Control Algorithms, Tuning, and Adaptive Gain Tuning in Process Control.

Alberto Arner Rovira
Louisiana State University and Agricultural & Mechanical College

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CONTROL ALGORITHMS, TUNING, AND ADAPTIVE
GAIN TUNING IN PROCESS CONTROL

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Chemical Engineering

by

Alberto Arner Rovira
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M.S., Louisiana State University, 1966
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ABSTRACT

Controller tuning relationships based on optimizing the response of a first order lag plus dead time process are developed for proportional-plus-integral (PI) and proportional-plus-integral-plus-derivative (PID) control algorithms. Minimum error integrals were used as criteria of performance for the feedback control loop. The relationships presented are shown to provide excellent response characteristics in particular for processes where little or no overshoot is desired.

The effect of applying the proportional action to the feedback variable in a PI control algorithm is analyzed and found to provide a more consistent response to set point changes and load changes than the conventional PI algorithm, tuned for either set point or load changes.

A technique of adaptive gain tuning for a PI controller is developed. The method is based on the use of sensitivity coefficient analysis to the identification of model parameters. The model chosen is a second order lag, and the technique is applied to a stirred tank chemical reactor temperature control system. The response of the proposed adaptive gain tuning technique proved to be superior to the response of the unadapted algorithm. Digital computers were used to simulate the control systems studied.
CHAPTER I

INTRODUCTION

The chemical industry has always been one of the most competitive and profitable sectors of our economy. In recent years the effect of high energy costs and shortages, foreign competition, and environmental and safety regulations have contributed to a significant decrease in profit potential. As a result of these pressures, improved design, operation and regulation of processes has become of utmost importance to chemical engineers.

The proliferation of process control computers and more recently the introduction of microprocessors have enhanced the capability of process and control engineers to study, develop and implement new and more sophisticated control techniques to achieve improved performance in the operation of process plants.

Recently, one of the most active research in the field of process control has been in the area of feedback control systems tuning, including related areas like adaptive, direct digital and nonlinear control. It is the purpose of this work to present and evaluate several techniques in the area of control algorithm tuning and adaptive gain tuning that may be applied to improve the performance of control loops in process plants. Chapter II is a comparative study of selected control algorithms with major emphasis in the sensitivity to parameter settings and their response performance to both set point and load (disturbance) changes. Most controller tuning relationships are based on optimizing...
the responses to load changes. In Chapter II significant dif-
ferences were found to exist between optimal tuning parameters for
set point vs. load changes. The advantages and disadvantages of both
types of tuning approaches are discussed in Chapter III. Tuning
relationships for proportional plus integral (PI) and proportional plus
integral plus derivative (PID) algorithms based on optimal set point
change response are presented for a first order lag plus dead time model.

A technique used to avoid fast rising and overshooting res-
ponses to set point changes is to apply the proportional action of a PI
or PID controller to the controlled variable as opposed to the error
signal which is a more conventional approach. Chapter IV presents a
comparison of both methods and discusses the response characteristics to
be considered in selecting one of them. Tuning relationships to be used
are also discussed.

Most chemical processes are of a nonlinear nature. As levels
of operation and/or process parameters change, controller parameters
have to be adjusted or retuned to maintain acceptable behavior of the
process. Chapters V and VI present techniques to compensate for these
nonlinearities. The first is a simple automatic tuning technique in which
a nonlinear process is characterized by a linear model at different
operating conditions. Tuning relationships for the model are then used
to adjust controller settings as variations in process conditions occur.

Chapter VI demonstrates an adaptive gain tuning technique for
automatically adjusting the gain of a PI or PID controller. It is based
on using parametric sensitivity theory to the identification of model
parameters. The method was applied to a stirred tank reactor control
system and its performance compared to the unadapted behavior of the system.

The results presented in this work were obtained by numerical simulation in a digital computer. The main programs used are included in the Appendix.
CHAPTER II
STUDY OF CONTROL ALGORITHMS: LINEAR AND NONLINEAR

Introduction
Many different types of algorithms and tuning techniques have been proposed in the literature (1-8) for conventional analog control and for direct digital control of processes. The ones most commonly used are the proportional, proportional plus integral and proportional plus integral plus derivative. These algorithms are very effective, linear, and easily implemented in analog hardware; which accounts for their popularity. With the introduction of the digital computer as a direct process controller a variety of control functions may easily be implemented, which previously had not been tried, in the hope of improving the control performance of the system. It is the purpose of this chapter to investigate and compare different algorithms which have been selected on the basis of their attractiveness and potential. The responses were analyzed for disturbance and set point changes. The effect of sampling time as well as the sensitivity to tuning parameters were also considered.

Control Loop
The algorithms were tested as part of a direct digital control loop as the one shown in Figure II-1. In this simplified unity feedback loop the controlled or feedback variable is fed into
Figure II-1. Direct Digital Control Loop
the computer at regular sampling intervals. The computer compares this feedback variable with the set point input producing an error signal. A control algorithm will calculate the new value of the manipulated variable. This position of the manipulated variable is held constant between sampling periods by a zero order hold. The transfer function of the zero order hold is written as:

\[ H(s) = \frac{1 - e^{-Ts}}{s} \]  

where: 
H(s) = zero order hold transfer function
T = sampling time
s = Laplace transform variable

The process model used is the familiar first order lag plus dead time which may be written as:

\[ G(s) = \frac{Ke^{-\theta_0 s}}{\tau s + 1} \]  

where: 
G(s) = process transfer function
K = process gain
\theta_0 = dead time
\tau = time constant

**Algorithms Considered**

The algorithms studied may be divided into two groups:

a) Algorithms which show steady state offset.

b) Algorithms which show no steady state offset.

The main difference between these two groups is that the
algorithms which do not exhibit an offset have an integral element which eliminates the steady state error.

The algorithms which show steady state offset considered are:

\[ m_n = K_c e_n \] \hspace{1cm} \text{proportional} \hspace{1cm} (II-3)

\[ m_n = K_c |e_n| e_n \] \hspace{1cm} \text{proportional squared} \hspace{1cm} (II-4)

\[ m_n = K_1 e_n + K_2 |e_n| e_n \] \hspace{1cm} \text{proportional plus proportional squared} \hspace{1cm} (II-5)

\[ m_n = K_1 e_n + K_2 m_{n-1} \] \hspace{1cm} \text{two parameter} \hspace{1cm} (II-6)

where: \( e_n \) = error at sampling interval \( n \)

\( m_n \) = manipulated variable at sampling interval \( n \)

\( K_i \) = controller constant

In the second group which does not show steady state offset the following algorithms were considered:

\[ m_n = K_c \left( e_n + \frac{1}{T_i} \sum e_n \right) \] \hspace{1cm} \text{PI} \hspace{1cm} (II-7)

\[ m_n = K_c \left( |e_n| e_n + \frac{1}{T_i} \sum |e_n| \right) \] \hspace{1cm} \text{P}^2 \hspace{1cm} (II-8)

\[ m_n = K_c \left( |e_n| e_n + \frac{1}{T_i} \sum |e_n| \right) \] \hspace{1cm} \text{P}^2 \text{I}^2 \hspace{1cm} (II-9)

\[ m_n = K_c \left( e_n + \frac{1}{T_i} \sum e_n + T_D \frac{(e_n - e_{n-1})}{T} \right) \] \hspace{1cm} \text{PID} \hspace{1cm} (II-10)

\[ m_n = K_1 e_n - K_2 e_{n-1} + (1 - K_3) m_{n-1} + K_3 m_{n-2} \] \hspace{1cm} \text{Mosler's} \hspace{1cm} (II-11)

\[ m_n = K_c \left( \text{sgn} e_n \sqrt{|e_n|} + \frac{1}{T_i} \sum e_n \right) \] \hspace{1cm} (II-12)
\[ m_n = K_c \left( \exp \left( \frac{|e_n|}{\tau_1} \right) e_n + \frac{1}{T_i} \sum e_n T \right) \]  \hspace{1cm} (II-13)

where:
\( T = \) sampling time
\( T_i = \) integral constant
\( T_D = \) derivative constant

**Performance Criteria**

Several authors (9,10,11) have recommended the use of error integrals as figures of merit or performance criteria in comparing control system responses. In this work the integral of the absolute value of the error (IAE) was used. Two different forms of this integral were used:

a) \[ IAE - 1 = \int_0^\infty |c - c(\infty)| \, dt \]  \hspace{1cm} (II-14)

b) \[ IAE - 2 = \int_{\theta_0}^\infty |c - c(\infty)| \, dt \]  \hspace{1cm} (II-15)

where
\( c = \) controlled variable
\( \theta_0 = \) dead time of system
\( t = \) time

The IAE \(- 2\) was used only in the study of the algorithms that show steady state offset.

**Sensitivity**

Controller parameters are very seldom tuned optimally. Lack of accuracy and approximations used in modeling the process may affect the values obtained from tuning relationships. Therefore, a quantity
which will measure the sensitivity of the response to controller tuning is desired. For purposes of this study we shall define sensitivity to controller parameters as:

\[
\frac{0.5(\text{IAE}_{a+0.05a} + \text{IAE}_{a-0.05a}) - \text{IAE}_a}{\text{IAE}_a} \times 100
\]  

(II-16)

This is the average deviation of the criterion function (IAE) when the parameter is 5% above and below its optimum value (a).

**Parameter Optimization**

The parameters of the different algorithms were optimized for a range of sampling times using a numerical Pattern Search version programmed by Moore (12). In order to simplify the characterization of the process, the model parameters were nondimensionalized. Therefore the first order lag plus dead time model may be characterized by the ratio of dead time to time constant \(\theta_o/\tau\) and the sampling time becomes the ratio of sampling time to time constant \(T/\tau\).

**Algorithms Which Show Steady State Error or Offset**

In this class the following algorithms were tested.

\[
m_n = K_c e_n \quad P
\]

(II-3)

\[
m_n = K_c |e_n| e_n \quad P^2
\]

(II-4)

\[
m_n = K_1 e_n + K_2 e_{n-1} \quad 2P
\]

(II-5)

\[
m_n = K_1 |e_n| e_n + K_2 e_n \quad PP^2
\]

(II-6)

For a process with \(\theta_o/\tau = 0.5\), Figure II-2 shows the optimum values of IAE - 1 for the response to a unit disturbance of the algorithms listed above as a function of sampling time. Figure II-3 shows...
Figure II-3. Optimum IAE-2 vs Sampling Time \((T/\tau)\). \(\theta_0/\tau = .5\)
an equivalent plot for IAE - 2. The proportional squared ($p^2$) proved to be the poorest and will not be discussed any further. The two parameter algorithm (2P) performed better than any other, especially at low sampling times; one drawback of this algorithm is that it is very sensitive to tuning. It may be noticed that for sampling times ($T/T_1$) greater than 1.6 the difference between the performance of the algorithms became very small. The proportional plus proportional squared algorithm performed better than the proportional algorithm; this algorithm may be useful for nonlinear processes.

