1981

Faraday Rotation Investigations of Surface Space-Charge Layers in Metal - Oxide - Semiconductor (Mos) Systems.

Gary L. Wallace
Louisiana State University and Agricultural & Mechanical College

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The Louisiana State University and Agricultural and Mechanical Col. Ph.D. 1981

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FARADAY ROTATION INVESTIGATIONS OF
SURFACE SPACE-CHARGE LAYERS IN
METAL-OXIDE-SEMICONDUCTOR (MOS) SYSTEMS

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Physics and Astronomy

by

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\[ 10^{-2}, 10^{-1}, 1, 2, 10, \lambda = 10^{-6} \text{ cm}, \]
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\[ n_2 = 5.3 \times 10^{16} \text{ cm}^{-3}, \quad \tau_1 = \tau_2 = 2.3 \times 10^{-12} \text{ s}, \quad \tau_e = 10^{40} \text{ s}, \]
\[ \lambda = 10^{-2} \text{ cm}, \quad B = 10^5 \text{ G and } \varepsilon_\lambda = 11.8. \] The vertical lines indicate the
corresponding values of \( \omega_1 = 9.26 \times 10^{12} \text{ s}^{-1} \) and \( \omega_2 = 4.20 \times 10^{12} \text{ s}^{-1} \) ..............................................

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\[
\begin{align*}
m_1 &= 0.19 \text{ m}, m_2 = 0.42 \text{ m}, \\
n_1 &= 6.7 \times 10^{15} \text{ cm}^{-3}, n_2 = 5.3 \times 10^{16} \text{ cm}^{-3}, \\
\tau_1 &= \tau_2 = 2.3 \times 10^{-12} \text{ s}, \lambda = 10^{-6} \text{ cm}, B = 10^5 \text{ G}, \epsilon_\lambda = \epsilon_\text{Si} = 11.8, \\
\epsilon_\text{SiO}_2 &= 3.81 \text{ and } \tau/\tau_e = 10^{-2}\text{, 1, } 10\ldots
\end{align*}
\]

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The free carrier Faraday effect is considered in both the Drude model and the Appel-Overhauser model for inversion layer electrons at the SiO$_2$-Si interface. An exact algebraic relation between the photon, cyclotron, collision and plasma frequencies is obtained which must be satisfied in order to achieve null rotation and null ellipticity for a single pass of the electromagnetic wave in the Drude model. Plots of the rotation versus angular photon frequency in the Appel-Overhauser model for various values of the electron-electron (e-e) scattering time $\tau_e$ are presented. These plots show that Faraday rotation measurements are a possible way of determining the strength of the e-e interaction as measured by $\tau_e$. The multiple pass Faraday rotation and ellipticity are shown to have a unique decomposition into components arising from the single pass effect, transmission effects at each boundary, and multiple reflection effects. Plots of the multiple pass rotation for various values of $\tau_e$ are also given. Finally, using null Faraday rotation we propose a method by which the various contributions to the multiple pass rotation may be measured.
I. INTRODUCTION

The surface space-charge layer on a semiconductor lies at the basis of modern microelectronics. The space-charge layer arises on the semiconductor surface when this surface is brought into contact with another substance, usually a metal, insulator or another semiconductor.

The most useful device used in studying surface physics of semiconductors under controlled conditions is the metal-insulator-semiconductor (MIS) system. This was first proposed by Moll and by Pfann and Garrett in 1959 as a voltage variable capacitor. We will be concerned with the metal-oxide-semiconductor (MOS) system, Figure 1. This system is of interest for both technological and basic scientific reasons. First of all, the MOS system is the gate structure for most of the insulated-gate field-effect transistors (IGFET) or the metal-oxide-semiconductor field-effect-transistor (MOSFET). Secondly, the MOS system is of interest from the viewpoint of basic physics research because most of the important parameters can be varied simply by "turning a knob", e.g. the charge carrier density can be varied continuously by about six orders of magnitude (10$^{-15}$ - 10$^{21}$ cm$^{-3}$). Thus, we have the possibility of studying many-body effects under controlled conditions.
The IGFET has been made from various semiconductors such as Si, Ge, and GaAs and insulators such as SiO$_2$, Si$_3$N$_4$, and Al$_2$O$_3$. We will concentrate our attention on the SiO$_2$-Si interface with p-type Si. The addition of the metal layer of thickness 20-50 Å, called the gate, on the oxide layer of thickness 1000-5000 Å, allows us to apply a voltage $V_g$, called the gate voltage, or equivalently an electric field, across the SiO$_2$-Si interface. The gate voltage determines the electron density in the space-charge layer.

If a small positive gate voltage ($V_g > 0$) is applied, the energy bands bend downward, and the majority carriers (holes) are depleted. This region is called the depletion layer and extends into the semiconductor to the order of $10^4$ Å. If $V_g > 0$ is large enough such that the conduction band edge $E_c$ crosses over the Fermi level $E_F$, the number of minority carriers (electrons) at the interface becomes larger than that of the majority carriers and therefore the surface is inverted. This region is called an inversion layer and extends into the semiconductor to about 100 Å. The case in which we are concerned is that of an n-type inversion layer in p-type Si, as shown in Figure 2.

In 1957, Schrieffer proposed that the electric field associated with an inversion layer was strong enough to produce a potential well whose width
Figure 1. The metal-oxide-semiconductor (MOS) system.

Figure 2. Depletion layer and an n-type inversion layer in a p-type semiconductor.
perpendicular to the interface, which we take as the z direction, was small compared to the wavelengths of the carriers. This observation lead, via Schrodinger's equation, to the quantization of the energy levels of the inversion layer electrons. These energy levels are grouped into what are called electric subbands, each corresponding to a quantized level for motion in the z direction and no restrictions on the motion in the xy-plane, the plane parallel to the interface. This two dimensional structure of the electron gas was confirmed experimentally in 1966 by Fowler et al.\(^8\) using the optical technique of cyclotron resonance.

Optical experiments can be divided naturally into two distinct classes, viz. interband and intraband effects. The interband transitions involve quantum states in two different energy bands while intraband transitions, or simply free carrier effects, involve only a single energy band. Our attention in this work will be confined to intraband effects and since the energy band gap\(^9\) of Si at 0° K is 1.16 eV (and 1.12 eV at 300° K), we are therefore restricted to photon energies in the range \(10^{-4}\) - 1 eV which is the mid-infrared, far infrared and microwave regions of the electromagnetic spectrum.

Cyclotron resonance has been the most useful and most frequently used magneto-optical tool since its discovery in the early fifties.\(^{10,11}\) It depends on the
application of an external dc magnetic field $\vec{B}$ which
impacts to the carriers a frequency, called the cyclotron
frequency given by $\omega_c = eB/m^*c$, in the plane perpendicular
to $\vec{B}$. $m^*$ is the effective mass of the charged carrier
and accounts for the environment, i.e. the background
lattice. If an electromagnetic wave of angular frequency
$\omega$ is sent through the system, the electron will oscillate
at the frequencies $\omega$ and $\omega_c$ simultaneously. When $\omega$ is
adjusted to equal $\omega_c$, resonant absorption occurs and the
electron will move in an orbit of increasing radius until
it collides after a time $\tau$, called the collision time,
with the lattice.

However useful cyclotron resonance experiments have
been, as exemplified by the existence of a two dimensional
electron gas, there are two shortcomings. First, in order
for the resonance to be easily observable the condition
$\omega_c \tau > 1$ must be maintained and hence has limited applic-
ability (for a proof of this point, see Appendix D).
Secondly, observations\(^{12-15}\) of cyclotron resonance has
lead to a dilemma.

In order to discuss this dilemma, we need a result
derived in 1967 by Stern and Howard\(^6\): at the Si(100)
surface, the energy levels can be grouped into two distinct
sets of overlapping subbands, denoted by $E_0, E_1', \ldots, E_0'$,
$E_1', \ldots$, with $E_0 < E_0'$. The situation where we have suffi-
ciently low temperature that only the lowest electric
subband $E_0$ is occupied by electrons is called the electric quantum limit.

By raising the temperature we can thermally induce partial occupation of the $E'_0$ subband. Now, since both the $E_0$ and $E'_0$ subbands are occupied, we are dealing with two distinct effective masses, $m_1$ and $m_2$. Therefore, since the cyclotron frequency is inversely proportional to the effective mass, we expect to see two cyclotron resonance peaks. The dilemma is that the experiments of Abstreiter et al., Allen et al., Künlbeck and Kotthaus, and Abstreiter et al. observed a single resonance peak at an effective mass value intermediate between $m_1$ and $m_2$. This dilemma lead Appel and Overhauser in 1978 to incorporate into the theory of cyclotron resonance a second electron system interacting with the first through an electron-electron collision time $\tau_e$, and is in good agreement with the above mentioned observations.

Since cyclotron resonance is limited in its applications by the condition $\omega_c \tau > 1$, we are motivated to study other magneto-optical effects. In particular, the Faraday effect, first observed in 1845 by Faraday, is independent of the requirement that $\omega_c \tau$ be greater than one, and thus is applicable in the region ($\omega_c \tau < 1$) where cyclotron resonance gives no information. It also provides useful complementary information even when cyclotron
resonance is easily observable.

In Section II, we define the Faraday geometry and consider the Faraday effect for the inversion layer at the SiO₂-Si interface in both the Drude and Appel-Overhauser models. Null Faraday rotation is shown to give information about the effective mass and the collision time. In Section III, we include the boundary effects and the multiple reflections effects into the Faraday rotation and ellipticity. Further, we obtain a unique decomposition of the multiple pass rotation and ellipticity into terms due purely to the single pass, boundary effects, and multiple reflection effects. Section IV is a discussion on how the single pass, boundary, and multiple reflection effects can be measured experimentally.
II. FARADAY EFFECT

A. Faraday Geometry

The situation shown in Figure 3 will be simply referred to as the Faraday geometry. It consists of a plane monochromatic wave of angular frequency $\omega$, linearly polarized in the $\hat{x}$ direction and propagating in the $\hat{z}$ direction

$$\vec{E} = E_\omega e^{i(k\cdot\vec{r}-\omega t)} \hat{x},$$

a uniform external dc magnetic field

$$\vec{B} = B\hat{z}, \quad B > 0,$$

and a plane-parallel substrate of thickness $d$ whose boundaries are located at $z = 0$ and $z = d$.

Decomposing the plane electromagnetic wave into its right and left circularly polarized components $\vec{E}_-, \vec{E}_+$, we have

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

where
Figure 3. The Faraday geometry.
\[ \hat{E}_+ = \frac{1}{\sqrt{2}} E \hat{e}^i (\hat{k} \cdot \hat{r} - \omega t) \hat{e}_+ \]

with

\[ \hat{e}_+ \equiv (\hat{x} + i\hat{y})/\sqrt{2} . \]

The single pass Faraday rotation \( \theta \) and the single pass ellipticity \( \delta \) are derived in Appendix A and are given by \( 19-21 \)

\[ \theta = \frac{\omega d}{2c} (n_+ - n_-) , \quad (1) \]

and

\[ \delta = \tanh \left\{ \frac{\omega d}{2c} (\kappa_- - \kappa_+) \right\} , \quad (2) \]

where \( c \) is the velocity of light in vacuo and \( n_+ \), \( \kappa_+ \) are the real and imaginary parts of the complex refractive indices \( n_+ + i\kappa_+ \) corresponding respectively to the left and right circularly polarized components of the initially linearly polarized wave. The complex refractive indices are related to the dielectric constants \( \varepsilon_+ \) by

\[ \varepsilon_+ \equiv \varepsilon'_+ + i\varepsilon''_+ \]

\[ = (n_+ + i\kappa_+)^2 \quad (3) \]
where the prime and double prime denote real and imaginary parts, respectively.

