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On the Thermodynamics of Degradation

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ON THE THERMODYNAMICS OF DEGRADATION

A Dissertation
Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
In partial fulfillment of the
Requirements for the degree of
Doctor of Philosophy

in

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# Table of Contents

ACKNOWLEDGMENTS .............................................................................................................. ii

ABSTRACT ..................................................................................................................................... v

INTRODUCTION ............................................................................................................................... 1
  I.1 Problem Statement .................................................................................................................. 1
  I.1 Overview of Dissertation ......................................................................................................... 2
  I.3 References .............................................................................................................................. 4

CHAPTER 1. THERMAL RESPONSE OF THE SYSTEM UNDERGOING FATIGUE .......... 5
  1.1 Introduction ........................................................................................................................... 6
     1.1.1 Parameters Influencing Fatigue ......................................................................................... 10
     1.1.2 Low-Cycle (LC) and High-Cycle (HC) Fatigue ................................................................. 11
     1.1.3 Variable Load Amplitude ............................................................................................... 12
     1.1.4 Stress-State: Torsion, Bending, Axial and Combined ...................................................... 13
     1.1.5 Environmental Effect ..................................................................................................... 14
  1.2 Summary of the Methods of Thermal Approach to Fatigue ............................................... 14
  1.3 Detailed Description of the Present Method ......................................................................... 15
  1.4 Experimental Method .......................................................................................................... 19
  1.5 Thermal Analysis ................................................................................................................... 23
  1.6 Discussion ............................................................................................................................... 25
     1.6.1 Fatigue Life Prediction .................................................................................................... 25
     1.6.2 The Fatigue Curve ......................................................................................................... 29
     1.6.3 Error Analysis .................................................................................................................. 32
  1.7 Conclusions ........................................................................................................................... 33
  1.8 References ............................................................................................................................... 34

CHAPTER 2. LIFE PREDICTION OF METALS UNDERGOING FATIGUE LOAD BASED ON THERMAL RESPONSE .......................................................... 37
  2.1 Introduction ........................................................................................................................... 38
  2.2 Experimental Apparatus and Procedure .............................................................................. 42
  2.3 Results and Discussion ......................................................................................................... 44
  2.4 Conclusions ........................................................................................................................... 49
  2.5 References ............................................................................................................................... 50

CHAPTER 3. THERMODYNAMICS OF FATIGUE FAILURE .......................................... 52
  3.1 Introduction ........................................................................................................................... 53
  3.2 Degradation-Entropy Generation (DEG) Theorem ............................................................... 57
  3.3 Experimentally-Verified Entropy-Fatigue Formulation ......................................................... 61
     3.3.1 Application to Torsion and Tension-Compression Fatigue ............................................. 62
     3.3.2 Application to Bending Fatigue ....................................................................................... 62
  3.4 Accelerated Testing ............................................................................................................... 69
  3.5 Damage Mechanics and Entropy ........................................................................................... 70
  3.6 References ............................................................................................................................... 74

CHAPTER 4. THE EFFECT OF SURFACE COOLING ON FATIGUE LIFE ............... 77
Abstract

All materials when subjected to fatigue loading are prone to failure if the number of cycles exceeds a certain level. Prediction of the number of cycles to failure is, therefore, of utmost importance in nearly all engineering applications. The existing methods for evaluating the fatigue life are tedious, expensive, and extremely time consuming as fatigue often takes many thousands to millions of cycles until failure occurs. Therefore, methods that can readily estimate the number of cycles to failure are highly desirable.

In this work, innovative solutions to fatigue problems are presented and their practical significance is discussed. The premise of this research is that the energy dissipation due to hysteresis effect manifests itself as heat, raising the temperature of the specimen. The temperature evolution during fatigue can be utilized as an index to assess the useful fatigue life and fast prediction of premature failure. Specifically, fatigue experiments of two types of metals (Aluminum 6061-T6 and Stainless Steel 304L) show that the slope of the temperature at the beginning of the test is intimately related to the fatigue life, thereby, it provides a fast prediction technique to assess failure.

An experimental investigation was conducted to study the effect of surface cooling on the improvement of fatigue life. Experiments show that the surface cooling has significant effect on the fatigue life. For example, 1000% improvement is observed for Steel 4145 undergoing rotating-bending fatigue. It is proposed that the concept of self-organization within the context of irreversible thermodynamics can be used to gain insight into the observed phenomenon. Further, it is shown that fatigue degradation and thermodynamic entropy generation are intimately connected and that their relationship can be used for prediction of failure and making fundamental advances in the study of fatigue without having to resort to traditional approaches that depend on empirical models.
Introduction
I.1 Problem Statement

Degradation due to alternating fatigue loading is one of the most predominate modes of failure in a diverse array of man-made components and natural systems. Needless to say, the frequent occurrence of fatigue damage in structures and devices has furnished the incentive for investigating its mechanism since the 1900s. A review of the massive number of publications in the fatigue area, especially in recent years during which fatigue has been under intensive research, reveals that challenges to modeling its mechanism have only been partially successful: no models can claim to be complete.

If some of the fatigue failures could be predicted and prevented through better lifetime prediction of structural performance, it would have a major impact on the economics of society. To just have an impression of how prevention of fatigue failure of structures and machinery components benefits the economic, the National Bureau of Standards issued Special Publication 647 that assessed the costs to the United States for material fractures for a single year (1978). The estimated total cost was $119 billion dollars per year or 4% of the Gross National Product. The publication authors believe that almost 1/3 ($35 billion per year) could be saved through the use of currently available technology and almost another 1/4 ($28 billion per year) could be saved through fracture-related research [1]. The capability of predicting the rate of decline in the residual strength and the remaining useful life of a machinery component is, therefore, of great interest.

The economy of North America is largely influenced by the performance of the hydrocarbon-based energy industry (oil and gas), which in turn is influenced by the performance of steel pipelines as the primary mean of transporting petroleum products, natural gas, etc [2]. Studies show that geotechnical movements impose large displacement on the pipelines leading to formation of fractures due to low cycle fatigue loadings. If fatigue failure happens, a serious
environmental hazard occurs and the pipeline operation has to be shut down for several days so as to repair the damaged portion. Therefore, a real-time fatigue monitoring technology is of paramount significance to predict the service life prior to fracture, so as to prevent the catastrophic failure.

Another concern in fatigue research community that is less dramatic, but still important and expensive in terms of energy consumption and time is the duration of fatigue experiments especially for high-cycle tests. Today most fatigue tests are performed on servo hydraulic test facilities. In some cases these tests have long test times and high costs because the machines have a high demand on energy and investment costs [3]. For example, in a typical fatigue test at 10 Hz of loading frequency, 8 weeks would be required to amass $5 \times 10^7$ cycles. For higher number of cycles, experiments may take several months to complete. It can be imagined that, for example, for high-cycle fatigue testing of full-scale prototype electric power consumption of the test facility rises significantly. Therefore, the time level and energy consumption are practically unacceptable and hence, fast prediction of fatigue failure is necessary.

In this research, we propose a novel technique for fast prediction of fatigue failure and prevention of premature failure based on the fundamental physics of fatigue damage.

I.2 Overview of Dissertation

First two chapters are devoted to introduction and thermal analysis of fatigue failure. For a system undergoing fatigue load, its thermal response to external loads is studied so as to assess the time to failure. A method and apparatus are proposed for predicting the service life of a metallic structure subjected to cyclic loading. Such objects experience fatigue, which can lead to failure after a number of loading cycles. The proposed method allows for an accurate prediction of the number of cycles to failure for a metallic structure by observing the slope of the rise in surface temperature of the object after the cyclic loading has begun. The method provides early
and accurate predictions of service life and does not require destructive testing. The method and apparatus of the present work may be installed on working equipment, thus providing service life predictions for materials in real world use. The method uses an empirically derived relationship that was confirmed using analytical relationships and material properties. The derived formula uses two constants that may be determined empirically using a described process. The constants also may be estimated mathematically. The apparatus may include a wireless temperature sensor mounted on the metallic object of interest and a data analysis unit to perform the needed calculations.

Thermal response of the system undergoing fatigue load is used for further analysis, comprising the assessment of damage parameter and failure detection based on the accumulation of plastic energy. These parts of analysis are done within a thermodynamic context using entropy as a natural time base of degradation.

In Chapter 3 fatigue problem is tackled within a thermodynamic framework. Toward that, it is shown that entropy —the most important quantitative means by which disorder can be measured— emerges in the development of thermodynamics of fatigue degradation. We present the proposition that the entropy and entropy generation of the fatigue system can be taken as the starting point for developing a thermodynamic framework capable of assessment of fatigue degradation. It is shown that the entropy production during fatigue process can serve as a measure of degradation. The thermodynamic entropy of metals undergoing repeated cyclic load reaching the point of fracture is a constant, independent of geometry, load, and frequency. That is, the necessary and sufficient condition for final fracture of a metal undergoing fatigue load corresponds to a constant irreversible entropy gain.

Based on the entropic approach, method of improving fatigue life of materials from the viewpoint of self-organization is discussed in Chapter 4. The results of a series of experiments reveal
significant improvement in fatigue life when the surface of the specimen is cooled. An explanation for the observed phenomenon is offered which utilizes the notion of self-organization within the context of irreversible thermodynamics. The potential of thermodynamic methods in selecting the materials with improved mechanical properties for fatigue are discussed.

Finally, in Chapter 5 future directions and recommendations are given. It will be discussed that the thermodynamics has potential for fundamental study of fatigue and for the development of new materials with improved mechanical properties.

I.3 References


Chapter 1: Thermal Response of the System Undergoing Fatigue*

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1.1 Introduction

All structures undergoing fatigue loading are prone to crack formation and its subsequent growth that increases with time. When a crack is formed, the strength of the structure or the component is decreased and can no longer function in the intended manner for which it was designed for. Moreover, the residual strength of the structure decreases progressively with increasing crack size. Eventually, after a certain time the residual strength becomes so low that the structure fails [1]. It is, therefore, of paramount importance to be able to predict the rate of decline in the component’s residual strength and the remaining life of the system.

Fracture mechanics is a branch of science that provides insights into the mechanism of failure and helps predict the service life of structures [1]. As depicted in Figure 1.1, several disciplines are involved in the development of fracture mechanics. At the right end of the scale is the engineering load-stress analysis. Applied mechanics covers the analysis of crack tip stress fields as well as the elastic and plastic deformations of the material in the vicinity of the crack. Material science concerns itself with the fracture processes on the scale of atoms and dislocations in the form of impurities and grains.

In order to make a successful use of fracture mechanics in an engineering application, it is essential to have some knowledge of the total field shown in Figure 1.1. Fatigue failure can occur only if—as a result of the presence of micro-cracks, local yielding, micro-cavities, etc.—the applied load produces an increase in the stress in a point (or a zone) of the material, with local values exceeding the elastic limit [2]. It is known that if the stress is static, the local plasticization and the redistribution of the stress onto the surrounding material does not generate any particularly critical condition and the material reaches failure only under decidedly greater loads. On the contrary, in the case of cyclic loading, where the stress is one of fatigue, the material arrives at the condition of local yielding (micro-plasticization) and a micro-crack is
generated. Hence, the repeated application of the stress leads to the crack propagating until, eventually, the condition of failure is reached and the specimen breaks.

Figure 1.1. Illustration of the broad field of fracture mechanics.

The thermoelastic effect, which governs the relationship between the temperature variation and stress (or strain) change in the elastic range, has been well documented, and has been utilized to characterize the elastic stress field. Different means—such as thermocouples, thermistors, and thermography techniques—have been employed to monitor the temperature changes during mechanical tests [3-6]. The thermoelastic stress analysis by thermography is now an advanced full-field stress measurement method. In materials undergoing cyclic loading, most of the dissipated energy due to hysteresis effects manifests itself as heat, and the heat is removed from the material by heat transfer.

Heat can be transferred by three processes: conduction, convection, and radiation. Conduction is the transfer of heat along a solid object. Convection transfers heat through the exchange of hot and cold molecules, e.g., air, water, etc. Radiation is the transfer of heat via electromagnetic (usually infrared, IR) radiation. Although these three processes can occur
simultaneously, it is not unusual for one mechanism to overshadow the other two. If the fatigue experiment is rapid enough, which is generally true for low-cycle fatigue testing, the temperature rise can be surprisingly high. For fatigue tests at 1,000 Hz, for example, the temperature could increase 200° to 400° K above the initial temperature, depending on the material tested and specimen geometry [3, 4].

The temperature evolution resulting from the heat generated during the fatigue process is utilized to monitor the fatigue-crack propagation [5-8], to measure the energy required to produce a unit area of a fatigue crack by propagation [8], to determine the endurance limit of some materials [10, 11], and to characterize the evolution of cumulative damage in the fatigue process [3, 4, 12, 13].

In the present chapter, a novel approach of nondestructive thermographic technique is used to characterize the fatigue behavior of metals. Specially, laboratory tests were conducted with Aluminum alloy and Stainless Steel under cyclic bending and torsion loads. The same trend is expected to persist in multiaxial loading involving the combination of bending, tension and compression as well as torsion. In the laboratory tests, detailed temperature distributions on the specimen surface, and temperature changes as a function of time (cycles) were obtained. A two-dimensional form of a thermal-mechanical coupling model for a low-cycle bending fatigue was formulated to ensure the validity of the experimental results and to provide insight into the complex fatigue behavior. The results of the experimental and analytical works were used to develop a new method for predicting the fatigue life. The predictions of temperature changes during fatigue were found to be in good agreement with the experimental results.

In materials undergoing cyclic loading, most of the dissipated energy due to hysteresis effect manifests itself as heat and causes an increase in the mean temperature. An abrupt temperature
rise in the first few cycles, followed by a steady state in later cycling, is a characteristic of metals that undergo the high-stress level fatigue testing.

In particular, we have determined that slope of the temperature-versus-time curve at the beginning of the test can be effectively utilized as an index for fatigue life prediction. This invention is expected to be applicable for the axial tension/compression loading and torsion of solid specimens of variety of shapes, as well as a thin-walled tube. Therefore, a temperature sensor, either contacting (e.g., thermocouple) or non-contacting (e.g., fiber optic, IR camera), can be used to measure the surface temperature of the specimen under cyclic loading. Test results obtained using the invention used a non-contacting sensor. In this arrangement, the need for measuring the dissipation energy due to plastic deformation from the hysteresis loop is eliminated. Also, this invention can provide an early prediction of the service life of machinery components under cyclic loading. The material properties and thermal boundary conditions are the input parameters and the service life time of the specimen is the output. Furthermore, for a system already in service, this device enables us to determine the remaining life.

Laboratory experimental results conducted at the LSU Center for Rotating Machinery have confirmed the validity of this invention for the case of cyclic bending and torsion loads. A thermographic technique that utilizes an IR-camera (i.e., non-contacting method) was used to measure the temperature increase in the specimen due to hysteresis heating during fatigue testing. However, fiber optic temperature sensors are available that can provide temperature data that can be recorded from a machine remotely.

A miniature electronic chip may be attached to the surface of a specimen under cyclic loading to measure its temperature and process the data to predict the onset of catastrophic failure. This device will be capable of measuring the slope of the temperature curve at the very early stages of the cyclic loading and rapidly estimate the specimen's fatigue life. For a new component, this
information would pertain to the *fatigue life*; for an existing machine in service, it would provide an estimate of the *remaining life*. This instrument provides a very fast and reliable method for the determination of the service life of the machinery components under cyclic loading and torsion. In practical applications, wireless technology provides compact, lightweight, reliable data transfer from the device that can be remotely monitored and processed in real time to predict the number of cycles for fatigue failure. An illustration showing the use of a wireless sensor and a data acquisition unit is shown in Figure 1.2.

![Diagram](image.png)

**Figure 1.2.** Block illustration of an embodiment of the fatigue detection unit.

### 1.1.1 Parameters Influencing Fatigue

Figure 1.3 shows a schematic of fatigue failure processes categorized based on the significance of the parameters affecting fatigue life. In this figure, parameters that influence fatigue life are categorized into four groups including: low- and high-cycle fatigue, the state of stress, loading amplitude & loading sequence, and environmental condition. In the present work, the effect of stress state is experimentally investigated. Experiments pertain to loads from low- to intermediate-fatigue cycles. The effects of environmental condition and loading sequence are also discussed.
1.1.2 Low-Cycle (LC) and High-Cycle (HC) Fatigue

In conventional fatigue analyses, to visualize the time-to-failure characteristic of a specific material, the familiar $SN$ curves are employed. A typical schematic of $SN$ is shown in Figure 1.4. Fatigue tests basically embrace two domains of cyclic stress or strain, differing distinctly in character. In each domain, failure occurs by different physical mechanisms. The first domain is restricted to lives less than $10^3$ or $10^4$ cycles (low-cycle) wherein significant plastic straining occurs and typically involves large applied stress amplitude, yielding relatively short life. The second domain is associated with the lives greater that $10^6$ cycles (high-cycle) wherein stresses and strains are largely confined to the elastic region. High-cycle fatigue is generally associated with low stress amplitude and long life, where most of the fatigue life is spent on crack initiation.
1.1.3 Variable Load Amplitude

Most applications in practice experience complex loading histories with variable amplitudes and frequencies. For example, a windmill blade experiences very complex combination of low- and high load amplitudes during its normal operation. Figure 5 shows a typical schematic of variable amplitude loading history. Realistic representation of service loads is a key ingredient of successful fatigue analysis or design. In general, therefore, the load history is pseudo-random. Traditionally, the so-called linear damage rule first proposed by Palmgren (1924) and later extended by Miner (1945) is used to account for the load-cycle variation. The Miner linear damage rule states that the damage fraction, $D_i$, at stress level $S_i$ is equal to the cycle ratio, $n_i/N_i$, where $n_i$ is the number of cycles and $N_i$ is the fatigue life at stress level of $S_i$. The fatigue criterion for variable-amplitude loading is defined as:

$$\sum \frac{n_i}{N_i} \geq 1$$  \hspace{1cm} (1.1)

It assumes that the life to failure can be estimated by summing the percentage of life used up at each stress level. The linear damage rule has two shortcomings. First, it does not consider the “sequence” effect: It is well known that the order in which cycles are applied has a significant effect on the stress-strain response of a material to applied loading, due to the nonlinear relationship between stress and strain (plastic material behavior). Second shortcoming is that the
linear damage rule is amplitude independent. It predicts that the damage accumulation is independent of the stress level. It is experimentally observed that at high strain amplitudes, a crack(s) will initiate after a few cycles, whereas at low strain amplitudes almost all of the life is spent in the crack initiation phase.

