2013

Parametric optimization of a single-tracked vehicle

Darrick Jason Berner
Louisiana State University and Agricultural and Mechanical College

PARAMETRIC OPTIMIZATION OF A SINGLE-TRACKED VEHICLE

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Master of Science in Mechanical Engineering

in

The Department of Mechanical Engineering

by
Darrick J. Berner
BS, Louisiana State University, 2002
December 2013

ACKNOWLEDGEMENTS

All the work presented in this thesis would not be possible if it were not for the help of so many who also sacrificed their time and support.

I would like to thank Dr. Pang for his patience and guidance thru the years, even after his retirement. Dr. Helms, thank you for helping me generate ideas and for all the interesting discussions we had in your office. Dr. Wahab, thank you for your recommendations and agreeing to stay on the committee in spite of the heavy burden. Dr. Ram, thank you for your quality instruction and taking time to answer my questions outside of class.

I would like to give a heartfelt thanks to Ms. Diane Morgan for ensuring all my forms were turned in on time and putting forth a tremendous effort keeping me on the straight and narrow path towards graduation. I certainly would not have come this far if it were not for your efforts. Ms. Telisha Snell, thank you for keeping me informed of LSU policy and responding, to all my inquiries in short order.

Thanks to all my supervisors, employees, and chain of command at LA Department of Transportation & Development for giving me time off to attend class and taking on the additional burden due to my absence.

I also would like to thank my family for inspiring my curious nature and encouraging me to go to college. Finally, I would like to thank my wife, Elizabeth, for her support and taking on the tremendous burden due to my continued absence, and my children, Johnathan and Katelyn, who I love most affectionately and were my inspiration for carrying on until the finish.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS ii
LIST OF FIGURES iv
ABSTRACT vi
CHAPTER 1: INTRODUCTION 1
CHAPTER 2: BACKGROUND 4
CHAPTER 3: ANALYSIS 12
CHAPTER 4: PARAMETRIC ANALYSIS OF SYSTEM 29
CHAPTER 5: DYNAMIC VIBRATION ABSORBER 37
CHAPTER 6: SENSITIVITY ANALYSIS 46
CHAPTER 7: CONCLUSION 48
REFERENCES 49
VITA 50

LIST OF FIGURES

Figure 1: Springs 6

Figure 2: Dampers 8

Figure 3: Dynamic Vibration Absorber Resonance 11

Figure 4: Half-Car Model 13

Figure 5: System Dynamic Response 19

Figure 6: Poles in S-Plane 20

Figure 7: 1st Mode 22

Figure 8: 2nd Mode 23

Figure 9: 3rd Mode 24

Figure 10: 4th Mode 25

Figure 11: Amplitude in Bounce Mode 26

Figure 12: Amplitude in Pitch Mode 26

Figure 13: Amplitude in Pitch Mode 27

Figure 14: Phase Angle in Pitch Mode 27

Figure 15: Mass Perturbation for Amp in Bounce Mode 29

Figure 16: Mass Perturbation for Amp in Pitch Mode 30

Figure 17: Mass Perturbation for Phase in Bounce Mode 30

Figure 18: Mass Perturbation for Phase in Pitch Mode 31

Figure 19: Spring Perturbation for Amp in Bounce Mode 32

Figure 20: Spring Perturbation for Amp in Pitch Mode 32

Figure 21: Spring Perturbation for Phase in Bounce Mode 33

Figure 22: Spring Perturbation for Phase in Pitch Mode 33

Figure 23: Damper Perturbation for Amp in Bounce Mode 34

Figure 24: Damper Perturbation for Amp in Pitch Mode 35

Figure 25: Damper Perturbation for Phase in Bounce Mode.. 35

Figure 26: Damper Perturbation for Phase in Pitch Mode... 36

Figure 27: Dynamic Response of System with Absorber 37

Figure 28: Absorber Spring Perturbation for Amp in Primary Bounce Mode 38

Figure 29: Absorber Spring Perturbation for Amp in Primary Pitch Mode 39

Figure 30: Absorber Spring Perturbation for Amp in Secondary Bounce Mode 39

Figure 31: Absorber Spring Perturbation for Phase in Primary Bounce Mode 40

Figure 32: Absorber Spring Perturbation for Phase in Primary Pitch Mode 40

Figure 33: Absorber Perturbation for Phase in Secondary Bounce Mode.................................... 41

Figure 34: Absorber Mass Perturbation for Amp in Primary Bounce Mode................................ 42

Figure 35: Absorber Mass Perturbation for Amp in Primary Pitch Mode................................... 42

Figure 36: Absorber Mass Perturbation for Amp in Secondary Bounce Mode............................ 43

Figure 37: Absorber Mass Perturbation for Phase in Primary Bounce Mode 43

Figure 38: Absorber Mass Perturbation for Phase in Primary Pitch Mode 44

Figure 39: Absorber Mass Perturbation for Phase in Secondary Bounce Mode 44

Figure 40: Front Spring Sensitivity.. 46

Figure 41: Mass Sensitivity 47

ABSTRACT

The purpose of a suspension system for a vehicle is to contribute to the handling and assist in isolating the occupants from vibrations due to road irregularities. Generally, these primary functions are often at odds so the goal is to design a suspension system that finds the appropriate compromise.

The focus of this thesis is to develop a two degree of freedom model and use parametric analysis to demonstrate an optimization technique by varying several geometric characteristics on a single-track vehicle. Furthermore, a dynamic vibration absorber will be added to the model to demonstrate its effect on the system. Also, out-of-plane motion will be discussed qualitatively.

After modeling the system in the symmetric (vertical) plane, the equations of motion can be found using rigid body dynamics. The frame and suspension can be considered as a rigid body connected to the wheels with elastic systems. Basically, the rigid body constitutes the sprung mass while the masses attached to the wheels constitute the unsprung masses. Then the expression can be linearized and converted into a state space matrix where the eigenvalues, eigenvectors, and natural frequencies can be extracted. The parametric analysis consists of a perturbation of one parameter and measuring the effect on the natural frequency. The results show that the system is most sensitive to mass perturbations, especially at resonance.

A dynamic vibration absorber is then attached to the system and subject to parametric analysis as well. The results show the system is most sensitive to variations in the forcing frequency on the primary mass. So a dynamic absorber would be more appropriate for a system subject to a single, fixed excitation frequency.

CHAPTER 1: INTRODUCTION

Motorcycles are single-track vehicles and therefore have some attributes in common with and different from dual-track vehicles such as cars, trucks, etc. Primarily, motorcycles lack lateral stability when stationary. However, they exhibit increased lateral stability as longitudinal velocity increases. The motions of a motorcycle can be grouped into two categories: those transverse to the central plane of symmetry, out-of-plane motions, and those within its central plane of symmetry, in-plane motions. Out-of-plane motions depict motorcycle stability, while in-plane motions are characterized by comfort and traction performance. More specifically, in-plane dynamics consists of rectilinear motion and suspension activation, which is the focus of this thesis.

The primary goals of a motorcycle suspension are to provide increased traction, decreased weight transfer, and rider comfort by reducing vibration. In addition, the suspension should be designed such that the natural frequency is well out of range of the anticipated driving frequency. Otherwise mechanical resonance will occur which will result in extremely high vibration amplitudes. Typical suspension parameters are stiffness, damping, sprung mass, unsprung mass, and tire properties. Sprung mass is the mass supported by the suspension (chassis, engine, rider, etc.), while the unsprung mass is the mass between the suspension and ground (tires, rims, brakes, etc.). Most vehicles are designed to minimize unsprung mass as much as feasible, especially vehicles which are focused on performance. In fact, sprung mass is usually an order of magnitude higher than unsprung mass, while the stiffness and natural frequency of the sprung mass is an order of magnitude lower than it. The damping value of the unsprung mass, i.e. the tire, is negligible [Wong, 477-480]. It is important to note, that while it is widely understood a suspension is designed to isolate the vehicle from the road profile, in reality it is actually designed the isolate the sprung mass from the unsprung mass. This is due to the fact that the unsprung mass is undamped. So minimizing its relative value will minimize its contribution to the vibration response of the damped sprung mass. Also, the tire traction diminishes with increased vertical load fluctuations which can rise if the wheel (sprung) mass is prominent in relation to the sprung mass. This is because the longitudinal forces, and lateral forces, generated by a tire are directly proportional to the normal load [Cossalter, 208]. Variations in normal tire load are directly proportional to the vertical tire deflection since a tire behaves like a spring reacting to vertical forces. However, smaller bumps in the road surface can be absorbed by the tires since the radial stiffness of the tires is much greater than the suspension stiffness. So their influence at low bump frequencies becomes negligible [Cossalter, 144]. Therefore an appropriately designed suspension should focus more on the larger bumps and maintaining traction while the tires can absorb the small amplitude fluctuations.

Lateral motion includes balancing, leaning, steering, and turning. In fact, these parameters exhibit a direct relationship during typical operation [Cossalter, 241]. This is precisely what sets a motorcycle, or single-tracked vehicle, aside from a dual-tracked vehicle such as a car or truck. Also, the polar moment of inertia has more of an effect on pitch for a single-tracked vehicle while having a more substantial effect on the steering of a dual-tracked vehicle. This is attributed to the fact that a turn from a motorcycle is not dialed in from the steering apparatus, but rather from lean and countersteering. Unfortunately, a motorcycle does not lend itself well to automated testing, which is impossible in some cases. This is why the list of performance specifications is typically much smaller for a motorcycle.

Longitudinal motion includes acceleration, stoppies (sudden stop with rear wheel off the ground), wheelies (sudden acceleration with front wheel off the ground), brake dive, and most suspension activation. In contrast to lateral motion, a motorcycle is longitudinally stable when stationary, but can become unstable under sufficient acceleration or deceleration. When analyzing weight transfer, Euler's Second Law, the conservation of linear and angular momentum principal, can be used to determine the ground reaction forces generated [Wikipedia.com].

An effective means of reducing the number of dependent coordinates is obtained by sharing the joints between rigid bodies. So in the plane of symmetry a motorcycle can be simplified to a rigid body connected to the wheels with an elastic suspension system. More specifically, it can be reduced to three (3) rigid bodies: chassis (frame, engine, rider, etc.), rear wheel, and front wheel. The chassis constitutes the sprung mass while the front and rear wheels are denoted as unsprung masses. In this thesis, the in-plane suspension motion will be analyzed as a two-degree of freedom model, excluding the unsprung mass, with a combination of vertical motion, bounce, and rotating motion, pitch. These two motions correspond to the vibrating modes in the symmetric plane. The focus will be on the motorcycle suspension parameters such as mass, dampers, and springs, which will be optimized using parametric analysis. In addition, a dynamic vibration absorber will be included in the analysis.

CHAPTER 2: BACKGROUND

The front and rear suspension systems on a motorcycle are quite different from each other. In addition, there is a wide range of variations depending on the class, make, and model. Aside from the off-road models, there are four (4) classes of motorcycles [Wikipedia.com]:

1. Standard: Also known as "traditional." This is the most versatile type and is targeted to a wide range of owners.
2. Cruiser: A relatively comfortable class, but has a large rake angle with high handlebars. It also has a lower center of gravity with a soft suspension system.
3. Sport: This has the most aggressive posture and is geared for performance. The suspension system is very stiff.
4. Touring: The most comfortable of all the classes and is intended for long-distance driving. It is a very heavy bike with a high center of gravity, especially when transporting luggage.

The most common front suspension configuration is the telescopic forks, which consists of two telescopic sliders running along the inner fork tube. The fork tubes are simply large hydraulic shocks with internal coil springs. They form a prismatic joint between the chassis and front wheel. The telescopic fork is characterized by low inertia around the axis of steering. However, high friction forces are encountered normal to the sliders during braking. Obviously the front forks are compressed during braking as the rear suspension is unloaded resulting in pitch. The compression of the front suspension depends on the angle between the resultant force and the direction of the suspension movement, which is at maximum compression when they are parallel. Therefore, in order to reduce dive or pitch, it is necessary to reduce the resultant force of the front braking and load transfer forces. This can be accomplished by modifying the steering axis, known as the angle of inclination. However, increasing the angle of inclination results in a reduction in stiffness and damping coefficients of the front suspension. Other limitations of front fork suspension are the high unsprung mass values and the difficulty in achieving progressive force/displacement values.

The traditional rear suspension is composed of a large fork made up of one or two trailing swing arms with at least one spring-damper unit, typically one per side, inclined at an angle with respect to the swing arm. Advantages of the swing arm include: ease of fabrication, low reaction forces transmitted to the chassis, and increased heat dissipation from the shock absorbers. Additionally, large amplitude of motion of spring-damper units nearly equal to the vertical amplitude of the wheel motion therefore causing high compression and extension velocities of the shock absorbers. Some limitations include the non-progressive force-displacement characteristics and the possibility of residual torsional stresses generated across the swing-arm due to inherent differences in spring characteristics.

A variation of the swing-arm suspension is the cantilever mono-shock system, characterized by one spring-damper unit. Its advantages over the swing-arm include ease of adjustment, reduced unsprung mass, high torsional and bending stiffness, and high vertical wheel amplitude. One shortcoming of the cantilever mono-shock system is that it does not permit progressive force-displacement. However, this can be addressed by adding a linkage to the system. These designs are typically based on a four-bar linkage and are distinguished by the different attachment points of the spring-damper unit. Modest unsprung masses are obtained, along with increased amplitudes, but greater reaction forces are exchanged among the various parts of the four-bar linkage.

For swing-arm suspension, an increase in spring inclination will result in greater stiffness. Also, the closer the spring attachment point moves toward the spring pivot, the higher the stiffness rate. In fact, with these modifications, the progressive behavior attainable with classic suspension can reach 50-60% at most.

One disadvantage of the swing arm is that oscillations will transmit fluctuations to the sprocket motion as well, and causes irregularities in the motor's rotation. If the sprocket is not concentric with the input sprocket, slip can be reduced.

Since the size and skill of the rider significantly affects the suspension response, most motorcycles are equipped with adjustable dampers and springs on the front and rear suspension. The choice of front and rear suspension characteristics depends on many parameters: rider weight, bike weight, position of center of gravity, load distribution, characteristics of stiffness, vertical damping of tires, geometry of motorcycle, road surface, power, technique, etc.

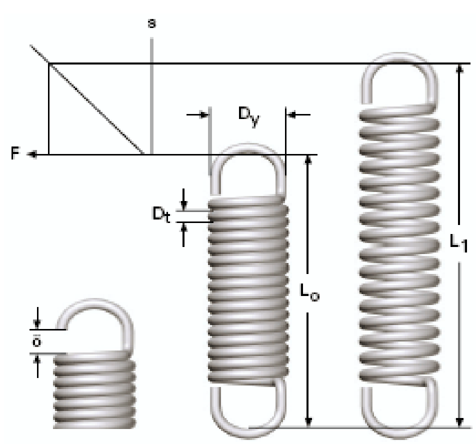

Figure 1: Springs

A mechanical spring is an elastic energy storing device that is generally assumed to have negligible mass and damping values. Even though actual springs are nonlinear, most practical applications assume a linear relation between spring deflection and applied force as given by the following equation:

$$F = k\delta = kx$$

where k is the spring stiffness or spring constant. Actually, the curve representing elastic force versus vertical displacement can be linear, in accordance with Hooke's Law. It can also follow a progressively increasing curve or a progressively decreasing curve, depending on the application. Low stiffness values can increase ride comfort by minimizing natural frequencies of motorcycle vibration modes, but cause wide variations in ride height and trim during acceleration and braking. On the other hand, high stiffness values reduce variations in ride height and trim, but lessen comfort and cause tire adherence problems during acceleration and braking. Therefore, a progressive suspension system is a good compromise. During small wheel travel, a progressive suspension will exhibit softer suspension while increasing stiffness to more rigid values when encountering high wheel travel. Additionally, increasing stiffness with deformation enables maintenance of more or less constant frequency of the in-plane vibration modes as mass increases.