Inverting this equation we find \( n_+^+ \), \( k_+^+ \) in terms of the real and imaginary parts of \( \varepsilon_+^+ \) as

\[
n_+^2 = \frac{1}{2} \left\{ \left[ (\varepsilon_+^+)' \right]^2 + \left( \varepsilon_+^+'' \right)^2 \right\}^{1/2} + \varepsilon_+^+', \tag{4}
\]

and

\[
k_+^2 = \frac{1}{2} \left\{ \left[ (\varepsilon_+^-)' \right]^2 + \left( \varepsilon_+^-'' \right)^2 \right\}^{1/2} - \varepsilon_+^-'. \tag{5}
\]

B. Null Faraday Rotation and Null Ellipticity

It is clear from Eqs. (1) and (2) that null Faraday rotation\(^2\) and null ellipticity\(^3\) are obtained whenever

\[
n_+ = n_-, \tag{6}
\]

and

\[
k_+ = k_- , \tag{7}
\]

respectively.

Squaring Eqs. (6) and (7) and making use of Eqs. (4) and (5) gives
\[
[(e_+^1)^2 + (\epsilon''_+)^2]^{1/2} - [(e_-^1)^2 + (\epsilon''_-)^2]^{1/2} = -(\epsilon'^+ - \epsilon'^-) \quad (8)
\]

and

\[
[(e_+^1)^2 + (\epsilon''_+)^2]^{1/2} - [(e_-^1)^2 + (\epsilon''_-)^2]^{1/2} = \epsilon'^+ - \epsilon'^-. \quad (9)
\]

From these equations we obtain the single general result\textsuperscript{23}

\[
\{(e_+^1)^2 - (\epsilon''_+)^2\}^2 = 4(\epsilon'^+ - \epsilon'^-)[(\epsilon''_+)^2 - (\epsilon''_+)^2] \quad (10)
\]

as the condition on \(\epsilon_+\) in order to achieve null Faraday rotation and null ellipticity. To proceed further we must make a specific choice for the dielectric constants \(\epsilon_+\).

We consider null rotation and null ellipticity in two models:

i) The classical Drude model for which we can obtain an exact algebraic relationship between the photon, cyclotron, collision and plasma frequencies as the condition to achieve null Faraday rotation and null ellipticity. Beyond this point we must resort to algebraic approximations and numerical calculations.

ii) The Appel-Overhauser model for which we can only obtain algebraic approximations and numerical results.
C. Drude Model

The equation of motion for an electron of charge \( q = -e(e>0) \) and mass \( m^* \), called the effective mass, driven by an electromagnetic field \( \vec{E}, \vec{H} \), and a static magnetic field \( \vec{B} \) is

\[
m^* \frac{d^2 \vec{v}}{dt^2} + m^* \vec{v} \times \vec{v} = -e(\vec{E} + \frac{\vec{v}}{c} \times \vec{H}) - e \frac{\vec{v}}{c} \times \vec{B}
\]

(11)

In the Faraday geometry the solution of Eq. (11), see Appendix C for details, for \( \sigma_+ \) is

\[
\sigma_+ = \frac{ine^2/m^*}{\omega^2 + \omega_c^2 + iv} \quad ,
\]

(12)

hence

\[
\epsilon_+ = \epsilon_l \{ 1 - \frac{\omega_p^2}{\omega (\omega + \omega_c + iv)} \}
\]

(13)

where \( \epsilon_l \) is the dielectric constant of the lattice, \( \omega_c = eB/m^*c \) is the cyclotron frequency, and \( v \) is the collision frequency. In addition

\[
\omega_p^2 = \frac{4\pi ne^2}{m^* \epsilon_l}
\]

(14)

is the plasma frequency with \( n = N^{3/2} \) where \( n, N \) are the electron density and surface density, respectively,
or alternatively \( n = N/d \), where \( d \) is the thickness of the charge layer. Thus, separating \( \varepsilon_\pm \) into its real and imaginary parts we obtain

\[
\varepsilon_+ = \varepsilon_\epsilon \left\{ 1 - \frac{\omega^2 (\omega + \omega_c)}{\omega [ (\omega + \omega_c)^2 + \nu^2 ]} \right\} ,
\]

and

\[
\varepsilon_- = \varepsilon_\epsilon \frac{\nu \omega^2}{\omega [ (\omega + \omega_c)^2 + \nu^2 ]} .
\]

Substituting Eqs. (15) and (16) into Eq. (10) and simplifying, we obtain the condition for null Faraday rotation and null ellipticity in the form of a quintic equation for \( x = \left( \frac{\omega}{\Omega} \right)^2 \):

\[
f(x) = 4x^5 + \{ 8 [2 (\frac{\nu}{\Omega})^2 - 1] - 3 (\frac{\omega}{\Omega})^2 \} x^4
- 8 (\frac{\omega}{\Omega})^2 [2 (\frac{\nu}{\Omega})^2 - 1] x^3 - 2 \{ 4 [2 (\frac{\nu}{\Omega})^2 - 1] + 3 (\frac{\omega}{\Omega})^2 \} x^2
- 4x + (\frac{\omega}{\Omega})^2 = 0 ,
\]

where

\[
\Omega = (\omega_c^2 + \nu^2)^{1/2} .
\]
The introduction of the frequency $\Omega$ is extremely useful, as will be apparent shortly. For now we simply note that $\varepsilon'_+ = \varepsilon'_-$ (but $\varepsilon'_+ \neq \varepsilon'_-$) when $\omega = \Omega$. Thus, instead of treating $\omega, \omega_P, \omega_c, \omega$ as our four basic independent variables, we found it more convenient to choose $\omega, \omega_P, \Omega, \omega_c$ (the choice $\omega, \omega_P, \Omega, \omega_c$ would be equally good). It is also useful to re-write Eq. (17) in the form:

$$f(x) \equiv 4x^5 + 8[2(\frac{\nu}{\Omega})^2-1]x^4 - 8[2(\frac{\nu}{\Omega})^2-1]x^3 - 4x$$

$$- \left(\frac{\omega_P}{\Omega}\right)^2 \{3x^4 + 8[2(\frac{\nu}{\Omega})^2-1]x^3 + 6x^2-1\} = 0 .$$

(19)

The fact that only an $\omega_P^2$ term occurs in $f(x)$ is a notable feature of this equation. We have investigated this quintic equation in $x$ algebraically, graphically and numerically, with the following conclusions:

(1) In general, we see by inspection of Eq. (17) that

$$f(\pm \infty) = \pm \infty ,$$

(20)

$$f(0) = \left(\frac{\omega_P}{\Omega}\right)^2 > 0 ,$$

(21)

$$f(1) = -16(\frac{\nu}{\Omega})^2 \left(\frac{\omega_P}{\Omega}\right)^2 < 0 ,$$

(22)
\( f(-1) = -16 \left( \frac{\omega}{\Omega} \right)^2 \left( \frac{\epsilon_0}{\Omega} \right)^2 < 0 , \) \hspace{1cm} (23)

and

\( f'(0) = -4 < 0 , \) \hspace{1cm} (24)

where the prime denotes differentiation with respect to \( x. \)

(2) For fixed \( v, \omega_p \) and \( \Omega, \) Eq. (17) has, for \( x = (t_0/f_t)^2, \) two complex roots, one negative root in the interval \((-1,0), \) and two positive roots, one in each of the intervals \((0,1) \) and \((1,\infty), \) see Appendix E for details. Of course, only the two positive roots are of potential physical significance. It turns out that only the larger positive root gives \( \theta = 0, \) while only the smaller positive root gives \( \delta = 0 \) when inserted into Eqs. (1) and (2). This is expected, since by inspection of Eqs. (8) and (9), we see that Eqs. (6) and (7) are not satisfied simultaneously, since this would imply \( \epsilon_+ = \epsilon_- \) which can only occur for \( \omega = \Omega, \) or \( x = 1, \) contradicting Eq. (22).

Thus, for null Faraday rotation to be obtained, we have the following conclusions\(^\text{22,23}:\)

(i) Only the larger positive root gives \( \theta = 0 \) when inserted into Eq. (1),

(ii) By inspecting Eqs. (20)-(24), the desired root always occurs for \( \omega > \Omega, \) or \( x > 1, \)
(iii) For $\omega_p/\Omega << 1$: $x \approx 1 + \omega_p^2/2\Omega^2$, so that $\theta = 0$ when

$$\omega \approx \Omega (1 + \frac{\omega_p^2}{4\Omega^2}) \approx \Omega,$$  \hspace{1cm} (25)

(iv) For $\omega_p/\Omega >> 1$: $x \approx \frac{3}{4} (\frac{\omega_p}{\Omega})^2$, so that $\theta = 0$ when

$$\omega \approx 0.866\omega_p >> \Omega.$$  \hspace{1cm} (26)

The corresponding results for null ellipticity are as follows\textsuperscript{23}:

(i') Only the smaller positive root gives $\delta = 0$ when inserted into Eq. (2),

(ii') By inspecting Eqs. (20)-(24), the desired root always occurs for $0<\omega<\Omega$ (or $0<x<l$),

(iii') For $\omega_p/\Omega << 1$: $x \approx \frac{1}{4} (\frac{\omega_p}{\Omega})^2$, so that $\delta = 0$ when

$$\omega \approx \frac{1}{2} \omega_p << \Omega,$$  \hspace{1cm} (27)

(iv') For $\omega_p/\Omega >> 1$: Consider the two functions $y = f(x,\omega_p)$ and $y = f(x,\omega'_p)$ for $\omega_p \neq \omega'_p \neq 0$. Using Eq. (17) we deduce that the intersection points of these curves are given by the real solutions of
\[ g(x) = 3x^4 + 8(2(\frac{\nu}{\Omega})^2 - 1)x^3 + 6x^2 - 1 = 0. \]  \hspace{1cm} (28)

Note that this is independent of both \( \omega_p \) and \( \omega_p' \). Hence, for all values of \( \omega_p \), the curves \( y = f(x, \omega_p) \), given by Eq. (17), intersect at the real solutions of Eq. (28).

The discriminant, \( D(g) \), of \( g(x) \) is given by

\[ D(g) = -6912 (\frac{\nu}{\Omega})^4 [1 - (\frac{\nu}{\Omega})^2]^2 < 0 \] \hspace{1cm} (29)

since \( 0 < (\frac{\nu}{\Omega})^2 < 1 \). Therefore, Eq. (28) has two complex roots and hence two real roots.

