Travel time tomographic imaging of the distribution of the effective stress in clean sand under a model footing

William M. Tanner

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TRAVEL TIME TOMOGRAPHIC IMAGING OF THE DISTRIBUTION OF THE EFFECTIVE STRESS IN CLEAN SAND UNDER A MODEL FOOTING

A Thesis

Submitted to the Graduate School of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering

in

The Department of Civil and Environmental Engineering

By
William M. Tanner
B.Sc. Louisiana State University, 2002
May, 2004
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ABSTRACT

The use of high-speed data acquisition systems, inexpensive and reliable transducers, and better models of interpretation have combined to make elastic wave tomographic imaging of geotechnical engineering systems easier to accomplish both in the laboratory and in the field. An important application of these developments is that the evaluation of states of effective stress in soils using images of elastic wave velocity distribution. As a consequence, it is possible to experimentally estimate the state of induced and in-situ effective stresses and to compare these results with established models of stress distribution based on the theory of elasticity (e.g., Boussinesq’s solution).

The effective stress versus shear wave velocity relationship follows a Hertz’ model. The parameters for this relationship are calibrated by testing the dry sand in both a modified triaxial cell and an oedometer cell hosting bender elements.

The long term objective of this research is to obtain a tomographic image of the states of in-situ and induced stresses in clean, dry sand underneath a model footing. The post calibration test program consists of a test cell that is capable of yielding a K₀-state of stress condition while allowing independent control of simulated overburden pressure and bearing pressure. The elastic waves will be generated and received using bender elements (i.e., bimorph piezoceramic crystals).

Justification of the travel time data from the test cell is made possible by a numerical integration of the Boussinesq solution for our stress conditions. Furthermore, velocity field images are presented as well as recommendations for improvements.
CHAPTER 1
INTRODUCTION

1.1 Motivation of Investigation

The motivation for this research stems from the potential of using elastic wave travel time tomographic imaging for monitoring the in-situ properties and behavior of geomaterials and their application on foundation engineering. Data from elastic wave propagation offers the potential to assess various types of systems including crack propagation in concrete due to over-stressing, internal void creation in reinforced concrete and pavement, and stress concentration and distribution in materials. In particular for this research, stress imaging can provide more detailed descriptions of the distribution of effective stresses in soils. This can be of great value in a range of geotechnical engineering problems from monitoring structures behavior to assess slope stability problems. Rather that relying on elastic solution estimates of these stresses, they can be assessed in-situ and also non-destructively. The ultimate hope is to one day see tomographic imaging as an integral part of pre, during and post construction evaluation.

1.2 Objective of this Research

The major objective associated with this research is the evaluation of the feasibility of rendering a tomographic imaging of the effective stress distribution by means of elastic wave propagation. To achieve this objective several tasks are performed. The first task is the evaluation of the shear-wave velocity – effective stress relation for the sandy soil used in this investigation. The second task is the collection of a shear-wave travel time data in a system with controlled state of the stress. The third task involves the development of robust model to justify and explain the collected data. The final task is
the attempt of obtaining a tomographic image of the distribution of the effective state of stress under a circular spread footing.

1.3. Organization of the Thesis

Chapter 2 presents wave propagation concepts in solids then reintroduces some of the concepts for particulate media. In this chapter the stress-shear wave velocity equation is given. This equation is central to this research. The remainder of this work focuses on attaining different portions of this equation.

Chapter 3 describes the methods and analysis for obtaining the calibration values for the stress-velocity equation. This chapter delineates three different tests that are conducted to this end. This is the first major task: to obtain calibration values for the stress-velocity equation.

Chapter 4 details the design and construction of a cell that enables the independent control of overburden pressure and bearing pressure applied through a model circular footing. Chapter 4 also describes the various experiments that are run in this cell. The data collected from these experiments are analyzed in later chapters. This partially fulfills the second and third tasks, the comparison of measured travel times to theoretical travel times and determination of an inverted velocity field.

Chapter 5 presents the implementation of the elastic solution. The Boussinesq’s solution for distribution of stress in a semi-infinite elastic medium underneath a point load is integrated around the surface of our model footing. The stress levels due to the Boussinesq problem are then added to the overburden stress for the completion of the theoretical effective stress field. This enables the calculation of theoretical travel times for comparison to measured travel times.
Chapter 6 implements an inversion algorithm for the calculation of a velocity field. This is the fourth and final task of the research program.

Chapter 7 summarizes the conclusions of the research and proposed tasks and objectives for future research endeavors.
CHAPTER 2
CONCEPTS OF WAVE PROPAGATION IN SOILS

2.1 Wave Propagation in Elastic Solids

If a certain location on a medium is subjected to an internal acceleration, the result will be the propagation of the perturbation throughout the entire medium. Such an action is termed a mechanical wave. Furthermore, if the medium through which the mechanical wave is traveling is elastic, then the particle motion due to the wave will displace and return to its original position once the wave has passed. This perturbation of propagation is known as an elastic waves. In an unbounded region of a body two modes of elastic wave propagation exist. The manner in which a particle is displaced relative to the direction of wave propagation is indicative of the type of wave moving through the body or on the surface of the body. One is the longitudinal or compression wave in which the particle displacement is in the direction of the wave propagation. The other is a transverse distortion wave where the particle motion is perpendicular to the direction of the wave propagation. At boundaries, several types of surface waves may exist, including: Raleigh waves, Stonley waves, and Love waves (Kolsky 1963; Achenbach 1975).

The differences in propagation between the different types of waves are revealed mathematically by the wave equation. Considering only one-dimensional propagation the differential form of the wave equation can be derived from Newton's Second Law and Hooke's Law and is presented as (Elmore and Heald, 1969)

\[
\rho \frac{\partial^2 u}{\partial t^2} = E \frac{\partial^2 u}{\partial x^2} \quad \text{(longitudinal waves)} \quad (2.1)
\]
\[
\rho \frac{\partial^2 \theta}{\partial t^2} = G \frac{\partial^2 \theta}{\partial x^2} \quad \text{(transverse waves)} \quad (2.2)
\]

where \(\rho\) is the medium density, \(u\) is the particle displacement, \(t\) is time, \(x\) is the Cartesian coordinate, \(E\) is Young's modulus of elasticity, and \(G\) is the shear modulus.

**2.2 Velocity of Wave Propagation in Elastic Solids**

Each mode of wave propagation has a different velocity for any given medium. The velocity of an elastic wave through a particular medium depends on the inertial properties and elastic properties of that medium. The elastic property of a material determines to what spatial extent the localized excitation is felt upon the instant of application. If a body were infinitely deformable, only the point of excitation would be displaced and the wave velocity would be zero. Likewise, if a body were infinitely rigid the entire body would displace synchronously with the point of excitation and the wave velocity would be infinitely large. The inertial properties of the medium offer some resistance to the particle passing on its energy to the adjacent particle. A body that has no inertia will propagate a wave instantly to the boundaries of the body. A body with infinite inertia will not propagate a wave as the velocity will be zero (Achenbach 1975). The velocities of longitudinal compression waves, \(V_p\), and transverse distortional waves, \(V_s\), are revealed in the wave equations and are presented in Equations 2.3 and 2.4

\[
V_p = \sqrt{\frac{M}{\rho}} \quad (2.3)
\]

\[
V_s = \frac{G}{\rho} \quad (2.4)
\]

Where \(M\) is the constraint modulus. The constraint modulus and Young's modulus are related by the Poisson’s ratio \(\nu\) as
\[
M = \frac{1 - v}{(1 + v)(1 - 2v)} E \tag{2.5}
\]

and the shear modulus is related to Young's modulus by

\[
E = 2(1 + v)G \tag{2.6}
\]

The above equations for velocity of a propagating wave are for a completely elastic medium. Under this condition, Equations 2.3 and 2.4 are pertinent and the material is said to be non-dispersive. If the medium is not elastic it is said to be dispersive and the velocity of propagation becomes frequency and amplitude dependent (Santamarina et al 2001).

2.3 Reflection and Transmission in Elastic Solids

When a wave traveling through a homogenous elastic solid medium encounters an interface between two materials with different properties some of the energy of the wave is reflected back into the incident medium and some of the wave energy is transmitted into the adjacent medium. The amplitude of the reflected and transmitted wave depends on the properties of the two media (elastic parameter and density) and the angle of the incident wave. Figure 2.1 shows a graphical representation of the reflection and transmission of incident compression and horizontally and vertically polarized transverse waves. In this figure subscript i denotes incident, r denotes reflection and t denotes transmission. Snell's Law relates angles of incidence, reflection and transmission and mode propagation velocities. Snell's law is (Richart et al 1970),

\[
\frac{\sin a}{V_{p1}} = \frac{\sin b}{V_{s1}} = \frac{\sin e}{V_{p2}} = \frac{\sin f}{V_{s2}} \tag{2.7}
\]

where angles a, b, e and f are the angles of incidence, reflection and transmission for each wave type.
In accordance with the principle of conservation of energy, the energy of the incident wave must be parcelled out to each of the resulting reflected and transmitted waves. Richart et al (1970) presented this distribution of energy in terms of amplitudes since the energy of an elastic wave is proportional to the square of the amplitude. For

**Figure 2.1** Reflections and transmissions of a) an incident compressional wave, b) an incident shear wave polarized vertically and c) an incident shear wave polarized horizontally.
each type of incident wave Equations 2.8 through 2.17 describe the distribution of energy into each reflection and transmission:

a) For an incident compression wave:

\[
(A - C)\sin a + D\sin b - E\sin e + F\cos f = 0 \tag{2.8}
\]

\[
(A + C)\cos a + D\sin b - E\cos e - F\sin f = 0 \tag{2.9}
\]

\[
-(A + C)\sin 2a + D\frac{V_{p1}}{V_{sl}}\cos 2b + E\frac{\rho_2}{\rho_1}\left(\frac{V_{s2}}{V_{sl}}\right)^2 V_{p1}\sin 2e - F\frac{\rho_2}{\rho_1}\left(\frac{V_{s2}}{V_{sl}}\right)^2 V_{p1}\cos 2f = 0
\]

\[
-(A - C)\cos 2b + D\frac{V_{sl}}{V_{p1}}\sin 2b + E\frac{\rho_2}{\rho_1}\frac{V_{p2}}{V_{p1}}\cos 2f + F\frac{\rho_2}{\rho_1}\frac{V_{s2}}{V_{sl}}\sin 2f = 0 \tag{2.11}
\]

b) For an incident transverse wave polarized in the vertical direction:

\[
(B + D)\sin b + C\cos a - E\cos e - F\sin f = 0 \tag{2.12}
\]

\[
(B - D)\cos b + C\sin a + E\sin e - F\sin f = 0 \tag{2.13}
\]

\[
(B + D)\cos 2b - C\frac{V_{sl}}{V_{p1}}\sin 2a + E\frac{\rho_2}{\rho_1}\frac{V_{s2}}{V_{sl}V_{p2}}\sin 2e - F\frac{\rho_2}{\rho_1}\frac{V_{s2}}{V_{sl}}\cos 2f = 0 \tag{2.14}
\]

\[
-(B - D)\sin 2b + C\frac{V_{p1}}{V_{sl}}\cos 2b + E\frac{\rho_2}{\rho_1}\frac{V_{p2}}{V_{sl}}\cos 2f + F\frac{\rho_2}{\rho_1}\frac{V_{s2}}{V_{sl}}\sin 2f = 0 \tag{2.15}
\]

c) For an incident transverse wave polarized in the horizontal direction:

\[
B - D - F = 0 \tag{2.16}
\]

\[
B - D - \frac{\rho_2}{\rho_1}\frac{V_{s2}}{V_{sl}}\frac{\cos f}{\cos b} = 0 \tag{2.17}
\]

where A is the amplitude of the incident compression wave, B is the amplitude of the incident transverse wave with either polarization, C is the amplitude of reflected compression wave, D is the amplitude of reflected transverse wave with either
polarization, \( E \) is the amplitude of transmitted compression wave, \( F \) is the amplitude of transmitted transverse wave with either polarization, \( \rho_1 \) is the density of medium 1 and \( \rho_2 \) is the density of medium 2.

Figure 2.2 Amplitudes of a) reflected compression wave, b) reflected shear wave with vertical polarization, c) transmitted compression wave and d) transmitted shear wave with vertical polarization (Richart et al 1970).

Figure 2.2 shows the amplitude of each reflected and transmitted wave due to an incident compression wave normalized with respect to the incident compression wave amplitude and how that ratio varies with the incident angle. Figure 2.3 shows the same plot for an incident transverse wave polarized in the vertical direction.
2.4 Attenuation of Waves in Elastic Solids

In general, the decay of waves as they travel through a body is due to geometrical spreading of the wavefront and material losses. However, in a perfectly elastic medium, any energy that is taken to move a particle as a wave passes will be regained as the
particle returns to its at rest condition. Therefore, the only attenuation that exists as a wave travels through a perfectly elastic medium is the attenuation due to wavefront spreading. For this type of attenuation, the ratio of amplitudes between two points is

\[ A_1 \cdot r_1^n = A_2 \cdot r_2^n \]  

(2.18)

where \( A_1 \) is the amplitude at point 1 which is located at distance \( r_1 \) from the source and \( A_2 \) is the amplitude at point 2 which is located at distance \( r_2 \) from the source, and \( n \) is an exponent that depends on the geometry of wave propagation front (\( n=0.5 \) for cylindrical wave front and \( n=1 \) for spherical wave front - Santamarina et al 2001). This equation is valid if the wavefront is spherical in form as is the case for a point source.

2.5 Wave Propagation in Soils

Soils have a very small elastic strain region and are inelastic outside this region. However, if the strain level in a soil mass is kept to a very low level the behavior can be assumed to be elastic. In this case, the equations of motion for waves in elastic solids are pertinent. Even for the case in which it is appropriate to consider the soil behavior to be elastic, one must still deal with the discrete nature of the solid phase of the soil. Hertz in 1881 turned his attention to the problem of two elastic spheres loaded axially, see Figure 2.4.

![Simple schematic of the classical Hertzian problem](image)

**Figure 2.4** Simple schematic of the classical Hertzian problem
The load-deformation response is naturally non-linear since an incremental vertical displacement caused by the load will encounter larger and larger circular cross sections at the interface. This interaction is shown in Figure 2.5.

![Figure 2.5 Non-linear force deformation response to the Hertzian problem (Santamarina et al 2001).](image)

In the course of his study, the following relationship is derived for the bulk modulus of a system of spherical particles under isotropic loading conditions.

\[
B = \frac{1}{2} \left[ \frac{2G}{3(1-\nu)} \right] \sigma_0^{\frac{1}{3}}
\]

(2.19)

where \( \sigma_0 \) is the confining pressure (Richart et al 1970). The bulk modulus is related to the elastic parameters constraint modulus \( M \), and the shear modulus \( G \) as a function of Poisson's ratio.

\[
M = \frac{3(1-\nu)B}{(1+\nu)}
\]

(2.20)

\[
G = \frac{3(1-2\nu)B}{2(1+\nu)}
\]

(2.21)
The substitution of Equation 2.19 and 2.20 into Equation 2.3 gives rise to the following expression for compression wave velocity (Richart et al 1970).

\[ V_p \propto \sigma_0^{1/6} \]  \hspace{1cm} (2.22)

2.5.1 Wave Velocity and Effective Stresses

Wave velocity depends both on the stiffness and density of the medium. These parameters are related to the effective stresses, the formation history, the degree of saturation, and the amount of cementation material in soils. In cases where the effect of saturation (i.e., fully saturated or fully dry soils) and cementation may be disregarded, simple equations may be derived to relate velocity and effective state of stresses (Duffy and Mindlin 1957; White 1983). These equations help evaluating velocities of wave propagation as power relations of the effective state of stress. Experimental studies have shown that the velocity of wave propagation in soils may be expressed in terms of the effective stress in the direction \( \sigma' || \) of wave propagation and in the direction of particle motion \( \sigma' \perp \) (Roesler 1979; Stokoe et al. 1991; LoPresti and O’Niel1991; Jardine et al. 2001; Fioravante and Capoferri 2001) as:

\[ V_p = \alpha \left( \frac{\sigma' ||}{\sigma_{\text{ref}}} \right)^{\beta} \]  \hspace{1cm} (2.23)

\[ V_s = \alpha \left( \frac{\sigma' ||}{\sigma_{\text{ref}}} \right)^{\beta ||} \left( \frac{\sigma' \perp}{\sigma_{\text{ref}}} \right)^{\beta \perp} \]  \hspace{1cm} (2.24)

where \( \alpha \) is the wave velocity at a mean effective confinement stress equal to 1 kPa and is unique for each mode of propagation, \( \beta, \beta || \) and \( \beta \perp \) are exponents that depend on the type of soils and its stress history, and \( \sigma_{\text{ref}} = 1 \) kPa is the reference pressure. If a shear wave is
propagating through a medium is subjected to isotropic stresses, $\sigma'_{\parallel} = \sigma'_{\perp} = \sigma'_0$, and $\beta_{\parallel} + \beta_{\perp} = \beta$. Physical meanings of $\alpha$ and $\beta$ parameters are discussed in greater detail in Chapter 3.

2.5.2 Damping and Effective Stresses

Even though the wave propagation is considered to be elastic, no material is perfectly elastic and some material or intrinsic losses will be experienced as well as geometric attenuation. In addition to velocity, the state of stress has influence on how the wave will attenuate in a soil mass. Santamarina et al. (2001) gives the following equation for the small strain damping ratio

$$D = \frac{\alpha_D}{\left( \frac{\sigma'_0}{1 \text{kPa}} \right)^{\beta_D}}$$  \hspace{1cm} (2.25)

where $\alpha_D$ is the damping value corresponding to 1 kPa confinement in Figure 2.6 , $\sigma'_0$ is the isotropic confinement and $\beta_D$ is an experimentally determined value. Figure 2.6 shows the relationship of damping ratio vs. confinement.

