Investigating conceptual teaching strategies in a computer based mathematics curriculum: the case of R2R

Haitham Sleiman Solh
Louisiana State University and Agricultural and Mechanical College, hsolh1@lsu.edu

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INVESTIGATING CONCEPTUAL TEACHING STRATEGIES IN A COMPUTER BASED MATHEMATICS CURRICULUM: THE CASE OF R2R

A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the Requirements for the degree of Doctor of Philosophy

in

The Department of Educational Theory, Policy, and Practice

by

Haitham Sleiman Solh
B.S., Lebanese University, 1998
M.Ed., University of Southern Mississippi, 2002
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ABSTRACT

This study investigated the various teaching strategies implemented by the teachers in a computer-based Mathematics course delivery model created by the LSU Math Department and applied in Louisiana High Schools. This model was referred to as R2R (Roadmap to Redesign). It was a course design that includes whole class instruction time (about 25% of class time) in addition to time spent with teacher assistance utilizing computer software (MathXL) to practice exercises, submit homework assignments, and take quizzes and tests. The study aimed to determine the proportion of teachers using the R2R model that opt to support a conceptual agenda in their whole class instruction, the proportion of teachers that had a procedural focus in their instruction, and the instructional methods used by each of these groups.

The study employed a mixed methods design, where both quantitative and qualitative data were collected. The quantitative data was collected through a survey instrument, and served to categorize teachers as potentially conceptual or potentially procedural, and determine the proportion of each category. The qualitative data were collected through observations and interviews, and served to portrait the characteristics of procedural and conceptual instruction.

The survey identified 7 out of 34 teachers as potentially oriented by conceptual goals in their R2R instruction, 8 teachers as procedurally oriented, and the remaining 19 teachers as intermediate in their orientation. Procedurally oriented teachers tended to demonstrate solution methods, focusing on the quickest and most efficient approach to solve a problem. Of three potentially conceptual teachers observed and interviewed in the study, none actually displayed instructional methods that aligned with their aspirations for student learning—all of them adopted procedural methods of instruction. However, one of
the teachers, observed a second time during a portion of the course following the completion of R2R goals, did conduct a conceptually oriented lesson.

The initial motivation for this study was to document the ways in which conceptually oriented teachers adapt the R2R format to meet their goals. The study results tentatively suggested that the time constraints imposed by the R2R curriculum render this instructional approach incompatible with conceptually oriented instruction.
CHAPTER 1

INTRODUCTION

In a technology-driven 21st century, new technologies are giving rise to major transformations in the mathematics education landscape. Opportunities are now available for students and teachers to engage in mathematical experiences that were inconceivable a few years ago. Generally, technology is a broad concept that deals with the usage and knowledge of tools and crafts. The Merriam-Webster dictionary offers a definition of the term “technology” as "the practical application of knowledge especially in a particular area" (Merriam-Webster Online Dictionary, 2009, ¶ 2). In this sense, technology refers to the tools and machines used to alleviate difficult tasks and solve real-world problems. It is worth noting that the tools and machines need not be material. Virtual technology, such as computer software, falls under this definition of technology.

Technology

Interest in the use of technology in teaching mathematics has grown steadily over the last twenty years. From powerful computation tools such as graphical calculators and desktop computers, to software programs permitting numerical or formal computation and geometrical manipulation, to internet-based activities that found their way to many mathematics classrooms, it has become clear that the traditional assumptions on how mathematics should be taught in the classroom are changing substantially. The National Council of Teachers of Mathematics (NCTM) argued for the use of technology in their most prominent publication, “Principles and Standards for School Mathematics” (2000). NCTM (2000) set a “technology principle,” stating that “Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and
enhances students' learning” (p. 24). NCTM (2000) emphasized the positive role of technology in the classroom by affirming that “students can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. They can work at higher levels of generalization or abstraction” (p. 24). NCTM (2000) standards emphasized the use of technology at most levels, as they recommended that students “develop fluency in operations with real numbers, vectors, and matrices, using technology for complicated cases” (p. 222). Students should also develop the ability to “judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology” (NCTM, 2000, p.296). Frederick Leung (2006) similarly reflects this emphasis in his claim that “the incorporation of information and communication technologies into mathematics classrooms is one of the most important themes in contemporary mathematics education” (p. 29).

The interest in the proper incorporation and use of technology in the classroom stems from the vast array of possibilities that technological tools seem to offer. Technology enables the exploitation of dynamic media like audio, video, and interactive software, allows students to collaborate in ways that were previously impossible, and provides tools to increase teacher productivity from lesson planning to record keeping. There are several pedagogical reasons given for the benefits of using technology in the classroom such as students’ engagement, interactivity, and students’ empowerment. Students using technology become active participants in the learning process instead of passive listeners. One-to-one technology enables the access, manipulation, and presentation of, and much more importantly, the creation of, multimedia rich descriptions and analyses. Technology can transform students from passive recipients of the teacher's knowledge to autonomous knowledge-constructors. Moreover, having students work in
partnerships or small groups with a computer based problem as a guide allows students to work collaboratively. From a teacher’s perspective, the advent of electronic technology can allow teachers and curriculum designers to focus more on mathematical ideas and devote less classroom time to the mastery of mechanical and computational skills. Calculators “reduce time spent on routine calculations and enrich students’ understanding of deep ideas” (Alper et al., 1996, p. 165). Technology also allows educators to capture the attention of students through interactive instructional activities.

Despite the huge potential of technology use in the classroom, NCTM (2000) warns that technology cannot replace the mathematics teacher, nor can it be used as a replacement for basic understandings and intuitions. The teacher must make prudent decisions about when and how to use technology, and should ensure that the technology is enhancing students’ mathematical thinking. (p. 24).

By highlighting the importance of the role of the teacher, NCTM is responding to a frequent charge leveled against technological innovations in mathematics education: that they exploit the capabilities of the technology rather than meet an instructional need. In other terms, the use of technological tools is often technology based rather than theory based. Simply put, in the rush to jump on the technology bandwagon, teachers often fail to gear the mathematical activity that uses technology to achieve targeted learning objectives. For instance, Fey (2006) charged that symbol manipulation tools (Derive, Matlab, Mathematica, etc.) used in math classrooms “are seldom used as aids in developing student understanding of symbolic forms and manipulations, or in solving problems that require such calculations” (p. 348). Earle (2002) summarized it best: “it is not about what technology by itself can do, but what teachers and learners may be able to accomplish using these tools” (p.10).
Procedural and Conceptual Instruction

If one were to concede that the potential for technology to enhance students’ understanding and engagement can be realized only if it is used appropriately, it becomes crucial to define what the appropriate use of technology is; several factors affect this determination, namely the quality of technological tool, its mathematical goals, and the implementation process.

NCTM (2000) affirmed that “Technology can help teachers connect the development of skills and procedures to the more general development of mathematical understanding” (p.26). This statement points to the intersection of the issue of technology use with another longstanding issue in mathematics education: the priority of conceptual and procedural emphases. Technology affects this discussion because technologies are designed to dramatically influence learning in both of these aspects. Hence, it is important to define these terms and determine how the interplay between them affects what is perceived as good instruction.

Hiebert and LeFevre (1986) defined conceptual knowledge to be knowledge “that is rich in relationships” and “can be thought of as a connected web” (p. 3); they claimed that the development of conceptual knowledge is achieved by the construction of relationships between pieces of information. In contrast, they defined procedural knowledge to be made up of two distinct parts: knowledge of the formal symbol representation system, that is to say knowledge of the symbols used to represent mathematical ideas and the acceptable form/order of writing such symbols, and knowledge of rules, algorithms, or procedures for completing mathematical tasks. (Hiebert and Lefevre, 1986, p. 6). Star (2005) challenged both definitions, claiming that in both cases, Hiebert and Lefevre (1986) departed from psychological perspectives on concepts and procedures by referring to only a particular subset of each type of knowledge. Only deep,
richly connected knowledge is considered conceptual for Hiebert and Lefevre (1986), and only knowledge that is superficial and poor in connections is labeled procedural. A detailed analysis of these terms is discussed in the next chapter.

The “respective roles of procedural and conceptual knowledge in students’ learning of mathematics continue to be a topic of animated conversation in the mathematics education community” (Star, 2005, p. 404). Some argue against de-emphasizing procedures over concepts (Klein 2003, Budd et. al. 2005), while others argue that procedural knowledge should play a secondary, supporting role to conceptual knowledge in students’ learning of mathematics (Pesek & Kirshner, 2000). Still, other researchers (Wu, 1999) labeled the conceptual-procedural dichotomy as “bogus,” since neither skills nor concepts can be taught or emphasized in complete isolation. From Hiebert and Lefevre’s (1986) perspective, the focal point of instruction needs to be conceptual knowledge, since “procedural knowledge that is informed by conceptual knowledge results in symbols that have meaning and procedures that can be remembered better” (p.16). This point of view is emphasized by Pesek and Kirshner (2000), who see that instruction “should involve students in reflecting, explaining, reasoning, connecting, and communicating” (p. 525). Thus, students should develop relational understanding, that is “understanding both what to do and why” (Skemp, 1987, p. 9). This point of view is not shared by Wu (1999), who boldly claimed that “deep understanding of mathematics ultimately lies within the skills” (p. 7). To add another dimension to the debate, the duality itself is now reconceptualized to attach a “quality” (superficial or deep) to the knowledge type (Baroody et. al., 2007). Star (2000) argued that the traditional usage of the terms procedural knowledge and conceptual knowledge obscures the myriad ways procedures and concepts can be known. He also added a depth dimension to Hiebert and LeFevre’s (1986) classification system to get levels of procedural knowledge.
In short, the conceptual and procedural distinction has received a great deal of discussion and debate in mathematics education through the years. Questions of how students learn mathematics—which precedes which, and what connections, if any, exist between the two types of knowledge—seem to be dominating the discussion on how to teach mathematics to serve the best interests of students. Simply put, the question is whether “developing skills with symbols leads to conceptual understanding, or whether the presence of basic understanding should precede symbolic representation and skill practice” (Sowder, 1998, ¶ 5).

Math Wars

The heated debate on what type of knowledge should be the focus of instruction escalated to affect national policy on mathematics education during a White House Reception for the National Association of Elementary School Principals and the National Association of Secondary School Principals in July of 1983. There, Ronald Reagan advocated the use of one of John Saxon’s Math books based on a skills-oriented incremental approach. In 1989, NCTM stimulated a movement with the publishing of “Curriculum and Evaluation Standards for School Mathematics,” which became the seed of the “reform” movement, with the aim of “making mathematics learning more substantial and engaging for students” (Kilpatrick, 2001, p.421). During the next few years, the teaching of mathematics became the subject of a heated controversy known as the “math wars.” Those who favored procedural knowledge (often labeled as traditionalists) feared that reform-oriented, “standards-based” curricula were superficial and undermined classical mathematical values, while “reformers” claimed that such curricula reflect a deeper, richer view of mathematics than the traditional curriculum. The types of curricula that emerged in the following years were very much affected by this
debate. Consequently, the type of knowledge that was the focal point in the classroom depended on whether the curriculum taught was “traditional” or “reform oriented.”

**Background and Rationale of the Study**

To tackle the problem, the researcher needed to find a curriculum or a curriculum instance that focused on one type of knowledge (procedural or conceptual), and that involved substantial use of technology. In 2006, the researcher was part of a team that conducted an evaluation study on “Teaching College Algebra using the R2R Format in 3 Baton Rouge Schools” (R2R stands for Roadmap to Redesign). The study, conducted for the Math Department at Louisiana State University, aimed at evaluating to what extent the R2R design was effective in facilitating students’ mastery of prescribed mathematical procedures, as well as to what extent was the R2R design helped students understand mathematics. It is important to briefly explain what the R2R design is before elaborating on the details of the study.

Louisiana State University (LSU) redesigned its College Algebra (MATH 1021) class to become a computer based course that requires students to attend only one lecture per week, and three hours of required lab time per week with teacher/tutor assistance available. The course was tailored for advanced high school math classes, and is currently offered and implemented in several schools in Louisiana, with many schools offering dual enrollment. The design includes whole class instruction time (about 25% of class time) in addition to time spent utilizing computer software (MathXL) with the teacher’s help to practice exercises, submit homework assignments, and take quizzes and tests.

At a first glance, R2R has the feel of a methodology that it is geared mainly towards skills building, since many of its features seem to be promoting procedural learning. For example, the quizzes and homework exercises are very similar to the examples given in class and online; homework can be submitted and re-submitted until
students “get it right,” and help tools (like “help me solve it” or “view an example” buttons) walk the students through step by step procedures. The various ways in which teachers tackle instruction in the “margins” of R2R (lecture time and one-on-one interaction with students), however, play an important role in determining their goals. The whole-class instruction time provides a wide range of options for the teachers. They may use it effectively to promote higher order thinking skills and teach for conceptual understanding. They can use creative strategies to challenge students’ thinking and engage them with problems, all while keeping up with the pace and demands of teaching the course, or they can use class time to support the procedural agenda already embedded in MathXL. It is not clear how many teachers, if any, actually teach to reach conceptual goals. Identifying those teachers, their strategies, and what common characteristics they share could prove to be a useful contribution to the growth and development of the R2R design. Identifying successful strategies that are deemed conceptually oriented can prove to be a substantial contribution to the debate of procedures vs. concepts.

Purpose and Design Overview

To achieve a better picture of the learning impact of R2R, the study investigated the strategies and techniques R2R teachers implemented in their whole-class instruction as well as in their one-on-one engagement with the students. The first phase of the study aimed at providing a panoramic view of the various strategies that teachers use in their classes. Illustrating these techniques served to accomplish two goals. It permitted:

a) Getting a rough count of the proportions of teachers that opt for conceptual and procedural agendas in response to software that is procedurally oriented; and,

b) Identifying the particular teachers who are adopting conceptual agendas as well as teachers who are procedurally oriented, for further, intensive study.
The second phase of the study focused on the latter group of teachers, and included the documentation and analysis of the varieties of methods these teachers used to support their teaching agendas, most importantly the strategies used to support conceptual understanding in the R2R environment. The research questions targeted were:

1. What proportion of teachers opt to support the procedural focus of R2R in their lecture periods, and what proportion use the lecture portion to insert a conceptual focus into the course?

2. What are the varieties of approaches that R2R teachers use to enhance the students’ conceptual understanding of the mathematical content?

3. If no teachers with a clear conceptual focus were found, then what characterizes the varieties of teaching techniques that the teachers employ in the margins of R2R?

To address the first question, a survey designed specifically for this study was set up for the teachers so they can elaborate on their teaching styles within R2R (describing a typical lecture, the perceived advantages they see in the approach, what problems they face, etc.). In addition to the survey, 7 classroom observations were conducted while taking extensive field notes that were analyzed later. The data obtained from the survey instrument served to answer the first question as well as isolate two groups of teachers for interviews and observations in the second phase. The interview and observation data served to answer the second question and alternatively the third one.

The model of the study was a sequential mixed method design adapted from Tashakkori and Teddlie’s (2003) “Handbook of Mixed Methods in Social and Behavioral Research.” It is a design in which the quantitative data is collected and analyzed first. The results of the data analysis are used to shape and collect the qualitative data, and inferences are drawn from both data sources.
The study started with a comprehensive literature review that included a detailed
discussion of the perceptions and the nuances about conceptual and procedural knowledge
as well as how each of these domains can be targeted by using technology. The next
chapter provided a thorough explanation of the methodology used to tackle the research
questions. In the following chapter, the results of the study were summarized, analyzed,
and interpreted. In the final chapter, the overall summary of the findings was discussed,
the limitations of the study were listed, and the possibilities for future research were
highlighted.
CHAPTER 2
LITERATURE REVIEW

Chapter Overview

This chapter discusses how each domain of knowledge (procedural and conceptual) has been articulated from the perspective of mathematics educators. A brief historical review introduces the origin and the essence of the debate over which type of knowledge should be the focus of instruction. From the days of McLellan and Dewey (1895), who argued for understanding, to Thorndike (1922), who presented the case for skill learning, to Brownell (1935), who advocated focusing on both realms, the chapter outlines the progress of the debate with Bruner (1960), Gagné (1977), Hiebert and LeFevre (1986), and Skemp (1987). The chapter then details the contemporary state of the debate through discussions between Hiebert (2004), Star (2005, 2007), and Baroody, et al. (2007), all of whom offered alternative reconceptualizations to the notion of procedural knowledge, and shifted the focus of the discussion to the relationship between skills and concepts.

The next portion of the chapter explores how the lack of consensus on this issue affects mathematics in the classroom. This section illustrates how the debate over the procedural-conceptual duality played an important part in the so-called “math wars” that have shaped the question of “how” and “what” mathematics should be taught in the classroom. The last portion of the chapter addresses how conceptuality can be enhanced using technology. An overview of the advantages of technology use is presented, and the rationale behind the use of technological tools in teaching for understanding is discussed from psychological as well as pedagogical points of view. The last paragraph lays out how these three issues are connected in the context of the study.
Historical Perspectives: The Early Years

According to Baroody (2003), researchers interested in the teaching and learning of mathematics “have long distinguished between knowledge memorized by rote and knowledge acquired meaningfully” (p. 4). Over the past century, many discussions in the mathematics education field stemmed from this distinction; even though the labels or terms used to describe each type of knowledge changed over time, there is little doubt that the debate was essentially about “the relationship between computational skill and conceptual understanding” and which should receive greater emphasis during instruction (Resnick & Ford, 1981, p. 246).

Hiebert and Lefevre (1986) date the debate back to the 19th century, when McLellan and Dewey (1895) presented a mathematics curriculum they felt would raise the level of understanding beyond that which existed in classrooms at the time (p.2). In 1895, McLellan and Dewey published “The psychology of number and its applications to methods of teaching arithmetic” in International Educational Series. In prefacing the chapter, the authors challenged the methods of teaching arithmetic at that time, warning of the danger in using what they labeled as a “fixed unit theory” to teach, claiming that “deadening the pupil’s mind and arresting its development” would occur if bad methods were used (p. v). They called for the joining of the “mechanical” side of training with the “intellectual” in such a form as to “prevent the fixing of the mind in thoughtless habits” (p. v). The book was followed in 1897 by an arithmetic book called “The Public School Arithmetic”, in which McLellan and Dewey (1897) claimed that the purpose of the book was “to help both teacher and pupil make the most of the unrivalled mental discipline that arithmetic affords” (p. vi). The book reflected Dewey’s hands-on approach to promoting learning through students' experiences.
McLellan and Dewey’s (1897) approach stemmed from Dewey’s belief that arithmetic can be most effectively taught within the context of use; arithmetic occurred in the course of building a club house or cooking or any other activity that the students would do in trying to satisfy their natural curiosity. This was a stark contrast to the dominant belief at that time that “mathematics instruction should focus on promoting the mastery of basic skills, not on cultivating understanding of mathematical concepts” (Baroody, 2002, p. 8). In the late 19th and early 20th century, the aim of instruction was routine, non-adaptive expertise. Proponents of what was known as “drill theory” (Smith 1921, Thorndike 1922) believed that learning could be adequately explained without referring to any unobservable internal states. In fact, they saw that the most efficient way to accomplish bond formation is through direct instruction and repetitive practice. This idea was emphasized through Thorndike’s (1922) “law of exercise”, which stated that connections become strengthened with practice and weakened when practice is discontinued. Basically, the tenets of “drill theory” were that students learn associations, or bonds, between unrelated stimuli, and the formation of such bonds does not require understanding.

Dissatisfied with the rationale of the drill theory, but also critical of Dewey’s approach (claiming it’s time consuming and that resulting knowledge is superficial), Brownell (1935) proposed a middle ground through his “meaningful learning theory.” In Brownell’s (1935) view, “instruction should focus on promoting the meaningful memorization of skills” (Baroody, 2002, p. 6). While Brownell (1935) recommended the judicious use of textbook-based and instruction and valued skill mastery, he recognized the value of building on students’ experiences in order to achieve conceptual understanding. Brownell (1935) argued that meaning is to be sought in the structure, organization, and the inner relationship of the subject itself. He viewed mathematics
(arithmetic in particular) as a closely knit system of ideas, principles, and processes, in which the true test of learning was not “mere mechanical facility in ‘figuring’ but an intelligent grasp upon number relations and the ability to deal with arithmetic situations with proper comprehension of their mathematical as well as their practical significance” (Brownell, 1935, p. 19). By not favoring procedural competence over the ability to understand content (and vice versa), Brownell (1935) may well have been the first to highlight the importance of both realms of knowledge, and to open the door to redefining mathematical competence as “knowing both what to do and why” (Skemp, 1978, p. 9).

In the following years, the debate over what form of knowledge should take central stage in instruction continued along the same trajectory. Bruner (1960) made a case for understanding with the constructivist theory, proposing that learning is an active process in which learners construct new ideas or concepts based upon their current or past knowledge. Bruner’s (1960) view of the learner is one that selects and transforms information, constructs hypotheses, and makes decisions, relying on a cognitive structure to do so. On the other hand, Gagné (1977) advocated skill learning with the publication of “Conditions of Learning.” In it, Gagné (1977) identified nine types, or levels, of learning, each requiring a different type of instruction based on a hierarchy from the simplest to the most complex task. The following decade marked a new perspective in how skills and concepts are perceived as domains of knowledge.

**Contemporary Views**

Procedures and concepts were viewed as separate entities that compete in the classroom setting until the early 1980s, when researchers like Resnick & Ford (1981) called for research efforts to focus on “how understanding influences learning of computational routine, and how skilled computation, in turn, affects conceptual learning” (Baroody, 2002, p. 8). The attention shifted towards the relationships between “conceptual
knowledge” and “procedural knowledge.” The publishing of “Conceptual and Procedural Knowledge: The Case of Mathematics” (Hiebert, 1986) was a landmark in redefining the two realms of knowledge and began to shed some light on the potential relationships between the two. These links between concepts and processes were re-examined in several publications over the following years, namely by Hiebert & Handa (2004), Star (2005, 2007), and Baroody, Feil, & Johnson (2007).

Definitions and Characteristics

“Conceptual and Procedural Knowledge: The Case of Mathematics” was one of the earliest and most recognized attempts of looking at the relationships between conceptual and procedural knowledge. Although treated as distinct, the two types of knowledge connected in mutually beneficial ways. In the chapter, “Conceptual Knowledge as a Foundation for Procedural Knowledge,” Carpenter (1986) characterized conceptual knowledge as knowledge that “involves a rich network of relationships between pieces of information, which permits flexibility in accessing and using information” (p. 113). Hiebert & LeFevre (1986) similarly defined conceptual knowledge as "knowledge that is rich in relationships” (p. 3), conceiving of it as a web in which individual facts and propositions are linked. Hiebert & Lefevre (1986) argued that “a unit of conceptual knowledge cannot be an isolated piece of information;” instead, it is by definition a part of conceptual knowledge only if the holder recognizes its relationship to other pieces of information” (p. 4). More up-to-date definitions of conceptual knowledge bear a significant resemblance to Hiebert & Lefevre’s (1986). McCormick (1997), for example, asserts that conceptual knowledge “is concerned with relationships among items of knowledge” (p. 141), while Rittle-Johnson & Alibali (1999) define it as “explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain” (p. 175).
As for procedural knowledge, Hiebert and Lefevre (1986) defined it as knowledge of the symbol representation system (the formal language of mathematics), and of the rules, algorithms, or procedures for completing mathematical tasks. Knowledge of the symbol representation system encompasses familiarity with the symbols as well as awareness of syntactic rules (that $a+b = c$ is a syntactically valid statement, while $a+ = bc$ is not). In the advanced levels, this knowledge includes awareness of syntactic configuration of formal proofs without necessarily knowing the content of the proof, just the validity of its structure. Knowledge of algorithms means the knowledge of the rules or procedures used to solve mathematical tasks. Algorithms are perceived as step-by-step instructions that are executed in a predetermined sequence, and prescribe how to complete tasks, moving in a sequential order from the given to the answer. Rittle-Johnson & Alibali (1999) produced a concise definition when they defined procedural knowledge as “action sequences for solving problems” (p. 175). For Haapasolu (2003), procedural knowledge “denotes dynamic and successful utilization of particular rules, algorithms or procedures within relevant representation forms” (p. 4). Haapasolu added that procedural knowledge requires not only knowledge of the objects being utilized, but also knowledge of format and syntax for the representational system(s) expressing them.

Hiebert & Lefevre (1986) explained that the development of conceptual knowledge requires the construction of relationships between pieces of information; this can occur by tying together existing pieces of information, or creating an appropriate connection between existing knowledge and new information. Moreover, they distinguished between a “primary” connection, where the pieces of information connected were at the same level of abstractness, and reflective, or abstract, connections in which digging deeper was required to find a link between what appears to be superficially different pieces of information. For procedural knowledge, they identified two types of procedures based on
the objects upon which the procedures operate. Procedures can operate on standard written symbols, whereby the task is to “transform the symbol expression from the given form to an answer form by executing a sequence of symbol manipulation rules” (Hiebert & Lefevre, 1986, p. 6). These procedures include problem solving strategies and geometric constructions.

From Hiebert & Lefevre’s (1986) perspective, the biggest difference between procedural knowledge and conceptual knowledge is that in procedural knowledge, only the relationship “after,” which is used to sequence sub-procedures, is the primary relationship (in a procedure, the sub-procedures are all linearly sequenced). In contrast, “conceptual knowledge is saturated with relationships of many kinds” (p. 8). What makes their characterization of conceptual and procedural knowledge different from earlier debates is that the authors looked at the potential relationships between the two aspects of knowledge rather than which one is more important. They argued that “mathematical knowledge, in its fullest sense, includes significant, fundamental relationships between conceptual and procedural knowledge” (Hiebert & Lefevre, 1986, p. 9). They maintained that students are not fully competent in mathematics if either kind of knowledge is deficient, or if there were no connections between the two domains. They argued that when such a connection is missing, students may be able to understand what concept is targeted by a problem but not know how to tackle it. Similarly, they may perform a series of procedures that lead to the correct answer but not understand the rationale behind the use of the procedure and the meaning behind the answer they found. Critical connections between concepts and procedures “would contribute in many other ways to the development of a sound knowledge base” (p. 9). In the following figure, Baroody (2003) summarized Hiebert & Lefevre’s (1986) characterizations of procedural and conceptual knowledge (p. 12).
Figure 2.1: Hiebert & Lefevre's Characterizations of Procedural and Conceptual Knowledge, adapted from Baroody (2003)
Reconceptualizing the Relationship between Procedural and Conceptual Knowledge

As stated earlier, what makes Hiebert & Lefevre’s (1986) characterization of conceptual and procedural knowledge different from earlier debates is that the authors looked at the potential relationships between the two aspects of knowledge rather than at which one is more important. The authors argued that students are not fully competent in mathematics if either kind of knowledge is deficient, or if there are no connections between the two domains. They maintained that conceptual connections can give meaning to symbols, and that competence can be achieved when meaningful symbol manipulation occurs rather than symbols being only visual patterns. According to Hiebert & Lefevre, this occurs when there is reference for symbols; usually in concrete or real world experience. Moreover, understanding how and why procedures work provides possibilities for procedures to become part of a network of information that is less likely to deteriorate, since relationships that are meaningful and highly organized can be easier to remember.

Currently, the mathematics education research community is moving further away from the procedural/conceptual dichotomy to reconceptualize the meaning and role of each type of knowledge. A lot of theory and research focused on the benefits of conceptual knowledge for procedural knowledge (Kilpatrick, Swafford & Findell, 2001), and, though less frequently, procedural knowledge for mathematical competence (Star, 2005). Several researchers have stressed the importance of exploring the relationships between concepts and procedures: Rittle-Johnson & Alibali (1999) stated that “these two types of knowledge lie on a continuum and cannot be separated” (p. 175), Baroody (2003) argued that “linking conceptual and procedural knowledge can greatly benefit the acquisition of the former, as well as the latter” (p. 13), and Hiebert & Handa (2004) maintained that current researchers are probing “how these kinds of knowledge… support, or interfere with each other as
students learn mathematics” (p. 1). Examining these relationships has led several researchers to question the long standing definitions of each realm of knowledge.

