.CS.CU.CV
COMMON/FLUNSD/NSND .NSED .NSEBD .NSESV .NSUGS .NSFGS
+ .NVND .NVED .NVED .NVESV .NVUGS .NVFGS .NSFFN
COMMON/SPCMN/ISPCM(160)
COMMON/TIME/T,DT,DTRMN,DTRMX,DTP,TPT,DTPR,T1LOAD,F1LOAD,
+ T2LOAD,F2LOAD,FSLP,FSLP,JUMP,IR,IWRT
COMMON/STASH/TST1,TST2,DDTR,PSRMX,SVMMX(10),PLSTRN(10)
+ .MXSTRS(10)
COMMON/CNSTTV/BOD(14)
COMMON/WRPTN/NXS,NELS,NWFP,NWR,NRBP,NRB,
S NRRW,NRRBP,NLWR,
S NHRWP,NLBRP,NHRBP,IPANIC
EQUIVALENCE(NBAND,NWR)(CD,FILNM(1))(CL,FILNM(2))
COMMON/DSCNTL/NLBDS,LBDU,LBDP,NBDU,NBDP,NDBD,NDBP,ZWTWP
DATA NWED/16/,NWED/16/,NWEBD/64/,NWEB/32/
DATA NWB/256/
DATA MSZ/3600/

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DATA MXNDS,NEL/NELS/300.00/
DATA FILNM(1),FILNM(2),FILNM(3),FILNM(10),FILNM(14)
DATA CB,CE,CF,CG,CH,CS,CU,CV
CALL PESTUP(1)

3000 JUMP = NXS/2
IR = IABS(IR)
CALL UPDBDF(BN,DN,FFN)
CALL DUDCRY(NXS,EGT,NMR,BN,NXS)
CALL UPDBDF(BN,DN,FFN)
CALL OUSVX(NXS,EGT,NMR,BN,DXS)

3100 IF(IR.LE.02) GOTO 5000
C PLASTIC/ELASTIC PROBLEM
READ(5,*)DTRMN,DTRM,N,TST1,TST2,DTM,TMR
DTST12 = TST2 - TST1
DTR = (DTRM-DTRMN)/DTST12
DT = ( TST12
IWRT = 00
WRITE(5,*)STARTUP, T,DTRMN,DTRX,TST1,TST2,DTM,TMR
WRITE(5,*)T,DTRMN,DTRX,TST1,TST2,DTM,TMR
C TOP OF PLASTIC LOOP
4000 DO 4100 I=1,NXS
4100 FN(I) = 0.0
C GLOBAL NODE FORCES ARE NULL TO ALLOW ADDING OF PLASTIC FORCES
CALL PLASTP( DN,FFN )
C IF(JUMP.LE.00) GOTO 5000
C CALL UPDBDF(BN,DN,FFN)
C CALL DUSVX(NXS,EGT,NMR,BN,DXS)
C IF(T.LT.TMR) GOTO 4000
IWRT = IRYT
CALL PUTOUT(BN,DXS,FPN)
TMR = TMR + DTMP
GOTO 4000
C END OF PLASTIC LOOP
5000 CALL DUSVX(NXS,EGT,NMR,BN,DXS)
C 5100 CALL PUTOUT(BN,DXS,FPN)
CALL PESTUPI(2)

STOP 'STAGE3'

END

STAGE 3 SETUP FOR PLASTIC/ELASTIC FEM PROGRAM

SUBROUTINE PESTUPI (KICK)

BYTE JOBNM(6), FILNM(14), CB, CD, CE, CF, CG, CL, CN, CS, CU, CV

COMMON/FLUNSD/NSND, NSED, NSEBD, NSESV, NSUGS, NSFGS
+ NVND, NVED, NVEBD, NVESV, NVUGS, NVFGS, NSFFN

COMMON/SPC M N /1 SPCMN(160)

