Thermodynamic Approach to Fatigue Failure Analysis in Metals and Composite Materials

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THERMODYNAMIC APPROACH TO FATIGUE FAILURE ANALYSIS IN METALS AND COMPOSITE MATERIALS

A Dissertation
Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in
The Department of Mechanical Engineering

by
Mehdi Naderi Abadi
B.S., Amirkabir University of Technology, Iran, 1997
M.Sc., Iran University of Science and Technology, Iran, 2002
August 2011
to

My Wife and My Daughter
Acknowledgements

I would like to thank the SONOTECH® for supplying the couplant needed for the acoustic emission tests in Chapter 6.

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# Table of Contents

Acknowledgement ........................................................................................................ iii

Abstract ......................................................................................................................... vii

Introduction .................................................................................................................. 1

I.1 Problem Statement ................................................................................................. 1
I.2 Overview of Dissertation ...................................................................................... 2
I.3 References .............................................................................................................. 5

Chapter 1: On the Thermodynamic Entropy of Fatigue Fracture ................................. 6

1.1 Introduction ........................................................................................................... 7
1.2 Experimental Procedure ..................................................................................... 9
1.3 Theory and Formulation ..................................................................................... 11
1.4 Numerical Simulation ....................................................................................... 16
1.5 Results and Discussion ...................................................................................... 19
1.6 Conclusions ........................................................................................................ 25
1.7 References ......................................................................................................... 25

Chapter 2: Real-time Fatigue Life Monitoring by Thermodynamic Entropy ................. 28

2.1 Introduction ......................................................................................................... 29
2.2 Theory and Methodology for Fatigue Life Monitoring with FFE ....................... 31
2.3 Experimental Apparatus and Structural Health Monitoring System ................ 34
2.4 Results and Discussion ...................................................................................... 36

2.4.1 Constant Amplitude Loading ...................................................................... 36
2.4.2 Variable Amplitude Loading ...................................................................... 37
2.5 Conclusions ....................................................................................................... 42
2.6 References ......................................................................................................... 43

Chapter 3: An Experimental Approach to Low-Cycle Fatigue Damage Based on Thermodynamic Entropy ................................................................................. 46

3.1 Introduction ......................................................................................................... 47
3.2 Experimental Procedure ................................................................................... 48
3.3 Theory and Formulation .................................................................................... 49

3.3.1 Entropy Law ................................................................................................. 49
3.3.2 Damage Parameter Evolution ..................................................................... 52
3.4 Results and Discussion ...................................................................................... 55
3.5 Conclusion .......................................................................................................... 58
3.6 References ......................................................................................................... 59

Chapter 4: A Thermodynamic Approach to Fatigue Damage Accumulation under Variable Loading ................................................................................................................ 62

4.1 Introduction ......................................................................................................... 63
4.2 Theory and Formulation ..................................................................................... 65

4.2.1 Damage Accumulation ................................................................................ 65
4.2.2 Entropy Production ..................................................................................... 68
4.3 Experimental Procedure ................................................................................... 69
Abstract

Fatigue is a dissipative process and must obey the laws of thermodynamics. In general, it can be hypothesized that the degradation of machinery components is a consequence of irreversible thermodynamic processes that disorder a component, and that degradation is a time dependent phenomenon with increasing disorder. This suggests that entropy—a fundamental parameter in thermodynamics that characterizes disorder—offers a natural measure of component degradation.

The majority of the existing methods for prediction of fatigue are limited to the study of a single fatigue mode, i.e., bending or torsion or tension-compression. Further, the variability in the duty cycle in a practical application may render many of these existing methods incapable of reliable performance.

During this research, we put forward the idea that fatigue is a degradation process and that entropy is the most suitable index for assessing degradation. That is, tallying irreversible entropy is more reliable and accurate than many of the other methods presented in the existing papers. We show that in processes involving fatigue, for a given material (metal and composite laminate), there exists a unique threshold of the cumulative thermodynamic entropy beyond which fatigue fracture takes place. This threshold is shown to be independent of the type of the fatigue process and the loading history. This exciting result is the basis of the development of a Fatigue Monitoring Unit (FMU) described in this research.

We also propose a general procedure for assessment of damage evolution based on the concept of entropy production. The procedure is applicable to both constant- and variable amplitude loading. Empirical relations between entropy generation and damage evolution for two types of metals (Aluminum 6061-T6 and Stainless steel 304) and a woven Glass/Epoxy composite laminate are proposed and their potential for evaluation of fatigue damage are investigated.
Introduction
I.1 Problem Statement

All structures and machinery components—the fuselage and wings of an aircraft, the blades of a helicopter and a windmill, down-hole drilling components and the like—are prone to degradation and failure will eventually occur if the designs are not able to maintain integrity. Failure due to fatigue—the progressive and localized structural damage that occurs when a material is subjected to repetitive cyclic loading—is thought to be the most common source of structural degradation, making fatigue diagnosis and prognosis capabilities highly desirable. Fatigue failure results in significant financial losses to the industry due to forced shutdown. Moreover, fatigue failure of machinery can be catastrophic to human life—both the operator and user—and often result in significant damage to the environment. Hence, the proper fatigue design can reduce the undesirable losses. The closer the fatigue failure analysis to the real products, the greater is the confidence in the resulting engineering designs.

The current methods for prediction of fatigue failure such as stress curves methods, cumulative damage models, cyclic plastic energy hypothesis, crack propagation rate models, and curve fitting on the limited laboratory data use deterministic theory and order-of-magnitude error analyses and require many constants that must be experimentally determined. Although Miner’s cumulative damage rule has a lot of industrial applications because of simplicity, its implementation requires a large safety factor in the design. Most importantly, the probabilistic nature of fatigue results in variability of the fatigue life. For example, two fatigue tests of the same material with the same conditions lead to different predictions for the number of cycles to failure. Hence, the randomness behavior of fatigue forces designers to implement large safety factor in their works, which often lead to costly and insufficient designs.
In general, fatigue degradation is an irreversible process which disorders a system and produces entropy in accordance with the second law of thermodynamics. Thus, entropy—a fundamental parameter in thermodynamics that characterizes disorder—offers a natural measure of component deterioration [1, 2]. Utilizing the degradation theorem recently published in [1], a thermodynamic approach to fatigue failure can be developed. Even though the number of fatigue failure prediction models is numerous, the relevant question in this dissertation is: can thermodynamic entropy predict the rate of fatigue degradation more accurately than the existing methods?

1.2 Overview of Dissertation

Permanent degradation is the result of an irreversible process accompanied by generation of entropy, as demanded by the second law of thermodynamics. Entropy is a fundamental parameter that can be used to quantify the behavior of irreversible degradation processes. The variation of the entropy, \( dS \), is the sum of two terms [3]

\[
dS = d_iS + d_eS
\]  

(1-1)

where \( d_iS \) is the entropy generated inside the system and \( d_eS \) represents the entropy supplied to the system. The second law of thermodynamics states that \( d_iS \) must be equal to zero for reversible processes and positive for irreversible processes; that is:

\[
d_iS \geq 0
\]  

(1-2)

In the thermodynamics of irreversible processes, the objective is to relate the internal entropy production to the various irreversible processes that may be occurring within the solid continuum as a system. Using the definition of Helmholtz free energy, \( \psi = u - \theta \), the entropy production Equations can be written as [4]

\[
\dot{\gamma} = \frac{w}{\theta} + \frac{k}{\theta^2} \theta_j \theta^j
\]  

(1-3)
where $\gamma$ is cyclic entropy production, $\theta$ represents the temperature, and $k$ is thermal conductivity. The parameter $w$ denotes cyclic strain energy. For metals cyclic strain energy is related to fatigue properties and is nearly constant during fatigue as follows [5]

$$w = AN_f^B$$  \hspace{1cm} (I-4)

where $A$ and $B$ are material constants, and $N_f$ is the number of cycles at failure.

Unlike metals, process of energy variation in composite material is not constant. In composite materials, the hysteresis energy variation is much more complicated than their metal counterparts due to matrix cracking, matrix/fiber delamination and fiber breakage [6] and to the best of our knowledge, there are no expressions available to relate cyclic hysteresis energy to composite fatigue properties. Therefore, in this research, experimentally determined the hysteresis energy is used for this purpose.

By having the hysteresis energy values for metals or composite materials, one can calculate entropy at the time $t_f$ when failure occurs by integration of Equation (I-3) as follows

$$\gamma_f = \int_0^{t_f} \left( \frac{w}{\theta} + \frac{k}{\theta^2} \theta \frac{\theta}{\theta} \right) dt$$  \hspace{1cm} (I-5)

A possible application of the proposed hypothesis of the constant entropy gain at failure is in the development of a methodology for prevention of the catastrophic failure of metals or composite laminates undergoing fatigue load. The entropy generation increases during the fatigue life toward a final value of $\gamma_f$, called fracture fatigue entropy (FFE). Thus, FFE can be utilized as an index of failure. As the entropy generation accumulates toward the FFE, it provides the capability of shutting down of the machinery before a catastrophic breakdown occurs.

The concept of thermodynamic entropy can be extended to the analysis of fatigue damage using continuum damage mechanics (CDM) which provides an effective approach for analyzing degradation stages in metals and composite materials. In CDM, a damage variable, D, in terms of
materials microstructures is defined to represent the deterioration of a material prior to initiation of macrocracks. Homogenous materials like metals have four stages of damage: accommodation stage, nucleation of dislocations, micro and macrocrack initiations, and macrocrack propagation [4, 7-8]. Entropy accumulates owing to progressive nature of irreversibility during each stage until it reaches the critical value of FFE.

In non-homogenous materials, particularly composite materials, damage stages and entropy accumulation have different trend to those of metals. Damage and entropy accumulation suddenly increases due to matrix cracking, fiber/matrix delamination, and fiber breakage of weak interfaces [6]. Then, a gradual increase of entropy accumulation continues as fatigue progresses. Finally, a sharp increase of damage due to fiber breakage takes place.

Thermodynamic approach to evaluate fatigue life using the first and the second law of thermodynamics is explained in more details in Chapter 1. Laboratory experiments conducted at LSU Center for Rotating Machinery (CeRoM) and theoretical and numerical accomplishments are briefly described in this chapter. A fatigue prevention unit (FPU) that was designed, built, and tested in CeRoM is described in Chapter 2. This unit uses FFE as a failure criterion to halt the operation of fatigue machine before complete failure occurs. Chapter 3 and 4 explain damage evolution using thermodynamic relationships for constant- and variable-amplitude loading, respectively. Thermodynamic analysis of fatigue prediction of composite materials is presented in Chapter 5 with starting formulation and analytical discussion. Then, in Chapter 6 and 7 fatigue damage formula is discussed for two different types of fatigue tests using thermography, acoustic emission, stiffness approach, energy method, and entropy concept. Theses analyses are useful for recognizing damage stages in composite laminates. In Chapter 8, fatigue life prediction based on entropy method is compared with stress- and energy- based criteria. Finally, the conclusions and future steps are discussed in Chapter 9.
I.3 References


Chapter 1: On the Thermodynamic Entropy of Fatigue Fracture*

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1.1 Introduction

All structures and machinery components undergoing fatigue loading are prone to crack formation [1] and its subsequent growth that increases with time. When a crack is formed, the strength of the structure or the component decreases and it can no longer function in the intended manner for which it was designed for. Moreover, the residual strength of the structure decreases progressively with increasing crack size. Eventually, after a certain time the residual strength becomes so low that the structure fails [2]. It is, therefore, of paramount importance to be able to predict the rate of decline in the component’s residual strength and the remaining life of the system.

Many researchers have attempted to develop methods for quantifying fatigue in order to predict the number of cycles to failure. Among them, Miner [3] pioneered the idea of quantifying fatigue damage based on the hypothesis that under variable amplitude loading, the life fractions of the individual amplitudes sum to unity. Later, Coffin [4] and Manson [5] independently proposed the well-known empirical law

\[
\frac{\Delta \varepsilon_p}{2} = c \varepsilon' (N_f)^c
\]

which relates the number of cycles to failure \( N_f \) in low-cycle fatigue regime to the amplitude of the applied cyclic plastic deformation, \( \Delta \varepsilon_p \). In this relationship, \( \varepsilon' \) and \( c \) are the specified mechanical properties. The role of energy dissipation associated with plastic deformation during fatigue loading as a criterion for fatigue damage was also investigated by Halford [6] and Morrow [7].

The energy approach for estimating the fatigue life of materials under cyclic loading tests has gained considerable attention by researchers [7-14]. Morrow’s paper [7] is representative of a pioneering work that takes into account cyclic plastic energy dissipation and fatigue of metals that undergo cyclic loading. He presented a descriptive theory of fatigue that uses the cumulative plastic strain energy as a criterion for fatigue damage and the elastic strain energy as a criterion for fracture. For fully reversed fatigue load, Morrow derived a relation for plastic strain energy
per cycle $W_p$ in terms of the cyclic stress-strain properties, applicable when plastic strain is predominant. Park & Nelson [11] proposed an empirical correlation for estimation of fatigue life taking into account the elastic strain energy, $W_e$, as well as plastic strain energy, $W_p$. In the high-cycle regime, plastic strains are usually quite small and the $W_p$ approach becomes computationally unreliable. Park & Nelson [11] proposed that the two energy terms, $W_p$ and $W_e$, must be combined into the total strain energy parameter, $W_t$. That is,

$$W_t = W_p + W_e = AN_f^α + BN_f^β$$  \hspace{1cm} (1.1)

where the constants $A$, $α$, $B$ and $β$ can be determined from a set of uniaxial fatigue test data that cover a sufficiently large number of cycles. The energy dissipation due to plastic deformation during fatigue is a fundamental irreversible thermodynamic process that must be accompanied by irreversible entropy gain.

Permanent degradations are the manifestation of irreversible processes that disorder a system and generate entropy in accordance to the second law of thermodynamics. Disorder in systems that undergo degradation continues to increase until a critical stage when failure occurs. Simultaneously with the rise in disorder, entropy monotonically increases. Thus, entropy and thermodynamic energies offer a natural measure of component degradation [11-17]. In this chapter the entropy rise in bending, torsion, and tension-compression fatigue of metallic components and particularly the entropy at the instance when failure occurs is quantified. According to Whaley [18], the entropy at the instant when fracture occurs can be estimated by integrating the cyclic plastic energy per temperature of material. We put forward the hypothesis that at the instance of failure, the fracture fatigue entropy (FFE) is constant, independent of frequency, load, and specimen size.
1.2 Experimental Procedure

A series of fatigue tests is performed to examine the validity of the proposed hypothesis. Three different stress states examined are: completely reversed bending, completely reversed torsion and axial loads. Tests are conducted with Aluminum 6061-T6 and Stainless Steel 304 specimens. The fatigue testing apparatus used is a compact, bench-mounted unit with a variable-speed motor, variable throw crank connected to the reciprocating platen, with a failure cut-off circuit in a control box, and a cycle counter. The variable throw crank is infinitely adjustable from 0 to 50.8 mm to provide different levels of stress amplitude. The same fatigue apparatus is used for applying torsion, bending and axial load using appropriate fixture.

Figure 1.1 shows a schematic of the experimental setup used for torsion tests. The torsional fatigue tests are made using a round bar specimen clamped at both ends and rotationally oscillated at one of the ends via a crank with specified amplitude and frequency. Bending fatigue tests involve a plane specimen clamped at one end and oscillated at the other end, which is connected to the crank. The tension-compression fatigue tests involve clamping a plate specimen at both ends in the grips and oscillating the lower grip at a specified amplitude and frequency. All tests are conducted by installing a fresh specimen in the apparatus, specifying the operating condition, and running continuously until failure occurs. All tests are run until failure, when the specimen breaks into two pieces.

High-speed, high-resolution infrared (IR) thermography was used to record the temperature evolution of the specimen during the entire experiment. The IR camera was a MIKRON M7500 with temperature range between 0 °C to 500 °C, resolution of 320 × 240 pixel, accuracy of ±2% of reading, sensitivity/NETD of 0.08 °C at 30 °C, and image update rate of 7.5 Hz. Before fatigue testing, the surface of the specimen is covered with black paint to increase the thermal emissivity of the specimen surface. Figure 1.2 shows surface temperature evolution of a series of bending fatigue tests at the clamped end where the specimen fractures. These tests pertain to subjecting
an Aluminum specimen to different stress amplitudes. It is to be noted that a persistent trend emerges from all the experiments. Initially, the surface temperature rises since the energy density associated with the hysteresis effect gives rise to generation of heat greater than the heat loss from the specimen by convection and radiation. Thereafter, temperature tends to become relatively uniform for a period of time until it suddenly begins to rise, shortly before failure occurs. Figure 1.2 also shows how the temperature of the specimen varies around a mean value. The rise of the mean temperature during fatigue test is due to the plastic deformation of the material. The oscillation of the temperature around mean value is caused by the thermoleastic effect [13, and 19].

![Experimental apparatus for torsion fatigue test](image)

**Figure 1.1.** Schematic of the experimental apparatus for torsion fatigue test.
Figure 1.2. Evolution of temperature in bending fatigue of Aluminum specimen at 10 Hz at different displacement amplitudes a)49.53 mm, b)48.26 mm, c)38.1 mm, d)35.56 mm. Temperature increases initially, levels off for a period, it suddenly rises just before fracture occurs.

1.3 Theory and Formulation

Description of the relevant irreversible processes requires formulating the first and second laws of thermodynamic as applicable to a system whose properties are continuous function of space and time. According to the first law of thermodynamics the total energy content \( E \) within an arbitrary control volume can change only if energy flows into (or out of) the control volume through its boundary:

\[
dE = dQ - dW \tag{1.2}
\]

where \( Q \) and \( W \) are heat flow and work across the boundary of the control volume. In terms of the specific quantities, the law of conservation of energy for a control volume can be written as [20]:

\[
\rho \frac{du}{dt} = -\text{div} J + \sigma : D \tag{1.3}
\]
where $\rho$ is density, $u$ is specific internal energy, $J_q$ is heat flux across the boundary, $\sigma$ is symmetric stress tensor, and $D$ is symmetric rate of deformation tensor.

The second law of thermodynamics (Clausius-Duhem inequality) postulates that the rate of entropy generation is always greater than or equal to the rate of heating divided by the temperature $T$ [21]. That is:

$$\frac{\rho ds}{dt} \geq -\text{div}(J_q/T)$$

where $s$ represents the specific entropy. The right hand side of Equation (1.4) can be written as:

$$\text{div}(J_q/T) = \text{div}J_q/T - J_q \cdot \text{grad} T/T^2$$

Substituting Equation (1.5) into Equation (1.4) and replacing $\text{div}J_q$ from Equation (1.3) yields:

$$\rho \frac{ds}{dt} + (\sigma : D - \rho \frac{du}{dt} - J_q \cdot \text{grad} T/T) / T \geq 0$$

Let $Ψ$ represents the specific free energy defined as [21]:

$$Ψ = u - Ts$$

Differentiating Equation (1.7) with respect to time $t$, and dividing the result by temperature $T$ yields:

$$-(dΨ/dt + s dT/dt)/T = ds/dt - du/(Tdt)$$

Considering Equation (1.8), the inequality (1.6) reads:

$$(\sigma : D - \rho (dΨ/dt + s dT/dt) - J_q \cdot \text{grad} T/T) / T \geq 0$$

For small deformations, the deformation rate tensor $D$ is replaced by $\dot{\varepsilon}$ which represents the total strain rate. The total strain is decomposed to plastic and elastic strain:

$$\varepsilon = \varepsilon_p + \varepsilon_e$$
The specification of the potential function (free specific energy $\Psi$) must be concave with respect to temperature $T$ and convex with respect to other variables. Also, potential function $\Psi$ depends on observable state variables and internal variables [21]:

$$\Psi = \Psi(\epsilon, T, \epsilon_p, \epsilon_v, V_k)$$  \hspace{1cm} (1.11)

where $V_k$ can be any internal variable.

By referring to Equation (1.10), strains are decomposed to $\epsilon - \epsilon_p = \epsilon_v$, so we can rewrite Equation (1.11) as:

$$\Psi = \Psi(T, \epsilon - \epsilon_p, V_k) = \Psi(T, \epsilon_v, V_k)$$  \hspace{1cm} (1.12)

Using the chain rule, the rate of specific free energy can be written as:

$$\frac{\partial \Psi}{\partial t} = \frac{\partial \Psi}{\partial \epsilon_v} \dot{\epsilon}_v + (\frac{\partial \Psi}{\partial T} \dot{T} + (\frac{\partial \Psi}{\partial V_k} \dot{V_k})$$  \hspace{1cm} (1.13)

After substitution of Equation (1.13) into Equation (1.9), we obtain:

$$\left(\dot{\epsilon}_v + \sigma : \dot{\epsilon}_p - \rho(\frac{\partial \Psi}{\partial T} + s) \dot{T} - \rho \frac{\partial \Psi}{\partial V_k} \dot{V_k} - J_q \cdot \text{grad} T / T \right) / T \geq 0$$  \hspace{1cm} (1.14)

For small strains, the following expressions define the thermoelastic laws [21]:

$$\sigma = \rho \frac{\partial \Psi}{\partial \epsilon_v}$$  \hspace{1cm} (1.15)

$$s = - \frac{\partial \Psi}{\partial T}$$  \hspace{1cm} (1.16)

The constitutive laws of Equations (1.15) and (1.16) arise from fulfillment of non-negative inequality of Equation (1.14). By defining thermodynamic forces associated with the internal variables [21] as follows:

$$A_k = \rho \frac{\partial \Psi}{\partial V_k}$$  \hspace{1cm} (1.17)

Hence, the Clausius-Duhem inequality is reduced to express the fact that volumetric entropy generation rate is positive:

$$\dot{\gamma} = \sigma : \dot{\epsilon}_p / T - A_k / T - J_q \cdot \text{grad} T / T^2 \geq 0$$  \hspace{1cm} (1.18)
Equation (1.18) is also interpreted as the product of generalized thermodynamic forces
\[ X = \{\sigma/T, A/T, \text{grad } T/T^2\} \]
and generalized rates or flows \( J = \{\dot{\varepsilon}_p, -\dot{\varepsilon}_q, -\dot{J}_q\} \), [22-24]:

\[ \gamma = \sum_k X_k J_k \quad \text{(1.19)} \]

Irreversible thermodynamics considers forces \( X \) as drivers of flows \( J \). Each \( J \) can depend on all forces [20] and intensive quantities (e.g., temperature \( T \)) associated with the dissipative process.

Equation (1.18) describes the entropy generation process which consists of the mechanical dissipation due to plastic deformation, nonrecoverable energy stored in the material, and the thermal dissipation due to heat conduction. For metals nonrecoverable energy represents only 5-10% of the entropy generation due to mechanical dissipation and is often negligible [6, 25, and 26]:

\[ \dot{A} \dot{V}_k/T \approx 0 \quad \text{(1.20)} \]

Therefore, Equation (1.18) reduces to:

\[ \gamma = \sigma : \dot{\varepsilon}_p/T - J_q : \text{grad } T/T^2 \geq 0 \quad \text{(1.21)} \]

The coupling of thermodynamics and continuum mechanics requires the selection of observable and internal variables [27]. In the present study, two observable variables: temperature \( T \) and total strain \( \varepsilon \) are chosen. By referring to Equation (1.2) and replacing \( \rho \, du/dt \) by the expression derived from \( u = \Psi + Ts \):

\[ \rho \left( \dot{\Psi} + T \dot{s} + \dot{T}s \right) = -\text{div} J_q + \sigma : D \quad \text{(1.22)} \]

Considering Equations (1.13), (1.15) and (1.16) and small deformations, Equation (1.22) yields:

\[ \sigma : \dot{\varepsilon} + A \dot{V}_k + \rho T \dot{s} = -\text{div} J_q + \sigma : \dot{\varepsilon} \quad \text{(1.23)} \]
By applying chain rule to Equation (1.16), we can express $s$ by:

$$s = -\frac{\partial^3 \Psi}{\partial \varepsilon \partial T} \varepsilon - \frac{\partial^3 \Psi}{\partial T^2} T - \frac{\partial^3 \Psi}{\partial V \partial T} V_k$$

Substitution of Equations (1.28), (1.16) and (1.17) into Equation (1.24) results in:

$$s = -\frac{\partial \sigma}{\rho \partial T} \varepsilon + \frac{\partial \varepsilon_s}{\partial T} T - \frac{\partial A}{\rho \partial T} V_k$$

By introducing the specific heat, $C = T(\partial \varepsilon / \partial T)$, using Equations (1.10), (1.20) and (1.25), and taking into account Fourier’s law ($J_q = -k \text{grad} T$), Equation (1.23) leads to [21]:

$$k \nabla^2 T = \rho C T - \sigma : \varepsilon_p - T \partial \sigma / \partial T : \varepsilon_e$$

where $k$ is the thermal conductivity.

Equation (1.26) shows the energy balance between four terms: transfer of heat by conduction ($k \nabla^2 T$), retardation effect due to thermal inertia ($\rho C T$), internal heat generation consisting of plastic deformation ($W_p = \sigma : \dot{\varepsilon}_p$)—which is responsible for mean temperature rise—and thermoelastic coupling term, $W_e = T \partial \sigma / \partial T : \dot{\varepsilon}_e$, which takes into account the thermoelastic effect (Figure 1.2).

