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Axis-switching in square coaxial jets

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AXIS-SWITCHING IN SQUARE COAXIAL JETS

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Master of Science in Mechanical Engineering

in

The Department of Mechanical Engineering

by

Victor Piffaut
Ecole Nationale Supérieure d’Ingénieurs de Constructions Aéronautiques, Toulouse 2003
December 2003
Dedication

As a tribute to the large international community at LSU trying to share their diversity with our fellow students and faculty, this thesis is dedicated to people who keep their eyes open to the world and don’t believe everything carried out by the media.

Could they be more and more in the United States.
Acknowledgements

This long journey through the Graduate School is reaching to an end, with the help of the following people:

Thanks to my parents who supported me through my years of college.

Thanks to Way-Ho Choy and Samuel Bonnafous, my two mentors on the project. Ross Rials, thank you for your help on the project and in English, the nice atmosphere in the lab and the pre-meeting waits. I thank particularly my advisor, Dr Nikitopoulos for providing me with directions and new ideas, and for introducing me to the wild world of turbulence…

I would like to thank all my friends who have been for me an international family and leave me so many memories: Amit, Chuck and Ryan for the homework done together, the entire two-phase lab crew for the coffee breaks and the entertainment, Jan for the crazy adventures (it could have been worse...), Christophe who guided me from my first week to the last, the whole French community from Guillaume, Sam and Yannick who welcomed me, to Fred and his many girlfriends, to Flo and her complaints, to Papi Steven for the plane flights and the political arguments, and to the new female generation, Lucie and Carole (good luck, girls!), the kala kala Greek community, especially Greg and Aspasia, the Lafayette boys Benoit and Zizou, Olivier and his good connections, Jason and his apartment complex, and Debbie, the perfect girl next door.

I would like to thank Louis and Lynn Leggio who have been a real host family.

I would never have gone through these two years in the USA without the French connection providing me with chocolate and other basic alimentation: my parents, my lovely sister Laetitia, the just-married Marie and Bruno, and Marie-Paule.
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Abstract

Airflow from single and coaxial contoured nozzles with a square cross section is studied using constant temperature anemometry. The inner to outer jet velocity ratio is 1.5. The Reynolds number of the outer jet based on the hydraulic diameter of the outer nozzle exit is 15,300. Results are compared to previous data obtained with circular nozzles under the same conditions.

Properties of the jets and their shear layers such as turbulence shear layer thicknesses and growth rates were derived from the velocity record. The mixing between the jets and with the external flow was found to be enhanced by passive forcing. Axis switching was observed in the near field of the jets.

Fourier space analysis was conducted. Energies of the preferred modes and their harmonics were computed in the shear layers. A sub-harmonic cascade was observed, evidence of non-linear interaction between the preferred mode and its sub-harmonics.
Chapter 1  Introduction

1.1  Literature Survey

Jet flows are widely used in engineering system, such as injectors. In most of them, mixing is the main purpose. That is why it is important to understand fully the dynamics of this mixing, and propose some means of control and improvement.

This study is an occasion to review the works done throughout the years by researchers on single and coaxial jets, and the means they used to control them, in order to improve the mixing with the ambient air and/or between the jets.

1.1.1  Jets

1.1.1.1  Free Jets

The Reynolds averaged Navier-Stokes equation for turbulent plane flow can be written in the jet axis direction, by basing the scaling analysis on the width of the jet as:

\[ U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + \frac{\partial}{\partial y} (u'v') = 0 \]

Where U and V are the mean velocity components, and u and v are the fluctuating components in the x and y directions respectively. The last term of this expression is linked with the Reynolds stress tensor, \(-\rho u'v'\). It represents the stress exerted by the turbulent fluctuation in the mean flow.

The free jet can be divided into two distinct regions. The first region is called the developing region or potential core. Close to the exit, in the near field region, the velocity distribution is strongly dependent on the nozzle exit conditions. The studies led by Wygnanski and Fiedler (1969) [73] show that the flow in this region behaves as inviscid, and its velocity remains 99% of the centerline exit velocity. This region usually ends after
about five exit-diameters from the nozzle exit. Similarity profiles have been proposed separately by Michalke [49] and Ho & Huerre [33] for the developing region. They both offered a hyperbolic tangent profile pattern shear layer.

Downstream, in the far field, the velocity profile has a gaussian shape. This second region is called the fully developed region. Velocity profiles and the shear stress component of the Reynolds tensor have been shown to be self-similar:

\[
\frac{U}{U_s} = f(y/l) \quad \text{and} \quad \frac{u'v'}{U_s^2} = g(y/l)
\]

Us (velocity at the axis) and l (half-width of the jet) are depending on x only: \( l \propto x \) and \( U_s \propto x^{-1} \)

Unlike laminar jets, turbulent jets are characterized by the existence of eddies. Eddies are mostly present at the interface region, and can be identified by high-level fluctuating vorticity. The characteristics of the flow will be different according to the size of the eddies. They are usually divided into two groups, large and small eddies.

The scale of large eddies is the flow width. In a boundary layer, the scale will be the momentum thickness \( \theta \), and in a jet, it will be its diameter D. An important parameter in turbulent flow that characterizes the shedding frequency of the eddies is the Strouhal number, \( St_\theta = \frac{f \cdot \theta}{\Delta U} \), where \( \Delta U \) is the velocity jump in the shear layer and \( f \) is the frequency of the preferred mode. The non-dimensional spatial growth rate and the phase velocity are found to vary with the Strouhal number. According to experimental studies and calculations on Orr-Sommerfeld stability, the Strouhal number in a shear
layer is in the range of 0.012 to 0.020. The early development of large-scale coherent structures depends on the initial conditions (Nikitopoulos and Liu 1987 [54]).

In the midfield region, however, the scaling parameter is the diameter of the jet D. The Strouhal number becomes \( St_D = \frac{f \cdot D}{U_s} \), where \( U_s \) is the velocity of the jet.

According to experimental studies and calculations on Orr-Sommerfeld stability, the Strouhal number in a jet is in the range of 0.2 to 0.5. A two-dimensional roll-up process is completed downstream of the location where the preferred mode reaches its maximum amplitude. A sub-harmonic component is then generated with lower amplitude and half of the frequency of the preferred mode. Large eddies lose their energy and break down into less organized structures and smaller in terms of frequency. A length parameter has been defined by Kolmogorov to characterize the size of the smallest dissipating eddies: the Kolmogorov microscale, \( \eta = \left( \frac{v}{\varepsilon} \right)^{1/4} \), where \( v \) is the dynamic viscosity and \( \varepsilon \) the dissipation rate of the large eddies.

Turbulent jet flows have high Reynolds number, and therefore are unstable. The instability results in amplification of any disturbance. The flow solution of the Navier-Stokes equation is known to be of the form of

\[
\vec{R} = [U, V, W, p] = [U(x, y, z, t), V(x, y, z, t), W(x, y, z, t), p(x, y, z, t)]
\]

A small disturbance vector can be introduced: \( \vec{\delta} = [\delta u, \delta v, \delta w] \). Assuming \( \vec{U} \) to be a function of y only and in the x-direction and neglecting non-linear terms, the Navier-Stokes equations become:
\[
\frac{\partial \tilde{u}}{\partial t} + U \frac{\partial \tilde{u}}{\partial x} + V \frac{\partial \tilde{u}}{\partial y} + \tilde{v} \frac{\partial U}{\partial y} = -\frac{1}{\rho} \frac{\partial \tilde{p}}{\partial x} + \nu \left( \frac{\partial^2 \tilde{u}}{\partial x^2} + \frac{\partial^2 \tilde{u}}{\partial y^2} + \frac{\partial^2 \tilde{u}}{\partial z^2} \right)
\]
\[
\frac{\partial \tilde{v}}{\partial t} + U \frac{\partial \tilde{v}}{\partial x} = -\frac{1}{\rho} \frac{\partial \tilde{p}}{\partial y} + \nu \left( \frac{\partial^2 \tilde{v}}{\partial x^2} + \frac{\partial^2 \tilde{v}}{\partial y^2} + \frac{\partial^2 \tilde{v}}{\partial z^2} \right)
\]
\[
\frac{\partial \tilde{w}}{\partial t} + U \frac{\partial \tilde{w}}{\partial x} = -\frac{1}{\rho} \frac{\partial \tilde{p}}{\partial z} + \nu \left( \frac{\partial^2 \tilde{w}}{\partial x^2} + \frac{\partial^2 \tilde{w}}{\partial y^2} + \frac{\partial^2 \tilde{w}}{\partial z^2} \right)
\]

A solution of the disturbance vector can be written, in cartesian coordinates, as:

\[
\tilde{r} = [\tilde{u}, \tilde{v}, \tilde{w}, \tilde{p}] = [\tilde{u}(y), \tilde{v}(y), \tilde{w}(y), \tilde{p}(y)]e^{i(kx + mz - \beta \omega)}
\]

Or in cylindrical coordinates:

\[
\tilde{r} = [\tilde{u}, \tilde{v}, \tilde{w}, \tilde{p}] = [\tilde{u}(r), \tilde{v}(r), \tilde{w}(r), \tilde{p}(r)]e^{i(\alpha x + m\phi - \beta \omega)}
\]

where \( \beta \) and \( m \) are real numbers, called circular frequency and wave number.

\( \alpha = \alpha_r + i\alpha_i \) is a complex number. \( \alpha_r \) is the axial wave number and \( \alpha_i \) the spatial growth rate. As the disturbance follows the behavior of \( e^{-\alpha_i x} \), a negative \( \alpha_i \) results in an unstable disturbance. \( m \) is defining the modes. \( m = 0 \) correspond to the axisymmetric mode, \( m = 1 \) correspond to the helicoidal mode.

Michalke (1971) [49] and Chan (1974) [10] first showed that large-scale structures can be well modeled by linear stability theory. Armstrong et al. (1977) [1] and Stromberg et al. (1980) [67] suggested that the large-scale structure of turbulence in a circular jet is dominated by the axisymmetric and the first order azimuthal components of turbulence. Plots of the spatial growth rate \(-\alpha_i \theta\) versus the Strouhal number \( \frac{\beta \theta}{U} \) have been provided by Cohen & Wygnanski [14] and Michalke & Herman (1982) [50]. They highlighted that the natural instability changes as the streamwise location varies. They also found the Strouhal number of the preferred mode to be in a range of 0.10 to 0.60 as Armstrong et al. suggested. Petersen and Samet [59] found an experimental value of 0.40
in agreement with this. However, amplification rates predicted by the linear stability theory are much larger than the measured values. Non-linear interactions are taking place between modes as revealed in shear layers by experiments lead by Ho and Huang (1982) [32]. Binary and trinary mode interactions in the mixing layer have been theoretically shown by Nikitopoulos and Liu [54] [55].

1.1.1.2 Annular Jet

Investigations of the annular jet have been carried out by Ko and Chan (1979) [40] and Ko and Lam (1984) [42]. We can distinguish three regions in the annular jet, as Ko & Chan proposed: an initial merging zone, an intermediate merging zone and a fully merged zone (Figure 1.1).

The initial merging zone starts at the exit of the nozzle and finishes at the end of the potential core. The potential core mixes with the outer mixing region in this zone and develops a toroidal vortex street similar to the one of the single jet. According to Ko & Chan’s works, the vortices travel downstream with a velocity of about 0.6 \( U_0 \) and the corresponding Strouhal number \( \frac{f_j \cdot D_0}{U_0} \) is about 0.6.

