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Developing Auxiliary Resource Materials to Support the EngageNY Geometry Curriculum

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DEVELOPING AUXILIARY RESOURCE MATERIALS TO SUPPORT THE ENGAGENY GEOMETRY CURRICULUM

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ABSTRACT

With the advent of current education reform, and the introduction of the Common Core State Standards for Mathematics, the present offerings of the geometry curriculum have become dated. One contribution to remedy this situation is a project by the state of New York called EngageNY. EngageNY is a common core aligned mathematics curriculum across all grades. The EngageNY Geometry Curriculum Module 1 is the basis from which this thesis was developed.

It is the purpose of this thesis to present a supplement to the EngageNY Geometry Module 1 Curriculum and to describe why it is advantageous to have such a supplement available. The intention of creating the problems presented in this thesis is to offer a set of high quality practice problems as an additional resource geometry teachers could use to complement their current curriculum.

After careful analysis of Module 1 of the EngageNY Geometry Curriculum, it became clear that extra practice problems were needed to augment the problem sets offered in each lesson of Module 1. Mathematics is a discipline that requires practice. The problems for this thesis provide a useful enrichment to enhance the EngageNY materials. In addition to extra practice, I believe that all levels of ability should be addressed in the geometry classroom, therefore differentiation played a key role in the development of the practice problems presented in the auxiliary resource materials. And, while EngageNY Geometry Module 1 is comprehensive in its offering of lesson activities, its contribution to providing plenty of assessment items is lacking. The problems developed for this thesis address this problem by providing a number of problems that reflect the current format of Louisiana standardized tests, namely real world application problems and multiple choice items.

The enrichment problems presented in this thesis were designed to reflect the ideas portrayed in EngageNY Geometry Module 1. I tried to stay true to the essence of each lesson of Module 1 but also kept in mind my objectives of providing problems for differentiation and application. The analysis of Module 1 began as I read through each lesson before teaching it. Notes were taken on student reactions to the lesson and on the timeliness of the lesson. I then noted whether the problem sets offered in Module 1 were adequate for extra practice and commented on their application to real world situations.

During the process of creating the practice problems, it was my aim to produce problems in a format that is highly conducive to learning. To achieve my goals, each problem was designed with an easy to read format, an accompanying diagram and a detailed solution designed to help teachers save time. I hope that this attention to detail will enhance and distinguish this work as a worthwhile, quality resource that will enhance any geometry teacher's classroom.

CHAPTER 1: INTRODUCTION AND STATEMENT OF PURPOSE

In recent years, teachers of mathematics had numerous resources upon which they could refer to when designing their daily, weekly, even yearly units and lesson plans. In most cases, a curriculum was determined and resources were purchased by the state or local districts to be distributed to the schools and teachers who would ultimately put them to use for planning and instructional support. More often than not, textbooks were the primary source of instructional resources that teachers relied upon to plan and administer their lessons. Many major mathematics textbook publishers offered state correlations specific to the state's standards making it easier to align the standards and sequence them for a teacher's lesson plans. With the textbook, an assortment of workbooks, assessment packages, planning resources and supplemental packages were offered to assist teachers in achieving their goal of supplying their students with a quality education. The internet has also been a rich and cost effective resource teachers have used for planning instruction.

With the current education reform, many of these resources have become obsolete, particularly in high school mathematics. With the adoption of the Common Core State Standards for Mathematics (CCSSM) critical curriculum changes are being made that affect the integrity of the current resources available. These changes have caused teachers to struggle to find meaningful, rigorous resources to help them plan and execute instruction.

One of the very few timely efforts being made to create a curriculum aligned to the CCSSM and provide resources for teachers that are written specifically for the new standards is an endeavor by the New York State Education Department (NYSED) called EngageNY. It is the purpose of this thesis to present a supplement to the EngageNY Geometry Module 1 curriculum and to describe why it is advantageous to have such a supplement available. This supplement to

the Geometry Module 1 could also serve as a template for the development of auxiliary materials for further modules in the EngageNY Geometry Curriculum. It is my belief that by augmenting the current EngageNY Geometry Curriculum with a set of high quality practice and assessment problems, teachers will have an additional resource that will help increase their students' retention levels and conceptual understanding of high school geometry.

CHAPTER 2: SURVEY OF GEOMETRY CURRICULUM

2.1 Teaching Geometry in Louisiana

I am currently in my 5th year teaching high school geometry. I teach at a small, rural, K-12 public magnet academy. The students in my geometry classes are highly motivated and express a great desire to learn geometry and are working hard to be successful in their studies of the subject.

In previous years, Louisiana geometry teachers were encouraged to follow the Louisiana Comprehensive Curriculum (LCC) for high school geometry. Since 2010, however, the LCC has been dismissed and Louisiana geometry teachers are on their own to find a viable curriculum that provides a rigorous plan of instruction that meets district and state goals. The state of Louisiana has adopted the Common Core State Standards for Mathematics (CCSSM) and it is an aim of both the Louisiana State Department of Education and also my district for teachers to use these standards when planning instruction. Mathematics teachers in my district are encouraged to use the Louisiana Department of Education's (LDOE) website, Louisiana Believes, specifically the Classroom Support Toolbox/Teacher Support Toolbox, when planning for instruction. One of the planning resources LDOE lists on their website is EngageNY. The following is an excerpt from this page titled Mathematics CCSS Planning Resources. "The New York State Education Department (NYSED) has engaged teachers, administrators, and education experts across New York and the nation in the creation of curriculum maps (scope and sequences), modules (units), and lessons with at least one module completed and posted for each grade/course at http://www.engageny.org/mathematics" (louisanabelieves, n.d.).

The Louisiana Department of Education has only one "Tier 1" recommendation for instructional materials at the time of this writing. This recommendation is Eureka Math, a

curriculum for mathematics that is the twin sister of the EngageNY curriculum modules. The fact that an official recommendation and mentioning of EngageNY as a teacher resource leads me to believe that LDOE has a strong faith in the work being produced by EngageNY as a worthwhile resource for Louisiana teachers to utilize. It is my desire to present the highest quality curriculum I could find as a teaching resource for instructional needs and I, too, agree with LDOE that EngageNY is at the top of the list.

As I began to plan for the 2013-2014 school year, I chose to follow the EngageNY Geometry curriculum. However, my planning and subsequent implementation of this curriculum exposed a limitation of Module 1 of the geometry curriculum. It needed more practice and assessment problems. A recent study by the Norwegian University of Science and Technology found that practice in mathematical tasks strengthen cognitive skills and results in a task being performed correctly upon subsequent attempts (H. Sigmundsson, *et al*., 2013). Extra practice was called for and I was soon put to task developing a set of problems that would complement EngageNY Geometry Modules to enhance my students' learning experience. In a 1987 study conducted by Zhu and Simon, the authors found that working problems and practicing from worked examples provided significant evidence of learning geometry (Zhu and Simon, 1987). It is my belief that students need to practice different types of problems in order to completely understand a concept. Therefore, a supplement to EngageNY was required to provide my geometry students the needed practice.

In addition to additional independent practice problems, I believe it is essential to continually assess my students based on the skills learned from the instructional content. Research has shown that practice testing improves student learning (Roediger, & Karpicke, 2006). Integrating problems and questions into formative and summative assessments that

reflect the design of our state's End of Course Test in Geometry is a strategy I believe is beneficial to my students. By doing so, my students become accustomed to the form and content for which they will be tested at the end of the year, thus providing an opportunity for building confidence.

2.2 Overview of High School Geometry

In general, the geometry courses currently being taught in American high schools are based upon Euclidean geometry. This branch of mathematics is founded on the works by a philosopher and mathematician named Euclid of Alexandria who lived approximately 2300 years ago in ancient Greece. Euclidean geometry (sometimes referred to as plane Euclidean geometry) is the study of geometry based on Euclid's undefined terms, definitions, postulates (including Euclid's fifth postulate, the parallel postulate), and common notions.

For the most part, the high school geometry curriculum in the US today follows an organization that follows the design prescribed by the vast majority of current geometry textbooks.

Current topics covered in most popular high school geometry textbooks include

- Points and Lines
- Logic (Deductive Reasoning, Inductive Reasoning)
- Angles
- Parallel and Perpendicular Lines
- Polygons (Triangles, Quadrilaterals, Trapezoids)
- Transformations, Translations, Reflections
- Triangle Congruence
- Similarity
- Measurement (Area, Arc Measure and Arc Length, Area of a Circle)
- Three Dimensional Figures
- Surface Area and Volume
- Coordinate Geometry

- Right Triangles and Trigonometry
- Circles

This list was compiled by reviewing nine geometry textbooks published from 1993 to 2014. The myriad of textbooks that possess these topics and the variety and inconsistent nature of standards adopted by each state comprise the basis upon which geometry curricula had been designed and implemented in the United States for the past few decades. This lack of cohesiveness (among other reasons which will be developed further in this paper) set the stage for the need of a common set of standards for all schools across the nation. Hence, The Common Core State Standards for Mathematics were conceived.

2.3 Overview of the Common Core State Standards

It has been said that curriculum in the United States has been "a mile wide and an inch deep" (Schmidt, *et al.*, 2002). This is partly due to districts being driven to create curricula designed around a textbook. When geometry textbooks offer 700+ pages (2005 Glencoe Geometry Student Edition) on a variety of the above mentioned topics, one could only hope that you would have enough time in the school year to cover even half of the book. Hence, the mile wide and inch deep outlook.

Education leaders across the United States have expressed concerns about the future of education in this country. As a result, a discussion proposing a shift in education policy, particularly a common set of standards written for reading and mathematics across all states in all grades was originated. One consequence of this proposal was a whirlwind of reform activity, specifically the creation of the Common Core State Standards Initiative.

With education reform in full gear, the stage was set to change the way mathematics has been taught for the past few decades. The Common Core State Standards for Mathematics was a result of this reform. The Common Core State Standards (CCSS), through an initiative generated

by education leaders across several states, were completed in 2009. With a new set of standards at hand, 43 states and five territories are making an effort to create a mathematics education for U.S. children that provides a deeper focus of the concepts taught rather than covering a multitude of information per subject. According to The Common Core State Standards Initiative,

> For more than a decade, research studies of mathematics education in high-performing countries have concluded that mathematics education in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on this promise, the mathematics standards are designed to address the problem of a curriculum that is "a mile wide and an inch deep" (corestandards.org/Math/, n.d.).

Louisiana is one of the 43 states that has opted in to adopt the CCSS and has been transitioning toward full implementation since 2010. The Common Core State Standards Initiative was motivated by the need to:

- provide a consistent set of expectations across the United States that will satisfactorily prepare students for entry into college and career training programs
- provide students with a consistent set of tools that will allow them to compete and collaborate with their peers anywhere in the world including their peers in the United States
- provide state educators with the tools necessary to collaborate on educational policy which includes the development of teaching materials and resources
- provide states with the tools necessary for the development of consistent assessment systems to measure student performance
- provide clarity and consistency of standards across the states
- provide content and rigor that will enable the students of the United States to compete with the top performing students of other nations in the world.

(corestandards.org/about-the-standards/development-process/, n.d.; and tncore.org/about_tn_standards.aspx, n.d.)

The Common Cores State Standards Initiative developed the standards for English/Language Arts and Mathematics. This thesis will focus only on the geometry domain of the Common Core State Standards for Mathematics, specifically, the congruence cluster of the CCSSM covered in Module 1 of the EngageNY Geometry Curriculum (a common core aligned curriculum).

2.4 Overview of the Geometry Domain of the CCSS

The following is an overview of the Geometry Domain of the CCSS in High School Mathematics:

<u>Congruence</u>
Experiment with transformations in the plane
Understand congruence in terms of rigid motions
Prove geometric theorems
Make geometric constructions

<u>Similarity, Right Triangles, and Trigonometry</u>
Understand similarity in terms of similarity transformations
Prove theorems involving similarity
Define trigonometric ratios and solve problems involving right triangles
Apply trigonometry to general triangles

<u>Circles</u>
Understand and apply theorems about circles
Find arc lengths and areas of sectors of circles

<u>Expressing Geometric Properties with Equations</u>
Translate between the geometric description and the equation for a conic section
Use coordinates to prove simple geometric theorems algebraically

<u>Geometric Measurement and Dimension</u>
Explain volume formulas and use them to solve problems
Visualize relationships between two-dimensional and three-dimensional objects

<u>Modeling with Geometry</u>
Apply geometric concepts in modeling situations
(corestandards.org/Math/Content/HSG/introduction, n.d.)

The high school geometry standards are the foundation for developing a rigorous, comprehensive course in geometry. Although there are several different types of geometries, the focus of the CCSSM for high school geometry is plane Euclidean Geometry. The Common Core State Standards for Mathematics were the basis of the development of the EngageNY/Eureka mathematics curriculum and more specifically its geometry domain.

2.5 Overview of EngageNY

EngageNY is a free, comprehensive, open access curriculum that was designed in response to meet the needs of educators who must address the education reform sweeping our country. With over 70,000 pages, EngageNY is one of the largest curriculum projects that has ever been undertake. This curriculum is a suggestion of how teachers *could* teach mathematics, not a prescription of how teachers *should* teach mathematics. Since this mathematics curriculum is so vast and comprehensive, it is the only one of its kind in the country. EngageNY provides educational resources that support the state's educational philosophy, particularly the initiative for college and career readiness. One key aspect of the EngageNY website is their wealth of Common Core aligned teaching and assessment materials. While there is a curriculum designed for each grade level in both English and Mathematics, this thesis will address only the geometry curriculum, specifically Module 1.

2.6 Overview of EngageNY Geometry Module 1 Curriculum

The EngageNY curriculum for geometry is divided into five learning units called modules. The curriculum for the EngageNY Geometry is as follows,

Module 1	Congruence, Proof, and Constructions
Module 2	Similarity, Proof, and Trigonometry
Module 3	Extending to Three Dimensions
Module 4	Connecting Algebra and Geometry through Coordinates
Module 5	Circles With and Without Coordinates
	(engageny.org/draft-new-york-common-core-geometry-overview, n.d.)

The EngageNY Geometry Curriculum is directly aligned to the CCSSM as evidenced by comparing the above modules to the Geometry Domain of the CCSSM. Each standard in the Geometry Domain is addressed in the Geometry Curriculum of EngageNY. Specifically, Module 1, Congruence, Proof, and Construction, is directly related to the Congruence cluster of

the Geometry Domain as each standard listed is addressed in this module. The table of contents from the EngageNY Geometry Module 1 Teacher Materials is as follows:

Topic A: Basic Constructions
- Lesson 1: Construct an Equilateral Triangle
- Lesson 2: Construct an Equilateral Triangle II
- Lesson 3: Copy and Bisect an Angle
- Lesson 4: Construct a Perpendicular Bisector
- Lesson 5: Points of Concurrencies

Topic B: Unknown Angles
- Lesson 6: Solve for Unknown Angles—Angles and Lines at a Point
- Lesson 7: Solve for Unknown Angles—Transversals
- Lesson 8: Solve for Unknown Angles—Angles in a Triangle
- Lesson 9: Unknown Angle Proofs—Writing Proofs
- Lesson 10: Unknown Angle Proofs—Proofs with Constructions
- Lesson 11: Unknown Angle Proofs—Proofs of Known Facts

Topic C: Transformations/Rigid Motions
- Lesson 12: Transformations—The Next Level
- Lesson 13: Rotations
- Lesson 14: Reflections
- Lesson 15: Rotations, Reflections, and Symmetry
- Lesson 16: Translations
- Lesson 17: Characterize Points on a Perpendicular Bisector
- Lesson 18: Looking More Carefully at Parallel Lines
- Lesson 19: Construct and Apply a Sequence of Rigid Motions
- Lesson 20: Applications of Congruence in Terms of Rigid Motions
- Lesson 21: Correspondence and Transformations

Topic D: Congruence
- Lesson 22: Congruence Criteria for Triangles—SAS
- Lesson 23: Base Angles of Isosceles Triangles
- Lesson 24: Congruence Criteria for Triangles—ASA and SSS
- Lesson 25: Congruence Criteria for Triangles—SAA and HL
- Lesson 26: Triangle Congruency Proofs—Part I
- Lesson 27: Triangle Congruency Proofs—Part II

Topic E: Proving Properties of Geometric Figures
- Lesson 28: Properties of Parallelograms
- Lessons 29: Special Lines in Triangles
- Lessons 30: Special Lines in Triangles

Topic F: Advanced Constructions
- Lesson 31: Construct a Square and a Nine-Point Circle
- Lesson 32: Construct a Nine-Point Circle

Topic G: Axiomatic Systems
- Lessons 33 - 34: Review of the Assumptions

(engageny.org/resource/geometry-module-1-overview, n.d.)

