An Explicit State Model of a Synchronous Machine - Transformer - Scr Bridge Unit.

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AN EXPLICIT STATE MODEL OF A SYNCHRONOUS MACHINE - TRANSFER - SCR BRIDGE UNIT.

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ABSTRACT

A rigorous mathematical model of a synchronous machine-transformer-thyristor bridge unit is presented in an explicitly expressed state-space form with coefficients which are explicit functions of conventional parameters. The developed model has a minimum number of state-variables for various operation modes of the bridge and its form is such that it can be readily interfaced with any type of dc network connected to the bridge. In addition, the model has the capability to consider different types of transformer winding connections. Several approximate models of reduced order are presented where each is expected to be sufficiently accurate in a certain time interval. Also, a method to account for the synchronous machine saturation is suggested. A digital computational procedure for simulating the developed models is included. The proposed simulation method is very versatile in its application since a priori knowledge of the dc network configuration is not required and it can cope with normal as well as complicated abnormal operating conditions of the bridge. The use of the model is demonstrated by considering the charging circuit of a high-power pulse generating system as the dc network. An illustrative set of simulation results is shown from which relevant information is obtained. Comparison of the simulation results of the full-order and the approximate models showed that the system
can be accurately represented by a third order reduced model for the short term response and by a second order reduced model for the long term response. The results also showed that the harmonic contents of the machine line currents are reduced by using a Δ-Y or Y-Δ transformer connection. Finally, it was found that the effect of the transformer magnetization current is negligible.
1. INTRODUCTION

Digital simulation is an established technique for the evaluation and analysis of power system dynamic behavior where the validity of the conclusions is based on the accuracy of the model used. However, there is always a conflict between the required accuracy and the acceptability of the computer time needed for the solution. Consequently, the development of an adequate and appropriate model structure for a particular application is of great importance. In addition, the availability of system data (in practice) for calculating the model coefficients must also be considered.

On synchronous machine dynamic analyses and modeling, a great deal of work has been done in the past. Due to the nonavailability of digital computers, the earlier techniques [1-4] were based on the development of closed form analytical solutions. However, in order to avoid the inherent complexity involved, simplifying assumptions such as negligible machine resistances and constant flux-linkages were made. Empirical formulas were then obtained by introducing decrement factors to account for resistances and flux-linkage variations.

As the demand for more accurate representation of synchronous machines and the significance of computer simulation became apparent, numerous model structures in
equivalent-circuit and state-space form surfaced. Some models such as the equivalent circuits developed by Rankin [5], and Jackson and Winchester [6], although resulting in a very accurate prediction, are far too complex for multi-machine behavior investigation, especially under unbalanced operating conditions. Further complications are also associated with these models because of the non-familiarity and availability of the circuit parameters. The state-space representation of machines has been suggested in a wide variety of forms [7-10], depending on the choice of the state variables, reference frame, and base values. The engineering application and the nature of the required data are the determining factors for the proper selection of a suitable model for a particular case of interest.

The investigation in this thesis concerns the operation of a synchronous machine whose output is directly or indirectly connected to a thyristor bridge. Such a system is used in a variety of applications such as ac exciters, H.V.D.C. systems, high power pulse loads, and variable speed motor drives. As a consequence, a considerable amount of work has been performed and published on this subject in recent years [11-17].

Closed form solutions describing the steady-state characteristics of controlled rectifier bridges fed by synchronous generators have been developed by numerous authors where, as in the case of synchronous machine modeling, simplifying assumptions had to be made. Franklin [11] assumed
negligible armature and rotor resistances. Bonwick [12] ignored the damper windings and considered a constant field flux-linkage. In both methods, besides the above-mentioned approximations, constant dc output current is assumed which results in limited applications. In the development presented by Abdel-Razek and Poloujadoff [13], the commutation time was assumed to be relatively short. All the winding resistances were first neglected after which their effects were introduced by approximate time functions.

Harashima et al. [14] developed a model in state-space form for a round rotor synchronous motor fed by a current-source inverter. The model was presented in the $\alpha$, $\beta$ reference frame with the field winding current assumed to be constant. This model has the disadvantage that the damping effect of the field winding is ignored. In addition, practicing engineers are not familiar with the machine data required for the model.

Other state-space models have been presented [15-17] where the variation of all machine winding variables and the effects of their resistances are considered. These models, although very comprehensive, involve time-varying matrices which depending on the integration method used in the simulation require at least one inversion at every time step, leading to increased computation time and inaccuracy, and possibly even computational instability.

The object of the present work is to develop a comprehensive state-model of a synchronous machine connected
through a three-phase transformer to a thyristor bridge
where the transformer is primarily used for matching the ma-
chine and dc network voltage ratings and reducing the high
current harmonics. The developed model should be in such a
form which can be readily interfaced with any type of dc
network connected to the bridge dc terminals. This can be
accomplished by developing an explicitly expressed state-
model with currents as state variables, since currents are
the variables which directly link the system components in-
cluding the dc network. The use of an explicit state-model
would also eliminate the need for any matrix inversion in
the digital simulation of the system. Finally, it is de-
sired that the model coefficients be expressed in terms of
a set of data which are readily available in practice. In
the modeling, filter networks which are conventionally in-
corporated to reduce the harmonic contents of the line cur-
rents are not considered since they are mainly used in
H.V.D.C. systems.
2. MACHINE MODELING

An explicit state-space model of a salient-pole synchronous machine with damper winding is developed in this chapter. The armature and rotor currents along with the speed are chosen as the state variables. The model is presented in the d,q as well as $\alpha,\beta$ reference frame where in either case all model coefficients are expressed in terms of the standard machine parameters.

For some applications, the full order machine model may not be desirable. Therefore, several approximate models reduced order are considered.

Also included in this chapter is the case where the synchronous machine is of the permanent-magnet type whose equations are compared with those with of the machine with field excitation. Relationships between the corresponding parameters of the two machine types are obtained. Finally, a method to include saturation effects in the synchronous machine simulation is presented.

2.1 Salient-Pole Synchronous Machine

In the generalized theory of electrical machinery [1], a three-phase salient-pole synchronous generator can be represented by six separate, but mutually coupled, circuits. These are three identical lumped windings, a, b, and c, symmetrically placed in the stator, a field winding $f$, and
two lumped equivalent damper windings \( k_d \) and \( k_q \) placed in the rotor. The winding distribution and the air gap shape are assumed to be such that the trigonometric Fourier Series expansions of the self and mutual inductances of the stator windings as a function of the rotor position angle contain no harmonics higher than the second order, and that the mutual inductances between the stator and rotor windings vary sinusoidally with the rotor position angle. Furthermore, it is assumed that the effects of saturation, hysteresis and eddy current are negligible. In the following sections, starting from the Park's equation of a salient-pole synchronous machine, several state-space models are obtained.

2.11 Park's Equation

It is well known that the equations for a synchronous machine can be greatly simplified if its phase variables are transformed from the stationary, \( a,b,c \) to the rotating \( d,q \) coordinate system. This transformation called the \( d,q \) or Park transformation [3,18] results in the following volt-amperere equations [1]:

\[
\begin{align*}
V_d &= -R_a i_d + \psi_d - \omega \psi_q \\
V_q &= -R_a i_q + \psi_q + \omega \psi_d \\
V_o &= -R_a i_o + \psi_o \\
V_f &= R_f i_f + \psi_f
\end{align*}
\]  

(2.1)
\[ 0 = R_{kd} i_{kd} + \psi_{kd} \]
\[ 0 = R_{kq} i_{kq} + \psi_{kq} \] (2.1)

All of the parameters and variables in equations (2.1) and hereafter are expressed in per-unit values. The per-unit system used here is based on equal mutual inductances between the three circuits in the d-axis and equal self inductances for the two circuits in the q-axis [19]. As a result of the chosen bases the flux linkages are related to the currents by

\[
\begin{bmatrix}
\psi_d \\
\psi_f \\
\psi_{kd} \\
\psi_q \\
\psi_{kq}
\end{bmatrix} = \frac{1}{\omega_0} \begin{bmatrix}
-X_d & X_{md} & X_{md} \\
-X_{md} & X_f & X_{md} \\
-X_{md} & X_{md} & X_{kd} \\
0 & -X_q & X_{mq} \\
0 & -X_{mq} & X_q
\end{bmatrix} \begin{bmatrix}
i_d \\
i_f \\
i_{kd} \\
i_q \\
i_{kq}
\end{bmatrix},
\]

(2.2)

and

\[ \psi_o = -\frac{1}{\omega_0} X_o i_o. \]

Note that the reactances used in (2.1) and (2.2) except for \( X_d \) and \( X_q \) are the primitive machine parameters. In practice, not all of these reactances can be directly determined from field tests. Appendix A shows the relationships between the primitive parameters and the conventional stand-
ard parameters [20].

The electromagnetic torque $t_e$ expressed in per-unit is

$$t_e = \psi_d i_q - \psi_q i_d.$$  \hspace{1cm} (2.3)

The equation for the mechanical motion in per-unit is given by

$$t_m - t_e = \frac{2H\omega_b^3}{\omega_o^2} \omega + D(\omega),$$ \hspace{1cm} (2.4)

where $t_m$ is the mechanical torque, $H$ and $D(\omega)$ are respectively the machine inertia constant and damping torque [21], $\omega_o$ is the per-unit electrical synchronous speed, and $\omega_b$ is the base speed in electrical radians per second.

2.12 State-Space Model in d,q Reference Frame

Depending on the selection of the state variables, a wide variety of machine models can be developed. The engineering application of the machine usually determines which model is most suitable for the particular case of interest. The two most obvious choices for the state variables in the electrical equations are the circuit flux-linkages or currents [9]. Since in this study a thyristor load is considered and the currents are the variables which directly link the machine with the dc network, the armature and rotor currents along with the speed $\omega$ are chosen as the state variables. Furthermore, the zero-sequence equation is not considered, but it can be readily included if it is required.
The state equations with these state variables are obtained by substitution of the flux-linkage equation (2.2) in (2.1) and (2.4), and rearranging the resultant equations with the standard parameters substituted for the primitive parameters.

\[
\begin{bmatrix}
\dot{i}_d \\
\dot{i}_q \\
\dot{i}_f \\
\dot{i}_{kd} \\
\dot{i}_{kq}
\end{bmatrix} =
\begin{bmatrix}
-1/\tau_d & \omega_{A_d} & A_{df}/\tau_d' & A_{dkd}/\tau_{do}' & \omega_{A_{dkq}} \\
-\omega_{A_qd} & -1/\tau_q & \omega_{A_qf} & \omega_{A_{qkd}} & A_{qkq}/\tau_{qo}' \\
-1/\tau_d & \omega_{A_d} & A_f/\tau_d' & A_{fkd}/\tau_{do}' & \omega_{A_{dkq}} \\
-1/\tau_d & \omega_{A_d} & A_{kdf}/\tau_d' & A_{kd}/\tau_{do}' & \omega_{A_{dkq}} \\
-\omega_{A_qd} & -1/\tau_q & \omega_{A_qf} & \omega_{A_{qkd}} & A_{kq}/\tau_{qo}'
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q \\
i_f \\
i_{kd} \\
i_{kq}
\end{bmatrix} +
\begin{bmatrix}
A_d \\
0 \\
A_v \\
A_v' \\
0
\end{bmatrix}
\begin{bmatrix}
1/\chi_d' \\
0 \\
1/\chi_q' \\
1/\chi_d' \\
0
\end{bmatrix}
\begin{bmatrix}
v_d \\
v_q
\end{bmatrix}
\tag{2.5}
\]

\[
\dot{\omega} = \frac{\omega_0}{2\bar{H}_b^3} \left\{ (X_d - X_q)i_d i_q + (X_q' - X_q'')i_d i_{kq} - (X_d - X_{d\ell})A_d i_q i_f - (X_d - X_{d\ell})A'_d i_q i_{kd} + \frac{\omega_0^2}{2\bar{H}_b^3} \left\{ \tau_m - D(\omega) \right\} \right\}
\]
Herein, the state variables $i'_{f}, i'_{kd}$, and $i'_{kq}$ are related to the original variables $i_{f}, i_{kd}$, and $i_{kq}$ in respective order by constant parameters $A_{d}, A'_{d}$, and $A_{q}$:

$$i'_{f} = \frac{i_{f}}{A_{d}},$$

$$i'_{kd} = \frac{i_{kd}}{A'_{d}},$$

and

$$i'_{kq} = \frac{i_{kq}}{A_{q}}.$$ \hspace{1cm} (2.6)

The newly introduced time constants $\tau_{d}$ and $\tau_{q}$ are defined as:

$$\tau_{d} = \frac{X''_{d}}{\omega_{o}R_{a}},$$

$$\tau_{q} = \frac{X'_{q}}{\omega_{o}R_{a}}.$$ \hspace{1cm} (2.7)

The A-coefficients listed in Appendix B as simple functions of the standard machine parameters are dimensionless and are therefore independent of the base system chosen.