As previously stated, the spring force will typically exhibit a nonlinear relation with the input deflection. However, in most practical applications, it can be approximated with a linear relationship provided the applied stress is within the elastic region for the spring.

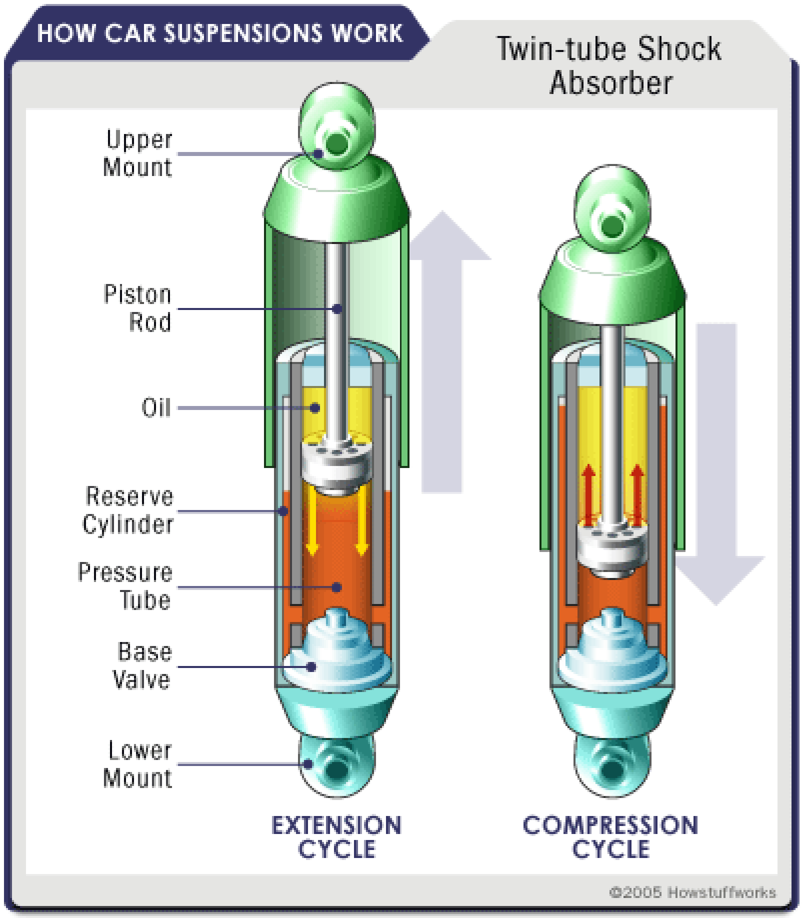

Figure 2: Dampers

A viscous damper is an energy dissipating device and is the most commonly used damping mechanism in vibrating systems. The damping force, which is nonconservative, is represented by the following relation:

$$F = c\dot{x} = cv$$

where c is the damping constant. Like the spring, the damper actually has a nonlinear relation with the applied force, but can be approximated as linear for practical applications. It is important to understand that a damper does not reduce the maximum response. Instead, the function of the damper is to reduce the number of oscillations in response to the forcing frequency. While the damping force is included in the free body diagram, it is actually nonconservative since it is path-dependent. Interestingly, while progressive springs are ideal, progressive damping curves are not since damping is too soft at low disturbances and too sharp at high disturbances (velocities) resulting in “hydraulic lock.” Cartridge forks have been offered and yield a linear relation between force and velocity. They have infinite tuning capabilities.

Typically, the damping coefficient in compression should be lower than in extension. This is due to the fact that when a wheel encounters a bump or step it must follow the profile of the obstacle without generating too much opposing forces, but when it encounters a pothole, it can jump over it with only a temporary loss of wheel contact with the road plane. Depending on motorcycle type and rider, the damping in compression is typically less than half that in the extension phase. For a comfortable ride, the average of the compression and extension coefficients must generally have a value around 30-35% with respect to critical damping. Reduced damping is equal to the product of the actual damping coefficient and the square of the velocity ratio. Therefore, if a linear shock absorber (constant coefficient) is used with a progressive suspension (increasing coefficient with wheel amplitude), the reduced damping is progressive because of the general progressive behavior of the suspension mechanism. The shock absorber’s degree of regressive rate must be made by accounting for the contrary effect generated by any progressive rate of suspension.

One additional feature worth mentioning is the motorcycle trim which is simply the configuration it acquires in various conditions during transient and steady-state motion. Trim is dependent upon the stiffness values for the front and rear suspension, applied forces to the motorcycle, and geometry of the drive chain and swing arm. In order to regulate the trim of the motorcycle, under variation of the load, preloading (spring pre-compression) can be used. With preloading, in order to obtain maximum amplitude greater forces must be applied. Also, it governs the maximum value of the wheel travel and the field of amplitude increases.

In some instances a vehicle may experience excessive vibrations so it may be appropriate to install a dynamic vibration absorber. A vibration absorber is a fairly recent invention that is simply a secondary mass-spring system attached the primary system. It is designed to cancel or minimize the steady-state vibrations of the primary system by vibrating out of phase with the excitation frequency. The force provided by the dynamic absorber is equal in magnitude and opposite in direction of the disturbing force as shown:

$$F_{absorber,cyclic} = kx = -F_0$$

So the stiffness of the absorber must be able to absorb the force and deflection from the input force on the primary system.

By introducing a dynamic vibration absorber to a single-degree-of-freedom system, with one natural frequency, it essentially becomes a two-degree-of-freedom system, with two new natural frequencies. So resonance is eliminated at the excitation frequency, but two resonances nearly as large are introduced that are within 25% of the natural frequency for the primary system. So it is imperative that the harmonic excitation must be well known, and not deviate from its excitation frequency. In fact, a dynamic absorber is typically appropriate for a system running at a constant frequency and the frequency is at resonance.

If the frequency range is too wide to be considered effective, it may be desirable to use a tuned mass damper (some sources reference it a damped dynamic vibration absorber), which is simply a dynamic vibration absorber with damping introduced. A tuned mass damper will greatly reduce the two introduced resonance peaks by a factor of ten, but it will not eliminate the response at resonance. This is because the equations of motion do not decouple, as will be shown later, and so the magnitude of the primary mass amplitude will not be zero. Here is an illustration of both a dynamic vibration absorber and several tuned mass dampers:

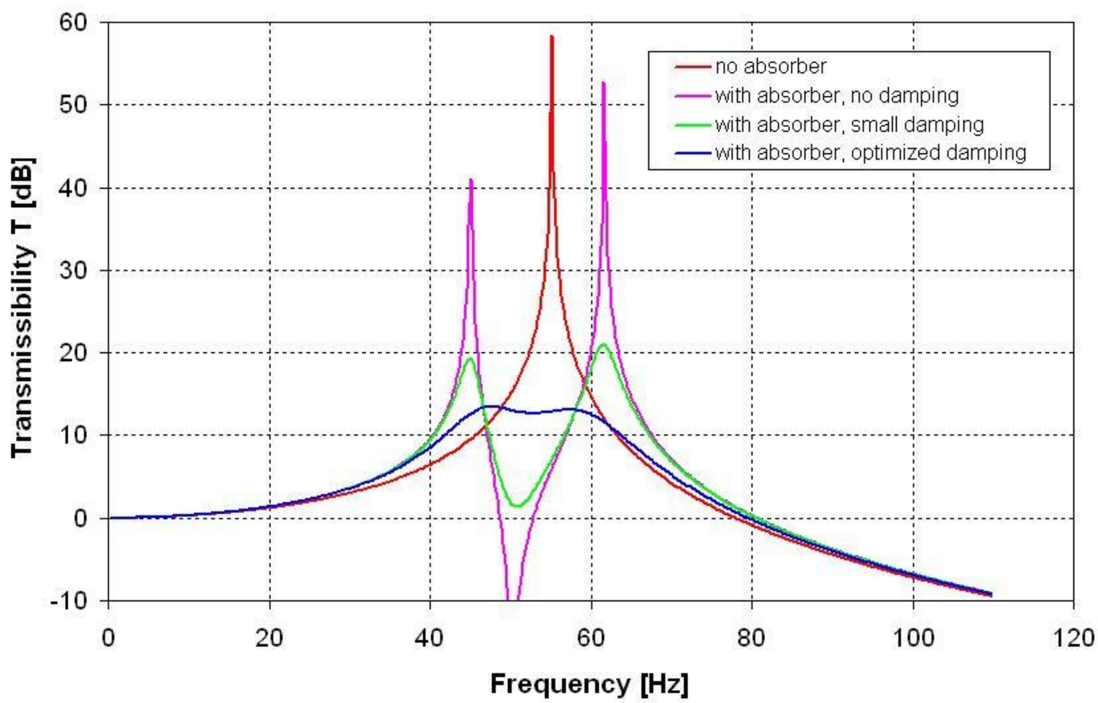

Figure 3: Dynamic Vibration Absorber Resonance

Note the tuned mass damper (damped absorber) is demonstrated by a smoother response curve.

CHAPTER 3: ANALYSIS

Vibrations in motorcycles are generally caused by the profile of the road, aerodynamics, and internal balancing such as engine and wheels. One technique is to decompose the elevation road profile into a series of sine waves, varying in amplitude and phase. This method is called the Power Spectral Density Function, which is a representation of the amplitude density versus path frequency. The sprung mass acceleration can then be determined by multiplying the Power Spectral Density Function by the square of the in-plane complex transfer function of the motorcycle. As with most rigid body models, the bounce and pitch modes are coupled. The bounce response is more pronounced at low frequencies, while the pitch is more pronounced at middle frequencies. Dips in the Power Spectral Density Function versus frequency plots can be attributed to wheelbase filtering which is closely related to p/V. Since any periodic function can be expressed as a summation of sine functions, the solution(s) presented in this thesis will be developed with one sinusoidal input.

A motorcycle in straight running can be characterized by more than ten (10) degrees of freedom to as simple as two (2) degrees of freedom. If lateral motion is included, and the motorcycle is running on an arbitrary path, a model can exceed fifty (50) degrees of freedom. However, a highly complex degree of freedom model does not give insight into the equation of motion and is difficult to use for development or demonstration, which is precisely why the focus will be on a two-degree of freedom model.

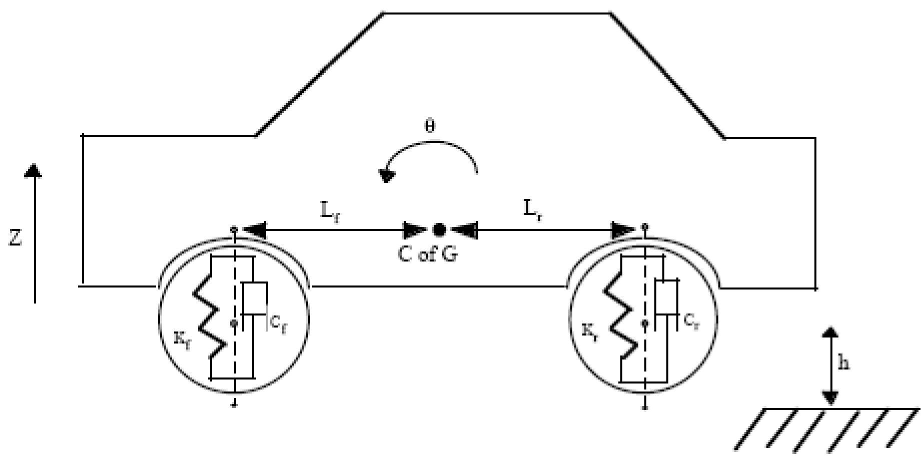

Figure 4: Half-Car Model

The free body diagram used in this thesis is nearly identical to the half-car model and has excluded the unsprung mass. This model is a useful tool as it will illustrate the two primary modes of vibration in the symmetric plane for both a car and a motorcycle:

1. Bounce: represented by a vertical coordinate, $\vec{y}$
2. Pitch: represented by an angular coordinate, $\vec{\theta}$

Both coordinates are captured by the *equations of motion* that are constructed by *Newtonian* or *Lagrangian Mechanics*. The equations of motion are second order, linear, differential equations used to describe the dynamics of a physical system, such as a motorcycle, with respect to time. The dynamic response, which will be analyzed and plotted, is a solution to this formula.

The difference between the half-car model and the free body representation of a motorcycle is the location of the plane of symmetry. The motorcycle suspension lies in the symmetric plane while a car needs to resolve its components into an equivalent system that is captured by it. As previously indicated, lateral dynamics is excluded from the half-car model, but these properties certainly contribute to the suspension activation, especially for the motorcycle.