Redefining Types of Knowledge

Star (2005) argued that the definitions of conceptual and procedural knowledge (as proposed by Hiebert & Lefevre in 1986) represent “a critical departure from psychological views of concepts and procedures” (p. 407), since these definitions only refer to the “quality” of the knowledge type. For instance, the term concept in itself implies connected knowledge, but Star (2005) points out that the “connections inherent in a concept may be only limited and superficial, or they may be extensive and deep” (p. 407). This implies that richly connected knowledge is only a particular subset of conceptual knowledge. Similarly, Star (2005) challenges Hiebert & Lefevre’s (1986) characterization of procedural knowledge as superficial and not rich in connections, arguing that there are “many different kinds of procedures, and the quality of connections within a procedure varies” (p. 407). While some procedures are executed in a pre-determined order, and may not be rich in connections (algorithms), other procedures are heuristics and, in Star’s (2005) view, “the execution of heuristics requires that one make choices; wise choices that indicate quite sophisticated and deep knowledge” (p. 407). Star (2005) concludes that Hiebert & Lefevre’s (1986) definitions of conceptual and procedural knowledge “suffer from an entanglement of knowledge type and knowledge quality” (p. 408). To disentangle those two characteristics, he proposes that both knowledge types may be either superficial or deep. By this characterization, the current usage of the terms conceptual knowledge and procedural knowledge would refer to only “deep conceptual” and “superficial procedural” knowledge, which leaves the door open for mathematics educators to explore new “types” of knowledge, like “superficial conceptual” and, more importantly, “deep procedural.” Deep procedural knowledge would be “knowledge of procedures that is associated with
comprehension, flexibility, and critical judgment and that is distinct from (but possibly related to) knowledge of concepts” (Star, 2005, p. 408). Star (2005) also considers “procedural flexibility” (p. 408) as an indicator of deep procedural knowledge. Flexibility describes the facility to choose appropriate procedures so that a maximally efficient solution can be generated. Separating these independent characteristics of knowledge (type versus quality) allows for the reconceptualization of procedural knowledge as potentially deep, and suggests the need for research on what it is and how it develops. In short, Star sees an intrinsic value in studying procedural knowledge as independent domain that goes beyond simple applications of algorithms.

Baroody, Feil, & Johnson (2007) agreed that identification of knowledge type should take place independently of the degree of connectedness, but proposed an alternative reconceptualization of procedural and conceptual knowledge. While they agreed that each type of knowledge may have a deep or superficial characterization, they disagreed with Star when it comes to characterizing procedural knowledge. The major difference between their perspective and Star’s (2005) views was the affirmation that deep procedural knowledge cannot exist without deep conceptual knowledge or vice versa. Baroody et.al. (2007) claim that superficial procedural knowledge may exist independently, however, deep procedural knowledge is not achieved in isolation of the presence of conceptual understanding. To them, the degree of connectedness, or mutual dependence, between conceptual and procedural knowledge ranges from no connection to rich connections depending on the level of structure between the two. The authors see that the depth of understanding entails “both the degree to which procedural and conceptual knowledge are interconnected, and the extent to which that knowledge is otherwise complete” (Baroody et. al., 2007, p. 123). Figure 2.2 illustrates Baroody, Feil, & Johnson’s reconceptualization.
Figure 2.2: The Mutually Dependent Relation between Procedural and Conceptual Knowledge as suggested by Baroody, Feil, & Johnson (2007, p. 124)
Star (2007) disagreed with the characterization of deep procedural knowledge in terms of the connection to conceptual knowledge. Baroody et al. (2007) claimed that a student can have deep knowledge of a procedure only when he/she knows “the conceptual basis for each of its steps” (p. 119). Star (2007) opposed the notion that procedures are understood only when conceptual knowledge become involved. While acknowledging that it is important to explore the connections to conceptual knowledge, Star (2007) maintained that “procedures can be known deeply, flexibly, and with critical judgment…and not necessarily as a result of connections to conceptual knowledge” (p. 133). He refused to associate all things shallow with procedures; “once understanding is involved, a link is immediately extended to concepts” (personal communication, March 2009), and stressed that “procedural knowledge is valuable in and of itself, not solely because of its connections with and integration to conceptual knowledge” (2007, p. 134).

The National Research Council’s synthesis of the literature on mathematics’ learning (Kilpatrick et al., 2001) concluded that “without sufficient procedural fluency, students have trouble deepening their understanding of mathematical ideas or solving mathematical problems. The attention they devote to working out results they should recall or compute easily prevents them from seeing important relationships” (p. 122). At the same time, the report cautions against students practicing procedures they do not understand. It is quite clear that while the mathematics education research community did not reach a consensus on re-defining procedural competence and conceptual knowledge, a continued purposeful dialogue about the depth of procedures and concepts and the relationships between each domain is useful, especially since the field of mathematics education has been marred by the inability to reach a middle ground between proponents of conceptual knowledge and advocates of procedural competence. This led to a highly
politicized debate known as the “math wars,” which affected national policy on what and how mathematics should be taught.

Math Wars

Background

The debate over “what” and “how” mathematics should be taught dates back to Dewey’s (1895) progressive approach and continues to receive attention throughout the twentieth century. Given that the purpose of this section is to review how the debate over the priority of procedural vs. conceptual knowledge became an issue of political importance, it is not essential to extensively discuss its development in the early part of the twentieth century. It is important, however, to note that two major waves of reform marked this era: the “new math” movement of the 1950s through the early 1970s and the standards-based movement of the past two decades. Kilpatrick (2001) argues that “although differing sharply in their approach to curriculum content, these reform efforts have shared the aim of making mathematics learning more substantial and engaging for students” (p. 421). The first reform attempt was triggered by the 1957 launch, and successful orbiting, of the Russian satellite Sputnik, which caused a panic in educational establishments. Americans felt they had fallen behind the Soviet Union, so the “Sputnik Crisis” triggered the introduction of the “New Math” approach that emphasized mathematical structure through abstract concepts like set theory and number bases. The difficulty of the “New Math” for kids, opposition from parents and teachers, and the failure to achieve substantial research and empirical outcomes all contributed to the fast decline of “New Math.” The sharp drop in standardized test scores induced a call from the National Council of Teachers of Mathematics for new directions in mathematics education. The report was entitled “An Agenda for Action” and it was released in 1980. In 1989, NCTM published its “Curriculum and Evaluation Standards for School
Mathematics,” a document commonly known as the NCTM Standards. Several researchers (Klein, 2003; Schoenfeld, 2004) attribute the publication of the Standards to the beginnings of the so-called math wars.

The Conflict

The NCTM Standards reinforced the general themes of progressive education by advocating student-centered, discovery learning. They were oriented toward five general goals for all students: “that they learn to value mathematics, that they become confident in their ability to do mathematics, that they become mathematical problem solvers, that they learn to communicate mathematically, and that they learn to reason mathematically” (NCTM, 1989, p. 5). Schoenfeld (2004) maintained that the Standards were “grounded in assumptions about learning being an active process rather than one of memorization and practice” (p. 266), as they, the Standards, stated that a constructive, active view of the learning process must be reflected in the way much of mathematics is taught. Thus, instruction should vary and include opportunities for appropriate project work; group and individual assignments; discussion between teacher and students and among students; practice on mathematical methods; exposition by the teacher. (NCTM, 1989, p. 10)

The Standards emphasized the role that students play in constructing their own learning, and maintained that this role can be strengthened and exploited to increase the number of students who are successful in mathematics classrooms.

In 1992, the California Department of Education published the “Mathematics Framework for California Public Schools” (also known as the Frameworks). Schoenfeld (2004) explained that “the Frameworks represented the next incremental step in the change agenda, grounded in the positive national reaction to the NCTM Standards and the growing research base on mathematical thinking and problem solving” (p. 271). Publishers, seeing the positive national response to the Standards, and seeing California as being on the leading edge of a national trend, created texts in line with their view of
Standards (and Frameworks)-based mathematics. In 1994, the California State Board of Education approved instructional materials consistent with the Frameworks. However, the new textbooks were radically different from the traditional orderly, sequential presentation of formulas and pages of practice problems familiar to parents. New texts featured “colorful illustrations, assignments with lively, fun names and sidebars discussing topics from the environment to Yoruba mathematics” (Rosen, 2000, p. 61).

It was not long until those textbooks were highly criticized and vehemently opposed. Often labeled as “Fuzzy math” and “New New math” (in reference to the failed New Math reform of the 1960s), these books induced a lot of concern regarding their perceived mathematical shortcomings. Klein (2003) argued that the “new mathematics books and curricula typically failed to develop fundamental arithmetic and algebra skills” (p. 200). Schoenfeld (2004) stated that “new materials and new practices raised concerns among some parents, some of whom enlisted outside help (from mathematicians, legislators, etc.) in combating the new practices and materials” (p. 273). This led to the birth of organizations that opposed the Standards movement, such as the “Mathematically Correct” group and “Save Our Children from Mediocre Math.” The work of those organizations and others “facilitated the evolution of the antireform collectives into a potent political force” (Schoenfeld, 2004, p. 273). In 1996, the California State Board of Education agreed to convene a new mathematics Frameworks writing team, and a new Frameworks document was approved in 1999. Ralston (2004) maintained that the 1999 frameworks focused on content, and “espoused a traditionalist perspective that would have been controversial in any case but was made more so because of the inability of progressives to comment on the final content standards document before it was adopted” (p.404). It was seen by many reformers as “a return to the basics, to the algorithmic skill-
and-drill classrooms of the past” (Wilson, 2003, p. 179). The traditionalist base in California became a foundation for a national antireform movement.

The preliminary report of the National Mathematics Advisory Panel (2007) indicated that

one aspect of the debate is over how explicitly children must be taught skills based on formulas or algorithms (fixed, step-by-step procedures for solving math problems) versus a more inquiry-based approach in which students are exposed to real-world problems that help them develop fluency in number sense, reasoning, and problem-solving skills. (p. 2)

In the reform approach, computational skills and correct answers are not the primary goals of instruction. Those who disagree with this philosophy (Klein, 2003, the Mathematically Correct group) maintain that students must first develop computational skills before they can understand concepts of mathematics. These skills should be practiced until they become automatic, and learning abstract concepts of mathematics depends on a solid base of knowledge of the tools of the subject. Klein (2003) argued that “the utilitarian justification of mathematics was so strong that both basic skills and general mathematical principles were to be learned almost invariably through ‘real world’ problems. Mathematics for its own sake was not encouraged” (p. 192). The NMAP report concludes that “teaching in very few classrooms would be characterized by the extremes of these philosophies. In reality, there is a mixing of approaches to instruction in the classroom, perhaps with one predominating” (p. 3).

The issue of “math wars” overlaps significantly with the characterization of procedural and conceptual knowledge as well as other salient issues. The focus on which realm (skills or concepts) should take precedence in instruction led to several years of heated debates that reflected negatively on the mathematics education field, from teachers and students, to policy makers. The proliferation of technology in the last 20 years or so added an additional realm to this debate. As Rubin (1999) explained, “the role of
technology in math education must be in service of goals we hold for student’s mathematical knowledge and expertise” (p. 1). Given the lack of consensus on what those goals are, it is important to review how technology use has affected teaching pedagogies and instructional goals.

Technology

Background

Technology casts a new light on education, particularly in the 21st century. A couple of decades ago, teachers had at their disposal a chalkboard, paper and pencils, compasses, rulers, protractors, and maybe some primitive calculator. Today advances in information and communication technology (ICT) “represent a fundamental paradigm shift in mathematics education” (Leung, 2006, p. 34). Graphic calculators, computers, software programs, and internet access are no longer rare commodities, but are, perhaps, available in most mathematics classes today. In fact, Mitani (2007) reported that in 1998, instructional computers in public schools were far less numerous than today. On average, the ratio of students to computers was 6.3 to 1 and there was an observable gap in students’ access to them by demographic characteristics of the school population. Eight years later, in 2006, 3.8 students share a single instructional computer, and students in all types of schools have gained much better access to instructional computers. Hoyles and Noss (2006) stated that

the sheer ubiquity of personal computers has brought about a cultural shift in how people think with and about computers. The possibilities are evident, and above all, the kinds of technological potential that are now emerging contain the seeds of radical change. (p. 2)
Today, the presence of a huge number of technological tools opens up the potential for change in the didactical field, and seems to challenge traditional assumptions on how mathematics should be taught in the classroom, making the incorporation of information and communication technologies into mathematics classrooms “one of the most important themes in contemporary mathematics education” (Leung, 2006, p.29). The importance of technology use falls in line with the earlier discussion on the realms of knowledge, as technological tools may be used to support conceptual as well as procedural agendas.

Definition and Rationale

The acronym most commonly used for technology or information technology is ICT. ICT stands for “Information and Communication Technology,” and refers in principle to all technologies used for processing information and communicating. According to Anderson (2008), “in most educational circles, it means computer technology, multimedia, and networking, especially the Internet” (p. 4). The scope of ICT is dynamic and continuously changes with the creation of new technologies; as Anderson (2008) points out, technology once referred to only hardware, however now it includes software techniques as well.

Dede (2008) argued that ICT “aid with representing content, engaging learners, modeling skills, and assessing students’ progress in a manner parallel to how a carpenter would use a saw, hammer, screwdriver, and wrench to help construct an artifact” (p. 43). The key point in his analogy is that tools make the job easier, and the result is of higher quality than otherwise possible without the tools. Keengwe and Onchwari (2008) stated that the potential for technology to positively impact students’ learning is realized by “providing a more active learning, more varied sensory, and conceptual modes; less mental drudgery; learning better tailored to individuals, and better aid to abstraction” (p.
Other pedagogical reasons given for the benefits of using technology in the classroom include students’ engagement, interactivity, and empowerment.

Technology has been introduced into classrooms because many researchers believe it has a great potential to improve the educational experience. Hinostroza et al. (2008) state that technology “can be considered to facilitate student learning, may change the curriculum and may improve teaching and learning… and may have a variety of impacts on learning, which includes achievement, competencies and student behavior” (p. 93). Moreover, the use of technology in the classroom can promote a student-centered instruction and may result in a shift from traditional “transmission” instruction to a more learner-centric model. As Hinostroza et al. (2008) point out, the rationale behind the use of technology in mathematics classes “is not the availability and affordability of sophisticated IT, but the ways this technology enables powerful learning situations that aid students in extracting meaning out of complexity” (p. 83). This does not, however, mean that technological tools are all geared towards a meaning-oriented pedagogy, as a variety of theoretical frameworks inform the use of technology in the math classroom.

Theoretical Perspectives

The earlier discussion on math wars established that the conflict between reform and traditional mathematicians revolved around the prioritization of concepts over skills and vice versa. This duality can be attributed to the theoretical assumptions that each side draws from, and it is the same duality that informs the choice and use of technological tools in mathematics classrooms.

Several conceptual frameworks can be used to describe the relationships between learning theories, pedagogical strategies, and information technology. Drawing on the earlier discussion on the procedural/conceptual duality, a clear influence of the behaviorist school of thought on the technological tools advocated by traditionalists can be detected.
Similarly, a strong sense of constructivism grounds a major portion of the reform research in the field of technology in math education. Dabbagh (2006) explained that the behaviorist theories of learning assume that knowledge is an absolute, reflecting universal truths about reality. Human behaviors, such as learning, are purposive, but are guided by unknowable inner states. Relationships between contextual instructional variables (stimuli) and observable, measurable student behaviors (responses) are the means to generate learning. Drawing on Dabbagh’s (2006) characterization, Dede (2008) indicated that, for behaviorists, “the purpose of education is for students to acquire skills of discrimination (recalling facts), generalization (defining and illustrating concepts), association (applying explanations), and chaining (automatically performing a specified procedure)” (p. 46). In short, the learner must know how to execute the proper response as well as the conditions under which the response is made. Knowledge and skills are transferred as learned behaviors, and internal mental processing is not considered part of instructional design or assessment. Student motivation to achieve these goals is extrinsic, “by associating pleasant stimuli with correct answers and neutral or even negative stimuli with incorrect responses” (p. 46). Behaviorism is generally attributed to the influence of Watson (1913), Thorndike (1913), and B.F. Skinner (1950).

**Constructivism**

Constructivist theories of learning assume that meaning is imposed by the individual rather than existing in the world independently. According to Dede (2008), “people construct new knowledge and understandings based on what they already know and believe, which is shaped by their developmental level, their prior experiences, and their socio-cultural background and context” (p. 50). In a constructivist environment, learning involves mastering authentic tasks in meaningful, realistic situations. Learners build personal interpretations of reality based on experiences and interactions with others,
creating novel and situation-specific understandings. Constructivist-based instruction can foster learning by “providing rich, loosely structured experiences and guidance that encourage meaning-making without imposing a fixed set of knowledge and skills” (Dede, 2008, p. 51). The theory of constructivism is generally attributed to the influence of Jean Piaget, who articulated the mechanisms by which knowledge is internalized by learners. Piaget’s constructivism was the theoretical assumption that during cognitive development, children acquire their knowledge through a process of creative invention. Under this assumption, knowledge is “constructed,” its acquisition is neither a process of discovering innate ideas nor a process of storing facts that are encoded in the environment. Piaget’s alternative for these two views is “the mechanism of interaction between heredity and experience that produce knowledge” (Inhelder & Sinclair, 1969, p. 42). For Piaget, children create knowledge as their biological predispositions interact with their experience. Each learner interprets experiences and information in the light of their extant knowledge, their stage of cognitive development, their cultural background, their personal history, and so forth. The key ingredient in this process, according to Piaget, is the “active self-discovery of discrepancies between current concepts and actual outcomes” (Brainerd, 2003, p.271). Initially, the child possesses some immature form of a concept that leads him/her to believe certain outcomes will occur, but when actual outcomes differ from those he predicted, and he recognizes that discrepancy on his own. The child then invents a new concept that is able to encompass actual outcomes. Cognition becomes an adaptive activity: “To be viable, a new thought should fit into the existing scheme of conceptual structures in a way that does not cause contradictions. If there are contradictions, either the new thought or the old structures are deemed to require changing” (Von Glasersfeld, 1995, p. 4).
Each of these theoretical perspectives was used as a rationale to advocate particular technology uses. For instance, Computer-Assisted Instruction (CAI) was a type of instructional technology closely associated with the behaviorist school of thought. Dede (2008) listed Atkinson (1968) and Suppes (1968) as the pioneers of computer-based instruction, as exemplified by the development of the PLATO and TICCIT CAI systems used in some schools in the 1970s. Larkin and Chabay (1992) argued that Computer Assisted Instruction “pushes the frontier of the best that can be done with current technology,” but it remains obvious that the early CAI programs could teach simple skills such as alternative algorithms for division (p. 2). According to Anderson (2008), CAI could effectively teach factual knowledge with the following cognitive attributes: “one right answer, basic mental processes primarily involving assimilation into memory” (p. 18). Dede (2008) explains that CAI as a pedagogical application is limited both in what it can teach and in the types of engagement it offers to learners (p. 47).

Similarly, constructivist pedagogical technologies span a wide range. For instance, the NCTM position statement (2008) on the role of technology in the teaching and learning of mathematics maintained that “calculators and other technological tools, such as computer algebra systems, interactive geometry software, applets, spreadsheets, and interactive presentation devices, are vital components of a high-quality mathematics education” (p. 1). An example that illustrates the role of technology in a constructivist environment is the Jasper Woodbury Problem Solving Series that the National Research Council advocated. It consists of 12 video-disc based adventures that focus on mathematical problem finding and problem solving. In this curriculum, Middle school students in math class view 15 min video adventures that embed mathematical reasoning problems in complex, engaging real-world situations.
Promises and Challenges

Kaput (1992) argued that the use of technology in support of instruction in the math classroom requires, in addition to general knowledge of the students’ forms of learning and interaction with the subject matter, “particular knowledge of the individual learner at each state in the interaction with the machine” (p.545). Technological tools allow for students to “make processes of thinking available as explicit objects for reflection” (Shaffer and Kaput, 2006, p.109). Students’ use of technology can make the thinking and problem-solving process more transparent to the teacher. For instance, digital files can provide documentation of the processes the student used to solve problems. It is often easier to store and retrieve these files electronically than on paper. This enables teachers to follow the students’ construction schema and gives a better understanding of the “students’ mathematics.” If students’ modes of thinking are clearer to the teacher, then he/she can challenge their current understanding by creating discrepancies to induce cognitive adaptation. Keeping in mind that, in a constructivist environment, “it is not productive for teachers and other adults to spell out discrepancies for children or to correct children’s ideas deliberately,” the role of the teacher becomes simply to facilitate discovery by providing the necessary resources and by guiding learners as they attempt to assimilate new knowledge to old and to modify the old to accommodate the new (Brainerd, 2003, p.271). A wide variety of technological tools is available (graphic calculators, spreadsheets, software, etc.) to help educators and a sense that these tools serve the constructivist view of teaching and learning is evident because “computers can undermine the didactic, lecture methodology, and, instead promote the student as a self-directed learner” (Matusevich, 1995, p. 3). As Collins (1991) put it, “using computers entails active learning, and this change in practice will eventually foster a shift in society's beliefs toward a more constructivist view of education” (p. 33). Kieran (2007) also
argued that “one of the research-tested advantages of the introduction of a variety of representations in technology-supported environments is that such approaches are enabling traditionally unsuccessful students to gain access to the problem-solving aspects of algebra” (p. 33).

Anderson (2008) stated that “ICT and the rapidly evolving knowledge society pose a difficult challenge to educators and policy makers” (p. 8). The co-existence, and periodic emergence of different perspectives about the role, benefits and problems of the use of ICT in education, generates a lot of debate around these issues and does not leave enough time to settle arguments and produce foundational ideas (Dillon, 2004). This occurs particularly in this area because “technology evolves/changes too rapidly; therefore, there are always ‘new technologies’ that entail new promises about impact in students’ learning, renewing expectations and possibilities” (Hinoztroza et al., 2008, p. 84). For example, Hinoztroza et al. (2008) point out that integrated learning systems (early 1990) were replaced by Web systems (late 1990), which in turn were replaced by learning objects (2002), which are now being replaced by software to be used in portable devices (2004) and classroom applications, such as smart boards and wearable technologies (2006).

**Technology and Math Wars**

It is worth noting that some of the battles in the math wars involved the use of technology, as newly installed political administrations defined new technology-related goals and proposed the use of “new technologies,” which in turn shifted researchers’ interest (or funding possibilities) so as to investigate these new proposals. The reform movement, headed by the NCTM, strongly advocated the use of graphing calculators and computer technology. As stated in the “technology principle” in the Standards documents, electronic technologies are essential tools for teaching, learning, and doing mathematics. They furnish visual images of mathematical ideas, they facilitate organizing and analyzing data, and they compute efficiently and accurately. They
can support investigation by students in every area of mathematics. (NCTM, 2000, p. 24)

NCTM claimed that when technological tools are available, students can focus on decision making, reflection, reasoning, and problem solving.

The emphasis on the use of certain aspects of technology, however, faced criticism from anti-reform advocates. Klein (2003) argued strongly against de-emphasizing basic calculation skills and over reliance on calculators in response to NCTM’s (2000) recommendation that “appropriate calculators should be available to all students at all times” (p. 25). Hoyles and Noss (2006) expressed their concern over the tendency of the students to use the power of the technology to avoid the cognitive load of “mathematical thinking,” pointing out that it is important not to transform the students from mathematical learners to mathematical users; the difference being that “learners need to search for and appreciate generality and structure, while users want simply to get a particular job done or a problem solved” (p. 3). Cuban (2001) maintained that the ineffective use of technology turned computers into “merely souped-up typewriters” (p. 69). Menoushahri (2004) pointed out that a common tendency in mathematics instruction is to focus on the power of the technology as a presentation device rather than as a discourse participant. In her study, she stated that “even though the technology served as a powerful information provider, it only became a learning medium when supported by appropriate teacher intervention and tasks” (quoted in Kieran, 2007, p. 731). Another frequent charge leveled against technological innovations in mathematics education is that they often seem to be designed to exploit the capabilities of the technology rather than meet an instructional need, that is, they are technology based rather than theory based. That happens when teachers do not gear the mathematical activity that uses technology towards achieving the learning objectives they are targeting. Fey (2006) remarked that symbol manipulation tools (Derive, Matlab, Mathematica, etc.) that are used in math classrooms “are seldom used as
aids in developing student understanding of symbolic forms and manipulations, or in solving problems that require such calculations” (p. 348).

The use of technology in mathematics classrooms has great potential to enhance students’ conceptual understanding, skills building, and engagement with content when properly planned and used. Both traditionalists and reformers have advocated technological tools that fall in line with their philosophies, but the mere use of technology is not enough to support a certain agenda. The orientation of the technological tools used, the instructional goals of educators, and the process of implementation all play a role in determining the relative success of the learning experience. As Earle (2002) succinctly put it, “it is not about what technology by itself can do, but what teachers and learners may be able to accomplish using these tools” (p. 10).
CHAPTER 3

METHODOLOGY

Chapter Overview

The methodology used for the study is described in this section. First, a brief summary of the R2R history is introduced, followed by the characteristics of the R2R design. After the introduction of the research questions, a detailed explanation of the sampling strategy and data collection tools is presented. The data analysis methods that served to analyze the research findings conclude the last section of the chapter.

The discussion in the previous chapter about procedural and conceptual knowledge concluded that each type of knowledge can be targeted using technological tools. Class structure and the instructional goals, however, are not only bound by technological tools, but also by teachers’ approaches. Educators can emphasize the skills targeted by a software program, or use techniques that help gear instruction towards understanding concepts. This study investigates the various techniques that may be used in parallel with a computer-based design in an advanced math curriculum. This model was initially designed for a university setting and was known as the Roadmap to Redesign (R2R). The study focused on the high school adaptation of the redesigned college algebra course at Louisiana State University.

History

Roadmap to Redesign was a Fund for the Improvement of Post Secondary Education (FIPSE) initiative whose goal was to establish an efficient means of spreading the ideas and practices of the Program in Course Redesign to additional institutions. Conducted from 2003 to 2006, the Program in Course Redesign was a Pew-funded effort created by the National Center for Academic Transformation (NCAT) that demonstrated how colleges and universities can redesign their instructional approaches using technology.
to achieve cost savings as well as quality enhancements (Twigg, 2007). Redesign projects focused on large-enrollment, introductory courses, which have the potential of impacting significant student numbers and generating substantial cost savings. As part of this initiative, Louisiana State University (LSU) redesigned its College Algebra (MATH 1021) course. Prior to the redesign, this 3 credit course was taught in a traditional format with three lectures per week. The goals of the redesign were to allow for reduced personnel, incorporate technology to grade student homework, provide consistent content presentation, and continue current success rates. In a nutshell, NCAT states that the main purpose behind the redesign efforts is “achieving improved outcomes at a reduced cost” (Twigg, 2007, ¶1).

For LSU, the new design was a computer-based instructional course delivery model that required students to attend only one lecture, and three hours of required lab time per week with teacher/tutor assistance available. During lab time, the students continue the rest of the required course work using computer software called MyMathLab, a program that was generated specifically for the course. The main components of evaluation were homework assignments, quizzes, and tests. Students doing homework have unlimited attempts prior to the due date, and software embedded tools like “Help me solve it,” “View an example,” “Video,” and “Textbook,” as well as teacher/tutor help are available. Students have ten attempts for each quiz, with a 75 minute time limit per quiz. Their best score is kept, and the quizzes are not proctored. Quiz tutorials are available for review. Finally, for the tests, students have one attempt (proctored) with a 90 minute time limit. An important feature of this design is the noticeable similarity between the “view an example” problems that are solved step-by-step for students, and the homework and quiz problems. This indicates the software program’s procedural orientation, as it prompts students to repeat procedures for different numbers.
Under the supervision of the LSU Math Department, the course is currently offered and implemented in several schools in Louisiana, with many schools offering it as dual enrollment at high school as well as college (LSU in 2007, Southeastern Louisiana University in 2008). The pilot course was offered in the fall of 2006 at three Baton Rouge high schools. It followed the same model described above, with the only difference being that the software used at the high schools was called MathXL (provided by Pearson Education). According to the Pre-Calculus Math Coordinator at LSU, Mrs. Phoebe Rouse, the idea of tailoring the program to become a dual enrollment high school course started in the spring of 2006, when the LSU lab school (U-High) approached LSU’s math department about the possibility of teaching College Algebra and Trigonometry as a concurrent enrollment class. The program of concurrent enrollment existed already between LSU and U-High for students who had an ACT score of 28. Mrs. Rouse explained that since the teachers at U-high were SACS certified and technically were LSU faculty, there was a new agreement between LSU and U-High to lower this threshold so that they can teach Math 1021 and Math 1022 for students who had an ACT score of 23. During the summer, the teacher from U-High was trained to do the course the same way LSU was doing it at the college level. Additionally, two teachers were interested in taking the same approach in their classes. Different models were tailored for each school: the Lab school students would follow the same criteria as LSU, where the overall grade would be determined by the weighted average of the assignments, quizzes, and tests. For the other schools, another format was developed: Students could take credit at LSU only if they achieved a 70% grade on their final exam. These models were applied throughout the whole 2006-2007 academic year. The following year, 14 schools were involved in this program. In the summer of 2008, a Louisiana Systemic Initiative Program (LASIP) grant
ensured that all teachers involved were properly trained and also received a stipend for participating in the workshops (85% participation was mandatory to get paid).

