1982

Analysis of the Instability of Laminar, Newtonian Liquid Jets in Air.

Warren Thorne Abbott

Louisiana State University and Agricultural & Mechanical College

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ANALYSIS OF THE INSTABILITY
OF LAMINAR, NEWTONIAN LIQUID
JETS IN AIR

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Chemical Engineering

by

Warren Thorne Abbott
B.S.Ch.E., Louisiana State University, 1977
M.S.Ch.E., Louisiana State University, 1979
August 1982
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DEDICATION

This dissertation is dedicated to my parents, for their love, help, and understanding during my years in graduate school.
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ABSTRACT

This dissertation results from an investigation into the instability of free jets of a Newtonian liquid (water) in air which are produced from long nozzles and have initially parabolic velocity profiles. This work includes an experimental investigation into both the jet length and the drop formation of such jets. This work also includes a numerical solution of the momentum equations in cylindrical coordinates. From the experimental work, a modification to existing theoretical equations has been made to account for the existence of local maxima in the length of laminar jets as their velocity is increased. An addition to the modification to account for the local minimum in the jet length as the jet velocity profile become turbulent is suggested. From the numerical simulation more insight into the greater instability of sinusoidal disturbances on the surface of laminar jets has been gained.
The instability of a cylindrical jet of fluid that is immiscible in the phase in which it is formed, such as a jet of water in air, is a phenomenon that is fundamental to a wide variety of physical and chemical processes. Such jets respond to the influence of the interfacial tension of the jet surface. Of primary importance are those processes that require a fine dispersion of one phase into a second immiscible phase. Examples of this type of process are the atomization of fuels, spray driers, and perforated plate extraction columns. In these examples, the dispersion results from either a primary instability of a jet or as a secondary effect following the formation of a column of liquid from a liquid sheet. Processes such as fiber spinning and devices such as ink jet printers and recorders also involve the stability of an immiscible liquid jet. Jet break up has also been suggested as a way to provide very uniform hydrogen isotope targets for laser-induced nuclear fusion (Schwenn and Sigel, 1975) and to produce uranium fuel pellets for fission reactors (Haas, Kitts, and Beutler, 1967).

Because of the importance of the phenomena of jet break up, there recently has been a considerable effort devoted to the stability analysis of laminar, Newtonian-liquid jets in...
air. The mathematical models that have been developed give results that are in good agreement with experimental data, with one exception: it is known that laminar jets produced by long nozzles are less stable than those produced by short nozzles and orifices. The present theories are unable to account for this difference. The purpose of this research is:

1) To investigate the phenomena associated with the instability of laminar, Newtonian-liquid jets in which the velocity profile is parabolic at the exit of the nozzle;
2) To explain why such jets are more unstable;
3) To provide a simple correlation to estimate drop sizes from and lengths of such jets from theory and experimental data; and
4) To provide insight into the mechanism of the instability of such jets through a numerical solution of the free jet momentum equation.
1.1 Previous Work

In 1856, Plateau first attributed the fundamental mechanism of the instability of a liquid jet in air to the interfacial tension of the jet fluid. In his analysis, Plateau determined that the free energy of a cylindrical column of fluid is reduced by an axisymmetric disturbance if the wave length of the disturbance is greater than the circumference of the fluid column. The instability results from the system's attempt to minimize the free surface energy by obtaining a minimum surface area per unit volume. Thus, the column breaks into spherical drops.

Rayleigh, in 1878, linearized the equations of motion for an inviscid fluid by neglecting all second and higher order terms to obtain a mathematical description of the break-up process. The linearized equations of motion, along with the appropriate boundary conditions, form a boundary value problem. The solution is a characteristic equation that relates the growth rate of a disturbance to its wave length. Rayleigh found that whereas all axisymmetric disturbances that have a wave length greater than the circumference of the undisturbed jet are unstable, there is a disturbance with a particular wavelength whose amplitude grows faster than all of the others. This disturbance ultimately dominates the break up of the jet, determining the length of the jet and the size of the drops that are formed at the break up point. Rayleigh's equation shows good agreement with experimental data in predicting values
of jet length and of drop size for water jets in air at low jet velocities, but shows poor agreement for both higher jet velocities and higher fluid viscosities.

Weber, in 1931, extended the results of Rayleigh by:

1) properly accounting for the viscosity of the jet fluid; and

2) considering the aerodynamic forces at the jet surface.

Weber's result shows excellent agreement with experimental data on low velocity jets over a wide range of fluid properties. However, the theory deviates markedly from experimental data for higher jet velocities (where aerodynamic effects are even more important). It has been observed that there exists a critical jet velocity, as shown in Figure 1-1, at which point the jet length no longer increases but rather decreases, with increasing jet velocity. Weber's theory is inconsistent in predicting these local maximum jet lengths and the associated critical jet velocity. In some experiments the predicted values were greater than the experimental values, while in others the predicted values were less.

In 1966, Grant and Middleman obtained jet length data for liquid jets with a wide range of fluid properties using nozzles with length to diameter ratios of about 100. They were unable to correlate their data using Weber's result. As before, in some cases the observed values of the maximum jet length and the critical velocity were greater than
Figure 1-1. A Typical Jet Length Curve for a Jet Produced from a Long Nozzle
predicted values, while in other cases they were less. An empirical modification of Weber's equation led to a good correlation of their data that were taken at standard pressures, but failed for their data that were taken at reduced ambient pressures. The jet lengths that were predicted by the modified theory at subatmospheric pressures were much greater than those observed experimentally.

Experiments by Fenn and Middleman, in 1969, for several fluids and for several nozzle diameters (with length to diameter ratios of about 100) over a range of reduced ambient pressures gave several interesting results. For a highly viscous fluid (\(\approx 50 \, \text{mPa} \cdot \text{s}\)) they found that the maximum jet length and critical velocity increased with decreasing ambient pressure, i.e. decreasing ambient density. This result was qualitatively, but not quantitatively, in agreement with Weber's theory. However, for a low viscosity fluid (\(\approx 1 \, \text{mPa} \cdot \text{s}\)) the relation between the jet length and the jet velocity was independent of the ambient pressure. Fenn and Middleman concluded that Weber's theory was not correct since it did not include the effects of the ambient gas, which they showed to contribute greatly to the instability of a liquid jet.

In 1972 and 1973, Phinney and Humphries obtained a number of measurements on the length of laminar, Newtonian-liquid jets from long nozzles and from sharp-edged orifices. They concluded that an additional destabilizing mechanism is important for jets formed by long...
nozzles. They attributed the source of this additional mechanism to an increase in the initial amplitude of the surface disturbance arising from "some Tollmien-Schlichting type of instability ..."

In 1974, Sterling and Sleicher showed that the deviation between the experimental measurements and the theoretical results of Weber are largely attributed to the manner in which the jets are formed. From measurements made on jets formed by short nozzles in which the velocity profile at the nozzle exit is nearly uniform, they showed that, for all cases, Weber's theory over-estimates the the aerodynamic effect. That is, the experimental maximum jet length and critical velocity in all cases are greater than predicted by Weber's equation. Furthermore, jets formed by long nozzles with fully developed laminar velocity profiles at the nozzle exit were found to be less stable than comparable jets with uniform velocity profiles. From this work it can be concluded that the difference in the stabilities of the two types of jets may be the result of either the length of the nozzle or the shape of the initial velocity profile.

Sterling and Sleicher demonstrated that as the viscosity decreases the difference in the behavior between jets from long nozzles and short nozzles increases. They attributed the enhanced instability of jets formed by long nozzles to the relaxation of the initial parabolic velocity profile. A semi-empirical modification of Weber's equation
to account for the effects of the viscosity of the surrounding media (the aerodynamic effects) shows excellent agreement with data that were obtained on jets for which the effects of the initial velocity profile were negligible. The relative motion between the jet and the surrounding air was found to enhance the instability, but to a degree much less than is predicted by Weber's equation. Measurements on both the jet length and the most unstable wave length compared well with values predicted by the modified theory.

Kitamura and Takahashi (1978) have performed numerous experiments on the effect of nozzle length on the instability of jets of fluids with various physical properties. The nozzle length had two effects. First, as the nozzle length was increased, the disturbances caused by the sudden contraction at the nozzle entrance were damped and the perturbation of the surface was reduced. Thus the jet was stable, that is longer. Second, as the nozzle length was increased, the surface velocity of the jet fluid became smaller, since the jet velocity profile became more parabolic. Kitamura and Takahashi concluded that for low velocity jets, the velocity profile has very little effect on the breakup length. This is in contrast to what Sterling and Sleicher observed. In Sterling and Sleicher's work, the nozzles had a smooth contraction at the nozzle entrance. Thus entrance effects were greatly reduced and the surface velocity played a more important role in the breakup. This would explain the difference in the two different results.
All of the theoretical work discussed above involved a solution of the linearized equations of motion. It is known that the growth of surface disturbances is a non-linear process, with generation of higher harmonics and feedback into the fundamental; these effects would be neglected in a linearized equation. Yuen (1968) has analyzed the non-linear effects of surface disturbances on the instability of capillary jets. His results gave good agreement with the linearized theories. Yuen's results predict that the depth of the trough of the disturbance will increase faster than the height of the crest. Also, his theory predicts that for longer wave lengths, undulations, or swelling, will occur in the troughs between the crests.

Rutland and Jameson (1971) performed several experiments to test the validity of Yuen's theory and were able to observe the swelling in the troughs. They observed that there never existed more than a single undulation in the troughs. But they observed in many cases that after the main drops broke off, the ligament caused from the undulation would breakup into many satellite droplets, indicating the presence of many oscillations in the ligament.

Numerical simulations of the free jet problem have been performed by many other researchers. The majority of the previous work involved the use of finite element schemes with the velocity as the dependent variable. Horsfall (1973) used a fixed grid finite difference scheme with
velocity as the dependent variable, but apparently had problems matching the boundary conditions. Shokoohi (1976) and Dutta and Ryan (1982) used orthogonal, curvilinear coordinates with the vorticity function and the stream function as the dependent variable. Dutta and Ryan studied viscoelastic fluids with a fixed relaxation distance to investigate the phenomena of die swell. Shokoohi used water in his simulations, but failed to show any jet swell or contraction, or velocity profile relaxation.

Keller, Rubinow, and Tu (1973) report that whereas most jet instability studies have been performed using temporal instability (how the disturbance amplitude grows with time), the jet should be analyzed using spatial instability (how the amplitude grows with distance from the nozzle exit).
1.2 Present State of Knowledge

From previous investigations, it is now clear that there are at least three mechanisms that contribute to the break up of laminar liquid jets in air. They are:

1) The action of surface tension;
2) The forces on the surface that arise from the relative motion between the jet and the air; and
3) A mechanism that is associated in some manner in which the jet is formed.

An adequate mathematical model that accounts for the first two mechanisms is available.

It is clear that the third mechanism has a destabilizing effect, and that it occurs only for laminar jets formed by long nozzles, that is, with initially parabolic velocity profiles. But, it remains to be determined whether the source of the enhanced instability arises from some Tollmien-Schlichting type of instability in the nozzle, as proposed by Phinney, or in the relaxation of the velocity profile, as proposed by Sterling and Sleicher. It must also be determined how this effect is to be treated in a modification of the theory so as to account for the enhanced instability. The approach of Phinney, to consider the amplitude of the initial disturbance as a variable, could well yield a correlation for the jet length, but not for the most unstable wave length (and thus the drop size). A more general approach would be to modify the characteristic equation so that the jet length and the most
unstable wave length can be predicted together. This is the approach taken in this research.

Before this research work was done, there were no data available to perform this modification. In this work, data were collected for jet length, drop size, and drop spacing for water jets in air. The data were taken using long, smooth entrance nozzles of varying diameters, all with length to diameter ratios of about 300, and at varying jet velocities. These data were used to estimate the values of parameters in a semi-empirical modification to Weber's equation used in the prediction of the jet length.
CHAPTER 2

Development of Mathematical Equations

2.1 Theoretical Equations

The equations in this section are derived in a cylindrical coordinate system \((r, \theta, z)\). The \(z\)-axis corresponds to the centerline of the jet; the positive \(z\)-direction is opposite to the flow of the jet. Axial symmetry is assumed so that \(\partial/\partial r\) at the centerline is equal to zero. If two dimensional flow in the \(r\) and \(z\) directions is assumed, then the angular component of the velocity is equal to zero and \(\partial/\partial \theta\) is equal to zero.

As mentioned in Chapter 1, Rayleigh discovered that the breakup of a cylindrical jet is caused by the growth of a small, periodic, axisymmetric disturbance on the surface of the jet (see Fig. 2-1). This disturbance must have a wavelength, \(\lambda\), that is greater than the circumference of the jet. We take the jet radius to be \(R = a + \eta\), where \(a\) is the undisturbed jet radius and the disturbance amplitude, \(\eta\), is assumed to be represented by the first term of a Fourier series:

\[
\eta = \text{Real}\{\eta_0 \exp\{bt + ikz\}\} \quad 2.1.1
\]

where \(\eta_0\) is the initial amplitude of the disturbance,
\(b\) is the exponential growth rate constant,
\(t\) is the time,
Figure 2-1. Illustration of an Axisymmetric Surface Disturbance.
\(i\) is the imaginary number \(\sqrt{-1}\),
\(k\) is the wave number of the disturbance \((2\pi/\lambda)\), and
\(z\) is the axial coordinate.

It can be seen from this equation that the break up of a jet is characterized by two parameters, the growth rate constant \(b\) and the wave number \(k\). The objective is to find an expression for \(b\) as a function of the wave number, the relative velocity between the jet and its surroundings, and the fluid properties of the jet and the surrounding gas.

To find this equation for the growth rate constant, \(b\), let us start with the continuity equation and the momentum equation:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad 2.1.2 \]
and

\[ \frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla p + \rho \mathbf{g} - \nabla \tau. \quad 2.1.3 \]

where \(\mathbf{v}\) is the jet velocity vector given by \(\mathbf{v} = u\mathbf{e}_r + v\mathbf{e}_z\),
\(u\) is the radial component of the jet velocity,
\(v\) is the axial component of the jet velocity,
\(\mathbf{e}_r\) is the unit vector in the radial direction,
\(\mathbf{e}_z\) is the unit vector in the axial direction,
\(p\) is the fluctuating component of the jet pressure,
\(\rho\) is the jet fluid density,
\(\tau\) is the stress tensor,
\(\nabla\) is the gradient operator,
\(\nabla \cdot\) is the divergence operator,
\(r\) is the radial coordinate, and
\( g \) is the acceleration due to gravity.

If we assume constant density, a Newtonian fluid with constant viscosity \((\nabla \cdot \mathbf{v} = -\mu \nabla^2 \mathbf{v})\), and neglect gravity and nonlinear terms \((\nabla \cdot \mathbf{v} \mathbf{v})\) we get:

\[
\nabla \cdot \mathbf{v} = 0
\]

2.1.4

and

\[
\frac{\partial \mathbf{v}}{\partial t} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{v}
\]

2.1.5

where \( \nu \) is the kinematic viscosity, \( \mu/\rho \).

The boundary conditions at the centerline of the jet result from the requirement for axial symmetry as mentioned above:

\[
\frac{\partial u}{\partial r} = 0, \quad \frac{\partial v}{\partial r} = 0, \text{ and } \frac{\partial p}{\partial r} = 0 \quad \text{at } r = 0.
\]

2.1.6
2.1.7
2.1.8

To ensure that the continuity equation is finite at the centerline, \( u \) must be equal to zero at \( r = 0 \).

The boundary conditions at the surface of the jet are:

1) No net flux of mass across the surface of the jet,

\[
u = \frac{\partial u}{\partial t}
\]

at \( r = a \);

2.1.9

2) The shear stress is equal to zero at the jet surface:

\[
\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} = 0
\]

at \( r = a \);

2.1.10

3) The normal stress at the surface must be continuous:

\[
(p - \rho) + 2\mu \frac{\partial u}{\partial r} = P - \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right)
\]

at \( r = a \),

2.1.11

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where P is the undisturbed pressure, given by $P = \frac{\sigma}{a}$,
$p$ is the fluctuating component of the jet pressure,
$\hat{p}$ is the fluctuating ambient gas pressure,
$R_1$ and $R_2$ are the orthogonal principal radii of
curvature of the jet surface,
a is the radius of the undisturbed jet, and
$\sigma$ is the surface tension.

It can be shown that $R_1$ and $R_2$ may be approximated to first
order in $\eta$ by:

$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{a} - (1 - k^2 a^2) \eta/a^2$.  \hspace{1cm} (2.1.12)

After substituting equation (2.1.12), into equation (2.1.11)
we get:

$(p - \hat{p}) + 2\mu \frac{\partial u}{\partial r} = \sigma (1 - k^2 a^2) \eta/a^2$ \hspace{1cm} at $r = a$.  \hspace{1cm} (2.1.13)

If we apply the continuity equation and the momentum
equation to the ambient gas, assuming that the gas is
inviscid ($\mu = 0$) and incompressible ($\hat{\rho} = $ constant), and that
the gas is moving with a velocity $V$ relative to the surface
velocity of the jet we get:

$\nabla \cdot \vec{v} = 0$  \hspace{1cm} (2.1.14)

and

$\frac{\partial \vec{u}}{\partial t} + V \vec{v} \cdot \nabla \vec{v} = -\frac{\nabla p}{\hat{\rho}}$.  \hspace{1cm} (2.1.15)

with the surface boundary condition:

$\theta = \frac{\partial \eta}{\partial t} + V \eta \frac{\partial \eta}{\partial z}$ \hspace{1cm} at $r = a$.  \hspace{1cm} (2.1.16)
where \( \mathbf{v} \) is the velocity vector of the ambient gas, given by
\[
\mathbf{v} = \mathbf{e}_r + \mathbf{e}_z,
\]
\( \mathbf{e}_r \) is the radial velocity of the ambient gas, and
\( \mathbf{e}_z \) is the axial velocity of the ambient gas.

Equation (2.1.15) gives the relationship for \( \rho \) in equation (2.1.13).

Expressing the ambient gas velocity, \( \mathbf{v} \), in terms of the ambient velocity potential, \( \phi \), yields:
\[
\nabla^2 \phi = 0 \quad 2.1.17
\]

and
\[
\partial \phi / \partial t + V \partial \phi / \partial z = -\rho/\rho \quad 2.1.18
\]

where \( V \phi = \mathbf{v} \). 2.1.19

The boundary condition becomes
\[
\partial \phi / \partial r = 3 \eta / \partial t + V \partial \eta / \partial z \quad \text{at} \quad r = a. \quad 2.1.20
\]

Let us assume that the ambient velocity potential, \( \phi \), may be expressed as the product of two functions:
\[
\phi(r,z,t) = \phi(r) \exp(\beta t + ikz). \quad 2.1.21
\]

Substituting equation (2.1.21) into equation (2.1.17) gives:
\[
r^2 d^2 \phi / dr^2 + rd \phi / dr - k^2 r^2 \phi = 0. \quad 2.1.22
\]

Equation (2.1.22) is a modified Bessel's differential equation of order zero. The solution is:
\[
\phi(r) = A_1 I_0(kr) + A_2 K_0(kr) \quad 2.1.23
\]
where $A_1$ and $A_2$ are arbitrary constants that are to be determined from the boundary conditions, and $I_0$ and $K_0$ are zeroth order modified Bessel functions of the first and second kind, respectively. Since $\tilde{\beta}(r)$ must have a finite solution in the region of the ambient gas ($a<r<\infty$), $A_1$ must be equal to zero, because $I_0(x)$ goes to infinity as $x$ goes to infinity. So equation (2.1.21) becomes:

$$\tilde{\beta}(r,z,t) = A K_0(kr) \exp(bt + ikz).$$  

The jet velocity can be expressed in terms of the velocity potential, $\phi$, and the stream function, $\psi$, as:

$$\mathbf{v} = \mathbf{v}_\phi + \mathbf{v}_\psi$$  

where $\mathbf{v}_\phi = -\nabla \phi$,$/r$,

$v_\phi$ is the irrotational component,

and $v_\psi$ is the rotational component.