The following zone is called the intermediate mixing zone. The jet vortices from the outer region now meet the wake vortices from the recirculating region, where the Strouhal number \( \frac{f_w \cdot D_0}{U_0} \) is about 0.3. The Strouhal number of the vortices from the outer mixing region is more important; thus they will be preferred and grow downstream as presented by Ko & Chan. The locus of the maximum mean velocity intercepts the jet centerline at a point called reattachment point.
Further downstream, about 1 to 1.5 \( D_0 \) from the reattachment point, the flow is fully developed. We are now in the fully merged zone. This zone is similar to a single jet. The curves \( U/U_{\text{max}} = \text{const.} \) are straight convergent lines in this zone.

### 1.1.1.3 Coaxial Jets

The main parameters of coaxial jets are the velocity ratio of the inner jet to the outer \( \lambda^{-1} \), the area ratio of the outer nozzle to the inner, and the Reynolds numbers based on the hydraulic diameters of the nozzles

\[
\begin{align*}
\text{Re}_{Dh_i} &= \frac{U_i \cdot Dh_i}{\nu}, \quad \text{Re}_{Dh_o} = \frac{U_o \cdot Dh_o}{\nu}
\end{align*}
\]

where \( \overline{U_i} \) and \( \overline{U_o} \) are the inner and outer bulk velocities respectively.

Early studies of circular coaxial jets (Forstall and Shapiro 1950 [19], Chigier and Beer 1964 [13], Williams et al. 1969 [72]) explored mean flow properties for various Reynolds numbers, various velocity ratios \( \lambda^{-1} \), and various circular geometries. Many experiments have been carried out for velocity ratios less than 1. An extensive presentation of these experiments with their respective parameters has been given by
Samuel Bonnafous [8] (see Table 1.1). Ko and Au (1985) [39] presented experimental results of mean velocity, turbulent intensity pressure measurements, and gave velocity and pressure spectra.

**Table 1.1 Works on coaxial jet flow (Samuel Bonnafous [8])**

<table>
<thead>
<tr>
<th>Author</th>
<th>Fluid</th>
<th>Do (mm)</th>
<th>Di (mm)</th>
<th>Dt (mm)</th>
<th>Dho (mm)</th>
<th>Area Ratio l-1</th>
<th>Uo (m/s)</th>
<th>Ui (m/s)</th>
<th>Re (Dho)</th>
<th>Re (Dhi)</th>
</tr>
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<tr>
<td>Dahm, Frieler, Tryggvason (1992) [17]</td>
<td>water</td>
<td>76.45</td>
<td>53.34</td>
<td>55.80</td>
<td>20.65</td>
<td>0.96</td>
<td>0.59-4.16</td>
<td>varying</td>
<td>0.11</td>
<td>varying</td>
</tr>
<tr>
<td>Ko, Au (1985) [39]</td>
<td>air</td>
<td>40.00</td>
<td>20.00</td>
<td>22.00</td>
<td>18.00</td>
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<td>20.00</td>
<td>22.00</td>
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<td>30.00</td>
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<td>50.00</td>
<td>15.00</td>
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</tr>
<tr>
<td>Bonnafous, S. Nikitopoulos DE (2001) [8]</td>
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<td>15.24</td>
<td>17.78</td>
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</tbody>
</table>

Velocity and turbulence in the near field have been studied experimentally for velocity ratios greater than 1 ($1.4 \leq l^{-1} \leq 3.3$) by Ko and Kwan (1976) [41] and Kwan and Ko (1977) [45].

For $l^{-1} < 1$, the coaxial jet presents the same three zones as the annular jet: initial merging, intermediate merging, and fully developed. Similarity of the mean velocity and turbulence intensity profiles has been found in each zone. However, only the mean velocity profiles seem to present similarity in the inner mixing region. The initial merging zone has a major effect on the downstream development of large-scale structures as shown by forcing experiments by Tang and Ko (1993) [68] for a velocity ratio of 0.3.
The outer mixing region presents many similarities to the single jet. The properties of the outer mixing region are quite independent of the velocity ratio $\lambda^{-1}$. In comparison to vortices in the inner mixing region, the ones in the outer mixing region have a lower frequency and a larger scale.

In the inner mixing region, a shed of vortices is observed. They develop alternatively from the inner and the outer lip of the inner nozzle rotating outward and inward respectively. According to the works done by Ko and Au (1985) [39], the behavior changes with $\lambda^{-1}$.

For $0.8 \leq \lambda^{-1} \leq 1.0$, the alternate shedding vortical structures are entrained outwards under the rotational motion of the outer structures that prevail.

For $\lambda^{-1} \leq 0.5$ the inner initial vortices develop from the outer lip of the nozzle and rotate inward towards the jet axis, in an opposite motion to the outer vortices. These vortices are known as coflowing-wake-vortices. When $\lambda^{-1}$ becomes small, the strength of these vortices decreases.

For $0.5 \leq \lambda^{-1} \leq 0.8$ both coflowing-wake-vortices and alternate shedding vortices are present. When $\lambda^{-1}$ grows, coflowing-wake vortices weaken and alternate shedding vortices increase in strength.

The large-scale vortical structures and their interactions have been shown by flow visualization by Dahm et al. (1992) [17]. Several $\lambda^{-1}$ have been investigated for low Reynolds numbers.

According to those observations, one can say that coaxial jets are the transition between single jet and annular jet.
1.1.2 Control

If the interaction between the jet and the surroundings is important for the development of large scales structures, the flow control also has a major role to play. There are two types of control: passive and active.

1.1.2.1 Passive Control

Passive control is the geometrical modification of the jet nozzle. Those modifications will affect the jet properties and thus the flow conditions. Pipe nozzles and contracting nozzle will present different properties. The effects of a contraction chamber before the exit of the nozzle have been extensively presented by Wai-Ho Choy [12].

Au and Ko [2] suggested that the thickness of the lip is another effective way to control the flow. For example in coaxial jets, a thinner lip will enhance the mixing rate. Noise suppression results depend also on vortices linked to the lip thickness. The exit of the nozzle itself can present various geometries. Many shapes have been carried out using regular geometry figures such as circle, ellipse, square, rectangle, triangle, etc.

1.1.2.2 Axis-switching

Works done on elliptic nozzles by Ho and Gutmark [31] and on rectangular nozzles by Hertzberg and Ho [30] for different aspect ratios have shown that the mixing rate is higher than in circular jets. According to Crow and Champagne (1971) [16], Brown and Roshko (1974) [9], and Hussain (1986) [35], the entrainment rate is controlled by large-scale coherent vortical structures. In non-axisymmetric jets, the improvement of the mixing rate is due to a deformation of the shear layer vortex rings, leading to axis switching: Grinstein and Kailasanath (1995) [25] found higher entrainment rates where the vortices roll up.
Axis switching is a change of the axial cross section of the jet, which can evolve through shapes similar to the initial one but with its axis rotated by an angle characteristic to the initial shape (Grinstein 2001 [21]). It has been observed in many laboratory experiments on non-circular jets as Sforza et al. (1966) [66], Sfeir (1979) [65], Krothapalli et al. (1981) [44], Husain and Hussain (1983) [34], Tsuchiya et al. (1986) [71], Ho and Gutmark (1987) [31], Gutmark et al. (1989) [28], Quinn and Militzer (1988) [61], Toyoda and Hussain (1989) [69], Hussain and Husain (1989) [36], Quinn (1992) [60], and more recently Bonnafoe (2001) [8]. This change is not due to helical turning of the jet column as Tsuchiya et al. and Ho and Gutmark explained.

A description of axis switching is given by Grinstein and De Vore (1992) [22] (see Figure 1.2). A vortex ring can be considered as a thin vortex tube. Its local velocity can be expressed as \( \tilde{u} \sim C \cdot \tilde{b} \log(1/\sigma) \), as presented by Batchelor (1967) [3]. \( C \) is the local curvature of the tube, \( \sigma \) its local cross-section, and \( \tilde{b} \) the binormal to the plane containing the tube. A higher curvature implies a higher local velocity. The change in the azimuthal velocity will result in deformation of the vortex ring structure. In the case of a square jet flow at the exit, the curvature moves to infinity at the corners while it is zero on the side. The corners of the ring will move faster in the streamwise direction. The ring deforms, and the corners will start moving inward of the jet while the sides move outward. The curvature of the side increases while the curvature of the corners decreases. The shape of the ring changes to eventually become a flat square with its axis switched by 45 degrees. One, several, or no axis switching can occur depending on the initial conditions.
Meanwhile, streamwise vorticity produced at the corners plays an important role as presented by Grinstein and De Vore. Liepmann and Gharib (1992) [47] demonstrated the importance of streamwise vortices in the entrainment process. The stretching of streamwise vorticity results in formation of braid vortices (or hairpin vortices) that interact with the vortex ring and provoke its breakdown, enhancing the transition to turbulence (Gutmark and Grinstein 1999 [26]). Zaman (1996) [74] suggested that streamwise vorticity can delay or accelerate axis switching according to the initial conditions. Grinstein (2001) [21] stated that the axis switching depends on a strong interaction between azimuthal and streamwise vorticity.

The occurrence of axis-switching, the number of switch-overs, and the distance from the exit of the first cross-over have been found to depend on several variables (Koshigoe et al. [43], Grinstein et al. [24]):

![Figure 1.2 Deformation of vortex rings (Grinstein and De Vore [22])](image)
- Aspect ratio of the exit of the nozzle in the cases of rectangular and elliptic nozzles
- Reynolds number and Mach number of the jet
- Initial conditions of the jet (azimuthal distribution of the momentum thickness, momentum thickness and initial turbulence level)

Krothapalli et al (1981) [44] found a linear relationship between the distance of the first cross-over from the exit and the aspect ratio of the nozzle for a rectangular jet based on their experiments and the ones carried out by Sforza and by Sfeir. This result is partially due to the variation of the distance for the width along the minor axis to overtake the width along the major axis. This relationship has been confirmed later by experiments by Tsuchiya (1985) [71] and Grinstein (1995). The latest added that the axis rotation period is also a linear function of the aspect ratio for rectangular and elliptic jets.

The azimuthal distribution of momentum thickness at the exit of the nozzle plays a major role in the occurrence of axis-switching as described by Grinstein et al [24]. If the initial shear layer grows at the same rate on the major and minor axis or if the initial momentum thickness on the major axis (corner for square nozzle) is greater than at the minor axis (flat side for square nozzle) then there will be no cross over. This is why axis-switching has been found to be faster for contoured nozzles than for orifices, and faster for orifices than for channel or pipe nozzles (see Krothapalli et al., 1981, Tsuchiya et al., 1985, Husain and Hussain, 1989 [36], Grinstein and Gutmark, 1995).

Faster growth on the flat sides (or small curvature) will be found for higher initial turbulence level and higher ratio of the equivalent diameter to the local momentum thickness of the shear layer $D_e/\theta$. Meanwhile, at the corner (or high curvature), the
stretching as described before will be faster if the jet structure is more coherent and the cross-section of the vortex is thinner as foreseen by Husain and Hussain (1989). Thus higher initial turbulence level and thinner momentum thickness at the exit of the nozzle will improve the axis-switching (Grinstein et al. [24]). This has been verified experimentally by Grinstein et al.

Finally Mach number and Reynolds number of the flow have a very weak influence on the axis-switching (Husain and Hussain, 1983 [34], Grinstein et al.).

1.1.2.3 Non-axisymmetric Nozzle Exit Shapes

Elliptic jets have been studied by Gutmark and Ho (1986) [27], who observed axis switching. The entrainment rate was found to be eight times higher than in circular jets. Flow visualization carried out by Hussain and Husain (1989) [36] gave evidence of a third switch over and identified braid vortices. Both studies showed that axis switching can be seen quite far downstream from the jet exit (up to 100 equivalent diameter).

The same conclusions have been drawn by Tsuchiya et al. (1986) [71] about rectangular jets. Krothapalli et al. (1981) [44] proposed a linear relationship between the distance from the nozzle exit to the first cross over point, and the aspect ratio of the nozzle. Laboratory studies have been carried out by Zaman [74] [75] on subsonic rectangular jets with varying aspect ratios.