Note that in (2.5), each coefficient is explicitly expressed in terms of the standard machine parameters. Although this model is time-invariant and under the assumption of constant rotor speed is also linear, its usage is complicated during unbalanced operation of the machine. Consequently, considering the essentially unbalanced load type in this study, a model most suitable for unbalanced operations of the machine is presented in the next section.
2.13 State-Space Model in $\alpha, \beta$ Reference Frame

In the stationary $\alpha, \beta$ reference frame the $\alpha$-axis coincides with the axis of phase winding $a$ whereas the $\beta$-axis is perpendicular to it [18]. Therefore, the displacement angle of the $\alpha$-axis from the $d$-axis is the rotor position angle $\theta$. Transformation of equation (2.5) from $d,q$ to $\alpha,\beta$ variables results in:

$$\begin{align*}
\dot{X} &= A_G X + B_G v_f + E_G v_{\alpha,\beta} \\
\dot{\omega} &= \frac{\omega_0}{2H\omega_b^3} \left\{ (X_d - X_q) \left[ (i_{\beta}^2 - i_{\alpha}^2) \sin 2\theta / 2 + i_{\alpha} i_{\beta} \cos 2\theta \right] + \\
&+ \frac{\omega_0^2}{2H\omega_b^3} \left\{ t_m - D(\omega) \right\} \right\}
\end{align*}$$

with

$$X = [i_{\alpha} \ i_{\beta} \ i_f' \ i_{kd} \ i_{kq}]^t$$

The matrices $B_G$, $E_G$, and $A_G$ are as follows:

$$B_G = \frac{\omega_0}{X_d} \begin{bmatrix} A_d \cos \theta \\ A_d \sin \theta \end{bmatrix}, \quad A_V = \begin{bmatrix} A_d' & A_d \\ 0 & 0 \end{bmatrix}, \quad E_G = -\frac{\omega_0}{X_d^2} \begin{bmatrix} X_d' \sin^2 \theta + X_q' \cos^2 \theta & (X_q' - X_d') \sin 2\theta / 2 \\ (X_d' - X_q') \sin 2\theta / 2 & X_q'' \sin^2 \theta + X_d'' \cos^2 \theta \end{bmatrix}$$

(2.9)
\[
A_c = \begin{bmatrix}
-\cos^2\theta/\tau_d - \sin^2\theta/\tau_q \\
\omega(A_{dq} - A_{qd})\sin\theta/2 \\
(1/\tau_q - 1/\tau_d)\sin\theta/2 - \\
\omega(1 - A_{dq}\cos^2\theta - A_{qd}\sin^2\theta) \\
(1/\tau_q - 1/\tau_d)\sin^2\theta/2 + \\
\omega(1 - A_{dq}\sin^2\theta - A_{qd}\cos^2\theta) \\
-\cos\theta/\tau_d - \omega A_{dq}\sin\theta \\
-\sin\theta/\tau_d + \omega A_{dq}\cos\theta \\
-\cos\theta/\tau_d - \omega A_{dq}\sin\theta \\
-\sin\theta/\tau_d + \omega A_{dq}\cos\theta \\
\sin\theta/\tau_d - \omega A_{dq}\cos\theta \\
-\cos\theta/\tau_q - \omega A_{dq}\sin\theta \\
\end{bmatrix}
\]

\[
\begin{aligned}
& A_d\cos\theta/\tau_d' - A_{kd}\cos\theta/\tau_d'' - A_{qk}\sin\theta/\tau_q' + \\
& A_{df}\sin\theta/\tau_d' + A_{kd}\sin\theta/\tau_d'' + A_{qk}\cos\theta/\tau_q' + \\
& -A_{df}\cos\theta/\tau_d' - A_{kd}\cos\theta/\tau_d'' - A_{qk}\sin\theta/\tau_q' + \\
& \omega A_{qf}\sin\theta - \omega A_{qd}\sin\theta + \omega A_{dk}\cos\theta
\end{aligned}
\]

(2.9)
Note that this model could have been obtained by direct transformation of a model with \( a,b,c \) variables into \( \alpha,\beta \) variables. However, the coefficients of the resulting model would contain design machine parameters which are not directly measurable from field tests and do not have a one-to-one relationship to the standard machine parameters.

### 2.2 Order Reduction of Machine Model

For certain types of synchronous machine analysis where the accurate behavior in only a particular interval is desired, it is advantageous to use an approximate state model of reduced order. The use of a simplified machine model is sometimes even required. For example, analyzing a relatively large power system, it is not practical to represent each machine in the system by its fully detailed model since that would result in a system model which is too large to handle.

The behavior of a synchronous machine after a sustained disturbance can be chronologically decomposed into the immediate, subtransient, transient, and steady states. In order to clarify what is meant by these states, the machine response to a sudden change in the armature currents, although well-known [1,2,21,22], will be briefly reviewed.

Immediately after the occurrence of a sudden change in the armature currents, currents are induced in the field and damper windings. These currents oppose the armature m.m.f. change, and the opposition is initially strong
enough to maintain the flux-linkage of every rotor circuit at its initial value. The immediate state interval is defined here as the interval in which the flux linkages of the rotor windings are approximately still equal to their initial values. The length of this period depends mainly on the machine size and external circuits, and can vary from a fraction of a cycle to as long as a few cycles.

As the change in the flux-linkages become appreciable, the change in the damper winding flux-linkages is noticed first since the time constants associated with them are relatively much shorter than the field winding time constant. The interval where the field winding flux-linkage is approximately still equal to its initial value is considered the subtransient interval.

As the fast decaying damper winding effects disappear, the only damping effect experienced by the armature is from the field winding. The transient interval is defined as that interval of the machine behavior where change in the field winding flux-linkage is taking place while the effects of the damper winding are negligible. When the effect of the field winding disappears, the machine approaches steady state where the damping of all rotor windings has no or negligible effect on the terminal behavior. In the following sections, an appropriate model for each of the aforementioned states is proposed.

2.21 Immediate State

Since in the immediate state, the rotor flux-
linkages remain practically unchanged, these flux-linkages are assumed to be constant in the analysis. The armature and rotor currents then change in a manner such that the field and damper winding flux-linkages are maintained at their initial values. Designating the initial values of the currents at $t = t_o$ and $\theta = \theta_o$ by $i_{t_o}^j$, $j = \alpha, \beta, d, q, f, kd, $ and $kq$, the flux-linkage equations (2.2) become

$$\psi_f = \psi_f^i = -\frac{X_d - X_{dl}}{\omega_o} (i_d^o - i_f^o/A_f - i_{kd}^o)$$

$$\psi_{kd} = \psi_{kd}^i = -\frac{X_d - X_{dl}}{\omega_o} (i_d^o - i_f^o - A_kd^i i_{kd}^o)$$

$$\psi_{kq} = \psi_{kq}^i = -\frac{X_d}{\omega_o} (A_q^i q^k + i_{kd}^o)$$

(2.10)

This constraint provides additional algebraic equations by which the field and damper winding currents can be calculated from the armature currents without having to solve them from the differential equations. The algebraic equations are obtained from (2.2):

$$i_f = A_d i_d + \frac{\omega_o A_d A_kd^i}{(X_d^i - X_{dl})} \left[ A_kd^i \psi_f^o - \psi_{kd}^o \right]$$

$$i_{kd} = A_d^i i_d - \omega_o A_d^i \left[ \frac{A_d}{(X_d^i - X_{dl})} \psi_f^o - \frac{1}{X_d^i} \psi_{kd}^o \right]$$

$$i_{kq} = A_q^i q^k + \frac{\omega_o}{X_q} \psi_{kq}^o$$

(2.11)

Substitution of (2.11) in the machine model (2.5) taking (2.6) into account yields the second order state model for the machine in $d, q$ variables:
\[
\begin{bmatrix}
  \dot{i}_d \\
  \dot{i}_q 
\end{bmatrix} = \begin{bmatrix}
  -1/\tau_{d0} & \omega A''_{dq} \\
  -\omega A''_{qd} & -1/\tau_{q0}
\end{bmatrix} \begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix}
\]

\[
\omega = \frac{\omega_0}{2H\omega_b^3} \left\{ (X''_d - X''_q) i_d i_q + \omega A_q \psi_{qk}^o i_d + \omega A''_d (A_d \psi_f^o - \psi_{kd}^o) i_q \right\} +
\]

\[
\omega^2 = \frac{\omega_0^2}{2H\omega_b^3} \left\{ t_m - D(\omega) \right\}
\]

where the newly introduced time constants are defined as follows:

\[
\frac{1}{\tau_{d0}} = \frac{1}{\tau_d} - \frac{A_{df}}{\tau_{d0}} - \frac{A_{dkd}}{\tau_{d0}}
\]

\[
\frac{1}{\tau_{q0}} = \frac{1}{\tau_q} - \frac{A_{qkq}}{\tau_{q0}}
\]

Transformation of equation (2.12) into the \( \alpha, \beta \) reference frame results in
2.22 Subtransient State

In the analysis of the subtransient state, the field winding flux-linkage, \( \psi_f \), is assumed to remain constant at its initial value as in (2.10) since its change is not appreciable. Substitution of \( \psi_f^o \) in (2.2) gives an algebraic
equation from which the field current can be calculated as a function of the direct-axis armature and damper winding currents:

\[ i_f = A'_f \left[ i_d - i_{kd} + \frac{\omega_o}{X_d - X_{dl}} \psi_f^o \right] \quad (2.15) \]

The machine model for the subtransient state is obtained by substitution of equation (2.15) in (2.5).

\[
\begin{bmatrix}
   i_d \\
   i_q \\
   i_{kd} \\
   i_{kq}
\end{bmatrix} =
\begin{bmatrix}
   -1/\tau_d + A'_{df}/\tau_{do} & \omega_{dq} & A_{dkd}/\tau_{do} - A'_{df}/\tau_{do} & \omega_{dkq} \\
   -\omega(A_{qd} - A'_{qf}) & -1/\tau_q & \omega(A_{qkd} - A'_{qf}) & A_{qkq}/\tau_{qo} \\
   -1/\tau_d + A'_{kdf}/\tau_{do} & \omega_{dq} & A_{kd}/\tau_{do} - A'_{kdf}/\tau_{do} & \omega_{dkq} \\
   -\omega(A_{qd} - A'_{qf}) & -1/\tau_q & \omega(A_{qkd} - A'_{qf}) & A_{qkq}/\tau_{qo}
\end{bmatrix}
\begin{bmatrix}
   i_d \\
   i_q \\
   i_{kd} \\
   i_{kq}
\end{bmatrix} +
\begin{bmatrix}
   A_d/\tau_{do} \\
   \omega_{qf} \\
   \psi_f^o + \omega_{qf} \\
   A_{qf} \\
   0 \\
   0 \\
   0 \\
   0
\end{bmatrix}
\begin{bmatrix}
   A_d \\
   1/\chi''_d \\
   0 \\
   1/\chi''_d \\
   0 \\
   0 \\
   1/\chi''_d \\
   1/\chi''_d \\
\end{bmatrix}
\begin{bmatrix}
   V_d \\
   V_q 
\end{bmatrix} \quad (2.16)
\]

\[
\dot{\omega} = \frac{\omega_o}{2H\omega_b^3} \left\{ (X'_d - X_q)i_d i_q + (X_q - X''_q)i_d i_{kq} - (X'_d - X_{dl})A'_d i_q i_{kq} - \omega_o \frac{(X_d - X'_d)}{(X_d - X_{dl})} \psi_f^o i_q \right\} + \frac{\omega_o^2}{2H\omega_b^3} \left\{ t_m - D(\omega) \right\}
\]
Transformation of the approximate model (2.16) into the $\alpha, \beta$ reference frame yields:

\[
\dot{X} = A_P X + B_P v_f + B_{\phi} \psi_f \tau + E_P v_{\alpha, \beta}
\]  

(2.17)

\[
\omega = \frac{\omega_0}{2H\omega_0^3} \left\{ (X_q - X_d') \left[ (i_\alpha^2 - i_\beta^2) \sin 2\theta / 2 - i_\alpha i_\beta \cos 2\theta \right] + (X_q - X_d') (i_\alpha \cos \theta + i_\beta \sin \theta) i'_{kd} + (i_\alpha \sin \theta - i_\beta \cos \theta) \right\} + \frac{\omega^2}{2H\omega_0^3} \left\{ t_m - D(\omega) \right\}
\]

where

\[
X = \begin{bmatrix} i_\alpha & i_\beta & i'_{kd} & i'_{kq} \end{bmatrix}^t.
\]

The coefficient matrices $E_P$, $A_P$, $B_P$, and $B_{\phi}$ are written below.

\[
E_P = \frac{\omega_0}{\chi_m} \begin{bmatrix}
X_d' \sin^2 \theta + X_d' \cos^2 \theta & (X_q' - X_d') \sin 2\theta / 2 \\
(X_q' - X_d') \sin 2\theta / 2 & X_q' \sin^2 \theta + X_d' \cos^2 \theta \\
X_q' \cos \theta & X_q' \sin \theta \\
-X_d' \sin \theta & X_d' \cos \theta
\end{bmatrix}
\]  

(2.18)
2.23 Transient State

For the transient state analysis, the effect of the damper windings is neglected since the damper winding currents have approximately zero values or zero averages with relatively low peak values and frequencies greater than the machine frequency.