Substituting equation (2.1.25) into the continuity equation, 2.1.4, gives:

$$\nabla^2 \phi = 0.$$  

As with the ambient velocity potential, $\tilde{\beta}$, let us express the jet velocity potential, $\phi$, as the product of two functions:

$$\phi(r,z,t) = \beta(r) \exp(bt + ikz).$$  

If equation (2.1.27) is substituted into equation (2.1.26), we get:
\[
\frac{r^2 d^2 \bar{z}}{dr^2} + r d\bar{z}/dr - k^2 r^2 \bar{z} = 0. \tag{2.1.28}
\]

Equation (2.1.28) is also a modified Bessel's differential equation of order zero. The solution to this equation is:

\[
\bar{z}(r) = B_1 I_0(kr) + B_2 K_0(kr). \tag{2.1.29}
\]

Since \( \bar{z}(r) \) must have a finite solution in the region of the jet (\( 0 < r < a \)), \( B_2 \) must be equal to zero, because \( K_0(x) \) goes to infinity as \( x \) goes to zero. After substitution, equation (2.1.27) becomes:

\[
\psi(r,z,t) = B I_0(kr) \exp(bt + ikz). \tag{2.1.30}
\]

Substitution of equation (2.1.25) into the linearized momentum equation, 2.1.5, and separation of the irrotational and rotational terms yields:

\[
\frac{\partial \psi}{\partial t} = -\frac{p}{\rho} \tag{2.1.31}
\]

and

\[
\frac{\partial \bar{B}}{\partial t} = \nu \nabla^2 \bar{B}. \tag{2.1.32}
\]

Using the expression for \( \bar{B} \), we obtain from equation (2.1.32)

\[
(1/\nu)\frac{\partial \psi}{\partial t} = \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} \tag{2.1.33}
\]

\[
= \nabla^2 \psi.
\]

Now, let us express the stream function, \( \psi \), as:

\[
\psi(r,z,t) = \psi(r) \exp(bt + ikz). \tag{2.1.34}
\]

Substituting equation (2.1.34) into equation (2.1.33) we
obtain:

\[ r^2 \frac{d^2 \Psi}{dr^2} - r \frac{d\Psi}{dr} - (k^2 r^2 + b/\nu) \Psi = 0. \quad 2.1.35 \]

Equation (2.1.35) is a first order modified transformed Bessel's differential equation with the following solution:

\[ \Psi(r) = C_1 r I_1(k_1 r) + C_2 r K_1(k_1 r) \quad 2.1.36 \]

where \( k_1 = k^2 + b/\nu \)

and \( I_1 \) and \( K_1 \) are first order modified Bessel functions of the first and second kind, respectively. Since \( \Psi(r) \) must be finite in the region of the jet \( (0 < r < a) \), \( C_2 \) must be equal to zero because \( K_1(x) \) goes to infinity as \( x \) goes to zero.

After substitution, equation (2.1.34) becomes:

\[ \psi(r,z,t) = C r I_1(k_1 r) \exp(bt + i kz). \quad 2.1.37 \]

The constants \( A, B, \) and \( C \) in equations (2.1.24), (2.1.30), and (2.1.37), respectively, must be determined from the boundary conditions. Substituting equation (2.1.25) into equations (2.1.9), (2.1.10), and (2.1.13) gives the appropriate boundary conditions for the jet in terms of \( \phi \) and \( \psi \):

\[ \frac{\partial \phi}{\partial r} + \left( \frac{1}{r} \right) \frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t} \quad \text{at } r = a, \quad 2.1.38 \]

\[ \left( \frac{1}{r} \right) \frac{\partial^2 \phi}{\partial z^2} - \left( \frac{\partial (\phi/r)}{\partial r} \right) = 0 \quad \text{at } r = a, \quad 2.1.39 \]

\[ 2 \mu \frac{\partial^2 \phi}{\partial r^2} = (\rho - \beta) + \sigma (1 - k^2 a^2) \frac{\eta}{a^2} \quad \text{at } r = a. \quad 2.1.40 \]
The pressure terms, $\delta$ and $\rho$, are given by equations (2.1.18) and (2.1.31), respectively, so that equation (2.1.40) becomes:

$$2\mu a^2 \phi /ar^2 + (a\phi /at - a^2/\partial t - V\partial \phi /\partial z) = \sigma (1-k^2a^2)\eta/az$$

at $r = a$. 2.1.41

Solving for $A$, $B$, and $C$, then substituting equations (2.1.24), (2.1.30), and (2.1.37) into equation (2.1.41) yields:

$$b^2 [\xi I_0(\xi)/(2I_1(\xi)) + \rho \xi K_0(\xi)/(2\rho K_1(\xi))]$$
$$+ b[2V\rho \xi^2 K_0(\xi)/(\rho a K_1(\xi))]i$$
$$+ \left(\mu \xi^2/\rho a^2\right)(2\xi I_0(\xi)/I_1(\xi) - 1$$
$$+ 2\xi^2(\xi I_0(\xi)/I_1(\xi) - \xi I_0(\xi)/I_1(\xi))/\left(\xi^2 - \xi^2\right))$$
$$= (\sigma/2\rho a^3)(1-\xi^2)\xi^2 + V^2\rho \xi^3 K_0(\xi)/(2a^2\rho K_1(\xi))$$

2.1.42

where $\xi = ka$

and $\xi^2 = \xi^2 + ba^2/\nu$.

Equation (2.1.42) is the desired equation for the growth rate constant. This equation predicts the growth rate of a small periodic disturbance as a property of the physical properties of the jet fluid, the jet diameter, and the inviscid ambient gas density and velocity. This equation does not include any information about the jet velocity profile, which is the topic of Section 2.2.

The results of all previous investigators can be shown to be special cases of equation (2.1.42). If $\rho$, $\mu$, and $V$ are set equal to zero, then equation (2.1.42) reduces to:
Equation (2.1.43) is the result which Rayleigh obtained in his solution for an inviscid jet acting only under the influence of surface tension.

If only \( \mu \) is set equal to zero, then Alterman's solution for a cylindrical vortex sheet with surface tension is obtained. The imaginary component of Alterman's solution contributes to the velocity of propagation of the disturbance along the jet. Calculations by Sterling and Sleicher in 1974 show that this propagation is negligible compared to the jet velocity.

If the imaginary term of equation (2.1.42) is neglected, and the following assumptions are made:

1) \( \bar{p}K_0(\xi)/(2pK_1(\xi)) \ll \xi I_o(\xi)/(2I_1(\xi)) \)

2) \( \xi \ll 1 \), so that \( \xi I_0(\xi)/(2I_1(\xi)) \approx 1 \)

3) \( \xi_i < 1 \), but \( \xi_i \neq \xi \)

then Weber's equation for the growth rate constant is obtained:

\[
b^2 + [3\mu \xi^2/(\rho a^3)]b = \left[\frac{\sigma}{(2\rho a^3)\xi^2}\right]b + \left[\frac{v^2\rho \xi^3}{(2\rho a^3)}\right]K_0(\xi)/K_1(\xi)
\]

(2.1.44)

Dividing equation (2.1.44) by \( \sigma/(2\rho a^3) \) yields the dimensionless form:

\[
b^2 + 6z \xi^2 \beta = \left[\frac{1-\xi^2}{\xi^2}\right] + C_1 \omega \xi^3 K_0(\xi)/K_1(\xi)
\]

(2.1.45)
or \[ \beta^2 + \beta F_1(Z, \xi) = F_2(\xi) + F_3(\hat{\Omega}_e, \xi) \] 2.1.46

where \( \beta = b\sqrt{2\rho a^2/\sigma} \), the dimensionless growth rate constant,

\( Z = \mu/\sqrt{\rho\sigma d} \), the Ohnesorge number \((= \sqrt{\hat{\Omega}_e/Re})\)

\( \hat{\Omega}_e = \hat{\rho}V^2d/\sigma \), the ambient Weber number,

\( F_1 = 6Z\xi^2, \)

\( F_2 = (1-\xi^2)\xi^2, \)

\( F_3 = C_1\hat{\Omega}_e\xi^3K_0(\xi)/K_1(\xi), \)

and \( C_1 \) is a constant that is equal to one in Weber's result, but which Sterling found by experimental means to be equal to about 0.175.
2.2 Empirical Equations

The purpose here is to develop simple empirical equations to predict the continuous length of a jet and the size of the drops that are formed subsequent to breakup.

We first relate jet length and drop size to the growth rate constant and wave number. Empirical equations for the growth rate constant and wave number are then presented, and an empirical equation for jet length follows. An extension of this empirical equation to account for the effects of velocity profile relaxation is then suggested.

2.2.1 Basic relationships

Consider equation (2.1.46), as derived in Section 2.1:

$$\beta^2 + \beta F_1(Z,\xi) = F_2(\xi) + F_3(\tilde{\omega},\xi).$$

2.2.1

If a jet is subjected to an axially symmetric surface disturbance of wave length $\lambda > 2\pi a$, then the jet will break up into sections of length $\lambda$. As discussed in Chapter 1, Rayleigh showed mathematically that there is a single disturbance of wave length $\lambda^*$ that will grow most rapidly. This disturbance will have a growth rate constant equal to $b^*$ (or $\beta^*$). If the initial amplitudes of all disturbances are of the same order of magnitude, the disturbance with wave length $\lambda^*$ will thus dominate the breakup process, and the jet will break into sections of length $\lambda^*$. This section will form a spherical drop of diameter $D$. We ignore, as a
first approximation, the volume of fluid contained in small satellite drops. The diameter of the drop, \( D \), can be calculated from the wave number of the disturbance as:

\[
D = 2a(1.5\pi/\xi^+) \quad 2.2.2
\]

where \( \xi^+ = k/a = 2\pi a/\lambda^+ \).

From equation (2.2.2) it can be seen that the primary drops formed from free jet breakup will be larger than 1.6765 jet diameters, since \( \xi \) must be less than 1 for breakup to occur. Conversely, the dimensionless wave number, \( \xi^+ \), may be determined from the diameter of the drops that are formed by:

\[
\xi^+ = 1.5\pi(d/D)^3 \quad 2.2.3
\]

Furthermore, the disturbance amplitude will grow to a magnitude equal to the jet radius, \( a \), in time \( t^+ \). Setting equation (2.1.1) equal to \( a \) at the time of the break up gives:

\[
a = \eta_0 \exp\{b^+t^+\} \quad 2.2.4
\]

and solving for \( t^+ \) yields

\[
t^+ = (1/b^+) \ln\{a/\eta_0\}. \quad 2.2.5
\]

The jet length, \( L \), can then be calculated from the jet bulk velocity, \( V \), and the time, \( t^+ \), required to achieve breakup

\[
L = Vt^+ = (V/b^+) \ln\{a/\eta_0\} \quad 2.2.6
\]
or in terms of dimensionless quantities

\[ \Lambda = -(\bar{W}_e / 2\beta^+) \ln(\alpha_0) \] \hspace{1cm} 2.2.7

where \( \Lambda \) is the dimensionless jet length to diameter ratio, and \( \alpha_0 \) is the dimensionless initial disturbance amplitude.

Equation (2.2.6) predicts that the jet length increases linearly with jet velocity, if \( \beta^+ \) is constant; this is in good agreement with experiments at small jet velocities. The initial disturbance parameter, \( \alpha_0 \), is a function of the experimental conditions, temperature, ambient pressure, and room vibrations, but may be considered to be constant over each experimental run (all ranges of jet velocity for the same nozzle diameter).

2.2.2 Behavior for small \( \bar{W}_e \)

Function \( F_3 \) in equation (2.2.1) arises from the aerodynamic interaction between the jet and the surrounding gas. When the inertia of the ambient gas can be neglected, that is for low jet velocities (\( \bar{W}_e < 1 \)), \( F_3 \) can be ignored. Straightforward differentiation of the remaining equation shows that \( \beta \) has a maximum value when

\[ \xi = (0.5/(1+3Z)) = \xi^* \] \hspace{1cm} 2.2.8

which gives the expression for the initial dimensionless growth rate constant, \( \beta^* \):

\[ \beta^* = 0.5/(1+3Z). \] \hspace{1cm} 2.2.9
Substituting equation (2.2.9) into equation (2.2.7) yields:

\[ \Lambda_0 = -\bar{\omega} (1+3Z) \ln(\alpha_0) \quad \text{2.2.10} \]

or solving for \( -\ln(\alpha_0) \)

\[ -\ln(\alpha_0) = \Lambda_0 / (\bar{\omega} (1+3Z)) \quad \text{2.2.11} \]

So, if we know the jet length at a velocity such that \( \tilde{\omega} < 1 \), we can calculate \( \ln(\alpha_0) \) from equation (2.2.11) to be used in equation (2.2.7). Anno estimates \( -\ln(\alpha_0) \) to be between six and twelve. Now solving equation (2.2.7) for \( \beta^+ \) gives:

\[ \beta^+ = -\left(\frac{1}{2\Lambda}\right) \ln(\alpha_0) \quad \text{2.2.12} \]

So, if we know the jet length at a particular velocity and the initial disturbance amplitude, we can estimate \( \beta^+ \).

2.2.3 Aerodynamic effects

In 1974, Sterling and Sleicher showed that excellent agreement with experimental data occurs when the constant, \( C_1 \), in \( F_3 \) in equation (2.2.1) is equal to 0.175. Using numerical methods they found that:

\[ \xi^* = \xi \exp\{0.035\tilde{\omega}\}, \quad \text{2.2.13} \]

and Mahoney and Sterling (1978) found that:

\[ \beta^* = 0.5f(\tilde{\omega}, Z) / (1+3Z) \quad \text{2.2.14} \]

where

\[ f(\tilde{\omega}, Z) = (1+1.96Z+2G(1-G^2+0.5C_1\tilde{\omega}(0.9G-0.2)) / (1+1.96z) \]

and

\[ G = (1/\sqrt{Z}) \exp\{0.035\tilde{\omega}\} \]
which yields:

\[ \Delta = -\text{We} (1+3Z) \ln(\alpha_0)/f(\text{We}, Z) \]  \hspace{1cm} 2.2.15

The jet length predicted by equation (2.2.15) shows good agreement with the experimental data of Phinney and Humphries for jets formed with an initially uniform velocity profile (from orifices or very short nozzles) or for jets formed by long nozzles when the Ohnezorge number is large. Data taken on jets from long nozzles and small values of Z are not correlated well by equation (2.2.15), and in all cases, the observed jet lengths are less than what are predicted by the equation.

2.2.4 Correlation for effects of velocity profile relaxation

Previous work by Grant and Middleman in 1966 gives evidence that the effects of velocity profile relaxation can be correlated by the addition of a new term to the characteristic equation for the exponential growth rate constant, equation (2.2.1). Their data were taken so as to ensure fully developed laminar flow at the nozzle exit. A modification to the characteristic equation, based on these data, leads to predicted jet lengths that agree well with experimental data at atmospheric pressure, but fails for data obtained at reduced ambient pressures.

Grant and Middleman's modified equation (2.2.1) by using the following expression for the term \( F_3 \):

\[ F_3 = \text{We} \xi^3 F(Z) \]
where \( F(Z) = C Z^{-1/2} \).

In theory, the term \( F_3 \) results from the inertia of the ambient gas. Grant and Middleman's modification, however, the term \( F_3 \) contains the Ohnesorge number, \( Z \), which depends only on the properties of the jet fluid. From this we can conclude that their modification is taking into account processes that are occurring within the jet and is independent of the ambient conditions. However, their expression for \( F_3 \) retains the ambient density, thus the correlation is lost when the ambient pressure is reduced, accounting for the fact that their modification did not work for reduced ambient pressure. Grant and Middleman's correlation could be accounted for by the relaxation of the initial parabolic velocity profile. From purely heuristic arguments, Sterling and Sleicher have shown that the velocity profile relaxation effects should increase in proportion to \( \text{We}/(Z+3Z^2) \), a conclusion in substantial agreement with Grant and Middleman's experimental results.

Rather than modify the aerodynamic term, \( F_3 \), with an arbitrary function of \( Z \) as Grant and Middleman did, another term can be added to the characteristic equation. That is, equation (2.2.1) becomes:

\[
\beta^2 + \beta F_1(Z, \xi) = F_2(\xi) + F_3(\text{We}, \xi) + F_4(\text{We}, Z, \xi)
\]

As discussed above, it is anticipated that \( F_4 \) will have the following form:

\[
F_4(\text{We}, Z, \xi) = \text{We} \left[ g(\xi) / (Z + 3Z^2) \right]
\]
If $\beta^+$ and $\xi^+$ are measured for various values of $We$, $\dot{We}$, and $Z$, the function $F_4$, and thus $g(\xi)$, can be determined from equation (2.2.16).

After the form of function $F_4$ is established for fully developed laminar flow, the effect of nozzle length can be accounted for by multiplying $F_4$ by a function of the nozzle length-to-diameter ratio. This function should go to unity for large values of the ratio (long nozzles), and should go to zero for small values (short nozzles and orifices).

The results of experimentally determining $F_4$ and $g(\xi)$ are presented in chapter 4.
2.3 Numerical Solution of the Navier-Stokes Equation

2.3.1 Formulation of the equations.

The basic assumptions which will be used in the development of the Navier-Stokes equation for computer simulation are:

1) Cylindrical coordinates \((r, \theta, z)\),
2) Two dimensional flow in \(r\) and \(z\) \((\partial/\partial \theta = 0)\),
3) Axisymmetric, \((\partial/\partial r = 0 \text{ at } r = 0)\),
4) Incompressible flow, \(\rho = \text{constant}\),
5) Newtonian fluid \((\tau = -\mu \nabla \cdot \nabla)\),
6) No surface charge, and
7) Gravity is negligible \((\bar{g} = 0)\).

If we start with the differential form of the continuity equation

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]

and make the assumptions of constant density and two dimensional flow, we get:

\[ \nabla \cdot \mathbf{v} = 0 \quad \text{or} \quad \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial v}{\partial z} = 0. \]

Now if we take the differential form of the Navier-Stokes momentum transport equation:

\[ \frac{\partial (\rho \mathbf{v})}{\partial t} + (\nabla \cdot \mathbf{v}) \mathbf{v} + \nabla p = \rho \bar{g} - \nabla \tau \]

and apply assumptions 1, 2, 4, 5, and 7, we have

\[ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + \left(\frac{1}{\rho}\right) \nabla p = \nu \nabla^2 \mathbf{v}. \]
The $r$-component of the momentum equation is:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial z} + \frac{1}{\rho} \frac{\partial p}{\partial r} = \nu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right).$$

2.3.3

The $z$-component of the momentum equation is:

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial z} + \frac{1}{\rho} \frac{\partial p}{\partial r} = \nu \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} \right).$$

2.3.4

Now, if we subtract the partial-with-respect-to-$r$ of the $z$-momentum equation from the partial-with-respect-to-$z$ of the $r$-momentum equation we get the vorticity transport equation:

$$\frac{\partial \omega}{\partial t} + \nabla \cdot \omega = \frac{u}{r} \omega - \frac{\omega}{r^2} = \nu (\nabla^2 \omega - \omega / r^2).$$

2.3.5

where $\omega$ is the $\theta$-component of the vorticity vector, given by:

$$\omega = [\nabla \times \mathbf{u}] = \frac{\partial u}{\partial z} - \frac{\partial v}{\partial r}. \quad 2.3.6$$

If $u$ and $v$ are expressed in terms of the stream function $\psi$ as

$$u = (1/r) \frac{\partial \psi}{\partial z}, \quad 2.3.7$$

and

$$v = -(1/r) \frac{\partial \psi}{\partial r}, \quad 2.3.8$$

and then substituted into equation 2.3.6 we obtain:

$$\omega = (1/r) \nabla^2 \psi. \quad 2.3.9$$
The purpose of using the vorticity transport equation is twofold. First, we eliminate the pressure term which is present in the momentum transport equation. Second, we now have only one differential equation in time instead of two. This will greatly improve the speed of computation over solving the r and z momentum equations. This was the approach that was taken by Shokoohi. After many computer runs, however, it was found that the form of the vorticity transport equation is inappropriate for the laminar jet problem. The initial parabolic velocity profile fails to relax.