![Figure 2.6 Damping ratio vs. confining pressure (Santamarina et al 2001).](image)

2.5.3 Wave Velocity and Void Ratio

Hardin and Richart (1963) studied the effects of void ratio on wave propagation. Their study utilized resonant column testing on Ottawa sands, crushed quartz sand and
crushed quartz silt. They concluded that void ratio was the most influential variable on wave velocity, varying very nearly linearly. Figure 2.7 shows the variation of both velocity of transverse waves and shear modulus with void ratio. Furthermore, Figure 2.7 presents findings for two types of particles shapes, round and angular.

**Figure 2.7** Shear wave velocity vs. void ratio for several confining pressures (Hardin and Richart 1963).

For the Ottawa sand, the empirical velocity equations are

\[
V_s = (170 - 78.2e)\sigma_0^{1/4} \quad \text{for} \quad \sigma_0 > 95\text{kPa}(2000\text{psf}) \quad (2.26)
\]

\[
V_s = (119 - 56e)\sigma_0^{3/10} \quad \text{for} \quad \sigma_0 < 95\text{kPa}(2000\text{psf}) \quad (2.27)
\]

where \(e\) is the void ratio and \(\sigma_0\) is effective confining pressure. Similarly, the equation of velocity for the crushed quartz sand is

\[
V_s = (159 - 53.5e)\sigma_0^{1/4} \quad \text{for all } \sigma_0 \quad (2.28)
\]
2.6 Summary

In this chapter wave propagation concepts are presented. The chapter begins with a brief overview of classical continuum mechanics including mode of propagation, velocity of each type of propagation, waveform reflection and transmission and geometric attenuation of waves in a perfectly elastic medium. These concepts pertain to the global behavior of a geo-system rather than considering the soil’s multi-phase nature.

Beginning with Section 2.5, some of the wave propagation concepts are reintroduced from the perspective of the discrete nature of a soil mass. The wave velocity concept is built upon the classical Herztian problem and results in Equation 2.22. Experimental studies have shown that the velocity can be related as a power relation of the effective state of stress. Equations 2.23 and 2.24 show these semi-empirical relations. Equation 2.24 is of particular importance regarding this research. In the next chapter, this equation is calibrated for our purposes.
CHAPTER 3
CALIBRATION OF THE EMPirical S-WAVE VELOCITY-EFFECTIVE STRESS RELATIONSHIP

3.1 Wave Velocity Measurements in Soils

Dynamic soil characterization tests are abundant (Woods 1978; Brocanelli and Rinaldi 1997), however few fall into the category of linear strain levels (Kramer 1996). Such small strain level tests include tests that generate shear deformations that are smaller than the threshold strain $\gamma_{th}<10^{-4}$, including resonant column tests and pulse impulse testing. This research will implement piezoceramic bimorphs as transducers that yield strain amplitudes below the threshold of elastic behavior (Brignoli et al. 1996). It is common procedure to modify the end platens of a standard triaxial cell to host the bender elements. Additionally, many other standard laboratory devices may be modified to accommodate the bender elements. This research utilized a triaxial cell, oedometer cell, and a modified pressure cell model each fitted with bender elements.

Flexural piezoceramic elements have been used in geotechnical applications for the past few decades (Shirley, 1978; Shirley and Hampton, 1978; Dyvik and Madshus, 1985; Thomann and Hryciw, 1990). The bending action of the element is a result of two thin piezoceramic plates which are glued to opposite sides of a conductive metal shim (refer to Figure 3.1). These plates extend and contract independently upon application of a voltage differential. The result of combining them is a flexing or bending type action when the plates are properly oriented with respect to each other.

Figure 3.1c shows that the cantilever action of the element can be accomplished in one of two ways.
Figure 3.1 Bender elements configuration and operation: (a) series bender element arrangement, (b) parallel bender element arrangement, and (c) electrical circuit and action (Morgan Electro Ceramics 2003).

The two methods differ in the polarization of the plates with respect to the middle metal shim. The series-arranged bender element has one plate with its negative side toward the metal shim while the other plate has its positive side toward the shim. The parallel-arranged bender element has the negative side of each plate positioned toward the metal shim. Parallel operation yields a tip displacement twice that of the series operation for the same applied voltage. This is because the full driving voltage is applied to each plate in the parallel case (Morgan Electro Ceramics 2003). Since this research is concerned with achieving the highest possible receiver response, the series operation is used. Reversing the polarity of the drive voltage causes the deflection to change direction, thus a dynamic action can be provided by a signal generator. The strain levels produced by this dynamic cantilever action are below the linear elastic strain limit of particulate media.

Because of the fact that the maximum displacement generated by bender elements are very low and the fact that they may be used as both actuator and sensor, bender
elements are well suited for geotechnical engineering research involving elastic wave propagation. However, the geo-system environment requires special preparation of the transducer. It must be shielded from water intrusion on to the contact points of the wires as well as grounded to minimize electromagnetic interference, see Figure 3.2. An example of data collected from bender elements, with and without grounding is shown in Figure 3.3. Santamarina et al (2001) recommend preparation techniques for geotechnical studies.

**Figure 3.2** Illustration of the sealing and shielding techniques used for this research

![Illustration of sealing and shielding techniques](image)

**Figure 3.3** Very large electromagnetic interference can mask the arrival of the mechanical wave. The electromagnetic interference is a capacitive discharge type curve.

Fiorvante and Capoferri (2001) and Blewett et al (1999; 2000) have illustrated some uses of bender elements in wave propagation studies on triaxial specimens. It is
common to modify the end caps of the triaxial specimen by machining a groove for the bender to rest in, while providing a cantilever reaction. The bender element is then slightly intrusive to the soil specimen. Zeng and Ni (1998) took a similar approach to placing bender elements within an oedometer cell. Their application involved installing a pair of bender elements in the side walls of the cell to measure the shear modulus in the horizontal plane as well as a pair in the end caps to measure the shear modulus in the vertical direction.

### 3.2 Physical Meaning of the $\alpha$ and $\beta$ Parameters

The $\beta$ parameter in Equations 2.23 and 2.24 is known for a limited set of theoretical situations. These equations are given again below.

\[
V_p = \alpha \cdot \left( \frac{\sigma_{\parallel}}{\sigma_{\text{ref}}} \right)^{\beta} \tag{2.23}
\]

\[
V_s = \alpha \cdot \left( \frac{\sigma_{\parallel}}{\sigma_{\text{ref}}} \right)^{\beta_i} \cdot \left( \frac{\sigma_{\perp}}{\sigma_{\text{ref}}} \right)^{\beta_{\perp}} \tag{2.24}
\]

Santamarina et al (2001) presents several theoretical values for the $\beta$ parameter. These values are outlined in Table 3.1.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 1/6$</td>
<td>For the case of Hertzian contact, perfect spheres of elastic material</td>
</tr>
<tr>
<td>$\beta = 1/4$</td>
<td>For the case of cone tip to plane contact</td>
</tr>
<tr>
<td>$\beta = 1/4$</td>
<td>For the case of plastic yielding at the contact of spheres</td>
</tr>
</tbody>
</table>

Therefore, the velocity-stress relationship derived from Hertzian contact and continuum mechanics must be re-calibrated for any given soil type where the overall contact behavior is not known. This is because even slight deviations from geometric
perfection cause the $\beta$ value for the Equation 2.22 to change significantly. This fact is illustrated nicely with the work done by Duffy and Mindlin (1957). Their results are shown in Figure 3.4.

![Figure 3.4: S-wave (?) velocity versus effective confining pressure for specimens made of steel bearings (open circles: 1/500th in dimension tolerance and open triangles: 1/100th in dimension tolerance - Duffy and Mindlin 1957)](image)

Duffy and Mindlin (1957) constructed two bars out of steel ball bearings in a face-centered cubic packing. One bar consisted of ball bearings with a diameter tolerance of five hundred thousandths of an inch while the other bar consisted of ball bearings with a tolerance of one hundred thousandths of an inch. As shown in Figure 3.4, the bar formed of the ball bearings with the smaller tolerance produced a velocity trend closer to the theoretical Hertzian velocity trend. Theoretical values can however serve as a general guide.

Whereas $\beta$ indicates the slope of the velocity-stress relationship and is dependent upon the geometry of the particle contacts, $\alpha$ is the value of the shear wave velocity at a mean confining stress of 1 kPa and is dependent on other physical properties of the soil. These include type of packing, fabric changes that occur when the state of stress is
changed and Poisson’s ratio. Santamarina et al. (2001; 2003) have experimentally shown that there is a relationship between $\alpha$ and $\beta$ for different types of soils. Figure 3.5 shows that for isotropically loaded soils as the $\alpha$ coefficient increases, the $\beta$ coefficient decreases. At the limit, cemented geomaterials such as intact rocks, the wave velocity is almost independent of the state of effective stresses (e.g., Mavko et al. 1998).

To determine $\alpha$ and $\beta$ values for the sand used in this research, tests are conducted in both a modified triaxial cell and a modified oedometer cell. The following sections present the methodology used for each as well as the results and discussion.

3.3 Calibration in a Modified Triaxial Cell

A deep groove is machined into each end platen of the triaxial cell. A coated and wired bender element is then placed into the groove in such a way as to form a small cantilever. Figure 3.6 shows a detail of the placement of the bender element in the end platen. Figure 3.7 shows the benders in relation to the whole specimen and test setup.

Figure 3.5 Relationship between $\alpha$ and $\beta$ for typical soils. Data from Santamarina et al (2001).
Figure 3.6 Sketch of placement of bender elements within the end platens of a triaxial cell. The vertical arrow represents the direction of shear wave propagation while the transverse arrow on the left indicate the direction of particle motion. The symbols below the vertical arrow on the right indicate that the direction of particle motion in the plane of the page.

The configuration shown in Figure 3.7 enables shear waves to propagate throughout the sand specimen. The bender element on the opposing end platen receives the mechanical disturbance and translates it back into an electrical signal. Figures 3.8 and 3.9 show the test setup.

Figure 3.7 Bender elements in the triaxial cell showing position and orientation.
Figure 3.8 View of test specimen within the triaxial cell.

Figure 3.9 Test setup showing load frame and pressure board.
3.3.1 Isotropic Stress Conditions

For verification of the experimental calibration method and comparison to published data, the triaxial specimen is subjected to a series of isotropic stresses. The procedure is as follows:

1. Secure the rubber membrane to the bottom end platen with an O-ring
2. Place the split mold around the bottom platen and stretch the membrane over the mold
3. Apply a slight vacuum to the cavity between the mold and membrane
4. Cut a slit in a piece of filter paper to allow the paper to slide down over the bender element and rest flush on the end cap. This is to prevent sand from falling down the hole in the epoxy filling. The hole allows for the application of suction directly to the specimen
5. Fill membrane with sand
6. Place top cap over specimen and membrane around top cap. Secure with an O-ring
7. Apply suction to the specimen and remove the mold
8. Place the cell over the specimen and fill cavity with water
9. Subject specimen to the following isotropic loads. 6.9, 13.8, 20.7, 27.6, 34.5, 68.9, 103.4, 137.9, 172.4, 206.8, 241.3, 275.8, 310.3, and 344.7 kPa. Likewise unload the specimen in the same manner. At each stress for both loading and unloading collect a waveform for data analysis

This procedure is applied to three sand specimens each with a different void ratio. The first had a void ratio of 0.69, the second had a void ratio of 0.77 and the third had a void ratio of 0.79. The different void ratios are achieved by compaction of the sand with vibration as the triaxial specimen mold is being filled. For each specimen, the shear wave
velocity corresponding to each stress increment is evaluated. Once the shear wave velocities have been determined, they are plotted against the effective confining stress. Figures 3.10 shows an example of waveforms for the test with an initial void ratio of 0.79. Figures 3.11, 3.12 and 3.13 show the shear wave velocity vs. isotropic loading and unloading results for each tested specimen. Details of the model production are given in Appendix A, Mathgram A.1.

Figure 3.10 Time-series traces for the specimen with an initial void ratio of 0.79 a) loading and b) unloading (fig. con’d).
Figure 3.11 Shear wave velocities for specimen with initial void ratio of 0.69.
Figure 3.12 Shear wave velocities for specimen with an initial void ratio of 0.77.

Figure 3.13 Shear wave velocities for specimen with an initial void ratio of 0.79.
Figures 3.11 through 3.13 are plotted in log scale so that the data trends are more apparent. A possible explanation for the loading being faster than the unloading is that for any given stress on the plot, the loading point represents the specimen with a lesser density than the unloading point. From the Equation 2.4 for continuum wave mechanics, a lesser density in any medium means a higher shear wave velocity. Furthermore, it is clear that a transition in the slope of both loading and unloading occurs when the log of the pressure value in kPa is approximately 1.5. Since the slope of the velocity pressure relationship is governed by particle contacts, the explanation of this phenomenon lies in the behavior of these contacts. Pre-transition pressure particle contacts behave elastically and post transition pressure particle contacts behave plastically (see also Fratta and Santamarina 2002).

3.3.2 Anisotropic Stress Conditions

To capture the effect of anisotropy on the sand, a series of alternating isotropic stress and anisotropic stress paths were used. The procedure used is as follows:

1. Assemble the specimen according to the same procedure for isotropic stress conditions.

2. Once the specimen is assembled and in the triaxial cell, isotropically pressurize the specimen to 10, 20, 30, 40, and 50 kPa, and collect waveform data at each stress level.

3. Leave $\sigma_3$ at 50 kPa and increase $\sigma_1$ to 70, 90, 110, and 130 kPa. Next, decrease $\sigma_1$ in like manner until isotropic stresses are restored. Collect waveform data at each loading and unloading point.

4. Isotropically pressurize the specimen from 50 to 60, 70, 80, 90, and 100 kPa and collect waveform data at each point.
5. Leave $\sigma_3$ at 100 kPa and increase $\sigma_1$ to 140, 180, 220, and 260 kPa. Decrease $\sigma_1$ in like manner until isotropic stresses are restored. Collect waveform data at each loading and unloading point.

6. Isotropically pressurize the specimen from 100 to 120, 140, 160, 180, and 200 kPa and collect waveform data at each point.

7. Leave $\sigma_3$ at 200 kPa and increase $\sigma_1$ to 280, 360, 440, and 580 kPa. Decrease $\sigma_1$ in like manner until isotropic stresses are restored. Collect waveform data at each loading and unloading point.

This procedure is applied to three sand specimens each with a different void ratio. The first has a void ratio of 0.62, the second has a void ratio of 0.71 and the third has a void ratio of 0.89. The different void ratios were achieved by compacting the sand with vibration as the triaxial specimen mold was being filled. Example traces from specimen with initial void ratio 0.62 are given in Figure 3.14.

**Figure 3.14** Velocity traces for specimen with initial void ratio of 0.62 a) isotropic compression and b) one CTC cycle is shown.
Anisotropic stresses follow the conventional triaxial compression line but do not reach the failure envelope. The stress path that the specimen was subjected to is shown in Figure 3.15. At each data point indicated by a box, waveform data is collected. On each conventional triaxial compression (CTC) line, waveform data was collected twice for every data point seen. This is to capture the hysteretic effects of the velocity-stress relationship during axial loading and unloading.

**Figure 3.15** Stress path taken for every anisotropic calibration test. Cambridge definitions are used for mean stress \((p=(\sigma_1+2\sigma_3)/3)\) and deviator stress \((q=\sigma_1-\sigma_3)\).

The next step is to calculate the shear wave velocity. Figures 3.16 through 3.18 show the shear wave velocities plotted against mean stress \((p)\) for each of the tests.

**Figure 3.16** Shear wave velocities in the triaxial chamber.
The isotropic compression data points have a shallower slope than the CTC data points. The trend of data points lying along the steeper slopes indicates the phase of testing in which $\sigma_1$ is changed while $\sigma_3$ is kept constant. Using the data in Figures 3.14 through 3.16, it is possible to calculate $\alpha$, $\beta_1$ (or $\beta_1^\perp$) and $\beta_3$ (or $\beta_3^\perp$) values in equation 2.24. Whereas the $\alpha$ and $\beta$ parameters for the isotropic stress tests could be evaluated simply by fitting a line through the data, the parameters needed for Equation 2.24 were evaluated by utilizing a least squares solution. Details of the solution can be found in the Appendix A in Mathgram A.2.
3.4 Modified Oedometer Testing

Calibration also took place in a modified oedometer cell. The plexiglass cell is prepared by cutting a pair of horizontal slits on opposing sides of the cell. Likewise a pair of vertical slits is cut into opposing sides of the cell rotated 90 degrees from the horizontal pair. Wired and coated bender elements are then secured into the slots with hot glue, see Figure 3.19

![Figure 3.19 Top view of the oedometer cell](image)

The cell is filled and loaded on a consolidation frame as seen in Figure 3.20. The stress is applied vertically through the end cap which rests on the soil surface. Therefore the vertical stress throughout the specimen corresponds to the applied vertical stress and the stresses in the horizontal plane correspond to the Ko condition. Since the shear wave velocity depends on the stress in the direction of wave propagation and the stress in the direction of particle displacement, the bender elements in the horizontal plane collect information regarding the vertical and horizontal stress. The bender elements in the vertical plane collect information regarding the horizontal stresses only. The procedure used with the oedometer calibration is:
1. Fill the cell with sand until the sand level is flush with the top lip of the cell.