As part of the study on algorithms which show steady state offset, the algorithm

$$m_n = K|e|^3e$$

(II-17)

was optimized for a unit step disturbance and a sampling time of 0.1 in order to have an indication of which would be the most desirable type of gain in a controller. It was found that $K = 1.1682$ and $a = -.7405$ and the algorithm showed a slight improvement over proportional control. Figure II-4a shows a plot of the manipulated variable as a function of the error. A proportional controller produces a straight line function.

Algorithms Which Show No Steady State Offset

Analysis of the responses of the algorithms which show no steady state offset described in equations II-7 to II-13 showed that for the first order lag plus dead time process studied, only four of these algorithms show adequate behavior. These are:

$$m_n = K_c \left( e_n + \frac{1}{T_1} \sum e_n T \right)$$

(II-7)
Figure II-4. Manipulated Variable vs Error for the Algorithm

\[ m = K|e|^a e. \]

Constants: \( K = 1.1682 \)
\( a = -0.7405 \)

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The last algorithm is equivalent to the one proposed by Mosler, et al (8), which in z-transform notation may be written as:

\[ D(z) = \frac{K_1 - K_2 z^{-1}}{(1 - z^{-1})(1 + K_3 z^{-1})} \]  

Following is a comparative discussion of these algorithms, which will be referred to as: P^2I, PID, and Mosler's. The PI algorithm was the basis of comparison since this is the most widely used algorithm. Responses to both disturbance and set point changes were compared.

1. Set point changes - The minimum IAE of the responses to set point changes are shown for the four algorithms under consideration in Figures II-5 and II-6, for \( \theta_0/\tau \) of .1 and .5 respectively, as function of the sampling time \( (T/\tau) \). Figures II-7, II-8, II-9 and II-10 show the optimum (IAE) responses of each algorithm for a process with \( \theta_0/\tau \) of .5, and values of sampling time of .1, .5, 1., and 2.

Table II-1 shows the sensitivities to controller parameters of each algorithm. The following observations based on these results may be made:

a) For sampling times greater than 2., the four algorithms performed similarly, meaning that the third parameter in PID and Mosler's algorithm may be omitted; thus, becoming a PI algorithm.
Figure II-5. Optimum IAE vs Sampling Time (T/r) for A Unit Step Change in Set Point, T_0r = 0.1
Figure 11-6. Optimum IAE vs Sampling Time (T/Tc) for a Unit Step Change in Set Point, T0/T = 0.5

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<table>
<thead>
<tr>
<th>Sampling Time</th>
<th>K_c</th>
<th>K_1</th>
<th>1/T_1</th>
<th>K_2</th>
<th>T_D</th>
<th>K_3</th>
</tr>
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<td>0.963</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>P^2I</td>
<td>16.40</td>
<td>19.82</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>PID</td>
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<td></td>
</tr>
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<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>P^2I</td>
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<tr>
<td></td>
<td>PID</td>
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<td>1.53</td>
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</tr>
<tr>
<td></td>
<td>Mosler</td>
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<td>Mosler</td>
<td>03.30</td>
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</tr>
</tbody>
</table>

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Figure II-7. Optimum IAE Responses to a Step Change in Set Point for a Sampling Time \((T/\tau)\) of 0.1.

Process \(\theta_o/\tau = 0.5\).
Figure XI-8. Optimum IAE Responses to a Step Change in Set Point for a Sampling Time \((T/\tau)\) of 0.5.

Process \(\theta_o/\tau = 0.5\).
Figure II-9. Optimum IAE Responses to a Step Change in Set Point for a Sampling Time (T/\tau) of 1.0.

Process \(\theta_0/\tau = 0.5\).
Figure II-10. Optimum IAE Responses to a Step Change in Set Point for a Sampling Time (T/τ) of 2.0.

Process θ₀/τ = .5.
b) The optimum responses of $P^2I$ showed little or no overshoot, but being a nonlinear algorithm, it was found to be sensitive to parameter variations.

c) Mosler's algorithm was best at sampling times in the vicinity of the dead time, but this was also an area of very high sensitivity to tuning.

d) For PI, PID and $P^2I$ there is a plateau in the IAE plots (Figures II-5 and II-6) after a sampling time of 1. This indicates that sampling less often will not cause deterioration of the response, and is due to the fact that in one sampling time, the response has almost reached steady state.

e) $P^2I$ with only two parameters performs as well as PID with three. The disadvantage of $P^2I$ being its nonlinearity.

2. Disturbances - The optimum values of the IAE of the responses to a unit step disturbance are shown in Figures II-11 and II-12 for $\theta_o/\tau$ of .1 and .5 respectively. Table II-2 shows the sensitivity to controller parameters and Figures II-13, II-14, II-15, and II-16 show optimal responses of each algorithm for sampling times of .1, .5, 1., and 2.. The following observations, very similar to the set point case, may be made:

a) For sampling times greater than 1.5 the IAE curves bunch together, indicating that nothing is gained by using a third parameter as in PID and Mosler's algorithms.

b) The optimal responses of $P^2I$ showed little or no undershoot.

c) Mosler's algorithm performed best at sampling times in the vicinity of the dead time, but was highly sensitive to tuning in this area.

A comparison of optimum IAE controller settings for disturbances and set point changes is shown in Tables II-3a,b,c and d, for PI, $P^2I$, PID, and Mosler's algorithms respectively. They are for a dead time $(\theta_o/\tau)$ of .5. It should be remembered that the
Figure II-11. Optimum IAE vs Sampling Time (T/\tau) for A Unit Step Disturbance. \( \theta_0/\tau = 0.1 \)
Figure II-12. Optimum IAE vs Sampling Time (T/τ) For A Unit Step Disturbance.

θ₀/τ = 0.5
TABLE II-2

Controller Parameter Sensitivity for Disturbances

<table>
<thead>
<tr>
<th>Sampling Time</th>
<th>$K_c/K_1$</th>
<th>$1/T_i/K_2$</th>
<th>$T_d/K_3$</th>
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</tr>
<tr>
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<td>0.62</td>
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</table>
Figure II-13. Optimum IAE Response to a Unit Step Disturbance for a Sampling Time (T/\tau) of 0.1. Process \( \Theta_0 / \tau = 0.5 \).
Figure II-14. Optimum IAE Responses to a Unit Step Disturbance for a Sampling Time \((T/\tau)\) of 0.5.

Process \(\theta_0/\tau = 0.5\).
Figure II-15. Optimum IAE Response to a Unit Step Disturbance for a Sampling Time (T/\tau) of 1.0.

Process \theta_o/\tau = 0.5.
Figure II-16. Optimum IAE Responses to a Unit Step Disturbance for a Sampling Time (T/τ) of 2.0. Process θ₀/τ = 0.5.
TABLE II-3a

Optimum IAE Parameter Settings for PI Controller.

\[ m_n = K(e_n + \frac{1}{T_i} \sum e_n T) \]

<table>
<thead>
<tr>
<th>Sampling Time</th>
<th>Set Point</th>
<th>Changes</th>
<th>Disturbances</th>
</tr>
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<td>.071</td>
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TABLE II-3b

Optimum IAE Parameter Settings for $P^2 I$ Controller

$$m_n = K(\frac{1}{T_i} \sum e_n T)$$

<table>
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<th>Sampling Time</th>
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<th>Changes</th>
<th>Disturbances</th>
</tr>
</thead>
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<td>$K$ $1/T_i$</td>
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<td>4.0</td>
<td>.071</td>
<td>3.558</td>
<td>.034 7.587</td>
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</table>
TABLE II-3c

Optimum IAE Parameter Settings for PID Controller

\[ m_n = K(e_n + \frac{1}{T_i} \sum e_n T + T_D \frac{(e_n - e_{n-1})}{T}) \]

<table>
<thead>
<tr>
<th>Sampling Time</th>
<th>Set Point</th>
<th>Changes</th>
<th>Disturbances</th>
</tr>
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TABLE II-3d

Optimum IAE Parameter Settings for Mosler's Controller

\[ m_n = K_1 e_n - K_2 e_{n-1} + (1 - K_3) m_{n-1} + K_3 m_{n-2} \]

<table>
<thead>
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<th>Changes</th>
<th>Disturbances</th>
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</table>

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sampling time and controller parameters have been nondimensionalized (i.e., process gain and time constant equal to 1.)

The differences between set point and disturbance tuning are considerable for every algorithm, PI showing the smallest differences. The differences in tuning parameters are larger at fast sampling rates, decreasing as sampling becomes less frequent. The case of the PI and PID algorithms will be discussed in greater detail in Chapter III.

Summary

Of the algorithms which show steady state offset only the ones with two parameters (PP and 2P) performed better than a proportional algorithm. It is questionable if the improvement in performance is large enough to justify going to a two parameter algorithm.

Of the algorithms which show no offset it was found that P²I, PID, and Mosler's performed as well or better than a PI algorithm. P²I has the main disadvantage of being nonlinear and therefore its tuning depends on the size of the disturbances. PID may be used at fast sampling rates with success. Mosler's algorithm should perform very well at sampling times somewhat larger than the dead time of the system in order to avoid the highly sensitive region in the vicinity of the dead time.

In the cases studied it was found that except for Mosler's algorithm at a sampling time equal to the dead time, the performance of the algorithms did not improve greatly over PI. This is due to the fact that in a linear first order lag plus dead time model the response is unaffected by disturbances a length of time equal to the dead time, therefore the disturbance will go undetected for that time.
plus a fraction of the sampling time (depending on when the disturbance entered the system). It is then very difficult to make the process return to steady state in a more efficient manner than PI does, using a conventional type of feedback controller. It may also be concluded that a given nonlinear algorithm may be used to improve the response of a given nonlinear process, but it is doubtful that much can be done with linear models.
REFERENCES


CHAPTER III
TUNING CONTROLLERS FOR SET POINT CHANGES

Introduction

Techniques for tuning controllers based on the open loop response of a system or process reaction curve were developed by Ziegler and Nichols (1) and later refined by Cohen and Coon (2) and Smith and Murrill (3). These methods were based on the use of the one quarter decay ratio of the response as the performance criterion. Lopez, et al (4) developed open loop tuning relationships based on the minimization of error integral criteria. Miller, et al (5) compared the above mentioned tuning techniques and concluded that techniques based on the use of error integrals as performance criteria are superior to others.

In his work Lopez developed tuning relationships which minimized the error integrals of the response to disturbance changes. Optimum settings for disturbance changes are not optimum for changes in set point; and in certain applications set point changes are rather common. It is the purpose of this chapter to develop controller tuning relationships based on the minimization of error integrals for set point changes. It will also be shown how these relationships may be used to tune both conventional and direct digital control loops.

Control Loop

This work will be concerned with single input-single output
control loops such as the one shown in the block diagram in Figure III-1. The process block includes the dynamics of sensing elements, valves, etc., in addition to the actual process. The controller block represents the control algorithm.