From the Free Body Diagram, the equations of motion can be determined by using the *Newtonian Method*:

$$m\ddot{y}_M + (c_F + c_R)\dot{y}_M + (-ac_F - bc_R)\dot{\theta}_M + (k_F + k_R)y_M + (-ak_F - bk_R)\theta_M = g\sin\omega t$$

$$I\ddot{\theta}_M + (-ac_F - bc_R)\dot{y}_M + (a^2c_F + b^2c_R)\dot{\theta}_M + (-ak_F - bk_R)y_M + (a^2k_F + b^2k_R)\theta_M = 0$$

To analyze in MATLAB, however, the equations of motion must be collected in a matrix:

$$\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix}\begin{pmatrix} \ddot{y}_M \\ \ddot{\theta}_M \end{pmatrix} + \begin{bmatrix} (c_F + c_R) & (-ac_F - bc_R) \\ (-ac_F - bc_R) & (a^2c_F + b^2c_R) \end{bmatrix}\begin{pmatrix} \dot{y}_M \\ \dot{\theta}_M \end{pmatrix}$$
$$+ \begin{bmatrix} (k_F + k_R) & (-ak_F - bk_R) \\ (-ak_F - bk_R) & (a^2k_F + b^2k_R) \end{bmatrix}\begin{pmatrix} y_M \\ \theta_M \end{pmatrix} = \begin{bmatrix} g \\ 0 \end{bmatrix}\sin(\omega t)$$

Substituting values into the *mass matrix*, M, *damping matrix*, C, and *stiffness matrix*, K, respectively:

$$M = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} = \begin{bmatrix} 570.0 & 0 \\ 0 & 2051.1 \end{bmatrix};$$

$$C = \begin{bmatrix} (c_F + c_R) & (-ac_F + bc_R) \\ (-ac_F + bc_R) & (a^2c_F + b^2c_R) \end{bmatrix} = \begin{bmatrix} 117.0 & 61.3 \\ 61.3 & 602.9 \end{bmatrix};$$

$$K = \begin{bmatrix} (k_F + k_R) & (-ak_F + bk_R) \\ (-ak_F + bk_R) & (a^2k_F + b^2k_R) \end{bmatrix} = \begin{bmatrix} 2763.4 & 1447.6 \\ 1447.6 & 14232.3 \end{bmatrix}$$

The matrix has the form of the general equations of motion relation:

$$M\ddot{\vec{x}} + C\dot{\vec{x}} + K\vec{x} = \vec{g}\sin(\omega t) + \vec{h}\cos(\omega t)$$

The solution is obtained by superimposing the results from two simpler solutions:

$$\vec{x}(t) = \vec{x}_P(t) + \vec{x}_H(t)$$

where $\vec{x}_P$ is the *particular solution* of:

$$M\ddot{\vec{x}}_P + C\dot{\vec{x}}_P + K\vec{x}_P = \vec{g}\sin(\omega t) + \vec{h}\cos(\omega t)$$

and $\vec{x}_H$ is the *general* or *homogeneous solution* of:

$$M\ddot{\vec{x}}_H + C\dot{\vec{x}}_H + K\vec{x}_H = 0$$

The homogeneous equation represents the free vibration of the system which decays over time, eventually returning to a static position. Therefore, the homogeneous response represents the transient response while the particular response is the steady-state response. The transient response for each root is determined solely by the location of the root [Hale, 77-80]. To obtain the *particular solution* the following relation is performed:

$$\vec{x}_P(t) = \vec{p}\cos(\omega t) + \vec{q}\sin(\omega t)$$

where $\vec{p}$ and $\vec{q}$ are constant vector amplitudes. Upon substituting this in the general equation of motion formula and factoring out like trigonometric terms, the equation can, again, be expressed in matrix form so the amplitudes can be determined:

$$\begin{bmatrix} K - \omega^2 M & \omega C \\ -\omega C & K - \omega^2 M \end{bmatrix} \begin{pmatrix} \vec{p} \\ \vec{q} \end{pmatrix} = \begin{pmatrix} \vec{h} \\ \vec{g} \end{pmatrix} \xrightarrow{yields} \begin{pmatrix} \vec{p} \\ \vec{q} \end{pmatrix} = \begin{bmatrix} K - \omega^2 M & \omega C \\ -\omega C & K - \omega^2 M \end{bmatrix}^{-1} \begin{pmatrix} \vec{h} \\ \vec{g} \end{pmatrix}$$

The *homogeneous solution*, which describes the system exclusively, can be obtained by substituting a solution of the form:

$$\vec{x}_H(t) = \vec{r}e^{s_j t}$$

into the *homogenous equation*:

$$M\ddot{\vec{x}}_H + C\dot{\vec{x}}_H + K\vec{x}_H = 0$$

to obtain the *quadratic eigenvalue problem*:

$$(s^2M + sC + K)\vec{r} = 0 \; where \; \vec{r} \neq 0$$

So the non-trivial solution is found by taking the determinate of a polynomial of degree 2*n* and solving for s:

$$det(s^2M + sC + K) = 0$$

The determinate will yield 2*n* roots s_j; *j=1,2,….2n*, which are called the eigenvalues or poles of the system. For each pole s_j there exists an eigenvector $\vec{r} \neq 0$ which satisfies the *quadratic eigenvalue problem*. The eigenvectors span the state space such that:

$$\vec{V} = \begin{bmatrix} \vec{r}_1 & \dots & \vec{r}_{2n} \\ s_1\vec{r}_1 & \dots & s_{2n}\vec{r}_{2n} \end{bmatrix}$$

is invertible. Then by superposition:

$$\vec{x}_H(t) = \sum_{j=1}^{2n} a_j \vec{r}_j e^{s_j t}$$

becomes the solution to the *homogeneous equation* which is expressed as a combination of 2*n* linear independent functions. The *eigenvector matrix*, $\vec{V}$, is invertible if the *eigenvalues* or *poles* are unique. That is $s_p \neq s_q$ for all $p \neq q$. In the event there are repeated *eigenvalues*, the appropriate solution is then:

$$\vec{x}_H(t) = P(t)\vec{r}e^{st}$$

where *P(t)* is a polynomial in *t*.

Now to evaluate the *eigenpairs*, which are the *eigenvalues* and *eigenvectors*, the identity:

$$\dot{\vec{x}}_H - \dot{\vec{x}}_H = 0$$

along with the *homogeneous equation*, can be rewritten in the form:

$$\begin{bmatrix} 0 & I \\ K & 0 \end{bmatrix}\begin{pmatrix} \vec{x}_H \\ \dot{\vec{x}}_H \end{pmatrix} - \begin{bmatrix} I & 0 \\ C & M \end{bmatrix}\begin{pmatrix} \dot{\vec{x}}_H \\ \ddot{\vec{x}}_H \end{pmatrix} = \begin{pmatrix} \vec{0} \\ \vec{0} \end{pmatrix}$$

So with the definitions:

$$\vec{z} = \begin{pmatrix} \dot{\vec{x}}_H \\ \vec{x}_H \end{pmatrix}; \vec{A} = \begin{bmatrix} 0 & I \\ K & 0 \end{bmatrix}; \vec{B} = \begin{bmatrix} I & 0 \\ C & M \end{bmatrix}$$

the first order realization of the *homogeneous equation* is given by:

$$\vec{A}\vec{z} - \vec{B}\dot{\vec{z}} = 0$$

Finally, a solution of the form:

$$\vec{z}(t) = \vec{v}e^{st}$$

where $\vec{v}$ is a constant vector and is used to obtain the *generalized eigenvalue problem*:

$$(\vec{A} - s\vec{B})\vec{v} = 0$$

The simpler solutions can be solved analytically. However, the more complicated solutions will need a program with a comprehensive algorithm, such as MATLAB. In MATLAB, the command:

$$[\vec{V}, S] = eig(\vec{A}, \vec{B})$$

returns the resultant values for the eigenvalue diagonal matrix and eigenvector matrix as such:

$$S = \begin{bmatrix} s_1 & & \\ & \ddots & \\ & & s_{2n} \end{bmatrix}; \vec{V} = \begin{bmatrix} \vec{r}_1 & \cdots & \vec{r}_{2n} \\ s_1\vec{r}_1 & \cdots & s_{2n}\vec{r}_{2n} \end{bmatrix}$$

so the values for the eigenpairs $(s_j, \vec{r}_j)$ are obtained. The final step is to impose the boundary conditions, which are typically known values:

$$\vec{x}(0) = \vec{x}_0 = \vec{p} + \sum_{j=1}^{2n} a_j \vec{r}_j \, ; \dot{\vec{x}}(0) = \vec{v}_0 = \omega\vec{q} + \sum_{j=1}^{2n} a_j \, s_j \vec{r}_j$$

and can be written in matrix form:

$$\vec{V}\vec{a} = \begin{pmatrix} \vec{x}_0 - \vec{p} \\ \vec{v}_0 - \omega\vec{q} \end{pmatrix}$$

where $\vec{V}$ is given by the eigenvector matrix and $\vec{a} = (a_1 \quad \cdots \quad a_{2n})^T$. Note that the coefficients a_j are determined by the boundary condition matrix when $\vec{V}$ is invertible. Now that the boundary conditions have been applied, it is possible to determine the *homogenous solution*, and by superimposing the results from the *particular solution* the *system response* can be obtained by the relation:

$$\vec{x}(t) = \vec{x}_p(t) + \vec{x}_H(t) = \vec{p}\cos(\omega t) + \vec{q}\sin(\omega t) + \sum_{j=1}^{2n} a_j \vec{r}_j e^{s_j t}$$

which can also be written as:

$$\vec{x}(t) = \vec{p}\cos(\omega t) + \vec{q}\sin(\omega t) + [\vec{r}_1 \quad \dots \quad \vec{r}_{2n}] \begin{bmatrix} e^{s_1 t} & & \\ & \ddots & \\ & & e^{s_{2n} t} \end{bmatrix} \begin{pmatrix} A_1 \\ \vdots \\ A_{2n} \end{pmatrix}$$

It is important to note that positively damped systems will exhibit a decayed oscillatory response.

Here is a plot of the dynamic response for the system (bounce-green; pitch-blue):

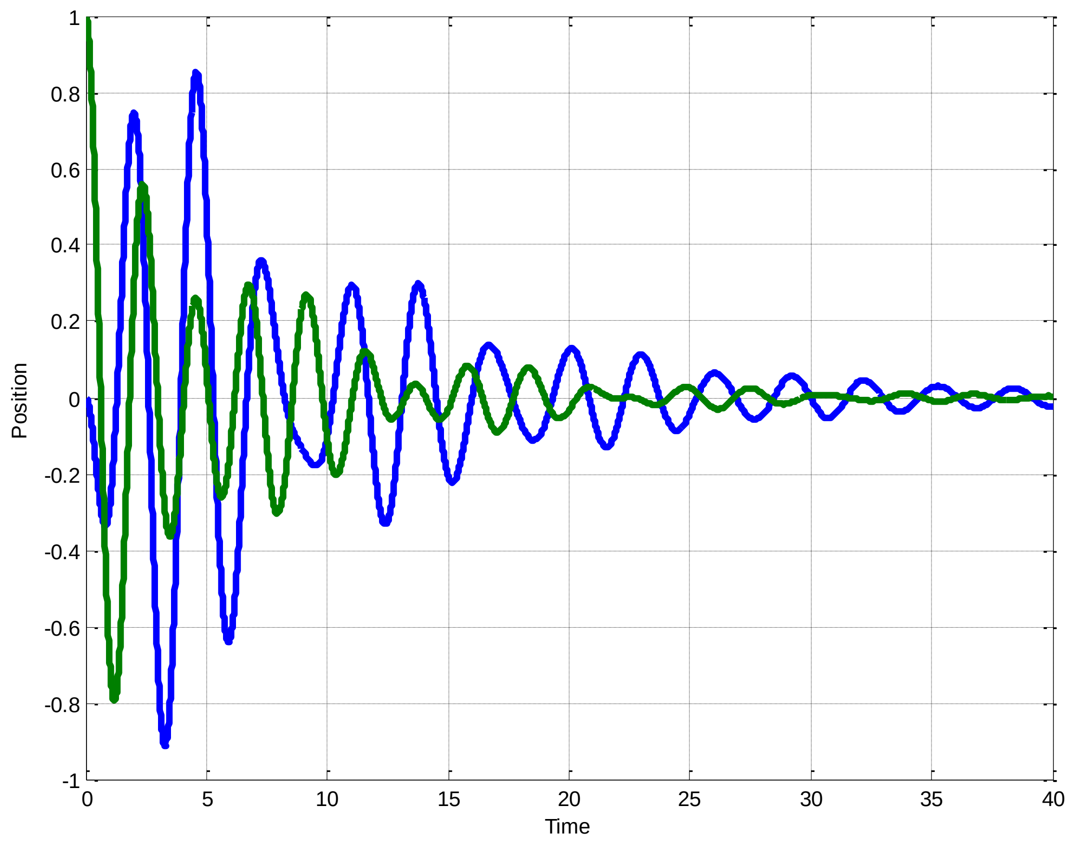

Figure 5: System Dynamic Response

Although, there is some local variation in the amplitudes, there is a general decay over time. The variations can be attributed to a poor selection of stiffness or damping values. The sensitivity from changes of these values will be illustrated in the upcoming parametric study. The transient system response is determined by the pole locations.

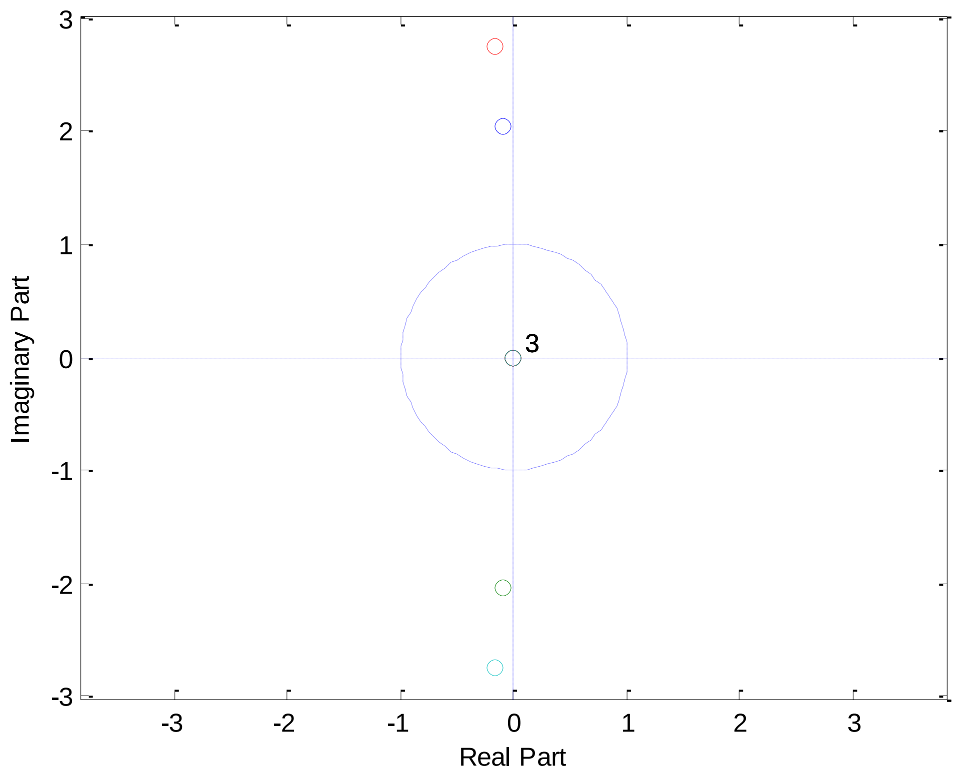

Figure 6: Poles in S-Plane

The eigenvalues or poles are:

1. J=1: -0.0888+2.0464i
2. J=2: -0.0888-2.0464i
3. J=3: -0.1608+2.7512i
4. J=4: -0.1608-2.7512i

These four (4) values form two (2) sets of complex conjugate pairs in the negative real region. The dominant roots are always closest to the imaginary axis since they are associated with the longest settling times and have the smallest damping ratios (the damping coefficient divided by the critical damping coefficient). The real part is indicative of the system stability, with negative values corresponding to decay and positive values corresponding to growth (unstable). Therefore, this system is stable and will decay over time. Generally, most passive vibration systems exhibit negative real parts. The natural frequencies as such:

$$s_{1,2} = \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)\omega_n = \pm i\omega_n (no\ damping)$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}\ where\ \omega_d = damped\ nat.frequency$$

The real part of a complex pole is associated with the system damping and the imaginary part is associated with the frequency. If the polynomial from the determinate matrix has real coefficients, the solution yields complex conjugate pairs. However, the system response is typically in real terms, but can be expressed as imaginary (numbers known as impedance) [Bishop and Dorf, 238-240].

A mode shape is a mathematical description of deflection and forms a pattern that describes the shape of vibration if the system were to vibrate only at the corresponding natural frequency [Inman, 317]. In an effort to be thorough, the real, imaginary, and absolute values will be plotted.