For the sake of completeness, we give the exact solution for the two real roots of Eq. (28):

\[ x = \frac{1}{3} \left\{ -[a - (a^2 + 9b - 3)^{1/2}] \right. \]

\[ + \left[ [a - (a^2 + 9b - 3)^{1/2}]^2 - 3[a^2 + 9b - 3]^{1/2}\right]^{1/2}\]

\hspace{1cm} \hspace{1cm} (30)

where

\[ a = 2[2(\frac{\nu}{\Omega})^2 - 1], \]

and

\[ b = \frac{1}{3} \left\{ 4(\frac{\nu}{\Omega})^2 \left[ 1 - (\frac{\nu}{\Omega})^2 \right]^{1/3} \right\}. \]
Inspection of Eq. (30) shows that Eq. (28) has one positive root and one negative root.

If \( h(\nu, \Omega, \omega_p) \) and \( H(\nu, \Omega, \omega_p) \) denote the smaller and larger positive roots of Eq. (17), respectively, so that

\[
\Omega < H(\nu, \Omega, \omega_p) < \infty, \quad \text{for all } \nu, \Omega, \omega_p,
\]  

(31)

and

\[
0 < h(\nu, \Omega, \omega_p) < \Omega, \quad \text{for all } \nu, \Omega, \omega_p,
\]  

(32)

it is easy to see that

\[
\lim_{\omega_p/\Omega \to 0} H(\nu, \Omega, \omega_p) = \Omega,
\]  

(33)

and

\[
\lim_{\omega_p/\Omega \to \infty} h(\nu, \Omega, \omega_p) = x_0^{1/2} \Omega,
\]  

(34)

where \( x_0 \) is given by the right hand side of Eq. (30).

Equation (28) and its results says that both \( h \) and \( H \) are strictly monotone increasing functions of \( \omega_p \) and therefore \( \Omega \) is a strict lower bound of the frequency for which null Faraday rotation is obtained, while \( x_0^{1/2} \Omega \) is a strict upper bound of the frequency for which null ellipticity is obtained.
From the results of this section we have that the photon frequencies $\omega$, for which the null Faraday rotation and null ellipticity conditions are satisfied are always distinct but fundamentally related by being solutions of Eq. (17).

In Figure 4 we present plots of $f(x)$ versus $x$. Table 1 is a list of $\omega/\Omega$ values for which null Faraday rotation is achieved, and Table 2 is a corresponding list of values for null ellipticity. Figure 5 is a plot of the Faraday rotation and Figure 6 is the corresponding plot of the ellipticity.

Suppose that we are in the regime where $\omega_p/\Omega << 1$. Then null Faraday rotation implies $\omega = \Omega$. Further, if the magnetic field is strong ($\omega_c >> \nu$), then $\omega = \omega_c$, from which the effective mass can be obtained. If we now go to a weak magnetic field ($\omega_c << \nu$), by "turning a knob", we obtain a measure of the collision frequency $\nu = 1/\tau$. If $\omega_c > \nu$, $\omega_c$ can be obtained from cyclotron resonance and $\nu$ can be calculated from $\nu = (\omega^2 - \omega_c^2)^{1/2}$ and furnishes a check on the collision frequency obtained from cyclotron resonance experiments.

Up to this point we have assumed that the electron density is constant. In the case of a charged plasma with varying density we thus have the complication of varying plasma frequency. For example, in the MOS system the density profile through the inversion layer must be
Figure 4. Plot of $f(x)$ versus $x$ for $\omega/\Omega = 3/5$, $\nu/\Omega = 4/5$ and for $\omega_p/\Omega$ values as indicated on the curves.
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Table 1. Values of $(\omega/\Omega)$ for which null Faraday rotation is obtained, for various values of $\omega_c$, $\nu$, and $\omega_p$. 
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Table 2. Values of $\omega/\Omega$ for which null ellipticity is obtained, for various values of $\omega_c$, $\nu$ and $\omega_p$. 
Figure 5. Plot of the Faraday rotation $\theta$ versus $\omega/\Omega$ for $\omega/\Omega = 3/5$, $\nu/\Omega = 4/5$, $\omega_p/\Omega = 2$ and $\lambda = 10^{-6}$ cm.
Figure 6. Plot of the ellipticity $\delta$ versus $\omega/\Omega$ for $\omega_c/\Omega = 3/5$, $\nu/\Omega = 4/5$, $\omega_p/\Omega = 2$ and $\lambda = 10^{-6}$ cm.
calculated theoretically. 6 Hence the calculation of a non-zero value for the Faraday rotation \( \theta \) is more involved and uncertain. However, if we select our parameters so that Eq. (25) holds we see that the condition for null Faraday rotation is only weakly dependent on \( \omega_p \). Furthermore, we have verified numerically that, for \( \omega_p \) values close to the value for which null Faraday rotation is obtained (\( \omega_p^{(0)} \) say), \( \theta \) is very small and proportional to \( (\omega_p - \omega_p^{(0)}) \). Thus, the optimum accuracy is achieved by selecting \( \omega_p^{(0)} \) to be a suitably weighted average of the plasma frequencies and also by taking \( (\omega_p^{(0)}/\Omega) \) to be as small as possible.

D. Appel-Overhauser Model

In order to explain observations of cyclotron resonance in electron inversion layers on Si, Appel and Overhauser 17 proposed a system consisting of two degenerate electron systems interacting with one another. Their results, for \( \varepsilon_+ \), stated in our notation is as follows:

\[
\varepsilon_+ = \varepsilon_\perp + i \frac{4\pi}{\omega_p} \sigma_+ ,
\]

where the conductivities are given by
\[ \sigma_+ = -\frac{eV}{\mathcal{M}} \left[ \frac{n_1}{m_1} p_+ + \left( \frac{1}{m_1} - \frac{1}{m_2} \right) \pi_+ \right], \] 

\[ n = n_1 + n_2 \] and \[ M = n_1 m_1 + n_2 m_2, \]

the indices indicate the electron systems, \( V \) is the volume under consideration, which we take to be 1, and \( m_i \) (\( i = 1, 2 \)), \( n_i \) are the effective masses and the carrier concentrations, respectively.

\[ p_+ = -\frac{eV}{D_+} \left[ n_1 \tau_1 \left[ 1 + \frac{\tau_2}{\tau_e} - i\tau_2(\omega + \omega_2) \right] 
+ n_2 \tau_2 \left[ 1 + \frac{\tau_1}{\tau_e} - i\tau_1(\omega + \omega_1) \right] \right], \] 

is related to the total momentum of the electron systems,

\[ \pi_+ = -\frac{eV}{D_+} \left\{ \frac{\mu \tau_1}{m_1} \left[ 1 - i\tau_2(\omega + \omega_2) \right] 
- \frac{\mu \tau_2}{m_2} \left[ 1 - i\tau_1(\omega + \omega_1) \right] \right\}, \] 

is related to the relative momentum of the electron systems, and

\[ D_+ = 1 + \mu \left( \frac{1}{n_1 m_1} \frac{\tau_1}{\tau_e} + \frac{1}{n_2 m_2} \frac{\tau_2}{\tau_e} \right) - \tau_1(\omega + \omega_1) \tau_2(\omega + \omega_2) 
- i\tau_1(\omega + \omega_1) \left( 1 + \frac{\mu}{n_2 m_2} \frac{\tau_2}{\tau_e} \right) 
- i\tau_2(\omega + \omega_2) \left( 1 + \frac{\mu}{n_1 m_1} \frac{\tau_1}{\tau_e} \right) \]
where $1/\mu = 1/n_1^{m_1} + 1/n_2^{m_2}$. $\omega_i$ are the cyclotron frequencies and $\tau_i$ the collision times.

Before resorting to numerical calculations, it is instructive, first of all, to examine different regimes which are amenable to algebraic calculations and have some interesting physical aspects.

Letting $\tau_e \to 0$, i.e., strong electron-electron (e-e) interaction ($\tau_1, 2/\tau_e \gg 1$), in Eq. (35), we obtain

$$\lim_{\tau_e \to 0} \epsilon_+ = \epsilon_\perp (1 - \frac{\omega^2_{pc}}{\omega (\omega + \omega_c + i\nu_c)})$$

(40)

where

$$\omega^2_{pc} = \frac{4\pi e^2 n}{m_c \epsilon_\perp}$$

$$\omega_c = \frac{eB}{m_c c}$$

$$\nu_c \equiv \frac{1}{M} \left( \frac{n_2}{\tau_2} m_2 + \frac{n_1}{\tau_1} m_1 \right)$$

(41)

and $m_c \equiv M/n$ is the concentration averaged mass. Hence, the electron systems "clump together" and become a single electron system with effective mass $m_c$, electron density $n$ and collision frequency $\nu_c$. This is formally identical to the free carrier Drude Model defined in Eq. (13).
Thus, all of our results on null Faraday rotation and null ellipticity in the Drude theory carry over to the Appel-Overhauser theory in this regime.

Making the further identifications, \( m_1 = m_2 = m^* \), \( \tau_1 = \tau_2 = \tau \) and \( n_1 = n_2 = \frac{1}{2} n \), we see that the Appel-Overhauser model reduces identically to the free carrier Drude model.

Letting \( \tau \to \infty \), i.e. weak e-e interaction \( (\tau_{1,2}/\tau < 1) \), we are dealing with two systems of non-interacting electrons and it follows that

\[
\varepsilon_+ = \varepsilon_{+1} + \varepsilon_{+2}, \quad (42)
\]

where

\[
\varepsilon_{+i} = \varepsilon \left\{ \frac{1}{2} - \frac{\omega^2}{\omega \{\omega + \omega_i + 1 \nu_i \}} \right\}, \quad i = 1, 2, \quad (43)
\]

and

\[
\omega^2_{pi} = \frac{4\pi n_i e^2}{m_i \varepsilon \varepsilon_0}, \quad (44)
\]

\[
\nu_i = 1/\tau_i. \quad (45)
\]

If in this regime, we make the further simplifying assumption that \( |\omega + \omega_i| >> \nu_i \), or equivalently, \( \text{Im } \varepsilon_{+i} << \text{Re } \varepsilon_{+i} \) so that \( \text{Re } \varepsilon_{+i} = n_{+i}^2 \), then the Faraday rotation becomes
\[ \theta = \theta_1 + \theta_2 \]  

(46)

where

\[ \theta_i = - \frac{\epsilon_i \delta \epsilon_i \omega_i^2 p_i}{2N_i c (\omega^2 - \omega_i^2)} \]  

(47)

with

\[ N_i = \frac{n_{+i} + n_{-i}}{2} . \]  

(48)

Simplifying Eq. (46) gives

\[ \theta = - \frac{\epsilon_i \delta \epsilon_i \omega_i^2 p_i}{2N_i c (\omega^2 - \omega_1^2)} \left\{ \frac{\omega_1 \omega_1^2 p_1 + \omega_2 \omega_2^2 p_2}{\omega^2 - \omega_1^2} \right\} \]  

(49)

where \( N = (N_1 + N_2)/2 \).