Square jets have been studied by Sforza et al. (1966) [66], duPlessis et al. (1974) [18] and Trentacoste and Sforza (1976) [70]. Their studies did not investigate precisely the square vortical interactions. Quinn and Militzer (1988) [61] compared square jets properties to circular jets properties. They found a faster spreading rate at similar distance from the exit of the nozzle and similar hydraulic diameter. Off-center velocity peaks have
been found by Quinn (1992) [60] for square slot. Simulations on square jets have been processed by Grinstein et al. (1995) [24] and Grinstein and De Vore (1996) [23]. Grinstein (2001) [21] found a significantly larger jet spreading for square jets than for rectangular jets with an aspect ratio of 2 or 3: rib pairs rather than single ribs, are present at the corners, and the vortex rings break down closer to the jet exit.

Isosceles and equilateral triangular jets have been studied by Schadow et al. (1988) [63]. As in square jets, mixing is improved by self-induction at the corners of the triangle. Koshingoe et al. (1988) conducted experimental and numerical studies. They showed differences between orifices, pipes and contoured nozzle exits and proposed conditions for axis-switching occurrence. Schadow et al. (1990) [64] found evidence of enhancement in fine scale mixing in the corners and large scale mixing on the flat sides.

Zaman [74] [75] proposed an alternative passive control: tabs at the exit of the nozzle. The perturbation introduced by the presence of tabs modifies the streamwise vorticity that either stops or promotes the axis switching.

1.1.2.4 Coaxial Non-Axisymmetric Jets

Few studies have been carried out on coaxial non-axisymmetric. Flow visualizations have been made by Bitting et al. (1997) [5] at low velocity ratio and various Reynolds number for various shapes of coaxial nozzle exits: square, triangular, lobed and axisymmetric. A comparison between square and circular coaxial jets at similar conditions for several velocity ratios has been presented by Bitting et al. (1998) [7] and followed by DPIV comparison (2001) [6]. They found that the internal unmixed region becomes smaller as the velocity ratio decreases. Flow visualization and spectra measurements by Nikitopoulos and Bitting (2000) [53] showed differences in initial
velocity profiles and mixing enhancement for coaxial square jets compared to circular. Bonnafous (2001) [8] presented experimental results on coaxial square jets with evidence of axis switching for a velocity ratio \( \lambda^{-1} = 0.5 \).

### 1.1.3 Active Control

Active control is a modification of the jet properties by forcing the flow at certain bands of frequencies. Excitation can result in an amplification of turbulence in the mixing layers of a jet, increasing the coherence of the vertical structures (Crow and Champagne 1971 [16], Moore 1977 [52], Hussain and Zaman 1981 [37]). The primary potential core can also be shortened. This is a sign of a faster mixing, as presented by Lepicovsky et al. (1985) [46].

There are two main ways to force the flow: mechanically, with a vibrating ribbon or an oscillating flap at the exit of the nozzle, or acoustically, with loudspeakers inside the nozzle. Many experimentalists, because of the wide range of frequencies that can be covered, have preferred the acoustical method. Exciting a turbulent jet at its preferred mode helps large structures to be more coherent.

Three input parameters are critical for active control:

- Frequency of excitation.
- Excitation level.
- Mode of excitation.

Theoretical analyses have been made, and quantitative data have been taken on forced axisymetric jets (Cohen and Wygnanski 1987 [15], Long and Petersen 1990 [48]). Experimental investigations of active controlled jets are presented by Zaman and Hussain (1980) [76].
1.1.3.1 Frequency of Excitation

A logical frequency that many experimentalists used is the natural frequency of the mixing layers. However interesting results have been obtained from sub-harmonic forcing both in numerical simulations (Patnaik et al. 1976 [57], Riley and Metcalfe 1980 [62]) and in experimental studies (Zangh et al. 1985 [77], Hajj et al. 1992 [29]). Theoretical studies on sub-harmonic forcing have been carried out by Kelly (1967) [38], Monkewitz (1988) [51] and Nikitopoulos and Liu (1987) [54].

1.1.3.2 Excitation Level

The excitation level needs to be correctly chosen. Crow and Champagne [16] characterized the excitation level in plotting the turbulent intensity of the flow at the centerline of the jet versus the forcing amplitude. The plot shows a quite linear relationship for a range of 0.1 to 1%. Then it reaches saturation. At a good excitation level, the velocity power spectrum will not differ from the unforced spectrum, but from a narrow band centered on the forcing frequency. If the excitation is too low, no control will be possible on the flow. If it is too high, the entire spectrum will suffer from non-linear distortion.

1.1.3.3 Modes

An excitation with helical waves can be done as Long and Petersen [48] performed for an axisymmetric jet. The shape of the velocity cross-section changes with the spinning mode number. It becomes elliptic for a combination of \( m = +1 \) and \( m = -1 \), and square for a combination of \( m = +2 \) and \( m = -2 \). Their results can then be compared with the passive control method. Pashereit et al. (1992) [56] showed visualization of patterns ensuing from secondary interaction of helicoidal and axisymmetric modes. Azimuthal
mode modulation can control cross sectional shape of an iso-velocity contour in the near-field (Cohen and Wygnanski [15]).

Acoustical forcing can also influence the pairing between fundamental and sub-harmonic vortices. A phase difference between the two frequencies will affect the pairing. The relative phase of the two modes is a determining factor of the intramodal energy transfer (Monkewitz 1988 [51]). The coalescence of the two vortices will be improved as the phase difference decreases. When the phase difference is $\pi$, no pairing occurs. A shredding interaction takes place instead.

Petersen and Clough (1992) [58] get a better resonant interaction in forcing the 3/2 harmonic, the fundamental and the sub-harmonic than the last two alone. A three-mode interaction has been theoretically explained by Nikitopoulos and Liu (2001) [55]: the three-mode interaction has a weak direct effect and a strong indirect effect on the flow.

1.2 Objectives

This study follows the work done by Way-Ho Choy and Samuel Bonnafous on coaxial jets. The aim is to study passive forcing on coaxial jet with a inner to outer velocity ratio of 1.5. Chapter 6 focuses on the comparison between airflows exiting square contour coaxial nozzles and circular contoured coaxial nozzles. Each jet needs to be studied separately. The outer jet has been studied by Samuel Bonnafous. Chapter 5 gives some complements on the outer single jet. The inner single jet is studied on Chapter 4. The testing facility, the hotwire equipment and the processing are described in Chapter 2. An analysis of the natural disturbances due to the nozzle cavities is presented in Chapter 3.
Chapter 2 Facility

2.1 Testing Facility

2.1.1 Coaxial Circular and Square Nozzles

This chapter will focus on the square coaxial nozzle. Details on the circular coaxial nozzle can be found in Wai-Ho Choy’s thesis [12].

Experiments have shown that initial conditions play a determining role for the mixing of the jet. Special care has been taken in the design of the nozzles. To be able to compare results from the square nozzles to those from the circular nozzles, the square nozzles have been designed to have the same hydraulic diameter as the circular nozzles.

The dimensions of the nozzle exits are shown on Figure 2.1.

![Figure 2.1 Geometric characteristics of the nozzle exits](image)

The hydraulic diameter is defined as four times the area to perimeter ratio:

\[ D_h = \frac{4 \cdot A}{P} \]. We get for the present nozzles:

\[ Dh_i = 0.6" = 15.24 \text{ mm} \]
\[ Dh_o = 0.8" = 20.32 \text{ mm} \]
Contoured nozzles have been used. The exit plane is the same for both inner and outer nozzles. The design has been described by Wai-Ho Choy et al. [11]. A matched third-degree polynomial with zero first derivative ends has been used to design the nozzle in order to avoid separation on the inner jet surface (see Figure 2.2). The contraction ratios for the nozzles are as follows:

- Inner nozzle: 25:1
- Outer nozzle: 13.6:1

The outer to inner exit area ratio is 4.89.

![Square nozzle contours](image)

**Figure 2.2 Square nozzle contours**

In order to encourage the mixing of the inner and outer jets, the lip between the two jets has been designed as thin as possible. A thinner lip is supposed to encourage the mixing between the inner and outer jet to happen closer to the exit of the nozzle in reducing the distance between the two jets.

Both nozzles are mounted together on the same body (Figure 2.3). The air is supplied from two distribution chambers through eight 3/8” Teflon tubes for the outer
and four 1” rubber pipes for the inner jet. The air goes through a series of 4 wire-mesh screens to damp large-scale motions and to reduce fluctuation levels.

![Figure 2.3 Square and circular coaxial nozzles](image)

Speakers can be added to the body for acoustic forcing. 4 holes on the side of the outer nozzle and one on the bottom of the inner nozzle have been designed for this purpose.

The body is mounted on a stand, aligned with precision with a rotary table that allows rotating the nozzles about the jet axis (Figure 2.4).
2.1.2 Air Supply

Air is supplied by a tank where dry air is compressed at 170 to 280 psi. Because of the fluctuations in the upstream pressure, a second tank has been added, where the air is stored at the constant pressure of 110 psi. The flow is then divided in two lines each supplying one nozzle. On each line, a regulator valve keeps the flow at 80 psi. The flow rate is measured by two Rosemount orifice flowmeters. The outer flowmeter has an orifice with a diameter 0.642” installed. The inner has an orifice with diameter 0.248” installed. Flow rates are controlled downstream of the line by a flow control valve. The air is then brought to a distribution chamber by a 1” rubber pipe, were it is divided trough four 1” rubber pipes for the inner, and eight 3/8” Teflon tubes for the outer. Figure 2.5 shows the air supply diagram.
Figure 2.5 Jet facility and forcing equipment
2.1.3 Forcing Equipment

To force the flow at a particular frequency, acoustic excitation has been chosen. A National Instruments AT-AO-10 waveform generator board can generate five different signals. The signal is sent to an amplifier. Two different amplifiers have been used: Amplifier #1 - Radio Shack Stereo Power P.A. Amplifier MPA200, and Amplifier #2 - Optimus Stereo Amplifier. Independent signals can be sent to five identical SANMING Electronics speakers (model S-75A, 75 Watts, 16 ohms). Four speakers (spk. 1 to 4) are set on the side of the outer nozzle oriented 90 degrees from each other, allowing azimuthal excitation. One speaker (spk. 0) is set at the bottom of the inner nozzle. A diagram of the forcing system is shown on Figure 2.5. In Crow and Champagne study [16], the excitation is done right at the exit of the jets whereas it is done inside the nozzle for the present layout. Thus particular care has been taken to evaluate the effect of the nozzle cavities on the excitation signal at the exit of the jets (i.e. the transfer function of the nozzles). The results are presented later in this thesis.

The excitation signal read at the exit of the nozzles is sampled by two means: microphones for acoustic response and a hotwire anemometer for velocity response. Two ¼” Brüel & Kjær microphones, type 4135 wired to two Brüel & Kjær NEXUS Microphone Conditioning Amplifier, allow for pressure measurements at the exit of the jets. The hotwire measurements are described in section 2.3.1.

2.2 Operating Conditions

Wai-Ho Choy’s experiments [12] on the axis-symmetric coaxial jet have been made at a modest Reynolds number. This study kept the same Reynolds number, \( \text{Re}_{\text{Db}} = 15,300 \) to have a consistent set of data to compare. The important parameter is
the ratio of the inner jet velocity to the outer jet velocity $\lambda^{-1}$. Three ratios have been studied by Wai-Ho Choy: 0.5, 1.0 and 1.5. Samuel Bonnafous [8] studied the square coaxial jet with a velocity ratio of 0.5. The present study is concerned with the square coaxial jet of a velocity ratio 1.5. The Reynolds numbers are based on the hydraulic diameter of the co-flow. There are three ways to characterize the Reynolds number and the velocity ratio: using the exit bulk velocities ($Re_{Dho} = \frac{U_o D_{ho}}{\nu}, \lambda^{-1} = \frac{U_i}{U_o}$), or using the centerline velocity at the exit ($Re_{Dho CL} = \frac{U_{o CL} D_{ho}}{\nu}, \lambda^{-1 CL} = \frac{U_{i CL}}{U_{o CL}$). The flow conditions for the present study are presented with the flow conditions for the corresponding circular jet experiment in Table 2.1. The experiments are made with air at 80 psi at temperatures varying from 21 to 27 degrees Celsius. The viscosity varies from $1.81 \times 10^{-5}$ to $1.85 \times 10^{-5}$ kg/m/s. The side plane and the diagonal plane were studied separately in two different experiments.