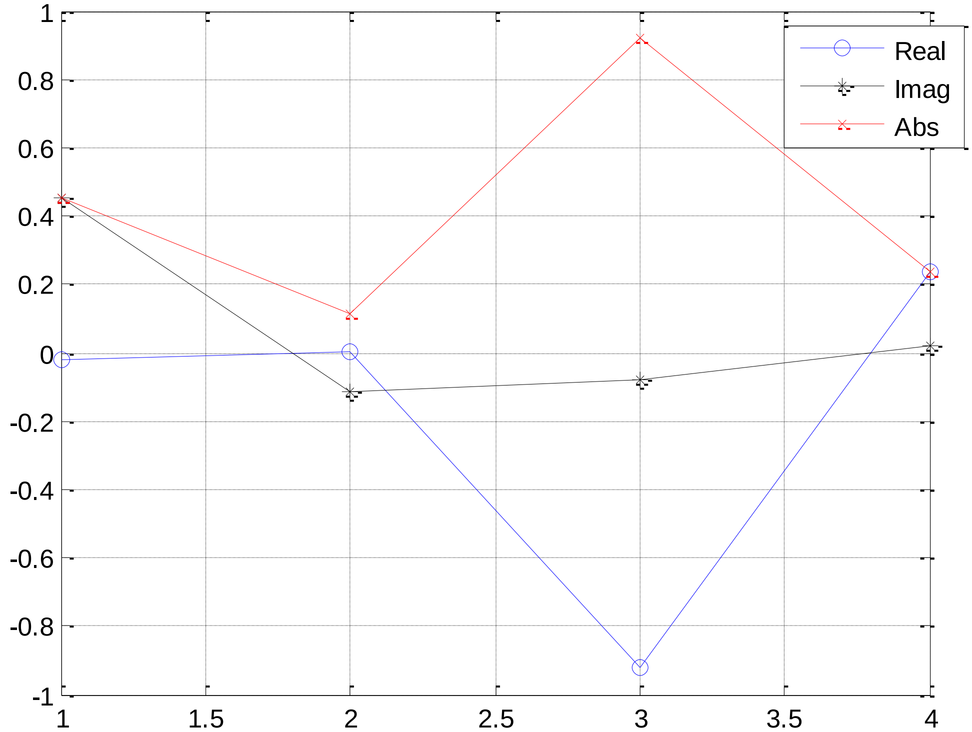

Figure 7: 1st Mode

The first modal values for v_1 are:

1. -0.0195+0.4504i
2. 0.0050-0.1157i
3. -0.9200-0.0800i
4. 0.2364+0.0206i

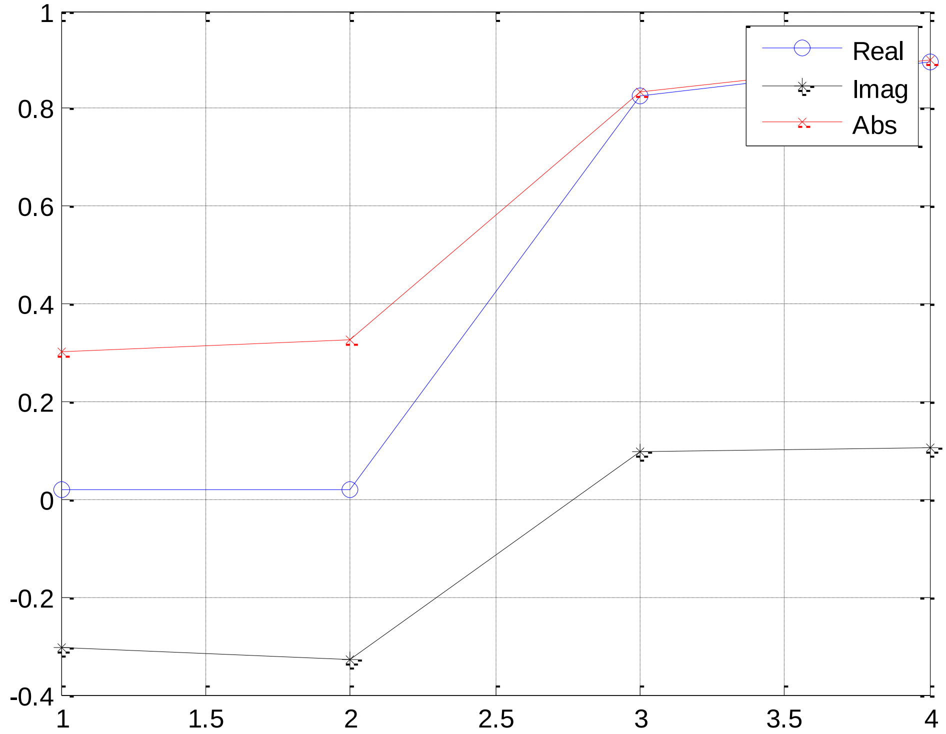

Figure 8: 2nd Mode

The second modal values for v_2 are:

1. 0.0176-0.3018i
2. 0.0191-0.3264i
3. 0.8275+0.0971i
4. 0.8950+0.1050i

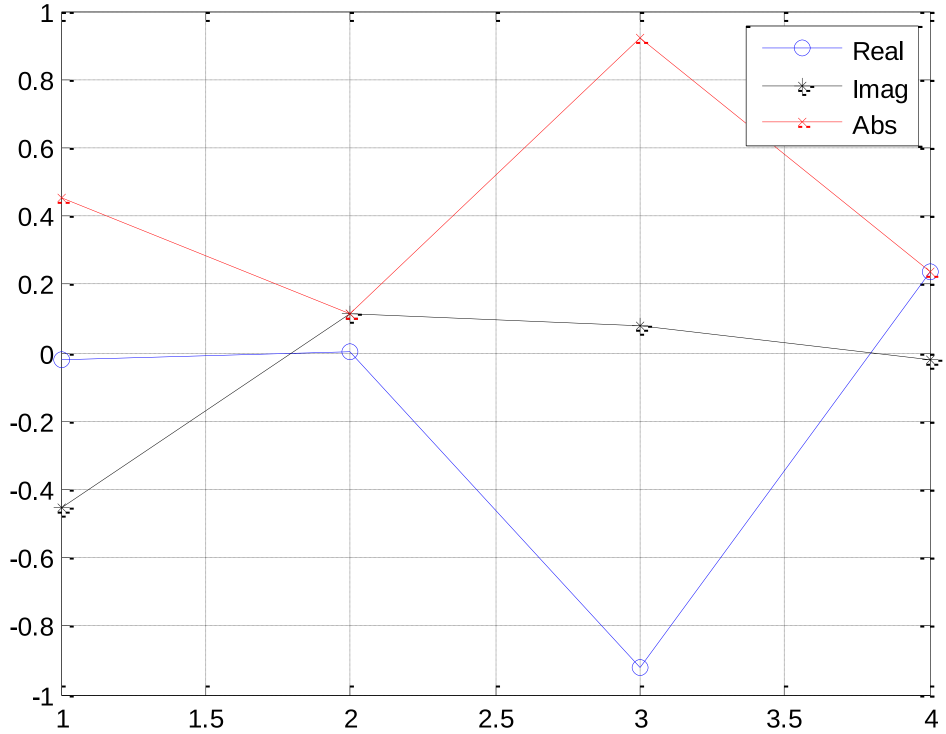

Figure 9: 3rd Mode

The third modal values for v_3 are:

1. -0.0195+0.4504i
2. -0.0050-0.1157i
3. -0.9200-0.0800i
4. -0.2364+0.0206i

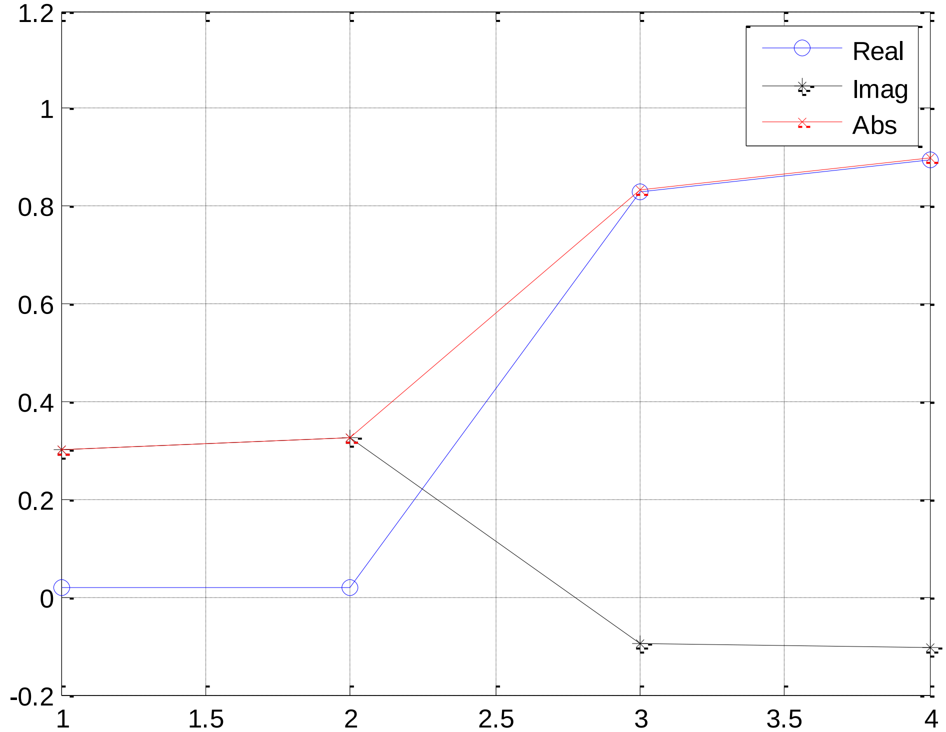

Figure 10: 4th Mode

The fourth modal values for v_4 are:

1. 0.0176+0.3018i
2. 0.0191+0.3264i
3. 0.8275-0.0971i
4. 0.8950-0.1050i

So now the magnitude and phase angle will be plotted over a range of natural frequencies to verify the value for the resonance frequency and to see if there are other peaks.

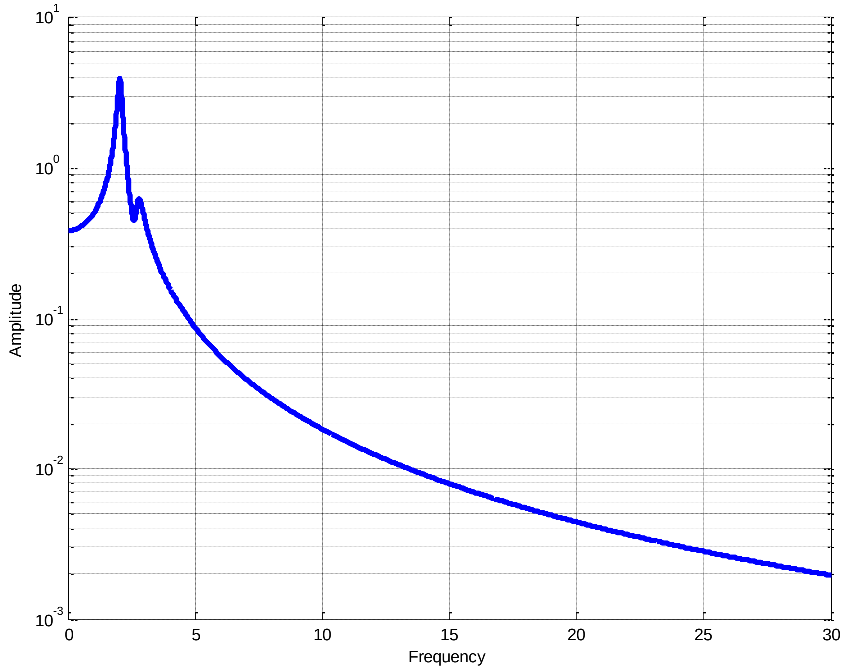

Figure 11: Amplitude in Bounce Mode

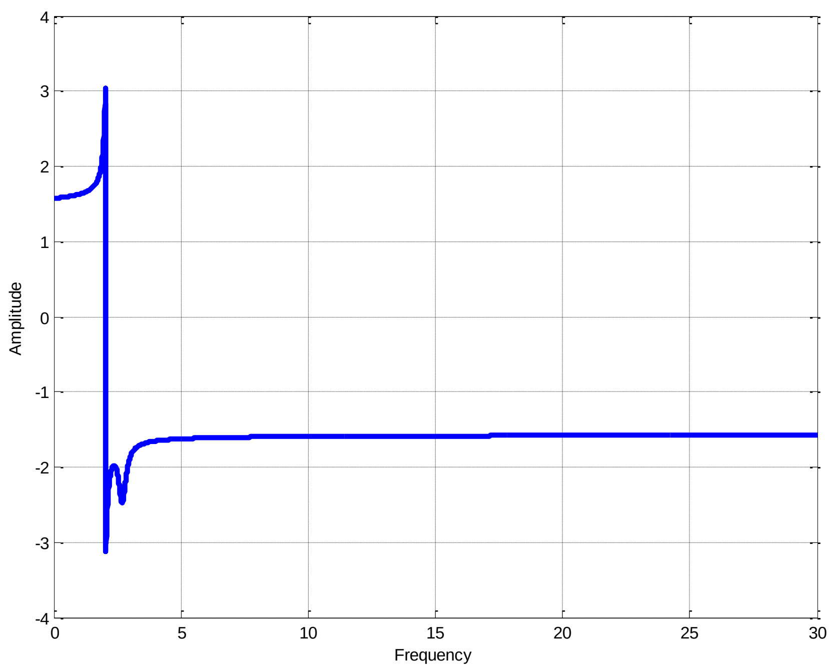

Figure 12: Amplitude in Pitch Mode

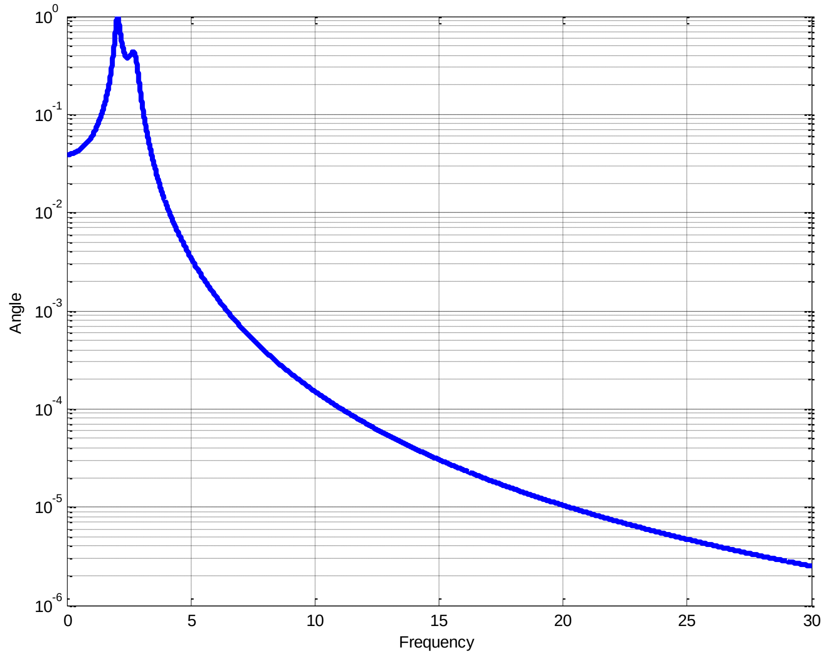

Figure 13: Amplitude in Pitch Mode

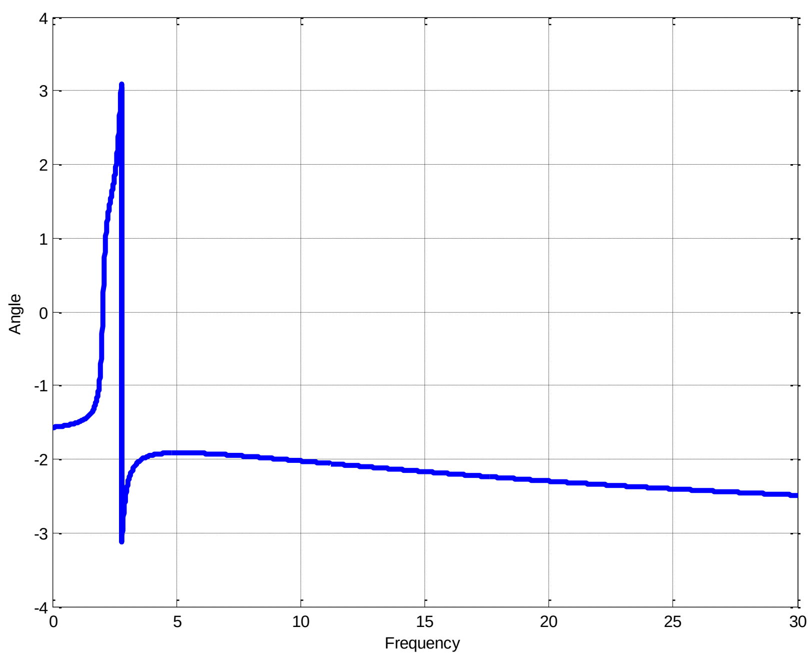

Figure 14: Phase Angle in Pitch Mode

As expected, the peaks occur at the values for the natural frequencies or resonance frequencies. The system is designed to avoid an excitation at these frequencies.