For photon frequencies \( \omega \), restricted to the range

\[ \omega_2 < \omega < \omega_1 , \]  

(50)

we see, using Eq. (47), that \( \theta_1 \) is positive and \( \theta_2 \) is negative. In fact \( \theta \) is actually zero for a photon frequency \( \omega \) given by

\[ \omega^2 = \omega_1 \omega_2 \left( \frac{\omega_2 \omega_1^2 + \omega_1 \omega_2^2 p_1}{\omega_1 \omega_2 + \omega_2 \omega_2^2 p_2} \right) , \]  

(51)

or
Selecting, as in Ref. 17, \( m_1 = 0.19 \text{ m} \) and \( m_2 = 0.42 \text{ m} \), where \( m \) is the free electron mass, it follows that

\[
\omega = \omega_1 \left( \frac{1 + \frac{n_2}{n_1}}{m_2/m_1 + \frac{n_2}{n_1}} \right)^{1/2}.
\] 

(52)

Now the values of \( n_1 \) and \( n_2 \) at the Si (100) surface are determined essentially by the energy difference between the two different sets of overlapping subbands, which in turn is determined by the temperature and the uniaxial stress. However, as noted in Ref. 15, the latter dependence is not known accurately enough to precisely predict the values of \( n_1 \) and \( n_2 \). However, estimates can be made and it is clear that if we vary \( \omega \) within the range given by Eq. (50) that a zero in \( \theta \) can be found provided we are in the weak coupling regime. In Fig. 7 we present the results of a numerical calculation, where we have chosen parameters for which the above algebraic treatment holds. The existence of the zero between \( \omega_1 \) and \( \omega_2 \) is striking and in agreement with the algebraic result given in Eq. (53). The other zeros in the vicinity of \( \omega_1 \) and \( \omega_2 \) are also expected, e.g. applying Eq. (25) to electron system 1 we have \( \theta = 0 \) when \( \omega = 9.54 \times 10^{12} \) and applying it to electron system 2 we have \( \theta = 0 \) when \( \omega = 4.79 \times 10^{12} \).
Figure 7. Plot of the Faraday rotation $\theta$ versus angular photon frequency $\omega$ for the Appel-Overhauser model, using parameters $m_1 = 0.19 \text{ m}$, $m_2 = 0.42 \text{ m}$, $n_1 = 6.7 \times 10^{15} \text{ cm}^{-3}$, $n_2 = 5.3 \times 10^{16} \text{ cm}^{-3}$, $\tau_1 = \tau_2 = \tau = 2.3 \times 10^{-12} \text{ s}$, $\tau_0 = 10^{40} \text{ s}$, $\lambda = 10^{-6} \text{ cm}$, $B = 10^5 \text{ G}$ and $\epsilon_\phi = 11.8$. The vertical lines indicate the corresponding values of $\omega_1 = 9.26 \times 10^{12} \text{ s}^{-1}$ and $\omega_2 = 4.20 \times 10^{12} \text{ s}^{-1}$. 
in good agreement with Figure 7.

The investigations of Ref. 17 indicate, at least for the experiments presently of interest, that the appropriate regime is either strong or intermediate coupling. Thus, we are motivated to investigate the dependence of $\theta$ on $\tau_e$ and, in particular, as we go from $\tau_e = \infty$ to smaller values at what value does the zero in $\theta$ disappear. Typical results are presented in Fig. 8. The following features are worthy of comment:

(i) the shape of the rotation versus frequency curve depends significantly on the value of $\tau/\tau_e$. In particular, as we go from small values of $\tau/\tau_e$ to larger values, two zeros in the curve disappear,

(ii) the relative minimum value of the rotation associated with weak e-e interaction changes into a relative maximum for strong e-e interaction,

(iii) $d\theta/d\omega$ evaluated at $\omega = \omega_1$ changes sign from positive in the weak coupling regime to negative in the strong coupling regime.

Thus, we have three potentially very useful methods for obtaining a handle on the strength of the e-e coupling.

Up to this point we have neglected boundary effects, i.e. we have set the Fresnel reflection and transmission coefficients identically equal to 0 and 1, respectively. When these conditions are relaxed the electromagnetic wave will undergo multiple internal reflections and
Figure 8. Plot of the Faraday rotation $\theta$ versus angular photon frequency $\omega$ for the Appel-Overhauser model, using the parameters $m_1 = 0.19 \, m$, $m_2 = 0.42 \, m$, $n_1 = 6.7 \times 10^{15}$ cm$^{-3}$, $n_2 = 5.3 \times 10^{16}$ cm$^{-3}$, $\tau_1 = \tau_2 = \tau = 2.3 \times 10^{-12}$ s, $\tau / \tau_e$ values $10^{-2}$, $10^{-1}$, 1, 2, 10, $\ell = 10^{-6}$ cm, $B = 10^5$ G and $\epsilon_0 = 11.8$. The vertical line indicates the corresponding value of $\omega_1 = 9.26 \times 10^{12}$ s$^{-1}$. 
contribute significantly to the measured Faraday rotation, e.g. White et al.\textsuperscript{27} have reported enhancement of the single pass value by a factor of 100. Thus, we must account for multiple internal reflections in the theory of the Faraday effect.
III. MULTIPLE REFLECTIONS

Donovan and Medcalf\textsuperscript{28} reformulated the theory of the free carrier Faraday effect in semiconductors for a vacuum-medium-vacuum system, taking into account multiple reflections of the beam in the medium. Here we generalize Donovan and Medcalf's results to the case of three distinct media and also extend their analysis in the sense of obtaining a unique decomposition of the multiple pass rotation, $\Theta$ say, which gives added physical insight into the various contributions to $\Theta$. The multiple pass ellipticity $\Delta$ is similarly treated.

Letting

$$ N_{j\pm} = n_{j\pm} + iK_{j\pm}, \quad j = 1, 2, 3 \quad (54) $$

be the complex refractive index of medium $j$, the Fresnel transmission and reflection coefficients\textsuperscript{29} at normal incidence are (Figure 9):

$$ r_{12\pm} = \frac{N_{2\pm} - N_{1\pm}}{N_{2\pm} + N_{1\pm}}, \quad t_{12\pm} = \frac{2N_{1\pm}}{N_{2\pm} + N_{1\pm}}, \quad t_{12\pm} + r_{12\pm} = 1, $$

$$ r_{23\pm} = \frac{N_{3\pm} - N_{2\pm}}{N_{3\pm} + N_{2\pm}}, \quad t_{23\pm} = \frac{2N_{2\pm}}{N_{3\pm} + N_{2\pm}}, \quad t_{23\pm} + r_{23\pm} = 1, $$

$$ r_{21\pm} = \frac{N_{1\pm} - N_{2\pm}}{N_{1\pm} + N_{2\pm}}, \quad t_{21\pm} = \frac{2N_{2\pm}}{N_{1\pm} + N_{2\pm}}, \quad t_{21\pm} + r_{21\pm} = 1. \quad (55) $$
Figure 9. Multiple internal reflections in a plane-parallel sample of thickness d. The angles $\phi_1$, $\phi_2$ and $\phi_3$ are shown non-zero for clarity.
It is convenient to rewrite Eqs. (55) in the form

\[
\begin{align*}
    r_{ij\pm} &\equiv |r_{ij\pm}| e^{i\xi_{ij\pm}} \\
    t_{ij\pm} &\equiv |t_{ij\pm}| e^{i\gamma_{ij\pm}}
\end{align*}
\]  

(56)

where

\[
\tan \xi_{ij\pm} = \frac{2(n_{ij\pm} - n_{ji\pm})}{n_{ij\pm}^2 - n_{ji\pm}^2 + \kappa_{ij\pm}^2 - \kappa_{ji\pm}^2},
\]

\[
\tan \gamma_{ij\pm} = \frac{n_{ij\pm} - n_{ji\pm}}{n_{ij\pm}(n_{ij\pm} + n_{ji\pm}) + \kappa_{ij\pm}(\kappa_{ij\pm} + \kappa_{ji\pm})}.
\]  

(57)

The pair of lower case Latin indices (ij) indicate that the electromagnetic wave is propagating from medium i in the direction of medium j.

The transmission amplitudes \( \tau_{n\pm} \) of the 2n-times reflected beam are

\[
\tau_{n\pm} = t_{12\pm} t_{23\pm} r_{23\pm} r_{21\pm} e^{i(2n+1)\delta_{\pm}}
\]  

(58)

where

\[
\delta_{\pm} \equiv \frac{\omega d}{c} (n_{2\pm} + i\kappa_{2\pm})
\]

\[
\equiv (\beta_{\pm} + i\alpha_{\pm}/2) d
\]  

(59)
is the change in phase of the corresponding beam introduced by transversing the sample (medium 2).

Summing all the multiply internally reflected amplitudes we obtain

\[ \tau_{\pm} \equiv \sum_{n=0}^{\infty} \tau_{n\pm}, \]

\[ = T_{\pm} e^{i\delta_{\pm}}, \quad (60) \]

where

\[ T_{\pm} \equiv \frac{t_{12\pm} t_{23\pm}}{1 - r_{23\pm} r_{21\pm} e^{2i\delta_{\pm}}}. \quad (61) \]

It is now convenient to define \( \eta_{\pm} \) by

\[ (1 - r_{23\pm} r_{21\pm} e^{2i\delta_{\pm}})^{-1} = |1 - r_{23\pm} r_{21\pm} e^{2i\delta_{\pm}}|^{-1} e^{i\eta_{\pm}} \quad (62) \]

so that

\[ \tan \eta_{\pm} = \frac{|r_{23\pm} r_{21\pm}| e^{-\alpha_{\pm} d} \sin(2\beta_{\pm} d + \xi_{21\pm} + \xi_{23\pm})}{1 - |r_{23\pm} r_{21\pm}| e^{-\alpha_{\pm} d} \cos(2\beta_{\pm} d + \xi_{21\pm} + \xi_{23\pm})}. \quad (63) \]
From the equation (see Appendix A, Eq. (A.5))

\[
\frac{\tau^-}{\tau^+} = \left| \frac{\tau^-}{\tau^+} \right| e^{-2i\theta},
\]

we obtain for the multiple pass Faraday rotation \( \theta \)

\[
\theta = \theta + \theta_T + \theta_{MR} \quad (64)
\]

where

\[
\theta = (\beta_+ - \beta_-)d/2
\]

is the single pass Faraday rotation,

\[
\theta_T \equiv \frac{1}{2}(\gamma_{12+} + \gamma_{23+} - \gamma_{12-} - \gamma_{23-}) \quad (65)
\]

is due purely to boundary effects and is independent of the sample thickness \( d \), and

\[
\theta_{MR} \equiv \frac{1}{2}(n_+ - n_-) \quad (66)
\]

is the contribution due purely to multiple reflection effects.