2. Place the pore stone and the top cap directly on the sand surface.

3. Apply the following vertical stresses to the specimen, 0, 25, 50, 75, 100, 150, 200, 250, 300, 350, 400, 300, 200, and 100 kPa. Collect waveform data for analysis at each stress increment.

This procedure was applied to one specimen that has an initial void ratio of 0.70.

Figure 3.20 a) Close-up of specimen in the cell on consolidation frame, b) loading frame.

Figure 3.21 shows the velocity profile for the horizontally polarized waves. The shear wave velocity here depends on the stresses in the vertical and horizontal direction.

Figure 3.22 shows the velocity profile for the vertically polarized waves. The velocity for this case depends on the stresses in the horizontal plane only.
Since the level of stresses which control the shear wave velocity for the horizontally polarized waves are lower, because of the $K_o$ condition, one would expect the maximum shear wave velocity to be lower. In fact, that is exactly what is shown by comparison of Figures 3.21 and 3.22. Mathgram A.3 has additional details.

Table 3.2 gives a summary of the $\alpha$, $\beta_{\text{parallel}}$ and $\beta_{\text{perpendicular}}$ that were used to create the models shown for the oedometer cell and triaxial cell experiments. The information given for the isotropic triaxial tests in Table 3.2 is plotted against published data for
varying types of soils and other particulate media in Figure 3.5. The capital (I) in Table 3.2 denotes an isotropic triaxial test where (A) denotes anisotropic triaxial test.

**Table 3.2** Summary of \( \alpha \) and \( \beta \) parameters

<table>
<thead>
<tr>
<th>Test Type</th>
<th>( \alpha ) (m/s)</th>
<th>( \beta_{par} )</th>
<th>( \beta_{perp} )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-Triaxial Cell</td>
<td>Pre-break Loading: 137.5 Pre-break Unloading: 100.1 Post-break Loading: 119.4 Post-break Unloading: 90.5</td>
<td>Pre-break Loading: 0.125 Pre-break Unloading: 0.189 Post-break Loading: 0.173 Post-break Unloading: 0.222</td>
<td></td>
<td></td>
</tr>
<tr>
<td>eo = 0.69</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I-Triaxial Cell</td>
<td>Pre-break Loading: 129.2 Pre-break Unloading: 98.4 Post-break Loading: 111.2 Post-break Unloading: 77.6</td>
<td>Pre-break Loading: 0.13 Pre-break Unloading: 0.172 Post-break Loading: 0.178 Post-break Unloading: 0.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>eo = 0.77</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I-Triaxial Cell</td>
<td>Pre-break Loading: 138.1 Pre-break Unloading: 105.5 Post-break Loading: 91.9 Post-break Unloading: 70.7</td>
<td>Pre-break Loading: 0.102 Pre-break Unloading: 0.151 Post-break Loading: 0.224 Post-break Unloading: 0.27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>eo = 0.79</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A-Triaxial Cell</td>
<td></td>
<td>0.147</td>
<td>0.024</td>
<td>0.171</td>
</tr>
<tr>
<td>eo = 0.62</td>
<td>123.98</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A-Triaxial Cell</td>
<td></td>
<td>0.146</td>
<td>0.012</td>
<td>0.158</td>
</tr>
<tr>
<td>eo = 0.71</td>
<td>128.52</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A-Triaxial Cell</td>
<td></td>
<td>0.171</td>
<td>-0.012</td>
<td>0.159</td>
</tr>
<tr>
<td>eo = 0.89</td>
<td>122.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oedometer Cell</td>
<td></td>
<td>0.17</td>
<td>0.02</td>
<td>0.19</td>
</tr>
<tr>
<td>horizontal pol.</td>
<td>122.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oedometer Cell</td>
<td></td>
<td>0.17</td>
<td>0.02</td>
<td>0.19</td>
</tr>
<tr>
<td>vertical pol.</td>
<td>120</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Even though Figures 3.21 and 3.22 look as if the data makes fits nicely with the model, the level of confidence is very low. The original traces did not have a clear shear wave arrival, for this case a travel time trend was overlaid on the data in an attempt to identify the proximity of the arrival (see Mathgram A-3). Even with the trend as a guide, the arrival is still unclear. For this reason, data for the oedometer cell has been deemed unreliable. The best results come from the two densest anisotropic triaxial cell tests. The trend for all three anisotropic triaxial tests, indicate that the predominate parameter is \( \beta_{par} \). This parameter has much more influence over the velocity of the shear wave than \( \beta_{perp} \).
3.5 Discussion

3.5.1 Velocity Analysis

For proper velocity calculations, two important pieces of information must be known. These are the distance the wave travels and the time it takes to travel that path. Investigations of the appropriate travel length were conducted by Viggiani and Atkinson, (1995) and Dyvik and Madshus (1985). The work by Viggiani and Atkinson (1995) involved varying lengths of triaxial cell specimens with bender elements mounted in the end caps. The results of this study indicate that the correct travel length is not the full triaxial specimen height but rather the distance from bender element tip to bender element tip. This is slightly less than the specimen height because the bender elements intrude a few millimeters into the soil. These conclusions agree with previous research. While the travel length is straightforward and easy to measure, the travel time of the wave is often more complex.

The interpretation of travel time can be difficult and sensitive to technique as documented by Viggiani and Atkinson (1995), Santamarina and Fam (1997), Ferreira (2003), Arulnathan et al (1998), among other leading researchers. As mentioned by Jovicic et al (1996), the main problem is the subjectivity involved when determining an arrival. The travel time for the purposes of this research is determined using either time shifted signals to calculate the difference in time between successive signals or by the intersection of tangents of zero and first slope. Other methods of determining the arrival time, such as cross correlation, yield physically impossible travel times. This is probably due to the frequency shift between successive signals caused by the change in effective stresses in the soil specimen (Santamarina and Fratta, 1998). Zeng and Ni (1998) indicate
that the size of the bender element also affects the clarity of the shear wave first arrival. They concluded that if the stress levels are high enough, 300 kPa for their experiment, the motion of the bender will be inhibited. To correct the problem, the researchers optimized the size of the bender elements by experimentation taking into account stress levels, mounting techniques, sample size and tip-to-tip distances.

3.5.2 Boundary Conditions

Part of the difficulty in identifying the wave arrival arises from the fact that the wave which arrives first may not necessarily be the sought after shear wave. Reflected compression waves and a component of the shear wave, the near field component which travels at the speed of a compression wave, both can interfere with the positive identification of the direct shear wave as indicated by Sanchez-Salinero et al (1986) and Jovicic et al (1996). Figure 3.23 shows the compression and shear wave paths in the oedometer cell.

![Figure 3.23](image)

**Figure 3.23** a) Direct shear and reflected compression wave paths for the bender elements in the vertical plane. b) Direct shear and reflected compression wave paths for the bender elements in the horizontal plane.

The determining factors of whether a reflected compression or a direct shear wave will reach the receiver first are the geometry of the cell, the orientation of the bender
elements (i.e., vertical or horizontal plane) and the Poisson’s ratio of the material being tested. The equation used to evaluate the ratio of travel times of compression and shear waves is Equation 3.1

\[
V_p^2 = \frac{V_s^2(v - 1)}{\left(v - \frac{1}{2}\right)}
\]  

(3.1)

Table 3.3 summarizes the results of the analysis on the oedometer cell using Equation 3.1. Values for Poisson’s ratio are taken to be within the range 0.10 to 0.20. This is a reasonable assumption of Poisson’s ratio for elastic wave propagation. In Table 3.3, \(t_p\) is the travel time of the compression wave and \(t_s\) is the travel time of the shear wave.

<table>
<thead>
<tr>
<th>Poisson’s ratio</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel time ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal Benders (t_p/t_s)</td>
<td>1.045</td>
<td>1.006</td>
<td>0.96</td>
</tr>
<tr>
<td>Vertical Benders (t_p/t_s)</td>
<td>1.084</td>
<td>1.043</td>
<td>0.995</td>
</tr>
</tbody>
</table>

The results of the analysis indicate that the first arrival will not be the shear wave but rather the compression wave that is reflected off of the walls of the plexiglass cell. In order to avoid this complication, it is recommended that the geometry of the cell be designed such that the reflected compression wave will not reach the receiver before the direct shear wave.

3.6 Summary

It is necessary to select an appropriate transducer for measuring states of stress. In light of the literature, bender elements are selected to provide a shear wave source and
receiver. Furthermore, the unknown parameters of Equation 2.24 (S-wave velocity versus effective stress) are discussed and physical meaning attached to each. The calibration of Equation 2.24 takes place in two different standard laboratory test cells that are modified to host the bender elements. These are a triaxial cell and an oedometer cell. For each type of test, the methodology used is presented as well as the results of each test. Values for the calibrated parameters are shown in Table 3.2. A discussion is provided acknowledging concerns of the directivity of the propagating waves and the determination of the first arrival.

Equation 2.24 has three parts. One part is the shear wave velocity, one part is the state of stress and the third part is the calibration parameters. With the calibration parameters assigned values based on the work discussed in this chapter, the next chapters will be concerned with obtaining either the shear wave velocity (Chapter 4) or the state of stress (Chapter 5).
CHAPTER 4
WAVE PROPAGATION TESTING
IN TOMOGRAPHRIC PRESSURE CELL

4.1 Introduction

Chapter 3 documents the calibration of the shear wave velocity versus state
stresses relation for a uniform fine sandy soil. Once the calibration is complete, it is only
natural to try to obtain data to evaluate the feasibility of rendering tomographic images of
the state of effective stresses. To accomplish this research goal, a pressure cell is
designed and built. This test cell permits not only controlling the simulated in-situ $K_o$
state of stresses but it also permits controlling the bearing pressure of a simulated spread
footing. Under these testing conditions, the shear wave data is collected at different
depths. The interpreted shear wave velocity is evaluated to monitor the induced effective
stresses. The description of the test set and the collected data is presented next. The long
term objective for the tests presented in this chapter is to combine the calibrated stress-
velocity equation with field measurements to image the effective state of stress via
inversion analyses.

4.2 Test Setup and Design

The testing setup for the monitoring the state of effective stresses by means of
shear wave propagation under a circular footing is described in this section. The pressure
cell is a large plexiglass cylinder with aluminum end caps (see figure 4.1). An inflatable
bladder is placed between the top of the soil and the bottom of the top end cap. This
bladder allows the application of a uniform bearing pressure. The bladder pressure is
forced down into the soil by the reaction of the top end cap. A small hole (15 mm
diameter) in the middle of the bladder allows the passage of a rod through the bladder and into the soil below. At the end of the rod, there is a model footing buried beneath the sand. The top end of the rod couples to a loading frame that provides the reaction for the bearing pressure. This bearing pressure is separate and independent of the overburden pressure supplied by the bladder. A picture of the cell is shown in Figure 4.1.

![Figure 4.1 Picture of the pressure cell, the sand specimen, and the load frame with the probing ring.](image)

To generate and monitor the propagation of shear waves, bender elements oriented in the vertical direction to produce horizontally polarized shear waves are aligned in two vertical columns one on either side of the footing. This bender-element configuration simulates typical cross-hole setups as shown in Figure 4.2. It is expected that the spacing shown in Figure 4.2 captures a large majority of the induced stress bulb.
created by the bearing pressure. Induced stress bulbs are discussed more thoroughly in Chapter 5. The test setup enables the measurement of shear wave velocity as a function of different stress states (i.e., Equation 2.24).

**Figure 4.2** Test setup showing configuration and orientation of bender elements.

Figure 4.2 shows how the bender elements are attached to so called “bender-element anchors”. Since the bender elements are not resting in a groove machined into the side of the plexiglass wall, they must be cantilevered in another way. The anchors are glued to the bender elements with approximately one half of the bender element protruding from the tip of the anchor. The horizontal pressure due to the $K_o$ condition acts over a larger area on the anchor than on the protruding tip of the bender element. The result of this is that a larger force acts on the anchor than on the bender thus producing a cantilever type reaction. The isolation of the bender elements from the side walls or end
caps prevents waves from spreading via the test cell in addition to propagating through the specimen. A detail showing the dimensions of the bender element and the anchor are shown in Figure 4.3.

![Bender Element and Anchor Diagram]

**Figure 4.3** Detail of bender element and anchor

### 4.3 Zero-Bearing Pressure Test

The purpose of this test is to capture the effects of constant overburden without the influence of bearing pressure. Waveform data are collected in the cross-hole testing format. The procedure is as follows:

1. Lower the load frame platform until there is no contact between the load cell and the rod through which bearing pressure is applied.

2. On the specimen used for test #4, inflate the bladder to 17.2, 34.5, 103.4, and 137.9 kPa. At each overburden pressure, collect waveform data for analysis.

The results of the zero bearing tests are shown in Figure 4.4. Results indicate that the velocity is fairly constant with depth and increases with increasing overburden pressure as expected, although there appears to be a slight drop in velocity with depth. This could be due to an arching effect of the stresses in developed by the bladder.
4.4 Cross-hole Testing

Four tests conducted in the cell shown in Figure 4.2 were evaluated in a cross-hole manner. The purposes of these tests are to demonstrate effects of boundary and scaling problems as well as to serve as a guide for the solution of the full tomographic test. The procedure for the cross-hole testing is as follows:

1. Fill the cell with sand. As the sand is being poured, place the bender elements in their prescribed locations.

2. Once the sand level reaches the prescribed depth for the footing to be placed, stop pouring sand and place the footing on the soil surface. Two tests use the 76.2 mm diameter footing and two tests use the 127 mm diameter footing.

3. Continue pouring sand until the lip of the plexiglass cell is encountered.

4. Place the deflated bladder on the lip of the plexiglass and the top end cap over the bladder.
5. Secure the top end cap by placing the threaded rods through both end caps and
    tightening down the machine nuts.
6. Connect the pressure line to the bladder and place the steel ball bearing in the pit on
top of the footing rod. This should be aligned vertically with the bottom of the load
cell.
7. Inflate the bladder with 16.5 kPa of pressure. This acts as simulated overburden
    pressure.
8. For the test using overburden pressure only, apply bladder pressures of 17.2, 34.5,
    103.4, and 137.9 kPa
9. For the tests using the 76.2 mm diameter footing, apply pressures of 48.7, 97.5, 146.3,
    195.1, 243.9, and 292.6 kPa. For the tests using the 127 mm diameter footing, apply
    pressures of 17.6, 35.1, 52.7, 70.2, 87.8, and 105.3 kPa.
10. Collect waveform data for each source location. For the cross-hole tests, only the
    receiver located in the same horizontal plane as the source is considered.
    Each footing size is tested twice, once with a dense specimen and once with a
    looser specimen. The first two tests use the 76.2 mm diameter footing and had an initial
    void ratio of 0.88 and 0.82, respectively. While the third and fourth test uses the 127 mm
diameter footing and had a void ratio of 0.95 and 0.75, respectively. As with the
    specimens in Chapter 3, the different void ratios were achieved by vibrocompaction.
    The velocities of the cross-hole tests can be seen in Figures 4.5 through 4.8. The
    velocity trends show a general decrease with depth in agreement with expectation due to
    theoretical stress distribution. Also, since the velocity is proportional to the stress,
increased stress levels via increased bearing pressures yield the increasing velocity trend.
It should be noted that the velocities are computed by assuming a linear wave travel path. In fact, the wave will follow the fastest rather than the shortest. Since stress conditions are not homogeneous, one may expect a non-linear travel path. This will be discussed further in Section 4.5

![Cross-hole testing: shear wave velocity versus depth results (initial void ratio $e_o=0.88$)](image)

**Figure 4.5** Cross-hole testing: shear wave velocity versus depth results (initial void ratio $e_o=0.88$)

Elastic solutions indicate that ten percent of the induced vertical stress will be felt by the soil at a depth of approximately 2.0 times the width of the footing on the axis of symmetry. At some depth therefore, the influence of the induced stress should be minimal. In this region, stresses are dominated by the simulated overburden and as a consequence the velocity of the wave at that point will depend mostly on the overburden stress rather than the induced bearing pressure. The expectation is that the velocity curves
will converge beyond the depth of influence of the induced bearing pressure. This trend is not clearly seen in Figure 4.5 although it appears that the slopes suggest such a point of convergence if extrapolated. In an effort to capture this effect more clearly, the deepest two sets of bender elements are spaced at a distance of 50.8 mm from the adjacent bender element rather than the 25.4 mm spacing separating the shallowest six. This is shown in the results of the remainder of the tests (Figures 4.6-4.8). In these figures, if a data point appears to be missing, indicated by no dotted line extending into the velocity graph, it is because the signal received from that bender element was not useful for data analysis. This event is caused by a variety of reasons, usually however from the grounding wire snapping off of the anchor or a short circuit.

Figure 4.6 Cross-hole testing: shear wave velocity versus depth results (initial void ratio $e_o=0.82$): a) loading and b) unloading (fig. con’d)
Figure 4.7 Cross-hole testing: shear wave velocity versus depth results (initial void ratio $e_0=0.95$): a) loading and b) unloading (fig. con’d)
Figure 4.8 Cross-hole testing: shear wave velocity versus depth results (initial void ratio $e_0=0.75$): a) loading and b) unloading (fig. con’d)
The results show a greater tendency for the velocities to converge at depth. This trend is especially apparent when comparing Figure 4.5 to figure 4.6a. In these tests, the 76.2 mm diameter footing is used however Figure 4.6a has a deeper set of bender elements.

**4.5 Full Tomographic Testing**

In addition to cross-hole testing, a full tomographic velocity profile was obtained. This test involved capturing waveform data on all eight receivers for a given source. This test generates much more information about the testing region than a cross-hole test. The data from this test is used to create the tomographic image of the state of stress in the specimen. Because of the principle of reciprocity, the entire medium can be tested with waveforms traveling in only one direction. Figure 4.9 shows the paths of wave travel for this experiment. Again, the assumption inherent in Figure 4.9 is that the wave path is linear. This is not strictly correct as the ray path with follow the Fermat’s principle.
Figure 4.9 Each wave path collects information from the specimen. Compared to figures 4.4 through 4.8, the information content is clearly higher for this type of test.