Now that the system and dynamic properties are known, a parametric study will be performed to illustrate the sensitivity in modifications to the suspensions parameters such as: inertia, stiffness, and damping. Perhaps a better understanding can be reached as to the contribution of each value to the system properties.

CHAPTER 4: PARAMETRIC ANALYSIS OF SYSTEM

The actual analysis will consist of perturbing each parameter 50% to 100% of its initial value, depending on practical values, while keeping the other terms constant. Each parameter will be varied for five (5) iterations and in equal increments. Only the front spring and shock will be perturbed so the effects on pitch can be demonstrated.

The first term to be perturbed will be the mass, m. It is important to note that variations in the mass will also be realized by the moment of inertia, I, since it is a function of mass as well as radius of gyration. So it is reasonable to expect changes in pitch since variations in inertia would certainly have an effect on vehicle rotation in the vertical plane.

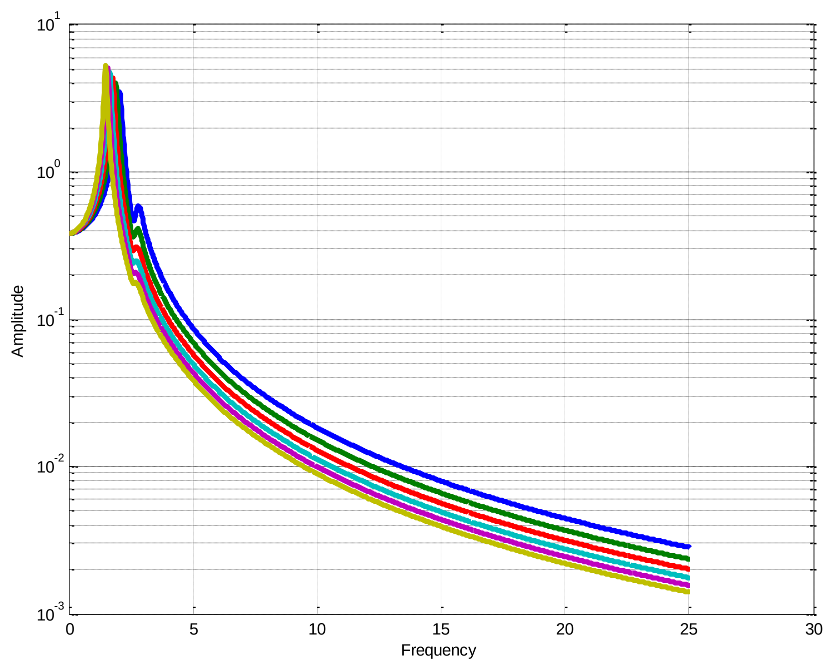

Figure 15: Mass Perturbation for Amp in Bounce Mode

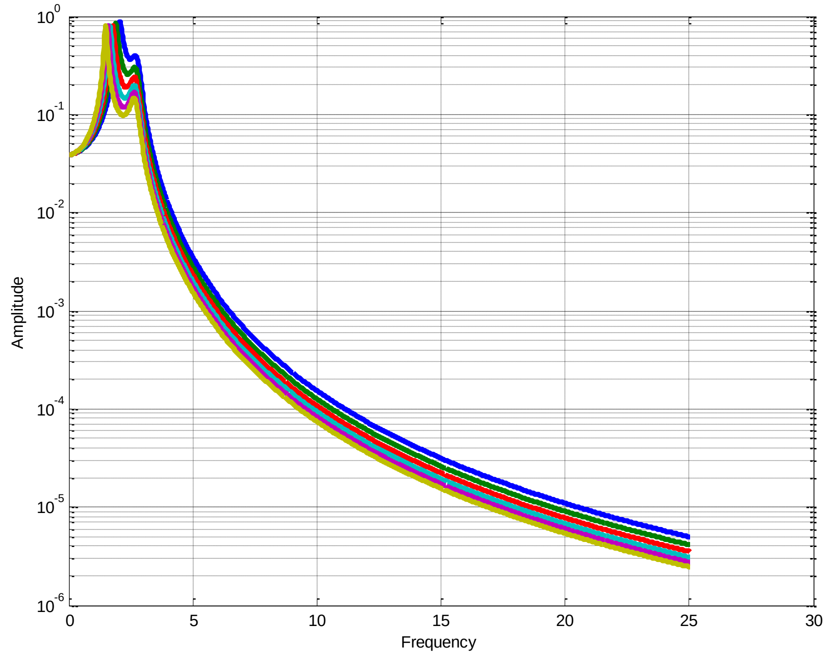

Figure 16: Mass Perturbation for Amp in Pitch Mode

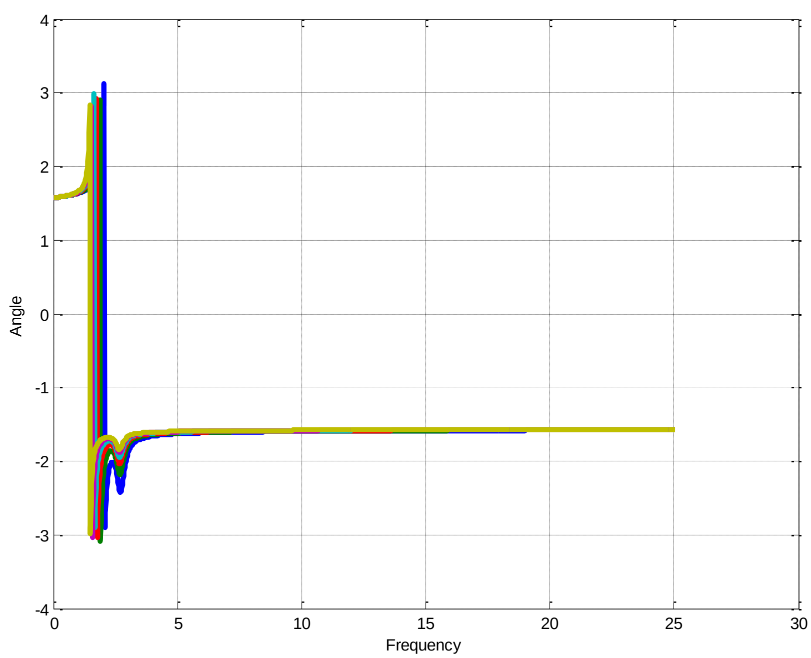

Figure 17: Mass Perturbation for Phase in Bounce Mode

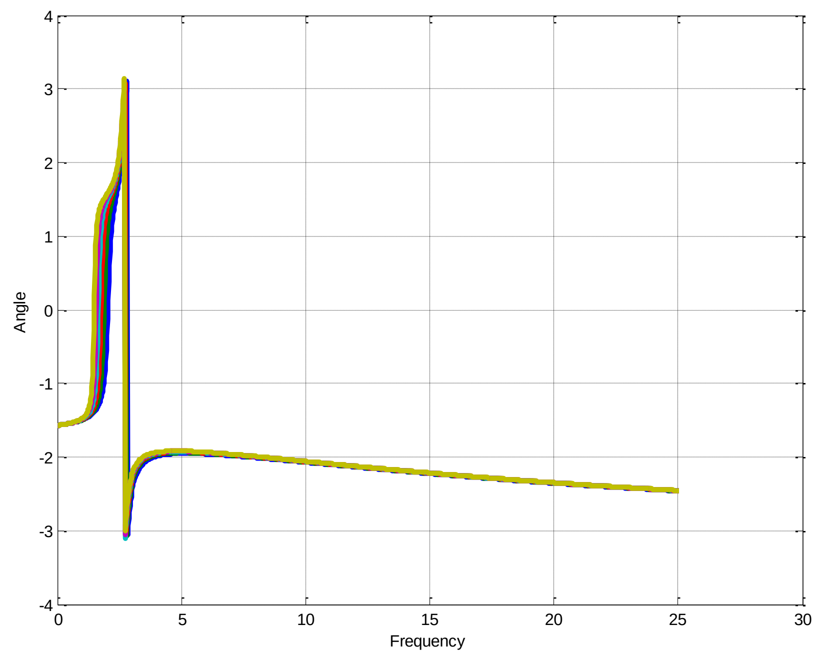

Figure 18: Mass Perturbation for Phase in Pitch Mode

As observed from the graphs, modifications of the mass shift the values vertically in the Amplitude Function Graphs, especially at resonance, while there is a more pronounced vertical shift in the Phase Function Graphs, especially near the values for the natural frequency. So there is an apparent correlation with changes in amplitude or magnitude to changes in mass, which is expected due the relation of the undamped natural frequency with mass. There is also a variation of the value for natural frequency in relation to the phase angle, but this could be from lag since the moment of inertia is effected by variations in mass.

Next, the front spring will be subject to a perturbation since it contributes more to the handling characteristics of the motorcycle. The results are shown.