The total energy generation in Equation (1.22) is the combination of elastic and plastic energy, $W_t = W_e + W_p$ for low and high-cycle fatigue [5, 6, and 9].

$$W_t = 2(1+\nu)\sigma f^{2b} N^{3b} / (3E) + 4\varepsilon_0 \left( \frac{l-n}{l+n} \right) \sigma_f^{1/(1+n')} \left( \sigma_f^{1/n'} \right)$$

where $n'$ is cyclic strain hardening exponent, $\varepsilon'_f$ is fatigue ductility coefficient, $\sigma_f$ denotes the fatigue strength coefficient, $\sigma_u$ represents the stress amplitude and $\nu$ is the Poisson’s ratio. The parameters $b$, $E$, and $N$ represent the fatigue strength coefficient, modulus of elasticity and the number of cycles to failure, respectively.
Since the temperature fluctuation caused by thermoelastic effect is small in comparison with mean temperature rise (Figure 1.2), the elastic part in Equation (1.26) can be neglected [13]. Therefore, Equations (1.20) and (1.25) can be simplified to:

\[
\rho C \dot{T} - k \nabla^2 T = W_p
\]  

(1.28)

\[
\gamma = W_p / T - J_q \cdot \text{grad} T / T^2 \geq 0
\]  

(1.29)

The fracture fatigue entropy (FFE) can be obtained by integration of Equation (1.29) up to the time \( t_f \) when failure occurs:

\[
\gamma_f = \int_0^{t_f} \left( W_p / T - J_q \cdot \text{grad} T / T^2 \right) dt
\]  

(1.30)

where \( \gamma_f \) is FFE. In low-cycle fatigue where the entropy generation due to plastic deformation is dominant and the entropy generation due to heat conduction is negligible, Equation (1.26) reduces to:

\[
\gamma_f = \int_0^{t_f} \left( W_p / T \right) dt
\]  

(1.31)

The experimental temperature, such as those shown in Figure 1.2, can be used to calculate FFE.

1.4 Numerical Simulation

Simultaneous solution of Equations (1.28) and (1.29) is necessary to determine the entropy generation. For this purpose, a commercial software package (Flexpde) which employs the finite element method to solve partial differential Equations is utilized.

(a) Computational Model

Three-dimensional models with ten-node quadratic tetrahedral elements and appropriate number of meshes for the specimens undergoing bending are developed. The corresponding number of finite elements for bending is 2709. Figure 1.3 shows geometry and finite element
meshes used for the specimen undergoing bending fatigue and because of symmetric condition, only half of the specimen is modeled.

A mesh dependency study was carried out to investigate the effect of the number of meshes on the calculated entropy generation from Equation (1.29). The results of the effect of mesh refinement for bending test of Al-6061 at 10 Hz and 49.53 mm displacement amplitude is shown in Table 1.1. It reveals that the calculated result for FFE is independent of mesh refinement.

![FEM model and associated mesh in bending fatigue.](image)

**Figure 1.3. FEM model and associated mesh in bending fatigue.**

<table>
<thead>
<tr>
<th>No. of mesh</th>
<th>FFE (MJm^-3K^-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2709</td>
<td>3.960</td>
</tr>
<tr>
<td>2897</td>
<td>3.955</td>
</tr>
<tr>
<td>5604</td>
<td>3.956</td>
</tr>
<tr>
<td>10771</td>
<td>3.954</td>
</tr>
<tr>
<td>15875</td>
<td>3.955</td>
</tr>
</tbody>
</table>

(b) Boundary Conditions

Figure 1.4 shows a two dimensional sketch of the computational model used for the bending load with the notations indicating the boundary conditions. A summary of boundary conditions is shown in Table 1.2. Different tip displacements amplitudes (25-50 mm) at different frequencies (6-18 Hz) are considered as the applied loads in the model. Boundary W1 exchanges heat to the surrounding by convection and radiation. Walls W2 are at room temperature, \( T_a \). Convective
heat transfer is assumed as the boundary condition on walls W3. The convective heat transfer coefficient $h$ is estimated using an experimental procedure which involves measuring the cooling rate of the specimen surface temperature after a sudden interruption of the fatigue test [Appendix A]. Surface emissivity, $\varepsilon_o$ is calculated to be 0.93 and $\sigma_o$ is the Stephan-Boltzman constant equals to $5.67 \times 10^{-8}$ (Wm$^{-2}$K$^{-4}$).

Walls W4 are associated with the glass wool insulation used in the experiments, thereby, zero heat flux at this boundary. The boundary W5 is considered as a symmetric boundary condition.

Thermal and mechanical properties of the materials are summarized in Table 1.3 [28, and 29]. Fatigue properties of the selected materials are based on the experimental studies of Wong [30] and Lin et al. [31].

---

**Figure 1.4. Schematic of 2-D model with boundary condition notations.**
Table 1.2. Boundary conditions

<table>
<thead>
<tr>
<th>boundary type</th>
<th>thermal condition</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1 Wall, convection &amp; radiation to air</td>
<td>$k \frac{\partial T}{\partial n} = h(T - T_a) + \sigma_n \varepsilon (T - T_a)$</td>
<td>$n$ is the normal to the wall</td>
</tr>
<tr>
<td>W2 Wall, constant $T$</td>
<td>$T = T_a$</td>
<td></td>
</tr>
<tr>
<td>W3 Wall, convection to air</td>
<td>$k \frac{\partial T}{\partial n} = h(T - T_a)$</td>
<td>$n$ is the normal to the wall</td>
</tr>
<tr>
<td>W4 Wall, insulation</td>
<td>$\frac{\partial T}{\partial n} = 0$</td>
<td>$n$ is the normal to the wall</td>
</tr>
<tr>
<td>W5 Wall, symmetric plane</td>
<td>$\frac{\partial T}{\partial n} = 0$</td>
<td>$n$ is the normal to the wall</td>
</tr>
</tbody>
</table>

Table 1.3. Material properties

<table>
<thead>
<tr>
<th>material</th>
<th>$k$ (Wm$^{-1}$K$^{-1}$)</th>
<th>$\rho$ (kg m$^{-3}$)</th>
<th>$C$ (Jkg$^{-1}$K$^{-1}$)</th>
<th>$\sigma'$ (Mpa)</th>
<th>$\varepsilon'$</th>
<th>$n'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-6061</td>
<td>164</td>
<td>2659</td>
<td>871</td>
<td>535</td>
<td>1.34</td>
<td>0.062</td>
</tr>
<tr>
<td>SS 304</td>
<td>16</td>
<td>7900</td>
<td>500</td>
<td>1000</td>
<td>0.25</td>
<td>0.171</td>
</tr>
<tr>
<td>Glass wool</td>
<td>0.037</td>
<td>200</td>
<td>0.66</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
</tbody>
</table>

1.5 Results and Discussion

The evolution of entropy generation is calculated for the entire fatigue life and then integrated over time to determine the entropy generated during fatigue process (Equation 1.30). Figure 1.5 shows comparison of numerical and experimental entropy generation based on Equations (1.30) and (1.31) for bending fatigue of Al 6061-T6 where frequency and displacement amplitude are 10 Hz and 49.53 mm, respectively. Small difference between the experimental result and numerical simulation is due to the fact that heat conduction is neglected in Equation (1.31). The final value of the entropy generation (about 4 MJ/m$^3$K for this test) is associated with the entropy at fracture when the specimen breaks into two pieces.

An uncertainty analysis is performed using the method of Kline and McClintock [32]. This method uses relative uncertainty in various primary experimental measurements to estimate the uncertainty of the final result. If the result of an experiment, $R$, assumed to be calculated from $M$ independent parameters, $z_1, z_2, \ldots, z_M$ then the uncertainty propagated into the result, $\delta R$ is:

$$\delta R = \sqrt{\sum_{i=1}^{M} \left( \frac{\partial R}{\partial z_i} \delta z_i \right)^2}$$  \hspace{1cm} (1.32)
where $\delta_{i}, \delta_{j}, \ldots, \delta_{m}$ are the uncertainties of the independent parameters. Using this method, the uncertainty in the calculated result $\delta R$ will have the same level as each of the individual parameters. Applying Equation (1.32) to Equation (1.31) gives:

$$\delta \dot{\gamma} = \frac{\partial \dot{\gamma}}{\partial T} \delta T$$  \hspace{1cm} (1.33)

Maximum error in calculating entropy based on uncertainty analysis is about $\pm 1\%$.

![Volumetric entropy generation evolution vs. number of cycles for Al 6061-T6 under bending test, frequency=10 Hz, displacement amplitudes=49.53 mm.](image)

Figure 1.5. Volumetric entropy generation evolution vs. number of cycles for Al 6061-T6 under bending test, frequency=10 Hz, displacement amplitudes=49.53 mm.

Figure 1.6 shows the results of experimental FFE for bending fatigue tests at different frequencies. Results of different displacement amplitudes and different thicknesses of specimen, i.e., 3, 4.82, and 6.35 mm are shown in this figure. Fracture fatigue entropy is found to be about 4 MJ/m$^3$K, regardless of the load, frequency and thickness of the specimen. It is to be noted that the results of seven sets of experiments presented in Figure 1.6 correspond to different combination of specimen thicknesses and operating frequencies. Also, experimental data are associated with the different displacement amplitudes ranging from 25 mm to 50 mm. The same concept for plotting experimental data is followed in Figures 1.7 and 8.
Figure 1.7 presents the results of experimental FFE plotted as a function of the fatigue life for bending and tension-compression tests for Al 6061-T6 specimens at 10 Hz. It is seen that the FFE is independent of the type of loading.

Figure 1.8 presents the results of entropy generation at failure for stainless steel 304 undergoing bending, and torsion fatigue tests. The results show that the entropy generation at the fracture point for SS 304 is about 60 MJ/m³K, independent of frequency and geometry. It is to be noted that the fatigue life of a specimen undergoing cyclic load is only weakly dependent on the test frequencies [7, and 33] up to 200Hz.

The results presented in Figures 1.6-8 demonstrate the validity of the constant entropy gain at failure for Aluminum and Stainless Steel specimens. The results reveal that the necessary and sufficient condition for final fracture of Al 6061-T6 corresponds to the entropy gain of 4 MJ/m³K regardless of the test frequency, thickness of the specimen and the stress state. For SS 304 specimens, this condition corresponds to entropy gain of about 60 MJ/m³K.

![Fracture Fatigue Entropy vs. No. of Cycles to Failure](image)

**Figure 1.6.** Fracture fatigue entropy vs. number of cycles to failure for different bending fatigue tests of Al 6061-T6 with different specimen thicknesses, frequencies and displacement amplitudes. Fracture fatigue entropy remains at roughly 4 MJ/m³ K regardless of the thickness load and frequency. Displacement amplitude is varied from 25 mm to 50 mm.
Figure 1.7. Experimental fracture fatigue entropy vs. number of cycles to failure for tension-compression, bending, and torsional fatigue tests of Al 6061-T6 at frequency 10 Hz. Fracture fatigue entropy remains at about 4 MJ/m³K for both tension-compression and bending fatigue. Displacement amplitude is varied from 25 mm to 50 mm.

Figure 1.8. Experimental fracture fatigue entropy vs. number of cycles to failure for bending and torsional fatigue tests of SS 304 for different loads (25-50 mm displacement amplitudes) and frequencies. Fracture fatigue entropy remains at about 60 MJ/m³K for tension-compression and bending and torsion fatigue.

A possible application of the proposed hypothesis of the constant entropy gain at failure is in the development of a methodology for prevention of the catastrophic failure of metals.
undergoing fatigue load. As demonstrated in this work (Figure 1.5), the entropy generation increases during the fatigue life toward a final value of $\gamma_f$. Thus, fracture fatigue entropy, FFE can be utilized as an index of failure. As the entropy generation accumulates toward the FFE, it provides the capability of shutting down of the machinery before a catastrophic break down occurs.

The concept of constant entropy gain at the fracture point, $\gamma_f$ assumes that thermodynamic condition associated with the entropy generation is identical during the fatigue process and varies only in the duration of the process. That is, failure occurs when

$$N = N_f, \quad \gamma = \gamma_f$$

(1.34)

Within the range of the experimental tests presented, $\gamma_f$ is only dependent upon the material and is independent of load, frequency and thickness. Therefore, the duration of the fatigue process varies depending on the operating conditions in order to satisfy the condition of Equation (1.34).

Based on this concept, one can conduct an accelerated failure testing scheme by increasing process rates $J$ while maintaining equivalent thermodynamic forces $X$ to obtain the same sequence of physical processes, in identical proportions, but at a higher rate. For example, by increasing frequency, the rate of plastic deformation $\dot{\varepsilon}_p$ increases and subsequently the rate of degradation increases while the duration of the test is shortens in order to satisfy Equation (1.34). This is in accordance with the accelerated testing procedure recently put forward by Bryant et al. [17] based on the thermodynamics of degradation.

Figure 1.9 shows the normalized entropy generation during the bending fatigue of SS-30 and Al 6061-T6 for different thicknesses, displacement amplitudes and frequencies. The abscissa of Figure 1.9 shows the entropy generation using Equation (1.30) and normalized by dividing by the entropy gain at the final fracture, $\gamma_f$. The ordinate shows the number of cycles normalized by
dividing by the final number of cycles when failure occurs. It can be seen that normalized entropy generation monotonically increases until it reaches the entropy at the failure point. Interestingly, a similar trend between normalized wear plotted against the normalized entropy was reported by Doelling et al. [16]. Their work resulted in prediction of flow of the Archard’s wear coefficient (Archard [34]) with remarkable accuracy.

![Normalized entropy generation vs. normalized number of cycles for bending fatigue of SS 304 and Al 6061-T6 for different thicknesses of specimen, displacement amplitudes and frequencies.](image)

**Figure 1.9.** Normalized entropy generation vs. normalized number of cycles for bending fatigue of SS 304 and Al 6061-T6 for different thicknesses of specimen, displacement amplitudes and frequencies.

The relation between the normalized cycles to failure and normalized entropy generation is approximately linear and can be described as follows:

\[
\frac{\gamma}{\gamma_f} \approx \frac{N}{N_f}
\]  

(1.35)

where \(\gamma_f\) is a property of material. Using Equation (1.34), the number of cycles to failure can be expressed as:

\[
N_f \approx \left(\frac{N}{\gamma}\right) \cdot \gamma_f
\]  

(1.36)
Equation (1.36) offers a methodology for prediction of the fatigue failure of a given material based on the measurement of the thermodynamic entropy generation. By having FFE $\gamma_f$ and calculating entropy generation $\gamma$ at a selected number of cycles $N$, the fatigue life $N_f$ of the specimen can be predicted. Calculation of the entropy generation $\gamma$ can be performed at the very beginning number of cycles of the test, thereby, providing an accelerated testing method for determination of fatigue failure.

1.6 Conclusions

A thermodynamic approach for characterization of material degradation is proposed which utilizes the entropy generated during the entire life of the specimens undergoing fatigue tests. Results show that the cumulative entropy generation is constant at the time of failure and is independent of geometry, load, and frequency. Moreover, it is shown that the fracture fatigue entropy (FFE) is directly related to the type of material. That is, materials with different properties such as Steel and Aluminum have a different cumulative entropy generation at the fracture point. Calculations show that entropy generation for Aluminum 6061-T6 is 4 MJ/m$^3$K and 60 MJ/m$^3$K for Stainless Steel 304. The implication of this finding is that by capturing the temperature variation of a system undergoing fatigue process, the evolution of entropy generation can be calculated during the fatigue life and then compared to the appropriate FFE for the material to assess the severity of degradation of the specimen. Also, a methodology is offered for prediction of the fatigue failure of a given material based on measurement of the thermodynamic entropy generation.

1.7 References


Chapter 2: Real-time Fatigue Life Monitoring Based on Thermodynamic Entropy*
2.1 Introduction

All machinery components—the fuselage and wings of an aircraft, the blades of a helicopter and a windmill, down hole drilling components and the like—are susceptible to fatigue and eventually fail when their structural integrity is compromised. Therefore, accurate assessment of fatigue is of paramount importance for maintaining safety in engineering systems. Of different types of structural damage, fatigue is thought to be the most common source of structural degradation [1], making fatigue diagnosis and prognosis capabilities highly desirable.

The open literature contains many studies pertaining to fatigue damage and life prediction theories with interest in continuous monitoring of structural health and integrity so that remedial actions can be taken to prevent failure [2-24]. To account for the accumulation of fatigue damage, the so-called Miner’s rule [7] is often applied. However, Miner’s rule does not consider the nonlinear accumulative rate of fatigue damage process [5]. To consider the effect of variable amplitude loadings, Li et al. [3] used the principle of continuum damage mechanics and developed a methodology for fatigue damage in bridge structures with a real-time, data-monitoring capability. Zheng et al. [14] studied the fatigue life of an old steel bridge by using specimens with original rivet central holes under two loading levels. Agerskov and Nielsen [15] experimentally investigated the fatigue damage accumulation in steel highway bridges under random loading based on the principles of fracture mechanics.

Fatigue damage accumulation can also be investigated using sensing devices such as acoustic sensors, eddy currents sensors (Meandering Winding Magnetometer sensors), ultrasonic sensors, and electromechanical sensors [16-19]. Moreover, thermal imaging techniques [20] are utilized for structural failure analysis.

The majority of the existing methods are limited to the study of a single fatigue mode, i.e., bending, torsion, or tension-compression. Further, the variability in the duty cycle in a practical application may render many of the existing methods incapable of reliable performance.
The present chapter attempts to predict the remaining service life of a component under fatigue loading by relating the fatigue damage, as an irreversible process, to the thermodynamic entropy. The premise of this chapter is that fatigue damage is an irreversible process involving degradation that causes disorder in the system and in accordance with the second law of thermodynamics [29] manifests itself in generation of entropy. As time progresses, disorder in the system continually increases until it reaches a critical stage when failure occurs. Simultaneously with the rise in disorder, entropy monotonically increases. Thus, as demonstrated in recent literature, entropy offers a natural measure of a component’s deterioration [27-29].

In a recent paper [29], the authors showed that the cumulative entropy production in metals subjected to cyclic loading—bending, torsion, and tension-compression—at the onset of fracture is constant. This so-called fracture fatigue entropy (FFE) is independent of geometry, load, and frequency. That is, the necessary and sufficient condition for final fracture of a metal undergoing fatigue load corresponds to a constant irreversible entropy gain. Specifically, an extensive set of experimental results involving bending, torsion, and tension-compression fatigue tests revealed that FFE is approximately $4 \pm 0.3$ MJ m$^{-3}$K$^{-1}$ for Al 6061-T6 and approximately $60 \pm 5$ MJ m$^{-3}$K$^{-1}$ for SS 304. Research shows that fatigue properties may vary with frequencies, environmental conditions, and temperature [30-33]. For example, Liaw et. al [30] reported that the fatigue life varies with changing the test frequencies and environmental conditions involving fatigue tests in air, nitrogen, and mercury. Kohout [31] reported that a decrease in testing temperature shifts the fatigue life toward higher fatigue strength and vice versa. Therefore, neglecting the variation of the fatigue properties with different test conditions can produce some errors in the calculation of FFE.

In this chapter we attempt to predict the remaining service life of a component under fatigue loading by FFE concept described in the chapter 1. A methodology for online monitoring of fatigue life in machinery components is presented that utilizes the accumulation of entropy to
assess the severity of degradation associated with fatigue. Using FFE concept, a prototype called Fatigue Monitoring Unit (FMU) is developed that automatically shuts down the machine prior to fatigue fracture based on a user-specified factor of safety\(^1\).

### 2.2 Theory and Methodology for Fatigue Life Monitoring with FFE

In accordance with the second law of thermodynamics in solids with internal friction the production due to plastic deformation and the thermal dissipation is given by the following relationship [29].

\[
\dot{\gamma} = \frac{w_p}{T} - \mathbf{J}_q \cdot \nabla T / T^2
\]  

(2.1)

where \(\dot{\gamma}\) represents the entropy production rate \((\dot{\gamma} \geq 0)\), \(\mathbf{J}_q\) denotes the heat flux, \(T\) is the surface temperature, and \(w_p\) is the cyclic plastic energy per unit volume which can be calculated from Morrow’s approximation [34-36].

\[
w_p = AN_f^{\gamma}
\]  

(2.2)

where \(N_f\) represents the number of cycle to failure, and constant \(A\) and \(\alpha\) are material specifications (see Table 2.1) and can be calculated from the following relationship [34 and 37].

\[
A = 2^{2+bc} \sigma'_e ' e^d \left(\frac{e - b}{e + b}\right) N_f^{\epsilon e}
\]  

(2.3)

\[
\alpha = 2 + b + c
\]  

(2.4)

where \(e'\) is the fatigue ductility coefficient, and \(\sigma'_e\) denotes the fatigue strength coefficient. Parameters \(b\) and \(c\) represent the fatigue strength coefficient, and fatigue ductility coefficient, respectively.

<table>
<thead>
<tr>
<th>Material</th>
<th>(A)</th>
<th>(\alpha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-6061</td>
<td>930.8</td>
<td>-0.964</td>
</tr>
<tr>
<td>SS 304</td>
<td>236</td>
<td>-0.501</td>
</tr>
</tbody>
</table>

\(^1\) A technology disclosure has been filed with the LSU Office of Intellectual Properties and a provisional patent application has been submitted.
In low-cycle fatigue where the entropy production due to plastic deformation is dominant, the entropy production due to heat conduction is negligible [29], Equation (2.1) reduces to:

\[ \gamma = \frac{\gamma_p}{T} \] (2.5)

The accumulation of entropy production can be obtained by integrating Equation (2.5):

\[ \gamma = \int_0^t \left( \frac{\gamma_p}{T} \right) dt \] (2.6)

At the onset of fracture, \( t = t_f \), the fracture fatigue entropy (FFE) for a given material corresponds to the value of \( \gamma_f \) and the corresponding number of cycle is \( N_f \). That is, FFE is defined by integration of Equation (2.5) up to the time \( t_f \) [29]:

\[ \gamma_f = \int_0^{t_f} \left( \frac{\gamma_p}{T} \right) dt \] (2.7)

where \( \gamma_f \) represents FFE.

Equation (2.7) shows that the entropy production continuously increases during the fatigue life until the final value is reached and failure take places. That is, failure occurs when

\[ N = N_f, \quad \gamma = \gamma_f \] (2.8)

To prevent fatigue failure, the system must be shut down prior to \( t_f \), the instant when fracture occurs. By continuously calculating \( \gamma \) using Equation (2.7) and comparing to FFE for the material being tested, one can assess the percentage of remaining life and halt the operation with a desired factor of safety, e.g., 10% remaining life, etc.

In the viewpoint of fatigue damage, the critical stage is the onset of macrocrack initiation [29]. Experiments show that the corresponding macrocrack propagation period can be identified by detecting the instant when temperature rises abruptly. Figure 2.1 shows the variation of temperature and the rate of temperature change with time (temperature slope) for Al 6061-T6.
specimen during the bending fatigue process. Temperature increases at the beginning of the test, and then tends to become flat for a major portion of the specimen’s life until it suddenly begins to rise, shortly before failure occurs. Simultaneous with temperature variation, the rate of the change of temperature with time decreases at the beginning of the test, tends to become approximately nil during the steady-state phase, and finally around 90% of fatigue life it begins to increase. To prevent fracture, the operation should be halted when

$$\frac{\gamma}{\gamma_f} \leq 0.9$$  \hspace{1cm} (2.9)

**Figure 2.1. The evolution of temperature and slope of the temperature where failure occurs at the displacement amplitude of 49.53 mm in bending fatigue of Aluminum specimen**

Due to probabilistic nature of fatigue, the exact number of cycles at the failure point is not known. However, research shows that for a given material, the FFE is a constant value independent of the loading sequence, loading type and frequency. Therefore, by monitoring the accumulation of entropy and comparing it to FFE, one can assess the remaining life of a specimen. Hence, Equation (2.9) is based on the entropy accumulation during a fatigue process and as soon as accumulated entropy reaches 90% of FFE, the system operation is halted.
A procedure based on Equations (2.7) and (2.9) is developed for the structural health monitoring which uses LabVIEW (Laboratory Virtual Instrumentation Engineering Workbench) programming environment. This software provides the necessary data acquisition from sensors and online calculations as well as online data analysis. The flow chart of the procedure is shown in Figure 2.2.

![Flow chart](image)

**Figure 2.2. A flow chart of procedure for the fatigue analysis**

### 2.3 Experimental Apparatus and Structural Health Monitoring System

A series of fatigue tests is performed to examine thermodynamic entropy concept. Specimens are made of Al 6061-T6 and SS 304. The fatigue testing apparatus is a compact, bench-mounted unit with a variable-speed motor, variable throw crank connected to the reciprocating platen with a failure cut-off circuit in a control box, and a cycle counter. A detailed description of the apparatus is reported in chapter 1.

The prototype of life prediction system consisting of fatigue monitoring unit (FMU) which measures the entropy accumulation, and the bending fatigue machine is shown in Figure 2.3. FMU prototype consists of a data acquisition module (OMB-DAQ-56), an electronic circuit with
a normally closed relay, a USB-9472 data acquisition module, and computer hardware. The control module of the fatigue test apparatus is described below.

Figure 2.3. Prototype of the fatigue monitoring unit (FMU) and bending fatigue test machine

The specimen’s thermal response is monitored via thermocouples placed near the clamped end. Since it is difficult to place the thermocouple at the exact location where failure occurs, two thermocouples are placed close to the clamped end to independently determine the entropy production. It is to be noted that increasing the number of thermocouples reduces the error in the calculation of entropy production. DAQ-56 transfers temperature data measured by the
thermocouples to the computer. Entropy production is individually calculated based on the temperature data of the thermocouples in the LabVIEW which analyzes data and the entropy production. The maximum of the calculated entropy based on the thermocouples data is chosen for comparing with FFE and assessing the remaining life. This corresponds to the minimum temperature reading of the thermocouples. Maximum error in temperature measurement based on uncertainty analysis is about \( \pm 2\% \) [38]. Fatigue life is then calculated and compared with 90\% of life which is defined as the threshold in the present study to stop the fatigue machine. Finally, as soon as the fatigue life falls below the threshold, a signal is sent via USB-9472 to the electronic circuit to switch off the fatigue machine.

2.4 Results and Discussion

This section presents the results of fatigue life monitoring for two types of bending and torsional fatigue tests of Al 6061-T6 and SS 304 at the frequency of 10 HZ and for different constant and variable amplitude loadings.

2.4.1 Constant Amplitude Loading

Different displacement amplitudes from 20 to 50 mm are applied to the specimens under bending and torsional fatigue tests. All tests are continued up to the Stage 1 where the fatigue machine is automatically shut down by FMU. Then, the machine is manually turned back on to continue the process until fracture occurs (Stage 2).