![Load-time display of service history](image)

**Figure 1.5.** Load-time display of service history.

1.1.4 Stress-State: Torsion, Bending, Axial and Combined

In many engineering applications, components are subjected to complex states of stress and strain. Complex stress-state in which the three principle stresses are nonproportional and/or their directions often change during a loading cycle. Fatigue under these conditions, termed *multiaxial fatigue*, is an important design consideration for reliable operation of many engineering components. Applied stresses on the components can vary from uniaxial to biaxial or more complex combination. For example, components such as crank shafts, propeller shafts, and rear axles are often subjected to combined bending and torsion load. Thermodynamic entropy accumulation during a fatigue process is basically referred to hysteresis heating of material due to plastic strain energy dissipation. It is discussed by Park and Nelson [22] that the effect of stress state on plastic strain energy can be effectively taken into account by introducing the so-called Multiaxiality Factor (MF). Effective plastic work per cycle $\Delta w^*$ is then defined by
multiplying the MF by plastic work per cycle, that is $\Delta w^* = MF\Delta w$. The effective plastic strain energy $\Delta w^*$ instead of plastic strain energy $\Delta w$ is to be utilized to account the effect of stress state on thermal response of the system undergoing multiaxial fatigue load.

### 1.1.5 Environmental Effect

Quantitative fatigue life predictions are hampered by many inter-connected factors influenced by environmental effects. For example, at elevated temperatures, the mean stress and thermomechanical effects are complex because of the interaction between creep, fatigue and environments. Some environmental conditions that influence the fatigue life of components and structures include: high-temperature and low temperature fatigue, corrosion fatigue in gaseous and liquid environment and surface treatment. Many fatigue life prediction models have been developed to assess the effect of environment on the life time of the samples, but almost all the models are empirical and lack of generality.

### 1.2 Summary of the Methods of Thermal Approach to Fatigue

The surface temperature is related to the number of cycles to failure. In particular, we have determined that the slope of the temperature curve at the beginning of the test can be effectively utilized as an index for fatigue life prediction. Using this technique, the remaining life of a machine can be predicted and catastrophic failure can be avoided. This technique can be applied by installing a sensor, in-situ, and testing the component while it is undergoing the fatigue load. The life expectancy estimate is obtained while the object is in use, which provides an advantage over techniques that require stopping the operation of the machine.

It is expected that this invention is applicable to other types of loadings like axial tension/compression loading, repeated bending and torsion of thin-walled tube.

Also, it is expected that a miniature wireless temperature sensor attached to the surface of the component under the fatigue load can be used to take data and the results can be monitored
remotely. In this fashion, temperature of the component is collected and at the same time transferred to a signal receiver. Other types of temperature sensors either contacting (e.g., thermocouple) or non-contacting (e.g., fiber optic, IR camera) can be used to measure the temperature of the surface of specimen under fatigue loading.

A data analyzer unit to convert the temperature data to fatigue-life span. Such a unit could calculate the slope of the temperature evolution at the beginning of the test and then convert the data to determine the service life of the component. This device may consist of an integrated electronic circuit which is programmed to process the temperature data. Processing the temperature data by this device may be carried out at the same time as the component is under the fatigue life. This type of arrangement is illustrated schematically in Figure 1.2.

1.3 Detailed Description of the Present Method
All materials when subjected to cyclic loading are prone to fatigue failure if the number of cycles exceeds a certain level. Prediction of the number of cycles to failure is, therefore, of utmost importance in nearly all engineering applications. The existing methods for evaluating the fatigue life require installing measuring instruments on a specimen and subject it to cyclic testing in a prescribed manner, e.g., bending or tension/compression. The process is tedious, expensive, and extremely time consuming as it often takes many thousands to millions of cycles until failure occurs. Furthermore, the length scale of the specimen can make instrumentation difficult and prohibitive in some cases, e.g., when one deals with miniature devices. In addition, the operating environment where the fatigue test is conducted is often different from what the unit experiences in practice. Therefore, a method that can readily estimate the number of cycles to failure is highly desirable. The purpose of the present study is to establish an effective and experimentally simple procedure to predict the fatigue life of the specimens in the early stage of the test, thereby preserving testing time, particularly when dealing with low-cycle fatigue.
Most of the dissipated strain energy during fatigue test is converted into heat, which manifests itself in the form of a change in temperature. The temperature evolution, however, is complicated and the associated data are hard to interpret. Complications arise only due to the interrelated effects of thermal and mechanical coupling, strain amplitudes, and loading histories, but also owing to multiple modes of heat transfer from the material to the environment [1]. A review of the published literature reveals that there are innovative techniques for assessment of various aspects of fatigue based on the temperature evolution resulting from the heat generation. Examples include monitoring fatigue-crack propagation [2-5], assessing the energy required to produce a unit area of a fatigue crack by propagation [6], determining the endurance limit of some materials [7, 8], and characterizing the evolution of cumulative damage in the fatigue process [9-12].

The energy approach for the purpose of estimating the fatigue limit of materials under cyclic loading tests is well documented in the literature [13-23]. Morrow’s paper [13] is representative of a pioneering work that takes into account cyclic plastic energy dissipation and fatigue of metals that undergo cyclic loading. He has developed a descriptive theory of fatigue that uses the cumulative plastic strain energy as a criterion for fatigue damage and the elastic strain energy as a criterion for fracture. For fully reversed fatigue load, Morrow [13] derived a relation between plastic strain energy per cycle in terms of the cyclic stress-strain properties, applicable when plastic strain is predominant. Jiang et al. [1] employed this plastic energy dissipation as the heat generation term in their analysis to determine a one-dimensional temperature distribution in an axial tension/compression fatigue test. They demonstrated that the temperature index, defined as the temperature difference between the steady state mean temperature and the temperature of initial stress-free stage, has a linear relationship with the dissipated energy density, $\Delta W$. Hence, they showed that the temperature index has a correlation with fatigue life and can be utilized for
the fatigue-life prediction. Park and Nelson [22] explained that the effect of stress state on plastic
strain energy can be effectively taken into account by introducing the so-called Multiaxiality
Factor (MF). Effective plastic work per cycle $\Delta W^*$ is then defined by multiplying the MF by
plastic work per cycle, that is $\Delta W^* = MF \Delta W$. The effective plastic strain energy $\Delta W^*$ instead of
plastic strain energy $\Delta W$ is to be utilized to account the effect of stress state on both temperature
profile and fatigue life. Nevertheless, it is interesting to note that the Multiaxiality Factor
becomes unity (MF=1) for uniaxial fatigue load. That is, $\Delta W^* = \Delta W$ considered for bending load
in the present study.

Fargione et al. [24] used a thermographic technique to observe the evolution of surface
temperature of specimens under fatigue load. In all of the tests performed, they observed that,
with stress level above the fatigue limit $\sigma_0$, the thermal variation underwent three distinct stages:
an initial increase followed by a steady state and, then an abrupt increase. They have postulated
that for low temperature differences (no more than 100 K), the heat transfer from the specimen to
the environment in the stabilized quasi-isothermal stage (stage 2) could be considered directly
proportional to the thermal difference $\Delta T$, and that the integral of the function $\Delta T = f(N)$ over
the fatigue life is a constant, representing a basic characteristic of a given material. Meneghetti
[14] studied the fatigue limit of stainless steel specimen under uniaxial tests based on the
experimental measurement of material thermal increment. He postulated that the energy
dissipated in a unit volume of a material as heat seems to be a more promising parameter for
fatigue characterization rather than the surface temperature. In fact, for a given material, loading
and mechanical boundary conditions, the energy dissipated in a unit volume of a material
depends only on the applied stress amplitude and load ratio undergoing a constant amplitude
fatigue test. Whereas the surface temperature depends also on the specimen geometry, test
frequency and the thermal boundary conditions that determine the rate of heat transfer from the material to the surroundings [14].

In the present study, a method has been developed to rapidly predict the number of cycles to failure in a system undergoing bending fatigue. The method is based on the evolution of surface temperature. The results of a series of bending fatigue experiments and theory reveal that the surface temperature of a specimen can be directly related to the number of cycles to failure. In particular, it is shown that the slope of the temperature plotted as a function of time at the beginning of the test can be effectively utilized as an index for fatigue life prediction. For a new specimen, this information would pertain to the fatigue life; for an existing sample in service, it would provide estimate of the remaining life. The advantage of this method with respect to the others is the capability of rapid life estimation. A temperature sensor—either contacting (e.g., thermocouple) or non-contacting (e.g., fiber optic, IR camera)—can be used to measure the surface temperature of specimen. Specifically, test results presented in this work involve the determination of the fatigue life using a non-contacting sensor, and much more advantageous than contacting methods such as those that requires installing strain gages. It is also shown that the fatigue test results of both Aluminum and Stainless Steel can be effectively represented by a single curve which gives the number of cycles to failure as a function of initial slope of the temperature evolution. The development of a universal fatigue curve representing different materials—based on experiments with aluminum and stainless steel—into a single universal curve is a new scientific contribution of interest to the research community. It is to be noted that the present method of prediction of fatigue failure is established by a series of laboratory fatigue tests during which the environmental condition does not change. For the processes involving fatigue with variable environmental condition, the application of the present method is limited since temperature measurements are influenced by thermal boundary condition. The present
method is greatly appreciated for the applications in which thermal conditions do not change during fatigue process. The study of the effect of environmental conditions on the temperature evolution and fatigue life of the specimens undergoing fatigue is not within the scope of present research.

1.4 Experimental Method

Figure 1.6 presents a schematic diagram of the experimental setup for cyclic bending fatigue tests. The apparatus is a compact, bench-mounted unit with a variable-speed motor, variable throw crank connected to the reciprocating platen, a failure cut-off circuit in a control box, and a cycle counter. The electric motor is located in a distance far from the specimen (as shown in Figure 1.6), therefore, the heat generated by the motor does not influence the heat generation due to material deformation. The variable throw crank is infinitely adjustable from 0 to 2 inches. Tests involved bending fatigue of a specimen clamped at one end, and oscillated at the other end with specified amplitude and frequency. All tests are performed at a constant frequency of 10 Hz at seven different stress amplitudes. All tests are run until failure, when the specimen brakes into two pieces. One end of the specimen is connected to the crank through three holes and the other end is clamped. To reduce the conduction heat transfer from the clamped end of the specimen to the grip, glass-wool insulation is used. The glass-wool insulation can be satisfactorily squeezed between the grips without causing any additional stress concentration. High-speed, high-resolution infrared thermography is used to record the temperature evolution of the specimen during the entire experiment. An IR camera (MIKRON M7500) with temperature range between 0 °C to 500 °C, resolution of 320 × 240 pixel, accuracy of ±2% of reading, sensitivity/NETD of 0.08 °C at 30 °C, and image update rate of 7.5 Hz is used. Two different types of material, Aluminum 6061 and Stainless Steel 304, are tested. Dimensions of the specimen are shown in Figure 1.7a. A typical thermographic image from an aluminum sample is shown in Figure 1.7b.
Before fatigue testing, the specimens are treated with a thin layer of black paint in order to reduce IR reflections and increase the thermal emissivity of the specimen surface. The emissivity of the specimen is determined by comparing the temperature measured by means of a thermocouple and by means of the infrared camera. The thermal resistance due to the use of paint on the surface of the specimen does not influence the fatigue life prediction since in the proposed method the rate of temperature rise is utilized instead of temperature itself.

**Figure 1.6.** Schematic of experimental apparatus.

**Figure 1.7.** (a) Aluminum Specimen, (b) thermographic image (all temperatures are in °C).
Figure 1.8 shows the temperature evolution of low-cycle fatigue tests for four different displacement amplitudes, \( \delta \). They pertain to subjecting an Aluminum specimen to different oscillation amplitudes at a fixed frequency (10 Hz in these experiments). Note that a persistent trend emerges from all the experiments. Starting from the ambient, initially, the temperature rises since the energy density increases with the hysteresis effect. Thereafter, temperature remains nearly constant until shortly before the failure and finally shows a rapid rise just prior to failure. For the same experimental conditions, a higher maximum stress level causes a higher steady-state mean temperature during fatigue tests, as illustrated in Figure 1.8. A similar temperature behavior is observed for axial cyclic loading by the other investigators [24].

![Temperature evolution graph](image)

**Figure 1.8. Evolution of temperature until failure occurs.**

Figure 1.9 shows a typical evolution of temperature vs. number of cycle at the point where the specimen breaks. The temperature evolution undergoes three stages: an initial increase (stage 1), steady-state (stage 2), and an abrupt increase prior to final failure (stage 3). The first stage of the temperature increase is limited to a very low number of cycles (in general, about 10% of the
entire lifespan of the specimen for loads not close to the yield stress [24]). Starting from the ambient, initially, the temperature rises since the energy generated is greater than the heat transferred out of the specimen. During the second stage, the cyclic stress and strain responses become stable: There is balance between the hysteresis-energy generation and heat dissipation as the mean temperature tends to steady state. The second stage, i.e. the period when the temperature tends to stabilize, varies considerably in the duration. For loads close to the yield stress, this stage is extremely short; for loads only slightly above the fatigue limit $\sigma_0$, it spans over almost the whole lifespan of the specimen. If the load is greater than the fatigue limit, the rate of increase in temperature in stage 1 and the steady-state temperature in stage 2 increases as the load increases. In the third stage—the stage where failure occurs—the temperature increases rapidly for comparatively very small number of cycles. In this stage, macrocracks are formed. It is generally accepted that when a macrocrack appears, there is large plastic deformation at the crack tip and the plastic work generated during this deformation is mostly converted to heat [13]. Consequently, temperature rises suddenly rapidly just before the fatigue failure occurs. Huang et al. [12] used the method of infrared-sensing technology to study the variation of infrared energy on some stainless and superalloys during a revolving bending fatigue test. They showed that there is a distinct temperature rise as the time draws near the point where fracture occurs. Their data indicated that a temperature rise can be detected at about 10 minutes prior to the fracture of a stainless steel (924) with high ductility, and about 5 minutes for superalloy (GH33) with lower ductility. This could be used as a warning of an imminent fracture, thereby, the estimation of the time to failure provides the capability of shutting down the machinery before a catastrophic break down occurs.
Figure 1.9. Typical temperature evolution for a low-cycle fatigue test.

1.5 Thermal Analysis

A thermal analysis has been carried out to predict the temperature of the specimen under the bending fatigue load. A two-dimensional heat conduction model is developed to analyze the problem. The present model is restricted to the isotropic bar with the constant thermal conductivity. It is analytically treated using the integral transform technique [25]. This technique is especially attractive for transient and steady-state heat conduction problems since it treats all space variables in the same manner and has no inversion difficulties as in the case of Laplace transformation because both the integral transform and inversion formula are defined at the onset of the problem. With the integral transform technique the solutions for the finite regions are in the form of infinite series. Details on the thermal analysis are presented in Appendix A. The solution of the temperature distribution inside the bar is found to be as:

$$T = T_0(\theta + 1)$$

(1.2)

where $T(x, y, t)$ is the temperature at point $(x, y)$, $t$ is time and $\theta$ can be found from Eq. (A.1) in Appendix A. Note that the room temperature is assumed to be constant and does not vary with space and time. Also, the convective heat transfer coefficient, $h_c$, over the fluctuating beam is
considered to be constant. It should be mentioned that one expects that the free end of the beam, where the vertical displacement is greater than the clamped end, should experience greater heat loss by convection. However, it is a good approximation for the fixed end of the beam, where heat transfer is primarily dominated by natural convection. The heat generation term inside the solid is associated with the plastic deformation and is assumed to be constant during fatigue life as discussed by Morrow [13]. The heat generation is mainly due to the inelastic deformation. Therefore, one may ignore the elastic strain and deal solely with the plastic strain. In this analysis, heat generation term is calculated using the expression derived for plastic strain energy per cycle $\Delta w$, by Morrow [13]:

$$\Delta w = \frac{4 \varepsilon'_f \left( \frac{1 - n'}{1 + n'} \right)}{\left( \sigma'_f \right)^{n'}} \sigma_a \left( 1 + n' \right)^{n'}$$

where $\sigma_a$ is the stress amplitude, $n'$ is the cyclic strain hardening exponent, $\varepsilon'_f$ and $\sigma'_f$ are cyclic ductility and strength of the material. As a first approximation, the stress amplitude is estimated from the relation known as the Basquin equation, gives the fatigue strength properties and takes the following form [26]:

$$\Delta \sigma / 2 = \sigma'_f \left( 2N_f \right)^b$$

where $\sigma'_f$ is the fatigue strength coefficient, $b$ is the fatigue strength exponent and $2N_f$ is number of reversals to failure (i.e., each $2N$ is one cycle). The values of the parameters in Eq. (1.3) and (1.4) are listed in Table 1.1.