We can see the reason why the vorticity transport equation does not work for the parabolic velocity profile case if we make the appropriate substitutions into equation (2.3.5). On substitution, we find that the right-hand-side of the equation is identically equal to zero and that the gradient term is equal to $u_0/r$. Therefore, the time derivative of the vorticity is identically zero, which implies that a parabolic velocity profile will not, on its own, change (relax) with time; this fact is in contradiction with observations and solution of the momentum equations. So instead of solving the vorticity equation as we would like to do, we must solve the two momentum equations simultaneously.

In order to solve the momentum equations, we must know the jet pressure profile as a function of $r$ and $z$. Since we have no such equation, we must assume that the $\partial P/\partial r$ and
3P/3z are negligible. This assumption is not bad. At first glance, one would assume that 3P/3z cannot be equal to zero, but after closer examination it is seen that the axial velocity of the jet fluid is the result of the momentum of the jet as it leaves the exit of the nozzle, and is not the result of an external pressure drop as is the case inside of the nozzle.

The solution procedure is to start with a continuous, infinitely long jet with a given velocity profile and a given surface perturbation at time equal to zero. Then as we step forward in time, we solve the r and z momentum equations simultaneously. As we proceed in time, we must also calculate the radius of the jet at each point in the z-direction from the kinematic boundary condition at the surface:

\[ \frac{\partial \eta}{\partial t} = u - v \frac{\partial \eta}{\partial z} \quad \text{at} \quad r = R \quad 2.3.10 \]
and \[ R = a + \eta \quad 2.3.11 \]

where \( \eta \) is the displacement of the surface from the undisturbed radius, \( a \), and \( R \) is the radius of the jet at \((z,t)\).

The equations developed above apply only to the interior of the jet, the boundary conditions which apply for the edges of the r,z region are covered in the next four sections.

2.3.2 Boundary conditions at z = 0, the nozzle exit
The region at $z=0$ is just at the end of the nozzle. It is assumed that the nozzle wall is rigid and that:
\[
\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial \eta}{\partial t} = 0.
\]
Thus $u$, $v$, and $\eta$ at $z=0$ are always equal to the initial conditions given in section 2.3.6.

2.3.3 Boundary conditions at the end of the jet, $z=Z$.

There are no suitable boundary conditions to be used at this point since the jet is assumed initially to be infinitely long, and $Z \ll \infty$. One possibility is to make $Z$ a multiple of the disturbance wavelength, $\lambda$. Since the velocity profiles are symmetrical about the nodal points, the velocity profile at $Z = n\lambda$ could be calculated from the previous nodal point. But this method imposes a restriction on values of $\lambda$ for a given $Z$.

An alternative procedure is to assume that the spatial derivatives are small and smooth enough that the values at $z=Z$ may be calculated from difference approximations of the axial derivatives. Special consideration must be given to the point at $r=R$ and $z=Z$. At this point the velocities are extrapolated from upstream values with the equation
\[
y(x+\Delta x) = 2y(x) - y(x-\Delta x)
\]

2.3.4 Boundary conditions at the centerline, $r=0$.

The boundary conditions at the centerline provide a great simplification of the equations in this region.

From the axial symmetry condition:
\[ u = 0 \quad \text{at } r = 0 \quad 2.3.13 \]

and
\[ \frac{3v}{3r} = 0 \quad \text{at } r = 0. \quad 2.3.14 \]

From equation 2.3.14 we get
\[ \frac{1}{r} \frac{3v}{3r} = \frac{3z}{r^2} \quad \text{at } r = 0. \quad 2.3.15 \]

2.3.5 **Boundary conditions at the surface, \( r = R \).**

There are two boundary conditions at the surface which must be included in the numerical solution. The first boundary condition is the condition that the normal stress at the surface must be equal to zero, which may be represented as:

\[
(p - \bar{p}) + \sigma((\frac{\partial u}{\partial r}) \cos^2 \alpha + (\frac{\partial v}{\partial z}) \sin^2 \alpha) = P - \sigma(\frac{1}{R_1} + \frac{1}{R_2})
\]

\[ \text{where } \cos^2 \alpha = \frac{1}{1 + (\frac{\partial r}{\partial z})^2} \]
\[ \sin^2 \alpha = \frac{(\frac{\partial r}{\partial z})^2}{1 + (\frac{\partial r}{\partial z})^2} \]
\[ P = \sigma / a \]
\[ \frac{1}{R_1} + \frac{1}{R_2} = 1 / R - \sigma (\frac{\partial^2 \eta}{\partial z^2}) / (1 + (\frac{\partial \eta}{\partial z})^2)^3 \]

If we assume that \( \bar{p} - p \ll P \), then we can use equation 2.3.16 to solve for the radial velocity at an imaginary point just outside of the jet surface, which may then be used to solve for the radial derivatives of the radial velocity at the surface.

The other boundary condition is that the shear stress at the surface is zero, or
From equation 2.3.17 we may solve for the axial velocity at an imaginary point just outside of the surface of the jet. With this value we may calculate \( \frac{\partial v}{\partial r} \) and \( \frac{\partial^2 v}{\partial r^2} \) at the surface.

2.3.6 Initial conditions, \( t = 0 \).

The following initial boundary conditions are assumed:

<table>
<thead>
<tr>
<th>in general</th>
<th>for parabolic profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta = f(z) ),</td>
<td>( \eta = f(z) )</td>
</tr>
<tr>
<td>( u = u(r,z) ),</td>
<td>( u = 0 ) for all ( z )</td>
</tr>
<tr>
<td>( v = v(r,z) ),</td>
<td>( v = \frac{1}{2}v_{avg}(1-(r/R)^2) ) for all ( z ).</td>
</tr>
</tbody>
</table>

Where \( v_{avg} \) is the average axial velocity of the jet at the exit of the nozzle (Re= \( v_{avg}d/\nu \)).

The surface disturbance may be imposed as either:

1) a spacial perturbation that is a function of axial distance, such as \( f(z) = \eta_0 \sin(kz) \), or

2) a temporal instability at the nozzle exit that is a function of time, such as \( f(0) = \sin(\omega t) \).

2.3.7 Numerical solution technique.

To solve the equations developed above, the method of finite differences is used. The jet is divided into a grid network of \( n_i \) points in the \( r \)-direction by \( n_j \) points in the \( z \)-direction; a point on this grid is specified by the pair \( (i,j) \), where \( iAx \) is the radial position and \( jAz \) is the axial position. In the numbering system convention used here, \( i = 1 \).
is the centerline, $i = n_i$ is the surface, $j = 1$ is the nozzle exit, and $j = n_j$ is the end of the jet. The time position is given by $k \Delta t$. The time derivatives are approximated by simple forward differences, while the interior and surface space derivatives are approximated by second order Crank-Nicholson central differences, and the end axial derivatives are approximated by second order Crank-Nicholson backward differences as discussed in section 2.3.3. The Crank-Nicholson technique is well documented in the literature (Carnahan 1969, Conte 1972, Kitter 1969) and will not be discussed here. Suffice it to say here that this technique improves the stability of the system, allowing larger time steps to be used, reducing the amount of computation time required. The finite difference equations used in this program are listed in Appendix A.

The equations are solved at each point in time using Gauss-Seidel iteration. This technique is also well documented in the literature (Carnahan 1969, Conte 1972, Kitter 1969) and will not be discussed here. Because of the closed form of the Crank-Nicholson/Gauss-Seidel method, the accuracy of the integration can be controlled better than with an open method, such as Runge-Kutta. It should be noted that this system of equations consists of a boundary value problem in the $r$-direction and initial value problems in the $z$-direction and in time. The boundary value aspect of the solution also makes solution by an open method more difficult.
The difficulty in solving this type of problem is amplified by the presence of the free surface of the jet. Since the outer radial boundary of the system (the jet surface) is a function of time, special consideration of the grid points at the surface must be taken into account. There are two possible methods of handling the free surface problem. The first method is illustrated in Figure 2-2. In this method the radial step size, \( \Delta r \), is constant for each axial position, which means that the number of radial grid spacings varies with axial position. This method allows for easy handling of the interior points, but does not handle the surface points well. If many grid points are used, then the surface may be approximated by the actual grid points that are in the region \( R \pm \frac{1}{2} \Delta r \), as shown by the solid line in Figure 2-2. If fewer grid points are used, then special care must be taken to approximate the surface at imaginary grid points, as shown by the dashed line in Figure 2-2. The major problem with this method is that in order to allow for the growth of the jet surface, many grid points must be set aside outside of the actual jet region. This then increases the storage space required without improving the accuracy of the solution. Another problem is that when fewer grid points are used, the approximation of the derivatives at the surface becomes more difficult and less accurate.

The other method sacrifices some computation time to achieve a more accurate solution. This method, shown in Figure 2-3, uses a constant number of grid points for each
Figure 2-2. Finite Difference Grid for a Variable Number of Radial Grid Points.
Figure 2-3. Finite Difference Grid for a Constant Number of Radial Grid Points.
axial position (conversely, $\Delta r$ is a function of the axial position). The major advantages of this method are more accurate evaluation of the surface derivatives and more efficient use of storage space. The major disadvantage is that the grid point $(i,j)$ is usually at a different radial position than the grid point $(i,m)$, $m \neq j$; what this means is that the gradients cannot be approximated simply from the differences in adjacent grid points. This problem is solved by using quadratic interpolation to evaluate the values at the appropriate radial position for the neighbors of each grid point; this is the source of the increase in computation time that was mentioned earlier. This is the method that was used in this research.

One other problem with solving these equations is that they are in cylindrical coordinates. The problem is that the $1/r$ factor present in many of the terms tends to infinity as the centerline is approached; this adds to the stiffness of the system. There are several ways to reduce this effect on the stiffness. One way is to model a jet with a large initial radius, so that small values of $r$ do not have to be used. Another way is to use larger step sizes (but this reduces the accuracy of the solution).

The results of the computer simulations are given in Chapter 4.
3.1 The Experimental Apparatus

The experimental apparatus that was used in this research is illustrated in Figure 3-1. This setup consists of the following units:

1) A storage tank to hold the water and to provide a constant head pressure.

2) A nitrogen cylinder with a pressure regulator to provide the system with a the pressure head which determines the jet velocity.

3) A calming chamber that is used to eliminate the effects of fittings between the tank and the nozzle. A schematic diagram of the calming chamber is shown in Figure 3-2. The effect of the flow straighteners and the wire meshes is to reduce turbulence at the entrance of the nozzle as much as possible.

4) A pressure gauge and mercury manometer located on the calming chamber in a position so as to measure the pressure at the entrance of the nozzle.

5) A Lure-lock fitting at the end of the calming chamber to allow quick and easy replacement of the nozzles.

6) Nozzles of different diameters, but all with length to diameter ratios of about 300.
Figure 3.1. The Experimental Apparatus.
Figure 3-2. Schematic Diagram of the Calming Chamber.
7) An optical detector, associated electronics, and a minicomputer to determine the size of the drops.

3.1.1 The optical detector

The optical detector was constructed by Robert Chow (1978) following the design of Ritter, Sterling, and Zinner (1976). The detector works on the principle that the voltage output of a photodiode is proportional to the amount of light that is incident on its surface. Conversely, if the diode is fully illuminated, then the reduction of the output voltage is proportional to the size of the shadow that passes over the diode. It is this latter principle that is employed in this detector.

Refering to Figure 3-3, if the thickness of the light field that falls on the diode surface is larger than the size of the object that is creating the shadow, then the output voltage will be an indication of the area of the object as it is projected onto the plane of the diode surface. This method would work well in determining the volume of spheres.

A problem with this method is that if more that one object (drop) crosses the light field at the same instant, there is no way to distinguish between the different objects. To overcome this problem (or to minimize the effect), the thickness of the light field that falls on the diode can be made much smaller than the smallest dimension of the object. Then the output signal will be approximately
Figure 3-3. Schematic Diagram of the Optical Detector.
proportional to the area of a thin slice of the drop as the drop crosses the light field. As the light field is made smaller and smaller, the diode output becomes proportional to a chord of the drop at time t. So the largest reduction of incident light would then correspond to the diameter of a spherical drop rather than to the cross-sectional area.

The optical detector is illustrated in Figure 3-4, and consists of the following parts:

1) A source of collimated light.

2) An entrance slit to restrict the width of the light field. In this experiment the entrance slit width was set at approximately 1.0 mm.

3) An exit slit to reduce the possibility that the detector will "see" more than one drop at a time, as well as to reduce the interference of stray and scattered light. In this experiment, the exit slit width was set at approximately 0.1 mm.

4) A 3 mm by 10 cm linear photodiode which converts the light signal incident on its surface to a corresponding dc voltage.

3.1.2 Electronic Signal Conditioner

The raw output of the optical detector in itself is unacceptable for use by the digital computer. The chore of monitoring the rate of change of the output voltage is too tedious, and would require a computer and an analog to digital convertor of one or two orders of magnitude faster.
Figure 3-4. A Photograph of the Optical Detector.
clock rate than the ones used in this experiment. The electronic signal conditioning circuit eliminates this problem by continuously approximating the derivative of the amplified diode output signal electronically. This circuit was first designed by T. J. Ouwerkerk (1981).

The raw signal from the diode is first inverted (so that increasing shadow sizes correspond to a signal that becomes more positive) and then pre-amplified. The signal may then optionally be further amplified and/or filtered to remove high frequency noise (cutoff is ~2 kHz) before it is sent to the rest of the circuit. Both the pre-amp and the optional amplifier have external gain controls which may be set by the operator to give any desired range of output. The signal is then sent to the derivative circuit.

When the derivative circuit determines that the rate of change of the signal is equal to zero, it triggers a pulse that initiates three other circuits. One of these circuits sends a TTL interrupt signal to the computer indicating that a peak maximum has been reached; another circuit maintains the signal that is available for the analog to digital convertor at the maximum signal amplitude; the third circuit continues to monitor the amplified diode output and reset the other circuits when the signal returns to the baseline (or more correctly falls below some pre-specified threshold value).

The signal conditioning circuit is shown in the block diagram in Figure 3-5, and in the schematic diagram in
Figure 3-5. Block Diagram of the Signal Conditioning Circuit.
Figures 3-6 and 3-7. The circuit parameters are set to provide the following voltage levels when the lamp voltage is set to 24.0 VDC:

1) TP1 is set to +2.00 VDC. This voltage is adjusted by potentiometer R1 (external).
2) TP2 is set to 0.00 VDC. This voltage is adjusted by potentiometer R2 (internal).
3) A/D output is set to -1.00 VDC. This voltage is adjusted by potentiometer R3 (internal).

Only the +2.00 volt signal at TP1 must be readjusted each time the equipment is cold-started. This parameter compensates for the change in the lamp output as the lamp ages, and for the change in the diode output as its properties change with use. Figure 3-8 shows a typical oscilloscope trace of the signal conditioning circuit outputs. Figure 3-9 shows the photodiode amplifier output for a typical stream of drops.

3.1.3 The Analog to Digital Convertor

An analog to digital convertor, ADC, is a device that converts an analog voltage signal to an equivalent digital integer value that can be used by a computer. The ADC used in this experiment is a Ratheon Miniverter model MADC12-06 (Hewlet-Packard Model 2310C). This ADC is a 64 channel (48 available) multiplexed device that will convert analog signals in the range of -10 volts to +10 volts to digital values of -2048 to +2047 (11 bits plus sign). The ADC has a
Figure 3-6. Schematic Diagram of the Amplifier Section of the Signal Conditioning Circuit.
Figure 3-7. Schematic Diagram of the Filter, Peak Detector, and Clamping Circuits of the SCC.
Figure 3-8. Sample Oscilloscope Tracing of the SCC Output
Figure 3-9. Sample Oscilloscope Tracing of the Amplifier Output for a Stream of Drops.
maximum scan rate of 35 kHz in the random access mode with a sample and hold aperture time of 100 nanoseconds. The ADC is completely controlled by the computer, and may be used on a priority interrupt system, that is, the computer may start the conversion process, then continue doing some other chore. When the conversion is finished an interrupt signal is sent to flag the computer that a digitized value is available. Programs that were used in this experiment, the computer continually polls the convertor for a completion signal rather than waiting for an interrupt signal; this avoids the complication that the computer may be performing some uninterruptable task when the ADC interrupt signal appears. This is necessary because the timing is critical and is possible because the computer is dedicated to this one experiment.

3.1.4 The Minicomputer

The minicomputer that was used in this experiment is a 16-bit word, Hewlet-Packard 2116B minicomputer with 32 k word (64 k byte) magnetic core memory. The instruction time for register instructions (such as SHIFT) is 1.6 microseconds; the instruction time for input/output and non-'indirect' memory reference (such as ADD) instructions is 3.2 microseconds (each indirect branch is another 1.6 microseconds). Associated with the minicomputer are the following pieces of equipment:

1) A Time Based Generator, TBG, that is used under
computer control to provide time delay intervals. The TBG was modified to provide a time measurement of 0 to 163.83 ± .01 milliseconds.

2) A Burrough's video display terminal, VDT, for operator interactive response, and for computer output.

3) An ARS-35 Teletype, TTY, that is used to produce hard copies of computer output when requested by the operator.

4) A 24 channel, multiplexed, 12-bit digital to ± 10 volt analog convertor, DAC, (Raytheon model MDAC12-06).

5) An Electronics Associate Inc. (EAI) X-Y Plotter that is operated under computer control through the DAC.

The minicomputer was used to collect and perform elementary statistical analysis of the drop size data. To collect the data, the computer continuously monitors the TTL trigger signal that indicates that a peak has been detected. When the trigger signal goes high (+2.0 to +5. VDC), the computer immediately issues the start command to the converter to digitize the signal detector output, reads the current value of the TBG, resets the TBG, then waits for the conversion complete (CC) signal. When the CC signal is received, the computer converts the ADC output to a signed integer which it then stores in a vector, and increments the vector pointer. Then it stores the time period from the TBG in another vector, increments that vector pointer, resets
the convertor circuits, then waits for the next trigger pulse. This procedure is repeated for the number of drops that the operator specified at the beginning of the run (a maximum of 10,000 drops).

After the specified number of data points are collected, the computer then produces a histogram that shows the relative number of drops that have a given diameter (see Fig. 3-10), and a histogram of the period between drops (see Fig. 3-11). Then with the use of a simple interactive FORTRAN program, the computer calculates the mean and the standard deviation of each peak of the histogram. With this program the user determines a separation point between each peak from the graphic output, and from this information, the program computes the mean and standard deviation of each peak.
Figure 3-10. Example of the Drop Size Distribution Plot Produced by the Minicomputer.
Figure 3-11. Example of the Drop Spacing Distribution Plot Produced by the Minicomputer.
3.2 Experimental Procedure

The procedure for experimentally determining the jet lengths and drop sizes is as follows:

1) The volumetric flow rate as a function of head pressure was determined for each nozzle. These data were then converted to mass average velocities. The data were obtained by setting a specific head pressure, then collecting the fluid in a graduated cylinder for a set period of time. The ratio of the amount of fluid collected to the time period gives volumetric flow rate. Dividing the volumetric flow rate by the cross-sectional area of the nozzle gives the mass average velocity at the nozzle exit. The average velocity versus pressure head are presented in Table B-1 and in Figure 3-12.

2) The jet length as a function of head pressure data were collected at the same time. These data were obtained by using a strobe light to "stop" the motion of the jet and the drops, then the distance from the exit of the nozzle to the end of the jet (the pinch-off point) was measured with a flexible ruler. The effect of gravity on the length of the jet is small when the jet velocity is large enough that there is only a small amount of curvature in the jet trajectory. The jet length versus velocity data are presented in Table B-2 and in Figure 3-13.

3) Finally the optical detector/computer setup was used
Figure 3-12. Experimental Velocity versus Head Pressure.
Figure 3-13. Experimental Jet Length versus Jet Velocity.
to obtain the average drop diameter and drop spacing for each nozzle at various flow rates (head pressures). Before each run the optical detector was calibrated by finding the zero and the span. The zero reading was taken with nothing in the light field. The span was obtained by dropping stainless steel balls of a known diameter through the light field, this gave an voltage (or ADC) output for a known diameter. The zero and span were then used in computing the drop diameter. The period between the drops was obtained directly from the Time Base Generator as described above. The drop size and drop spacing versus velocity data are presented in Table B-3 and in Figures 3-14 and 3-15, respectively.
Figure 3-14. Experimental Drop Size versus Jet Velocity.
Figure 3-15. Experimental Drop Spacing versus Jet Velocity.
3.3 The Simulation Package

The FORTRAN source code given in Appendix C was used to solve the differential equations that were derived in Section 2.3. Briefly, the program consists of a MAIN program and the following subroutines: INITAL, VELOCT, RADIUS, and INTERP. The main routine:

1) Reads in the various input data that are required to define a run;
2) Writes out the input data;
3) Calls subroutine INITAL to set up the initial profiles;
4) Integrates the time dependent differential equations by calling VELOCT and RADIUS at each time step;
5) Determines from information returned from VELOCT and RADIUS if the time step size should be reduced;
6) Writes intermediate values to temporary data sets which are used later in making the graphical output.