Figure 2.4 shows the SS 304 temperature evolution during the torsional test under two different constant amplitude loadings (35.56 and 33.02 mm). Surface temperature is seen to rapidly increase at the beginning of the test and then it approaches to a stationary value until it is suddenly stopped by FMU based on the defined threshold. During the degradation process, entropy accumulation is calculated in FMU and compared with FFE. As long as accumulated entropy is below the threshold (in this work 90 \% of life), the specimen under fatigue is on the safe side. Note that the threshold is completely a user-defined parameter and can be set to
accommodate a desired factor of safety. The fatigue test is continued until failure occurs and the specimen breaks into two pieces, Stage 2. The fatigue machine is switched off with 10% error in case (a) and 12% error in case (b). To reduce error, thermocouples must be placed as close as possible to location where the failure takes place.

![Graph showing temperature evolution vs. fatigue life for torsion test of SS 304 under constant load.](image)

**Figure 2.4.** Temperature evolution vs. fatigue life for torsion test of SS 304 under constant load. a) 35.56 mm displacement amplitude, b) 33.02 mm displacement amplitude. As soon as accumulated entropy reaches to 90% of life, fatigue monitoring unit shuts down the fatigue machine, Stage 1. The operation is halted within 10% error in case (a) and 12% error incase (b).

### 2.4.2 Variable Amplitude Loading

To investigate the role of variable load and load sequencing effects on the proposed methodology, some tests are carried out under two-stage cyclic loading sequences: High-to-Low load and Low-to-High load. All tests are performed up to the Stage 1 where the load sequence is changed, and then continued to the threshold point at which the fatigue machine is stopped, Stage 2. Finally, the tests are resumed and operation continued until fracture occurs (Stage 3.)

Figure 2.5 depicts the temperature variation versus fatigue life of Aluminum specimen under two different variable amplitude loadings: 44.45 to 36.83 mm as High-to-Low amplitude and
36.83 to 44.45 mm as Low-to-High amplitude. Entropy is accumulated up to Stage 1, which corresponds to the instant when the load was changed. Then, the accumulated entropy is added to the accumulated entropy of Stage 1. As soon as the threshold is reached, the fatigue machine is automatically shut down. Continuing the experiment up to Stage 3, eventually failure occurs and the specimen breaks into two parts.

![Temperature evolution vs. fatigue life for bending test of Al 6061-T6 under variable load](image)

**Figure 2.5.** Temperature evolution vs. fatigue life for bending test of Al 6061-T6 under variable load. a) High-to-Low amplitude (44.45 to 36.83 mm), b) Low-to-High amplitude (36.83 to 44.45 mm). As soon as accumulated entropy reaches to 90% of life, fatigue monitoring unit shuts down the fatigue machine, Stage 2. The operation is halted within 10% error in case (a) and 12% error incase (b).

The ratio of the number of cycle at the threshold point \( (N_t) \), to that of complete failure occurs \( (N_f) \) is reported in Table 2.2 and 2.3 for Al 6061-T6 and SS 304 of bending and torsional tests, respectively. In some tests, the operation of the machine is stopped close to the 10% of remaining life. However, in some tests the reaming life has some errors within the range of 5% to 15% error.
Table 2.2 The ratio of the number of cycle at the calculated stop point to the final number of cycle for Al 6061-T6

<table>
<thead>
<tr>
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<th>$N_r$</th>
<th>$N_f$</th>
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<td>Bending Fatigue Test</td>
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<td>92</td>
</tr>
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</tr>
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<td>6400</td>
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<td></td>
<td>2600</td>
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<td>76</td>
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Table 2.3 The ratio of the number of cycle at the calculated stop point to the final number of cycle for SS 304

<table>
<thead>
<tr>
<th>Test Type</th>
<th>$N_r$</th>
<th>$N_f$</th>
<th>$%N_r/N_f$</th>
</tr>
</thead>
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<td>Bending Fatigue Test</td>
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<td>9000</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>8400</td>
<td>9000</td>
<td>93</td>
</tr>
<tr>
<td></td>
<td>10600</td>
<td>12000</td>
<td>88</td>
</tr>
<tr>
<td></td>
<td>11000</td>
<td>13400</td>
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<td>6600</td>
<td>84</td>
</tr>
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<td></td>
<td>11556</td>
<td>13800</td>
<td>83</td>
</tr>
<tr>
<td></td>
<td>7700</td>
<td>7900</td>
<td>97</td>
</tr>
<tr>
<td></td>
<td>7200</td>
<td>8700</td>
<td>82</td>
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<tr>
<td></td>
<td>11500</td>
<td>14000</td>
<td>82</td>
</tr>
<tr>
<td></td>
<td>11524</td>
<td>13300</td>
<td>86</td>
</tr>
</tbody>
</table>
Figure 2.6 and 2.7 show the estimated life versus experimental life for Al 6061-T6 and SS 304 under bending and torsional fatigue test. The data are scattered around the 45 degree solid line and limited in the safety factor band (dashed line). The results reveal that FFE concept yields promising results in the prevention of fatigue with structural health monitoring. The results presented in these figures highlight that tallying entropy can be used to halt the operation of fatigue machine before the complete failure takes place within about 20% error for Al 6061-T6 and 15% error for SS 304.

![Graph showing estimated life vs. experimental life for bending and torsion fatigue of Al 6061-T6](image_url)

**Figure 2.6. Estimated life vs. experimental life for bending and torsion fatigue of Al 6061-T6**
Figure 2.7. Estimated life vs. experimental life for bending and torsion fatigue of SS 304

Figure 2.8 shows the experimental fatigue life of the present work with that of [39] for Al-6061-T6. Figure 2.9 also shows comparison of fatigue design curve obtained from the results of entropy criterion presented in Table 2.2 and that of [39] for Al 6061-T6. The results of fatigue tests stopped by entropy concept and those of complete failure are converted to the stress amplitude based on the following relationship proposed by [39].

\[ \sigma = \frac{14479}{\sqrt{N_f}} + 96.5 \]  \hspace{1cm} (2.10)

where \( \sigma \) is the stress amplitude.

The design curve of [39] is for the safety factor of 2 and the current design curve safety factor is approximately 1.1. It should be noted that the fatigue design curve based on entropy accumulation helps halting the operation of a component with the maximum usage of life before catastrophic failure takes place.
Figure 2.8. Comparison of experimental fatigue life of the current work with that of [39] based on stress approach.

Figure 2.9. Comparison of fatigue design curve based on the experimental results of entropy method and the results of [39] based on stress approach.

2.5 Conclusions

The monitoring of the accumulation of entropy leading to the final fracture can be used as an effective tool to prevent fatigue failure. This requires the knowledge of the amount of
accumulated entropy at the failure point called fracture fatigue entropy (FFE). This information is available for Al 6061-T6 and SS 304. A prototype of fatigue monitoring unit (FMU) is developed as a structural health monitoring system installed on the existing fatigue machine. By capturing the temperature variation of the specimen undergoing fatigue process, the evolution of entropy generation is calculated in real time. The comparison of accumulated entropy with FFE allows for removing the specimen from service before fatigue fracture occurs. Results are presented for both constant and variable loads. In the laboratory prototype, an OMB-DAQ-56 is used to receive the temperature signal from the thermocouple and to provide an appropriate temperature-based input signal to the computer by means of LabVIEW software code. The results of a series of laboratory fatigue tests show that FFE concept can be used to effectively halt the operation of the system. It is shown that the method is capable of handling both constant and variable amplitude loadings.

2.6 References


Chapter 3: An Experimental Approach to Low-Cycle Fatigue Damage Based on Thermodynamic Entropy*

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3.1 Introduction

Fatigue fracture is one of the main failure mechanisms of structural elements subjected to cyclic loading. Depending on the functions of the component and its usage in service, the impact of fatigue on the overall system can change from a minor degradation to catastrophic failure [1]. Therefore, understanding the evolution of fatigue and prediction of fatigue failure are important fields of research for development of new design as well as assessment of the remaining life of structures in service [2]. In this prospective, continuum damage mechanics (CDM) provides an effective approach for studying the fatigue damage evolution process.

In CDM a damage variable, $D$, in terms of material microstructures is defined to represent the degradation of a material state prior to the initiation of macrocracks [3-5]. This variable should be consistent with the fatigue damage mechanism of the material, and measured by an appropriate experimental procedure. The literature contains several approaches for measuring the damage variable. Some methods use the changes in dynamic responses, i.e. stress and strain [6-8] to determine the damage variable while others measure changes in the mechanical properties, i.e., the elastic modulus and the tensile strength [6, 9, and 10], or changes in physical properties, i.e., thermal and electric properties [6, and 11]. Most of these methods are deduced from the theories of creep and plastic damage and lack physical basis of fatigue damage accumulation. Hence, their applications to fatigue damage problems have not been entirely satisfactory [4, and 12].

Since the accumulation of damage is a dissipative process, it must obey the laws of thermodynamics [13, and 14]. In general, it can be hypothesized that the degradation of machinery components is a consequence of irreversible thermodynamic processes that disorder a component, and that degradation is a time dependent phenomenon with increasing disorder [15]. This suggests that entropy — a fundamental parameter in thermodynamics that characterizes disorder — offers a natural measure of component degradation [16].
Utilizing a general theorem developed by Bryant et al. [17] that relates entropy generation to irreversible degradation via generalized thermodynamic forces and degradation forces, Amiri et al. [19] recently developed an experimental approach to evaluate critical fatigue damage value based on the concept of entropy flow. They determined the entropy flow in different specimens subjected to bending fatigue and evaluated the critical damage value. In that study, the entropy flow was experimentally measured to link the irreversible degradation caused by bending fatigue to the entropy flow. They showed that the damage parameter can be evaluated based on the entropy supplied to the surrounding. This approach required measuring the surface temperature and estimating the convective heat transfer coefficient.

In this chapter, a more general approach utilizing entropy generation is developed. The cumulative plastic strain energy is used as a criterion for fatigue damage [20, and 21]. A series of experimental measurements involving low cycle bending, torsion and tension-compression fatigue is presented which relates the fatigue damage variation to entropy generation. Inferred thermographic technique which is being widely utilized in the evaluation of fatigue processes [22-29] is used to evaluate fatigue characteristic.

### 3.2 Experimental Procedure

Fatigue tests are conducted with Aluminum 6061-T6 and Stainless Steel 304 specimens. Detail description of the fatigue testing apparatus is reported in chapter 1, and 2 The details of the number of specimens in each type of fatigue tests are shown in Table 3.1.

<table>
<thead>
<tr>
<th>Material</th>
<th>Test Type</th>
<th>Number of fatigue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-6061</td>
<td>Bending</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Torsion</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Tension-Compression</td>
<td>2</td>
</tr>
<tr>
<td>SS 304</td>
<td>Bending</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Torsion</td>
<td>4</td>
</tr>
</tbody>
</table>
High-speed, high-resolution infrared thermography is used to record the temperature evolution of the specimen during the entire experiment. A thermocouple is used to measure the ambient temperature, and then it is attached on the surface of the specimen to calibrate IR camera at the beginning of the test. Figure 3.1 reports an example of the surface temperature evolution of Steel specimen in bending test at the clamped end and in torsion test around the middle of the specimen where fracture occurs. Surface temperature is seen to rapidly increase at the beginning of the test since the energy density associated with the hysteresis effect gives rise to generation of heat greater than the heat loss from the specimen by convection and radiation. Then energy generation balances the energy dissipation, thereby temperature reaches a steady state condition. Temperature suddenly begins to rise shortly before failure occurs.

![Figure 3.1. Evolution of temperature vs. fatigue life of torsion and bending fatigue of Steel specimen at 10 Hz and different displacement amplitudes a) torsion test, $\delta = 35.56 \text{ mm}$, b) torsion test, $\delta = 33.02 \text{ mm}$, c) bending test, $\delta = 49.53 \text{ mm}$, d) bending test $\delta = 45.72 \text{ mm}$. Temperature increases initially, reaches steady state for a period, suddenly rise before fracture occurs](image)

3.3 Theory and Formulation

3.3.1 Entropy Law

The Clausius-Duhem inequality states that in solids with internal friction all the deformations cause positive entropy generation rate [6]:

---

49
\[
\dot{s}_i = \frac{1}{T} \sigma : \dot{\varepsilon}_p - \frac{1}{T} A_k \dot{V}_k - \frac{1}{T^2} q \cdot \nabla T \geq 0
\] (3.1)

where \( \dot{s}_i \) is entropy generation rate, \( \sigma \) is the stress tensor, \( \dot{\varepsilon}_p \) is the plastic strain rate, \( T \) is the absolute temperature, \( V_k \) can be any internal variable such as damage and hardening, \( A_k \) are thermodynamic forces associated with the internal variables, and \( q \) is the heat flux. Equation (3.1) describes the entropy generation process which consists of the mechanical dissipation due to plastic deformation (\( W_p = \sigma : \dot{\varepsilon}_p \)), nonrecoverable energy stored in the material (\(- A_k \dot{V}_k\)), and the thermal dissipation due to heat conduction (\(- \frac{1}{T^2} q \cdot \nabla T\)). For metals nonrecoverable energy represents only 5-10% of the entropy generation due to mechanical dissipation and is often negligible [6]:

\[ A_k \dot{V}_k \approx 0 \] (3.2)

Therefore, Equation (5.1) yields to:

\[
\dot{s}_i = \frac{W_p}{T} - \frac{1}{T^2} q \cdot \nabla T
\] (3.3)

where \( W_p \) is the cyclic plastic energy determined by Morrow’s cyclic plastic energy dissipation formula given below [19, and 30]:

\[
W_p = 4\varepsilon'_f \left( \frac{1-n'}{1+n'} \right) \sigma_u^{(1+n')/(1-n')} \left( 2\sigma'_f \varepsilon'_f \left( \frac{1-n'}{1+n'} \right) 2N_f \right)^{n'+c}
\] (3.4)

where \( n' \) is cyclic strain hardening exponent, \( \varepsilon'_f \) is the fatigue ductility coefficient, \( \sigma'_f \) denotes the fatigue strength coefficient, \( \sigma_u \) represents the stress amplitude, \( N_f \) is the final number of cycles when failure occurs, \( b \) is fatigue strength exponent and \( c \) is fatigue ductility exponent.

Material properties are summarized in Table 3.2 [31-34].

In processes involving low-cycle fatigue, the entropy generation due to plastic deformation is dominant and the entropy generation due to heat conduction is negligible. The corresponding
entropy generation phenomenon starts slowly and grows. During the early stage of fatigue entropy generation due to plastic deformation is still a dominant effect. Therefore, Equation (3.3) reduces to:

\[
 s_i = \frac{W_p}{T} \tag{3.5}
\]

Therefore, the total entropy generation can be obtained by integration of Equation (3.5) up to the time \( t_f \) when fracture occurs:

\[
 s_g = \int_0^{t_f} \left( \frac{W_p}{T} \right)
\]

where \( s_g \) is the total entropy generation at the onset of fracture.

The experimental temperature, such as those shown in Figure 5.1, can be used to calculate total entropy generation.

<table>
<thead>
<tr>
<th>Table 3.2. Material properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>Al-6061</td>
</tr>
<tr>
<td>SS 304</td>
</tr>
</tbody>
</table>

The relation between normalized entropy generation and normalized cycles to failure

Figure 3.2 shows normalized entropy generation \((s_i/s_g)\) versus normalized cycles to failure \((N/N_f)\). It is interesting to note that similar trends for normalized entropy flow plotted against normalized cycles to failure in bending fatigue and for normalized wear plotted against the normalized entropy flow were reported by Amiri et al. [18] and Doelling et al. [16], respectively. Similarly, the current experimental measurements confirm the correlation between fatigue degradation and entropy generation, \( s_i \). The relation between the normalized cycles to failure and normalized entropy generation is approximately linear and can be described as:

\[
 \frac{s_i}{s_g} = \frac{N}{N_f} \tag{3.7}
\]
Entropy generation in Equation (3.7) is calculated using Equation (3.4). In the sections that follow, it is shown that how this linear relationship in Equation (3.7) between the normalized entropy generation and normalized cycles to failure is used to evaluate the damage parameter.

![Figure 3.2](image)

**Figure 3.2.** Normalized entropy generation vs. normalized number of cycles for bending fatigue of SS-304 for different displacement amplitudes

### 3.3.2 Damage Parameter Evolution

Several approaches are available to assess fatigue damage [2, 6, 9, 11, 16, and 18]. Of particular interest in this chapter is the work of Duyi and Zhenlin [2] who developed a simple but realistic method for analyzing the evolution of the low-cycle fatigue damage based on the reduction of static toughness, which is defined as the area of stress-strain curve, and the plastic strain energy during fatigue failure. According to their work, a general logarithmic expression describing the damage evolution can be postulated as follows:

\[
D = A + B \ln \left( 1 - \frac{N}{N_f} \right) 
\]

(3.8)

where \(D\) is dimensionless damage parameter, and \(A\) and \(B\) are material parameters.

Equation (3.8) is expressed in terms of material static toughness as [2]:

\[
D_N = 1 - \frac{U_{r(N)}}{U_{r_0}} = \frac{D_{N_f}}{\ln N_f} \ln \left( 1 - \frac{N}{N_f} \right) 
\]

(3.9)
where \( U_{r_0} \) (volumetric energy) is the static toughness of the intact material, \( U_{r(N)} \) (volumetric energy) is static toughness of the material pre-fatigued with \( N \) cycles. \( D_{N_f-1} \) denotes the critical value of the damage variable at which the material pre-fatigued with \( N_f - 1 \) cycles is tensioned to final rupture under cyclic stress amplitude, and can be expressed by:

\[
D_{N_f-1} = 1 - \frac{\sigma_a^2}{2EU_{r_0}}
\]  

(3.10)

where \( \sigma_a \) is the cyclic stress amplitude, \( E \) is the elastic modulus.

According to Equation (3.9), damage variable is a function of the \( N/N_f \) ratio, i.e.,

\[
D = f\left(\frac{N}{N_f}\right)
\]

(3.11)

Using the degradation-entropy generation theorem developed by Bryant et al. [19], one can develop a relationship between the degradation caused by cumulative damage and the entropy generation in the form of:

\[
D = f(s_i)
\]

(3.12)

Taking advantage of Equation (3.7), this following relationship holds:

\[
D = A + B \ln (1 - s_i/s_{c_i})
\]

(3.13)

Failure occurs when \( D \) reaches its critical value, \( D_c \) \( (D_c \leq 1) \), [14]). At the onset of failure, Equation (3.13) leads:

\[
D_c = D_o + B \ln (1 - s_{c_o}/s_{c_i})
\]

(3.14)

where \( D_o \) is the initial damage and the \( s_{c_i} \) is the critical value of entropy generation at the time when temperature starts rising just after steady state phase. In order to find the onset of rapid temperature rise after steady state phase in the experiment, a program developed in FlexPDE (a commercial software for solving partial differential equation) is employed which calculates the slope of the temperature based on the temperature data recorded from the experiment throughout
the fatigue life. Figure 3.3 shows a typical evolution of the slope of the temperature versus normalized number of cycles (normalization with respect to the number of cycles to failure) for Aluminum 6061-T6 specimens subjected to different bending load amplitudes. This graph reveals that as the temperature approaches to steady state phase, the slope of the temperature drops quickly and remains almost zero during this phase for around 90% of the fatigue life, after which it starts to increase rapidly. The onset of abrupt increase in the temperature slope (or temperature by itself) is considered as the critical condition for evaluation of $s_k$ in Equation (3.14).

Solving Equation (3.14) for $B$ and substituting into Equation (3.13) yields:

$$D = D_0 + \frac{(D_e - D_0)}{\ln(1 - s_c/s_e)} \ln(1 - s_c/s_e)$$

Equation (3.15) as a measure of degradation satisfies two important properties of a convex function as follows [35]:

1) A differentiable function of one variable (here damage; $D$) is convex on an interval if and only if its derivative is monotonically non-decreasing on that interval; that is,

$$\frac{\partial D}{\partial s} > 0$$

2) A twice differentiable function of one variable is convex on an interval if and only if its second derivative is non-negative, i.e.

$$\frac{\partial^2 D}{\partial s^2} > 0$$
Figure 3.3. The evolution of slope of the temperature where failure occurs at different load amplitudes in bending fatigue of Aluminum specimens

3.4 Results and Discussion

The variation of the damage parameter for different displacement amplitudes and different fatigue tests (bending, torsion, and tension-compression) are plotted in Figure 3.4 for Aluminum 6061 along with the Equation (3.15). The results of present work are compared with the results based on the work of Duyi and Zhenlin [2] and Amiri et al. [18]. The specimens are initially free of defect, $D_o = 0$. Also, at the critical point where temperature starts to shoot up normalized entropy, $s_u/s_f$, is considered 0.9, as an indication of 90% of fatigue life. The results show that for different fatigue tests, the present experimental work is in good agreement with Duyi and Zhenlin [2] and the work of Amiri et al. [18]. Small discrepancy between damage evolution based on entropy flow of the work of Amiri et al. [18] and present experimental work can be explained as follows. The degradation-entropy generation theorem ([17]) proves that: (a) the degradation rate $\dot{w} = \sum_i \dot{w}_i$ is a linear combination $\dot{w} = \sum B_i \dot{S}_i$ of the components of entropy generation $\dot{S}_i = \sum X_i^j J_i^j$ of the dissipative processes $p_i$ where $J_i^j$ are generalized rates or flows; (b) the generalized degradation forces $Y_i^j$ are linear functions $Y_i^j = B_i X_i^j$ of the generalize
thermodynamic forces $X_i$ and (c) the proportionality factors $B_i$ are the degradation coefficients given by $B_i = \frac{\partial w}{\partial S}$. In Amiri et al. [18], the entropy flow was utilized as an indication of entropy generation. This required estimation of the convective heat transfer coefficient which can play a role in the damage evolution during fatigue process. The present formulation is based on the entropy generation and does not explicitly require the knowledge of convective heat transfer coefficient. The maximum uncertainty predicted by Amiri et al. [18], $\pm 7.8\%$, whereas as in the present work it is $\pm 1\%$.

Figure 3.5 shows the evolution of damage variable for Stainless Steel at different displacement amplitudes and different fatigue tests. The results of present work are compared with the results based on the work of Duyi and Zhenlin [2] and Amiri et al. [18]. As shown in this figure, in the early stage of fatigue life, damage increases monotonically, then the slope increases with increasing the number of cycles followed by a sudden rise near the critical number of cycles defined as 90% of fatigue life. This figure also shows that entropy generation can be utilized to evaluate damage evolution during fatigue process. It can be seen that the results of present work are in good agreement with the results of Duyi and Zhenlin [2] and Amiri et al. [18].

Damage evolution versus normalized entropy generation $(s_i/s_g)$ of Aluminum and Stainless Steel is plotted in Figures 3.6 and 3.7 for different displacement amplitudes and fatigue tests, respectively. They confirm the good correlation between fatigue degradation and entropy generation based on Equation (3.15). They also reveal that thermodynamics entropy, an index of degradation, can be utilized to measure damage during fatigue process.
Figure 3.4. Damage variable of present experimental work, the work of Duyi and Zhenlin (2001) and the work of Amiri et al. (2009) for Aluminum at a) bending test, $\delta = 49.53\,\text{mm}$  
  b) bending test, $\delta = 44.45\,\text{mm}$  
  c) torsion test, $\delta = 40.64\,\text{mm}$  
  d) tension-compression test, $\delta = 38.1\,\text{mm}$

Figure 3.5. Damage variable of present experimental work, the work of Duyi and Zhenlin (2001) and the work of Amiri et al. (2009) for Stainless Steel at a) torsion test, $\delta = 40.64\,\text{mm}$  
  b) bending test, $\delta = 48.26\,\text{mm}$  
  c) bending test, $\delta = 43.18\,\text{mm}$  
  d) torsion test, $\delta = 30.48\,\text{mm}$
Figure 3.6. Damage evolution vs. normalized entropy generation of Al 6061-T6 for different displacement amplitudes and fatigue tests.

Figure 3.7. Damage evolution vs. normalized entropy generation of SS 304 for different displacement amplitudes and fatigue tests.

3.5 Conclusions

A series of low cycle bending, torsion and tension-compression fatigue experiments is performed to evaluate the damage parameter based on thermodynamics entropy generation. Infrared thermographic technique is used to capture the surface temperature evolution of the specimen under fatigue process, and entropy generation is evaluated using the surface temperature of the specimen. Entropy generation is utilized as an effective tool to measure damage as a degradation
of material under fatigue process. The primary result is a strong correlation between normalized number of cycle and entropy generation. It is shown that the normalized cycles-to-failure has a linear relationship with the normalized entropy generation. Based on the experimental results and cyclic plastic energy approximation, a new damage variable is defined as a function of entropy generation of the form \( D = D_0 + \frac{(D_e - D_0)}{\ln(1 - s/s_0)} \ln(1 - s/s_0) \). It is revealed from this equation that the irreversible thermodynamic entropy can be utilized to measure degradation of a system under bending, torsion and tension-compression fatigue with excellent comparison to the experimental measurements.

3.6 References


Chapter 4: A Thermodynamic Approach to Fatigue Damage Accumulation under Variable Loading*

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4.1 Introduction

Many parts of structures and machinery components are subjected to variable-amplitude loadings under normal service conditions and are susceptible to fatigue damage. In general, fatigue damage in metals involves four stages: The first stage involves the accommodation or nucleation stage where dislocations take place when permanent slip bands are formed [1-6]. The second stage involves the nucleation of dislocations into micro-cracks [1-6]. In the third stage, the micro-cracks tend to orient themselves perpendicular to the direction of the maximum shear stress and consequently move toward that direction [5]. Macro-cracks are then formed when the size of a micro-crack significantly grows and generates a high stress concentration at its front. The fourth stage of fatigue damage involves the propagation of the macro-crack, characterized by striation and beach-mark formation [1-11]. A macro-crack rapidly propagates, breaking the metal into two or more pieces.