We now define the single pass Faraday rotation with boundary effects, \( \theta_T \) say, as
In Figure 10 we present a plot of $\theta$, $\theta_T$ and $\theta$ versus $\log \omega$. Figure 11 is a plot of the Faraday rotation $\theta$ for various values of $\tau/e$. We explicitly point out that the decomposition of $\theta$ given in Eq. (64) is independent of the model used for the dielectric constants $\varepsilon_{\pm}$. It is also noteworthy that the boundary effects $\theta_T$ can be decomposed into effects at each boundary, i.e.

$$\theta_T = \theta_1 + \theta_2$$  \hspace{1cm} (68)$$

where

$$\theta_1 \equiv \frac{1}{2}(\gamma_{12}+\gamma_{12}^-)$$  \hspace{1cm} (69)$$

is due to interface 1 (see Fig. 9) and

$$\theta_2 \equiv \frac{1}{2}(\gamma_{23}+\gamma_{23}^-)$$  \hspace{1cm} (70)$$

is due to interface 2. Hence

$$\theta = \theta + \theta_1 + \theta_2 + \theta_{MR}.$$  \hspace{1cm} (71)$$
Figure 10. Plot of $\theta$, $\theta_T$ and $\theta$ versus log $\omega$ for the Appel-Overhauser model, using the parameters $m_1 = 0.19 \text{ m}$, $m_2 = 0.42 \text{ m}$, $n_1 = 6.7 \times 10^{15} \text{ cm}^{-3}$, $n_2 = 5.3 \times 10^{16} \text{ cm}^{-3}$, $\tau_1 = \tau_2 = 2.3 \times 10^{-12} \text{ s}$, $\tau_e = 1040 \text{ s}$, $\ell = 10^{-2} \text{ cm}$, $B = 10^5 \text{ G}$ and $\varepsilon_\ell = 11.8$. The vertical lines indicate the corresponding values of $\Omega_1 = 9.26 \times 10^{12} \text{ s}^{-1}$ and $\Omega_2 = 4.20 \times 10^{12} \text{ s}^{-1}$. 
Figure 11. Plot of the Faraday rotation $\Theta$ versus $\log \omega$ for the Appel-Overhauser model, using parameters $m_1 = 0.19 \, m$, $m_2 = 0.42 \, m$, $n_1 = 6.7 \times 10^{15} \, \text{cm}^{-3}$, $n_2 = 5.3 \times 10^{16} \, \text{cm}^{-3}$, $\tau_1 = \tau_2 = \tau = 2.3 \times 10^{-12} \, s$, $\ell = 10^{-6} \, \text{cm}$, $B = 10^5 \, \text{G}$, $\epsilon_{\ell} = \epsilon_{\text{Si}} = 11.8$, $\epsilon_{\text{SiO}_2} = 3.81$ and $\tau/\tau_e = 10^{-2}$, 1, 10.
Since we are dealing with a system with many variable parameters it is clear that a variety of other curves could be presented. However, we feel it is best to defer any further elaborations until some experimental Faraday rotation results have been obtained.

Donovan and Medcalf\textsuperscript{28} define a "non-reflection" case by setting $T_+ = T_-$. Using Eqs. (8), (12) and (13), we see that this condition is true only when

$$\theta_T + \theta_{MR} = 0,$$

so that

$$\theta = \theta,$$

which in our terminology is the single pass Faraday rotation. They state that\textsuperscript{28} "Above the microwave region $\theta$ varies rapidly with $\omega$ and oscillates about the non-reflection curve". Using the decomposition Eq. (64) and Eq. (67) we can extend this statement as follows: The Faraday rotation $\theta$ oscillates about the curve $\theta_T$ over all frequencies $\omega$. See Figure 10.

The multiple pass ellipticity $\Delta$, accounting for multiple reflections, is
\[ \Delta = \frac{|T_+| e^{-\alpha_+ d/2} - |T_-| e^{-\alpha_- d/2}}{|T_+| e^{-\alpha_+ d/2} + |T_-| e^{-\alpha_- d/2}}. \]  

(72)

Similar to the decomposition (Eq. (64)) for the Faraday rotation \( \theta \), we have for the ellipticity

\[ \Delta = \tanh(\delta_S + \delta_T + \delta_{MR}). \]  

(73)

where

\[ \delta_S \equiv \tanh^{-1} \delta \]

\[ = \frac{\omega d}{2c} (\kappa_+ - \kappa_-) \]  

(74)

with \( \delta = \tanh \delta_S \) as the single pass ellipticity,

\[ \delta_T \equiv \frac{1}{2} \ln \left| \frac{t_{12} - t_{23}^-}{t_{12}^+ + t_{23}^+} \right| \]  

(75)

is due purely to boundary effects and is independent of the sample thickness \( d \), and

\[ \delta_{MR} \equiv \frac{1}{2} \ln \left| \frac{1 - r_{23}^- r_{12}^+ e^{2i\delta_+}}{1 - r_{23}^- r_{12}^- e^{2i\delta_-}} \right|. \]  

(76)
is due purely to multiple reflection effects.

Similar to the definition $\Theta_T$ we now define the single pass ellipticity with boundary effects, $\Delta_T$ say, as

$$\Delta_T \equiv \tanh(\delta_S + \delta_T) .$$

In Figure 12 we present a plot of $\delta$, $\Delta_T$ and $\Delta$ versus $\log \omega$. 
Figure 12. Plot of $\delta$, $\Delta_T$, and $\Delta$ versus $\log \omega$ for the Appel-Overhauser model, using the same parameters as in Figure 10.
IV. AN APPLICATION OF NULL FARADAY ROTATION AND NULL ELLIPTICITY

It is well known\textsuperscript{30,31} that the use of wedge shaped substrates for the sample eliminates the multiple reflection effects.

Figure 13. Substrate with constant thickness $d'$, $d''$ in the regions AB and CD, respectively, and wedge shaped in the region BC.

Thus, in the region BC of the sample shown in Fig. 13, $\theta$ reduces to

$$\theta = \theta + \theta_T.$$
If the frequencies $\omega$ and $\omega_c$ are adjusted so that
$\theta$ remains constant in the region BC, then we are assured
that $\theta = 0$ since $\theta$ is linearly d-dependent while $\theta_T$ is
d-independent. Hence $\theta = \theta_T$ and we obtain an experimental
determination of the transmission effect $\theta_T$. Now, in the
plane-parallel sections, AB of thickness $d'$ and CD of
thickness $d''$ for the same choice of $\omega$ and $\omega_c$, we again have
$\theta = 0$ and $\theta$ reduces to $\theta = \theta_T + \theta_{\text{MR}}$. However the value of
$\theta_T$ is known from above and may be subtracted out, giving
a measure of the multiple reflection contribution $\theta_{\text{MR}}$ for
the plane-parallel substrates of thickness $d'$ and $d''$.

It is clear that the preceding paragraphs on null
Faraday rotation are also true for null ellipticity if
we replace $\theta$ by $\tanh^{-1}\Delta$, $\theta$ by $\delta_S$, $\theta_T$ by $\delta_T$ and $\theta_{\text{MR}}$ by $\delta_{\text{MR}}$.

We explicitly point out that these applications of
null Faraday rotation and null ellipticity are independent
of the model chosen for the dielectric constants $\varepsilon_\pm$ since
every model involves the parameters $\omega$ and $\omega_c$. Thus, they
can be applied to systems other than the SiO$_2$-Si system.
V. CONCLUSIONS

We have investigated the electron inversion layer at the SiO$_2$-Si interface using both the classical Drude model and the Appel-Overhauser model.

In the case of a single pass of an electromagnetic wave, we have shown that there is one photon frequency for which null Faraday rotation is achieved and one for null ellipticity in the Drude theory. These two frequencies are always distinct but fundamentally related by being roots of a quintic equation.

For the Appel-Overhauser theory, obtaining a corresponding polynomial for the null Faraday effects has proved intractable at the present time. However, since this model reduces identically to the Drude model, there is at least one photon frequency for which null Faraday rotation is achieved. Plots of the rotation versus photon frequency are given for various values of the e-e coupling strength as measured by the parameter $\tau_e$. In the case of non-interacting electron systems, three zeroes of the rotation are found. Two of these are expected from applying the Drude model to each of the electron systems, the third zero is characteristic of the existence of two electron systems and is intermediate in value between the other two. It is also shown that the shape of the rotation versus photon frequency curves gives us a method by which we can determine
whether the electron systems are weakly, intermediate, or strongly coupled by the parameter $\tau_e$.

When boundary and multiple reflection effects are taken into account, a unique decomposition of the multiple pass rotation and ellipticity into contributions from the single pass, boundary effects, and multiple reflections effects is obtained. Plots of the multiple pass Faraday rotation are given and are found to be remarkably similar in shape to the single pass case.

Future work in this area is to include the metal layers of the MOS system. This work is in progress.
REFERENCES

5. B. D. McCombe, Far Infrared Optical and Magneto-Optical Studies of Si Space Charge Layers


TABLE 3

Useful Constants

<table>
<thead>
<tr>
<th>Physical Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron charge, $-e$</td>
<td>$-4.802 \times 10^{-10}$ esu</td>
</tr>
<tr>
<td>Electron rest mass, $m_e$</td>
<td>$9.107 \times 10^{-28}$ gms</td>
</tr>
<tr>
<td>Plank's constant, $h$</td>
<td>$6.624 \times 10^{-27}$ erg·sec = $4.136 \times 10^{-15}$ eV·sec</td>
</tr>
<tr>
<td></td>
<td>$1.054 \times 10^{-27}$ erg·sec = $6.583 \times 10^{-16}$ eV·sec</td>
</tr>
<tr>
<td>Speed of light, $c$</td>
<td>$2.998 \times 10^{10}$ cm/sec</td>
</tr>
<tr>
<td>Boltzmann's constant, $k_B$</td>
<td>$1.380 \times 10^{-16}$ erg/°K = $8.617 \times 10^{-5}$ eV/°K</td>
</tr>
<tr>
<td>Wavelength associated with 1 eV</td>
<td>$1.2396 \times 10^{-4}$ cm</td>
</tr>
<tr>
<td>Frequency associated with 1 eV</td>
<td>$2.418 \times 10^{14}$/sec</td>
</tr>
<tr>
<td>Temperature associated with 1 eV</td>
<td>$1.1605 \times 10^4$ °K</td>
</tr>
<tr>
<td>1 angstrom, $\AA$</td>
<td>$10^{-8}$ cm</td>
</tr>
<tr>
<td>1 micron, $\mu$</td>
<td>$10^{-4}$ cm</td>
</tr>
</tbody>
</table>
APPENDIX A
DETAILS OF THE DERIVATION OF THE SINGLE PASS FARADAY ROTATION AND SINGLE PASS ELLIPTICITY

Consider a monochromatic plane wave of frequency $\omega$ linearly polarized in the x direction and propagating in the z direction

$$\hat{E}_0 = E_0 e^{i \tau x},$$

where $\tau \equiv k \cdot r - \omega t$ and for convenience we assume $E_0$ real. The left and right circularly polarized components of $E_0$ are

$$\hat{E}_\pm = (\epsilon_\pm \cdot \hat{E}_0) \hat{\epsilon}_\pm$$

$$= \frac{1}{\sqrt{2}} E_0 e^{i \tau} \epsilon_\pm,$$

so that

$$\hat{E}_0 = \hat{E}_+ + \hat{E}_-.$$

Neglecting the boundary effects at $z=0$ and $z=d$, the transmitted waves $\hat{E}_t^\pm$ are given by

$$\hat{E}_t^\pm = e^{i \delta} \hat{E}_\pm.$$
with amplitudes

\[ E^t_\pm = \frac{1}{\sqrt{2}} e^{i\delta_\pm} E_0 e^{i\tau} \]

where the complex phase shift due to transversing the sample of thickness \( d \) is

\[ \delta_\pm = \frac{\omega d}{c} (n_\pm + i\kappa_\pm) \]

\[ = (\beta_\pm + i\alpha_\pm/2)d \].