This test is also carried out on the same specimen presented in Figure 4.8. This is a time saving step and because the specimen had already been stressed and relaxed, it was necessary to increase the overburden pressure to ensure that the soil was in a normally consolidated state. The level of overburden pressure equaled the highest level of overburden applied to the specimen in Figure 4.4 (overburden pressure only). From this point, bearing pressures were applied. The procedure is:

1. Inflate the bladder to a pressure of 137.9 kPa. This is equal to the maximum stress the soil has felt thus far.

2. Apply bearing pressures of 0, 70, 140, and 280 kPa. For each of these pressures collect waveform data. Each source will have eight corresponding receiver waveforms. The testing region will be as shown in Figure 4.9. Velocity results of the tomographic test by source number are presented in Figures 4.10 through 4.13.
Figure 4.10 Average velocity profile results for source 1 and 2
Figure 4.11 Average velocity profile results for source 3 and 4
Figure 4.12 Average velocity profile results for source 5 and 6
Figure 4.13 Average velocity profile results for source 7 and 8
4.6 Discussion

4.6.1 Boundary Conditions

As in the case of the oedometer and triaxial cells in Chapter 3, the test cell described here must be analyzed for the reflection of compression waves and its relative travel time with respect to the direct shear wave. The analysis is carried out as before, utilizing Equation 3.1. However, in the oedometer and triaxial cells, waveforms traveled in only one or two planes whereas for the full tomographic test waveforms traveled in 64 different planes. Considering only the extreme cases greatly reduces number of analyses that must be performed. The result depends on the geometry of the cell in the plane of wave propagation and the Poisson’s ratio. The minimum travel length for either the compression or shear wave is in the horizontal plane, thus for the analysis a cross section of the test cell may be considered. Figures 4.14 and 4.15 show the wave path in the cell for both the minimum and maximum travel path, respectively.

Figure 4.14 Path taken by a direct shear wave and a reflected compression wave for the minimum travel distance (Source 1 and Receiver 1).
Results of the analysis are displayed in Table 4.1. In Table 4.1, $t_p$ is the travel time of the compression wave and $t_s$ is the travel time of the shear wave. The conclusion of this analysis is that for the minimum travel distance, the shear wave will arrive before the compression wave. For the maximum travel length, the compression wave will arrive slightly ahead of the direct shear wave. Therefore, care must be taken in the determination of the shear wave arrival for the case of long travel lengths. Compression waves with multiple reflections will arrive late enough to exclude their consideration. Note that the compression wave arrival comes earlier as the value of Poisson’s ratio increases. At a Poisson’s ratio of 0.5, a shear wave cannot propagate and the travel time ratio will be infinitely large.
Table 4.1 Directivity analysis of pressure cell

<table>
<thead>
<tr>
<th>Poisson’s ratio/ Travel time ratio</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum travel length (t_p/t_s)</td>
<td>1.57</td>
<td>1.52</td>
<td>1.45</td>
</tr>
<tr>
<td>Maximum travel length (t_p/t_s)</td>
<td>0.973</td>
<td>0.936</td>
<td>0.894</td>
</tr>
</tbody>
</table>

4.6.2 Scale Effects

There are mainly two scale effects: one related to the size of the footing and its stiffness with respect to the stiffness of the soil, and the effect of the size of the footing and the cell in relation to the particle. The second scale effect is deemed negligible.

The Boussinesq solution for stress distribution below a footing assumes that the material below the footing reacts to the applied load with a uniform pressure diagram (i.e., a flexible footing). This is also the assumption in the conventional rigid method of foundation design (Bowles 1988; Das 1999). The Winkler solution uses classical beam mechanics to formulate solutions for deflection, slope, moment and shear. It is indicated that the variable $\beta$ in Equation 4.1, arising from the Winkler solution, indicates whether the foundation chosen should be considered rigid or flexible (Bowles 1988; Das 1999).

$$\beta = \sqrt[4]{\frac{B_1 k}{4EI}}$$  \hspace{1cm} (4.1)

where $E$ is the modulus of elasticity of foundation material, $I$ is the moment of inertia of cross section of equivalent beam, $B_1$ is width of cross section of equivalent beam and $k$ is coefficient of subgrade reaction. Based on the material properties and geometry of the equivalent beams of both the 76.2 mm and 127 mm diameter footings used in this study, the $\beta$ values indicate that it is appropriate to consider the footings as rigid. Therefore, proper consideration for the stress distribution must be considered in the analysis.
Chapter 5 will take a more detailed look at the implications of a rigid versus a flexible footing with regard to the stress distribution.

4.7 Summary

In this chapter, detailed descriptions are given for the test cell that is used to capture shear wave velocities as a function of induced and in-situ stresses. Three types of tests are run. One type is a zero bearing test in which only the in-situ stresses are felt by the sand, one type is a cross hole test in which in-situ and induced stresses are applied independently and the third is a full tomographic test in which in-situ and induced stresses are also applied independently.

The travel times measured from the cross hole tests is needed to compare to the calculated travel times produced as a result of knowing the stress field (Chapter 5). This is for justification of the data presented in this chapter.

The full tomographic test is needed to produce a full velocity field (Chapter 6). This velocity field is necessary for the rendering of a state of stress image.

Furthermore, a discussion is presented which acknowledges boundary conditions and scale effects of the testing cell.
CHAPTER 5
EVALUATION OF INDUCED STRESSES UNDER A FOOTING
FOR THE INTERPRETATION OF S-WAVE VELOCITY

5.1 Introduction

The interpretation of the S-wave velocity data under a footing depends both in the in-situ and induced stresses. For the case study presented in this thesis, the in-situ stresses may be evaluated from the overburden stresses obtained from the applied bladder pressure and an assumed Ko-condition. The induced stresses must be evaluated from elastic solutions.

Authors over the years have published several models including the Boussinesq’s solution, Gibson’s model and Holl’s model (Poulos and Davis 1974). All these models permit the evaluation of the induced stresses caused by a point load on the surface of a semi-infinite medium that is linear, elastic, and isotropic. The difference between these models is that one assumes that the materials is homogeneous (Boussinesq’s solution), linearly increasing modulus of elasticity (Gibson’s models), or power increasing modulus of elasticity (Holl’s model).

This thesis will make use of the Boussinesq’s solution to evaluate the induced stresses under the footing. This solution is favored over the other solution because of its simpler implementation and because it provides a first good estimate to the distribution of induced stresses under circular footings.

5.2 In-situ and Induced Stresses

Stresses in any geo-system have one of two sources. One is the self weight, or in-situ, stress of the soil and the other is due to applied external, or induced, stresses. The
combination of these two stresses forms the total stresses. Typically, the total stresses are shared between the soil particles and the pore water. The amount of stress felt by the soil particles can be computed by Equation 5.1:

\[ \sigma' = \sigma - u \]  

(5.1)

in this equation \( u \) is the pore water pressure, \( \sigma \) is the total stress and \( \sigma' \) is the effective stress felt by the soil skeleton. Equation 5.1 is valid for full pore saturation. If the geo-system has no pore water, as in the case of this research, Equation 5.1 reduces to Equation 5.2.

\[ \sigma' = \sigma \]  

(5.2)

That is, the stress felt by the soil therefore is equal to stress caused by the total weight applied above the point under evaluation. For a homogenous soil, the vertical in-situ stress is calculated by Equation 5.3.

\[ \sigma_v = \gamma z \]  

(5.3)

where \( \gamma \) is the unit weight of the soil, and \( z \) is the depth to the point in question. In the case of zero lateral soil movement the horizontal in-situ stress is obtained by multiplying the vertical in-situ effective stress by the value of the lateral earth pressure coefficient at rest, \( K_0 \). This value represents the ratio of horizontal to vertical in-situ stresses. For normally consolidated soils the lateral earth pressure coefficient is often computed using Equation 5.4.

\[ K_0 = \left[ 1 - \sin(\phi) \right] \frac{OCR}{\sin(\phi)} \]  

(5.4)

where \( \phi \) is simply the friction angle of the material and OCR is the over-consolidation stress of the soil. In the case of a normally consolidated soil, the OCR is equal to one and Equation reduces to be a function of the friction angle only.
Induced stresses in a soil due to external pressures are usually computed using elastic solutions. These solutions assume that the soil is semi-infinite, homogenous, linear, elastic and isotropic. Boussinesq in 1885 presented a solution for the stress in an elastic medium due to a point load on the surface. This solution is applied to this research in the following section.

5.3 Evaluation of Induced Stresses: Boussinesq’s Solution

Poulos and Davis (1974), among many others, present the equations for stress in the Boussinesq’s problem. These solutions can be seen in Equations 5.5 through 5.7.

\[
\sigma_v = \frac{3Pz^3}{2\pi \nu^2 R^5} \tag{5.5}
\]

\[
\sigma_r = -\frac{P}{2\pi \nu^2} \left[ -\frac{3r^2z}{R^3} + \frac{(1-2\nu)R}{R+z} \right] \tag{5.6}
\]

\[
\sigma_\theta = -\frac{(1-2\nu)P}{2\pi \nu^2} \left[ \frac{z}{R} - \frac{R}{R+z} \right] \tag{5.7}
\]

In these equations, \(P\) is the point load, \(\nu\) is the Poisson’s ratio, and every other variable is as defined in Figure 5.1. These equations are useful only for the specific case of a point load. For other loading conditions, i.e. circular footings, square footings, strip footings or any other non-point load, Equations 5.5 through 5.7 must be integrated over the surface in question. Many researchers and authors have presented the result of the three common cases of circular, square and strip footings. For example, Lambe and Whitman (1963), Poulos and Davis (1974), and McCarthy (1998) have all displayed graphical and/or tabular solutions for various loading cases.

This research is concerned only with the solution for a circular footing. Furthermore, for the purpose of knowing the induced stresses along any given shear wave
path, we have developed our own solution for the circular footing and compared its graphical results to published results from the authors mentioned above.

![Diagram](image)

**Figure 5.1** Nomenclature for the Boussinesq’s solution presented in equations 5.5 through 5.7, $\sigma_r$ is the radial stress and $\sigma_\theta$ is the circumferential stress

Rather than performing a true integration of the Boussinesq’s solution in which the area is divided into an infinite number of infinitesimally small sectors, a numerical integration of the area in which larger sectors are summed together is implemented. Each sector is resolved into a point force located at its centroid and Boussinesq’s solution applied to every point in the soil below. Details of the integration can be seen in Mathgram A.4. Figure 5.2a shows the vertical induced stress as computed by our integration. For comparison, the vertical induced stress as presented by Lambe and Whitman (1963) is given in Figure 5.2b. Similarly Figures 5.3a shows the variation of the vertical induced along the horizontal axis, compared to Poulos and Davis (1974) in Figure 5.3b.
Figure 5.2 Boussinesq’s solution for the vertical induced stress due to a circular footing from a) numerical integration performed for this study and b) solution presented in Lambe and Whitman (1969).

Figure 5.3 Boussinesq’s solution for the vertical stress due to a circular footing stress displayed with ratio of induced to applied vertical stress on the x-axis: a) numerical integration and b) solution presented in Poulos and Davis (1974).
Figure 5.2 a and b displays the well known “pressure bulb” while Figure 5.3a and b display the percent of surface contact pressure on the x-axis and distance away from center in the horizontal direction in number of radii on the contours. Plots for both the induced stresses in the x and y directions are given in Figures 5.4 and 5.5.

**Figure 5.4** x-direction induced stresses a) numerical integration and b) Solution presented Poulos and Davis (1974 – Solution for Poisson’s ratio ν=0.5).

**Figure 5.5** y-direction induced stresses obtained by numerical integration of the Boussinesq’s solution.
Comparison plots for Figures 5.4 and 5.5 are hard to come by in the literature. Most authors present only the vertical induced stress field for purposes of settlement calculations. Poulos and Davis (1974) have the most extensive information regarding this solution found thus far. For this study, the bender elements lie in the xz plane (y = 0). Therefore, Figure 5.4 shows the stresses in the x-direction, or direction parallel to shear wave propagation for a few chosen distances of offset radii. Figure 5.5 shows the stresses in the y-direction, or direction perpendicular to shear wave propagation. The calculated elastic stress field is needed check the velocity field image, shown in Chapter 6. In other words, the calculated elastic stress field enables calculation of the velocity field for comparison with the measured velocity field. If the calculated velocity field and the measured velocity field do not match, we can begin to relax the assumption of purely linear wave paths in accordance with Fermat’s Principle. Travel times for non-linear travel paths can be modeled until a match is attained. In this way, the collected data can be justified.

5.4 Rigid vs. Flexible Footings

The Boussinesq’s solution to induced stresses assumes that a uniform soil pressure will develop directly underneath the footing. This is a good assumption when the foundation is said to be flexible. In Section 4.5, the model footing was treated as an equivalent beam section of a mat foundation so that the Winkler analysis presented in Bowles (1988) and Das (1999) could be applied. This analysis indicates that the footing may in fact behave rigidly. Under this type of behavior the soil reaction pressure may not be uniform as assumed by Boussinesq’s solution. Muki (1960) considered this problem. The distribution of stress for this case is shown in Figure 5.5.
When this is the stress distribution, the resolved force on the surface due to each discretized sector will not be proportional only to the size of the sector but also to its location away from the center of the footing. While it is fully realized that the pressure distribution may not be uniform, the analysis for this study will only assume a uniform soil pressure.
5.5 Evaluation of Travel Times

The elastic solution enables the approximate calculations of shear wave travel times by combining the calibrated velocity-stress equation (Equation 2.24) and the pixelized version of the stress distribution (see Mathgram A.5). This involved taking half of the region (dark shade) under the footing and subdividing it into a matrix of pixels, 3 across and 10 down, see Figure 5.6.

Figure 5.6 Subdivision of one-half of testing region into a matrix of pixels.

Each pixel is then assigned one stress value for the x direction stress (parallel) and y direction stress (perpendicular) to the direction of wave propagation assuming straight rays. The assigned value corresponds the mean value of the pixel. This is permitted by the elastic solution given in Section 5.3. Figure 5.7 a) and b) show the region in Figure
5.6 with color-coded stress values. Red indicates highest induced level of stress and blue indicates the lowest induced level of stress.

\[ \Delta \sigma_x \quad \Delta \sigma_y \]

**Figure 5.7** a) x-direction induced stresses and b) y-direction induced stresses calculated using the Boussinesq’s solution. Only the right-hand of the stressed areas are presented.

The cross-hole travel times are then calculated by summing the time a wave would spend crossing each pixel. Equation 5.8 shows the basic relationship used for this calculation.

\[ t = \sum \frac{L_i}{V} \]  \hspace{1cm} (5.8)
where $L_i$ is the length of the ray in pixel $i$ and $V$ is shear wave velocity obtained with the calibrated Equation 2.24. This equation is presented below for clarity.

\[ V = \alpha \left( \frac{\sigma_x'}{1\text{kPa}} \right)^{\beta_{\text{par}}} \left( \frac{\sigma_y'}{1\text{kPa}} \right)^{\beta_{\text{perp}}} \]  

(5.9)

In equation 5.9, $\sigma_x'$ and $\sigma_y'$ are not only the induced stresses but the combination of the induced and in-situ stress. The calibration parameters are assigned values of $\alpha = 130 \text{ m/s}$, $\beta_{\text{par}} = 0.15$ and $\beta_{\text{perp}} = 0.012$.

By comparing the theoretical travel times to the measured travel times, possible limitations in our analysis can be exposed. Comparison data used for this section are the cross-hole tests described in Section 4.3. Section 4.3 presents four cross-hole tests. Two are tested with a footing that has a diameter of 76.2 mm, and two are tested with a footing that has a diameter of 127 mm. For each footing size, travel times for a dense and a loose specimen are measured. However, only the tests corresponding to initial void ratios of 0.75 and 0.82 are used for comparison here as the theoretical model is based on the assumptions of Boussinesq’s solution and cannot address the issue of varying void ratios. These particular two tests are chosen because they are closest together in density. Figure 5.8 presents the theoretical and measured travel times for the test with corresponding void ratio of 0.82. Figure 5.9 presents the theoretical and measured travel times for the test with the corresponding void ratio of 0.75. The theoretical travel times converge well with depth indicating a fall off of the influence of induced stresses. The measured data does not display this behavior quite as apparently although there is some amount of converging behavior. This could be an error due to the cell boundaries and due to assumption of the straight rays (this latter situation will be further explored in Chapter 6).
Figure 5.8 Measured cross-hole travel times compared to theoretical travel times (footing diameter $D=76.2$ mm).

Figure 5.9 Measured cross-hole travel times compared to theoretical travel times (footing diameter $D=127$ mm).
5.6 Summary

It is necessary to integrate the Boussinesq’s solution for a point load over the surface of a circle. This provides the solution for the stress distribution under the model footing. All three normal stresses are computed in this way. Comparisons from the literature are provided to ensure that the integration is performed correctly. These stress levels must be combined with the overburden pressure provided by the inflated bladder to obtain the solution for the stress levels felt by the soil in the test cell.

It is acknowledged that while the Boussinesq’s solution assumes uniform soil pressure distribution, the physical reality of the system may be different. Since the system may be considered rigid (see Section 4.6) a different distribution may be in effect (Section 5.4). However the Boussinesq’s solution is carried out for this research and other solutions may be needed in the future (see Chapter 7).