The literature contains rich volumes of predictive models for characterizing fatigue damage. Among them, Palmgren [12] and Miner [13] have reported pioneering research work on the concept of fatigue damage accumulation. Specifically, they established the idea of linear cumulative fatigue damage based on the hypothesis that the sum of the life fractions of the individual amplitudes is unity. However, the application of these rules to the prediction of fatigue life is often unsatisfactory since they do not consider the nonlinear accumulative rate of fatigue damage process under variable amplitude loading [14, 15].

Many researchers have subsequently attempted to improve upon the Palmgren and Miner’s rule. Among them, Manson and Halford [16-19] developed the double-linear damage rule approach. Other researchers [20-24] have developed fatigue damage theories based on the plastic strain energy or the total strain energy that takes into account the nonlinearity in fatigue damage under variable amplitude loadings. More recently, continuum damage mechanics (CDM) approach has been applied to evaluate cumulative fatigue damage [25-28]. Other noteworthy
experimental and theoretical studies on the concept of cumulative damage and the fatigue crack growth under variable amplitude loading include the works of Duyi, and Zhenlin [29], Varvani-Farahani et al. [30], Jono [31], Liu and Sankaran [32]. However, to date, the complexity of variable-loading fatigue damage has hampered the development of a reliable design methodology for structural components [14].

In this chapter we treat the variable-loading fatigue damage using the concept of thermodynamic entropy. In general, fatigue damage is an irreversible process involving degradation that causes disorder in the system and produces entropy in accordance with the second law of thermodynamics. This suggests that entropy — a fundamental parameter in thermodynamics that characterizes disorder— offers a natural measure of degradation [33]. To this end, Bryant et al. [34] developed a general theorem for characterizing the degradation behavior of dissipative processes via entropy. The degradation-entropy generation theorem relates entropy generation to irreversible degradation, via generalized thermodynamic forces $\mathbf{X}_i$ and degradation forces $\mathbf{Y}_i$. According to the degradation-entropy theorem it is established that:

(i) the degradation rate $\dot{w} = \sum \dot{w}_i$ is a linear combination $\dot{w} = \sum B_i \dot{S}_i$ of the components of entropy production $\dot{S}_i = \sum_j X'_i J'_j$ of the dissipative processes $p_i$, where $J'_j$ are generalized rates or flows;

(ii) the generalized degradation forces $\mathbf{Y}_i$ are linear functions $Y'_i = B_i X'_i$ of the generalize thermodynamic forces $X'_j$, and (iii) the proportionality factors $B_i$ are the degradation coefficients given by $B_i = \partial \dot{w}_i / \partial \dot{S}_i |_{\dot{w}_i}$.

Utilizing the degradation theorem, we recently developed an experimental approach for evaluation of fatigue damage based on the entropy production in processes involving bending, torsion, and tension-compression under the constant-amplitude loading [35]. In addition, in an extensive set of experiments with two different metals, Naderi et al. [36] showed that by tallying the entropy production up to the fracture point, one arrives at a unique constant—a so-called
fracture fatigue entropy (FFE)—independent of geometry, load, and frequency. In other words, the necessary and sufficient condition for a metal to fracture is that its accumulated entropy production reaches a certain level.

In this study, we extend the analysis presented in [35] to develop a method for assessing the fatigue life of a component subjected to variable-amplitude loading histories. A series of experimental measurements involving low-cycle bending fatigue is presented that shows the validity of the approach.

4.2 Theory and Formulation

The formulation involves evaluation of damage accumulation and entropy production. For the damage accumulation, we make use of the recent developments in continuum damage mechanics (CDM). The basic approach in CDM is to introduce a damage variable into the formulation to represent the deterioration of a material state prior to the initiation of macro-cracks [37]. According to CDM, failure occurs when the damage parameter, $D$, reaches a critical value. This critical value is not necessarily the onset of fracture [28]. Chaboche [37] describes this condition as the breaking point of the continuum volume element. It corresponds to the point where damage becomes localized and leads to the initiation of a macro-crack [38]. The relationship between the accumulation of damage and thermodynamic entropy is described in the following sections.

4.2.1 Damage Accumulation

Damage starts with slip followed by fine cracks that are visible only at high magnifications. These micro-cracks grow and eventually become visible with the naked eye. Finally, micro-cracks coalesce into macro-sized cracks and fracture occurs [5, 39].

The accumulation of damage can be viewed as a degradation process and using the degradation-entropy production theorem [34] one can derive the following relationship between
the degradation caused by the cumulative damage and the entropy production [35].

\[ D = A + B \ln (1 - s/s_g) \]  

(4.1)

where \( A \) and \( B \) are material parameters, \( s \) represents the entropy production during fatigue life, and \( s_g \) is the total entropy production at the onset of fracture. A brief explanation of entropy production calculation during fatigue life is provided in Section 4.2.2 and further details can be found in [36]. Under a constant amplitude loading, Equation (4.1) reads:

\[ D = \frac{D_c}{\ln(1 - s_c/s_g)} \ln(1 - s/s_g) \]  

(4.2)

where \( D_c \) represents the critical damage value, i.e. the onset of macro-crack initiation (\( D_c \leq 1 \), [38]), and \( s_c \) is the critical value of the entropy production at the onset of temperature rise just before the complete failure and the beginning of macro-crack initiation [40].

In the derivation of Equation (4.2), it is assumed that material is free of initial defect with no entropy history and that the initial damage value is zero (\( D_0 = 0 \)). However, most components subjected to variable amplitude loading have “entropy memory.” To account for history effect, let \( s_0 \) represent the initial value for entropy production corresponding to the initial damage of \( D_0 \).

Substituting these values in Equation (4.1) yields:

\[ D_0 = A + B \ln (1 - s_0/s_g) \]  

(4.3)

Similarly, the relationship between the critical damage, \( D_c \), and its critical entropy production \( s_{ic} \) now reads:

\[ D_c = A + B \ln (1 - s_c/s_g) \]  

(4.4)

Solving Equations (4.3) and (4.4) for \( A \) and \( B \) and substituting into Equation (4.1) yields

\[ D = D_0 + \frac{(D_c - D_0)}{\ln \left( \frac{1 - s_c/s_g}{1 - s_0/s_g} \right)} \ln \left( \frac{(1 - s/s_g)}{(1 - s_0/s_g)} \right) \]  

(4.5)
Equation (4.5) provides a relationship for the damage evolution with entropy history effect. This equation can be further extended to an n-stage loading sequence. Hence, the $k^{th}$ stage of an $n$-stage loading can be described as follows.

$$D_k = D_{N_k-1} + \frac{(D_e - D_{N_k-1})}{\ln \left( \frac{(1-s_e/s_g)}{(1-s_{k-1}/s_g)} \right)} \ln \left[ \frac{(1-s/s_g)}{(1-s_{k-1}/s_g)} \right], \text{ for } k = 1 \ldots n \quad (4.6)$$

where $s_{k-1}$ represents the accumulated entropy at $(k-1)^{th}$ stage.

Finally, summing up the damage variation of every stage, the total damage evolution of $n$-stage loading is:

$$D = \sum_{k=1}^{n} \left[ D_{N_k-1} + \frac{(D_e - D_{N_k-1})}{\ln \left( \frac{(1-s_e/s_g)}{(1-s_{k-1}/s_g)} \right)} \ln \left[ \frac{(1-s/s_g)}{(1-s_{k-1}/s_g)} \right] \right], \text{ for } k = 1 \ldots n \quad (4.7)$$

Figure 4.1 presents the schematic of the fatigue damage evolution plotted as a function of the normalized number of cycle. This figure is based on Equation (4.6) under variable-amplitude loading, where $N_f$ denotes the number of cycles to failure. The degradation process continues with the existing micro-cracks and their clustering as well as with nucleation and propagation of new micro-cracks [5, 39, 41]. Meanwhile, entropy accumulates due to the progressive nature of irreversibility during each stages of the loading. Thus, degradation grows and entropy generation piles up from the beginning to the onset of Stage 2. At the beginning of Stage 2, the number and the size of the initial micro-cracks—characterized by $D_1$—and the amount of the accumulated entropy, $s_1$, increases. As the process continues, more micro-cracks grow in the plane of maximum shear stress [5] and the end of Stage 2 becomes the initial condition for the third stage. Finally, during the last loading stage, as soon as macro-crack is created, its growth continues along planes of maximum tensile stress up to the complete failure.
**Figure 4.1.** Schematic of damage evolution vs. normalized entropy production during multistage loading.

### 4.2.2 Entropy Production

According to the Clausius-Duhem inequality, in solids with internal friction the entropy production due to plastic deformation and the thermal dissipation have the following relationship [36].

\[
\dot{s} = w_p / T - J_q \cdot \text{grad} T / T^2
\]  

(4.8)

where \( \dot{s} \) represents the entropy production rate (\( \dot{s} \geq 0 \)), \( J_q \) denotes the heat flux, \( T \) is the surface temperature, and \( w_p \) is the cyclic plastic energy per unit volume which can be calculated using the Morrow’s approximation [19, 42, 43]:

\[
w_p = A N_f^{\alpha}
\]  

(4.9)

where constant \( A \) and \( \alpha \) are the material specifications (see Table 4.1) and can be calculated from the following relationship [42, 43].

\[
A = 2^{2-\psi c} \sigma_f' c_f' (\frac{c-b}{c+b}) N_f^{\psi c}
\]  

(4.10)
\[ \alpha = 2 + b + c \]  \hspace{1cm} (4.11)

where \( \varepsilon' \) is the fatigue ductility coefficient and \( \sigma' \) denotes the fatigue strength coefficient.

Parameters \( b \) and \( c \) represent the fatigue strength coefficient, and the fatigue ductility coefficient, respectively.

**Table 4.1. Fatigue properties [43, 52]**

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-6061</td>
<td>930.8</td>
<td>-0.964</td>
</tr>
<tr>
<td>SS 304</td>
<td>236</td>
<td>-0.501</td>
</tr>
</tbody>
</table>

In a low-cycle fatigue where the entropy production due to plastic deformation is dominant, the entropy production due to heat conduction is negligible [36], and Equation (4.1) reduces to:

\[ s = \frac{w_p}{T} \]  \hspace{1cm} (4.12)

The accumulation of the entropy production can be obtained by integrating Equation (4.5):

\[ s = \int_0^t \left( \frac{w_p}{T} \right) dt \]  \hspace{1cm} (4.13)

At the onset of fracture, i.e. when \( t = t_f \), the fracture fatigue entropy (FFE) for a given material corresponds to the value of \( s_g \) and the corresponding number of cycle is \( N_f \) [36].

The entropy production is calculated based on the temperature measurements and the cyclic energy approximation, Equation (4.12). Equation (4.7) is then used to determine fatigue damage evolution. The following section describes experimental procedure for measuring temperature during multistage loading.

**4.3 Experimental Procedure**

Figure 4.2 presents a schematic diagram of the experimental setup for cyclic bending fatigue tests. The apparatus is a compact bench-mounted unit with a variable speed motor, variable throw crank connected to the reciprocating platen, with a failure cut-off circuit in a control box and a cycle counter. The variable throw crank is adjustable from 0 to 2 inches. For bending fatigue tests, the specimen is clamped at one end, and oscillated at the other end with specified
amplitude and frequency. Fatigue tests are run until failure occurred, that is until the specimen breaks into two pieces. Glass wool insulation is used to reduce the heat conduction from the clamped end of the specimen to the grip. High-speed, high-resolution infrared (IR) thermography is used to record the temperature evolution of the specimen during the entire experiment. The IR camera is a MIKRON M7500 with temperature range between 0 °C to 500 °C, resolution of 320 × 240 pixel, accuracy of ±2% of reading, sensitivity/NETD of 0.08 °C at 30 °C, and image update rate of 7.5 Hz. Before fatigue testing, the surface of the specimen is covered with black paint to increase the thermal emissivity of the specimen surface. Fatigue tests conditions are summarized in Table 4.2.

![Figure 4.2. Schematic diagram of the experimental setup in bending](image)

<table>
<thead>
<tr>
<th>Table 4.2. Fatigue tests conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatigue tests</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Bending</td>
</tr>
</tbody>
</table>
Figure 4.3 shows a typical evolution of the temperature plotted as a function of the number of cycles for an Aluminum 6061-T6 specimen at the frequency of 10 Hz and displacement amplitude of 49.53 mm. Examination of Figure 4.3 reveals that the temperature evolution undergoes three distinct phases: an initial increase (Stage I), steady-state (Stage II), and a drastic increase prior to failure (Stage III). The first stage of the temperature evolution is limited to a very low number of cycles—in general, about 10% of the entire lifespan of the specimen [46]. In this phase, the initial rise in temperature represents the material’s response to the sudden movement of dislocations and defects accompanied with surface intrusion and extrusion [3-6, 39]. During the second stage, the cyclic stress and strain response becomes stable. As a result, the hysteresis energy generation and heat dissipation are in balance and temperature becomes steady. It is worthwhile to mention that in this stage, a slight decrease in temperature is observed. This can be attributed to heat loss to the surroundings [45] due to the creation of new surfaces as a result of micro-cracks formation [46]. Hence, in the process of heat balance between the hysteresis-energy generation and dissipation to the surroundings, the surface temperature of the specimen decreases slightly [45, 47]. In the third phase which last within 5-10% of the entire lifespan, the temperature rises rapidly after a comparatively small number of cycles until failure occurs due to large plastic deformation caused by stress concentrations at the crack tips [48-51].

Figure 4.4 shows an example of the surface temperature evolution of SS 304 specimen subjected to a three-stage loading (50.8 mm to 46.99 mm to 44.45 mm). The temperatures were measured near the clamped end of the specimen where fracture occurs. The surface temperature is seen to rapidly increase at the beginning of the test due to inelastic effect [50] up to the point where the load is switched to a lower value. During the first loading period, first, slip bands occur followed by fine micro-cracks. Since the switching the load must be manually applied, the specimen’s surface temperature drops.
Figure 4.3. Typical temperature evolution during constant load. During the first stage, temperature drastically increases due to inelastic effect. Then, during equilibrium stage (stage II) temperature slightly decreases due to the formation of new surfaces as a result of micro-cracks creation. Finally, temperature abruptly increases according to the large plastic deformation at the tip of the macro-crack.

Figure 4.4 Temperature evolution vs. fatigue life for bending test of SS 304 under three-stage variable load (50.88 mm to 46.99 mm to 44.45 mm) at frequency 10 HZ. During the first stage, temperature increases up to the point where the load is switched to the lower value. During the second stage temperature starts to increase and then tend to a stationary value until the beginning of the third stage where the load is changed. At this stage the specimen experiences another temperature increase and stationary value until it suddenly begin to rise, shortly before failure occurs. The beginning of each load follows with temperature increase to up the stationary temperature due to inelastic effect as a result of material resistance to deformation. Total process of fatigue damage consists of micro-crack initiation, micro-crack propagation, macro-crack initiation and macro-crack propagation.
4.4 Results and Discussion

To investigate the role of variable loading effects on the crack initiation and propagation based on the accumulation of entropy generation, we present the results of a series of multistage loading tests (with both two- and three-stage loading sequences) and make use of the general fatigue damage formula, Equation (4.7), and the entropy generation relationship, Equation (4.12), to evaluate the evolution of damage.

4.4.1 Multistage Loading

This section presents the results of fatigue damage evolution for bending fatigue tests of Al 6061-T6 and SS 304 at frequency 10 HZ under both two- and three-stage amplitude loading sequences.

4.4.1.1 Two-Stage Loading

In order to examine the validity of damage evolution relation based on the entropy production (Equation 4.7) under variable loading, Bhattacharya & Ellingwood [28] developed the following fatigue damage relationship applicable to two-stage loading sequence.

\[
D = 1 - (1 - D_0) \prod_{k=1}^{n_1} f(\varepsilon_k, \Omega) \prod_{k=n_1+1}^{n_2} f(\varepsilon_k, \Omega) \tag{4.14}
\]

where \( n_1 \) and \( n_2 \) represent the number of cycle during stage 1 and 2, respectively. \( \varepsilon_k \) denotes the strain limit in cycle \( k \). \( \Omega = \{E, \sigma', \epsilon', n', S_e\} \) represent a set of material parameters. Specifically, \( \sigma' \) and \( \epsilon' \) are the fatigue strength and the fatigue ductility coefficient, \( n' \) represents the cyclic strain hardening exponent, and \( E \) and \( S_e \) are the elastic modulus and the endurance limit, respectively. The function \( f(\varepsilon_k, \Omega) \) is defined as follows.

\[
f(\varepsilon_k, \Omega) = \frac{\frac{1}{l + n'} \Delta\varepsilon_{\text{tot}}^{\text{tot}} - \Delta\varepsilon_0^{\text{tot}} \Delta\varepsilon_0 + C_k}{\frac{1}{l + n'} \Delta\varepsilon_{\text{tot}}^{\text{tot}} - \Delta\varepsilon_0^{\text{tot}} \Delta\varepsilon_0 + C_k} \tag{4.15}
\]

where
\[ C_k = \frac{3}{4} \frac{\sigma_f}{K'} - \frac{\Delta e^{s_{n'}}_{pl}}{I + n'} + \Delta e^{s_{n'}}_{pl} \Delta e^{s_{n'}}_{pl} \]  

(4.16)

\[ \Delta e_{pm}, \Delta e^{s_{n'}}_{pl}, \text{ and } \Delta e^{s_{n'}}_{pl} \text{ are obtained in terms of from } \Delta \sigma_m \text{ and } \Delta \sigma_f \text{ which are maximum and minimum nominal stress.} \]

\[ \Delta e_{pm} = \left( \frac{\Delta \sigma_m}{K'(I - D_{k-1})} \right)^{\frac{1}{n'}} \]  

(4.17)

\[ \Delta e^{s_{n'}}_{pl} = \left( \frac{\Delta \sigma_{\delta}}{K'(I - D_{k-1})} \right)^{\frac{1}{n'}} \]  

(4.18)

\[ \Delta e^{s_{n'}}_{pl} = \left( \frac{\Delta \sigma_{\delta}}{K'(I - D_{k-1})} + \frac{S_e}{K'} \right)^{\frac{1}{n'}} \]  

(4.19)

where \( D_{k-1} \) is the damage value in cycle \( k - 1 \).

The material properties are reported in Table 4.3 [52-55].

<table>
<thead>
<tr>
<th>Material</th>
<th>( E ) (Gpa)</th>
<th>( K' ) (Mpa)</th>
<th>( S_e ) (Mpa)</th>
<th>( \sigma'_f ) (Mpa)</th>
<th>( \varepsilon'_f )</th>
<th>( n' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al 6061-T6</td>
<td>69</td>
<td>426</td>
<td>93</td>
<td>535</td>
<td>1.34</td>
<td>0.062</td>
</tr>
<tr>
<td>SS 304</td>
<td>183</td>
<td>1660</td>
<td>128</td>
<td>1000</td>
<td>0.25</td>
<td>0.171</td>
</tr>
</tbody>
</table>

Figure 4.5 shows damage variation versus fatigue life of Al 6061-T6 subjected to a loading amplitude of 41.91 to 34.29 (mm) and 34.29 to 41.91 (mm) 10 Hz. The specimens are initially free of micro-cracks and defects (i.e., \( D_0 = 0 \)), and thus free of entropy history, i.e. \( s_0 = 0 \). Degradation process starts with both slip bands intrusions and extrusions on the surface of the specimen. Then, the micro-cracks initiated in local slip bands tend to coalesce and grow along planes of maximum shear stress [5, 39, 56-58]. As soon as the load changes, the existing micro-cracks tend to move along new planes of maximum shear stress. Consequently, the trend of degradation changes and a knee point appears in the damage evolution. Simultaneous with damage process, entropy accumulates in every cycle. As cycling progresses, micro-cracks...
combine and create a major crack known as critical damage initiation or critical entropy production [36, 38]. Finally fracture takes place after macro-crack propagation.

It should be noted that the loading sequence has a major influence on the total fatigue life. At a high stress level, a great deal of total fatigue life is spent in the growth of micro-cracks and macro-cracks. At a low stress level, a large fraction of total fatigue life is associated with the nucleation of cracks and micro-cracks growth [39]. Hence, the specimen subjected to a Low-to-High loading experiences longer life than that of a specimen subjected to High-to-Low [29, 38]. The results show that the present experimental work is in good agreement with the results based on Bhattacharya & Ellingwood [29] theory. The maximum error based on uncertainty analysis is about ±1%.

The variation of the damage parameter versus the fatigue life for two-stage loading is plotted in Figure 4.6 for SS 304 based on Equation (4.7) and (4.14). The two stages are: (a) High-to-Low from 46.99 to 36.83 mm and (b) Low-to-High from 36.83 to 46.99 mm. The results presented in Figure 4.6 reveal that the damage caused by the High-to-Low sequence is more severe than that of the Low-to-High sequence. This is predicted both by the CDM approach of Bhattacharya & Ellingwood (Equation 4.14) and the present entropy approach. Clearly, the duration of fatigue crack from the inception to failure depends on the stress level. When the stress is changed from high to low, Figure 4.6 (a), the amount of entropy production reduces due to a lower plastic energy released at the crack tip. On the contrary, in Figure 4.6 (b), degradation starts with a low-load associated with lower entropy production. With changing to the high-load level, the entropy production per cycle increases due to high stress concentration at the crack tip. The duration of the process of slip bands intrusions and extrusions and creation of dislocation during High-to-Low load is shorter than that of Low-to-High [5, 39]. Hence, fatigue life of the Low-to-High load level is longer than that of the High-to-Low loading sequence.
Figure 4.5. Damage evolution vs. fatigue life of present experimental work and the work of Bhattacharya & Ellingwood [38] for Al 6061-T6 at a) High-to-Low load: 41.91 to 34.29 (mm) b) 34.29 to 41.91 (mm). Damage accumulates up to the knee point. At this point entropy production and damage serves as an initial condition for the next stage of the loading.

Figure 4.6. Damage evolution vs. fatigue life of present experimental work and the work of Bhattacharya & Ellingwood [38] for SS 304 at a) High-to-Low load: 46.99 to 36.83 (mm) b) 36.83 to 46.99 (mm). Damage accumulates up to the knee point. At this point entropy production and damage serves as an initial condition for the next stage of the loading.

4.4.1.2 Three-Stage Loading

Fatigue tests under three-stage loading for different displacement amplitudes of Al 6061-T6 and SS 304 are carried out to examine the validity of damage evolution-based entropy production
formula. Two different loading sequences are: High-to-Middle-to-Low loading amplitude and Low-to-Middle-to-High amplitude loading.

Figure 4.7 depicts the evolution of damage versus the fatigue life of Al 6061-T6 for 46.99 to 39.73 to 31.75 (mm) and 31.75 to 39.73 to 46.99 (mm). Entropy progressively increases from the beginning of the test and what is accumulated at the end of the first stage becomes the initial condition for the second stage (i.e., the first knee point) and so forth. At the beginning of each knee point, the fatigue damage behavior varies due to the change of stress level and the entropy production rate, i.e., in Figure 4.7(b), micro-crack growth rate, the degree of irreversibility and entropy production per cycle accelerates due to the decrease of strain energy per cycle. In contrast, in Figure 4.7(a) the rate of degradation decelerates due to the transition to a lower load level. It is also to be noted that changing the load level varies the stress concentration at the tip of the cracks. Hence, the amount of plastic energy, which affects entropy production rate, changes. It can be seen that the proposed damage evolution model based on entropy production is capable of predicting-degradation process of the specimen under multistage loading.

### 4.4.2 Universal Damage Evolution

In this section we develop a universal damage evolution model based on the concept of the thermodynamic fatigue fracture recently introduced in [36]. We show how the concept of tallying the entropy production during the fatigue process can be utilized to assess degradation. According to Naderi et al. [36], simultaneous with the rise of degradation, entropy continuously increases towards the fatigue fracture entropy (FFE). Research shows that regardless of the type of the mechanical fatigue process (i.e, bending or torsion, or tension-compression), FFE is constant unique to the material. Specifically, for Al 6061-T6, $\text{FFE} \approx 4 \text{ MJ m}^{-3}\text{K}^{-1}$ and for SS 304, $\text{FFE} \approx 60 \text{ MJ m}^{-3}\text{K}^{-1}$.

Damage evolution versus normalized entropy production is plotted in Figure 4.8 and 4.9 for Al 6061-T6 and SS 304, respectively. These results are normalized with respect to the FFE.
values for AL6061-T6 and SS 304. These results have two important implications: a) fatigue damage accumulation is independent of load, frequency, and geometry; and b) damage behavior under variable loading has no knee point and independent of multistage loading and loading sequence. As damage progresses toward the final fracture, the accumulated entropy monotonically increases to the fracture fatigue entropy (FFE) which is unique for a given material [36]. Note that cumulative entropy production over individual amplitudes sums to unity: 

$$\sum_{k=1}^{n} s_k / s_g = 1.$$  

This concept can be utilized as a criterion for fatigue damage monitoring and preventing the system under cyclic loading before failure occurs.

![Figure 4.7. Damage evolution vs. fatigue life of present experimental work for Al 6061-T6 at a) High-to-Middle-to-Low load: 46.99 to 39.73 to 31.75 (mm) b) Low-to-Middle-to-High: 31.75 to 39.73 to 46.99 (mm). Damage accumulates up to the knee point. At this point entropy production and damage serves as an initial contention for the next stage of loading.](image-url)
Figure 4.8. Damage evolution vs. normalized entropy production (normalization with respect to FFE) different fatigue tests (bending, torsion, and tension-compression test) with different loads, thicknesses, loading sequences and frequencies for Al 6061-T6. Damage evolution-based entropy production is independent of load, frequency, and loading history and has no knee point under variable amplitude loading.