The control algorithms which will be tuned are the proportional plus integral (PI) and the proportional plus integral plus derivative (PID). In both cases the controllers are assumed to be "ideal", i.e. no interaction between modes and no lags associated with them. An ideal PID controller may be represented in transfer function form by the following equation:

\[
\frac{M(s)}{E(s)} = K_c \left(1 + \frac{1}{T_i s} + T_D s\right)
\]

where: \( M(s) \) = manipulated variable
\( E(s) \) = error
\( K_c \) = controller gain
\( T_i \) = reset time
\( T_D \) = derivative time
\( s \) = Laplace transform variable

Open loop tuning methods are based on the characterization of the process reaction curve, the response of the process to a step change in the controller output or manipulated variable. The process may be represented by a first order lag plus dead time:

\[
\frac{Output}{Input} = \frac{Ke^{\tau s}}{\tau s + 1}
\]
Figure III-1. Block Diagram of Basic Control Loop.
where: $K =$ process gain

$\tau =$ time constant

$\theta_o =$ dead time

We may calculate the model parameters $(K, \tau, \theta_o)$ from the process reaction curve by the method shown in Figure III-2.

**Performance Criteria**

Error integral criteria have been recommended (5,6,8) for the analysis and comparison of controller performance. In this work two error integrals will be used:

1. The integral of the absolute value of the error, $IAE$.

$$IAE = \int_0^\infty |e(t)| dt$$

2. The integral of the time and the absolute value of the error, $ITAE$.

$$ITAE = \int_0^\infty t|e(t)| dt$$

**Tuning Relationships**

As stated earlier in this work the tuning relationships will be based on optimizing the response to a step change in set point as opposed to a step change in disturbance. This problem is one of finding the minimum value of the integral criteria with respect to the controller parameters. In other words the integral criteria $\phi$ is a function of $K_C$, $T_i$, $T_D$ for a given process and the optimal controller parameters are those that minimize $\phi$. This optimization of the
controller parameters was accomplished by using a numerical Pattern Search version programmed by Moore (7). The calculations were performed in a digital computer.

To generalize the results the parameters have been non-dimensionalized and the controller settings will be expressed as functions of the ratio of dead time to time constant \((\theta_0/\tau)\). Figures III-3 and III-4 show the tuning relationships for PI controllers and Figures III-5 and III-6 for PID controllers. Empirical equations have been fitted to these curves and the resulting equations are shown in Table III-1.

**Application to Direct Digital Control**

Consider the block diagram of a typical direct digital control loop shown in Figure III-7a. Notice that the sampler and the zero order hold are the main difference from the conventional analog loop considered up to now. Moore (7) showed that the sampler and hold could be approximated by a dead time equal to one half the sampling time of the sampler. The effect of this approximation is shown in the equivalent diagram of Figure III-7b.

Based on this approximation the tuning technique developed for continuous controllers in this article may be extended to the more complex sampled data case. This is accomplished by considering an "effective dead time" for direct digital control loops. This "effective dead time" is the sum of the process model dead time plus one half the sampling time of the loop, i.e.

\[ \theta' = \theta_0 + \frac{1}{2} T_s \]
Figure III-3. Optimum Settings for Proportional Plus Integral (PI) Controller. IAE Criterion.
Figure III-4. Optimum Settings for Proportional plus Integral (PI) Controller. ITAE Criterion.
Figure III-5. Optimum Settings for Proportional Plus Integral Plus Derivative (PID) Controller. IAE Criterion.
Figure III-6. Optimum Settings for Proportional Plus Integral Plus Derivative (PID) Controller. ITAE Criterion.
TABLE III-1
Tuning Equations for Set Point Changes

Controller Algorithm: \[ M(S) = K_c \left( 1 + \frac{1}{T_i s} + T_d s \right) E(s) \]

Tuning Equations: \[ KK_c = a(\theta_0/\tau)^b \]
\[ \frac{T}{T_i} = c + d(\theta_0/\tau) \]
\[ \frac{T_d}{\tau} = e(\theta_0/\tau)^f \]

<table>
<thead>
<tr>
<th>Controller</th>
<th>Criterion</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
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<tr>
<td>PI</td>
<td>IAE</td>
<td>.758</td>
<td>-.861</td>
<td>1.02</td>
<td>-.323</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>ITAE</td>
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<td>-.916</td>
<td>1.03</td>
<td>-.165</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PID</td>
<td>IAE</td>
<td>1.086</td>
<td>-.869</td>
<td>.74</td>
<td>-.13</td>
<td>.348</td>
<td>.914</td>
</tr>
<tr>
<td>PID</td>
<td>ITAE</td>
<td>.965</td>
<td>-.85</td>
<td>.796</td>
<td>-.1465</td>
<td>.308</td>
<td>.929</td>
</tr>
</tbody>
</table>
Figure III-7a. Block Diagram of Typical Digital Control Loop with a Sampler and a Zero Order Hold.

Figure III-7b. Block Diagram of Equivalent Continuous Control Loop. The Dead Time Approximates the Sampler and the Zero Order Hold.
Using this effective dead time we may find the control parameters in the same figures and equations used for the continuous case (Figures III-3 to III-6, Table III-1).

Comparison of Set Point vs. Disturbance Tuning

The difference between set point and disturbance tuning parameters for a PI controller based on the IAE criterion is shown in Figure III-8. Values of controller gain ($K_c$) are larger for disturbance tuning, and values of reset time ($T_i$) are smaller (larger $1/T_i$). The difference is the largest for small $\theta_o/\tau$, decreasing as $\theta_o/\tau$ increases.

The responses to a set point change for a first order lag plus dead time with $\theta_o/\tau = .5$ are shown in Figure III-9a for a PI controller. Figure III-9b shows the responses for a disturbance change. As expected, set point tuning is best for set point changes, and disturbance tuning is best for disturbance changes. Neither method may be judged better than the other since they are based on different criteria. Processes where seldom set point changes occur should be tuned for disturbances. Processes where frequent set point changes occur, as is often the case in batch processes or where supervisory control is used, should be tuned accordingly. In other words a compromise should be reached on what is best for each particular case in question. Figures III-10a and III-10b show the same conclusions for a PID controller.

Figures III-11 and III-12 are equivalent to III-9 and III-10, but for $\theta_o/\tau = .2$. In this case the difference in tuning methods is appreciably larger since $\theta_o/\tau$ is smaller.
Figure III-8. Difference Between Set Point and Disturbance Tuning Parameters for a PI Controller. IAE Criterion.
Figure III-9a. Response to a Set Point Change. Disturbance vs. Set Point Tuning. PI Controller. IAE Criterion.
Figure III-9b. Response to a Disturbance Change. Disturbance vs. Set Point Tuning. PI Controller. IAE Criterion.
Figure III-10a. Response to a Set Point Change. Disturbance vs. Set Point Tuning. PID Controller. ITAE Criterion.
Figure III-10b. Response to a Disturbance Change. Disturbance vs. Set Point Tuning. PID Controller. ITAE Criterion.
Figure III-11a. Response to a Set Point Change. Disturbance vs. Set Point Tuning. PI Controller. IAE Criterion.
Figure III-11b. Response to a Disturbance Change. Disturbance vs. Set Point Tuning. PI Controller. IAE Criterion.
Figure III-12a. Response to a Set Point Change. Disturbance vs. Set Point Tuning. PID Controller. ITAE Criterion.
Figure III-12b. Response to a Disturbance Change. Disturbance vs. Set Point Tuning. PID Controller. ITAE Criterion.

θ_0/τ = .2
REFERENCES


CHAPTER IV
EFFECT OF APPLYING THE PROPORTIONAL ACTION
TO THE FEEDBACK VARIABLE IN A PI ALGORITHM

Introduction
The difference existing between the controller tuning parameters obtained by optimizing the response to disturbances and the parameters obtained by optimizing the response to set point changes has been shown in Chapter III. In a proportional plus integral (PI) algorithm, the proportional action may be applied to the error signal or to the feedback variable (1,2). Both variations respond in the same manner to disturbances, but they respond differently to changes in set point. In this chapter the responses of both algorithms to the different types of inputs will be studied.

Control Loop
A typical single input-single output direct digital control feedback loop is shown in Figure IV-1. The loop differs from a continuous control system by the introduction of the samplers and zero order hold. The computer, represented by the dotted block, comprises the comparator and the control algorithm.

Control Algorithms
An ideal continuous proportional plus integral control algorithm may be written as

\[ m = K_c \left[ e + \frac{1}{T_i} \int e dt \right] \]
where: \( m \) = manipulated variable

\( e \) = error signal

\( t \) = time

\( K_c \) = controller gain

\( T_i \) = reset or integral time

When used in direct digital control, the discrete form of this algorithm is:

\[
m_n = K_c \left[ e_n + \frac{1}{T_i} \sum_{i=0}^{\infty} e_n \Delta t \right]
\]

where: \( \Delta t \) = sampling time

\( n \) = sampling interval

A variation of the common PI algorithm described above is to apply the proportional action to the feedback or controlled variable (\( c \)) instead of to the error signal (\( e \)). This algorithm which will be called proportional on the controlled variable plus integral (PCI) may be expressed as:

\[
m_n = K_c \left[ -c_n + \frac{1}{T_i} \sum_{n=1}^{\infty} e_n \Delta t \right]
\]

where: \( c \) = controlled or feedback variable

It may be noted that both algorithms are equivalent when no set point changes occur. It is when set point changes take place that these two algorithms behave differently.

Process

A first order lag plus dead time model will be used to test the algorithms; i.e.,
\[ G(s) = \frac{K e^{-\theta_0 s}}{\tau s + 1} \]

where:  
- \( G(s) \) = process transfer function  
- \( K \) = process gain  
- \( \tau \) = time constant  
- \( \theta_0 \) = dead time  
- \( s \) = Laplace transform variable

For convenience the process gain and time constant will be considered equal to one. This is equivalent to non-dimensionalization of the equation. The process may then be characterized by the ratio of dead time to time constant (\( \theta_0 / \tau \)).

**Performance Criterion**

The integral of the absolute value of the error (IAE) will be used as the criterion of performance. It may be expressed as:

\[ \text{IAE} = \int_{0}^{\infty} |e| \, dt \]

**Results and Discussion**

The parameters which minimize the integral of the absolute error (IAE) of the responses to disturbance and set point changes were found for both PI and PCI algorithms. These optimal parameters are plotted in Figure IV-2a and IV 2b for a range of \( \theta_0 / \tau \) from 0.1 to 1.0, and a sampling time of 0.1. Since disturbance tuning is the same for both algorithms only one curve is shown on each graph corresponding to disturbances, while two curves are needed for set point tuning: one for each algorithm. It may be noticed that the set point tuning curves
Figure IV-2a - Optimum Parameters for PI and PCI Algorithms, based on the Response to Disturbance and Set Point Changes. IAE Criterion. Sampling Time of 0.1.
Figure IV-2b - Optimum Parameters for PI and PCI Algorithms, based on the Response to Disturbance and Set Point Changes. IAE Criterion. Sampling Time of 0.1.
for the PCI algorithms are much closer to the disturbance tuning curve than those for the PI algorithm. Therefore, if both a PI and a PCI algorithms are tuned for disturbances, the PCI will be closer to optimal set point tuning than the PI algorithm. It may also be pointed out that as $\theta_0/\tau$ increases, the parameters are less dependent on the type of tuning for both algorithms, and the differences between disturbance and set point tuning become smaller.

Responses to a step change in set point and disturbance for a process with a $\theta_0/\tau$ of 0.2 are shown in Figures IV-3 to IV-6. Figures IV-3 and IV-5 show the responses of a PI algorithm to disturbance and set point changes respectively for both types of tuning. Figures IV-4 and IV-6 show equivalent responses for a PCI algorithm.