Geometry Module 1 represents the significant shifts made in the traditional geometry curriculum as defined by the Common Core State Standards for Mathematics. Module 1 investigates transformations of the plane and the importance of transformations when defining congruence. It was the introduction of this module that made it evident that the CCSSM presented a different way of looking at high school geometry.

CHAPTER 3: INSPIRATION FOR THE AUXILIARY PRACTICE PROBLEMS

3.1 Inspired by EngageNY

The enrichment problems created for this thesis were originally designed as a supplemental resource to complement the EngageNY curriculum. By creating a catalog of problems comparable to the problems offered by EngageNY, I now have resources that would expand the current offerings of the curriculum I am using in my geometry class.

As the development of the problems evolved, I felt that other geometry teachers could find it to be a beneficial additional resource to enhance their current geometry curriculum regardless of their adoption of the EngageNY curriculum. It also became clear that this resource could be utilized not only by teachers but also by students, parents or anyone else interested in supplementing their knowledge and learning of high school geometry as these problems can be used independently of EngageNY. The goal of the auxiliary resource materials is to provide further study and additional challenges on the topics presented by EngageNY Module 1 Geometry curriculum.

Since geometry is a state tested course in Louisiana (students in public school must pass the Geometry End of Course Test (EOC) to receive credit to graduate), my students are very concerned about passing this test with the highest possible score. They not only want to master the few problems they had time to do in class, but also want to see many different types of problems related to the standard or standards taught that day. "Most mathematics lessons require student follow-up to refine and sharpen their newly acquired skills" (Posamentier, *et al*., 2006, p. 27). Posamentier also states that, "student work done outside of the classroom helps to develop independent thought and creative thinking skills." The lessons in the EngageNY Geometry Module 1 are certainly rich in instructional content but somewhat weak in heterogeneous

independent practice problems. It was this need for extra material to supplement the in-class examples and homework problem sets of EngageNY that inspired the idea for this thesis and the development of the auxiliary practice problems and applications for high school geometry.

As a high school mathematics teacher, I am constantly searching for resources that address the needs of my students. These practice problems can be used by geometry teachers to supplement their instructional models, provide a ready resource for homework/independent practice problems, offer additional problems for assessment, or to use as an extra resource to help create a differentiated classroom. A solution guide is also available that will not only supply the answers to all of the practice problems, but also provide detailed, worked out solutions, a feature that will save valuable time for the teacher. The solution guide will also be useful for parents since each problem is fully explained with diagrams enabling the parent to help his/her child succeed in mastering the problem when needed.

3.2 Reasons to Supplement EngageNY

At the beginning of the 2013-2014 school year my district implemented a series of initiatives for implementing the Common Core State Standards. Few of these proposals were directed at the high school mathematics teacher, as there were very few common core resources available. One suggestion was to follow the EngageNY curriculum as an instructional resource.

I implemented EngageNY immediately as school began in August, 2013. I soon discovered that I would need extra resources to fulfill the needs of my students. This is not to say that EngageNY Geometry is not comprehensive, if anything, it is a highly comprehensive geometry curriculum. But as previously mentioned, it was the lack of heterogeneous practice materials that gave me pause. I have taught these same students since the 8^{th} grade and one characteristic I have come to recognize (and common to most) is that they desire reinforcement

in several different ways especially before a grade is applied. Our high school students have time in their daily schedule to prepare and practice for state administered tests. Since geometry is a state tested course, preparing for the EOC is essential for our students. One way to achieve this goal is through practice problems of the material learned in class. Since "teaching the test" is not a goal of mine, reinforcement of daily lesson material is one way to provide the extra practice that support the lesson's objectives and gives my students confidence to move on to the next topic since mastering a variety of problems will build assurance levels to accomplish this goal.

Unfortunately, after using the EngageNY Geometry curriculum for a few weeks, I found the following disadvantages:

- Practice problems are available for each lesson but practice sets are somewhat limited.
- Lack of assessments and assessment problems. There are only two assessments per module: a mid module assessment and end of module assessment. Module 1 consists of 34 lessons. Therefore, there is a need of more frequent assessments to measure student progress throughout the modules.
- Not in a user friendly format. Some of the material is printed in an 8 point font!
- Not portable or readily accessible (must have computer and internet access).
- Neither student nor parent friendly. Much of the material is highly technical and contains function notation that would be difficult for a student to comprehend without guidance.
- Does not provide an acceptable format for students who are absent and need to make up work. Presenting the material "as is" to a student who missed the lesson is not recommended.

Therefore, this list was the impetus to create a set of problems in a user-friendly format, readily accessible and portable for the student or teacher.

CHAPTER 4: THE DEVELOPMENT OF THE PRACTICE PROBLEMS

4.1 Analyzing EngageNY Geometry Module 1

After deciding to undertake the task of producing a set of practice problems to supplement EngageNY, I began by analyzing each lesson of the EngageNY Geometry curriculum. I also took notes during the execution of each lesson in the classroom (as most teachers do) as to what worked well, what conceptual problems my students were having, and what was deficient. It should be noted that the EngageNY Geometry curriculum is very rigorous, and this alone presented a great challenge to me in designing problems that could meet this standard of rigor. My task was to try to develop problems for students of all levels of ability to supplement their classroom experience and also have problems for independent practice. I wanted a mixture of rigor such that the struggling students in the class could build up their confidence to move to the next level of rigor without giving up. Thus, the practice problems evolved into a mixture of exercises designed for a differentiated set of students while still providing rigor and alignment to the Common Core State Standards for Mathematics.

Each problem included in the auxiliary material of this thesis was developed from the analysis of EngageNY lessons and problem sets and also from my reflections on the daily activities that I call "Notes".

4.2 Module 1 Lesson Analysis and Lesson Reflections

This section includes lesson summaries and lesson reflections that aided in the development of the enrichment problems. Included are the Lesson Summaries, Notes and Problem Set Comments for each lesson listed. It should be noted that some lessons may be summarized singularly, others may be grouped together in their summaries and notes. As Module 1 itself is quite long, this is primarily to be as succinct as possible. Also, at the end of the

Problem Set Comments a statement will be provided on whether practice for real world application or word problem was available in the lesson. For brevity, this statement will read RW application provided in this lesson or RW application not provided in this lesson. The inclusion of this reason is a direct result of Louisiana's high stakes testing as most of the questions on the state's Geometry End of Course Test are word problems based on application or real world situations. Therefore, it is desirable to include practice on these types of problems to achieve success on the Geometry End of Course Test.

Upon implementation, and during the module analysis, I noticed that there was a shortage of formal assessment items provided by Module 1. There are only two formal assessments provided in this module, a mid-module assessment and end of module assessment. Teachers, however, may choose to use problems from selected problem sets to assess student understandings, although this is not implied by EngageNY.

Preliminary Analysis: I did not start to develop any auxiliary problems until a few lessons had been completed in my classroom. I have tried to follow the themes of EngageNY lessons and problem sets but have decided to include only problems requiring a solution or task to complete. From my preliminary examination, I felt that Euclid's undefined terms and a few basic definitions should be revisited. I also decided that a rudimentary introduction to basic geometry information was needed. Therefore, I created a vocabulary and notation page and two introductory lessons (Lesson A and Lesson B) that precede Lesson 1.

Lesson 1 Summary

- Brainstorming activity where students are presented with a problem and are asked to position three people who are playing catch the same distance apart. Questions asked:

Where to place them, How to figure it out precisely, and What tool to use. Key idea: how do we get precision of measurement?

- Lesson 1 introduces students to construction with compass and straight edge. Key idea: discover the equilateral triangle.
- Euclid's Proposition 1 (part I) - this next activity asks the students to read Euclid's Proposition 1 (taken from the Elements) and annotate it as they are reading it.
- Euclid's Proposition 1 (part II) - take the annotations and revise to create a step by step process on how to construct an equilateral triangle
- Euclid's Proposition 1 (part III) - compare student steps with Euclid's
- Euclid's Proposition 1 key ideas - communicate precisely, understand mathematical terms and vocabulary

Lesson 1 Notes:

- Day 1: This lesson could not be completed in one class period. We only had time to finish Sitting Cats. Students had trouble with the compass and straightedge construction. I should have anticipated this and had a whole class period to practice constructing circles with the compass. Most are not very dexterous at all. I had to personally help each student with the compass even though I modeled how to construct circles using one. One student adjusted the compass such that the distance between the two given cats was the radius, but did not make the connection that a circle was to be drawn, perhaps since we did not have enough time for her to explore and discover this.
- Day 2: Reading and annotating Euclid took much longer than the 12 minutes suggested. We read Euclid aloud and made the annotations during the reading, but I had them working together in pairs to complete translating the annotations using a step-by-step

process. Most students developed a good set of instructions in constructing an equilateral triangle. Finished the day with return to the sitting cats example with newly set of instructions and more experience with compass. Several students realized after reading Euclid what to do to solve the sitting cat problem. By the end of class all had correct circles drawn and equilateral triangles. A better success today.

- Day 3: Geometry Assumptions suggested time allotment 7 minutes - actual time 20 minutes. Vocabulary suggested time 3 minutes - actual time 15 minutes. Students' behavior deteriorated during this segment of the lesson --- they did not like this part of the lesson at all! I asked students to read aloud (taking turns)-- I must find a different approach for technical information especially on 5th day of school! Exit ticket: similar to sitting cats but students must create the problem simulating a real life situation then communicate in writing how to solve the problem using equilateral triangle. Students enjoyed working with the compasses again and were quite successful on the exit ticket. Problem Set given as Homework.

  Problem Set Comments: Since the homework problem set was designed to be assigned after the mastery of the lesson, it took three days until I assigned these problems. On the fourth day most students brought these problems to the Math Time Study Room and completed them there before class. There were only three problems in the Lesson 1 problem set. RW application provided in this lesson**.**

Lesson 2 Summary

- Lesson 2 is a continuation of constructing equilateral triangles but expands on the Standards for Mathematical Practice (3) to critique the reasoning of others and Standards

for Mathematical Practice (6) to attend to precision by communicating clearly and precisely by asking student to write the steps to construct equilateral triangles.

- Students are to discover how a lack of precision affects the outcome of a construction. The importance of vocabulary is stressed.
- Students test each other's written instruction (step by step process) for precision (precision of written instruction is a desired outcome). Discussion is to elicit correct mathematical vocabulary - no pronouns allowed.
- Activity to construct equilateral triangles following a certain requirement and then write out instructions on how this was accomplished.
- Activity to construct a hexagon and write a set of instructions using precise mathematical language. Exit ticket - determine if given triangle is an equilateral triangle and justify conclusion.

Lesson 2 Notes:

- Day 1: This is a very good lesson for reinforcing communication. We spent much more time on the critique than suggested (my decision as I thought it very beneficial). The students were engaged and each wanted to participate. Some students worked on homework together, therefore we chose one of the group for critique. The discussion was very successful for MP3 and showing the importance of precision when writing instructions or explaining how to do something
- We also had time to discuss the importance of using correct mathematical language and vocabulary. Some students commented that the papers with "math" vocabulary had easier to follow instructions than those whose papers lacked precise mathematical language.

- At this point we were running out of time so we reviewed the vocabulary in Lesson 1 and I called for volunteers to choose a word and restate in their own words but with the restriction they had to use mathematical language. Everyone saw that if they could restate the vocabulary word in their own words using mathematical language, then they got pretty close to the precise definition given! It was a good day in geometry class.
- Day 2: Spent the entire class on the two construction exercises. Students learned from the previous day how important it is to write precise instructions. Students were very successful with this activity. Everyone got the exit ticket right and correct justification. Problem Set Comments: There is only one problem in the problem set. The problem is a writing assignment. RW application not provided in this lesson.

Lesson 3 & 4 Summary

- By Lesson 4, students have experience with the compass and straightedge. Lessons 3 & 4 involve more practice with construction. Lesson 3 has students copying and bisecting an angle. A sorting activity in Lesson 3 involves students being able to place the steps to copy an angle in exact order. In Lesson 4, students construct a perpendicular bisector. Students must master this skill. The perpendicular bisector is a very important construction with regards to transformations. Also in Lesson 4, students are asked to divide a given line segment into four equal segments based on their knowledge learned in the lesson.
- Important vocabulary: midpoint, straight angle, right angle, equidistant, and perpendicular.

Lesson 3 & 4 Notes:

- These lessons were quite the success. We finished the entire lesson and had time to

finish the problem set for Lesson 3 and finish two of the three problems in the problem set for Lesson 4. Copying the angle was straight forward and the sorting activity was a good reminder of the preciseness required when giving instructions. In Lesson 4, I gave no hints to the division of the line into equal segments and some students had a hard time starting, some knew that perpendicular bisector was the key and were finished within minutes. Of course once the first person finished, the late starters "figured it out" and got busy!

Problem Set Comments: There are five angles to be copied in Problem Set 3 and only three problems in Problem Set 4. Students completed Problem Set 3 in class and completed two problems of Problem Set 4 in class. RW application provided in these lessons.

Lesson 5 Summary

- Students are to construct a perpendicular bisector using a string and pencil, then compare to compass and straight edge construction. The lesson shifts to points of concurrency. This lesson is heavy with new vocabulary and applying learned constructions particularly the angle and perpendicular bisectors.

  Key terms/ideas: concurrent, points of concurrency, incenter, and circumcenter

  The problem set is a chart with geometry facts, diagrams and abbreviations which will be used for proofs involving unknown angles, etc.

Lesson 5 Notes

- After reading through this lesson, I decided to skip the string and pencil construction so that we would have time to complete and discuss the chart assigned as a problem set.
- Day 1: This lesson took longer that suggested, we did not get to the chart even though

we dropped the string and pencil construction.

- Students were engaged with the constructions, but upon questioning I found they were getting the terms mixed up (inscribed, circumscribed, incenter, circumcenter). More time needs to be spent on inscribed/circumscribed polygons.
- Day 2: We spent 15 minutes on reviewing the constructions and terms from yesterday (with a whiteboard exercise). This seemed to help the students understand the terms associated with points of concurrency.
- We completed the chart and discussed the importance of the facts listed (the next few lessons begin proofs of unknown angles).

  Problem Set Comments: I found the assignment of the chart inappropriate as an assignment for independent study. There should have been problems associated with the points of concurrency assigned for this lesson, which is exactly what I assigned through an online education software program our district purchases. RW application not provided in this lesson**.**

Lessons 6, 7, & 8 Summary

- All three of these lessons involve solving for unknown angles with the associated terminology introduced.
- Key terms: adjacent angles, vertical angles, straight angles, angles at a point, supplementary angles, complementary angles, transversal, corresponding angles, alternate interior angles, alternate exterior angles, auxiliary line, exterior angles of a triangle.
- Lesson 6 involves solving for angles of lines that intersect at a point. Students must

provide a reason for the solution using proper terminology. This lesson involves using algebra skills.

- Lesson 7 introduces the transversal over parallel lines. Congruent is mentioned casually, as in "corresponding angles are congruent", but defining congruent is not approached in this lesson.
- Lesson 8 introduces the unknown angle of triangles.

Lessons 6, 7, & 8 Notes

- These three lessons took the allotted time and students were engaged all three days. Students have had experience with these type problems before (in 8^{th} grade) and enjoy applying their algebra skills.
- Although EngageNY (Lesson 8) identifies the two angles opposite the exterior angle as opposite interior angles of a triangle, I introduce remote angles to my students for two reasons: (1) I don't want the students to confuse them with alternate interior angles from Lesson 7; (2) I'm not sure what the Louisiana EOC will use since in previous years we have used the term "remote angles" when referring to the two angles in a triangle opposite the exterior angle. We spend some time on these terms so that there is no confusion.

  Problem Set Comments: We had time all three days to do the problem sets in class although they were more complex than the problems offered in the lesson. Three problems in Lesson 6, four problems in Lesson 7 and three problems in Lesson 8. RW application not provided in these lessons.

Lessons 9, 10, & 11 Summary

- Students begin writing proofs to justify the relationships between lines and angles. Lesson 9 introduces deductive reasoning, but does not dwell on the subject as in previous years when logic and reasoning was a large part of the geometry curriculum.
- Lesson 9 involves angle relationships of basic configurations of angles with triangles included.
- Lesson 10 introduces the auxiliary line in proof and uses angle configurations involving the transversal much more than in Lesson 9.
- Lesson 11 involves configurations of angles with a transversal and brings in the theorems related to this type of situation (such as vertical angles theorem and alternate interior angles theorem).

Lessons 9, 10, & 11 Notes

- The students were given the chart with angle facts to start and used it as a "cheat sheet". Each lesson was completed in the allotted time. The students displayed a dependency on me and on their cheat sheet. At one point during Lesson 11, I asked them to put the chart away so that I could see how much they could do independently. Many students could not get started and kept asking me for help. This showed me that critical thinking needs to be worked on so that they can complete a proof 100% on their own. I determined that much more practice was needed.