COMMON/TIME/T, DT, DTRMN, DTRMX, DTPR, DTPR, T1LOAD, T0LOAD,
+ T2LOAD, F2LOAD, FSLF, FSLP, JUNF, IRUN, IMRT

COMMON/STASH/TST1, TST2, DDT, PSLNX, PVNX(10), PLSTRN(10)
+ MXSTRS(10)

COMMON/CNSTTV/E00(14)

COMMON/WRKPTN/NXS, NELS, NWBP, NMBP, NMRB, NRB
$ NRRB, NHRBP, NLRWP
$ NHP, NHPBP, NHRBP, IPANIC

EQUIVALENCE(MBAND, NMR), (CB, FILNM(1)), (CL, FILNM(2))

COMMON/DSCNTL/NLBD, LBD, LBFD, NBUD, NBDF, ADTWP

DATA NWED/16/, NWEVD/16/, NWEVD/64/, NVEBD/32/
DATA NWBP/256/
DATA MSZ/3600/
DATA NMXNOS/3200/00/
DATA FILNM(1), FILNM(14), FILM(2), FILM(10), FILM(14)
+ / "0" + "1" + "2" + "3" + "4" + "5" + "6" + "7" + "8" + "9"/
DATA CB, CE, CF, CG, CL, CN, CS, CU, CV
+ /"B" + "E" + "F" + "G" + "H" + "S" + "U" + "V"/

GOTO (1000,6000), KICK

1000 READ(5,1)JOBNM, IRUN, NSND, NSED, NSEBD, NSUGS, NSFFN, IRYT, IPANIC

IF (IRUN.EQ.00) RETURN

DO 1050 J=1, 9
1050 FILNM(J+03) = JCBNM(J)