Course Design

Currently, 46 teachers are applying this course design in their Advanced Math classes. The course is designed to include whole-class instruction time and lab time. During the whole-class instruction time, which constitutes about 25% of the overall class time, the teacher plays the role of a lecturer, solving example problems and highlighting concepts, problem solving techniques, and any issues that may come up when students do their homework and prepare for their quizzes and tests. The remaining class time is spent in a “lab” setting, where students work homework problems, practice tests, and quizzes on their computers. Each student is given a username and password to log on to his/her MathXL page. Once logged in, the student can see what portion of the course he/she has completed, his/her previous grades, and what parts are available for them to work on. The students have access to the quizzes and homework assignments as long as the teacher is keeping them “open”, however, when the assignment becomes unavailable, students can no longer make changes, but they can still review their work and check their previous mistakes. A typical student access page looks like the following figure:

![MathXL Student Page Screen Shot](image-url)

Figure 3.1: MathXL Student Page Screen Shot
During lab time, students can inquire about problems and ask for help from the teacher (who is usually moving around the class to monitor students’ progress) as well as their peers. They can also use the various help tools provided by the software, such as “View an example” and “help me solve it.” A key feature in the software is that the “View an example” button opens a similar question in a new window. Given the resemblance between the question that the students are required to solve and the detailed example provided for them by the software, students can simply follow the steps explained for them in details while performing calculations for different sets of numbers. Figure 3.2 shows an example where the problem is displayed first, followed by figure 3.3, where the “View an example” help window provides assistance.

**Figure 3.2: MathXL homework problem**
Match the function \( f(x) = x^2 - 8x + 7 \) to one of the given graphs.

In order to determine the correct graph, we begin by completing the square on the right side.

Add and subtract the square of half the \( x \)-coefficient, \( \left( \frac{-8}{2} \right)^2 = 16 \), in order to complete the square. Then simplify.

\[
f(x) = x^2 - 8x + 16 + 7 - 16
\]
\[
= (x - 4)^2 - 9
\]

Therefore, the graph of \( f \) can be obtained by shifting the graph of \( y = x^2 \) 4 units to the right and 9 units down. The correct graph is shown on the right.

Figure 3.3: “View an Example” Display for the problem in Figure 3.2

Instructor and software help is available during homework and class time, but not during assessments (quizzes and tests). During quizzes, teachers may offer some clarifications, but the software tools are not available for students to use.

As indicated earlier in this chapter, the “View an example” button simply leads the students in a step-by-step procedure they are able to duplicate. Because the example is displayed in a pop up window, all the student has to do is to repeat the steps using the numbers in the problem he/she is asked to solve. This is clearly reflected in the following example (Figures 3.4 and 3.5).
Figure 3.4: MathXL Homework Problem

Suppose that \( f(x) = 4x - 16 \) and \( g(x) = -3x + 8 \).

(a) Solve \( f(x) = 0 \).
(b) Solve \( f(x) > 0 \).
(c) Solve \( f(x) = g(x) \).
(d) Solve \( f(x) \leq g(x) \).
(e) Graph \( y = f(x) \) and \( y = g(x) \) and label the point that represents the solution to the equation \( f(x) = g(x) \).

(a) For what value of \( x \) does \( f(x) = 0 \)?

\[ x = \ldots \] (Type an integer or a simplified fraction)

Figure 3.5: “View an example” Display for the Problem in Figure 3.4

(a) To solve \( f(x) = 0 \), set \( f(x) = 4x - 8 \) equal to 0 and solve for \( x \).

\[
\begin{align*}
4x - 8 &= 0 \\
4x &= 8 \\
&\text{Add 8 to both sides.}
\end{align*}
\]
The previous examples illustrate the extent to which MathXL reinforces procedural tendencies. The students can mimic a procedure several times until they are capable of reproducing this procedure in a different setting (in a quiz for example). MathXL, however, is a support tool for a teacher in the class. Teachers have the freedom to steer their instruction in a direction similar to the software (training students to detect and replicate procedures), or to use the lecture portion to engage the students in conceptually oriented instruction, where students conjecture, propose methods to solve a problem and test their hypotheses, and link previously learned concepts to the newly formed ones. That is why the techniques teachers use in the R2R setting become an important indicator of the quality of instruction the students are receiving in this design.

**Study Objectives**

The study aimed at investigating the various strategies and techniques that teachers, working in the high school format of the R2R approach, are implementing in their whole-class instruction as well as in their one-on-one engagement with the students. The objectives of the study were to determine the characteristics of these strategies, and to analyze each in terms of its conceptual affordances.

The first portion of the study aimed at providing a panoramic view of the various strategies that teachers are using. Illustrating the various techniques that teachers choose to adopt in response to procedurally oriented software enabled the researcher to categorize the teachers as procedurally oriented or conceptually oriented. Furthermore, it made the distinction between “procedural” and “conceptual” within the R2R model clearer, since, as indicated earlier, the complexity of the interplay between concepts and skills makes it difficult to qualify a teaching instance as purely conceptual or procedural. After the categorization of the teachers’ strategies as conceptually oriented or procedurally oriented, the second phase of the study highlighted the conceptually oriented strategies in order to
find common characteristics among these techniques. The following research questions were targeted:

1. What portion of the teachers using the R2R design is oriented by conceptual goals?
2. How do conceptually identified teachers structure their instruction to achieve their goals?
3. What instructional patterns, if any, emerge across the conceptual methods?
4. If no conceptual teachers were found, then what varieties of instructional strategies are the teachers implementing in the R2R classrooms?

The possibility of not finding conceptually oriented teachers was taken into consideration from the onset of the study, given that not every “conceptually-identified” teacher (from the survey) may turn out to be a conceptually-oriented teacher in the class. In addition, the teachers’ understanding of conceptually oriented instruction varies immensely, and that plays an important role in orienting their classroom practices. Provided no conceptually oriented teachers were indentified, then the study would focus on profiling the various categories of teaching styles that the research unveils. That would include profiling procedurally oriented teachers to summarize their approaches and highlight the nuances between them, as well as describing in details any other emerging categories of teaching methods that the researcher would find. For instance, if some teachers employed some form of conceptual teaching in their classes, but the distinction between procedures and conceptual agendas is not clear throughout the lesson, then a new category (quasi-conceptual) would emerge, and through analysis of lessons transcripts and interviews, a detailed profile for the teachers who fit in that category would be created, and an overall characterization of this category would be detailed.
Study Design

This study utilizes a mixed methods approach. Mixed methods research is formally defined as “the class of research where the researcher mixes or combines quantitative and qualitative research techniques, methods, approaches, concepts or language into a single study” (Johnson & Onwuegbuzi, 2004, p. 17). Patton (1990) explained that multiple sources of information should be sought and utilized because no single source of information can be trusted to provide a comprehensive evaluation. Studies that rely solely on one method can be vulnerable to errors that may be inherent to that method. By incorporating a combination of observations, interviews and document analysis the researcher is capable of cross-validating the findings to achieve a better understanding of the topic.

This study is characterized by a naturalistic design, a design that Patton (2002) defined as a “discovery oriented” approach that minimizes investigator manipulation of the study setting and “places no prior constraints on what the outcomes of the research will be” (p.39). Specifically, the study employed a sequential mixed model design (Cresswell, 2002; Tashakkori and Teddlie, 2003). Cresswell (2002) explained that “a mixed model approach is one in which the researcher tends to base knowledge claims on pragmatic grounds (e.g., consequence oriented, problem-centered, and pluralistic)” (p. 19). Such a design employs strategies of inquiry that involve collecting data either simultaneously or sequentially to best understand research problems. The data collection “involves gathering both numeric information (e.g., on instruments) as well as text information (e.g., on interviews) so that the final database represents both quantitative and qualitative information” (Cresswell, 2002, p. 20). Cresswell et al. (2003) identified the traits of a “sequential” design as typically using qualitative results to assist in explaining and interpreting the findings of quantitative outcomes. The initial quantitative phase of a study
may be used to characterize individuals along certain traits of interest related to the research questions. These quantitative results can then be used to guide the purposeful sampling of participants for a qualitative study. In this case, the quantitative data was used to categorize participating teachers in order to identify a sample of conceptually oriented teachers. Tashakkori & Teddlie (2003) also defined a “sequential” mixed model design as one that involves “one type of question, two types of data that are collected in sequence (with one being dependent on the other) and analyzed accordingly, and one type of inference at the end” (p. 687). In this type of design, the first strand is an exploratory study that includes data collection, data analysis and inference in one approach, while the second strand involves new data collection, analysis, and inference. In this study, the first strand data is an online survey designed specifically for this study with the purpose of coding the answers to identifying teachers who have the highest percentage of answers that reveal conceptual tendencies. The second strand aims at getting a better understanding of the approaches of the identified conceptual teachers. Class transcripts, notes, and teacher interviews form the bulk of the data in this strand. Finally, meta-inferences can be made based on the nature of inferences in the two strands of the study. If no conceptually oriented teachers are identified, in-depth analysis of the teaching strategies employed in the margins of R2R becomes the objective of the second strand. For the purposes of this study, the margins of R2R are defined as all interactions between the teacher and the students that take place in the lecture portion of the lesson, as well as the one-to-one exchanges between the teacher and a student during a lab session. In that case, the analysis of the lessons transcripts and teacher interviews covers a sample of the teachers that includes conceptually-identified teachers, procedurally oriented teachers, and any emerging category in-between.

Figure 3.6 illustrates the sequential model design as it applies to this study.
Objective: Identify conceptually oriented teachers

Data Collection: Conduct the online survey

Data Analysis: Analyze the survey responses

Inference: Categorizing teachers based on orientation

Meta-Inference: Profiling conceptual (or emerging) categories of teachers and documenting approaches employed in R2R

Objective: Identify characteristics of conceptually oriented instruction

Data Collection: observations, interviews and videotaping

Data Analysis: analyze notes, lesson transcripts and interviews

Inference: Portraying conceptual teaching strategies

Figure 3.6: Sequential Mixed Model Design

Adapted from Tashakkori & Teddlie (2003, p. 686)
Data Collection

Qualitative and quantitative data were collected for this study. The phases of the research and the purpose of the instrument used are summarized in the following table:

<table>
<thead>
<tr>
<th>Step</th>
<th>Instrument</th>
<th>Data collected</th>
<th>Purpose of data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conducting survey</td>
<td>Online survey</td>
<td>Teachers’ responses to the survey</td>
<td>Categorizing teachers based on orientation as Conceptual (C), Procedural (NC), or Quasi-Concept (QC)*</td>
</tr>
<tr>
<td>Identifying conceptually</td>
<td>Survey results</td>
<td>Frequency table, cross tabulation</td>
<td>Selecting teachers for observations and interviews</td>
</tr>
<tr>
<td>oriented teachers</td>
<td>analysis by</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SPSS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>Audio tape,</td>
<td>Detailed thick descriptions</td>
<td>Characteristics of conceptual teaching</td>
</tr>
<tr>
<td></td>
<td>notepad</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interviews</td>
<td>Interview guide</td>
<td>Teachers’ responses</td>
<td>Characteristics of conceptual teaching</td>
</tr>
</tbody>
</table>

Table 3.1: Data Collection Methods and Objectives

* Quasi-Conceptual categorizes a teacher whose instructional agenda is neither dominantly conceptual nor necessarily purely procedural. It could also refer to teachers who are not clear on what conceptual teaching is and how it is implemented.

Quantitative Component

The first strand of this study was the quantitative portion, which consisted of a survey, designed specifically for this research, sent to all the teachers that were using the high school R2R modified model. The survey required detailed answers about their values, strategies and approaches (Appendix A). It included 24 total questions that focused on demographics (7 questions); teachers’ general beliefs about teaching/learning (5 questions); their normal practices of teaching prior to R2R (2 questions); their reaction to and practices within R2R (7 questions), and their perception of R2R and how to improve it (3 questions). The survey was set up online, and teachers were asked to participate in the study by email. A reminder was sent to those who did not respond after one week. All communication with the teachers was approved by their program supervisors. The
quantitative component served as a vehicle to identify those teachers who profess to be conceptually oriented. It also yielded an overall breakdown of the proportion of teachers opting for procedural or conceptual approaches. It is worth noting here that categorizing teachers based on the survey results is only in reference to their thoughts and ideals. It remained to be studied whether their practices were conforming to their perspectives. Hence, the label attached to those teachers was “potentially-conceptual.”

Qualitative Component

Once potentially-conceptual oriented teachers were identified, the second phase of the study (the qualitative component) began with contacting those teachers to set up possible observations and interview dates. Given that this contact occurred towards the end of the fall semester, some teachers agreed to be interviewed prior to observations, while others allowed the researcher to observe their classes then conduct an interview at a later date. The purpose of this phase was to provide an in-depth analysis of the techniques used by those teachers in order to identify the characteristics of conceptually oriented strategies, and how those educators are incorporating such strategies within the margins of R2R – like, for example, the time the teachers spend lecturing and/or interacting with the students away from the computer. This component included conducting focused, non-participant observations during which extensive notes were taken, audio-taping of some classes, and conducting several teacher interviews. For purposes of pursuing the alternative route of this study, observations and interviews also included teachers who were identified as procedural for the purpose of documenting the strategies and techniques they employed in their classes too. This component served to portray the range of methods used by different teacher categories. The next section includes a detailed explanation of the various data collection methods used.
Sampling

Sampling “involves selecting units (events, people, settings, etc.) in a manner to maximize the researcher’s ability to answer the questions that are set forth in a study” (Tashakkori and Teddlie, 2003, p.715). The target participants in the first phase of study were all 46 teachers that were teaching college algebra for high school students using the R2R design. These were divided into two groups based on the model they were following: 29 teachers who were following the “Early Start” model associated with Southeastern Louisiana University (where students received either a Pass or Audit as their final grade), and 17 teachers who were following the LSU-Alexandria model (where students received a standard letter grade evaluation at the end of the course).

Based on the observations and the survey responses, a purposeful sample was selected for further observations and interviews. Tashakkori & Teddlie (2003) defined a purposeful sampling as “selecting specific units, or types of units, based on a specific purpose rather than randomly” (p. 713). Patton (2002) explained that purposeful sampling’s power “derives from the emphasis on in-depth understanding” (p.46). The sample in the study was selected based on the teachers’ responses to the survey, and it included teachers who were projected to be conceptually oriented based on their answers, as well as teachers who were identified to have procedural tendencies.

Observations

Focused observations, as Spradley (1980) explained, are based on narrow structural questions, and ask for details about the structure of particular domains of interest. Three teachers were selected for observation based on their responses to the survey that indicated an interest in, and intention to, enhance students’ understanding of the content rather than just perfecting the performance of procedures. During the observations, field notes were taken on the behavior and activities of the teacher and
students in the classroom. The observational protocol followed was dividing the note pages down the middle to separate descriptive notes (description of setting and participants, account of activities) from reflective notes (researcher’s personal thoughts and impressions). These notes provide good foundation for qualitative analysis, since “good description takes the reader into the setting being described” (p. 437). Non-participant observations were conducted to ensure no interference with the teachers’ pedagogies. Also, observations were conducted for three teachers whose answers revealed procedural tendencies. These observations aimed at documenting the varieties of procedural instruction that accompany the R2R design.

**Interviews**

The six teachers selected for observations were also selected to be interviewed, however only 5 were interviewed for this study, as one teacher declined. Interviewing is “a meeting of two persons to exchange information and ideas through questions and responses, resulting in communication and joint construction of meaning about a particular topic” (Janesick, 2004, p. 30). According to Patton (2000), “we interview people to find out from them those things we cannot directly observe” (p.340), since observations cannot clearly determine feelings, thoughts, and intentions of the people observed. Thus, the purpose of the interview is to enter into the other person’s perspective. For interview purposes, an “interview guide” was used in this research (Appendix B). The interview guide is defined as a “list of questions or issues that are to be explored in the course of an interview” (Patton, 2002, p. 343).

Interviews started with brief explanation and instructions for the interviewee, followed by the key research questions, and then some probes to follow key questions. The seed questions in the interview guide used in this study explored the teacher’s vision of teaching for conceptual understanding, the challenges to teach conceptually within the
margins of R2R, and specific teaching instances in which the teacher addressed conceptual goals. The length of the interviews ranged from 25 to 50 minutes. The purpose of the interviews was to get an in-depth look at the teachers’ choice of strategies, the rationale behind these choices, and the effects of the R2R design (if any) on their approach in teaching college algebra. Just like observations, interviews were conducted to cover the range of teacher categories. Three interviews were conducted with the teachers who were identified to be conceptually oriented. Then, 2 additional interviews were conducted with teachers who were categorized as procedural, in order to get a perspective on their classroom practices. The interviews were audio-taped, then carefully transcribed and analyzed.

Data Analysis

Both qualitative and quantitative data analysis procedures were used. Teddlie and Onwuegbuzie (2003) defined mixed methods data analyses as “the use of quantitative and qualitative analytical techniques, either concurrently or sequentially, at some stage beginning with the data collection process, from which interpretations are made in either a parallel, an integrated, or an iterative manner” (p. 352).

For the survey instrument, questions that indicated the teachers’ orientations and practices were selected for the purpose of analysis. These questions covered teachers’ beliefs on problem solving approaches, priority of skills over concepts or vice versa, students’ group work and in-class dialogue, teaching practices prior to R2R, and describing a typical lecture in the R2R design. 8 of those questions had “restricted choice” answers, and 2 were open-ended. The multiple choice questions were analyzed by assigning a code for each answer: Answers labeled as “C” indicated a conceptual tendency; answers labeled as “NC” indicated a lack of conceptuality, and answers that
indicated a possibility for conceptuality were labeled as “QC” (Quasi-conceptual). For instance, in answering the question about the best problem-solving strategies, respondents could choose: “Comparing and contrasting different solution strategies is what helps students understand a problem” (which would indicate a conceptual tendency and will get a “C”); or “Multiple solution strategies may exist for a certain problem. This provides the teacher with the opportunity to have students appreciate the easiest, most efficient method” (which indicates some confusion as to what conceptuality in terms of the teacher taking the lead in deciding what is the most efficient method, thus will get a “QC”); or finally, “It’s best to focus on the one approach that enables the students to solve a problem efficiently and correctly. Introducing alternative methods is likely to confuse students” (which was coded a “P”). In addition to the analysis of the multiple choice questions, a rubric was created for the analysis of the open-ended, subjective question. The question selected for this analysis asked participants to describe a typical lecture in terms of what the teacher and students normally do. Generally, an answer was given a “C” (for conceptual) if it included indications that the teacher was not demonstrating procedures for students, and that connections to previously explained concepts were established. In contrast, an answer would be deemed “P” if it clearly reflected common procedural practices such as doing an example on the board and asking students to do similar problems. An answer would get a “QC” if it reflected a teacher’s aspiration towards conceptuality but at the same time a failure to determine how to implement conceptual strategies. After all the data were compiled, the teachers’ answers were analyzed in SPSS to determine the categories for each range of scores. Teachers were categorized as potentially-conceptual, quasi-conceptual, or potentially-procedural.
For the purpose of analyzing interview transcripts, lesson transcripts, and observations notes, a case study approach was used. Case study involves “a detailed description of the setting or individuals, followed by analysis of the data for themes or issues” (Cresswell, 2002, p. 191). According to Patton (2002), the purpose within this approach is to “gather comprehensive, systematic, and in-depth information about each case of interest” (p.447). Each teacher was considered to be a separate case, and analysis of the lesson transcripts and interview of the teacher led to the construction of an individual case for each teacher. Once each case was written up, an inductive analysis was conducted to compare and contrast individual cases. Patton (2002) defined an inductive approach as one that begins by “constructing individual cases, without pigeon holing or categorizing those cases…Once that is done, cross-case analysis can begin in search of patterns and themes that cut across individual experiences” (p.57).

In this study, the qualitative analysis began by transcribing the teachers’ interviews and observations, and organizing the field notes. An in-depth analysis was then conducted for each teacher, based on the transcripts of his/her lesson and interview. A closer look at each case enabled further analysis that led to a number of “themes.” The themes were analyzed for individual cases, then a cross-case pattern analysis followed to compare and contrast the characteristics of the teaching strategies discussed. Finally, the results obtained were discussed in terms of their relevance to past research findings and possible implications for the future. The following chapter contains the discussion of the findings.
CHAPTER 4
RESULTS AND DATA ANALYSIS

Chapter Overview

This chapter contains the results obtained from the various data collection methods as well as the subsequent analysis. Two types of data (quantitative and qualitative) were collected and analyzed separately. In the conclusions section, inferences drawn from each source of data contribute in portraying the characteristics of the various categories of teaching strategies.

Quantitative Data

As indicated earlier, one of the objectives of the study was to determine what portion of the teachers using the R2R design are oriented by conceptual goals. For this purpose, a survey was created and set up online, and invitations for the teachers participating in the R2R design were sent. The survey consisted of 3 sections, the first one addressed teachers’ demographics, while the other two sections inquired about teachers’ beliefs and practices. In the following sections of this chapter, the teachers’ responses are documented and analyzed. Then, teachers are categorized as conceptual, quasi-conceptual, or procedural based on their responses. As indicated earlier, the “Quasi-Conceptual” categorization refers to the teachers who may have aspirations towards conceptuality, but have not fully embodied a conceptual teaching agenda. Quasi-conceptual teachers have not reached a full understanding of what conceptual teaching is and how it is practiced.

Demographics

74% of the teachers invited to participate (34 out of 46) voluntarily responded to the survey. The participants were teachers from various schools across Louisiana who taught advanced Math courses using MathXL. 97% of the participating
teachers were certified to teach mathematics (one teacher was not certified). The data for this portion of the study were collected over the fall of 2008 and the spring of 2009. The following tables categorize respondents based on age, sex, highest level of education, and number of years teaching mathematics.

<table>
<thead>
<tr>
<th>Age</th>
<th>25 or below</th>
<th>26-30 years</th>
<th>31-35 Years</th>
<th>36-40 years</th>
<th>41-45 years</th>
<th>46-50 years</th>
<th>51 or above</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response Count</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>8</td>
<td>5</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>Response Percent</td>
<td>5.9</td>
<td>2.9</td>
<td>14.7</td>
<td>23.5</td>
<td>14.7</td>
<td>11.8</td>
<td>26.5</td>
</tr>
</tbody>
</table>

Table 4.1: Teachers’ Distribution Based on Age

<table>
<thead>
<tr>
<th>Sex</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response Count</td>
<td>12</td>
<td>22</td>
</tr>
<tr>
<td>Response Percent</td>
<td>35.3</td>
<td>64.7</td>
</tr>
</tbody>
</table>

Table 4.2: Teachers’ Distribution Based on Sex

<table>
<thead>
<tr>
<th>Sex</th>
<th>Bachelor</th>
<th>Masters</th>
<th>Specialist</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response Count</td>
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<td>17</td>
<td>1</td>
</tr>
<tr>
<td>Response Percent</td>
<td>47.1</td>
<td>50</td>
<td>2.9</td>
</tr>
</tbody>
</table>

Table 4.3: Teachers’ Distribution Based on Highest Level of Education Attained

<table>
<thead>
<tr>
<th>Years</th>
<th>1st</th>
<th>1-3 years</th>
<th>4-6 Years</th>
<th>7-9 years</th>
<th>10-12 years</th>
<th>More than 12 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response Count</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>17</td>
</tr>
<tr>
<td>Response Percent</td>
<td>2.9</td>
<td>5.9</td>
<td>8.8</td>
<td>11.8</td>
<td>20.6</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 4.4: Teachers’ Distribution Based on Number of Years Teaching Mathematics

As the data show, the majority of teachers who participated in the study were experienced (70% have taught mathematics for 10 years or more, 82% have been teaching mathematics for more than 7 years). Also, it is worth noting that 50% of them hold a Masters degree in Mathematics or related field.
Coding Responses

The teachers’ responses were collected online. The relevant questions that pertain to teachers’ beliefs and practices were identified. These questions inquire about the teachers’ perspectives on issues of problem solving in mathematics, priority of concepts or skills in instruction, classroom dialogue, practices in introducing new material to students, and the perceived advantages and/or disadvantages of the R2R approach in terms of bettering students’ skills or conceptual understanding. Eight relevant questions were selected for the quantitative analysis. The remaining questions (that were not selected for the analysis) were not designed to probe the teachers’ strategies. Their purpose was to collect some information on the teacher's expectations from the R2R design, the problems they may have faced applying it, and their suggestions for its improvement.

Seven questions had 3 possible answers. For each one of these questions, an answer was coded as either a “C” for Conceptual, a “QC” for Quasi-Conceptual, or “P” for Procedural (Appendix C). In addition, one of these questions afforded respondents an open-ended response option. In this case, the teachers’ written answers were also coded using the same criteria (C, QC, or P). The coding was based on the extent to which the answer contained elements of conceptuality (soliciting students’ ideas, discussing their strategies, linking current topic to previous knowledge). For instance, when asked about the precedence of skills over concepts (or vice versa), one teacher answered, “This depends. One can and should teach fractional concepts way before teaching the facts. On a different level, it is nearly impossible to work with rational expressions without some rote factual foundation.” This answer was labeled as “QC”, since the teacher expresses some conceptual orientation, but sees conceptual teaching as topic-dependent.
The last question asked teachers to describe a typical lecture in their classes in terms of what they and the students do. The answers for this question were labeled using the same criteria as the teacher-generated answers. For instance, a teacher answered this question as follows:

When I introduce the continuous compound interest formula \( A = Pe^{rt} \), my students read background information (an article) about natural numbers, then my students and I have a discussion about the article they just read. I then prompt the students’ prior knowledge of another formula \( A = P(1 + \frac{r}{n})^{nt} \). Together, we derive from it the formula \( A = Pe^{rt} \) by letting \( n \to \infty \). We then compare and contrast the two formulas before I ask the students to work on their own on some problems. This answer was coded as a “C”, since the teacher engaged students in a classroom discussion that required their input in generating a formula rather than simply writing it down. By doing so, the teacher also opened the door for the students to understand the connection between the current topic and previously taught concepts. On the other hand, one of the answers was, “I model the examples for the students. Students take notes and additional practice is provided to the students before working on the computer.” This answer was coded as a “P”, given the inclination of the teacher to demonstrate then ask the students to duplicate a method or a procedure. A few answers indicated some elements of conceptuality but also some procedural tendencies. For instance, one of the answers a teacher provided was:

I usually begin with questions on the previous lesson or unusual problems that the students had. I then present the lesson, sometimes using investigation, but the students really prefer for me to develop the topic with examples and let them get to work.

This answer shows that the teacher values linking students’ previous knowledge to the lesson he/she is explaining, and that he/she uses investigation as a possible method of delivery. But the teacher also mentions that students prefer a traditional procedural approach in which he/she demonstrates via examples in order for the students to replicate the procedure on similar problems. Hence, it is not clear whether a conceptual agenda is
the main goal of instruction. In this case, the answer is coded as a “QC”, since it shows that the teacher has some conceptual tendencies along with some procedural ones, and it is not clear which agenda is the focus of the instruction. All given answers and their assigned codes are listed in Appendix D.

Scores and Categories

Once all the answers were coded, the codes were converted to scores for each individual respondent. An initial count of each answer type was done; then the teacher’s score was calculated according to the following formula:

\[ \text{Score} = (\# \text{ of “C” answers } \times 2) + (\# \text{ of “QC” answers } \times 1) + (\# \text{ of “P” answers } \times 0). \]

Teachers’ scores ranged from 3 to 12, with a mean of 6.76, and a standard deviation of 2.61. Table 4.5 summarizes the frequencies of the obtained scores:

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>Percent</th>
<th>Cumulative Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>5.9</td>
<td>5.9</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>17.6</td>
<td>23.5</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>14.7</td>
<td>38.2</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>17.6</td>
<td>55.9</td>
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<td>7</td>
<td>3</td>
<td>8.8</td>
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<td>Total</td>
<td>34</td>
<td>100.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5: Frequencies of Teachers’ Scores

The next step was to divide the teachers who participated in the survey into categories based on their scores. Since answers coded as conceptual are worth 2 points each, a maximum score of 16 was possible. Given that the purpose of the instrument was to determine potentially conceptual teachers, it seemed reasonable to label the top 20% (roughly) of the teachers as conceptual, and the bottom 20% (also approximately) as procedural. This corresponds to a score of 10 and above for conceptual teachers, and a
score of 4 or below for procedural teachers. The teachers who scored between 5 and 9 were labeled as quasi-conceptual. The data in Table 4.1 show that eight teachers scored 4 points or below, while 7 teachers scored 10 points or above. The remaining 19 teachers’ scores ranged between 5 points and 9 points.

The findings in this section indicate that 20.6% of the teachers (7 out of 34) using the R2R design are oriented by conceptual goals, while 23.5% of this sample (8 out of 34) have strong procedural tendencies. The remaining teachers fall into the quasi-conceptual category. This category includes the teachers who do not have a clear distinction between “conceptual” and “procedural” in their teaching agendas. In other terms, these teachers may be sympathetic towards conceptual agendas, but are generally confused about what they are and how to implement them. These data serve to answer the first research question, which required finding what portion of the teachers using the R2R design are oriented by conceptual goals. The next section discusses the strategies employed by the teachers who were identified as conceptually oriented, as well as the approaches taken by the procedurally-identified teachers. A profile is created for each individual teacher, and the overall characterization of each category is discussed in terms of similarities and differences of the teachers’ strategies.