  Problem Set Comments: There are a total of seven problems for three days (3, 2, & 2, respectively). I find this very inadequate based upon my students need for extra practice in this area. On the fourth day and fifth day of this series of lessons we spent both class

periods working on angle proofs obtained from other sources. RW application not provided in these lessons**.**

Lessons 12, 13, 14, & 15 Summary

- Lesson 12 introduces transformations of the plane. Students begin with a partner activity asking one student to describe a transformation printed on a card while the other student tries to draw the object moving across the card.
- Vocabulary and terminology is introduced in Lesson 12 such as transformations as functions, basic rigid motion, pre-image, image, line of reflection, center of rotation, angle of rotation, distance and angle preserving.
- Lesson 13 introduces rotations as rigid motion transformations. Function notation is introduced. Students practice finding the angle of rotation as well as the center of rotation and practice construction of a rotation and construction finding the center of rotation.
- Lesson 14 involves reflections and finding and constructing the line of reflection as well as reflecting an object across the line of reflection
- Lesson 15 introduces symmetry and its relationship to rotations and reflections.

Lessons 12, 13, 14, & 15 Notes

- Lesson 12, Day 1: For Lesson 12, the card activity sounded like a good idea until we actually tried it. The students became frustrated because they had very little information to work with since the activity is performed before the lesson about transformations is actually given. Their experience with transformation was in 8th grade and involved the coordinate plane so I believe they were expecting something familiar. Finally we did the

activity as a class with the video camera and this was a little better. The class was over at this point.

- Lesson 12, Day 2: We continue with Lesson 12 and discuss the important vocabulary and concepts relating to rigid motion transformations stressing that rigid motion transformations are functions and what that means. We use an online program to get familiar with the terms and see the transformations "in action". They told me this helped them very much to visualize how the card activity should have worked for them.
- Lesson 12, Day 3: We practice the problems in Lesson 12 with a few additional problems I made for them.
- Lesson 13, Day 1: For Lesson 13 we spent the entire class on a discussion of function notation for rotations. As an activity, I gave the students several cards with an angle of rotation about a point (or some cards had a labeled picture) and had them write each one in function notation. This was really successful, for all students passed the assessment with 100% at the end of class.
- Lesson 13, Day 2 & 3: We continued the lesson as written: practicing the constructions with compass and straightedge and the perpendicular bisector. Students were successful in finding the center of rotation for the most part, but this part of the lesson took much longer than planned, but I went ahead and spent two days so that their experience could grow.
- Lesson 14, Day 1: Lesson 14 introduced reflections. I did not tell them that the perpendicular bisector of the line connecting two points is the line of reflection but happily this discovery was made by most of the students. Again, we spent a considerable amount of time on the function notation and I had the student write each reflection with

the correct notation. Due to the pace at which my students perform constructions, this lesson took two days.

- Lesson 14, Day 2: The entire class period was spent on the exercise problems, exit ticket and problem set involving construction of the line of reflection or reflecting an object over a given line of reflection.
- Lesson 15, Day 1: Lesson 15, Symmetry. The first day was spent on the discussions about symmetry especially the fact that two reflections across intersecting lines produces a rotation. I also added order and magnitude of rotational symmetry since Louisiana's Geometry EOC may have these terms included on the test.
- Lesson 15, Day 2: We spent the entire class constructing and identifying lines of symmetry and working the problems in the exercises and problem set.

  Problem Set Comments: There were two problems in Lesson 12, but surprisingly six problems in Lesson 13 involving construction, so this was more than adequate for the lesson (although we did the problems in class). There were four problems in Lesson 14 and six problems in Lesson 15. We did all problem sets in class, so for independent study I assigned similar problems that were obtained from other sources. RW application not provided in these lessons.

Lesson 16 Summary

- This lesson involves translations. Although students translated objects in the coordinate plane in 8^{th} grade, this lesson introduces translations along a vector (a given direction and distance). To do this student must construct parallel lines using a compass and straight edge.

- Students also translate objects in the coordinate plane and then must draw vectors associated with these translated objects.

Lesson 16 Notes

- Day 1: This lesson took two days to complete. The five minute construction of parallel lines took much longer and we did not have time to finish the discussion.
- Day 2: The students have become a bit better at construction, but the 8 minute exercise still took 20 minutes for everyone to finish. After the class exercises were completed, we proceeded to complete the problem set.

  Problem Set Comments: Since most students do not have a compass at home, I decided not to assign the remaining problems in the problem set for independent study. There are seven problems in the problem set, and although I would have liked to have more time for practice, we weren't able to complete all of the problems in class. RW application not provided in this lesson**.**

Lesson 17 Summary

- The focus of Lesson 17 is the perpendicular bisector and what it means in terms of rigid motion and its distance preserving property.
- Student must understand that all points of a transformation are equidistant from the perpendicular bisector.
- Students are asked to analyze the line of reflection and discuss the requirement of equidistance from the points of the pre-image and image to the line of reflection (the perpendicular bisector).

Lesson 17 Notes

- This lesson was a very good lesson to use as a tool for assessment. The lesson is heavy

with discussion and questioning so I asked the students to reflect independently before open discussion of each section. This worked out very well; the students who were confused about distance preserving or the perpendicular bisector as the line of reflection were able to express their difficulties on paper as well as putting down resolutions during open discussion.

Problem Set Comments: There is one single problem in the problem set asking students to create two problems for other students to solve involving reflections and rotations. RW application not provided in this lesson.

Lesson 18 Summary

- The focus of this lesson is parallel lines, specifically the parallel postulate and its implications.
- Students are asked to discuss the meaning of parallel and then are asked to construct parallel lines using rigid motion (rotation).
- Students are introduced to proof by contradiction in this lesson.
- Students must prove lines are parallel using the parallel postulate.

Lesson 18 Notes

- This lesson was quite rigorous and therefore took much longer than anticipated by the authors. We spent four days on this lesson.
- Day 1: Students are separated into four groups and discuss what it means for lines to be parallel. They use chart paper to display their ideas and then all groups compare results with each other.
- Day 2: Students construct parallel lines according to the lesson instructions in Example 2. The class began a discussion of the proof in Example 3 but did not have time to

finalize their thoughts on its meaning.

- Day 3: The discussion of the proof in Example 3 is continued from the day before and then we moved on to Examples 4 & 5.
- Day 4: Examples 6 & 7 were discussed and I assigned the exit ticket and problem set as independent classwork/homework. Many students came to me for extra help on the problem set proofs indicating confusion on how to start and write the proofs.

  Problem Set Comments: There are ten high quality problems in the problem set which I felt was appropriate practice for this lesson. RW application not provided in this lesson.

Lesson 19 Summary

- The focus of this lesson is congruence. In this lesson, students will study the relationship between rigid motion and congruence.
- By the properties of rigid motion, students define congruence.
- Students analyze congruence in terms of rigid motion (rotation, reflection, translation).

Lesson 19 Notes

- Upon returning to familiar territory (rigid motion) students seem much more engaged with this lesson than with the previous lesson.
- Students enjoy construction and have become quite proficient with the compass.
- This lesson was a complete success; students show understanding of congruence as it relates to rigid motion.

  Problem Set Comments: There are only two problems in the problem set which the students completed in class. RW application not provided in this lesson.

Lessons 20, 21, & 22 Summary

- These three lessons focus on the relationship between correspondence and congruence.

- The key idea in Lesson 20 is correspondence of parts of two figures (congruent and not congruent).
- Lesson 21 demonstrates congruence by rigid motion transformation. Students list correspondences of parts of two figures after a rigid motion has been applied.
- Lesson 22 begins a series of congruence criteria for triangles. The focus of this lesson is side-angle-side congruence criteria (SAS) for triangles.

Lessons 20, 21, & 22 Notes

- Students have an understanding of corresponding parts from previous grades so they were eager to contribute during Lesson 20.
- In Lesson 21, students seemed a bit overwhelmed so I decided to use a geometry software program to demonstrate the transformations. This helped the students to visualize how a transformation gives rise to a congruence. The lesson took two days given the alteration of the lesson.
- Since the geometry software program was so successful in Lesson 21, we used it again for Lesson 22 to visualize the transformations that prove the SAS criterion as given in the lesson. I think this helped most students complete the proofs offered in the lesson exercises and problem set.

  Problem Set Comments: Number 4 on the problem set for Lesson 20 prevented them from completing the lesson in class and caused the students to complain about the monotony of the problem. Otherwise the problem sets for two of the lessons were adequate, with the exception of Lesson 21 for which there were only two problems given. RW application not provided in these lessons.

Lessons 23, 24, 25, 26, & 27 Summary

- Lesson 23 takes the isosceles triangle and compares proof by transformation to proof by side-angle-side (SAS) criterion.
- Lesson 24 has students proving the angle-side-angle (ASA) and side-side-side (SSS) congruence criteria for triangle congruence.
- Lesson 25 continues with triangle congruence proofs by introducing two more congruence criteria: side-angle-angle (SAA) and hypotenuse-leg (HL).
- Lessons 26 & 27 recap the previous four lessons with a series of proofs in which students must apply the five congruence criteria learned in Lessons 22-25.

Lessons 23, 24, 25, 26, & 27 Notes

- The proof by transformation in Lesson 23 is pretty straight forward and students follow the proof easily. The SAS proof in the same lesson got a slow start by some students since I required them to complete the proof independently. About half of the class completed the proof correctly without assistance.
- Lesson 24 begins with the congruence proof for ASA using a rigid motion. As we worked through the proof, I noticed that the students became restless. I stopped in the middle of the proof for discussion and they expressed concern about not understanding the proof and why the wording was so different from the previous proofs on SAS. I had them re-write the proof in their own words using the given proof as a guide. This seemed more effective than just working through the proof as it was written.
- In Lesson 25, students were fascinated by the discovery that SAA was an extension of the SAS criterion. Students also quickly understood that HL was also an extension of the

SAS criterion due to the fact that the triangles must be right triangles to apply this criterion.

- The patty paper exercise in Lesson 25 to show that SSA cannot prove congruence was very good and the students showed complete understanding why SSA cannot be used as a criterion for congruence.
- I combined Lesson 26 and Lesson 27 into a two day activity. On the first day, I chose six of the proofs to be part of a stations activity. The statements and reasons for each proof were typed on a separate card and placed (mixed up) in a baggie. Students were paired up to match each statement to its corresponding reason. The second day consisted of having the students complete the remaining proofs on paper without assistance. Needless to say, they preferred the stations activity with its extra help but were ultimately successful on most of the remaining proofs.

  Problem Set Comments: The number of problems in each problem set is inadequate for mastery of these skills. There are a total of 14 practice problems (5, 3, 4, 1, & 1 respectively) for all five lessons. RW application not provided in these lessons.

Lesson 28 Summary

- Lesson 28 uses triangle congruence criteria to prove that the properties of parallelograms hold true.

Lesson 28 Notes

- Students are familiar with parallelograms in a basic way from their understandings of these quadrilaterals in previous grades. Therefore, the proofs offered in this lesson engage the students without overwhelming them. The only difficulty was encountered on Example 4 which presented quite a challenge to the students.

Problem Set Comments: There are five proofs in this problem set which are enough of a challenge to be suitable practice for this objective. RW application not provided in this lesson.

Lessons 29 & 30 Summary

- These two lessons focus on mid-segments and medians in triangles. The proofs in these two lessons are designed to emphasize the properties of these two special lines in triangles.

Lessons 29 & 30 Notes

- Students enjoyed these two lessons. The combination of construction and questioning to complete the first proof in Lesson 29 provides an engaging activity for the students as evidenced by their absolute attention and participation.

  Problem Set Comments: There is very little opportunity for practice with these two problem sets, as there are only five problems provided between the two lessons. RW application provided in these lessons.

Lessons 31 & 32 Summary

- Lessons 31 & 32 have students constructing a square, triangle and a nine point circle.
- Students must communicate the steps of construction precisely so that their instructions can be followed successfully by other students.
- The construction of the nine-point circle provides further exploration of the centers of triangles and their properties.

Lessons 31 & 32 Notes

- Students are becoming very proficient with compass and straight edge constructions. These two lessons are a complete success in that regard. Students are engaged and on

task during the initial constructions. However, constructing the midpoints of the altitudes on the triangles (for the nine point circle) proved to be the point at which the students began to lose focus. Only a couple of determined students labored on without complaint.

- Since the compass and straightedge approach to the nine point circle can prove to be tedious (and messy), I decided to extend the activity using a geometry software program. This approach was received more positively than the manual approach.

  Problem Set Comments: Again, there is very little opportunity for practice with these two problem sets, as there are only two practice problems provided between the two lessons. RW application not provided in these lessons.

Lessons 33 & 34 Summary

- The final two lessons in Module 1, Lessons 33 & 34, list the justifications used in the proofs and other activities given in this module. These two lessons are the study of axiomatic systems.
- The list of justifications is comprehensive and provides the students with a handy reference of the principles and properties of the geometry studied thus far.

Lessons 33 & 34 Notes

- Students complete the tables in these two lessons by filling in the Notes/Solutions column for each entry.
- The two tables were divided evenly among four groups and each group given the task of completing the Notes/Solution column for six entries. Each group transferred their table to chart paper and presented their work to the class. Discussions on each presentation followed. Students completed their personal table during the presentation/discussion. This approach was a success, as these students work very well in groups.

- The proofs and questions provided were completed independently (in class) after the presentations ended.

  Problem Set Comments: The problem sets for these two lessons were sufficient in providing a rudimentary practice, but insufficient as a review of the entire module. RW application not provided in these lessons N/A.

4.3 Organization and Design of the Practice Problems

After the analysis of Module 1 was complete I began the process of designing the practice problems. The organization and design of the practice problems is intended to be as simple as possible to read and follow. Each practice lesson consists of four sections: the objectives of the practice lesson, the relevant vocabulary of each practice lesson, an introduction to provide background information and define vocabulary that will help the student achieve the objectives of the practice lesson, and finally the practice section that will supply the practice problems of each practice lesson. After careful review of dozens of textbook and workbook formats, I designed a format that provided ease of reading and ease of locating relevant information. The number and title of each practice lesson is found at the top of each page for easy reference to EngageNY. The objectives are emphasized at the top of the page within a text box. A list of relevant vocabulary for each lesson follows the objectives. The introduction section follows the listed vocabulary and ranges from a few sentences to a whole page depending upon the complexity of the lesson, the amount of required background information needed, or the number of vocabulary terms needing to be defined. Following the introduction is the practice section. Each practice section contains problems that satisfy the lesson's objectives. There is not a set number of practice problems for each lesson. Some lessons will have a few problems while others may have several problems available for practice.

Since the geometry curriculum is rich in visual content, many problems were developed from a diagram that I created based on the analysis of each lesson and then the problem itself evolved from the diagram. Almost every problem in the auxiliary resource materials (including solutions) is accompanied by a diagram. These diagrams were created by a variety of methods. Most diagrams were created with Geometer's Sketchpad®, although some diagrams were created with Microsoft Word® drawing tools, Microsoft Paint® (both used with permission from Microsoft®), open source images captured from the internet, or a combination of any or all four methods. All mathematical content utilized Microsoft Equation Editor® (used with permission from Microsoft®) and Design Science Math Type Equation Editor™ (an add on application used in Microsoft Word®).

CONCLUSION

In conclusion, the analysis of Module 1 lead me to believe that there was a need for extra practice problems for the following reasons:

1. The number of problems in the problem sets was insufficient. Most problem sets were completed in class.
2. The problem sets did not provide many opportunities for differentiation.
3. The number of real world applications and word problems was inadequate.
4. The number of formal assessment items was deficient.

These reasons guided the rationale of creating the enrichment exercises provided in this thesis. It was my intention to develop a set of high quality practice problems that teachers of geometry could find beneficial to use to supplement their implementation of the EngageNY Geometry Curriculum. This supplement provides extra practice in all concepts offered in Module 1 of the EngageNY Geometry Curriculum. It was not only the extra practice that I felt was needed, but a need of problems that could be used for differentiation in the classroom.

The problems included in this thesis vary in degree of difficulty and scope, thus providing the necessary differentiation that teachers need when providing for their students' needs. Also, I am interested in the adaptation of geometry problems that apply to real world situations, therefore it was important to include in the practice problems applications to real world situations. Many of the Geometry End of Course test questions are constructed of word problems that apply to real world situations, therefore, a number of this type of problem are included in the auxiliary resource materials. I believe that students need this type of practice to improve their ability to master geometry.

Upon further reflection, I have concluded that a set of extra practice problems for all modules in the EngageNY Geometry Curriculum would be advantageous for teachers to have. The development of extra practice problems for these modules could be a worthwhile endeavor. With time constraints put upon teachers and their busy schedules, having this extra resource to use for independent student work, differentiation in the classroom, and extra assessment items at their immediate disposal could be a valuable and time saving resource.