READ(5,*), NSFGS, NSESV

WRITE(6,1)JOBNM, IRUN, NSND, NSED, NSEBD, NSUGS, NSFFN, NSFGS, NSESV

IF (IRUN.LT.00) READ(5,1)(FILNM(J), J=1, 9)
C
IF( NSNO LE.00 ) GOTO 1060
C
FILNM(11) = CN
FILNM(12) = CD
FILNM(13) = CD
C
OPEN
* (UNIT=NSND, NAME=FILNM, ACCESS='DIRECT*, ASSOCIATEVARIABLE=NVND,
  RECORDSIZE=NVND, TYPE='OLD*, DISP='KEEP*)
C
1060 IF( NSED LE.00 ) GOTO 1070
C
FILNM(11) = CE
FILNM(12) = CL
FILNM(13) = CD
C
OPEN
* (UNIT=NSED, NAME=FILNM, ACCESS='DIRECT*, ASSOCIATEVARIABLE=NVED,
  RECORDSIZE=NVED, TYPE='OLD*, DISP='KEEP*)
C
1070 IF( NSEBD LE.00 ) GOTO 1080
C
FILNM(11) = CE
FILNM(12) = CS
FILNM(13) = CV
C
OPEN
* (UNIT=NSESV, NAME=FILNM, ACCESS='DIRECT*, ASSOCIATEVARIABLE=NVESV,
  RECORDSIZE=NVESV, TYPE='UNK AeN*, DISP='KEEP*)
C
1080 IF( NSES V LE.00 ) GOTO 1090
C
FILNM(11) = CE
FILNM(12) = CS
FILNM(13) = CV
C
OPEN
* (UNIT=NSESV, NAME=FILN M, ACCESS='DIRECT*, ASSOCIATEVARIABLE=NVESV,
  RECORDSIZE=NVESV, TYPE='UNK AeN*, DISP='KEEP*)
C
1090 READ(5,*) NXS, NELS, NW, NWB, NLBS, NPBDs
WRITE(6,*) ' DATA SIZE AND CONTROL P A R M S'
WRITE(6,*) NXS, NELS, NW, NWB, NLBS, NPBDs
C
NRRB = NWB/NW
NRP = NSZ/NW
NP = NWB/NRRB
NP = WND( NRP, NLBS )
NR = NRRBNRP
NWP = NWPNWP
C
IF( NSUGS LE.00 ) GOTO 1100
C
FILNM(11) = CU
FILNM(12) = CG
FILNM(13) = CS
C
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OPEN
  \{UNIT=NSFGS\*NAME=FILNM\*ACCESS='DIRECT'\*ASSOCIATEVARIABLE=NVFGS,
  \} RECORDSIZE=NWLB\*TYPE='OLD'\*DISP=KEEP
C
1100 IF (NSFGS LE 00) GOTO 3000
C
FILNM(11) = CF
FILNM(12) = CG
FILNM(13) = CS
C
OPEN
  \{UNIT=NSFGS\*NAME=FILNM\*ACCESS='DIRECT'\*ASSOCIATEVARIABLE=NVFGS,
  \} RECORDSIZE=NWLB\*TYPE='UNKNOWN'\*DISP=KEEP
C
3000 RETURN
C
6000 WRITE(7,*) 'CLOSING DATA SETS'
C
IF (NSED GT 00) CLOSE(UNIT=NSED)
IF (NSEBD GT 00) CLOSE(UNIT=NSEBD)
IF (NSESV GT 00) CLOSE(UNIT=NSESV)
IF (NSUGS GT 00) CLOSE(UNIT=NSUGS)
IF (NSFGS GT 00) CLOSE(UNIT=NSFGS)
C
RETURN
C
1 FORMAT(1X,6A1,12I5/1X,15I5)
2 FORMAT('* SIZE OF WORKING ARRAY INSUFFICIENT,MSZ,M3END,NWRP,315')
3 FORMAT(12X, 15I5 )
C
END
C
END
C
TRUNCATING EXPONENTIAL FUNCTION SUBPROGRAM
C
FUNCTION TEXP( T )
C
TEXP = T
IF ( TEXP LE 70.0 ) GOTO 010
TEXP = 2.51544E30
RETURN
C
010 IF ( TEXP GE -70.0 ) GOTO 020
TEXP = 0.0
RETURN
C
020 TEXP = EXP( TEXP )
RETURN
C
END
SUBROUTINE DUSVX(NX,A,M,TF,X,F)
C
REAL A(1),X(1),F(1)
LOGICAL TF(1)
REAL*8 SD