Figure IV-4 shows that there is almost no difference between disturbance and set point tuning for a PCI algorithm when the system is subject to disturbances, while Figure IV-3 shows that PI responds much slower to disturbances when set point tuned than when tuned for disturbances. Figure IV-5 shows the large overshoot inherent in PI algorithms tuned for disturbances when a set point change occurs. This overshoot is not present in the response of a PCI algorithm as shown in Figure IV-6, but a slower rising response is noticed.

Tuning relationships have been developed for PI controllers based on minimizing error integrals by Lopez, et al (3,4) for disturbances and in Chapter III of this work for set point changes. Lopez's tuning relationships also apply for a PCI controller since they are based on the response to disturbances. From the closeness of the parameters discussed above, it seems that the development of set point tuning relationships for PCI algorithms is not justified, and the
Figure IV-3 - Response to a step disturbance.

PI Algorithm tuned for set point and for disturbances.
Figure IV-4 - Response to a step disturbance.

PCI Algorithm tuned for set point and for disturbances.
Figure IV-5 - Response to a step change in set point.
PI Algorithm tuned for set point and for disturbances.
Figure IV-6 - Response to a step change in set point. PCI Algorithm tuned for set point and for disturbances.

Set Point Tuning - IAE = .457

Disturbance Tuning - IAE = .595
tuning relationships developed by Lopez for disturbances are good enough for any types of input when a PCI controller is used.

Summary

The following points should be made before concluding this discussion:

a) PCI seems to be a more consistent algorithm, tuning being less dependent on the type of input considered than a PI controller.

b) The PCI algorithm may be implemented in continuous control systems.

c) The principle of applying the proportional action to the feedback or controlled variable may also be applied to proportional-plus-integral-plus-derivative (PID) algorithms with similar results.

d) The PCI algorithm is not recommended for use in the slave controller of a cascade system, because in this case it is desired that the proportional action respond immediately to the set point change.
REFERENCES


CHAPTER V

AUTOMATIC TUNING TECHNIQUE

Introduction

The tuning of process controllers in the chemical industries is usually done manually in the field or in the control room by instrument engineers and technicians. In many processes where levels of production are changed fairly often, the controller parameters have to be readjusted or retuned after each one of these changes due to the nonlinear nature of most processes. A method is proposed herein, where control parameters are tuned automatically when changes in level of production occur. The method is easily implemented when direct digital control is being used. A small variation may also be implemented in continuous conventional analog control.

The method proposed is simulated as part of a chemical reactor temperature control system, different alternatives are also evaluated.

Outline of the Method

In the proposed automatic parameter adjusting technique outlined here, a nonlinear process is characterized by a linear model at different levels of production. With sufficient points, the parameters of the linear model may be expressed as functions of the level of production. This may be accomplished by fitting the data points with an empirical equation, or by using tables and linear interpolation between points. The model used may be one obtained from experimental plant
tests, as the ones used in this work. Tuning relationships for the model being used must be available.

With the information described above stored in the control computer, and sampling the variable which defines the level of production; i.e., feed flow rate, as soon as a change in level of production is noticed, the model parameters may be calculated for this new level and from the tuning relationships the new control parameters obtained and adjusted. It will be noted that the controller parameters may be stored directly as functions of the level of production, therefore saving storage and one step in the calculation procedure; on the other hand, tuning relationships stored may be used for several loops.

An explanation of the chemical reactor model in which the proposed method is simulated, the control strategy, and analysis of the performance of the method follows.

Process

The process for evaluating the proposed control parameter adjustment scheme is the continuous stirred tank chemical reactor system shown in Figure V-1. Feed at a flow rate $W$, temperature $T_i$, and concentration $C_{A_0}$ enters the reactor of volume $V_r$. Two moles of component A react to produce one mole of B according to the equation:

$$A \rightarrow \frac{1}{2} B$$

The reaction is second order, irreversible, and exothermic. The rate of reaction being given by

$$r_A = kC_A^2$$
Parameters:

\( W = 50 \text{ lb/min} \)
\( T_i = 198^\circ \text{F} \)
\( C_{Ao} = 0.5975 \text{ lb-mole/ft}^3 \)
\( T_{Wi} = 80^\circ \text{F} \)
\( V_t = 13.38 \text{ ft}^3 \)
\( A = 100 \text{ ft}^2 \)
\( V_B = 8.64 \text{ ft}^3 \)
\( U = 25 \text{ BTU/hr} \ ^{\circ} \text{F} \text{ ft}^2 \)
\( \rho_t = 55 \text{ lb/ft}^3 \)
\( \rho_w = 62.4 \text{ lb/ft}^3 \)
\( C_p = 0.9 \text{ BTU/lb} \ ^{\circ} \text{F} \)
\( C_{pw} = 1 \text{ BTU/lb} \ ^{\circ} \text{F} \)
\( \Delta H = -6000 \text{ BTU/lb-mole} \)
\( k = k_0 e^{-a/T} \)
\( k_o = 8.33 \times 10^8 \text{ ft}^3/\text{lb-mole min} \)
\( a = 14000^\circ \text{R} \)

Figure V-1. Stirred Tank Chemical Reactor
The product stream leaves at a temperature $T$ and concentration $C_A$. To remove the heat evolved by the reaction, cooling water is circulated through a tube bundle, water entering at temperature $T_w^i$ and leaving at $T_w^o$.

The basic assumptions made in the development of the mathematical model are:

1. The contents of the tank are perfectly mixed, so that the concentration and temperature of the reacting mass and exit stream may be considered equal.

2. The contents of the tube bundle are recirculated fast enough so that the temperature of the cooling water may be considered uniform and equal to the exit water temperature.

3. The physical properties of the different streams, and the tank level are considered to be constant.

4. Heat losses to the surroundings and heat generated by the impeller are negligible.

5. There is no change in volume due to the reaction.

6. The valve and temperature sensor dynamics may be represented by first order lags with time constants of $\tau_1$ and $\tau_2$ minutes respectively.

The equations which describe the system are:

Material Balance on A:

$$\frac{dC_A}{dt} = \frac{W}{V_tP_t} (C_{Ao} - C_A) - kC_A^2$$

Heat Balance on reactor:

$$\frac{dT}{dt} = \frac{W}{V_tP_t} (T_I - T) + \frac{(-\Delta H)}{\rho C_p} kC_A^2 - \frac{UA}{V_tP_tC_p} (T - T_w^o)$$

Heat Balance on tube bundle:

$$\frac{dT_w^o}{dt} = \frac{M}{V_BP_w} (T_w^i - T_w^o) + \frac{UA}{V_BP_tC_w} (T - T_w^o)$$
Valve dynamics:

\[ M(s) = \frac{1}{\tau_1 s + 1} \text{MV}(s) \]

Temperature sensor dynamics:

\[ T_R(s) = \frac{1}{\tau_2 s + 1} \text{T}(s) \]

The controller acting on the cooling water flow rate is an "ideal" proportional plus integral controller which may be represented as:

\[ \text{MV}(s) = K_c \left(1 + \frac{1}{\tau_i s}\right) E(s) \]

Performance Criteria

The performance criterion used to evaluate and compare the control parameter adjustment scheme is the integral of the time weighted absolute error, namely:

\[ \text{ITAE} = \int_{0}^{\infty} t|e|dt \]

The ITAE criterion was selected since it produces less oscillations in the response and in general lower values of gain and \(1/T_i\), when compared to the integral of the absolute value of the error.

Characterization of the Process

Two different models were used to characterize the reactor system considered. Both of these methods are based on the response to a step change in the manipulated variable, or process reaction curve.
A process reaction curve was obtained at each of the three levels of production considered, namely 50%, 75% and 100%. The two models obtained were the popular first order lag plus dead time model, and the second order lag plus dead time model approximation proposed by Meyer (2). The values of the model parameters are shown in Table V-1. The variation of these parameters from level to level points out the nonlinearity of the reactor system. These parameters may be fitted with an equation or used in table form, interpolating for intermediate points.

**Tuning Relationships**

Another reason which supports the choice of the models discussed above is that tuning relationships are readily available for both of them. Lopez, et al (1) and Lopez (3) developed tuning relationships based on the optimization of error integrals.

For the first order plus dead time model, empirical equations for the gain and the reset time based on minimizing the integral of time weighted absolute error (ITAE) are (1):

\[
K_c = \frac{859}{K} \left(\frac{\theta_o}{\tau}\right)^{-0.977}
\]

\[
\frac{1}{T_i} = \frac{674}{\tau} \left(\frac{\theta_o}{\tau}\right)^{-0.68}
\]

For the second order plus dead time model equations are not available and graphs (3) have to be used.

**Results**

Table V-2 shows the values of the control parameters for a PI controller at three different levels of production (50%, 75% and 100%) as obtained by:
**TABLE V-1**

**MODEL PARAMETERS**

First order lag plus dead time model

\[ G(s) = \frac{K e^{-\theta_0 s}}{s + 1} \]

<table>
<thead>
<tr>
<th>Feed Flow Rate (lb/hr)</th>
<th>K</th>
<th>( \theta_0 )</th>
<th>( \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>.782</td>
<td>3.43</td>
<td>15.41</td>
</tr>
<tr>
<td>75</td>
<td>.3185</td>
<td>2.7</td>
<td>12.55</td>
</tr>
<tr>
<td>100</td>
<td>.1305</td>
<td>2.25</td>
<td>9.25</td>
</tr>
</tbody>
</table>

Second order lag plus dead time model

\[ G(s) = \frac{k_0 \omega_n^2 e^{-\theta_0 s}}{s^2 + 2\zeta \omega_n s + \omega_n^2} \]

<table>
<thead>
<tr>
<th>Feed Flow Rate (lb/hr)</th>
<th>K</th>
<th>( \theta_0 )</th>
<th>( \zeta )</th>
<th>( \omega_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>.782</td>
<td>1.1</td>
<td>1.18</td>
<td>.118</td>
</tr>
<tr>
<td>75</td>
<td>.3185</td>
<td>1.0</td>
<td>1.1</td>
<td>.154</td>
</tr>
<tr>
<td>100</td>
<td>.1305</td>
<td>0.85</td>
<td>0.97</td>
<td>.197</td>
</tr>
</tbody>
</table>
### TABLE V-2

**CONTROL PARAMETER SETTINGS**

**a. Based on First Order Lag Plus Dead Time Model and Tuning Relationships**

<table>
<thead>
<tr>
<th>Feed Flow Rate (lb/hr)</th>
<th>$K_C$</th>
<th>$\frac{1}{T_i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>4.73</td>
<td>.0875</td>
</tr>
<tr>
<td>75</td>
<td>11.3</td>
<td>.1065</td>
</tr>
<tr>
<td>100</td>
<td>25.3</td>
<td>.1375</td>
</tr>
</tbody>
</table>

**b. Based on Second Order Lag Plus Dead Time Model and Tuning Relationships**

<table>
<thead>
<tr>
<th>Feed Flow Rate (lb/hr)</th>
<th>$K_C$</th>
<th>$\frac{1}{T_i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>10.25</td>
<td>.063</td>
</tr>
<tr>
<td>75</td>
<td>17.3</td>
<td>.082</td>
</tr>
<tr>
<td>100</td>
<td>39.8</td>
<td>.089</td>
</tr>
</tbody>
</table>