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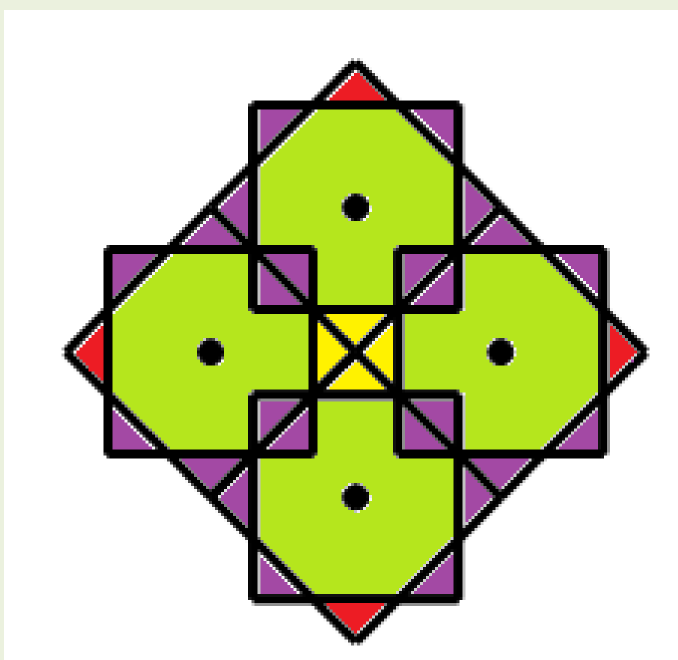
Practice Problems and Applications Auxiliary Resources to Supplement EngageNY Geometry



Module 1

Introduction

These problems are intended to provide extra practice for students studying a high school course in geometry. This project was originally designed to provide supplementary exercises for those students whose districts have chosen the EngageNY High School Geometry curriculum, but is also well suited as a set of problems for any student wishing to enhance his or her geometry education and thus, can be used independently of EngageNY.
The goal of this material is to provide further study and additional challenges on the topics presented by EngageNY Module 1 Geometry curriculum therefore, teachers may find it a beneficial additional resource to enhance their current geometry curriculum.

Organization of Enrichment Problems and Applications

The practice lessons for Module 1 will consist of two introductory lessons and 34 practice lessons with the following design:

Objectives
Vocabulary
Introduction
Practice

Objectives will state what the student will do during the progression of the lesson.

Vocabulary will *list* the relevant vocabulary used in the lesson. Since vocabulary is not unique to any individual lesson, there will be a glossary incorporated at the end of the book that will provide definitions of all vocabulary listed in each lesson. You will notice that some lessons will be absent of a vocabulary section as the key words may have been introduced in a previous lesson.

Introduction this section will introduce the topics covered in the practice lesson. Some lessons may provide background information, defined vocabulary, or other related material that may help the student accomplish the objectives of the exercise.

Practice will provide the problems related to the objectives of the lesson.

Note: Solutions for the practice lessons will be provided in the teacher's edition.

Disclaimer: Assume that all figures, shapes, diagrams, etc. are NOT drawn to scale.

Contents

Practice Lesson 24 Congruence Criteria for Triangles ASA and SSS

Practice Lesson 25 Congruence Criteria for Triangles SSA and HL

Practice Lesson 26 Triangle Congruency Proofs

Practice Lesson 27 Triangle Congruency Proofs Part II

Practice Lesson 28 Parallelograms

Practice Lesson 29 Special Lines in Triangles: Mid-segments

Practice Lesson 30 Special Lines in Triangles: Medians

Practice Lesson 31 Construct a Square

Practice Lesson 32 Construct a Nine Point Circle

Practice Lesson 33 Review of Assumptions Part I

Practice Lesson 34 Review of Assumptions Part II

Vocabulary and Notation

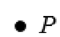
Objectives
- Build a vocabulary of fundamental geometric terms in order to communicate mathematically and effectively
- Identify and model commonly accepted notation used in geometry

Why study vocabulary?
Knowledge of geometry vocabulary is an essential component of learning geometry. Building a vocabulary of geometric terms is a powerful tool students can utilize to become proficient in the study of geometry. Learning to communicate verbally and mathematically in relation to geometry requires an expansive set of terms and phrases that is much broader than any other discipline of high school mathematics.
Before we get started we must clarify the meaning of the following few words.

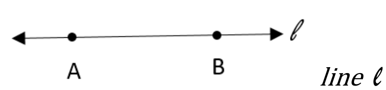
Definition - the accepted meaning of a term.
Undefined terms - terms that cannot be precisely defined therefore can only be explained by examples and descriptions.
Assumption - that which is accepted as true without proof.
Postulate - a postulate is a basic assumption accepted as true but cannot be proven.
Theorem - a proven fact using undefined terms, postulates and other theorems.

The fundamentals of geometry are based upon the following **undefined** terms. It should be noted that these terms are simply concepts, and that other geometric ideas depend on their existence.

Point - indicates position or location and has no length, width, or depth



Line - a straight, infinite set of points that has length but no width or depth. A line extends in both directions.



Plane - an infinite set of points that has length and width but no depth (i.e., it extends in all directions)

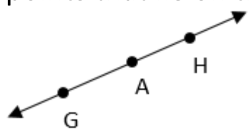
Distance along a Line and **distance along a circular arc** are undefined until a unit distance is determined or selected. The resulting distance is a length.

The following list consists of basic geometric terms and the vocabulary used in the first few lessons. As this book progresses, vocabulary will be added and addressed as needed.

Line segment - part of a line consisting of two points (called endpoints), and the set of all points between them.

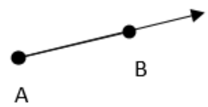
line segment $\overline{AB}$

collinear - three or more points that lie on the same line.



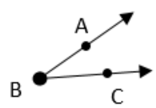
G, A, and H are collinear points

ray - a part of a line consisting of a given point (called the endpoint) and the set of all points on one side of the endpoint. A ray extends in one direction only.



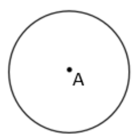
ray $\overrightarrow{AB}$

angle - the union of two rays having the same endpoint (called a vertex). The rays are called the sides of the angle.



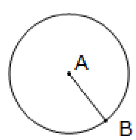
ways to name an angle:
$\angle B$, *or* $\angle ABC$, *or* $\angle CBA$

circle - the set of all points in a plane that are a fixed distance from a point in the center.



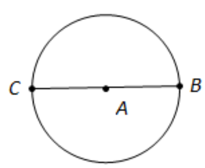
Circle A
also denoted $\odot A$

radius - the distance from the center of the circle to any point on the circle.



radius $\overline{AB}$

diameter - the distance across a circle through its center point.



diameter $\overline{CB}$

Introductory Lesson A Naming and Identifying Basic Geometric Figures

Objectives
- Identify and name geometric figures associated with lines and angles
- Explore collinearity

Introduction

This introductory lesson will offer exercises that will re-introduce topics previously learned in elementary and middle school. This lesson will also provide practice with the concept of collinearity.
Students should be familiar with **points, lines, angles,** and **rays**.

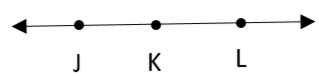
Practice

1. Draw, label, and name a ray with endpoint B and another point C.

Solution: A ray should be named with the endpoint listed first.

ray BC or $\overrightarrow{BC}$

2. List all of the line segments can be named using the points on the following line.



Solution: $\overline{JK}$, $\overline{KL}$, and $\overline{JL}$ (the line segments can also be named in reverse order $\overline{KJ}$, $\overline{LK}$, $\overline{LJ}$).

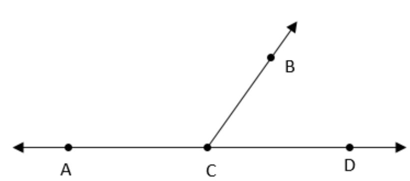
3. One name for the following figure is $\overleftrightarrow{AB}$. List all other names that apply.

Solution: Line ℓ, line AB, line BA, and $\overleftrightarrow{BA}$

4. a. List all of angles found in the following figure:

Solution: ∠ACB, ∠BCA, ∠BCD, ∠DCB, ∠DCA, ∠ACD

b. Name one ray in the above figure.

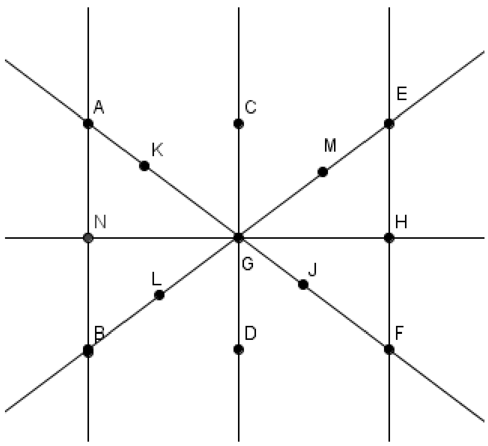


Solution: $\overrightarrow{CB}$ (other solutions: $\overrightarrow{CA}$, $\overrightarrow{CD}$)

5. Three or more points that are part of one line are called **collinear**. One example where the use of collinear points would be applicable is the design of words spelled out by a marching band on a field, where the band members are lined up in single file as points on a line. Give another example where the concept of collinear points would be used in a real world application.

Solution: Answers will vary. Some examples are light bulbs in a road side sign, cabbage plants in a farm field, the port holes on a cruise ship, the windows on a jet liner, the number keys on a calculator, etc.

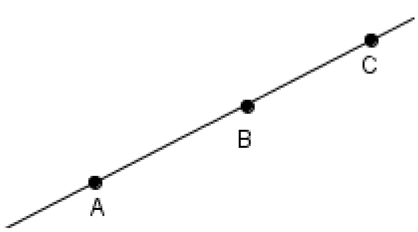
6. Name all collinear points on the diagram below:

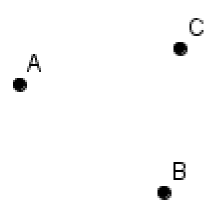


Solution: points A, N, B are collinear; points C, G, D are collinear; points E, H, F are collinear;
points A, K, G, J, F are collinear; points B, L, G, M, E are collinear; points N, G, H are collinear.

7. If collinearity is defined as three or more points on one line in a plane, then how many lines can contain three distinct points? Explain your reasoning clearly and concisely.

Solution: Given three distinct points in the plane, there may be no lines or one line. If the case is one line, the three points are collinear as in the diagram below left. If the case is no lines, then the three points are non-collinear as in the diagram below right.





Introductory Lesson B Introduction to Classical Construction with Compass and Straightedge

Objectives

- Use a compass and straight edge to construct a line segment and a circle

Vocabulary:	Classical Construction	compass
	straightedge	arc

Introduction

In geometry, **classical constructions** utilize two tools, the compass and straightedge. The **compass** is an instrument or tool used for drawing circles and arcs. It comes in a variety of styles, a few are represented below:


Image 1


Image 2


Image 3

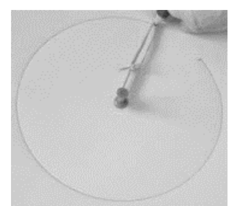


Image 4

A **straightedge** is a tool with an edge free from curves such as a ruler, triangle, or T-square. A true straightedge will have no markings, so if a ruler is used, disregard any such marks and use it for its straightedge only. A few straightedge tools are shown below:


Image 5


Image 6


Image 7


Image 8

To become successful in your geometric constructions, you must practice with your compass and straightedge. Holding your compass takes a gentle approach with a flexible wrist. The compass is designed to construct circles. Most constructions in this book will be based on the construction of the circle, so practice making different sized circles until you are proficient with the compass and the feel of the compass is natural in your hand.

The constructions in this lesson should be made with **compass and straightedge only**.

Note: Constructions shown in the solutions section will be shown once; i.e., after a perpendicular bisector is constructed, then future requests to create perpendicular bisectors will be referred to only and not shown in a step by step manner. **Also note**: most constructions

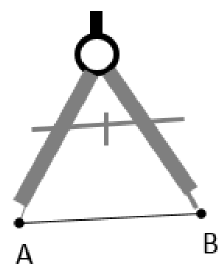
will be shown with an **arc** (a portion of the circumference of a circle). Please be mindful that all constructions can be similarly drawn with a complete circle, since a true construction is that of the circle as opposed to the arc (which will be shown primarily to eliminate clutter on the page).

Practice

1. Practice with your compass and straightedge by constructing circles with radii of differing lengths. For each circle, draw the radius from the center point to a point on the circle.

2. Copy a line segment.
   Given: line segment $\overline{AB}$ and Point P (at right).
   Copy the line segment $\overline{AB}$ so that you create line segment $\overline{PQ}$.

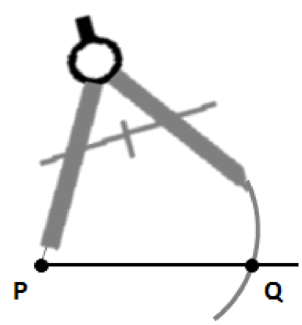
Solution: Place compass point on Point A and stretch out until lead is on Point B.



With your straightedge, draw a reference line through Point P and extend out longer than your intended line segment.

Without changing the compass width, place compass point on Point P and construct an arc across the reference line. Place a point on the intersection of the arc and the line and label it Q.

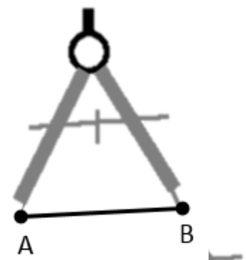
$\overline{PQ}$ is a copy of line segment $\overline{AB}$.



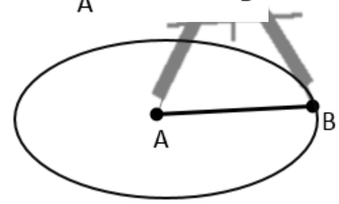
3. Construct a circle with radius $\overline{AB}$.

Solution: Place compass point on Point A and stretch out so that the lead is on Point B.

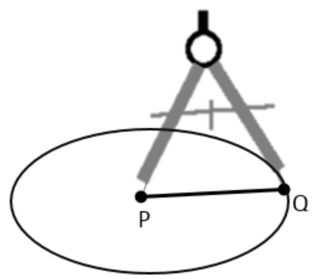
Swing the compass around to make a complete circle starting and stopping at Point B.



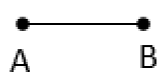
4. Construct a circle whose radius $\overline{PQ}$ is the same length as the given line segment $\overline{AB}$.



Solution: Copy line segment $\overline{AB}$. Label the new line segment $\overline{PQ}$. Construct the circle with radius $\overline{PQ}$.

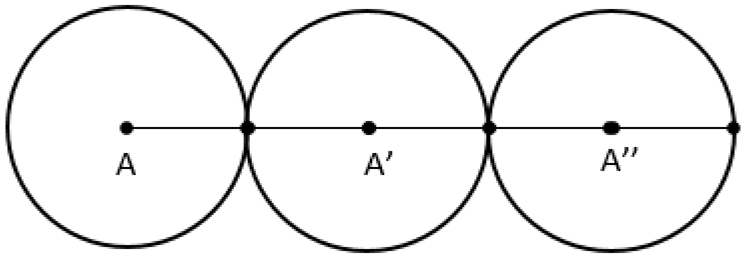


5. Construct three circles within a larger circle such that the length of the diameter of the larger circle will be equal to the combined diameters of the smaller circles minus the length of line segment $\overline{AB}$.

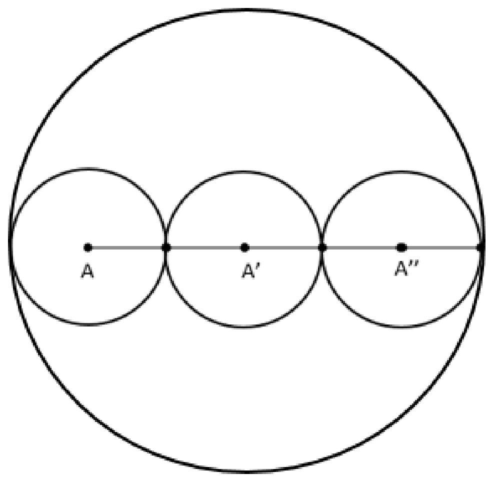


Solution: Copy line segment $\overline{AB}$ five times such that Point A of each successive segment lies directly on Point B of the previous segment.

Place the compass on Point A and construct a circle with center A. Construct the second circle with center A' and the third circle with center A''.



Now construct a circle with center A’ and radius $\overline{A'B''}$.