C INITIALIZE A FOR REDUCTION OF F
C
CALL RDWRT(A,NLRA,NMRA,NX,03)
C
LKZ = MP*NLRA-01
M = MP-01
LK = L*N+K-LKZ
C
L = NX
C
1000 LMM = MAX(01*L-M)
IF (LMM$GE$NLRA) GO TO 1010
CALL RDWRTH(A,NLRA,NMRA,NX,03)
LKZ = MP*NLRA-01
C
1010 IF(TF(L)) GO TO 2000
SD = F(L)
JMN = L+01
C
IF (JMN$GT$NX) GO TO 1022
C
ML = M+L-LKZ
JNX = MNO(NX,L+M)
C
DD 1020 J = JMN,JNX
SD = SD - A(ML+J)*DBLE(X(J))
1020 CONTINUE
1022  JMN = MAX0(JM1,L-M)
    JMX = L-01
C    IF( JMX.LT.JMN ) GOTO 1030
C    DO 1025 J=MJN,JMX
C      IF( .NOT.TF(J) ) GOTO 1025
C      SD = SD - A(MJ+L-LKZ)*DBLE(X(J))
    1025  CONTINUE
C    1030  X(L)=SD
C    2000  L=L-01
C      IF(L.GE.01) GO TO 1000
C      REDUCTION OF F STORED IN X
C      BEGIN BACK SUBSTITUTION AND CALCULATION OF
C      UNKNOWN X AND UNKNOWN F
C      L=01
C    3000  LMM=MAX0(JM1,L-M)
C      IF(L.LE.NHRA) GO TO 3010
C      CALL RDWR(A,NLRA,NHRA,LMM,04)
C      LKZ=MPE*NLRA-01
C    3010  IF(TF(L)) GO TO 3011
C      SD=DBLE(X(L))/A(MP*L-LKZ)
C      GO TO 3012
C    3011  SD=-DBLE(X(L))*A(MP*L-LKZ)
C    3012  JMX=L-01
C      IF(JMX.LT.LMM) GO TO 3025
C      DO 3020  J=LMM,JMX
C      IF(TF(L)) GO TO 3015
C      IF(TF(J)) GO TO 3020
C    3015  SD=SD-A(MJ+L-LKZ)*DBLE(X(J))
C    3020  CONTINUE
C    3025  IF(TF(L)) GO TO 3030
C      X(L)=SD
C      GO TO 3033
C      CATCH THE KNOWN X'S IN THE KNOWN EQUATION
C    3030  JMX = MIN0(L+N,NX)
JMN = L+1
C IF( JMN.GT.JMX ) GOTO 3032
C DO 3031 J=JMN, JMX
C IF( .NOT.TF(J) ) GOTO 3031
C SD = SD - A(M*L+J-LKZ)*DBLE(X(J))
C 3031 CONTINUE
C 3032 FL = SD
 GO TO 3040
C 3033 IF(JMX.LT.LNN) GO TO 3040
C DO 3038 J=LNN, JMX
IF( .NOT.TF(J) ) GO TO 3038
F(J) = F(J) + A(M*J+L-LKZ)*X(L)
3038 CONTINUE
C 3040 L=L+1
 IF(L.GE.NX) GO TO 3000
 C X CONTAINS SOLUTION, F CONTAINS FORCES
 C RETURN
 END
C SUBROUTINE DUDCR (NX, A, PP, TF, NRP)
C REAL*4 A(1)
 LOGICAL*1 TF(1)
 REAL*8 SD, DD
C DATA EPS/1.E-10/
C SUBSCRIPT L*K = M*L+K=LKZ OR L+L = MP*L-LKZ
C MP=01
C CHECK TO SEE IF THIS IS A REPAIR OR A NEW MATRIX
C IF(NRP.GE.NX) GO TO 2000
C REPAIR. GET NEW DATA.
C CALL RDRRT(A, NLRWP, NHRWP, NRP,-5)
LZ=MP*ALRP=01
C K=MINC(NRP+M, NX)
C 1000 IF(TF(K)) GO TO 1900
C L=NRP
 LNN=MAX2(L, K-M)
C
1010 IF(L*LT=L*M) GO TO 1900
   C IF(L*LT=NLWLP) CALL RDWRT(A,NLWLP,NHRWP,L+5)
   C IF(TF(L)) GO TO 1800
   C SD=A(K*L+K-LKZ)
   J=K+01
   JMX=MINO(L+M,NX)
   C 1020 IF(J.GT.JMX) GO TO 1700
   C IF(TF(J)) GO TO 1600
   SD=SD-DBLE(A(L*M+J-LKZ))*A(K*M+J-LKZ)
   1600 J=J+01
   GO TO 1020
   C 1700 A(L*M+K-LKZ)=SD
   C 1800 L=L-01
   GO TO 1010
   C 1900 K=K-01
   IF(K.GT=NRW) GO TO 1000
   C REDUCTION REPAIR COMPLETE GO INTO NORMAL REDUCTION
   C GO TO 2100
   C GET NEW DATA FOR START OF CHERRY REDUCTION
   C 2000 CALL RDWRT(A,NLWLP,NHRWP,NX,2)
   LKZ=NP*NHRWP-01
   K=NX
   C C START OF REDUCTION LOOP
   C 2100 CONTINUE
   IF(TF(K)) GO TO 3000
   L=K
   C C BRANCH CONTROL FOR L=K
   C ASSIGN 2850 TO LUMP
   ASSIGN 2303 TO JUMP
   C LMN=MIXO(K-M,01)
   C 2200 IF(L*LT=LMN) GO TO 3000
   IF(L*GE=NLWLP) GO TO 2202
   C CALL RDWRT(A,NLWLP,NHRWP,L+2)
   LKZ=NP*NHRWP-01
   C 2202 IF(TF(L)) GO TO 2900
   SD=A(K*L+K-LKZ)
   J=K+01
   JMX=MINO(L+M,NX)
C 2300 IF(J.GT.JMX) GO TO 2800
IF(TF(J))GOTO 2305
DD=AM*J-L-KZ)
C GO TO JUMP, (2303,2304)
C 2303 A(MK+J-LKZ)=DD/A(MP+J-LKZ)
C 2304 SD=SD-DD*A(MK+J-LKZ)
C 2305 J=J+01
GO TO 2300
C 2800 A(MK+K-LKZ)=SD
C GOTO LUMP, (2850,2900)
C 2850 ASSIGN 2900 TO LUMP
ASSIGN 2304 TO JUMP
C DUMP THE DIAG TERM SO THAT USER WONT DESPAIR
C MPKLZ = MP*K-LKZ
C TEST THE DIAG TERM HERE
C IF(ABS(A(MPKLZ)).GE.EPS) GOTO 2900
C BAD, SO DOCTOR IT UP AND FORGE AHEAD
C WRITE(6,*),'BAD DIAG*,K,A(MPKLZ)
A(MPKLZ)=SIGN(EPS,A(MPKLZ))
C 2900 L=L-01
GO TO 2200
C 3000 K=K-01
IF(K.LE.01) GO TO 2100
C CALL RDWR(Time,A,NLRWP,NHRWP,0,2)
C RETURN
C END
SUBROUTINE AUTO(NNDS,XN,BN,DN,FN,NU,EGT,MXNDS,MXELS)
C ROUTINE FOR GENERATING FEM MESH FOR THE TWO DIM MODEL
C OF THE GEN ELEC STRESS CONCENTRATION SPECIMEN