<table>
<thead>
<tr>
<th></th>
<th>$\varepsilon'_f$</th>
<th>$\sigma'_f$ (MPa)</th>
<th>$n'$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum 6061</td>
<td>0.35</td>
<td>689</td>
<td>0.03</td>
<td>-0.094</td>
</tr>
<tr>
<td>Stainless Steel 304</td>
<td>0.55</td>
<td>1660</td>
<td>0.28</td>
<td>-0.15</td>
</tr>
</tbody>
</table>
Figure 1.10 shows the temperature distribution for Aluminum sample at three different stress amplitudes (beam tip displacements). It can be seen that the analytical solution accurately captures both the initial rise and the steady state temperature. The surface temperature of the specimen suddenly increases before failure occurs. This can be attributed to the formation of macrocrack at the onset of the fracture point. When macrocrack occurs the plastic deformation at the crack tips is large. The larger the plastic deformation, the larger is the energy dissipation and the associated temperature rise. This has not been considered in our analytical model; hence the formulation presented here is restricted to stages 1 & 2.

![Comparison of experimental results and analytical solution](image)

**Figure 1.10. Comparison between experimental results and analytical solution.**

### 1.6 Discussion

#### 1.6.1 Fatigue Life Prediction

An idealization of the experimental temperature evolution is shown in Figure 1.11. It reveals that the surface temperature increases during the first stage of the test (stage 1), then remains almost constant until shortly before the failure (stage 2) and finally increases abruptly prior to failure, (stage 3). For loads greater than the fatigue limit, the stabilization temperature in stage 2 or
temperature gradient in stage 1, $R_0$ are higher the greater the load with respect to the fatigue limit [24].

![Figure 1.11. Rate of temperature in stage 1 and stabilized temperature in stage 2 increase as the stress level increases.](image)

Heat generation term, $\Delta w$ on the other hand, can be related to the number of cycles for fatigue failure, $2N_f$. According to [13], the fatigue life in terms of the dissipated energy density per cycle is:

$$\Delta w = 4e'_f \sigma'_f \left( \frac{c-b}{c+b} \right) (2N_f)^{b+c}$$  \hspace{1cm} (1.5)

In this equation all the parameters except $\Delta w$ and $N_f$ are the properties of material, as defined earlier. Substituting heat generation term in dimensionless form $\varphi = \Delta w \frac{a^2}{kT_0}$, from Eq. (1.5) into Eq. (1.3), and then taking the time derivative of temperature Eq. (1.2), yields a relationship between the rate of temperature rise in the stage 1 and the number of cycles for fatigue failure, $2N_f$ as follow:

$$\left. \frac{\partial T}{\partial t} \right|_{\tau=0, \eta=b/a} = \frac{aT_0}{a^2} \left. \frac{\partial \theta}{\partial \tau} \right|_{\tau=0, \eta=b/a}$$  \hspace{1cm} (1.6)
\[
\frac{\partial T}{\partial t} \bigg|_{t=0, \ y=b} = f\left(2N_f\right) \tag{1.7}
\]

\[
\frac{\partial T}{\partial t} \bigg|_{t=0} = R_0 = 4c' \sigma'_f \left(\frac{c-b}{c+b}\right)\left(2N_f\right)^{c'} \cdot \Psi \tag{1.8}
\]

where

\[
\Psi = \frac{4\sigma_f \sigma}{a^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[ \left( \nu_n^2 + B^2 \left( \frac{b}{a} + \frac{B}{\nu_n^2 + B^2} \right) + B \right)^{-j} \right] \cdot \sin \beta_m \xi \cdot 
\]

\[
\cdot \left( \nu_n \cdot \cos \nu_n \eta + B \cdot \sin \nu_n \eta \right) \int_{\eta=0}^{b/a} \sin \beta_m \xi' \cdot \left( \nu_n \cdot \cos \nu_n \eta' + B \cdot \sin \nu_n \eta' \right) \, d\xi \, d\eta' = \text{const.} \tag{1.9}
\]

The number of cycles for fatigue failure can be expressed as a function of slope of the curve using Eqs. (1.7) and (1.8):

\[
2N_f = \left[ \frac{R_0}{4c' \sigma'_f \left(\frac{c-b}{c+b}\right) \cdot \Psi} \right]^{1/b+c} = c_1 R_0^{c_2} \tag{1.10}
\]

where

\[
c_1 = \left[ \frac{1}{4c' \sigma'_f \left(\frac{c-b}{c+b}\right) \cdot \Psi} \right]^{1/b+c} \tag{1.11a}
\]

\[
c_2 = \frac{1}{b+c} \tag{1.11b}
\]

The parameter $R_0$ represents the gradient of the temperature evolution at the beginning of test (stage 1). In the experiment, the $R_0$ is determined from a curve fit through the 10-15 seconds of the recorded data which is equivalent to 100-150 loading cycles with frequency of 10 Hz. In the
present study, the data recording interval is one second, therefore, every 10 loading cycles temperature data is recorded.

Figure 1.12 shows the predicted number of cycles to failure plotted against the temperature slope for Aluminum specimen undergoing bending load. The ordinate of Figure 1.12 shows the number of cycles for the complete failure of the specimen, $2N_f$, and the abscissa shows the slope of the temperature curve at the beginning of the test, $R_0$. As the slope increases the number of cycles to failure decreases, i.e., a larger initial slope corresponds to a larger stress amplitude and thus a shorter fatigue life would be expected. The initial slope of the experimental temperature curve is calculated using a curve fit chart to the very early temperature data points and evaluating time derivative of the obtained curve fit at the beginning of the test.

![Figure 1.12. Fatigue failure prediction as a function of slope for Aluminum.](image)
1.6.2 The Fatigue Curve

Equation (1.10) is of the form of \( 2N_f = c_1 R_\theta^{c_2} \) with two constants \( c_1 \) and \( c_2 \) determined analytically from Eqs. (1.11). Analytical values of constant \( c_2 \) for Aluminum 6061 and Stainless Steel 304 are -1.3 and -1.35, respectively. However, examination of 21 different Aluminum and Steel materials shows that \( c_2 \) changes within the range of -1.1 to -1.35. Aluminum 6061 and Stainless Steel 304 are quite different metals in many respects, yet the \( c_2 \) value for these metals is nearly identical, so it was held constant for the present analysis.

In the present study, attempts are made to represent the results of Aluminum and Steel with a single, universal fatigue life curve. To generate such a curve, the value of \( c_2 \) is set to be -1.22 in order to consolidate both fatigue curves of Aluminum and Stainless Steel into a single curve. An error analysis associated with this approximation has been carried out and discussed in the following section. Having the value of \( c_2 \) fixed, experimental data points for each material are plotted in diagram with \( y \)-axis as the number of cycles to failure, \( 2N_f \) and \( x \)-axis as the slope of the temperature, \( R_\theta \). A curve of the form \( 2N_f = c_1 R_\theta^{c_2} \) with \( c_2 = -1.22 \) is fitted to the experimental data points. The value of constant \( c_1 \) is changed until a curve which best fits the experimental data is obtained. Constant \( c_1 \) was found to be 204 for Aluminum 6061 and 14102 for Stainless Steel 304. Figure 1.13 shows the experimental results for the number of cycles to failure plotted against the temperature slope for Steel specimen. A curve with constants \( c_1 = 14102 \) and \( c_2 = -1.22 \) is fitted to the experimental data as described above.
Figure 1.13. Fatigue life as a function of slope of the temperature for SS 304L.

Since the constant $c_2$ is assumed to be identical for both Aluminum and Steel, it is possible to find a unique curve that describes the fatigue behavior of both materials based on the temperature evolution. In order to find such a curve, $2N_f$ in Eq. (1.10) was divided by $c_1$ and the results for both materials are plotted in Figure 1.14. It can be seen that the resulting curve closely approximate the experimental results of both Aluminum and Steel. This figure reveals that for a given initial slope and known material constant $c_1$, the results apply to both Aluminum and Steel specimens, hence the generality of the analysis. The results presented in Figure 1.14 are the normalized number of cycles to failure, i.e., number of cycles to failure divided by the constant $c_1$, $2N_f/c_1$. To make use of the Figure 1.14, for a given slope of the temperature $R_0$ on the $x$ axis, the value on the $y$ axis should be multiplied by constant $c_1$ and find the associated number of cycles to failure $2N_f$. The loading frequency and the environmental temperature were kept constant during all tests. It is worthwhile to note that the experimental work of Tobushi et al. [27] shows a very weak dependence of fatigue life on loading rate for frequencies up to 200Hz. Also, Morrow [13] explains that the size and shape of hysteresis loop
are only weakly dependent on test frequency. That is, the fatigue life is weakly dependent on test frequency within the range of his work. However, for very high test frequencies (i.e. over 200Hz) or environmental temperatures, fatigue life of the specimen will be influenced. Moreover, by increasing convection heat transfer coefficient both the surface temperature and fatigue life will be changed [27]. It is shown by Tobushi et al. [27] that in the region of low-cycle fatigue below $10^4$ cycles, the fatigue life in water is larger than that in air. This is due to the fact that the convection heat transfer coefficient in water is much greater (typically of the order of $10^2$) than that in air. Higher convection heat transfer coefficient results in lower temperature of the specimen and consequently, larger fatigue life. Experiments should be carried out to investigate the effect of these conditions which is not the purpose of this work. Also, the correlation presented in Figure 1.14 is for initially intact specimens. If the material has experienced any kind of fatigue loads prior to the test, its thermal behavior may be different due to the presence of defects and structural damages.

Figure 1.14. Unique curve for prediction of fatigue failure based on slope for Aluminum and Stainless Steel.
1.6.3 Error Analysis

With any experimental investigation there is always a certain degree of unavoidable uncertainty. An uncertainty analysis is performed using the method of Kline and McClintock [28]. This method uses the relative uncertainty in various primary experimental measurements to estimate the uncertainty of the final result. If the result of an experiment, \( R \), assumed to be calculated from \( M \) independent parameters, \( z_1, z_2, \ldots, z_M \) then the uncertainty propagated into the result, \( \delta R \) is:

\[
\delta R = \sqrt{\sum_{i=1}^{M} \left( \frac{\partial R}{\partial z_i} \delta z_i \right)^2}
\]  

(1.12)

where \( \delta z_1, \delta z_2, \ldots, \delta z_M \) are the uncertainties of the independent parameters. Using this method, the uncertainty in the calculated result \( \delta R \) will have the same level as each of the individual parameters. Applying Eq. (1.12) to Eq. (1.10) gives:

\[
\delta(2N_f) = \sqrt{\left( \frac{\partial (2N_f)}{\partial c_1} \delta c_1 \right)^2 + \left( \frac{\partial (2N_f)}{\partial R_0} \delta R_0 \right)^2 + \left( \frac{\partial (2N_f)}{\partial c_2} \delta c_2 \right)^2}
\]

(1.13)

The sensitivity coefficients in Eq. (1.13) can be obtained by differentiating Eq. (1.10) with respect to \( c_1, c_2 \) and \( R_0 \):

\[
\frac{\delta(2N_f)}{2N_f} = \sqrt{\left( \frac{\partial c_1}{c_1} \right)^2 + \left( \frac{c_2}{R_0} \delta R_0 \right)^2 + \left( \ln(R_0) \delta c_2 \right)^2}
\]

(1.14)

A simple calculation shows that the uncertainty associated with the first term in Eq. (1.14) is negligible. A typical calculation for steel specimen with \( c_1 = 14102 \), \( c_2 = -1.22 \) and \( R_0 = 1.5 \) results in:

\[
\frac{\delta(2N_f)}{2N_f} = \sqrt{5 \times 10^{-7} + 2.78 \times 10^{-7} + 6.6 \times 10^{-7}} = \pm 5.4\%
\]
uncertainty associated with the number of cycles to failure. Error analysis using Eq. (1.14) shows that the dominant sources of error are second and third terms in square root and are relevant to the temperature slope value, $\theta_R$. For either very low or very high temperature slopes, the associated error will be large. Very low values of temperature slope are relevant to the very low displacement (load) amplitudes and consequently high-cycle fatigue while, the results of the analyses presented in this work are intended for low-cycle fatigue. For the low-cycle fatigue, an equation of the form $2N_f = c_1R_0^{c_2}$ with $c_2$ as a preset exponent fits the experimental data quite well.

As stated before, in the development of the model it is assumed that elastic strain is negligible in comparison to plastic strain. Therefore, the simulations are generally appreciated whilst dealing with low-cycle fatigue tests. This assumption is for simplification of the mathematical modeling presented merely for better understanding of to the nature of the experimental trends. Hence, the implication of the assumptions of discarding elastic strain does not influence the experimental results.

1.7 Conclusions

An experimental and theoretical study is carried out to investigate the temperature evolution of Aluminum 6061 and Stainless Steel 304L specimens undergoing fully reversed bending. Surface temperature is continuously recorded using thermography technique. An analytical thermal analysis is carried out employing cyclic plastic heat dissipation as a heat source. Experiments show that the surface temperature of a specimen under fatigue bending undergoes three stages: an initial increase, a steady state and an abrupt increase just before the final fracture. The analyses reveal that the surface temperature is related to the number of cycles to failure. In particular, it is shown that slope of the temperature curve at the beginning of the test (first stage) can be effectively utilized as an index for fatigue life prediction. The experimental data for both
Aluminum and Stainless Steel are correlated with a unique curve which gives the number of cycles to failure as a function of slope of the temperature evolution. A universal curve is obtained that relates a given initial temperature slope to the fatigue life where applies to both Aluminum and Steel specimens. This method offers a very rapid technique to predict the fatigue life of a specimen undergoing cyclic load. Once the fatigue test starts running, the rate of temperature rise is measured in early stage of the test and the life time is estimated, thereby preserving testing time and preventing catastrophic failure. In spite of the advantages, the application of the present method in the fatigue processes with variable environmental condition is limited due to the influence of thermal conditions on the temperature measurements.

1.8 References


Chapter 2: Life Prediction of Metals Undergoing Fatigue Load Based on Thermal Response*

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2.1 Introduction

In the present chapter, the method and apparatus for prediction of fatigue failure presented in the preceding chapter is extended to embrace the effect of the stress state on the thermal behavior of the material under fatigue load. Aluminum Alloy 6061 and Stainless Steel 304 are selected as testing materials and specimens are subjected to completely reversed torsion load. Similar to what presented in Chapter 1, a thermographic technique is used to measure the temperature increase of the specimen due to hysteresis heating during the fatigue testing. Experimental results indicate that the initial rate of temperature rise as a function of time can be utilized as an index for prediction of fatigue life. An empirical correlation of the form \( N_f = c_1 R_{\theta}^{c_2} \) with constants \( c_1 \) and \( c_2 \) is derived that relates the rate of temperature rise, \( R_{\theta} \), at the beginning of the test to the number of cycles to failure, \( N_f \). It is shown that \( c_1 \) is dependent upon the material properties and stress state whilst \( c_2 \) is a constant. Experimental results are consolidated into a single curve which gives the time to failure as a function of initial slope of temperature rise, thereby enabling fast prediction of fatigue failure.

Strength of structures and components when subjected to repeated loading decreases as cracks tend to form in response to fluctuating stress and strain. Continuation of applied cyclic load results in increasing the crack size and decreasing the residual strength of the material to the extent that the structure or component can no longer function in the intended manner for which it was designed. Eventually, the residual strength becomes so low that the failure of the structure becomes imminent [1]. The capability of predicting the rate of decline in the residual strength and the remaining useful life of a component is, therefore, of great interest.

It is well documented that the most of the energy dissipated in a material exposed to cyclic loading manifests itself as heat and causes an increase in the temperature of the specimen. Experiments show that particularly when metals are subjected to high-stress cyclic testing, their
temperature rises significantly during the first few cycles. This is a characteristic of metals experiencing hysteresis heating. Thus, the relationship between temperature and fatigue characteristics has become a subject of considerable interest [2-9]. Methodologies are developed to relate the temperature rise of a material during fatigue process to life duration. Among them are the works of [2-5] summarized in Figure 2.1.

Huang et al. [2] showed the rate of temperature rise of the material with high ductility near the end of the fatigue testing is related to the life of the fatigue fracture. Their experimental results revealed that very close to the final failure there is a sharp increase of temperature (see Figure 2.1) just after a steady-state condition. This temperature rise is associated with initiation and propagation of macrocrack [9] and imminent fracture; hence, this information can be used to terminate the operation of a machinery to prevent the catastrophic failure. The relation of the rate of temperature rise, $\Delta T$, versus fatigue life was given as:

$$\frac{\Delta T}{\Delta t} = C' \cdot \exp\left(\frac{G}{(N_f)^b}\right)$$

(2.1)

where $t$ is time and $C'$, $G$ and $b$ are constants depending on the properties of the material and the test conditions.