Not drawn to scale

Practice Lesson 1 Construct an Equilateral Triangle

Objectives
- Use a compass and straight edge to construct an equilateral triangle
- Apply the properties of equilateral triangles to solve real world problems

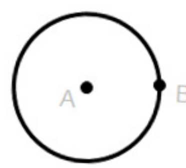
Vocabulary: equilateral triangle

Introduction

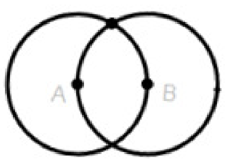
In elementary grades, students classified triangles by angle size (acute, obtuse, right) and by sides (scalene, isosceles, equilateral). In this lesson we will construct equilateral triangles and show how the properties of equilateral triangles can be used in real world applications.

Practice

1. An **equilateral triangle** is a triangle in which all three sides are of equal length and all three angles have equal measure (specifically, 60° each). Construct three circles so that each circle passes through the centers of the other two circles. Write clear and concise directions that describes the construction process. Describe how the resulting polygon is an equilateral triangle.

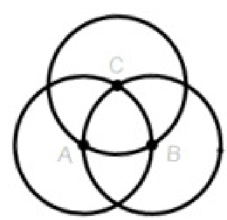


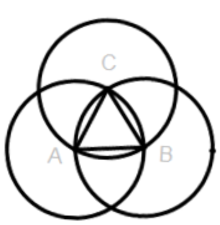
Solution: Choose any length for the radius. Construct a circle with center A, radius $\overline{AB}$.



Without adjusting the compass, construct a second circle with center B, radius $\overline{BA}$. (Note: the length of $\overline{AB}$ is equal to the length of $\overline{BA}$).

At one of the intersections of the two circles place point C. Construct a third circle with center C, radius $\overline{CA}$. (Note: the length of $\overline{CA}$ is the same as the length of $\overline{AC}$).

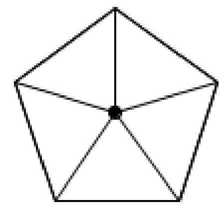




Since point C is on both Circle A and Circle B, its distance from the center of each circle is the same. Similarly, point B is on Circle A and Circle C, so its distance from the center of each circle is the same. Likewise, point A is on both Circle B and Circle C, so its distance from the center of each circle is the same. So now we have three circles in which all radii are of equal length.

Since all three radii have the same length, then connecting the points with line segments will result in a triangle with all three sides of equal length, thus, creating an equilateral triangle.

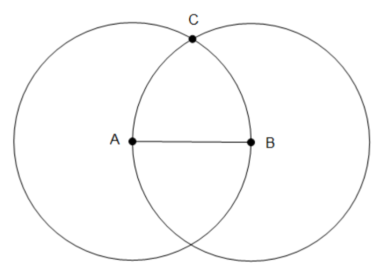
2. The figure shown at the right is an example of five copies of a triangle which fit around a point in a plane.
   a. Explain why the triangles in this figure cannot be equilateral.
   b. How many equilateral triangles will fit around a point in a plane? Explain your answer verbally and show your answer visually with a constructed example.



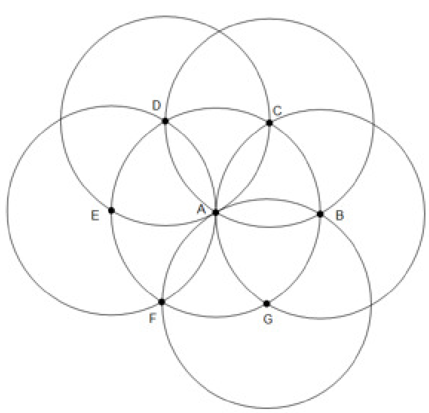
Solution:

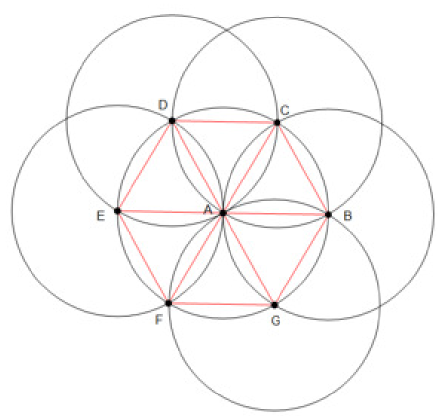
a. There are 360° around a point. The figure above consists of 5 angles around a point and 360 ÷5 = 72. An equilateral triangle does not contain a 72° angle, therefore, the triangles cannot be equilateral.
b. Since there are 360° around a point, there will be six equilateral triangles that will fit around a point. To find the number of degrees that each vertex of the equilateral triangle will have, divide 360 by 60 since each angle of an equilateral triangle measures 60°. The result is six-60° angles that will fit around a point.

To construct the figure with six equilateral triangles around a point, a center point should be identified. Call the center A. Then a radius length should be determined. Using a compass, construct a circle with point B as a point on Circle A. Now we have a circle with fixed radius length $\overline{AB}$. Without adjusting the compass, create a circle with center B and point A as a point on Circle B with the fixed radius length $\overline{AB}$. Label the intersection of the two circles with a point (point C as in the diagram at right)



Next, construct a circle with center C and point A on Circle C. Label the intersection of Circle C and Circle A as point D. Continue around Circle A in this manner until 6 circles have been constructed with point A on each circle, as shown below, left. With a straightedge, draw a line segment connecting all of the points and a line segment from each point to point A as shown below, right:





3. The map below is an aerial map of rural Texas. AT&T would like to add another cell tower that is north and equidistant from the current towers they already have in place (Tower A and Tower B). The two towers in place service the area west of Talbert Ranch Road and the county west of Valley Mills. Show the location where the new tower should be to satisfy the required location. Give clear and concise instructions on how you determined the new location.

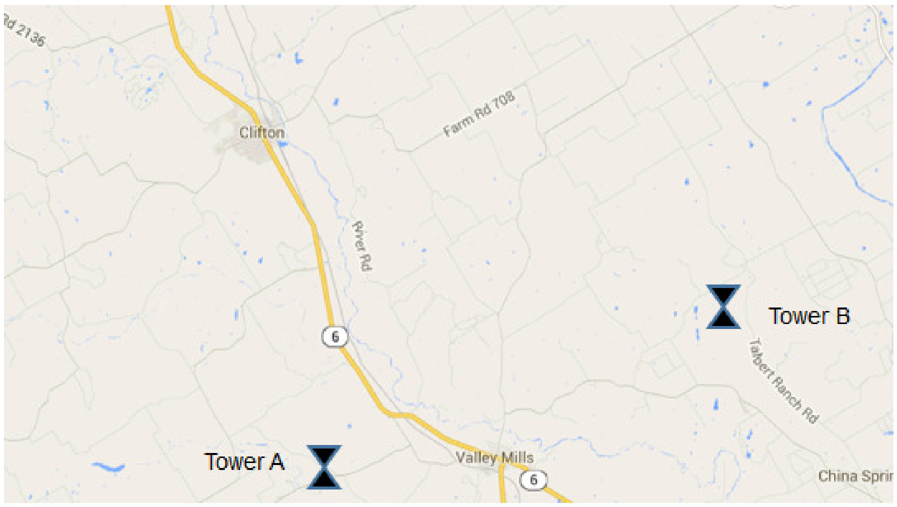


Image 9

Solution: Construct an equilateral triangle with Valley Mills and China Springs at two of the three vertices.
Step 1: Construct a circle with Tower A as center and Tower B on the circle.
Step 3: Locate the intersection of these two circles to the north of the two towers and mark a point. Label it point C. This is where the location of the new cell tower should be.
Step 4. Connect Tower A and Tower B with a line segment and connect each cell tower to point C to show the equilateral triangle created (as shown below):

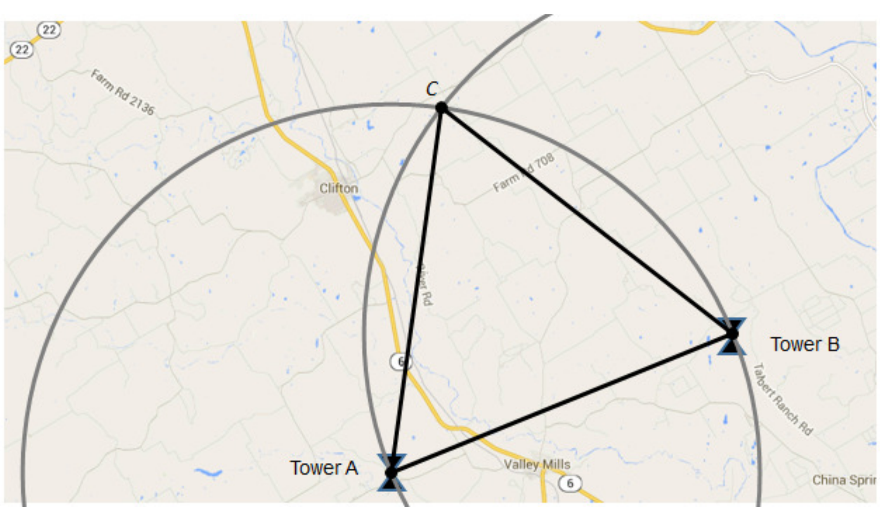


Image 10

Practice Lesson 2 Extensions of the Equilateral Triangle

Objectives

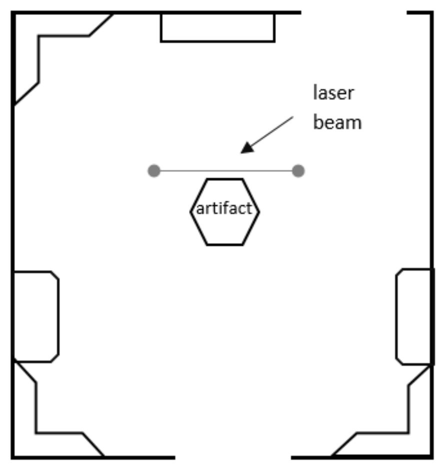
- Construct equilateral triangles
- Apply the properties of equilateral triangles to real world situations

Introduction

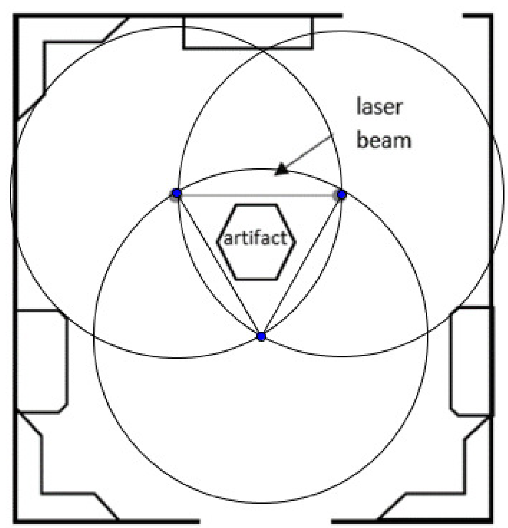
Students take their skills and knowledge about equilateral triangles to another level by applying what they have learned to solve problems that arise in real situations. Students will continue to use their compass and straight edge for their constructions in this lesson.

Practice

1. A museum is acquiring a very rare and valuable artifact. The most effective security protocol is to place laser beams in a triangular pattern surrounding the artifact. A diagram of the room where the artifact is displayed is shown at right. With a compass and straight edge, show where the laser beams should intersect so that the three beams are exactly the same length.



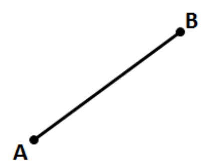
Solution: Constructing an equilateral triangle will ensure that the laser beams are of equal length. Construct the first circle with its center at the left endpoint of the existing laser beam and its radius the length of the laser beam. Construct the second circle with its center at the right endpoint of the existing laser beam and its radius the length of the laser beam. Construct the third circle with its center at the intersection of the two previous circles choosing an endpoint of the laser beam as a point on this circle thus creating a radius to either endpoint (to ensure the artifact is within the triangle, choose the center of this circle to be the bottom intersection of the two circles). Connect the three points using a straightedge.

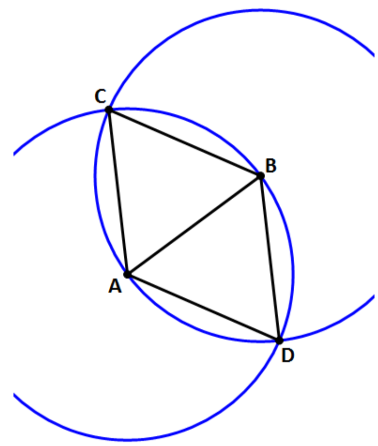


2. Describe a situation in which the properties of the equilateral triangle may be useful.

   Solution: Answers may vary. One such situation would be in architecture. The Geodesic Dome is one such example. Concert speaker set-up would be another useful application with the speakers on the stage as two of the vertices and the mixer board in the center of the room equidistant away from the stage as the third vertex.

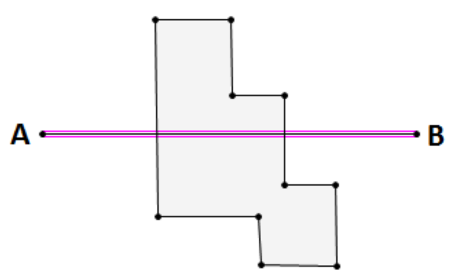
3. Trace line segment $\overline{AB}$. Construct an equilateral triangle with $\overline{AB}$ as one side. How many different triangles could you construct with side $\overline{AB}$?

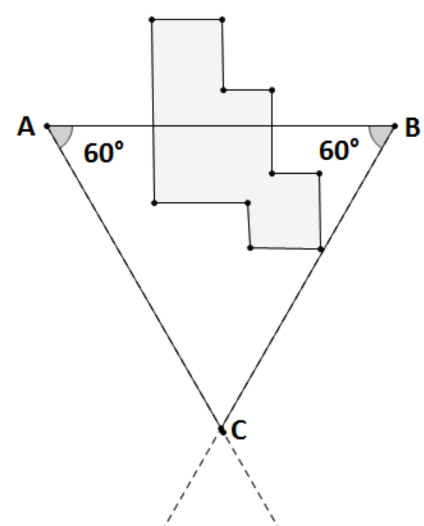




   Solution: Construct circle with center A. Construct a second circle with center B. Place a point at the intersections located above and below the line segment (labeled C and D on the diagram at right). Two different equilateral triangles can be constructed with side $\overline{AB}$.

4. Surveyors use equilateral triangles to measure distances around obstructions. A surveyor needs to measure the distance from point A to point B. Find the point where he needs to place his instrument in order to find the measure of $\overline{AB}$.

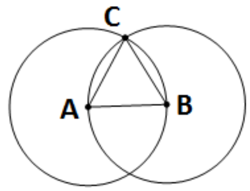




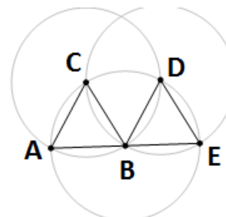
   Solution: Since surveyors use equilateral triangles to measure distances, then the distance $\overline{AB}$ would be the same as the distance $\overline{AC}$, created by construction of an equilateral triangle. The surveyor should first place his instrument at A and measure a 60° angle (since equilateral triangles have three 60° angles) with vertex A and extend a straight line out from A at this angle. He should then repeat this at B. Where the two lines intersect is point C. Then the surveyor should measure the distance from A to C and that would be the distance from A to B.

5. A gardener has three herb beds she wishes to plant in a design that consists of three equilateral triangles laid out in such a way that all three triangles share a common vertex. Construct such a design and give a step by step explanation of the construction.

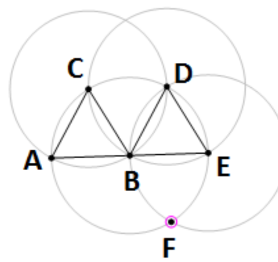
Solution:



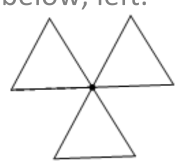
Step 1: Construct a circle with center A and radius AB.

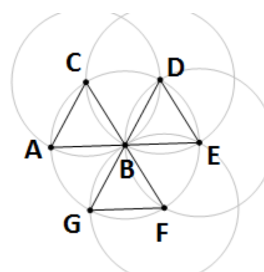


Step 2: Construct a second circle with center B and radius BA. Label the intersection of the circles C.
Step 3: Join AC and BC to create and equilateral triangle.
Step 4: Construct a circle with center B and radius BC.
Step 5: Construct a circle with center C and radius BC. Label the intersection of the two circles D.
Step 6: Construct a circle with center D and radius BD. Label the intersection of circle B and circle D with point E.