C O.B............................ C O-B
C . . C.O-C.B IS CENTER LINE OF SPECIMEN
C . . IN DIR OF LOADING
C . . CA-C IS CENTER LINE ACROSS DIR OF LOAD
C . . D IS RADIUS OF SEMICIRCLE CUT OUT
C . . B IS HALF LENGTH OF CONSTANT THICKNESS
C . . PART OF SPECIMEN
C . .
C C . . .......
C . . .......
C LOAD
C . . DIRECTION
C . .
C O.O. CA.O......A. ......... C O .

C XN = NODE COORDINATES
C DN = NODE DISPLACEMENTS
C FN = NODE APPLIED FORCES
C NU = NODES MAKING UP AN ELEMENT
C EGT = ELEMENT ELASTIC PARAMETERS
C BN = BOOLEAN VAR TELLING THAT NODE DOF IS TIED OR FREE ( T OR F )
C FLUNSD = FORTRAN LOGICAL UNIT NUMBERS FOR SAVING DATA

REAL*4 FCLD(IO),FCSTR(20),YSTR(20)
REAL* A
XN(MAXNDS)
DN(MAXNDS)
FN(MAXNDS)
EGT(3,MXELS)
EGT(3)
INTEGER*2 NU(MAXELS),LDND(10)
REAL*4 SPCM(16,5)
INTEGER*1 BN(MAXNDS),KNRFND