Thus, the total transmitted wave is

\[ \hat{E}^t = \hat{E}^t_+ + \hat{E}^t_- \]

\[ = \hat{E}^{t*}_x + \hat{E}^{t*}_y \]

where the complex \( x \) and \( y \) components \( \hat{E}^t_x, \hat{E}^t_y \) are

\[ \hat{E}^t_x = \frac{1}{2} (e^{i\beta_+d} e^{-\alpha_+d/2} + e^{i\beta_-d} e^{-\alpha_-d/2}) E_0 e^{i\tau} \]

and

\[ \hat{E}^t_y = -\frac{i}{2} (e^{i\beta_+d} e^{-\alpha_+d/2} - e^{i\beta_-d} e^{-\alpha_-d/2}) E_0 e^{i\tau} \].
Defining the quantities

\[ \beta \equiv (\beta_+ + \beta_-) d/2 \]

and

\[ \theta \equiv (\beta_+ - \beta_-) d/2 \]

\( \tilde{E}_x \) and \( \tilde{E}_y \) may be written as

\[
\tilde{E}_x = \frac{i}{2} (e^{i\theta} e^{-\alpha d/2} + e^{-i\theta} e^{\alpha d/2}) E_0 e^{i(\tau + \beta)},
\]

\[
\tilde{E}_y = -\frac{i}{2} (e^{i\theta} e^{-\alpha d/2} - e^{-i\theta} e^{\alpha d/2}) E_0 e^{i(\tau + \beta)}.
\]

Expanding the complex exponentials and taking the real parts to obtain the physical wave, we obtain

\[
E_x \equiv \text{Re} \tilde{E}_x = \frac{1}{2} \{ (e^{-\alpha d/2} + e^{-\alpha d/2}) \cos \theta \cos(\tau + \beta) \\
- (e^{-\alpha d/2} - e^{-\alpha d/2}) \sin \theta \sin(\tau + \beta) \} E_0,
\]

and
\[ E_y \equiv \Re \text{e}^t 
\]
\[
= \frac{1}{2} \left\{ (e^{\alpha_+d/2} - e^{\alpha_-d/2}) \cos \theta \sin(\tau+\beta) 
\right. \\
\left. + (e^{\alpha_+d/2} + e^{\alpha_-d/2}) \sin \theta \cos(\tau+\beta) \right\} E_0 .
\]

Defining
\[
a \equiv \frac{1}{2} (e^{\alpha_+d/2} + e^{\alpha_-d/2}) E_0 
\]
\[
b \equiv \frac{1}{2} (e^{\alpha_+d/2} - e^{\alpha_-d/2}) E_0 
\]

we can write

\[ E_x = a \cos \theta \cos(\tau+\beta) - b \sin \theta \sin(\tau+\beta) \]
\[ E_y = a \sin \theta \cos(\tau+\beta) + b \cos \theta \sin(\tau+\beta) \]

We now define \( E'_x, E'_y \) by

\[ E'_x \equiv E_x \cos \theta + E_y \sin \theta \]
\[ E'_y \equiv -E_x \cos \theta + E_y \sin \theta . \]

Hence
\[ E'_x = a \cos(t + \beta) \]

\[ E'_y = b \sin(t + \beta) \]

and therefore

\[ \frac{E'_x^2}{a^2} + \frac{E'_y^2}{b^2} = 1. \]

Thus, the transmitted wave is elliptically polarized with semimajor axis \( a \), semiminor axis \( b \), and inclined at an angle \( \theta \) with respect to the positive \( x \)-axis with \( \theta \) positive in the counterclockwise direction. Collecting these results, we have

\[ \theta = \frac{\omega d}{2c} (n_+ - n_-) \]  \hspace{1cm} (A.1)

is the single pass Faraday rotation and

\[ \delta = e^{-\alpha_+ d/2} \frac{e^{-\alpha_- d/2}}{e^{\alpha_+ d/2} + e^{-\alpha_- d/2}} \]

\[ = \tanh \left[ \frac{\omega d}{2c} (\kappa_- - \kappa_+) \right] \]  \hspace{1cm} (A.2)

is the single pass ellipticity defined as the ratio of the semiminor axis to the semimajor axis.
We have proved a special case of a more general theorem: If the transmitted wave is

\[ \hat{E}_t^{\pm} = \hat{\tau}_ \pm \hat{E}_0 \]

then the Faraday rotation \( \Theta \) and ellipticity \( \Delta \) are

\[ \Theta = - \frac{1}{2} \arg(\tau_- / \tau_+) \quad (A.3) \]

and

\[ \Delta = \frac{1 - \tau^-}{1 + \tau^-} \quad (A.4) \]

where

\[ \frac{\tau_-}{\tau_+} = r e^{-2i\Theta} \quad (A.5) \]
For the sake of definition, we define an electromagnetic field in vacuo as the pair of vectors $\hat{\mathbf{E}},\hat{\mathbf{B}}$, called the electric vector and magnetic induction vector, respectively.

In order to describe the field in the presence of a dielectric medium, we must define a second set of vectors, viz. $\mathbf{J}$ the electric current density, $\mathbf{D}$ the electric displacement current, and $\mathbf{H}$ the magnetic vector.

These five vectors are related by a set of four simultaneous partial differential equations called Maxwell's equations, which in Gaussian units are

\begin{align*}
\nabla \times \mathbf{H} - \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} &= \frac{4\pi}{c} \mathbf{J}, \quad (B.1) \\
\nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} &= 0, \quad (B.2) \\
\n\nabla \cdot \mathbf{D} &= 4\pi \rho, \quad (B.3) \\
\n\nabla \cdot \mathbf{E} &= 0. \quad (B.4)
\end{align*}

Eqs. (B.1) and (B.2) describe the time evolution of the electromagnetic field while Eqs. (B.3) and (B.4) describe boundary conditions. Eq. (B.3) is regarded as the
defining equation for the electric charge density $\rho$ and Eq. (B.4) implies that no free magnetic monopoles exist.

Eqs. (B.1)-(B.4) relate the five basic vectors $\vec{E}$, $\vec{H}$, $\vec{D}$, $\vec{B}$ and $\vec{J}$ defining the electromagnetic field. To have a unique determination of the field, we must add to these equations relations which describe the behavior of the dielectric under the influence of the field. These relations are known as the constitutive relations:

$$\vec{J} = \sigma^\uparrow \vec{E}$$  \hspace{1cm} (B.5)

$$\vec{D} = \varepsilon^\uparrow \vec{E}$$  \hspace{1cm} (B.6)

$$\vec{B} = \mu^\uparrow \vec{H}$$  \hspace{1cm} (B.7)

where $\sigma^\uparrow$ is the conductivity tensor, $\varepsilon^\uparrow$ is the lattice dielectric tensor, and $\mu^\uparrow$ is the magnetic permeability tensor. We shall consider only non-magnetic media for which $\mu^\uparrow$ is equal to $\mathbb{1}$, the unit tensor, so that $\vec{B} = \vec{H}$.

Using the constitutive relations and Eq. (B.2) to eliminate $\vec{H}$, Eq. (B.1) may be written

$$\nabla^2 \vec{E} - \nabla (\nabla \cdot \vec{E}) = \frac{1}{c^2} \varepsilon^\uparrow \frac{\partial^2 \vec{E}}{\partial t^2} + \frac{4\pi}{c^2} \sigma^\uparrow \cdot \frac{\partial \vec{E}}{\partial t}.$$  \hspace{1cm} (B.8)

Assuming a plane wave solution of the form
\[ \vec{E} = \vec{E}_0 \ e^{i(\vec{k} \cdot \vec{r} - \omega t)}, \quad (B.9) \]

Eq. (B.8) becomes

\[ \kappa^2 \vec{E} - \vec{k}(\vec{k} \cdot \vec{E}) = \frac{\omega}{c^2} \vec{\epsilon} \cdot \vec{E}, \quad (B.10) \]

where

\[ \vec{\epsilon} \equiv \epsilon_{\parallel} + i \frac{4\pi}{\omega} \vec{\sigma} \quad (B.11) \]

is the dielectric tensor of the lattice plus a system of free carriers.

For a medium whose crystalline structure is of the cubic class $\epsilon_{\parallel}$ is diagonal,\(^3^2\) ie.

\[ \epsilon_{\parallel} = \epsilon_{\parallel} \quad (B.11) \]

where $\epsilon_{\parallel}$ is a scalar, and since we are restricting our attention to intraband effects, $\epsilon_{\parallel}$ may be taken as real.

For a cubic semiconductor in a uniform constant external magnetic field $\vec{B}$ which coincides with one of the crystalline axis, \([001]\), \([111]\), or \([011]\), the conductivity tensor will have the form\(^3^3\)
\[ \hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \]  \hspace{1cm} (B.12)

\( \sigma_{zz} \) is the conductivity at zero magnetic field. For the directions \([001]\) and \([111]\), \( \sigma_{xx} = \sigma_{yy} \).

From the equation

\[ \det(\hat{\sigma} - \lambda) = 0 \]

we obtain the eigenvalues \( \lambda \) of \( \hat{\sigma} \) as

\[ \lambda = \frac{1}{2} (\sigma_{xx} + \sigma_{yy}) \pm i\sqrt{\frac{1}{4}(\sigma_{xx} - \sigma_{yy})^2} \]  \hspace{1cm} (B.13)

We will now assume that \( \hat{B} = B\hat{z}, B > 0 \) and hence the crystalline axis \([001]\), so that \( \sigma_{xx} = \sigma_{yy} \). Under this assumption the eigenvalues \( \lambda \) simplify, which we denote simply by \( \sigma_{\pm} \), to

\[ \sigma_{\pm} = \sigma_{xx} \pm i\sigma_{xy} \]  \hspace{1cm} (B.14)

\( \sigma_{\pm} \) corresponds to left and right circularly polarized waves.

Eq. (B.12) gives the form of \( \hat{\sigma} \) with respect to the set of unit base vectors \( \{\hat{x}, \hat{y}, \hat{z}\} \). If we transform to
the equivalent set

\[ \{ \epsilon_\pm, z \} , \]

where

\[ \epsilon_\pm \equiv (\hat{x} \pm i\hat{y})/\sqrt{2} , \]

\( \sigma \) is diagonalized and has the form

\[ \sigma = \begin{pmatrix} \sigma_\pm & 0 & 0 \\ 0 & \sigma_\pm & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} . \]  \hspace{1cm} \text{(B.16)}

Considering Eqs. (B.11) and (B.12), it is clear that the form of \( \sigma \) is

\[ \sigma = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ -\epsilon_{xy} & \epsilon_{xx} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix} . \]  \hspace{1cm} \text{(B.17)}

and has the eigenvalues
\[ \epsilon_{\pm} = \epsilon_{xx} \pm i\epsilon_{xy}, \]
\[ = \epsilon_{\parallel} + i \frac{4\pi}{\omega} \sigma_{\parallel}, \quad (B.18) \]

and

\[ \epsilon_{zz} = \epsilon_{\parallel} + i \frac{4\pi}{\omega} \sigma_{zz}, \]

with respect to the base set \{\epsilon_{\parallel}, z\}, \epsilon becomes

\[ \epsilon_{\rightarrow} = \begin{pmatrix}
\epsilon_{\parallel} & 0 & 0 \\
0 & \epsilon_{\parallel} & 0 \\
0 & 0 & \epsilon_{zz}
\end{pmatrix}. \quad (B.19) \]

Thus, if the plane wave, Eq. (B.9), is either right or left circularly polarized and propagating along \( \hat{B} = \hat{B}_z \), so that \( \hat{E}, \hat{k} \), then Eq. (B.10) becomes

\[ k_{\pm}^2 = \frac{\omega^2}{c^2} \epsilon_{\pm}. \quad (B.20) \]