The solution to the induced pressure distribution allowed for the calculation of theoretical travel times which are then compared to measured travel times. It is possible that any discrepancies between the two may be attributed to the assumption of a linear travel path in the calculation of theoretical travel times and the boundaries of the cell.
CHAPTER 6

IMPLEMENTATION OF THE INVERSION ALGORITHM FOR
THE EVALUATION OF EFFECTIVE STATE OF STRESS

6.1 Tomographic Imaging

Tomographic imaging is a powerful tool for the non-destructive assessment of bodies. While tomography has been widely used in the medical field for several decades, it has been slow to develop in civil engineering. Obstacles are the size of civil engineering systems, which for example are often much larger than a patient in a hospital, the limited angles of illumination and the cost of imaging an entire system (Fernandez and Santamarina 2003). However despite these obstacles, great potential remains for tomographical studies.

6.2 Linear Inversion Algorithms

The boundary measurements, the travel times, can be inverted to obtain information from the space across which each waveform travels. The act of sending waves through the medium is referred to as illumination (Prada et al 2000). Unfortunately, it is often very difficult if not impossible to obtain complete 360 degree illumination with geotechnical engineering systems. For the purposes of this research only one plane is illuminated, the plane corresponding to the xz plane with its origin located at the center point on the underside of the footing and containing the bender elements.

The first step in the inversion procedure is to subdivide the region between the bender elements in the cell described in chapter 4 into a matrix of pixels (Santamarina and Fratta 1998, Prada et al 2000, Fernandez and Santamarina 2003). Each pixel has its
own parameter such as position, velocity or slowness (velocity\(^{-1}\)) and induced stress. The calibration parameters \(\alpha\), \(\beta_{\text{par}}\) and \(\beta_{\text{per}}\) (in the S-wave velocity versus effective stress relation - Equations 2.23 and 2.24) are common to every pixel as they are dependent on the material of the medium rather than the spatial variation of the pixels. The travel time of a ray \(i\) is simply the summation of the product of the length \(L_{i,k}\) of the ray through each pixel \(k\) and the slowness of each corresponding pixel.

\[
t_i = \sum_k L_{i,k}s_k
\]  

(6.1)

The creation of the matrix of pixels is accomplished via an inversion algorithm developed by Santamarina and Fratta (1998). Details of this algorithm are shown in Mathgram A6. The position of each pixel relative to source and receiver bender elements is shown in Figure 6.1.

![Figure 6.1 Spatial distribution of pixels.](image)

Once the matrix of pixels is formed, the algorithm then computes the travel length matrix \(L\) (i.e., ray tracing) by computing the length that each ray will travel through each
pixel. The assumption with this algorithm is that the rays will follow a straight path.

Figure 6.2 shows the information content of each pixel. The lighter colored pixels represent high information content regions, many rays pass through that pixel, whereas the darker pixels represent low information content, few rays pass through that pixel. The image is not completely symmetric because some of the rays were eliminated from consideration due to the fact that they yielded physically unacceptable travel times.

![Ray density of each pixel.](image)

**Figure 6.2** Ray density of each pixel.

By reading the measured travel times from the experiment described in Section 4.4, the pixelized velocity field can be solved inversely via Equation 6.2 (Prada et al. 2000):

\[
\mathbf{s}^{(\text{predicted})} = \mathbf{L}^{\text{generalized inverse}} \mathbf{t}^{(\text{measured})}
\]

Inversion algorithm

\[
(6.2)
\]
where $s$, $L$ and $t$ are as previously defined. There are several least square solutions which can be applied to Equation 6.2, a comprehensive list can be found in Santamarina and Fratta (1998). For the purposes of this research, the Regularized Least Squares Solution is implemented. This choice of solutions enables the smoothing, or averaging of the second derivative (Prada et al 2000). Smoothing is physically acceptable in that the velocity may not jump instantaneously from point to adjacent point, the change must be gradual. This solution is

$$s_{\text{estimate}} = \left( L^T L + \lambda R^T R \right)^{-1} L^T t_{\text{measured}}$$

(6.3)

where $\lambda$ is the regularization coefficient and $R$ is the regularization operator. The regularization operator is generated by another algorithm developed by Santamarina and Fratta (1998) and is seen in Mathgram A.7. This operator provides the smoothing of the second derivative in the inverted velocity field image. The regularization coefficient must be optimized by monitoring at the same time the residual error and the difference between the maximum and minimum inverted velocity values (Figure 6.3). The residual error is the difference between the calculated travel time and the measured travel time:

$$E = t^{\text{measured}} - t^{\text{calculated}}$$

(6.4)

where the calculated travel time $t^{\text{calculated}}$ is determined as:

$$t^{\text{calculated}} = L s^{\text{predicted}}$$

(6.5)

The optimal regularization coefficient is found as a compromise between the minimizing the error and smoothing the variation of the inverted velocities (Santamarina and Fratta 1998). This is the value used for the tomographic inversion of velocities for all bearing pressures, $\lambda_{\text{op}}$ and $R$ remain constant throughout the inversion. As indicated by Figure 6.3 a and b, the $\lambda_{\text{op}} = 0.178$ m$^2$, see also Mathgram A.8.
Figure 6.3 a) Error between calculated and measured travel times and b) minimum and maximum inverted velocities with respect to regularization coefficient for all four bearing pressures.

It is probable that the error between measured and calculated travel times is due to model error (i.e., the assumed linear travel path used for the calculated travel times). The physical reality is that the travel path is probably somewhat non-linear according to Fermat’s Principle as discussed in Section 4.4. Thus for the data to be fully justified, other methods of determining travel length need to be implemented. A non-linear ray tracing algorithm may fit the data better. While it is acknowledged that travel paths may be non-linear, only the linear ray tracing algorithm is implemented.

Figure 6.4 shows each measured travel time relative to its calculated travel time. When travel length is either minimum or maximum, the calculated travel times seems to deviated from measured travel times. Travel lengths in between the extremes match the calculated and measured travel times well. Ray numbers are assigned according to Figure
6.4 a and b and continue in this manner until all rays are numbered. Some rays have been excluded because they yielded physically inaccurate travel times (refer to Chapter 5).

Figure 6.4 a) and b) show the definition of ray numbers. Additional numbering continues in this manner. c) Comparison of measured (symbols) and calculated (lines) travel times.

Bearing pressures:
- q = 0 kPa
- q = 70 kPa
- q = 140 kPa
- q = 280 kPa

Note: Trends are separated for ease of viewing. For this reason, numbers are not displayed on the vertical axis.
Figure 6.5 Velocity field image for all four bearing stresses.

The inverse of the slowness given in Equation 6.3 is the velocity. The velocity fields are shown in Figure 6.5. The results meet expectations in that the velocity decreases with depth (the induced stresses become smaller with depth, see Sections 4.4, 4.5 and 5.3) and with increasing bearing pressure (see Equation 2.24).

6.3 Addressing the Straight Ray Assumption

Since it is suspected that wave travel may be in a curved fashion, Figure 6.6 shows a qualitative rendering of the testing region and curved wave paths. This is strictly for illustration. The rays will bend toward the regions of higher stress. Since the highest stress level is directly beneath the footing, the rays will have an upward curvature as shown.
Figure 6.6 Qualitative rendering of curved ray paths and induced pressure bulbs.

The ray bending occurs not only due to the heterogeneity of the stress field but it is also exacerbated by the stress induced anisotropy (Santamarina and Fratta 1998; Fratta et al. 2001; Fernandez and Santamarina 2003). Rays in this sense will favor not only traveling close to the pressure bulb but they will also favor rays that are aligned towards the vertical direction as they capture greater stresses and therefore greater s-wave velocity (Equation 2.24).
6.4 Discussion of Tomographic Imaging Results

The velocity field is necessary so that each pixel can be assigned a slowness value. This is a prerequisite step in the tomographic inversion of the stress field. Because of the polarization of the shear waves, only two of the three normal stresses can be imaged via inversion. These two stresses, referred to radial and circumferential stresses (see Chapter 5), both lie in the horizontal plane. For the imaging of the vertical stress, the polarization of the wave needs to be rotated 90 degrees. This is accomplished simply by orienting the bender elements in the horizontal plane rather than the vertical.

6.5. Summary

This chapter opens by briefly presenting the potential of tomographic imaging within the field of civil engineering. The methods used for the tomographic inversion of the data collected for this research is then covered. Algorithms available from Santamarina and Fratta (1998) are used to develop the matrices needed to implement the Regularized Least Squares Solution. The choice of this particular solution enables the smoothing of the second derivative. This is acceptable physically because velocity changes must be gradual since the stress change is also gradual. The ray tracing algorithm assumes linear wave paths. Since the stress levels are known to vary throughout the specimen and that Fermat’s Principle applies, the validity of this assumption needs to be further investigated (see Chapter 7).
CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

This thesis presents the results of travel time tomography for the evaluation of the distribution of the effective stresses under a model of circular spread footings. Limitations in the data and methods of analysis are identified.

The development of the methodology for the tomographic imaging involves several steps, including calibration of the shear-wave velocity versus effective state of stress relationship, the development of a physical model of a shallow foundation system, the collection of elastic wave propagation data and the inversion analysis of the data.

This calibration of the shear wave velocity-stress equation (equation 2.24) is accomplished in the triaxial cell (Chapter 3). While the oedometer cell can be used, the determination of the first arrival is difficult due to the nearly simultaneous arrival of the P-and S-wave from different travel paths. Other difficulties in the analysis of the data in the oedometer cell are that several arrivals are captured at the receiver: signals coming from the plexiglass (both compression and shear waves), the reflected compressive wave, the direct shear wave, and the electromagnetic interference. The calibration parameters for the anisotropic test yield the range of values that may be used in the inversion algorithms. It is found that the exponent in the direction of wave propagation $\beta_{\text{par}}$ has a much greater influence on the velocity of shear wave propagation than the exponent in the direction parallel to particle motion $\beta_{\text{perp}}$. Isotropic compression triaxial calibration yield values for $\alpha$ and $\beta$ that match well with those published in the literature and fall along the trend associated with the $\alpha$, $\beta$ relationship.
Chapter 4 presents the test cell and the collected cross-hole and tomographic imaging data. The test cell includes independent control of overburden pressure and bearing pressure while preventing lateral displacement (simulated \( K_0 \)-condition). Under the simulated footing sixteen bender-elements sources and receivers are placed to generated and monitor shear waves. Velocities are captured for cross-hole and full tomographic tests. The zero-bearing test evaluates the effects of only bladder pressure on the sand specimen. In this test, velocity is expected to be constant with depth assuming full transfer of the bladder pressure vertically into the soil. The results indicate that the velocity drops slightly with depth. This implies less stress is felt by the soil at depth. This could be due to an arching effect of the stresses in the soil. The effect is that the stresses arch away from the bladder horizontally and vertically. The stresses then terminate on the cell wall and increase the friction between the soil and cell wall.

Chapter 5 presents the theoretical and numerical tools needed to evaluate the shear wave data. The induced stresses in the soil are obtained using the Boussinesq’s solution. The induced stresses are then coupled with the overburden stresses to obtain the theoretical stress field for the test region. This information permits the calculation of theoretical cross-hole travel times for comparison with measured cross-hole travel times. This analysis is performed to expose possible errors in our analysis and to justify our collected travel time data. While the theoretical travel times converge with depth due to diminishing induced stresses, the measured travel times show a lack of convergence with depth. This apparent inconsistency may also be attributed to arching of stresses in the cell. Other discrepancies between the theoretical and measured travel times may be due to the assumption of a linear wave travel path in the calculation of the theoretical travel
times. It is strongly expected that the wave rays travel not in a linear path, as assumed for our analysis, but rather in at least a slightly non-linear fashion in accordance with Fermat’s Principle (Fernandez and Santamarina 2003).

The full tomographic test data from Chapter 4 is used in Chapter 6 to calculate a velocity field. This is accomplished via inversion algorithms and the Regularized Least Square Solution, which permits smoothing of the second derivative of the solution. The inverted velocity field matches well with expectations of the velocity trend with increased stress and depth. This provides a pixelized representation of the velocity throughout the test region. The combination of this velocity field and the calibration parameters makes the rendering of the state of stress image possible.

The data from the full tomographic test contain rays whose velocity depends on stresses in all directions (x, y, and z). The stress in the direction parallel to wave propagation will have a component from both the x and z directions following not only the heterogeneity but also the anisotropy of the state of effective stresses.

7.2 Recommendations and Future Work

While advances are made throughout the course of this research, future work and constructive criticism are needed. It is recommended that any future calibration or tomographic testing take place in a cell that has been fully analyzed for first arrival prior to testing. Furthermore it is also recommended that any bender element testing avoid anchoring the benders directly to the housing of the test cell unless there is no rigid connection from one bender to the other, as in the case of the triaxial specimen.

Improvements also need to be made to the test cell described in Chapter 4. Currently, there is no way to assess any problems such as slightly non-vertical force
(eccentricity) on the model footing. There are a few options, first a linear bearing can be installed over the hole in the top end cap. This will reduce any friction on the rod providing the bearing pressure and help to ensure the load is purely vertical.

Second, a sheet of pressure film may be placed flush with the bottom end cap to measure the average effective stresses in the cell. A flexible layer will need to be resting on top of the pressure film to provide a coupling between the soil and the pressure film. This is needed because if the soil grains bear directly on the pressure film, the film will reveal only highly localized pressures at the point of grain contacts rather than the pressure being felt by the system as a whole.

Third, the model footing should be redesigned with varying thicknesses and diameters. This change in geometries will help assessing the variation in soil pressure development due to varying levels of footing flexibility. These three steps should help in assessing the presence of uniform soil pressure development directly underneath the footing.

Lastly, the height to diameter ratio of the cell should be decreased. This action should reduce the effect of arching. If the diameter of the cell becomes larger than can be serviced by the load frame, alternative loading mechanisms need to be developed.

A new inversion model needs to be implemented that takes into consideration the curvature in travel paths (Figure 6.6). This model must incorporate the heterogeneity and anisotropy in the effective stress field. The tomographic solution for such a problem becomes non-linear.

While the work completed and discussed represents a significant advance, the rendering of the state of stress image was not accomplished. However, all necessary
components are given, the inverted velocity field and the soil calibration. Future work will render the updated inversion algorithm and the image of the stresses that are studied in this research.
REFERENCES


APPENDIX A
MATHGRAMS

Mathgram A-1 Triaxial calibration for isotropic states of stress
Reading in raw data for analysis

A0 := READPRN("PRINT_00-3.txt")
A1 := READPRN("PRINT_01-3.txt")
A2 := READPRN("PRINT_02-3.txt")
A3 := READPRN("PRINT_03-3.txt")
A4 := READPRN("PRINT_04-3.txt")
A5 := READPRN("PRINT_05-3.txt")
A6 := READPRN("PRINT_06-3.txt")
A7 := READPRN("PRINT_07-3.txt")
A8 := READPRN("PRINT_08-3.txt")
A9 := READPRN("PRINT_09-3.txt")
A10 := READPRN("PRINT_10-3.txt")
A11 := READPRN("PRINT_11-3.txt")
A12 := READPRN("PRINT_12-3.txt")
A13 := READPRN("PRINT_13-3.txt")
A14 := READPRN("PRINT_14-3.txt")
A15 := READPRN("PRINT_15-3.txt")
A16 := READPRN("PRINT_16-3.txt")
A17 := READPRN("PRINT_17-3.txt")
A18 := READPRN("PRINT_18-3.txt")
A19 := READPRN("PRINT_19-3.txt")
A20 := READPRN("PRINT_20-3.txt")
A21 := READPRN("PRINT_21-3.txt")
A22 := READPRN("PRINT_22-3.txt")
A23 := READPRN("PRINT_23-3.txt")
A24 := READPRN("PRINT_24-3.txt")
A25 := READPRN("PRINT_25-3.txt")
A26 := READPRN("PRINT_26-3.txt")
Indices and constants

\[ N := \text{rows}(A0) \]
\[ k := 0..26 \]
\[ i := 0..N-1 \]
\[ n := 1..26 \]
\[ a := 10 \]
\[ b := .05 \]
\[ c := .025 \]

\[ \Delta t := \left( A0_{1001,0} - A0_{1000,0} \right) s \]
\[ \Delta f := \frac{1}{N \Delta t} \]
\[ \Delta t = 1 \times 10^{-6} s \]
\[ \Delta f = 500Hz \]

Creation of matrix “d” holding the time signal for each isotropic confining pressure

\[ d^{(0)} := A0^{(2)} \]
\[ d^{(1)} := A1^{(2)} \]
\[ d^{(2)} := A2^{(2)} \]
\[ d^{(3)} := A3^{(2)} \]
\[ d^{(4)} := A4^{(2)} \]
\[ d^{(5)} := A5^{(2)} \]
\[ d^{(6)} := A6^{(2)} \]
\[ d^{(7)} := A7^{(2)} \]
\[ d^{(8)} := A8^{(2)} \]
\[ d^{(9)} := A9^{(2)} \]
\[ d^{(10)} := A10^{(2)} \]
\[ d^{(11)} := A11^{(2)} \]
\[ d^{(12)} := A12^{(2)} \]
\[ d^{(13)} := A13^{(2)} \]
\[ d^{(14)} := A14^{(2)} \]
\[ d^{(15)} := A15^{(2)} \]
\[ d^{(16)} := A16^{(2)} \]
\[ d^{(17)} := A17^{(2)} \]
\[ d^{(18)} := A18^{(2)} \]
\[ d^{(19)} := A19^{(2)} \]
\[ d^{(20)} := A20^{(2)} \]
\[ d^{(21)} := A21^{(2)} \]
\[ d^{(22)} := A22^{(2)} \]
\[ d^{(23)} := A23^{(2)} \]
\[ d^{(24)} := A24^{(2)} \]
\[ d^{(25)} := A25^{(2)} \]
\[ d^{(26)} := A26^{(2)} \]