COMMON /SPCMN/ A,B,C,D,RA,RB,RC,E,EG,PLT
COMMON/FLUNSD/NSND,NSED,NSEBD,NSESV,NSUGS,NSFGS
DATA T-ZERO/0.
,NA,RA,B,NB,RB,NC,RC,D,E,EG,PLT
B99 = B99 .99
C01 = C01 .01
IF(PLT .LE. 0.0 ) GOTO 990
CALL PLTSET
CALL PLTSCL(0.0,C01,0.0,B0.5,0.5,0.5*PLT*C0.5+PLT*B)
990 CONTINUE
EG =ELASTIC MODULUS=MODULUS OR RIGIDITY
C DEFAULT= 30. ,12. , 1.0
DO 1000 J=1,2
DO 1000 I=1,MXNDS
DN(I,J)=ZERO
FN(I,J)=ZERO
1000 CONTINUE
BN(I,J)=.FALSE.
C INITIALIZE FOR GE ELEMENT MESH
C GET SPACING PARAMETERS
C
CALL SFCNCG(NA,RA,RDA,RZA)
CALL SFCNCG(NB,RB,RDB,RZB)
CALL SFCNCG(NC,RC,RDC,RZC)
C
NOEL = 00
NAP = NA + 01
NBF = NB + 01
NCF = NC + 01
FCBF(NBF) = 0.0
FCF(NAP) = 0.0
CA = C-A
DCA = D-CA
DD = D*DCA
NLD = 00
C ESTABLISH FIRST ROW PARAMETERS
I1 = 01
I2 = NAP
DO 1500 BN(2*I) = .TRUE.
TD = A
RZ = CA
CAC = 1.0
SAC = 0.0
ASSIGN 2090 TO IFRST
ASSIGN 2200 TO IBDR
ASSIGN 3100 TO KICK
2000 R = RZ
DR = TDR*RZA
C
THIS LOOP GENERATES A RADIAL ROW OF NODES
IF( I2.GT.MXNDS ) I2 = MXNDS
DO 2100 I = I1, I2
XN(1,I) = R*CAC
XN(2,I) = R*SAC
R = R + DR
GOTO IFRST, (2090,2100)
2090 FCST(I) = DR
2100 DR = RD#DR
IF( PLT.LE.0.0 ) GOTO 2110
C
WRITE(6,1)(I,XN(1,I),XN(2,I),I = I1,I2)
FORMAT( 5(14,1P E11.3,E10.3 ) )
1
2110 GOTO IBDR, (2200,2300)
2200 BN(1,I2) = .TRUE.
C
BRANCH TO ASSIGNED LABEL OF KICK
C
2300 GOTO KICK, (3100,3200,3700)
C
3100 ASSIGN 3200 TO KICK
C
KICK IS 3100 ONLY ONCE
C
ASSIGN 2100 TO IFRST
ASSIGN 3150 TO KKBK
ASSIGN 3500 TO JUMP
C

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CC = CC
DC = CC*RB
DB = BB*RB
BB = DB

3150 J1 = I1
J2 = I2
I1 = I2 + 01
I2 = I1 + NA
RBC = SQRT(BB*BB + CC*CC)
SAC = BB/RBC
CAC = CC/RBC
RZ = -DCA*CAC
RZ = RZ*SQRT(RZ*RB + DD)
TDR = RB - RZ
C
NOW GENERATE THE RADIAL ROW
C
GOTO 2000
C
RETURN BY KICK BEING 3200
3200 GOTO JUMP, (3500, 3600, 3610)
C
3500 DB = DB*PB
BB = BB + DB
C
IF( BB < BB ) GOTO 4100
C
BB = BB
ASSIGN 3600 TO JUMP
GOTO 4100
C
3600 ASSIGN 2300 TO IBDR
ASSIGN 3610 TO JUMP
C
3610 NLD = NLD + 01
LND(NLD) = I2
C
IF( CC < CC ) GOTO 3700
C
FCLD(NLD) = DC
CC = CC - DC
DC = DC*RD
GOTO 4100
C
3700 ASSIGN 4150 TO KKBK
C
4100 CALL RUELGN(NOEL,NU,EVL,E,EXELS,I1,I2,J1,J2,XN,PLT)
C
GOTO KKBK, (3150, 4150)
C
4150 NNDS = I2
C
C
CALCULATE LOADING FACTORS
C
C5 = 0.5/C
DCM = FCLD(1)
FCLD(1) = DCM*C5
C
DO 4160 I = 2, NCP
C
DC = FCLD(I)
FCLD(I) = (DCM+DC)*C5
DCM = DC