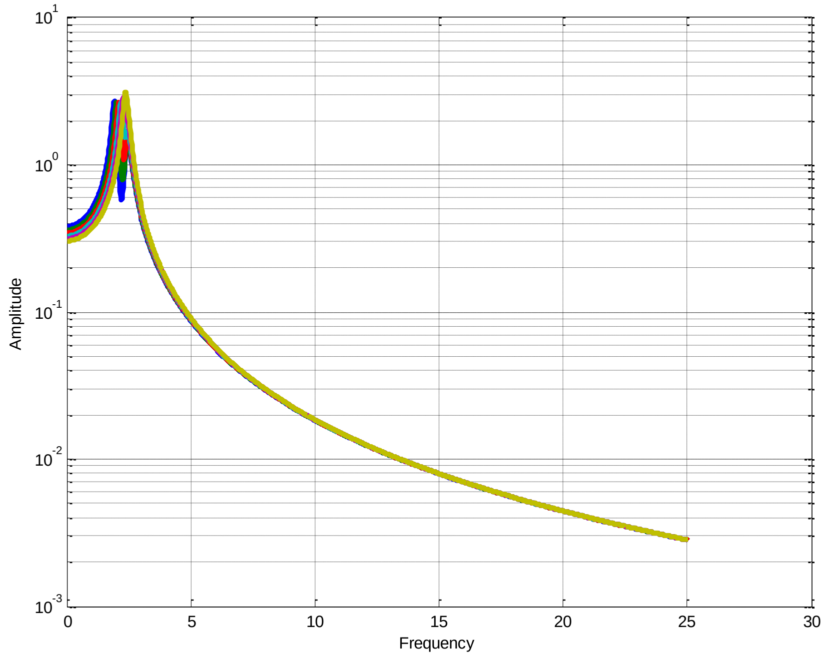

Figure 19: Spring Perturbation for Amp in Bounce Mode

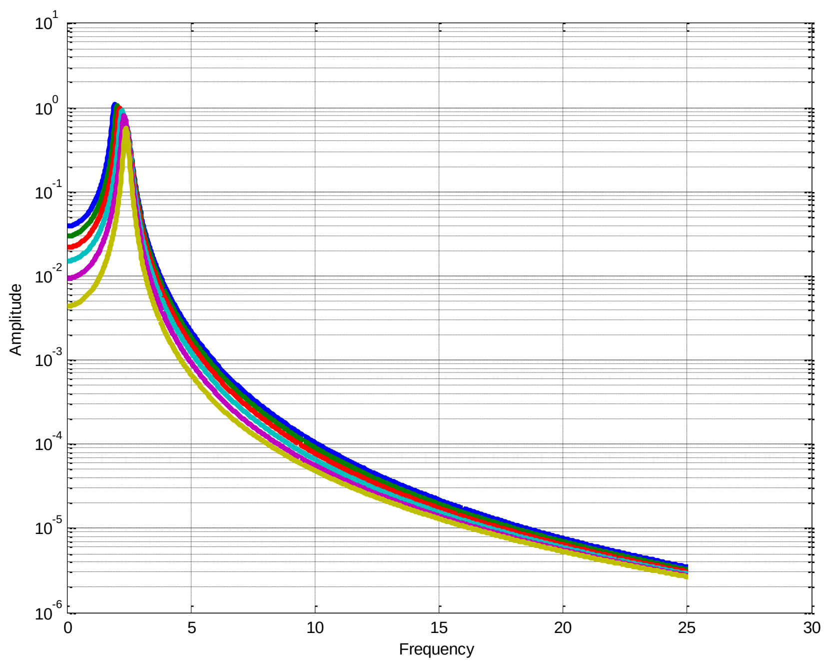

Figure 20: Spring Perturbation for Amp in Pitch Mode

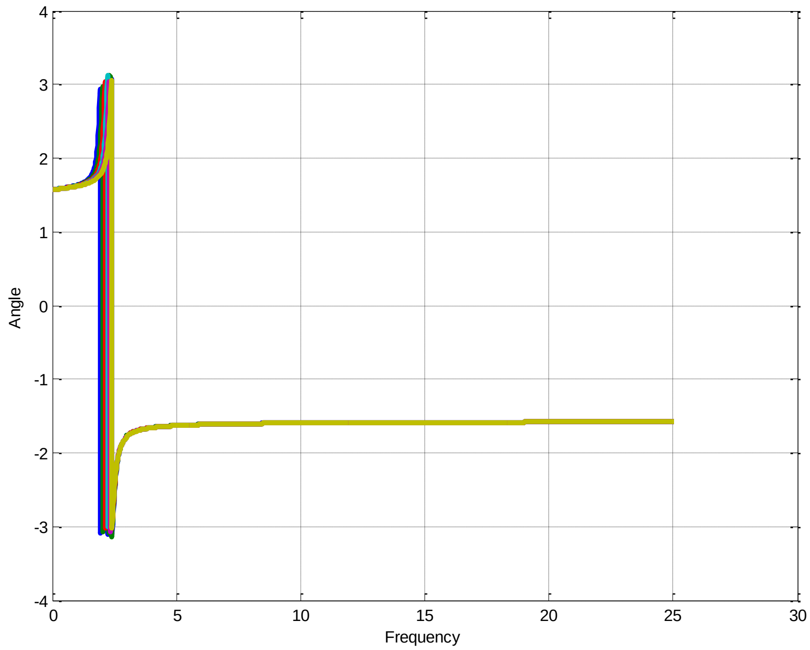

Figure 21: Spring Perturbation for Phase in Bounce Mode

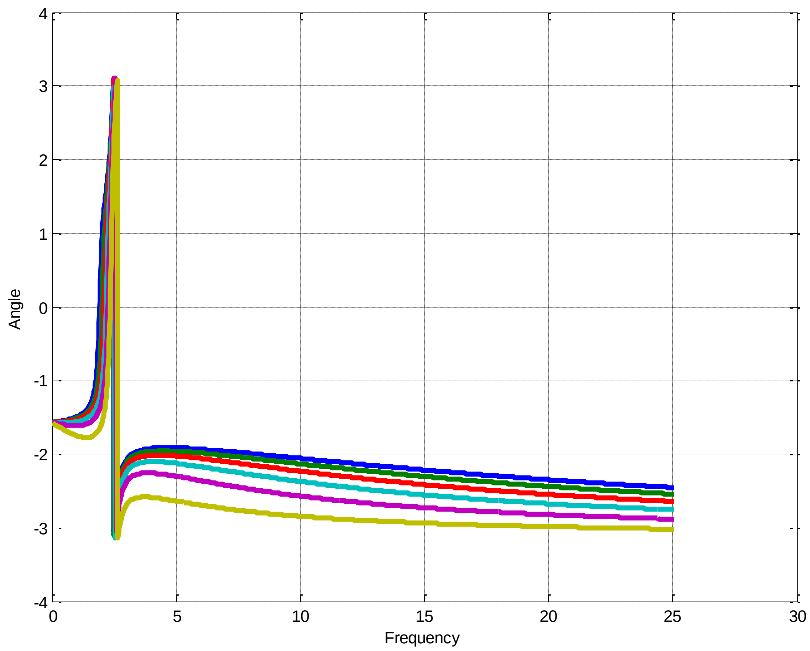

Figure 22: Spring Perturbation for Phase in Pitch Mode

As with mass, stiffness has a direct effect on the natural frequency which exhibits a vertical shift in the graph in the amplitude functions and somewhat in the phase response. It should be noted, the rear springs are typically 60% stiffer than the front springs. This is intended to prevent high pitch values due to a lag in response since the front tires would be disturbed before the rear tires (assuming the vehicle has forward velocity).

Finally, the front damper will be subject to perturbation. The results are shown in the following figures:

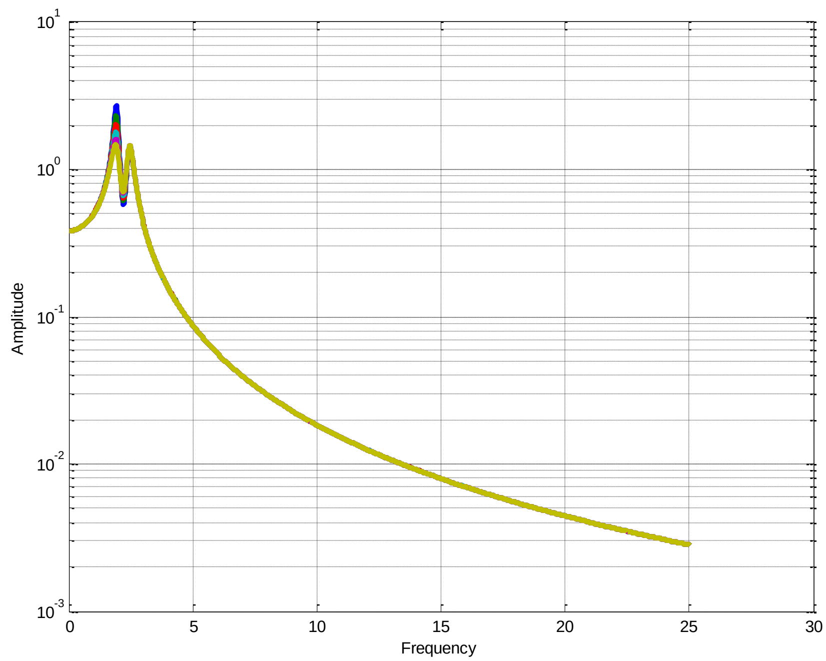

Figure 23: Damper Perturbation for Amp in Bounce Mode

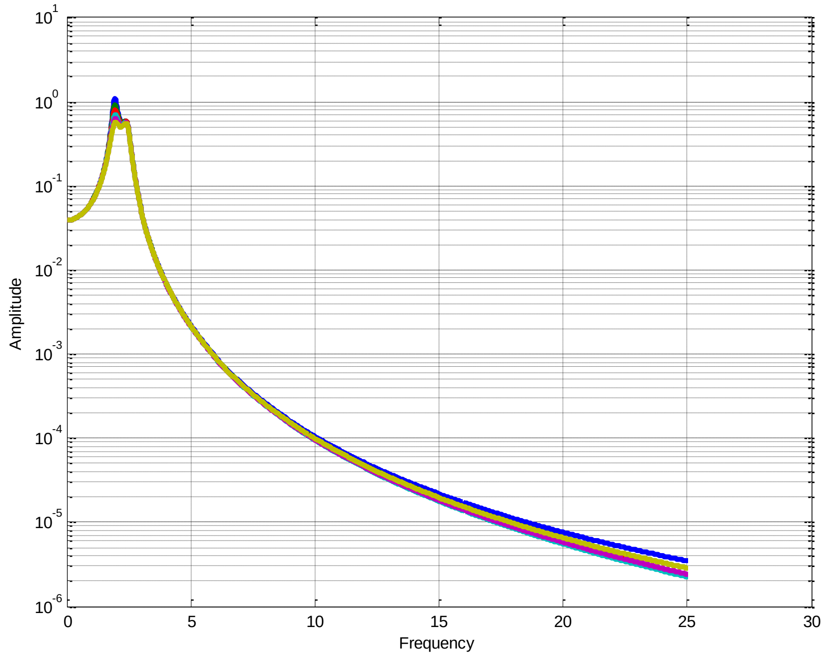

Figure 24: Damper Perturbation for Amp in Pitch Mode

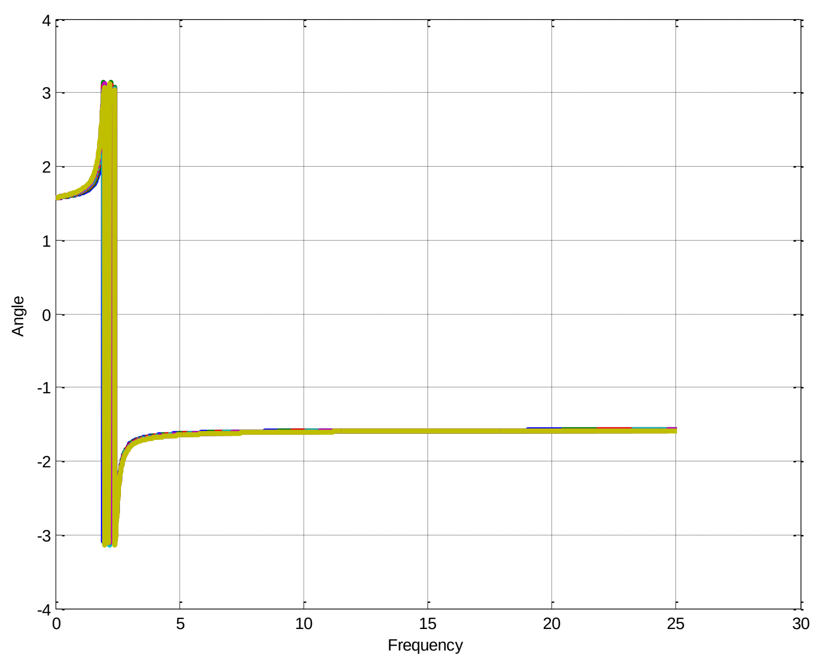

Figure 25: Damper Perturbation for Phase in Bounce Mode

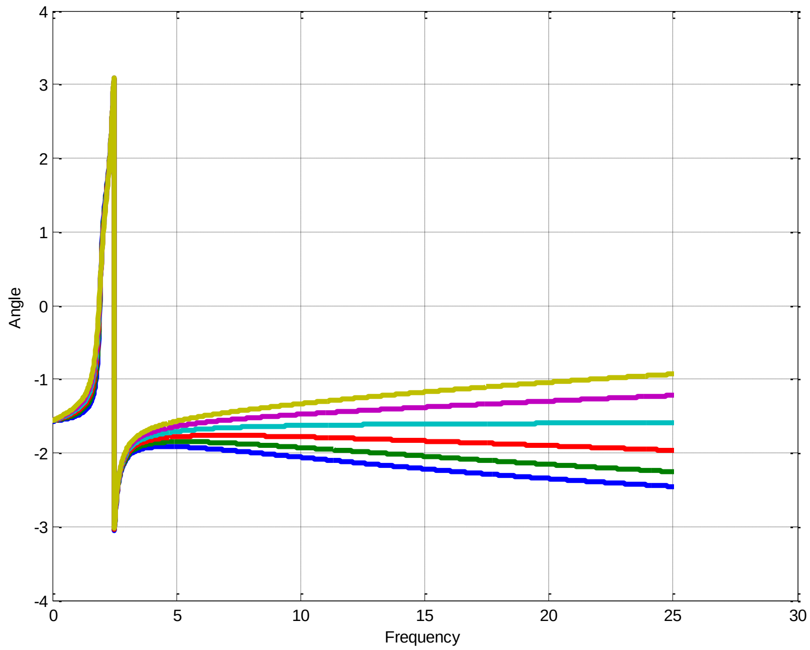

Figure 26: Damper Perturbation for Phase in Pitch Mode

All the graphs for the damper perturbations yield very little change, except at extreme values for frequency. This seems justified since there is very little difference in damped frequency and natural frequency. In terms of handling, low damping values correlate with poor resonance control, but do provide isolation from high excitation frequencies. Conversely, increased damping yields improvement in resonance control at the expense of high frequency isolation from road input.

CHAPTER 5: DYNAMIC VIBRATION ABSORBER

Finally, a dynamic absorber was investigated in MATLAB. According to theory, the mass of the absorber should be between 10 and 25% of the primary system mass. The value for the spring is determined as such:

$$k_{absorber} = m_{absorber}\omega^2$$

Here is the response at those values:

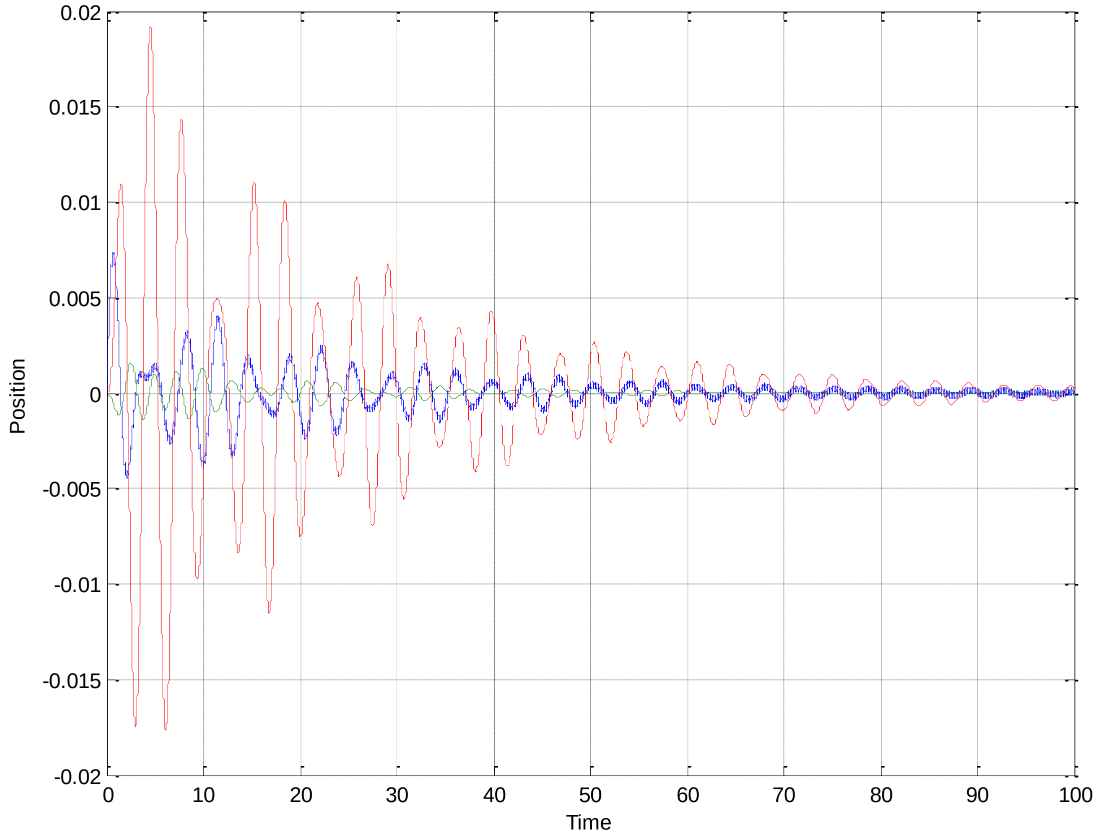

Figure 27: Dynamic Response of System with Absorber

This is expected since the absorber (red) is canceling out the amplitude of the bounce (green). The response of the pitch (blue) is also reduced since it is coupled with bounce. However, typically a single vibration absorber is designed to modify only a single node.