Under this condition, the machine electrical state variables are the armature and field currents only. The approximate model for the transient state can then be readily obtained in the \(d,q\) or \(\alpha,\beta\) reference frame by deleting the damper winding currents in (2.5) or (2.8).

2.24 Steady State

In the steady state, it is assumed that the values of the field and damper winding currents or their average values are equal to \(V_f/R_f\) and zero respectively. In addition, the variations of these currents from their average values are relatively small with frequencies greater than the machine frequency so that their effects on the terminal behavior are not noticeable. Therefore, constant field current and zero damper winding currents are assumed in the analysis.

With these assumptions, the armature currents are the only electrical state variables in the machine equations. The approximate state model in the \(d,q\) or \(\alpha,\beta\) reference frame can be directly found from (2.5) or (2.8) by substituting \(V_f/A_d R_f\) for \(i_f^r\) and zero for \(i_{kd}^r\) and \(i_{kq}^r\).
2.3 Permanent-Magnet Synchronous Machine

With the recent development of permanent-magnet (p.m.) alloys, especially the Alnico family, and voltage control availability offered by high power SCR'S, p.m. synchronous machines with a wide range of power ratings have assumed a more important role in power system applications. Since no external excitation, slip rings, brushes, and so forth are required, these machines are more compact and have lower cost and higher efficiency and reliability than the conventional type machines. These attractive features have made it desirable to develop models for the p.m. machines in the same form as the standard models of the conventional machines. The characteristics of a p.m. machine [23-27] are first briefly reviewed and its behavior is then modeled.

The rotor cross-section of a typical two-pole salient-pole p.m. machine is shown in Fig. 1. The armature winding arrangement is of the conventional three-phase type.

![Figure 1. Rotor Cross-Section](image-url)
The rotor structure is held together by a steel sleeve (non-magnetic) shrunk around it. The permanent-magnet consists of cast blocks which are arranged side by side to provide optimal magnetic properties. In order to protect the magnets from severe demagnetizing effects of a sudden short-circuit current or any other transient armature currents, a damper winding in the form of copper segments is fitted around and between the magnets. Steel pole tips support the magnet and maintain a tight magnetic circuit.

To ensure consistent performance, the magnets are stabilized by subjecting them to a demagnetizing force greater than any expected in service. The operating performance of the p.m. machine (in general) is therefore virtually the same as the conventional type operating at a constant field current. After stabilization, the permanent magnet operates on an approximately straight line in the demagnetization section of the B/H characteristics [23]. The volt-ampere equations in the d-q reference frame can be written as [25]

\[
\begin{align*}
V_d &= -R_a i_d - L_{d1} i_d + L_{md} i_{d1} + L_{md} i_{kd} - \omega (-L_{q1} i_q + L_{mq} i_{kq}) \\
V_q &= -R_a i_q - L_{q1} i_q + L_{mq} i_{kq} + \omega (-L_{d1} i_d + L_{md} i_{d1} + L_{md} i_{kd} + \psi_{pm}) \\
0 &= V_{d1} = R_{d1} i_{d1} + L_{md} i_{d1} + L_{md} i_{kd} \\
0 &= V_{kd} = R_{kd} i_{kd} - L_{md} i_d + L_{md} i_{d1} + L_{kd} i_{kd} \\
0 &= V_{kq} = R_{kq} i_{kq} - L_{mq} i_q + L_{q} i_{kq}
\end{align*}
\]
where $i_{dl}$ is the direct-axis damper winding current and $i_{kd}$ and $i_{kq}$ represent the abundant rotor body eddy currents in the direct and quadrature-axis respectively. The flux linkage $\psi_{pm}$ is constant and is equal to $ANB_0$ where $B_0$ is the useful magnetic flux density, $A$ is the permanent magnet cross-sectional area, and $N$ is the equivalent turns of the d-axis armature winding. Note that the per-unit system used here is the same as the one employed in Section (2.11).

Let a current $i_f$ be defined such that

$$i_{dl} = i_f - I_{pm}$$

where $I_{pm}$ is a constant current ($I_{pm} = 0$) taken equal to $\psi_{pm}/L_{md}$. Since $\omega\psi_{pm}$ is the open-circuit voltage $E$, $I_{pm} = E/\omega L_{md}$. Substitution of $i_{dl}$ from (2.20) into (2.19) results in the following:

$$V_d = -R_a i_d - L_{d} \dot{i}_d + L_{md} \dot{i}_f + L_{md} \dot{i}_{kd} - \omega(-L_{q} \dot{i}_q + L_{mq} \dot{i}_{kq})$$

$$V_q = -R_a i_q - L_{q} \dot{i}_q + L_{mq} \dot{i}_{kq} + \omega(-L_{d} \dot{i}_d + L_{md} \dot{i}_f + L_{md} \dot{i}_{kd})$$

$$V_f = R_{dl} \dot{i}_f - L_{md} \dot{i}_d + L_{dl} \dot{i}_f + L_{md} \dot{i}_{kd}$$

$$0 = R_{kd} \dot{i}_{kd} - L_{md} \dot{i}_d + L_{md} \dot{i}_f + L_{kd} \dot{i}_{kd}$$

$$0 = R_{kq} \dot{i}_{kq} - L_{mq} \dot{i}_q + L_{q} \dot{i}_{kq}$$

where the constant voltage $V_f$ is defined as $R_{dl} I_{pm}$. The permanent-magnet field system therefore resembles an electrically excited field winding of resistance $R_{dl}$, and inductance $L_{dl}$, to which is connected a constant dc source.
voltage $V_f$ of zero internal impedance. Comparison of equation (2.21) and the conventional machine equation reveals the following equalities between the parameters of the two machine types.

$$L_f = L_d$$

$$R_f = R_d$$

Thus, the machine models given in Section 2.1 can also represent a p.m. machine where the coefficients are found from the relationships given in (2.22).

2.4 Saturation Effect

Saturation presents complications in synchronous machine analysis since it introduces non-linearity in the machine equations. A large variety of techniques has been published [28-34] to account for saturation. Slemon [28] developed nonlinear equivalent circuits for saturated synchronous machines. The saturable regions of the machine are first identified, then each is represented by a non-linear reactance in the steady-state equivalent circuits. Although the evaluation of the nonlinear reactances is rigorously explained, the method is only applicable for steady-state analysis.

Garg [29] developed a comprehensive method to include saturation. Expressions were derived for the machine primitive inductances in terms of machine dimensions and magnetic characteristics. In this approach, the machine
permeabilities are recalculated at every digital simulation time instant from the knowledge of the machine flux-linkages and the magnetization characteristics of the armature and rotor core materials. The effect of saturation is therefore accounted for by updating the inductances instant by instant. The use of this technique, although very accurate and independent of the machine state behavior, is restricted due to the nonavailability of detailed machine dimensions and magnetization characteristics. Furthermore, because of the complexity of the nonlinear inductance expressions and their recalculation at every integration time step, the method involves exorbitant use of computer time.

The most widely suggested method [30-34] to account for saturation is by using a multiplier, generally known as saturation factor. This technique is utilized in the following recommended procedure. For a salient-pole synchronous machine, let the saturation factors be defined as

\[ k_i = \frac{\psi_i^s}{\psi_i^u}, \quad (i = d, q, f, kd, and kq) \]

where the superscripts s and u respectively refer to saturated and unsaturated values of the flux-linkages. Since the saturation on the quadrature axis is usually slight, the values of \( k_q \) and \( k_{qq} \) can be assumed unity. In obtaining the values of \( k_d \), \( k_f \) and \( k_{kd} \), it is desired that only the open-circuit curve be used since in practice it is the only saturation curve available. The nonlinear value of \( k_f \) is therefore approximated from the open-circuit charac-
teristic while \( k_d \) and \( k_{kd} \) are obtained from \( k_f \):

\[
k_d = N_d k_f
\]

\[
k_{kd} = N_{kd} k_f
\]

where \( N_d \) and \( N_{kd} \) depend on the armature and rotor circuit equivalent turn ratios and magnetic properties. Note that in obtaining \( k_f \), the effects of the armature and damper currents are ignored since the open-circuit characteristic is only dependent on the field current. A higher accuracy can be achieved using a family of zero power-factor characteristics where the armature current effect is accounted for. However, these curves are usually not available.

The parameters \( N_d \) and \( N_{kd} \) are in general nonlinear. If it is assumed that the degrees of saturation in the armature, field and damper flux-linkage paths are linearly proportional to each other, then the values of \( N_d \) and \( N_{kd} \) become constants and can be made equal to unity by appropriate readjustment of the per-unit base system. Under these assumptions, the saturation effect can be included by simply multiplying the direct-axis reactances in the flux-linkage equation (2.2) by \( k_f \).

In the digital simulation program, \( k_f \) is updated at every time step and remains fixed during the intertime step intervals. This staircase representation of \( k_f \) is numerically justified if the step size is sufficiently small. However, the technique can be modified by introducing \( k_f \) as a state variable.
3. TRANSFORMER MODELING

In this chapter, a set of equations describing the performance of a three-phase transformer with different types of winding connections is developed. The equations are written in an input-output relationship form so that they can be easily coupled with the equations of the components connected to the transformer. In the transformer equations, the transformer connection is reflected by a number of constants whose values depend on the connection type.

Two cases are considered. First, the equations are presented for a transformer with finite magnetization reactance. The transformer equations with infinite magnetization reactance, i.e., negligible magnetization current, are then obtained as a special case. In both cases, the transformer iron losses are assumed to be negligible and no grounding is available, i.e., no possible path for the flow of zero-sequence currents. However, the zero-sequence equations can be readily considered for grounded wye-connected transformer windings [35].

3.1 Finite Magnetization Reactance

The general three-phase transformer arrangement with labeling (in per unit) is shown in Fig. 2, where the primary and secondary windings can be connected in wye-wye, wye-Delta, Delta-wye or Delta-Delta.
The equations for each of these connections are first obtained in a, b, c variables, then transformed into $\alpha, \beta$ variables. The resulting transformer equations for any connection type can be written as

$$v_{\alpha, \beta} = r_t i_{\alpha, \beta} + \ell_t i_{\alpha, \beta} + R i_{\alpha', \beta'} + C v_{\alpha', \beta'}, \quad (3.1)$$

and

$$i_{\alpha', \beta'} = -\frac{1}{\tau_t} i_{\alpha', \beta'} + C_t i_{\alpha, \beta} - \frac{c_5}{k_{22}} v_{\alpha', \beta'}. \quad (3.2)$$
Herein,

\[ \ell_{22} = \ell_2 + \ell_m, \]

\[ \tau_t = \ell_{22}/\tau_2, \]

\[ r_t = c_0 r_1, \]

\[ e_t = c_0 (\ell_1 + \frac{\ell_2 \ell_m}{\ell_{22}}). \]

\[ C = \frac{\ell_m}{\ell_{22}} \begin{bmatrix} c_1 & -c_2 \\ c_2 & c_1 \end{bmatrix}, \]  \hspace{1cm} (3.3)

and

\[ R = \frac{\ell_m \tau_2}{\ell_{22}} \begin{bmatrix} c_3 & c_4 \\ -c_4 & c_3 \end{bmatrix}. \]

where the values of the \( c \)-constants which depend on the transformer connection type are listed in Table 1.

<table>
<thead>
<tr>
<th>Transformer Connection</th>
<th>( c_0 )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
<th>( c_4 )</th>
<th>( c_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y-Y</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Y-( \Delta )</td>
<td>1</td>
<td>( \sqrt{3/2} )</td>
<td>1/2</td>
<td>1/2</td>
<td>-1/2( \sqrt{3} )</td>
<td>1/3</td>
</tr>
<tr>
<td>( \Delta )-( \Delta )</td>
<td>1/3</td>
<td>1</td>
<td>0</td>
<td>1/3</td>
<td>0</td>
<td>1/3</td>
</tr>
<tr>
<td>( \Delta )-Y</td>
<td>1/3</td>
<td>( \sqrt{3/2} )</td>
<td>-1/2</td>
<td>1/2</td>
<td>1/2( \sqrt{3} )</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1. \( c \)-Constants for Different Transformer Connections
The transformer magnetization currents $i_{m\alpha,\beta}$, considered here as the internal transformer variables, are also functions of the transformer connection type:

$$i_{m\alpha,\beta} = \frac{22}{\xi_m} \left[ C^t i_{\alpha,\beta} - C i_{\alpha',\beta'} \right]$$

### 3.2 Negligible Magnetization Current

The magnetization currents of medium and large size transformers in power system modeling are usually neglected due to their very small and negligible effect on the overall terminal behavior. Neglecting the magnetization current amounts to letting $\xi_m$ approach infinity in equations (3.1) through (3.3) resulting in:

$$v_{\alpha,\beta} = r_t i_{\alpha,\beta} + \ell_t i_{\alpha,\beta} + C v_{\alpha',\beta'} \tag{3.4}$$

$$i_{\alpha,\beta} = C i_{\alpha',\beta'} \tag{3.5}$$

Now,

$$r_t = c_o (r_1 + r_2),$$

$$\ell_t = c_o (\ell_1 + \ell_2),$$

and

$$C = \begin{bmatrix} c_1 & -c_2 \\ c_2 & c_1 \end{bmatrix}.$$

The $c$-constants have the same values as those listed in Table 1.
4. THYRISTOR BRIDGE MODELING

A model for a three-phase thyristor bridge which can be used for any external electrical operating conditions is developed in this chapter. The method applied here is different from those used in several studies performed and published concerning bridge converter modeling [36-41]. The model is in a form that can be easily combined with the external network equations resulting in a minimum overall model order.