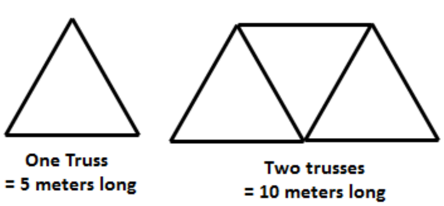


Step 7: Connect BE and BD to create the 2nd equilateral triangle.
Step 8: Construct a circle with center E radius EB. Label the intersection of circle E and circle B with point F
Step 9: Construct a circle with center F radius B. Label the intersection of circle B and circle F with point G.
Step 10: Draw line segments GF, GB, and BF to create the 3rd triangle. The completed design is below, left.



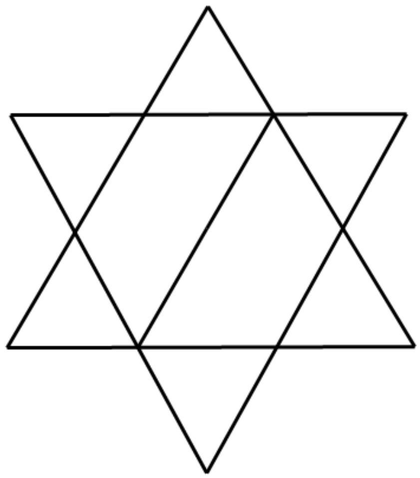


6. The truss of a Warren Bridge is created by equilateral triangles welded to a horizontal frame similar to the example shown at right. If the span of a bridge is 800 meters long, how many triangles would the engineers need to include in one side of the design if each truss measures 5 meters long (as shown in the diagram below)?

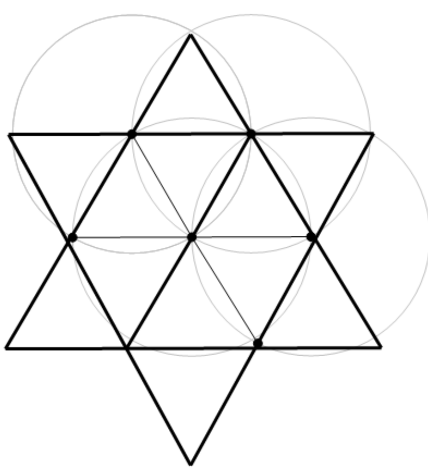


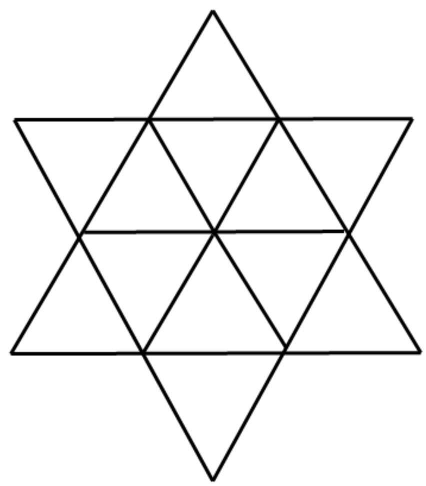
Solution: By the diagram on the right, three equilateral triangles create two trusses (two on bottom, one on top) so, 800 ÷10 = 80 meters of a two-truss structure. Multiply 80 x 3 = 240. Therefore it takes 240 equilateral triangles to create one side of the bridge.

7. Equilateral triangles are popular among quilt designers. The design below is incomplete. How many triangles need to be added so that the entire design is comprised of the same sized patch? Complete the construction.



Solution: The entire design will have 12 same sized equilateral triangles, so six need to be added.





Practice Lesson 3 Copy and Bisect an Angle

Objectives
- Use a compass and straight edge to copy an angle
- Use a compass and straight edge to bisect an angle

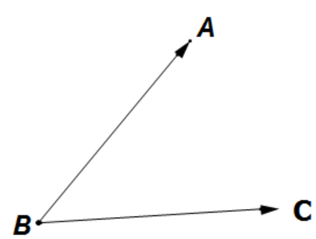
Vocabulary: bisect congruent
angle bisector

Introduction

In this lesson students will use a compass and straight edge to copy and bisect an angle. The term congruent must be defined at this time. When two angles have the same measure they are said to be **congruent**. To copy an angle with compass and straightedge means to construct an angle that is congruent to an existing angle for which we do not know the angle measure. Similarly, to bisect an angle with a compass and straight edge is to create two congruent angles from one existing angle without knowing the angle measure.

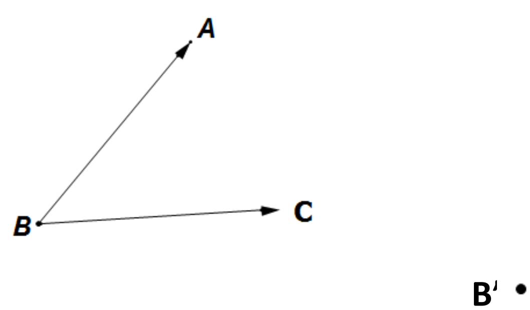
Practice

1. Copy ∠ABC (shown below) using only a compass and straightedge.

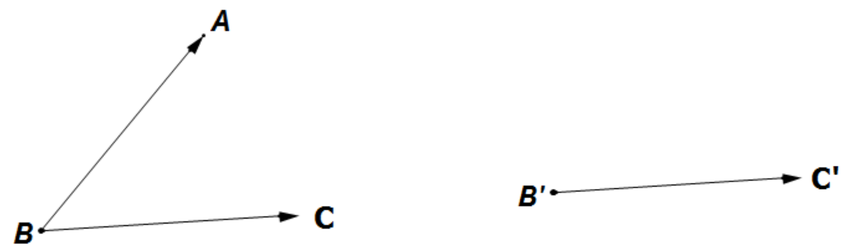


Solution:

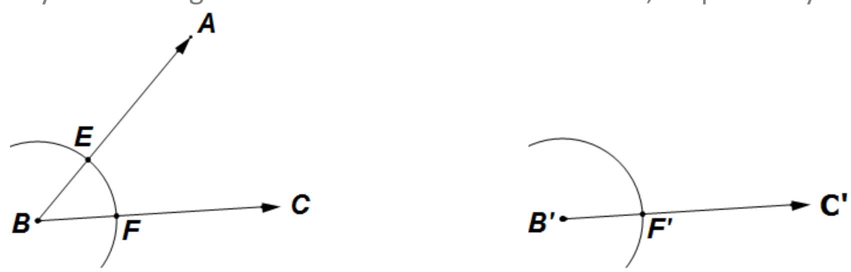
Step 1: Construct a point B' not on ∠ABC that will be the vertex of the new angle.



Step 2: From B', construct a ray $\overrightarrow{B'C'}$, the length or direction does not matter.

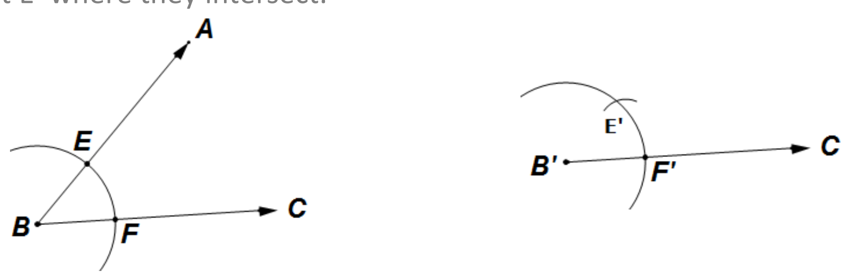


Step 3: Place the compass on point B and extend out to a little past C. Make an arc across both rays of the angle. Label the intersections E and F, respectively.

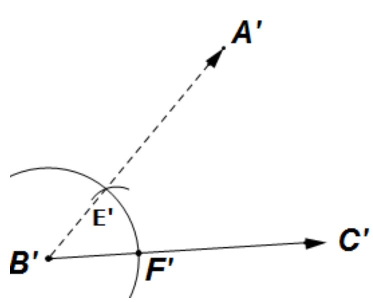


Step 4: Without changing the compass width, place the point of the compass on Point B' and create an arc of similar size across $\overrightarrow{B'C'}$. Label the intersection of $\overrightarrow{B'C'}$ and the arc F'.

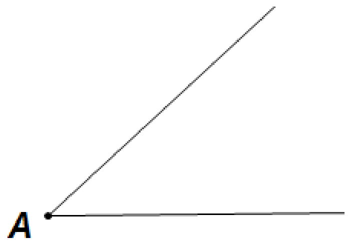
Step 5: Set the compass on Point F and adjust the width to Point E. Without changing this setting, place the compass on Point F' and construct an arc across the previous arc, creating Point E' where they intersect.



Step 6: With your straightedge, construct a ray from B' through E'.

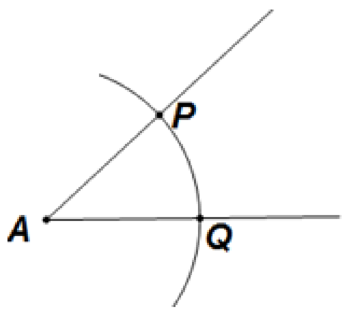


2. Bisect the angle below.

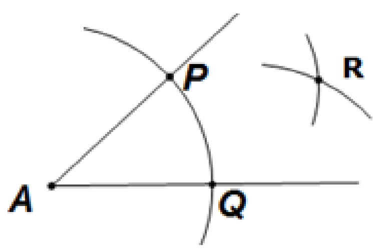


Solution:

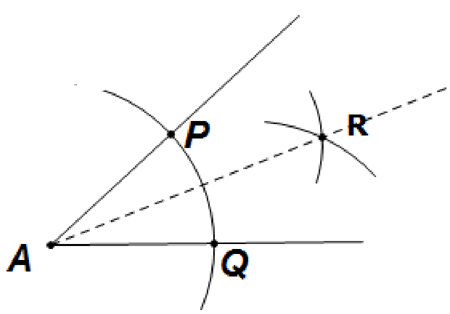
Step 1: Set the compass on Point A and construct an arc across both sides of ∠A. Label each intersection P & Q as shown.



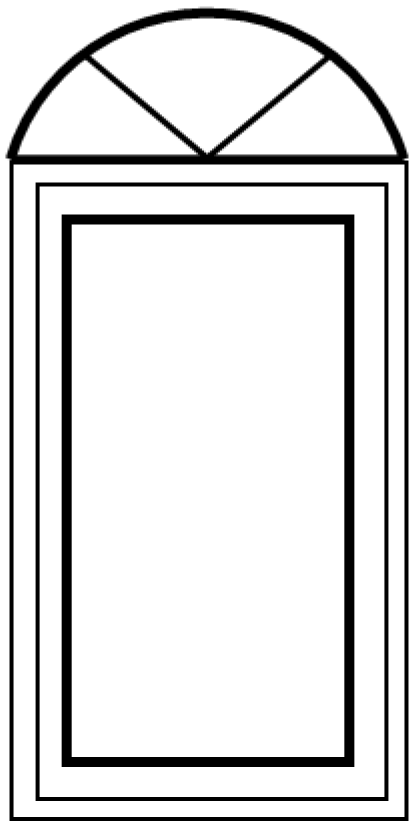
Step 2: Set the compass at Point Q and draw an arc that is inside ∠A. Without changing the width of the compass, place the compass at Point P and draw an arc that is inside ∠A and intersects the previous arc. Label the intersection R.

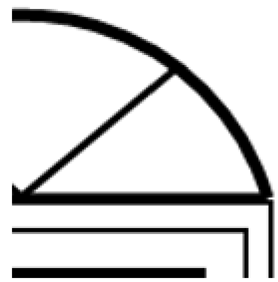


Step 3: Use a straightedge to construct a line through AR



3. The window shown below (left) has an arched detail composed of three angles, two of which are congruent. The diagram, below right, is a template of one of the acute angles in the semi-circle. Give a complete set of instructions on how to duplicate the angle in the template when no measurements are given.





Solution:

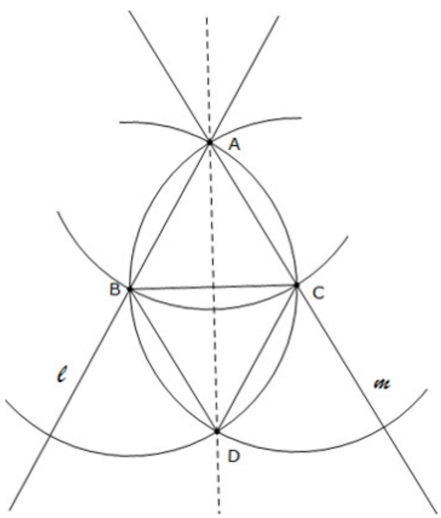
1. Label the vertex of the angle to be copied ∠C
2. Construct a point not on ∠C and label it G. This will be the vertex of the new angle.
3. Construct a ray of any length or direction from Point G. T
4. With a compass, place the point of the compass on Point C and adjust the width a convenient distance from the vertex.
5. With the compass still on Point C, construct an arc across both sides of the angle. Label each intersection of the arc and sides D and E respectively.
6. Without adjusting the compass, move the compass to Point G and construct a similar arc across the ray. Label the intersection of the arc and ray Point H.
7. Move the compass to Point D and adjust the width to Point E.
8. Without adjusting the width, place the compass on Point H and construct an arc across the previous arc.
9. Label the intersection Point J.
10. With a straight edge, construct a ray from Point G to Point J.
    ∠JGH is now a duplicate of ∠ECD.

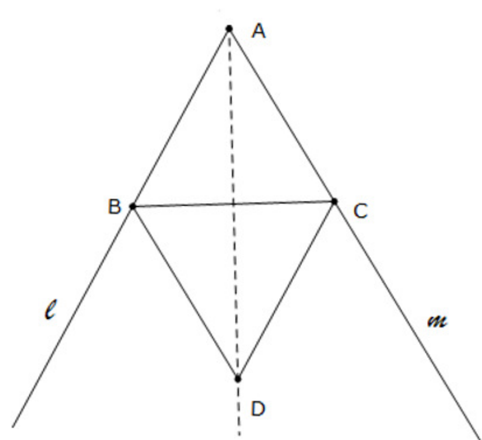
4. Construct an equilateral triangle. Demonstrate how you could construct an angle bisector of an angle of the equilateral triangle that would also bisect an angle outside the triangle. Provide a detailed set of instructions so that someone may mimic this construction.

Solution:

1. Place point B and point A at any location in the plane
2. Construct a line, ℓ, through point B and point A
3. Construct a circle with center A and radius AB
4. Construct a circle with center B and radius BA
5. At the intersection of the two circles, place point C
6. Construct a circle with center C and radius CA
7. At the intersection of Circle B and Circle C place point D
8. Construct a line, m, through point A and point C
9. Draw a line segments joining BC, CD, and BD. This is the equilateral triangle
10. Bisect $\angle$BAC by constructing a line through points A and D

See the construction (left) and finished diagram (right) below.





$\angle$BAC is bisected as well as $\angle$BDC

Practice Lesson 4 Construct a Perpendicular Bisector

Objectives

Use a compass and straightedge to construct:

- a perpendicular line to a point not on the line
- a perpendicular line to a point on a line
- a perpendicular bisector of a line segment

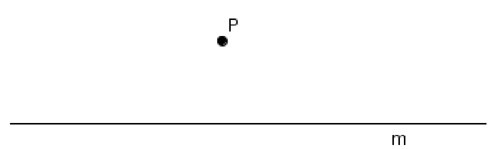
Vocabulary: perpendicular line, perpendicular bisector, midpoint, congruent segments, Perpendicular Bisector Theorem, equidistant

Introduction

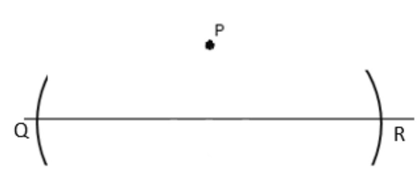
In this lesson we will use the construction of the perpendicular bisector to explore the special properties that it provides. Students should already know that perpendicular means "at right angles to" (**perpendicular line**: a line is perpendicular to another line if the angles created by their intersection is 90°). This lesson will also introduce the **Perpendicular Bisector Theorem,** a very important theorem in the study of geometry states, if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment. All constructions in this section require the use of a compass and straightedge. Knowledge of midpoint, congruent segments, and equidistant is important. The **midpoint** of a line segment is the point that lies halfway between the endpoints of that line segment. Congruent segments are said to be congruent if they have equal measure. When one or more objects have the same distance from another object, they are said to be **equidistant** from that object. For example, the endpoints of a line segment are **equidistant** from the midpoint of the line segment.

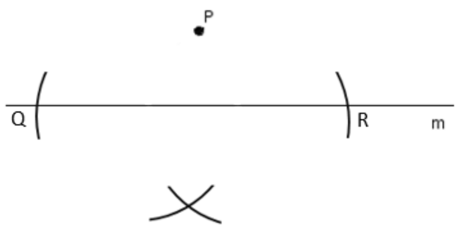
Practice

1. Given line m and point P, construct a perpendicular line to line m and through point P. Write the steps of your construction in a clear and concise manner so that anyone who reads them will be able to perform the construction.