<table>
<thead>
<tr>
<th>Table 2.1 Flow conditions</th>
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<tbody>
<tr>
<td><strong>Flow rate $Q$ (SCFM)</strong></td>
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<tr>
<td><strong>Bulk velocity $U$ (m/s)</strong></td>
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<tr>
<td><strong>Max velocity $U_{max}$ (m/s)</strong></td>
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<tr>
<td><strong>Centerline velocity $U_{CL}$ (m/s)</strong></td>
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<tr>
<td>$U_e = \frac{(U_{CL,o} + U_{CL,i})}{2}$</td>
</tr>
<tr>
<td>$Re_{Dho}$</td>
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<tr>
<td>$Re_{Dho CL}$</td>
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<tr>
<td>$\lambda^{-1}$</td>
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<tr>
<td>$\lambda^{-1 CL}$</td>
</tr>
</tbody>
</table>
2.3 Data Acquisition

2.3.1 Constant Temperature Anemometry

The only means of velocity measurement used in this study is constant temperature anemometry. TSI hot film probes were used, connected to a TSI IFA 300 hot-wire acquisition system. The temperature of the hot film is kept constant by varying the current through the film. Output voltages are recorded and then processed using a calibration polynomial. Each probe used was calibrated regularly before a period of use. A thermocouple connected to the acquisition system measured the temperature of the airflow upstream of the flow meter allowing for temperature corrections due to departure from the calibration conditions. 65,536 measurements are taken at each location, at a sampling rate of 50,000 Hz. Aliasing is prevented by a 20,000 Hz low pass filter.

A TSI 1210-T1.5 single hot wire was used in the axis-symmetric jet measurements. A TSI 1241-10 X-hot film was used for the square jet measurements. The X-wire was chosen as it gives two components of the velocity. Results were obtained of the main velocity component (the axial component), and the radial component in the plane of measurement. A hot film has been preferred to a hot wire after carrying some comparative tests, because of its robustness.

2.3.2 Traverse System

The measurements are taken in vertical planes, passing through the centerline of the jet. A two-degree-of-freedom traversing system allows moving the probe in the axial (stream wise) direction (x-direction) and span wise (y-direction) of the jet airflow, covering the xy plane. Stepping motors controlled by the IFA 300 system give a 0.05 mm positioning precision. After each move, a 0.6 second delay is applied before taking
data. For the axis-symmetric jet the scan was in the diametric plane. For the square jets, the scan were made in the planes of symmetry, were the azimuthal component of the velocity could be assumed negligible: the median plane, or side plane (SC), and the diagonal plane (SD) (Figure 2.6).

![Figure 2.6 Scanning planes for the circular and the square jet](image)

The measurement grid is made of 21 levels perpendicular to the centerline for the circular jet, and 27 for the square jet, from 1 mm \((0.05D_{ho})\) at the exit to 210 mm \((10.33D_{ho})\) in the far field. The probe scans the whole span of the jet at each of these levels. The number of points at each level and their spacing vary depending on the complexity of the jet from about 400 points in the near field to about 100 in the far field.

### 2.4 Data Processing

A file containing the voltages measured by both hot-film probes and the related time is generated by the IFA 300. This file is processed by the software to get the velocity information according to the calibration file for each point in space. A computational process is then applied using a set of Fortran programs developed in-
house. The velocity is then known in the three-dimensional space \((x, y, t)\). Mean and fluctuating RMS velocities are obtained through time averaging.

An error can be made if the measured velocity \((u, v)\) is not perfectly aligned with the nozzle axis and does not correspond to the actual \((U, V)\) velocity (see Figure 2.7). A geometric correction is applied:

\[
U = u \cos(t_0) + v \sin(t_0) \\
V = -u \sin(t_0) + v \cos(t_0)
\]

![Figure 2.7 Correction angle](image)

For a specific shear layer, the velocity jump \(\Delta U = U_{\text{max}} - U_{\text{min}}\) can be obtained from the measurements. The locations where the velocity drops by 10, 50 and 90% of the velocity jump are interpolated (respectively \(Y_{01}, Y_{05}\) and \(Y_{09}\)). The shear layer thickness is then defined as \(\delta = |Y_{01} - Y_{09}|\). The momentum thickness is calculated using the formula:

\[
\theta = \int_{Y_{01}}^{Y_{09}} \left( \frac{U(y) - U_{\text{min}}}{\Delta U} \right) \left( 1 - \frac{U(y) - U_{\text{min}}}{\Delta U} \right) dy
\]

A parallel processing is computed in the Fourier space. The data obtained by the IFA 300 are a sample of 32,768 points at 50,000 Hz for a span of 0.65536 seconds. A fast
Fourier transform is computed using 32 blocks of 10,000 points with a sampling rate of 50,000 Hz, and a resolution of 5 Hz. The spectrum of the velocity is now known in the three dimensional space \((x, y, f)\).

Information about the energies is also derived. The energy of a mode is computed, integrating at each point \((x, y)\) of the measurement plane, the square of the Fourier coefficient over a narrow frequency band (+/-5% around the peak):

\[
E_f = E_{f_0}(x, y, f) = \int \left( \frac{A(x, y, f_0)}{A_0} \right)^2 df
\]

where \(A_0 = \sqrt{\frac{1}{\Delta f} \int A^2 df}\) is the integrated energy over the entire spectrum at the exit center line point.

The energy of each mode is then integrated over the shear layer regions for each streamwise location \(x\). For a specific frequency \(f_s\) in the region \(R(x)\), the integration is computed as follows:

\[
E_{R(x),f_s} = E_{R(x),f_s}(x) = \frac{1}{\delta(x)} \int_{0.95f_s}^{1.05f_s} \int_{R(x)} A^2(x, y, f_s) dy df
\]

The integration is scaled by the shear layer thickness at the specific streamwise location \(\delta(x)\). The energy density contains both coherent and random contributions of the wave \(f_s\). In the shear layer, \(R(x)\) has been defined as the region where the velocity is in the range of \([0.1(U_{\text{max}} - U_{\text{min}}) - 0.9(U_{\text{max}} - U_{\text{min}})]\).
Chapter 3  System Identification of the Square Coaxial Nozzles

Single and coaxial jets are studied in the frequency domain further in this thesis. It is therefore important to distinguish the jet flow natural disturbances from those due to the nozzle interior configuration. A transfer function of the nozzles is necessary to isolate the natural frequencies of each nozzle.

3.1 Operating System

As described in section 2.1.3, a set of four speakers in the outer nozzle, and one speaker in the inner make forcing experiments possible.

Uniform white noise is generated by a National Instrument AT-AO-10 waveform generator through a Labview program. The signal is sent to each speaker after amplification.

A microphone is placed on the side of the nozzle exit at 1mm downstream to read the output signal. After amplification, the output signal is recorded with a Digital Analyzer SRS 785 simultaneously with the input signals.

3.2 Theoretical Model

The system on Figure 3.1 can be seen as a five inputs / single output model as represented Figure 3.2.

The output \( y(t) \) may be considered as the sum of all unmeasured output signals \( y_i(t), i = 0,1,2,3,4 \) and an uncorrelated output noise, \( n(t) \).

\[
y(t) = \sum_{i=0}^{4} y_i(t) + n(t)
\]

Its finite Fourier transform can be written as:
Figure 3.1 System description (from Samuel Bonnafous [8])
Figure 3.2 Five inputs / single output model diagram

\[ Y(f) = \sum_{i=0}^{4} Y_i(f) + N(f) \]

Each unmeasured output signal is a function of the corresponding input signal:

\[ Y_i(f) = H_i(f)X_i(f) \]

The output signal becomes in phase space:

\[ Y(f) = \sum_{i=0}^{4} H_i(f)X_i(f) + N(f) \]

The Fourier transforms \( X_i(f) \) and \( Y(f) \) for records \( x_i(t) \) and \( y(t) \) of length \( T \) are:

\[ X_i(f) = \int_{0}^{T} x_i(t)e^{-j2\pi ft} dt \]
\[ Y(f) = \int_{0}^{T} y(t)e^{-j2\pi ft} dt \]

Autospectral and cross-spectral density functions are defined as:

\[ G_{ii}(f) = G_{x_i,x_i}(f) = \lim_{T \to \infty} \frac{2}{T} E\left[X_i(f)\right]^2 \]
\[ G_{ij}(f) = G_{x_i,x_j}(f) = \lim_{T \to \infty} \frac{2}{T} E\left[X_i^*(f)X_j(f)\right] \]
\[ G_{yy}(f) = \lim_{T \to \infty} \frac{2}{T} E\left[|Y(f)|^2\right] \]
\[ G_{iy}(f) = \lim_{T \to \infty} \frac{2}{T} E\left[X_i^*(f)Y(f)\right] \]

Where \( E[ \ ] \) is the expected value operation taken over a finite number of elements:
\[ E[A(f)] = \frac{1}{N} \sum_{k=1}^{N} A_k(f) \]

\( G_{iy}(f) \) and \( G_{yy}(f) \) can be expanded as:
\[ G_{iy}(f) = \sum_{j=0}^{4} H_j(f)G_{ij}(f) + G_{in}(f) \]
\[ G_{yy}(f) = \sum_{i=0}^{4} \sum_{j=0}^{4} H_i^*(f)H_j(f)G_{ij}(f) + G_{nn}(f) + \sum_{i=0}^{4} H_i^*(f)G_{in}(f) + \sum_{j=0}^{4} H_j(f)G_{nj}(f) \]

The noise \( N(f) \) is assumed to be uncorrelated with the five inputs, thus:
\[ G_{in}(f) = G_{ni}(f) = 0 \]
\[ G_{iy}(f) = \sum_{j=0}^{4} H_j(f)G_{ij}(f) \]
\[ G_{yy}(f) = \sum_{i=0}^{4} \sum_{j=0}^{4} H_i^*(f)H_j(f)G_{ij}(f) + G_{nn}(f) \]

The five inputs are mutually uncorrelated to each other as they can be controlled separately. Thus \( G_{ij}(f) = 0 \), for \( i \neq j \). The correlation functions then simplify as:
\[ G_{iy}(f) = H_iG_{ii}(f) \]
\[ G_{yy}(f) = \sum_{j=0}^{4} |H_i(f)|^2 G_{ii}(f) + G_{nn}(f) \]

The system can then be considered a collection of single-input / single-output models. Each transfer function can be written as:
\[ H_i(f) = \frac{G_{iy}(f)}{G_{ii}(f)} \]
The ordinary coherence function $\gamma_{iy}$ is defined by:

$$
\gamma_{iy}^2(f) = \frac{|G_{iy}(f)|^2}{G_{ii}(f)G_{yy}(f)}
$$

Where for all $f$:

$$
0 \leq \gamma_{iy}^2(f) \leq 1
$$

For an ideal case of linear system without noise, the coherence function will be unity. It will be zero if $x_i(t)$ and $y(t)$ are completely unrelated. Low values indicate non-linearity or excessive noise.

### 3.3 Experimental Conditions and Settings

The facility used to send forcing acoustic waves to speakers, to read them and record them from a microphone has been described in section 2.1.3. This paragraph will focus on the protocol and settings only.