Jiang et al. [3] showed that the difference between steady-state and ambient temperature can be correlated with the fatigue life (Figure 2.1). They reported the following relation between the fatigue life, $N_f$, and the temperature index, $\Delta T_{\text{ind}}$, defined as the temperature difference between steady-state temperature and the temperature of the initial stress-free stage for an axially loaded specimen:

$$(N_f)^m = C \Delta T_{\text{ind}}$$

(2.2)

where $m$ and $C$ are material constants. Using this relationship, they recommend that the temperature during the steady-state condition can be used as a warning of impending failure.
Fargione et al. [4] presented a procedure for analyzing the fatigue behavior of an axially loaded specimen based on its temperature evolution. They proposed that the integration of the area under the temperature rise over the entire number of cycles of a specimen remains constant, thus representing a characteristic of the material:

\[ \phi = \int_0^{N_f} \Delta T dN = \text{constant} \]  \hspace{1cm} (2.3)

where \( \phi \) is a constant regardless of the stress amplitude. Therefore, with a specified value of \( \phi \) for a given specimen or mechanical component, the entire fatigue curve can be obtained. In their study, the fatigue curve is defined as the locus of the tips of temperature profiles at the end of the tests.

Also shown in Figure 2.1, is a recent study of the thermal behavior of the different materials—Aluminum 6061 and Stainless Steel 304—when subjected to cyclic fatigue bending load [5]. A fatigue criterion is proposed that relates the cycles to failure to the initial rate of temperature rise of tested materials.

The use of the temperature evolution for the study of fatigue has been quite versatile in recent years. Propagation of fatigue crack [10-13], determination of the endurance limit of some materials [6], and characterization of the evaluation of cumulative damage in fatigue processes [7, 14, 15] are examples of the pertinent recent publications in the open literature.

Different means—such as thermocouples, thermistors, and thermography techniques—have been employed to monitor the temperature changes during mechanical tests [10, 11, 14, 15]. In the present work, a thermographic infrared imaging system is used to characterize the fatigue failure of specimens subjected to cyclic torsion tests. This work is a companion to the fatigue analysis performed by Amiri and Khonsari [5] to assess the failure of materials undergoing bending cyclic load.
Figure 2.1. Summary of the works on fatigue prediction based on temperature.

Of interest in the present study is the investigation of the effect of stress state on fatigue criterion proposed in the previous work. Completely reversed torsional load is applied to round specimens made of Aluminum 6061 and Stainless Steel 304. Results of the experiments reveal that the surface temperature of a specimen undergoing cyclic torsional load can be directly related to the number of cycles to failure. Further, it is shown that the fatigue test results of both Aluminum 6061 and Stainless Steel 304 undergoing torsion can be effectively represented by a
single curve which gives the number of cycles to failure as a function of initial slope of the temperature evolution.

2.2 Experimental Apparatus and Procedure

The apparatus used for fatigue tests is a LFE-150 (Life, Fatigue, Endurance) testing machine, designed for testing any specimen requiring oscillatory or reciprocating actuation. The description of the apparatus is given in previous chapter. Briefly, the machine provides constant-amplitude cyclic motion, adjustable from 0 to 50.8 mm to provide different levels of stress amplitude. The same fatigue machine is used for applying torsion and bending load using appropriate fixture. The torsional fatigue testing attachment is designed to test torsion specimen in fatigue with twist angle up to $22^\circ$ ($\pm11^\circ$). The machine is equipped with a failure cut off circuit in a control box, and a cycle counter to count the number of cycles. Figure 2.2 shows a photograph of the experimental setup used in this study to applying cyclic torsion load. Torsional tests involve a round bar tapered in middle and clamped at both ends. One end rotationally oscillates with specified amplitude and the other end is fixed. An extensive set of experiments has been performed that involves testing two types of materials of interest. The specimens used are fabricated from Aluminum Alloy 6061 and Stainless Steel 304. Considering torsion of Aluminum and Stainless Steel, 13 tests are conducted at different level of stress amplitudes and constant frequency of 10 Hz. All tests are performed with a fresh specimen and are run until failure, when the specimen breaks into two pieces.

Full field surface temperature was monitored by means of an infrared camera MIKRON M7500 with temperature range between 0°C to 500°C, resolution of $320 \times 240$ pixel, accuracy of $\pm2\%$ of reading, sensitivity/NETD of 0.08°C at 30°C, and image update rate of 7.5 Hz. Before testing, the specimens are covered with black paint to increase the emissivity of the specimen surface. Dimensions of the specimens used for torsion tests are shown in Figure 2.3a.
Based on the ASTM STP 566, torsion fatigue specimens generally follow one of two basic designs. In the first type of design, the specimens contain a parallel length with tangentially blending fillets, whereas in the second design the specimens have a continuous radius. In the present work, specimens are fabricated in accordance to the first type of design. The design of the gripping ends of the specimens and the dimensions of the specimens are recommended by the manufacturer of the fatigue testing apparatus. A typical thermographic image of specimen during fatigue test is shown in Figure 2.3b.

![Experimental apparatus for torsion fatigue test.](image)

Figure 2.2. Experimental apparatus for torsion fatigue test.

![Specimen for torsion, Thermographic image of specimen under torsion](image)

Figure 2.3. (a) Specimen for torsion, (b) Thermographic image of specimen under torsion (all temperatures are in ºC).
2.3 Results and Discussion

Experimental study reveals that a metal undergoing fatigue test is subjected to an increase of the surface temperature, such that the higher the applied load the higher the hysteresis heating and the associated temperature rise. Figure 4 shows the results of bending tests for two displacement amplitudes of $\delta_1 = 44.45$ and $\delta_2 = 49.53$ mm for Aluminum specimen taken from Ref. [5]. A higher displacement amplitude corresponds to a higher applied load. According to [5], the temperature evolution during low-cycle bending fatigue undergoes three distinct stages: an initial increase during the first phase of the test (Phase 1), followed by a fairly flat temperature profile representing a “quasi-steady” operation (Phase 2), and finally a rapid increase immediately prior to failure (Phase 3). These three distinct phases are illustrated in Figure 2.4. In Figure 2.4, the rate of temperature rise in first phase is identified by two slopes $R_{01}$ and $R_{02}$ corresponding to stress levels $\delta_1$ and $\delta_2$, respectively. During the second phase there is a balance between hysteresis heating and heat transfer to environment, thereby, steady-state temperature. As the load increases, the initial slope of temperature in first phase and the steady-state temperature in second phase increases. In the third phase—the stage where failure occurs—the temperature increases rapidly for comparatively very few number of cycles. It is generally accepted that in third phase macrocracks appear and because of large plastic deformation at the crack tip temperature rises rapidly just before the fatigue failure occurs.

This distinct trend of the temperature evolution has led researches to develop innovative ideas for detecting failure of the specimen undergoing cyclic load. In particular, researchers [2, 3, 9] have been able to relate the temperature evolution in second or third phase to onset of fatigue failure as discussed earlier.
Figure 2.5 shows the present results of torsional tests for two twist angles of $\gamma_1 = 6^\circ$ and $\gamma_2 = 7.5^\circ$ for Aluminum specimen. It can be seen that the steady-state temperature in the second phase is absent for the case of torsion fatigue test. The temperature rises in the first phase, followed by the second phase during which the rate of temperature rise is smaller and finally rapid increase of temperature in third phase. Based on this observation, the steady-state temperature does not necessarily occur during a fatigue test. Moreover, temperature rise at the end of the fatigue test occurs more rapidly and failure prediction will be unsafe particularly when testing a high frequency fatigue. Thus, an alternative approach is desirable.

The present experimental approach provides an alternative technique to predict the fatigue life of specimens considering the initial slope of the temperature evolution in the first phase. The first phase is defined as a period of increase in surface temperature of the specimen. Test results show that the number of cycles for failure, $N_f$, can be correlated to the slope of the temperature curve in the first phase, $R_0$, as follow:

$$N_f = c_1 R_0^{c_2}$$  \hspace{1cm} (2.4)
where \(c_1\) and \(c_2\) are constants. The empirical correlation given by Eq. (2.4) confirms that the constant \(c_2\) is nearly identical for both Aluminum 6061 and Stainless Steel 304 subjected to torsion or bending load. The value of \(c_2\) is found to be \(c_2 = -1.22\) for the best curve fitting. It is to be noted that the value of \(c_2\) is identical for different load amplitudes. The \(c_1\) value is derived empirically by plotting the data points using Eq. (2.4) with constant \(c_2 = -1.22\), to obtain the best curve fit. The values of constant \(c_1\) for Aluminum and Stainless Steel are listed in Table 2.1. Values of \(c_1\) for bending load is also presented in Table 2.1 for the comparison. It is to be noted that the correlation found for torsion test is of the same form of correlation presented for bending load by [5].

![Figure 2.5. Temperature evolution of two tests for torsion fatigue.](image)

**Table 2.1. Values of constant \(c_1\) for different materials and loading conditions.**

<table>
<thead>
<tr>
<th></th>
<th>Bending</th>
<th>Torsion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum 6061</td>
<td>204</td>
<td>8120</td>
</tr>
<tr>
<td>Stainless Steel 304</td>
<td>14102</td>
<td>183830</td>
</tr>
</tbody>
</table>
Figure 2.6 shows the results of experiments along with the empirical correlation of the form of Eq. (2.4). The abscissa of Figure 2.6 shows the initial slope of the temperature rise during first phase. The ordinate shows the number of cycles to failure. Correlation of the form $N_f = c_1 R_0^{c_2}$ is also presented in Figure 2.6 which best fits the experimental data. The $c_1$ value is strongly dependent on the material and the stress state; see Table 2.1. Since constant $c_2$ is almost identical for both materials and both torsion and bending loads, the results of $N_f/c_1$ versus $R_0$ consolidated in the form of a single curve.

![Figure 2.6. Fatigue failure prediction as a function of the slope of the temperature increase during the first phase.](image)

Figure 2.7 shows the general fatigue-life curve as a function of slope of the temperature rise during the first phase. The results of both Aluminum 6061 and Stainless Steel 304 subjected to torsion load are plotted in this figure. The results of bending fatigue tests taken from [5] are also presented in the same figure for comparison. Figure 2.7 reveals that regardless of the stress state,
the results of the fatigue life against initial slope comparatively follow the universal curve for both selected materials. The collapse of the behavior of both materials and stress states onto a single correlation curve is of interest to the research community. Interestingly, it is shown that despite of difference in temperature evolution of materials when subjected to torsion and bending, the initial slope of temperature rise is a characteristic of materials and can be utilized for prediction of fatigue life. As expected, increasing the slope results in decreasing the fatigue life since a steeper temperature slope corresponds to a larger applied load and consequently lower fatigue life. Having the surface temperature or particularly slope of the temperature rise at the very beginning of the fatigue test, the fatigue life of a specimen will be predicted, thereby preserving testing time. Also, in the present methodology for prediction of fatigue life measurements are made using non-contacting thermography technique which is of actual practical value compared to alternative contacting methods—such as those that require installing strain gages.

![Graph showing the universal curve for prediction of fatigue failure based on initial slope.](image)

**Figure 2.7. Universal curve for prediction of fatigue failure based on initial slope.**
It is to be noted that loading frequency is kept constant at 10 Hz during all tests. Many researchers have studied the effect of loading frequency on fatigue behavior of metals, among them are [16-18]. Results of the work of Tobushi et al. [16] reveal that the fatigue life is independent of frequencies up to 200 Hz. However, for very high frequencies (i.e. over 200 Hz) fatigue life can be influenced. Also fatigue tests are conducted in air at room temperature. Investigation of the effect of environment on fatigue behavior of metals has been of interest for many researchers. For example, Christ [19] investigated the effect of environmental temperature on the fatigue life of Stainless Steel AISI304L in air and vacuum. Study of the effect of loading frequency and environment on the fatigue life of the specimen is beyond the scope of the present work. Also, the correlation presented in Figure 2.7 is for initially intact specimens. If the material had experienced any kind of fatigue loads prior to the test, its thermal behavior would have been different due to the presence of defects and structural damages.

2.4 Conclusions

In summary, all metals when subjected to hysteresis heating effect are prone to fatigue and failure will result if the number of cycles exceeds a certain level. The energy dissipated due to hysteresis heating causes an increase in the temperature of the specimen. A thermographic infrared detection system has been used to measure the temperature evolution during the fatigue testing of Aluminum alloy 6061 and Stainless Steel 304 specimens undergoing torsion and bending load. Three distinct phases of temperature profile for a specimen undergoing bending load were observed: an initial temperature rise, a steady-state phase, and an abrupt increase of temperature. It was observed that for the specimen undergoing torsion the temperature change during the second phase gradually increases so the steady-state temperature does not necessarily occur in fatigue tests. Nevertheless, the results of experiments show that for both torsion and bending, the slope of temperature rise in the first phase is a characteristic of metals that undergo fatigue
testing. The slope of the temperature rise in the first phase of temperature profile can be effectively utilized to develop a model of predicting time of failure. Results of a series of torsion and bending experiments are used to present an empirical correlation which gives the fatigue life of a specimen as a function of slope of the temperature rise in first phase. The proposed empirical correlation is of the form of \( N_f = c_1 R_0^{c_2} \) with two constants \( c_1 \) and \( c_2 \). It is shown that results of the experiments can be described with a simple curve called universal fatigue curve. Measuring the slope of the temperature rise at the beginning of a fatigue test \( \theta R \), it can be utilized to effectively predict the life \( N_f \) of a specimen in both torsion and bending load, thereby preserving testing time.

2.5 Reference


Chapter 3: Thermodynamics of Fatigue Failure
3.1 Introduction

In Chapters 3 and 4, degradation processes and are studied within a thermodynamic framework, particularly, using the concept of entropy as an index of degradation. In the present chapter, fatigue failure is studied within a thermodynamic context. It is shown that the entropy can be utilized as an index of deterioration of metals undergoing fatigue loads. A technique is presented to predict the failure of structures subjected to cyclic load. Damage parameter is defined in this chapter via entropy concept.

Most structural components are prone to degradation and failure will eventually occur if they do not maintain their required structural integrity. Mechanical failure involves an extremely complex interaction of load, frequency and environmental conditions. Loads may be monotonic, steady, variable, uniaxial or multiaxial; environmental effects can vary drastically for one application to another; and processes may involve chemical reactions. Further, the interaction of the operating conditions, geometrical variations, and material properties create a wide range of synergetic complexity and variety of failure modes in all fields of engineering, Stephens et al. [1]. In metals alone, the possible mechanical failure modes can fall into one of the following categories: excess deformation, ductile fracture, brittle fracture, impact loading, creep, relaxation, thermal shock, wear, buckling, corrosion and fatigue. Among them, 50 to 90 percent of all mechanical failures are classified as fatigue failure, Stephens et al. [1].

Fatigue failure occurs when the strength of a structure and/or its components decrease when subjected to repeated loading and cracks tend to form in response to fluctuating stress and strain. Eventually, continuation of applied cyclic load results in increasing the crack size and decreasing the residual strength of the material to the extent that the structure or component can no longer function in the intended manner for which it was designed for. In the end, the residual strength becomes so low that the failure of the structure becomes imminent, Broek [2]. The knowledge
and the capability of predicting the rate of decline in the residual strength and the remaining useful life of a component are, therefore, necessary at the design stage to guard against premature fatigue failure.

Fatigue life of a structure or component is mainly divided into two periods: (i) the crack initiation period and (ii) the crack growth period. Crack nucleation and microcrack growth occur in the first period, primarily at the material surface. The second period starts when the fatigue crack penetrates into the material subsurface. Major part of the life of component or structure is expended in the second period. The growth of the fatigue crack thus depends upon the crack growth resistance of the material as a bulk property, environmental condition, stress-state and service load history. Therefore, a methodology for prediction of fatigue life which encompasses these factors is highly desirable.

Nearly all existing testing methods for evaluating the fatigue life require instrumenting a specimen and subject it to cyclic testing in one *prescribed manner*, e.g. bending alone or tension/compression alone. The process is tedious, expensive, and extremely time-consuming as it often takes many thousands to millions of cycles until failure occurs. Furthermore, the *length scale* of the test specimen can make it difficult and often prohibitive to instrument in some applications, for example, when one deals with miniature devices. In addition, the operating environment in which the specimen is tested is often different from what the unit experiences in practice.

The premise of this chapter is that fatigue is a degradation process and the entropy generation due to material damping during fatigue and fracture is a fundamental thermodynamic process. We propose to develop an appropriate fatigue degradation theory using the concept of entropy—which physicists sometimes refer to as the “arrow of time”—as a natural time base. In processes involving fatigue, damage occurs as a result of local accumulation of plastic strain energy. Since
plastic deformation is irreversible, it must be accompanied by irreversible entropy production. During each fatigue cycle, plastic work and strain energy are dissipated as fatigue crack forms and propagates in the structure. Our objective is to develop the fundamental science that can appropriately describe fatigue processes in terms of dissipation process functions $p_i = p_i(\zeta_i)$ based on the irreversible entropy generation in order to assess damage accumulation in a component or a structure subjected to fatigue load.