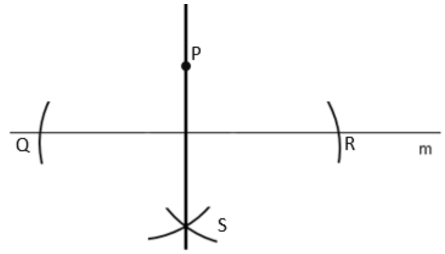
Solution: Place your compass point on point P and extend the compass beyond line m. Construct two arcs across line m so that there is an arc on the left and an arc on the right. Label the intersections of line m and the arcs points Q and R respectively.





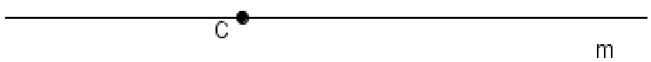
Place the point of the compass on point Q and construct an arc below the line. Move the compass to point R and construct an arc across the arc you made for point Q.

Label the intersection of the arcs point S. With your straightedge, draw a line from P to S.

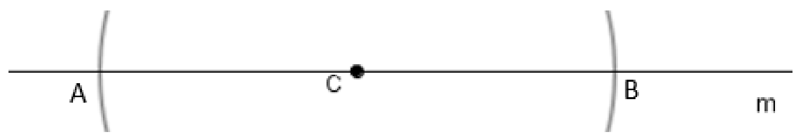


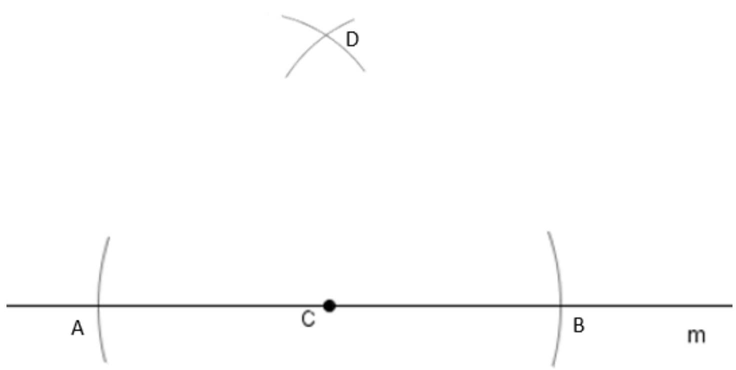
The line PS is perpendicular to line m.

2. Given line m and a point on the line, point C, construct a perpendicular line to point C. Write the steps of your construction in a clear and concise manner so that anyone who reads them will be able to perform the construction.



Solution: Place the compass point on point C and construct two arcs across line m, one to the left of C and one to the right of C. Label these intersections point A and point B respectively.

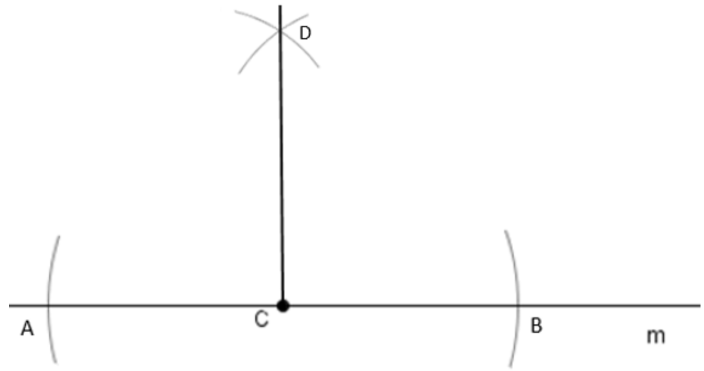




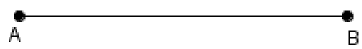
Place the compass point of point A and adjust it so that it is halfway between point A and point C. Construct an arc above the line. Without changing the width of the compass, place the compass point on point B and construct an arc across the previously made arc. Label the intersection point D.

With a straightedge, draw a line from point C to point D.

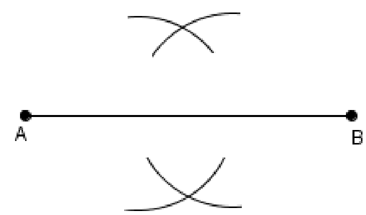
The line CD is perpendicular at C.



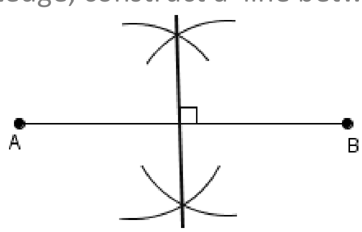
3. Construct the perpendicular bisector to the line segment AB. Explain your construction clearly and concisely.



Solution: Place the compass point on point A and adjust a little past half the length of AB. Construct an arc above and below the line segment. Without changing the compass width, place the compass point on point B and construct an arc above and below AB crossing the previously made arcs.



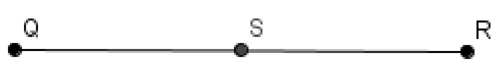
With a straightedge, construct a line between the two arc intersections.



The line is the perpendicular bisector of AB.

4. Find the midpoint of line segment $\overline{QR}$. Explain how the midpoint divides $\overline{QR}$ into two congruent segments.

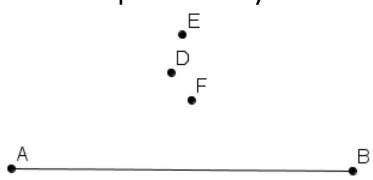
Solution: Find the perpendicular bisector of the segment $\overline{QR}$. The intersection of the bisector and $\overline{QR}$ will be the midpoint of the line segment. S is the midpoint of line segment $\overline{QR}$. Since $\overline{QR}$ was bisected, then by the definition of perpendicular bisector, $\overline{QS}$ = $\overline{SR}$ and therefore, $\overline{QS} \cong \overline{SR}$.



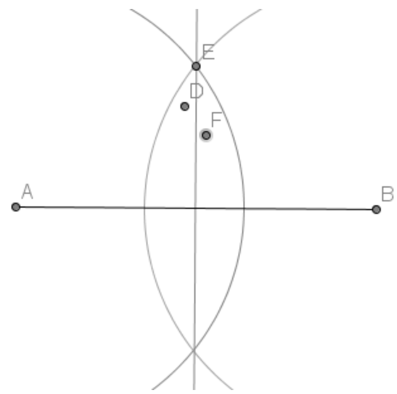
5. The **Perpendicular Bisector Theorem** states that if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment. Explain in a clear and concise manner the meaning of this statement.

Solution: Since the perpendicular bisector divides the line segment into two equal parts, then the perpendicular bisector is an equal distance from each endpoint of the segment. Therefore, any point on the bisector is an equal distance from each end point of the line segment. (A demonstration of this can be seen by using a geometry software program).

6. Which point (D, E, or F) is exactly the same distance from point A as it is to point B? Show and explain how you know.



Solution: Construct the perpendicular bisector of $\overline{AB}$. Since point E lies on the perpendicular bisector of $\overline{AB}$ (and points D and F do not), point E must be the same distance to point A as it is to point B as stated by the Perpendicular Bisector Theorem (given in problem 5).



7. The board of city planners of Los Angeles, CA decided that an emergency clinic was needed that would be exactly the same distance from the two elementary schools, Canfield Avenue Elementary School and Crescent Heights Boulevard Elementary School, as shown on the map below. Both schools have rear access to Airdrome Street. The property available for sale and development is the area bordered by Airdrome St., S. Canfield Ave., W. Pico Blvd. and S. LaCienega Blvd. There is no property available on Airdrome St.

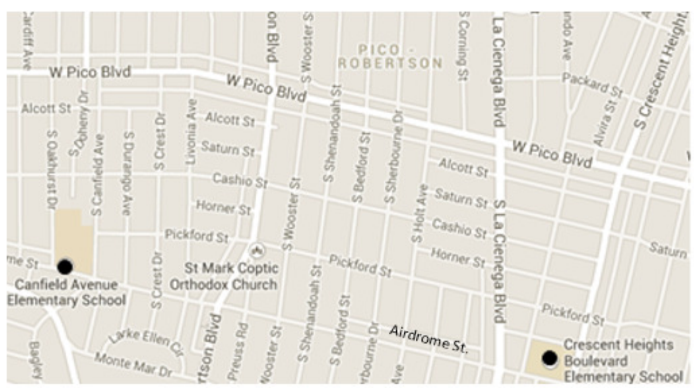


Image 12

Determine the best location of the clinic that would satisfy the board's requirement of being exactly the same distance from each school.

Solution: The location of the clinic should be on the corner of Pickford St. and S. Shenandoah St. since this location is on the perpendicular bisector of the segment of Airdrome St. between the two schools. Since no property is available on Airdrome St., the next closest property on the perpendicular bisector of Airdrome St. inside the required boundaries is the intersection of Pickford St. and S. Shenandoah St. (shown at point E).

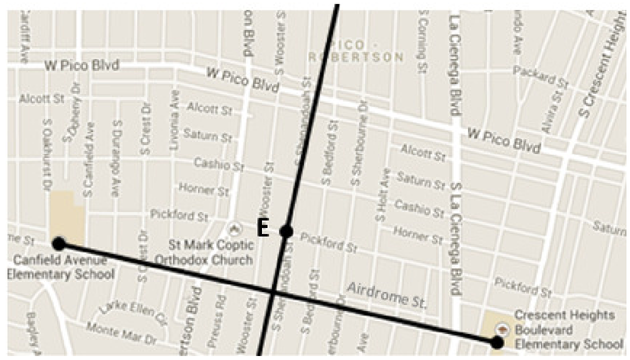
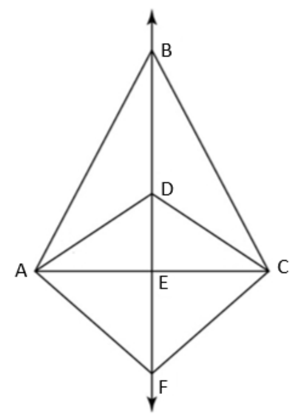


Image 13

8. Δ ADE is congruent to ΔCDE. Points B, D, E and F are collinear. Show that $\overline{BF}$ is the perpendicular bisector of $\overline{AC}$. (Note: the symbol $\cong$ will be used to denote "is congruent to")



Solution: Since Δ ADE $\cong$ ΔCDE, then $\overline{AE} \cong \overline{CE}$. That makes point E the midpoint of $\overline{AC}$. Also, since Δ ADE $\cong$ ΔCDE, the length of $\overline{AD}$ is equal to the length of $\overline{CD}$ so point D is equidistant from the endpoints of $\overline{AC}$. Therefore, by the Perpendicular Bisector Theorem, D must be a point on the perpendicular bisector of $\overline{AC}$. Since B, D, E, and F are collinear, then point B lies on the same line as point D. Therefore, $\overline{BF}$ is the perpendicular bisector of $\overline{AC}$.

Practice Lesson 5 Points of Concurrency of a Triangle

Objectives
- Use a compass and straight edge to construct the points of concurrency of a triangle
- Identify and describe the points of concurrency of a triangle

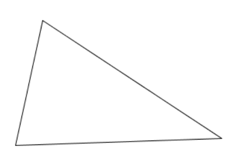
Vocabulary:	concurrent	points of concurrency	median
	centroid	altitude	orthocenter
	incenter	circumcenter	inscribe
	circumscribe		

Introduction

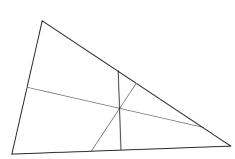
Students will use constructions developed in the previous two lessons (Practice Lesson 3 and Practice Lesson 4) to explore the points of concurrency of triangles. Students should be familiar with the terms median and altitude of a triangle. A **median** is a segment connecting any vertex of a triangle to the midpoint of the opposite side. An **altitude** of a triangle is a line segment connecting a vertex to the line containing the opposite side and perpendicular to that side. This lesson will introduce the topic of concurrency. When three or more lines meet at a single point, they are said to be **concurrent**. In a triangle, the three medians, three perpendicular bisectors, three angle bisectors, and three altitudes are each concurrent. **Points of concurrency** are intersections of three lines constructed from certain parts of a triangle (such as each side or vertex) and refer to the various "centers" of the triangle. Construction of the points of concurrency will introduce the topics of incenter, circumcenter, centroid, orthocenter, circumscribed circles, and inscribed circles. The **incenter** of a triangle is the point where the three angle bisectors meet. This point is the same distance from each of the three sides of the triangle. The **circumcenter** of a triangle is the point where the three perpendicular bisectors meet. This point is the same distance from each of the three vertices of the triangles. The **centroid** of a triangle is the point where the three medians meet. This point is the center of mass for the triangle. The **orthocenter** of a triangle is the point where the three altitudes meet, making them concurrent. This lesson is intended to provide practice and verification that these "centers" are present in all triangles, regardless of size or shape.

Practice

1. The triangle at right is an acute triangle. Construct the perpendicular bisectors of each side of the triangle.



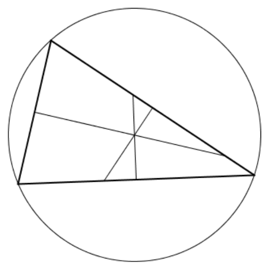
Solution: Construct a perpendicular bisector of each side of the triangle.



a. What do you notice about these three lines?

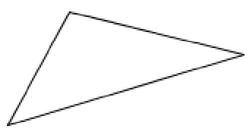
Solution: They intersect at one point inside the triangle.

b. With your compass, place the point on the intersection of the three lines and adjust the width so that it reaches to one vertex. Draw the circle. How is the triangle related to the circle?

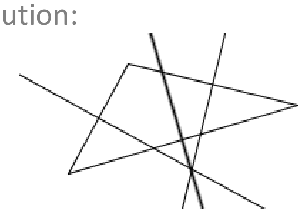


Solution:
The triangle is related to the circle by the fact that each vertex is on the circle and the center of the circle is the intersection of the three lines.

2. The triangle below is an obtuse triangle. Construct the perpendicular bisectors of each side of the triangle.



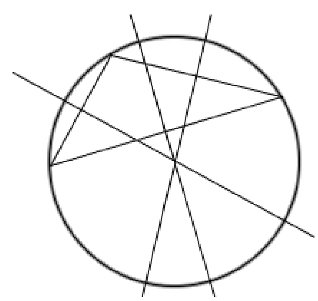
Solution:



a. What do you notice about these three lines?

Solution: They intersect at one point outside the triangle.

b. With your compass, place the point on the intersection of the three lines and adjust the width so that it reaches to one vertex. Draw the circle. How is this triangle related to the circle?



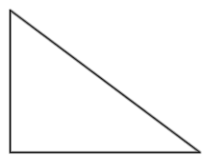
Solution: The triangle is related to the circle by the fact that each vertex is on the circle and the center of the circle is the intersection of the three lines.

3. The intersection of the perpendicular bisectors of the sides of a triangle is called the **circumcenter** of the triangle which is a point of concurrency of the triangle. Make a conjecture about the circumcenter of a *right triangle*.

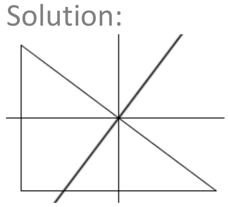
   Solution: The circumcenter of a right triangle must be on one of the sides of the triangle, since a right triangle is neither acute nor obtuse.

4. Notice the circles in 1 and 2. We say a circle is **circumscribed** about a triangle when the circle is constructed around the triangle and passes through all its vertices.

   a. Construct the circumcenter of the following right triangle:

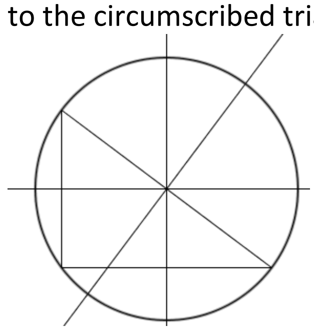
   

   Solution:

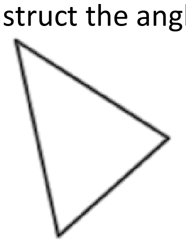
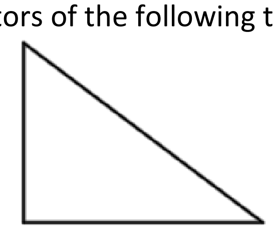
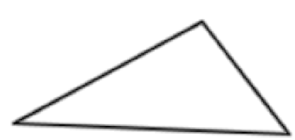
   The circumcenter is on the hypotenuse of the right triangle as shown above.

   b. Construct a circle about the triangle in part A. Can you identify an important property that is present in the construction of the circumcenter? (Hint: relate the circumcenter to the circumscribed triangle).