**c. Based on Minimizing ITAE for a 10% Upward Step Change in Feed Flow Rate at Each different Level**

<table>
<thead>
<tr>
<th>Feed Flow Rate (lb/hr)</th>
<th>$K_C$</th>
<th>$\frac{1}{T_i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>10.9</td>
<td>.047</td>
</tr>
<tr>
<td>75</td>
<td>18.24</td>
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</tr>
<tr>
<td>100</td>
<td>40.8</td>
<td>.089</td>
</tr>
</tbody>
</table>

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Figure V-2b. - Adjusted response. First order lag plus dead time tuning.
Figure V-2d - Adjusted response. Second order lag plus dead time tuning.
Figure V.2e - Unadjusted response, Second order lag plus dead time tuning.
Figure V-2F - Adjusted response. Optimum parameters for a 10% step change in feed flow rate.
Figure V-2g - Unadjusted response. Optimum parameters for a 10% step change in feed flow rate.
**TABLE V-3**

**ITAE FOR UPWARD SERIES OF STEPS**

a. Using first order lag plus dead time tuning

<table>
<thead>
<tr>
<th></th>
<th>50-75%</th>
<th>75-100%</th>
</tr>
</thead>
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<tr>
<td>Adjusted</td>
<td>2110</td>
<td>1109</td>
</tr>
<tr>
<td>Unadjusted</td>
<td>1124</td>
<td>894</td>
</tr>
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</table>

b. Using second order lag plus dead time tuning

<table>
<thead>
<tr>
<th></th>
<th>50-75%</th>
<th>75-100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjusted</td>
<td>706</td>
<td>338</td>
</tr>
<tr>
<td>Unadjusted</td>
<td>594</td>
<td>1162</td>
</tr>
</tbody>
</table>

c. Using settings from optimization of the ITAE of the response to a 10% step in feed flow rate

<table>
<thead>
<tr>
<th></th>
<th>50-75%</th>
<th>75-100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjusted</td>
<td>606</td>
<td>352</td>
</tr>
<tr>
<td>Unadjusted</td>
<td>933</td>
<td>1869</td>
</tr>
</tbody>
</table>
a) Using the first order lag plus dead time model and corresponding ITAE tuning relationships.

b) Using the second order lag plus dead time model and corresponding ITAE tuning relationships.

c) By minimizing the ITAE of the response of the system to a 10% upward step change in feed flow rate.

From this table it can be seen that the second order lag plus dead time model yields parameter values much closer to the parameters obtained by optimizing the response of the system to a 10% step change in feed flow rate, than those obtained by using the first order lag plus dead time model.

A sequence of two step changes in feed flow rate were applied to the reactor system to compare the use of the parameter adjustment technique with the unadjusted case for each one of the three sets of parameters under consideration.

The sequences of steps taken are shown in Figures V-2a and V-3a. The first sequence is from 50% to 75% and then from 75% to 100% level of production. A time of 120 minutes elapses between steps so that steady state may be reached. The second sequence is in the reverse direction; i.e., from 100% to 75% and then from 75% to 50% level of production.

Figures V-2b, V-2d and V-2f show the responses of the automatically adjusted control loop for the upward series of steps, while Figures V-2c, V-2e and V-2g show the unadjusted case. Table V-3 shows the values of the integral of time weighted absolute error (ITAE) for each one of the steps. The use of the adjustment technique proved to be very good when the second order model or the optimization parameters were used. When used with the first order model there is a deterioration of control for the first step but an improvement in the second
step. This poor performance is attributed to the fact that the first order lag plus dead time model does not approximate the dynamics of the reactor system with sufficient accuracy.

For the downward series of steps, the responses for the adjusted and unadjusted cases are shown in Figures V-3b to V-3g, and values of the ITAE criterion are shown in Table V-4 for each step. The responses of the unadjusted system turned out to be unstable, showing that some form of tuning adjustment is necessary, either manually by operators or automatically as proposed in this article. The responses of the automatically adjusted system turned out to be very good, and again the second order lag plus dead time tuning proved to be better than the first order lag plus dead time tuning.

Conclusions

From the results of this study, two basic conclusions may be drawn. The first is the superiority in this case of the second order lag plus dead time model over the first order lag plus dead time model, and the closeness of the predicted parameters from the second order model to those obtained by the optimization of the response. The first order model has been very popular in use by the control engineers, and the second order model has been generally rejected since it introduces one extra parameter. From this work it appears that it should perhaps be considered since increase in accuracy of results may pay for the little extra work in its use.

The second conclusion is that it is necessary in many cases to retune control parameters when production changes occur, and a simple adjusting technique, like the one discussed, may be developed.
This technique may be easily implemented in direct digital control computers and also in continuous analog control installations.
Figure V-3a - Downward Series of Steps in Level of Production
Figure V-3b - Adjusted response. First order lag plus dead time tuning.
Figure V-3c - Unadjusted response. First order lag plus dead time tuning.
Figure V-3g - Unadjusted response. Optimum parameters for a 10% step change in feed flow rate.
TABLE V-4

ITAE FOR DOWNWARD SERIES OF STEPS

a. Using first order lag plus dead time tuning

<table>
<thead>
<tr>
<th></th>
<th>100-75 %</th>
<th>75-50 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjusted</td>
<td>2394</td>
<td>3505</td>
</tr>
<tr>
<td>Unadjusted</td>
<td>unstable</td>
<td>unstable</td>
</tr>
</tbody>
</table>

b. Using second order lag plus dead time tuning

<table>
<thead>
<tr>
<th></th>
<th>100-75 %</th>
<th>75-50 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjusted</td>
<td>803</td>
<td>1488</td>
</tr>
<tr>
<td>Unadjusted</td>
<td>unstable</td>
<td>unstable</td>
</tr>
</tbody>
</table>

c. Using settings from optimization of the ITAE of the response to a 10% step in feed flow rate

<table>
<thead>
<tr>
<th></th>
<th>100-75 %</th>
<th>75-50 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjusted</td>
<td>701</td>
<td>1212</td>
</tr>
<tr>
<td>Unadjusted</td>
<td>unstable</td>
<td>unstable</td>
</tr>
</tbody>
</table>
REFERENCES


CHAPTER VI

ADAPTIVE GAIN TUNING USING PARAMETER SENSITIVITY COEFFICIENTS

Introduction

The nature of most chemical processes is that their response characteristics vary both with time and operating conditions. The inherent linearities of most conventional controllers require that controller parameter changes or retuning be performed to compensate for the process nonlinearities. In Chapter V a simple automatic tuning technique was presented in which the model parameters and controller settings were identified a priori from the responses of the process to step changes at different levels of operation. In this chapter a real time adaptive gain tuning technique is presented.

The method is based on Tomovic's (1) application of sensitivity coefficients to the identification of model parameters, also referred to as the problem of inverse sensitivity.

The adaptive technique will be applied to a chemical reactor temperature control system, and the model reference will be a second order system.

The purpose of this work is to investigate the use of the identification technique and its applicability to adaptive control.
Theory of Inverse Sensitivity

Given a dynamic system

\[ F(\dot{x}, \ddot{x}, x, t, q_i) = 0 \]  \hspace{1cm} (VI-1)

where: \( x \) = independent variable
\( t \) = time
\( q_i \) = parameters; \( i = 1, \ldots, m \)

the sensitivity or influence coefficients are defined as:

\[ u_{q_i} = \frac{\partial x}{\partial q_i} \]  \hspace{1cm} (VI-2)

where: \( u_{q_i} \) = sensitivity coefficient of parameter \( q_i \)
\( x \) = system output
\( q_i \) = model parameter

If we designate the variations of the system caused by parameter changes as:

\[ \Delta x = x(t, q_{i0} + \Delta q_i) - x(t, q_{i0}) \]  \hspace{1cm} (VI-3)

then we can say that:

\[ \Delta x = f(\Delta q_i) \quad i = 1, 2, 3, \ldots, m \]  \hspace{1cm} (VI-4)

or formally:

\[ \Delta q_i = f^{-1}(\Delta x) \]  \hspace{1cm} (VI-5)

This is a statement of the problem of inverse sensitivity which relates the system parameter variations to the deviations of the system output from the ideal system or model. In his work, Tomovic (1) proposes a method to solve this problem of inverse sensitivity. It is shown pictorially in Figure VI-1, and is based in comparing the output of the dynamic system with the output to the model to produce the function \( \Delta x \) in equation VI-5.
Figure VI-1. Tomovic's Method for Calculating $\Delta x$ in The Method of Inverse Sensitivity.
Since there is no one-to-one solution to equation VI-5, the solution will be accomplished by reducing the problem to one of optimization. Tomovic proposes optimizing the expression:

\[ I = \text{Opt} (\Delta x + \Delta q) \]  

(VI-6)

where \( \Delta x \) is the difference between system and model as shown in Figure VI-1, and \( \Delta q \) will be the linear approximation of the system:

\[ \Delta q = \sum_{i=1}^{m} \frac{\partial x}{\partial q_i} \cdot \Delta q = \sum_{i=1}^{m} u_{i0} \cdot \Delta q_i \]  

(VI-7)

where \( u_{i0} \) are by definition the sensitivity coefficients of the system parameters. By substituting VI-7 into VI-6 and using the minimum of the integral of the square of the error over an interval of time \( T \) as the optimization criteria, we get:

\[ I_{\text{min}} = \int_{0}^{T} \{ \Delta x(t) + (u_{i1} \Delta q_1 + ... u_{m1} \Delta q_m) \}^2 dt \]  

(VI-8)

Differentiating with respect to each parameter, for a three parameter system, Equation VI-8 reduces to a system of algebraic equations

\[ C_{i1} \Delta q_1 + C_{i2} \Delta q_2 + C_{i3} \Delta q_3 = b_i \quad i = 1,2,3 \]  

(VI-9)

where:

\[ b_i = - \int_{0}^{T} \Delta x(t) u_i(t) dt \]  

(VI-10)

and

\[ C_{ij} = C_{ji} = \int_{0}^{T} u_i(t) \cdot u_j(t) dt \quad \text{for } i = 1,2,3 \]  

(VI-11)

The system of algebraic equations in VI-9 can be solved for the parameter variations \( \Delta q_i \) which may then be used to update the model parameters. The linear approximation used in equation VI-7 will place restrictions in the region of convergence. In cases in which the starting points of the model parameters are outside this region.
of convergence, a steepest descent method proposed by Marquardt (2) will be used. In this method a constant factor $\lambda$ is added to the diagonal elements of the matrix described by Equations VI-9. We may express this in matrix notation as:

$$ (C + \lambda I) \Delta Q = B \quad \text{(VI-12)} $$

As the value of $\lambda$ increases a gradient method is obtained. For $\lambda$ equal to zero, the system of equations reduces to the original set of equations (VI-9).