Now to illustrate the effect of perturbations on the vibration absorber parameters, both the mass and spring values of the absorber will be modified. First the values of the spring will undergo a perturbation:

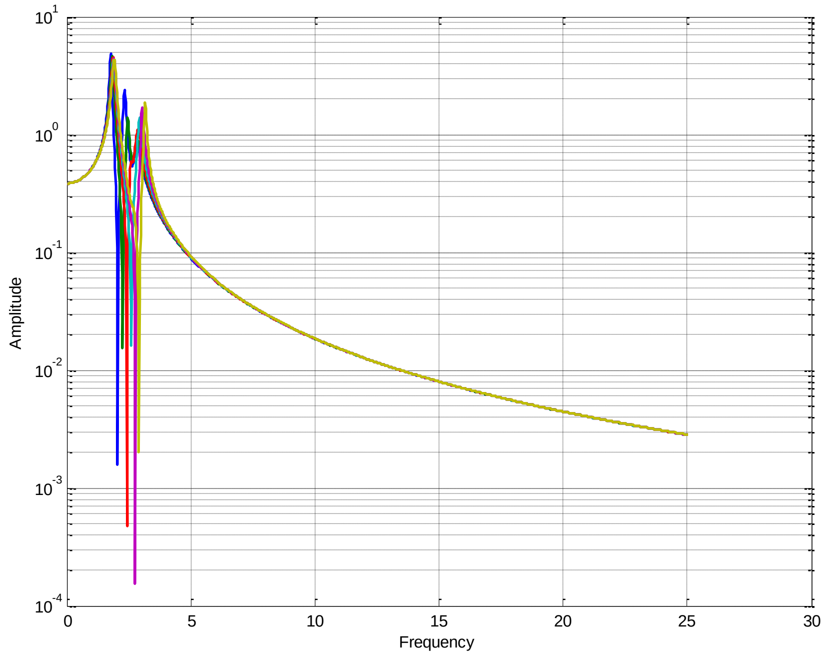

Figure 28: Absorber Spring Perturbation for Amp in Primary Bounce Mode

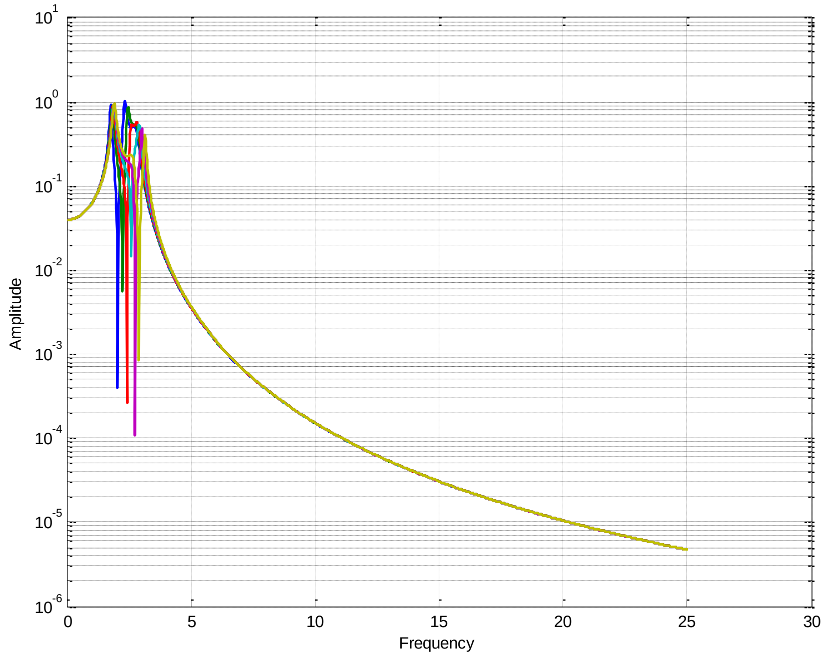

Figure 29: Absorber Spring Perturbation for Amp in Primary Pitch Mode

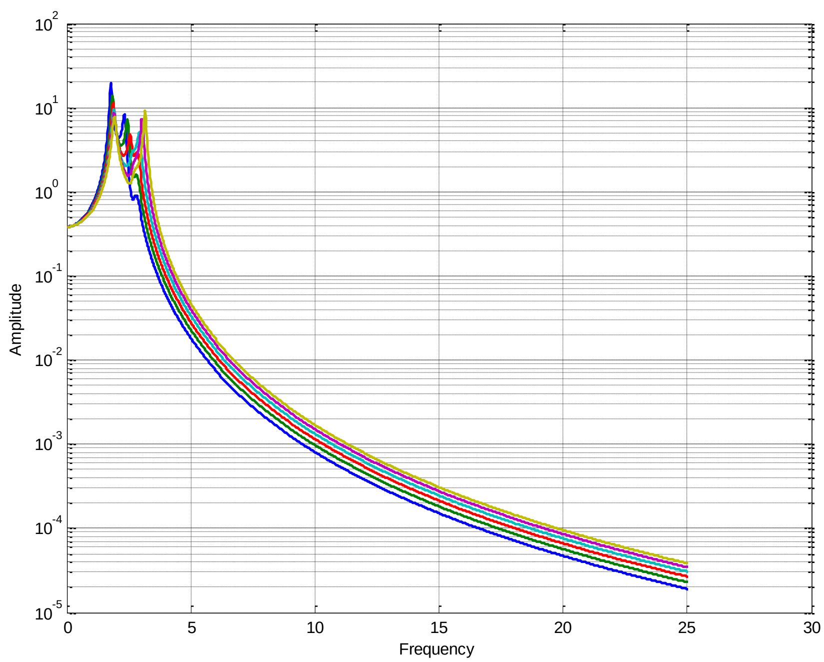

Figure 30: Absorber Spring Perturbation for Amp in Secondary Bounce Mode

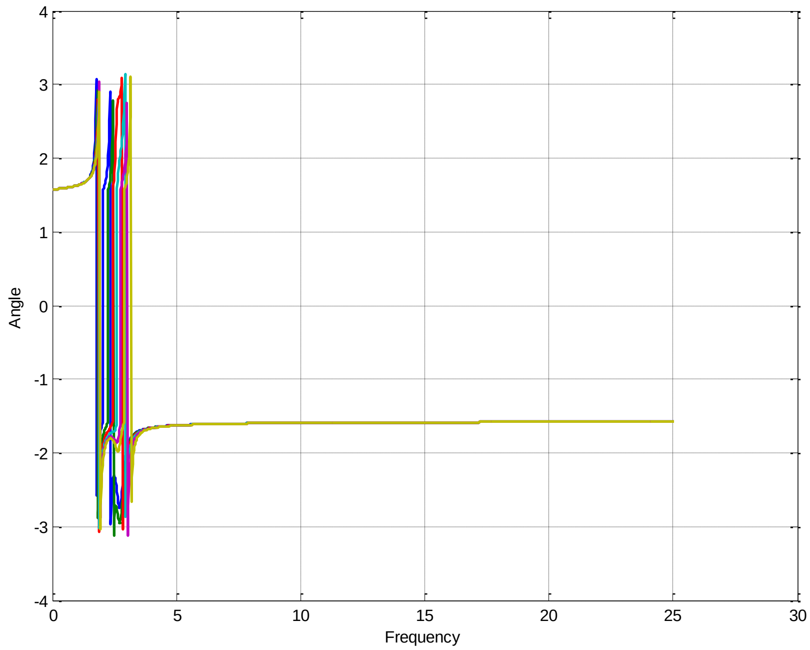

Figure 31: Absorber Spring Perturbation for Phase in Primary Bounce Mode

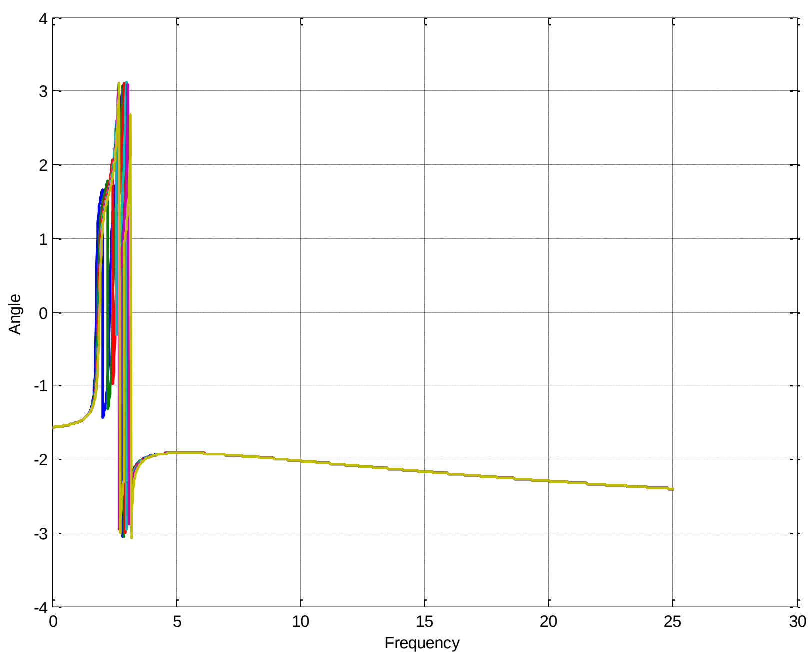

Figure 32: Absorber Spring Perturbation for Phase in Primary Pitch Mode

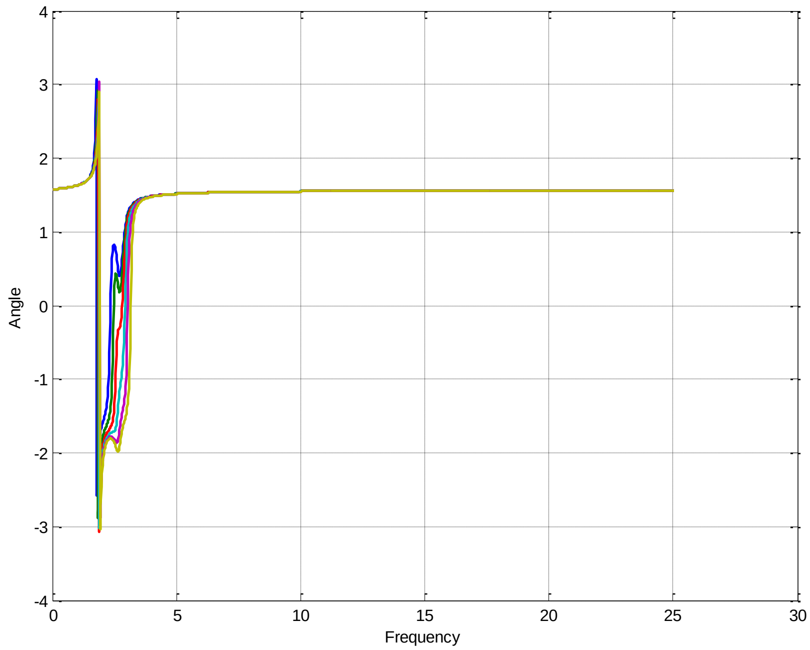

Figure 33: Absorber Perturbation for Phase in Secondary Bounce Mode

So it appears changes in the stiffness for the secondary system (absorber) correlate with local variations in the values for natural frequency which agrees with the equation for resonance. As with a single-degree-of-freedom system, increases in stiffness correlates with an increase in the undamped natural frequency.

Now the change in response due to a perturbation of the secondary absorber mass will be observed.