For the sake of clearness, some of the main characteristics of the rectification process, although well-known [42-44], are summarized. For simplicity, thyristor ideal switching is assumed in this study, i.e., inverse current and forward voltage drop are completely neglected.

A thyristor starts to conduct (ignite) as soon as the anode voltage (with respect to cathode) is positive and the gate voltage exceeds a critical voltage level. However, once it has been triggered on, it will remain on even if the gate signal is removed. Thyristor conduction is terminated when the anode current goes to zero. Therefore, thyristor extinction depends primarily on the external electric circuits.

Although each of the thyristors in the three-phase
thyristor bridge system operates as discussed earlier, the analysis of the bridge operation is more involved since there exists $2^6$ possible combinations of the thyristor operation modes. In the following sections, the operation of the thyristor bridge is analyzed and modeled.

4.1 Input-Output Equation

A model for a three-phase thyristor bridge in an input-output relationship form which is versatile in its scope of use is to be obtained. The thyristor bridge circuit diagram and variable notations are shown in Fig. 3.

![Figure 3. Three-Phase Thyristor Bridge Circuit Diagram](image-url)
In general, the physical process in a three-phase bridge connected thyristors represents a sequence of unsymmetrical switchings involving various combinations of simultaneously conducting thyristors. Depending on the number of simultaneously conducting thyristors, the bridge operation can be categorized into normal or abnormal operating conditions.

In the normal operating condition, the number of simultaneously conducting thyristors is either two or three. Consequently, two operation intervals (namely, conduction and commutation intervals) exist during this condition. Due to the importance of these two intervals, the thyristor bridge model for each one of them is presented in separate subsections.

In the abnormal operating condition there is no power flow through the bridge. The possible number of simultaneously conducting thyristors is none or larger than three. When none of the thyristors is conducting, the bridge acts as an open-circuit, i.e., $i_a = i_b = i_c = i_L = 0$. When more than three thyristors are conducting, however, the bridge acts as a short-circuit, i.e., $v_a = v_b = v_c = v_L = 0$. Note that under these operating conditions no relationship exists between the external ac and dc networks, i.e., these external circuits behave independently with respect to each other.
4.11 Conduction

During a conduction interval, the dc network is connected to two phases of the ac network while its third phase is open, i.e., the ac electrical network is operating as a single-phased system. The two thyristors involved in this period can be any pair of the six thyristors as long as they do not belong to the same phase.

Due to the type of unbalanced operation of the external ac network, the $\alpha,\beta$ variables are chosen for the ac terminal phase variables. The input-output relationship for all possible combinations is thus given by

\[ v_\ell = k_\alpha v_\alpha + k_\beta v_\beta \]

(4.1)

\[ i_\ell = i_\alpha/k_\alpha = i_\beta/k_\beta \]

where the values of the constants $k_\alpha$ and $k_\beta$, listed in Table 2, depend on which pair of thyristors is conducting.

It is evident from the bridge model (4.1) that the ac terminal currents are dependent, that is,

\[ i_\alpha = \frac{k_\alpha}{k_\beta} i_\beta. \]

(4.2)

Note that since the constant $k_\beta$ never assumes zero value, expression (4.2) can be used for any thyristor combination listed in Table 2.
### Table 2. $k$-Constants for Conduction

<table>
<thead>
<tr>
<th>Combination Code</th>
<th>Thyristors Conducting</th>
<th>$k_\alpha$</th>
<th>$k_\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1, 2</td>
<td>$\sqrt{3/2}$</td>
<td>$\sqrt{1/2}$</td>
</tr>
<tr>
<td>2</td>
<td>2, 3</td>
<td>0</td>
<td>$\sqrt{2}$</td>
</tr>
<tr>
<td>3</td>
<td>3, 4</td>
<td>$-\sqrt{3/2}$</td>
<td>$\sqrt{1/2}$</td>
</tr>
<tr>
<td>4</td>
<td>4, 5</td>
<td>$-\sqrt{3/2}$</td>
<td>$-\sqrt{1/2}$</td>
</tr>
<tr>
<td>5</td>
<td>5, 6</td>
<td>0</td>
<td>$-\sqrt{2}$</td>
</tr>
<tr>
<td>6</td>
<td>6, 1</td>
<td>$\sqrt{3/2}$</td>
<td>$-\sqrt{1/2}$</td>
</tr>
</tbody>
</table>

### Table 3. $k'$-Constants for Commutation

<table>
<thead>
<tr>
<th>Combination Code</th>
<th>Thyristors Conducting</th>
<th>$k'_\alpha$</th>
<th>$k'_\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1, 2, 3</td>
<td>$\sqrt{1/6}$</td>
<td>$\sqrt{1/2}$</td>
</tr>
<tr>
<td>2</td>
<td>2, 3, 4</td>
<td>$-\sqrt{1/6}$</td>
<td>$\sqrt{1/2}$</td>
</tr>
<tr>
<td>3</td>
<td>3, 4, 5</td>
<td>$-\sqrt{2/3}$</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>4, 5, 6</td>
<td>$-\sqrt{1/6}$</td>
<td>$-\sqrt{1/2}$</td>
</tr>
<tr>
<td>5</td>
<td>5, 6, 1</td>
<td>$\sqrt{1/6}$</td>
<td>$-\sqrt{1/2}$</td>
</tr>
<tr>
<td>6</td>
<td>6, 1, 2</td>
<td>$\sqrt{2/3}$</td>
<td>0</td>
</tr>
</tbody>
</table>
4.12 Commutation

In a commutation interval, three thyristors (each belonging to a different phase of the ac network) are conducting simultaneously. Under this condition, two of the ac terminal phases are short-circuited and connected through the dc network to the third ac phase.

Considering the type of unbalanced operation during the commutation interval, the $\alpha, \beta$ variables are chosen to describe the ac terminal behavior (the same as for the conduction interval). The input-output relationship for all possible thyristor combinations can be written in the following form:

$$v_\beta = v_\alpha / k'_\alpha = v_\beta / k'_\beta$$

(4.3)

$$i_\alpha = k'_\alpha i_\alpha + k'_\beta i_\beta$$

where the values of the constants $k'_\alpha$ and $k'_\beta$, listed in Table 3, depend on the combination of three conducting thyristors.

From the bridge model (4.3) it can be seen that the ac terminal voltages are related during this interval by a constant, i.e., $v_\beta = \frac{k'_\beta}{k'_\alpha} v_\alpha$. Since the constant $k'_\alpha$ never assumes a value equal to zero, this expression for $v_\beta$ can be used for any thyristor combination given in Table 3.
4.2 Rectifier Control

As mentioned earlier, an idealized thyristor with a positive anode voltage ignites as soon as a positive dc voltage with respect to the cathode, called the gate signal, is applied to the gate. The gate signal can delay the thyristor ignition by an angle $\alpha$ designated as the ignition delay angle. Since variation of $\alpha$ affects the output voltage level of the thyristor, $\alpha$ is used as a basic means of output voltage and power control.

In the three-phase thyristor bridge operation with equal firing interval assumption, a new thyristor is fired every 60 electrical degrees where the firing order is shown in Fig. 3 as the thyristor numbers. The gate signals are ideally in the form of pulses where their length can be as long as 180° [43]. Although a relatively short gate pulse is desirable in order to reduce the triggering circuit power consumption, certain thyristor bridge loads may require long gate signals.

The gate excitation circuit has to be independent of voltage transients or load variations. Consequently, it is usually fed by an infinite ac source through a gating transformer. However, in isolated systems such as a thyristor bridge fed by a synchronous generator where the generator output is not synchronized with an infinite ac source, the generator position can be used as a synchronizer for the control pulse generator. In such an operation, called rotor-position control, the beginning of each gate signal
pulse is physically triggered by the position of the rotor, where the detectors are located 60 electrical degrees apart. Thus, at every 60 electrical degrees, a thyristor is triggered in the firing order as indicated in Fig. 3.
5. COMPOSITE MODEL

The models of the synchronous machine, transformer, and thyristor bridge developed in the previous chapters are to be combined such that the composite system model will be applicable with any dc network type. Explicit models are first obtained for the machine-transformer (MT) unit which are then combined with the bridge equations. Assuming a general dc network model, the necessary equations which interface the model with the machine-transformer-bridge (MTB) model are developed. Finally a flow chart is presented to indicate the computational steps in the computer program.

5.1 Machine-Transformer Equation

The MT unit is shown in Fig. 4 where the terminal voltages and currents are indicated by the \( \alpha, \beta \) and \( \alpha', \beta' \) subscripts.

![Diagram of Machine-Transformer Unit](image)

Figure 4. Machine-Transformer Unit and Labeling
In combining the synchronous machine and transformer equations (respectively in Chapters 2 and 3), the machine mechanical equation remains unchanged as given in (2.8).

5.11 Finite Transformer Magnetization Reactance

First, the voltages \( v_{\alpha,\beta} \) from the transformer equation (3.1) are substituted in the machine electrical equations (2.8). After collecting the coefficients of the armature current derivatives \( i_{\alpha,\beta} \) of the resulting equations on the left hand side of the equation, the coefficient matrix of the machine state variable derivatives is obtained. Premultiplying the equations by the inverse of this matrix and subsequent substitution of \( i_{\alpha,\beta} \) in the remaining transformer equation (3.2) result in the explicit state equation of the MT unit:

\[
\dot{X} = AX + BV_f + Ev_{\alpha,\beta}, \tag{5.1}
\]

where

\[
X = [i_\alpha, i_\beta, i'_f, i'_k, i'_d, i'_{\alpha'}, i'_{\beta'}]^t.
\]

The elements of the matrices \( A(7 \times 7) \), \( B(7 \times 1) \), and \( E(7 \times 2) \) are given in Appendix C as functions of the machine and transformer parameters and the transformer connection constants.

5.12 Negligible Transformer Magnetization Current

By letting \( \lambda_m \) in (5.1) approach infinity or by direct combination of equations (2.8), (3.4), and (3.5), the equation of the machine-transformer unit is obtained:
\[
\dot{X} = A_T X + B_T v_f + E_T v_{\alpha\beta'}
\]  
(5.2)

where

\[
X = [i_{\alpha}, i_{\beta}, i_{f}, i_{kd}, i_{kq}]^t,
\]

and the matrices \(A_T\), \(B_T\), and \(E_T\) are listed below.

\[
A_T = \begin{bmatrix}
-c_a^2/\tau_d - c_{\beta}^2/\tau_q & c_a c_{\beta}(1/\tau_q - 1/\tau_d) & c_a A_{df}/\tau_d & c_a A_{kd}/\tau_d & -c_{\beta} A_{qkd}/\tau_q \\
-\omega c_a c_{\beta}(A_{dq} - A_{q-d}) & -\omega(1 - c_a^2 A_{dq} - c_{\beta}^2 A_{q-d}) & -\omega c_{\beta} A_{dq} & -\omega c_{\beta} A_{qkd} & +\omega c_{\alpha} A_{kq} \\
c_a c_{\beta}(1/\tau_q - 1/\tau_d) & -c_{\beta}^2/\tau_d - c_a^2/\tau_q & c_{\beta} A_{df}/\tau_d & c_{\beta} A_{kd}/\tau_d & +c_{\alpha} A_{qkd}/\tau_q' \\
+\omega(1 - c_{\beta}^2 A_{dq} - c_a^2 A_{q-d}) & +\omega c_a c_{\beta}(A_{dq} - A_{q-d}) & +\omega c_a A_{dq} & +\omega c_{\alpha} A_{qkd} & +\omega c_{\beta} A_{kq} \\
-c_a/\tau_d - \omega c_{\beta} A_{dq} & -c_{\beta}/\tau_d + \omega c_a A_{dq} & A_f/\tau_d & A_{kd}/\tau_d & \omega c_{\alpha} A_{kq} \\
-c_a/\tau_d - \omega c_{\beta} A_{dq} & -c_{\beta}/\tau_d + \omega c_a A_{dq} & A_{kd}/\tau_d & A_{kd}/\tau_d & \omega A_{kq} \\
c_{\beta}/\tau_q - c_a A_{q-d} & -c_a/\tau_q - \omega c_{\beta} A_{q-d} & \omega A_{qf} & \omega A_{q kd} & A_{kq}/\tau_q \\
\end{bmatrix}
\]

\[
B_T = \begin{bmatrix}
c_{\alpha} A_{d} \\
c_{\beta} A_{d} \\
\omega A_{V} \\
A_{V} \\
0 \\
\end{bmatrix} , \quad E_T = -\omega \begin{bmatrix}
c_{\alpha}/X''_d + c_{\beta}/X'_q \\
c_{\alpha} c_{\beta}(1/X''_d - 1/X'_q) \\
c_{\alpha}/X''_d \\
- c_{\beta}/X'_q \\
c_{\alpha}/X'_q \\
\end{bmatrix} 
\]
(5.3)
The $c_{\alpha}$ and $c_{\beta}$ coefficients appearing in (5.3) are functions of the transformer $c$-constants:

$$c_{\alpha} = c_1 \cos \theta + c_2 \sin \theta$$
$$c_{\beta} = c_1 \sin \theta - c_2 \cos \theta$$

(5.4)

It is noted that the effect of the transformer resistance $r_t$ and inductance $l_t$ is the same as adding a series impedance to the machine armature circuit. The standard machine impedances are then modified as shown at the end of Appendix C.