Finally, in order to calculate the coefficients of Equation VI-9 we must know the sensitivity coefficients of the model parameters. In this work a second order system will be used as the mathematical model:

$$ \ddot{y} + A \dot{y} + By = K \cdot f(t) \quad \text{(VI-13)} $$

Taking partial derivatives with respect to each parameter and inverting the order of differentiation we arrive at the set of differential equations:

$$
\begin{align*}
\ddot{u}_A + A \dot{u}_A + Bu_A &= 0 \\
\ddot{u}_B + A \dot{u}_B + Bu_B &= -y \\
\ddot{u}_K + A \dot{u}_K + Bu_K &= f(t)
\end{align*}
\quad \text{(VI-14)}
$$

with initial conditions, $u_i(0) = u_i'(0) = 0$. A numerical solution of these equations simultaneously with Equation VI-13 will provide the parameter sensitivity coefficients required to calculate the coefficients of Equation VI-9.
Identification of a Second Order System

In order to verify the applicability of the identification technique, similar second order systems were used for both process and model. The process equation used was:

\[ x + Ax + Bx = Ky \]  

(VI-15)

with parameters \( A = .16 \)

\( B = .01 \)

\( K = .01 \)

To test convergence, different starting points were chosen for the model parameters and a step change in the input (y) was used for excitation. Iterative computations of new model parameters were performed at time intervals of 4 minutes. The results are shown in Table VI-1.

For most cases convergence occurred before the response reached steady state usually within 5 iterations. In one case, where the starting points were quite far from the actual process parameters, convergence was not achieved except when using Marquardt's gradient method. Trial and error produced a value of \( \lambda \) equal to 0.5 as being adequate for convergence. These results validate the applicability of the use of sensitivity coefficients to continuous identification of process parameters.

Application to Adaptive Gain Tuning

Having proven the effectiveness of the method of inverse sensitivity to the identification of model parameters, a technique for applying this concept to adaptive gain tuning of a control system was developed.

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### TABLE VI-1

Parameter Identification of a Second Order System

Process: \( \ddot{x} + A\dot{x} + Bx = Ky \)

where \( K = .01 \)

\( A = .16 \)

\( B = .01 \)

<table>
<thead>
<tr>
<th>Starting Values</th>
<th># of Intervals For Convergence</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K )</td>
<td>( A )</td>
</tr>
<tr>
<td>.1</td>
<td>.16</td>
</tr>
<tr>
<td>.009</td>
<td>.13</td>
</tr>
<tr>
<td>.001</td>
<td>.13</td>
</tr>
<tr>
<td>.01</td>
<td>.16</td>
</tr>
<tr>
<td>.01</td>
<td>.16</td>
</tr>
<tr>
<td>.01</td>
<td>.03</td>
</tr>
<tr>
<td>.01</td>
<td>.25</td>
</tr>
<tr>
<td>.006</td>
<td>.13</td>
</tr>
</tbody>
</table>

(1) Marquardt’s Method Required for Convergence. \( \lambda = .5 \)
This technique is also based on using a second order system as the model reference in the identification algorithm but allowing only the gain of the model \( (K) \) to vary. In this fashion the identification is reduced to one parameter. The value of this parameter is then used to adjust the gain or proportional action of a proportional plus integral (PI) control algorithm that controls the manipulated variable of the process. The reasons or assumptions for developing this technique follow:

- reducing the identification of model parameters to the process gain simplifies the computations to be performed in real time by a computer.

- the greatest return or benefit in the dynamic response of a system is obtained by maintaining a constant loop gain.

- in tuning a PI controller for a second order system, the proportional action is influenced more by the system gain than by other system parameters.

- the system gain varies more than the other parameters for most systems. As will be shown later, this is the case for the reactor control system to which the technique will be applied.

The proposed adaptive gain tuning method is illustrated in Figure VI-2. Inverse sensitivity is used to identify the model gain \((K)\) in real time from the difference between process and model outputs \((\Delta x)\). The model is updated with the new value and a new controller gain \((K_c)\) is calculated from it.
Calculate Controller Gain

Identify Model Gain

Controller

Set Point

Model

Process

Figure VI-2. Adaptive Gain Tuning Technique
Reactor Control System

The process selected for testing the proposed adaptive gain tuning technique was the simulated stirred tank chemical reactor temperature control system described in Chapter V. The reader is referred to Chapter V for a detailed discussion of the process and the equations that describe it.

In order to select initial parameters for the identification of the model, the method developed by Meyer (3) for approximating process response with second order systems was used. Step changes in cooling water flow rate were applied to the reactor and from the open loop response, Meyer's graphical method was used to calculate the second order model parameters. Table VI-2 shows the parameters calculated at three temperature levels. In this table it can be noticed that the process gain (K) exhibits variations (non-linearities) much larger than the other two parameters (ξ and W_n), lending support to the assumption made previously.

Averages of these parameters were used as starting points in the identification of the process. As a result of several test runs it was found that values of W_n = 0.08 and ξ = 1.0 could be used satisfactorily over the temperature range of operation.

The temperature controller algorithm used in the reactor system is the velocity form of the standard PI algorithm:

\[ m_n - m_{n-1} = K_c \left( e_n - e_{n-1} + \frac{1}{T_i} \ast e_n \ast \Delta T \right) \]  

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TABLE VI-2

Model Parameters From Reactor Response to a
Step Change in Flow Rate at Different Temperature Levels Using Meyer's Approximation

Model Equation: \[ \frac{C(s)}{R(s)} = \frac{K W_n^2}{s^2 + 2\xi W_n + W_n^2} \]

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>K</th>
<th>( \xi )</th>
<th>( W_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>178</td>
<td>.23</td>
<td>.90</td>
<td>.10</td>
</tr>
<tr>
<td>198</td>
<td>.97</td>
<td>1.01</td>
<td>.09</td>
</tr>
<tr>
<td>218</td>
<td>1.65</td>
<td>1.25</td>
<td>.08</td>
</tr>
</tbody>
</table>

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where \( m_n \) = manipulated variable

\( e_n \) = error signal

\( K_c \) = controller gain

\( T_i \) = integral time

The controller gain \( K_c \) was adjusted from the model parameters using an equation of the form

\[
K_c = \frac{C_1}{K}
\]  

(VI-17)

where \( C_1 \) = constant

\( K \) = model gain

The integral parameter \( (1/T_i) \) was kept constant since the contributing parameters to the process time constants (\( \xi \) and \( \nu_n \)) were not allowed to vary. Values of \( C_1 \) and \( 1/T_i \) were estimated from the controller settings used for the reactor system in Chapter V and from optimization of the response to a 5°F step change in set point.

Results

The adaptive gain tuning technique was tested by applying a series of step changes in the temperature set point of the reactor system. Figure VI-3 shows the responses to step changes from 175°F to 188°F and from 188°F to 198°F. For comparison purposes, both the adapted and the unadapted cases were plotted. The adaptive technique showed improvement over the unadapted case as shown by the integral of the absolute value of the error (IAE) values: 142 vs 201 for the first
Figure VI-3. Response to Step Changes in Temperature

Adapted: $I_AE = 114$

Unadapted: $I_AE = 163$

$K_c = 3.7/K$

$1/T_1 = 0.05$

$I_AE = 142$

$T_1 = 30.1$

$T_1 = 200.1$

$T_1 = 300.1$

$T_1 = 375.1$
step, and 114 vs 163 for the second step. Figure VI-4 shows a plot of the manipulated variable (cooling water rate) for the steps. It can be seen that the adapted case controlled the water flow rate in a smoother fashion. Note that after the initial effect of the set point change, the adaptive case did not overcool the reactor; this was due to the lower controller gain calculated because of the increased process gain at higher temperatures.

Figure VI-5 shows the responses for larger step changes: from 175°F to 198°F and from 198°F to 218°F. The results are similar to the ones discussed for the first series of steps.
Figure VI-4. Manipulated Variable Response

Unadapted

Adapted

Cooling Water Flow Rate (lb/min)

TIME (MINUTES)

0 100 125 150 175 300 325 350 375

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Figure VI-5. Response to Step Changes in Temperature
REFERENCES


CHAPTER VII
CONCLUSIONS AND RECOMMENDATIONS

This work was intended to present and evaluate control algorithms, tuning techniques, and their use in industrial control applications. Chapters II, III, and IV covered the performance of control algorithms and tuning techniques. Chapters V and VI discussed how to automatically retune controllers when used in nonlinear processes, Chapter VI addressing a new adaptive gain tuning technique using inverse sensitivity theory. As a result of this work the following general conclusions are presented:

- Sensitivity of control algorithm response to tuning parameters is a very important consideration in algorithm selection. Although some algorithms like P²I and Mosler showed an improvement over conventional PI and PID algorithms when optimally tuned, they also showed a sensitivity to parameter settings that may not be desirable in practical applications.

- Tuning parameters developed from optimizing responses to step changes in set point can differ significantly from those developed by optimizing response to disturbance changes. The set point tuning relationships presented in Chapter III are more conservative than disturbance tuning relationships, and are recommended for use in processes where frequent set point changes occur, or when overshoot in the response is not desirable.
- The Proportional on the Control Variable plus Integral (PCI) algorithm was found to be less dependent on the type of tuning than the PI algorithm, and produced more consistent responses for both types of excitation, showing little or no overshoot to set point changes.

- The adaptive gain tuning method proposed in this work using inverse sensitivity theory proved to be an effective technique for automatically adjusting the controller gain in nonlinear processes.

Based on this work, it is recommended that additional research be conducted to expand the use of inverse sensitivity to identify and adapt all three parameters of the second order model, and also include adjustments to the reset and rate parameters of the controller.

Finally, recognizing that computer simulation is one step removed from actual process plant operation, it is felt that the true test of the techniques proposed in this work will come when they are implemented in a live process control environment.
APPENDIX A

Computer Programs
MAIN PROGRAM TO CALCULATE OPTIMUM CONTROLLER SETTINGS. ----

CALLS 'PATTERN' - WHICH IN TURN CALLS 'PRCC'

DIMENSION P(6),STEP(5)
COMMON DT,DTME,STME,TAU,NPST,NOT1,NSTM,NTME,NPARMT
NPARAT=3
NPARMT=2
I0=1
I0=0
NRD=3
NRD=4
DT=.005
DTME=0.
STME=.1
TAU=0.
S1AX=4.
P(1)=1.
P(2)=.5
DJ 100 KK=1,10
DTME=DTME+1
NPST=STME/DT+0.1
N)DME=DTME/DT+0.1
TFIN=20.
ANTHE=TFIN/STME
NTIME = ANTIME
NSTILE = IFIN/STMC+0.5
CALL PATTERN(F,STEP,N1,D,IC,PHI)
WRITE(6,8)STMC,DTMC,TAU,(P(I),I=1,NDARMT),PHI
8 FORMAT(1X,3F10.4,3F15.5,F20.8)
100 CONTINUE
STOP
END
SUBROUTINE PATTERN(F, ISTEP, NRD, I0, CJST)
DIMENSION P(6), STLP(3), R1(1000), J2(1000), T(1000), S(1000)
C:: WHY NP
C------ STARTING POINT
L = 1
ICK = 2
ITTER = 0
DO 5 I = 1, NP
P1(I) = P(I)
J2(I) = P(I)
T(I) = P(I)
5 S(I) = STEP(I) * 10.
C------ INITIAL BOUNDARY CHECK AND COST EVALUATION
CALL BOUNDS(F, IOUT)
IF (IOUT LE 0) GOTO 10
IF (ICK LE 0) GOTO 6
WRITE (6, 1005)
WRITE (6, 1000) (J, P(J), J = 1, NP)
RETURN
6 RETURN
10 CALL PROC(F, C1)
IF (ICK LE 0) GOTO 11
WRITE (6, 1001) ITTER, C1
WRITE (6, 1000) (J, P(J), J = 1, NP)
C------ BEGINNING OF PATTERN SEARCH STRATEGY
11 DO 99 INRD = 1, NRD
99 DO 12 I = 1, NP
12 S(I) = S(I) / 10.
IF (ICK LE 0) GOTO 20
WRITE (6, 1003)
WRITE(6,1002) (J*S(J), J=1, NP)