To determine the transfer functions $H_i$, an experiment is run sending a white noise signal as $x_i(t)$, while $x_j(t) = 0$ for $j \neq i$. In other terms all speakers are off but one. The white noise is generated at a rate of 25,000 pts/s and at amplitude of 0.5V. The microphone records $y(t)$ through a Bruel and Kjaer Conditioning amplifier with the following settings:

- Low pass filter: 22.4 kHz
- High pass filter: 20 Hz
- Amplification: 100 V/Pa

$x_i(t)$ and $y(t)$ are recorded simultaneously by the SRS digital analyzer with the following settings:
Time record of each ensemble element: $T = 0.250 s$, 
- Number of points in each time record: $n = 4000 \text{ pts}$,
- Sampling rate: $f_s = \frac{n}{T} = 16,380 \text{ pts/s}$.

This study focuses its interest in frequencies below $f_{\text{max}} = 7800 \text{ Hz}$. According to Nyquist’s theorem, the sampling rate must be more than twice $f_{\text{max}}$. This theorem is satisfied here.

The data are averaged over $N = 100$ number of elements.

The digital analyzer performs the Fourier transform with a span of 12.8 kHz, with 16 Hz increments.

3.4 Results

The speakers 1 to 4 exciting the outer nozzle are similar and placed symmetrically. Thus they are expected to present the same transfer function. $H_i(t), i = 1,2,3,4$ are plotted in Figure 3.3 (a) with the transfer function of the speaker exciting the inner nozzle, $H_0(t),(b)$. The similarity of the outer speaker is verified for most of the bandwidth.

Coherence functions of one outer speaker and the inner speaker are presented in Figure 3.3 (c) and (d). Since the frequency response of each outer speaker has just been seen to be very similar, only one coherence function and one unwrapped phase are presented. The coherence function is close to unity for a frequency higher than 500 Hz. Below this frequency, the correlation between the input signal and the output signal is weak and the system is not linearly ideal.
Figure 3.3 Transfer functions of each speaker exciting the nozzles, \( i = 1,2,3,4 \) (a), \( i = 0 \) (b). Coherent functions of speaker 1 (c) and 0 (d). Unwrapped Phase of speaker 1 (e) and 0 (e)
3.5 Validity of the Model

To validate the five-inputs / one-output model, the same white noise signal $x(t)$ is sent to each speaker with amplitude of 0.5 V.

![Diagram](image)

**Figure 3.4 Five-inputs / single-output system with same input $x(t)$**

The system becomes:

![Diagram](image)

**Figure 3.5 Five-inputs / single-output system with same input $x(t)$**
The autospectral density functions $G_{xx}(f)$ and $G_{yy}(f)$, and the cross-spectral density function $G_{xy}(f)$ are computed. The transfer function and coherence function are deduced from:

$$H_i = \frac{G_{xy}(f)}{G_{xx}(f)}$$

$$\gamma_{xy}^2(f) = \frac{|G_{xy}(f)|^2}{G_{xx}(f)G_{yy}(f)}$$

$H_i(f)$ is plotted in Figure 3.6 with $\sum_{i=0}^{4} H_i(f)$. The superposition of the two curves validates the five-inputs / one-output model on most of the bandwidth. The unwrapped phase of the excitation with the four outer speakers $\Phi_i(f)$ is shown on Figure 3.7.
3.6 System Reduction and Transfer Functions

The system can be characterized more precisely if the effect of the speakers can be removed from the transfer function. Because the speakers are identical, they are expected to have the same transfer function $H_s(f)$. With help of an anechoic chamber, it has been verified and this transfer function has been identified.

Each $H_i(f)$ can now be described by the product of the transfer function $H_s(f)$ of the speaker by the transfer function of the corresponding nozzle. Since all the outer nozzle frequency responses are similar, a general outer transfer function can be defined:

$$H_{outer}(f) = \frac{H_1(f)}{H_s(f)} = \frac{H_2(f)}{H_s(f)} = \frac{H_3(f)}{H_s(f)} = \frac{H_4(f)}{H_s(f)}$$

The inner transfer function can be defined as well:

$$H_{inner}(f) = \frac{H_0(f)}{H_s(f)}$$
The new model becomes:

\[ x_3(t) \rightarrow H_s(f) \rightarrow H_{inner}(f) \rightarrow n(t) \]

\[ x_1(t) \rightarrow H_s(f) \rightarrow H_{outer}(f) \rightarrow y(t) \]

\[ x_2(t) \rightarrow \Sigma \rightarrow H_s(f) \rightarrow \Sigma \]

\[ x_3(t) \rightarrow H_{outer}(f) \rightarrow \Sigma \]

\[ x_4(t) \rightarrow \Sigma \rightarrow H_{inner}(f) \rightarrow \Sigma \]

**Figure 3.8 Simplified model diagram**

\( H_{inner}(f) \) and \( H_{outer}(f) \) now reveal the natural frequencies of the nozzle cavities.

Figure 3.9 presents the poles and possible zeroes of both transfer functions. The unwrapped phases of both transfer function is presented in Figure 3.10.
Figure 3.9: Poles and zeroes of the inner and outer transfer functions.
Figure 3.10 Unwrapped phases of the inner and outer transfer functions
Chapter 4  Passive Forcing on Inner Single Jet

To understand the coaxial jet, it is important to first study each flow separately. The outer jet with a velocity of 14.2 m/s has been studied by Samuel Bonnafous [8]. The inner jet with a velocity of 19.7 m/s is now studied using the same facility.

4.1  Time Space Results

4.1.1  Initial Conditions

The initial conditions were tailored to match Wai-Ho Choy’s benchmark experiments on the circular coaxial jets, for comparison purposes [12]. The measurements have been made in two planes: the median (SC) and the diagonal (SD). Table 4.1 gives some information about the present study and Wai-Ho Choy’s study.

<table>
<thead>
<tr>
<th></th>
<th>Single circular inner jet (Choy [12])</th>
<th>Single square inner jet (side)</th>
<th>Single square inner jet (diagonal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Re}_e ) (m/s)</td>
<td>20,750</td>
<td>19,930</td>
<td>19,260</td>
</tr>
<tr>
<td>( U_e ) (m/s)</td>
<td>21.35</td>
<td>19.71</td>
<td>19.60</td>
</tr>
<tr>
<td>Maximum velocity ( U_{\text{max}} ) (m/s)</td>
<td>21.93</td>
<td>20.07</td>
<td>20.18</td>
</tr>
<tr>
<td>Momentum thickness ( \theta_o ) (mm)</td>
<td>0.1360</td>
<td>0.1450</td>
<td>0.2407</td>
</tr>
<tr>
<td>Shear layer thickness ( \delta ) (mm)</td>
<td>0.505</td>
<td>0.636</td>
<td>0.836</td>
</tr>
<tr>
<td>( Y_{0.5} / D_{hi} )</td>
<td>0.485</td>
<td>0.500</td>
<td>0.661</td>
</tr>
<tr>
<td>( D_{hi} / 2\theta_e )</td>
<td>56.03</td>
<td>52.55</td>
<td>31.62</td>
</tr>
</tbody>
</table>
The momentum thickness is of the same order for the square and the circular jet. It is higher in the diagonal plane because of the more extensive viscous region in the corner inside the nozzles. The relatively high ratio \( D_{hi} / 2 \theta_e \) shows that the shear layer is very thin compared to the width of the jet. The ratio of the initial momentum thickness at the corner to the initial momentum thickness on the flat side is \( \theta_{0,SD} / \theta_{0,SC} = 1.66 \), greater than \( \sqrt{2} \). The non-uniform initial momentum thickness distribution may alter the presence of axis-switching.

The velocity profiles scaled by the centerline velocity at \( x/D_{hi} = 0.066 \) for the three cases have been plotted on Figure 4.1. The spanwise coordinate \( y \) has been scaled by the diagonal plane geometrical factor \( \sqrt{2} \) for comparison with the other profiles. The profiles indicate reasonably similarity between the square and the circular jets.

Figure 4.1 Initial mean velocity profiles
4.1.2 Streamwise Velocity

Figure 4.3 shows the velocity profiles of the square jet in both planes with Wai-Ho Choy’s results on the circular jet. The shear regions of the jet quickly become larger than the circular jet, and better mixing is implied. The streamwise component of the velocity has been mapped in the median plane (SC) and the diagonal plane (SD) for the square jet (Figure 4.4). Contours of the circular jet in the radial plane are opposed for comparison. Unlike the in the circular jet, a velocity deficit in the potential core is noticeable in both planes of the square jet before 3.5 hydraulic diameters. This deficit corresponds to a deformation of the width of the jet. As shown in Figure 4.2, the velocity half-width is increasing in the side plane, leading to a swelling of the global section of the jet. The width of the jet increases in the side plane and the potential core widens (Figure 4.4). The width decreases in the diagonal plane and the potential core narrows. This results in a crossing over at 1.35 diameters. The trend is then inverted until the section becomes azimuthally uniform. This phenomenon called axis-switching can also be viewed in the velocity profiles of Figure 4.3. A detailed description of axis-switching has been given in section 2.1.1.
Figure 4.3 Mean streamwise velocity profiles
Figure 4.4 Streamwise velocity contours for square (a and b) and circular (c) jets
4.1.3 Spanwise Velocity

Nicely symmetrical axial and span wise velocity profiles at the exit of the nozzle are shown in Figure 4.5. The span wise velocity is weaker in the diagonal plane, which may be explained by the contraction of the jet width in the diagonal plane and the expansion of the jet width in the side plane. Off-center axial-velocity peaks due to vena-contracta effects are also present in Quinn’s experiments [60]. However, as in Samuel Bonnafous’ experiments, the off-center peaks are weaker than in Quinn’s results. Quinn uses a higher Reynolds number (about 184,000) than the one for the present study. The peaks do not exceed 3 % in the present study.

Figure 4.6 shows contours of the spanwise velocity. A nice symmetry is observed in the side plane (SC). However the diagonal plane presents a small asymmetry for \(1D_{hi} \leq x \leq 3D_{hi}\). The third component of the velocity has been assumed to be negligible on the diagonal plane (see 2.3.2). However, just off the diagonal, the flow is strongly three-dimensional. Thus the smallest misalignment can affect the symmetry. After analyzing the data of the single jet, the alignment procedure has been improved and the symmetry is better for the coaxial jet (see Figure 6.6).

The expansion of the jet in the side plane is clearly shown on Figure 4.6-b. On the diagonal plane, the jet has a tendency to shrink in the initial zone. After 3 hydraulic diameters, the jet generally expends in both planes.

4.1.4 Turbulence Quantities

The streamwise turbulence intensity \(\sqrt{u'^2} / U_e\) is mapped in both side (Figure 4.7-a) and diagonal (Figure 4.7-b) planes. Some profiles have been extracted in Figure 4.8.
Figure 4.5 Spanwise and streamwise velocity profiles at $x / D_{hi} = 0.066$

Figure 4.6 Spanwise velocity contours for square single jet
The same general trends are observed in both planes. The spanwise turbulence intensity \( \sqrt{v'^2} / U_e \) and the Reynolds stress tensor \( \tau / U_e^2 = -u'v' / U_e^2 \) are plotted in Figure 4.7-c and Figure 4.7-d respectively. All turbulence intensities show peaks where the local shear in the mean stream wise velocity is found. The shear layer is responsible for high production of turbulence. The results agree with Quinn’s experimental results.