The scientific outcome of the present chapter will be a general method for prediction and treatment of fatigue problem applicable to distinctive features, types, rates, and sequences of cyclic loading (i.e. low and high-cycle fatigue, multiaxial loading, etc.). This involves formulation of a generalized “degradation force” in terms of its generalized “thermodynamic force” counterpart.

Fatigue problems have been under intensive research for many decades, but progress has been at best incremental. In addition, there is a great need for prediction of behavior and the reliability of micro-devices, a subject that is poorly understood. By taking advantage of the fundamental principles of irreversible thermodynamics, it is possible to develop the foundation for analyzing problems involving degradation in a manner which can result in major impact on the science and technology. Further, structured methods for constructing degradation models that remove many of the existing limitations and novel accelerated testing procedures for failure can be developed. The need for generalization and unifying principles that relate entropy to dissipative processes that cause degradation as well as development of methodology for accelerated testing are intellectually stimulating and potentially transformative. Our research goals include:

- Field unification by using entropy as a natural “time base” and unifying principle. Note that in these formulations entropy is necessarily an implicit variable. These formulations could lead to structured ways to interpret results of accelerated testing;
Formulation of relationship between entropy and fatigue and damage;

Experimentations to verify theory;

Development of an accelerated testing methodology; and

Development of the implementation techniques for structural health monitoring.

The science base that underlies modeling and analysis of machine reliability has remained substantially unchanged for decades. A significant gap exists between available machinery technology and science to capture degradation dynamics in a modeling paradigm suitable for early failure prediction, structured development of accelerated failure testing, and control of machine maintenance. This chapter focuses upon a theoretical and empirical investigation of degradation in machinery components, to advance scientific understanding of degradation dynamics. Proposed research focuses upon:

1) Developing a thermodynamic characterization of degradation dynamics which employs entropy, thermodynamic disorder, as the fundamental measure of degradation.

2) Developing methods of measurement of state variables that are functionally related to entropy which is, necessarily, an implicit system variable.

The thermodynamic state of a system can be described in terms of system energies and entropy. Entropy measures disorder of a system, and increases with disorder. We propose thermodynamically based models of fatigue degradation based on consideration of the irreversible effects caused by dissipative phenomena. Recently, a general theorem that relates entropy generation to irreversible degradation, via generalized thermodynamic forces $X_i$ and degradation forces $Y_i$ has been developed by Bryant et al. [3]. The Degradation-Entropy Generation (DEG) theorem proves that: (i) the degradation rate $\dot{w} = \sum_i \dot{w}_i$ is a linear combination $\dot{w} = \sum_i B_i \dot{S}_i$ of the components of entropy production $\dot{S}_i = \sum_i X_i / J_i$ of the
dissipative processes $p_i$ where $J_i^j$ are generalized rates or flows; (ii) the generalized degradation forces $Y_i^j$ are linear functions $Y_i^j = B_i X_i^j$ of the generalize thermodynamic forces $X_i^j$, and (iii) the proportionality factors $B_i$ are the degradation coefficients given by $B_i = \partial w/\partial S \bigg|_{p_i}$. In what follows we describe the DEG theorem and address how it characterize a system undergoing fatigue load and accelerated testing.

3.2 Degradation-Entropy Generation (DEG) Theorem

Recently a team of researchers at Center for Rotating Machinery (CeRoM) at Louisiana State University has developed novel methods for prediction of failure of components undergoing fatigue load and components subjected to fatigue. In the present research, predictive methods are developed within a thermodynamic framework of degradation processes. First, a general thermodynamics framework of degradation processes is developed which formulates degradation processes in terms of the irreversible entropy produced. The fundamental theory of Degradation-Entropy Generation theorem first developed by Bryant et al. [3], establishes a general relationship between rate of degradation of systems and the rate at which entropy is produced by underlying dissipative irreversible processes. It is the novelty of this theorem to characterize the degradation processes such as wear, fretting wear and fatigue without complications associated with phenomenological models which are limited primarily to the specific system being examined. The Degradation-Entropy Generation theorem also offers a methodology for use in accelerated degradation testing. The need for generalization and unifying principles that relate entropy to dissipative processes that cause degradation as well as development of methodology for accelerated testing are discussed.

Suppose a degradation mechanism consists of $i = 1,2,\ldots,n$ dissipative processes $p_i$, where each $p_i = p_i(\xi_i)$ could describe an energy, work, or heat characteristic of the process, and depends on a
set of time dependent variables \( \zeta_i^j, \ j=1,2,...m_i \). To accumulate effects of the processes on overall degradation or aging, define degradation measure
\[
w = w\{ p_i(\zeta_i^j) \} = w(\zeta_i^j), \quad i =1,2,...,n; \quad j=1,2,...,m_i
\]
which must depend on all \( \zeta_i^j \) of the \( n \) processes \( p_i \). Any dissipative process \( p_i \) must produce an irreversible entropy \( S_i' = S_i'\{ p_i(\zeta_i^j) \} \), characterized by the same set of variables \( \zeta_i^j \). Here “prime” denotes irreversible entropy, subscript \( i \) references specific process \( p_i \), and superscript \( j \) indicates which \( \zeta_i^j \) of process \( p_i \). In accordance with the 2nd law, the degradation mechanism must generate total irreversible entropy
\[
S' = S'\{ p_i(\zeta_i^j) \} = \sum_i S_i', \quad i =1,2,...,n; \quad j=1,2,...,m_i.
\]

The 2nd law mandates nonnegative entropy generation and the sum over the dissipative processes. The rate of degradation \( \frac{dw}{dt} \) can be determined by applying the chain rule to Eq. (3.1):
\[
\frac{dw}{dt} = \sum_i \sum_j \left( \frac{\partial w}{\partial \zeta_i^j} \frac{\partial \zeta_i^j}{\partial p_i} \right) \frac{\partial \zeta_i^j}{\partial t} = \sum_i w_i = \sum_i \sum_j Y_i^j J_i^j. \tag{3.3}
\]

Rate of entropy \( \frac{dS'}{dt} \), which is entropy generation, via Eq. (3.2) and the chain rule is
\[
\frac{dS'}{dt} = \sum_i \sum_j \frac{\partial S'}{\partial \zeta_i^j} \frac{\partial \zeta_i^j}{\partial p_i} \frac{\partial \zeta_i^j}{\partial t} = \sum_i \hat{S}_i' = \sum_i \sum_j X_i^j J_i^j. \tag{3.4}
\]

In Eqs. (3.3) and (3.4), \( \hat{S}_i' \) and \( \hat{w}_i \) denote entropy generation and degradation rate contributions arising from specific process \( p_i \). For stationary systems, or systems near equilibrium, irreversible thermodynamics [4-6] expresses entropy generation \( \hat{S}_i' = X_i^j J_i^j \) as the product of a generalized force \( X_i^j \) and a generalized rate or flow \( J_i^j \), which, comparing the sums in Eq. (3.4), \( X_i^j = \partial S'/\partial p_i(\partial p_i/\partial \zeta_i^j) \) and \( J_i^j = \partial \zeta_i^j/\partial t \). Likewise, \( \hat{w}_i = Y_i^j J_i^j \) in Eq. (3.3), with \( Y_i^j = \partial w/\partial p_i(\partial p_i/\partial \zeta_i^j) \). Since the 2nd law mandates non-negative entropy generation \( \hat{S}_i' \geq 0 \) for any
process (indexed by \(i\)), signs in Eq. (3.4) of multiplying factors must be identical, i.e., \(\text{sgn}(X_i') = \text{sgn}(J_i')\). Equations (3.3) and (3.4) share rate factors \(J_i'\). Irreversible thermodynamics considers forces \(X_i'\) as drivers of flows \(J_i'\). Each \(J_i'\) can depend on all forces [7] and intensive quantities (e.g., temperature \(T\)) associated with the dissipative process via [5]:

\[
J' = J'(X^1, X^2, \ldots; T) \approx \sum_q L_{qj} X^q
\]

where subscript \(i\) was dropped in Eq. (3.5) for clarity. For systems near equilibrium or stationary— at equilibrium entropy production \(dS'/dt = 0\); stationary systems have total entropy change \(dS/dt = dS'/dt + dS_c/dt = 0\), where \(dS_c/dt\) is entropy flow—relation Eq. (3.5) is invertible and usually assumed linear [5]. Non-negative entropy production demands symmetric, positive definite \(L_{qj}\). Applications of Eqs. (3.4) and (3.5) have explained diverse phenomena—thermoelectric effect, diffusion [5-6], and phase changes, among others—and have given an alternate derivation of Kirchoff’s voltage law for resistive networks.

Equation (3.4) can be constructed via methods of thermodynamics [4-6]. Klamecki [8-11] did this for cases of tribological interest. Since Eqs. (3.3) and (3.4) share \(J_i'\)’s, an entropy production analysis—obtained by constructing Eq. (3.4)—can elucidate the rates associated with degradation Eq. (3.3). In analogy to generalized thermodynamic force \(X_i' = \partial S'/\partial p_i \partial p_i/\partial \zeta_i'\) in Eq. (3.3), we call \(Y_i' = \partial w/\partial p_i \partial p_i/\partial \zeta_i'\) in Eq. (3.4) “generalized degradation force” and define ratio:

\[
G_i \triangleq \frac{Y_i'}{X_i'} = \left. \frac{\partial w}{\partial S'} \frac{\partial S'}{\partial p_i} \right|_{p_i} = \left. \frac{\partial w}{\partial p_i} \frac{\partial p_i}{\partial \zeta_i'} \right|_{p_i} \quad (3.6)
\]

which exists, since \(X_i' \neq 0\) except when the system is not degrading. The last term in Eq. (3.6) means \(\partial w/\partial S'\) with process \(p_i\) active, which suggests \(G_i\) measures how entropy generation and
degradation interact on the level of processes \( p_i \), rather than the process variables \( \zeta_i \). The preceding formulations suggest an approach for degradation analysis:

1. From knowledge of the degradation mechanism, list the irreversible processes \( p_i = p_i(\zeta_i) \) and their variables \( \zeta_i \). Often, the process \( p_i \) can be energy dissipated, and may be posed in terms of lost work, heat transferred, or a thermodynamic energy, e.g., internal energy or Gibbs free energy.

2. Using Eq. (3.3), obtain an expression for the degradation rate \( dw/dt \).

3. Obtain entropy generation \( dS'/dt \) of Eq. (3.4) via irreversible thermodynamics. Historically, this has involved applying laws of thermodynamics to a control volume about the body in question. Klamecki [8-11] presents examples relevant to tribology.

4. Via Eq. (3.4), obtain process rates \( J_i^j \). Also, compare Eqs. (3.3) and (3.4), since rates \( J_i^j \) are common.

5. Via Eqs. (3.3), (3.4), and (3.6), get thermodynamic forces \( X_i^j \) and degradation forces \( Y_i^j \).

   By measuring coefficients \( G_i \) of Eq. (3.6), \( Y_i^j = \partial w/\partial p_i (\partial p_i/\partial \zeta_i) \) can be related to \( X_i^j = \partial S'/\partial p_i (\partial p_i/\partial \zeta_i) \). For \( X_i^j \) and \( G_i \), if process \( p_i \) is an energy dissipated, then definition of entropy suggests \( \partial S'/\partial p_i = 1/T_i \), where \( T_i \) is a temperature.

6. If needed, via Eq. (3.5), relate rates \( J_i^j \) to thermodynamic forces \( X_i^j \).

All systems must obey conservation of energy, stated by the first law of thermodynamics

\[
dE = dQ - dW + \sum \eta_k dN_k \quad (3.7a)
\]

where \( E \) is internal energy, \( Q \) and \( W \) are heat flow and work across the boundary of the relevant control volume, and \( \eta_k \) and \( N_k \) are chemical potential and number of moles of species \( k \). A change in entropy

\[
dS = dS' + dS_c \quad (3.7b)
\]
consists of a reversible change \( dS_e \) from entropy flow, and an irreversible change from entropy generation \( dS' \). We note that entropy flow arises from heat transfer via heat flow \( dQ \) and matter flow \( \sum \eta_k dN_k \)

\[
TdS_e = dQ + \sum \eta_k dN_k \tag{3.7c}
\]

Change in number of moles

\[
dN_k = d'N_k + d_e N_k \tag{3.7d}
\]

consists of a chemical reaction term \( d'N_k \) and a matter transport term \( d_e N_k \). At equilibrium, a system’s entropy is maximum and entropy production ceases: \( dS'/dt = 0 \) [6]. A system produces entropy (\( dS'/dt > 0 \)) until equilibrium. Stationary systems have \( dE = 0 \) and \( dS = 0 \).

Our interest is the irreversible effects of dissipation caused by work of non-conservative forces. This dissipated work must eventually diffuse through heat flow \( dQ \) and/or mass flow \( \Sigma \eta_k dN_k \), as an entropy flow \( dS_e/dt \). Via Eq. (3.7), the irreversible entropy produced can be linked to the work dissipated [5, 11, 12]. Open systems demand balancing flows of entropy, heat, work, energy, and mass over a control volume about the degrading body or system, to construct open system counterparts of entropy generation Eq. (3.4), and possibly degradation Eq. (3.3).

### 3.3 Experimentally-Verified Entropy-Fatigue Formulation

An appropriate fatigue degradation theory is proposed using “arrow of time” entropy as a natural time base. Accumulated damage and degradation can be related via the DEG theorem to irreversible entropy. Progression of fatigue [13] is normally rated in terms of number of stress cycles \( N \). With approximately the same work \( \delta W \) dissipated per cycle, counting stress cycles is equivalent to tallying total energy \( N \delta W \) dissipated, or total entropy \( \approx N \delta W / T \) generated. For any cyclic irreversible process, entropy monotonically increases each cycle. If temperature \( T \) is known, entropy changes can be estimated via Eq. (3.2), and the process could be modeled via...
methods of previous section. Tallying irreversible entropy should be more accurate than counting cycles, since the energy per cycle $\delta W$ will not remain constant throughout the fatigue process. Furthermore, it is not restricted on the application of a constant load since it deals with accumulation of entropy as a function of time.

3.3.1 Application to Torsion and Tension-Compression Fatigue

We will complement the fatigue-entropy theory with experiments. Fatigue failure can occur only if—as a result of the presence of micro-cracks, local yielding, and micro-cavities—a fatigue crack coalesces and propagates. The applied load produces an increase in the stress about a point (or zone) of the material, with local values exceeding the elastic limit [19]. It is known that if the stress is static, the local plasticization and redistribution of the stress onto the surrounding material does not generate any particularly critical condition, and the material reaches failure only under decidedly greater loads. For cyclic loading characteristic of fatigue, when the material arrives at the condition of local yielding (micro-plasticization) and a micro-crack forms, repeated application of the same stress induces the crack to propagate until, in time, the lengthened crack sufficiently weakens the structure and the specimen breaks.

3.3.2 Application to Bending Fatigue

During each fatigue cycle, plastic work and strain energy are dissipated by fatigue crack formation and propagation in the structure. We will formulate the dissipation process functions $p_i = p_i(\zeta_i)$ for these mechanisms, and apply the methods of previous section. As fatigue cracks form and propagate, a component’s strength $S_N$ decays via a power law according to the number of cycles $N$ logged: $S_N = S_u N^\alpha$, $\alpha < 0$, where $S_u$ is reference strength [14]. In light of the preceding paragraph, we note that strength $S_N$ diminishes as irreversible entropy $S'$ increases, making $S_N$ a candidate degradation measure $w$. We will investigate other degradation measures, such as the damage parameter [15-18] as it is used in damage mechanics. The equations
governing the system processes, written in terms of energy statements, can be embedded into the failure law via the energy-entropy relations of previous section. Based on experiments presented later in this chapter, we anticipate formulating these equations in terms of field quantities such as temperature and entropy density. We anticipate subsuming of classical fatigue models—and indeed, other models of material degradation—into a general thermodynamic framework using the DEG theorem.

It is interesting that the temperature evolution resulting from the heat generated during the fatigue process can be utilized to monitor fatigue-crack propagation [20-23]; measure the energy required to produce a unit area of a fatigue crack by propagation [24]; determine the endurance limit of some materials [25, 26]; and characterize evolution of cumulative damage in the fatigue process [27-30]. When a material is subjected to cyclic deformation, mechanically induced strain-energy dissipation is strongly dependent on the magnitude and history of the cyclic stress and on the coupling between mechanical and thermal effects. Most of the dissipated strain energy is converted into heat, manifested by changes in temperature.

The premise of this research—that processes involving degradation are naturally related to entropy generation—was tested in the LSU Center for Rotating Machinery, as a proof of concept for this proposal. The proposed apparatus is described in the preceding chapters.