Original time signals
Travel time calculations
First travel time taken from initial time trace and trigger
\[ t_{t0} := 0.000898 \text{sec} - 0.000414 \text{sec} \quad t_{t0} = 4.84 \times 10^{-4} \text{ s} \]

Subsequent travel times calculated by shifting the signals
\[ \Delta t_{t1} := 3.1 \times 10^{-5} \text{ sec} \quad t_{t1} := t_{t0} - \Delta t_{t1} \quad t_{t1} = 4.53 \times 10^{-4} \text{ s} \]
\[ \Delta t_{t2} := 2.6 \times 10^{-5} \text{ sec} \quad t_{t2} := t_{t1} - \Delta t_{t2} \quad t_{t2} = 4.27 \times 10^{-4} \text{ s} \]
\[ \Delta t_{t3} := 2.2 \times 10^{-5} \text{ sec} \quad t_{t3} := t_{t2} - \Delta t_{t3} \quad t_{t3} = 4.05 \times 10^{-4} \text{ s} \]
\[ \Delta t_{t4} := 1.6 \times 10^{-5} \text{ sec} \quad t_{t4} := t_{t3} - \Delta t_{t4} \quad t_{t4} = 3.89 \times 10^{-4} \text{ s} \]
\[ \Delta t_{t5} := 4.8 \times 10^{-5} \text{ sec} \quad t_{t5} := t_{t4} - \Delta t_{t5} \quad t_{t5} = 3.41 \times 10^{-4} \text{ s} \]
\[ \Delta t_{t6} := 2.4 \times 10^{-5} \text{ sec} \quad t_{t6} := t_{t5} - \Delta t_{t6} \quad t_{t6} = 3.17 \times 10^{-4} \text{ s} \]
\[ \Delta t_{t7} := 1.6 \times 10^{-5} \text{ sec} \quad t_{t7} := t_{t6} - \Delta t_{t7} \quad t_{t7} = 3.01 \times 10^{-4} \text{ s} \]
\[ \Delta t_{t8} := 1.2 \times 10^{-5} \text{ sec} \quad t_{t8} := t_{t7} - \Delta t_{t8} \quad t_{t8} = 2.89 \times 10^{-4} \text{ s} \]
\[ \Delta t_{t9} := 9 \times 10^{-6} \text{ sec} \quad t_{t9} := t_{t8} - \Delta t_{t9} \quad t_{t9} = 2.8 \times 10^{-4} \text{ s} \]
Distance between bender element tips

length of bender element in top platen \( l_t := 10.3\text{mm} \)

length of bender element in bottom platen \( l_b := 10.0\text{mm} \)

average height of specimen \( h_{ave} := 105.7\text{mm} \)

tip to tip distance \( l_{tt} := h_{ave} - l_t - l_b \) \( l_{tt} = 85.4\text{mm} \)
Creation of confining pressure vector

\[
\begin{align*}
\sigma_0 & := 1 \text{ psi} \\
\sigma_1 & := 2 \text{ psi} \\
\sigma_2 & := 3 \text{ psi} \\
\sigma_3 & := 4 \text{ psi} \\
\sigma_4 & := 5 \text{ psi} \\
\sigma_5 & := 10 \text{ psi} \\
\sigma_6 & := 15 \text{ psi} \\
\sigma_7 & := 20 \text{ psi} \\
\sigma_8 & := 25 \text{ psi} \\
\sigma_9 & := 30 \text{ psi} \\
\sigma_{10} & := 35 \text{ psi} \\
\sigma_{11} & := 40 \text{ psi} \\
\sigma_{12} & := 45 \text{ psi} \\
\sigma_{13} & := 50 \text{ psi} \\
\sigma_{14} & := 45 \text{ psi} \\
\sigma_{15} & := 40 \text{ psi} \\
\sigma_{16} & := 35 \text{ psi} \\
\sigma_{17} & := 30 \text{ psi} \\
\sigma_{18} & := 25 \text{ psi} \\
\sigma_{19} & := 20 \text{ psi} \\
\sigma_{20} & := 15 \text{ psi} \\
\sigma_{21} & := 10 \text{ psi} \\
\sigma_{22} & := 5 \text{ psi} \\
\sigma_{23} & := 4 \text{ psi} \\
\sigma_{24} & := 3 \text{ psi} \\
\sigma_{25} & := 2 \text{ psi} \\
\sigma_{26} & := 1 \text{ psi}
\end{align*}
\]

Height correction due to strain of triaxial specimen

\[ l_{\text{corr}} := \text{READPRN"newheight3.txt"} \cdot \text{cm} \]

Calculation of velocity

\[ V_{\text{corr},k} := \frac{l_{\text{corr}}}{t_k} \]

Velocity vs. stress and curve fit
\[
V_{\log} := \log\left(\frac{V_{\text{corr}} \cdot s}{m}\right)
\]
\[
\sigma_{3\log} := \log\left[\frac{\sigma_3}{\left(1.0 \times 10^3\right) \text{Pa}}\right]
\]

\[
q := 0..3 \quad \sigma_{3\log_q} := \sigma_{3\log_q} \quad V_{\log_q} := V_{\log_q}
\]
\[
\alpha_1 := 10 \quad \alpha_1 = 137.51
\]
\[
\beta_1 := \text{slope}(\sigma_{3\log_q}, V_{\log_q}) \quad \beta_1 = 0.125
\]

\[
r := 4..13 \quad \sigma_{3\log_r-4} := \sigma_{3\log_r} \quad V_{\log_r-4} := V_{\log_r}
\]
\[
\alpha_2 := 10 \quad \alpha_2 = 119.39
\]
\[
\beta_2 := \text{slope}(\sigma_{3\log_r}, V_{\log_r}) \quad \beta_2 = 0.173
\]

\[
t := 13..22 \quad \sigma_{3\log_t-13} := \sigma_{3\log_t} \quad V_{\log_t-13} := V_{\log_t}
\]
\[
\alpha_3 := 10 \quad \alpha_3 = 90.525
\]
\[
\beta_3 := \text{slope}(\sigma_{3\log_t}, V_{\log_t}) \quad \beta_3 = 0.222
\]

\[
u := 23..26 \quad \sigma_{3\log_u-23} := \sigma_{3\log_u} \quad V_{\log_u-23} := V_{\log_u}
\]
\[
\alpha_4 := 10 \quad \alpha_4 = 100.136
\]
\[
\beta_4 := \text{slope}(\sigma_{3\log_u}, V_{\log_u}) \quad \beta_4 = 0.189
\]

\[
i := 0..3
\]
\[
ii := 4..13
\]
\[
iii := 14..22
\]
\[
iii := 23..26
\]
Mathgram A-2 Triaxial calibration anisotropic states of stress

Reading in raw data files

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<th>t := (A0)</th>
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Time series

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<td>A2</td>
</tr>
<tr>
<td>d</td>
<td>:=</td>
<td>A3</td>
<td>A4</td>
</tr>
<tr>
<td>d</td>
<td>:=</td>
<td>A5</td>
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A29 := READPRN"PRINT_29-10.txt"
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A31 := READPRN"PRINT_31-10.txt"
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A37 := READPRN"PRINT_37-10.txt"
A38 := READPRN"PRINT_38-10.txt"

\sigma_{30} := 10 \text{kPa} \quad \sigma_{10} := 10 \text{kPa} \quad \sigma_{311} := 50 \text{kPa} \quad \sigma_{111} := 60 \text{kPa} \quad \sigma_{322} := 100 \text{kPa} \quad \sigma_{122} := 160 \text{kPa}
\sigma_{31} := 20 \text{kPa} \quad \sigma_{11} := 20 \text{kPa} \quad \sigma_{312} := 50 \text{kPa} \quad \sigma_{112} := 50 \text{kPa} \quad \sigma_{323} := 100 \text{kPa} \quad \sigma_{123} := 140 \text{kPa}
\sigma_{32} := 30 \text{kPa} \quad \sigma_{12} := 30 \text{kPa} \quad \sigma_{313} := 60 \text{kPa} \quad \sigma_{113} := 60 \text{kPa} \quad \sigma_{324} := 100 \text{kPa} \quad \sigma_{124} := 120 \text{kPa}
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\sigma_{34} := 50 \text{kPa} \quad \sigma_{14} := 50 \text{kPa} \quad \sigma_{315} := 80 \text{kPa} \quad \sigma_{115} := 80 \text{kPa} \quad \sigma_{326} := 120 \text{kPa} \quad \sigma_{126} := 120 \text{kPa}
\sigma_{35} := 50 \text{kPa} \quad \sigma_{15} := 60 \text{kPa} \quad \sigma_{316} := 90 \text{kPa} \quad \sigma_{116} := 90 \text{kPa} \quad \sigma_{327} := 140 \text{kPa} \quad \sigma_{127} := 140 \text{kPa}
\sigma_{36} := 50 \text{kPa} \quad \sigma_{16} := 70 \text{kPa} \quad \sigma_{317} := 100 \text{kPa} \quad \sigma_{117} := 100 \text{kPa} \quad \sigma_{328} := 160 \text{kPa} \quad \sigma_{128} := 160 \text{kPa}
\sigma_{37} := 50 \text{kPa} \quad \sigma_{17} := 80 \text{kPa} \quad \sigma_{318} := 100 \text{kPa} \quad \sigma_{118} := 120 \text{kPa} \quad \sigma_{329} := 180 \text{kPa} \quad \sigma_{129} := 180 \text{kPa}
\sigma_{38} := 50 \text{kPa} \quad \sigma_{18} := 90 \text{kPa} \quad \sigma_{319} := 100 \text{kPa} \quad \sigma_{119} := 140 \text{kPa} \quad \sigma_{330} := 200 \text{kPa} \quad \sigma_{130} := 200 \text{kPa}
\sigma_{39} := 50 \text{kPa} \quad \sigma_{19} := 80 \text{kPa} \quad \sigma_{320} := 100 \text{kPa} \quad \sigma_{120} := 160 \text{kPa} \quad \sigma_{331} := 200 \text{kPa} \quad \sigma_{131} := 240 \text{kPa}
\sigma_{310} := 50 \text{kPa} \quad \sigma_{110} := 70 \text{kPa} \quad \sigma_{321} := 100 \text{kPa} \quad \sigma_{121} := 180 \text{kPa} \quad \sigma_{332} := 200 \text{kPa} \quad \sigma_{132} := 280 \text{kPa}
\sigma_{333} := 200 \text{kPa} \quad \sigma_{133} := 320 \text{kPa}
\sigma_{334} := 200 \text{kPa} \quad \sigma_{134} := 360 \text{kPa}
\sigma_{335} := 200 \text{kPa} \quad \sigma_{135} := 320 \text{kPa}
\sigma_{336} := 200 \text{kPa} \quad \sigma_{136} := 280 \text{kPa}
\sigma_{337} := 200 \text{kPa} \quad \sigma_{137} := 240 \text{kPa}
\sigma_{338} := 200 \text{kPa} \quad \sigma_{138} := 200 \text{kPa}
Original time signals

\[ N := \text{rows}(A0) \quad i := 0..N-1 \quad b := .05 \]

Hydrostatic loading velocity traces

CTC loading and unloading, \( \sigma_3 = 50 \) kPa
Hydrostatic compression velocity traces

CTC loading and unloading, $\sigma_3=100$ kPa

- $\sigma_3=60$ kPa
- $\sigma_3=70$ kPa
- $\sigma_3=80$ kPa
- $\sigma_3=90$ kPa
- $\sigma_3=100$ kPa

- $\sigma_1=120$ kPa
- $\sigma_1=140$ kPa
- $\sigma_1=160$ kPa
- $\sigma_1=180$ kPa
- $\sigma_1=160$ kPa
- $\sigma_1=140$ kPa
- $\sigma_1=120$ kPa
- $\sigma_1=100$ kPa
Hydrostatic compression velocity traces

CTC loading and unloading, \(\sigma_3=200\, \text{kPa}\)

- \(\sigma_3=120\, \text{kPa}\)
- \(\sigma_3=140\, \text{kPa}\)
- \(\sigma_3=160\, \text{kPa}\)
- \(\sigma_3=180\, \text{kPa}\)
- \(\sigma_3=200\, \text{kPa}\)

- \(\sigma_1=240\, \text{kPa}\)
- \(\sigma_1=280\, \text{kPa}\)
- \(\sigma_1=320\, \text{kPa}\)
- \(\sigma_1=360\, \text{kPa}\)
- \(\sigma_1=240\, \text{kPa}\)
- \(\sigma_1=200\, \text{kPa}\)
Travel time taken directly from traces

\[
\begin{bmatrix}
0.000442 \\
0.000408 \\
0.000387 \\
0.000370 \\
0.000358 \\
0.000345 \\
0.000328 \\
0.000324 \\
0.000327 \\
0.000333 \\
0.000343 \\
0.000335 \\
0.000326 \\
0.000319 \\
0.000313 \\
0.000301 \\
0.000293 \\
0.000288 \\
0.000281 \\
0.000285 \\
0.000288 \\
0.000281 \\
0.000285 \\
0.000288 \\
0.000281 \\
0.000285 \\
0.000288 \\
0.000281 \\
0.000285 \\
0.000277 \\
0.000272 \\
0.000262 \\
0.000256 \\
0.000256 \\
0.000252 \\
0.000247 \\
0.000258 \\
0.000261 \\
0.000261 \\
0.000267 \\
0.000278
\end{bmatrix}
\]

\[tt := 0.000293\] s
Evaluation of S-wave velocity vs. state of stress

\[ k := 0.34 \quad r := 0.1 \]

Tip to tip distance:

\[ L := (\text{READPRN}("\text{newheight10.txt}") - \text{mm}) \]

Measured shear wave velocity:

\[ V_k := \frac{L_k}{t_k} \]

Evaluation of velocity-stress semi-empirical relation (by least square solution):

Evaluation of matrix parameters:

\[ H_{k,r} := 1 \quad H_{k,1} := \log \left( \frac{\sigma_{1k}}{1 \text{kPa}} \right) \quad H_{k,2} := \log \left( \frac{\sigma_{3k}}{1 \text{kPa}} \right) \]

\[ \log V_k := \log \left( \frac{V_k}{m} \right) \]

Least square solution:

\[ \text{Sol} := \left( H^T \cdot H \right)^{-1} \cdot H^T \cdot \log V \quad \text{Sol} = \begin{pmatrix} 2.109 \\ 0.146 \\ 0.012 \end{pmatrix} \]

Results:

\[ \alpha := 10 \text{ m/sec} \quad \alpha = 128.519 \text{ m/s} \]

\[ \beta_1 := \text{Sol}_1 \]

\[ \beta_3 := \text{Sol}_2 \]

Semi-empirical equation:

\[ V_m := \alpha \cdot \left( \frac{\sigma_{1k}}{1 \text{kPa}} \right)^{\beta_1} \cdot \left( \frac{\sigma_{3k}}{1 \text{kPa}} \right)^{\beta_3} \]

Comparison of our results to Santamarina's proposed a and b relation:

\[ \beta_1 + \beta_3 = 0.158 \]

\[ 0.36 - \frac{\alpha}{700} = 0.176 \]
Mathgram A-3 Oedometer calibration
Horizontally polarized S-wave propagation – Vertically oriented bender elements

Reading raw data files

\[ \begin{align*}
A0 & := \text{READPRN}("\text{PRINT}_02-1V\.txt") & \phantom{\text{READPRN}} & H_0 := A0 \\
A1 & := \text{READPRN}("\text{PRINT}_03-2V\.txt") & \phantom{\text{READPRN}} & t_1 := A1 \\
A2 & := \text{READPRN}("\text{PRINT}_06-1V\.txt") &\phantom{\text{READPRN}} & H_2 := A2 \\
A3 & := \text{READPRN}("\text{PRINT}_07-2V\.txt") & \phantom{\text{READPRN}} & t_3 := A3 \\
A4 & := \text{READPRN}("\text{PRINT}_10-1V\.txt") & \phantom{\text{READPRN}} & H_4 := A4 \\
A5 & := \text{READPRN}("\text{PRINT}_11-2V\.txt") & \phantom{\text{READPRN}} & t_5 := A5 \\
A6 & := \text{READPRN}("\text{PRINT}_14-1V\.txt") & \phantom{\text{READPRN}} & H_6 := A6 \\
A7 & := \text{READPRN}("\text{PRINT}_15-2V\.txt") & \phantom{\text{READPRN}} & t_7 := A7 \\
A8 & := \text{READPRN}("\text{PRINT}_18-1V\.txt") & \phantom{\text{READPRN}} & H_8 := A8 \\
A9 & := \text{READPRN}("\text{PRINT}_19-2V\.txt") & \phantom{\text{READPRN}} & t_9 := A9 \\
A10 & := \text{READPRN}("\text{PRINT}_22-1V\.txt") & \phantom{\text{READPRN}} & H_{10} := A10 \\
A11 & := \text{READPRN}("\text{PRINT}_23-2V\.txt") & \phantom{\text{READPRN}} & t_{11} := A11 \\
A12 & := \text{READPRN}("\text{PRINT}_26-1V\.txt") & \phantom{\text{READPRN}} & H_{12} := A12 \\
A13 & := \text{READPRN}("\text{PRINT}_27-2V\.txt") & \phantom{\text{READPRN}} & t_{13} := A13 \\
A14 & := \text{READPRN}("\text{PRINT}_30-1V\.txt") & \phantom{\text{READPRN}} & H_{14} := A14 \\
A15 & := \text{READPRN}("\text{PRINT}_31-2V\.txt") & \phantom{\text{READPRN}} & t_{15} := A15 \\
A16 & := \text{READPRN}("\text{PRINT}_34-1V\.txt") & \phantom{\text{READPRN}} & H_{16} := A16 \\
A17 & := \text{READPRN}("\text{PRINT}_35-2V\.txt") & \phantom{\text{READPRN}} & t_{17} := A17 \\
A18 & := \text{READPRN}("\text{PRINT}_38-1V\.txt") & \phantom{\text{READPRN}} & H_{18} := A18 \\
A19 & := \text{READPRN}("\text{PRINT}_39-2V\.txt") & \phantom{\text{READPRN}} & t_{19} := A19 \\
A20 & := \text{READPRN}("\text{PRINT}_42-1V\.txt") & \phantom{\text{READPRN}} & H_{20} := A20 \\
A21 & := \text{READPRN}("\text{PRINT}_43-2V\.txt") & \phantom{\text{READPRN}} & t_{21} := A21 \\
A22 & := \text{READPRN}("\text{PRINT}_46-1V\.txt") & \phantom{\text{READPRN}} & H_{22} := A22 \\
A23 & := \text{READPRN}("\text{PRINT}_47-2V\.txt") & \phantom{\text{READPRN}} & t_{23} := A23 \\
A24 & := \text{READPRN}("\text{PRINT}_50-1V\.txt") & \phantom{\text{READPRN}} & H_{24} := A24 \\
A25 & := \text{READPRN}("\text{PRINT}_51-2V\.txt") & \phantom{\text{READPRN}} & t_{25} := A25 \\
A26 & := \text{READPRN}("\text{PRINT}_54-1V\.txt") & \phantom{\text{READPRN}} & H_{26} := A26 \\
A27 & := \text{READPRN}("\text{PRINT}_55-2V\.txt") & \phantom{\text{READPRN}} & t_{27} := A27 \\
\end{align*} \]