Qualitative Data

In the following section, qualitative data obtained from observations and interviews are analyzed. Since the survey results categorized teachers as conceptual, quasi-conceptual, or non-conceptual based on their overall responses, teachers from each category were selected for observations and interviews. It is worth noting here that the purpose of the quantitative portion was to identify potentially-conceptual teachers in order to study them more closely in the qualitative part of the study. Moreover, one of the goals
of the study was to get as clear a picture as possible of the differences between conceptually and procedurally oriented teachers. Therefore, the focus of the qualitative portion of the study was on the teachers who scored highest (conceptually identified) or lowest (procedurally identified). The teachers who evinced aspects of both conceptual and procedural teaching agendas were not profiled in the study.

A total of 4 observations (for three conceptually-identified teachers) and 3 interviews were conducted. It is worth noting that one teacher declined to be observed and interviewed. Additionally, three procedural teachers were observed and two were interviewed. For each category of teacher, individual teachers’ data were carefully analyzed, and a teacher profile was created. Each profile included an analysis of the teacher’s ideals gleaned from his/her responses to the survey and interview questions. Then, analysis of the teacher’s observed class was conducted to characterize his/her approach in the classroom. Finally, a cross-case analysis was done to enable the researcher to highlight the similarities and differences across the various teachers’ approaches within a given category.

Procedurally-Identified Teachers

For each case, the analysis of the survey and interview responses is followed by the analysis of the classroom practices, and a new categorization of the teacher is deduced based on both analyses. The next section discusses, in detail, the approaches adapted by three procedurally identified teachers in order to compare and contrast the strategies used in their classrooms.

Teacher P1

This teacher was identified as procedurally oriented based on her answers to the survey. A 30-minute interview was conducted with her, and one of her classes was observed, audio-taped, and transcribed.
Survey and Interview Responses for Teacher P1

The teacher’s responses to the survey indicated a procedural tendency; she stated that a student has understood a concept if he/she solved a problem correctly, and that “basic mathematical facts need to be emphasized prior to the introduction of concepts.” She also described her teaching approach as “illustrating a method or procedure and then have students practice it with several examples,” claiming that this occurs about 75% of the class time. Some of her answers, though, did not have a purely procedural orientation. She stated that “students’ ideas should be solicited, but the teacher should correct their mistaken interpretations,” and that multiple solution strategies may exist for a problem, but she usually uses the easiest and most efficient method. In her interview answers, the teacher described her normal teaching approach:

I write notes that cover general ideas…For example, I would give them examples on linear equations, I would give them the definition of what linear equation is, and show the different forms of it, pick a formula and go with it, and then from there I would work examples.

This answer indicates that the teacher considers herself the source of the information that will be dispensed to the students, so she plays a big role in most of the lesson phases, from giving definitions to solving the examples and problems. Also, she stated that she prefers starting with simple examples and gradually increases the level of difficulty: “I’m probably spending more time on some of the easier examples than I’m supposed to…I can’t do the hardest ‘solve the rational equation’ till I do a couple of easy ones.” The teacher stated that this approach is the only possible route for her due to time constraints.

There’s not a lot of time for them to go to the board because I am basically working bell to bell…Because of time constraints, there’s not a lot of discovery type of activities that I used to do in advanced math….There’s times when you need a couple of more days on this.

The teacher acknowledged the students’ dependence on the software helps: “You can tell they’re not understanding something when they quickly use the ‘help me’ key all the time,
which means they really don’t know what they’re doing, they’re just mimicking what’s on the screen.” The issues that the teacher raised (lack of time, acknowledging the dependence on software help) explain the teacher’s statements on how to efficiently show students solution strategies. To check whether the ideals she talked about are reflected in her classroom practices, one of her classes was observed and audio-taped.

Classroom Observation for Teacher P1

The teacher started the class with a quiz on a previous lesson. The students did the quiz on paper, without computer help. Then, the teacher summarized the required material for the final exam, and proceeded to the explanation of the lesson. She reminded the students of the standard formula for a circle, and solved an example where the radius and the center were given, and the equation of the circle was required. She solved another example where the students were required to convert a given equation of a circle to its standard form by completing the square. The teacher then solved several examples where she identified graphs of polar equations and converted them to the Cartesian form. The examples increased in difficulty, and the last one required completing the square to determine the center and the radius of the given circle. The last portion of the lesson was spent on testing for symmetry for some polar equations, and determining the names and shapes of the graphs of these equations.

Emerging Themes and Analysis for Teacher P1

The targeted lesson’s objectives were to identify the graphs of polar equations and convert them to the Cartesian form, and to test for symmetry in polar equations. To reach the objectives, the teacher mainly demonstrated to the students how to perform the conversions and graph the equations. She started with a reminder of the equation of
circles, but when a student did not remember the equation, she gave a numerical example to illustrate how to write a standard equation of a circle:

Teacher: All right, now you said you forgot, so here’s an example: They told you r was 5, and the center (-3, 6), write the equation. Well, I’m going to refer to my standard form: I’m going to write my equation first, plug in the -3. Now the -3 and the negative is going to be a positive, and I end up with a positive. My 6, I’m going to plug it in, and my 5 for the r. So my equation is $(x+3)^2 + (y-6)^2 = 25$. Is that clear?

This example shows the teacher plugging in the numbers in the equation and performing calculations. This approach can teach a student how to identify the center and the radius of a circle, but it does not address the student’s question from a conceptual standpoint, as it does not explain how the standard equation of a circle is derived.

Similarly, the next example reflects the teacher’s tendency to show the students the steps needed to answer a question. The question asked the students to convert an equation from the general form to the standard form, and the teacher did the following:

Teacher: This is a step by step explanation. So, what do I do? First, I pair my $x^2$ and $4x$ together, and I leave a blank here next to the $4x$, if you remember the rules for completing the square, and $y^2$ is paired with $6y$, and here’s the blank. Now whatever I’m going to do to do to one side of the equal sign, I’m going to have to do for the other side. So I put two blanks on the other side. Remember, the c term is $\frac{1}{2}$ of $b^2$ and $\frac{1}{2}$ of $a^2$, so what’s half of 4?

Student: 2

Teacher: squared?

Student: 4

Teacher: half of 6?
Student: 3
Teacher: squared?
Student: 9.
Teacher: But whatever I put here and here (points at the left side of the equation)…
Student: You got to put on the other side.
Teacher: Ok, so now we factor this, going back to our Algebra 2. So \((x+2)^2 + (y–3)^2 = 1\). Of course, one squared is one. So my \(r\) is 1, and my \(h\) and \(k\) are -2 and 3.

In this dialogue, two characteristics of this teacher’s approach are reflected: First, the focus on the “step-by-step” explanation confirms the teacher’s tendency to show the students how to do a procedure. Second, the students’ role in the process was to answer simple numerical questions. In fact, this pattern is repeated in several other instances of classroom interaction, as it is clear in the following dialogue:

Teacher: We’re given in this case \(\theta = \pi/4\), so remember the formula, just to refresh your memory, \(\tan \theta = y/x\). I’m just going to transform the given equation to something I know, and \(\tan\) both sides. So what’s \(\tan\) of \(\pi/4\)?
Student: One.

Teacher: One, because it’s \(\frac{\sqrt{2}}{2}\) divided by \(\frac{\sqrt{2}}{2}\), so it’s 1. So now this looks familiar, right? But now I’m going to find the equation in rectangular form, so I’ll change \(\tan \theta\) to \(y/x\), so I’ll put instead of it \(y/x\), and it’ll be \(y/x = 1\). Now I want to get that \(x\) out of the denominator, so I multiply both sides by \(x\), \(x\) cancels out, and \(y\) equals what?
Student: \(x\).
Teacher: Now if you remember how to graph it from Algebra 1, what is the slope and the y-intercept?
Students: ...

Teacher: Remember, the equation is $y = mx + b$. So the slope is 1, and the $y$-intercept is…?

Student: Zero.

Teacher: That’s it, zero.

In both dialogues, the teacher gave the students the rule or formula, started to apply the rule using the given numbers, and asked the students to provide the answers to questions like “what is 3 squared?” and “what’s the tan of $\pi/4$?” The teacher did not explain the meaning or the rationale behind the procedures (like how the standard equation of the circle was derived). The students did not have a say in the choice of strategy or in the selection of the formula to be used. Moreover, the fast pace of instruction did not allow for time for the teacher to check for students’ understanding of about the nature of the problem, or for the students to replicate the explained procedure. In summary, the teacher’s ideals and tendencies highlighted in the interview and survey were on display during a good portion of her class. She showed the students a number of examples in a step-by-step explanation format. Her approach in this class is characterized as procedural.

Teacher P2

This teacher was identified as procedurally oriented based on her answers to the survey. She did not have time to do an interview, but allowed one of her classes to be observed and audio-taped.

Survey Responses for Teacher P2

The teacher’s responses to the survey indicated a predominantly procedural orientation. For her, basic mathematical facts need to be emphasized prior to the introduction of concepts, understanding a concept is reflected in a student’s ability to solve a problem, and students’ ideas may be solicited, but it’s up to the teacher to correct their
mistaken interpretations. A couple of answers, however, reflect potentially conceptual
tendencies. For instance, when asked about the best solution strategy, she stated that
“comparing and contrasting different solution strategies is what helps students understand
a problem.” She also maintained that she sometimes introduces material by posing a
problem and paving the way for students to solve it by giving hints. However, her overall
categorization based on the survey responses was procedural. It remained to be seen if any
conceptual tendencies can be detected in her classroom practices.

Classroom Observation for Teacher P2

A sheet that contained notes about the lesson was handed to the students at the
beginning of the class, and spaces were left on the sheet so that students could take notes
or solve examples. The class started with the teacher defining a vector, explaining its
properties, and the terminology associated with it (initial point, terminal point). She then
asked the students about the difference between a vector and a ray. After a small
discussion, the students were unable to reach the answer, so the teacher stated that length
or magnitude is the property that vectors have but rays do not. The teacher went on to
explain graphically how to add two vectors, subtract two vectors, and how to graph a
multiple of a vector. The teacher proceeded to define unit vectors $i$ and $j$ and asked the
students to draw them. She then explained how to find a position vector when given the
coordinates of two points, and gave the students an example to work on their own. The
teacher moved on to explain how to calculate the magnitude of a vector, and how to use
the magnitude to find a unit vector for a given vector, and gave the students another
example to solve. She started to explain how to split a position vector into the sum of two
vectors (one on the x-axis and one on the y-axis) when the class was abruptly ended with a
fire alarm.
Emerging Themes and Analysis for Teacher P2

The objectives of this lesson were enabling the students to add and subtract vectors, find multiples of a vector, position vectors, unit vectors, and the magnitude of a vector. To reach the objectives, the teacher used mostly demonstrations combined with some instances of lecturing. The teacher initially started by asking for students’ feedback in comparing vectors and rays, but then proceeded to explain the difference through lecturing. She then showed the students how to add vectors, and used this information to subtract vectors:

Teacher: Now let’s say I have a vector V and I have a vector W, and I want to draw a picture of vector V+W. So what I can do is draw vector V, then from the arrow, we start with the other vector W. Now where do you think we would have vector V+W?

Students: Here.

Teacher: It goes from the left to the right. The initial point is here (points at start of vector V). They may ask you to recognize in MathXL some vectors drawn. Now we have vector V – W. So that would be the same as if you have a vector V + (-W). And so we have V going in any direction. Now if we have W going up, then (-W) would go down. Then, where would be V + (-W)?

A key point in the explanation was the conclusion that V – W is the same as V + (–W), which was stated by the teacher. The teacher demonstrated the addition and subtraction process to the student, but she did not try to ask the students to attempt to draw V – W based on their knowledge of how to draw the vector V+W. Other instances reflect this tendency to show the students how to answer the given questions: When asked to draw the 2V vector, the teacher stated that it’s V+V and proceeded to draw it on the board. Also, the teacher explained how to find a position vector as follows:
Teacher: Now the next important thing it says is we often give a vector as a position vector. A position vector has an initial point at (0,0) and a terminal point at \((a,b)\), all right? We can write a position vector as \(ai + bj\), MathXL writes it as \(ai + bj\), all right? That’s a position vector. Now we’re going to find a position vector \(PQ\) and write it as something \(i + something j\), \(ai + bj\), if \(P (6, 2)\) and \(Q (4,3)\). All right? Find me the position vector \(PQ\). So we’re looking to find \(a\) and \(b\). So what do we do? We’re going to find \(4 – 6\) first, because it’s terminal – initial, and to find the \(b\), we do \(3 – 2\). So the position vector is \(-2i + j\). Got it?

In this excerpt, it appears that the teacher is interested in showing the students the steps needed to find a position vector, and that the question “So what do we do?” is not meant to solicit students’ ideas. Another interesting observation was the teacher’s tendency to provide correct answers when students committed mistakes: For instance, when a student misconstrued \(2V\) as \(V \times V\), the teacher replied: “Not \(V \times V\). It’s \(V + V\),” and towards the end of the lesson, when an answer was given in a fractional format \((-\frac{4i + 3j}{5})\), the teacher asked the students to write it in an \(ai + bj\) format, and when a student suggested multiplying, the teacher responded: “I don’t multiply. I want to write it as \(ai + bj\). So how do I write it? All you have to do is separate.”

The teacher’s questions seemed geared towards getting specific pieces of information from the students. Once she showed the students how to find something, like a unit vector or a position vector, she followed her explanation with an example for the students to work, and then checked the answer with them. It is clear that her approach was based on providing the students with the definitions, demonstrating certain procedures, and allowing the students to replicate those procedures. These are the characteristics of a procedural approach that aims at training students to replicate certain steps when faced with similar problems.
Teacher P3

This teacher was identified as potentially-procedural. One of his lecture classes was observed, and a 30-minute interview was conducted with him.

Survey and Interview Responses for Teacher P3

This teacher was identified as somewhat procedurally oriented based on his answers to the survey questions: He strongly agreed that solving problem means that the student understood the concept, and he believed that basic mathematical facts need to be emphasized prior to the introduction of concepts. When asked about his approach, he replied that he employs several strategies, one of which is “illustrating/discussing a method and then having the students practice it.” However, he did state that he sometimes poses a problem and paves the way for students to solve it by giving hints. He also mentioned that he solicits students’ ideas, but often corrects their mistaken interpretations. The last two answers indicate quasi-conceptual tendencies. The teacher’s answers to the interview questions reflected some conceptual orientations. In his interview, the teacher stated that he believed his instruction to be geared for conceptual development, which he defined as the “connection throughout basics, set up axioms, agree on notions, and from that, you build the concepts based on that. It’s the connectivity between the math concepts, the notions, what they cover.” He also maintained that one of the challenges he faced was dealing with procedurally-oriented software:

I had to build the concepts. The software is not concept oriented. We don’t use textbooks, and supposedly there’s enough material online to justify not having an online textbook, but the reality is it’s just solved examples, which is basically a procedure, there is no conceptual development.

Interestingly, the teacher revealed that because time is a major constraint for him to teach the way he wants to, he opted to take his students to the lab only twice a week and increase lecture time instead. Overall, there were some differences between his survey
answers and his interview responses in terms of his teaching approach. It was necessary to attend one of his classes to get a better feel for his instructional approach.

Classroom Observation for Teacher P3

The data for this observation come mostly from the researcher’s field notes due to an equipment malfunction. The recorder was set to voice operated recording mode so it was only active when someone spoke loud or in its proximity. A few dialogues were recorded, but not the whole session.

The teacher started the class by outlining what sections will be covered in the next couple of weeks, and listing the requirements for the final exam. Working on the overhead projector, he reminded the students of the properties of a Cartesian plane, and how to determine the location of a point on it. He then explained that the location of a point in a polar plane is expressed using different variables, namely $r$ and $\theta$. He listed the equations for conversion from the polar form to the rectangular (Cartesian) form of an equation. He solved an example that asked for identifying and graphing a polar equation ($r = 4$). After converting the polar equation to the Cartesian form, the teacher graphed the resulting equation, and repeated the same procedure for another equation ($r \cos\theta = -3$). He then asked a student to come to solve another example ($r = 4 \sin\theta$). The student had some difficulty starting the solution process, so the teacher asked for students’ suggestions, and a student suggested multiplying both sides by $r$. The student at the projector did a couple of steps then made a mistake in transferring a term from one side of the equation to the other side. The teacher promptly corrected the mistake, asking the student to erase what he has written, and dictated what the student needed to write. The student continued to work on the problem while the teacher walked around in the class checking on the other students’ work. He then told the student what to write to complete the solution process. Once this example was completed, the teacher asked another student to solve another
example \( r = \frac{1}{3 - 2 \cos \theta} \) on the projector. The starting point was suggested by the teacher, so the student did the first step (cross multiplication), and then the teacher asked him to use the conversion rules \( x = r \cos \theta \) and \( r = \sqrt{x^2 + y^2} \) to change the equation to the Cartesian form. The student struggled to continue from there on, so the teacher asked the other students to help him while he continued to walk around in the class. The student spent about 15 minutes doing calculations and trying to rearrange the equation while he often followed his classmates’ suggestions. Towards the end of the problem, the teacher completed the calculation and stated that the resulting equation is the equation of an ellipse.

Emerging Themes and Analysis for Teacher P3

The targeted lesson’s objective was to identify the graphs of polar equations and convert them to the Cartesian form. The teacher started by listing the equations that can be used for conversion, and proceeded to solve an example. However, he did not ask the students to solve a similar example; instead, he asked one of the students to come to the projector and attempt to solve a slightly different problem. The student struggled to complete the solution process, and ended up taking cues from the teacher and his classmates. The teacher mainly asked the student to write the steps necessary to solve the problem. Also, the other students’ contributions were generally for identifying the next steps or correcting a mistake their classmate did. Taking these factors into consideration, it appears that the teacher was attempting an ineffective form of procedural instruction: He wanted a student to solve the problem in front of his classmates, but in selecting a different problem type, he neither made the process a repetitive practice activity, nor he provided opportunities for fruitful discussions. He wanted the students to be involved in the solution process, since he asked them to help their classmate, but there was no dialogue among
students to justify the choice of steps. In terms of effectiveness, the length of the calculations made some students lose interest in the problem (in fact one student stated that he is totally lost!).

Overall, the teacher’s approach is closest to a procedural one. In his approach, he recalled on a student to demonstrate a procedure with the help of his classmates, but the help in this observed class came in the form of stating the next step of the procedure or correcting a mistake. The teacher tried to involve the students by delegating the task of solving to them, but their involvement may not have been necessarily productive.

Cross-Case Analysis for Procedurally Identified Teachers

A cross-case analysis requires the researcher to divide the data by type across all cases investigated and examine each carefully. When a pattern from one data type is corroborated by the evidence from another, the finding is stronger. When evidence conflicts, deeper probing of the differences is necessary to identify the cause or source of conflict. This cross-case analysis will examine the consistency (or lack thereof) between the teacher’s answers and practices, the role of the teacher, and the role of the student. A summary statement on the instructional flow gives the overall characterization of procedural teaching in the margins of R2R.

Consistency between Questionnaire Data and Teacher Practices

In general, most of the teachers’ ideals expressed in their survey responses were reflected in their practice. Teacher P1 maintained that she mostly illustrated a method or a procedure in order for the students to practice it later, and she adopted this approach throughout most of her class. In general, her answers and her approach were consistent. Teacher P3’s responses to the survey and interview indicated some procedural tendencies as well as some conceptual ideals: He articulated the meaning of teaching for concepts well, and explained how he often poses problems then pave the way for students to solve
them. However, he also maintained that basic skills need to be introduced prior to concepts. In his class, he attempted to engage the students and ask them to provide feedback in helping a classmate, which is an approach conceptual teacher often follow. However, he emphasized completing the procedures and getting the correct answer in the class by constantly providing missing steps and correcting the student’s mistakes. Interestingly, the issues he brought up in his answers to the interview questions (developing concepts through connections, setting up axioms, and the connectivity between notions) were not detected in his instruction. Overall, the type of instruction observed in his class aligns itself with procedural ideals more than conceptual ones.

Teacher P2 had some discrepancies between her responses and her practices: She stated that “comparing and contrasting different solution strategies is what helps students understand a problem” (survey response), but this was not a component of her instruction. Also, she claimed that students’ ideas should form the focus of the class discussion, but most of the class discussion focused on her solving examples rather than soliciting students’ ideas. Her answer “Basic mathematical facts need to be emphasized prior to introduction of concepts” was consistent with her practices during the observed class.

Role of the Teacher

In the three cases discussed so far, the teacher was the main source of information during the lesson, though the methods of delivering this information varied. Generally, the teachers demonstrated the procedures for the students, but there were differences in the tasks assigned for the students after the explanation. One teacher (Teacher P1) solved an example for the students, but did not let them attempt similar exercises, while another (Teacher P2) attempted to verify whether the students are capable to replicate the procedure through one exercise that was attempted immediately after the example given by the teacher. The third teacher (Teacher P3) asked the students to solve a slightly
different example than the one he solved. All three teachers did not ask for students to explain or justify their approaches. When a student made a mistake, the teacher normally corrected it and moved on. Finally, all three teachers did not have a conceptual focus in their instruction, as they did not attempt to make connections between polar coordinates and geometry (Teacher P1 and P3), or provide a rationale for the formula used to calculate the magnitude of a vector or a unit vector (Teacher P2), the teachers simply showed the students effective methods to solve the examples, and in some cases, students’ concerns were remedied with a reminder to refresh their memories. While slight differences exist between the 3 teachers’ approaches, they are all centered on teacher as the provider of the correct and efficient way of solving problems.

Role of the Students

In general, the students’ role was to follow the teacher’s explanation and, replicate the procedure on another example. Generally, the students observed step-by-step explanations, and were asked to respond to some simple questions, such as calculations (Teacher P1) or understanding the meaning of expressions like 2V and 4V (Teacher P2). They sometimes asked about a formula they did not remember, or uttered an incorrect answer, and their teacher stated the formula or corrected the mistake. It seemed that the students had a slightly different role in Teacher P3’s class, as they were asked to help their classmate in solving the problem. However, they were simply telling the student at the projector what to write. Overall, the students’ involvement in all three classes observed was minimal and restricted to a few simple tasks.

Characterization of Procedurally Oriented Teaching

The main observed feature of procedurally oriented teaching was the control of the teacher over various aspects of instruction. Generally, the teacher decided on a solution process, completed the steps to solve the example, and provided the students often with a
practice example. The teacher asked the students to replicate the demonstrated procedure by solving an example similar to the one he solved. However, the teacher did not explain the conceptual underpinnings of problems, and focused on showing the students the best and most efficient approach to solve them. As for students, they were generally passive, often copying notes or attempting to solve slightly modified examples. Their occasional input was restricted to answering numerical questions or stating the next step of the procedure. When a student made a mistake, instant correction took precedence over discussion and justification.

**Conceptually-Identified Teachers**

As stated earlier, the purpose of the questionnaire was to determine the portion of the teachers who identify themselves as conceptually oriented in their instruction. A teacher, however, may not fully embrace a conceptual agenda for several reasons, the most important of which is his/her ability to correctly characterize and carry out conceptual teaching. As clarified in Chapter 2, there is a complex interplay between concepts and procedures, and teachers may have difficulties separating the two domains in certain situations. Moreover, concepts are sometimes confused in practice with meaningful procedures. In a study conducted by Simon et al. (2000), it was found that teachers struggle to develop pedagogical approaches to meet the challenges posed by the mathematics education reform. As Cady (2006) explained, “there is more to implementing the Standards in a classroom than using manipulatives and asking ‘why’ questions” (p. 460) Hence, an important part of this study was to reconcile the teachers’ theoretical perspectives on teaching for concepts (as reflected in their oral and written answers) with their practical application of those perspectives in class (as seen in the observations). Therefore, the target research foci were identified for teachers classified as potentially-conceptual:
* If the teacher’s lesson analysis reveals a conceptual agenda, then how does a conceptually-oriented teacher structure his/her instruction to achieve conceptual goals?

* Otherwise, if a teacher’s practices do not conform with his/her conceptual goals, then how is that teacher’s practice characterized?

Teacher C1

This teacher was identified as potentially-conceptual based on his answers to the survey. One of his lecture classes was observed, and a 30-minute interview was conducted with him.

Survey and Interview Responses for Teacher C1

The teacher’s answers to the questionnaire reflected a conceptual orientation: He did not assume that solving a problem correctly means that the student understood the concept behind the problem. He maintained that skills should not be practiced until there is a sound conceptual foundation, and he objected to the students’ replicating a procedure in an example “instead of understanding the conceptual basis behind it.” He also thought that while students’ ideas need to be solicited, it is important for him, as their instructor, to correct their mistaken interpretations. During the interview, this teacher stressed that conceptual development is a part of his instructional agenda, saying that he always urges the students to “take the time to focus on the conceptual basis of the problems instead of simply solving examples.” He explained the conceptual focus as

the math behind the formula, how these things go together…showing them how a certain formula came about from something else they know from before. Those are things that helped me when I was in school, and I think they help my students too.

He then re-stated his position on students solving repetitive practice problems, saying that in an exam, if you have no idea of the concept behind a problem, then a small change in the problem is going to throw you off. So if you never stopped to think
about the concept, about what you are trying to accomplish….the concepts are always your ultimate goal.

He acknowledged that this is a challenge for him, citing the similarity between the examples the students solve and the ones provided to them by the software: “The examples are nice to outline the concept…But they’re too quick to provide a similar example for them, and then they get into the habit of changing the numbers from this example to this example.” Speaking from the student’s perspective, the teacher said that the similarity in the examples makes it very easy to replicate a procedure. Expressing the students’ typical perspective, he said, “I never thought about anything that I was doing, and I don’t have to think about anything I was doing.” He also listed connecting ideas and anticipating “what comes next” as part of the students’ conceptual development he targets in class. He also stressed the importance of “finding a way to somehow get them (students) to be active, not just sitting there copying line after line.” To do so, he asks his students what he referred to as a “leading type of question, so that they can demonstrate why they’re doing the next step.”

The survey data revealed that this teacher is primarily conceptually oriented, and his interview answers reflect a desire to steer his students towards understanding concepts. However, the teacher expressed concerns about the course design, from choice of examples – “they’re too quick to provide a similar example for them, and then they get into the habit of changing the numbers” – to the time allotted for instruction – “Time is definitely an issue here, because the students don’t have the background…I would love to have more time in here!” He also acknowledged that he faces difficulty in motivating the students to think about the concepts behind the problems. These can be interpreted as indicators that his instruction may not adhere to his ideals, given that he portrayed teaching for concepts in this design as a challenge that arises from time and software constraints.
Classroom Observation for Teacher C1

The class started with the teacher listing the requirements for an upcoming test, and asking the students whether they have specific problems that they needed help with. Several students indicated a compound interest problem. The teacher started by asking students to identify what each number in the given problem represents. The teacher then wrote the formula and plugged in the given numbers to obtain the equation $\frac{5}{3} = (1.015)^t$.

He then pointed out why the students may have found this problem “tricky,” since the variable they are required to find is in the exponent. The teacher then explained the step-by-step procedure of solving this equation by applying the natural logarithm to both sides of the equation, then alternatively by applying the base 10 logarithm to both sides. A similar example was then discussed, as the teacher demonstrated how to solve a “decay” problem to find the time $t$ in the formula $A = A_0 e^{kt}$. The teacher then moved on to a new section on systems of linear equations. He started with an example ($3x - y = 2$ and $x + y = 10$) and asked for students’ feedback on the appropriate method he should follow to solve the example. After listing all possible approaches, the teacher chose the elimination method, claiming it is the most suitable choice in this case, and making an argument against the use of substitution. A student then asked how he can determine which method to use when faced with a problem, so the teacher explained through examples when the use of each of those methods (substitution and elimination) is preferable, and proceeded to solve an example where neither method is preferred, using Kramer’s rule. Throughout the solution process of the examples, the teacher basically explained and wrote the steps, while asking the students to perform some of the calculations. The students were mostly copying the steps from the board.
Emerging Themes and Analysis for Teacher C1’s Lesson

Several objectives were the focus of instruction during the lesson. First, the teacher wanted to review the procedure to solve a certain problem type (radioactive decay) while focusing also on calculator use skills. A part of the explanation suggested an interest in enabling students to draw a parallel between the compound interest formula and the decay formula. For the remainder of the lesson, the objective was to review all possible methods to solve a system of linear equations in two unknowns. Throughout the lesson, part of the objective became teaching students to use certain preferred methods with certain problem types, such as using substitution when one of the equation is in \( y = \) or \( x = \) format. All those objectives left room for the teacher to structure his instruction for conceptual understanding, notably by linking the various solution methods to the notions of functions and multiple representations. However, Teacher C1’s lesson was consistently narrowly focused on the procedural method to be acquired by the student.

The teacher’s instruction had very few elements, if any, of conceptually oriented instruction. By doing most of the talking in class, he reduced the students’ role to answering specific questions, most of which were geared to prompt the next step of the procedure. For instance, the teacher wrote an example’s equation \((500 = 300(1+0.06/4)^t)\), and proceeded as follows:

Teacher: So the amount of money you accumulated over a certain period of time is this (points at the 500), this is the initial amount of money, the deposit (points at the 300), and 0.06 is what?

Student: Interest Rate.