5.2 Machine-Transformer-Bridge Equation

The composite system block diagram with the terminal variable labeling is shown in Fig. 5.

As described in Chapter 4, the performance of a system involving a bridge converter can be categorized into normal and abnormal operating conditions. The system equations under normal operation (conduction or commutation) with finite and infinite transformer magnetization reactance are given in the following sections. For the abnormal operating
condition, the system equations for the open or short-circuits are readily obtained by substituting respectively zero for the terminal currents $i_{\alpha', \beta'}$ or voltages $v_{\alpha', \beta'}$ in the MT equations in Section 5.1.

5.21 Conduction Interval

In combining the MT equation with finite magnetization reactance (5.1) and the thyristor bridge equation (4.1), the $\alpha, \beta$ subscripts of the voltages and currents in (4.1) are changed to $\alpha', \beta'$. Because of the dependency which exists during the conduction interval between $i_{\alpha'}$ and $i_{\beta'}$ ($i_{\alpha'} = k_{\alpha} i_{\beta'}/k_{\beta}$), the order of the composite system model is reduced by one, i.e., the terminal current $i_{\ell} = i_{\beta'}/k_{\beta}$ is considered as a state variable while $i_{\alpha'}$ is eliminated. The resulting system equation is then as follows:

$$
\dot{X} = A' X + B' v_f + E' v_{\alpha' \beta'},
$$

with

$$
v_{\alpha'} = D' X + n'_f v_f + n'_\ell v_\ell
$$

and

$$
v_{\beta'} = -(k_{\alpha}/k_{\beta}) v_{\alpha'} + (1/k_{\beta}) v_\ell
$$

Herein,

$$
X = [i_{\alpha} \ i_{\beta} \ i_{f} \ i_{k_d} \ i_{k_q} \ i_{\ell}]^t
$$

where the elements of the matrices $A'(6 \times 6)$, $B'(6 \times 1)$, $E'(6 \times 2)$ and $D'(6 \times 1)$, and the $n'$-coefficients are given in Appendix D. Note that the $v_{\alpha'}$ expression in (5.6) is obtained from the relationship between $i_{\alpha'}$ and $i_{\beta'}$, while $v_{\beta'}$ is found from (4.1).
The composite model when the transformer magnetization reactance approaches infinity can be obtained from the general system equation (5.5) or directly from the combination of (4.1) and (5.2). The resulting system model becomes

\[ \dot{\mathbf{X}} = A'_r \mathbf{X} + B'_r \mathbf{v}_f + E'_r \mathbf{v}_{\alpha\beta'} \quad (5.7) \]

with

\[ \mathbf{v}_{\alpha'} = D \mathbf{X} + n_f \mathbf{v}_f + n_l \mathbf{v}_l \quad (5.8) \]

\[ \mathbf{v}_{\beta'} = -(k_\alpha/k_\beta) \mathbf{v}_{\alpha'} + (1/k_\beta) \mathbf{v}_l \]

Herein,

\[ \mathbf{X} = [i_l \quad i_f' \quad i_{kd} \quad i_{kq}]^t \]

where the matrices \( A'_s \), \( B'_s \), \( E'_s \) and \( D \), and the \( n \)-coefficients are listed below.

\[ A'_r = \begin{bmatrix}
-(c_\alpha g_\beta \tau_q + c_\beta g_\alpha /\tau_d')/k_\beta & c_\beta A_{df}/k_\beta & c_\beta A_{dkd}/k_\beta & c_\alpha q k q /k_\beta \\
\omega (k_\alpha - c_\alpha g_\alpha A_{qd} + c_\beta g_\beta A_{dq}) /k_\beta & \omega c_\alpha A_{q\beta}/k_\beta & \omega c_\alpha A_{qkd}/k_\beta & \omega c_\alpha A_{dkq}/k_\beta \\
-g_\alpha /\tau_d + \omega g_\beta A_{dq} & A_f /\tau_d' & A_{fkd} /\tau_d' & \omega A_{dkq} \\
-g_\alpha /\tau_d + \omega g_\beta A_{dq} & A_{kdf} /\tau_d' & A_{kd} /\tau_d' & \omega A_{dkq} \\
-g_\beta /\tau_d - g_\alpha A_{qd} & \omega A_{qf} & \omega A_{qkd} & A_{kq} /\tau_d' \\
\end{bmatrix} \quad (5.9) \]
\[
B_r = \begin{bmatrix}
\omega \\
A_V \\
A'_V \\
0
\end{bmatrix}, \quad E_r = -\omega
\]

\[
D^t = \frac{k_\beta}{g}
\begin{bmatrix}
g_\alpha g_\beta (1/\tau_q - 1/\tau_d)
g_\beta A_d / \tau^t_d - \omega g_\alpha A_q f \\
g_\beta A_d / \tau^t_d - \omega g_\alpha A_q k d \\
-g_\alpha A_q k q / \tau^t_q - \omega g_\beta A_d k q
\end{bmatrix}
\]

\[
n_f = \omega k_\beta g_\alpha A_d / g \ X'_d
\]

\[
n_q = \omega (c_\alpha g_\alpha / X''_q - c_\beta g_\beta / X''_d) / g
\]

with

\[
g_\alpha = c_\alpha k_\alpha + c_\beta k_\beta
\]

\[
g_\beta = c_\alpha k_\beta - c_\beta k_\alpha
\]

and

\[
g = \omega (g_\alpha^2 / X''_q + g_\beta^2 / X''_d)
\]

The \(c_\alpha\) and \(c_\beta\) parameters are given in (5.4) in terms of the \(c\)-constants.
5.22 Commutation Interval

Priming the subscripts $\alpha, \beta$ in the thyristor bridge equation (4.3) and subsequent substitution of $v_{\alpha'}$ and $v_{\beta'}$ in terms of $v_{\ell}$ in the MT equation (5.1) results in the following MTB equation where the transformer magnetization reactance is considered finite:

$$\dot{X} = A_X X + B v_f + F v_{\ell} \quad (5.10)$$

with

$$i_{\ell} = k'_\alpha i_\alpha + k'_\beta i_{\beta'}$$

In (5.10), $X$, $A(7 \times 7)$, and $B(7 \times 1)$ are the same as those in (5.1), while the elements of the newly defined matrix $F(7 \times 1)$ are given in Appendix D.

With the assumption of negligible transformer magnetization current, the composite system model can be obtained by letting $\ell_m$ in (5.10) approach infinity or by direct substitution of (4.3) into (5.2). This results in

$$\dot{X} = A_r X + B_r v_f + F_r v_{\ell} \quad (5.11)$$

with

$$i_{\ell} = k'_\alpha i_\alpha + k'_\beta i_{\beta'}$$

Herein, $X$ is the same as in (5.2), $A_r(5 \times 5)$ and $B_r(5 \times 1)$ are given in (5.3), and
where

\[ g'_\alpha = c'_\alpha k'_\alpha + c'_\beta k'_\beta \]

\[ g'_\beta = c'_\alpha k'_\beta - c'_\beta k'_\alpha \]

Recall that \( c_\alpha \) and \( c_\beta \) which are functions of the \( c \)-constants (5.4) determine the transformer connection type and the \( k \)-constants indicate the thyristors conducting.

5.3 DC Network and Interface Equations

The dc side of the thyristor bridge is connected to a network which generally can be of any type and also can include dc source voltages.

![Figure 6. DC Network Block Diagram](image-url)
Let the state equation of the network as shown in Fig. 6 be given as

$$ \dot{X}_k = A_{dc} X_k + B_{dc} U_k$$  \hspace{1cm} (5.13)

where $A_{dc}$ and $B_{dc}$ are the coefficient matrices and $U_k$ is the input.

The dc network equation (5.13) together with the ac network equations in Section 5.2 give the overall system model. The equation needed to interface the two network equations depends on whether the terminal voltage $v_k$ and current $i_k$ in (5.13) are state or input variables. The interface equation for each of the possible conditions is given below.

a. $v_k \in X_k$ and $i_k \in U_k$: No interface equation is required in this case since there is no repetition of state variables in the two network equations.

b. $i_k \in X_k$ and $v_k \in X_k$ or $U_k$: Equation (5.13) can be rewritten as

$$ \dot{X}'_k = G X'_k + H i_k + K U'_k$$  \hspace{1cm} (5.14)

and

$$ v_k = J X'_k + r i_k + l \dot{i}_k + I U'_k$$  \hspace{1cm} (5.15)

where $X'_k$ includes all of the dc network state variables except $i_k$. The terminal voltage $v_k$ can be part of $X'_k$ or $U'_k$. 
The interface equation for the conduction interval is developed by first substituting the expression for $i_L$ from the MTB equation (5.5) or (5.7) into (5.15). The resulting $V_L$ expression is then substituted in the $v_{\alpha',\beta'}$ expressions (5.6) or (5.8). After collecting the $v_{\alpha',\beta'}$ terms, the algebraic equation can be written in the following form.

$$v_{\alpha' L} = P X + Q X_L' + M v_f + N U_L' . \quad (5.16)$$

This equation interfaces the ac network equation (5.5) or (5.7) to the dc network equation (5.14) with $v_L$ replaced by $k_\alpha v_{\alpha'} + k_\beta v_{\beta'}$.

For the commutation interval, $i_L$ in equation (5.15) is first replaced by $k_\alpha i_{\alpha'} + k_\beta i_{\beta'}$, the $i_{\alpha'}$ and $i_{\beta'}$ expressions from (5.10) or (5.11) are then substituted in this equation. The resulting equation which links (5.14) to the MTB equation (5.10) or (5.11) can be written as

$$v_L = P' X + Q' X_L' + M' v_f + N' U_L' . \quad (5.17)$$

c. $v_L$ and $i_L \in U_L$: The dc network equation for this case is of the same form as equation (5.14) and (5.15) with the term $\ell i_L$ eliminated. The interface equation for the conduction period is obtained by substitution of $v_L$ from (5.15) in (5.6) or (5.8). The resulting algebraic equation is of the same form as (5.16). For the commutation period, the interface equation is (5.15).

For illustration purposes, a RLC load (Fig. 7) which will be used in the simulation test case in Chapter 6
is now considered.

Figure 7. RLC Load

The RLC load equations can be written in a form similar to equations (5.14) and (5.15) of case b with \( v_c \) and \( i_\ell \) \( \in \mathbb{X}_\ell' \), and \( v_\ell \) \( \in \mathbb{U}_\ell' \)

\[
\dot{v}_c = -(1/R_c C_\ell) v_c + (1/C_\ell) i_\ell \\
v_\ell = v_c + R_\ell i_\ell + L \dot{i}_\ell
\] (5.18) (5.19)

The transformer magnetization current is considered negligible for this example. Thus, the overall system model for the conduction interval is (5.7) and (5.18), and for the commutation interval (5.11) and (5.18) where the coefficients of the interface equations (5.16) and (5.17) are obtained as described in case b and listed in Appendix F.
5.4 Computational Flow Chart

A flow chart for digital simulation of the overall system is shown in Fig. 8. The flow chart basically contains four separate computational regions for the conduction, commutation, open-circuit, and short-circuit operating conditions of the system. It also indicates the basic computational logics pertinent in the transition between the four regions.

In the preparation process, the generator, transformer, and load parameters, transformer connection type, step length $\Delta t$, total simulation time $t_{\text{end}}$, and thyristor ignition delay angle $\alpha$ are inputed. The $\theta$-independent parts of the coefficients of the system models (5.5)-(5.17) are then calculated. The initial values of the state variables and the thyristor combination code $I_{\text{cc}}$ are obtained. The value of $I_{\text{cc}}$ which is incremented rotationally from 1 to 6 determines the thyristors conducting and their corresponding values $k_{\alpha,\beta}$ and $k'_{\alpha,\beta}$ (Tables 2 and 3). The initial value of $I_{\text{cc}}$ depends on the value of $\alpha$ and the initial rotor position angle $\theta_0$. For the Y-Y or A-A connections, if $\theta_0$ is for example equal to 90 electrical degrees (thus no-load voltage $v_0^* = 0$), $I_{\text{cc}}$ is initially equal to 3 for $\alpha \leq 60^\circ$ and 2 for $60^\circ < \alpha \leq 120^\circ$. Note that to obtain the same steady-state dc voltage level, there is a $30^\circ$ advancement or retardation in $\alpha$ for the A-Y and Y-A connections respectively.

The interface logics shown in the flow chart indicate the necessary conditions for the start or termination of
Figure 8. Computational Flow Chart

START

Parameters & Constants

I.C. t=0, J=0

Conduction Interval Eqns

Open-CKT Eqns

J=J+1

I_{cc} = I_{cc} + 1

is

Y

N

is

v_n' > v_n

\omega t = \alpha + J\pi / 3

Y

N

is

Y

N

Commutation Interval Eqns

To 4 (o.c.)