20 IFAIL=0.0
C----Perturbation About T
   T(I)=T(I)+S(I)
   IC=IC+1
   CALL BOUNDS(F, IOUT)
   IF(IOUT.GT.0) GO TO 23
   call proc(p,c2)
   L=L+1
   IF(IC.LT.3) GO TO 22
   WRITE(6,1002)(L,C2)
   WRITE(*,1000)(J,P(J), J=1, NP)
22 IF(C1-C2).GE.23.25
23 IF(IG.GE.2) GO TO 24
   S(I)=-S(I)
   GO TO 21
24 IFAIL=IFAIL+1
   P(I)=T(I)
   GO TO 30
25 T(I)=P(I)
   C1=C2
30 CONTINUE
   IF(IFAIL.LT.1) GO TO 35
   IF(ICK.EQ.2) GO TO 30
   IF(ICK.EQ.1) GO TO 35
   CALL proc(t,c2)
   L=L+1
IF (10.LT.2) GOTO31
WRITE(6,1002)L,C2
WRITE(6,1000)(J,T(J),J=1,NP)
31 IF (C1-C2).LE.32,24,34
32 ICK=1
DO 33 I=1,NP
B1(I)=B2(I)
P(I)=B2(I)
33 T(I)=B2(I)
GOTO20
34 C1=C2
35 IB1=0
DO 39 I=1,NP
32(I)=T(I)
IF (ABS(B1(I)-B2(I))*LT.*.01*ABS(S(I))) IB1=IB1+1
39 CONTINUE
IF (IB1.EQ.NP) GOTO90
ICK=0
ITCR=ITCR+1
IF (ICK.LT.2) GOTO40
WRITE(6,1091)ITCR,C1
WRITE(6,1090)(J,T(J),J=1,NP)
C-----ACCELERATION STEP
40 SJ=1.0
DO 45 I=1,11
45 SJ=1.0
42 P(I)=T(I)
SJ=SJ-.1
CALL BOUNDS(I, IOUT)
IF(IOUT.LT.1)GOT046
IF(I1.EQ.11)ICK=1
45 CONTINUE
46 DO47 I=1, NP
47 H1(I)=R2(I)
GOT020
90 DO91 I=1, NP
91 T(I)=B2(I)
99 CONTINUE
 DO100 J=1, NP
100 P(I)=T(I)
       IF(I0.LE.0)RETURN
 WRITE(6, 1034)L,C1
 WRITE(6, 1030)(J,P(J), J=1, NP)
 RETURN
1009 FORMAT (5X, 5(I7, E13.6))
1001 FORMAT (/1X14HITTERATION NO., I3/5X, 5HCOST=, E15.6, 20X,
1 13HPARAMETERS)
1002 FORMAT (10X3HNO., I4, 1X5HCOST=, E15.6)
1003 FORMAT (/1X20*STEP SIZE FOR EACH PARAMETER )
1004 FORMAT (1H13FANSWERS AFTER , I3, 2X, 2HFUNCTIONAL EVALUATIONS //
1 5X5HCOST=, E15.6, 20X, 13HOPTIMAL PARAMETERS )
1005 FORMAT (1H135*INITIAL PARAMETERS JXT OF BOUNDS )
END
SUBROUTINE BOUNDS(X,I)
DIMENSION X(I)
I=0
IF(X(1) .LT. 0.001) GC11:0
RETURN
10 I=1
RETURN
END
SUBROUTINE 'PROC' CALLED BY 'PATERN' ---
CALCULATES IAE FOR FIRST UNDER PLUS DEAD TIME PROCESS RESPONSE

SUBROUTINE PROC(P, PHI)
DIMENSION C(1000), P(1)
COMMON DT, DELAY, STIME, TAU, NPST, NTME, NSTME, NTIME, NPARAT
PHI=0.
CSS=1.
CSS=0.
A=0.
CAB=0.
TIME=0.
A=0.
E=1.
EE=1.
U=111=1, NTME
11 C(1)=0.
J=1
CX=0.
U=12K=1, NSTME
C=0=C(J)
TIME=TIME+CT
E=C(J)
DAK=-P(1)*(CD-CRB)+P(2)*E
CDB=CB
AM=AM+DAM
AX=AX+1.
DO 12 I=1,NPST
CX=CX+(AM-CX)*DT
C(J)=CX
J=J+1
PHI=PHI+ABS(CSS-CX)*T
12 IF(J*GT*N)THE J=1
RETURN
END
SUBROUTINE PROC(P, PHI)
DIMENSION P(C), C(S)
COMMON DT, DELAY, STIME, TAU, N, STIME, NSTIME, NPAR
CSS=1.
CSS=0.
PHI=0.
AK=0.
AC=EXP(-STIME)
K=DELAY/STIME
D=DELAY-STIME*FLOAT(K)
DEL=STIME-D
AJ=EXP(-DEL)
C1=0.
J=K+1
C(J)=0.
DU1=1.
1 C(J+1)=0.
A=0.
STM=1./STIME
DU7=1.
NSTIME
I=J-K+1
E=C(I)-1.
A=A+E*STIME
DAK=P(1)*((C(I)-C(I-1))+P(0)*E)
AK=AK+DAK
I=J+2
AK2=-AK
AK1=C1-AK2
C(I)=AK2+AK1+AD
C1N=C1
C1=AK2+AK1*AC
AK2=AK2-C55
IF(((C1-CSS)*(CIN-CSS).LT.0.) GOTO2
PHI=PHI+ABS(AK1*(1.-AC)*AK2*STIME)
GOTO7
2 ZTIME=ALOG(-AK1/AK2)
AZ=EXP(-ZTIME)
PHI=PHI+ABS(AK1*(1.-AZ)*AK2*ZTIM1)
ZABS(AK1*(AZ-AC)
1+AK2*(STIME-ZTIME))
7 CONTINUE
RETURN
END
PROGRAM 'SENSE' ---
-USED TO CALCULATE THE SENSITIVITY TO PARAMETER TUNING
-CALLS 'PROC'

DIMENSION P(6),PP(6),
PHJ(6),PHL(6)
COMMON DT,SIME,NPSI,NDIME,NSTME
READ(5,6) ANRM
6 FORMAT(13)
DTME=0.5
DT=0.05
TFIN=40.
11 READ(5,8) IK,SIME,P(1),P(2),P(3)
8 FORMAT(12,4F10.0)
NPSI=SIME/DT+0.1
NDIME=SIME/DT+0.1
NSTME=TFIN/SIME+0.8
WRITE(6,10) SIME
10 FORMAT(1H0, 6HSTIME=,F4.2)
CALL PROC(P,PHI)
PHI=PHI
WRITE(6,30) (P(I),I=1,NPARMT),PHI
30 FORMAT(1X,1F20.6)
DU40I=1,NPARMT
40 PP(1)=P(1)
DO100 JI=1,NPARMT
  P(JI)=PP(JI)+1 &P(JI)
CALL PROC(P,PHI)
```
WRITE(6,30) (P(I),I=1,NPARMT),PHI
PHU(JI)=PHI
P(JB)=PP(JB)-1*PP(JI)
CALL PREC(F,PHI)
WRITE(6,30) (P(I),I=1,NPARMT),PHI
PHL(JB)=PHI
P(JB)=PP(JB)
SENS=(.5*(PHU(JJ)-PHI)**5*(PHL(JJ)-PHI))/PHI**100.
100 WRITE(6,?1) SENS
21 FORMAT(1X,SHECNS=,'F13.5)
   IF(IK.EQ.4)GOTO51
   GO TO 11
51 CONTINUE
STOP
END
```
SUBROUTINE CALLED BY 'SENSE'
- CALCULATES IAC OF RESPONSE OF FIRST ORDER SYSTEM

SUBROUTINE PROC(P,PHI)
DIMENSION P(6), C(100)
COMMON DT, STME, NPST, NDTME, NSTME
PHI=0.
E=0.
DO 11 I=1, NDTME
  1 C(I)=J.
  J=1
  CX=0.
  A=0.0
  AMO=0.
  AMO0=0.
  EE=0.
  STM=1./STME
  DUA2=1.*STME
  E=-C(J)

ALGORITHM
AM=P(1)*E+P(2)*EE+(1.-P(3))*AM0+P(4)*AM00
AM00=AM0
AM0=AM
EE=EE
AMM=AM+1.
DO21=1.*NPST
\[ C_X = C_X + \left( A_X - C_Y \right) \times DT \]
\[ C_Y = C_Y + \left( A_Y + C_Y \times DT \right) \times DT \]
\[ J = J + 1 \]
\[ \text{IF} \left( J > 0 \right) \text{RETURN} \]
\[ \text{END} \]
SUBROUTINE CALLED BY *PATERN*
- CALCULATES IAE FOR REACTOR RESPONSE
- USED TO FIND OPTIMUM CONTROLLER SETTINGS FOR REACTOR