A flow chart of the simulation package is given in Figure 3-16.

Subroutine VELOCT solves the radial and axial momentum equations using the Crank-Nicholson/Gauss-Siedel technique that was discussed in Section 2.3. It should be noted that this routine uses forward difference approximations of the axial derivatives for surface points that lie on the trailing edge of a surface disturbance and uses backward difference approximations for surface points that lie on the
Start

Read and write input data

VELOCT

Use Gauss-Siedel iteration to solve the velocity equations

RADIUS

Use Gauss-Siedel iteration to solve for the jet radius

Has the jet pinched off?

Yes

Write Results

Stop

No

t = t + \Delta t

Reduce step size?

Yes

\Delta t = \Delta t / 2

No

Figure 3-16. Simulation Package Flow Chart
leading edge. All central difference approximations are used for all other points.

Subroutine RADIUS computes the disturbance amplitude at time $t$ from the kinematic boundary condition. If the change in the amplitude at any point from one time step to the next is greater than 0.2, then RADIUS flags the MAIN routine that the time step size should be reduced. If the amplitude becomes less than one hundredth of the original undisturbed jet radius, then RADIUS flags MAIN that the jet has essentially pinched-off and to halt the simulation.

Subroutine INTERP is uses quadratic interpolation to approximate the velocities between radial grid points which are used in the finite difference equations as discussed in Section 2.3.

The input data that are required for the program are: undisturbed jet radius, simulation length, initial disturbance amplitude, disturbance wave length or frequency (one or the other, not both), jet fluid viscosity and density, interfacial tension, number of radial and axial grid points initial time step size, the maximum number of time steps to allow, convergence tolerances for VELOCT and RADIUS (usually 0.001), the minimum allowable time step size, and various debugging and operational flags that are discussed in Appendix C.

Many preliminary test cases were run to determine the applicable range of the program. In order to reduce the stiffness of the system of equations, it was found that the
Initial jet radius should be greater than or equal to 5 mm (this allows the user to use a larger time step size, in order to use less computer time without having the errors in the computation grow exponentially). If no surface disturbance is to be used, then 51 is the minimum number of radial grid points that may be used, and still have reliable answers. If a surface disturbance is used, then as few as 21 radial grid points may be used, although better results are obtained with more grid points. In all cases, the minimum number of axial grid points is 51. It should be noted that the number of differential equations that must be solved may be calculated from the following equation

\[ nd = (2^{ni} + 1) \times nj. \]

So, for a 51 by 51 grid there are 5302 differential equations that must be solved at each time step. Furthermore, the integration is an iterative procedure, and on the average, each time step requires about 5 iterations to converge, depending on the time step size (smaller step size gives fewer iterations and visa versa). From this we see that it is essential to find the minimum number of grid points and the maximum time step size that may be used and still give accurate results.
CHAPTER 4

Results

The results of fitting an empirical equation to the experimental data as well as example test cases using the simulation package are presented in this chapter.

4.1 Experimental Results

A plot of the ratio of the drop size to the nozzle diameter versus Reynolds number is given in Figure 4-1. Each data point on the plot represents the average of drop diameters measured at various distances from the breakup point; the solid line represents the empirical equation that fits the data. From this figure we can see that the drop size for this range of Reynolds number is a linear function of the Reynolds number, the drop diameters decreasing with increasing Reynolds number. The following equation gives a simple empirical relationship between the drop diameters and the Reynolds number:

\[ \frac{D}{d} = (1.968 - 4.731 \times 10^{-4} \text{Re}). \]  

4.1.1

It should be noted that this equation only applies to water jets, since the only parameters that were changed were the nozzle diameter and the jet velocity.

It was expected that all of the primary drops would be larger than 1.68 jet diameters, but from Figure 4-1 we see
Figure 4-1. Dimensionless Drop Size versus Reynolds Number
that at the higher Reynolds numbers the actual drop diameters are much smaller than this value. The reason for the smaller than expected drop sizes at the higher velocities is that the jet contracts in the axial direction, and the contraction increases with increasing jet velocity.

The ambient Weber number for all of these data points was below one. From previous discussions we know that at for Weber numbers below one the aerodynamic effects are negligible. The results of previous investigators using short nozzles indicate that the drop size is not a function of the jet velocity. So the results given in this paper are new, and have not been reported in the literature.

The drop size data were to be used to calculate the most unstable disturbance wave length, $\xi^+$, from equation (2.2.3). But this equation assumes that the jet is not contracting. Anno, on the other hand uses the jet diameter at the breakup point rather than the undisturbed jet diameter to calculate $\xi^+$. From the data presented here, it appears that this must be the case.

Since the actual jet diameter at the break up point was not measured, the drop size data was not used to find a correlation between $\xi^+$ and the jet velocity. Alternatively, it was assumed that $\xi^+$ is constant and can be calculated from the low velocity data where the jet contraction is small.

The ratio of the jet length to the undisturbed jet diameter, $\Lambda$, versus the jet Reynolds number data are plotted
as stars in Figure 4-2; the solid line represents the empirical equation given later. We can see from this figure that for Reynolds number between about 300 and 1000, $A$ is a linear function of the Reynolds number. In the range 1000 to about 1500 $A$ changes parabolicly passing through a maximum that appears to be the same for both nozzles. From this fact, and from equation (2.2.7) we see that $\beta^*$ must be proportional to $R_{e^2}$. Then from equation (2.2.16) we see that $F_4$ must be proportional to $R_{e^4}$. Also, from Figure 4-2, we see that the effect of the parabolic relationship disappears as $R_{e}$ becomes larger (as the jet approaches the turbulent flow region, $R_{e} \geq 2100$). It is known from other researchers that, once in the turbulent region, the jet length continues to grow more or less as a linear function of velocity. From this we assume that $\beta^*$ must then approach a constant value at the higher Reynolds numbers. So, from the above observations the following empirical equation is postulated:

$$\begin{align*}
F_4 &= g(R_{e}, Z)/ (1 + 3Z) \\
g(Re, Z) &= P_1 + P_2 (P_3 - Re) 
\end{align*}$$

Equation 4.1.2 applies only to the laminar region, the following form of $g(Re, Z)$ applies to both the laminar and the turbulent regions:
Figure 4-2. Dimensionless Jet Length versus Reynolds Number
\[ g(Re, Z) = P_1 + P_2 (P_3 - Re)^4 (\tan^{-1} \left( \frac{(P_4 - Re) Z}{\pi/2} \right) + 1) / 2 + P_5 (\tan^{-1} \left( \frac{P_7 (Re - P_6)}{\pi/2} \right) + 1) / 2 \]  \hspace{1cm} 4.1.3

The dashed line in Figure 4-2 represents the results of setting \( F_4 \) equal to zero. It can be seen that this term does represent the effects of the velocity profile relaxation of jets produced from long nozzles.

The parameters in equation (4.1.2) were determined from the experimental data by minimizing the following sum of squared errors equation with a pattern search routine:

\[ E^2 = \sum \left( \frac{y_{obs} - y_{calc}}{y_{obs}} \right)^2 \]  \hspace{1cm} 4.1.2

The experimentally determined values, as well as a parametric study of the effect of each variable, are given in Table 4-1. The parametric study consists of perturbing each parameter by ±10% and noting the change in \( E^2 \). From the relative change in \( E \) for each parameter we can get a feel for the sensitivity of the function \( g \) to each of the seven parameters. Looking at Table 4-1 we can see that \( P_1, P_3, \) and \( P_4 \) produce the largest change in the sum of squared errors. Since these parameters, from equation (4.1.2) control the position and shape of the peak of the jet length curve, it is reasonable to assume that these parameters might be functions of the properties of the jet and the nozzle, for example they might be functions of the Ohnesorge number. From Figure 4-2 we can see that equation (4.1.2) gives a reasonably good fit to both the 0.25 mm nozzle and the 0.51 mm nozzle data.
### TABLE 4-1

Parametric Study of the Constants for Equation 4.1.1

<table>
<thead>
<tr>
<th>$E^2$</th>
<th>$P_1$</th>
<th>$P_2/10^{-10}$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
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<tr>
<td>.3509</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>53.11</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>.2305</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>52440.</td>
<td>*</td>
</tr>
<tr>
<td>.2304</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>42900.</td>
<td>*</td>
</tr>
<tr>
<td>.2305</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>3.347</td>
</tr>
<tr>
<td>.2304</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>2.738</td>
</tr>
</tbody>
</table>

**Note:** $E^2$ is given by equation (4.1.2).

"*" indicates that the parameter has the same value as the base case.
4.2 Simulation Results

Before discussing the test cases of the simulation runs a brief discussion of the graphical output in necessary. The each figure consists of four plots:

1) jet radius as a function of axial position,
2) radial velocity at the surface as a function of axial position,
3) axial velocity at a given distance from the nozzle exit as a function of jet radius, and
4) axial velocity at the surface as a function of axial position.

Each line on the plots represents the profile a given point in time. Recalling from Section 2.3, the initial velocity profile is assumed to be parabolic. So from the plots we see how the velocities and the surface change with both position and time.

The first five test cases that are presented here do not have any externally supplied surface disturbances. Case 1 is the inviscid case (viscosity equal to zero) with a fluid surface tension equal to that of water. In this case there was no relaxation of the parabolic velocity profile and there was no contraction of the jet radius. This indicates that this condition is meta-stable (that is, as long as there is no disturbance to the surface, the jet will maintain a constant radius). There is no figure for this case since all of the variables remained at their initial
values.

Case 2 is for the viscous jet (viscosity equal to that of water) with no surface tension. In this case, the jet velocity profile relaxed with time (and distance) and again the surface did not contract, since there is no initial radial velocity and no surface tension. Therefore, this situation is also meta-stable. There is no figure for this case since the relaxation profiles are identical to those shown in the figure for case 3.

Case 3 is for a jet of water (both viscosity and surface tension). The results of a 500 ms simulation run are presented in Figure 4-3. Although the jet contraction was too small to be seen in the simulated jet figure, the plot of the radial velocity versus jet distance clearly indicates that, because of the negative velocities, the jet is contracting. From this plot it can be seen that the radial velocity is increasing in magnitude exponentially from which we can conclude that the jet will eventually pinch off at some finite distance from the nozzle exit. From the plot of the axial velocity profile at the end of the jet, we can see that the velocity profile has contracted slightly in the time span of the simulation. From the plot of the axial velocity at the surface, we can see that the rate at which the jet surface is approaching its steady state velocity is decreasing exponentially.

Figure 4-4 is the simulation results of case 4. In this case, the surface tension was reduced by a factor of
Simulation Time = 500.E-03 s
Undisturbed Jet Radius = 5.00E-03 m
Disturbance Amplitude = 0.00E+00 m (0.00E+00)
Disturbance Wave Length = 1.00E+30 m (2.00E+32)
Disturbance Wave Number = 6.28E-30 1/m (3.14E-32)
Disturbance Frequency = 0.00E+00 1/s
Average Exit Velocity = 200.E-03 m/s
Surface Tension = 72.8E-03 Nt/m (or kg/s²)
Jet Fluid Viscosity = 1.00E-03 Pa s (or kg/m s)
Jet Fluid Density = 999.E+00 kg/m³
Jet Reynolds Number = 1.99E+03
Jet Weber Number = 5.49E+00
Jet Ohnesorge Number = 1.18E-03
ni = 51, nj = 51

Figure 4-3. Simulation Run for Water (Low Viscosity and High Surface Tension), No Surface Disturbance.
Figure 4-3 (cont'd).
Simulation Time = 500.E-03 s

Undisturbed Jet Radius = 5.00E-03 m
Disturbance Amplitude = 0.00E+00 m (0.00E+00)
Disturbance Wave Length = 1.00E+60 m (2.00E+62)
Disturbance Wave Number = 62.8E-75 1/m (2.27E-76)
Disturbance Frequency = 0.00E+00 1/s
Average Exit Velocity = 200.E-03 m/s

Surface Tension = 7.28E-03 Nt/m (or kg/s²)
Jet Fluid Viscosity = 1.00E-03 Pa s (or kg/m s)
Jet Fluid Density = 999.E+00 kg/m³
Jet Reynolds Number = 1.99E+03
Jet Weber Number = 5.49E+01
Jet Ohnesorge Number = 3.72E-03
ni = 51, nj = 51

Figure 4-4. Simulation Run for Low Surface Tension and Low Viscosity, No Surface Disturbance.
Figure 4-4 (cont'd).
ten to show the effect of surface tension in the contraction rate. The result of this reduction can be seen in the plot of the radial velocity. Here we see that the radial velocity at 25 jet diameters is nine orders of magnitude less than the radial velocity in case 3. So we see from this that the radial velocity, and thus the jet contraction, is a strong function of the surface tension.

In case 5 the viscosity was increased by a factor of ten while the surface tension was that of water. The results of case 5 are shown in Figure 4-5. For this case we can see that the amount of relaxation is very great, in fact, in the same amount of time, this jet has almost assumed a plug flow profile while cases 3 and 4 were still almost parabolic. We see from the radial velocity plot that this case will contract less than the low viscosity and high surface tension case, but more than the low viscosity and low surface tension case.

The rest of the cases that follow all have a sinusoidal surface disturbance of a fixed wave length. In all of these cases, the disturbance wave lengths were relatively large, about 40 jet diameters. These relatively large wave lengths were used because better and faster simulation could be obtained than with smaller wave lengths. (To simulate the smaller wave lengths more grid points are needed, thus increasing the computation time.) Various physical properties are changed so as to illustrate the their effect on the breakup process. It should be noted that because of
Simulation Time = 500E-03 s

Undisturbed Jet Radius = 5.00E-03 m
Disturbance Amplitude = 0.00E+00 m (0.00E+00)
Disturbance Wave Length = 1.00E+30 m (2.00E+32)
Disturbance Wave Number = 62.8E-75 1/m (2.27E-76)
Disturbance Frequency = 0.00E+00 1/s
Average Exit Velocity = 200.E-03 m/s

Surface Tension = 72.8E-03 N/m (or kg/s²)
Jet Fluid Viscosity = 10.0E-03 Pa s (or kg/m s)
Jet Fluid Density = 999.E+00 kg/m³

Jet Reynolds Number = 1.99E+02
Jet Weber Number = 5.49E+00
Jet Ohnesorge Number = 1.18E-02

ni = 51, nj = 51

Figure 4-5. Simulation Run For High Surface Tension and High Viscosity, No Surface Disturbance.
Figure 4-5 (cont'd).
the computer time that is required for a simulation of a water jet (low viscosity) to reach steady state, all of the following simulation runs were made with the disturbance applied at \( t = 0 \). Because of this, the surface velocity remains very small, and therefore there is very little propagation of the disturbance down the length of the jet. So the jet length is only a very weak function of the jet velocity.

It is possible, with the existing program, to run the simulation with no disturbance until the steady state profile is obtained for the region in question. Then the program may be restarted using the steady state profile as the initial profile with the desired surface disturbance. This procedure must be repeated for each new jet fluid physical property, thus increasing the cost the if the simulation package is to be used to predict the jet lengths of many different fluids.

In case 6 we have the case of a jet of water with a surface disturbance that has an initial disturbance amplitude that is nine orders of magnitude smaller than the undisturbed jet radius. The average jet velocity is such that the jet Reynolds number is approximately 2000. From Figure 4-6, we see that the jet required about 352 ms to breakup. This is the base case to which the remaining cases will be compared.

In case 7 the surface tension was increased by a factor of ten. We see from Figure 4-7 that the breakup time is
Simulation Time \( = 352.0 \times 10^{-3} \text{s} \)

Undisturbed Jet Radius \( = 5.00 \times 10^{-3} \text{ m} \)
Disturbance Amplitude \( = 5.00 \times 10^{-12} \text{ m} \) (or \( 1.00 \times 10^{-9} \))
Disturbance Wave Length \( = 400.0 \times 10^{-3} \text{ m} \) (or \( 8.00 \times 10^{1} \))
Disturbance Wave Number \( = 15.7 \times 10^{0} \text{ 1/m} \) (or \( 7.85 \times 10^{-2} \))
Disturbance Frequency \( = 0.00 \times 10^{0} \text{ 1/s} \)
Average Exit Velocity \( = 200.0 \times 10^{-3} \text{ m/s} \)

Surface Tension \( = 72.8 \times 10^{-3} \text{ Nt/m} \) (or \( 72.8 \times 10^{-3} \text{ kg/s}^{2} \))
Jet Fluid Viscosity \( = 1.00 \times 10^{-3} \text{ Pa s} \) (or \( 1.00 \times 10^{-3} \text{ kg/m s} \))
Jet Fluid Density \( = 999.0 \times 10^{0} \text{ kg/m}^{3} \)

Jet Reynolds Number \( = 1.9 \times 10^{3} \)
Jet Weber Number \( = 5.49 \times 10^{0} \)
Jet Ohnesorge Number \( = 1.18 \times 10^{-3} \)
ni = 26, nj = 101

Figure 4–6. Simulation Run for a Water Jet with a Fixed Wave Length Sinusoidal Disturbance.
Figure 4-6 (cont'd).
Simulation Time = 136.0E-03 s

Undisturbed Jet Radius = 5.00E-03 m
Disturbance Amplitude = 5.00E-12 m (1.00E-09)
Disturbance Wave Length = 400.0E-03 m (8.00E+01)
Disturbance Wave Number = 15.7E+00 m^-1 (7.85E-02)
Disturbance Frequency = 0.00E+00 1/s
Average Exit Velocity = 200.0E-03 m/s

Surface Tension = 728.0E-03 N/m (or kg/m s^2)
Jet Fluid Viscosity = 1.00E-03 Pa s (or kg/m/s)
Jet Fluid Density = 999.0E+00 kg/m^3

Jet Reynolds Number = 1.99E+03
Jet Weber Number = 5.49E+01
Jet Ohnesorge Number = 3.72E-04

ni = 26, nj = 101

Figure 4-7. Simulation Run for a Jet Fluid of Very High Surface Tension with a Sinusoidal Disturbance.
Figure 4-7 (cont’d).
considerably less, as we would expect. Notice that the breakup point is shorter than the for case 6, this is because the axial velocity at the surface is less since the breakup time is less.

In case 8, not shown, the viscosity was increased by a factor of ten. As expected the breakup time is greater. Also, since the relaxation is faster, the breakup point is further down stream.
CHAPTER 5

Conclusions and Recommendations

5.1 Conclusions

From the results that are given in Chapter 4, the form of a working empirical equation has been found that gives satisfactory prediction of the jet length of laminar jets of Newtonian fluids produced from long nozzles. The original equation (4.1.3) for both laminar and turbulent flow contained an eighth parameter that was to correspond to the Ohnesorge number, thus reducing the number of parameters by one and improving the fit of the equation. We would like to think that if this is the correct equation, then some physical significance can be placed on each of the unknown parameters. For example, $P_1$ and $P_5$ might be linked to the minimum Reynolds number needed for jetting to occur (refer to Fig. 1-1). The parameter $P_3$ obviously is the critical Reynolds number corresponding to the peak in the jet length curve. But if there is any way of theoretically determining $P_3$ is not obvious, and may not be possible; remember that the critical Reynolds number for the transition from laminar to turbulent flow must be determined experimentally.

The evidence is pointing more and more towards the relaxation of the velocity profile as the cause of the increased instability of the jet. From the results of the computer simulations, it appears that one of the reasons
why laminar jets are more unstable is that the axial velocity at the surface, which is responsible for propagating the surface disturbance down the jet remains small for a significant distance from the nozzle. Since the disturbance is moving slowly at the beginning of the jet, the unstable feedback is amplified in this region, making the disturbance amplitude grow faster than if the surface velocity were larger.
5.2 Recommendations

The following recommendations result from observations made in using the present experimental apparatus and from results of the computer simulations.