Number of points and indices:
\[
\begin{align*}
& N := \text{rows}(A0) & i := 0.. N - 1 & u := 0.. N - 1 & ii := 0.. 75C \\
& M := \text{cols}(H) & k := 0.. M - 1 & k1 := 0.. 2.. M - 1 & r := 0.. 13
\end{align*}
\]

Signal corrections:
\[
\begin{align*}
H_{ii,k} & := H_{ii,k} \\
H_{d,i,k1} & := H_{i,k1} - H_{i,k1} \\
H_{nd,i,k1} & := H_{i,k1} - H_{i,k1}
\end{align*}
\]
Separation between bender element tips: \[ L := 48 \text{ mm} \]

Vertical stresses: \[ \sigma'_v := \begin{pmatrix} 5 \\ 25 \\ 50 \\ 75 \\ 100 \\ 150 \\ 200 \\ 250 \\ 300 \\ 350 \\ 400 \\ 300 \\ 200 \\ 100 \end{pmatrix} \text{kPa} \]

Modeled shear wave velocity:
\[ \alpha = 122.5 \frac{\text{m}}{\text{s}} \quad \beta_{\text{par}} = 0.17 \quad \beta_{\text{per}} = 0.02 \]

\[ V_{\text{sh}} := \alpha \left( \frac{K_0 \cdot \sigma'_v}{1 \text{kPa}} \right)^{\beta_{\text{par}}} \left( \frac{K_0 \cdot \sigma'_v}{1 \text{kPa}} \right)^{\beta_{\text{per}}} \]

Calculated travel times:
\[ t_r := \frac{L}{V_{\text{sh}}} \]

Original time signals, subtraction of two signals removes electromagnetic noise
Mathgram A-4 Integration of the Boussinesq’s solution for a circular footing

kPa := 10^3 Pa

Radius footing:
R := 1.5 in

Applied bearing pressure
q := 50 kPa

Poisson's ratio:
ν := 0.5

for comparison with published data

Finite sectors definitions
M := 16
i := 0..M – 1
N := 12
k := 0..N – 1

θ_k := 2·π·k + 0.5
2N

r_i := R·i + 0.5
M

Finite sectors:

x_{i,k} := r_i·cos(θ_k)

y_{i,k} := r_i·sin(θ_k)

Δr := r_2 – r_1
Δθ := θ_2 – θ_1

Δr = 2.381 × 10^{-3} m
Δθ = 15 deg

Finite sectors area:

ΔA_{i,k} := Δr·r_i·Δθ

Force in each finite sector:
P_{i,k} := q·ΔA_{i,k}

Centroid location of each finite sector
Vertical Stresses:

\[ \sigma_z(x_n, z_j) := 2 \sum_{i} \sum_{k} \left[ \frac{3P_{i,k} \cdot z_0^3}{2\pi \sqrt{z_0^2 + (x_0 - x_{i,k})^2 + (y_{i,k})^2}} \right]^5 \]

Definition of coordinates:

\[ \text{NP} := 100 \]

\[ n := 0..\text{NP} \quad x_0 := 6R \cdot \frac{n}{\text{NP}} \]

\[ j := 0..\text{NP} \quad z_0 := 10R \cdot \frac{j}{\text{NP}} \]

Vertical induced stresses

\[ \sigma_{znj} := \sigma_z(x_{n}, z_{0\cdot\text{NP} - j}) \]

\[ \sigma_{zp_{sm}} := \text{submatrix}(\sigma_z, 0, 100, 0, 99) \]
Radial and tangential stress components along the xz-plane (yo = 0):

\[ \cos(xo, zo) := \sum_i \sum_k \left[ \frac{-P_{i,k}}{2\pi} \left( \frac{zo^2 + (xo - \xi_{i,k})^2 + (\eta_{i,k})^2}{zo^2 + (xo - \xi_{i,k})^2 + (\eta_{i,k})^2} \right) \right] \]

\[ \sin(xo, zo) := \sum_i \sum_k \left[ \frac{-P_{i,k}}{2\pi} \left( \frac{zo^2 + (xo - \xi_{i,k})^2 + (\eta_{i,k})^2}{zo^2 + (xo - \xi_{i,k})^2 + (\eta_{i,k})^2} \right) \right] \]
Net radial and tangential stresses for a point on the x-axis:

\[
\sigma_{n, j} := \sigma(x_{n, j}, z_{NP - j})
\]

\[
\sigma_{sm} := \text{submatrix}(\sigma, 0, NP, 0, NP - 1)
\]

\[
\sigma_{y, j} := \sigma(y_{0, j}, z_{NP - j})
\]

\[
\sigma_{ysm} := \text{submatrix}(\sigma, 0, NP, 0, NP - 1)
\]
**Mathgram A-5 Theoretical Travel Time Calculation**

- **Radius footing:** \( R := 2.5 \text{ in} \)
- **Applied bearing pressure (to be multiplied by vector containing stress values):** \( q := 1 \text{kPa} \)
- **Poisson's ratio:** \( v := 0.15 \)
- **Finite sectors definitions**
  - \( M := 16 \)
  - \( N := 12 \)
  - \( \theta_k := 2 \cdot \pi \cdot k + 0.5 \)
  - \( r_i := R \cdot i + 0.5 \)
- **Finite sectors x and y coordinates:**
  - \( x_{i,k} := r_i \cos(\theta_k) \)
  - \( y_{i,k} := r_i \sin(\theta_k) \)
- **Finite sectors area:** \( \Delta A_{i,k} := \Delta r_i \cdot \Delta \theta \)
- **Force in each finite sector:** \( P_{i,k} := q \cdot \Delta A_{i,k} \)

![Centroid location of each finite sector](image)

- Centroid location of each finite sector

\[ kPa := 10^3 \cdot Pa \]

\[ \Delta r := 3.969 \times 10^{-3} \text{ m} \]

\[ \Delta \theta = 15 \text{deg} \]
Definition of coordinates:

\[ \text{NP} := 100 \]
\[ n := 0..\text{NP} \]
\[ x_n := 6R \frac{n}{\text{NP}} \]
\[ j := 0..\text{NP} \]
\[ z_j := 10R \frac{j}{\text{NP}} \]

Vertical Stresses:

\[ \sigma(z_n, z_j) := 2 \sum_i \sum_k \left[ \frac{3P_{i,k} z_0^3}{2\pi \left[ \sqrt{z_0^2 + \left( x_i - x_{i,k} \right)^2 + \left( y_{i,k} \right)^2 \right]^5} \right] \]

\[ \sigma_{zp_{n,j}} := \sigma(\frac{x_n - 6R}{n_{NP}}, \frac{z_0 - 10R}{j_{NP}}) \]

\[ \sigma_{zp_{sm}} := \text{submatrix}(\sigma_{zp}, 0, 100, 0, 99) \]
Radial and tangential stress components along the xz-plane (yo)}
Net radial and tangential stresses for a point on the x-axis:

\[ \sigma_n, j := \sigma(x_n, z_{0 \cdot NP - j}) \]
\[ \sigma_{sm} := \text{submatrix}(\sigma, 0, NP, 0, NP - 1) \]

Assigning one x-stress value to any given pixel:
\[ \Delta \sigma_{x u, v} := \text{mean}(\text{submatrix}(\sigma_x, 10 \cdot v, 9 + 10 \cdot v, 10 \cdot u, 9 + 10 \cdot u)) \]

Assigning one y-stress value to any given pixel:
\[ \Delta \sigma_{y u, v} := \text{mean}(\text{submatrix}(\sigma_y, 10 \cdot v, 9 + 10 \cdot v, 10 \cdot u, 9 + 10 \cdot u)) \]
Pressures used for this test: \[ q_b := \begin{pmatrix} 17.56 \\ 35.11 \\ 52.67 \\ 70.23 \\ 87.79 \\ 105.34 \end{pmatrix} \]

Calibration parameter: \[ \alpha := 130 \frac{m}{s} \]
\[ \beta_{\text{par}} := 0.15 \]
\[ \beta_{\text{perp}} := 0.012 \]

Index used for pressures: \[ r := 0..5 \]

Theoretical Travel Time Calculation

\[ t_{u,rr} := 2 \sum_v \frac{0.0217m}{\alpha \cdot \left( \frac{16.5 \text{kPa} \cdot 0.6 + \Delta \sigma_{u,v} \cdot q_b_{rr}}{1 \text{kPa}} \right)^{\beta_{\text{par}}} \cdot \left( \frac{16.5 \text{kPa} \cdot 0.6 + \Delta \sigma_{y,v} \cdot q_b_{rr}}{1 \text{kPa}} \right)^{\beta_{\text{perp}}}} \]
**Mathgram A-7 Straight ray tracing algorithm**

**ORIGIN:= 1**

**fn= tracing algorithm.mcd**

**Straight Ray Tracing Algorithm** (Santamarina and Fratta 1998)

**Input x and y coordinates of the source and receiver for each of the m rays**

\[
XY := \text{READPRN}("travel time - q=280 kPa.txt")
\]

**Number of pixels across:** \( nh := 5 \)

**Number of rays**

\( m := \text{rows}(XY) \)

\( m = 56 \)

\( i := 1..m \)

**Extracting coordinates of sources and receivers:**

\( xs_i := XY_{i,1} \)

\( ys_i := XY_{i,2} \)

\( xr_i := XY_{i,3} \)

\( yr_i := XY_{i,4} \)

**Coordinates of top-left corner:**

\( Xtop := \min(xs) \)

\( Ytop := \min(ys) \)

**Coordinates of bottom-right corner:**

\( Xbot := \max(xs) \)

\( Ybot := \max(ys) \)

\( \Delta x := \frac{Xbot - Xtop}{nh} \)

\( \Delta x = 0.026 \)

\( nv := \lceil \frac{Ybot - Ytop}{\Delta x} \rceil \)

\( nv = 9 \)

\( R := \frac{\Delta x}{\sqrt{\pi}} \)

\( Ytop := Ytop - 0.5[ nv \cdot \Delta x - (Ybot - Ytop) ] \)

\( Ybot := Ytop + nv \cdot \Delta x \)

**Final space coordinates:**

\( Xtop = 0 \)

\( Ytop = 0.023 \)

\( Xbot = 0.13 \)

\( Ybot = 0.257 \)

**Computation of x and y coordinate for the center of each of the nh.nv pixels:**

\( j := 1..nh \cdot nv \)

\( x_j := Xtop + \left( j - \text{floor} \left( \frac{j - 1}{nh} \right) \cdot \Delta x - 0.5 \right) \cdot \Delta x \)

\( y_j := Ytop + \left( \text{floor} \left( \frac{j - 1}{nh} \right) + 0.5 \right) \cdot \Delta x \)

**Computation of rays slopes:**

\( \tan \alpha_i := \frac{ys_i - yr_i}{(xs_i - xr_i) + 10^{-6}} \)

**Computation of distances**

\[
LN_{i,j} := \frac{(ys_i - y_j) + (x_j - xs_i) \cdot \tan \alpha_i}{\sqrt{(\tan \alpha_i)^2 + 1}}
\]

**Computation of travel lengths**

\[
l_{i,j} := \text{if} \left[ LN_{i,j} < R, 2 \cdot \sqrt{R^2 - (LN_{i,j})^2} \right] \]

**Correction for true length:**

\[
L_{true,i} := \sqrt{(xs_i - xr_i)^2 + (ys_i - yr_i)^2} \]

\( L_{true,i} := \sum_{j} l_{i,j} \cdot \text{lcorn}_{i,j} := l_{i,j} \cdot \text{Lest}_{i} \)

**Distribution of information:**

\[
\Psi_j := \sum_{i} \text{lcorn}_{i,j} \quad u := 1..nh \quad v := 1..nv \quad \text{Image} \Psi_v, u := \Psi_{u+nb \cdot (v-1)}
\]

**Output. Travel length matrix:**

WRITEPRN("L5x9-280kPa.txt") □ lcorn

WRITEPRN("xy5x9.txt") □ xy
Comments:
The optimization of the equivalent radius was based on min and max error and L2 norm comparing the computed length $L_{est}$ and the true Pythagorean length.
It shows that the radius $R$ that gives the same area [$R = \Delta x / \sqrt{\pi}$] is quasi-optimal (a denominator 1.78 is slightly better)
Mathgram A-8 Generator of Regularization Matrix

**Generator of Regularization Matrices** (Santamarina and Fratta 1998)

**ORIGIN**: 1

**Number of pixels in horizontal and vertical directions**:

\[ \text{nh} := 5 \]
\[ \text{nv} := 9 \]
\[ j := 1 \ldots \text{nh} \]
\[ i := 1 \ldots \text{nv} \]

**Definitions of coordination**:

\[ p_{i,j} := (i - 1) \cdot \text{nh} + j \]
\[ a_{i,j} := \text{if} \{ i = 1, p_{2,j} \cdot p_{i-1,j} \} \]
\[ b_{i,j} := \text{if} \{ i = \text{nv}, p_{\text{nv}-1,j} \cdot p_{i+1,j} \} \]
\[ l_{i,j} := \text{if} \{ j = 1, p_{i,j+1} \cdot p_{i,j-1} \} \]
\[ r_{i,j} := \text{if} \{ j = \text{nh}, p_{i,j-1} \cdot p_{i,j+1} \} \]

**Define kernel**:

\[ \text{kc} := -4 \]
\[ \text{ka} := 1 \]
\[ \text{kb} := 1 \]
\[ \text{kl} := 1 \]
\[ \text{kr} := 1 \]

**Computation of matrix R**:

\[ x := 1 \ldots \text{nh} \cdot \text{nv} \]
\[ y := \text{nh} \cdot \text{nv} \]
\[ R_{x,y} := 0 \]
\[ R_{x,x} := R_{x,x} + \text{kc} \]
\[ R_{x,a_x} := R_{x,a_x} + \text{ka} \]
\[ R_{x,b_x} := R_{x,b_x} + \text{kb} \]
\[ R_{x,l_x} := R_{x,l_x} + \text{kl} \]
\[ R_{x,r_x} := R_{x,r_x} + \text{kr} \]

**Store matrix R**:

\[ \text{WRITEPRN} ("\text{R5x9.txt}") \]
Mathgram A-9 Tomographic Inversion Algorithm

**Reading data:**

\[ d_1 := \text{READPRN}("travel time - q=0 \text{kPa}.txt") \]
\[ d_2 := \text{READPRN}("travel time - q=70 \text{kPa}.txt") \]
\[ d_3 := \text{READPRN}("travel time - q=140 \text{kPa}.txt") \]
\[ d_4 := \text{READPRN}("travel time - q=280 \text{kPa}.txt") \]

**Source-receivers coordinates:**

\[ x_{s1} := d_1^{(1)} \cdot m \]
\[ y_{s1} := d_1^{(2)} \cdot m \]
\[ x_{r1} := d_1^{(3)} \cdot m \]
\[ y_{r1} := d_1^{(4)} \cdot m \]
\[ x_{s2} := d_2^{(1)} \cdot m \]
\[ y_{s2} := d_2^{(2)} \cdot m \]
\[ x_{r2} := d_2^{(3)} \cdot m \]
\[ y_{r2} := d_2^{(4)} \cdot m \]
\[ x_{s3} := d_3^{(1)} \cdot m \]
\[ y_{s3} := d_3^{(2)} \cdot m \]
\[ x_{r3} := d_3^{(3)} \cdot m \]
\[ y_{r3} := d_3^{(4)} \cdot m \]
\[ x_{s4} := d_4^{(1)} \cdot m \]
\[ y_{s4} := d_4^{(2)} \cdot m \]
\[ x_{r4} := d_4^{(3)} \cdot m \]
\[ y_{r4} := d_4^{(4)} \cdot m \]

**Travel time:**

\[ t_1 := d_1^{(5)} \cdot \text{sec} \]
\[ t_2 := d_2^{(5)} \cdot \text{sec} \]
\[ t_3 := d_3^{(5)} \cdot \text{sec} \]
\[ t_4 := d_4^{(5)} \cdot \text{sec} \]