As mentioned earlier, temperature rise of a specimen undergoing cyclic loading is principally due to the energy generation and dissipation of the plastic work. The energy approach for estimating the fatigue life of materials under cyclic loading tests has gained considerable attention by researchers [31-37]. Morrow’s paper is representative of a pioneering work that takes into account cyclic plastic energy dissipation and fatigue of metals that undergo cyclic loading. He presented a descriptive theory of fatigue that uses the cumulative plastic strain energy as a criterion for fatigue damage and the elastic strain energy as a criterion for fracture.
For fully reversed fatigue load, Morrow derived a relation for plastic strain energy per cycle \( W_p \) in terms of the cyclic stress-strain properties, applicable when plastic strain is predominant. Park & Nelson [34] proposed an empirical correlation for estimation of fatigue life taking into account the elastic strain energy \( W_e \) as well as plastic strain energy \( W_p \). In the high-cycle regime, plastic strains are usually quite small and the \( W_p \) approach becomes computationally unreliable. Park & Nelson [34] proposed that the two energy terms, \( W_p \) and \( W_e \), must be combined into the total strain energy parameter \( W_t \),

\[
W_t = W_p + W_e = AN_f^\alpha + BN_f^\beta
\]

(3.8)

where the constants \( A, \alpha, B \) and \( \beta \) can be determined from a set of uniaxial fatigue test data that cover a sufficiently large number of cycles. The energy dissipation due to plastic deformation during fatigue is a fundamental irreversible thermodynamic process that must be accompanied by irreversible entropy gain. For the purpose of the present work, we considered the energy dissipation due to plastic deformation to assess for the evaluation of the entropy generation. Since are experiments encompass a range from low to intermediate cycle fatigue tests, plastic energy dissipation is solely used for the analysis. Hence, energy dissipation due to elastic deformation is neglected. In low-cycle fatigue where the entropy generation due to plastic deformation is dominant and the entropy generation due to heat conduction is negligible, entropy generation can be found from the following equation:

\[
\gamma_f = \int_0^{t_f} \left( \frac{W_p}{T} \right) dt
\]

(3.9)

where \( \gamma_f \) is the entropy at the time when specimen breaks, \( W_p \) is the heat dissipation of plastic work, \( t_f \) is the time of fracture and \( T \) is the temperature. The evolution of entropy generation is calculated for the entire fatigue life and then integrated over time to determine the entropy generated during fatigue process (Eq. 3.9).
Figure 3.1 shows comparison of numerical and experimental entropy generation for bending fatigue of Al 6061-T6 where frequency and displacement amplitude are 10 Hz and 49.53 mm, respectively. Small difference between the experimental result and numerical simulation is due to the fact that heat conduction is neglected in Eq. (3.9), but not in simulation. The final value of the entropy generation (about 4 MJ/m$^3$K for this test) is associated with the entropy at fracture when the specimen breaks into two pieces. An uncertainty analysis is performed using the method of Kline and McClintock [38]. Maximum error in calculating entropy based on uncertainty analysis is about $\pm 1\%$.

![Graph showing Volumetric Entropy Generation vs. Number of Cycles for Al 6061-T6 under Bending Test, Frequency=10 Hz, Displacement Amplitudes=49.53 mm.](image)

**Figure 3.1. Volumetric entropy generation evolution vs. number of cycles for Al 6061-T6 under bending test, frequency=10 Hz, displacement amplitudes=49.53 mm.**

Figure 3.2 presents the results of entropy at fracture plotted as a function of the fatigue life for bending and tension-compression fatigue tests for Al 6061-T6 specimens at 10 Hz. It is seen that the entropy is independent of the type of loading. Figure 3.3 presents the results of entropy generation at the fracture point for stainless steel 304 undergoing bending, and torsion fatigue tests. The results show that the entropy generation at the fracture point for SS 304 is about 60 MJ/m$^3$K, independent of frequency and geometry.
Figure 3.2. Entropy vs. fatigue life for different fatigue tests of Al 6061-T6 at frequency 10 Hz. Fracture fatigue entropy remains at about 4 MJ/m\(^3\)K for both tension-compression and bending fatigue.

Figure 3.3. Entropy vs. fatigue life for different fatigue tests of SS 304 for different loads, frequencies and tests. Fracture fatigue entropy remains at about 60 MJ/m\(^3\)K for tension-compression and bending and torsion fatigue.

The results presented in Figures 3.2 & 3.3 demonstrate the validity of the constant entropy gain at the fracture point for Aluminum and Stainless Steel specimens. The results reveal that the necessary and sufficient condition for final fracture of Al 6061-T6 corresponds to the entropy gain of 4 MJ/m\(^3\)K regardless of the test frequency, thickness of the specimen and the stress state. For SS 304 specimens, this condition corresponds to entropy gain of about 60 MJ/m\(^3\)K. The concept of constant entropy gain at the fracture point, \(\gamma_f\) assumes that thermodynamic condition
associated with the entropy generation is identical during the fatigue process and varies only in the duration of the process. That is, failure occurs when

\[ N = N_f, \quad \gamma = \gamma_f \]  

(3.10)

A possible application of the proposed hypothesis of the constant entropy gain at the final fracture is in the development of a methodology for prevention of the catastrophic failure of metals undergoing fatigue load. As the entropy generation accumulates toward the \( \gamma_f \), it provides the capability of shutting down of the machinery before a catastrophic break down occurs. This treatment can be easily applied to very small (miniature) or large systems, where current methods of cyclic fatigue testing using existing techniques can be expensive, tedious, or even impractical. This technology can be used to rapidly determine the expected fatigue life of a new component, or the remaining fatigue life an existing structure or component.

Based on the concept presented by Eq. (3.10), one can conduct an accelerated failure testing scheme by increasing process rates \( J \) while maintaining equivalent thermodynamic forces \( X \) to obtain the same sequence of physical processes, in identical proportions, but at a higher rate. For example, by increasing frequency, the rate of plastic deformation \( \dot{\varepsilon}_p \) increases and subsequently the rate of degradation increases while the duration of the test is shortened in order to satisfy Eq. (3.10). This is in accordance with the accelerated testing procedure recently put forward by Bryant et al. [3] based on the thermodynamics of degradation.

Figure 3.4 shows the normalized entropy generation during the bending fatigue of SS-30 and Al 6061-T6 for different thicknesses, displacement amplitudes and frequencies. The abscissa of Figure 3.4 shows the entropy generation using Eq. (3.9) and normalized by dividing by the entropy gain at the final fracture, \( \gamma_f \). The ordinate shows the number of cycles normalized by dividing by the final number of cycles when failure occurs. It can be seen that normalized entropy generation monotonically increases until it reaches the entropy at the failure point.
Interestingly, a similar trend between normalized wear plotted against the normalized entropy was reported by Doelling et al. [39]. Their work resulted in prediction of flow of the Archard’s wear coefficient [40] with remarkable accuracy.

![Graph](image)

**Figure 3.4. Normalized entropy generation vs. normalized number of cycles for bending fatigue of SS 304 and Al 6061-T6 for different thicknesses of specimen, displacement amplitudes and frequencies.**

The relation between the normalized cycles to failure and normalized entropy generation is approximately linear and can be described as follows:

\[
\frac{\gamma}{\gamma_f} \approx \frac{N}{N_f}
\]  \hspace{1cm} (3.11)

where \(\gamma_f\) is a property of material. Using Eq. (3.11), the number of cycles to failure can be expressed as:

\[
N_f \approx \left(\frac{N}{\gamma}\right) \cdot \gamma_f
\]  \hspace{1cm} (3.12)

Equation (3.12) offers a methodology for prediction of the fatigue failure of a given material based on the measurement of the thermodynamic entropy generation. By having FFE, \(\gamma_f\) and calculating entropy generation \(\gamma\) at a selected number of cycles \(N\), the fatigue life \(N_f\) of the
specimen can be predicted. Calculation of the entropy generation \( \gamma \) can be performed at the very beginning number of cycles of the test, thereby, providing an accelerated testing method for determination of fatigue failure.

### 3.4 Accelerated Testing

Accelerated testing consists of test methods that accelerate the degradation of component or structure for predicting longer life based on its shorter life. We propose to demonstrate accelerated testing of degradation. As mentioned earlier, Eqs. (3.3) and (3.4) have common rate factors \( J_i/ = \partial \zeta_i/ \partial t \) which depend on system parameters \( \zeta_i \). If the flow rates \( J_i/ \) are judiciously chosen, the rate of degradation \( dw/dt \) in Eq. (3.3) can be observed without waiting long times. In general, accelerated failure testing schemes could be based on EqS. (3.3)-(3.7), wherein we increase process rates \( J_i/ \) to accelerate the test, while maintaining “equivalent” forces \( X_i/ \) and \( Y_i/ \) to obtain the same sequence of physical processes, in identical proportions, but with higher rate. Simply increasing rates \( J_i/ \) may alter the physical processes. For example, moderate heat hatches an egg; if heating is accelerated without maintaining “equivalent” forces, the egg cooks. Via Eq. (3.5), altering any \( J_i/ \) could change every \( X_i/ \). As part of this research, we will investigate what accelerated conditions achieve “equivalent” forces. One possibility is to keep overall \( dw/dt \) and \( dS'/dt \) in the same proportion, subject to elevated rates \( J_i/ \) in Eq. (3.5). The key to accelerated testing is measurement of relations Eqs. (3.5) and (3.6).

As demonstrated by experiments and represented by Eq. (3.10), fracture will occur if the entropy accumulation reaches the fracture entropy \( \gamma_f \). Based on this concept, one can conduct an accelerated failure testing scheme by increasing process rates \( J \) while maintaining equivalent thermodynamic forces \( X \) to obtain the same sequence of physical processes, in identical proportions, but at a higher rate. For example, by increasing frequency, the rate of plastic deformation \( \varepsilon_p \) is increased and subsequently the rate of degradation increases while the duration
of the test is shortened in order to satisfy Eq. (3.10). This is in accordance with the accelerated testing procedure recently put forward by Bryant et al. [3] based on the thermodynamics of degradation.

### 3.5 Damage Mechanics and Entropy

Permanent degradation is the result of an irreversible process accompanied by generation of entropy, as demanded by the second law of thermodynamics. Entropy is a fundamental parameter that can be used to quantify the behavior of irreversible degradation processes. The variation of the entropy, $dS$, is the sum of two terms [41]:

$$dS = dS_i + dS_e$$  \quad (3.13)

where $dS_i$ is the entropy generated inside the system and $dS_e$ represents the entropy supplied to the system. The second law of thermodynamics states that $dS_i$ must be equal to zero for reversible processes and positive for irreversible processes; that is:

$$dS_i \geq 0$$  \quad (3.14)

In the thermodynamics of irreversible processes, the objective is to relate the internal entropy production to the various irreversible processes that may be occurring within the solid continuum as a system. For this purpose, Eqs. (3.13) and (3.14) can be written in the volumetric forms [41] as:

$$\rho \frac{ds}{dt} = - \text{div}(J_{s,\text{tot}}) + \dot{\gamma}$$  \quad (3.15)

where

$$S = \int_{V} \rho s dV$$  \quad (3.16)

$$\frac{dS_e}{dt} = - \int_{\Omega} J_{s,\text{tot}} d\Omega = \int_{V} \text{div}(J_{s,\text{tot}}) dV$$  \quad (3.17)
\[ \frac{dS_i}{dt} = \int \dot{\gamma} dV \]  

(3.18)

where \( s \) is the entropy per unit mass and \( \rho \) denotes the density, \( J_{s,\text{tot}} \) is the total entropy flow per unit area and unit time, \( d\Omega \) is the element of surface area, and \( \dot{\gamma} \) represents the entropy source strength or entropy production per unit volume per unit time.

The Clausius-Duhem inequality states that in solids with internal friction all the deformations cause positive entropy production rate [42]:

\[ \dot{s}_i = \frac{I}{T} \sigma : \dot{\varepsilon}_p - \frac{I}{T} A_k \dot{V}_k - \frac{I}{T^2} \vec{q} \cdot \text{grad} T \geq 0 \]  

(3.19)

where \( \sigma \) is the stress tensor, \( \dot{\varepsilon}_p \) is the plastic strain rate, \( T \) is absolute temperature, \( V_k \) can be any internal variable such as damage and hardening, \( A_k \) are thermodynamic forces associated with the internal variables and \( \vec{q} \) is the heat flux. There is no objective way of choosing the nature of the internal variable, \( V_k \), that is best suited to the study of a phenomenon. The choice is dictated by experience, physical feeling and very often by the type of application [43]. In deriving the Clausius-Duhem inequality it is assumed that variables \( A_k \) associated with the internal variables are defined by the specification of the thermodynamic potential \( \Psi(T, \varepsilon_e, V_k) \) with \( \varepsilon_e \) as elastic strain. Hence, \( s, \sigma \) and \( A_k \) constitute the associated variables. Equation (3.17) is also interpreted as the product of generalized thermodynamic forces \( X = \{\sigma, A, \text{grad} T\} \) and generalized rates or flows \( J = \{\dot{\varepsilon}_p/T, -\dot{V}_k/T, -\vec{q}/T^2\} \):

\[ \dot{s}_i = \sum_k X_k \cdot J_k \]  

(3.20)

Irreversible thermodynamics considers forces \( X \) as drivers of flows \( J \). Each \( J \) can depend on all forces [44] and intensive quantities (e.g., temperature \( T \)) associated with the dissipative process.
The entropy flow $s_e$ is the part of total entropy $s$ which is measured on the surfaces of the body while the entropy generation $s_i$ is measured inside the volume of the solid material. Due to the complexity associated with the assessment of terms in Eq. (3.19) such as internal variable, the entropy flow can be conveniently evaluated and linked to the damage for processes involving degradation using Eq. (3.17). In the present study, entropy exchange between the system and the surroundings is used as a connection between damage and degradation process. Hence, the entropy flow to the environment, $(dS_e/dt)$, was determined by experimentally measuring the surface temperature of the specimen. From experimental temperature, one can evaluate the rate of entropy flow to the surrounding by simple temperature variation:

$$\dot{s}_e = h(T - T_a) = h\left(I - \frac{T_a}{T}\right)$$  \hspace{1cm} (3.21)

where $h$ is the convective heat transfer coefficient. Figure 3.5 shows the normalized entropy flow during the fatigue life until failure. The abscissa of Figure 3.5 shows the entropy calculated using Eq. (3.21) and normalized by dividing by the maximum value at the onset of failure. The ordinate shows the number of cycles normalized by dividing by the final number of cycles when failure occurs. It can be seen that normalized entropy monotonically increases until it reaches the maximum at the failure point. The relation between the normalized cycles to failure and normalized entropy is approximately linear and can be described as follows:

$$\frac{s_{e,\text{max}}}{s_f} \approx \frac{N}{N_f}$$  \hspace{1cm} (3.22)

The relationship between entropy flow and number of cycles for failure is in accordance to the DEG theorem which states that the degradation forces $Y_{i,j}$ are linear functions $Y_{i,j} = B_i X_{i,j}$ of the generalize thermodynamic forces $X_{i,j}$. The proportionality factors $B_i$ are the degradation coefficients given by $B_i = \partial w/\partial S_{i,p}$, where $w$ is the degradation measure, $S$ is the irreversible
entropy, and the variable $p_i$ represents the dissipative processes. The relationship $B_i = \partial w / \partial S|_{p_i}$ means $\partial w / \partial S$ with process $p_i$ active, which suggests $B_i$ measures how entropy generation and degradation interact on the level of process $p_i$. Irreversible thermodynamics suggest that $B_i$ can depend on intensive variables such as temperature $T$, [3].

![Diagram](image)

**Figure 3.5. Normalized number of cycle vs normalized entropy.**

Entropy flow is directly related to the surface temperature of the specimen, $T$. According to literature [45], damage variable is a function of the $N/N_f$ ratio, i.e. $D = f(N/N_f)$. Hence, following Eq. (3.22), damage variable can be defined as $D = f(s_e/s_f)$. A logarithmic expression for the damage variable is expressed in terms of the entropy flow as follows:

$$D = \frac{-1}{\ln(s_f)} \ln \left( \frac{1 - \frac{s_e}{s_f}}{s_f} \right)$$

(3.23)

where $D$ is the damage parameter, $s_e$ ($Jm^3K^{-1}$), is the volumetric entropy and $s_f$, ($Jm^3K^{-1}$), is the maximum entropy at the fracture point. The correlation expressed in Eq. (3.23) enables the assessment of damage evolution during a fatigue process based on entropy measurement. The advantage of entropy-damage correlation presented in Eq. (3.23) with respect
to other damage evolution prediction models such as Miner rule is that Eq. (3.23) enables the prediction of critical damage variable. Critical damage variable or critical condition in fatigue process is referred to as a condition wherein the operation of the machinery or component is not reliable and the final fracture is imminent. Critical damage variable $D_c$ is associated with the critical entropy generation, $s_c$.

### 3.6 Reference


Chapter 4: The Effect of Surface Cooling on Fatigue Life
4.1 Introduction

A series of fully reversed cyclic load is conducted to investigate the effect of surface cooling on the fatigue life of metallic specimens. The experiments involve rotating-bending of Stainless Steel 304L and Steel 4145. External cooling is provided via a vortex tube at a constant cooling rate. The results of experiments reveal significant improvement in fatigue life when the surface of the specimen is cooled. An explanation for the observed phenomenon is offered which utilizes the notion of self organization within the context of irreversible thermodynamics.

Environmental effects on fatigue life of metals have been widely investigated for many years with particular interest in elevated or cryogenic temperatures [1-5], corrosive operating condition [6-13], vacuum or pressurized environment [14-17]. All of the factors strongly affect the fatigue life of metals.