J=J+1, J_c = 0

J=J-l

I_{cc} = I_{cc} + 1

Short-CKT Eqns

To 4 (o.c.)

J=J+1

J=J+1

N

N

Y

N

Y

N

\omega t = \alpha + J\pi / 3

N

is

N

\omega t = \alpha + J\pi / 3

N

is

N

is

Y

N

\omega t = \alpha + J\pi / 3

N

is

Y

N

is

Y

N

is

Y

N

is

Y

N
each computational region as explained in the following.

a. The conduction computational region is terminated and followed by the commutation region when a new thyristor is triggered on. After every 60 electrical degrees advancement of the rotor \( \theta = \alpha + j\pi/3; j = 0,1,2,\ldots \), a new thyristor is fired and it will be ignited when the voltage across it is checked to be positive. At any time, however, the conduction region is terminated and followed by the open-circuit region if the load current \( i_L \) goes to zero.

b. The commutation continues as long as the current in the outgoing thyristor, \( i_{I_{cc}} \), has not reached zero and \( \theta < \alpha + J\pi/3 \). In case a new thyristor is fired before \( i_{I_{cc}} \) has become zero, the value of \( v_L \) (which is also the negative of the voltage across the new thyristor) is then checked. If \( v_L \) is positive, commutation is continued and the value of \( J_c \) which indicates the number of new thyristors fired in this interval is increased by one. Otherwise, \( v_L < 0 \), the commutation interval is terminated and followed by the short-circuit region. In the normal operating condition, however, the commutation interval ends and followed by the conduction interval, i.e., when \( i_{I_{cc}} \) goes to zero, \( i_L \neq 0 \), and \( J_c = 0 \). Note that the combination code \( I_{cc} \) is incremented (1 to 6, then back to 1) only when \( i_{I_{cc}} = 0 \).

c. The short-circuit region lasts as long as more than three thyristors are conducting. The integer \( J_s \) which has
an upper limit equal to 3 indicates the number of thyristors conducting in excess of 3. This integer which is initially equal to \( J_C \) is incremented by one whenever a new thyristor is fired in this region. The short-circuit region is terminated and followed by commutation when \( i_{I_{CC}} \) reaches zero, \( i_Z \neq 0 \), and \( J_S = 0 \).

d. The open-circuit computational region lasts until the GT terminal no-load voltage, \( v'_L = k_\alpha v'_\alpha + k_\beta v'_\beta \), is equal to or greater than the load terminal open-circuit voltage \( v_L \). This region is always followed by conduction.
6. MODEL APPLICATION EXAMPLE

As an application example, the generator-transformer-bridge (GTB) unit is used as a regulated dc power source in a network configuration as shown in Fig. 9 for generating repetitive high-voltage, high-power pulses.

![Figure 9. High-Power Pulse Generating System](image)

When a trigger pulse turns \( S_c \) on, charging of capacitor \( C \) is initiated. The capacitor leakage resistance is represented by \( R_c \) and the inductance of a resonance inductor by which the charging time can be adjusted is designated by \( L \) with its resistance \( R_L \). After \( C \) is charged up to the desired voltage level, \( S_c \) is turned off and \( S_d \) turned on, resulting in the discharge of energy from \( C \) through a pulse forming network into the load after which the conditions are reset for the next cycle.

The interaction between the regulated dc supply and the RLC network is independent of the remaining system and will be analyzed in this chapter. Because of the independency,
it is assumed that the $S_c$ gate signal is never removed and also that $S_d$ is never fired. This assumption permits the state variables to reach the steady state from which useful conclusions can be drawn.

6.1 System Data

For this text case, an aircraft generator with manufacturer listed parameters (except for the armature leakage reactance $X_{dl}$ whose value was unknown and was approximated) given in Table 4 was used. Using the generator ratings for the system base, the resistance and reactance of the transformer were respectively assumed equal to 0.0003 pu and 0.0237 pu.

$$
\begin{align*}
X_d &= 2.10 \quad X_q = 0.786 \quad \tau_{do} = 0.2075 \\
X_d' &= 0.216 \quad X_q' = 0.107 \quad \tau_{do}' = 0.00726 \\
X_d'' &= 0.1863 \quad X_{dl} = 0.04 \quad \tau_{qo}'' = 0.0460 \\
R_a &= 0.0189
\end{align*}
$$

Table 4. Data in Per-Unit of a 120 KVA, 208V, 400 Hz Synchronous Machine (Base Time = 1).

Assuming a pulse cycle of 10 ms with a pulse energy of 1 kilojoules at rated generator voltage results in $C = 8.33 \times 10^{-3}$ pu with an estimated leakage resistance $R_c = 1.2 \times 10^5$ pu. The dc charging current $i_d$ will vary approximately as a half-wave sinusoid, starting at zero and after reaching a maximum decays back to zero. If the desired charging time is around 5 ms, the resonance inductance $L = 2.363 \times 10^{-4}$ pu. Note that the value for $L$ is an approximate value because of the sinusoidal assumption of the charging current and estimation of the equivalent inductance of the GTB unit.
6.2 Simulation Results

A computer program was developed to simulate the behavior of the subsystem consisting of the GTB unit and RLC network. The program in double precision was set up in the manner described in Section 5.4 where negligible generator saturation and transformer magnetization current were assumed. Since the generator for this type of application is usually equipped with a sufficiently large flywheel and also since the duration of the charging period is relatively short, the generator speed during the charging period does not change appreciably and was considered constant. For the integration of the differential equations, the fourth order Runge-Kutta method was employed. The integration step-length was taken equal to one electrical degree (6.944 ms).

The program was run starting from no-load steady-state with 1 pu generator voltage corresponding to

\[ V_f = \frac{(X_d - X_{d'}^*)E^0}{\omega_e(X_d - X_{d'}^*)^T_{do}} = 0.0021 \text{ pu} \quad i_f = 0.4854 \text{ pu}. \]

At time \( t_0 = 0 \), with an assumed initial rotor position angle of -30°, \( S_c \) is turned on and the capacitor \( C \) begins to charge up from zero value. The capacitor voltage \( v_c \) can reach its maximum value equal to the peak value of the line-to-line voltage (=\( \sqrt{2} \) pu) when the bridge firing angle \( \alpha = 0 \) and \( R_c \to \infty \). When \( \alpha \) is larger than or equal to 120°, \( v_c \) takes on its minimum value of zero (no charging).
For $0^\circ \leq \alpha \leq 120^\circ$, the values of $v_c$ obtained from the computer simulation results with the full-order model and Δ-Y transformer connection are given in Table 5 and plotted in Fig. 10 (after 3 cycles or 7.5 ms, and 100 cycles or 250 ms, of charging). Selected simulation plots for three values of $\alpha$ ($0^\circ$, $35^\circ$, and $70^\circ$) are shown in Figs. 11, 12 and 13 respectively. These plots indicate the variation of the generator terminal voltages $v_a$, $v_b$ and $v_c$, rotor currents $i_f$, $i_{kd}$ and $i_{kq}$, armature currents $i_a$, $i_b$ and $i_c$, GTB unit terminal voltage $v'_q$, capacitor voltage $v_c$, and load current $i_l$ as functions of time where because of lack of available space, selected cycles of more significance are only shown. Some detailed numerical results concerning these plots are given in Table 6.

![Figure 10. Capacitor Voltage vs Firing Angle (Full-Order Model, Δ-Y Connection).](image)
<table>
<thead>
<tr>
<th>α (degrees)</th>
<th>( v_c ) after 3 cycles</th>
<th>( v_c ) after 100 cycles</th>
<th>( i_g )</th>
<th>( i_f )</th>
<th>( i_{kd} )</th>
<th>( i_{kq} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.35</td>
<td>1.36</td>
<td>3.6</td>
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<td>0.56</td>
<td>3.4</td>
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<tr>
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<td>0</td>
<td>0.485</td>
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</tr>
</tbody>
</table>

Table 5. Per-Unit Values of Variables as Functions of \( \alpha \) (Full-Order Model, Δ-Y Connection).

<table>
<thead>
<tr>
<th>α (degrees)</th>
<th>No. of cycles to reach first zero value of ( i_g )</th>
<th>% pu values after 100 cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( i_g ) (peak)</td>
</tr>
<tr>
<td>0</td>
<td>2.43</td>
<td>0.51</td>
</tr>
<tr>
<td>35</td>
<td>2.66</td>
<td>0.89</td>
</tr>
<tr>
<td>70</td>
<td>2.44</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Table 6. Summary of Numerical Results (Full-Order Model, Δ-Y Connection).
Figure 11. Response Curves Using Full-Order Model and Δ-Y Connection for \( \alpha = 0^\circ \)
Figure 12. Response Curves Using Full-Order Model and Δ-Y Connection for $\alpha=35^\circ$
Figure 13. Response Curves Using Full-Order Model and Δ-Y Connection for $\alpha=70^\circ$
Figs. 14, 15 and 16 indicate the responses of the full-order model with $\alpha = 35^\circ$ for Y-Y, Y-$\Delta$ and $\Delta$-$\Delta$ transformer connections respectively. Comparison of these responses and that with $\Delta$-Y connection (Fig. 12) indeed shows that the generator line-current harmonic contents are reduced when $\Delta$-Y or Y-$\Delta$ connections are used. Furthermore, no noticeable difference can be seen in the system behavior between the $\Delta$-Y and Y-$\Delta$ connections, and also between the Y-Y and $\Delta$-$\Delta$ connections.

Finally, the computer program was modified to include the transformer magnetization current. The simulation results, however, indicated no noticeable change in the system response and therefore are not shown.
Figure 14. Response Curves Using Full-Order Model and Y-Y Connection for $\alpha=35^\circ$
Figure 15. Response Curves Using Full-Order Model and Y-Δ Connection for $\alpha = 35^\circ$
Figure 16. Response Curves Using Full-Order Model and Δ-Δ Connection for \( \alpha = 35^\circ \)
6.3 Reduced Order Simulation

System simulations were performed for different approximate generator models developed in Chapter 2 with Δ-Y transformer connection and α = 35°. A reduced value for the capacitor leakage resistance, \( R_C = 2.0 \) pu, was used so that the steady-state load current \( i_\phi \) does not become too small and better comparison of the models with the full-order model (Fig. 17) can be made.

Figs. 18 through 21 show the system behavior using the approximate generator models where some of the numerical results are summarized in Table 7. Each figure shows selected response intervals (in cycles) during which comparison of the accuracy of the model with the full-order model can be best described. As expected, the system response using the immediate-state approximate model (Fig. 18) matches almost perfectly during the first few cycles (up to 3) with the full-order model response, see Fig. 17. For the subtransient-state model (Fig. 19), however, satisfactory accuracy is obtained up to the 5th cycle. Both approximate models result in higher values for the line voltages and currents than those with the full model after the 5th cycle.

Comparison of the simulation results for the transient-state and full-order models indicates discrepancies in the performance during the first few cycles, but practically no difference after the fifth cycle. The response of the steady-state approximate model, Fig. 21, shows a reasonably good accuracy after the fifteenth cycle.
<table>
<thead>
<tr>
<th>Generator model</th>
<th>Max. value of $i_L$ (pu)</th>
<th>Avg. pu values after 100 cycles</th>
<th>Best accuracy interval (cycles)</th>
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<tbody>
<tr>
<td>Full-order</td>
<td>2.7</td>
<td>0.63</td>
<td>0.33</td>
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<tr>
<td>Immediate-state</td>
<td>2.7</td>
<td>0.99</td>
<td>0.49</td>
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<tr>
<td>Subtransient-state</td>
<td>2.8</td>
<td>1.24</td>
<td>0.62</td>
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<tr>
<td>Transient-state</td>
<td>1.6</td>
<td>0.60</td>
<td>0.30</td>
</tr>
<tr>
<td>Steady-state</td>
<td>0.65</td>
<td>0.62</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Table 7. Response Comparison for Different Models, Δ-Y connection $\alpha = 35^\circ$, $R_c = 2.0$ pu.
Figure 17. Response Curves Using Full-Order Model and Δ-Y Connection for $\alpha=35^\circ$ and $R_c=2.0\,\text{pu}$
Figure 17. Continued
Figure 18. Response Curves Using Immediate-State Model and Δ-Y Connection for α=35° and $R_c=2.0 \text{pu}$
Figure 19. Response Curves Using Subtransient-State Model and Δ-Y Connection for α=35° and R_c=2.0 pu
Figure 20. Response Curves Using Transient-State Model and Δ-Y Connection for $\alpha=35^\circ$ and $R_c=2.0$ pu
Figure 21. Response Curves Using Steady-State Model and Δ-Y Connection for α=35° and R_c=2.0 pu
7. CONCLUSIONS

The dynamic characteristics of a synchronous machine connected to a bridge converter through a three-phase transformer have been analyzed and comprehensively modeled. A number of approximate machine models was proposed by the use of which the system order is reduced, i.e., an order reduction of three for the immediate-state and steady-state models, two for the transient-state model, and one for the sub-transient model. A digital computational procedure was presented for the simulation of the developed models. Relative to the simulation methods proposed by other authors, the procedure presented here has the following features.