The measurement of the diameter of the primary drops to be used in calculating the disturbance wave number assumes that the drops are spherical when they pass through the detector. Of course this is not true. The drops will more closely resemble ellipsoids. If a second optical detector were mounted perpendicular to the first one, then two semi-major axes could be measured, and the volume of the drop could be calculated much more accurately. From the drop volume the disturbance wave number may then be calculated.

To make the drop size data more useful it is necessary to be able to measure the jet diameter near the breakup point. One way to do this is to photograph the jet, from the nozzle exit to the breakup point. After enlarging the photograph, it would then be possible to measure the jet diameter as a function of distance from the exit. Alternatively, the optical detector may be used to measure the jet diameter directly. The current circuit design cannot be used to perform this measurement because of the circuitry that is used to maintain the baseline. If a simple DC amplifier circuit is added, then the jet diameter can be determined by the minicomputer.

To add in the functional dependency of the disturbance
wave length or frequency, data must be collected for jets in which an artificial disturbance is imposed on the jet. With forced disturbances, the effect of the disturbance wave length can be separated from the effect of the jet Reynolds number, since it has been shown that for the free jet (no artificial disturbances) the wave length is a function of the jet velocity.

The optical detector can also be used to determine the average jet length by monitoring the output of the DC amplifier discussed in the previous paragraph. When the RMS level drops to 50% of the constant voltage output, it is then assumed that the average breakup point has been reached (the jet is continuous for 50% of the time at this point). This can be monitored either directly with a voltmeter, or by the minicomputer.

In order to test whether equation 4.1.1 has any physical significance, experiments should be made with fluids of varying physical properties. This would help to determine the exact functional dependence of the equation on the Ohnesorge number.

The extent of the velocity profile relaxation could be inferred from experimental run using nozzles of varying lengths. By varying the length of the nozzle, the velocity profile at the exit can be changed. It is expected that the shorter the nozzle the more stable would be the jet, so that the relaxation term, $F_4$, would have to be multiplied by a new term, $F_5$, that would represent the velocity profile at
the exit. We would expect $F_5$ to have the following form:

$$F_5 = 1 - \exp(-P_8 (l/d))$$  \hspace{1cm} 5.2.1

where $l$ is the nozzle length and $d$ is the nozzle diameter and $P_8$ is an empirical constant. The term $F_5$ with this equation will go to zero for orifices and to one for long nozzles.

To give further credence to the belief that the increased instability of long nozzle jets are more unstable experiments need to be conducted with very long nozzles, greater than the $300 \ l/d$ ratios that were used here. If, as Phinney suggested, the increased instability is the result of the presence of Tollmein-Schlichting waves that are produced by the nozzle walls, then the effect of these waves should be a function of the length of the nozzle. It would be expected that the longer the nozzle, then the greater the instability. If there is no significant decrease in the jet lengths with the longer nozzles at the same Reynolds number, then it must be assumed that the velocity profile relaxation is the contributing factor to the instability, not the Tollmein-Schlichting waves.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition, units</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>The uniform undisturbed jet radius at time= 0, m.</td>
</tr>
<tr>
<td>b</td>
<td>The complex frequency of the disturbance, s^{-1}.</td>
</tr>
<tr>
<td></td>
<td>(Real part= growth rate.)</td>
</tr>
<tr>
<td></td>
<td>(Imaginary part= propagation rate.)</td>
</tr>
<tr>
<td>C_i</td>
<td>A dimensionless constant in Weber's equation.</td>
</tr>
<tr>
<td>d</td>
<td>The internal diameter of the nozzle, m.</td>
</tr>
<tr>
<td>D</td>
<td>The diameter of a drop, m.</td>
</tr>
<tr>
<td>( \hat{e}<em>r, \hat{e}</em>\theta, \hat{e}_z )</td>
<td>The unit vectors in the r-, ( \theta )-, and z-directions, respectively.</td>
</tr>
<tr>
<td>( \ddot{g} )</td>
<td>The acceleration due to gravity, m/s^2.</td>
</tr>
<tr>
<td>i</td>
<td>The imaginary number ( \sqrt{-1} ).</td>
</tr>
<tr>
<td>k</td>
<td>The wave number of the surface disturbance, m^{-1}.</td>
</tr>
<tr>
<td>l</td>
<td>The length of the nozzle, m.</td>
</tr>
<tr>
<td>L</td>
<td>The continuous jet length, m.</td>
</tr>
<tr>
<td>p</td>
<td>The fluctuating component of the pressure, Pa.</td>
</tr>
<tr>
<td>P</td>
<td>The non-fluctuating component of the pressure, Pa.</td>
</tr>
<tr>
<td>r</td>
<td>The radial coordinate, m.</td>
</tr>
<tr>
<td>R</td>
<td>The radius of the jet at any axial position, m.</td>
</tr>
<tr>
<td>R_1, R_2</td>
<td>The principal radii of curvature of waves of frequency b, m.</td>
</tr>
<tr>
<td>Re</td>
<td>The dimensionless Reynolds number (=( \rho v_{avg}d/\mu )).</td>
</tr>
<tr>
<td>t</td>
<td>Time, s.</td>
</tr>
<tr>
<td>u</td>
<td>The radial component of the velocity, m/s.</td>
</tr>
<tr>
<td>v</td>
<td>The velocity vector, m/s.</td>
</tr>
<tr>
<td>v</td>
<td>The axial component of the velocity, m/s.</td>
</tr>
</tbody>
</table>
\( v_{avg} \) The average axial velocity at the nozzle exit.

\( V \) The jet velocity relative to the ambient gas, m/s.

\( We \) The dimensionless Weber number (\( = \rho v^2 d/\sigma \)).

\( z \) The axial coordinate, m.

\( Z \) The dimensionless Ohnesorge number (\( = \sqrt{We/Re} \)).

\( Z \) An arbitrary finite distance in the axial direction.

**Greek Symbols**

\( \alpha \) The dimensionless initial disturbance amplitude.

\( \beta \) The dimensionless growth rate constant (\( = b\sqrt{2\rho a^2/\sigma} \)).

\( \eta \) The disturbance amplitude, m.

\( \eta_0 \) The initial disturbance amplitude, m.

\( \Theta \) The angular coordinate, rad.

\( \lambda \) The disturbance wave length, m.

\( \Lambda \) The dimensionless jet length to jet diameter ratio.

\( \mu \) The absolute viscosity, Pa·s.

\( \nu \) The kinematic viscosity, (\( = \mu/\rho \)) m²/s.

\( \xi \) The dimensionless wave number (\( = ka \)).

\( \pi \) The constant \( \approx 3.1415926 \).

\( \rho \) The density, kg/m³.

\( \sigma \) The interfacial tension, Pa·m.

\( \tau \) The stress tensor, Pa.

\( \phi \) The velocity potential, m²/s.

\( \psi \) The stream function, m³/s.

\( \omega \) The angular vorticity function, s⁻¹.
Differentiation Symbols

\[ \nabla \] The gradient operator, m\(^{-1}\).

\[ \nabla \cdot \] The divergence operator, m\(^{-1}\).

\[ \nabla \times \] The curl operator, m\(^{-1}\).

\[ \nabla^2 \] The Laplacian operator, m\(^{-2}\).

\[ \frac{\partial}{\partial r} \] The partial derivative with respect to r.

\[ \frac{\partial}{\partial t} \] The partial derivative with respect to t.

\[ \frac{\partial}{\partial z} \] The partial derivative with respect to z.

Special symbols

A circumflex (\(^\sim\)\) over a variable indicates that the variable refers to the ambient fluid.

A superscript plus sign (\(^+\)) indicates that the variable refers to the most unstable disturbance.

() and [] are used to indicate a grouped quantity, either for clarity or to indicate multiplication.

{} is used to indicate that a variable is a function of the enclosed parameters, for example: \(y(x)\) means that \(y\) is a function of \(x\).
LIST of REFERENCES


Horsfall, F., Polymer, 14, p262 (1973).


Phinney, R. E., AIChe J., 18, p452 (1972).


Plateau, M. T., Phil. Mag., 12, p286 (1856).

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APPENDIX A

The Finite Difference Form of the Free Jet Equations
A = νΔt/(Δr)²
B = νΔt/r²
C = νΔt/2rΔr
D = ½(u_{i,j,k} + u_{i,j,k+1})Δt/Δr
E = νΔt/(Δz)²
F = ½(v_{i,j,k+1} + v_{i,j,k})Δt/Δr
Gr = 2A + 2E + B
Gz = 2A + 2E
H = σ/2ν

Note: Unless specified otherwise, the equation for v_{i,j,k+1} is the same as for u_{i,j,k+1} if the term Gr is replaced by Gz (and u is replaced by v).

For Internal Points

\[ u_{i,j,k+1} = [(2-Gr)u_{i,j,k} + (A+C-D)(u_{i+1,j,k} + u_{i-1,j,k}) + (A-C+D)(u_{i,j+1,k} + u_{i,j-1,k}) + (E-F)(u_{i,j+1,k} + u_{i,j-1,k})]/(2+Gr) \]

At the Centerline, i=1

\[ u_{i,j,k+1} = 0 \]

\[ v_{i,j,k+1} = [(2-2E-4A)v_{i,j,k} + 4A(v_{i+1,j,k} + v_{i-1,j,k}) + (E-F)(v_{i,j+1,k} + v_{i,j-1,k}) + (E+F)(v_{i,j+1,k} + v_{i,j-1,k})]/(2+2E+4A) \]

if j=nj then
\[ \begin{align*} 
v_{i,j,k+1} &= [(2-3F+E-4A)v_{i,j,k} + 4A(v_{i+1,j,k} + v_{i,j+1,k}) \\
&+ (4F-2E)(v_{i,j-1,k} + v_{i-j,j,k}) \\
&+ (E-F)(v_{i,j-z_j,k} + v_{i,j-z_j,k})]/(2+2E+4A) \\
\end{align*} \]

At the Surface, \( i=j \)

\[ \begin{align*} 
\alpha\eta/\alpha z &= \frac{1}{4}(\eta_{j+1,j,k+1} + \eta_{j+1,j,k} - \eta_{j-1,j,k+1} - \eta_{j-1,j,k})/\Delta z \\
\alpha^2 \eta/\alpha z^2 &= \frac{1}{4}(\eta_{j+1,j,k+1} + \eta_{j+1,j,k} - 2(\eta_{j,j,k+1} + \eta_{j,j,k}) + \eta_{j-1,j,k+1} + \eta_{j-1,j,k})/(\Delta z)^2 \\
\alpha v/\alpha z &= \frac{1}{4}(v_{i,j+1,j,k+1} + v_{i,j+1,j,k} - v_{i,j-1,j,k+1} - v_{i,j-1,j,k})/\Delta z \\
q &= 1 + (\alpha\eta/\alpha z)^2 \\
\alpha u/\alpha r &= -(3v/3z)(3\eta/3z)^2 + Hq/R - h(1/r - (1/q)\alpha^2 \eta/\alpha z^2) \\
\alpha v/\alpha r &= -\alpha u/\alpha z = -(u_{i,j,k+1} - u_{i,j-1,k+1})/2\Delta z \\
\end{align*} \]

\[ \begin{align*} 
\alpha u/\alpha r &= \frac{1}{2}(4u_{i,j,k+1} - u_{i-1,j,k+1} + 2(\alpha u/\alpha r)\Delta r) \\
\end{align*} \]

Points on the Leading Edge of a Crest

\[ \begin{align*} 
\alpha v/\alpha z &= \frac{1}{4}[3(v_{i,j,k+1} + v_{i,j,k}) - 4(v_{i,j-1,j,k+1} + v_{i,j-1,j,k}) \\
&+ (v_{i,j-z_j,k+1} + v_{i,j-z_j,k})]/\Delta z \\
Gr &= 2A-E+3F+B \\
Gz &= 2A-E+3F \\
\end{align*} \]

\[ \begin{align*} 
u_{i,j,k+1} &= [(2-Gr)u_{i,j,k} + (A+C-D)(u_{i+1,j,k} + u_{i,j+1,k}) \\
&+ (A-C+D)(u_{i-1,j,k} + u_{i-1,j,k}) \\
&+ (4F-2E)(u_{i,j-1,k} + u_{i,j-1,k}) \\
&+ (E-F)(u_{i,j-z_j,k} + u_{i,j-z_j,k})]/(2+Gr) \\
\end{align*} \]
Points on the Trailing Edge of a Crest

\[ \delta v/\delta z = \mu \left[ -3(v_{i,j,k+1} + v_{i,j,k}) + 4(v_{i,j+1,k+1} + v_{i,j+1,k}) ight. \\
\left. - (v_{i,j+2,k+1} + v_{i,j+2,k}) \right] / \Delta z \]

Gr = 2A - E - 3F + B

Gz = 2A - E - 3F

\[ u_{i,j,k+1} = ((2-Gr)u_{i,j,k} + (A+C-D)(u_{i+1,j,k} + u_{i+1,j,k})) \]
\[ + (A-C+D)(u_{i-1,j,k} + u_{i-1,j,k}) \]
\[ - (4F+2E)(u_{i,j+1,k} + u_{i,j+1,k}) \]
\[ + (E+F)(u_{i,j+2,k} + u_{i,j+2,k}) \] / (2 + Gr)

Equations to Calculate the Surface

A = \%(u_{n,i,j,k+1} + u_{n,i,j,k}) \Delta t

B = \%(v_{n,i,j,k+1} + v_{n,i,j,k}) \Delta t / \Delta z

In General

\[ \eta_{j+1} = \eta_{j} + A - B(\eta_{j+1} + \eta_{j+2} - \eta_{j-1} - \eta_{j-2}) \]

At the Nozzle Exit, j=1

\[ \eta_{1,j+1} = \left[ (1+3B)\eta_{1,j} + A - B(4(\eta_{z,k+1} + \eta_{z,k}) - (\eta_{3,k+1} + \eta_{3,k})) \right] / (1-3B) \]

At the End, j=n

\[ \eta_{n,j+1} = ((1-3B)\eta_{n,j} + A + B(4(\eta_{n,j-1,k+1} + \eta_{n,j-1,k}) \\
\left. - (\eta_{n,j-2,k+1} + \eta_{n,j-2,k}) \right) / (1+3B) \]
APPENDIX B

Experimental Data
TABLE B-1
Experimental Jet Velocity Data

<table>
<thead>
<tr>
<th>Nozzle ID (mm)</th>
<th>Pressure (psi)</th>
<th>Flow rate (ml/s)</th>
<th>Velocity (m/s)</th>
<th>Reynolds Number</th>
<th>Weber Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>5.04</td>
<td>.0616</td>
<td>1.26</td>
<td>314</td>
<td>5.45</td>
</tr>
<tr>
<td>0.25</td>
<td>10.0</td>
<td>.124</td>
<td>2.53</td>
<td>631</td>
<td>22.0</td>
</tr>
<tr>
<td>0.25</td>
<td>15.0</td>
<td>.181</td>
<td>3.69</td>
<td>920</td>
<td>46.8</td>
</tr>
<tr>
<td>0.25</td>
<td>20.0</td>
<td>.230</td>
<td>4.68</td>
<td>1170</td>
<td>75.2</td>
</tr>
<tr>
<td>0.25</td>
<td>25.0</td>
<td>.273</td>
<td>5.56</td>
<td>1390</td>
<td>106.</td>
</tr>
<tr>
<td>0.25</td>
<td>30.0</td>
<td>.314</td>
<td>6.40</td>
<td>1600</td>
<td>141.</td>
</tr>
<tr>
<td>0.25</td>
<td>35.0</td>
<td>.358</td>
<td>7.28</td>
<td>1810</td>
<td>182.</td>
</tr>
</tbody>
</table>

Flowrate = 0.08213√Pressure - 0.1367
Velocity = 20.37*Flowrate
Reynolds Number = 249.3*Velocity
Weber Number = 3.434*(Velocity)²

<table>
<thead>
<tr>
<th>Nozzle ID (mm)</th>
<th>Pressure (psi)</th>
<th>Flow rate (ml/s)</th>
<th>Velocity (m/s)</th>
<th>Reynolds Number</th>
<th>Weber Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.51</td>
<td>3.03</td>
<td>.253</td>
<td>1.24</td>
<td>631</td>
<td>11.3</td>
</tr>
<tr>
<td>0.51</td>
<td>5.04</td>
<td>.402</td>
<td>1.97</td>
<td>1000</td>
<td>27.2</td>
</tr>
<tr>
<td>0.51</td>
<td>7.03</td>
<td>.543</td>
<td>2.66</td>
<td>1350</td>
<td>49.6</td>
</tr>
<tr>
<td>0.51</td>
<td>9.02</td>
<td>.653</td>
<td>3.20</td>
<td>1630</td>
<td>71.7</td>
</tr>
<tr>
<td>0.51</td>
<td>9.50</td>
<td>.673</td>
<td>3.29</td>
<td>1670</td>
<td>75.8</td>
</tr>
<tr>
<td>0.51</td>
<td>11.2</td>
<td>.761</td>
<td>3.73</td>
<td>1900</td>
<td>97.5</td>
</tr>
</tbody>
</table>

Flowrate = 0.3151√Pressure - 0.2959
Velocity = 4.895*Flowrate
Reynolds Number = 508.6*Velocity
Weber Number = 7.349*(Velocity)²
### TABLE B-2
Experimental Jet Length Data

<table>
<thead>
<tr>
<th>Nozzle ID (mm)</th>
<th>Pressure (psi)</th>
<th>Velocity (m/s)</th>
<th>Jet Length (mm)</th>
<th>Reynolds Number</th>
<th>Weber Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>5.30</td>
<td>1.07</td>
<td>5.0</td>
<td>266</td>
<td>3.91</td>
</tr>
<tr>
<td>0.25</td>
<td>7.66</td>
<td>1.85</td>
<td>11.0</td>
<td>460</td>
<td>11.7</td>
</tr>
<tr>
<td>0.25</td>
<td>9.72</td>
<td>2.43</td>
<td>15.0</td>
<td>606</td>
<td>20.3</td>
</tr>
<tr>
<td>0.25</td>
<td>11.6</td>
<td>2.91</td>
<td>19.0</td>
<td>726</td>
<td>29.1</td>
</tr>
<tr>
<td>0.25</td>
<td>13.6</td>
<td>3.38</td>
<td>22.0</td>
<td>844</td>
<td>39.3</td>
</tr>
<tr>
<td>0.25</td>
<td>15.0</td>
<td>3.69</td>
<td>25.0</td>
<td>921</td>
<td>46.9</td>
</tr>
<tr>
<td>0.25</td>
<td>16.8</td>
<td>4.07</td>
<td>27.0</td>
<td>1015</td>
<td>56.9</td>
</tr>
<tr>
<td>0.25</td>
<td>18.2</td>
<td>4.35</td>
<td>29.0</td>
<td>1090</td>
<td>65.0</td>
</tr>
<tr>
<td>0.25</td>
<td>20.2</td>
<td>4.73</td>
<td>31.0</td>
<td>1180</td>
<td>77.0</td>
</tr>
<tr>
<td>0.25</td>
<td>22.5</td>
<td>5.15</td>
<td>30.0</td>
<td>1280</td>
<td>91.1</td>
</tr>
<tr>
<td>0.25</td>
<td>25.0</td>
<td>5.58</td>
<td>29.0</td>
<td>1390</td>
<td>107.</td>
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<tr>
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<td>30.0</td>
<td>6.38</td>
<td>25.0</td>
<td>1590</td>
<td>140.</td>
</tr>
<tr>
<td>0.25</td>
<td>35.1</td>
<td>7.12</td>
<td>19.0</td>
<td>1780</td>
<td>174.</td>
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<tr>
<td>0.25</td>
<td>44.9</td>
<td>8.43</td>
<td>16.0</td>
<td>2100</td>
<td>244.</td>
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<tr>
<td>0.25</td>
<td>49.1</td>
<td>8.94</td>
<td>17.0</td>
<td>2230</td>
<td>274.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nozzle ID (mm)</th>
<th>Pressure (psi)</th>
<th>Velocity (m/s)</th>
<th>Jet Length (mm)</th>
<th>Reynolds Number</th>
<th>Weber Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.51</td>
<td>1.67</td>
<td>.541</td>
<td>10.0</td>
<td>275</td>
<td>2.15</td>
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<tr>
<td>0.51</td>
<td>2.05</td>
<td>.760</td>
<td>15.0</td>
<td>387</td>
<td>4.24</td>
</tr>
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<td>2.28</td>
<td>.881</td>
<td>20.0</td>
<td>448</td>
<td>5.70</td>
</tr>
<tr>
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<td>2.73</td>
<td>1.10</td>
<td>28.0</td>
<td>559</td>
<td>8.89</td>
</tr>
<tr>
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<td>3.19</td>
<td>1.31</td>
<td>35.0</td>
<td>664</td>
<td>12.5</td>
</tr>
<tr>
<td>0.51</td>
<td>4.19</td>
<td>1.71</td>
<td>45.0</td>
<td>869</td>
<td>21.5</td>
</tr>
<tr>
<td>0.51</td>
<td>5.05</td>
<td>2.02</td>
<td>52.0</td>
<td>1030</td>
<td>29.9</td>
</tr>
<tr>
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<td>57.0</td>
<td>1160</td>
<td>38.4</td>
</tr>
<tr>
<td>0.51</td>
<td>6.57</td>
<td>2.51</td>
<td>57.0</td>
<td>1270</td>
<td>46.1</td>
</tr>
<tr>
<td>0.51</td>
<td>7.46</td>
<td>2.76</td>
<td>55.0</td>
<td>1410</td>
<td>56.2</td>
</tr>
<tr>
<td>0.51</td>
<td>8.46</td>
<td>3.04</td>
<td>36.0</td>
<td>1550</td>
<td>67.8</td>
</tr>
<tr>
<td>0.51</td>
<td>9.51</td>
<td>3.31</td>
<td>29.0</td>
<td>1680</td>
<td>80.4</td>
</tr>
<tr>
<td>0.51</td>
<td>11.7</td>
<td>3.83</td>
<td>27.0</td>
<td>1950</td>
<td>108.</td>
</tr>
<tr>
<td>0.51</td>
<td>12.8</td>
<td>4.07</td>
<td>28.0</td>
<td>2070</td>
<td>122.</td>
</tr>
<tr>
<td>0.51</td>
<td>13.8</td>
<td>4.28</td>
<td>31.0</td>
<td>2180</td>
<td>135.</td>
</tr>
</tbody>
</table>