The wave number \( k \) is related to the complex refractive index \( n+ik \) by
\[ k = \frac{\omega}{c} (n + i\kappa) \]
\[ \equiv \beta + i\alpha/2 \]  \hspace{1cm} (B.21)

where

\[ \beta \equiv \frac{\omega}{c} n , \]  \hspace{1cm} (B.22)
\[ \frac{\alpha}{2} \equiv \frac{\omega}{c} \kappa . \]  \hspace{1cm} (B.23)

\( n \) is the real refractive index and is related to the phase velocity, \( v_p \) in the medium by \( v_p = c/n \). \( \kappa \) is the extinction coefficient and is related to the absorption coefficient \( \alpha \) by \( \alpha = 2\omega\kappa/c \). Hence, from Eq. (B.20), we have

\[ \varepsilon_\pm = (n_\pm + i\kappa_\pm)^2 . \]  \hspace{1cm} (B.24)
Here we give the details of the derivation of the Drude model for the dielectric constants \( \varepsilon_\pm \). The harmonic bounding term \( \omega_0 \) associated to interband effects is taken non-zero. The electrons, with charge \( q = -e \) and an effective mass which accounts for the background lattice, are driven by either a left or right circularly polarized electromagnetic wave \((\mathbf{E}_\perp, \mathbf{H}_\perp)\) propagating in the \( \hat{z} \) direction and a uniform constant external magnetic field \( \mathbf{B} = B\hat{z}, B > 0 \). The equation of motion is

\[
m\ddot{\mathbf{r}} + m\dot{\mathbf{v}} + m\omega_0^2\mathbf{r} = -e(\mathbf{E}_\perp + \frac{\mathbf{H}_\perp}{c} \times \mathbf{H}_\perp) - e\frac{\mathbf{H}_\perp}{c} \times \mathbf{B}
\]

(C.1)

where \( c \) is the speed of light in vacuo, \( \nu \) is the collision frequency, \( \mathbf{r} \) is the position vector. At this point, most authors\(^{34-37}\) drop the term due to the magnetic component \( \mathbf{H} \) of the wave since its magnitude is \( \nu/c \) times the electric component term. However, we shall retain this term and show that under the above restrictions it does not contribute to the dielectric constants. We shall discuss this point in more detail at the end of the derivation.
If we set

\[ \hat{E}_\omega(x, t) = E_\omega(x, t) e^{\pm i \omega t}, \]

then \( \hat{H}_\omega(x, t) \) is given by

\[ \hat{H}_\omega(x, t) = \pm i H_\omega(x, t) e^{\pm i \omega t}. \] (C.3)

where

\[ H_\omega(x, t) = \sqrt{\varepsilon} E_\omega(x, t) \]

Assuming a solution of the form \( \vec{r} = \vec{r}_0 e^{-i \omega t} \), Eq. (C.1) becomes

\[ (-\omega^2 - i \omega v + \omega_o^2) \vec{r} = -\frac{e}{m^*} \vec{E}_\omega + i \frac{e\omega}{m^* c} \vec{r} \times \vec{H}_\omega + i \frac{e\omega}{m^* c} \vec{r} \times \vec{B}. \] (C.4)

Expanding the cross products and separating into scalar equations gives

\[
\begin{bmatrix}
-\omega^2 - i \omega v + \omega_o^2 & -i \omega c y & +i \omega_H z / \sqrt{2} & = \frac{e}{\sqrt{2m^*}} E_\omega \\
i \omega c x & +(-\omega^2 - i \omega v + \omega_o^2) y & +i \omega_H z / \sqrt{2} & = \frac{i e}{\sqrt{2m^*}} E_\omega \\
i \omega_H x / \sqrt{2} & +i \omega_H y / \sqrt{2} & +(-\omega^2 - i \omega v + \omega_o^2) z & = 0
\end{bmatrix}
\] (C.5)
where we have define

\[ \omega_c \equiv \frac{eB}{m^*c}, \]  

(C.6)

and

\[ \omega_H \equiv \frac{eH}{m^*c}. \]  

(C.7)

Solving these for \( \hat{r} \), we obtain

\[ \hat{r} = \frac{e/m^*}{\omega(\omega + \omega_c + i\nu) - \omega_0^2} \hat{E} . \]  

(C.8)

Hence, the dipole moment contributed by one electron is

\[ \hat{p} = -e\hat{r} = \frac{-e^2/m^*}{\omega(\omega + \omega_c + i\nu) - \omega_0^2} \hat{E} . \]  

(C.9)

If \( n \) is the electron density, then the total dipole moment per unit volume \( \hat{p} \) is

\[ \hat{p} = n\hat{p} = \frac{ne^2/m^*}{-\omega(\omega + \omega_c + i\nu) + \omega_0^2} \hat{E} . \]  

(C.10)
Hence, using \( \mathbf{J} = \frac{\partial \mathbf{P}}{\partial t} \), we obtain

\[
\sigma_{\pm} = \frac{-i\omega n e^2/m^*}{-\omega(\omega \pm \omega_c + i\nu) + \omega_o^2},
\]

and therefore, using Eq. (B.18)

\[
\varepsilon_{\pm} = \varepsilon_\perp \left\{ 1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_c + i\nu) - \omega_o^2} \right\},
\]

where

\[
\omega_p^2 = \frac{4\pi n e^2}{m^* \varepsilon_\perp}.
\]

Setting \( \omega_o = 0 \), since we are only interested in intraband effects, we obtain the Drude model for a system of free carriers:

\[
\varepsilon_{\pm} = \varepsilon_\perp \left\{ 1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_c + i\nu)} \right\}.
\]

In order to point out the assumptions made in deriving the conductivity tensor from the classical equation of motion Eq. (C.1) we here give the essential equations in the derivation of the conductivity tensor from the Boltzmann transport equation and explicitly list the assumptions made. For full details of the derivation, we refer the reader to Blatt or Palik and Furdyna.
The Boltzmann equation is

\[
\frac{\partial \tilde{p}}{\partial t} \cdot \nabla_\tilde{p} f + \tilde{v} \cdot \nabla_\tilde{r} f + \frac{\partial f}{\partial t} = (\frac{\partial f}{\partial t})_{\text{collision}}
\]  \hspace{1cm} (C.15)

where \(\nabla_\tilde{p}\) and \(\nabla_\tilde{r}\) are the gradient operators in momentum and coordinate space, respectively. \(\tilde{v}\) is the carrier velocity and \(f(p, v, t, E, H, B)\) is the carrier distribution function under the applied external fields, \(E\), \(H\) and \(B\).

We now make the following assumptions:

(a) We restrict ourselves to a single energy band with an isotropic effective mass \(m^*\) and described by the energy-momentum relation

\[
E = \frac{|\tilde{p}|^2}{2m^*}
\]

where \(E\) is the carrier energy and \(\tilde{p}\) its momentum. The carrier velocity \(\tilde{v}\) is given by \(\nabla_\tilde{p} E\).

(b) The external fields are sufficiently weak so that

\[
f = f_0 + f_1
\]  \hspace{1cm} (C.16)

where

\[f_1 \ll f_0\]
and

\[ f_0 = \{1 + \exp[(E-E_F)/k_B T]\}^{-1} \]

is the Fermi-Dirac distribution which describes the carrier system when no external fields are applied. \( E_F \) is the Fermi energy and \( k_B T \) the thermal energy. \( f_1 \) is the perturbation due to the non-zero external fields.

(c) The background lattice is taken into account by the relaxation-time approximation defined by

\[ \frac{\partial f}{\partial t}_{\text{collision}} = - \frac{f_1}{\tau} . \]  

(C.17)

Under the assumptions a, b, c, Eq. (C.15) can now be written as

\[-e(\dot{E} + \frac{\dot{V}}{c} x \hat{H} + \frac{\dot{V}}{c} x \hat{B}) \cdot \nabla_p f + \vec{V} \cdot \nabla_r f = - \frac{f_1}{\tau} . \]  

(C.18)

We now assume:

(d) The magnetic component \( \hat{H} \) of the electromagnetic field is neglected.

(e) The system response \( f_1 \), due to the external fields, varies as \( \exp[i(\vec{k} \cdot \vec{r} - \omega t)] \).
(f) We retain only the smallest non-zero contribution of each term in Eq. (C.18), viz.

\[-e\hat{E} \cdot \nabla \phi = -e\hat{\nabla} \cdot \hat{E} \frac{\partial \phi^0}{\partial E},\]

\[-e(\hat{\nabla} \times \hat{B}) \cdot \nabla \phi = -e(\hat{\nabla} \times \hat{B}) \cdot \nabla \phi^1,\]

\[\frac{\partial \phi}{\partial t} = -i\omega \phi^1,\]

\[\hat{\nabla} \cdot \nabla \phi = i(\hat{\nabla} \cdot \hat{k}) \phi^1.\]

(g) The term due to \(\nabla \phi^1\) is neglected compared with \(\partial \phi/\partial t\), i.e. \(|\nabla \cdot \hat{k}| \ll \omega\). If \(\phi\) is the angle between \(\hat{v}\) and \(\hat{k}\), and \(\lambda\) the wavelength of the electromagnetic wave, this condition becomes \(|v_k \cos \theta| \ll v \ll \omega\) so that \(2\pi v/\lambda \ll \omega\) or \(v \ll \omega\).

Under the assumptions d, e, f, g, Eq. (C.18) becomes

\[-e(\hat{\nabla} \times \hat{B}) \cdot \nabla \phi^1 + f_1 \hat{\nabla} \cdot \hat{E} \frac{\partial \phi^0}{\partial E} = -\frac{f_1}{\tau_\omega} \quad \text{(C.19)}\]

where

\[\tau_\omega = \frac{1}{1-i\omega \tau}.\]
(h) We assume a solution of Eq. (C.19) of the form

\[ f_1 = f - f_0 = -\nabla \cdot \psi \frac{\partial f_0}{\partial \mathbf{E}} \]  

(C.20)

where \( \psi \) is the unknown to be determined. Thus Eq. (C.19) becomes

\[ \psi + \frac{e \tau}{m^* c} \mathbf{B} \times \psi = e \tau \mathbf{E} . \]  

(C.21)

Using Eq. (C.21), its dot product with \( \mathbf{B} \), and its cross product with \( \mathbf{B} \), we can solve for \( \psi \) and obtain

\[
\begin{align*}
\psi &= \frac{-e \tau [\hat{\mathbf{E}} + \frac{e \tau}{m^* c} \mathbf{B} \times \mathbf{E} + \left(\frac{e \tau}{m^* c}\right)^2 \mathbf{B} (\mathbf{B} \cdot \mathbf{E})]}{1 + \frac{e \tau}{m^* c} \mathbf{B}^2} 
\end{align*}
\]

(C.22)

where \( \mathbf{B} = |\mathbf{B}| \). Thus we obtain the current \( \mathbf{j} \) as

\[ \mathbf{j} = -\frac{2e}{\hbar^3} \int \mathbf{v} f \, d^3p . \]  

(C.23)