Teacher: It’s the interest rate. Ok, the 4 represents what?

Student: 4 times.

Teacher: In other terms the number of times per year interest is calculated. Right?
Student: Yeah.

In this case, the teacher did not question the student’s answer (4 times) and quickly interpreted it as the number of times per year interest is calculated. He then asked the students how they would go about solving this equation, but he preceded the question by steering them towards the approach they need to take:

Teacher: You need to be thinking in terms of inferences, in other words, this is an exponential term, if I need to get at this exponent here, I need to be thinking in terms of logarithms, logarithms are the inverse of exponential, right? Remember, what you need to do to one side has to be done to the other side too… What would you do?

Naturally, a student responded by suggesting taking the logarithm of both sides. Finally, even when a student made an error in the procedure, the teacher corrected the error and moved on without probing the student’s thought process that led to the mistake:

Teacher: So this is the natural log of 5/3, equals 4t times the natural log of 1.015

\[ \ln \left( \frac{5}{3} \right) = 4t \cdot \ln 1.015 \]. What do you do to both sides?

Student: Divide by 1.015?

Teacher: Why would you divide by 1.015? You want to divide both sides by, let’s say, 4 times Ln 1.015 to get t.

Though the teacher did on one or two occasions ask for a justification of an answer, it was mainly a validation of a step of the procedure that the students were applying. For instance, in a decay problem, the teacher explained the half-life of an element, and proceeded to write a problem on the board. The problem started with 50 g of an element whose half-life is 150 years. Writing 50 g \( \rightarrow \) … (after 150 years), he asked:

Teacher: What is the next number that follows?

Student: 25
Teacher: Yeah, why 25? I don’t see a 25 here.

Student: Because it’s half-life.

Finally, while the teacher stated in the interview that he wanted the students to think about their answers, he did not invite them to do so. Instead, he simply made a statement about the reasonableness of an answer without soliciting this conclusion from the students:

The answer is 221. It all makes sense, because I know it’s going to take 150 years to decay to 25 grams, so if it’ll be decaying to 18 grams, then it should take a slightly longer period of time.

In some portions of the lesson, the teacher’s dominance of the flow of the class extended to associating specific methods to certain systems of equations problems: “I’m not going to use the elimination method because if I add these two together, neither $x$ nor $y$ cancels out.” Instead of giving the students the choice of selecting a method, knowing that all of them will lead to the answer when followed correctly, he basically trained them to link a certain format in the problem to a preferred method; “You can automatically rule out the substitution method because I’m not told $x = $ or $y = $.” His approach did not take into account students’ individual learning styles, as perhaps visual learners would prefer solving a system by graphing rather than doing an algebraic procedure. In addition, he did not take the time to ascertain whether students understand the relation between solution approaches. The students’ understanding of solving systems of equation would be reduced to detecting a pattern in order to apply what would be a routine procedure: “you have to think about what is the best method to solve.”

Overall, despite survey answers suggesting that this teacher has conceptual tendencies, his approach in the lesson observed was procedurally-oriented. He demonstrated solution processes while seeking very little meaningful feedback from the
students. He steered the students towards detecting visual patterns that trigger a solution procedure, and did not explore any conceptual foundations for solving linear systems. It is worth noting that his instruction can prove to be very effective in training the students in solving certain systems of equations, but it’s clear that the students are simply following procedures rather than understanding the underlying concepts.

Teacher C2

This teacher was identified as potentially-conceptual based on her answers to the survey. One of her lecture classes was observed, and a 30-minute interview was conducted with her.

Survey and Interview Responses for Teacher C2

The teacher’s answers to the questionnaire and the survey reflected, to a certain extent, a conceptual orientation. She made it clear in both instruments that she did not believe solving a problem automatically means that the student understands the concept behind it. In fact, she complained that the students who practice a lot in the R2R format learn how to detect a problem type and associate it with a procedure: “They have learned to memorize how to get the answer without mastering the concept.” She maintained that “skills should not be practiced until there is a strong conceptual foundation” in her survey answers, and re-emphasized this point when she explained her teaching approach in the interview: “I explain the concept behind the problem, because I don’t believe they could understand how to set it up without the concept.” That goes along with her description of a typical class, where she would

  give them a problem that I wanted them to try to work on their own, without any background knowledge… after they try it, I try to get them to answer me and tell me why, what they think, without me supplying the answer. And then after that we go into the method of solving the problem.
Moreover, she determines the extent of the students’ understanding if “they can explain the concept back to me, whether they can understand methodology (procedure) or not is not important,” which indicates a priority of concepts over skills. However, she did acknowledge during the interview that “most of the problems in MathXL are about methodology, not conceptual.” This means that she sees a limitation in the extent that she can be “conceptually oriented” in the MathXL environment. The type of questions asked, and the time allotted to instruction were the two major reasons she cited for her inability to ensure all her students understood concepts: “I do my best to explain it, I put it in their terms, in common applications, but that still does not guarantee understanding, and unfortunately I got to keep going.”

Classroom Observation for Teacher C2

The data for this observation come from the researcher’s field notes. The teacher declined to give permission to record her class, stating that she would feel uncomfortable.

The teacher started the class with a bell ringer when she gave the students a linear equation in one variable and asked them what kind of answers they would get and how they would be derived. She then gave a linear equation in two variables and repeated the same question. When the students replied that both $x$ and $y$ needed to be found, she proceeded to start the lesson, which was about finding the solution for a system of equations in two unknowns. She started with an example ($2x + y = 5$ and $-4x + 6y = -2$), and asked the students what would constitute a solution for the system, and when a student answered that it’s also an $(x, y)$ ordered pair, she pointed out that it would be the ordered pair that would solve both equations. The teacher then asked the students to take a few minutes to solve the system without providing them with a hint or demonstrating the solution process. The teacher walked around among the students until they all (or almost all) finished, and she asked for a volunteer to go to the board and write his solution. The
student used the elimination method and wrote the solution procedure on the board. Then the teacher asked the students to verify their answers. She also asked if other methods can be used to solve the system. Another student volunteered to solve it using the substitution method. The teacher then asked her to interpret her findings graphically – explaining what a solution in the form \((a,b)\) means on a graph. The teacher concluded that the ordered pair solution is also a location of a point on the coordinate plane where the graphs of the two equations would meet. The teacher then asked the students to look at a word problem about a movie theater that sold two types of tickets and generated certain revenue. The students were required to find how many tickets of each type were sold. The teacher read the problem to the students and, through a series of short questions, set up the system of equations. She then solved the system using the elimination method. The last problem discussed in this class was a system that had no solution, so after the students tried to work it out, the teacher eventually stated that not all systems will lead to a solution, and that this type of systems is called an “inconsistent” system of equations.

Emerging Themes and Analysis of Teacher C2’s Lesson

The teacher introduced the topic by linking it to previously discussed notions, and led the students to think about the type of answers (ordered pair) they get from a system of equations as opposed to a linear equation in one variable. Also, the teacher made a connection between the numerical solution the students got and the location of this solution in a Cartesian plane. By doing that, she gave the numerical values the students obtained a meaningful reference on the graph. This can be seen as an attempt to make students think about how to interpret a solution for a system of two equations. Also, the teacher allowed the students to work on problems on their own. It is possible that the students have seen this type of problems in earlier courses, but the teacher did not attempt to start the class with a reminder on how to approach such problems. These indicators
suggest that the teacher may not be adopting a purely procedural approach. In the last
portion of the lesson, the teacher took a different direction. First she took the lead in
setting up the system of equations pertaining to the word problems. Then, she solved an
“inconsistent” system on the board, explained to the students the terminology she used,
and concluded that not all systems have a solution. With the movie tickets problems, she
suggested assigning $x$ as the number of tickets sold at a certain price and $y$ as the number
of tickets sold at the discounted price. She then asked the students if they can set up an
equation, reminding them that the total number of tickets is given. When the first equation
($x+y = \ldots$) is set, she then proceeded to explain how to get the second equation that
pertains to the revenue, and explained that the resulting system can be solved using any
method discussed earlier. It did not appear that the teacher was providing the students with
anything more than a demonstration on how to tackle such a problem, as there was no
discussion on where the revenue would come from. The teacher simply wrote the
equations on the board and proceeded to the solution. Moreover, she did not ask the
students to interpret the system they solved at the end of the class, but rather stated that the
systems that do not have a solution are called “inconsistent.”

The lesson’s objective was for the students to learn to apply two types of
procedures (substitution and elimination) to solve a system of linear equations in two
variables. It did not appear that the teacher had a conceptual agenda throughout the lesson,
even though she solicited students’ ideas and tried to steer them towards thinking about
the meaning behind a solution. She shifted towards demonstrating how to solve a word
problem towards the end of the class. Also, the teacher seemed eager to “show” the
students how to approach word problems rather than having them work on their own. This
falls in line with her expressed concern for classroom time during the interview.
Moreover, the teacher did not revisit the connection between the numerical solution and
the graphical representation on the last system (the inconsistent system), though she did attempt to link graphical and numerical solutions in earlier problems. Overall, the teacher seems to have some conceptual interests that she pursued only in limited fashion by making juxtapositions and references to connections. However, she did not organize instruction to solicit and work on students’ ideas, and oriented most of the lesson toward procedural goals. Her approach was closest to procedurally-oriented instruction.

Teacher C3

This teacher was identified as potentially-conceptual based on her answers to the survey. A 40-minute interview was conducted with her, and two of her classes were observed, audio-taped, and transcribed. The first class was an R2R regular class, while the second was a transitional class between the Algebra and the Trigonometry portions of the course. The stark difference between the teacher’s approaches in each class is highlighted at the end of the analysis.

Survey and Interview Responses for Teacher C3

The starting point of analysis for this case was the teacher’s perspective on conceptual teaching and the characterization of her own teaching, as displayed in her answers to the survey and the interview questions. In the survey, she maintained that “comparing and contrasting different solution strategies is what helps students understand a problem,” that “students’ ideas are solicited, but the teacher corrects their mistaken interpretations,” adding, “skills should not be practiced until there is a strong conceptual foundation.” During the interview, when asked to characterize her teaching, she labeled herself as follows:

I am somewhat conceptual, as I integrate conceptual teaching strategies. If the students don’t understand where the new concepts are coming from, and they have
a hard time making connections, I am trying to make a connection as I explain why
and how they need to understand the concept.

She also stressed the importance of the students’ need to reflect on answers in her class,
stating that “mechanics don’t necessarily mean that they understand what they’re doing,
sometimes they let the formula drive them. Plug it in, complete and get the answer. They
don’t think whether the answer is reasonable or not.”

It is already established that data from the survey show that this teacher’s thoughts
and ideals align with conceptual goals. Her answers to the interview questions also
revealed a conceptual orientation, particularly in her emphasis on how important it is that
students reflect on their answers, get involved in the teaching-learning process, and
establish connections with previously explained concepts. It was important to verify to
what extent these ideals are present in practice. Hence, two 50-minute classes were
observed, taped, and transcribed. An in-depth look at these observations follows.

Teacher C3 Classroom Observations

Lesson 1

The teacher started the class by asking students to compare the polar system
of coordinates to the Cartesian system in terms of their differences. Students came up with
a number of answers that the teacher acknowledged, however she continued to look for
“something that is so important, why the polar system is so different than the rectangular
system.” A student came up with the idea of “multiple points,” and the teacher clarified it
by explaining that it’s not “multiple points” but rather different representations for the
same point. She then introduced a new type of equation (in terms of $r$ and $\theta$, the variables
in the polar system of coordinates), and discussed each equation with the students to
determine what the equation represents and how to convert it to rectangular coordinates.
The first example was relatively simple ($r = 3$). When asked to identify the kind of
equation, a student mentioned the term “radius,” so the teacher asked the students about what comes to their minds when they hear this term, and students were quick to respond “circle.” The teacher then proceeded to graph a circle of radius 3. She then asked the students to convert this equation to the rectangular system, and a student suggested using $x = r \cos \theta$. The teacher acknowledged that there is a bit of relevance between the student’s suggestion and the equation she was looking for, and repeated the question again to give students a chance to respond again. She then wrote the standard equation of a circle on the board to remind students of it. When the students did not remember seeing this equation, she asked them to list all the relations between polar and rectangular variables, and proceeded to compare one of those equations ($r = \sqrt{x^2 + y^2}$) to the given problem. This enabled the students to solve and find an equivalent for the equation $r = 3$ in the rectangular system. A similar process was done on another equation, $\theta = 45^\circ$, then there was a gradual increase in the level of difficulty to get to the last example, which was converting $r = 4 \sin \theta$. This example gave the students a lot of difficulty, first in terms of knowing what to do to convert, as they suggested several possible avenues, but could neither justify their choice of strategy nor carry it out to a correct final answer. Moreover, the teacher spent some time re-explaining the technique used to simplify the resulting equation (completing the square). Previously-prepared notes were displayed for students to copy, and then the teacher did a couple of examples to remind the students of the steps needed to perform this procedure.

Emerging Themes and Analysis for Teacher C3 Lesson 1

The overall objective of the lesson was for students to convert equations from the polar system to the Cartesian system. To do so, the teacher led the students in a series of questions and answers, tapping into their pre-existing knowledge from the previous lesson. While no repetitive practice is involved here (examples given differed in
their difficulty as well as the type of equation needed to solve each example), the lesson appears to be geared mainly to teach students conversion methods. Had the teacher exploited the parallel between geometric constructs and polar coordinates, perhaps the lesson could have had a much more conceptual orientation. However, it appears that the lesson’s main objective was teaching routine conversion techniques as well as reviewing the “completing the square” approach (though reviewing this procedure was not planned ahead of time, it became an emerging objective prompted by the students’ inability to remember it).

Looking at the teacher’s approach, a few elements that characterize conceptual teaching were present during the lesson. The teacher constantly invoked students’ prior knowledge and asked for their feedback. At the start of the class, she asked: “What comes to your mind about the difference between the rectangular system and the polar system?” Throughout the class, she continued to put them in charge of the solution process. Questions like, “If I ask you to write an equation, what would you write?” “How do we go about this one?,” and “What can we do from here?” were often asked by the teacher. However, she was mostly targeting a specific piece of information – a formula, a definition, or a term that would enable her to explain (or re-explain) how to do the procedure at hand. For instance, several differences between the two coordinate systems were listed by some students, but the teacher kept asking for more until she got a response from one of the students that had a resemblance with what she had in mind. When the student replied “multiple points,” she reinterpreted this term as “multiple representations” and explained this difference through a brief lecture, then used this lecture as a gateway to start solving some examples. At various points during the lesson, she tried to pave the way for the students to reach an answer through hints and reminders: “Remember, when we talked about radius though?,” “You remember we talked about the equation of a circle?,”
and “Let me give you an idea then” were some of the statements made by the teacher in the course of solving examples. She did not link polar coordinates to geometric conceptions, and did not structure the class to enable students to deduce conversion rules rather than to simply memorize and apply them. Overall, the teacher’s approach had more elements of procedural demonstration than conceptually-oriented instruction.

Lesson 2

During this lesson, the teacher was attempting, in her own words, “to develop the fundamental foundation for trigonometry.” Unlike the previous class, she started the class with a question that requires reflective thinking, asking students whether having two pieces of information (2 sides, 2 angles, a side and an angle) is enough to “solve” a right-angled triangle (find the lengths of all sides and the measures of all angles). She then engaged the students in a discussion about what possible two pieces of information can be used, providing an opportunity for them to suggest possible avenues and ask questions about alternate possibilities. The discussion shifted from right-triangles to triangles in general. First, a student suggested the possibility of having the lengths of two sides and one angle. The teacher asked the students several times if this option is a viable one and why/why not. When faced with a lack of response, she proceeded to demonstrate through a counter example, where two triangles can have two sides congruent to each other respectively, and one angle of the first (not included between the two sides) congruent to an angle in the second. She drew the following figure:

![Figure 4.1: Figure drawn by teacher C3.](image-url)
In this figure, triangles ACB and ADB have the angle \( \angle A \) as a common angle and the side \( AB \) as a common side. Moreover, sides \( BC \) (in triangle ACB) and \( BD \) (in triangle ADB) are also congruent. Hence, two sides and one angle of the first triangle are respectively congruent to two sides and one angle of the second triangle. However, triangles ACB and ADB themselves are not congruent to each other, since one of them is included in the other one. The teacher chose this specific example in response to a students’ answer – that any given two sides result in solving the triangle – in order to disconfirm the answer with a counter-example.

Then, the teacher asked for other possibilities that cannot be used to prove two triangles congruent or solve a triangle, and a student suggested the AAA (angle- angle-angle) approach. The teacher used the same approach (providing a counter-example) to demonstrate that AAA is indeed not a valid method to prove congruence. She drew two “30°-60°-90°” triangles with different side lengths. The teacher then asked the students to try to remember the ratios of sides in a 30°-60°-90° triangle, and a student managed to do so with the help of the teacher. Then, the teacher used this information (by doubling each side of one triangle to get the other one) to deduce that two 30°-60°-90° triangles with the same ratios are not congruent. Once those possibilities were eliminated, the teacher proceeded to solve an example where an angle and a side were known, and used the Pythagorean Theorem and the angle sum theorem to find the length of the other sides and the measure of the remaining angle. During the last 20 minutes of the class, students did a hands-on activity, where they were asked to step outside the class and measure the distance between them and a tall object, then use a protractor to find the angle of elevation, in order to be able to use both collected pieces of information to calculate the height of the object.
Emerging Themes and Analysis for Teacher C3 Lesson 2

The class reflected a contrasting approach with the one taken by the teacher in the first lesson. As a first observation, the lesson does not appear to be skills-oriented, as at no point students were asked to repeatedly practice routine problems that would reinforce a skill. Indeed, there are several indicators to suggest that a conceptual agenda drives this lesson, namely the conceptual connection between the initial question and the activities used to confirm or disconfirm possibilities (all related to triangle congruence).

The teacher explored, in this case, the relationship between conditions for triangle congruence and the rigidity of a triangle. She tried to lead the students toward the insight that the conditions of congruence (like having the angle between the two sides in an SAS congruence) can only come from the fact that the triangle is “fixed” at its joints. So, if one cannot make different triangles from a given set of side lengths, then the conditions to prove triangle congruence are met. The continuous back and forth between solving triangles and proving triangles congruent is a clear indicator that the teacher wanted the students to understand more than just a procedure to solve a triangle, and wanted them to reach this conclusion on their own. The teacher focused on soliciting reflective answers from the students as well as establishing the connection between mathematical concepts. Both objectives reflect a conceptual agenda. It is worth noting that the teacher did not attempt such an approach in the previous lesson, though a mathematical connection could have easily been established (the geometry behind polar coordinates).

The second focal point here is analyzing the extent to which a conceptual agenda was maintained throughout the lesson. Looking at the lesson progression, one can notice that the teacher initially engaged the students in a task designed to elicit their understanding of previously explained concepts. Her initial question, whether any two pieces of information are enough to solve a right triangle, invited the students to think
about justifying a “yes” or “no” choice. Throughout the first few minutes of the lesson, she kept asking the students for feedback and justification:

Teacher: What would you like to suggest?

Student: Two sides.

Teacher: Any two sides?

Students: Yeah, ok.

Teacher: All right. So let’s put it this way.

[Teacher selects two sides that do not include the known angle]

Student: We can’t do just any two sides!

Teacher: Why couldn’t we? Why couldn’t we?

The teacher continued to ask for reasons why any two sides could not be used to solve the triangle. When faced with a lack of response from the students, however, she switched to the role of lecturer, and explained through the counter example listed above (Figure 4.1) how the knowledge of the lengths of two sides in not enough. In this particular instance, the teacher’s shifted from a student-centered approach, where “learning is occasioned by the student’s self-discovery of discrepancies that arise in engaging with an environment” (Kirshner, 2008), to a teacher-centered approach, where the teacher explains the “mature” form of the concept directly through a lecture. It is worth noting that both approaches aim at fostering students’ conceptual understanding.

Throughout the lesson, further instances in which Teacher C3 resorted to lecturing to convey conceptual content occurred. For example, she asked the students what combination of sides and angles (ASA, SAS, AAA…) is not enough to prove two triangles congruent. Initially, she tried to get a justified response from the students, as it shows in this dialogue:
Teacher: Which one of these should not be there when we’re proving 2 triangles congruent?

Student: Angle-Angle-Angle

Teacher: Right. Why?

Student: Well, you don’t have a side. It’ll be difficult to compare them.

Teacher: What does that mean? You’ve got to explain this so that everybody can understand

Student: If you have a 30-60-90 triangle, with sides like 3, 4, 5, you can still have another 30-60-90 triangle with different sides.

Teacher: I totally agree with you. I think I know what you’re trying to say…… though when you have a 3-4-5 triangle, which is a right triangle, it is not a 30-60-90 triangle. But you’re getting there.

But when the teacher asked the responding student and others to elaborate on this conclusion, they were not responsive, so she presented a possible explanation for them through an example:

Teacher: Let me just draw a picture and maybe it will jog your memory. Let me just draw a 30-60-90 triangle. Do you remember anything about the lengths of the sides in this type of triangles?

Student: I remember something like if the side opposite to 90 degrees is 2, then the side opposite to 30 degrees is…uh…

Teacher: One, that’s right. And the other one is $\sqrt{3}$. Very good. We’re going somewhere. We know that the 30-60-90 triangle is special in having this ratio. So now, I am going to expand each side twice as long. If I increase this length, that means I have to increase this one too. Remember, we talked about ratio, right? It’s multiple; you multiply or divide to maintain it…
Finally, when the sides of the bigger triangle were calculated, the teacher formulated the conclusion for the students.

Teacher: Now let’s compare the angles. It’s the same thing, isn’t it? This is the reason you guys cannot use angle-angle-angle to prove that two triangles are congruent to each other, because it could be that they are congruent, and it could be that they are similar.

Providing disconfirming cases, cases where the two given pieces of information are not enough to solve a triangle, the teacher quickly solved an example with the help of the students. She did not, however, attempt to solve other variations (for example when 2 sides are given), which is an indication that the lesson is not geared towards procedural competence.

The hands-on activity at the end of the lesson required the students to practically measure distance from an object and the angle of elevation, then use those measurements to calculate the height of an object. Several ideas were introduced during the explanation of the activity, like the equality of the angle of elevation and the angle of depression, and its relation to alternate interior angles. While it was an interesting activity that the students were eager to do, it appeared to be designed to create students’ interest more than apply the concept explained in the lesson. The teacher demonstrated to the students how to do the measurement and what each number represented, and then sent them out in groups to do the activity. Without observing the discussion about the activity (that probably took place in the next class), one cannot predict the objective the teacher was targeting with this activity.

In conclusion, several elements of conceptual teaching are present throughout the lesson; the teacher kept engaging the students, asking for justification, and connecting the lesson to previously discussed concepts. When students’ response became minimal, she
shifted to lecturing, which is still a conceptual approach, given that she wanted to provide
them with content they were unable to reach on their own. The trade-off in this approach is
that understanding the lecture requires metacognitively sophisticated students that are
capable of reshaping their understanding of the concept through the lecture. Still, the
purpose of the teacher’s lectures was to induce conceptual understanding, and in that
regards, it is fair to say that the teacher’s approach is a conceptually oriented one.

Teacher C3 Overall Characterization Based on Lessons 1 and 2

In conclusion, the teacher’s claim to be conceptually oriented in her instruction
(interview) is not always validated by her practice, since there is a clear difference
between her approach in the first lesson and the second one. Her approach was closer to
procedural instruction during the first lesson, since she constantly demonstrated how to
approach a problem without making the connection to the concept behind it. The teacher’s
lack of conceptual focus is on display when she prescribes practice as an antidote to
misunderstanding: “I don’t think you guys misunderstood at this point, but if you don’t do
any practice, then a month later you will be asking me why is y here. I want to make sure
we get it.” Clearly, “getting it” as a result of practice is not a product of a conceptual
agenda. In her second lesson, however, there is a clear conceptual focus that she pursued
throughout most of the class, first by initiating a meaningful classroom discussion, getting
students’ feedback, and asking them to justify their answers (a student centered conceptual
instruction), then by switching to a lecture mode when she no longer received responses
from the students (a teacher centered conceptual instruction). The interpretation of the
observed differences is discussed at the end of the following section.
Cross-Case Analysis for Conceptually Identified Teachers

The analysis examines the conceptually identified teachers’ questionnaire and interview responses, the role of the students in the observed classes, and the role of the teachers. The final portion of the analysis highlights the inconsistencies between the teachers’ responses and their classroom practices.

Overview of the Questionnaire and Interview Responses

The three teachers studied in this section were identified as potentially-conceptual based on their questionnaire and interview responses. They all stressed the importance of conceptual development, and objected to students’ replication of procedures without understanding the underlying concepts and reflecting on the answers. They voiced their concerns in their interviews: “There’s always the aspect of connecting ideas and also anticipation. That’s part of their conceptual development too, is anticipating what comes next” (Teacher C1); “the time I spend on concepts, it’s valuable to them because they understand why we’re doing it, and why the solutions are here and there” (Teacher C2); and “whether the answer is reasonable or not, that’s where conceptual orientation comes in. If they understand the problem conceptually, when they get a strange looking answer, they can immediately question it” (Teacher C3).

All three teachers expressed a concern for the whole class instruction time: “I try to keep to the same routine because the time I have is limited” (Teacher C1); “I do feel time is an issue, I felt I needed more classroom time” (Teacher C2); and “I also have an obstacle in knowing how much time I have, I need to convey the most important points to my students” (Teacher C3). They also complained about the software and the resemblance between the solved examples and the required problems,

I think a lot of the problems with the R2R for example or MathXL is that they’re too quick to provide a similar example for them, and then they get into the habit of:
Here’s a similar example, I (the student) am changing the numbers from this example to this example. (Teacher C1)

That’s one issue I have with the software, it’s that the students are led, they become dependent on seeing the exact types of problems over and over again, and then when they get to the test, they get something different. (Teacher C2)

Role of the Students

In general, the students’ role (for teachers C1, C2, and the first lesson of teacher C3) included following the teacher’s explanation and providing some feedback. The level of feedback was somewhat higher than simply answering numerical questions (as it was the case with procedural teachers), but it was not a sign of a focused discussion intended to challenge their preconceptions. Teacher C1 required the students’ feedback in completing procedures, and provided them with all the necessary information to conclude the next step of a procedure, or to explain why they came up with a particular numerical answer. Teacher C2 tried to invoke students’ prior knowledge, however, there was a sense of disconnect between her initial questions and the focus of the lesson. The students attempted solutions on the board and explained their approaches, but it was clear that they were capable of solving systems of equations using prescribed procedures. The teacher read, explained, and set up the equations for the word problem, leaving only the solution process to the students. Finally, Teacher C3 students were the most involved in the learning process, having to provide answers, justify their answers, and propose solution strategies. In Teacher C3’s procedural class, the students were required to be constantly interacting with the teacher, as she kept asking them what to do next and how to do it. They did eventually have to go back to taking notes and following procedures, but that was not the case in the second class. In the conceptually oriented class, the students were not only required to answer – “Any talk is better than no talk at all” – but they were also
required to justify their answers – “What does that mean? You’ve got to explain this so that everybody can understand.” They were encouraged to participate in the class, and their ideas were valued even if they gave some incorrect responses, “I totally agree with you. I think I know what you’re trying to say… I want you to know though, that when you have a 3-4-5 triangle, which is a right triangle, it is not a 30-60-90 triangle. But you’ve made a good point, you’re getting there.”

Role of the Teacher

Each case presented a slightly different role of the teacher in class. Teacher C1 opted for detailing step-by-step procedures, expressing a strong preference for certain solution approaches, and seeking little feedback from the students. Teacher C2 appeared to be looking to connect the current topic to the students’ prior knowledge, and attempted to create a meaningful relation between the solution obtained as an ordered pair and the graphical representation of the solution. However, she did not probe this connection throughout the lesson. During the second part of her class, she switched to demonstration as she read a word problem and explained how to set up the equations leading to solving it. Her approach involved the students a little, but her role involved dispensing information and displaying procedures more than initiating discussions and seeking connections.

As for teacher C3, some common characteristics of her role as a teacher were present in both classes: Invoking students’ prior knowledge, asking for feedback, and asking for suggestions on how to proceed in solving examples. The biggest difference in the classes observed was the lack of conceptual focus in her first class and the presence of a clear conceptual agenda in her second one. In the first class, she sought specific pieces of information, and proceeded to demonstrate how to use this information to complete a procedure and solve an example. The classroom discussion revolved around a term she wanted to hear or an answer she sought to find, an indication for her to continue the
solution process. In her second class, she managed to structure the discussion around getting feedback from the students, then challenge their answers (with a counter example) and allow them to struggle to find the rationale behind their answer. She kept asking for justification for their answers, laying the ground for them to figure out the link between the conditions of congruence and the rigidity of the triangle.

Inconsistency between Questionnaire Data and Teacher Practices

The first six lessons described in the study (Teachers P1, P2, P3, C1, C2, and lesson 1 for C3) all had a procedural agenda. While this was not a surprising result for the procedurally identified teachers, it certainly was not an expected outcome when observing teachers who claim to be conceptually oriented.