### 4.1.5 Shear Layer Evolution

The evolution of the shear layer growth rates are shown in Figure 4.9 for the side (SC) and diagonal (SD) planes of the square nozzle and the diametric plane of the circular nozzle. The velocity half-width \( Y_{05} \) grows faster in the side plane close to the exit, while it decreases (negative growth) in the diagonal plane. The growth of \( Y_{05} \) is directly related to the entrainment. Thus a better entrainment is observed on the flat side than at the corner. After 1.7 diameters the half-width growth rate decreases quickly in the side plane while increasing and becoming positive in the diagonal plane. This behavior is related to axis-switching.

The shear layer thickness grows faster in the side plane, but reaches a higher growth rate in the diagonal plane. The shear layer thickness has a greater growth rate for both square planes than in the circular jet. This is a sign of a better mixing. The square jet spreads faster overall than the circular jet as already noticed by Quinn.

The spanwise locations where the axial velocity is 10% and 90% of the maximum velocity are designated as \( Y_{01} \) and \( Y_{09} \) respectively. Their growth rates are shown in Figure 4.9. \( Y_{01} \) grows faster, indicating more entrainment in the slower part of the shear layer.
Figure 4.7 Turbulence intensities. $\sqrt{\overline{u'^2}} / U_e$ in the side (a) and diagonal (b) planes, 
$\sqrt{\overline{v'^2}} / U_e$ (c) and $-\overline{u'v'} / U_e^2$ (d) in the side plane
Figure 4.8 Turbulent velocity profiles in both planes
Figure 4.9 Derivative of $\delta$, $Y_{01}$, $Y_{05}$ and $Y_{09}$ with respect to $x$ for the square and circular single inner jet
4.2 Fourier Space Results

4.2.1 Spanwise Evolution of the Frequency Spectra

More information can be obtained in Fourier space. A Fast Fourier transform has been applied to the velocity data record (see section 2.4).

The energy carried by a particular wave is proportional to the square of the corresponding Fourier coefficient amplitude $A(x, y, f)$. Thus the logarithm of $A$ has been plotted as a function of the spanwise direction $y$ and the frequency $f$ for various axial locations $x_0$. The amplitude has been scaled by the integrated energy over the entire spectrum at the exit center line point: $A_0 = \sqrt{\frac{1}{\Delta f} \int A^2 df}$. Figure 4.10, Figure 4.11 and Figure 4.12 show the plots for six different locations for the circular single jet in the diametric plane and the square single jet on the side and the diagonal planes respectively.

Frequency peaks are easy to identify on these plots. Their spanwise location and their evolution appear clearly. Particularly, the natural frequency of each region will be studied on these figures. The natural frequencies of the outer, middle and inner shear layer will be called $f_o$, $f_m$ and $f_i$ respectively.

According to linear stability theory, the spatial growth rate of the initial disturbances varies with the Strouhal number in the early stage of the shear layer development. The Strouhal number based on the exit diameter $St_{D_h} = \frac{f \cdot D_h}{U_{max}}$ can be used to describe “jet-like” modes, while the Strouhal number based on the momentum thickness of the shear layer $St_{\theta} = \frac{f \cdot \theta_i}{U_{max}}$ can be used to describe the shear layer modes.
It is however critical to understand the role of the nozzle cavities as discussed in Chapter 3. A strong disturbance due to the nozzle will be amplified even if it is not a shear layer mode, and similarly a shear layer mode can be absent from the spectra if it falls in a zero of the nozzle cavity. The nozzle cavity acts as a pre-amplifier and it is important to keep an eye on the transfer function plotted at the end of Chapter 3 when making comments on the following figures.

The circular jet plots come from data processed by Wai-Ho Choy. The contour at the first height, \( x/D_{hi} = 0.07 \), corresponds to the initial disturbance. At \( x/D_{hi} = 0.52 \) the most amplified mode appears to be \( f_i = 2400Hz \). Its maximum Strouhal number based on the momentum thickness is \( St_{\theta_i} = 0.015 \), which falls in the range [0.012-0.02] predicted by the theory. At \( x/D_{hi} = 1.51 \), \( f_i \) has already disappeared to let \( f_i / 4 = 550 - 600Hz \) dominate the shear layer and penetrate the jet. This mode is present until \( x/D_{hi} = 4.92 \), where the jet mode appears around 400Hz. The Strouhal number of this mode based on the exit hydraulic diameter is \( St_{D_h} = 0.27 \), which is in the range [0.2-0.5] predicted by the theory.

A very similar behavior is observed in the square jet. The most amplified frequency is now \( f_i = 2910Hz \), with a Strouhal number of \( St_{\theta_i} = 0.019 \) in the range of prediction. The cascade is quicker than for circular jets: \( f_i / 4 = 725Hz \) already appears at \( x/D_{hi} = 0.59 \), and \( f_i / 8 = 365Hz \) starts dominating the shear layer and the jet at \( x/D_{hi} = 1.51 \). This mode corresponds to the jet mode, with a Strouhal number of \( St_{D_h} = 0.26 \). This is probably the reason why it is the only mode in the jet and stays far
Figure 4.10 Contours of spectra plotted along the spanwise direction for circular single jet in diametric plane.
Figure 4.11 Contours of spectra plotted along the spanwise direction for square single jet in side plane
Figure 4.12 Contours of spectra plotted along the spanwise direction for square single jet in diagonal plane
downstream in the spectra. At $x/D_{hi} = 0.59$, strong peaks can be noticed at 2480 Hz and 1840 Hz corresponding to peaks in the transfer function (Figure 3.9).

The energy of the preferred mode and its first three sub-harmonics has been computed, integrating at each point $(x, y)$ of the measurement plane the square of the Fourier coefficient over the frequency as described in section 2.4. The results are plotted in Figure 4.13 for the side plane and in Figure 4.14 for the diagonal plane. These figures show the evolution of the modes in space.

Both planes show the same behavior. The preferred mode $f_i$ appears very early and lasts less than one hydraulic diameter (about 140 momentum thicknesses). The first sub-harmonic $f_i/2$ is present also early, but lasts longer up to 2 or 3 hydraulic diameters (about 350 momentum thicknesses). The second sub-harmonic $f_i/4$ appears after the first hydraulic diameter, and stays for $1 \leq x/D_{hi} \leq 6$ ($140 \leq x/\theta_i \leq 840$). The jet region is dominated by the third sub-harmonic $f_i/8$, which starts in the shear layer at $x/D_{hi} = 2$ ($x/\theta_i = 280$), and spills in the jet core region for $3 \leq x/D_{ho} \leq 6$ ($420 \leq x/\theta_i \leq 840$).

### 4.2.2 Streamwise Energy Evolution of the Modes

The energy of each mode has been integrated over the shear layer region for each streamwise location $x$ as specified in section 2.4. Figure 4.15, Figure 4.16 and Figure 4.17 present the energy integral of the preferred mode of the shear layer and its three first harmonics for the circular jet in the diametric plane and the square jet in both side and diagonal plane respectively. (a) shows the streamwise evolution of the energy. (b) is the same energy scaled by its value at the exit of the nozzle, $E_{exit}$, so that the initial value of
Figure 4.13 Energy distribution of the modes \( f_i, f_i/2, f_i/4, f_i/8 \) on the side plane of the square single jet.
Figure 4.14 Energy distribution of the modes $f_i, f_i/2, f_i/4, f_i/8$ on the diagonal plane of the square single jet
each curve is 1. (d) is a logarithm version of (b) for the initial region where the axial coordinate is scaled by the momentum thickness. (c) gives the value of the energy of each disturbance at the exit, $E_{exit}$. In the initial region of the jet, the linear instability theory is assumed to apply. The energy follows the relationship $E(x) \propto e^{-\alpha x}$ where $-\alpha_i$ is the spatial growth rate. However the behavior in the early stages of the shear layer development depends on the initial conditions like the initial modal energies and phases [55]. Non-linear interaction between the modes can occur and some modes may not present a linear behavior.

Similarities can be observed for the three planes of study. Plot (a) concords the conclusions of section 4.2.1. The successive growth of the modes from the preferred mode to the third sub-harmonic is noticeable for the circular jet and the diagonal plane of the square jet. $f_i/8$ dominates the shear layer after two hydraulic diameters, to reach its maximum at about $x/D_{hi} = 4.5$. $f_i$ has the highest growth rate at the exit. It is more obvious on (d). However, $f_i/2$ is the mode, which sees the highest relative amplification. The linear growth rate in the initial region is very clear for the circular and the square jet in the side plane (d). However, it is not so clear in the diagonal plane. At the corner of the nozzle, three-dimensional effects take place. Thus the linear stability theory may not apply at the corner. It is worth noticing that in the square jet the exit energy of the $f_i$ and $f_i/2$ waves are two orders of magnitude less than $f_i/4$ and $f_i/8$ waves, which is not the case for the circular jet. In the circular jet, a high level is noticed for $f_i/4$. In the square jet, $f_i/4$ decreases quicker to let $f_i/8$ dominate, as the jet mode.
Figure 4.15 Streamwise amplification of the waves \( f_i, f_i/2, f_i/4, f_i/8 \) in the shear layer region scaled by the average energy at the exit (a), scaled by the energy of the corresponding wave at the exit (b) and (d). Energy density at the exit in the shear layer region (c). Circular coaxial jet in the diametric plane.
Figure 4.16 Streamwise amplification of the waves $f_i, f_i/2, f_i/4, f_i/8$ in the shear layer region scaled by the average energy at the exit (a), scaled by the energy of the corresponding wave at the exit (b) and (d). Energy density at the exit in the shear layer region (c). Square coaxial jet in the side plane.
Figure 4.17 Streamwise amplification of the waves $f_i, f_i/2, f_i/4, f_i/8$ in the shear layer region scaled by the average energy at the exit (a), scaled by the energy of the corresponding wave at the exit (b) and (d). Energy density at the exit in the shear layer region (c). Square coaxial jet in the diagonal plane.
Chapter 5  Passive Forcing on Outer Single Jet

The outer single jet for a square nozzle has been studied by Samuel Bonnafous [8] both in the time domain and in Fourier space. His results referred to Wai-Ho Choy’s study on the similar outer single jet for a circular nozzle [12]. To complete their work, the energy evolution of the modes is studied here.

5.1 Initial Conditions

Like an annular jet, the outer jet has two shear layers: the “outer shear layer” (OSL) at the outer lip of the nozzle, the “middle shear layer” (MSL) at the inner lip of the nozzle. However, the present study focuses on the outer shear layer which is of great interest for the following chapter. Table 5.1 gives the initial conditions for the outer shear layer of the outer circular and square jet.

<table>
<thead>
<tr>
<th></th>
<th>Circular</th>
<th>Square side</th>
<th>Square Diagonal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reynolds number, Re_o (D_{ho}U_c / ν)</td>
<td>15,300</td>
<td>15,300</td>
<td>15,300</td>
</tr>
<tr>
<td>Velocity jump across the shear layer, ΔU_o (m/s)</td>
<td>13.39</td>
<td>14.40</td>
<td>14.40</td>
</tr>
<tr>
<td>Initial momentum thickness, θ_e (mm)</td>
<td>0.189</td>
<td>0.173</td>
<td>0.320</td>
</tr>
<tr>
<td>D_{ho} (mm)</td>
<td>20.32</td>
<td>20.32</td>
<td>20.32</td>
</tr>
</tbody>
</table>

5.2 Shear Layer Frequencies

The preferred modes of the outer shear layer and their respective harmonics of each jet have been revealed in each study. They are recalled in Table 5.2.

\[ \text{St}_{θ_e} = \frac{f \cdot θ_e}{ΔU_o} \] is the Strouhal number of the preferred mode for the outer shear layer. The Strouhal number calculated for the circular jet is \( \text{St}_{θ_e} = 0.019 \). The Strouhal
number calculated for the square jet is \( St_{\phi} = 0.016 \). Both results fall in the range of \([0.012 \text{–} 0.02]\) predicted by the theory.