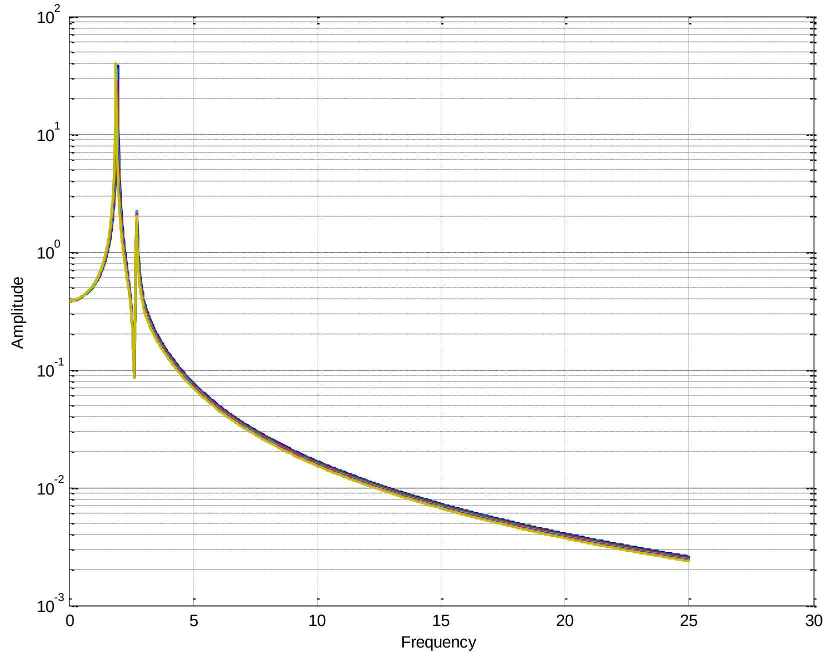

Figure 34: Absorber Mass Perturbation for Amp in Primary Bounce Mode

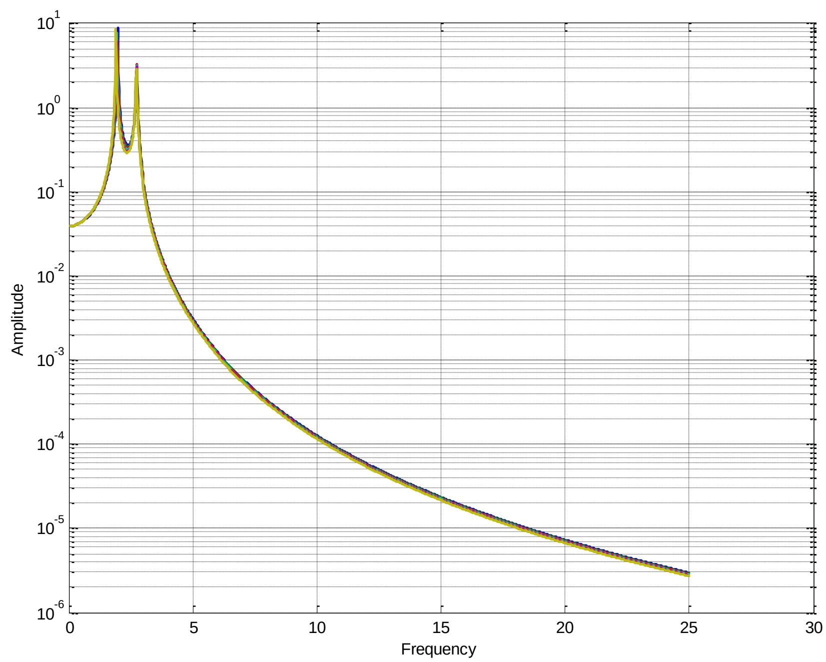

Figure 35: Absorber Mass Perturbation for Amp in Primary Pitch Mode

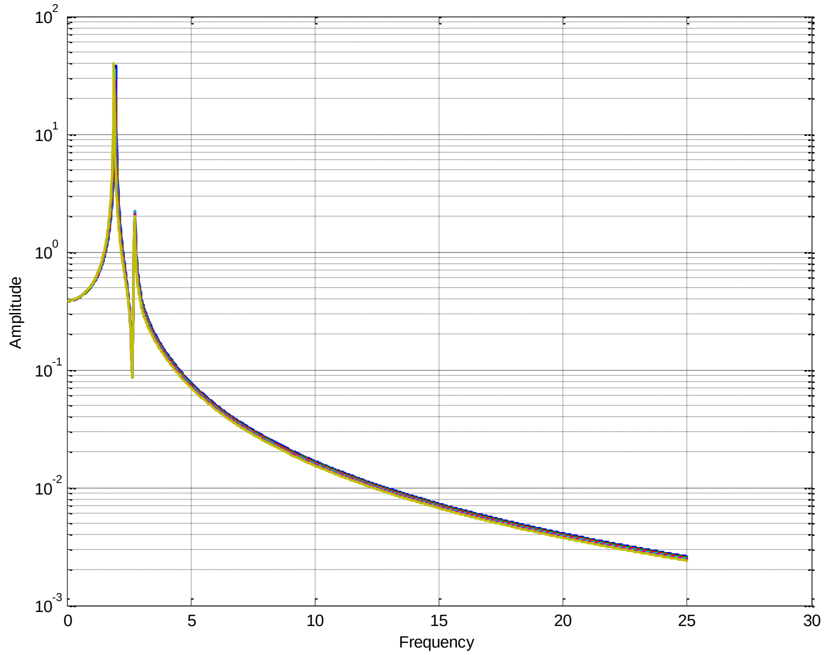

Figure 36: Absorber Mass Perturbation for Amp in Secondary Bounce Mode

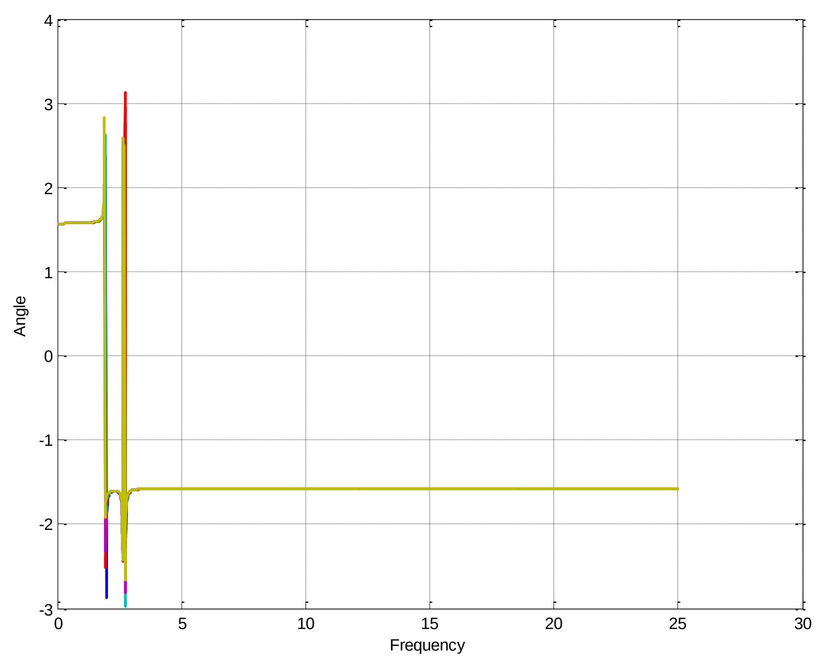

Figure 37: Absorber Mass Perturbation for Phase in Primary Bounce Mode

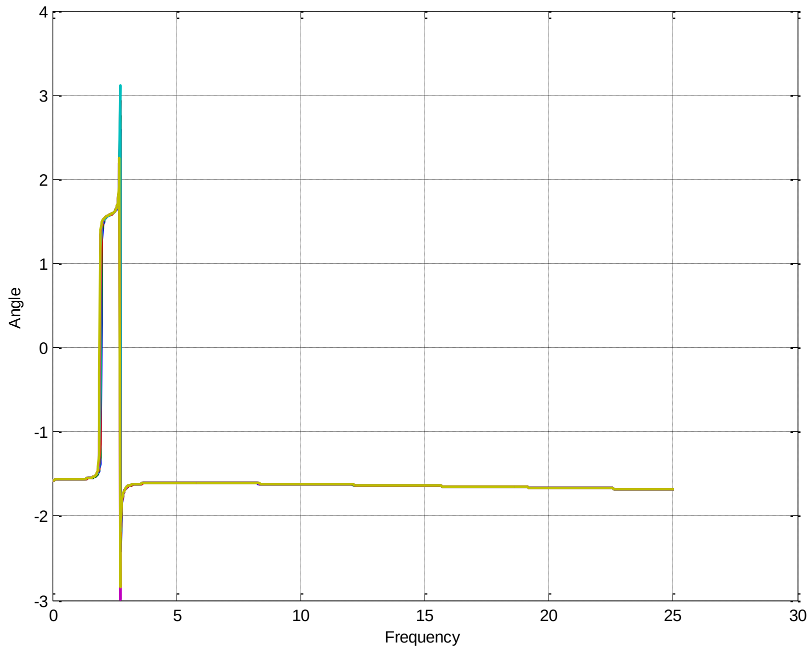

Figure 38: Absorber Mass Perturbation for Phase in Primary Pitch Mode

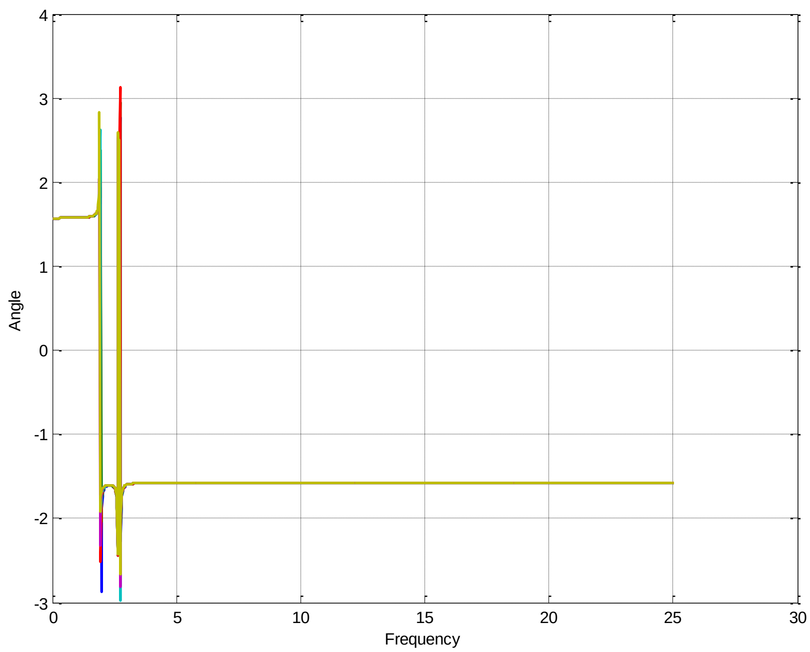

Figure 39: Absorber Mass Perturbation for Phase in Secondary Bounce Mode

So the secondary mass perturbations did not seem to affect the system response which is to be expected since, typically, a wide range of secondary mass (5% to 25%) values fall within normal values for the dynamic absorber.

The simplest method for attaching a dynamic absorber to the primary system is by fastening it directly to the frame as a cantilevered mass. Although crude, this method is quite common, especially for electronic components. The traditional method of suspending mass from a spring would obviously be inappropriate for a vehicle subject to pitch, roll, and yaw. A sliding mass would introduce damping through friction and the intent of this thesis is to minimize the damping value for the dynamic absorber.

CHAPTER 6: SENSITIVITY ANALYSIS

In order to more accurately assess the results from the parametric studies, sensitivity analysis is an appropriate method to check or verify the results. The methodology is reasonably intuitive. The sensitivity index θ_i, which results from a Taylor Series Expansion, relates how changes in values, x_i, affect R. The uncertainty of the variables to the result yield an estimate given by [Beasley and Figliola, 161-162]:

$$u_R = \pm\sqrt{\sum_{i=1}^{L}\left(\theta_i u_{x_i}\right)^2} = \pm\left[\sum_{i=1}^{L}(\delta R_i)^2\right]^{1/2} \quad (P\%)$$

Here is the sensitivity analysis for the front spring

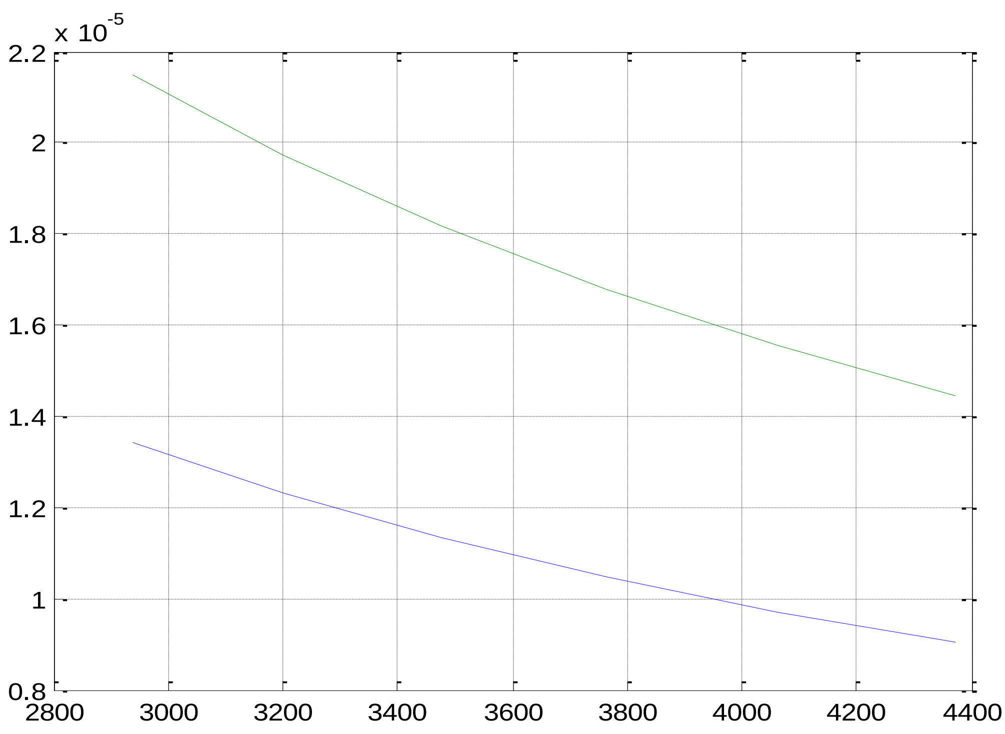

Figure 40: Front Spring Sensitivity

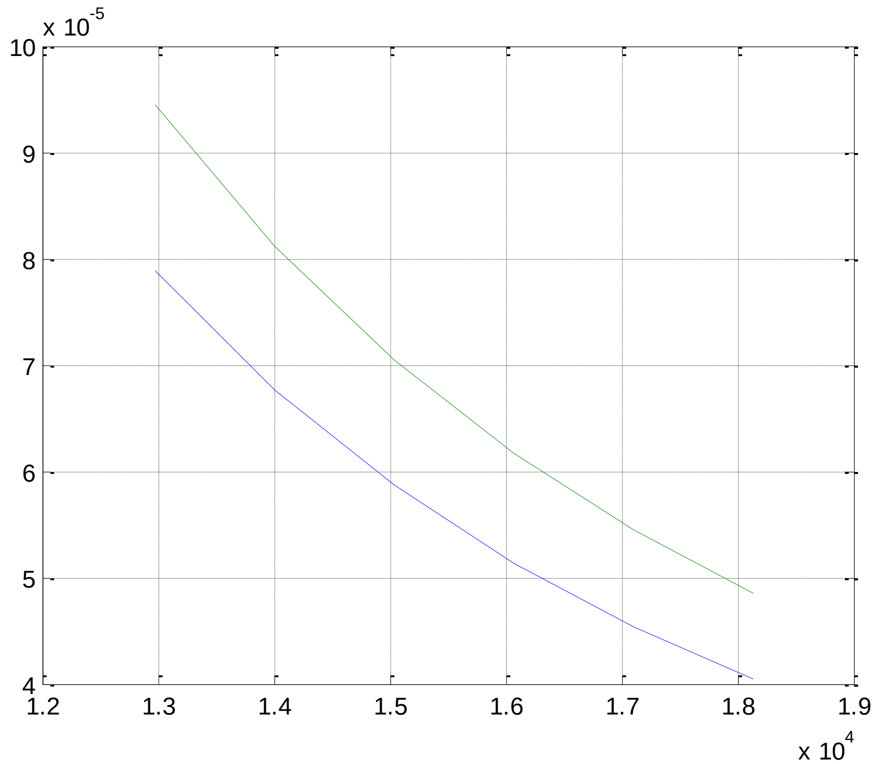

Figure 41: Mass Sensitivity

High sensitivity correlates with high values for the graph slope. As expected the results show there was a subtle effect of the mass on the system response. As indicated from the graphs, the system was most sensitive to changes in the values for the mass and inertia. This had much more of an effect on the pitch due to the lag in response from front to rear wheels.

The sensitivity graphs are not shown for the dynamic absorber since it is only indirectly related to parametric perturbations. The response of the absorber is more heavily dependent upon the excitation frequency. The values for the mass and stiffness of the absorber depend heavily on the excitation frequency, as stated previously.

CHAPTER 7: CONCLUSION

Suspension response is a key performance characteristic to the dynamics of a motorcycle. This is a highly obscure and specialized field of study. There has been extensive research performed over the years in various fields. This thesis focuses on a small niche within this field.

A closer look at the analysis shows that the perturbation of the mass realized the largest change in values. Not only was there a change in amplitude, but the natural frequency was modified as well. This is in contrast to the damping values which demonstrated very little change in system response. This would indicate that the values for the damping ratios are small.

The results from the system with the dynamic vibration absorber, which is designed to absorb the majority of the bounce from the primary system, were as expected. There was a subtle correlation with changes in parameters, but the system is highly sensitive to changes in the input frequency. So it is a useful tool for a narrow range of excitations.

However, since a vehicle is subject to varying frequencies a dynamic absorber, which is designed for a single frequency without too much deviation, may not be appropriate for a single-tracked vehicle unless it is to be designed for an environment where the excitation frequency is well known, such as a track. Perhaps a tuned mass damper would be more appropriate for this application. A tuned mass damper has been used in the past on high-performance vehicles and is certainly a better fit for the application.

REFERENCES

Beasly, Donald and Figliola, Richard. *Theory and Design for Mechanical Measurements, 3rd Ed.* New York: John Wiley & Sons, Inc., 2000.

Bishop, Robert and Dorf, Richard. *Modern Control Systems, 9th Ed.* New Jersey: Prentice Hall, 2001.

Cossalter, Vittore. *Motorcycle Dynamics, 2nd Ed.* Italy, 2006.

Greenwood, Donald. *Principles of Dynamics, 2nd Ed.* New Jersey: Prentice Hall, 1988.

Hale, Francis. *Introduction to Control System Analysis and Design, 2nd Ed.* New Jersey: Prentice Hall, 1988.

Hassul, Michael and Bahram, Shahian. *Control System Design Using MATLAB.* New Jersey: Prentice Hall, 1993.

Inman, Daniel. *Engineering Vibration, 3rd Ed*. New Jersey: Prentice Hall, 2008.

Kreyszig, Erwin. *Advanced Engineering Mathematics, 9th Ed*. New Jersey: John Wiley & Sons, Inc., 2006.

Rao, Singiresu. *Mechanical Vibration, 3rd Ed*. New York: Addison-Wesley Pub. Co., 1995.

"Types of Motorcycles." Wikipedia.com. Last modified November 13, 2013. http://en.wikipedia.org/wiki/Types_of_motorcycles

Waldron, Kenneth J. and Kinzel, Gary L. *Kinematics, Dynamics, and Design of Machinery*. New York: John Wiley and Sons, Inc., 1999.

Wong, J. Y. *Theory of Ground Vehicles, 4th Ed*. New Jersey: John Wiley & Sons, Inc., 2008.

Wright, Warren and Zill, Dennis. Advanced Engineering Mathematics, 5th Ed. Burlington: Jones & Bartlett Learning, 2014.

VITA

Darrick Jason Berner was born in Baltimore, Maryland, in May 1975 to Conrad Berner and Kim Chambers and was raised nearby. He is the oldest out of five siblings, three brothers and a sister. After spending 3 years in the military, he moved to Baton Rouge, Louisiana to attend college. Upon graduating with his Bachelor of Science in Mechanical Engineering degree in May, 2002, he began working at the Louisiana Department of Transportation & Development, where he has been for eleven years. During his employment with the state, he decided to pursue a Master of Science in Mechanical Engineering and hopes to graduate in December 2013. He has a lovely wife, Elizabeth, and two beautiful children, Johnathan and Katelyn.