The difference between the conceptually-identified teachers’ theoretical perspectives and their actual practices can be interpreted in several ways: It is possible that the teachers investigated did not have a good understanding of what conceptual instruction entails, or may not interpret conceptual teaching within the same frameworks of the researcher. Also, there is a possibility that the setting they worked in (the R2R model) was not compatible with conceptually oriented instruction. The case of teacher C3 may be seen as a supporting evidence of this incompatibility. This teacher expressed a clear conceptual orientation in the interview and questionnaire, and she was observed teaching in a fashion that was consistent with conceptual goals in the non-R2R-constrained class. However, when teaching a formal R2R class, she adopted a completely procedural agenda. These contrasts, in addition to the concern over time constraints and software issues, suggest the possibility that the R2R model may be inconsistent with conceptually-oriented instruction. These interpretations represent seed hypotheses for possible further studies.
CHAPTER 5
CONCLUSIONS, DISCUSSIONS, AND RECOMMENDATIONS

Review of the Goals

The study analyzed the teaching strategies employed by the teachers implementing the R2R design, a computer based design that includes whole class instruction (25% of class time) in addition to mandatory lab time spent utilizing computer software (MathXL) to practice and submit homework assignments with the teacher’s help, and take quizzes and tests. MathXL is a procedurally-oriented software program that permits students to submit homework assignments as many times as they want, repeating similar exercises until they are satisfied with the outcome. In addition, it permits them to repeat a quiz as many as ten times, each time with a slightly modified set of problems. MathXL also provides students with help tools (like “help me solve it” or “view an example” buttons) that walk the students through step by step procedures.

The study aimed first at providing a panoramic view of the various strategies that teachers used in their R2R classes. The first phase of the study involved conducting an online survey among the 46 teachers that were following the R2R design. The questionnaire (Appendix A) probed the teachers’ ideals and beliefs, and inquired about their practices in general as well as within the R2R design. The goals of the first phase were:

- to identify the teachers in the sample polled that opt for a conceptual agenda in response to software that is procedurally oriented (Math XL)
- to identify the teachers that opt for a procedural agenda in response to MathXL, and
- to determine the proportion of each category.
The second phase of the study aimed at documenting and analyzing the varieties of methods each category of teachers (the conceptually-identified and the procedurally-identified) used to support their teaching agendas. The data collected for this phase included classroom observations and teacher interviews. The goal of this portion of the study was finding patterns and common characteristics of conceptually-oriented strategies as well as procedurally-oriented ones. Three teachers in each category were observed and interviewed, and individual case studies were created for each teacher. A cross-case analysis of each category was conducted. The conclusions reached are outlined and discussed below.

Summary of Results

34 out of 46 (74%) teachers implementing the R2R design responded to the survey. The teachers’ responses were collected. Each answer was coded as “C” if it reflected a conceptual orientation, “P” if it reflected a procedural orientation, or “QC” if it showed uncertainty about conceptual teaching. For instance, in responding to the question of how the teacher typically introduces new material, the answer “Allowing students to attempt a new problem type without showing them a routine method” was labeled as “C”, since this approach conforms with conceptual ideals, while the answer “Illustrating/discussing a method or procedure and then have students practice it with several examples” was coded as “P”, since this indicates procedural tendencies. The full range of answers with their respective codes (Appendix C) was checked for accuracy through peer examination. Each teacher’s answers were then converted to scores by assigning 2 points for a “C” answer, 1 point for a “QC” answer, and 0 points for a “P” answer. Based on their scores, 7 out of 34 teachers (20.6%) were identified as potentially-conceptual, 8 out of 34 teachers (23.5%) were identified as potentially-procedural, and the remaining 19 teachers were labeled as quasi-conceptual.
Once categories of teachers were identified, three teachers of each category were selected for observations and interviews. Each potentially conceptual teacher from the selected sample was interviewed and observed teaching an R2R class, and one of the teachers (labeled C3 in the study) was observed teaching a non-R2R class. Moreover, each potentially procedural teacher was observed teaching an R2R class, and two of those teachers were interviewed. For each category of teachers, an account of the questionnaire and interview responses is given, the followed by a summary of their classroom practices.

Potentially-Procedural Teachers’ Questionnaire and Interview Data

In general, procedurally identified teachers shared some common ideals that were expressed in their survey as well as interview responses. For instance, Teacher P1 maintained in the questionnaire that she mostly illustrated a method or a procedure in order for the students to practice it later (75% of the time), and expressed a few points in the interview that go hand in hand with this characterization, when she explained that she spends plenty of time on “some of the easier examples” and “basically work bell to bell.” Teacher P3 maintained in the questionnaire that basic mathematical facts need to be emphasized prior to introduction of concepts, and stated in the interview that in the skills part of his instruction, like solving equations, “there’s really more demonstration and less conceptual connections.” Additionally, Teacher P3 expressed some conceptual ideals in his interview answers: He stated that conceptual development means “connection throughout the basics, setting up axioms, …and building the concepts based on that,” and argued for the importance of promoting classroom dialogue among students, stating that “there needs to be a dialogue between the students. Only then you can pinpoint what they really know.”
Potentially-Procedural Teachers’ Classroom Observation Data

Generally, potentially-procedural teachers’ classroom observations highlighted some common practices as well as some methodological differences. The three teachers observed mainly selected certain examples to start the class with, showed the students how to solve each example by going through a step by step explanation, and corrected the students’ mistakes whenever they occurred. The following excerpts reflect some of the common approaches the teachers in those classes took.

- Teacher P1: (writes the equation $x^2+y^2+4x-6y+12=0$) For this equation now, you have to complete the square, since this is given to me in the general form. This is a step by step explanation. So, what do I do? First, I pair my $x^2$ and $4x$ together, and I leave a blank here next to the $4x$, if you remember the rules for completing the square, and $y^2$ is paired with $6y$, and here’s the blank. Now whatever I’m going to do to do to one side of the equal sign, I’m going to have to do for the other side. So I put two blanks on the other side.

- Teacher P2: Now we’re going to find a position vector $PQ$ and write it as something $i$ + something $j$, $ai + bj$, if $P$ (6, 2) and $Q$ (4,3). All right? Find me the position vector PQ. So we’re looking to find $a$ and $b$. So what do we do? We’re going to find $4-6$ first, because it’s terminal – initial, and to find the b, we do $3-2$. So the position vector is $-2i + j$. Got it?

Also, all three teachers corrected the students’ mistakes promptly whenever they occurred, sometimes by asking the students to go back to a certain formula or rule, other times by simply pointing out the mistake. The following excerpts reflect this observation.

- Teacher P1: What’s your $h$ and $k$?

Student: It’s 1, right?
Teacher P1: Well, go back to that formula sheet. It says 
\((x-h)^2 + (y-k)^2 = r^2\), and you’re given 
\(x^2 + y^2 = 9\), so there’s nothing next to the \(x\) and \(y\). So the center is \((0,0)\).

What do we call that?

Student: The origin.

- Teacher P2: From the initial point here to the terminal. Now we want to draw \(2V\).

What is \(2V\)?

Student: \(V\) times \(V\).

Teacher P2: Not \(V\) times \(V\), \(V+V\).

- Teacher P2: So the unit vector will be \(\frac{-4i + 3j}{5}\). So what do we do with it? The vector is written as, just like the complex number, \(ai + bj\).

Student: Multiply.

Teacher P2: I don’t multiply. I want to write it as \(ai + bj\). So how do I write it? All you have to do is separate

Some differences in the approaches of the potentially-procedural teachers were observed by looking at the tasks assigned for the students after the explanation. One teacher (Teacher P1) solved examples and demonstrated procedures throughout the whole class, while another (Teacher P2) gave the students a similar exercise for them to attempt every time she completed explaining a procedure. The third teacher (Teacher P3) asked the students to solve a slightly different example than the one he solved.

Compatibility of Procedural Teachers’ Answers with their Practices

Generally, the teachers’ responses to the typical lecture description survey question did not provide a lot of data to compare their practices with their ideals: Teacher P1 explained that she lectures for 2 days a week, while the students go to the lab for 3 days.
Teacher P2 explained that she generally introduced one or more sections in order to “develop the concepts and work example problems.” Teacher P3 stated that his lesson normally included presentation of the context of the problem and discussion of the major theorems involved. However, in the interview answers, teacher P1 described her class in a manner consistent with her observed approach: “I write notes to cover the general ideas. For example, I would give them examples on linear equations, the definition of what a linear equation is, and show the different forms of it, pick a formula and go with it, and then from there we would work with it.” Teacher P3 described his instruction as dependent on what “phase” of the course he was in: “At the beginning, it’s mostly explaining where it came from, and then when it gets to the skills part, like solving equations, there’s really more demonstration and less conceptual connections unless there is a problem.” His classroom observation reflected practices that were consistent with the ones he described in the interview, given that he allowed students to demonstrate procedures during his class.

Potentially-Conceptual Teachers’ Questionnaire and Interview Data

The teachers’ answers to the questionnaire reflected a conceptual orientation: All three answered that solving a problem correctly does not mean that the student understood the concept behind the problem. Two of them maintained that skills should not be practiced until there is a sound conceptual foundation, and one of them stated that concepts can be taught regardless of the skill level. All three put an emphasis on soliciting students’ ideas, however only one of them answered that “students’ ideas, right or wrong, form the focus of class discussions”, while the other two stated that “students’ ideas are solicited, but the teacher corrects their mistaken interpretations.” In their interview answers, they stressed the importance of conceptual development: Teacher C1 stressed that he highlights the conceptual basis of the problems: “It’s how the things go together, if
you have a formula, where it came from, you know. It’s showing them how a certain
formula came about from something they know from before.” Teacher C2 expressed her
notion of conceptual development as “to show them the theory behind the problem that
they’re working, and why we are seeking to find a way to work these problems, and using
this method, why it was developed, where it’s relevant in real life to them.” All three
teachers objected to students’ replication of procedures without understanding the
underlying concepts and reflecting on the answers. They voiced their concerns in their
interview responses: “There’s always the aspect of connecting ideas and also anticipation.
That’s part of their conceptual development too, is anticipating what comes next” (Teacher
C1); “the time I spend on concepts, it’s valuable to them because they understand why
we’re doing it, and why the solutions are here and there” (Teacher C2); and “whether the
answer is reasonable or not, that’s where conceptual orientation comes in. If they
understand the problem conceptually, when they get a strange looking answer, they can
immediately question it” (Teacher C3).

Potentially-Conceptual Teachers’ Classroom Observations Data

The potentially-conceptual teachers’ classroom practices revealed two types of
practices: The first type was observed during the three teachers’ R2R classes, while the
other type of instruction was seen in Teacher C3’s non-R2R class. Each type of practices
is detailed next.

Classroom Observations for the R2R Classes

The classroom practices observed had some common characteristics as well
as a few notable differences. Generally, teachers in those classes often detailed some step-
by-step procedures, and corrected students’ misconceptions. Some of the teaching
instances that reflect these practices are highlighted in the quotes below:
- Teacher C1: you want to try to find a way to combine the two equations so that either $x$ or $y$ cancels out, and you can set it up either as addition or subtraction. The goal is to either add or subtract these equations so that $x$ or $y$ gets cancelled. Notice how $y$ cancels out when I add them. So what’s $3x+x$? …

- Teacher C1: So the equation becomes $\ln \frac{5}{3} = 4t \ln (1.015)$. Now, what do I do to both sides?

Student: Divide by 1.015?

Teacher: Why would I divide by 1.015? You want to divide both sides by, let’s say, 4 times $\ln 1.015$ to get $t$, right?

- Teacher C3: Can anyone tell me the equation of this in rectangular system? This is your polar equation $r = 3$.

Student: $x = r \cos \theta$

Teacher: Ok, you want to use that conversion equation. There’s a bit of relevance there, but I want you to remember, we talked about the equation of a circle. (Writes $(x-h)^2 + (y-k)^2 = r^2$). Remember? It’s not coming back to you?

A common practice during all those classes was involving students, though the level of involvement varied: Teacher C1 asked students to produce answers to numerical questions and complete the following steps of procedures: “So what’s $3x+x$?...So $x$ is equal to what?...How do I use this information to find $y$?...What do I do to both sides?” Teacher C2 asked students to solve problems on the board and explain the solution process to the rest of the class. Teacher C3 asked often students to find a way to get an answer, but proceeded to explain the method when students did not find the correct path: She started off with questions likes “So my question is how do I get from here to
here?...So what do I do here?...” then tried to clarify issues and give hints: “Let me give you an idea then… I’m going to draw the line, this is 45°…”

Classroom Observation for Teacher C3’s non-R2R Class

The teacher started the class with a thought-provoking question that prompted the students to participate: “So I’m asking, as long as I give you two pieces of information, is every triangle solvable?” The students were given time to think about the answer and were asked to justify it: When a student expressed that not any two sides are enough to solve a triangle, he was asked to explain why. The teacher encouraged the students to elaborate on their answers, and provided some positive feedback: “I totally agree with you. I think I know what you’re trying to say.” At some point, the students did not find a correct approach, so the teacher proposed a counter-example and asked the students whether their hypothesis works in that particular instant: “Let me just draw a picture and maybe it will jog your memory. Let me just draw a 30°-60°-90° triangle.” The teacher then proceeded to work the example with the students while constantly asking for their input. The teacher then stated the conclusion from the result of the counter example, and asked whether the students recognized why this conclusion was reached.

Compatibility of Conceptual Teachers’ Answers with their Practices

During their interviews, the potentially-conceptual teachers were asked to describe a typical lecture in R2R. Their answers were not generally compatible with their practices. Teacher C1 explained that he usually “outlines any relevant formula that they need to know, so that they have them down for the class period, because I want them to be thinking about where they might come into play later, you know, thinking ahead just a little bit. I usually outline the formulas anyway, then depending on what the lesson is, because they’re always changing, I always think it’s important for them to see things
algebraically, numerically, and graphically.” However, this was not reflected in his classroom instruction, as he was not exposing the students to a variety of methods. Teacher C2 also expressed a focus on conceptual development in the interview, but that was not present in her instruction. Teacher C3’s described lesson had more details on the content of the class than the method of delivery, so it was not clear whether her instruction was consistent with her explanation: “I introduce the sin of 30° or 45° and such, I go back to geometry, geometry of a 45, 45, 90 and 30, 60, 90 triangles. So we talk about the ratio in those special triangles. I actually draw a triangle, an isosceles right triangle on the board, and ask my students to name it specifically. I make sure they understand why it’s called an isosceles right triangle.”

Conclusions

The focus of the study was identifying the characteristics of conceptually-oriented instruction as opposed to the practices of procedurally-oriented instruction. Understanding differences between teachers’ expressed tendencies and actual practices emerged as an additional focal point of the analysis.

Characteristics of Procedurally-Oriented Instruction

The results showed that the potentially-procedural teachers following the R2R design had procedurally oriented ideals that were reflected in their classroom instruction. The presence of several procedural indicators in the observations data helped reaching this conclusion. Generally, the teacher in this group was always the source of information in starting a problem, deciding on the approach, setting up equations, and completing procedures. The teacher also corrected the students’ mistakes promptly whenever they occurred, and asked the students to answer numerical questions and complete procedures. This type of instruction is closest to what Hiebert & Lefevre (1986) referred to as “chains of prescription for manipulating symbols” (p.8). A missing component of this instruction
was the justification, the “why” factor, the explanation of the rationale behind taking a certain approach to solve an example. As Star (2005) explained, demonstrating meaningless procedures without “figuring out” the logic behind them leads to superficial procedural knowledge (as opposed to deep procedural knowledge that is associated with comprehension and critical judgment). Students in these classes are learning by “habituation.” Habituation is the learning of routine problems through “repeated practice [that] leads to gradual adjustment to task constraints” (Kirshner, 2002, p.50). The basic premise of habituated learning is succinctly summarized by Saxon: “You learn to work problems by working them repetitively, over a long period of time” (John Saxon, quoted in Hill, 1993, p.26). The type of instruction observed in the procedurally-identified teachers’ classes suggests that they were following a “teacher centered” habituationist pedagogy, an approach in which the teacher is responsible for creating the tasks that are meant to engage the students. This is confirmed by the teachers’ tendency, throughout all the observed classes, to select the problem, the solution strategy, and the solution process.

Characteristics of Conceptually-Oriented Instruction

The results show that the potentially-conceptual teachers following the R2R design had conceptually-oriented ideals (detected in their responses to the survey and interview questions) that were NOT reflected in their classroom practices. The common themes in the answers provided by this group of teachers were the focus on establishing meanings behind the problems and soliciting students’ ideas. Surprisingly, neither theme was detected in their classroom instruction: The teachers were mainly providing solution steps, and students’ ideas were not sought and discussed often. This discrepancy will be discussed later on in details.
One of the teachers (Teacher C3) was observed during a non-R2R class. The characteristics of her instruction during that class are detailed next.

Teacher C3’s non-R2R class contained a number of conceptual indicators: The teacher had a clear idea of how to structure her instruction to achieve conceptual goals. The students were requested to think about the content of examples, and their prior knowledge was often solicited and connected to the current example, as the teacher often build up solutions from existing and related notions. It is worth noting that the presence of conceptual indicators like invoking students’ prior knowledge, asking for feedback, and asking for suggestions on how to proceed in solving examples is not enough to characterize instruction as conceptual.

What enabled labeling teacher C3’s non R2R class was the presence of a conceptual focus in the lesson. In this case, the teacher attempted to establish a link between the conditions of congruence and the rigidity of a triangle. She wanted the students to understand that the conditions of congruence stem from the fact that a triangle is a “rigid” polygon, fixed at the joints, and when two triangles have equal side lengths respectively, then they must be congruent (unlike quadrilaterals for examples, where a square and a rhombus can have equal side lengths respectively but they are definitely not congruent). In that regard, the conceptual instruction observed in her class is characterized by the presence of an underlying concept (the origin of the conditions of congruence) that the teacher tried to enable students to understand. To do so, she designed activities, questions, and examples that provoked the students’ thoughts and challenged their current understanding of the topic. Through probing questions, the teacher tried to lead the students to discover discrepancies, think about content, and link the problem to earlier knowledge.
The teacher often attempted to guide the students with questions and hints. At a certain point during the instruction, she shifted to the direct explanation of the concept by lecturing. This approach can be used to get across conceptual content to students only if they are meta-cognitively sophisticated, that is students who are capable of noticing the difference between the explained concepts and their own understanding, and can work on their own to resolve those differences.

Compatibility of R2R and Conceptually Oriented Instruction

As stated in the methodology chapter, one of the objectives was to investigate the strategies teachers choose to adopt in response to procedurally oriented software. Given the procedural orientation of MathXL, finding teachers who were able to counter the tendencies of the software with conceptually oriented instruction was expected to be a challenge. The pool of potentially conceptual teachers consisted of 7 teachers who scored between 10 and 12 points (out of 16) on their questionnaire answers. One of those teachers declined further involvement with the study, which left the researcher with six cases to be studied. Two of those cases were teachers located in Alexandria, under the supervision of LSU-A department of Math, and followed a similar design to R2R in their courses. That narrowed the potential conceptual cases to four, so three of those four teachers (who were observed and interviewed in the study) constituted a respectable percentage of the potentially-conceptual population. It was concluded that the first two teachers (C1 and C2) followed a procedural approach in their classes, which contradicted the orientation they expressed in writing (questionnaire) and verbally (interview). The case of teacher C3 was specifically significant, since the teacher adopted a procedural approach in one of her classes, a regular R2R class, but was clearly oriented by a conceptual agenda in the other observed lesson, which was not bound by the same R2R constraints. The second observed lesson was done during a transition period between the Algebra portion (Math 1021) and
the trigonometry portion (Math 1022) of R2R. The teacher was, according to her interview, developing the fundamental foundations for the trigonometry portion.

In addition, one of the challenges of teaching for conceptual understanding is that the enormous effort the teacher needs to make to orchestrate a conceptually oriented lesson properly. The teacher usually needs first to probe the students’ current understanding of a concept, then design tasks that challenge their current conceptions, and attempt to lead them on a hypothetical learning path to refine their understanding. This is all done through questioning, reasoning, and justifying strategies. Incorrect student strategies play as important a role as their correct approaches, and the teacher need to be able to let the students reach a decision on the soundness or incorrectness of an approach. This type of instruction is more difficult to implement with tight time constraints, since various ideas and unexpected issues emerge during classroom discussions.

Given that all the conceptually identified teachers expressed a concern for the lack of time for whole class instruction, the findings of the study suggest that the conceptually-identified teachers’—those expressing a clear understanding of and orientation toward conceptual instruction in their questionnaire and interview responses—consistent failure to enact a conceptual agenda in the observed classes, owes to the time constraints of instruction imposed by the R2R curriculum approach. This is a possible interpretation for the discrepancy between the teachers’ responses to the survey and interview questions and their classroom practices. That the sole conceptually oriented lesson observed in this study occurred for Teacher C3 during a portion of the course after R2R obligations already had been met is another indicator that supports this conclusion. However, other interpretations for this discrepancy, such as teachers’ misunderstanding of conceptual teaching practices, are possible. This provides the ground for further exploration in future studies.
The established procedural orientation of MathXL, the limited instruction time, and the observed difference in one teacher’s orientation when the R2R constraints were removed present a valid argument for the incompatibility of R2R with conceptually oriented instruction. It is possible that the current design of the R2R course delivery system leaves little room for the teachers to structure their instruction to attain conceptual goals, but, as stated earlier, this hypothesis needs further probing to enable its generalizability.

Implications and Limitations

Implications for Research and Practice

The study presents several opportunities to explore from both research and practice perspectives. From a theoretical perspective, the findings of this study can be extended to probe the current debate on procedural and conceptual knowledge. As discussed in the second chapter, proponents of procedural knowledge argue for investigating and promoting “deep procedures,” that is procedures that are rich in connections, independent of concepts, and “associated with comprehension, flexibility and critical judgment” (Star, 2005, p.408). The R2R design presents a possible avenue to explore whether constructing meaning for the procedures would have a positive effect on the students’ ability to choose the appropriate and effective procedure for solution without depending on visual clues or demonstrations. This could require some change in the software design to include various approaches for solving a problem.

Moreover, the study established that teachers who were identified as potentially conceptual did not have a conceptual agenda while teaching in the R2R design. This conclusion can spawn a number of related studies. For instance, a possible explanation for this result may be teachers’ understanding, or lack thereof, of the subtleties of conceptual instruction. Further studies can investigate teachers’ understanding of conceptual
instruction through surveys, interviews, and observations. A possible avenue is to interview conceptually aspiring teachers prior to an observation, then follow up with further interviewing that probes the specific instances observed in their instruction.

Limitations

This section discusses the various limitations of the study. The basic question addressed here is to what extent one can trust the findings. Various research strategies used in the study are outlined, and possible improvements are suggested.

In any educational study, the validity of the results is a concern. Validity is generally concerned with the study’s success at measuring what the researcher set out to measure. As Merriam (1998) explained, validity deals with the question “Do the findings capture what is really there? Are investigators observing and measuring what they think they are measuring?” (p. 166). Since the study employed a mixed methods approach, there were threats to both internal validity (validity of the inferences) and external validity (validity of the generalizations and conclusions). Tashakkori and Teddlie (2003) proposed the term “inference quality” to include both types of validity mentioned here. Basically, a researcher needs to check the degree to which the interpretations and conclusions made meet the acceptable standards of rigor and trustworthiness.

Several strategies were used to ensure the validity of the study: First, multiple data sources (instrument, observations, and interviews) were used to generate the results. This technique, known as “triangulation”, ensures that the explanation of the phenomenon studies is not the product of one method or one piece of evidence. That is why in the procedural teachers’ cases, the analysis of their practices confirmed the conclusions reached from analyzing their questionnaire and interview data. The combination and comparison of the three data sources enabled reaching a well grounded conclusion. As for conceptually identified teachers, data from their questionnaire and interview responses
highlighted their concerns for time constraints and the software orientation. Thus, when observations data suggested the use of procedural approaches in their classes, it enabled the researcher to find a reasonable ground to interpret the findings. Finally, Teacher C3’s practices in a non-R2R lesson confirmed the identification made as a result of the instrument and interview data.

In addition to triangulation, member checks were conducted. Member checking “involves asking participants and other members of the social scene to check on the accuracy of the themes, interpretations, and conclusions.” (Teddlie & Tashakkori, 2009, p. 295). In this study, the lessons observed and transcribed were coded, and the generated codes were checked by the teachers to ensure that the researcher’s interpretation matched the teacher’s intentions. Since one of the teachers (whose lesson was transcribed) was not available for member checking, the inability to check the accuracy of the coding and conclusions about her lesson is a limitation of the study.

A third technique used to verify the validity of the results was peer examination or “peer debriefing” (Teddlie & Tashakkori, 2009, p. 295). As Merriam (1998) summarized, this technique involves “asking colleagues to comment on the findings as they emerge.” (p. 169). In this study, two graduate students (Ph.D. candidates) looked at the coding of the classes’ transcripts and read a summary of the research findings and conclusions. Each made some suggestions regarding the coding, and both agreed that the conclusions drawn appear to be adequately deduced from the data. Moreover, the coding of the instrument answers was checked and approved by the major professor supervising the study.

These techniques contributed in making the findings of the study credible. However, just like any research, a number of steps could have been taken to increase the trustworthiness of the findings, namely refining the instrument used and collecting more data. The questionnaire used had a number of questions that were relevant to individual
teachers’ profiles, but those questions were not helpful for the analysis of teachers’ orientations. In retrospect, the researcher could have field tested the instrument a few times before conducting the study. It was only tested once prior to its administration.

Moreover, a number of observations (Teachers C1, C2, and P1) were done about 2 weeks prior to the course final exam. This raises questions as to whether the pressure to complete the required material prior to the final exam has affected the teacher’s instruction. This could have skewed the conclusion drawn from the available data. Also, a number of additional classes could have been observed, especially for Teacher C3. However, time constraints played a big role in limiting the number of possible observations, as many of the classes observed were towards the end of a semester or a course.
REFERENCES


Section I

1. Age
   a) 25 or below
   b) 26 – 30 years
   c) 31 – 35 years
   d) 36 – 40 years
   e) 41 – 45 years
   f) 46 – 50 years
   g) 51 or above

2. Sex
   a) Male
   b) Female

3. Highest level of education attained
   a) Bachelor degree
   b) Masters degree
   c) Specialist degree
   d) Ph.D.
   e) Other (please specify) _________________________

4. Subjects in which you are certified
   a) Mathematics
   b) Science
   c) Other subjects (please specify)____________________________
   d) I am not certified

5. Number of years teaching mathematics
   a) 1st year
   b) 1 to 3 years
   c) 4 to 6 years
   d) 7 to 9 years
   e) 10 to 12 years
   f) More than 12 years

6. Number of years at current job
   a) 1st year
   b) 1 to 3 years
   c) 4 to 6 years
   d) 7 to 9 years
   e) 10 to 12 years
   f) More than 12 years
Section II

1. Mathematics is about problem solving. If a student can solve a type of problem correctly every time, that indicates to me he/she has understood the content
   a) Strongly agree
   b) Agree
   c) Neither agree or disagree
   d) Disagree
   e) Strongly disagree

2. I believe that
   a) Skills need to be emphasized prior to introduction of concepts
   b) Concepts can be taught regardless of skill acquisition level
   c) Skills should not be practiced until there is a sound conceptual foundation

3. I believe that
   a) Comparing and contrasting different solution strategies is what helps students understand a problem.
   b) Multiple correct solution strategies may exist for a certain problem. This provides the teacher with the opportunity to have students appreciate the easiest, most efficient method.
   c) It’s best to focus on the one approach that enables students to solve a problem efficiently and correctly. Introducing alternative methods is likely to confuse students.

4. Classroom dialogue is most effective for student learning if
   a) Students’ ideas, correct or incorrect, form the focus of class discussions.
   b) Students’ ideas are solicited, but the teacher corrects their mistaken interpretations
   c) The teacher presents the correct ideas in a clearly organized and deliberate fashion.