- Explicitly expressed state equations
- Minimum number of state-variables for various bridge operating modes
- Required data are conventional
- Capability of considering different transformer winding connections
- Capability of including transformer magnetization current
- Capability of coping with normal as well as complicated abnormal operating conditions of the bridge
- Ready adaptability for any application
- Relatively short computer CPU time (0.5-1.0 ms per integration time step for the case studied)
Based on the simulation results of the application example, relevant and useful information concerning the system performance by using the full-order and the approximate models has been obtained. Comparison of the short-term responses for the immediate-state or subtransient-state models and the full-order model indicated a good correlation of the generator and bridge output variables. For the long-term response, the transient-state model was found to be the most accurate approximate model. The steady-state model, however, proved to be the least accurate not only for the short-term response but also for the long-term response if compared with the transient-state model, especially for the generator output voltages. These results suggest the use of the immediate-state model for the short-term response and the transient-state model for the long-term response. However, for any other particular application, the system responses using the approximate models have first to be compared with the rigorous solution after which the appropriate approximate model can be selected based on the required degree of accuracy of the response in the interval of interest.

In the application example, it was observed that the use of a Δ-Y or Y-Δ connection for the transformer windings reduces the harmonic contents of the generator line currents. Also, the transformer magnetization current was found to have negligible effect on the system behavior. It is reasonable to expect that these statements generally hold for most applications.
REFERENCES


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APPENDIX A

STANDARD MACHINE PARAMETERS

The standard machine parameters are those defined in the IEEE test code [20] along with the armature leakage reactance \( X_{d\ell} \) in [45]. For a salient-pole synchronous machine, the standard parameters in per-unit, \( R_a, X_d, X_d', X_d'', X_{d\ell}, X_q, X_q', \tau_{do}, \tau_{do}' \) and \( \tau_{qo}'' \), and the primitive parameters in per-unit [19], \( R_a, X_{md} (= X_{df} = X_{dkd} = X_{fkd}), X_f, X_{kd}, X_{kq}, X_{mq} (= X_{kq}), X_q (= X_{kq}), R_f, R_{kd} \) and \( R_{kq} \), are related as follows:

\[
X_{md} = X_d - X_{d\ell}
\]

\[
X_f = \frac{(X_d - X_{d\ell})^2}{(X_d - X_d')}
\]

\[
X_{kd} = \frac{(X_d' - X_{d\ell})^2}{(X_d' - X_d'') + (X_d - X_d')}
\]

\[
X_{kq} = X_{d\ell}
\]

\[
X_{mq} = \sqrt{X_q^2 - X_q X_{q'}}
\]

\[
R_f = \frac{(X_d - X_{d\ell})^2}{\omega_0 (X_d - X_d') \tau_{do}}
\]

\[
R_{kd} = \frac{(X_d' - X_{d\ell})^2}{\omega_0 (X_d' - X_d'') \tau_{do}''}
\]

\[
R_{kq} = X_q / \omega_0 \tau_{qo}''
\]

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APPENDIX B

A-COEFFICIENTS

The dimensionless A-coefficients of the resultant models are listed below.

\[ A_{dq} = \frac{x_q}{x_d''} \]
\[ A_{df} = -(x_d'' - x_d\ell)^2\frac{(x_d - x_d')/x_d' (x_d' - x_d\ell)^2}{X_d(X_d - x_d\ell)} \]
\[ A_{dkd} = -(x_d' - x_d\ell)/x_d'' \]
\[ A_{dkq} = -(x_q - x_q')/x_d'' \]
\[ A_{qd} = x_d/x_d'' \]
\[ A_{qf} = (x_d'' - x_d\ell)(x_d - x_d')/x_d''(x_d' - x_d\ell) \]
\[ A_{qkd} = (x_d - x_d\ell)(x_d' - x_d'')/x_d''(x_d' - x_d\ell) \]
\[ A_{qkq} = -(x_q - x_q')/x_q'' \]
\[ A_{fkd} = x_d\ell(x_d' - x_d'')/x_d'(x_d'' - x_d\ell) \]
\[ A_{kdf} = x_d\ell(x_d'' - x_d\ell)(x_d - x_d')/x_d''(x_d' - x_d\ell)^2 \]
\[ A_d = (x_d - x_d')(x_d' - x_d\ell)/(x_d - x_d\ell)(x_d' - x_d\ell) \]
\[ A_d' = (x_d' - x_d\ell)/(x_d' - x_d\ell) \]
\[ A_q = (x_q - x_q')/x_q \]
\[ A_f = A_{kdf} - (X_d - X_{dL})/(X'_d - X_{dL}) \]

\[ A_{kd} = -X'_d/X''_d \]

\[ A_{kq} = -X_q/X'_q \]

\[ A_v = (X'_d - A_d X_{dL})/(X'_d - X_{dL}) \]

\[ A'_v = -A_d X_{dL}/(X'_d - X_{dL}) \]

The additional A-coefficients which are introduced for the approximate models are given below.

\[ A''_{dq} = X''_q/X'_d \]

\[ A'_{df} = -A_qf/A''_{dq} \]

\[ A''_{df} = A'_d A'_{df} \]

\[ A_{dkd} = 1 - A'_d (X''_d - X_{dL})/(X'_d - X_{dL}) \]

\[ A'_{qd} = X'_d/X''_q \]

\[ A''_{qd} = X'_d/X''_q \]

\[ A'_{qf} = (X_d - X'_d)/X''_q \]

\[ A'_{kdf} = A_{kdf} (X'_d - X_{dL})/(X'_d - X_{dL}) \]

\[ A'_f = (X_d - X'_d)/(X_d - X_{dL}) \]

\[ A''_f = -A_f (X'_d - X''_d)/(X'_d - X_{dL}) \]

\[ A'_{kd} = (X'_d - X_{dL})/A'_d (X_d - X_{dL}) - A'_{kd} \]
APPENDIX C

Elements of the matrices A, B, and E.

\[ A = \{a_{ij}\}, \quad i,j = 1,2,...,7 \]

\[ a_{11} = -\cos^2\theta/\tau_d - \sin^2\theta/\tau_q - \omega(A_{dq} - A_{qd})\sin2\theta/2 \]

\[ a_{12} = (1/\tau_q - 1/\tau_d)\sin2\theta/2 - \omega(1 - A_{dq}\cos^2\theta - A_{qd}\sin^2\theta) \]

\[ a_{13} = A_{df}\cos\theta/\tau'_{do} - \omega A_{qf}\sin\theta \]

\[ a_{14} = A_{dkd}\cos\theta/\tau'_{do} - \omega A_{qkd}\sin\theta \]

\[ a_{15} = -A_{dkq}\sin\theta/\tau''_{qo} + \omega A_{dkq}\cos\theta \]

\[ a_{16} = -\cos^2\theta/\tau_{d1} - \sin^2\theta/\tau_{q1} - (1/\tau'_{q1} - 1/\tau'_{d1})\sin2\theta/2 \]

\[ a_{17} = -\cos^2\theta/\tau'_{d1} - \sin^2\theta/\tau'_{q1} + (1/\tau_{q1} - 1/\tau_{d1})\sin2\theta/2 \]

\[ a_{21} = (1/\tau_q - 1/\tau_d)\sin2\theta/2 + \omega(1 - A_{dq}\sin^2\theta - A_{qd}\cos^2\theta) \]

\[ a_{22} = -\sin^2\theta/\tau_d - \cos^2\theta/\tau_q + \omega(A_{dq} - A_{qd})\sin2\theta/2 \]

\[ a_{23} = A_{df}\sin\theta/\tau'_{do} + \omega A_{qf}\cos\theta \]

\[ a_{24} = A_{dkd}\sin\theta/\tau'_{do} + \omega A_{qkd}\cos\theta \]

\[ a_{25} = A_{qkq}\cos\theta/\tau''_{qo} + \omega A_{dkq}\sin\theta \]

\[ a_{26} = \sin^2\theta/\tau'_{d1} + \cos^2\theta/\tau'_{q1} + (1/\tau_{q1} - 1/\tau'_{d1})\sin2\theta/2 \]

\[ a_{27} = -\sin^2\theta/\tau_{d1} - \cos^2\theta/\tau_{q1} + (1/\tau'_{q1} - 1/\tau'_{d1})\sin2\theta/2 \]
\( a_{31} = -\cos\theta/\tau_d - \omega A_{dq}\sin\theta \)

\( a_{32} = -\sin\theta/\tau_d + \omega A_{dq}\cos\theta \)

\( a_{33} = A_f/\tau'_{do} \)

\( a_{34} = A_{fkd}/\tau''_{do} \)

\( a_{35} = \omega A_{dkq} \)

\( a_{36} = -\cos\theta/\tau_{d1} + \sin\theta/\tau'_{d1} \)

\( a_{37} = -\sin\theta/\tau_{d1} - \cos\theta/\tau'_{d1} \)

\( a_{41} = a_{31} \)

\( a_{42} = a_{32} \)

\( a_{43} = A_{kdf}/\tau'_{do} \)

\( a_{44} = A_{kd}/\tau''_{do} \)

\( a_{45} = \omega A_{dkq} \)

\( a_{46} = -\cos\theta/\tau_{d1} + \sin\theta/\tau'_{d1} \)

\( a_{47} = -\sin\theta/\tau_{d1} - \cos\theta/\tau'_{d1} \)

\( a_{51} = \sin\theta/\tau_q - \omega A_{qd}\cos\theta \)

\( a_{52} = -\cos\theta/\tau_q - \omega A_{qd}\sin\theta \)

\( a_{53} = \omega A_{qf} \)

\( a_{54} = \omega A_{qkd} \)
\[ a_{55} = A_{kq}/\tau_{q0} \]
\[ a_{56} = \sin\theta/\tau_{q1} + \cos\theta/\tau_{q1} \]
\[ a_{57} = -\cos\theta/\tau_{q1} + \sin\theta/\tau_{q1} \]
\[ a_{61} = -c^{'}_\alpha \cos\theta/\tau_d - c^{'}_\beta \sin\theta/\tau_q + \]
\[ \omega(c^m_{2d}/l_{22} - c^{'}_\alpha A_{dq} \sin\theta + c^{'}_\beta A_{qd} \cos\theta) \]
\[ a_{62} = -c^{'}_\alpha \sin\theta/\tau_d + c^{'}_\beta \cos\theta/\tau_q - \]
\[ \omega(c^m_{1d}/l_{22} - c^{'}_\alpha A_{dq} \cos\theta - c^{'}_\beta A_{qd} \sin\theta) \]
\[ a_{63} = c^{'}_\alpha A_{df}/\tau_{d0} - \omega c^{'}_\beta A_{qf} \]
\[ a_{64} = c^{'}_\alpha A_{dkd}/\tau_{d0} - \omega c^{'}_\beta A_{qkd} \]
\[ a_{65} = -c^{'}_\beta A_{qkq}/\tau_{q0} + \omega c^{'}_\alpha A_{dkq} \]
\[ a_{66} = -c^{'}_\alpha (\cos\theta/\tau_{d1} - \sin\theta/\tau_{d1}) - \]
\[ c^{'}_\beta (\sin\theta/\tau_{q1} + \cos\theta/\tau_{q1}) - 1/\tau_t \]
\[ a_{67} = -c^{'}_\alpha (\sin\theta/\tau_{d1} + \cos\theta/\tau_{d1}) + \]
\[ c^{'}_\beta (\cos\theta/\tau_{q1} - \sin\theta/\tau_{q1}) \]
\[ a_{71} = -c^{'}_\beta \cos\theta/\tau_d + c^{'}_\alpha \sin\theta/\tau_q + \]
\[ \omega(c^m_{1d}/l_{22} - c^{'}_\beta A_{dq} \sin\theta - c^{'}_\alpha A_{qd} \cos\theta) \]
\[ a_{72} = -c'_\beta \sin \theta / \tau_d - c'_\alpha \cos \theta / \tau_q + \]
\[ \omega (c'_2 \ell_m / \ell_{22} + c'_\beta A_{dq} \cos \theta - c'_\alpha A_{qd} \sin \theta) \]

\[ a_{73} = c'_\beta A_{df} / \tau'_d + \omega c'_\alpha A_{qf} \]

\[ a_{74} = c'_\beta A_{df} / \tau'_d + \omega c'_\alpha A_{qf} \]

\[ a_{75} = c'_\alpha A_{kq} / \tau'_q + \omega c'_\beta A_{dkq} \]

\[ a_{76} = -c'_\beta (\cos \theta / \tau_{dl} - \sin \theta / \tau_{dl'}) + \]
\[ c'_\alpha (\sin \theta / \tau_{ql} + \cos \theta / \tau_{ql'}) \]

\[ a_{77} = -c'_\beta (\sin \theta / \tau_{dl} + \cos \theta / \tau_{dl'}) - \]
\[ c'_\alpha (\cos \theta / \tau_{ql} - \sin \theta / \tau_{ql'}) - 1 / \tau_t \]