### Table 5.2 Outer shear layer frequencies for circular and square outer jets

<table>
<thead>
<tr>
<th>Order</th>
<th>Circular</th>
<th>Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental ( f_o )</td>
<td>1350</td>
<td>1330</td>
</tr>
<tr>
<td>First sub-harmonic ( f_o/2 )</td>
<td>675</td>
<td>665</td>
</tr>
<tr>
<td>Second sub-harmonic ( f_o/4 )</td>
<td>340</td>
<td>330</td>
</tr>
<tr>
<td>Third sub-harmonic ( f_o/8 )</td>
<td>170</td>
<td>165</td>
</tr>
</tbody>
</table>

#### 5.3 Modes Evolution

The energy \( E_{OST,f_i} \) has been computed for the preferred mode and its three first sub-harmonics for each nozzle as described in section 2.4. The results are presented in Figure 5.1, Figure 5.2 and Figure 5.3 (a) for the circular jet and the square jet in side and diagonal planes respectively. The same energy is plotted but scaled by its value at the exit of the nozzle in (b) and (d). Therefore the initial value of each curve is 1. The energy is presented in a logarithm scale in (d) in order to see the linear instability behavior at the exit: \( E(x) \propto e^{-\alpha_l x} \) where \(-\alpha_l\) is the spatial growth rate. The energy at the exit of each wave is presented in (d).

The most amplified mode for the circular jet is 1350 Hz. It has the highest initial growth rate on Figure 5.1 (b). The amplification rate is progressively lower for the sub-harmonics. The mode with the maximum amplification is \( f_o/8 \). The maximum is reached at \( x/D_{ha} = 5 \cdot f_o/4 \) reaches its maximum before \( f_o/8 \), at \( x/D_{ha} = 2.8 \). This mode has the highest energy at the exit, about two orders of magnitude higher than \( f_o \).
The behavior of the square jet in the side plane is similar to the circular jet. \( f_o / 8 \) and \( f_o / 4 \) reach their peaks further downstream (\( x / D_{ho} = 4.4 \) and \( x / D_{ho} = 3.2 \) respectively). The energy at the exit of the \( f_o / 8 \) wave is about two orders of magnitude higher than the others. As seen in Samuel Bonnafous thesis, this mode is the strongest mode further downstream as a “jet-like” or “wake-like” mode. Thus, the high exit energy may be due to pressure feedback from the middle field.

In the diagonal plane, the behavior is more complex. In the corner, the flow is three-dimensional, and the linear stability theory may not be valid. Similarly, the preferred mode sees the greatest growth, followed successively by its sub-harmonics. However, non-linearity appears strongly in Figure 5.3 (d). \( f_o / 4 \) and \( f_o / 8 \) meet their first peaks at the same location as in the side plane. \( f_o / 8 \) sees a second peak further down in the far field. Like in the side plane, \( f_o / 8 \) has a greater level at the exit.
Figure 5.1 Circular jet. Streamwise energy amplification of the waves \( f_o/8, f_o/4, f_o/2, f_o \) in the outer shear layer region, (a), scaled by its energy at the exit (b) and (d). Energy density at the exit in the outer shear layer region, (c)
Figure 5.2 Square jet, side plane. Streamwise energy amplification of the waves $f_0/8, f_0/4, f_0/2, f_0$ in the outer shear layer region, (a), scaled by its energy at the exit (b) and (d). Energy density at the exit in the outer shear layer region, (c)
Figure 5.3 Square jet, diagonal plane. Streamwise energy amplification of the waves $f_{o}/8, f_{o}/4, f_{o}/2, f_{o}$ in the outer shear layer region, (a), scaled by its energy at the exit (b) and (d). Energy density at the exit in the outer shear layer region, (c)
Chapter 6  Passive Forcing on Coaxial Jets

As Samuel Bonnafous studied the square coaxial jet with a velocity ratio of 0.5 [8], here the square coaxial jet with a velocity ratio of 1.5 is studied. In the first part of this chapter, the time record will be studied. A close look will be put on axis-switching occurrence. The second part focuses on frequency properties in the Fourier space.

6.1  Time Space Results

6.1.1  Initial Conditions

The velocities and Reynolds numbers concerning the studied coaxial jet are given in section 2.2. The inner to outer velocity ratio is 1.5. The outer jet Reynolds number is about 15,300 as in Samuel Bonnafous study. The flow conditions have been given in Table 2.1. Table 6.1 gives information on the initial conditions and regarding the shear layers. The shear layer of interest for this study are the outer shear layer (OSL) and the inner shear layer (ISL). The middle shear layer disappears quickly after few hydraulic diameters.

<table>
<thead>
<tr>
<th></th>
<th>Coaxial circular jet (Choy [12])</th>
<th>Coaxial square jet (side)</th>
<th>Coaxial square jet (diagonal)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ISL</td>
<td>OSL</td>
<td>ISL</td>
</tr>
<tr>
<td>Velocity jump across the shear layer $\Delta U$ (m/s)</td>
<td>21.91</td>
<td>13.55</td>
<td>21.76</td>
</tr>
<tr>
<td>Initial momentum thickness $\theta_e$ (mm)</td>
<td>0.102</td>
<td>0.175</td>
<td>0.107</td>
</tr>
<tr>
<td>Initial shear layer thickness $\delta_e$ (mm)</td>
<td>0.644</td>
<td>0.809</td>
<td>0.560</td>
</tr>
</tbody>
</table>
The momentum thickness ratio of the diagonal to the side plane is
\[ \frac{\theta_{0,S_D}}{\theta_{0,SC}} = 2.57 \] in the outer shear layer and \[ \frac{\theta_{0,S_D}}{\theta_{0,SC}} = 1.55 \] in the inner shear layer. Again the non-uniform azimuthal distribution of the momentum thickness may impair axis-switching occurrence. The velocity profiles scaled by the centerline velocity at \( x/D_{hi} = 0.049 \) for the three cases have been plotted on Figure 6.1. The spanwise coordinate \( y \) has been scaled by the diagonal plane geometrical factor \( \sqrt{2} \) for comparison with the other profiles. The profiles indicate reasonably good similarity between the square and the circular jets. Off-center peaks are observed for each set like in the single jet. The inward slope of the outer jet has already been observed by Samuel Bonnafous and Wai-Ho Choy for various experiments with this present nozzle design. It seems dependant on the nozzle design as the slope is in the opposite direction on J. Bitting’s experiment with another design [4].


6.1.2 Streamwise Velocity

The streamwise velocity profiles have been plotted on Figure 6.2 for several streamwise locations $x_0$ for the square jets in both planes and circular jet. The middle shear layer of the diagonal scan presents a velocity deficit. This deficit was present in the square outer jet and the coaxial jet with a velocity ratio of 0.5. Samuel Bonnafous showed experimentally that this deficit is very local at the corner ($\pm 3^\circ$). It is very likely due to three dimensional effects in this region [8].

The velocity profiles give a first idea of the jet evolution. The middle shear layer vanishes quickly, and at $x/D_{ho} = 2.5$ only the inner and the outer shear layer are present. At $x/D_{ho} = 5.9$ the profiles look like a single jet and the velocity at the centerline starts decreasing. Comparing the results for the square jet at the exit and at $x/D_{ho} = 2.5$, an inversion seems to have occurred between the side and the diagonal profile. The side plane velocity profile is larger after 2.5 hydraulic diameters, and stays larger until the far field where the jet has become azimuthally uniform. This result predicts axis-switching.

Streamwise velocity contour plots are shown Figure 6.3. The potential core of the inner jet lasts far in the mid-field. It is longer for the square jet (until about 8.5 hydraulic diameters) than in the circular jet (until 7.5 hydraulic diameters).

It is worth noticing that the potential cores of both outer and inner jets expand in the side plane while it shrinks in the diagonal plane. The inner jet reaches its broader point in the side plane at $x/D_{ho} = 2$ and the outer jet at $x/D_{ho} = 2.5$. 

Figure 6.2 Mean streamwise velocity profiles
Figure 6.3 Streamwise velocity contour plots
6.1.3 Shear Layer Evolution

Figure 6.4 and Figure 6.5 show the streamwise evolution of the outer and inner shear layer growth rates respectively. The shear layer thickness $\delta$ and the spanwise location where the velocity equals 10% ($Y_{0.1}$), 50% ($Y_{0.5}$) and 90% ($Y_{0.9}$) of the velocity jump have been derived with regards to the streamwise direction $x$.

The outer shear layer grows faster in the side plane than in the diagonal plane. $d\delta/dx$ is related to the mixing in the shear layer. Thus there is more mixing at the flat side than at the corner of the nozzle. The velocity half-width grows dramatically in the side plane while decreasing in the diagonal plane, following the conclusions obtained earlier from Figure 6.3. This trend seems to inverse after 2 hydraulic diameters. $dY_{0.1}/dx$ and $dY_{0.9}/dx$ give information about the propagation of the shear layer outward and toward the flow respectively. A negative growth rate of $Y_{0.9}$ means a development of the shear layer towards the flow, while a positive growth rate of $Y_{0.1}$ means a development of the shear layer outward the flow. In the diagonal plane, the shear layer initially develops toward the flow and rapidly shifts outward as the total thickness grows and the shear layer slows its inward motion. In the side plane, however, the shear layer develops quickly outward as it moves away from the core, indicating a high entrainment of external air on the flat side.

The inner shear layer widens in both planes. After the middle shear layer vanishes (at about 1 hydraulic diameter), the inner shear layer grows faster on the side than in the corner, indicating a better entrainment on the flat side. The velocity half-width decreases in the diagonal plane and the shear layer moves inward. The velocity half-width increases
Figure 6.4 Derivative of $\delta$, $Y_{01}$, $Y_{05}$ and $Y_{09}$ with respect to $x$ for the square and circular coaxial jet. Outer shear layer
Figure 6.5 Derivative of $\delta$, $Y_{01}$, $Y_{05}$ and $Y_{09}$ with respect to $x$ for the square and circular coaxial jet. Inner shear layer
in the side plane for $1 \leq x / D_{ho} \leq 5$ and the shear layer moves outward. The velocity half-width growth tends to zero before the outer and the inner shear layer merge. The growth rates of $Y_{01}$ and $Y_{09}$ indicate that the inner shear layer develops towards the core of the inner jet in the diagonal plane. It develops towards the outer jet in the side plane. The inner shear layer is actually developing towards the high speed flow in the diagonal plane and toward the low speed flow in the side plane.

### 6.1.4 Axis-Switching Occurrence

Figure 6.6 shows the spanwise component of the velocity in both planes. In the diagonal plane, the spanwise velocity is strongly directed inward for both inner and outer shear layer and throughout the outer jet in the initial zone. In the side plane, the outer shear layer moves away from the inner core, with a highest velocity for $1.5 \leq x / D_{ho} \leq 4$, before the outer and inner shear layers merge. The middle shear layer tends to move towards the outer flow before it vanishes. The inner shear layer has an outward component with a level comparable to its inward component in the diagonal plane.

The velocity half-width is a good indicator of the evolution of the shear layer. It has been plotted in Figure 6.7. The axis-switching is very obvious, as the $Y_{05}$ curves of the side plane and the diagonal plane cross each other. The cross-over in the outer shear layer happens for $x / D_{ho} = 1.5$ at about the same distance from the exit as in the single outer jet or the coaxial jet with $\lambda^{-1} = 0.5$. The inner shear layer sees the cross over at the same location. The cross-over of the coaxial jet with $\lambda^{-1} = 0.5$ happened at 3.5 hydraulic diameters [8] and was not as clear as in the present study. Here the mechanism seems to be stronger and quicker.
Figure 6.6 Spanwise velocity contours

Figure 6.7 Half-width evolution in the coaxial square jet
6.1.5 Turbulence

The root mean square value has been computed on each point of the flow in both planes and presented in Figure 6.8. The turbulence level increase as the flow evolves and the shear layer develops downstream. At the exit the turbulence is higher at the lips of the nozzle, where the shear layer starts. The outer shear layer is about twice as more turbulent than the inner shear layer. In the midfield there is more mixing between the outer flow and the ambient air. In the far field, after the outer and the inner shear layer merged, the turbulence moves inward in the inner jet. The turbulence level in the outer shear layer and the inner jet is higher in the side plane than in the diagonal plane.