Note: The velocities, Reynolds number, and Weber number were calculated from the correlation equations given in Table 2.
TABLE B-3
Experimental Drop Size and Drop Spacing Data

<table>
<thead>
<tr>
<th>Nozzle ID (mm)</th>
<th>Pressure (psi)</th>
<th>Velocity (m/s)</th>
<th>Drop Size (mm)</th>
<th>Drop Spacing (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>5.0</td>
<td>0.956</td>
<td>0.475</td>
<td>2.15</td>
</tr>
<tr>
<td>0.25</td>
<td>10.</td>
<td>2.51</td>
<td>0.418</td>
<td>1.65</td>
</tr>
<tr>
<td>0.25</td>
<td>20.</td>
<td>4.70</td>
<td>0.348</td>
<td>1.10</td>
</tr>
<tr>
<td>0.25</td>
<td>30.</td>
<td>6.38</td>
<td>0.301</td>
<td>0.743</td>
</tr>
<tr>
<td>0.51</td>
<td>2.5</td>
<td>0.990</td>
<td>0.867</td>
<td>1.94</td>
</tr>
<tr>
<td>0.51</td>
<td>5.0</td>
<td>2.00</td>
<td>0.750</td>
<td>1.39</td>
</tr>
<tr>
<td>0.51</td>
<td>7.5</td>
<td>2.76</td>
<td>0.661</td>
<td>0.947</td>
</tr>
<tr>
<td>0.51</td>
<td>10.</td>
<td>3.43</td>
<td>0.612</td>
<td>0.683</td>
</tr>
</tbody>
</table>

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APPENDIX C

The Simulation Program
THIS IS THE MAIN ROUTINE USED IN SOLVING THE TIME VARYING AXIAL-SYMMETRIC JET PROBLEM WITH AN AXIAL SURFACE DISTURBANCE. THE DISTURBANCE MAY BE EITHER
1) A STANDING WAVE BY SPECIFYING THE INITIAL DISTURBANCE AMPLITUDE (AMPLO) AND THE DISTURBANCE WAVE LENGTH (WAVLEN)
2) A PROPAGATING WAVE BY SPECIFYING AMPLO AND THE DISTURBANCE FREQUENCY (FREQ)

THIS ROUTINE READS AND PRINT THE INPUT DATA, AND CALLS ALL OF THE ROUTINES THAT ARE USED IN THE SOLUTION, AS WELL AS THE ROUTINES THAT ARE USED IN THE GRAPHICAL OUTPUT.

THE SUBROUTINES THAT ARE CALLED ARE:
- INITIAL-USED TO SETUP THE INITIAL JET VELOCITY PROFILES AT TIME= 0
- VELOCT(ENTRY POINT VELOC)-USED TO SOLVE THE TIME DEPENDENT, R AND Z MOMENTUM EQUATIONS WITH FREE JET SURFACE BOUNDARY CONDITIONS
- RADIUS(ENTRY POINT RADIU) USED TO CALCULATE THE POSITION OF THE JET SURFACE FROM THE KINEMATIC BOUNDARY CONDITION
- PLOTLK(ENTRY POINT PLOTL)-USED TO PLOT THE VELOCITY AND SURFACE PROFILES AT TIME T.

THE INPUT DATA ARE ENTERED THROUGH A READ(6,INPUT) WHERE INPUT IS THE NAME OF THE NAMELIST THAT CONTAINS THE NAMES OF THE INPUT VARIABLES.

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>DESCRIPTION</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>NI</td>
<td>NUMBER OF RADIAL GRID POINTS</td>
<td></td>
</tr>
<tr>
<td>NJ</td>
<td>NUMBER OF AXIAL GRID POINTS</td>
<td></td>
</tr>
<tr>
<td>NK</td>
<td>NUMBER OF TEMPORAL GRID POINTS</td>
<td></td>
</tr>
<tr>
<td>RMAX</td>
<td>RADIUS OF THE UNDISTURBED JET</td>
<td>M</td>
</tr>
<tr>
<td>ZMAX</td>
<td>JET LENGTH TO EXAMIN</td>
<td>M</td>
</tr>
<tr>
<td>DELT</td>
<td>TIME STEP SIZE</td>
<td>SEC</td>
</tr>
<tr>
<td>AMPLO</td>
<td>INITIAL DISTURBANCE AMPLITUDE</td>
<td>M</td>
</tr>
<tr>
<td>WAVLEN</td>
<td>WAVE LENGTH OF THE DISTURBANCE</td>
<td>M</td>
</tr>
<tr>
<td>FREQ</td>
<td>FREQUENCY OF THE DISTURBANCE</td>
<td>1/SEC</td>
</tr>
<tr>
<td>VAVG</td>
<td>AVERAGE VELOCITY AT THE EXIT</td>
<td>M/SEC</td>
</tr>
<tr>
<td>VISC</td>
<td>VISCOSITY OF THE JET FLUID</td>
<td>PA*S</td>
</tr>
<tr>
<td>SURF</td>
<td>INTERFACIAL SURFACE TENSION</td>
<td>NT/M</td>
</tr>
<tr>
<td>RHO</td>
<td>DENSITY OF THE JET FLUID</td>
<td>KG/M³</td>
</tr>
<tr>
<td>VTOL</td>
<td>CONVERGENCE TOLERANCE FOR VELOCITY</td>
<td></td>
</tr>
<tr>
<td>RTOL</td>
<td>CONVERGENCE TOLERANCE FOR RADIUS</td>
<td></td>
</tr>
<tr>
<td>VRW</td>
<td>S.O.R. PARAMETER FOR R-MOM. EQN.</td>
<td></td>
</tr>
<tr>
<td>VZW</td>
<td>S.O.R. PARAMETER FOR Z-MOM. EQN.</td>
<td></td>
</tr>
<tr>
<td>RW</td>
<td>S.O.R. PARAMETER FOR KINEMATIC B.C.</td>
<td></td>
</tr>
<tr>
<td>ITMAX</td>
<td>MAXIMUM NUMBER OF ITERATIONS</td>
<td></td>
</tr>
<tr>
<td>DTMIN</td>
<td>MINIMUM VALUE FOR TIME STEP SIZE</td>
<td>SEC</td>
</tr>
<tr>
<td>IDBUGM</td>
<td>DEBUG OPTION FOR THE MAIN ROUTINE</td>
<td></td>
</tr>
<tr>
<td>IDBUGV</td>
<td>DEBUG OPTION FOR ROUTINE VELOCT</td>
<td></td>
</tr>
</tbody>
</table>
IDBUGR  DEBUG OPTION FOR ROUTINE RADIUS
RESTRT  RESTART FLAG (T OR F)
SAVE    SAVE FLAG (T OR F)

THE DEBUG OPTIONS ARE:
0= DEBUG OFF
1= PRINT ALL VALUES AT EVERY ITERATION
2= CALL PLOT ROUTINE AT EVERY ITERATION
3= PRINT OUT CONVERGENCE INFORMATION AT EVERY ITERATION
   (IF THE NUMBER OF ITERATIONS IN VELOCT OR RADIUS
   EXCEEDS 10, THE THE CORRESPONDING OPTION 3 IS USED
   UNTIL THE ROUTINE CONVERGES OR EXCEEDS THE MAXIMUM
   NUMBER OF ITERATIONS, AT WHICH POINT THE ROUTINE
   STOPS)

IF SAVE=T, THEN ALL OF THE NECESSARY VALUES NEEDED TO
RESTART THE PROGRAM AT THE LAST TIME STEP ARE SAVED ON
I/O UNITS 8 AND 10. THIS ALLOWS THE PROGRAM TO BE RUN
IN A SERIES OF SHORT JOBS, INSTEAD OF ONE LONG JOB.
THE INFORMATION THAT WILL BE REQUIRED TO PRODUCE THE
THE GRAPHICAL OUTPUT WILL BE SAVED ON I/O UNIT 9; THESE
NUMBER MAY THEN BE READ BY ROUTINE "MPLTO" TO PRODUCE
THE DESIRED PLOTS AT ANY TIME AFTER THE PROGRAM HAS
FINISHED.
(FOR IBM THE DATASETS THAT ARE CONNECTED TO THESE FILE
NUMBERS SHOULD HAVE THE FOLLOWING DCB ATTRIBUTES:
RECFM=FB,LRECL=3600,BLKSIZ=3600,DSORG=PS; THE DATASET
DISPOSITION MUST BE SET EQUAL TO "SHR")

IF SAVE=F, THEN NO VALUES WILL BE SAVED AND THE GRAPHICAL
OUTPUT WILL BE PRODUCED WHILE THE PROGRAM IS RUNNING,
EACH PLOT WILL CONSIST OF THE PROFILES FOR 10 TIME
STEPS, THE FIRST PROFILE IN THE GROUP WILL BE PLOTTED
WITH THE LETTER A, THE LAST PROFILE WILL BE PLOTTED
WITH THE LETTER J

IF RESTRT=T, THEN THE PROGRAM WILL READ ALL OF THE VALUES
NEEDED TO RESTART THE PROGRAM AT THE PREVIOUS TIME STEP
FROM UNIT 8 AS SAVED BY A PREVIOUS RUN WITH THE OPTION
SAVE=T. (IF AN ERROR OCCURS IN READING FROM UNIT 8,
THE PROGRAM AUTOMATICALLY SWITCHES TO UNIT 10, THIS
AVOIDS THE SITUATION WHERE THE PROGRAM MIGHT NOT
COMPLETE WRITING TO UNIT 8 BECAUSE OF AN ABNORMAL
TERMINATION)
(THE DATASET DISPOSITION FOR THE DATASET CONNECTED TO
UNIT 9 MUST BE "MOD")

IF RESTRT=F, THEN THE PROGRAM USES THE INFORMATION READ
BY "INPUT" TO START THE SIMULATION AT TIME=0
(THE DATASET DISPOSITION FOR THE DATASET CONNECTED TO
UNIT 9 MUST BE "SHR")

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C NOTE: WAVLEN AND FREQ ARE MUTUALLY EXCLUSIVE
C
C---------------------------------------------------------------
REAL*4 R,Z,FLOAT
LOGICAL RESTRT, SAVE, SKIP
C---------------------------------------------------------------
DIMENSION VR(52,151), VZ(52,151), ACCVR(151), R(154),
& OLDVR(52,151), OLDVZ(52,151), ACCVZ(151), Z(154),
& AMPL(151), OLDA(151), DELR(151)
C
C NROWS MUST BE EQUAL TO THE NUMBER OF ROWS IN THE VELOCITY
C ARRAYS
C NCOLS MUST BE EQUAL TO THE NUMBER OF COLUMNS IN THE
C VELOCITY ARRAYS
DATA NROWS/52/, NCOLS/151/
C---------------------------------------------------------------
COMMON /JETF/ DELZ, DELT, TIME, AMPLO, VRW,
& VZW, VAVG, VISC, VISCK, SURF, RMAX, ZMAX,
& RHO, WAVNUM, WAVLEN, VTOL, RTOL, RW, FREQ
C---------------------------------------------------------------
COMMON /JETI/ NI, NIP1, NIL, NJ, NJ1,
& NK, IFLAG, IDBUGV, IDBUGR, RESTRT, SAVE
C---------------------------------------------------------------
NAMELIST /INPUT/ NI, NJ, NK, RMAX, ZMAX,
& VRW, VTOL, AMPLO, VAVG, VISC, SURF, RHO,
& VZW, RTOL, WAVLEN, DELT, RESTRT, SAVE, EXPFAC,
& RW, IDBUGM, IDBUGV, IDBUGR, FREQ, IFLAG, DTMIN
C
C SETUP THE INITIAL DEFAULT VALUES FOR THE VARIABLES IN
C NAMELIST "INPUT"
C
RESTRT= .FALSE.
SAVE= .FALSE.
NI= 51
NJ= 51
NK= 100
IDBUGM= 0
IDBUGV= 0
IDBUGR= 0
RMAX= 1.D-3
ZMAX= 1.D-1
DTMIN= 1.D-5
VTOL= 1.D-3
RTOL= 1.D-3
VAVG= .1D-0
VISC= 1.D-6
RHO= 1.D3
AMPLO= 0.DO
EXPFAC= 0.DO
VRW= 1.D0
VZW= 1.D0
RW= 1.D0

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FREQ = 0. DO
ITMAX = 100

READ(5, INPUT)
WAVNUM = 1. D30
IF(WAVLEN .GT. 1. D30) WAVLEN = 1. D30
IF(WAVLEN .NE. 0. DO) WAVNUM = 6. 283 192 D0/WAVLEN
IF(WAVLEN .EQ. 1. D30) WAVNUM = 0.
IF(FREQ*WAVNUM .EQ. 0. DO) GO TO 20
WRITE(6, 10)
10 FORMAT(' ***** FREQ AND WAVNUM ARE MUTUALLY EXCLUSIVE',
       & ' E ither set FREQ equal to zero, or set',
       & ' WAVLEN to greater than 1.530')
STOP
20 IF(NI .GT. NROWS-1) NI = NROWS-1
IF(NJ .GT. NCOLS) NJ = NCOLS
NI1 = NI-1
NIP1 = NI+1
NJ1 = NJ-1
NK1 = NK-1
ANI1 = FLOAT(NI1)
DELZ = ZMAX/FLOAT(NJ1)
DR = RMAX/FLOAT(NI1)
VISCK = VISC/RHO
WRITE(6, 30) NI, NJ, NK, DR, DELZ,
       & DELT, RMAX, ZMAX, VAVG, VISC, WAVLEN,
       & VTOL, VISCK, AMPLO, RTOL, RHO, WAVNUM,
       & VZW, SURF, FREQ, VRW, IDBUGV, IDBUGR,
       & RW, ITMAX, RESTRT, SAVE
30 FORMAT(' THE INPUT DATA IS'/
       & ' NI=', I12, ', NJ=', I12, ', NK=', I12/
       & ' DELR=', E12.4, ', DELZ=', E12.4, ', DELT=', E12.4/
       & ' RMAX=', E12.4, ', ZMAX=', E12.4, ', VAVG=', E12.4/
       & ' VISC=', E12.4, ', WAVLEN=', E12.4, ', VTOL=', E12.4/
       & ' VISCK=', E12.4, ', AMPLO=', E12.4, ', RTOL=', E12.4/
       & ' RHO=', E12.4, ', WAVNUM=', E12.4, ', VZW=', E12.4/
       & ' SURF=', E12.4, ', IDBUGV=', I12, ', IDBUGR=', I12,
       & ' RW=', E12.4, ', ITMAX=', I12, ', RESTRT=', I12,
       & ' SAVE=', I12)

DIAM= 2. DO*RMAX
RE= DIAM*VAVG/VISCK
WE= 1. D30
IF(SURF .NE. 0.) WE= RHO VAVG VAVG*DIAM/SURF
OHN= 1. D30
IF(SURF .NE. 0.) OHN= VISC/DSQRT(RHO*SURF*DIAM)
WRITE(6, 40) RE, WE, OHN, WAVNUM
40 FORMAT(' THE REYNOLDS NUMBER IS ', 1PE12.4/
       & ' THE WEBER NUMBER IS ', E12.4/
       & ' THE OHNESORGE NUMBER IS ', E12.4/
       & ' THE WAVE NUMBER IS ', E12.4)

DO 50 J=1, NJ
50 DELR(J)= DR
INDB= 8
K0= 1

C------------------------------------------------------------------------
IF(RESTART) GO TO 80
TIME= 0.