(i) Assuming \( \tau \) is an even function of energy only, we obtain \( \mathbf{j} \) in the form

\[ \mathbf{j} = c_1 \mathbf{E} + c_2 (\mathbf{B} \cdot \mathbf{E}) \mathbf{B} + c_3 \mathbf{E} \times \mathbf{B} \]

and hence the conductivity tensor \( \mathbf{\sigma} \) in component form
\begin{equation}
\sigma_{ij} = C_1 \delta_{ij} + C_2 B_i B_j + C_3 \epsilon_{ijk} B_k
\end{equation}

where \(i,j,k = 1,2,3\) (or \(x,y,z\)), \(\delta_{ij}\) is the Kronecker symbol, \(\epsilon_{ijk}\) the permutation tensor with \(\epsilon_{123} = 1\), and

\[
C_1 = -\frac{e^2}{3\pi m^*} \left(\frac{2m^*}{\mu^2}\right)^{3/2} \int \frac{\tau_{\omega}}{1+\omega^2 c^2} \frac{E^{3/2}}{B^2} \frac{\partial f_0}{\partial E} dE
\]

\[
C_2 = -\frac{e^2}{3\pi m^*} \left(\frac{2m^*}{\mu^2}\right)^{3/2} \frac{1}{B^2} \int \frac{\omega c^2}{1+\omega^2 c^2} \frac{E^{3/2}}{c^2} \frac{\partial f_0}{\partial E} dE
\]

\[
C_3 = -\frac{e^2}{3\pi m^*} \left(\frac{2m^*}{\mu^2}\right)^{3/2} \frac{1}{B^2} \int \frac{-\omega c^2}{1+\omega^2 c^2} \frac{E^{3/2}}{c^2} \frac{\partial f_0}{\partial E} dE
\]

To evaluate the integrals in Eq. (C.25), we assume finally

(j) \(T = 0^\circ K\) so that

\[
\frac{\partial f_0}{\partial E} = -\delta (E-E_F)
\]

Hence, Eq. (C.24) becomes
where

\[ n = \frac{2}{h^3} \int f d^3 p \]

is the carrier density.

Setting \( \mathbf{B} = B z \), \( B > 0 \) and calculating \( \sigma_\pm = \sigma_{xx} \pm i \sigma_{xy} \), we obtain Eq. (C.11) with \( \omega_o = 0 \).

Thus, calculating the conductivity tensor from the classical equation of motion Eq. (C.1) incorporates the assumptions a-j listed above.

The assumptions (a), (d) and (g) are non-relativistic in nature. If an electron at rest absorbs a 1 eV photon, which is the maximum photon energy of which we are concerned, its velocity is \( v = 5.931 \times 10^7 \) cm/sec. Thus \( v/c = 1.98 \times 10^{-3} \) and hence relativistic effects are negligible. Condition (d), neglection of the magnetic component \( \mathbf{H} \) of the electromagnetic wave, is thus shown to be non-trivial as it was in Eq. (C.1). In fact, Bloembergen \(^{39} \) has shown that this term leads to the generation of higher harmonics in non-linear optics.

Assumptions (b), (e) and (f) are related to the strength of the external fields. These conditions are
satisfied since the photon energies which we are concerned with (E<1 eV) are not strong enough to produce interband transitions.

Assumptions (c) and (i) are valid if the change in energy of an electron per collision with the lattice is small compared with the thermal energy $k_B T$. For a 1 eV electron colliding with an isolated Si atom at rest the change in electron energy is $\Delta E = 3.5 \times 10^{-15}$ eV. At $1^\circ K$, $k_B T = 8.617 \times 10^{-5}$ eV. Thus, these conditions are satisfied since the experiments are performed at 1-4° K, eg. Fowler et al. used $T = 1.34^\circ K$. Hence, we also have assumption (j) approximately satisfied.

Finally, assumption (h) is a mathematical change of variables, and hence holds no physical content.
APPENDIX D
CYCLOTRON RESONANCE

Consider a plane monochromatic electromagnetic wave, propagating in the \( \hat{z} \) direction in a dielectric, whose electric vector is given by

\[
\hat{E} = E_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \hat{x}.
\]

The time-averaged power dissipated per unit volume is

\[
P = \frac{1}{2} \text{Re} <\mathbf{J} \cdot \hat{E}^*>
\]

\[
= \frac{1}{2} \text{Re} \sigma_{xx} |E_0|^2.
\]

From Eq. (B.14) we have

\[
\sigma_{xx} = \frac{1}{2} (\sigma_+ + \sigma_-)
\]

and using Eqs. (C.11) and (C.13), Eq. (D.1) becomes

\[
P = \frac{\omega^2 p}{8\pi} \frac{[1+(\omega^2 - \omega_c^2)\tau^2] \cdot |E_0|^2}{[1+(\omega^2 - \omega_c^2)\tau^2]^2 + 4\omega^2 c^2 \tau^2}
\]

The relative power \( P_r \) dissipated is defined by
\[ P_r = \frac{P}{P_0} = \frac{1 + (\omega^2 + \omega_c^2)\tau^2}{[1 + (\omega^2 - \omega_c^2)\tau^2]^2 + 4\omega_c^2\tau^2} \]  

where \( P_0 \) is the power dissipated when \( \omega = \omega_c = 0 \).

A resonance peak is said to exist if the graph of \( P_r \) versus \( \omega \) has a relative maximum. Setting

\[ \frac{dP_r}{d\omega} = 0 \]

we find that the frequency \( \omega \) where the resonance peak occurs is a solution of

\[ \omega^2 \tau^2 = \omega_c^2 \tau^2 - [(1 + \omega_c^2 \tau^2)^{1/2} - \omega_c \tau]^2 \]  

Thus, the condition to have a non-trivial resonance peak (i.e. \( \omega > 0 \)) is

\[ \omega_c \tau > \sqrt{3}/3 \approx 0.577 \]  

For the resonance peak to occur at \( \omega = \omega_c \) it is clear that

\[ \omega_c \tau \gg 1 \]
Here we prove that the quintic \( f(x) = 0 \) given in Eq. (17) has three real roots. We find it convenient to rewrite \( f(x) = 0 \) as a quintic in \( y = \omega^2 \) as follows:

\[
g(y) = 4y^5 + [8(\nu^2 - \omega_c^2) - 3\omega_p^2]y^4 - 8\omega_p^2(\nu^2 - \omega_c^2)y^3
\]

\[
-2(\omega_c^2 + \nu^2)^2[4(\nu^2 - \omega_c^2) + 3\omega_p^2]y^2
\]

\[
-4(\omega_c^2 + \nu^2)^4y + \omega_p^2(\omega_c^2 + \nu^2)^4 = 0 , \quad (E.1)
\]

hence

\[
f(x) = \Omega^{-10} g(y) .
\]

Therefore \( f(x) = 0 \) if and only if \( g(y) = 0 \).

Corresponding to Eqs. (20)-(24), we have

\[
g(\pm \infty) = \pm \infty , \quad (E.2)
\]

\[
g(0) = \omega_p^2 \Omega^8 > 0 , \quad (E.3)
\]

\[
g(\Omega^2) = -16\nu^2 \omega_p^2 \Omega^6 < 0 , \quad (E.4)
\]
For fixed \( \omega_c, v \) and \( \omega_p \), we see by inspection of Eqs. (E.2)-(E.5) that \( f(x) \) has at least three real roots, two positive roots and one negative root in the intervals \((1, \infty)\), \((0, 1)\) and \((-1, 0)\), respectively.

To prove that these are the only three real roots, we apply Descarte's Rule of Signs stated as follows:

**Theorem.** The number of positive real roots of \( P(x) = 0 \), where \( P(x) \) is a polynomial with real coefficients, is equal to the number of variations in sign occurring in \( P(x) \), or else is less than this number by an even integer. The number of negative real roots of \( P(x) \) is equal to the number of variations in sign occurring in \( P(-x) \), or else is less than this number by an even integer.

In Table 4 we list relations among \(-3\omega_p^2, 3\omega_p^2, 4(v^2-\omega_c^2)\) and \(8(v^2-\omega_c^2)\) which gives all possible sign combinations for the coefficients in \( g(y) \), the signs of these coefficients, and the possible number of real positive and negative roots.
TABLE 4

<table>
<thead>
<tr>
<th>Case</th>
<th>Signs of Coefficients of g(y)</th>
<th>Possible Number of Positive Roots</th>
<th>Possible Number of Negative Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3u^2_p &lt; 0 &lt; 3u^2_p &lt; 4(v^2 - w^2_p) &lt; 8(v^2 - w^2_c)</td>
<td>++++++</td>
<td>2,0</td>
</tr>
<tr>
<td>2</td>
<td>-3u^2_p &lt; 0 &lt; 4(v^2 - w^2_p) &lt; 3u^2_p &lt; 8(v^2 - w^2_c)</td>
<td>+0+++</td>
<td>2,0</td>
</tr>
<tr>
<td>3</td>
<td>-3u^2_p &lt; 0 &lt; 4(v^2 - w^2_p) &lt; 3u^2_p &lt; 8(v^2 - w^2_c)</td>
<td>++++++</td>
<td>2,0</td>
</tr>
<tr>
<td>4</td>
<td>-3u^2_p &lt; 0 &lt; 4(v^2 - w^2_p) &lt; 8(v^2 - w^2_c) &lt; 3u^2_p</td>
<td>++0++</td>
<td>2,0</td>
</tr>
<tr>
<td>5</td>
<td>-3u^2_p &lt; 0 &lt; 4(v^2 - w^2_p) &lt; 8(v^2 - w^2_c) &lt; 3u^2_p</td>
<td>++++++</td>
<td>4,2,0</td>
</tr>
<tr>
<td>6</td>
<td>8(v^2 - w^2_p) &lt; -3u^2_p &lt; 4(v^2 - w^2_c) &lt; 3u^2_p</td>
<td>++0++</td>
<td>4,2,0</td>
</tr>
<tr>
<td>7</td>
<td>8(v^2 - w^2_p) &lt; 4(v^2 - w^2_c) &lt; -3u^2_p &lt; 3u^2_p</td>
<td>++++++</td>
<td>4,2,0</td>
</tr>
</tbody>
</table>

Table 4. Possible real roots of the quintic equation g(y) = 0, or equivalently f(x) = 0.
Computer analysis shows that the discriminant in all cases is negative. Hence, for every case $f(x) = 0$ has two complex and thus three real roots.
VITA

Gary Lynn Wallace was born in Roanoke, Virginia on April 4, 1948. He attended Roland E. Cook Elementary School and was graduated from William Byrd High School in August 1967. From 1967 to 1972 he attended the Florida Institute of Technology in Melbourne, Florida and received a Bachelor of Science degree in Physics in June, 1972, a Master of Science degree in Mathematics in September, 1972, and a Master of Science degree in Physics in June, 1974. From 1972 to 1975 he was on the faculty of the Department of Mathematical Sciences at the Florida Institute of Technology. From 1975 to 1977 he was enrolled in the Department of Mathematical Sciences at the New Mexico State University in Las Cruces, New Mexico. He enrolled in 1977 as a graduate student in the Department of Physics and Astronomy at Louisiana State University where he is presently a candidate for the Doctor of Philosophy degree in Physics.
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Major Field: Physics

Title of Thesis: Faraday Rotation Investigations of Surface Space-Charge Layers in Metal-Oxide-Semiconductor (MOS) Systems

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