Figure 4.9. Damage evolution vs. normalized entropy production (normalization with respect to FFE) of different fatigue tests (bending, torsion, and tension-compression test) with different loads, thicknesses, loading sequences and frequencies for SS 304. Damage evolution-based entropy production is independent of load, frequency, and loading history and has no knee point under variable amplitude loading.
4.5 Conclusions

A series of low-cycle bending, torsion and tension-compression fatigue experiments is performed to evaluate the general damage evolution-based on the thermodynamic entropy production relation under constant and variable amplitude loading. Infrared thermographic technique is used to capture the surface temperature evolution of the specimen under fatigue process, and entropy production is evaluated using the surface temperature of the specimen. Entropy production is utilized as an effective tool to measure damage as a degradation of material under fatigue process. The primary result is a strong correlation between degradation and entropy production under multistage variable loading. It is shown that damage variation curve plotted against entropy production is free of knee point, independent of load, frequency, load history, and geometry, and has a universal behavior for Al 6061-T6 and SS 304 as the materials under test in this work. Based on the experimental results, a general damage variable is defined as a function of entropy production of the form

\[ D = \sum_{k=1}^{n} \left( D_{n_{k-1}} + \frac{(D_c - D_{n_{k-1}})}{\ln \left( \frac{1-s_{g1}}{1-s_{gk}} \right)} \ln \left( \frac{1-s_{gk}}{1-s_{g1}} \right) \right) \].

It is revealed from this equation that the irreversible thermodynamic entropy can be utilized to measure the degradation of a system under variable loading with excellent comparison to the experimental measurements. As a result, it is shown that the sum of the entropy fractions of the individual amplitudes is unity and the universal trend of damage evolution can be applied for structural monitoring system in order to halt the operation of the system under fatigue prior to failure.

4.6 References


Chapter 5: Thermodynamic Analysis of Fatigue Failure in a Composite Laminate
5.1 Introduction
Composite materials—owing to high strength and stiffness, low weight, and high fatigue life—are widely used in the industry and their applications are steadily growing. These materials are anisotropic and inhomogeneous and thus exhibit significantly more complicated behavior than their metal counterparts, particularly during repetitive cyclic loadings. In general, the fatigue degradation behavior of a composite laminate is characterized by a combination of matrix cracking, delamination, fiber/matrix debonding, and fiber breakage [1-2]. Thus, the complexities of multi-mode fatigue damage mechanisms in composite laminates have hampered the development of a general fatigue failure criterion [2].

A survey of literature reveals that there are a number of notable research works devoted to the prediction of fatigue life in composite laminates. One of the earliest models of fatigue life prediction is due to the work of Hashin and Rotem [3] followed by Sims and Brogdon [4] who based their analyses on the conventional S-N curve method. They replaced the static strength failure criterion with the fatigue strength failure criterion and related it to the number of cycles to failure. Fawaz and Ellyin [5] reported a semi-log linear relationship between the applied stress and the number of cycles to failure. Their analysis was capable of predicting the fatigue life of a composite laminate for different load ratios and load directions by the use of a reference S-N curve. According to Philippidis and Vassilopoulos [6], however, this criterion is very sensitive to the choice of the reference curve. Huang [7] presented a fatigue life prediction model for plain-woven glass fabric-reinforced polyester composites subjected to biaxial repetitive loading. There are a number of other investigators who have also attempted to characterize the fatigue failure based on S-N curve method for woven fabric-reinforced composites. See, for example, Owen and Griffiths [8], Amijima et al. [9], and Khan et al. [10].
One of the promising approaches for estimating the fatigue life of composite materials is based on energy consideration. Ellyin and EL-Kadi [11] postulated that the strain energy can be utilized to predict fatigue failure and showed that the fatigue life is related to the cyclic energy—a combination of number of cycles at failure and material constants—through a power-law relation. Using the critical plane approach (the plane in which cracks initiate and propagate), Plumtree and Cheng [12] and Peterman and Pulmtree [13] developed a fatigue failure criterion model. Natarajan et al. [14] also put forward a fatigue model based on the strain energy density by performing tension-tension and bending fatigue experiments on fiber-reinforced polymeric composites. Later, Shokrieh and Taheri-Behrooz [5] proposed a fatigue life model based on the energy method for unidirectional polymer composite laminates under tension-tension and compression-compression fatigue loading. To account for the effect of off- and on-axis strain energy on fatigue life, Varvani-Farahani et al. [2] proposed a fatigue life model for three different damage stages: matrix cracking, fiber matrix interface delamination, and fiber breakage.

Most of the Fatigue failure prediction models discussed above are typically performed under constant amplitude and simplified loading conditions. To date, a unified fatigue criterion for a composite laminate subjected to variable loading remains lacking.

The premise of this chapter is that the strain energy density associated with permanent degradation during fatigue must be accompanied by irreversible entropy gain in accordance with the second law of thermodynamics [16]. Thus, it is hypothesized that entropy—a fundamental parameter that characterizes disorder—offers a natural measure for assessment of deterioration [17-18]. More recently, Naderi et al. [19] have demonstrated that by tallying the entropy production in two metallic specimens (Al 6061-T6 and SS 304) up to the fracture point, one arrives at a unique material property—a so-called fracture fatigue entropy (FFE). Within a range of experimental conditions, this parameter was found to be independent of geometry, load, and frequency. In other words, the necessary and sufficient condition for a metal to fracture due to
cyclic loading is that its accumulated production of entropy reaches a certain level. In this paper, we quantify the entropy gain in a displacement-controlled bending fatigue and load-controlled tension-tension fatigue in a Glass/Epoxy (G10/FR4) laminate subjected to either constant- or variable-amplitude loading and determine the associated FFE.

The outline of this chapter is as follows. In section 5.2, we present the fundamental of the theory associated with entropy accumulation and introduce the concept of FFE. In section 5.3, experimental procedure to assess fatigue life prediction using entropy concept is presented. Results of entropy calculation are discussed in section 5.4, followed by the conclusions.

5.2 Theory and Formulation

The first law of thermodynamics states that the internal energy within an arbitrary control volume changes only if energy flows into (or out of) the control volume (Figure 5.1) [20] in accordance to the following expression:

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt}$$  \hfill (5.1)

where \(E\) is total energy, \(t\) denotes time, \(Q\) and \(W\) are heat flow and work across the boundary of the control volume, respectively.

For an open system, which is allowed to exchange heat with its environment (Figure 5.1), the total exchange of entropy \(dS\) consists of two terms: the entropy flow into or out of the system, \(dS\), and the entropy production within the system, \(dS\) [21-22], i.e.,

$$\frac{dS}{dt} = \frac{dS}{dt} + \frac{dS}{dt}$$  \hfill (5.2)

The entropy exchange with the surroundings, \(dS\), can be either positive or negative. But according to the second law of thermodynamics (Clausius-Duhem inequality) the rate of entropy generation within the system must be non-negative, i.e.,

$$dS \geq 0$$  \hfill (5.3)
In terms of the specific quantities, Equations (5.1) and (5.2) can be written as [19]:

\[
\frac{\rho}{dt} = -\nabla \cdot J^q + w
\]  

(5.4)

\[
\dot{\rho} = -\nabla \cdot J^s + \gamma
\]  

(5.5)

where \( \rho \) is the density, \( u \) denotes specific internal energy, \( J^q \) is heat flux across the boundary, and \( w \) is the rate of volumetric work of permanent deformation. The parameter \( s \) represents the total entropy per unit mass. \( J^s \) is the entropy flow and \( \gamma \) denotes the volumetric entropy production.

Using the definition of Helmholtz free energy, \( \Psi = u - \theta s \), the energy balance and the entropy production Equations (5.4) and (5.5) can be rewritten as [19].

\[
\rho \left( \dot{\Psi} + \dot{s} + \dot{\theta} \right) = -\nabla \cdot J^q + w
\]  

(5.6)

\[
\dot{\gamma} = \frac{w}{\theta} - \frac{\rho \left( \dot{\Psi} + \dot{s} + \dot{\theta} \right)}{\theta} - \frac{\mathbf{J}^s \cdot \nabla \theta}{\theta^2} \geq 0
\]  

(5.7)

where \( \theta \) represents the temperature.
By introducing the specific heat capacity, $c$, and the Fourier’s heat conduction law (\( J^t = -k \nabla \theta \)) into Equations (5.6) and (5.7), one can obtain the general form of the heat conduction equation and entropy production inequality as follows [19]:

$$k\dot{\theta} + \omega = \rho c \dot{\theta}$$  \hspace{1cm} (5.8)

$$\dot{\gamma} = \frac{\omega}{\dot{\theta}} + \frac{k}{\theta^2} \dot{\theta} \dot{\theta}_\gamma \geq 0$$  \hspace{1cm} (5.9)

where $k$ is the thermal conductivity.

Equation (5.9) consists of a combination of two terms: the mechanical dissipation due to permanent deformation ($\dot{\gamma}_{\text{mech}} = \frac{\omega}{\dot{\theta}}$) and the thermal dissipation due to heat conduction ($\gamma_{\text{cond}} = \frac{k}{\theta^2} \dot{\theta} \dot{\theta}_\gamma$).

The fracture fatigue entropy (FFE), $\gamma_f$, can be obtained by integrating Equation (5.9) up to the time $t_f$ when failure occurs [19]:

$$\gamma_f = \int_0^{t_f} \left( \frac{\omega}{\dot{\theta}} + \frac{k}{\theta^2} \dot{\theta} \dot{\theta}_\gamma \right) dt$$  \hspace{1cm} (5.10)

Equation (5.10) reveals that the cyclic hysteresis energy ($\omega$) is a crucial parameter for determining FFE. In metals, $\omega$—the so-called cyclic plastic energy—is a function of fatigue parameters such as cyclic strain hardening, fatigue ductility coefficient, and fatigue strength coefficient. To estimate the FFE for a metallic component, one can make use of the cyclic plastic energy formula such as the Morrow’s equation [23] for $\omega$. Theoretical procedure and experimental verification of thermodynamic fracture entropy was recently reported by the authors [19, 24]. However, in composite materials, the hysteresis energy variation is much more complicated than their metal counterparts and, to the best of our knowledge, there are no expressions available to relate cyclic hysteresis energy to composite fatigue properties.
Therefore, in this chapter, experimentally determined the hysteresis energy is used for this purpose. The detailed of this approach is described in Section 5.4.1.1.

5.3 Material and Experimental Procedure

The material studied is Glass /Epoxy (G10/FR4) which is an unbalanced woven fabric composite with plain weave and aligned configuration and consists of a continuous filament glass cloth with an epoxy resin binder. The plain woven glass fabric is stacked in fifteen and twenty four layers with two different thicknesses of 3 and 4.85 mm. Each woven layer has two unidirectional layers stacked in [0°/ 90°]. This type of a composite offers high tensile and flexural strength (see Table 5.1) and thus finds use in a variety of applications such as electrical equipment, aerospace structures, and rocket structural components. Specimens are prepared with on- and off-axis stacking sequences. In on-axis stacking, the warp and weft directions are aligned with the load direction. The former is called lengthwise (0°) while the latter is called crosswise (90°). In off-axis stacking, the angle between the warp and the load direction is (α =15°, 30°, 45°, 60° and 80°, in this study); See Figure 5.2a. Figure 5.2a also presents a schematic diagram of the experimental setup for tension-tension fatigue test. The apparatus used is MTS 810 servohydraulic single actuator. Sinusoidal fatigue loads are defined in Multipurpose TestWare (MPT) software which controls the fatigue test and applies with a frequency in the range of 5 to 15 Hz and a load ratio of 0 and 0.1. Constant and variable loads are applied in both the on-axis (0 and 90°) and the off-axis (15, 30, 45, 60 and 80°) directions. Variable loading tests include both high-to-low and low-to-high sequences.

Figure 5.2b shows the specimen dimensions used in a fully reversed bending fatigue test. The apparatus is a compact, bench-mounted unit with a variable-speed motor, variable-throw crank connected to the reciprocating platen, with a failure cut-off circuit in a control box, and a cycle counter. The variable throw crank is infinitely adjustable from 0 to 50.8 mm to provide different levels of stress amplitude.
Table 5.1. Mechanical properties of G10/FR4

<table>
<thead>
<tr>
<th></th>
<th>Lengthwise</th>
<th>Crosswise</th>
<th>Lengthwise</th>
<th>Crosswise</th>
<th>Lengthwise</th>
<th>Crosswise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile Strength (MPa)</td>
<td>275</td>
<td>240</td>
<td>380</td>
<td>310</td>
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<td>15</td>
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<tr>
<td>Flexural Strength (MPa)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic Modulus in Flexure (GPa)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Figure 5.2a-b Specimen geometry and schematic diagram of the experimental setup in tension-tension fatigue test and specimen geometry in bending fatigue test (all dimensions are in mm).

Test conditions are summarized in Table 5.2. Tension-tension fatigue tests are carried out with a total of 62 constant loads and 16 variable loads (high-to-low and low-to-high) with 3- and 4.85 mm-thick specimens in the range of 2 to 10 KN load amplitude. For the bending fatigue tests, 16 constant amplitude loads with the 3 mm-thick specimens are performed.
Table 5.2. Fatigue Test conditions

<table>
<thead>
<tr>
<th>Fatigue Test</th>
<th>Frequency (Hz)</th>
<th>Specimen Thickness (mm)</th>
<th>Number of Tests</th>
<th>Load Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension-Tension</td>
<td>5, 10, 15</td>
<td>3, 4.85</td>
<td>62, 10</td>
<td>2-10 (KN)</td>
</tr>
<tr>
<td>Bending</td>
<td>10</td>
<td>3</td>
<td>16</td>
<td>-</td>
</tr>
</tbody>
</table>

The instrumentation includes a high-speed, high-resolution infrared (IR) thermography used to record the temperature evolution of the specimen during the entire experiment. The IR camera is a MIKRON M7500 with temperature range between 0 °C to 500 °C, resolution of 320 × 240 pixel, accuracy of ±2% of reading, sensitivity/NETD of 0.08 °C at 30 °C, and image update rate of 7.5 Hz. Before fatigue testing, the surface of the specimen is coated with black paint to increase the thermal emissivity of the specimen surface.

5.4 Results and Discussion

A series of tension-tension and bending fatigue tests are performed under constant and variable amplitude loading. All tests are conducted until fracture occurs to determine the fatigue fracture entropy (FFE) associated with each specimen. In the following sections the results of tension-tension and bending fatigue tests are discussed.

5.4.1 Tension-Tension Fatigue

In the following sections, we present the results of experimentally-obtained hysteresis energy and temperature evolution that are necessary for thermodynamic analysis followed by experimental entropy generation evaluation.

5.4.1.1 Hysteresis Energy

Figure 5.3 shows the area within the hysteresis loop—the so-called strain energy—increases during a series of tension-tension fatigue tests. The hysteresis area of each cycle is obtained using a MATLAB™ code that utilizes the experimental load and displacement data between
loading and unloading paths. Typical experimentally-determined hysteresis curves at 5% of the total life, 50% of the total life, and just before failure are also presented in Figure 5.3. Examination of the strain energy reveals three distinct stages. The first stage is limited to about 5 to 10% of the total life; during the second stage, gradual and slow increase in strain energy occurs and lasts about 70-75% of total life; and the third stage the fibers tend to break and high strain energy is released.

![Hysteresis Energy per Cycle](image)

**Figure 5.3** Accumulation of hysteresis energy per cycle during tension-tension fatigue test at different constant load amplitudes, frequency of 10 Hz, and $R = 0$. The hysteresis energy evolution has three phases: initial increase, gradual and slow rise and abrupt rise.

### 5.4.1.2 Temperature, Entropy Evolution, and Entropy Production

Figure 4 shows the surface temperature recorded using an IR camera plotted versus the number of cycles-to-failure for a series of tension-tension fatigue test at frequency of 10 Hz and $R=0$. The temperature plotted in this figure is measured at the cross section where failure occurs. It shows that, under the conditions tested, temperature increases rapidly over the first 20% of the total life, followed by a more gradual increase. Finally, a sudden increase of temperature takes
place and the specimen fractures. A similar trend of the temperature evolution for woven and Polypropylene/Glass fiber composite is reported by [25-26].

![Graph showing temperature increase](image)

**Figure 5.4** Evolution of surface temperature increase during tension-tension fatigue test at different constant load amplitudes, frequency of 10 Hz, and load ratio $R=0$. Temperature increases initially, continues with slow and steady increase, and rises prior to failure.

We now turn our attention to the entropy production rate and entropy accumulation calculated based on Equations (5.9) and (5.10) for tension-tension fatigue test. The results of entropy generation are plotted in Figures 5.5a-b along with the corresponding SEM images of the surface of the specimen as well as the fracture cross section shown in Figures 5.5c-g. Examination of Figures 5.5a-b reveals that similar to the temperature rise, the entropy production undergoes three separate stages: an initial rise (Stage I), slow and steady increase (Stage II), and a drastic increase prior to failure (Stage III). During Stage I, entropy accumulates due to three mechanisms: the high energy released at the tips of micro-cracks in multiple locations in the matrix between layers, debonding at the cross-over points between warp and weft fibers and matrix, and the breakage of some fibers with low strength. Degradation grows and entropy generation piles up from the beginning of the test and continues to the onset of Stage II. Due to a comparatively small number of cycles in this phase, the accumulation of entropy is not
appreciable. As soon as the deterioration in the material reaches the saturation level, the trend of degradation changes. At this stage, the process of entropy production reduces due to a lower rate of heat generation and a slower damage process. However, entropy accumulation in Stage II is more pronounced than that of the previous stage due to a higher span of fatigue life there (around 55% of total life).

At the beginning of Stage III, the fatigue damage behavior and the degree of irreversibility vary due to the change of degradation modes from matrix cracking to fiber breakage. More energy is released due to the occurrence of fiber breakage during this stage. Both entropy production and its accumulation also increase with time until failure occurs. It is also to be noted that changing the degradation mode affects the heat dissipation rate and consequently the rate of entropy production.

The SEM images of Figs. 5c-g also show the details of matrix cracking, debondings, and delamination of the weft and the warp strands. Figures 5.5c-d show the SEM images of the specimen surface captured after completing approximately 85% of the total life and Figures 5.5e-f show the SEM images of the specimen taken after about 95% of total life is depleted. The SEM image of the fracture cross section is shown in Figure 5.5g. Examination of Figures 5.5c-d reveal that some of the debonded regions join each other at interface between the warp and the weft yarns. Debondings continue to propagate and delaminations take place between warp and weft bundles at the fabric cross-over points as well as between the adjacent layers in the Glass/Epoxy laminate. As fatigue process progresses to around 95% of the total life (Figures 5.5e-f), breakages at the delaminated areas occur and fiber strands are separated from the surface. Debondings and delaminations are the origin of more cracks in the pure matrix region. These cracks also propagate into some of the fiber bundles as shown in the SEM image of the fracture cross section (Figure 5.5g).
Figure 5.5a-d. a) Entropy production during tension-tension fatigue test of 5.25KN load amplitude at the frequency of 10 Hz, and $R = 0$, respectively. The entropy production undergoes three stages: initial increase, gradual and slow rise and abrupt rise. b) Entropy accumulation. Entropy accumulates during fatigue until reaches the critical value at the fracture. c-d) SEM image of the surface of the specimen around 85% of total life. e-f) SEM image of the surface of the specimen around 95% of total life. g) SEM image of the fracture cross section of woven Epoxy/Glass laminate. SEM images show different failure mechanism such as matrix cracking, debonding between fiber strands and matrix, delamination between warp and weft fiber strand at the cross-over points, and fiber breakage.
(Figure 5.5 continued)

Connection of debonding area at the cross-over points of weft and warp yarns

Debonding area between warp or weft strands and matrix

Connection of debonding area at the cross-over points of weft and warp yarns

Debonding area at the cross-over points of weft and warp yarns

Cracking at cross-over points

Debonding area and matrix cracking

Matrix

(c)

(d)
(Figure 5.5 continued)

- **Figure 5.5 (e)**
  - Fiber bundle breakage
  - Delamination area at the cross over points of weft and warp yarns

- **Figure 5.5 (f)**
  - Delamination and fiber breakage
  - Warp fibers
  - Weft fibers
  - Debonding and matrix cracking
Figures 5.6 a-c depict the results of FFE versus number of cycles for tension-tension fatigue tests. Tests are carried out for two different load ratios of $R = 0$ and 0.1. The results of entropy accumulation subjected to variable loading (high-to-low and low-to-high load) are also shown in Figures 5.6a-c. It is to be noted that according to an order of magnitude analysis, entropy generation owing to the mechanical dissipation is dominant and the entropy generation due to heat conduction is negligible (See Appendix B). Hence, in the calculation of FFE, heat conduction effect is neglected. The Results cover different frequencies within the range of 5 to 15 Hz, constant- and variable- load amplitude, and off-axis angles of 0, 15, 30, 45, 60, 80, and 90°. The results show that FFE at the fracture point for G10/FR4 is approximately 0.8 MJm$^{-3}$K$^{-1}$ for 90°, 1.0 MJm$^{-3}$K$^{-1}$ for 0° and 1.0 MJm$^{-3}$K$^{-1}$ for off-axis angle (15, 30, 45, 60, and 80°). Within the range of operating conditions tested, FFE is independent of frequency, load and fiber orientation.
Figure 5.6 a-c Fracture fatigue entropy versus the number of cycles to failure for different tension-tension fatigue tests of Glass/Epoxy (G10/FR4) laminate with different specimen thicknesses, frequencies and loads. a) Fracture fatigue entropy in crosswise (90°) direction. Constant loads are in the range of 4.9 KN-9.5 KN and variable loads are 5.15 KN to 6KN and 6 KN to 5.15 KN. b) Fracture fatigue entropy in lengthwise (0°) direction. Constant loads are in the range of 5 KN-13.5 KN and variable loads are 5.7 KN to 6 KN and 6 KN to 5.7 KN. Fracture fatigue entropy is within the range of 1 to 3 MJm⁻³K⁻¹. c) off-axis fracture fatigue entropy of 15°, 30°, 45°, 60° and 80°. Loads are in the range of 2.5KN to 5KN.
The accumulated fracture energy (total hysteresis energy) and FFE are plotted in Figures 5.7a-b for all the experimental tests performed at different load ratios, load amplitudes, and load angles. The accumulated fracture energy (AFE) is calculated by summing up the hysteresis energy per cycle to the total number of cycles at failure \( w_i = \frac{N_f}{\sum w_j} \). Figures 7a-b show that the predictions of the fatigue life based on AFE and FFE results have similar trends. However, the cumulative entropy concept distributes the experimental data within relatively narrower limits regardless of the load angles. Figure 5.7a also reveals that AFE of on-axis and off-axis angles close to warp and weft directions have different trends than those of the off-axis angles far from warp and weft directions. Failure in the weft and the warp directions and in off-axis angles close to on-axis are mostly dominated by the normal stress. However, the shear failure is mostly dominated by shear stress at the angles far from the warp and the weft directions.

In Figure 5.7b, the experimental data obtained from entropy concept are distributed around two relatively narrow limits, regardless of load angle and load sequence. The reason is that in
contrast to the accumulated fracture energy, thermodynamic entropy incorporates both the energy concept and temperature effect in its formulation (Equation 5.10).

Figure 5.7 Comparison of fatigue life prediction models at various load ratios (R=0, and 0.1), and load/fiber angles ($\alpha = 0, 15, 30, 45, 60, 80, 90^\circ$). a) Accumulated fracture energy concept. b) Fracture fatigue entropy criterion.
5.4.2 Bending Fatigue

A series of fully reversed bending fatigue tests is carried out for two different fiber orientations (0 and 90°) at the frequency of 10 Hz. In these series of tests, the dissipated strain energy—which is later utilized to determine the entropy accumulation—is estimated using an experimental procedure described by [27-28]. This procedure involves measuring the cooling rate after a sudden interruption of the fatigue test. Following the work of Meneghetti [27] and Meneghetti and Quaresimin [28], the statement of energy balance reads:

\[ w = h + \rho c \frac{\partial \theta}{\partial t} + \dot{e}_p \]  

(5.12)

where \( w \) is the input mechanical energy. The parameter \( h \) denotes the rate of the dissipated thermal energy in a unit volume due to conduction, convection and radiation. The remaining two terms on the right-hand side account for the rate of variation of internal energy which consists of two terms. The first term depends on the temperature variation of the material. The second term, \( \dot{e}_p \), is the rate of variation of the so-called stored energy of cold work, which is the part of the mechanical input energy responsible for creation of new surfaces, internal cracks, and changes in material micro-structures [29-32].

Referring to Figure 5.8, upon suddenly stopping the fatigue test at time \( t^* \) when the surface temperature reaches \( \theta^* \), the mechanical input power and the rate of variation of cold work, \( \dot{e}_p \), in Equation (12) drop to zero. Hence, just after \( t = (t^*)^+ \) Equation (5.12) can be written as:

\[- h = \rho c \frac{\partial \theta}{\partial t} \bigg|_{t=t^*} \]  

(5.13)

Measuring the temperature change as a function of time during the cooling period after stopping the fatigue test provides the right-hand-side of Equation (5.13). The thermal energy per unit volume per cycle, \( q \), is:
\[ q = \frac{h}{f} \]  

(5.14)

where \( f \) is the frequency of the test.

![Diagram](https://via.placeholder.com/150)

**Figure 5.8** Determination of dissipated thermal energy by experimental measurement of the cooling rate.

### 5.4.2.1 Validation of Hysteresis Energy Estimation

In order to examine the validity of Equations (5.12) to (5.14) for determining the hysteresis energy, we first apply the procedure described above to a tension-tension fatigue test and then compare the results to the hysteresis energy obtained directly using the MTS instrument.

Figure 5.9a shows the strain energy evolution of tension-tension fatigue test at the frequency of 10 Hz, \( R = 0 \), and 5.75 KN load amplitude. The curve-fitting toolbox of MATLAB\textsuperscript{TM} is used to generate the cooling curves from which the rate of drop in temperature, \( \frac{\partial \theta}{\partial t} \), is calculated and used in Equation (5.14) to evaluate the cyclic strain energy. The results show that dissipated energy obtained by stopping the fatigue test is in good agreement with the experimental results of MTS. Since the bending fatigue apparatus is not capable of measuring hysteresis loop, the method is applied for bending tests to determine the strain energy per cycle for different series of tests.
Figure 5.9a Comparison of cyclic strain energy based on the MTS results and the results of measuring the cooling curve for tension-tension fatigue test at the frequency of 10 Hz, R=0, and 5.75 KN load amplitude. 9b Dissipated energy variation during bending fatigue of G10/FR at the frequency of 10 Hz and displacement amplitude of 38.1 mm.