C CALL INITIAL(VR, VZ, NROWS, DELR, AMPL, EXPFAC)

C IF(.NOT. SAVE) GO TO 90
REWIN 9
WRITE(9,180) DELZ, DELT, AMPLO, VRW, VZW, VAVG,
& VISC, VISCK, SURF, RMK, ZMAX, ZHOU, WAVNUM,
& WAVLEN, VTOE, RTOL, RW, FREQ, RESTART, SAVE,
& NI, NIP1, NI1, NJ, NJ1, NK, ITMAX,
& IFLAG, IDBUGV, IDBUGR
GO TO 90
C------------------------------------------------------------------------

60 WRITE(6,70)
70 FORMAT(’***ERROR IN READING FROM UNIT 8, SWITCHING’,
& ’ TO UNIT 10 ***’)
INDB= 10

80 CONTINUE
READ(INDB,180,END=60) KO, TIME, DUMMY, M, N,(DELR(J),
& AMPL(J), ACCVR(J), ACCVZ(J),
& (VR(I,J), VZ(I,J),I=1,M),J=1,N)
IF(DELT .GT. O.DO) DELT= DUMMY
DELT= DABS(DELT)
NIP1= M
NI= M - 1
NI1= NI - 1
NJ= N
NJ1= NJ - 1

C------------------------------------------------------------------------
90 IF(IDBUGM .NE. 1 ) GO TO 160
WRITE(6,100)
100 FORMAT(’INITIAL BOUNDARY CONDITIONS’/)
WRITE(6,110) (DELR(J),J=1,NJ)
110 FORMAT(’ DELR=’,1P8E15.7)
DO 120 I=1,NI
120 WRITE(6,130) (VR(I,J), J=1,NJ)
130 FORMAT(’ VR=’,1P8E15.7)
DO 140 I=1,NI
140 WRITE(6,150) (VZ(I,J), J=1,NJ)
150 FORMAT(’ VZ=’,1P8E15.7)

C------------------------------------------------------------------------
160 CONTINUE

C CALL EACH SUBROUTINE TO PERFORM INITIALIZATION CHORES

C CALL VELOCT(VR, OLDVR, VZ, OLDVZ, NROWS, DELR, AMPL, OLDA)
CALL RADIUS(VR,OLDVR,VZ,OLDVZ,NROWS,DELR,AMPL,OLDA)
DO NOT CALL PLOTTING Routines IF SAVE WAS SPECIFIED

INTEGRATION LOOP STARTS HERE

IF(.NOT. SAVE) CALL PLOTLK(VR,VZ,NROWS,AMPL,DELR,R,Z)

INTEGRATION LOOP STARTS HERE

SKIP= RESTRT
DO 270 K=K0,NK
C PUT VALUES FROM PREVIOUS TIME STEP INTO OLDA, OLDVR, OLDVZ
IF(.NOT. SAVE) CALL PLOTL
C PUT VALUES FROM PREVIOUS TIME STEP INTO OLDA, OLDVR, OLDVZ
DO 170 J=1,NJ
AMP= AMPL(J)
OLDA(J)= AMP
DELR(J)= (RMAX + AMP)/ANI1
DO 170 I=1,NIP1
OLDVR(I,J)= VR(I,J)
OLDVZ(I,J)= VZ(I,J)
170 CONTINUE
IF(DELT .LT. DTMIN) GO TO 280
IF(.NOT. SAVE .OR. SKIP) GO TO 200
SAVE INTERMEDIATE VALUES
REWIND 8
WRITE(8,180) K, TIME, DELT, NIP1, NJ, (DELR(J),
& AMPL(J), ACCVR(J), ACCVZ(J),
& (VR(I,J), VZ(I,J),I=1,NIP1), J=1,NJ)
REWIND 10
WRITE(10,180) K, TIME, DELT, NIP1, NJ, (DELR(J),
& AMPL(J), ACCVR(J), ACCVZ(J),
& (VR(I,J), VZ(I,J),I=1,NIP1), J=1,NJ)
SAVE VALUES TO BE USED LATER FOR PLOTTING
WRITE(9,180) TIME,NI,NJ,(DELR(J),AMPL(J),VR(NI,J),
& VZ(NI,J),J=1,NJ),(VZ(I,NJ),I=1,NI)
180 FORMAT(9(50A8))
WRITE(6,190) K
190 FORMAT(' INTERMEDIATE VALUES WERE SAVED FOR TIME STEP',
& I4)
200 KFLAG = 0
CALL VELOC(KFLAG)
KFLAG = -1--VELOCITIES ARE TOO LARGE, GO TO ACCEL. STEP
KFLAG = 0--VELOC CONVERGED
KFLAG = 1--REDUCE STEP SIZE
IF (KFLAG) 280, 240, 210
210 DELT = DELT/2.
WRITE (6, 220) DELT
220 FORMAT ('
REDUCE STEP SIZE, DELT=', 1PE12.4)
DO 230 J = 1, NJ
AMPL(J) = OLD(A(J)
DO 230 I = 1, NIP
VR(I, J) = OLDVR(I, J)
230 VZ(I, J) = OLDVZ(I, J)
IF (DELT .LT. DTMIN) GO TO 280
GO TO 200
MFLAG = 0
CALL RADIU(MFLAG)
MFLAG = -1--PINCH OFF HAS OCCURRED
MFLAG = 0--RADIU CONVERGED
MFLAG = 1--REDUCE STEP SIZE
IF (MFLAG .GT. 0) GO TO 210
IF (MFLAG .LT. 0) GO TO 330
TIME = TIME + DELT
WRITE (6, 250) TIME
250 FORMAT (' ', TIME = ', 1PE12.4)
SKIP = .FALSE.
COMPUTE ACCELERATION VALUES AT THE SURFACE
DO 260 J = 1, NJ
ACCVR(J) = (VR(NI, J) - OLDVR(NI, J))/DELT
260 ACCVZ(J) = (VZ(NI, J) - OLDVZ(NI, J))/DELT
270 CONTINUE
MFLAG = -1
GO TO 330
MFLAG = 0
GO TO 300
ACCELERATION STEP
280 CONTINUE
WRITE (6, 290)
290 FORMAT (' ', / BEGIN ACCELERATION STEP | ', 1PE12.4)
300 K = K + 1
TIME = TIME + DELT
IF (.NOT. SAVE) GO TO 310
REWIND 8
WRITE(8,180) K, TIME, DELT, NIP1, NJ, (DELR(J),
AMPL(J), ACCVR(J), ACCVZ(J),
(VR(I,J), VZ(I,J), I=1,NIP1), J=1,NJ)
REWIND 10
WRITE(10,180) K, TIME, DELT, NIP1, NJ, (DELR(J),
AMPL(J), ACCVR(J), ACCVZ(J),
(VR(I,J), VZ(I,J), I=1,NIP1), J=1,NJ)
WRITE(9,180) TIME, NI, NJ, (DELR(J), AMPL(J), VR(NI,J),
VZ(NI,J), J=1,NJ), (VZ(I,NJ), I=1,NI)
C-----------------------------------------------
310 DO 320 J=1,NJ
  OLDVR(NI,J)= VR(NI,J)
  OLDVZ(NI,J)= VZ(NI,J)
  VR(NI,J)= ACCVR(J)*DELT + OLDVR(NI,J)
  VZ(NI,J)= ACCVZ(J)*DELT + OLDVZ(NI,J)
  AMP= AMPL(J)
  DELR(J)= (RMAX+AMP)/ANI1
  OLDA(J)= AMP
  MFLAG= 0
  CALL RADIU(MFLAG)
330 IF(.NOT. SAVE) CALL PLOTL
  IF(MFLAG) 360,300,340
340 DELT= DELT/2.DO
  DO 350 J=1,NJ
    VR(NI,J)= OLDVR(NI,J)
    VZ(NI,J)= OLDVZ(NI,J)
    AMPL(J)= OLDA(J)
    MFLAG= 0
  CONTINUE
  GO TO 310
C-----------------------------------------------
360 IF(.NOT. SAVE) CALL PLOTT
  WRITE(6,370)
370 FORMAT(' JETCALC IS FINISHED')
  STOP 10
END
SUBROUTINE VELOCT

(C  THIS ROUTINE SOLVES THE R- AND Z- COMPONENTS OF
C  THE JET FLUID VELOCITY BY INTEGRATING THE R AND Z
C  MOMENTUM EQUATIONS SIMULTANEOUSLY IN A 2 SPACE DIMENSION
C  AND TIME (R,Z,T)
C  SINCE THE PROBLEM IS A BOUNDARY VALUE PROBLEM IN THE R-
C  DIRECTION, THE SOLUTION REQUIRES A TRIAL-AND-ERROR
C  INTEGRATION TECHNIQUE
C  THE METHOD USED IN THIS PROGRAM IS A CRANK-NICHOLSON
C  TYPE APPROXIMATION TO THE SPACIAL DERIVATIVES AT THE
C  IMAGINARY POINT T+0.5*DT; THE ACTUAL INTEGRATION IS
C  PERFORMED WITH GAUSS-SEIDEL ITERATION. THIS ENSURES
C  THAT ALL OF THE BOUNDARY CONDITIONS WILL BE MET.
C
C  IN THIS ROUTINE I=1 CORRESPONDS TO THE CENTERLINE
C  J=1 CORRESPONDS TO THE NOZZLE EXIT
C  I=NI CORRESPONDS TO THE SURFACE
C  J=NJ CORRESPONDS TO SOME DISTANCE
C  DOWNSTREAM
C
C  NI= NUMBER OF POINTS IN THE R-(I-) DIRECTION
C  NJ= NUMBER OF POINTS IN THE Z-(J-) DIRECTION
C
C IMPLICIT REAL*8 (A-H,O-Z)
REAL*4 FLOAT
LOGICAL RESTRT, SAVE

DIMENSION VR(NROWS,1), VZ(NROWS,1), AMPL(1),
  OLDVR(NROWS,1),OLDVZ(NROWS,1), OLDA(1),DELRI)

COMMON /JETF/ DELZ, DELT, TIME, AMPLO, VRW,
    & VZW, VAVG, VISC, VISCK, SURF, RMAX, ZMAX,
    & RHO, WAVNUM, WAVLEN, VTOL, RTOL, RW, FREQ

COMMON /JETI/ NI, NIP1, NJ, N11, NJ, NJ1,
    & NK, ITMAX, IFLAG, IDBUGV, IDBUGR, RESTRT, SAVE

COMMON /CTRCOM/ DELVR, DELVZ, I, J, IM1, IP1, JM1, JP1, L

C  INITIALIZE VELOCT
C
VELMAX= 50.DO
J0= 2
I0= 1
DZ= DELZ
CALL TRAPVE(DELR)
GO TO 310

C ENTRY VELOC(KFLAG)
KOUNT= 0
IDBUG2= 0
10 JFLAG= 0
    IR= 1
    JR= 1
    BIGR= 0.DO
    IZ= 1
    JZ= 1
    BIGZ= 0.DO
C---------------------------------------------------------------------
    DO 150 J=JO,NJ
      JP1= J+1
      JM1= J-1
      DR= DELR(J)
      DRZ= DR/DZ
C---------------------------------------------------------------------
    DO 150 I=IO,NI
    VRSTAR= VR(I,J)
    WRSTAR= (1.DO-VRW)*VRSTAR
    VZSTAR= VZ(I,J)
    WZSTAR= (1.DO-VZW)*VZSTAR
    OVR= OLDVR(I,J)
    OVZ= OLDVZ(I,J)
C---------------------------------------------------------------------
    IF(J .EQ. 1 ) GO TO 170
      IP1= I+1
      IM1= I-1
      IM2= I-2
      R= DR
      IF(IM1 .NE. 0) R= FLOAT(IM1)*DR
    RDR= 1.D0/(R*DR)
    RDZ= 1.D0/(R*DZ)
C---------------------------------------------------------------------
C  A THROUGH G ARE DIMENSIONLESS
C    C= VISCK*RDR*DELT
C    A= C/DR*R
C    B= C/R*DR
C    C= .5D0*C
C    CZ= C/DZ*DR
C    D= .25D0*(VRSTAR+OVR)/DR*DELT
C    E= VISCK/(DZ*DZ)*DELT
C    F= .25D0*(VZSTAR+OVZ)/DZ*DELT
C    GZ= 2.DO*(A+E)
C    GR= GZ + B
C  H IS IN M/S
H= .5D0*SURF/VISC
C---------------------------------------------------------------------
C FOR CENTERLINE POINTS GO TO 910
C IF(I .EQ. 1 ) GO TO 910
C FOR END POINTS GO TO 31
C IF(J .EQ. NJ) GO TO 31
C FOR SURFACE POINTS GO TO 900
C IF(I .EQ. NI) GO TO 900
C---------------------------------------------------------------------
C C EVALUATE THE VELOCITIES IN THE INTERIOR
KF = 0
CALL INTERP(VR,NROWS,DELR,I,J,NIP1,NJ,-1,VRM1,KF)

C IF NO POINT AT J-1, THEN GO TO 50
IF(KF .NE. 0) GO TO 50
CALL INTERP(OLDVR,NROWS,DELR,I,J,NIP1,NJ,-1,OLVRM1,KF)
CALL INTERP(VZ,NROWS,DELR,I,J,NIP1,NJ,-1,VZM1,KF)
CALL INTERP(OLDVZ,NROWS,DELR,I,J,NIP1,NJ,-1,OLVZM1,KF)
CALL INTERP(VR,NROWS,DELR,I,J,NIP1,NJ,1,VRP1,KF)

IF NO POINT AT J+1, THEN GO TO 31
IF(KF .NE. 0) GO TO 31
CALL INTERP(OLDVR,NROWS,DELR,I,J,NIP1,NJ,1,OLVRP1,KF)
CALL INTERP(VZ,NROWS,DELR,I,J,NIP1,NJ,1,VZP1,KF)
CALL INTERP(OLDVZ,NROWS,DELR,I,J,NIP1,NJ,1,OLVZP1,KF)

DELVR= \begin{align*}
& \frac{(2 \cdot 0.0 - GR) \cdot OVR + (A + C - D) \cdot (VR(IP1,J) + OLDVR(IP1,J)) + (A - C + D) \cdot (VR(IM1,J) + OLDVR(IM1,J)) + (E - F) \cdot (VRP1 + OLVRP1) + (E + F) \cdot (VRM1 + OLVRM1)}{2 \cdot 0.0 + GR} \\
& \frac{(2 \cdot 0.0 - GZ) \cdot OVZ + (A + C - D) \cdot (VZ(IP1,J) + OLDVZ(IP1,J)) + (A - C + D) \cdot (VZ(IM1,J) + OLDVZ(IM1,J)) + (E - F) \cdot (VZP1 + OLVZP1) + (E + F) \cdot (VZM1 + OLVZM1)}{2 \cdot 0.0 + GZ} \end{align*}

GO TO 120

C------------------------------------------------------------------------------------------------------------------
C EVALUATE THE VELOCITIES AT THE CENTER-LINE (I=1)
C------------------------------------------------------------------------------------------------------------------

910 CONTINUE
IF(J .EQ. NJ) GO TO 920
DELVR= 0.0D0
GZZ= 2.0D0*(E+2.0D0*A)
DELVZ= \begin{align*}
& \frac{(2 \cdot 0.0 - GZZ) \cdot OVZ + 4.0D0 \cdot A \cdot (VZ(IP1,J) + OLDVZ(IP1,J)) + (E - F) \cdot (VZ(I, JP1) + OLDVZ(I, JP1)) + (E + F) \cdot (VZ(I, JM1) + OLDVZ(I, JM1))}{2 \cdot 0.0 + GZZ} \end{align*}

GO TO 120

C------------------------------------------------------------------------------------------------------------------
C I=1, J=NJ
C------------------------------------------------------------------------------------------------------------------

920 CONTINUE
DELVR= 0.0D0
GZZ= 3.0D0*F-E+4.0D0*A
DELVZ= \begin{align*}
& \frac{(2 \cdot 0.0 - GZZ) \cdot OVZ + 4.0D0 \cdot A \cdot (VZ(IP1,J) + OLDVZ(IP1,J)) + (4.0D0*F-2.0D0*E) \cdot (VZ(I, JM1) + OLDVZ(I, JM1)) + (E - F) \cdot (VZ(I, J-2) + OLDVZ(I, J-2))}{2 \cdot 0.0 + GZZ} \end{align*}

GO TO 120

C------------------------------------------------------------------------------------------------------------------
C 2
C EVALUATE THE VELOCITIES AT THE SURFACE (I=NI)

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C

900 CONTINUE
DADZ=.25D0*(AMPL(JP1)+OLDA(JP1)-AMPL(JM1)-OLDA(JM1))
& /DELZ
DADZ2= 0.DO
IF( DABS(DADZ) .GE. 1.D-35) DADZ2= DADZ*DADZ
D2ADZ2= .5D0*( OLDA(JP1)-2.DO*OLDA(J)+OLDA(JM1)
& + AMPL(JP1)-2.DO*AMPL(J)+AMPL(JM1))/(DZ*DZ)
IF( D2ADZ2 .LT. 1.D-35) D2ADZ2= 0.DO
930 CONTINUE

C

A PEAK

KF= 0
CALL INTERP(VR,NROWS,DELR,I,J,NIP1,NJ,-1,VRM1,KF)
IF(KF .NE. 0) GO TO 50
CALL INTERP(OLDVR,NROWS,DELR,I,J,NIP1,NJ,-1,OLVRM1,KF)
CALL INTERP(VZ,NROWS,DELR,I,J,NIP1,NJ,-1,VZM1,KF)
CALL INTERP(OLDVZ,NROWS,DELR,I,J,NIP1,NJ,-1,OLVZM1,KF)
CALL INTERP(VR,NROWS,DELR,I,J,NIP1,NJ,1,VRP1,KF)
IF(KF .NE. 0) GO TO 30
CALL INTERP(OLDVR,NROWS,DELR,I,J,NIP1,NJ,1,OLVRP1,KF)
CALL INTERP(VZ,NROWS,DELR,I,J,NIP1,NJ,1,VZP1,KF)
CALL INTERP(OLDVZ,NROWS,DELR,I,J,NIP1,NJ,1,OLVZP1,KF)
DVZDZ= .25D0*( VZP1 +OLVZP1 -VZM1 -OLVZM1)/DZ
IF( DVZDZ .LT. 1.D-20) DVZDZ= 0.DO
TERM1= (1.D0+DADZ2)
DVRDR= -DVZDZ*DADZ2
& + H*TERM1/RMAX - H*(1.D0/R-D2ADZ2/TERM1)*DSQRT(TERM1)
DVRDR= H*((R-RMAX)/(RMAX*R) + D2ADZ2)
VR(IP1,J)= (4.DO*VRSTAR-VR(IM1,J)+2.DO*DVRDR*DR)/3.DO
GZ= 2.DO*(A+E)
GR= GZ+B
DELVR= ( (2.DO-GR)*OVR
& + (A+C-D)*(VR(IP1,J)+OLDVR(IP1,J))
& + (A-C+D)*(VR(IM1,J)+OLDVR(IM1,J))
& + (E-VRP1+OLVR1)
& + (E-(VRM1+OLVRM1)
& )/(2.DO+GR)
VZ(IP1,J)=(4.DO*VZSTAR-VZ(IM1,J)-(VRP1-VRM1)/DELZ*DR)
& /3.DO
DELVZ= ( (2.DO-GZ)*OVZ
& + (A+C-D)*(VZ(IP1,J)+OLDVZ(IP1,J))
& + (A-C+D)*(VZ(IM1,J)+OLDVZ(IM1,J))
& + (E-F)*(VZP1+OLVZP1)
& + (E+F)*(VZM1+OLVZM1)
& )/(2.DO+GZ)
GO TO 120

C

SINGULAR SURFACE POINT

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20 CONTINUE
   CALL TRACK(20, I, J, .FALSE., 'VELOCT ', 0.D0)
   IF(J .EQ. NJ) GO TO 25
   DELVR = .5D0*(VR(I,JM1)+VR(I,JP1))
   DELVZ = .5D0*(VZ(I,JM1)+VZ(I,JP1))
   GO TO 120
25 CONTINUE
   DELVR = 2.D0*VR(I,JM1)+VR(I,J-2)
   DELVZ = 2.D0*VZ(I, JM1)+VR(I,J-2)
   GO TO 120