In \( [a_{ij}] \), the time-constant parameters are defined as follows:

\[ \tau_{dl} = \frac{X'_d \ell_{22}}{\omega_0 c'_3 \ell_m \tau_2} \]

\[ \tau_{ql} = \frac{X'_q \ell_{22}}{\omega_0 c'_3 \ell_m \tau_2} \]

\[ \tau'_{dl} = \frac{X'_d \ell_{22}}{\omega_0 c'_4 \ell_m \tau_2} \]

\[ \tau'_{ql} = \frac{X'_q \ell_{22}}{\omega_0 c'_4 \ell_m \tau_2} \]
\[ B = \{b_i\}, \quad i = 1,2,\ldots,7 \]

\[ b_1 = \omega A_d \cos \theta / X'_d \]

\[ b_2 = \omega A_d \sin \theta / X'_d \]

\[ b_3 = \omega A_v / X'_d \]

\[ b_4 = \omega A'_v / X''_d \]

\[ b_5 = 0 \]

\[ b_6 = \omega c'_\alpha A_d / X''_d \]

\[ b_7 = \omega c'_\beta A_d / X''_d \]

\[ E = \{e_{ij}\}, \quad i = 1,2,\ldots,7; \quad j = 1,2 \]

\[ e_{11} = -\omega c'_\alpha \cos \theta / X''_d - \omega c'_\beta \sin \theta / X''_q \]

\[ e_{12} = -\omega c'_\beta \cos \theta / X'_d + \omega c'_\alpha \sin \theta / X'_q \]

\[ e_{21} = -\omega c'_\alpha \sin \theta / X''_d + \omega c'_\beta \cos \theta / X''_q \]

\[ e_{22} = -\omega c'_\beta \sin \theta / X''_d - \omega c'_\alpha \cos \theta / X''_q \]

\[ e_{31} = -\omega c'_\alpha / X''_d \]

\[ e_{32} = -\omega c'_\beta / X''_d \]

\[ e_{41} = e_{31} \]

\[ e_{42} = e_{32} \]

\[ e_{51} = \omega c'_\beta / X''_q \]
\[ e_{52} = -\omega \cdot \frac{c_{\alpha}'}{\mathcal{X}_q} \]
\[ e_{61} = -\omega \cdot \frac{c_{\alpha}''}{\mathcal{X}_d} - \omega \cdot \frac{c_{\beta}'}{\mathcal{X}_q} - \frac{c_5}{\mathcal{X}_{22}} \]
\[ e_{62} = \omega \cdot \frac{c_{\alpha}'}{\mathcal{X}_d} (\frac{1}{\mathcal{X}_q} - \frac{1}{\mathcal{X}_d}) \]
\[ e_{71} = e_{62} \]
\[ e_{72} = -\omega \cdot \frac{c_{\alpha}''}{\mathcal{X}_d} - \omega \cdot \frac{c_{\alpha}'}{\mathcal{X}_q} - \frac{c_5}{\mathcal{X}_{22}} \]

where
\[ c_{\alpha}' = \frac{\mathcal{L}_m}{\mathcal{X}_{22}} (c_1 \cos \theta + c_2 \sin \theta) \]
\[ c_{\beta}' = \frac{\mathcal{L}_m}{\mathcal{X}_{22}} (c_1 \sin \theta - c_2 \cos \theta) \]

Remark: In all expressions for \([a_{ij}], [b_i], \text{and} [e_{ij}],\) the standard machine reactances and the armature resistance have to be augmented by the transformer reactance \(x_t = \omega \cdot \mathcal{L}_t\) and resistance \(r_t\) respectively, i.e.,

\[ X_{dl} = X_{dl} + x_t \]
\[ X_d = X_d + x_t \]
\[ X_d' = X_d' + x_t \]
\[ X_d'' = X_d'' + x_t \]
\[ X_q = X_q + x_t \]
\[ X_q' = X_q' + x_t \]
and

\[ R_a = R_a + r_t. \]

Note that these modifications of the machine impedances do not affect the values of \( \tau'_{do} \), \( \tau''_{do} \), and \( \tau''_{qo} \), but they must be considered in the calculation of the A-coefficients and the time-constants \( \tau_d \), \( \tau_{dl} \), \( \tau'_{dl} \), \( \tau_q \), \( \tau_{ql} \), and \( \tau'_{ql} \).
APPENDIX D

ELEMENTS OF MATRICES A', B', D', AND E'

The elements of the matrices A', B', D', and E' as functions of the elements of the matrices A, B, and E given in Appendix C are as follows:

\[ A' = \{a'_{ij}\}, \quad i,j = 1,2,\ldots,6 \]

\[ a'_{ij} = a_{ij}, \quad i,j = 1,2,\ldots,5 \]

\[ a'_{i6} = k_\alpha a_{i6} + k_\beta a_{i7}, \quad i = 1,2,\ldots,5 \]

\[ a'_{6j} = a_{7j}/k_\beta, \quad j = 1,2,\ldots,5 \]

\[ a'_{66} = (k_\alpha/k_\beta)a_{66} + a_{77} \]

\[ B' = \{b'_i\}, \quad i = 1,2,\ldots,6 \]

\[ b'_i = b_i, \quad i = 1,2,\ldots,5 \]

\[ b'_6 = b_7/k_\beta \]

\[ D' = \{d'_j\}, \quad j = 1,2,\ldots,6 \]

\[ d'_j = k_\beta(k_\beta a_{6j} - k_\alpha a_{7j})/h', \quad j = 1,2,\ldots,5 \]

\[ d'_6 = k_\beta(k_\beta^2 a_{67} + k_\alpha k_\beta(a_{66} - a_{77}) - k_\alpha^2 a_{76})/h' \]

\[ E' = \{e'_{ij}\}, \quad i = 1,2,\ldots,6; \quad j = 1,2 \]
\[ e'_{ij} = e_{ij}, \ i = 1, 2, \ldots, 5; \ j = 1, 2 \]

\[ e'_{6j} = e_{7j}/k_\beta, \ j = 1, 2 \]

The \( n \)-coefficients appearing in equation (5.6) are

\[ n'_f = k_\beta (k_\beta e_6 - k_\alpha e_7)/h' \]

\[ n'_k = (k_\beta b_{62} - k_\alpha b_{72})/h' \]

where

\[ h' = k_\alpha k_\beta (b_{62} + b_{71}) - k_\beta^2 b_{61} - k_\alpha^2 b_{72} \]
APPENDIX E

ELEMENTS OF MATRIX F

\[ F = [f_i], \ i = 1, 2, \ldots, 7 \]

\[ f_1 = -\omega \left( h^\alpha \cos \theta / X'_d + h^\beta \sin \theta / X'_q \right) \]

\[ f_2 = -\omega \left( h^\alpha \sin \theta / X'_d - h^\beta \cos \theta / X'_q \right) \]

\[ f_3 = -\omega \ h^\alpha / X'_d \]

\[ f_4 = f_3 \]

\[ f_5 = -\omega \ h^\beta / X'_q \]

\[ f_6 = -\omega \left( c^\alpha h^\alpha / X'_d - c^\beta h^\beta / X'_q \right) - c^5 k^\alpha / \ell_{22} \]

\[ f_7 = -\omega \left( c^\beta h^\alpha / X'_d + c^\alpha h^\beta / X'_q \right) - c^5 k^\beta / \ell_{22} \]

where

\[ h^\alpha = c^\alpha k^\alpha + c^\beta k^\beta \]

\[ h^\beta = c^\alpha k^\beta - c^\beta k^\alpha \]
APPENDIX F

INTERFACE EQUATION COEFFICIENTS

The elements of the coefficient matrices and parameters of the interface equations (5.16) and (5.17) for the RLC load and GTB unit with negligible transformer magnetization current are listed below.

\[ P = [p_{ij}], \ i = 1,2; \ j = 1,2,3,4 \]

\[ p_{11} = -\left( n_\alpha g_\alpha/\tau_d + n_\beta g_\beta/\tau_q - k_\beta R_e/L \right)/n_o + \omega (k_\alpha n_2 - k_\beta n_1 + n_\alpha g_\beta A_{dq} - n_\beta g_\alpha A_{qd})/n_o \]

\[ p_{12} = (n_\alpha A_{df}/\tau_{do} + \omega n_\beta A_{qf})/n_o \]

\[ p_{13} = (n_\alpha A_{dkd}/\tau_{do} + \omega n_\beta A_{qkd})/n_o \]

\[ p_{14} = (n_\beta A_{qkq}/\tau_{dq} + \omega n_\alpha A_{dkq})/n_o \]

\[ p_{21} = n_2 p_{11}/n_1 - \nu L (c_\beta g_\alpha/\tau_d + c_\alpha g_\beta/\tau_q - k_\beta R_e/L)/n_1 + \omega \nu L (k_\alpha + c_\beta g_\beta A_{dq} - c_\alpha g_\alpha A_{qd})/n_1 \]

\[ p_{22} = n_2 p_{12}/n_1 + \nu (c_\beta A_{df}/\tau_{do} + \omega c_\alpha A_{qf})/n_1 \]

\[ p_{23} = n_2 p_{13}/n_1 + \nu (c_\beta A_{dkd}/\tau_{do} + \omega c_\alpha A_{qkd})/n_1 \]

\[ p_{24} = n_2 p_{14}/n_1 + \nu (c_\alpha A_{qkq}/\tau_{dq} + \omega c_\beta A_{dkq})/n_1 \]

\[ Q = [q_i], \ i = 1,2 \]
\[ q_1 = \frac{n_3}{n_0} \]

\[ q_2 = \frac{n_2 n_3}{n_0 n_1} + \frac{k_\alpha}{n_1} \]

\[ M = [m_i], \quad i = 1, 2 \]

\[ m_1 = \frac{\omega_x n_\alpha a_d}{n_0 x_d''} \]

\[ m_2 = \frac{\omega_x a_d (n_2 n_\alpha/n_0 + L c_\beta)}{n_1 x_d''} \]

\[ N = [0] \]

with

\[ n_\alpha = n_1 c_\alpha + n_2 c_\beta \]

\[ n_\beta = n_2 c_\alpha - n_1 c_\beta \]

\[ n_0 = \omega \omega L \left( c_\alpha^2 + c_\beta^2 \right)^2 / x_d'' q + \omega \omega (g_\beta^2 / x_d'' + g_\alpha^2 / x_q'') \]

\[ n_1 = \omega \omega L \left( c_\alpha^2 / x_d'' + c_\beta^2 / x_q'' \right) + k_\beta^2 \]

\[ n_2 = \omega \omega L c_\alpha c_\beta (1/x_q'' - 1/x_d'') - k_\alpha k_\beta \]

\[ n_3 = \omega \omega c_\alpha g_\alpha / x_q'' - \omega \omega c_\beta g_\beta / x_d'' \]

\[ p' = [p'_j], \quad j = 1, 2, \ldots, 5 \]

\[ p'_1 = L \left( c_\alpha g_\alpha' / \tau_d + c_\beta g_\beta' / \tau_q - k_\alpha^l R / L \right) / n_4 - \omega L \left( k_\beta^l - c_\beta g_\alpha A_d q - c_\alpha g_\beta A_q d \right) / n_4 \]

\[ p'_2 = L \left( c_\beta g_\alpha' / \tau_d + c_\alpha g_\beta' / \tau_q - k_\beta^l R / L \right) / n_4 + \omega L \left( k_\alpha^l - c_\alpha g_\alpha A_d q - c_\beta g_\beta A_q d \right) / n_4 \]
\[ p_3' = -L \left( \frac{g'_{\alpha} A_{df}}{t'_{do}} + \omega g'_{\beta} A_{qf} \right) \]

\[ p_4' = -L \left( \frac{g'_{\alpha} A_{kd}}{t'_{do}} + \omega g'_{\beta} A_{kd} \right) \]

\[ p_5' = -L \left( \frac{g'_{\beta} A_{qk}}{t'_{qo}} + \omega g'_{\alpha} A_{dkq} \right) \]

\[ Q' = [q'] \]

\[ q' = 1/n_4 \]

\[ m' = -\omega_0 L \frac{g'_{\alpha} A_d}{n_4 x'_d} \]

\[ N' = [0] \]

with

\[ n_4 = 1 + \omega_0 L \left( \frac{g'^2_{\alpha}}{x'^2_d} + \frac{g'^2_{\beta}}{x'^2_q} \right) \]
VITA

Farrokh Shokooh was born on September 17, 1949, in Tehran, Iran. He is the son of Hajeh (Farijon) Barkatian and Dr. Yahya Shokooh. He attended elementary and secondary schools in Tehran, graduating from Azar High School in May 1967. In September of 1968, he entered Louisiana State University in Baton Rouge, LA, USA, and received his Bachelor and Master of Science degrees in Electrical Engineering in May of 1972 and 1975 respectively. He has held teaching and research assistantships from September of 1972 to the present. He is member of Eta Kappa Nu, Tau Beta Pi, Phi Kappa Phi, and Omicron Delta Kappa honor societies and a student member of IEEE. Presently, he is a candidate for the degree of Doctor of Philosophy in Electrical Engineering.
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Title of Thesis: An Explicit State Model of A Synchronous Machine-Transformer-SCR Bridge Unit

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Dean of the Graduate School

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Date of Examination:
May 10, 1979