5.4.2.2 Hysteresis Energy in Bending Fatigue

The results of cyclic dissipated energy obtained by stopping the fatigue test versus number of cycles to failure is plotted in Figure 5.9b for bending fatigue test at the frequency of 10 Hz and displacement amplitude of 38.1 mm. The results of dissipated energy for different series of bending fatigue for two different on-axis fiber directions along with temperature data are then used to calculate the FFE presented in Figure 5.10. The abscissa of Figure 5.10 shows the entropy generation at fracture, $\gamma_f$, and the ordinate shows the number of cycles to failure. Examination of the results reveals that by using the FFE parameter, all the results of the experimental data merge together to within a relatively narrow upper and lower limit, independent of load amplitude and fiber orientation. It is shown that the FFE for Glass/Epoxy (G10/FR4) laminate is approximately 3.0 MJm$^{-3}$K$^{-1}$ for 90° and 4.2 MJm$^{-3}$K$^{-1}$ for 0°.
The results presented in Figs. 6 and 10 demonstrate the validity of the hypothesis that the cumulative entropy at failure for Glass/Epoxy (G10/FR4) laminate for tension-tension and bending fatigue tests are confined within a relatively narrow upper and lower bands. The results reveal that the necessary and sufficient condition for final fracture of Glass/Epoxy corresponds to the entropy gain of approximately 0.8 MJm⁻³K⁻¹ for 90°, 1.0 MJm⁻³K⁻¹ for 0° and 1.0 MJm⁻³K⁻¹ for off-axis angle (15, 30, 45, 60, and 80°) in tension-tension fatigue tests and approximately 3.0 MJm⁻³K⁻¹ for 90°, 4.2 MJm⁻³K⁻¹ for 0° in bending fatigue tests.

A possible application of the proposed hypothesis of the constant entropy gain at failure is in the development of a methodology for prevention of the failure of composite laminates subjected to fatigue load. As demonstrated in this work (Figures 5.6 and 5.10), the entropy generation increases during the fatigue life toward a final value of $\gamma_f$. Thus, fracture fatigue entropy, FFE can be utilized as an index of failure. Successful implementation of this concept for monitoring the fatigue fracture of metals has been recently demonstrated [24].
5.5 Conclusions

A thermodynamic approach for characterization of woven Glass/Epoxy (G10/FR4) laminate degradation is proposed which utilizes the entropy generated during the entire life of the specimens undergoing tension-tension and fully-reversed bending fatigue tests subjected to constant and variable loadings. Results show that, within the range of operating conditions tested, the cumulative entropy generation is independent of load amplitude, load sequence, fiber orientation, and frequency. The results also reveal that fatigue failure prediction by tallying entropy criteria distributes all experimental data within two relatively narrow limits for both constant load and variable amplitude load (high-to-low and low-to-high load). Both accumulated fracture energy and accumulated fracture entropy have the similar variation and can be considered as a methodology for prediction of fatigue life. However, the results of entropy accumulation tend to be more unified than that of accumulated fracture energy due to the consideration of the temperature in its formulation (Equation 5.10). The implication of these findings is that the evolution of entropy generation can be utilized to assess the severity of degradation of the specimen and guard against fatigue failure.

5.6 References


Chapter 6: Dissipated Thermal Energy and Damage Evolution of Glass/Epoxy Using Infrared Thermography and Acoustic Emission
6.1 Introduction

The application of composites as a high-performance engineering material is becoming widespread owing to their high stiffness and strength. While parts made of composites are often superior to many other types of materials, they are not immune from degradation and damage. Of particular interest in this chapter is to investigate the characteristic of composite material subjected to fatigue brought about by cyclic loading.

Fatigue damage is an irreversible process which deteriorates the material properties and progressively grows toward a critical condition when failure occurs [1-3]. Relevant to composites materials, there are many experimental [4-22, 27, 31-32, 36-49], theoretical, and numerical [8, 13, 15, 23-26, 28-35] approaches for assessment of fatigue damage. Some methods rely on the assumed damage variable i.e., the degradation of Young’s [15-16, 24-29, 39, 47-49], residual strength degradation [11, 33-35], dissipated mechanical and thermal energy [4-9, 12-14, 30-32, 49] to quantify damage state while others use the adopted investigation technique i.e., thermography and acoustic emission [10, 17-23, 36-38, 40-46] to guard against fatigue failure.

Many non-intrusive methods — eddy current, optical holography, ultrasonic resonance, X-ray, etc.— are available for detecting voids and defects in composite material. Among them, for example, acoustic emission (AE) is utilized as an effective damage evaluation technique which detects failure events as they take place. In addition to AE monitoring, infrared thermography (IR) technique is intensively used for the detection of different damage modes [7, 10, 36, 37]. These powerful techniques are also capable of detecting the location where the failure is likely to occur. Nevertheless, a survey of the open literature reveals that the majority of their applications have been limited to primarily tension-compression fatigue studies and fewer studies are available for flexural fatigue damage evolution and their characterization [10, 11].

In the current study, as an alternative damage assessment method to conventional NDT techniques, the dissipated energy approach is utilized to evaluate the fatigue damage in
composite material. Most of the mechanical work, determined by the area under the hysteresis loop, is the dissipated thermal energy responsible for temperature rise inside the composite laminate. Given that measurements of the hysteresis loops are not always straightforward, an empirical method introduced in [50] is utilized to quantify the dissipated thermal energy in specimens subjected to cyclic loading. In this procedure, the cooling rate of the surface temperature at different number of cycles is measured from which the specific heat loss per cycle is determined.

In this chapter, dissipated thermal energy and damage evolution during bending fatigue for Epoxy/Glass (G10/FR4) composite laminate are calculated based on the temperature drop rate by stopping the fatigue test at various number of cycles. Damage evolution is investigated using the acoustic emission and thermography techniques, the results of which corroborate the findings from the dissipative thermal energy study.

6.2 Theory and Formulation
The following section presents the theory and formulation for characterizing dissipated energy and the estimation of the degradation process using the principles of thermodynamics.

6.2.1 Dissipated Thermal Energy
Structural composite degradation under fatigue loading can be examined by analyzing the dissipated energy evolution in the material. In accordance with the second law of thermodynamics, fatigue damage is an irreversible process related to the dissipated energy. A part of the total mechanical energy is the stored energy composed of two parts: a recoverable elastic part and an irreversible inelastic part. The heat generated raises the temperature of the body and exchanges with the surrounding [4, 9, 13].

According to the law of conservation energy, the internal energy within an arbitrary control volume changes is given by the following equation (Figure 6.1):
\[
\frac{dU}{dt} = \frac{dW}{dt} - \frac{dQ}{dt} \tag{6.1}
\]

where \( W \) is mechanical work, \( Q \) represents dissipated thermal energy, \( U \) is the internal energy, and \( t \) denotes time.

In terms of volumetric quantities, Equation (6.1) can be written in the following form [51]:

\[
w = q + \rho \frac{du}{dt} \tag{6.2}
\]

where \( w \) is the rate of volumetric mechanical work, \( q \) represents the rate of volumetric dissipated thermal energy, \( \rho \) is the density, and \( u \) denotes the specific internal energy.

The time rate of internal energy, \( \rho \frac{du}{dt} \), is composed of two terms: (i) a term, \( q_t \), that depends on the temperature variation of the material in the volume \( V \), and (ii) a term associated with the so-called stored energy of cold work, \( e_{stored} \), as a part of the mechanical work responsible for creation of new surfaces, internal cracks, changes in material micro-structures [4, 52-54].

\[
\rho \frac{du}{dt} = q_t + e_{stored} = \rho c \frac{dT}{dt} + e_{stored} \tag{6.3}
\]

where \( c \) denotes specific heat, and \( T \) represents temperature.

Therefore, Equation (6.2) reads:

\[
w = q + \rho c \frac{dT}{dt} + e_{stored} \tag{6.4}
\]

The dissipated energy (Equation 6.4) is estimated using an experimental procedure described by Meneghetti [50] which involves measuring the cooling rate after a sudden interruption of the fatigue test. Referring to Figure 6.2, upon suddenly stopping the fatigue test at the time \( t^* \) when the surface temperature value reaches \( T^* \), the temperature profile of the specimen drops. Since the variation of the specimen’s temperature field and the room temperature are insignificant over \( t^* \), the dissipated thermal energy, \( q \) in a differential time just before and after \( t^* \) is the same.
Hence, after the interruption of the test, the mechanical input power, $w$, and the rate of the damage energy, $e_{\text{stored}}$, in Equation (6.4) drops to zero [50]. Consequently, just after $t^*$ (that is $t = (t^*)^*$) Equation (6.4) reduces to:

$$-\rho c \frac{\partial T}{\partial t} = q$$  \hspace{1cm} (6.5)

where the thermal energy $q$ per unit volume is dissipated by conduction, convection, and radiation. As the dissipated thermal energy at the section when failure takes place is approximated by Equation (6.5), the thermal energy per unit volume per cycle, $H$, can be simply evaluated by knowing the test frequency:

$$H = \frac{q}{f}$$  \hspace{1cm} (6.6)

where $f$ is the frequency of the test.

---

Figure 6.1 Energy balance for specimen under fatigue loading
Figure 6.2 Determination of dissipated thermal energy by experimental measurement of the cooling rate

6.2.2 Damage Evolution

In this study, the dissipated energy method is utilized to identify damage progression during bending fatigue of Epoxy/Glass using following relation [5]:

$$D = \frac{H - H_0}{H_f - H_0}$$  \hspace{1cm} (6.7)

where $D$ ($0 \leq D \leq 1$, [5]) is the damage parameter. $H_0$ the presents initial value of the dissipated energy and $H_f$ is the final value of the dissipated energy at the time when failure occurs.

When the applied load is of the fully reversed bending type, the initial value of heat dissipation is zero and Equation (6.7) reads:

$$D = \frac{H}{H_f}$$  \hspace{1cm} (6.8)

6.3 Experimental Procedure

The material studied is Epoxy/Glass G10/FR4 which is a laminate composite consisting of a continuous filament glass cloth material with an epoxy resin binder. This type of material offers high tensile and flexural strength (see Table 6.1), and thus find use in a variety of applications such as electrical equipment, aerospace structures, and rocket structural components.
For bending fatigue tests, specimens are prepared with a laminate thickness of 3 mm with their fibers oriented in 0° and 90° directions.

<table>
<thead>
<tr>
<th>Table 6.1. Mechanical properties of G10/FR4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile Strength (MPa)</td>
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<td>Lengthwise</td>
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</tbody>
</table>

Figure 6.3 presents a schematic diagram of the experimental setup for cantilever reverse-bending fatigue tests. The apparatus is a compact bench-mounted unit with a variable speed motor, variable throw crank connected to the reciprocating platen, with a failure cut-off circuit in a control box and a cycle counter. The variable throw crank is adjustable from 0 to 50 mm. For bending fatigue tests, the specimen is clamped at one end and oscillated at the other end with specified amplitude and frequency. Dimensions of the specimen are shown in Figure 6.4. The specimens are manufactured with tapered widths to produce nominally constant stress along the test section in accordance with the ASTM STP standard. High-speed, high-resolution infrared (IR) thermography is used to record the temperature evolution of the specimen during the entire experiment. The IR camera is a MIKRON M7500 with temperature range between 0 °C to 500 °C, resolution of 320 × 240 pixel, accuracy of ±2% of reading, sensitivity/NETD (noise-equivalent temperature difference) of 0.08 °C at 30 °C, and image update rate of 7.5 Hz. Before fatigue testing, the surface of the specimen is covered with black paint to increase the thermal emissivity of the specimen surface.

The acoustic emission system employed consists of a PCI-2, a two-channel AE system on a PCI card, which is capable of sampling at a rate of 10^6 sample/sec. A wide band, 100-900 KHz, sensor is also mounted on the other side of the specimen, where is sufficiently far from the specimen oscillations. The sensor whose diameter is 19.02 mm is installed on the left side of the specimen (Figure 6.3) and is attached via ECHOGEL grade 30a gel type ultrasonic couplant. The
sensor is firmly clamped to the specimen during the experiment. Pre-amplification of 40 db is applied and recording threshold is set to 45 db. Peak Definition Time (PDT), Hit Definition Time (HDT) and Hit Lockout Time (HLT) are set to 50, 200, 300, respectively. The extracted AE features are counts, rise time, and signal strength.

Figure 6.3  Schematic diagram of the experimental setup in bending

Figure 6.4  Geometry of the specimen used for bending fatigue test. All dimensions are in mm (ASTM STP 566).
In the following sections experimental results of acoustic emission and Infrared thermography are utilized to study degradation evolution during bending fatigue test.

### 6.3.1 Infrared Thermography

Figure 6.5a shows the surface temperature recorded using an IR camera (along the dashed line in Figure 6.5b) versus normalized number of cycles. Normalization \( \frac{N}{N_f} \) is performed with respect to the number of cycles at failure \( (N_f) \). The specimen (G10/FR4) is subjected to the frequency of 10 Hz and displacement amplitude of 40.64 mm for which a total number of cycles to failure of 4000 is obtained. Examination of Figure 6.5 reveals that the temperature evolution undergoes three separate stages: an initial rise (Stage I), slow and steady increase (Stage II), and a drastic increase prior to failure (Stage III). During the first stage, temperature increases due to the occurrence of micro-cracks in multiple locations in the matrix, debonding at the weak interface between fibers and matrix, and breaking of some fibers with low strength. This stage is limited to a low number of cycles, typically 10-20% of the entire lifespan of the specimen \([2, 6, 7, 15-17]\). As the fatigue process progresses, the crack density in the matrix reaches a “saturation level,” where the damage growth becomes stable \[17\]. The existing cracks grow toward the fiber/matrix interface where cracks cannot cross the high strength fiber and begin to bifurcate in two directions. One moves along the fiber and the other runs around the fiber with a slow and gradual propagation \[16\]. In the third phase, which lasts approximately 10-20% of the entire lifespan, the temperature rises rapidly after a comparatively small number of cycles until failure occurs due to fiber breakage. It is worth to note that the mechanical properties of the laminate are temperature dependent and are expected to change if temperature rises far beyond the room temperature.
Figure 6.5a Evolution of surface temperature profile of Epoxy/Glass during bending fatigue at the frequency of 10 HZ and displacement amplitude of 40.64 mm revealing three separate stages: an initial rise (Stage I), slow and steady increase (Stage II), and a drastic increase prior to failure (Stage III) Figure 6.5b IR image of the surface temperature at Stage III identified in Figure 6.5a. Normalization ($N/N_f$) is performed with respect to the number of cycles at failure ($N_f$).
6.3.2 Acoustic Emission

Figures 6.6a and b show the cumulative rise-time and cumulative signal strength of the acoustic emissions plotted versus the normalized number of cycles (normalized with respect to number of cycle at failure) for two bending fatigue tests with displacement amplitude of 35.56 mm and 38.1 mm and a frequency of 10 Hz. The rise-time is defined as the time between the beginning of the AE hit and its peak amplitude and the signal strength is obtained via integration of rectified voltage signal over the duration of the signal. The results of these figures also reveal three distinct emission regimes. In the first stage (Stage I of Figures 6.6a and b) the AE events belong to cyclic deflection of the laminate as well as weak points of the material from all parts of the specimen [16]. The duration of the first stage depends on load level of the experiment and lasts about 10-20 % of the total life. Eventually as more cycles pass, the interface between the fibers and matrix breaks and matrix cracking occurs (Stage II of Figures 6.6a and 6b) [16]. Higher applied loads reduce the duration of this stage. The second stage, which incorporates a major fraction of the component life, has a positive slope whose value depends on the number of acoustic events. The third stage, in which fiber breakage occurs, is associated with the high rate of acoustic events and continues until the complete failure of the component.

6.3.3 Comparison of Acoustic Emission and Thermography

Figures 6.7a-b compare the temperature profile and AE cumulative counts. This experiment is carried out on a Glass/Epoxy laminate at the frequency of 10 Hz and displacement amplitude of 38.1 mm. The results of both the AE and the thermography tests exhibit a similar trend. In both cases, the Stage I lasts for an approximately 20% of the total life.
Figure 6.6a  AE Cumulative rise time vs. Normalized number of cycle (normalized with respect to the number of cycle at failure) of Epoxy/Glass laminate at frequency of 10 HZ and displacement amplitude of 35.56 mm. The short stage (Stage I) of AE rise time is followed by a relatively steady increase with a suddenly rise close to the failure. 6.6b AE Cumulative signal strength vs. Normalized number of cycle of Epoxy/Glass laminate at frequency of 10 HZ and displacement amplitude of 38.1 mm.
Figure 6.7 Surface temperature evolution, (a) and AE cumulative counts, (b) respect to the normalized number of cycle for 92% of total life. The results of both AE and IR thermography demonstrate the same damage evolution trend.

Figure 6.8a shows Scanning Electron Microscopy (SEM) image of the surface of the specimen during the first stage of the same test shown in Figure 6.7. As the operation continues, more acoustic events exceed the threshold and as a result the cumulative counts increase. Such
events represent matrix cracking and delamination within the laminate. The transition from Stage I to II, can be easily observed from the rate of the temperature and AE cumulative counts.

To further distinguish the nature of damage in different stages, an SEM image from the second stage of the laminate’s life is presented in Figure 6.8b in which matrix cracking and separation of matrix and fiber is detectable. This image is taken at 40% of the total life. Transition to Stage III occurs when the cumulative number of cycles approaches 90% of the total life. This can be easily recognized from the temperature profile and the cumulative AE counts plots in Figures 6.7a and 6.7b. The SEM image of the final stage, Figure 8c, depicts the fibers breakage of the laminate. After the 90% of the total life, the amount of emissions severely increases.

![SEM image of the surface of Epoxy/Glass during different stages of fatigue life.](image)

(a)

**Figure 6.8** SEM image of the surface of Epoxy/Glass during different stages of fatigue life. 

a) 5% fatigue test (Stage I). b) 40% of total life (Stage II). c) 90% of fatigue life at which temperature profile and acoustic behavior shoot up due to fiber breakage and macro-crack growth (Stage III).
6.4 Results and Discussions

Figure 6.9 presents different cooling curves within seconds after stopping the fatigue test at different number of cycles (26, 65, and 92% of total life) to investigate different cooling rate and
variation of dissipated energy. As cracks grow inside the composite and energy increases, more heat generates and temperature increases accordingly. Hence, halting the tests at various number of cycles results in different cooling rates. The curve-fitting toolbox of MATLAB is used to generate the cooling curves from which the rate of drop in temperature, \( \frac{\partial T}{\partial t} \) (or \( \theta \)), is calculated and are used in Equation (6.5) to evaluate the dissipated energy.

Figure 6.10 shows the dissipated energy evolution versus number of cycle for two different fatigue loadings. Due to the fact that the calculation of the dissipated thermal energy from experimentally measured cooling rates is not valid when the macro crack initiates at the surface [50], the dissipated thermal energy at the end of the specimen life —where macro cracks appear on the surface— cannot be estimated by calculating of the cooling rate. Hence, all the experimental data points except the last point in this figure are obtained using Equations (6.5) and (6.6), after interruption of the test and prior to abrupt temperature rise (Stage III of Figure 6.5). The sudden temperature increase indicates the large amount of dissipated energy is released due to macro-cracks propagation and fiber breakage.

Experimental results of Giancane et al.’s work [5] on composite laminate shows that Stage III of the dissipated energy starts around 90% of total life (\( N_f \)). In [5], it is observed that the value of the dissipated thermal energy before the abrupt increase equals about 75% of the \( H_f \). Therefore, based on the observations of tension-tension tests, the final value of \( H_f \) is estimated as approximately 1.43 times the value measured at about 85% of the total fatigue life. Then, to calculate the heat dissipated energy from the experimentally obtained number of cycles, an analytical model proposed by Wu and Yao [15] defined in Equation (6.9) is employed.

\[
H = H_f \left[ I - \left( 1 - \left( \frac{N}{N_f} \right)^b \right)^4 \right]
\]  

(6.9)
where \( H \) represents dissipated energy during fatigue. \( N \) and \( N_f \) are the number of cycles during fatigue and final failure, respectively. The value of \( A \) and \( B \) are model parameters (Table 6.2).

| Table 6.2. Fatigue parameters of Glass/Epoxy |
|-----------------|----------------|---------|
| \( A \)         | \( B \)         | \( N_f \) |
| 0.35            | 0.45            | 4000    |
| 0.32            | 0.38            | 6800    |

Figure 6.9 Normalized temperature curve vs. time acquisition after the fatigue test has been suddenly stopped at different number of cycles (26, 65 and 92% of total life), frequency of 10 Hz, and displacement amplitude of 40.64 mm. Temperature slope increases as fatigue continues and hence, dissipated energy expands during fatigue.
Figure 6.10 Dissipated energy variation during bending fatigue of G10/FR for two different displacement amplitudes at Frequency of 10 Hz.: a) displacement amplitude of 40.64 mm. b) displacement amplitude of 38.1 mm.
Figure 6.11 shows the variation of the damage parameter plotted as a function of the normalized number of cycles (normalized with respect to the number of cycle at failure) for two different loads at 10 Hz. The damage values are obtained by substituting the calculated dissipated thermal energy (Figures 6.11a and 6.11b) into Equation (6.8). The three previously defined degradation stages are detected in damage trend for two displacement level of 38.1 and 40.64 mm. The results show that as the 90% of total life is reached, the damage parameter approaches about 0.75. At this point the damage parameter experiences a rapid rise before the specimen fails. The results of experimental and analytical damage reveal that this damage parameter is independent of the load and the number of cycle.

![Figure 6.11 Damage evolution during bending fatigue of G10/FR for two different displacement amplitudes](image)

**Figure 6.11 Damage evolution during bending fatigue of G10/FR for two different displacement amplitudes**

Figure 6.11 shows a unified evolution of damage for different load levels. Based on these results and having the values of constants A and B under the conditions tested, one can calculate the damage value for a given load for when no experimental results are available. It should be noted
that more tests are recommended in order to determine more accurate values for A and B constants.

6.5 Conclusion

In this study, dissipated energy and damage accumulation is characterized using the infrared thermography technique for fully reversed bending of Epoxy/Glass. Acoustic emission technique is also used as a damage detection technique to verify the dissipated energy evolution and damage process. The temperature results in characterizing the degradation progression is corroborated by AE results and both methods show the damage evolution in a same manner. The results indicate the existence of three distinct stages during the lifespan of the laminate. The first of which is healthy period of almost 20% of the total life. In this phase of operation matrix cracking occurs at weak points of the material. A second stage is observed in which debonding and fiber matrix delamination take place. The final phase of operation is along with an abrupt temperature and acoustic descriptors increase due to fiber breakage and continue until the complete failure of the component. SEM images captured at three stages, in different cycles of operation, show the damage growth during degradation. The accumulated dissipated energy is obtained using an experimental procedure by stopping the fatigue test in various cycles and measuring the rate of the temperature gradient. Taking advantage of the calculated heat lost inside the material, damage progression is estimated during bending fatigue of Epoxy/Glass. This study proposes a relatively simple method based on heat dissipated energy to determine the degradation evolution inside a sample composite laminate. One can use the same procedure in order to characterize the damage stages in an application with the desired loading condition.

6.6 References


Chapter 7: Fatigue Damage Characterization of a Woven Glass/Epoxy Laminate
7.1. Introduction

While composite materials are often superior to their metal counterparts, they are not exempt from deterioration and damage. Of the different types of structural degradation, fatigue damage is one of the main failure mechanisms associated with composite materials when subjected to repetitive, cyclic loading.

Fatigue damage and failure mechanisms of composite laminates involve a combination of different mechanisms such as matrix cracking, fiber/matrix debonding, and fiber breakage. These mechanisms are considerably more complex than their metal counterparts wherein a main crack starts and propagates until fracture takes place. In composite materials, many micro-cracks initiate at the early stage of the fatigue evolution and result in different types of fatigue damage growth [1-2]. This complexity associated with their multi-damage mechanisms has hampered the development of a general fatigue degradation criterion [2-4].

There have been many attempts at describing the degradation mechanisms in composite materials. Some investigators use the reduction of stiffness [5-14], while others use strain energy variation [15-18]. Still others use non-destructive methods like thermography to monitor how damage occurs during cyclic loading [8-11, 19-21]. A comprehensive review of fatigue damage in composite laminates is available in the work of Degrieck and Paepegem [2] and Sendeckyj [22].

All of the above-mentioned methods —stiffness, energy, and thermography method— confirm the existence of three distinct phases in the evolution of the damage process. These are: an abrupt initial phase due to matrix cracking, a slow and gradual increase phase associated with fiber/matrix interfacial debonding, and finally the fiber breakage phase. Physically, the duration of each phase depends on the type of laminate and fiber orientation. However, examination of the published results reveals that the durations of three regimes evaluated by surface temperature measurement and stiffness reduction method are not identical [9-11]. For example, the initial
phase of damage based on the results of temperature evolution method is about 10-15% of the total life. However, 5-10% of total life for initial phase is reported when one uses the stiffness reduction method [9-11]. The differences observed between the correlation of damage behavior based on the stiffness reduction and the temperature rise calls for the development of an alternative approach for investigating the fatigue degradation in a composite laminate.

Generally, it can be hypothesized that the degradation of structural components is a consequence of irreversible thermodynamic processes that disorder a component, and that deterioration is a time-dependent phenomenon with increasing disorder [23]. This suggests that entropy — a fundamental parameter in thermodynamics that characterizes disorder— offers a natural measure of component degradation [24].