The evolution can also be tracked on Figure 6.9 as the spanwise profiles of the turbulent velocity are plotted for several streamwise locations \( x_0 \). At the exit, three peaks
Figure 6.9 Turbulent velocity profiles
are clearly present, one at each shear layer. The turbulence grows first in the inner shear layer and reaches a stable level at $x / D_{ho} = 0.39$. Then the outer shear layer sees a dramatic increase in turbulence. It becomes the most turbulent at $x / D_{ho} = 0.98$. It dominates further down, and absorbs the middle shear layer peak at $x / D_{ho} = 2.02$. The inner peak stays until $x / D_{ho} = 5.91$ where the inner shear layer merges with the outer. At the end of the potential core of the inner jet, the two remaining peaks tend to move towards the inner jet and show a tendency to merge in the far field.

### 6.2 Fourier Space Results

#### 6.2.1 Span Wise Evolution of the Frequency Spectra

The amplitudes of the Fourier coefficients have been plotted in Figure 6.10, Figure 6.11 and Figure 6.12 for the square jet in the side and in the diagonal plane and the circular jet respectively at four different locations. Two ranges of frequency is presented: $0 \leq f \leq 10,000$ and $0 \leq f \leq 1,000$.

At the exit, a strong peak appears in the inner shear layer around 3700 Hz at $St_{\theta_i} = \frac{f\theta_i}{\Delta U_i} = 0.0178$ for the square jet, $St_{\theta_i} = 0.0185$ for the circular jet. Both Strouhal numbers fall in the range $[0.012 - 0.02]$ predicted by the linear stability theory. As expected, the Strouhal numbers are similar for both configurations.

The first harmonic of the inner preferred mode is present after 0.3 hydraulic diameters. The first sub-harmonic starts also to appear. In the outer shear layer, a peak is
Figure 6.10 Contours of spectra plotted along the spanwise direction for square coaxial jet in side plane, $A_0 = 2.66 \times 10^{-3}$
Figure 6.11 Contours of spectra plotted along the spanwise direction for square coaxial jet in diagonal plane, $A_0 = 2.78 \times 10^{-3}$. 
Figure 6.12 Contours of spectra plotted along the spanwise direction for circular coaxial jet in diametric plane, $A_b = 4.50 \times 10^{-3}$
seen around 1350 Hz at $St_{\theta_o} = \frac{f\theta_o}{\Delta U_o} = 0.0166$ for the square jet and $St_{\theta_o} = 0.0174$,
similar for both geometries, and similar to the outer single jet.

At $x/D_{ho} \approx 1.8$, $f_o/8 = 165Hz$ dominates the outer shear layer and starts propagating in the outer jet and the inner jet. $2f_i$ disappeared in the inner jet, and $f_i$ starts to vanish.

Further down, when the inner and outer shear layer have merged, at about 5 hydraulic diameters, the flow is totally dominated by $f_o/8$. This mode corresponds to a Strouhal number based on the exit diameter of $St_{D_{ho}} = \frac{f_o D_{ho}}{U_{o,max}} = 0.23$ and $St_{D_{ho}} = 0.25$ for the square and the circular jet respectively, corresponding to the jet mode of the outer flow.

Frequency spectra have been plotted for the square jets at two streamwise locations: $x/D_{ho} = 0.05$ and $x/D_{ho} = 0.39$, in the middle of both inner and outer shear layer, in both side and diagonal planes. The dominating modes and their evolutions can be observed. In particular, a strong peak can be noticed in the outer shear layer at $f_i/2$.

6.2.2 Shear Layer Modes

The energies of the preferred modes and their harmonics have been computed as presented in section 2.4. Contours showing the energies of the inner modes in the side and the diagonal plane are shown in Figure 6.14 and Figure 6.15 respectively and the energies of the outer modes in Figure 6.16 and Figure 6.17. The evolution of the modes is similar in both planes.
Figure 6.13 Frequency spectra in the inner shear layer, on the side (a) / diagonal (b) planes, in the outer shear layer, on the side (c) / diagonal (d) planes
Figure 6.14 Energy distribution of the modes $f_i, f_i/2, f_i/4, f_i/8$ on the side plane of the square coaxial jet
Figure 6.15 Energy distribution of the modes $f_i, f_i/2, f_i/4, f_i/8$ on the diagonal plane of the square coaxial jet.
Figure 6.16 Energy distribution of the modes $f_a, f_a/2, f_a/4, f_a/8$ on the side plane of the square coaxial jet
Figure 6.17 Energy distribution of the modes $f_o, f_o/2, f_o/4, f_o/8$ on the diagonal plane of the square coaxial jet
The inner preferred mode $f_i$ appears very early and quickly looses 30% of its initial energy after 3 hydraulic diameters. $2f_i$ appears also early and vanishes after only 4 hydraulic diameters. The sub-harmonics arrive later in the inner shear layer and are strong in the outer shear layer. $f_i/8$ has a high energy in the outer jet after $x/D_{ho} = 1$ and propagates in the whole flow when the inner and the outer shear layer merge.

The outer preferred mode $f_o$ reaches a peak very early in the outer shear layer at $x/D_{ho} = 0.4$ and decreases quickly, while $f_o/2$ then $f_o/4$ and $f_o/8$ dominate at 0.9, 1.6 and 2.6 hydraulic diameters respectively. All the modes in the outer shear layer see a decrease in their energy when the inner and the outer shear layer merge. They increase again after the merging.

### 6.2.3 Streamwise Evolution of the Modes

The energy of each mode has been integrated across the corresponding shear layer as presented in section 2.4. Figure 6.18, Figure 6.19 and Figure 6.20 present the energy integral of the preferred inner shear layer mode, the first harmonic and its first three sub-harmonics for the circular jet in the diametric plane and the square jet in both side and diagonal plane respectively. (a) shows the streamwise evolution of the energy. (b) is the same energy scaled by its value at the exit of the nozzle, $E_{exit}$, so that the initial value of each curve is 1. (d) is a logarithm version of (b) for the initial region. The scaling factor is then the momentum thickness. (c) gives the value of the energy at the exit. $E_{exit}$ is the disturbance level of each wave at the exit.

In the circular jet, the energy at the exit of the nozzle of the mode $f_i$ is two orders of magnitude greater than the energy of the other modes. Having such initial energy, it
has a much stronger peak than the other modes. However, its relative amplitude is weaker, and $2f_i$ dominates on Figure 6.18 (b). $f_i$ reaches its maximum quickly (at $x/D_{ho} = 0.25$) and decreases very fast. $2f_i$ reaches then its maximum. A second growth is observed after the merging with the middle shear layer. The flow is then dominated by the sub-harmonics. After three millimeters ($x/\theta_{r,e} = 30$) the evolution of the mode is non-linear. The transition must happen before this location.

The side plane of the square coaxial jets shows a behavior similar to the circular jets. The main difference is a stronger $f_i/2$ mode, reaching a first peak when $2f_i$ has the highest amplitude ($x/D_{ho} = 0.4$).

The diagonal plane shows more differences. The energy at the exit of the first harmonic is also two orders of magnitude smaller than the preferred mode, but the energies at the exit of the sub-harmonics are only one order of magnitude smaller. The preferred mode has a smoother peak, further downstream (at $x/D_{ho} = 0.64$). The harmonic $2f_i$ is not shown in Figure 6.20 (b), its very low energy at the exit result in very high scaled values, which would compress the scale on the plot making information on the sub-harmonics unreadable. Similarly to the single jets, the linear behavior is not observed in the diagonal plane, probably due to three-dimensional flow at the corners.

In the outer shear layer (Figure 6.21, Figure 6.22 and Figure 6.23) results are very similar to the single outer jet (see section 5.3). The inner flow does not affect the outer shear layer mode evolution in the early stages of development. A nice cascade is observed from $f_o$ to $f_o/8$ in the circular coaxial jet. The same cascade happens in the square coaxial jets, in both planes. The energy at the exit of the third sub-harmonic $f_o/8$
is one order of magnitude higher than the energies of the other modes for the circular jets, and two orders of magnitude in the square jets. It can be interpreted as pressure feedback from the far field (after merging of the shear layers) where \( f_o / 8 \) dominates the flow.

Figure 6.18 Streamwise amplification of the waves \( f, f / 2, f / 4, f / 8 \) in the inner shear layer region scaled by the average energy at the exit (a), scaled by the energy of the corresponding wave at the exit (b) and (d). Energy density at the exit in the shear layer region (c). Circular coaxial jet in the diametric plane.
Figure 6.19 Streamwise amplification of the waves \( f_i, f_i/2, f_i/4, f_i/8 \) in the inner shear layer region scaled by the average energy at the exit (a), scaled by the energy of the corresponding wave at the exit (b) and (d). Energy density at the exit in the shear layer region (c). Square coaxial jet in the side plane.
Figure 6.20 Streamwise amplification of the waves $f_i, f_i/2, f_i/4, f_i/8$ in the inner shear layer region scaled by the average energy at the exit (a), scaled by the energy of the corresponding wave at the exit (b) and (d). Energy density at the exit in the shear layer region (c). Square coaxial jet in the diametric plane.
Figure 6.21 Streamwise amplification of the waves $f_\alpha, f_\alpha/2, f_\alpha/4, f_\alpha/8$ in the outer shear layer region scaled by the average energy at the exit (a), scaled by the energy of the corresponding wave at the exit (b) and (d). Energy density at the exit in the shear layer region (c). Circular coaxial jet in the diametric plane.
Figure 6.22 Streamwise amplification of the waves $f_0, f_0/2, f_0/4, f_0/8$ in the outer shear layer region scaled by the average energy at the exit (a), scaled by the energy of the corresponding wave at the exit (b) and (d). Energy density at the exit in the shear layer region (c). Square coaxial jet in the side plane.
Figure 6.23 Streamwise amplification of the waves $f_o, f_o/2, f_o/4, f_o/8$ in the outer shear layer region scaled by the average energy at the exit (a), scaled by the energy of the corresponding wave at the exit (b) and (d). Energy density at the exit in the shear layer region (c). Square coaxial jet in the diagonal plane.
Conclusion

The present study is a good complement to previous works done by Wai-Ho Choy [12] and Samuel Bonnafous [8] on coaxial square and circular jets, as the initial conditions are in good agreement.

This work gives evidence of axis-switching in coaxial square jets with an inner to outer velocity ratio of 1.5. This result is obtained in both inner and outer shear layer despite the low azimuthal coherence of momentum thickness at the exit of the nozzles. A 45-degree axis-switching occurs in both shear layers at 1.5 hydraulic diameters from the exit of the nozzle.

The variation of the initial conditions in the inner jet has little effect on the outer shear layer. The axis-switching occurrence and the outer shear layer mode evolution are similar for each velocity ratio. In the inner shear layer, axis-switching is clearer and happens earlier than with a velocity ratio of 0.5.

Better mixing has been obtained with a square geometry than a circular geometry. The potential core of the square jets deforms as the axis-switching phenomenon occurs. These conclusions agree with Samuel Bonnafous’ work on a velocity ratio of 0.5.

Similar behavior between both geometries has been observed in the frequency domain for the inner and outer single jets. A frequency cascade is present in the side plan of the square jet. However, the early evolution of the shear layer modes at the corners is strongly non-linear.

Non-linearity was more obvious in the inner shear layer of the coaxial square jet were the preferred mode has a strong energy in the early stages of development.
References


Vita

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