SUBROUTINE PROC(P, PHI)
DIMENSION P(6)
COMMON NPARM1, W, CA, T, TH, TF, PT, PHI0, PCA, WFR, WFA

PHI=0.0
PT=173.0
PCA=0.291
PTWO=116.29
TREAD=178.0
WFR=70.864
TF=173.0
TF=183.0
PT=193.0
WFR=28.77
PCA=26.57
PTWO=132.644
TREAD=199.0
TF=203.0
PT=215.0
WFR=13.2474
PCA=0.1752
PTWO=114.7088
TREAD=219.0
TF=213.0
\[ \text{XFR A = WFR} \]
\[ \text{A WFR = WFR} \]
\[ \text{T A U 1 = 2 5} \]
\[ \text{T A U 2 = 4} \]
\[ \text{U = 2 5} \]
\[ \text{A B = 1 0 0} \]
\[ \text{C P R = 3} \]
\[ \text{T I = 1 9 8} \]
\[ \text{V T = 1 3 4 8} \]
\[ \text{DEN Y = 5 5} \]
\[ \text{DH = 1 2 0 0 0} \]
\[ \text{DN = 6 0 0 0} \]
\[ \text{C A U = 8 5 7 5} \]
\[ \text{A K U = 3 3 J E 1 6} \]
\[ \text{A K A = 1 4 0 0 0} \]
\[ \text{C P A = 1} \]
\[ \text{V G = 3 6 4} \]
\[ \text{D E N w = 6 2 4} \]
\[ \text{T W I = 3 0} \]
\[ \text{D T = 0 1} \]
\[ \text{D T = 0 0 0 5} \]
\[ \text{S M E = 0 T} \]
\[ \text{A 1 = (1 - E X P (- C T / T A U 1))} \]
\[ \text{A 2 = E X P (- C T / T A U 1)} \]
\[ \text{B 1 = (1 - E X P (- C T / T A U 2))} \]
\[ \text{B 2 = E X P (- D T / T A U 2)} \]
\[ \text{E = 0} \]
\[ \text{E = 0} \]
\[ \text{A = 0} \]
TIME=0
J=0
K=0
AH3=V/(VT*DENR)
ABH=DH/(DENR*CPR)
CHC=J*AB/(60*VT*DENR*CPR)
ADJ=V/(VT*DENR)
BU=J*AB/(60*VBDENR)
1 CONTINUE
  J=J+1
  TIME=TIME+DT
  EE=EE
  E=TF-2EAD
  PHI=PHI+ANS(E)*DT
  A=A+.5*(E+EE)*STME
  SAFR=E(1)*(E-CEE*P(2)*CEE*DT)
  AWRF=AWRF-CAWRF
  IF (AWRF LT 0.1) AWRF=0.1
  WFR=AWRF+(AWRF-WFR)*DT/TAU1
  IF (WFR LT 0.1) WFR=0.1
  IF (WFR GT 120.) WFR=120.
  ABU=WFR/(VE*ENW)
  APT=PT+460.
  RA=AC(A *EXP(-AKA/APT))
  CA=PCA*(AMB*(CAO-PCA)-RA*PCA+PCA)*PT
  TW0=TW0+(ARK*(TW1-TW0)+3*BU*(PT-PTWO))*DT
  T=PT+(AH3*(TI-PT)+EH)*RA*PCA*PCA-CHC*(PT-PTWO)*DT
  PT=T
  PCA=CA
PTWO=TWO
TREAD=TREAD+(T-TREAD)*DT/TAU2
IF(J.LT.50)GOTO1
J=0
K=K+1
IF(K.LT.360)GOTO1
RETURN
END
IDENTIFICATION OF A SECOND ORDER PROCESS USING
SENSITIVITY COEFFICIENTS
TO VERIFY METHOD

IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION F(6)
DIMENSION A(2,3),B(3),DEL(3),U(3)
EQUIVALENCE (E(1),E1),(D(2),B2),(J(3),J3),(A(1,1),A11),(A(1,2),
1A12),(B(1,3),A13),(B(2,1),A21),(B(2,2),A22),(B(2,3),A23),
2(B(3,1),A31),(B(3,2),A32),(B(3,3),A33)
WLAMB=0.0
TS=4.0
TT=0.1

MODEL PARAMETERS
AM=0.1
BM=0.01
GM=0.006

PROCESS PARAMETERS
AP=.16
BP=0.01
GP=0.01
XR=5.0
X=0.0
XX=0.0
XXX=XX
Z=0.
ZZ=0.000
ZZZ=0.000
DT=.001
DT2=DT*DT
STME=DT
UUU2=0.
UU3=0.
UUU.1=0.
DO 70 IA=1,1AM
AL1=AM*DT
AL3=G*AM*DT2
DEN=1+AM*DT+EM*DT2
IJ=IJ+1
TIME=TIME+DT
EE=E
E=XR-X
SUM=SU+5*(E+EC)*S14E
XIN=P(1)*(E+F(2)*S14E)
X=(AL3*XIN+(2*+AL1)*XX-XXX)/XDEN
XX=XX
X=X
Z=(AL3*XIN+(2*+AL1)*ZZ-ZZZ)/ZDEN
ZP=(Z-ZZ)/CT
ZZZ=ZZ
ZZ=Z
DN=Z-X
U(1)=(-ZP*DT2+(2*+AL1)*UU1-UUU1)/ZDEN
U(2)=(-ZP*DT2+(2*+AL1)*UU2-UUU2)/ZDEN
U(3)=(XIN*DT2+(2*+AL1)*UU3-UUU3)/ZDEN
UUU1=UU1
UU1=U(1)
UUU2=UU2
UU2=U(2)
UUU3=UU3
UU3=U(3)
DO 31 I=1,3
31 H(I)=H(I)+DN*U(I)*CT
33 CONTINUE
23 WRITE(6,9) DELT,DEL(1),DEL(2),DEL(3),XIN
  9 FORMAT(1X,SD15.7)
94 WRITE(6,9) TIME,X,Z,U(1),U(2),U(3),AM,DM,CM
  9 FORMAT(1X,9F5.5)
  IF (TIME.GT.900.) GOTO 99
  IF (KKK.LT.20) GOTO 1
  KKK=0
  IF (TIME.GT.100.) GOTO 74
  IF (TIME.GT.50.) GOTO 75
GOTO 1
74 XR=15.
  GOTO 1
75 XR=10.
  GOTO 1
99 CONTINUE
STOP
END
PROGRAM FOR ADAPTIVE GAIN TUNING METHOD

IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION P(6)
IWT=0
IND1=0
IND2=0
PHI1=0.0000
PHI2=0.0000
GAMIN=0.2000
GAMAX=10.0000
DFG=1.3000
TS=6.0000
UIAS=0.4000
TT=0.0500
SI=1.0000
WN=0.0000
GAIN=0.35000
TSP1=1.78500
TSP2=1.90000
TSPFF=1.93000
LEXP=0
PTPD=0.00
PTPD=0.0000
W=50.000
WFR=70.36400
<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AK4</td>
<td>1.4003</td>
</tr>
<tr>
<td>CPW1</td>
<td>1.000</td>
</tr>
<tr>
<td>W1N1</td>
<td>0.000</td>
</tr>
<tr>
<td>T1E1</td>
<td>1.000</td>
</tr>
<tr>
<td>T1E2</td>
<td>0.000</td>
</tr>
<tr>
<td>D1</td>
<td>0.000</td>
</tr>
<tr>
<td>D1E1</td>
<td>0.000</td>
</tr>
<tr>
<td>D1E2</td>
<td>0.000</td>
</tr>
<tr>
<td>E</td>
<td>0.000</td>
</tr>
<tr>
<td>T2</td>
<td>0.000</td>
</tr>
<tr>
<td>T2E1</td>
<td>0.000</td>
</tr>
<tr>
<td>T2E2</td>
<td>0.000</td>
</tr>
<tr>
<td>K</td>
<td>0.000</td>
</tr>
</tbody>
</table>

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IAM=TS/DT+1*6-1
IAM=TS/2.0+6.1
IA=TS+0.1DC
KKK=KKK+1
DO IF I=1, I=IAM
ZULO=Z
Z2PLO=ZP
DO 70 IA=1,1000
AL1=2.0*WS1*WSN*DT
AL2=WSN*DT*DT
AL3=AL2*GAIN
VEN=1.0DO0+AL1+AL2
IJ=IJ+1
TIME=TIME+DT
EE=E
E=IF=READ
DAWRP=P(1)*(E-EE+P(2)*E*DT)
AWRP=AWRP-DAWRP
IF(AWRP.0,0,000) AWZ=R=1.0000
XIN=AWFR-*FRF
XIN=-XIN
XXO=(XIN-XXIN)/DT
XXN=XXI
#FR=FR+(AWFR-AWRF)*DT/TAU1
AUU=FR/((UE*ENW)
APT=DT+60.0C0
K=AKC+EXP(-AKA/AF1)
CA=PCA+(A36+(CAO-PCF)-RA*PCA*PCF)*DT
TW=PTW+(ABY*(TWI-PTW)+B3Y*(DT-PTW))DT
$T = PT + (AL) \times (TI - PT) + BH \times PA \times PCA \times \frac{PCA - QC \times (PT - PT \times C)}{C T}$

$PT = T$
$PCA = CA$
$PT \times U = TW$
$T REAC = TRAC + (T - T REAC) \times DT / TAU 2$
$T) = (T REAC - T REAC) \times DT$
$T RDEL = T REAC$
$Z = (AL) \times X IN + (2 \times 000 + AL) \times Z = Z Z \times Z / D E 1$
$Z D = (Z - Z Z) \times D T$
$Z Z Z = Z Z$
$Z = Z$
$T RDEL = T REAC \times T R E A C$
$P T R D = P T R D$
$P T R D = T R D E L$
$U C 1 C = (AL) \times X IN + (2 \times 000 + AL) \times U U U 1 - U U J 1 / D E N$
$U U U 1 = J U 1$
$U U 1 = U C 1 C$
$IF (T I M E \times G T \times 300) G O T 0 1 4$
$IF (T I M E \times G T \times 100) G O T 0 1 2$
$G O T 0 5$
$2 4 \Phi I N = \Phi I N + D A B S (E) \times DT$
$G O T 0 5$
$4 2 \Phi I N = \Phi I N + C A B S (E) \times DT$
$5 \text{ CONTINUE}$
$7 0 \text{ CONTINUE}$
$D 3 = D L D - T R D E L$
$D G = D G + D D \times U C 1 C$
$A G = A G + U C 1 C \times U C 1 C$
$7 1 \text{ CONTINUE}$
DGAIN=-DG/AG
IF (TIME*GT.20.0) IT=2
IF (IT.EQ.0) GOTO6
DGAIN=DGAIN*DFG
AQ=DA83(DQ)
IF (ADQ.LT.8IAS) GO TO 3
DGAIL=0.95000
IF (DGAIN.GT.DGAIL)DGAIN=DGAIL
IF (DGAIN.LT.-DGAIL)DGAIN=-DGAIL
GAIN=GAIN+DGAIN
IF (GAIN.LT.CNMIN)GAIN-CNMIN
IF (GAIN.GT.CNMAX)GAIN=CNMAX
3.3 CONTINUE
IF (TIME.GT.300.) GOTO 12
IF (TIME.GT.100.) GOTO 11
GOTO6
1.2 IF (INJ2.GT.1) GOTO 14
IND2=2
WFRK=-20.77
UU1=0.
UUU1=0.
PTRO=P1D+TRE-TSPFE
P3TRO=P3TDE+TRE-TSPFE
TRE=TSPFE
TF=TSPFE
KKK=0
14 CONTINUE
GOTO6
11 IF (IND1.GT.1) GOTO 13
VITA

Alberto Arner Rovira was born in Santiago de Cuba, Cuba on January 30, 1943. He obtained his elementary and secondary education at the Colegio de Dolores in Santiago de Cuba graduating in 1959.

He attended the Universidad de Oriente in Cuba for one year and subsequently transferred to Louisiana State University where he received his Bachelor of Science Degree in Chemical Engineering in May 1964. He was awarded a Master of Science Degree in Chemical Engineering in May 1966.

In October 1979, he married the former Jo Eva Peak of Walker, Louisiana.

From 1969 to the present he has been employed by the International Business Machines Corporation and is currently a Systems Engineering Manager in Houston, Texas.
EXAMINATION AND THESIS REPORT

Candidate: Alberto Arner Rovira

Major Field: Chemical Engineering

Title of Thesis: Control Algorithms, Tuning, and Adaptive Gain Tuning in Process Control

Approved:

[Signatures]

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:

December 12, 1980