5. Prior to R2R, I would typically introduce new material by
   a) Illustrating/discussing a method or procedure and then have students practice it with several examples
   b) Posing a problem then paving the way for students to solve it by giving hints and suggesting possible starting points
   c) Allowing students to attempt a new problem type without showing them a routine method, and then having them discuss their solution approaches
   d) Other approach (please specify)

6. In addition to your typical introduction to new material (question 4 i), do you sometimes use other approaches? If so, indicate which methods (a, b, c, or d), and the approximate percentage of time for each.
Section III

1. Is this your first semester teaching using the R2R approach?
   a) Yes
   b) No

2. How did you become involved in the R2R program?
   a) Volunteered after hearing about it from a friend/coworker
   b) Volunteered after I came across information about it online
   c) It was mandated by the school/school district
   d) Other (Please specify)_______________________________

3. When you signed up (or were assigned) for R2R (prior to adopting it), did you expect this approach to be_______ (you may indicate more than one).
   a) Helpful for students because it is engaging
   b) Helpful for students because it is geared towards enhancing their content knowledge
   c) Helpful at some level but not helpful on some others (please explain)
   d) Not helpful because of its structure (class time vs. computer time)
   e) Not helpful because of its content (topics, homework problems, quizzes…)

4. What perceived benefits, if any, do you see for using the R2R approach?
   a) Increases students’ participation in learning
   b) Increases students’ enjoyment of learning
   c) Better for students’ understanding of the content
   d) Better for students’ mastery of skills
   e) Less work for the teacher in preparation and planning
   f) Other (please specify) _________________________________

5. What perceived disadvantages, if any, do you see for using the R2R approach?
   a. Decreases students’ participation in learning
   b. Decreases students’ enjoyment of learning
   c. Is not helpful in enabling students’ understanding of the content
   d. Is not helpful in enabling students’ mastery of skills
   e. More work for the teacher in preparation and planning
   f. Other (please specify) _________________________________
6. The R2R approach requires students spending about 75% of the time working on their computers in class. Are you satisfied with the time allotted for the teacher within this approach? Why or why not?
   a) Yes,______________________________________________
   b) No,______________________________________________

7. Describe a typical lecture time in your class in terms of what you and the students regularly do.
   ______________________________________________________

8. What problems, if any, did you have with the way R2R is implemented? (You may check more than one)
   a) Software issues such as __________________________________
   b) Teaching strategies issues such as ___________________________
   c) Content issues such as _____________________________________
   d) Other (Please specify) _____________________________________

9. The R2R approach…
   a) did not meet my expectations
   b) met my expectations
   c) exceeded my expectations
   Comment (optional) __________________________________________

10. If you were the “R2R Czar”, how would you organize the program to more effectively meet its objectives?
   _____________________________________________________________________
APPENDIX B

INTERVIEW GUIDE / INTERVIEW TRANSCRIPTS

Interview Guide

1. Your responses to the questionnaire suggested an interest in students' conceptual development in math rather than just their skill with routine problems. Would you say this is a correct characterization of your teaching goals? What does conceptual development in math mean to you?

2. Teaching for concepts always is challenging, but perhaps this is especially true for R2R instruction. What sorts of special challenges related to conceptual goals do you find associated with the R2R model?

3. Please describe for me what is it that you do in a typical R2R class and how it links to your conceptual goals. Describe a lesson that you’ve given and addressed in it conceptual goals.

4. How do students interact with you and with each other during the class?

5. How do you verify that the students have understood a concept?
Teacher P1

H.S.: So, I’m going to tell you briefly about my study. I’m trying to look at the teaching strategies within the margins of R2R. So, in the beginning, if you can tell me what are the things you do in class in a typical normal lecture day?

T.: Basically, what I have done is I’ve gone through and looked at all the homework in a particular section, and then I write notes that cover general ideas for that. For example I would give them examples on linear equations. I would give them the definition of what linear equation is, and show the different forms of it, pick a formula and go with it. And then from there we would work examples, we talked about slopes and were trying to interpret what is the slope and what is the y-intercept. So, I have, because of our time constraints, I feel like there’s a lot of material to cover, especially in a high school setting, where they’re very distracted with football games and pep rallies and everything. I have photocopied out my notes, and the examples we do are in the book. That way they’re not worried about writing when I’m explaining the concept, they could watch me, they could ask questions. And then it comes time to work examples, if they know it, they can work on ahead, if they don’t know it, they can watch me, and then I give them time to fill it in.

H.S.: Can you tell me a little about the students’ interaction in class? Do they help each other…?

T.: Absolutely. I am amazed at how well they have taken to this. They’re more than willing to help each other in the lab. In fact, many times they’ll turn to a fellow student
and ask for help, and then if they can’t figure it out they’ll ask me. Now I have a few students that of course always ask me, just because you always have those kids, they won’t take anyone’s opinion but mine. But they work together all the time, and they do the job, they really do.

H.S.: When they ask you, do you kind of re-iterate what you were saying, or do you try to get them to think about it?

T.: I try to have them “pull it out” if I can. But sometimes they are absolutely lost, and so they do need directions, and it’s like “what do you think we should do next” type of question. But I try to pull it out.

H.S.: Do they come and solve problems on the board?

T.: Not really, because I have found… I think I probably teach too much. I’m probably spending more time maybe on some of the easier examples than I’m supposed to. Because they told us this summer, you give the concept and you do the hardest problem …[unclear]. But I found I can’t do this because I don’t have just the cream of the crop students, I have regular advanced math kids, and some of them barely scraped by Algebra II with a “C”, and I can’t do the hardest “solve the rational equation” till I do a couple of easy ones, so they can take the concept and try the hard ones. So, there’s not a lot of time for them to go to the board because I am basically working from bell to bell, and most of their showing is when they’re working with somebody next to them when they’re in the lab. And that is a problem because in a regular class, and that’s what I used to do when went over homework, I made them put them up, because they see me work, and they need to see somebody else work. I don’t have time to do that.
H.S.: Ok. Your answers to the questionnaire showed that you’re not interested in just skills. So to you, mathematically speaking, how can you tell that a student understood a concept?

T.: Well, because they’ve taken maybe a simple example I have done in class and have been able to tie in…And that’s something I do like about this program, it ties in. Right now, at the end of the course, it’s using substitution that they’ve learned in chapter 1, and they can take that, and take their factoring, and they know by looking at it, the type of problem it is, the basic skills that they have learned before that they may need as the underlying basis for what they are doing, they can tie all those things together. That’s how I tell they understood the concept.

H.S.: Were there any special challenges related to teaching for conceptual understanding that you found within this model? Did the R2R model put any obstacles in your objectives to teach for concepts?

T.: I think the time constraints. Because of time constraints, there’s not a lot of discovery type of activities that I used to do in advanced math, especially with the trigonometry, like constructing a graph from a unit circle. I’m not going to have time to do that kind of stuff, and this goes to Algebra too.

H.S.: Are you going to have more time in the trig?

T.: I don’t think so. I’m thinking maybe at the beginning. But I think they need me there when they’re starting the homework.
H.S.: Basically, you have a certain number of questions that need to be covered for the homework assignment, and then there is a quiz, and then you move on to the next section, and it accumulates till there is a test. So, the time constraints you mentioned, does it mean that you don’t determine your own pace?

T.: It has to be done by the end of the semester, and if I don’t get to it, they won’t have the final exam and they won’t get the college credit.

H.S.: So are there any trade-offs when it comes to exploring a certain topic in depth with the students at a certain time?

T.: Yes, absolutely. There’s times when you probably need a couple of more days. I really need a couple of more days on this, but I can’t do it because I’ll have a test scheduled in two weeks, and I have to do this, this, and that before the test.

H.S.: So how can you tell for a certain topic? Can you give me a specific example?

T.: Because when they are… For example, we just took the test on logarithms and my students did horribly on this test, and I had a feeling they were going to, because when they’re doing their homework and their quizzes, they can have their notebooks open, and you can tell when they’re not understanding something when they quickly use the “help me” key all the time, which means they really don’t know what they’re doing, they’re just mimicking what’s on the screen. So they make it through the quiz that way, because they have their notebooks in front of them. And I keep telling them that when you do a practice
test, you need to close the notebook to tell what you know from what you don’t know. And they were barely getting through it, so I knew we probably needed another week, absolutely we needed another week.

H.S.: Is there anything else you can tell me about how R2R is affecting, positively or negatively, your teaching approach? The whole structure of the program (besides the time).

T.: The main negative I can say is the time and the lack of it. I think it’s got many more positives: I think my students are retaining much more than they ever had in the past, because they’re forced to do their homework, even if they spent no time at home, and they worked only in the lab with me, they’re forced to do it because I’m constantly circulating, and if you’re digressing into something else, I’ll catch you and you’re back at it again. They’re helping each other and when they help each other they’re learning, whether they want to or not. They’re very excited that they’re college kids in a high school setting, so they’re very excited.

H.S.: Ok, I guess that’s just it. Thank you for your time.

T.: Ok.
Teacher P3

H.S.: So, I told you briefly about the study earlier. My first question for you is would you say that conceptual development is the most important goal of your instruction?

T.: yes.

H.S.: So what does conceptual development mean to you?

T.: It means connection throughout basics, set up axioms, agree on notions, and from that, you build the concepts based on that. It’s the connectivity between the math concepts, the notions, what they cover, the whole material connectedness, and where you can go from one concept to another, how the links are developed.

H.S.: Ok. Within the classroom you also have to attend to skills too. How do you do that and what proportion of the class time does it usually take?

T.: Even the skills are related to concepts, so when I attend to skills it’s more rooted in concepts. It’s almost a habit that there is a constant revision and upgrading, even when you’re doing concepts. You have to always upgrade and review.

H.S.: Ok. Teaching for concepts is always a challenge, but where there any special or additional challenges that you found by using this model?

T.: I had to build the concepts. The software is not concept oriented. We don’t use textbooks, and supposedly there’s enough material online to justify not having an online
textbook, but the reality is it’s just solved examples, which is basically a procedure, there is no conceptual development. If the kids have a textbook, conservatively 25% will read the beginning, how’s this related, what has it made me think, where did this come from. There is no such thing in the exercises.

H.S.: So how did this affect your instruction?

T.: It takes longer. I have to explain everything without a textbook. Before that, I could depend on a textbook to supplement what I say. Now there is no supplement, I have to do it all. If I don’t do it, it won’t get done. Some concepts…it’s ok, because somewhere along the line somebody explained it, like quadratic, they’ve dealt with it before. But for most of them, logarithmic functions, exponential functions, decay, this is the first time they see it, and this is so hard because I have to explain everything from the beginning, and that takes a long time. Now they supposedly, the way the program is set up, have to be in the lab 3 days a week. I take them 2 days a week and that’s too much. Because 3 days a week I lecture to get the concepts, the ideas to their heads, and 2 days in the lab is what they get. You have to do this because they’ll memorize it and a week from now, they just won’t know it. It’s like they’ve never done it!

H.S.: So you’re increasing the amount of time for lecturing. Is that benefiting your students more?

T.: It is. Because they throw the question and it’s answered right or wrong! Somebody has to chime in, because the class discussion, that’s missing. That’s important, because there
needs to be a dialogue between the students. Only then you can pinpoint what they really know.

H.S.: Ok. Maybe you can describe for me what is it that you do in a typical class and how it links to your conceptual goals.

T.: Well, it depends on which “phase” you’re in. In the introductory phase, I was doing something that they already know. Like for example exponential functions, you start with where it came from, compound interest, then you build that to the number $e$, then the general exponential function, then we build to the inverse of the exponential function, laying the ground to what’s coming in the logarithmic function, then you move to the exponential equations. At the beginning, it’s mostly explaining where it came from, and then when it gets to the skills part, like solving equations, there’s really more demonstration and less conceptual connections unless there is a problem. For example, if an equation looks like $xe^{ax}$, they sit there and stare at it, and I say the solution is easy, because we have 2 factors, so how does that relate to the graph, and what do you remember about it. It’s just going back, because the ground has already been laid, so they relate the equation to the big picture. It’s more like connecting the concepts. So they know what the graph of a function looks like, domain and range, at that point there’s a picture and they’re not just doing an equation. So this works out real good.

H.S.: Ok. Can you tell me a little about the students’ interaction in the class?

T.: It’s really not competitive. It’s more like a sense of personal pride, some of them don’t really care, but most of the time in my class the interaction is very little threatening.
H.S.: I meant mathematically.

T.: They tutor each other when somebody needs help, when somebody goes on the board most of the time one or more student will help, calculations are done by the class. They also love to show each other how to do things, they support each other. If somebody has a misconception, they’ll be glad to show him how things are done.

H.S.: So how do you check for understanding? How do you know that your students got a concept?

T.: There are different ways for that. First, when they answer the questions, the questions of the homework, right? I also ask what is it that they understood, what is it that is going on, until I pinpoint what is the concept the problem is about. Sometimes I deliberately give a wrong answer, and if they agree with me, then there is a problem. So they know they don’t have to agree with me all the time.

H.S.: Is there anything else you can tell me about the program and how it pertains to teaching students conceptually?

T.: I don’t have the flexibility to do what I want to do.
Teacher C1

H.S.: Basically, your answers to the questionnaire suggest that you are interested in the students’ conceptual development. Would you say that this is a good characterization of your teaching?

T.: Yes of course

H.S.: So what does conceptual development in Math mean to you?

T.: Well, I always stress to them that I feel like they would have a greater understanding of these things if they took the time to focus more on the conceptual basis of these problems, you know, examples, story problems and things like that. But the challenge is the concepts are really dry. It’s like the conceptual basis behind a lot of the formulas and the definitions and how these things go together, and if you have a formula, where it came from. You know, showing them how a certain formula came about from something they know from before. Those are things that I feel helped me as I was going through school, things I was interested in. I think we get in the habit of, when they’re in the lower levels algebra type of classes, that they start to feel that Math is understood through examples. They don’t understand that…You will show them an example, you go through a study guide, you try the best you can to give them an example that is relevant, but when they take the exam, if they have absolutely no idea about the concept behind the problem, then a little small change in the problem is going to throw them off, and if they don’t stop to think about what the concept is, what one is trying to accomplish…I would say the conceptual basis behind the math and the theory behind the math is the ultimate goal. I mean, the examples are nice to outline the concept, but the concepts is where the understanding is. I mean with
the examples they kind of follow along, and I think a lot of the problems with the R2R for example or MathXL, is that they’re too quick to provide a similar example for them, and then they get into the habit of: Here’s a similar example, I (the student) am changing the numbers from this example to this example, and I never thought about anything that I was doing, and I don’t have to think about anything that I’m doing. That’s one of the things that you have to try to get them out of the habit of, because if they’re changing the numbers from this to this, then they never thought about what they’re trying to do. But it’s always a challenge, because like I said, there’s a different way of looking at Math through concepts.

H.S.: So if I understand correctly, thinking about what they’re doing is to you getting the concept? Is there more to conceptual development than thinking about what they’re doing?

T.: Well, there’s always the aspect of connecting ideas and also anticipation. That’s part of their conceptual development too, is anticipating what comes next, thinking about how a problem would change and what that means, or if you’re solving for a variable in this position as opposed to another position. So tying concepts together, and things like that.

H.S.: Ok. Teaching for concepts is always challenging. What sort of challenges related to getting students to understand concepts did you face while working in R2R?

T.: Well, the way I often presented to students is through examples, almost like my job is go through a set of examples, but you always have to try to find a way to integrate that theory into the examples, and I try to find a way to make the students active in Math, not just sitting there copying line by line by line. You know, I always keep in the back of my
mind the question I think it’s most important that they ask themselves as they go through that type of problems. So, I ask them a leading type of question, to demonstrate that they know what they’re doing, not just this is how it’s done. There’s always that challenge. You have very limited amount of time of classroom when you have interaction with them as a group, since in the lab, it’s more of an individual basis, and you can’t go through a whole lesson with them individually, you know. It’s just helping them here and there. And I don’t disagree that it’s an effective way to introduce things through examples. But they get lost in the examples, and you have to find a way to explain why you’re doing what you’re doing. But they always get in the habit of “show me an example”, they’ll have the formula, this and that. I guess the challenge is to find a way to incorporate the conceptual stuff into what you’re doing in the example so that they see how it’s applied, how a formula works, etc… I have to find a way to ask them why, have them be able to tell you why. I got to the point where I felt I had to find somebody in that room to tell me how to move forward, so that I know they were thinking about what is the next logical step.

H.S.: Was time a factor? Did you feel the allocated time is appropriate?

T.: Time is definitely an issue here. You know, my students don’t have like the background to retain Algebra 2, so in Advanced Math, you’re taking this further, you expect them to know certain things, you know I tell them we did this so many times so I expect you to know that. I would love to have more time in here. Last year when I did it, I spent more time in the class and less time in the lab. As a result, you have to factor that some of the kids don’t have a computer at home, so you have to weigh that in too. So I try to find a kind of routine, and sometimes I wish I had more time for some things. In
the end, I would like to have a couple of more weeks! Time is always an issue, and I don’t know the solution for that.

H.S.: Ok. I’d like to get a little bit more into some details. Could you be kind enough to describe for me a lesson that you’ve given, in which you addressed conceptual goals? You can give me examples or talk to me in general how you target conceptual understanding through the lesson.

T.: Well, usually, I try to keep to the same routine, because the time I have is limited. Usually the way that I would start any class is I will usually outline any relevant formula that they need to know, so that they have them down for the class period, because I want them to be thinking about where they might come into play later, you know, thinking ahead just a little bit. I usually outline the formulas anyway, then depending on what the lesson is, because they’re always changing. I always think it’s important for them to see things algebraically, numerically, and graphically, and to be able to go between all three, especially when they talk about taking algebraic concepts and talking about them in terms of graphs. The visual people can see it, but it’s like a foreign language for some. So as we go through, I give them a short introduction, a sentence or two, maybe a definition, and then usually we just start with some sort of example that they would see. Then depending on the type of problem I integrate definitions or concepts, like “to do this here, you’re going to have to draw upon something you knew before…” So we kind of go through and the targets of the section, we hit them one at a time, any information that is relevant, any definition that is relevant to that specific skill, you know, if they need to see it visually as a graph, this how we check, anything that is relevant.
Teacher C2

H.S.: I am not going to take a lot of your time, just a few questions. Now based on your answers to the survey, it shows that you are interested in the students’ conceptual development. Would you say that this is a good characterization of your teaching? If so, then what does conceptual development mean to you?

T.: I would say that’s true. The conceptual development is for me to show them the theory behind the problem that they’re working, and why we are seeking to find a way to work these problems, and using this method, why it was developed, where it’s relevant in real life to them. The reason I do this for them is because I get a lot more… I would say participation from them when it’s relevant. And speaking as a former student, it didn’t make sense to me when the teacher would ask me word problems and give me no applications. In MathXL, there are a couple of applications in each section, but not a lot, and some of those applications are higher order, which they’ll get to once they master the lower ones, but I want to show them the basic concepts, you know, why do we need logarithms, or why do we need this and that. I want to give them a real life example and explain the theory behind it. That’s why.

I do spend a lot of time on skills though, after I introduce the concept. I introduce the concept with why, how it is, how it’s used, but then I do spend a lot of time on skills.

H.S.: Thank you. My next question is that the idea of teaching for conceptual understanding is always challenging. Particularly within this model (R2R), what sorts of challenges did you find associated with teaching for concepts in R2R?
T.: well, what I found is that the ones who really really study and get on the practice test a lot, they have learned to memorize when they see what kind of problem how to get the answer without mastering the concept. For instance, there was one where it was required to isolate the variable, and I said: What if they don’t ask you to isolate this variable? They ask you to isolate this other thing? And with the quizzes and assignment, it was always the same type of problems, so they had learned to stick that one in, spend two seconds on it, and move on. Sure enough, one of them stopped me today after the exam, and said: You were right, Mrs…., they didn’t ask us to get the variable in the denominator, they asked us to get the one in the numerator, and it was raised to an exponent, and I’m glad that you reminded us that’s not necessarily what’s going to come on your test. You can’t just memorize the kind of answers. For instance, there’s one with a perfect square denominator, and they’ve learned throughout practice tests and quizzes, it always turned out to be, their solutions would come out to be restricted because it’s at the bottom. But that’s not necessarily always true, so I gave them an example of why that’s not necessarily true. That’s one issue I have with the software, it’s that the students are led, they become dependent on seeing the exact types of problems over and over again, and then when they get to the test, they get something different, they get a kind of a curve ball, and they’re like: I’ve never seen this one before. Well, you did, but you just memorized how to get the answer for the question you did. They’re memorizing these particular problems instead of grasping the whole “what if” concept. I warned them, you guys are memorizing the practice test, you’re taking it like 10 times and memorizing it, you’re going to come to the final and be like “OH!”

H.S.: Ok. If you can briefly tell me how generally a lesson is conducted, and how do you see it addressing conceptual goals?
T.: I conduct my class somewhat like a regular high school class because that’s what my students are used to, but I also try to bridge them from traditional high school teaching to college education because they aren’t used to the lecture model, they’re used to a bell ringer and multiple activities. I try to bridge that but I still use a bell ringer, I do the practice for success, we do some problems, and then I talk about the concept, what we’re learning that day, and then I try to show them a real world application of it. It might be very simple, just so they get where it’s coming from. Then I take some problems, the higher order problems in MathXL, then we break those down and try to figure out how to do them. I guess, in the grand scheme of things, the time I spend on concepts is not that great, but it’s valuable to them because they understand why we’re doing it, and why the solutions are here and here, and why we need to develop this methodology. That’s it, I think.

H.S.: Thanks. I would like to get into some more details if you don’t mind. When you said you talk to them about concepts, how do you go about doing that? Do they take notes or is there a discussion, or…?

T.: We usually engage in discussions. For instance, quadratics, I talked to them about what a quadratic equation was, and some real life representation of it, even though they’ve seen it in Algebra 2. Some of the students memorize the skill and they don’t understand the concept behind it. So we would do something like that, then we do a real life application of it, and then I’ll give them a problem that I wanted them to try them to work on their own, without any background knowledge. They do have some background knowledge from Algebra 2, but some of them don’t get it. When we look at a graphical representation and say this is where they maximize profit, you know, you can see it clearly in a parabola.
And then from there after they try it I try to get them to answer me and tell me why, what they think, without me supplying the answer. And then after that we go into the method of solving the problem.

H.S.: Thank you. Maybe you can tell me how you students interact with one another in the class.

T.: It’s almost like a family environment, they act like brothers and sisters, and we’re all a team. They’re all like a family, and they want to help each other. When somebody comes up with a question, they don’t bat it down, they don’t, uh…Say somebody asked a conceptual question, “why this happens?” the other students aren’t negative to this student. They’re respectful of each other; they’re very helpful with each other, and if one of them doesn’t get it right, they lean over and try to show him how to do it. I like them a lot, it’s kind of them vs. the machine.

H.S.: Ok. In the lab, whenever they ask for your help, do they ask about how to do the problem or the concept behind it? And how do you go about responding?

T.: If it’s an application problem, I explain the concept behind the problem, because I don’t believe they could understand how to set it up without the concept. For instance, there was a system of equations about this lady who wanted a certain number of plates, and there was two sets of plates, and she needed to spend a certain amount of money. So I kind of asked them how many equations we need to set it up if we got two things going on, like how many plates and how much money to spend. But mostly, it’s more time methodology, to be honest with you. Most of the questions are about methodology
because most of the problems in MathXL are about methodology, not conceptual. So when an application problem comes up and they don’t know what to do, I make sure to explain the concept, hoping that they will recognize it and be able to apply it when other types of applications come up.

H.S.: How can you tell in class that they got a concept?

T.: It’s kind of an informal assessment. I can tell by them nodding their heads, raising their hands to give the answer, you know, they are interested. If I call on non-volunteers, and they can explain the concept to me, because there are plenty of volunteers, but when the non-volunteers are also understanding the concept, I believe then that it sunk in. Now, because of this course, I can’t necessarily always wait for everyone to get the concept. I do my best to explain it, I put it in their terms, in common applications, but that still does not guarantee, and unfortunately I got to keep going, so…

H.S.: Is time an issue in this design?

T.: I do feel time is an issue, I felt I needed more classroom time, not a lot, but when we’re in the lab, it’s all about the method, they really aren’t concerned when they’re in front of their computers, I don’t have their attention. In the back of their minds, they always want to get through the assignment. But I’m hoping I’ll get better in allocating my classroom time, because in the classroom, I’ve got their full attention.

H.S.: Could you describe a teaching instance that you were proud of, something that stayed as a good memory?
T.: Let me think, uh. It’s not really one in particular, but sometimes you get a student that is challenged, they’re not the best, but when they can explain the concept back to me, whether they can understand methodology or not is not important, we can work on the methodology later. I have some particular students in my head that have had a “light bulb” moment when they got it, and they were so happy and pleased.

H.S.: I guess that’s it. I appreciate your help.

T.: No problem.
Teacher C3

H.S.: I only have a few questions for you, probably won’t take more than 30 minutes. I am looking at teachers’ strategies that accompany, or are in the margins of, the R2R design. A lot of the teachers try to gear their teaching towards students getting concepts. So my first question for you is: Do you characterize your teaching as conceptual? And if yes, how?

T.: Well, I am somewhat conceptual, as I integrate conceptual teaching strategies. It’s because if the students don’t understand where the new concepts are coming from, and they have a hard time making connections from one to the next and so on. So whenever it’s possible, I am trying to make a connection as I explain why they need to understand the concept behind. So yes, I am.

H.S.: So, what does conceptual development in math mean to you?

T.: Ok. Just looking at an example, for instance, as kids were going through the MathXL program and discussing functions, particularly exponential function and exponential decay. Now there’s a question about half-life. Now most of the students are very good at all the mechanics. Mechanics means that usually they don’t understand what they’re doing, they let the formula drive them. Plug it in, complete and get the answer. They usually don’t understand the justification behind the use of the formula, and if they make a mistake, whatever the calculator displays is for them the answer. They don’t think whether the answer is reasonable or not. That’s where conceptual orientation comes in. If they understand the problem conceptually, when they get a strange looking answer, they can immediately question it “you know, something is not right here, therefore I may have
made a mistake somewhere”. So for example, if some sort of radioactive material’s half life is 100 years, so if the question happens to be how many grams are left from a 10 g sample of this material after 75 years? Conceptual students understand that the answer has to be between 5 and 10. But a student who is not conceptual and made a mistake and the answer came out to be 4 or 3 grams, they will put down that answer and move on. That’s the difference between conceptual students and non-conceptual ones. Do you agree with that?

H.S.: I do.

T.: And before I usually introduce material, what I usually do is, if possible, I try to teach them to look at the big picture. Ok, we have 10 g that will become 5 g after 100 years, what will they become after another 100? But it’s not always going to be a nice number like 100 years or 200 years. What if I wanted to know how much is left after 95 years? The conceptual students know when their answers make sense.

H.S.: True. This leads immediately to my next question. I would like you to describe for me in details what you normally do in a lecture in a class. Walk me through a 50 minutes lecture class if possible. And if you want, you can take a specific topic or a specific lecture and tell me about it.

T.: Well, actually what I can do is to talk about the lesson I gave yesterday. In my Advanced Math I class, what I’m doing is advanced math material in the class not necessarily college algebra, which is a trig. portion, because most of my students will be taking trigonometry. Right now what I’m doing is developing the fundamental foundation
of trigonometry, so the lesson I gave yesterday was like on special angles. The way that I actually introduce the sin of 30° or 45° and such, I go back to geometry, geometry of a 45, 45, 90 and 30, 60, 90 triangles. So we talk about the ratio in those special triangles. I actually draw a triangle, an isosceles right triangle on the board, and ask my students to name it specifically. Most of the time they get it right, sometimes they don’t. If they do get it, I make sure they understand why it’s called an isosceles right triangle. Like a right triangle is like a last name, so if you say a right triangle you don’t know what kind it is, like my last name is Lowery, if someone said Lowery, you don’t if they meant me, or my husband, or even my children. However, my first name definitely points out exactly who I am, so that’s where isosceles is coming from, it’s like also a first name. From there, I talk about corresponding angles and corresponding sides. And then I’m going to call one of the legs of the right isosceles triangle as one unit of something, since the other leg is congruent to it, it is also one unit of something. Now, there’s a theorem that says if you know the two sides of any right triangles, you can find the third side. So leg₁ squared + leg₂ squared is equal to hypotenuse squared. So 1² + 1² = 2, and so the sides are in the ratio of 1:1:√2. And then from there, I introduce the unit circle. The definition of a unit circle is a circle of radius one. And then I want to make a √2 out of a 1 (?), that means I have to divide by √2. So then I develop the sin of 45 as √2 / 2, the cos of 45 as √2 / 2, and so on and so forth. From there, I do the 30, 60, 90 triangle, and so on.

H.S.: Great. Thank you. Maybe you can tell me now how your students interact with each other in class, especially during a lecture. Do they help each other and how, generally speaking.
T.: Now, I do make a point that when I lecture, I ask my students that if they have a question, they should write it down immediately when the question pops up. I have a tendency to lose the train of thoughts if the students ask while I’m lecturing, so when students are raising their hands in the middle of my lecture, I say, whatever you are thinking right now, I want you to write it down so you won’t forget, and let me finish what I want to finish. I’m used to having their undivided attention as much as possible. Just give me about 5 minutes, I say, but I want you to write your question down, because if you don’t, you’re going to forget it. So write it down, and then I move on to what I need to do, and once my lecture is finished, we do have a discussion, question/answer session. If anybody have a question they’re not clear on, it’s at the time when I answer. Most of the time, whenever students have a question, they say “oh, by the way, you already answered my question”. That’s how I usually approach it. And then, once my lecture is done, they do an assignment that I make them do independently. I give them 5 minutes to do it independently, but 90% of the time they can help each other if they choose. But there are some students who like to do things by themselves because they need to pay attention. So they come to the other students and [not clear!]. So I give my students the choice to work by themselves or work together. Obviously, when students are doing the work, I go around to make sure they know what they’re doing. And answer their questions.