Environmental condition can affect the fatigue crack growth rate as it is influenced by surface condition [18-20] and subsequently, can affect the fatigue life. For example, Tobushi et al. [20] studied the influence of air and water atmosphere, temperature, strain amplitude and rotational speed on the fatigue life of a shape-memory alloy wire subjected to rotating-bending fatigue. They showed that in low-cycle fatigue below $10^4$ cycles, the fatigue life in water is longer than that in air. Water was capable of maintaining the wire temperature at a constant level so that the yield stress was constant. But, in the region of high-cycle fatigue above $10^5$ cycles, the fatigue life in water was shorter than that in air, mainly due to the introduction of corrosion fatigue in water.

Hirano et al. [21] investigated the effects of water flow rate on fatigue life of carbon steel in simulated light-water reactor (LWR) environment. They tested carbon steel at $289\,^\circ\text{C}$ for various dissolved oxygen contents (DO) at strain rates of 0.4, 0.01, and 0.001 percent per second (%/s). Their experimental results showed that at the strain rate of 0.01 %/s, the fatigue life increased
with increasing the flow rate under all DO conditions. Specifically, they reported that the fatigue life at a 7 m/s flow rate was about three times longer than that at a 0.3 m/s flow rate. This increase in fatigue life was attributed to increase in the crack initiation life and small-crack propagation life.

In the present work, we investigate the influence of surface cooling on the life of specimens undergoing rotating-bending fatigue test. Two different materials, Stainless Steel 304L and Steel 4145 are tested. Results are compared with the case of fatigue at room temperature. Significant improvement in fatigue life is observed. An explanation for the observed phenomenon that utilizes the concept of self-organization associated with irreversible thermodynamics is offered.

4.2 Experimental Procedure and Materials

Figure 4.1 presents a schematic of the experimental setup for rotating-bending apparatus. The unit is a compact bench-mounted unit with a variable speed motor with a failure cut-off circuit in a control box and a cycle counter. The test stress level is determined by selecting a percentage (%) of the tensile strength of the test material and converting that value into a bending moment. After the specimen is mounted into the collets, the speed control is adjusted to bring the machine up to the desired speed and the poise weight is then positioned on the calibrated beam to the bending moment previously calculated and locked into place. When the specimen fractures, the machine automatically stops and the number of cycles to failure is read from the cycle counter (digital indicator up to 99,999,900). Results obtained by testing specimens at various loads provide the necessary data for plotting stress vs. number of cycles to fatigue (SN) curve.

All tests are performed with a fresh specimen and run until failure, when the specimen breaks into two pieces. Infrared (IR) thermography was used to record the temperature evolution of the specimen during the entire experiment. The IR camera was a MIKRON M7500 with temperature range between 0°C to 500°C, resolution of 320×240 pixels, accuracy of ±2% of reading,
sensitivity/NEDT of 0.08°C at 30°C, and image update rate of 7.5 Hz. Two different types of materials, Stainless Steel 304L and Steel 4041 were tested. See Tables 4.1 and 4.2 for the properties [22]. Dimensions of the specimen are shown in Figures 4.2. Test section of the specimens are polished longitudinally, progressing through 0, 00 and 000 emery paper to remove nicks, dents, scratches and circumferential tool marks from the surface. Before fatigue testing, the surface of each specimen was covered with black paint in order to reduce IR reflections and increase the thermal emissivity of the specimen surface.

![Schematic diagram of the experimental setup for rotating-bending.](image)

**Figure 4.1. Schematic diagram of the experimental setup for rotating-bending.**

![Geometry of the specimens used for fatigue tests.](image)

**Figure 4.2. Geometry of the specimens used for fatigue tests (All dimensions are in mm).**
A vortex tube is used to supply cold air over the surface of the specimen during fatigue test. Temperature and flow are adjustable over a wide range using the control valve on the exhaust. In the present experiments the vortex tube produces the temperature range from -10°C to +18°C whilst the room temperature is 25°C. The vortex tube is aligned in such a way that the whole specimen surface is cooled by air flow. The temperature and velocity of the air flow are measured by means of thermocouple and digital anemometer, respectively, to calculate the cooling capacity.

### Table 4.1 Component elements properties of the materials. Note that the numbers with “<=” show the maximum value.

<table>
<thead>
<tr>
<th>Material</th>
<th>Composition, %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
</tr>
<tr>
<td>SS304L</td>
<td>&lt;=0.03</td>
</tr>
<tr>
<td>S4145</td>
<td>0.43-0.48</td>
</tr>
</tbody>
</table>

### Table 4.2 Mechanical and thermal properties of the materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Ultimate Strength</th>
<th>Yield Strength</th>
<th>Modulus of Elasticity</th>
<th>Thermal Conductivity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MPa</td>
<td>MPa</td>
<td>GPa</td>
<td>W/mK</td>
</tr>
<tr>
<td>SS 304L</td>
<td>564</td>
<td>210</td>
<td>193-200</td>
<td>16.3 @ 100°C</td>
</tr>
<tr>
<td>S 4145</td>
<td>1061</td>
<td>951</td>
<td>205</td>
<td>42.6 @ 0-100°C</td>
</tr>
</tbody>
</table>

### 4.3 Results and Discussion

4.3.1 Experimental Results

A series of rotating-bending fatigue tests is performed to investigate the effect of surface cooling on the fatigue life of the Stainless Steel 304L and Steel 4145. First, tests are carried out without surface cooling at different stress amplitudes for both materials. Figure 4.3 shows the evolution of temperature rise for Steel 4145 for different stress amplitudes. Note that a persistent trend emerges in all the experiments. Starting from the ambient, initially, the temperature rises since
the energy density increases with the hysteresis effect. Thereafter, temperature remains nearly constant until shortly before failure and finally experiences a rapid rise just prior to failure. It is worth noting, however, that the steady temperature phase is absent in the cases when the specimen is subjected to high stress levels. This is due to the fact that at a high stress the life of the samples is short and the material does not have enough time to dissipate heat to the surroundings. The greater the level of stress, the higher is the mean temperature during fatigue tests; see Figure 4.3.

![Temperature rise vs. number of cycles for Steel 4145 specimens without surface cooling](image)

**Figure 4.3. Evolution of the temperature rise for Steel 4145 specimens without surface cooling at the different stress amplitudes.**

Figure 4.4 shows the results of the surface temperature of two specimens under fatigue load with and without surface cooling. The material is Steel 4145 and the stress amplitude is $\sigma=719$ MPa. The cooling rate is kept constant at $Q=34.7$ W. It can be seen that the surface temperature in presence of the cooling decreases significantly. It also shows that in this particular case the fatigue life increased by 47%.
Figure 4.4. Temperature rise for Steel 4145 specimens with and without surface cooling at the same stress amplitude of $\sigma=719$ MPa.

The cooling rate of air flowing on to the specimen surface, $Q$, is calculated based on the mass flow rate of the air jet and its temperature, i.e., $Q=\rho V A c_p(T_{\text{out}}-T_0)$, where $\rho$ is the air density, $V$ is the air velocity, $A$ represents the cross sectional area of the vortex tube, $c_p$ is the specific heat of air, $T_{\text{out}}$ denotes the temperature of outlet air and $T_0$ is the room temperature. The temperature and velocity of the air at vortex tube outlet are -6.11 °C and 38.6 m/s, respectively.

The SN curves for fatigue life with surface cooling and without are shown in Figures 4.5 and 4.6 for Stainless Steel 304L and Steel 4145, respectively. Each data point is the average of two tests repeated under the same testing condition. It can be seen that the effect of fatigue life enhancement with cooling decreases as the stress level increases, i.e., the surface cooling appears to have a larger influence on the relatively high-cycle tests compared with those of the low-cycle fatigue tests. It is also interesting to note that the cooling increases the fatigue limit of Stainless Steel 304L from 405 MPa to 450 MPa, as shown in Figure 4.5.
Figure 4.5. Effect of surface cooling on fatigue life of Stainless Steel 304L undergoing rotating-bending fatigue.

Figure 4.6. Effect of surface cooling on fatigue life of Steel 4145 undergoing rotating-bending fatigue.

Figure 4.7 compares improvement in the fatigue for both materials in terms of the percentage of increase in life with and without cooling. It can be seen that cooling is more beneficial when
the stress amplitude is low. That is, surface cooling is more effective for high cycle fatigue rather than low cycle fatigue for the constant cooling rate considered in this study. For example, at \( \sigma = 495 \) MPa, the improvement in life is about 100%, whereas at \( \sigma = 450 \) MPa is about 1000%.

Also, the results show that the effect of surface cooling on fatigue life improvement in the case of Stainless Steel 304L is more profound than that of Steel 4145. To illustrate the different behavior of both steel when subjected to the fatigue load, the stress-life diagram of both materials is plotted in Figure 4.8 for comparison. This figure shows the distinct difference in SN curves of both materials as one compares the slopes of the curves. Given a number of cycle to failure, the stress amplitude required to fatigue the Stainless Steel 304L is much lower than that of Steel 4145.

Figure 7. Increase in fatigue life as a function of stress amplitude for Stainless Steel 304L and Steel 4145.
4.3.2 Thermodynamics Considerations

To gain insight into the understanding of the experimental results we resort to the notion of self-organization within the context of irreversible thermodynamics. We begin by first reviewing some of the pertinent literature on the self-organization relative to fatigue failure.

4.3.3 Effect of Electric Current on Fatigue

Conrad et al. [23] studied the effect of relatively high density electric current pulses on the fatigue life of Copper. Their experiments involved the rotating-bending fatigue of cold-drawn tough pitch Copper rod of 12.5 mm in diameter with a stated purity of 99.9%. The current density was about $1.3 \times 10^4$ A/cm$^2$ for the period of 100 $\mu$s with 2 current pulses per second. The frequency of the tests was 50 Hz and the tests were performed at 300 K. They showed that the fatigue can be improved by a factor of 1.3-3 by supplying electric current pulses. Figure 4.9 compares the results of the application of stress-cycle with and without application of electrical pulse. Figures 4.9a and b show the results for two different grain sizes. While the effect of electric current pulses of both grain sizes is noticeable, the procedure is more effective for lower stress amplitudes.
Conrad et al. [23] postulated that by applying the electric current, the number of cycles required for initiation of the microcracks increases, thus resulting in an increased in fatigue life and a decreased in the tendency for intergranular cracking. Physically, they attributed this effect to the increase in homogenization of slip, i.e., the decrease in spacing and width of persistent slip bands [23, 24]. They suggested that homogenization of slip may be caused by interaction between electrons and dislocations. From thermodynamic viewpoint, the homogenization of slip is analogous to the formation of the dissipative structures as a consequence of self-organization. There are other pertinent works on the effect of electric current of fatigue life of metals; see for example [25-27].

![Figure 4.9. Effect of electric current pulses on fatigue life on copper with purity of 99.9%](image)

- **(a)** Material with grain size=45 μm
- **(b)** Material with grain size=30 μm.

It is to be mentioned that the work of El Latif [25] on the effect of pre-application of high density a.c. on fatigue life of mild steel specimens showed significant reduction of endurance limit. That is, in his experiments, by applying current prior to the test, the fatigue life of the steel specimen decreased, and that the higher the imposed a.c. the lower the fatigue life. However, it is worth noting that El Latif [25] applied a.c. and mechanical load separately. Therefore, his testing procedure was completely different from the work of Conrad et al. [23] where both current and
load were applied simultaneously. El Latif attributed the unfavorable effect of a.c. on the fatigue life to the Joule heating and subsequent induced thermal stress which in turn degrades the material.

### 4.3.4 Effect of Magnetic Field on Fatigue

Another technique for enhancement of fatigue life is by application of magnetic field on a specimen undergoing fatigue load. Experimental work of Yong et al. [28] showed that by applying an alternating magnetic field the life of a specimen subjected to cyclic fatigue substantially increased. Their experiments were conducted using a uniaxial fatigue testing apparatus with smooth rod specimens made of A3 steel. The load ratio was $R=0.01$ and frequency $f=25$ Hz. Two series of tests were carried out with and without magnetic field. Specimens were placed inside a solenoid and magnetic field was formed by passing electrical current through the solenoid. The experiments were repeated three times at each stress level; see Table 4.3 [28].

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Cycle to failure, $N_t$</th>
<th>Average $N_t$</th>
<th>Increase in life, %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without magnetic field</td>
<td>With magnetic field</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>77880</td>
<td>77840</td>
<td>94050</td>
</tr>
</tbody>
</table>

It was observed that the fatigue life of the specimens was significantly improved under the application of magnetic field. The percentage of increase in fatigue life reported was 269%. While no thermodynamic analysis was presented, Yong et al. [28] attributed this effect to the formation of dissipative structures and self-organization during the course of the fatigue.

Some other pertinent works on the effect of magnetic field of fatigue life of metals can be found in [29-37].
4.3.5 Self-Organization during Fatigue

When a system reaches the equilibrium state, the entropy and associate disorder is maximum. It follows, therefore, that to increase the orderliness, the system should deviate from equilibrium state. According to Prigogine [37], to drive the system further far from equilibrium, the fluctuations from the average state should be above a certain critical value. For example, in a fatigue process, self-organization may occur above some critical density of the dislocations [38]. The source of fluctuation can be an external element(s) acting on the system to bring about deviation from the average state.

Self-organization is directly associated with the mechanism of formation of dissipative structures, which in a fatigue process of metals corresponds to the arrangement of new patterns of material’s microstructure formed during the structural transformation. More specifically, if we define structural transformation as the reconfiguration of microstructure during loading and unloading, then, the transformation can be examined by the forming and the movement of the partial dislocations and the accumulation of the strain energy [38]. If during a fatigue process dissipative structures are formed, the system entropy will decrease and this consequently results in slowing down the fatigue damage. It is, however, to be noted that the deviation of the system far from equilibrium is not the only necessary condition for the formation of dissipative structures. According to the Prigogine’s theorem [37] the emergence of dissipative structures in a system obeying linear laws is impossible. It is known that each thermodynamic flux can depend on the forces of all active processes in the system. However, if the dependency manifests itself in a linear manner, the possibility of formation of dissipative structures is unlikely. For example, in a heat conduction process, heat flux is linearly dependent upon the gradient of temperature via Fourier’s law. Hence, it is theoretically impossible to attain ordered behavior in a medium merely through the linear heat conduction process. Similarly, in a purely elastic deformation of
material (such as very high-cycle fatigue), expressed by linear relations between forces and fluxes, it is unlikely to achieve the increase in order as a consequence of deformation. On the contrary, consider the low- and intermediate-cycle fatigue, where the plastic deformation is dominant and the fluxes do not depend linearly on forces. In this case, the irreversible process of plastic deformation may result in formation of different patterns in material’s structure with improved properties (e.g., hardening effect). Therefore, to develop self-organized behavior we should take into account non-linear, non-equilibrium characterization of the domain.

As mentioned earlier, the dissipative structures are formed when the system deviates from average state. This can be induced externally via magnetic field, electric current, or through environmental condition in order to drive the system far away from the equilibrium or stationary state, i.e., when the system is no longer stable. The determination of the stability of non-equilibrium stationary state can be performed based on the Lyapunov’s theory of stability [39]. If a non-equilibrium stationary system loses its stability, the Lyapunov function \((\delta^2 S)/2\) should satisfy the following inequality:

\[
\frac{1}{2} \delta^2 S < 0
\]  

(4.1)

where \(S\) is the entropy of the system.

It can be shown that [39]:

\[
\frac{1}{2} \frac{d}{dt} (\delta^2 S) = \sum_k \delta X_k \delta J_k
\]  

(4.2)

where \(\delta X_k\) and \(\delta J_k\) are the deviation of thermodynamic forces and fluxes from stationary state. Eq. (4.2) is called the excess entropy generation. If the excess entropy production is non-negative, the given state of the system is stable; otherwise, the system loses its stability. We seek to show that through the application of the excess entropy production a thermodynamic system in general, and a fatigue system in particular, can achieve self-organization.
Consider a system with two active dissipative processes. Application of Eq. (4.2) yields:

\[
\frac{1}{2} \frac{d}{dt} \left( \delta^2 S \right) = \delta X_k \delta J_k = \delta X_1 \delta J_1 + \delta X_2 \delta J_2
\]  

(4.3)

The variation of external elements can drive the system far from average state and contributes to entropy generation by the product of the forces with associate fluxes. In general, if we denote the force and the flux of external element by \( X_2 \) and \( J_2 \), respectively, the entropy generation during a fatigue process can be written as:

\[
\frac{dS}{dt} = X_1 J_1 + X_2 J_2
\]

(4.4)

where \( X_1 = \sigma/T \), \( J_1 = \dot{\varepsilon}^p \) denote the plastic deformation force and flows, respectively. The \( \sigma \) and \( \dot{\varepsilon}^p \) denote the stress amplitude and the rate of plastic strain. The product of \( X_2 J_2 \) represents the contribution of the external element. If self-organization occurs, the entropy generation is minimal in a stationary state and the fatigue damage is retarded. Note that the flux of the external element, \( J_2 \), is the only variable which is controlled by the operator, i.e., it can be the supplied electric current, supplied magnetic field intensity of the rate of surface cooling. Assume that an electrical current, \( J=I \), is supplied with voltage \( X=V \). The contribution of this external element on the entropy generation is \( VI/T \). The minimal entropy generation requires:

\[
\frac{d}{dJ_2} \left( \frac{dS}{dt} \right) = \frac{d}{dJ_2} (X_1 J_1) + X_2 = 0
\]

(4.5)

Note that the external force, \( X_2 \), is assumed to be fixed and not influenced by the change in external flow, \( J_2 \). For example, one can supply a fixed voltage to the specimen under fatigue load while the electric current can change. Integrating Eq. (4.5) yields:

\[
\int_{(X_1J_1)_0}^{X_1J_1} d(X_1J_1) = -\int_{J_2=0}^{J_2} X_2 dJ_2
\]

(4.6)

where \( (X_1J_1)_0 \) represents the state of the system in absence of the external flux, i.e., when \( J_2=0 \). Eq. (4.6) now becomes:
\[ X_1 J_1 = (X_1 J_1)_0 - X_2 J_2 \]  

(4.7)

For simplification, assume that the force \( X_1 \) is not influenced by change in external flux (i.e., \( X_1 = (X_1)_0 \)). In the case of a fatigue problem, for example, this would imply that the external flux does not affect the stress distribution in the specimen. Therefore, Eq. (4.7) yields:

\[ J_1 = (J_1)_0 - \frac{X_2}{X_1} J_2 \]  

(4.8)

Equation (4.8) reveals that by increasing the external flux \( J_2 \), the flux \( J_1 \) decreases which, in turn, results in a decrease in the rate of fatigue damage. Figure 4.10 shows the variation of \( J_1 \) as a function of \( J_2 \). This figure shows that, theoretically, by increasing \( J_2 \) the fatigue damage can be slowed down, and even eliminated. While it is practically impossible to eliminate the fatigue damage, this simplified analysis illustrates the possibility of slowing down the rate of fatigue damage by the use of an external element such as electric current, magnetic field and surface cooling.