In this chapter, we propose a thermodynamically-based damage model for capturing the fatigue characteristics of a woven Glass/Epoxy (G10/FR4) laminate. We compare the results of a series of experiments to those of the existing methods for damage evaluation such as stiffness reduction, hysteresis energy variation, and surface temperature measurement methods. It is shown that the characterization of fatigue damage evolution based on the entropy concept is a promising approach for distinguishing different damage stages.

7.2. Theoretical Background of Fatigue Damage Evolution

In the following sections we begin by first reviewing the stiffness and energy criteria for characterizing fatigue damage and, then, go on to show how the thermodynamic entropy can be utilized for prediction of degradation in a Glass/Epoxy laminate subjected to cyclic loading.

7.2.1 Stiffness Approach

Monitoring the stiffness degradation in real time is a common quantitative measure of fatigue damage in a composite laminate. The premise of this approach is that cumulative damage generally results in the reduction in the material’s stiffness. For modeling purposes, a so-called damage parameter, $D_e^*$, is defined in terms of stiffness as follows [25].
\[
D^*_k = 1 - \frac{E}{E_0}
\]  
(7.1)

where \(E_0\) is the initial virgin material’s stiffness and \(E\) is the damaged material’s stiffness.

Experimental results show that the final measured stiffness when failure occurs is: \(1 - \frac{E_f}{E_0}\) instead of being unity [5]. To take this into account, a new cumulative damage parameter is defined as [5, 26]:

\[
D_k = \frac{E - E_0}{E_f - E_0}
\]  
(7.2)

Cumulative damage in Equation (7.2) is in the range of 0 to 1.

### 7.2.2 Hysteresis Energy Concept

Monitoring hysteresis energy is another method for assessment of fatigue damage in a composite material. As the cyclic fatigue process continues, the hysteresis area under the cyclic loading curve, \(H\), i.e., the dissipated energy, tends to increase. Using this property, a degradation index, \(D_H\), can be defined as follows [6,15].

\[
D_H = \frac{H - H_0}{H_f - H_0}
\]  
(7.3)

where \(H\) is the hysteresis energy which changes as a function of life during the fatigue process, \(H_0\) represents the initial strain energy and \(H_f\) is the final hysteresis energy at the instant when failure occurs. The initial strain energy for tension-tension fatigue test can be evaluated using the following expression [16]:

\[
H_0 = \frac{F^2 L}{2AE_0}
\]  
(7.4)

where \(F\) is the mean applied load (the average of maximum and minimum load), \(L\) is the specimen length, and \(A\) is the area of the specimen’s cross section.
7.2.3 Thermodynamic Approach

Fatigue damage is an irreversible process and produces entropy according to the second law of thermodynamics. The variation of entropy, $dS$, is the sum of two terms [27]:

$$dS = d_S + d_S$$

(7.5)

where $d_S$ represents the exchange entropy (either positive or negative) with the surroundings and $d_S$ is the entropy generation within the control volume. The second law of thermodynamics (Clausius-Duhem inequality) postulates that entropy generation must be non-negative, i.e.,

$$d_S \geq 0$$

(7.6)

In terms of the specific quantities, Equation (7.6) can be written as [27]:

$$\dot{\gamma} = \frac{d\gamma}{dt} = \frac{H}{T} + \frac{k}{T^2} T_j T_j \geq 0$$

(7.7)

where $k$ is the thermal conductivity, $T$ represents temperature, $t$ is time, and $\dot{\gamma}$ is entropy production rate.

Equation (7.7) consists of the mechanical dissipation due to permanent deformation ($\dot{\gamma}_{\text{mech}} = \frac{H}{T}$) and the thermal dissipation due to heat conduction ($\dot{\gamma}_{\text{cond}} = \frac{k}{T^2} T_j T_j$). In low-cycle fatigue the entropy generation due to mechanical dissipation is dominant and the entropy generation due to heat conduction is negligible [28]. Hence, Equation (7.7) reduces to

$$\dot{\gamma} = \frac{H}{T}$$

(7.8)

Similar to the damage accumulation of stiffness and hysteresis energy methods, an entropy-based fatigue damage criterion is proposed as follows.

$$D_s = \frac{\dot{\gamma} - \dot{\gamma}_0}{\dot{\gamma}_f - \dot{\gamma}_0}$$

(7.9)
where $\dot{\gamma}_f$ represents the entropy production just before the failure occurs and $\dot{\gamma}_0$ is the initial entropy production, i.e.,

$$\dot{\gamma}_0 = \frac{H_0}{T}$$  \hspace{1cm} (7.10)

Experimental temperature and hysteresis energy can be used in Equation (9) to quantify fatigue damage.

### 7.3. Material and Experimental Procedure

The material studied is Glass/Epoxy (G10/FR4) which is an unbalanced woven fabric composite with plain weave and aligned configuration consisting of a continuous filament glass cloth with an epoxy resin binder. The plain-woven glass fabric is stacked in fifteen layers within the thickness of 3 mm. Each woven layer has two unidirectional layers stacked in $[0^\circ/90^\circ]$. This type of composite offers high tensile and flexural strength, and thus finds use in a variety of applications such as electrical equipment, aerospace structures, and rocket structural components. Figure 7.1a shows the schematic of two layers of plain-woven Glass/Epoxy laminate with aligned configuration. Specimens are prepared with on- and off-axis stacking sequences. In on-axis stacking, the warp and weft directions are aligned with the load direction. The former is called lengthwise ($0^\circ$) while the latter is called crosswise ($90^\circ$). In off-axis stacking, the angle between the warp and the load direction is $\theta$ ($15^\circ$, $80^\circ$ in this study) as shown in Figure 7.1b. Fatigue tests are carried out with MTS 810 servohydraulic single actuator. Constant sinusoidal fatigue loads are applied at the frequency of 10 Hz.

The instrumentation includes high-speed, high-resolution infrared (IR) thermography which is used to record the temperature evolution of the specimen during the entire experiment. The IR camera is a MIKRON M7500 with temperature range between $0^\circ$C to $500^\circ$C, resolution of $320 \times 240$ pixel, accuracy of $\pm 2\%$ of reading, sensitivity/NETD of 0.08 $^\circ$C at 30 $^\circ$C, and image
update rate of 7.5 Hz. Before fatigue testing, the surface of the specimen is covered with black paint to increase the thermal emissivity of the specimen surface.

Figure 7.1a-b. a) Schematic representation of plain-woven Glass/Epoxy with aligned configuration. b) The specimen used in tension-tension fatigue test (all dimensions are in mm).

7.4. Results and Discussion

A series of tension-tension, load-controlled fatigue tests are performed at 10 Hz and zero load ratio $R$, defined as the ratio of minimum to maximum load. The specimens’ thickness is 3 mm with different fiber orientations ($\theta = 0, 15, 80,$ and $90^\circ$). In the following sections the experimental results of stiffness, hysteresis energy, temperature as well as entropy are presented and different damage models are compared with the entropy-based damage criterion.

7.4.1 Stiffness Deterioration and Hysteresis Energy Variation

Two important experimental parameters—stiffness and hysteresis energy—are plotted in Figures 7.2a and 7.2b versus the normalized number of cycles (with respect to the number of
cycles to failure) with 5.25 KN of on-axis (0°) and 4.25 KN of off-axis (80°) cyclic loading at the frequency of 10 Hz. About 20% reduction of stiffness \( \frac{E_o - E_f}{E_o} \) is seen in the Figure 7.2a.

The hysteresis energy is calculated by measuring the area of stress-strain hysteresis loop using MATLAB\textsuperscript{TM}. Stiffness is calculated using the linear regression of unloading hysteresis curve. In Figures 7.2a and 7.2b, there exist three degradation stages: In Stage I, a significant reduction in stiffness and an increase in hysteresis energy occur. During Stage II, stiffness slowly decreases in a linear fashion and hysteresis energy steadily increases. In Stage III, stiffness drops and hysteresis energy increases in a non-linear manner prior to failure. The explanation for different damage mechanisms in each stage has been discussed for woven laminates in literature [11, 29-30]. In Stage I, micro-cracks form in multiple locations in the matrix; debonding occurs in warp or weft at the interface between fibers and matrix; and weft and warp fibers with low strength break. As the fatigue process progresses, the crack density in the matrix and in fibers reaches a “saturation level,” where the damage growth becomes stable. During Stage II, more debonding in fiber strands and delaminations between warp and weft at the cross-over points take place. The interlacing nature of woven composite causes a gradual and slow trend in degradation process during this stage. Material’s degradation continues in Stage III due to the fiber breakage before fracture occurs.

Examination of Figures 7.2a and 7.2b reveals that the durations for the above-mentioned stages in the results of stiffness and hysteresis energy variation of both the on- and off-axis fiber orientation are not identical, especially in Stages II and III. The reason is that for off-axis angles, once a crack is formed, its tip is subjected to an opening mode normal to fibers and sliding mode parallel to the fibers. Depending on the off-axis angle, the stress magnitude in the fiber direction may not be sufficient to break the fibers and thus fibers can tolerate the load during Stage II. Fiber breakage occurs when the debondings connect with each other, leading to more
delaminations between the warp and the weft strands. Therefore, Stage II of the off-axis fatigue damage lasts long enough until the woven Glass/Epoxy laminate losses its load-carrying capacity remarkably. On the contrary, in an on-axis fatigue, most of the load is absorbed by the warp or the weft fibers and majority of the matrix cracks and the debondings in warp and weft yarns tend to propagate normal to the fibers in the transverse direction. Hence, due to the high magnitude of the load acting on the fibers, some of the weft or warp fibers that are debonded at the cross-over points begin to fail. Stage II of the off-axis and the on-axis fatigue damage last about 70% and 60% of the total life, respectively.

7.4.2 Temperature Evolution

Figure 7.3 shows the evolution of the surface temperature of two fatigue tests corresponding to Figure 7.2. During Stage I, temperature increases due to the frictional energy between the fiber and the matrix, and energy is released at the tip of the micro-cracks as they propagate during the cyclic operation. As soon as the damage in the specimen reaches its saturation level, Stage II starts and results in the balanced condition between energy generation and heat dissipation. In Stage III, an abrupt increase in temperature is observed following the fiber breakage and failure occurs soon thereafter. The similar observation for temperature is reported by [8-11] for woven laminates.

The temperature increase in both on- and off-axis in Stage I is about the same, and lasts approximately 30% of the total life. The linear and steady behavior of Stage II for the off-axis lasts longer than that of the on-axis laminate. The reason can be attributed to a different failure mechanism described as follows. When the specimen is subjected to an on-axis loading, the normal load is mainly absorbed by the fibers and micro-cracks tend to open in the transverse direction. However, in the case of the off-axis loading, the main fatigue damage mechanism is the matrix/fiber debonding due to shear stress. Therefore, as a result of the high-strength capacity of the fibers in tolerating the applied load in off-axis laminate, Stage III is postponed.
Figure 7.2. Stiffness and hysteresis energy evolution vs. normalized number of cycle (normalization with respect to the number of cycles at failure) for 5.25 KN of on-axis and 4.25 KN of off-axis at the frequency of 10 HZ. There are three distinct stages: In Stage I, stiffness reduces and energy increases rapidly due to matrix cracking. During Stage II, stiffness and dissipated energy grow steadily. Finally, in Stage III, fiber breakage takes place. About 20% of reduction in stiffness is seen in the 7.2a.
The difference in the duration of three stages among the result of temperature profile, hysteresis energy, and stiffness plot can be explained as follows. The increase in the surface temperature of the specimen results primarily from the friction energy between fibers and fiber/matrix interfaces and in part from the energy release at the tip of the cracks or mechanical energy. However, the reduction in the stiffness and the increase in the hysteresis energy are due to the deformation energy. An insufficient duration in the loading and unloading process, low thermal conductivity of woven Glass/Epoxy laminate, and low convection heat transfer coefficient prevent the system from reaching the steady thermal balance between the specimen and the surroundings. Therefore, very little heat is transferred during each fatigue cycle and because of the phase lag between the heat generation and dissipation, the balance between the heat generated per cycle and the heat lost to the surroundings delays the transition from one stage to the next.

Figure 7.4 shows the temperature evolution profile and temperature maps obtained by thermography technique at various stages of the evolution from 10% to 98% of the specimen life. Tests are carried out at 10 Hz and subjected to the load amplitude of 4.9 KN along the weft direction. Each figure shows the measured temperature profile at three different sections of the specimen. Figure 4a shows that during at least 10% of the life the temperatures at different sections are nearly identical, indicating relatively uniform axial temperature distribution. The ordinate of Figures 7.4b-g is the normalized temperature (with respect to the maximum and minimum temperature, \( \frac{T - T_{\text{min}}}{T_{\text{max}} - T_{\text{min}}} \)). Figure 7.4b shows the temperature map and the temperature profile of the specimen around 10% of the total life. The boundary of the large zone of the same color is identified by the dashed curve. During the early stage of fatigue, damage areas are randomly distributed in the specimen. This is brought about by the breakage of weak fibers and matrix cracking at the weak interfaces. Hence, the temperature profiles of the three sections
overlap each other. As cyclic load continues to 30\% of the total life (Figure 7.4c), the dashed zone area reduces and temperature difference between Section 7.3, and Section 7.1 and section 7.2 increases. As it can be seen from Figure 7.4a, the temperature map around 30\% of the total life is in Stage II. The area contained within the dashed curve from 30\% to 60\% of the total life does not change significantly. It should be noted that the observed slow- and linear damage is the characteristic of Stage II due to interlacing nature of woven fibers. During 60\% to 75\% of the total life, the temperature plot and the area enclosed by the dashed curve are changed remarkably due to the initiation of fiber breakage, which is the characteristic of the final stage. Also to be noted is that in Stage III of Figure 7.4a, the temperature variation is no longer linear. As the fatigue process continues from 75\% to 98\% of total life (Figures 7.4e-g), more fiber breakage takes place and the size of the cracks increases. Note that the fracture area is limited to the Section 1 where temperature is highest. The region enclosed by the dashed curve, representing a region of the high temperature, shrinks. In fact, the zone with the highest temperature corresponds to where fracture occurs.

Damage evolution discussion in the above sections reveals that the occurrence of three different phases during degradation based on stiffness, energy, and temperature methods do not necessarily yield identical correlations. In the following section, we apply the entropy concept which contains the combined effect of stiffness, energy and temperature to demonstrate an enhanced method for assessment of fatigue degradation in a Glass/Epoxy laminate.
Figure 7.3 Temperature evolution vs. normalized number of cycle (normalization with respect to the number of cycles at failure) for 5.25 KN of on-axis and 4.25 KN of off-axis at the frequency of 10 HZ. There are three distinct stages: In Stage I, temperature increases rapidly due to permanent deformation. During Stage II, temperature grows slowly and linearly. Finally, in Stage III, fiber breakage takes place.

Figure 7.4a-g Temperature map obtained by thermography technique for tension-tension fatigue with zero load ratio and 4.9KN load amplitude. a) temperature profile for the whole fatigue life at three different locations. b-g) comparison of temperature evolution at various percentage of life (%10, 30, 60, 75, 90, and 98 of total life)
(Figure 7.4 continued)
Figure 7.4 continued

(e)

(f)

(g)
7.4.3 Entropy Generation

Hysteresis energy and temperature profile are utilized in order to calculate the amount of entropy generation during each cycle based on Equation (7.8). The result of entropy generation corresponding to Figure 7.3 is plotted in Figure 7.5a along with the SEM images of the fracture surface in Figures 7.5b and 7.5c. Entropy generation also evolves in three stages with an initial stage where matrix cracking and the breakage of weak fibers occur. This stage lasts about 10% of total life. In this phase, the rate of entropy generation is high due to the high degree of associated disorders. During this stage, debondings take place in the weft and warp yarns close to the cross-over point. Stage II exhibits a linear and slow increase in entropy that lasts about 60% of the total life for off-axis fiber orientation. While, for the on-axis loading, this stage lasts around 50% of total life.

The SEM images of the fracture surfaces (Figures 7.5b and 7.5c) show the different types of degradation mechanisms in a woven Glass/Epoxy laminate. More debonding between weft and warp yarns and matrix take place during Stage II. These debondings and delaminations are the origin of more matrix cracks which propagate into some of the fiber bundles. Delaminations occur at the cross over points of the weft and the warp fibers. During Stage III, high amount of entropy is generated due to a relatively large amount of energy release as a result of the breakage of the fibers. Note that estimation of entropy generation during the fatigue process requires incorporating strain energy, stiffness, and temperature.

7.5 Damage Evolution

In this section, we examine the accumulation of fatigue damage using the thermodynamic entropy concept and compare with other methods. The results pertain to fatigue damage evolution of a series of tension-tension fatigue tests of woven Epoxy/Glass (G10/FR4) laminate for various load angles (0, 15, 80, and 90°).
Figure 7.5a) Entropy generation vs. normalized number of cycle (normalization with respect to the number of cycles at failure) for 5.25 KN of on-axis and 4.25 KN of off-axis at the frequency of 10 HZ. 5b and 5c) SEM image of the fracture surface of woven Epoxy/Glass laminate. SEM images show different failure mechanism such as matrix cracking, debonding between fiber strands and matrix and delamination between warp and weft fiber strand at the cross-over points.
(Figure 7.5 continued)

Figures 7.6a-c depict the evolution of fatigue damage for different load amplitudes calculated using Equations (7.2), (7.3), and (7.9). In Figure 6a the damage parameter is plotted based on the stiffness reduction method while in Figure 7.6b damage evolution is determined based on the hysteresis energy variation. Figure 7.6c presents the results of damage evolution obtained using the entropy generation method. The three stages described in the previous sections can be observed in each of these figures. Damage stages strongly depend on the fiber orientation. Most of the degradation in off-axis fiber orientation occurs in Stage II. However, when dealing with the on-axis fiber orientation, Stage II and Stage III have the dominant effect on the life span.

Table 7.1 summarizes the percentage of different damage stages obtained from stiffness, hysteresis energy, temperature, and entropy methods. The results show that the duration of Stage I in off-axis load condition is longer than that of in on-axis load condition. On the contrary, the extent of Stage III in off-axis orientation is shorter than that of the on-axis fiber orientation. It is worthwhile to mention that the duration of the degradation stages obtained by the entropy
concept lies between those obtained using the hysteresis energy and the temperature method. The reason is related to the dependency of thermodynamic entropy to both energy and temperature. The degradation-entropy generation theorem [23] proves that the rate of degradation has a linear relationship with entropy production via the thermodynamic flows and forces. Therefore, the fatigue damage equation, Equation (9), is directly linked to the thermodynamic entropy via the hysteresis energy.

![Figure 7.6a-c](image)

**Figure 7.6a-c** Damage parameter versus number of cycle to failure for four different fatigue test at the frequency of 10 HZ based on stiffness reduction, hysteresis energy evolution and entropy generation methods, respectively.
(Figure 7.6 continued)

(b)

(c)
Table 7.1. Comparison of different stages

<table>
<thead>
<tr>
<th>Direction</th>
<th>Load (KN)</th>
<th>Stiffness Criterion</th>
<th>Energy Method</th>
<th>Entropy Approach</th>
<th>Temperature Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Stage I (%)</td>
<td>Stage III (%)</td>
<td>Stage I (%)</td>
<td>Stage III (%)</td>
</tr>
<tr>
<td>0°</td>
<td>5.5</td>
<td>10</td>
<td>50</td>
<td>4</td>
<td>47</td>
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<tr>
<td></td>
<td>5.7</td>
<td>5</td>
<td>35</td>
<td>3</td>
<td>50</td>
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<td>6</td>
<td>47</td>
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<td>Avg.</td>
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<td></td>
<td>Avg.</td>
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<td>80°</td>
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<td>30</td>
</tr>
<tr>
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<td>Avg.</td>
<td>11.3</td>
<td>23.6</td>
<td>11.3</td>
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7.5. Conclusions

A series of load-controlled, tension-tension fatigue tests with zero load ratio and various load/fiber orientations is performed to study the fatigue damage mechanisms based on four different methods, namely stiffness reduction, hysteresis energy, temperature variation and thermodynamic entropy. The use of entropy for characterization of fatigue damage in a composite presented here is a new method. It utilizes the evolution of both the hysteresis energy and the temperature to evaluate fatigue damage. The primary result is that degradation undergoes three different stages, namely Stage I with matrix cracking, and fiber/matrix debonding, Stage II with fiber/matrix delamination and matrix cracking, and Stage III with fiber breakage. The life span of these stages strongly depends on the fiber orientation. The duration of each stage based
on thermodynamic entropy approach encompasses the effects of different stages of stiffness, hysteresis energy, and temperature evolution methods. Based on the experimental results, a damage variable as a function of entropy production of the form

\[ D_s = \frac{\dot{\gamma} - \dot{\gamma}_0}{\dot{\gamma}_f - \dot{\gamma}_0} \]

is introduced. This equation shows that the irreversible thermodynamic entropy can be utilized to assess the degradation of a system under repetitive cyclic loading.

### 7.7 References


Chapter 8: A Comprehensive Fatigue Failure Criterion Based on Thermodynamic Approach
8.1 Introduction

The establishment of a unified fatigue failure criterion subsuming on-axis, off-axis, load ratio, constant load, variable load, and load sequence in a universal manner is highly desirable to facilitate the design of composite structures subjected to repetitive loading. In comparison with metals, the fatigue behavior of composites is considerably more complicated due to the interaction of multitude of the processes such as matrix cracking, fiber/matrix delamination, and fiber breakage [1-4].

Generally, major fatigue life prediction models for composites are based on either stress or energy criterion. Hashin and Rotem [5] and Sims and Brogdon [6], for example, were among the earliest researchers who introduced a stress criterion for the analysis of the fatigue failure in composites. Rotem and Nelson [7] also developed a method to predict S-N curves for composite laminates based on a fatigue failure criterion similar to the Goodman diagram[8]. Fawaz and Ellyin [9] developed a stress-based model to predict the fatigue life of a composite laminate at different fiber directions. Kawai [10] also proposed a stress-based fatigue life prediction criterion for an off-axis unidirectional composite subjected to constant-amplitude loading by normalizing the applied stress with respect to static strength. Huang [11], Owen and Griffiths [12], Amijima et al. [13], and Khan et al. [14] have attempted to characterize the fatigue failure based on S-N curve method for woven fabric-reinforced composites.

Research reveals, however, that the stress-based criterion results in a large scatter of data. To overcome this shortcoming, many researchers have attempted to make use of a more unified fatigue failure criterion based on the strain energy that incorporates both the stress and the strain terms. Ellyin and EL-Kadi [15] utilized the strain energy to predict fatigue failure and related the total fatigue life to the total energy through a power-law relation. Later, EL-Kadi and Ellyin [16] proposed a fatigue failure criterion for a unidirectional composite laminate taking into account the effect of fiber orientation angle. Peterman and Cheng [2] and Peterman and Pulmtree [1]
used the critical plane (the plane in which cracks initiate and propagate) approach to develop a fatigue failure criterion model that incorporates the energy density into the analysis. Subsequently, Shokrieh and Taheri-Behrooz [3] developed a strain energy-based fatigue life model for unidirectional polymer composite laminates under different stress ratios and off-axis loading.

Although the energy-based models are capable of taking into account the effect of angle direction and the stress ratio, the associated cluster of experimental data cannot be contained within a reasonable upper and lower band. Further, this approach is not suitable for handling the fatigue problems that involve variable loading. The premise of this paper is that fatigue damage is an irreversible process that causes disorder in the system and manifests itself in generation of entropy in accordance with the second law of thermodynamics [17-20]. As time progresses, disorder in the system continuously increases until it reaches a critical stage when failure occurs. Simultaneously with the rise in disorder, entropy monotonically increases. Thus, as demonstrated in recent literature, entropy offers a natural measure of a component’s deterioration [17-20].

The aim of this chapter is to compare the fatigue life prediction methods that use stress and energy approaches with the entropy accumulation concept. For this purpose, an extensive set of tension-tension fatigue tests are performed with constant- and variable-amplitude loading (High-to-low and Low-to-High) and different load ratios and load sequences, frequencies, and fiber angles. The results show that entropy accumulation method has the advantage of incorporating both stress, strain, energy concept in a unified manner and is found to be more promising as compared with other fatigue life evaluation methods.

8.2 Material and Experimental Procedure

The material studied is Glass /Epoxy (G10/FR4) with high tensile and flexural strength (see Table 8.1) used in electrical equipment, aerospace structures, and rocket structural components. The Glass /Epoxy laminate is an unbalanced woven fabric composite with plain weave and
aligned configuration consisting of a continuous filament glass cloth with an epoxy resin binder that is cured under heat and pressure to form solid shapes (Figure 8.1a). This type of laminate is a fiberglass laminate, containing layers of fiberglass sheets. Those fiberglass sheets are stacked up and positioned in a fixture. A grade #10 (i.e., G10) epoxy is injected in to the layers of fiberglass sheets, heated and pressed at high pressure. FR4 is a fire retardant grade of G10. The plain woven glass fabric is stacked in fifteen layers for the thicknesses of 3 mm. Each woven layer has two unidirectional layers stacked in \([0^\circ/90^\circ]\). Specimens are prepared with two different stacking sequences. One is on-axis stacking in which the warp or weft directions are aligned with the load direction. The warp and the weft directions are called the lengthwise \((0^\circ)\) and the crosswise \((90^\circ)\), respectively. The other stacking sequence is off-axis stacking in which the angle between the warp and the load direction is \(\theta(15^\circ, 30^\circ, 45^\circ, 60^\circ\) and \(80^\circ\) in this study) (Figure 8.1b).

High-speed, high-resolution infrared (IR) thermography is used to record the temperature evolution of the specimen during the entire experiment. The IR camera is a MIKRON M7500 with temperature range between \(0^\circ\)C to \(500^\circ\)C, resolution of \(320 \times 240\) pixel, accuracy of \(\pm 2\%\) of reading, sensitivity/NETD of \(0.08\) °C at \(30\) °C, and image update rate of \(7.5\) Hz. Before testing, the surface of the specimen is coated with black paint to increase the thermal emissivity of the specimen surface.