H.S.: Generally, teachers face questions from students like “I didn’t get it”, or whenever they’re solving a problem (the students), they produce an incorrect answer or an incorrect step. How do you deal with that in class?

T.: When students tell me “I don’t get it”, mathematically it’s easy for me to deal with that, because I can see my students’ ideas at work. The student who doesn’t get it, I can
work with him one on one. If the person cannot take the first step, I try to give them
[inaudible], I ask the student to read the question first, and if it happens to be a word
problem, they have to come up with an equation, and then solve it. So how many things
are we looking at here, how many variables must be there, how many unknowns are there,
sort of a guide for the person to do. Now if the students do the work but somehow got the
incorrect answer, then I say “let me see your work”, and then I look at the first step and
check, and I ask if they see any mistake from the first step to the second step, and so on.
They usually figure out what’s wrong, like “I left a sign out”, so I see “there you go, that’s
your mistake”.

H.S.: Did teaching in the R2R put some constraints on you? If so, could you tell me what
some of those constraints were in terms of teaching for concepts?

T.: Ok. When my students realized that they will be doing all their work on the computer,
that was a very unusual situation, unlike any other math class that they have done. They
were very excited about it. Now, at the beginning, usually it’s more of a review material.
But some students, their knowledge is not 100%, but they think they know, so they don’t
take my lecture seriously, they only pretend that they are paying attention. So, I give them
a problem, they do their work, and somehow when they check their answer, it’s not
correct. And once we go through several sections, particularly new material, and they
realize that they spend hours and hours on the computer and they’re not very productive,
that’s when they realize how important it is for them to see my lecture part. They actually
made a comment about how they like it when I introduce new material when you teach us,
so they don’t spend as much time [?]. So at the beginning, they thought “we know what
you’re doing, let’s just get on the computer”, they’re just so anxious to do that. But the
thing was, since they didn’t understand the type of question, they realize how much time they’re spending on a similar question, they give me more credit. They realize that computer is fine, but the teacher cannot be replaced.

H.S.: Ok, you explained to me this from the students’ perspective. What about for you as a teacher? Did you have to make any changes to your teaching style based on the R2R design?

T.: Well, one thing I can think of is, you know how computer is very straight forward. Well, in most parts, I like it because the grade the students are getting is very objective. But the teacher can be very subjective, knowing that one student is very flexible, but the other is not, I have a tendency to give more credit to the student I think is capable, I might think the other one just got lucky. So when I give them partial credit, I may not give exactly the same partial credit. I like that [objectivity of grading] in computers. What I don’t like about it is that the computer is so unforgiving, sometimes I work all the questions and I get all the answers, but when transferring [the answer to the computer] I may leave a sign out or misplace a number. My students were frustrated when taking a test because they were doing things on paper and pencil ok, and once they finish they have to move the answer, they have to type the answer in the computer, and in the process they might make a mistake and lose it all. That is I think the students’ obstacle. I also have an obstacle in knowing how much time I have, I need to convey the most important points to my students, and then while they’re exercising, I cannot spend more time with them one-on-one. But I really don’t see too much obstacles at this point.
T. final comments: Well, I tell my students that I understand exactly what it’s like to be struggling through when you are doing your best but not getting the expected outcome. I sympathize with my students, and I tell them that if you feel like you did your best, whether your score is 75, 85, or 95, you ought to be happy with that. That’s what matters. My students understand that I don’t expect them to know everything. The important is that you did your best.

H.S.: I guess that’s it. Thank you for your time.
APPENDIX C

SURVEY ANSWERS WITH THEIR RESPECTIVE CODES

1. Mathematics is about problem solving. If a student can solve a type of problem correctly every time, that indicates to me he/she has understood the content.

   Strongly agree ➜ ➜ ➜ ➜ P
   Agree ➜ ➜ ➜ ➜ P
   Neither agree or disagree ➜ ➜ ➜ ➜ QC
   Disagree ➜ ➜ ➜ ➜ C
   Strongly disagree ➜ ➜ ➜ ➜ C

2. I believe that

   Skills need to be emphasized prior to introduction of concepts ➜ ➜ ➜ ➜ P
   Concepts can be taught regardless of skill acquisition level ➜ ➜ ➜ ➜ QC
   Skills should not be practiced until there is a sound conceptual foundation ➜ ➜ ➜ ➜ C

3. I believe that

   Comparing and contrasting different solution strategies is what helps students understand a problem. ➜ ➜ ➜ ➜ C
   Multiple correct solution strategies may exist for a certain problem. This provides the teacher with the opportunity to have students appreciate the easiest, most efficient method. ➜ ➜ ➜ ➜ QC
   It's best to focus on the one approach that enables students to solve a problem efficiently and correctly. Introducing alternative methods is likely to confuse students. ➜ ➜ ➜ ➜ P

4. Classroom dialogue is most effective for student learning if

   Students’ ideas, correct or incorrect, form the focus of class discussions. ➜ ➜ ➜ ➜ C
   Students’ ideas are solicited, but the teacher corrects their mistaken interpretations. ➜ ➜ ➜ ➜ QC
   The teacher presents the correct ideas in a clearly organized and deliberate fashion. ➜ ➜ ➜ ➜ P

5. Prior to R2R, I would typically introduce new material by
Illustrating/discussing a method or procedure and then have students practice it with several examples ➔ P

Posing a problem then paving the way for students to solve it by giving hints and suggesting possible starting points ➔ QC

Allowing students to attempt a new problem type without showing them a routine method, and then having them discuss their solution approaches ➔ C

6. What perceived benefits, if any, do you see for using the R2R approach?

- Increases students’ participation in learning
- Increases students’ enjoyment of learning
- Better for students’ understanding of the content ➔ P
- Better for students’ mastery of skills ➔ C
- Less work for the teacher in preparation and planning
- Other (please specify) _________________________________

7. What perceived disadvantages, if any, do you see for using the R2R approach?

- Decreases students’ participation in learning
- Decreases students’ enjoyment of learning
- Is not helpful in enabling students’ understanding of the content ➔ C
- Is not helpful in enabling students’ mastery of skills ➔ P
- More work for the teacher in preparation and planning
- Other (please specify) _________________________________

8. Describe a typical lecture in your class in terms of what you and the students regularly do.

(Answers and codes are in Appendix D)
### APPENDIX D

**LECTURE DESCRIPTIONS ANSWERS AND CODES**

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<th>Teacher’s Answer</th>
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<tbody>
<tr>
<td>1. When I introduce the continuous compound interest formula ( A = Pe^{rt} ), my students read background information (an article) about natural numbers, then my students and I have a discussion about the article they just read. I then prompt the students’ prior knowledge of another formula ( A = P(1 + \frac{r}{n})^{nt} ). Together, we derive from it the formula ( A = Pe^{rt} ) by letting ( n \rightarrow \infty ). We then compare and contrast the two formulas before I ask the students to work on their own on some problems</td>
<td>1. C</td>
</tr>
<tr>
<td>2. I try to cover the basics (equations, reasoning…); then I try to cover some problems students might see in the homework assignments.</td>
<td>2. P</td>
</tr>
<tr>
<td>3. I model the examples for the students. Students take notes, and additional practice is provided to the students before working on the computer.</td>
<td>3. P</td>
</tr>
<tr>
<td>4. Because we do not have textbooks, I provide students with all formulas, definitions, and theorems. I also provide sample problems. I then work with the students to complete sample problems and answer questions pertaining to samples. Because we only have 55 minutes per class period, we usually do not have time to begin course work until the next day</td>
<td>4. P</td>
</tr>
<tr>
<td>5. Lecture for 52 minutes.</td>
<td>5. P</td>
</tr>
</tbody>
</table>
6. Presentation of the social need and historical context that led to the development of the mathematics concept, where the concept stands in the field today, modern application that use the conceptual theory developed. Then, discussion of the layout of the unit’s instructional development, pointing out the major theorems that will be explored and why. Then begin the unit by discussion and conceptual building from the foundation up by reminding students of prerequisite skills and concepts and reviewing these where needed. Students work problems individually and on the overhead. Discuss solution methods to help students overcome difficulties and gain confidence in the knowledge and conduct informal formative assessment.

7. Question/Answer session and selected MathXL problems via Promethean Board to sample types of problems the students will see, directions on how to input answers.

8. Introduce a topic. Define all the terminology and work examples. Assign students certain problems to do to check for understanding. Give homework problems.

9. I usually begin with questions on the previous lesson or unusual problems that the students had. I then present the lesson, sometimes using investigation, but the students really prefer me to explain the topic with examples and let them work.

10. I spend two or more days going over the lecture notes. I spend the other days working one-on-one with students as they do their homework. I try to explain thoroughly because I have a class of 5, which enables a lot of one-on-one time.
11. I generally try to show the information before they take the test.

12. Typical lecture time has been spent primarily on discussing the new concepts that will be featured on the computer portion of the class. We start with a review of prior concepts, practice and model, do an activity, then use the other days to work in the lab.

13. At the beginning of the class, I cover the notes or basic concepts/rules/formulas for today’s topics. We then work on selected homework problems.

14. I introduce the basic concepts involved within the particular type of problem. Since most of the information is repetitious from Algebra II, I work the examples with some input from the students, then they begin to work online.

15. Twice a week, I introduce a lesson/section. The students take notes on how to solve the problems and we practice a few on paper. The students help each other understand the concept with my guidance. Sometimes I give them a problem and ask them to discuss with a neighbor how they would go about solving it. This leads to further discussion.

16. I introduce one or more sections and we develop the concepts and work example problems.

17. Bell ringer for 5 minutes. Discussion of the most recently due assignment (teacher addresses student questions). Lecture on prior knowledge that is needed for new concept. Explanation of the new concept. Example problems.
18. Go over the basic concepts of the section, then allow the kids to start working. Answer questions when they come up.

19. I spend two full class periods per week lecturing.

20. I lecture 2 days a week for 50 minutes and the students attend the lab 3 days.

21. I engage students in notes where I will work some problems then they will work similar problems. There is also class discussion.

22. During lecture time, I discuss the main ideas or concepts in a section and how they are related to previous concepts. I then help them work the more difficult examples as a guided practice.

23. Lecture time is devoted to highlighting key concepts, defining all relevant terms, going over the difficult examples, discussing how problems/concepts are related to something they have discussed before, and asking the students to describe the conceptual basis behind the section instead of limiting the understanding to the specific examples.

24. I attempt to introduce the underlying concepts, relating them to prior content (including previous courses). Once that is established I demonstrate how these concepts may be applied in a problem situation. Finally, I spend a little time preparing them for what they might expect in terms of their assignment.
25. College Algebra is a repeat of Algebra I and Algebra II in high school. So all I've had to do was work several examples and remind them about the concepts they learned in previous classes. I have not had to teach any new concepts.

26. Short explanation. Multi-media projection of lesson questions. Examples worked on board as we go through the lesson.

27. Typically five days are given per lesson. One of these days is dedicated to lecture. Because of the limited class time, most of the lecture is done by direct instruction. I'll mostly present the new information and occasionally ask for student feedback.

28. Material is presented and examples are worked.

29. I broadly describe the topics and present key concepts along with some fundamental skills that may be needed that are not otherwise frequently used enough to be second nature to the student. Having taught these same students since the tenth grade (geometry, algebra II and Advanced Math) I highlight items that I know are different than the approach they have seen before and those that I know they have not seen. When using an example, I select the most difficult they may encounter as there is not enough time to start easy and work up. The students ask questions and many will already be ahead of the pace and have some MathXL specific questions. Initially more on how to correctly enter answers; now, rarely.

30. I introduce each chapter by writing its title on the board. As I go through the chapter teaching the necessary topics, students interact with me. They ask questions which I try to answer quickly. Most of them take detailed notes of examples I put on the board.
31. I generally explain a broad mathematical principle and illustrate it using a concrete example problem. While working the example problem, I invite student involvement. We pause frequently for questions. Students take notes so that they may use them as they complete homework problems later.

32. Lecture 20% computer lab 80%

33. Quick overview of topics covered in sections. Questions that will give most trouble are emphasized

34. Lecture for 2 days and computer lab for the remaining days
APPENDIX E
SURVEY ANSWERS DATA TABLES

1. Age

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 25</td>
<td>5.9</td>
<td>2</td>
</tr>
<tr>
<td>26 – 30</td>
<td>2.9</td>
<td>1</td>
</tr>
<tr>
<td>31 – 35</td>
<td>14.7</td>
<td>5</td>
</tr>
<tr>
<td>36 – 40</td>
<td>23.5</td>
<td>8</td>
</tr>
<tr>
<td>41 – 45</td>
<td>14.7</td>
<td>5</td>
</tr>
<tr>
<td>46 – 50</td>
<td>11.8</td>
<td>4</td>
</tr>
<tr>
<td>51 or above</td>
<td>26.5</td>
<td>9</td>
</tr>
</tbody>
</table>

2. Sex

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>35.3</td>
<td>12</td>
</tr>
<tr>
<td>Female</td>
<td>64.7</td>
<td>22</td>
</tr>
</tbody>
</table>

3. Highest level of education attained

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bachelor degree</td>
<td>47%</td>
<td>16</td>
</tr>
<tr>
<td>Masters degree</td>
<td>47%</td>
<td>16</td>
</tr>
<tr>
<td>Specialist degree</td>
<td>2.9%</td>
<td>1</td>
</tr>
<tr>
<td>Ph.D.</td>
<td>0%</td>
<td>0</td>
</tr>
<tr>
<td>Other</td>
<td>2.9</td>
<td>1</td>
</tr>
</tbody>
</table>
4. Your degree is in _________________________

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics</td>
<td>20.6%</td>
<td>7</td>
</tr>
<tr>
<td>Mathematics Education</td>
<td>44.1%</td>
<td>15</td>
</tr>
<tr>
<td>Science</td>
<td>8.8%</td>
<td>3</td>
</tr>
<tr>
<td>Science Education</td>
<td>2.9%</td>
<td>1</td>
</tr>
<tr>
<td>Education Administration</td>
<td>2.9%</td>
<td>1</td>
</tr>
<tr>
<td>Administration and Supervision</td>
<td>2.9%</td>
<td>1</td>
</tr>
<tr>
<td>Engineering</td>
<td>5.9%</td>
<td>2</td>
</tr>
</tbody>
</table>

5. Subjects in which you are certified

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics</td>
<td>97%</td>
<td>33</td>
</tr>
<tr>
<td>Science</td>
<td>14.7%</td>
<td>5</td>
</tr>
<tr>
<td>Other</td>
<td>26.4%</td>
<td>9</td>
</tr>
<tr>
<td>I am not certified</td>
<td>2.9%</td>
<td>1</td>
</tr>
</tbody>
</table>

6. Number of years teaching Mathematics

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st year</td>
<td>2.9%</td>
<td>1</td>
</tr>
<tr>
<td>1 to 3 years</td>
<td>5.9%</td>
<td>2</td>
</tr>
<tr>
<td>4 to 6 years</td>
<td>8.8%</td>
<td>3</td>
</tr>
<tr>
<td>7 to 9 years</td>
<td>11.8%</td>
<td>4</td>
</tr>
<tr>
<td>10 to 12 years</td>
<td>17.6%</td>
<td>6</td>
</tr>
<tr>
<td>More than 12 years</td>
<td>52.9%</td>
<td>18</td>
</tr>
</tbody>
</table>
7. Number of years at current job

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st year</td>
<td>5.9%</td>
<td>2</td>
</tr>
<tr>
<td>1 to 3 years</td>
<td>23.5%</td>
<td>8</td>
</tr>
<tr>
<td>4 to 6 years</td>
<td>17.6%</td>
<td>6</td>
</tr>
<tr>
<td>7 to 9 years</td>
<td>14.7%</td>
<td>5</td>
</tr>
<tr>
<td>10 to 12 years</td>
<td>8.8%</td>
<td>3</td>
</tr>
<tr>
<td>More than 12 years</td>
<td>29.4%</td>
<td>10</td>
</tr>
</tbody>
</table>

8. Mathematics is about problem solving. If a student can solve a type of problem correctly every time, that indicates to me he/she has understood the conceptual underpinnings of the problem type.

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strongly Agree</td>
<td>32.3%</td>
<td>11</td>
</tr>
<tr>
<td>Agree</td>
<td>32.3%</td>
<td>11</td>
</tr>
<tr>
<td>Neither Agree nor Disagree</td>
<td>11.8%</td>
<td>4</td>
</tr>
<tr>
<td>Disagree</td>
<td>23.5%</td>
<td>8</td>
</tr>
<tr>
<td>Strongly Disagree</td>
<td>0%</td>
<td>0</td>
</tr>
</tbody>
</table>

9. I believe that

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic mathematical facts need to be emphasized prior to introduction of concepts</td>
<td>73.5%</td>
<td>25</td>
</tr>
<tr>
<td>Concepts can be taught regardless of skill acquisition level</td>
<td>17.6%</td>
<td>6</td>
</tr>
<tr>
<td>Skills should not be practiced until there is a sound conceptual foundation</td>
<td>11.8%</td>
<td>4</td>
</tr>
<tr>
<td>None of the above captures my viewpoint</td>
<td>2.9%</td>
<td>1</td>
</tr>
</tbody>
</table>
10. I believe that

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comparing and contrasting different solution strategies is what helps students understand a problem</td>
<td>67.6%</td>
<td>23</td>
</tr>
<tr>
<td>Multiple correct solution strategies may exist for a certain problem. This provides the teacher with the opportunity to have students appreciate the easiest, most efficient method</td>
<td>26.4%</td>
<td>9</td>
</tr>
<tr>
<td>It’s best to focus on the one approach that enables students to solve a problem efficiently and correctly. Introducing alternative methods is likely to confuse students</td>
<td>5.9%</td>
<td>2</td>
</tr>
</tbody>
</table>

11. I believe that

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students learn math best when they work together to discover mathematical ideas</td>
<td>17.6%</td>
<td>6</td>
</tr>
<tr>
<td>Students may learn from working together on a problem for a short period, but there’s no substitute for individual practice to consolidate skills and concepts</td>
<td>79.4%</td>
<td>27</td>
</tr>
<tr>
<td>It’s not very productive for students to work together during mathematics class time</td>
<td>2.9%</td>
<td>1</td>
</tr>
</tbody>
</table>
12. Classroom dialogue is most effective for student learning if

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students’ ideas, correct or incorrect, form the focus of class discussions</td>
<td>17.6%</td>
<td>6</td>
</tr>
<tr>
<td>Students’ ideas are solicited, but the teacher corrects their mistaken interpretations</td>
<td>58.8%</td>
<td>20</td>
</tr>
<tr>
<td>The teacher presents the correct ideas in a clearly organized and deliberate fashion</td>
<td>23.5%</td>
<td>8</td>
</tr>
</tbody>
</table>

13. Prior to adopting this course design, I would typically introduce new material by

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Illustrating/discussing a method or procedure and then have students practice it with several examples</td>
<td>41.1%</td>
<td>14</td>
</tr>
<tr>
<td>Posing a problem then paving the way for students to solve it by giving hints and suggesting possible starting points</td>
<td>55.9%</td>
<td>19</td>
</tr>
<tr>
<td>Allowing students to attempt a new problem type without showing them a routine method, and then having them discuss their solution approaches</td>
<td>14.7%</td>
<td>5</td>
</tr>
<tr>
<td>Other approach</td>
<td>5.9%</td>
<td>2</td>
</tr>
</tbody>
</table>
14. In addition to your typical introduction to new material, do you sometimes use other approaches? If so, indicate which methods and the approximate percentage of time for each.

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Making a connection from my students’ prior knowledge to a new topic that I am about to introduce.</td>
<td>2.9%</td>
<td>1</td>
</tr>
<tr>
<td>Manipulative Assisted Investigations</td>
<td>5.9%</td>
<td>2</td>
</tr>
<tr>
<td>I typically present the new material in an organized fashion, starting with a basic problem.</td>
<td>2.9%</td>
<td>1</td>
</tr>
<tr>
<td>Pose a problem to hook the students, try to bring in the real world, and let students take an initial guess on topic.</td>
<td>2.9%</td>
<td>1</td>
</tr>
<tr>
<td>I will occasionally use all methods listed in different situations (25%-illustrating, 10%-student attempts).</td>
<td>2.9%</td>
<td>1</td>
</tr>
<tr>
<td>Give them discovery activities so that they can develop their own rules, strategies.</td>
<td>2.9%</td>
<td>1</td>
</tr>
<tr>
<td>Reviewing prior knowledge that is the foundation for new concepts - 15% of class time</td>
<td>2.9%</td>
<td>1</td>
</tr>
<tr>
<td>I do all methods, it just depends on the material</td>
<td>2.9%</td>
<td>1</td>
</tr>
<tr>
<td>Giving a worksheet which will guide them to make conclusions, letting them teach themselves the concepts by looking for patterns</td>
<td>2.9%</td>
<td>1</td>
</tr>
<tr>
<td>Applications that capture the attention of the students</td>
<td>5.9%</td>
<td>2</td>
</tr>
<tr>
<td>Use of the calculator to demonstrate solutions to problems</td>
<td>2.9%</td>
<td>1</td>
</tr>
<tr>
<td>Illustrating/discussing a method or procedure then have students practice it with several examples</td>
<td>11.8%</td>
<td>4</td>
</tr>
</tbody>
</table>
15. Is this your first semester teaching using the R2R teaching approach?

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>61.8%</td>
<td>21</td>
</tr>
<tr>
<td>No</td>
<td>38.2%</td>
<td>13</td>
</tr>
</tbody>
</table>

16. How did you become involved in this design?

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volunteered after hearing about it from a friend/coworker</td>
<td>38.3%</td>
<td>13</td>
</tr>
<tr>
<td>Volunteered after I came across information about it online</td>
<td>8.8%</td>
<td>3</td>
</tr>
<tr>
<td>It was mandated by the school/school district</td>
<td>26.4%</td>
<td>9</td>
</tr>
<tr>
<td>Other (Please specify)</td>
<td>26.4%</td>
<td>9</td>
</tr>
</tbody>
</table>

17. When you signed up (or were assigned) to use this course design, did you expect this approach to be ________________________________
(You may indicate more than one)

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helpful for students because it is engaging</td>
<td>61.7%</td>
<td>21</td>
</tr>
<tr>
<td>Helpful for students because it is geared towards enhancing their content knowledge</td>
<td>58.8%</td>
<td>20</td>
</tr>
<tr>
<td>Helpful at some level but not helpful on some others</td>
<td>11.8%</td>
<td>4</td>
</tr>
<tr>
<td>Answer</td>
<td>Response Percent</td>
<td>Response Count</td>
</tr>
<tr>
<td>-----------------------------------------------------------------------</td>
<td>------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>Increases students’ participation in learning</td>
<td>94.1%</td>
<td>32</td>
</tr>
<tr>
<td>Increases students’ enjoyment of learning</td>
<td>38.2%</td>
<td>13</td>
</tr>
<tr>
<td>Better for students’ understanding of the content</td>
<td>50%</td>
<td>17</td>
</tr>
<tr>
<td>Better for students’ mastery of skills</td>
<td>55.9%</td>
<td>19</td>
</tr>
<tr>
<td>Less work for the teacher in preparation and planning</td>
<td>17.6%</td>
<td>6</td>
</tr>
<tr>
<td>Other (Please specify)</td>
<td>17.6%</td>
<td>6</td>
</tr>
</tbody>
</table>
19. What perceived disadvantages, if any, do you see for using the R2R approach? (You may check more than one)

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decreases students’ participation in learning</td>
<td>5.9%</td>
<td>2</td>
</tr>
<tr>
<td>Decreases students’ enjoyment of learning</td>
<td>5.9%</td>
<td>2</td>
</tr>
<tr>
<td>Is not helpful in enabling students’ understanding of the content</td>
<td>29.4%</td>
<td>10</td>
</tr>
<tr>
<td>Is not helpful in enabling students’ mastery of skills</td>
<td>29.4%</td>
<td>10</td>
</tr>
<tr>
<td>More work for the teacher in preparation and planning</td>
<td>5.9%</td>
<td>2</td>
</tr>
<tr>
<td>Other (Please specify)</td>
<td>23.5%</td>
<td>8</td>
</tr>
</tbody>
</table>

20. The current approach suggests students spending 75% of the time working on their computers in class. Are you satisfied with the time allotted for the teacher within this approach?

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes, I am satisfied</td>
<td>76.5%</td>
<td>26</td>
</tr>
<tr>
<td>No, I am not satisfied</td>
<td>23.5%</td>
<td>8</td>
</tr>
</tbody>
</table>

21. Describe a typical lecture time in your class in terms of what you and the students do regularly. [Answers are in Appendix D]

22. What problems, if any, did you have with the way the approach is implemented? (You may check more than one)

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Software issues</td>
<td>41.1%</td>
<td>14</td>
</tr>
<tr>
<td>Content issues</td>
<td>23.5%</td>
<td>8</td>
</tr>
<tr>
<td>Teaching strategies issues</td>
<td>11.8%</td>
<td>4</td>
</tr>
<tr>
<td>Other issues</td>
<td>47.1%</td>
<td>16</td>
</tr>
</tbody>
</table>
23. The current course design

<table>
<thead>
<tr>
<th>Answer</th>
<th>Response Percent</th>
<th>Response Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>did not meet my expectations</td>
<td>11.8%</td>
<td>4</td>
</tr>
<tr>
<td>met my expectations</td>
<td>50%</td>
<td>17</td>
</tr>
<tr>
<td>exceeded my expectations</td>
<td>38.2%</td>
<td>13</td>
</tr>
</tbody>
</table>

24. If you were the R2R Czar, how would you organize it so that it would meet its objectives more effectively?

Answers:
1. Because lots of materials are relatively new concepts to high school students, I need to spend more time for the lesson delivery in the classroom than it would be in a College Algebra Class in the university. Consequently, my students have to spend many hours either at home or after school for doing their assignments. This cause a problem because I teach in a low SES school where about 75% of our students live in the households the below poverty level. Thus, not every student has a computer and working internet connection at home. Since they have to spend a good many hours outside of the classroom, it would be beneficial if my students can check out the laptop computer. In addition, this is a technology incorporated program and, of course, losing an internet connection while they are doing the assignment is counter-productive.
2. Don't think so.
3. Students with different learning styles need more
4. It cannot be resuscitated!
5. No
6. It is fine the way it is.
7. Make sure that the classes were small enough to ensure that students’ needs were met. I am still waiting to see the end results before I make my determination. I would allow for more credit to students who put a (instead of a ] in the interval notation. I would continue to make use of the practice tests. This is a great tool.
8. I would make the homework more difficult and a little more weighty with respect to the grade. I would give a larger range of problems. I would break test 1 up or make it less percentage to keep students from feeling defeated from the beginning.
9. Curtailing some of the homework, providing a list of prerequisites that should be met before proceeding with the course.
10. The only thing I would change would be to cover a few less topics and go for a little more understanding in some areas.
11. The only issue I see lies in the fact that the students will not work several of the same type of problem in the homework. Therefore, the students do not see some of the more difficult problems that may show up on the test.
12. None
13. I'm not sure what if anything I would change
14. Get some instructor input on what types of questions should go into the test pool.
   Make sure each type is specifically covered in assignments and quizzes. I am so
   grateful to Southeastern for making practice tests available. This is an awesome
   tool for those committed to mastering the material.
15. Assist in planning the entire semester as far as breaking down (with rough
   estimates) what content will be taught in specific weeks.
16. Everything is well organized.
17. Add more applications to relate the concepts to practical aspects of the math.
18. In my case, I would like to give students an extra class period for lab work. This
   way I could take an extra day for lecture, and students would have more time to
   work on the computers at school.
19. Students really need more problems to work.
20. I think the course design meets LSU's objectives and, accordingly, would not
   organize it differently. Any reorganization would be addressed at the overall
   process approach versus data approach and that is another issue altogether. I do not
   know if MathXL can be utilized for the data approach.
21. I'd find some way to get a math aide in each classroom. One of our biggest
   problems is there is not enough of me to go around when students are working on
   computers.
22. Course is ok in design
23. I think the course is great
VITA

Haitham Sleiman Solh is a native of the southern city of Saida, Lebanon. He graduated with honors from Rafic Hariri High School in 1991, where he also earned his French Baccalaureate Diploma. He attended the American University of Beirut then transferred to the Lebanese University in Beirut, where he earned his Bachelor of Science degree in chemistry in 1998. Haitham attended the University of Southern Mississippi in Hattiesburg, Mississippi, where he earned his Master of Education in the summer of 2002. He began the doctoral program at Louisiana State University, Baton Rouge, in the summer of 2005.

Haitham has taught chemistry and mathematics for 8 years at the Middle and High School levels in Lebanon, Saudi Arabia, and United Arab Emirates. He also taught developmental math courses at American University in Dubai for 4 years. He is a member of a number of professional organizations, most notably the National Council of Teachers of Mathematics (NCTM) and the American Education Research Association (AERA).

Haitham currently resides in Baton Rouge pending the completion of his doctorate. His permanent place of residence is Saida, Lebanon.