![Figure 4.10. Effect of external flux on the in self-organized system.](image)

Turning our attention, now, to the fatigue problem with cooling, for simplicity, let us assume that the plastic deformation is the only independent source of energy dissipation in the system. We cool the surface of the specimen close to the room temperature. Therefore, we assume that
all the heat generated by the plastic deformation is removed out by conduction. Considering that, we can write the entropy production as:

\[
\frac{d_iS}{dt} = JX = (-k \nabla T) \nabla \left( \frac{1}{T} \right)
\]  
(4.9)

where thermodynamic flux can be defined as:

\[
J = -k \nabla T = \sigma \dot{\varepsilon}_p
\]  
(4.10)

From Eq. (4.10), one can solve for the temperature gradient \( \nabla T = -\sigma \dot{\varepsilon}_p / k \). Thermodynamic force is:

\[
X = \nabla \left( \frac{1}{T} \right) = -\frac{\nabla T}{T^2} = \frac{\sigma \dot{\varepsilon}_p}{kT^2}
\]  
(4.11)

Substitution of Eqs. (4.10) and (4.11) into Eq. (4.9) yields:

\[
\frac{d_iS}{dt} = \frac{(\sigma \dot{\varepsilon}_p)^2}{-kT^2}
\]  
(4.12)

Equation (4.12) represents the entropy generation. Since thermal conductivity, \( k \), is always positive, Eq. (4.12) is also positive. This is in accordance with the second law of thermodynamics.

For the system to lose its stability, Eq. (4.1) should be satisfied, viz:

\[
\frac{1}{2} \frac{d}{dt} \left( \delta^2 S \right) = \delta X \delta J = \left( \frac{\partial X}{\partial T} \right) \left( \frac{\partial J}{\partial T} \right) \left( \delta T \right)^2 < 0
\]  
(4.13)

In derivation of Eq. (4.13) we have assumed that the only parameter which changes during the process is temperature, \( T \). Thermodynamic flux, \( J \), and force, \( X \), are influenced by temperature.

Substitution of \( J \) and \( X \) from Eq. (4.10) and (4.11) into Eq. (4.13) results:

\[
\frac{1}{2} \frac{d}{dt} \left( \delta^2 S \right) = \delta X \delta J = \frac{\sigma^2}{kT^2} \left( \frac{\partial \dot{\varepsilon}_p}{\partial T} \right) \frac{\dot{\varepsilon}_p}{k} \frac{\partial k}{\partial T} \left( \delta T \right)^2 < 0
\]  
(4.14)

The only possible condition that ensures that Eq. (14) is negative is:
\[
\frac{\partial k}{\partial T} \frac{\partial \dot{e}_p}{\partial T} > 0
\] 

(4.15)

Therefore, the fatigue system can lose its stability if the thermal conductivity and the rate of plastic strain simultaneously increase or decrease with temperature. The increases of the rate of plastic strain with temperature can result in thermal stresses and can accelerate the fatigue failure. The increase in thermal conductivity helps the system to reduce the unfavorable effect of temperature on the rate of plastic deformation.

Generally, it is possible to draw an important conclusion that the process of self-organization during fatigue is possible if one or more independent processes, except plastic deformation itself, are affecting the system. That is, to initiate self-organization and for dissipative structures to form at least two dissipative processes are required. For example, in the abovementioned cases the primary process is plastic deformation while a supplementary process (electric current, magnetic field or surface cooling) is needed for initiation of the self-organization.

Gershman and Bushe [40] discuss that by increasing the number of independent processes in a system, the possibility of initiation of self-organization increases. Increasing the number of interacting processes in the system leads to a more complex system.

Fox-Rabinovich et al. [41] theoretically study the probability of the occurrence of the self-organization under complex conditions. They postulate that the probability, \( P \), of losing stability of a complex system with \( N \) simultaneously processes can be evaluated from: \( P=1-1/(2N) \). Therefore, by increasing the number of processes, \( N \), the system’s complexity increases which in turn leads to a probable initiation of self-organization. Based on this finding, simultaneous application of the external elements mentioned earlier (electric current, magnetic field and surface cooling) increases the possibility of formation of dissipative structure during fatigue process and leads to enhanced fatigue life.
4.4 Conclusions

In summary, this chapter presents the results of a series of fatigue tests of stainless steel 304L and Steel 4145 specimens subjected to rotating-bending fatigue load. In order to study the effect of environment, we used a vortex tube to blow cold air over the surface of the specimen. Samples were cooled continuously during the fatigue test. Results show that the surface cooling has significant effect on the fatigue life of the specimens. A thermodynamic analysis is presented to gain insight into the self-organization process. It is shown that the notion of self-organization can potentially explain the experimental results. Further studies are required to investigate the effect of surface cooling on different materials and subjected to different loading conditions.

4.5 References


Chapter 5: Future Directions and Recommendations
5.1 Recommendations

The thermodynamic analysis of fatigue failure presented in this research requires further development to ensure that the results can be generalized to fatigue of different types of loading, different environmental condition and different materials. For example, the fatigue life prediction model based on the slope of the temperature demands more study to investigate the effect of multiaxial loading on the slope.

Advances in technology in recent years, for example, demand devices of incredibly small sizes with the representative length scale on the order of micro- and nano-meter. Demonstrably the occurrence of any nanoscale defect during the operation could result in breakdown of the device. Yet conventional theoretical and experimental approaches developed for bulk material may not be practical when dealing with micro and nano-scale fatigue problems. This calls for the introduction of the need for hierarchical multiscale modeling to complement the traditional for the study of material degradation. Defects develop initially at the atomic scale due to cyclic slip and they grow to a form of microscopic fatigue damage commonly known as microplasticity, that accumulates irreversibility form micro- to macro-cracks.

Also, entropy approach to the fatigue failure is, in fact, established for low to intermediate-cycle fatigue tests. The problem of relating entropy to high-cycle fatigue failure is still remained unsolved. The high-cycle fatigue is associated with a class of fatigue problems where the stress amplitude is comparatively low, usually below the endurance limit. In this situation, bulk of the specimen undergoes elastic deformation where the entropy generation due to plasticity is almost negligible. This calls for the introduction of hierarchical multiscale modeling.

Dissipative feature of materials during deformation can be attributed to dislocation. The mechanism of the energy dissipation in metals is based on the concept of dislocation, thus, mechanical damping in metals is intimately connected to the moving dislocations. The motion of
dislocations leads to redistribution of stresses in the material which, in turn, results in temperature change at any locations. The resulting temperature gradient drives the heat flow and causes mechanical damping. This damping depends on the applied stress and the density of dislocations. The notion of microplasticity in high-cycle fatigue and the associated investigation of thermodynamic degradation offer a new and exciting research task for years to come.

Furthermore, one of the biggest challenges in fatigue analysis is the estimation of the useful remaining life of the material. When an equipment operates, in the field for an extended period of time, it is most desirable to know how much of fatigue life is left for such a piece of equipment or part. This information, if it could be accurately predicted — that is: scientifically through non-destructive testing — one can improve the risk of field failure by many fold. Both time and resources could be saved if a method for determining the extent of fatigue damage and an estimation of remaining fatigue life could be established.

The proposed thermodynamics analysis in this study offers a research program that aims at further development of these technologies and their extension to assess remaining fatigue life.

It is, also, worth mentioning that the thermodynamics analysis of cooling presented in Chapter 4 merely established a criterion for the self-organization and fatigue life improvement. However, future studies can be directed toward experimental verification of the theory of self-organization during cooling fatigue. Further studies are needed to investigate the effect of surface cooling on different materials and subjected to different loading conditions.

In the present study we showed that fatigue degradation and entropy generation are intimately related and that their relationship can be used for prediction of failure and making fundamental advances in the study of fatigue without having to resort to traditional approaches that depend on empirical models. It would be beneficial for the future studies to discuss that many widely used empirical correlations for fatigue analysis can be arrived at by consideration of irreversible
thermodynamics taking into account entropy generation as a degradation index. For example, in what follows, a physical construal for linear fatigue damage hypothesis known as the Miner’s rule is presented which takes into consideration the notion that metals fail upon accumulating a finite amount of entropy.

5.2 Development of Entropy-Fatigue Formulation

Consider a specimen subjected to a series of stress level \( \sigma_i \), \( i=1, 2, ..., \) and let \( D \) represents the material degradation defined as the ratio of accumulation of entropy generation divided by the total entropy generation up to the final failure, viz.,

\[
D = \frac{\gamma_1 + \gamma_2 + \gamma_3 + \cdots}{\gamma_f}
\]

(5.1)

where \( \gamma_1, \gamma_2, \gamma_3, ..., \) are the entropy generations at stress levels \( \sigma_1, \sigma_2, \sigma_3, ..., \) respectively.

Employing Eq. (3.9) from Chapter 3, \( \gamma_i \) can be written as:

\[
\gamma_i = \left(\frac{\Delta w_p}{T}\right)i N_i
\]

(5.2)

where the subscript \( i=1, 2, ..., \) corresponds to stress level \( \sigma_i \), and \( N_i \) denotes the number of cycles elapsed at corresponding stress level. As mentioned in Chapter 3, the total entropy generation \( \gamma_f \) is a material property and independent of stress level. Therefore, the following relationship can be obtained from Eq. (5.2):

\[
\gamma_f = \left(\frac{\Delta w_p}{T}\right)i N_{f,i} = \left(\frac{\Delta w_p}{T}\right)2 N_{f,2} = \left(\frac{\Delta w_p}{T}\right)3 N_{f,3} = \cdots
\]

(5.3)

where \( N_{f,i}, N_{f,2}, N_{f,3}, ..., \) are the fatigue lives from constant stress amplitude at stresses \( \sigma_1, \sigma_2, \sigma_3, ..., \) respectively. Substituting Eqs. (5.2) and (5.3) into Eq. (5.1), yields:

\[
D = \frac{\left(\frac{\Delta w_p}{T}\right)i N_i}{\left(\frac{\Delta w_p}{T}\right)i N_{f,i}} + \frac{\left(\frac{\Delta w_p}{T}\right)2 N_2}{\left(\frac{\Delta w_p}{T}\right)2 N_{f,2}} + \cdots = \frac{N_i}{N_{f,i}} + \frac{N_2}{N_{f,2}} + \cdots = \sum_{i} \frac{N_i}{N_{f,i}}
\]

(5.4)
Failure occurs when the accumulation of the entropy generation reaches its maximum, i.e., $\gamma_f$. This condition corresponds to $D=1$. Therefore, from Eq. (5.4) it follows:

$$\sum_i \frac{N_i}{N_{f_i}} = 1$$

(5.5)

which represents the linear fatigue damage hypothesis known as the Miner’s rule.

This derivation implies that empirical correlations for fatigue prediction can be subsumed into a more general thermodynamic analysis of the system taking into account the entropy generation. This conclusion reveals that fatigue deterioration and entropy are intimately related and their relationship can be put to use for prediction of material fatigue failure.
Appendix A: Conduction Heat Transfer Analysis

For simplicity, the fluctuating beam is modeled as a finite rectangle $0 \leq x \leq a$, $0 \leq y \leq b$ with $a$ as length and $b$ as thickness of the beam. The boundary conditions are shown in Figure A.1. The beam is initially at room temperature, $T_0$. At time $0 < t$, heat is generated within the solid beam at a rate $g(x, y, t)$.

Figure A.1. A fluctuating beam is modeled as a finite rectangle.

The governing equation, boundary and initial conditions for this problem is as follow:

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{g(x, y, t)}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad 0 \leq x \leq a, \ 0 \leq y \leq b, \ 0 < t \quad (A.1)
\]

\[
T(0, y, t) = T_0, \ T_y(a, y, t) = 0 \quad (A.2)
\]

\[
-k T_x(x, 0, t) + hT = hT_0, \ k T_y(x, b, t) + hT = hT_0 \quad (A.3)
\]

\[
T(x, y, 0) = T_0 \quad (A.4)
\]

Letting $\theta = (T - T_0)/T_0, \ \xi = x/a, \ \eta = y/a, \ \tau = \alpha t/a^2, \ \phi = ga^2/kT_0$ and $B = ha/k$, one arrives at the following dimensionless form of Eqs. (A.1)-(A.4):

\[
\frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2} + \phi = \frac{\partial \theta}{\partial \tau} \quad 0 \leq \xi \leq 1, \ 0 \leq \eta \leq b/a, \ 0 < \tau \quad (A.5)
\]

\[
\theta(0, \eta, \tau) = 0, \ \theta(1, \eta, \tau) = 0 \quad (A.6)
\]

\[
-\theta_x(\xi, 0, \tau) + B \theta = 0, \ \theta_x(\xi, b/a, \tau) + B \theta = 0 \quad (A.7)
\]
\[ \theta(\zeta, \eta, 0) = 0 \]  
(A.8)

The solution of the present problem is [25]:

\[ \theta(\zeta, \eta, \tau) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} e^{i(\beta_m \eta + \nu_n \xi)} \cdot K(\beta_m, \zeta) \cdot K(\nu_n, \eta) \int_{\tau' = 0}^{\tau} e^{i(\beta_m \eta + \nu_n \xi)} \cdot A(\beta_m, \nu_n, \tau') \cdot d\tau' \]  
(A.9)

where

\[ A(\beta_m, \nu_n, \tau') = \int_{\zeta = 0}^{\infty} \int_{\eta' = 0}^{\infty} K(\beta_m, \zeta') \cdot K(\nu_n, \eta') \cdot \varphi(\zeta', \eta', \tau') \cdot d\zeta' d\eta' \]  
(A.10)

where \( K(\beta_m, \zeta) \) and \( K(\nu_n, \eta) \) are kernels with their associated eigenvalues \( \beta_m \) and \( \nu_n \) along \( x \) and \( y \) axis, respectively. If the heat generation term, \( \varphi(\zeta, \eta, \tau) \), is only a function of space variable, i.e. \( \varphi(\zeta, \eta, \tau) \), Eqs. (A.9) and (A.10) can be combined as:

\[ \theta(\zeta, \eta, \tau) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\lambda_m^2} \left[ I - e^{-\lambda_m^2 \tau} \right] \left[ (\nu_n^2 + B^2) \left( \frac{b}{a} + \frac{B}{\nu_n^2 + B^2} \right) + B \right]^{-1} \cdot \sin \beta_m \zeta \cdot \left( \nu_n \cdot \cos \nu_n \eta + B \cdot \sin \nu_n \eta \right) \int_{\xi' = 0}^{\infty} \int_{\eta' = 0}^{\infty} \sin \beta_m \zeta' \cdot (\nu_n \cdot \cos \nu_n \eta' + B \cdot \sin \nu_n \eta') \varphi(\zeta', \eta') \cdot d\zeta' d\eta' \]  
(A.11)

where \( \lambda_m^2 = \beta_m^2 + \nu_n^2 \).

The final solution could be found from:

\[ T = T_0 (\theta + 1) \]  
(A.12)
Appendix B: Permission from the Journals

Permission for the Paper in Chapters 1
Permission for the Paper in Chapters 2

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Mehdi graduated with a bachelor degree from the Mechanical Engineering Department of Iran University of Science and Technology in 2003. Then, the same year, he pursued his master’s education at University of Tehran, one of the most prestigious schools in the country. He graduated as the first rank alumnus with a master’s degree from the Department of Mechanical Engineering in 2006. After completing master degree, he moved to the United States to pursue his doctorate in 2006. He joined Professor Khonsari’s research group, a leader in tribology and rotating machinery, at Louisiana State University in Baton Rouge, and started his doctoral work on developing a novel technique for prediction fatigue failure of metals. He was awarded the distinction of outstanding research assistant for the year 2008-2009 by the Mechanical Engineering Department at Louisiana State University. Mehdi’s research interests include Heat Transfer, Thermodynamics, Tribology and Mechanics of Solids. Mehdi currently lives in Baton Rouge, Louisiana.