C------------------------------------------------------------------
C
C LEADING EDGE
C
------------------------------------------------------------------
30 CONTINUE
   IF(J .EQ. 2) CALL TRACK(30, I, J, .FALSE., 'VELOCT ', 0.D0)
C
C THIS ENTRY POINT IS FOR NON-SURFACE GRID POINTS THAT
C DO NOT HAVE A POINT AT J+1
C
31 CONTINUE
   KF = 0
   CALL INTERP(VR, NROWS, DELR, I, J, NIP1, NJ, -1, VRM1, KF)
   IF(KF .NE. 0) GO TO 20
   CALL INTERP(OLDVR, NROWS, DELR, I, J, NIP1, NJ, -1, OLVRM1, KF)
   CALL INTERP(VZ, NROWS, DELR, I, J, NIP1, NJ, -1, VZM1, KF)
   CALL INTERP(OLDVZ, NROWS, DELR, I, J, NIP1, NJ, -1, OLVZM1, KF)
   IF(J .EQ. 2) GO TO 20
   CALL INTERP(VR, NROWS, DELR, I, J, NIP1, NJ, -2, VRM2, KF)
   IF(KF .NE. 0) GO TO 20
   CALL INTERP(OLDVR, NROWS, DELR, I, J, NIP1, NJ, -2, OLVRM2, KF)
   CALL INTERP(VZ, NROWS, DELR, I, J, NIP1, NJ, -2, VZM2, KF)
   CALL INTERP(OLDVZ, NROWS, DELR, I, J, NIP1, NJ, -2, OLVZM2, KF)
   IF(I .NE. NI) GO TO 35
   CALL INTERP(VR, NROWS, DELR, I, J, NIP1, NJ, -1, VRM1, KF)
   IF(KF .NE. 0) GO TO 20
   CALL INTERP(OLDVR, NROWS, DELR, I, J, NIP1, NJ, -1, OLVRM1, KF)
   CALL INTERP(VZ, NROWS, DELR, I, J, NIP1, NJ, -1, VZM1, KF)
   CALL INTERP(OLDVZ, NROWS, DELR, I, J, NIP1, NJ, -1, OLVZM1, KF)
   IF(J .EQ. 2) GO TO 20
   CALL INTERP(VR, NROWS, DELR, I, J, NIP1, NJ, -2, VRM2, KF)
   IF(KF .NE. 0) GO TO 20
   CALL INTERP(OLDVR, NROWS, DELR, I, J, NIP1, NJ, -2, OLVRM2, KF)
   CALL INTERP(VZ, NROWS, DELR, I, J, NIP1, NJ, -2, VZM2, KF)
   CALL INTERP(OLDVZ, NROWS, DELR, I, J, NIP1, NJ, -2, OLVZM2, KF)
C IF NOT A SURFACE POINT, SKIP TO 35
IF(I .NE. NI) GO TO 35
C FOR SURFACE POINTS, CALCULATE VR & VZ AT NI+1
DZ = .25D0*(3.D0*(VZSTAR+OVZ)
   -4.D0*(VZM1+OLVZM1)+VZM2+OLVZM2)/DZ
IF(DVZDV.LT.1.D-35) DVZDZ= 0.D0
TERM1= (1.D0+DADZ2)
DVRDR= -DVZDZ*DADZ2
& +(H*TERM1/RMAX - H*(1.D0/R-D2ADZ2/TERM1)*DSQRT(TERM1))
VR(IP1,J)= (4.DO*VRSTAR-VR(IP1,J)+2.DO*DVRDR*DR)/3.DO
VZ(IP1,J)= ( 4.DO*VZSTAR-VZ(IP1,J))
& +(3.DO*VRSTAR-4.DO*VRM1+VRM2)/DELZ*DR)/3.DO
35 CONTINUE
   GZ= 2.DO*A-E+3.DO*F
   GR= GZ+B
   DELVR= ((2.DO-GR)*OVR
   & +(A-C-D)*(VR(IP1,J)+OLDVR(IP1,J))
   & +(A-C+D)*(VR(IM1,J)+OLDVR(IM1,J))
$$\begin{align*}
\delta & + (4.00 \times F - 2.00 \times E) \times (V_{RM1} + OLV_{RM1}) \\
\delta & + (E - F) \times (V_{RM2} + OLV_{RM2}) \\
\delta & + (2.00 + G) \\
\Delta L_{VZ} & = \frac{(2.00 - G) \times OLVZ}{(2.00 + G)} \\
\delta & + (A + C - D) \times (V_{Z(IP1,J)} + OLVZ_{IP1,J}) \\
\delta & + (A - C + D) \times (V_{Z(IM1,J)} + OLVZ_{IM1,J}) \\
\delta & + (4.00 \times F - 2.00 \times E) \times (V_{ZM1} + OLVZ_{M1}) \\
\delta & + (E + F) \times (V_{ZM2} + OLVZ_{M2}) \\
\delta & + (2.00 - GZ) \times OLVZ \\
\delta & + (A + C - D) \times (V_{Z(IP1,J)} + OLVZ_{IP1,J}) \\
\delta & + (A + C - D) \times (V_{Z(IP1,J)} + OLVZ_{IP1,J}) \\
\delta & + (E - F) \times (V_{Z(IP1,J)} + OLVZ_{IP1,J})
\end{align*}$$

IF (AMPLO .EQ. 0.00 .AND. I .EQ. NI .AND. J .EQ. NJ)
$$\delta \quad \text{DELVR} = VR(I,J)$$
GO TO 120

C TRAILING EDGE

50 CONTINUE
IF (J .EQ. 2) CALL TRACK(50, I, J, .FALSE., 'VELOCT ', 0.00)
KF = 0
CALL INTERP (VR, NROWS, DELR, I, J, NIP1, NJ, 1, VRP1, KF)
IF (KF .NE. 0.0) GO TO 20
CALL INTERP (OLDVR, NROWS, DELR, I, J, NIP1, NJ, 1, OLVRP1, KF)
CALL INTERP (VZ, NROWS, DELR, I, J, NIP1, NJ, 1, VZP1, KF)
CALL INTERP (OLDVZ, NROWS, DELR, I, J, NIP1, NJ, 1, OLVPZP1, KF)
IF (J .EQ. NJ) GO TO 20
CALL INTERP (VR, NROWS, DELR, I, J, NIP1, NJ, 2, VRP2, KF)
IF (KF .NE. 0.0) GO TO 20
CALL INTERP (OLDVR, NROWS, DELR, I, J, NIP1, NJ, 2, OLVRP2, KF)
CALL INTERP (VZ, NROWS, DELR, I, J, NIP1, NJ, 2, VZP2, KF)
CALL INTERP (OLDVZ, NROWS, DELR, I, J, NIP1, NJ, 2, OLVPZ2, KF)
C IF NOT A SURFACE POINT, GO TO 55
IF (I .NE. NI) GO TO 55
C FOR SURFACE POINTS, CALCULATE VR & VZ AT NI+1
$$\begin{align*}
DVZDZ & = 0.25D0 \times (-3.00 \times (VZ_{STARR} + OLVZ(I,J)) \\
& + 4.00 \times (VZP1 + OLVZP1) - (VZP2 + OLVZP2)) / DZ \\
\delta & + H \times TERM1 / RMAX - H \times (1.00 / D2ADZ2 / TERM1) \times DSQR(TERM) \\
VR(IP1,J) & = (4.00 \times VRSTARR - VR(IM1,J) + 2.00 \times DVRDR \times DR) / 3.00 \\
VZ(IP1,J) & = (4.00 \times VZSTARR - VZ(IM1,J) + 3.00 \times VRP1 + VRP2) / DELZ \times DR) / 3.00
\end{align*}$$
55 CONTINUE
$$\begin{align*}
GZ & = 2.00 \times A - E - 3.00 \times F \\
GR & = GZ + B \\
\Delta L_{VR} & = (2.00 - GR) \times OLVZ \\
\delta & + (A + C - D) \times (VR(IP1,J) + OLVR(IP1,J)) \\
\delta & + (A - C + D) \times (VR(IM1,J) + OLVR(IM1,J)) \\
\delta & - (4.00 \times F + 2.00 \times E) \times (VRP1 + OLVRP1) \\
\delta & + (E + F) \times (VRP2 + OLVRP2) \\
\delta & + (2.00 + GR) \\
\Delta L_{VZ} & = (2.00 - GZ) \times OLVZ \\
\delta & + (A - C + D) \times (VZ(IP1,J) + OLVZ(IP1,J))
\end{align*}$$

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GO TO 120
170 CONTINUE

C 7
C EVALUATE THE VELOCITIES AT J=1
C
DELVR= 2.D0*VR(I,2)-VR(I,3)
DELVZ= 2.D0*VZ(I,2)-VZ(I,3)
C----------------------------------------------

120 CONTINUE
IF(OVR .EQ. 0.DO) GO TO 121
IF(DABS((DELVR-OVR)/OVR) .LT. 1.D-10) DELVR= OVR
121 IF(OVZ .EQ. 0.DO) GO TO 122
IF(DABS((DELVZ-OVZ)/OVZ) .LT. 1.D-10) DELVZ= OVZ
122 VRS= WRSTAR + VRU*DELVR
      VZS= WZSTAR + VZW*DELVZ
      VR(I,J)= VRS
      VZ(I,J)= VZS
      IF(J .EQ. NJ) GO TO 150
      IF(DABS(VRS) .GT. VELMAX) GO TO 315
      VRC= VRS-OVR
      IF(VRS .NE. 0.DO .AND. DABS(OVR) .GT. 1.D-5*VAVG)
      & VRC= VRC/OVR
      IF(VRC .GT. 25.DO) GO TO 200
      IF(DABS(VZS) .GT. VELMAX) GO TO 315
      VZC= VZS-OVZ
      IF(VZS .NE. 0.DO .AND. DABS(OVZ) .GT. 1.D-5*VAVG)
      & VZC= VZC/OVZ
      IF(VZC .GT. 25.DO) GO TO 200
      ERRORR= VRS - VRSTAR
      IF(DABS(VRSTAR) .GT. 1.D-10 .AND. VRS .NE. 0.DO)
      & ERRORR= ERRORR/VRSTAR
      IF(ERRORR .LE. BIGR) GO TO 123
      IR= I
      JR= J
      BIGR= ERRORR
123 IF(ERRORR .GT. VTOL) JFLAG= 1
      ERRORZ= VZS - VZSTAR
      IF(DABS(VZSTAR) .GT. 1.D-10 .AND. VZS .NE. 0.DO)
      & ERRORZ= ERRORZ/VZSTAR
      IF(ERRORZ .LE. BIGZ) GO TO 124
      IZ= I
      JZ= J
      BIGZ= ERRORZ
124 IF(ERRORZ .GT. VTOL) JFLAG= 1
150 CONTINUE
C------------------------------------------------------------------
IF(IDBUGV .EQ. 3 .OR. IDBUG2 .EQ. 3)
& WRITE(6,151) IR,JR,BIGR,IZ,JZ,BIGZ
151 FORMAT(‘<<<<<<<<<<< BIGR(‘,I3,’,’I3,’)=’,1PE12.4,
& ’ BIGZ(‘,I3,’,’I3,’)=’,E12.4)
IF(IDBUGV .EQ. 2) CALL PLOTL
C------------------------------------------------------------------
IF(JFLAG .EQ. 0 .AND. KOUNT .GT. 0) GO TO 220
KOUNT = KOUNT + 1
IF(KOUNT .EQ. ITMAX) GO TO 230
IF(KOUNT .GT. 21) GO TO 200
GO TO 10
200 WRITE(6,201) I,J,VRC,VZC
201 FORMAT(I=',I3,' J=',I3,' VRC=',1 PE 12.4,' VZC=',E12.4
KFLAG= 1
GO TO 310
220 CONTINUE
WRITE(6,221) KOUNT
221 FORMAT(‘ » » » » » » » IN VELOCT, ’,I2,’ ITERATIONS.’
IF(AMPLO .EQ. 0.0D0) VR(NI,NJ)= VR(NI,NJ1)
310 RETURN
C------------------------------------------------------------------
315 CONTINUE
WRITE(6,316) I,J,VRS,VZS
316 FORMAT(I=',I3,' J=',I3,' VRS=',1 PE 12.4,' VZS=',E12.4
KFLAG= -1
GO TO 310
C------------------------------------------------------------------
230 WRITE(6,240) KOUNT
240 FORMAT(‘ NO CONVERGENCE IN VELOCT AFTER’,I4,
& ’ ITERATIONS’) STOP 41
END
SUBROUTINE TRAPVE(DELR)
C------------------------------------------------------------------
REAL*8 DELR,DELR,DELVZ
DIMENSION DELR(1)
COMMON /CTRCOM/ DELVR,DELVZ,I,J,IM1,IP1,JM1,JP1,L
RETURN
ENTRY TRAPV(M)
WRITE(6,1) M
1 FORMAT(‘/’ TRAP IN VELOCT AT ‘,I3)
WRITE(6,2) I,J,DELR(JM1),DELR(J),DELR(JP1)
2 FORMAT(‘ (‘,I3,’ ,’I3,’), DELR=’,1PE12.4)
STOP 42
END

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SUBROUTINE RADIUS
& (VR,OLDVR,VZ,OLDVZ,NROWS,DELR,AMPL,OLDA)
C THIS ROUTINE COMPUTES RADIUS OF THE JET,
C USING THE SURFACE BOUNDARY CONDITION
C-
IMPLICIT REAL*8 (A-H,O-Z)
REAL*4 FLOAT
LOGICAL RESTRT, SAVE
C-
DIMENSION VR(NROWS,1), VZ(NROWS,1), AMPL(1),
& OLDVR(NROWS,1), OLDVZ(NROWS,1), OLDA(1),DELR(1)
C-
COMMON /JETF/ DELZ, DELT, TIME, AMPLO, VRW,
& VZW, VAVG, VISC, VISCK, SURF, RMAX, ZMAX,
& RHO, WAVNUM, WAVLEN, VTOL, RTOL, RW, FREQ
C-
COMMON /JETI/ NI, NIP1, NI1, NJ, NJ1,
& NK, ITMAX, IFLAG, IDBUGV, IDBUGR, RESTRT, SAVE
C-
COMMON /RTRCOM/ A,I,J,IP1,IM1,JP1,JM1,ITOP
C-
C INITIALIZE RADIUS
ANI1= FLOAT(NI1)
PI2= 6.283185308D0
GO TO 140
C-
ENTRY RADIU(MFLAG)
IDBUG2= 0
KOUNT= 0
C-
10 CONTINUE
JFLAG= 0
BIG= 0.
JBIG= 1
C-
DO 60 J=1,NJ
  DR= DELR(J)
  JP1= J+1
  JM1= J-1
  OA= OLDA(J)
  ASTAR= AMPL(J)
  WASTAR= (1.-RW)*ASTAR
  A= .5*(VR(NI,J)+OLDVR(NI,J))/DELT
  B= .125*(VZ(NI,J)+OLDVZ(NI,J))/DELZ*DELT
C-
IF(J .EQ. 1 )  GO TO 30
IF(J .EQ. NJ) GO TO 20
C-
AMP2= B*( AMPL(JP1)+OLDA(JP1)-(AMPL(JM1)+OLDA(JM1)))
OAMPJ= OLDA(J)
IF(OLDA(JP1) .GT. OAMPJ .AND. OLDA(JM1) .GT. OAMPJ)
& AMP2= 0.DO
AMP= OA + A - AMP2

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GO TO 40
C---------------------------------------------
20 CONTINUE
   AMPL = (A - B*( (AMPL(J-2)+OLDA(J-2))
& - 4.*(AMPL(JM1)+OLDA(JM1)) )
& + (1.-3.*B)*OA )/(1.+3.*B)
GO TO 40
C---------------------------------------------
30 CONTINUE
   AMPL = (A + B*( (AMPL(J+2)+OLDA(J+2))
& - 4.*(AMPL(JP1)+OLDA(JP1)) )
& + (1.+3.*B)*OA )/(1.-3.*B)
   IF(FREQ .NE. O.DO) AMP = AMPLO*DSIN(FREQ*TIME)
C---------------------------------------------
40 CONTINUE
   AMP = WASTAR + RW*AMP
   IF(0A .EQ. 0.DO) GO TO 50
   IF(DABS((AMP-OA)/OA) .LT. 1.D-13) AMP = OA
50
   AMPL(J) = AMP
   IF(J .EQ. 1 ) GO TO 60
   ERROR = AMP - ASTAR
   IF(DABS(ASTAR) .GT. 1.D-8*RMAX .AND. AMP .NE. O.DO)
&      ERROR = ERROR/ASTAR
   ERROR = DABS(ERROR)
   IF(ERROR .GT. RTOL) JFLAG = 1
   IF(ERROR .LT. BIG) GO TO 60
   BIG = ERROR
   BIGA = ASTAR
   JBIG = J
60 CONTINUE
C---------------------------------------------
   IF(JFLAG .EQ. 0 .AND. KOUNT .NE. 0) GO TO 90
   KOUNT = KOUNT + 1
   IF(KOUNT .GT. 10) IDBUG2 = 3
   IF(IDBUGR .EQ. 1) WRITE(6,150) ( J , AMPL( J ),OLDA( J ) , J= 1,N
   IF(IDBUGR .EQ. 2 .OR. IDBUG2 .EQ. 3) CALL PLOTL
   IF(IDBUGR .EQ. 3 .OR. IDBUG2 .EQ. 3)
&      WRITE(6,80) KOUNT, JBIG, BIG, AMPL(JBIG), BIGA
    C---------------------------------------------
   IF(KOUNT .LT. ITMAX) GO TO 10
   WRITE(6,70)
    70 FORMAT( ' IN RADIUS, THE MAXIMUM NUMBER OF ITERATIONS',
&       ' HAS BEEN REACHED.' )
    80 FORMAT( ' IN RADIUS, ITERATION ',I3,' BIG(',I3,',')=',
&       1PE12.4,' AMP=',E12.4,' ASTAR=',E12.4)
   STOP 61
C---------------------------------------------
90 CONTINUE
   DO 110 J=1,NJ
      Z= FLOAT(J-1)*DELZ
      AMP = AMPL(J)
      OAMP = OLDA(J)
      CHANGE = AMP - OAMP
IF(AMP*OAMP .LT. 0.) GO TO 110
IF(DABS(OAMP) .GT. 1.D-5*RMAX .AND. AMP .NE. 0.)
& CHANGE = CHANGE/OAMP
& CHANGE = DABS(CHANGE)
IF(CHANGE .GT. .2) GO TO 120
RAD = RMAX + AMP
IF(RAD/RMAX .GE. .04) GO TO 110
C------------------------------------------------------------------
WRITE(6,100) Z,RAD,TIME
100 FORMAT( ' JET PINCH-OFF HAS OCCURED AT Z=',1PE12.5,
& ' R=',E12.4,', TIME=',E12.4)
RAD = 0.
MFLAG = -1
GO TO 140
110 CONTINUE
GO TO 140
C------------------------------------------------------------------
120 WRITE(6,130) J,AMP,OAMP,CHANGE
130 FORMAT( ' AT J=',I3,' AMP=',1PE12.4,' OAMP=',E12.4,
& ' CHANGE=',E12.4)
MFLAG = 1
C------------------------------------------------------------------
140 RETURN
150 FORMAT( ' AMPL(',I3,')=',1PE12.4,' OLA=',E12.4)
END

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SUBROUTINE INTERP
   (ARRAY,NROWS,DELR,I,J,NI,NJ,K,VALUE,IFLAG)
C
C THIS SUBROUTINE INTERPOLATES BETWEEN TWO COLUMNS OF AN
C ARRAY AT THE ELEMENT (I,J)
C VALUE RETURNS WITH THE INTERPOLATED VALUE FROM THE J+K
C COLUMN
C QUADRATIC INTERPOLATION IS USED
C
C------------------------------------------------------------------
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION ARRAY(NROWS,1), DELR(1)
C------------------------------------------------------------------
JK= J+K
DR= DELR(J)
R= FLOAT(I-1)*DR
IF(JK .LT. 1) JK= 1
IF(JK .GT. NJ) JK= NJ
DRK= DELR(JK)
RK= FLOAT(NI-1)*DRK
C------------------------------------------------------------------
IF(R .GT. RK) GO TO 15
IK= R/DRK + 1.5
C------------------------------------------------------------------
IF(IK .EQ. NI) GO TO 16
IF(IK .GT. NI) GO TO 15
IF(IK .LT. 1) GO TO 25
C------------------------------------------------------------------
RK= FLOAT(IK-1)*DRK
X= R-RK
Y= X/DRK
VALO= ARRAY(IK,JK)
VALP= ARRAY(IK+1,JK)
C------------------------------------------------------------------
IF(IK .EQ. 1) GO TO 100
VALM= ARRAY(IK-1,JK)
VALUE= VALO + .5*(VALP - VALM)*Y
   + .5*(VALP - 2.*VALO + VALM)*Y*Y
GO TO 99
C------------------------------------------------------------------
100 Y= Y-1.
   VALM= ARRAY(IK+2,JK)
   VALUE= VALP + .5*(VALM - VALO)*Y
   + .5*(VALM - 2.*VALP + VALO)*Y*Y
GO TO 99
C------------------------------------------------------------------
15 IFLAG= 1
16 VALUE= ARRAY(NI,JK)
GO TO 99
C------------------------------------------------------------------
25 IFLAG= 2
   VALUE= ARRAY(1,JK)
99 RETURN
END
About the Author

The author was born the fifth of six sons to Harry B. Abbott, Sr. and Loie Martin Harris Abbott in the Baton Rouge General Hospital, on the twelfth day of the year nineteen hundred fifty-six. Without too much difficulty the author passed through the same elementary, junior high, and senior high schools that all of his brothers attended. In June nineteen hundred seventy-four, the author started his undergraduate studies in Chemical Engineering at the Baton Rouge campus of Louisiana State University. After three and one half years he graduated with a Bachelor of Science degree in Chemical Engineering, and after having been enticed into continuing his education at Louisiana State University, he began his graduate work that eventually led to the degree of Doctor of Philosophy in Chemical Engineering.
EXAMINATION AND THESIS REPORT

Candidate: Warren T. Abbott

Major Field: Chemical Engineering

Title of Thesis: Analysis of the Instability of Laminar Newtonian Liquid Jets in Air

Approved:

[Signature]
Major Professor and Chairman

[Signature]
Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:

July 21, 1982