Figure 8.1c presents a schematic diagram of the experimental setup for tension-tension fatigue test. Tests are carried out with MTS 810 servohydraulic single actuator. First, static tests are performed to determine the laminate’s ultimate tensile strength and modulus of elasticity. Next, extensive set of experiments are performed using a sinusoidal load applied at the frequency of \(10\) Hz and load ratios, \(R\), of 0 and 0.1. Finally, constant- and variable-load amplitudes (high-to-low and low-to-high) are performed in the load-controlled mode.
Figure 8.2a shows a typical temperature variation during fatigue life of Glass/Epoxy laminate for 2.785 KN load amplitude, zero load ratio, and the frequency of 10 Hz. Temperature rapidly increases at the early stage of fatigue life due to matrix cracking, weak fiber breakage, and delamination at the weak fiber/matrix interface. As soon as the damage reaches its saturation level, a steady and slow increase starts followed by sudden temperature rise due to fiber breakage.

Figure 8.2b shows a typical hysteresis energy evolution versus number of cycles for the same fatigue test of Figure 2a. The hysteresis area of each cycle is obtained using a MATLAB™ code that utilizes the experimental load and displacement data. Cyclic energy undergoes three regimes: rapid increase, slow and steady increase, and sudden rise before failure.

<table>
<thead>
<tr>
<th>Tensile Strength (MPa)</th>
<th>Modulus of Elasticity (GPa)</th>
<th>Shear Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0° 90° 15° 30° 45° 60° 80°</td>
<td>0° 90° 15° 30° 45° 60° 80°</td>
<td>170</td>
</tr>
<tr>
<td>375 320 253 190 210 235 234</td>
<td>19 17 15.5 7 3 6 15</td>
<td></td>
</tr>
</tbody>
</table>

**Table 8.1. Mechanical properties of G10/FR4**

![Diagram](image)

**Figure 8.1** a) Schematic of laminate configuration. b) Schematic of fiber orientation. c) Experimental setup in Tension-tension fatigue test.
(Figure 8.1 continued)

![Test Setup]

IR Camera

Hydraulic Grip

Specimen

Figure 8.2. a) Temperature variation and b) Hysteresis energy evolution versus fatigue life of Glass/Epoxy laminate for 2.785 KN load amplitude, the load ratio of zero and the frequency of 10 Hz.
8.3 Theoretical Background

In the following sections, we begin by presenting the energy criterion for predicting fatigue life followed by the thermodynamic entropy concept, introduced here as a new fatigue life prediction model.

8.3.1 Energy Approach

The energy concept has the advantage of taking into account both the stress and the strain field [2-3, 16] by directly analyzing the linear elastic stress-strain curve and the hysteresis loop (Figure 8.3). In the following sections both concepts are reviewed.

8.3.1.1 Linear Elastic Strain Energy-Life (W-N) Concept

Shokrieh and Taheri-Behrooz [3] proposed a fatigue life model based on the strain energy concept applied to on- and off-axis, unidirectional polymer-composite laminates subjected to tension-tension and compression-compression fatigue loading. Assuming that the stress-strain relation is elastic, they normalized the strain energy (normalized with respect to the maximum monotonic strain energy, i.e., the product of the maximum monotonic stress and strain) to indirectly take into account the fiber orientation angle. It should be noted that the linear stress-
strain assumption overestimates the elastic strain energy for the off-axis stacking with non-linear behavior. This method incorporates stress and strain terms and can be expressed as follows [3]:

$$\Delta W = \frac{1}{X\varepsilon_{1u}} (\sigma_{1\text{max}}\varepsilon_{1\text{max}} - \sigma_{1\text{min}}\varepsilon_{1\text{min}}) + \frac{1}{Y\varepsilon_{2u}} (\sigma_{2\text{max}}\varepsilon_{2\text{max}} - \sigma_{2\text{min}}\varepsilon_{2\text{min}}) + \frac{1}{S\varepsilon_{6u}} (\sigma_{6\text{max}}\varepsilon_{6\text{max}} - \sigma_{6\text{min}}\varepsilon_{6\text{min}}) \quad (8.1)$$

where $\Delta W$ is the normalized linear elastic strain energy (normalized with respect to monotonic strain energy), $\sigma_{1\text{max}}$, $\sigma_{2\text{max}}$, $\sigma_{6\text{max}}$, $\sigma_{1\text{min}}$, $\sigma_{2\text{min}}$, $\sigma_{6\text{min}}$ are the maximum and minimum stress components, and $\varepsilon_{1\text{max}}$, $\varepsilon_{2\text{max}}$, $\varepsilon_{6\text{max}}$, $\varepsilon_{1\text{min}}$, $\varepsilon_{2\text{min}}$, $\varepsilon_{6\text{min}}$ are the corresponding maximum and minimum strain components in the fiber directions (see Figures 8.3 and 8.4). $X$, $Y$, and $S$ are the maximum static strength in length and crosswise directions and shear strength (see Table 8.1 for their values). $\varepsilon_{1u}$, $\varepsilon_{2u}$, $\varepsilon_{6u}$ are the ultimate strain in the monotonic test. Taking advantage of the transformation relationships between the on- and off-axis stress and using the linear elastic assumption [3, 16], Equation (8.1) becomes

$$\Delta W = \frac{1 + R}{1 - R} \Delta \sigma \left( \frac{\cos^2 \theta}{X^2} + \frac{\sin^2 \theta}{Y^2} + \frac{\sin^2 \theta \cos^2 \theta}{S^2} \right) \quad (8.2)$$

where $R$ is the stress ratio ($= \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}$), $\Delta \sigma$ represents the stress range ($= \sigma_{\text{max}} - \sigma_{\text{min}}$), and $\theta$ denotes the angle between the lengthwise fibers and the load direction.

Having the stress range from experiment enables one to calculate the elastic strain energy density at different number of cycles at failure ($N_f$).

8.3.1.2 Hysteresis Energy-Life (H-N) Concept

The hysteresis area under the cyclic loading ($H$) — a quantity which represents the dissipated energy — increases during the fatigue process owing to the permanent deformation. This energy has two parts: the first part is released during the unloading process and the second part is the stored energy responsible for creation of new cracks and the propagation of the existing cracks.
The cyclic hysteresis energy is directly calculated from the experimental stress and strain in real time using a MATLAB\textsuperscript{TM} code as follows (Figure 8.2b).

\[ H = \sum_{i=1}^{n} (\sigma_i - \sigma_{i-1})(\varepsilon_i - \varepsilon_{i-1}) \]  

(8.3)

where \( n \) is the number of points acquired per cycle during fatigue. The total hysteresis energy or accumulated fracture energy, \( H_n \), at failure is obtained by summing up the hysteresis energy per cycle to the total number of cycles to failure, \( N_f \).
\[ H_i = \sum_{j=1}^{N_i} \left( \sum_{t=1}^{i} (\sigma_i - \sigma_{i-1})(\varepsilon_i - \varepsilon_{i-1}) \right) \]  \hspace{1cm} (8.4)

### 8.3.2 Thermodynamic Approach

The second law of thermodynamics (Clausius-Duhem inequality) postulates that the rate of entropy is always greater than or equal to the rate of heating divided by the temperature.

Utilizing the thermodynamic relationships, the entropy production can be stated as [19, 22].

\[ \dot{\gamma} = \frac{H}{T} + \frac{k}{T^2} T_i T_j \geq 0 \]  \hspace{1cm} (8.5)

where \( k \) is the thermal conductivity and \( T \) represents the temperature. Equation (8.5) —the entropy generation inequality—consists of the mechanical dissipation due to permanent deformation \( \dot{\gamma}_{\text{mech}} = \frac{H}{T} \) and the thermal dissipation due to heat conduction \( \dot{\gamma}_{\text{cond}} = \frac{k}{T^2} T_i T_j \).

Utilizing the entropy concept, the authors of the present paper demonstrated that for two metals, Al 6061-T6 and SS 304, by tallying the entropy production up to the fracture point, one arrives at a unique material property—a so-called fracture fatigue entropy (FFE)—that is independent of geometry, load, and frequency [19]. This property was utilized as a structural health monitoring mechanism in [23] to halt the operation of a machine prior to fracture at a user-specified percentage of the remaining life. The present paper seeks to extend the application from metal to a composite laminate. It was shown [19] that in low-cycle fatigue, entropy production due to mechanical dissipation is dominant. Similarly, in the present paper, according to an order of magnitude analysis (See Appendix B for details), the entropy generation due to heat conduction is shown to be negligible. It should be noted that the cyclic hysteresis energy \( (H) \) is a crucial parameter for determining FFE. In metals, cyclic hysteresis energy is a function of fatigue parameters such as cyclic strain hardening, fatigue ductility coefficient, and fatigue strength coefficient. To estimate the FFE for a metallic component, one can make use of the cyclic plastic...
energy formula such as the Morrow’s equation [25]. However, in contrast to a metallic component, the hysteresis energy variation for a composite material is more complex and, to the best of our knowledge, no expressions are available to relate the composite fatigue properties to the hysteresis energy. In this work we shall rely on the experimental measurement of the hysteresis energy. Experimental cyclic hysteresis energy data and surface temperature of the specimen at the section where failure takes place are utilized in Equation 8.5 to evaluate cyclic entropy production. FFE is then determined by integrating cyclic entropy production in real time.

8.4 Results and Discussion

Figure 8.5 shows the plot of stress-strain for Glass/Epoxy (G10/FR4) laminate subjected to a series of static tests for both on- and off-axis directions. While the stress-strain relationships for on-axis tests show that they are nearly linear before the ultimate failure occurs, in the off-axis tests a significant nonlinearity appears following the initial linear response up to only about 1% strain. When dealing with on-axis loading, the load is primarily supported by the fibers and failure stress is that of the normal stress. However, in the off-axis loading, the predominant failure mechanism is shear stress and the material behavior is governed by a combination of nonlinear elasticity and damage in the matrix and the yarns [25-26].

Figure 8.6 presents the variation of modulus of elasticity of Glass/Epoxy (G10/FR4) laminate at different fiber angles. It shows that as the off-axis angle increases, the magnitude of elastic modulus decreases up to 45°. Beyond 45°, the elastic modulus starts to increase. It is to be noted that the elastic modulus for off-axis angles can be predicted using the classical laminated plate theory. However, the present work is an experimental investigation without resorting to the calculation of elastic modulus using the classical laminated-plate theory.
Figure 8.5 Stress-strain relationship for on and off-axis for Glass/Epoxy laminate at room temperature.

Figure 8.6 The modulus of elasticity variation as off-axis angles changes for woven Glass/Epoxy laminate.

An extensive set of fatigue tests is carried out for Glass/Epoxy laminate at different stress ratios ($R=0$, and 0.1), load angles ($\theta = 0, 15, 30, 45, 60, 80, 90^\circ$) at the frequency of 10 Hz. Figures 8.7a-e present the results of stress, energy, and entropy method. As shown in Figure 8.6a, experimentally obtained S-N data for Glass/Epoxy laminate indicates that the fatigue life is
strongly dependent upon the fiber load angle and the stress ratio and that a relatively large data scatter is seen in the S-N. The normalized strain energy for the same experimental results obtained using Equations (8.2) to (8.4) are plotted against the number of cycles at failure in Figure 8.7b. In comparison with the S-N, the W-N method collapses all the data to within a relatively narrow band. A least square relation fitted to the experimental results is:

$$\Delta W = 1.2064 N_{f}^{-0.189}$$  \hspace{1cm} (8.6)$$

with the goodness of $R_g^2 = 0.8$.

Figure 8.7c shows the plot of cyclic hysteresis energy at the time when failure occurs. It has a similar trend with strain energy method and the best fitting line with the goodness of $R_g^2 = 0.82$ is:

$$H = 4.8262 N_{f}^{-0.444}$$  \hspace{1cm} (8.7)$$

Figure 8.7d shows the accumulation of hysteresis energy obtained using Equation (8.4) up to the time when failure takes place. It can be seen that the experimental data for the load/fiber angles close to crosswise angle ($90^\circ$) and lengthwise angle ($0^\circ$) are distributed around a straight line of

$$H_f = 2.4413 N_{f}^{0.5199}$$  \hspace{1cm} (8.8)$$

with the goodness of $R_g^2 = 0.83$. However, the fatigue life behavior of Glass/Epoxy laminate with high fiber/load angle ($30, 45, 60^\circ$) tends to have relatively different distribution described by the following relationship.

$$H_f = 1.2415 N_{f}^{0.5497}$$  \hspace{1cm} (8.9)$$

with the goodness of $R_g^2 = 0.88$. That is, the total hysteresis energy for $30, 45, \text{ and } 60^\circ$-specimens tends to increase with a different trend than that of $0, 15, 80, 90^\circ$-specimens. The reason can be attributed to the fatigue properties of Glass/Epoxy laminate at different load/fiber angles. The plot of stress–strain at the load angle close to crosswise and lengthwise direction is almost linear and failure is dominated by the normal stress. However, the shear failure is mostly
dominated by shear stress at the angles 30, 45, and 60° with the extreme non-linear stress–strain behavior.

A unified fatigue plot of fracture fatigue entropy versus number of cycle at failure obtained on the thermodynamic entropy concept, described in section 8.3.2, is shown in Figure 6e. It is worth pointing out that cumulative entropy production criterion collapses all the experimental data including constant loading, variable loading (High-to-Low (H-L) and Low-to-High (L-H)), load ratios, load amplitudes, and fiber orientations to within a relatively narrow band. Thus, the thermodynamic entropy method shows a much better capability of correlating fatigue data obtained under various load ratios and load angles than other methods.

The results of Figure 8.7e reveal that the necessary and sufficient condition for the final fracture of Glass/Epoxy laminate corresponds to the entropy gain of approximately between the lower band (0.4 MJ m⁻³K⁻¹) and upper band (2.5 MJm⁻³K⁻¹) for various on- and off-axis angles.

![Figure 8.7](image)

**Figure 8.7** Comparison of fatigue life prediction models at various load ratios (R=0, and 0.1), and load/fiber angles (θ = 0, 15, 30, 45, 60, 80, 90°). a) stress life approach. b) elastic strain energy method. c) cyclic hysteresis energy concept. d) accumulated fracture energy method. e) fracture fatigue entropy criterion. H-L: High-to-Low load, L-H: Low-to-High load.
(Figure 8.7 continued)

(b)

(c)
(Figure 8.7 continued)

A possible application of the method presented is in the prevention of the catastrophic failure of composite laminates subjected to fatigue. Ideally, the FFE concept can be implemented in a finite element (FE) code that couples structural analyses to heat transfer simulations. Cyclic entropy
production can then be evaluated using Equation (5). By accumulating of the cyclic entropy production in real time and comparing to FFE of the material, the remaining life can be estimated.

In general, a comprehensive fatigue failure criterion based on the thermodynamic entropy enables engineers to simply estimate the life of composite laminate under any combination of fiber orientation and stress ratio and to implement a system to automatically halt the operation of composite components before catastrophic failure occurs. An application of this concept for metallic specimens has already been demonstrated [23].

8.5 Conclusions

A series of load-controlled tension-tension fatigue tests with different load ratios and various load angles is performed to study the fatigue life based on four different methods: stress life (S-N), linear elastic strain energy (W-N), hysteresis energy (H-N), and thermodynamic entropy (E-N) method. The E-N method which is a new technique, utilizes the concept of fracture fatigue entropy. The results show the capability of the entropy method to predict fatigue life for various stress ratios and fiber orientations. The results show that fatigue life based on the stress consideration alone is sensitive to the load ratio, the load amplitude and the fiber orientations. As a result, the associated fatigue data have a great amount of scatter. Although the fatigue life prediction based on energy method collapses the data to within narrower limits, the scatter in the data still remains. Among the mentioned methods, the plot of fracture fatigue entropy versus fatigue life is more unified compared to stress-life and energy-life plots and has the following features:

(i) It is developed based on the thermodynamic entropy incorporating the concept of degradation theory [17] which relates deterioration of components to entropy generation via thermodynamic forces and thermodynamic flows.
(ii) It incorporates stress, strain, and energy terms and provides results that are much less scattered compared to that of the other methods such as S-N, W-N, and H-N.

(iii) The method presented takes into account the mean stress effect, and different fiber orientation angles as well as constant and variable loading.

8.6 References


Chapter 9: Conclusions and Future Steps
9.1 Conclusions

The present research establishes a new thermodynamic-based approach to predict the fatigue life and assess fatigue damage of metals and composite laminates subjected to constant and variable loading (High-to-Low and Low-to-High). Results show that the cumulative entropy generation is constant at the time of failure and is independent of geometry, load, and frequency. Moreover, it is shown that the fracture fatigue entropy (FFE) is directly related to the type of material. That is, materials with different properties such as Steel, Aluminum and Epoxy/Glass (G10/FR4) composite laminate have a different cumulative entropy generation at the fracture point.

The implication of this finding is that by capturing the temperature variation of a system undergoing fatigue process, the evolution of entropy generation can be calculated during the fatigue life and then compared to the appropriate FFE for the material to assess the severity of degradation of the specimen. A prototype of fatigue monitoring unit (FMU) is developed as a structural health monitoring system installed on the existing fatigue machine. By capturing the temperature variation of the specimen undergoing fatigue process, the evolution of entropy generation is calculated in real time. The results of a series of laboratory fatigue tests show that thermodynamic entropy-based fatigue life prediction curve is more unified than stiffness-, stress- and energy-based fatigue life prediction curves.

Furthermore, empirical relationships between damage and entropy production are developed for both metals and composite laminate. Fatigue damage relationships directly linked to the thermodynamic entropy via hysteresis energy, and temperature are more general than the other methods of fatigue damage such as stiffness-, stress- and energy-based fatigue damage methods.

Two different fatigue damage behaviors in metal and composite laminate exist. In metals a main crack starts and propagates until fracture takes place. However, fatigue damage and failure mechanisms of composite laminates involve the combination of different mechanisms such as matrix cracking, fiber/matrix debonding, and fiber breakage. Theses mechanisms are
considerably more complicated than their metal counterparts. In composite materials, degradation undergoes three different stages, namely Stage I with matrix cracking, Stage II with fiber/matrix interfacial debonding and matrix cracking, and Stage III with fiber breakage. Two non-destructive methods —Infrared thermography and acoustic emission— are utilized to characterize damage progression of Epoxy/Glass composite laminate during repetitive loading.

9.2 Future Steps

The following recommendation should be considered to extend and improve the present work.

- In real applications, structures are subjected to multiaxial fatigue. Therefore, more practical applications should be studied.
- Since experimental investigations are not always cost effective and laboratory tests cannot prototype the actual size and geometry of components and structures, an effective numerical method should be considered to further investigate the current approach.
- In this study, mainly low-cycle fatigue is investigated. However, most machinery components have a high-cycle fatigue life. In thermodynamic analysis of fatigue, temperature rise also is a key point which is negligible in high cycle fatigue. Therefore, more studies are required to investigate the current method for high cycle fatigue.
Appendix A: Heat Transfer Convection Estimation

The convective heat transfer coefficient was estimated using an experimental procedure described by Meneghetti [13] which involves measuring the cooling rate after a sudden interruption of the fatigue test. The statement of energy balance can be applied to the reduced section length of the specimen, as shown in Figure A.1:

\[
\rho c_v H_c \varepsilon d + \left( \rho c_p \frac{\partial T}{\partial t} + \dot{E}_p \right) = \left( H_{cd} + H_{cv} + H_{v} \right) + \int \sigma_i d e_i
\]  

(A.1)

The term on left hand side of the above equation is the input mechanical energy with \( f \) as frequency. \( H_{cd}, H_{cv}, \) and \( H_{v} \) represent the thermal power dissipated in a unit volume of material due to conduction, convection and radiation, respectively. The last term on the right hand side is the rate of variation of internal energy which itself consists of two terms. The first one depends on the temperature variation of material, while the second one, \( \dot{E}_p \) is the rate of variation of the so-called stored energy of cold work, which is that part of the mechanical input energy responsible for changes in material microstructure (re-arrangement of crystal imperfections, persistent slip band formation, etc.) leading to the initiation of fatigue micro-cracks [13].

Referring to Figure A.1, upon suddenly stopping the fatigue test at the time \( t' \) when the surface temperature has reached the stationary value \( T' = T_{stat} \), the mechanical input power and the rate of variation of cold work, \( \dot{E}_p \) in Equation (A.1) drops to zero. Hence, just after \( t' \) (that is \( t = (t')^+ \)) Equation (A.1) can be written as:

\[
\left. \rho c_p \frac{\partial T}{\partial t} \right|_{t=t'} = -(H_{cd} + H_{cv} + H_{v}) \]

(A.2)

where \( H_{cd} = -k \nabla^2 T \), \( H_{cv} = h_c (T - T_u)p/A \), \( H_{v} = h_v (T - T_o)p/A \), \( k \) is thermal conductivity and \( A, p \) are the cross section area and the perimeter of the cross section area as shown in Figure A.2. Since the thickness of the beam is small compared to the other dimensions, the variation of temperature across the thickness is assumed negligible. Consequently, the heat dissipation due to
conduction has been evaluated by measuring the surface temperature. Similarly, the radiation heat dissipation can be evaluated by considering the surface temperature and emissivity. The specimen surface was covered with a black paint with surface emissivity of $\varepsilon = 0.93$. Once the fatigue test is suddenly stopped, continuous recording of full field temperature provides us with the temperature evolution at each location on specimen. Then, the term on the left hand side of the Equation (A.2) can be evaluated at any point of interest on the specimen from the cooling curve. In this work, the area near the clamped end of the specimen where fracture occurs is of interest. The first term on the right hand side of the Equation (A.2) $H_{cd}$ is associated with conduction heat transfer and is evaluated by calculating the second derivative of temperature along $x$ the axis as shown in Figure A.2. It is to be noted that the variation of temperature across the thickness and width of the specimen is neglected. Therefore, temperature distribution along the $x$ axis on the mid-width line as shown by dashed line in Figure A.2 is obtained. Having obtained the temperature distribution along the $x$ axis, the second derivative of the temperature can be calculated at each location. The second term on the right hand side of Equation (A.2) $H_{cv}$ is associated with the convection heat transfer which is unknown and needs to be calculated from Equation (A.2). The third term on the right hand side of Equation (A.2) $H_{ir}$ is evaluated having determined the surface temperature distribution, the radiation heat transfer $h_r$, and the surface emissivity $\varepsilon$. Once the slopes of the cooling curve, $H_{cd}$ and $H_{ir}$ are evaluated, the convective heat transfer $H_{cv}$ can be calculated using Equation (A.2). Following this procedure, the heat transfer coefficient near the clamped end of the beam was found to be $h_r = 25 \text{ W/m}^2\text{K}$. It is to be noted that for constant heat transfer coefficient $h$ and constant thermal conductivity $k$, as the ratio of $A/p$ increases the heat transfer capacity by conduction becomes more significant with respect to heat transfer capacity by convection and radiation.
Figure A.1. Determination of heat transfer coefficient by experimental measurement of the cooling rate

Figure A.2. Reduced section length of the specimen with rectangular cross section having area $A$ and perimeter $\rho$. 
Appendix B: An Order of Magnitude Analysis

An order of magnitude analysis is performed to show that entropy generation due to mechanical dissipation is dominant and that entropy generation due to heat conduction is negligible.

Referring to Equation (B.1)

\[
\dot{\gamma} \approx \frac{w}{\theta} + k \frac{\Delta \theta}{\Delta y}^2 \approx \dot{\gamma}_{\text{mech}} + \dot{\gamma}_{\text{cond}}
\]  
\[
\text{(B.1)}
\]

where \( \Delta \theta \) represents the temperature difference between two cross sections at a distance, \( \Delta y \) with one of the sections being the failure section (Figure B.1). The tension-tension fatigue test of 5 KN load amplitude, the load ratio of zero, and the frequency of 10 Hz is considered for this analysis. The hysteresis energy data obtained using MTS instrument, temperature data form IR camera and \( \Delta y = 1.2 \text{ mm} \) are substituted in the Equation (B.1) to calculate the effect of mechanical dissipation and thermal dissipation on the entropy production. Table B.1 summarizes the scale analysis in details.

![Figure B.1 Schematic of temperature gradient in scale analysis](image)

Table B.1. Order of magnitude of tension-tension fatigue test

<table>
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<tr>
<th>% of total life (N_t)</th>
<th>( \Delta \theta ) (°C)</th>
<th>( \theta ) (°C)</th>
<th>( w ) (KJ m(^{-3}))</th>
<th>( \frac{\dot{\gamma}<em>{\text{mech}}}{\dot{\gamma}</em>{\text{cond}}} )</th>
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<tr>
<td>90</td>
<td>3</td>
<td>62.5</td>
<td>75</td>
<td>150</td>
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Examination of the results show that entropy generation due to heat conduction is negligibly small to that of the mechanical dissipation. Hence, the Equation (B.1) reduces to

\[ \gamma \approx \frac{w}{\theta} \approx \gamma_{\text{mech}} \quad \text{(B.2)} \]
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Vita
Mehdi Naderi Abadi was born in September 1975 in Neka, north of Iran. He went to the primary and high schools which were located in his hometown. He finished high school in 1992.

He was accepted in mechanical engineering at Amirkabir University of Technology in Tehran in 1992 and finished his bachelor’s degree in March 1997. After getting his bachelor’s degree, Mehdi went to military services for two years. Again, he was accepted in Iran University of Science and Technology to pursue his education in mechanical engineering. Simultaneously, he got a job in Azar Ab Industries Company as a design engineer of pressure vessel and heat exchanger. He finished his master’s degree in June 2002. His master’s thesis was on the numerical analysis of flow and heat transfer in horizontal channels. Mehdi then continued to work in Azar Ab Industries Company for three years more and switched his job as a consultant engineer in Namvaran Engineering and Management.

He moved to USA in October 2006 after he was accepted in the doctoral program in Louisiana State University. He finished his studies under the supervision of Prof. M. M. Khonsari who is the director of Center for Rotating Machinery (CeRoM). The main focus of his doctoral research has been on thermodynamic analysis of fatigue failure.