CONTINUE

CALCULATE THE FACTORS FOR THE MIDLINE STRESS HERE

C5 = 0.5*EG(3)
DAM = FCSTR(1)
YSTR(1) = XM(1,1)+0.25*DAM - C
FCSTR(1) = DAM*C5

DO 4170 I=2,NA

DA = FCSTR(I)
FCSTR(I) = (DA+DAM)*C5
YSTR(I) = XM(1,1) - C
DAM = DA

CONTINUE

FCSTR(NA) = DA*C5
YSTR(NA) = -DA*0.25

NODS = NNDS
NELS = NOEL

WRITE THE NODE DATA TO NSND.

IF( NSND.LE.00 ) RETURN

DO 4200 I=1,NNDS

WRITE(NSND'1)(XM(J,1),DN(J,1),FN(J,1),J=1,2),(BM(J,1),J=1,2)

WRITE INTO THE BACK END OF NSND THE DATA OF COMMON/SFCMN/.

DO 4300 I=1,NRC

WRITE(NSND'NRC)(SFCM(J,1),J=1,16)

RETURN

END

SUBROUTINE SPCNG(N,RX,RS,RMT)

THIS ROUTINE DETERMINES THE TERM TO TERM RATIO FOR A
GEOMETRIC SERIES TO GIVE A NUMBER OF STEPS WITH A FIRST TO
LAST RATIO GIVEN. IT ALSO DETERMINES THE INITIAL STEP SIZE
TO GET THE SEQUENCE STARTED SO THAT IT ENDS UP WITH A GIVEN SPAN
N = NUMBER OF STEPS IN THE SEQUENCE
RX = LAST/FIRST
RS = STEP TO STEP RATIO. STEP(I+1)=STEP(I)*RS
RMT = RATIO OF FIRST STEP TO SUM OF STEPS. STEP(1)=RMT*SUM

NOW DOWN TO BUSINESS
IF( N.LE.01 ) GOTO 2000
RS = RX**((1.0/(N-01))
RMT = 0.0
R = 1.0
DO 1000 I=1,N
RMT = RMT + R
1000 R = R*RS
RMT = 1.0/RMT

THAT'S ALL FOLKS
RETURN

END
VITA

Faysal A. Kolkailah was born on June 3, 1947, in Tanta, Egypt. He graduated from Iben Khaldoun High School in Cairo, Egypt in 1964. He received his B. S. in Aerospace Engineering from Cairo University in August, 1970.

From September 1970 until January 1977 he was employed by the Egyptian Airline Corporation as an Aeronautical Engineer in the Training and Research Department.

In March, 1977 he started a program of study at University of Cincinnati which led to his receiving the Master of Science degree in Aerospace Engineering in June, 1978. In September 1978 he began a course of study toward a doctorate degree in Aerospace Engineering at University of Cincinnati. After his wife's acceptance into the doctoral program in English at Louisiana State University, he transferred to the Mechanical Engineering Department at Louisiana State University in January 1980. While working towards his M.S. and Ph.D. degrees, he served as a teaching and research assistant at Aerospace Department (University of Cincinnati) and Mechanical Engineering Department (Louisiana State University). Upon completion of this research the doctoral degree in Mechanical Engineering was granted in August, 1982.

He married Miss Rasha Eldessouky in July 19, 1973, and they have two daughters, Noha and Nahlah.
EXAMINATION AND THESIS REPORT

Candidate: Faysal Kolksilah

Major Field: Mechanical Engineering

Title of Thesis: A Finite Element Analysis of High Temperature Fatigue and Creep at a Stress Concentration

Approved:

[Signatures]

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:

July 19, 1982