**Travel length:**

\[ L_1 := \text{READPRN}("L5x9-0kPa.txt") \cdot m \]
\[ i_1 := 1..\text{cols}(L1) \]
\[ k_1 := 1..\text{rows}(L1) \]
\[ L_2 := \text{READPRN}("L5x9-70kPa.txt") \cdot m \]
\[ i_2 := 1..\text{cols}(L2) \]
\[ k_2 := 1..\text{rows}(L2) \]
\[ L_3 := \text{READPRN}("L5x9-140kPa.txt") \cdot m \]
\[ i_3 := 1..\text{cols}(L3) \]
\[ k_3 := 1..\text{rows}(L3) \]
\[ L_4 := \text{READPRN}("L5x9-280kPa.txt") \cdot m \]
\[ i_4 := 1..\text{cols}(L4) \]
\[ k_4 := 1..\text{rows}(L4) \]

**Regularization matrix:**

\[ R := \text{READPRN}("R5x9.txt") \]

**Regularization coefficient:**

\[ u := 1..30 \]
\[ \lambda_u := 10^4 \cdot m^2 \]

**Inversion algorithm**

\[ s_1^{(u)} := \left( L_1^T \cdot L_1 + \lambda_u R^T \cdot R \right)^{-1} \cdot L_1^T \cdot t_1 \]
\[ V_{1_{i_{1}}, u} := \left( s_1^{(i_{1}, u)} \right)^{-1} \]
\[ s_2^{(u)} := \left( L_2^T \cdot L_2 + \lambda_u R^T \cdot R \right)^{-1} \cdot L_2^T \cdot t_2 \]
\[ V_{2_{i_{2}}, u} := \left( s_2^{(i_{2}, u)} \right)^{-1} \]
\[ s_3^{(u)} := \left( L_3^T \cdot L_3 + \lambda_u R^T \cdot R \right)^{-1} \cdot L_3^T \cdot t_3 \]
\[ V_{3_{i_{3}}, u} := \left( s_3^{(i_{3}, u)} \right)^{-1} \]
\[ s_4^{(u)} := \left( L_4^T \cdot L_4 + \lambda_u R^T \cdot R \right)^{-1} \cdot L_4^T \cdot t_4 \]
\[ V_{4_{i_{4}}, u} := \left( s_4^{(i_{4}, u)} \right)^{-1} \]
Calculated travel times and residual errors:

\[ tc_1 := L_1 s_1 \]
\[ tc_2 := L_2 s_2 \]
\[ tc_3 := L_3 s_3 \]
\[ tc_4 := L_4 s_4 \]

\[ E_{u_k} := \sum_{k} (t_{k} - tc_{k1},u)^2 \]

\[ \lambda_{op} = 0.178m^2 \]
APPENDIX B

TIME SERIES

Original time series for triaxial tests – Isotropic compression tests

Initial Void Ratio = 0.69
Initial Void Ratio = 0.77
Initial Void Ratio = 0.79

Hydrostatic loading velocity traces

Hydrostatic unloading velocity traces

Amplitude (mV)

Time (sec)
Original time series for triaxial tests – Anisotropic compression tests

Initial Void Ratio = 0.71

Hydrostatic loading velocity traces

CTC loading and unloading, sigma3=50 kPa

Initial Void Ratio = 0.71

Hydrostatic loading velocity traces

CTC loading and unloading, sigma3=50 kPa
Hydrostatic compression velocity traces

CTC loading and unloading, sigma3=100kPa
Hydrostatic compression velocity traces

Time (sec)

Amplitude (mV)

sigma3=120 kPa
sigma3=140 kPa
sigma3=160 kPa
sigma3=180 kPa
sigma3=200 kPa

CTC loading and unloading, sigma3=200kPa

Time (sec)

Amplitude (mV)

sigma1=240 kPa
sigma1=280 kPa
sigma1=320 kPa
sigma1=360 kPa
sigma1=320 kPa
sigma1=280 kPa
sigma1=240 kPa
sigma1=200 kPa
Initial Void Ratio = 0.62

Hydrostatic loading velocity traces

CTC loading and unloading, sigma3=50 kPa

- Time (sec)
- Amplitude (mV)

- trigger
- sigma3=10 kPa
- sigma3=20 kPa
- sigma3=30 kPa
- sigma3=40 kPa
- sigma3=50 kPa

- sigma1=60 kPa
- sigma1=70 kPa
- sigma1=80 kPa
- sigma1=90 kPa
- sigma1=80 kPa
- sigma1=70 kPa
- sigma1=60 kPa
- sigma1=60 kPa
- sigma1=50 kPa
Hydrostatic compression velocity traces

CTC loading and unloading, \( \sigma_3 = 200 \text{kPa} \)

\( \sigma_3 = \{120, 140, 160, 180, 200\} \text{kPa} \)

\( \sigma_1 = \{240, 280, 320, 360\} \text{kPa} \)
Initial Void Ratio = 0.89

Hydrostatic loading velocity traces

CTC loading and unloading, sigma3=50 kPa

sigma1=60 kPa
sigma1=70 kPa
sigma1=80 kPa
sigma1=90 kPa
sigma1=80 kPa
sigma1=70 kPa
sigma1=60 kPa
sigma1=50 kPa
Hydrostatic compression velocity traces

Amplitude (mV)

Time (sec)

CTC loading and unloading, sigma3=100kPa

Amplitude (mV)

Time (sec)
Hydrostatic compression velocity traces

CTC loading and unloading, $\sigma_3=200\,\text{kPa}$
Original time series for oedometer tests

Horizontally polarized waves

<table>
<thead>
<tr>
<th>Amplitude (mV)</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.18</td>
<td>0</td>
</tr>
<tr>
<td>0.16</td>
<td>0</td>
</tr>
<tr>
<td>0.14</td>
<td>0</td>
</tr>
<tr>
<td>0.12</td>
<td>0</td>
</tr>
<tr>
<td>0.10</td>
<td>0</td>
</tr>
<tr>
<td>0.08</td>
<td>0</td>
</tr>
<tr>
<td>0.06</td>
<td>0</td>
</tr>
<tr>
<td>0.04</td>
<td>0</td>
</tr>
<tr>
<td>0.02</td>
<td>0</td>
</tr>
<tr>
<td>0.00</td>
<td>0</td>
</tr>
</tbody>
</table>

- Red: trigger
- Blue: 0 kPa vertical pressure
- Green: 25 kPa vertical pressure
- Magenta: 50 kPa vertical pressure
- Cyan: 75 kPa vertical pressure
- Brown: 100 kPa vertical pressure
- Black: 150 kPa vertical pressure
- Red: 200 kPa vertical pressure
- Blue: 250 kPa vertical pressure
- Green: 300 kPa vertical pressure
- Magenta: 350 kPa vertical pressure
- Cyan: 400 kPa vertical pressure
- Brown: 300 kPa vertical pressure
- Black: 200 kPa vertical pressure
- Red: 100 kPa vertical pressure
- Magenta: arrival time model
Vertically polarized waves

<table>
<thead>
<tr>
<th>Amplitude (mV)</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.18</td>
<td>0</td>
</tr>
<tr>
<td>0.16</td>
<td>2 \times 10^{-4}</td>
</tr>
<tr>
<td>0.14</td>
<td>4 \times 10^{-4}</td>
</tr>
<tr>
<td>0.12</td>
<td>6 \times 10^{-4}</td>
</tr>
<tr>
<td>0.10</td>
<td>8 \times 10^{-4}</td>
</tr>
</tbody>
</table>

trigger
0 kPa vertical pressure
25 kPa vertical pressure
50 kPa vertical pressure
75 kPa vertical pressure
150 kPa vertical pressure
Did not save
250 kPa vertical pressure
100 kPa vertical pressure
300 kPa vertical pressure
350 kPa vertical pressure
400 kPa vertical pressure
200 kPa vertical pressure
300 kPa vertical pressure
arrival time model
Cross hole tests
Footing Diameter = 7.62 cm, Initial Void Ratio = 0.75

17.2 kPa overburden

34.4 kPa overburden

- tigger
- source 1 to receiver 1
- source 2 to receiver 2
- source 3 to receiver 3
- source 4 to receiver 4
- source 5 to receiver 5
- source 6 to receiver 6
- source 7 to receiver 7
- source 8 to receiver 8
68.9 kPa overburden

Time (sec)  Amplitude (mV)

0.001 0.0015 0.002 0.0025 0.003 0.0035 0.004 0.0045 0.005

137.9 kPa overburden

Time (sec)  Amplitude (mV)

0.001 0.0015 0.002 0.0025 0.003 0.0035 0.004 0.0045 0.005

tigger

source 1 to receiver 1
source 2 to receiver 2
source 3 to receiver 3
source 4 to receiver 4
source 5 to receiver 5
source 6 to receiver 6
source 7 to receiver 7
source 8 to receiver 8
Cross hole tests
Footing Diameter = 7.62 cm, Initial Void Ratio = 0.88

Traces for 97.54 kPa bearing load

Traces for 146.31 kPa bearing load
Traces for 195.08 kPa bearing

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Amplitude (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.005</td>
</tr>
<tr>
<td>0.002</td>
<td>0.004</td>
</tr>
<tr>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>0.004</td>
<td>0.002</td>
</tr>
<tr>
<td>0.005</td>
<td>0.001</td>
</tr>
<tr>
<td>0.006</td>
<td>0.000</td>
</tr>
<tr>
<td>0.007</td>
<td>0.000</td>
</tr>
<tr>
<td>0.008</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Traces for 243.85 kPa bearing

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Amplitude (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.005</td>
</tr>
<tr>
<td>0.002</td>
<td>0.004</td>
</tr>
<tr>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>0.004</td>
<td>0.002</td>
</tr>
<tr>
<td>0.005</td>
<td>0.001</td>
</tr>
<tr>
<td>0.006</td>
<td>0.000</td>
</tr>
<tr>
<td>0.007</td>
<td>0.000</td>
</tr>
<tr>
<td>0.008</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Traces for 292.62 kPa bearing

Amplitude (mV)

Time (sec)

0.005
0
−0.005
−0.01
−0.015
−0.02

0 0.5 10−4 0.0001 0.0005 0.001 0.0015 0.002

trigger
source 1 to receiver 1
source 2 to receiver 2
source 3 to receiver 3
source 4 to receiver 4
source 5 to receiver 5
source 6 to receiver 6
source 7 to receiver 7
source 8 to receiver 8
Footing diameter = 7.62 cm, initial void ratio = 0.82

Traces for 48.77 kPa bearing

Traces for 97.54 kPa bearing

- trigger
- source 1 to receiver 1
- source 2 to receiver 2
- source 3 to receiver 3
- source 4 to receiver 4
- source 5 to receiver 5
- source 6 to receiver 6
- source 8 to receiver 8
Traces for 146.31 kPa bearing

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Amplitude (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.001</td>
<td>0</td>
</tr>
<tr>
<td>0.002</td>
<td>0</td>
</tr>
<tr>
<td>0.003</td>
<td>0</td>
</tr>
<tr>
<td>0.004</td>
<td>0</td>
</tr>
<tr>
<td>0.005</td>
<td>0</td>
</tr>
</tbody>
</table>

Traces for 195.08 kPa bearing

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Amplitude (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.001</td>
<td>0</td>
</tr>
<tr>
<td>0.002</td>
<td>0</td>
</tr>
<tr>
<td>0.003</td>
<td>0</td>
</tr>
<tr>
<td>0.004</td>
<td>0</td>
</tr>
<tr>
<td>0.005</td>
<td>0</td>
</tr>
</tbody>
</table>

Legend:
- red: trigger
- blue: source 1 to receiver 1
- green: source 2 to receiver 2
- pink: source 3 to receiver 3
- cyan: source 4 to receiver 4
- brown: source 5 to receiver 5
- black: source 6 to receiver 6
- dark red: source 8 to receiver 8
Traces for 243.85 kPa bearing

Traces for 292.62 kPa bearing
Footprint diameter = 12.7 cm, initial void ratio = 0.95

Traces for 17.56 kPa bearing

<table>
<thead>
<tr>
<th>Amplitude (mV)</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-0.005</td>
<td>0.001</td>
</tr>
<tr>
<td>-0.01</td>
<td>0.002</td>
</tr>
<tr>
<td>-0.015</td>
<td>0.003</td>
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<tr>
<td>-0.02</td>
<td>0.004</td>
</tr>
<tr>
<td>-0.015</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Traces for 35.11 kPa bearing

<table>
<thead>
<tr>
<th>Amplitude (mV)</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-0.005</td>
<td>0.001</td>
</tr>
<tr>
<td>-0.01</td>
<td>0.002</td>
</tr>
<tr>
<td>-0.015</td>
<td>0.003</td>
</tr>
<tr>
<td>-0.02</td>
<td>0.004</td>
</tr>
<tr>
<td>-0.015</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Legend:
- red: trigger
- blue: source 1 to receiver 1
- cyan: source 2 to receiver 2
- magenta: source 3 to receiver 3
- light blue: source 4 to receiver 4
- brown: source 6 to receiver 6
- black: source 7 to receiver 7
- dark red: source 8 to receiver 8
Traces for 52.67 kPa bearing

Traces for 70.23 kPa bearing

Time (sec)

Amplitude (mV)

trigger
data for different source to receiver combinations.
Traces for 87.79 kPa bearing

Traces for 105.34 kPa bearing
Traces for 70.23 kPa unloading

Traces for 35.11 kPa unloading

- trigger
- source 1 to receiver 1
- source 2 to receiver 2
- source 3 to receiver 3
- source 4 to receiver 4
- source 6 to receiver 6
- source 7 to receiver 7
- source 8 to receiver 8
Footing diameter = 12.7 cm, initial void ratio = 0.73

Traces for 17.56 kPa bearing

Amplitude (mV)

Time (sec)

0 0.001 0.002 0.003 0.004 0.005 0.006

0

Traces for 35.11 kPa bearing

Amplitude (mV)

Time (sec)

0 0.001 0.002 0.003 0.004 0.005 0.006

0

trigger
source 1 to receiver 1
source 2 to receiver 2
source 3 to receiver 3
source 4 to receiver 4
source 5 to receiver 5
source 6 to receiver 6
source 7 to receiver 7
source 8 to receiver 8
Traces for 52.67 kPa bearing

Traces for 70.23 kPa bearing

Time (sec)
Amplitude (mV)

(trigger)
source 1 to receiver 1
source 2 to receiver 2
source 3 to receiver 3
source 4 to receiver 4
source 5 to receiver 5
source 6 to receiver 6
source 7 to receiver 7
source 8 to receiver 8

Amplitude (mV)

0.002
0.004
0.006
0.008
0.01
0.012

0

0.002
0.004
0.006
0.008
0.01
0.012

0

0.002
0.004
0.006
0.008
0.01
0.012

0

0.002
0.004
0.006
0.008
0.01
0.012

0

0.002
0.004
0.006
0.008
0.01
0.012

0

0.002
0.004
0.006
0.008
0.01
0.012

0

0.002
0.004
0.006
0.008
0.01
0.012

0

0.002
0.004
0.006
0.008
0.01
0.012

0

0.002
0.004
0.006
0.008
0.01
0.012

0

0.002
0.004
0.006
0.008
0.01
0.012

0

0.002
0.004
0.006
0.008
0.01
0.012

0

0.002
0.004
0.006
0.008
0.01
0.012

0
Traces for 87.79 kPa bearing

Traces for 105.34 kPa bearing

Time (sec)

Amplitude (mV)

trigger
source 1 to receiver 1
source 2 to receiver 2
source 3 to receiver 3
source 4 to receiver 4
source 5 to receiver 5
source 6 to receiver 6
source 7 to receiver 7
source 8 to receiver 8
Full Tomographic Test

0 kPa bearing/Source #1

0 kPa bearing/Source #2

<table>
<thead>
<tr>
<th>Amplitude (mV)</th>
<th>Trigger</th>
<th>Receiver 1</th>
<th>Receiver 2</th>
<th>Receiver 3</th>
<th>Receiver 4</th>
<th>Receiver 5</th>
<th>Receiver 6</th>
<th>Receiver 7</th>
<th>Receiver 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.02</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.03</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.04</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.05</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Time (sec)
0 kPa bearing/Source #3

0 kPa bearing/Source #4
140 kPa bearing/Source #1

Time (sec)

Amplitude (mV)

0 2 \times 10^{-4} 4 \times 10^{-4} 6 \times 10^{-4} 8 \times 10^{-4} 0.001 0.0012 0.0014

0.05 0.04 0.03 0.02 0.01 0 0.01

trigger receiver 1 receiver 2 receiver 3 receiver 4 receiver 5 receiver 6 receiver 7 receiver 8

140 kPa bearing/Source #2

Time (sec)

Amplitude (mV)

0 2 \times 10^{-4} 4 \times 10^{-4} 6 \times 10^{-4} 8 \times 10^{-4} 0.001 0.0012 0.0014

0.05 0.04 0.03 0.02 0.01 0 0.01

trigger receiver 1 receiver 2 receiver 3 receiver 4 receiver 5 receiver 6 receiver 7 receiver 8

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VITA

William Tanner was born in Wichita Falls, Texas on September 20, 1978. He received his Bachelor’s of Science in Civil Engineering from Louisiana State University in May of 2002 where he also competed in intercollegiate athletics. In August of 2002 he entered the Graduate program for Civil Engineering also at Louisiana State University. He received his Master of Science in Civil Engineering from Louisiana State University in May of 2004. He was married to Michelle Coryell on August 16th, 2004. They intend to relocate to Atlanta, Georgia for work and graduate school for his wife.