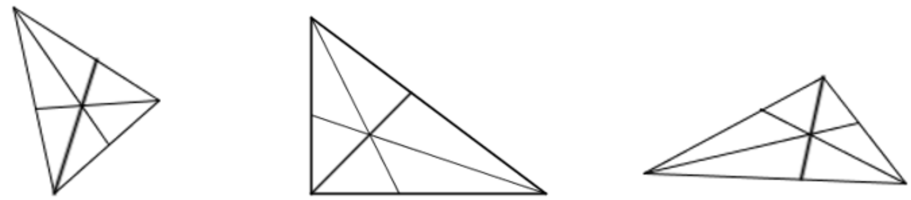
   

   Solution: Since each vertex is on the circle, and the circumcenter is the center of the circle by construction, then the distance from the circumcenter to each vertex is a radius of the circle; therefore, each vertex of the triangle is equidistant from the circumcenter of the triangle.

5. Construct the angle bisectors of the following triangles (an acute, right, and obtuse).

Solution:

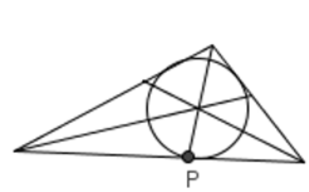


a. The intersection of the angle bisectors of the sides of a triangle is called the **incenter** of the triangle which is a point of concurrency of the triangle. Make a conjecture about the incenter of all triangles.

Solution: The incenter of all triangles is inside the triangle.

6. For each triangle above in question number 4, place a point on the intersection of an angle bisector and any side (call this point P). Now construct a circle for each triangle with the incenter as the center of the circle and point P on the circle. What do you notice about these circles?

Solution: The circles are inside the triangles



7. We say a circle is **inscribed** in a triangle when the circle is constructed inside the triangle and the circle touches each side of the triangle. What can we say about the incenter of these triangles?

Solution: The incenter is equidistant from the sides of the triangle.

8. A grocery chain in the Midwest currently services the cities of DE Moines, IA, Lincoln, NB, and Topeka, KS from its Indianapolis, IN distribution center (640 miles east of Lincoln, NB). The logistics department of the chain has put in a request to build a new distribution center that could better serve these three growing communities. The president of the company has offered a bonus to the employee who offers the best solution to the problem. The requirements for the proposal are as follows:

a. the proposal should have a diagram on the map below
b. the proposal should have a written explanation as to why the location is the most optimal

c. the proposal should be written in a clear and concise manner using appropriate mathematical language.

Image 14

Solution: The most optimal location for the new service center would be very near the town of Maryville, MO. As you can see from the diagram below, Maryville (point D on the map) is the circumcenter of the triangle that includes Lincoln, Topeka, and DE Moines as it vertices. Since the circumcenter is equidistant from the vertices of a triangle, Maryville is the most efficient location for the new distribution center.

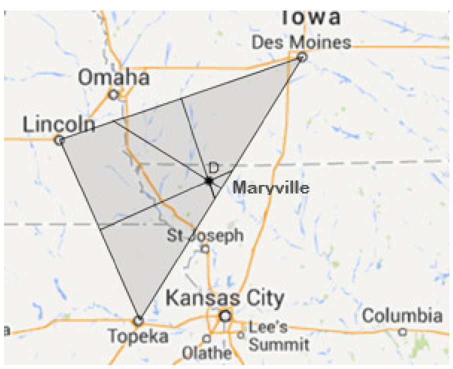


Image 15

9. After careful examination of problem 8, discuss the limitations upon which a true solution to this problem would present a completely different outcome.

Solution: Answers may vary. Possible limitations could be that there are no viable roads to the new location, the fictional distances may be concurrent, but there could always be real obstacles which could prevent a timely delivery of goods from the warehouse, such as restricted right of ways across properties, no bridge access along the route, hilly terrain to slow down speed, zoning laws preventing commercial construction in the area, etc.

10. The diagram below shows Melanie's new kitchen. The triangle represents the paths used to move from the sink, range, and refrigerator. Melanie wants to put an island in the center of these paths. Which point of concurrency should she use to calculate the exact center of the triangle? Explain your reasoning and provide an example to support your conclusion.

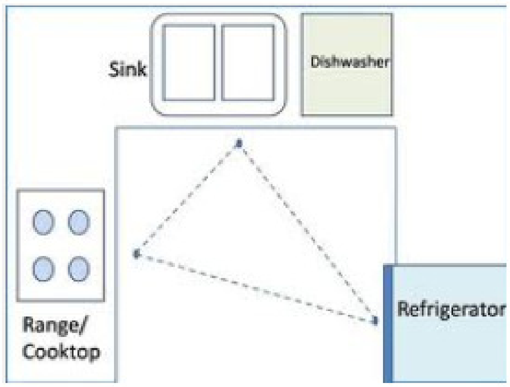


Image 16

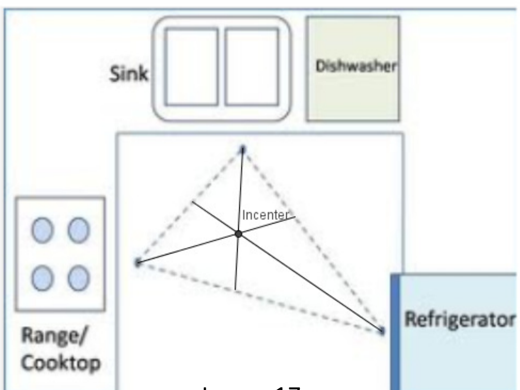


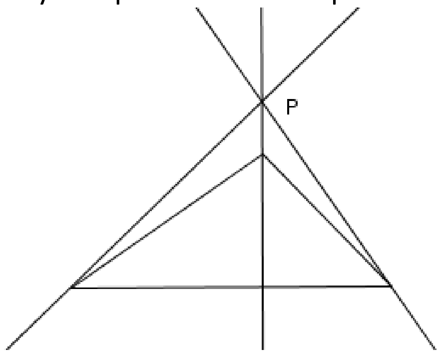
Image 17

Solution: Melanie should use the incenter as the point of concurrency to use for her calculations. The incenter is the center of a circle that touches each side of a triangle. The point at which this occurs is a perpendicular distance from the incenter to each side of the triangle. Since perpendicular distance is the shortest distance to a point, the island should be placed at the incenter to be the most efficient along the paths of the work triangle.

11. We have constructed and solved problems requiring the circumcenter and incenter of triangles. Name the other two points of concurrency and discuss their properties.

Solution: Two other points of concurrency are the centroid and the orthocenter.
The **centroid** is the point of concurrency where the medians of a triangle meet. A **median** of a triangle is a line segment joining a vertex to the midpoint of the opposite side. The centroid is the center of gravity of the triangle, i.e. it is the point at which the triangle is perfectly balanced. The centroid is always inside the triangle.
The **orthocenter** is the point of concurrency of the altitudes of the triangle. If the triangle is acute the orthocenter is inside the triangle, if the triangle is obtuse the orthocenter is outside the triangle, if the triangle is a right triangle, the orthocenter lies on the right angle.

12. The diagram below shows a triangle with a point of concurrency P. Describe the point of concurrency and provide an example of why it could be useful in a real world situation.



Solution: Point P is the orthocenter of the triangle. The orthocenter could be useful in surveillance. A camera could be positioned at point P so that optimal viewing of the triangular area could be attained.

Practice Lesson 6 Introduction to Angle Relationships

Objectives
- Describe and apply the properties of angles and angle pairs
- Find the measure of unknown angles
- Prove theorems about angles

New Vocabulary:	complementary angles	supplementary angles
	linear pair	straight angle
	adjacent angles	vertical angles
	Angle Addition Postulate	

Introduction

In this lesson, students will explore special pairs of angles and the theorems and proofs involving the relationships of these angles. Angle pairs are of particular importance in the study of geometry and students will find that pairs of angles are related in special ways. Students will also use their algebra skills when working to solve unknown angle problems, so students should be proficient in solving algebraic problems involving one variable. Students will begin to state reasons for their solutions to a problem. This will be their introduction to formal proof. In order to find the measure of unknown angles, definitions of the vocabulary in this lesson must be established.

The definitions are as follows:

a. **Adjacent angles** are two angles that have the same vertex, share a common side and have no interior points in common.
b. If two angles are adjacent, the two sides that are not shared are called the **exterior sides** of the adjacent angles.
c. Two angles are **supplementary** if the sum of their measures is 180°.
d. Two angles are **supplementary** if the exterior sides of a pair of adjacent angles form a straight line.
e. Two angles are **complementary** if the sum of their measures is 90°.
f. The angle measure of a straight line is 180°. A **straight angle** is a straight line, thus its measure is 180°.
g. A **linear pair** of angles are two adjacent angles whose non-common sides form opposite rays. (Opposite rays form a straight line therefore, the measure of the angles of a linear pair sum to 180°).
h. **Vertical angles** are formed by two intersecting lines. Vertical angles share the same vertex but do not share a common side. (Vertical angles are on opposite sides of the vertex). Vertical angles are congruent.
i. The **sum of the three angle measures of any triangle** is 180°.
j. The **sum of the measures of all adjacent angles** formed by three or more rays with the same vertex is 360° (angles around a point)

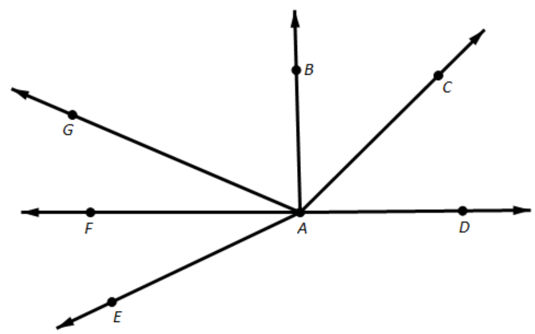
Practice

1. What is the difference between a linear pair and supplementary angles?

   Solution: a linear pair are two angles that are adjacent (share a common ray) and supplementary (the sum of their measures equal 180°), but supplementary angles are any two angles whose measures sum to 180° and not necessarily adjacent.

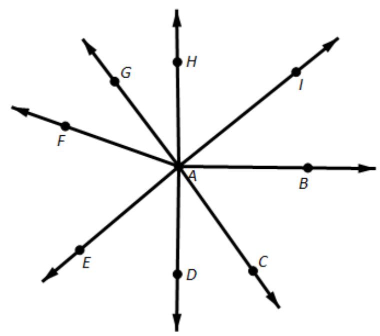
2. Use the diagram at right to answer a-d. ∠BAD is a right angle. ∠BAF is a right angle.

   a. Name all linear pairs.
   b. Name a pair of complementary angles
   c. Name a pair of adjacent angles
   d. Name an angle supplementary to ∠DAC



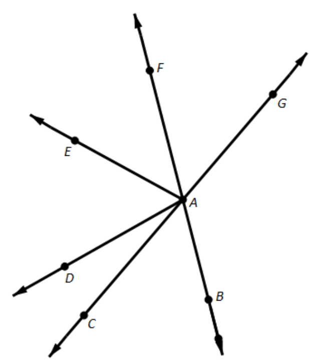
   Solution:
   a. ∠FAG & ∠GAD; ∠FAB & ∠BAD; ∠FAC & ∠CAD
   b. ∠FAG & ∠GAB; ∠BAC & ∠CAD
   c. ∠EAF & ∠FAG; ∠FAG & ∠GAB; ∠GAB & BAC ∠BAC & CAD
   d. ∠CAF

3. Refer to the diagram at right.
   ∠EAI ⊥ ∠GAC. ∠HAD,∠EAI, & ∠GAC are straight angles
   Describe how ∠HAI and ∠EAG are supplementary but not a linear pair.



   Solution: a linear pair consists of two adjacent angles that are also supplementary. Since ∠HAI & ∠EAG are not adjacent, they are not a linear pair. However, ∠HAI & ∠IAD are both supplementary and a linear pair. Since ∠IAD and ∠GAD

4. The **Angle Addition Postulate** states that if $\overrightarrow{AOB}$ is on the interior of ∠AOC, then m∠AOB + m∠BOC = m∠AOC.
   Use the Angle Addition Postulate to find the following angles in the diagram at right. ∠FAB & ∠GAC are straight angles.

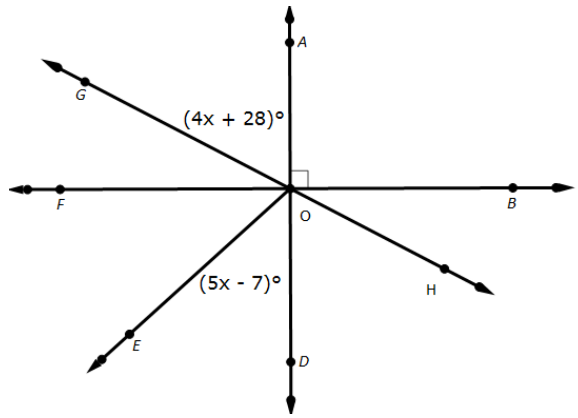


   a. If m∠EAG = 102°, what is m∠EAC?
   b. If m∠DAC = 20°, what is m∠EAD?
   c. If m∠FAE = 47°, what is m∠BAC?

   Solution:
   a. It is given that m∠GAC = 180°, and m∠EAG = 102°, therefore, m∠EAC = 78° since ∠EAG & ∠EAC are supplementary.

b. m∠EAD = m∠EAC - m∠DAC. Therefore, m∠EAD = 58°, since 78 - 20 = 58.
c. m∠GAF = m∠GAE - m∠FAE; so m∠GAF = 55°, since 102 - 47 = 55. Since ∠GAF and ∠BAC are vertical angles (which are congruent), then m∠BAC = 55°.

5. Use the Angle Addition Postulate to find the measure of all the angles in the diagram shown below. Given: ∠BOF & ∠AOD are straight angles; m∠AOB = 90°.

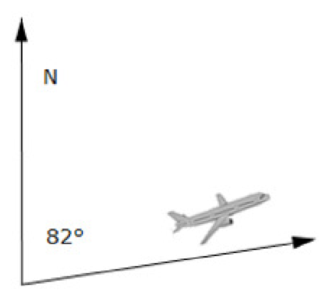


Solution:
m∠AOB = 90° (Given)
m∠AOF = 90° (∠AOB & ∠AOF are supplementary angles)
m∠AOG = 68° (using the Angle Addition Postulate, m∠AOG + m∠GOF = 90° ; therefore solving for x we find that m∠AOG = 68°).
Similarly, m∠GOF = 22° (90 - 68).
m∠HOD = 68° since it is vertical to ∠AOG.
m∠DOE = 43°(by substituting the value found for x)
m∠EOF = 47° since ∠DOE and ∠EOF are complementary angles.

6. Upon approaching JFK International airport, the pilot of a passenger aircraft has been instructed to land on a runway that is 90° east of due north. The flight path of the airplane is 82° SE of due north (see the diagram shown below).

   a. How many degrees must the pilot turn (and in which direction) to land on the specified runway?

   b. Suppose the runway is 90° west of due north. How many degrees must the pilot turn the plane to land on this runway?



Solution:
a. The pilot needs to land on a runway that is 90° from due north. Therefore, the pilot needs to turn SE by 90 - 82, which is 8° to land on the specified runway.
b. The pilot must make the 8° correction plus a complete turn of 180° clockwise to turn around. The pilot therefore, must turn SW by 188°.

Practice Lesson 7 Parallel Lines

Objectives

- Identify the angle relationships formed by parallel lines and a transversal
- Name angles formed by parallel lines and a transversal
- Use the special relationships formed by parallel lines cut by a transversal to find unknown angle measures

New Vocabulary: parallel, transversal, corresponding angles, alternate interior angles, alternate exterior angles

Introduction

Lines in a plane are either parallel or they intersect. Two lines are **parallel** if they lie in the same plane and do not intersect. When two parallel lines are intersected by a third line (called a **transversal**), then special relationships are formed by the eight angles created by the intersection. In this lesson students will identify, name and use these special relationships to prove the lines intersected by a transversal are parallel as well as proving angles in these relationships are congruent or supplementary.

Definition of terms:

a. **corresponding angles** are angles that that occupy the same relative position at each intersection where a straight line crosses two others.

b. when two lines are crossed by another line (a **transversal**), the pairs of angles on opposite sides of the transversal but inside the two lines are called **alternate interior angles**.

c. when two lines are crossed by another line (a transversal), the pairs of angles on opposite sides of the transversal and outside (on the exterior of) the two lines are called **alternate exterior angles.**

d. **vertical angles** are a pair of non-adjacent angles formed by the intersection of two straight lines. Two angles are **vertical angles** if their sides form two pairs of opposite rays. (**Vertical Angles Theorem**: vertical angles are congruent).

Properties of angles created by a transversal crossing parallel lines are:

e. If two parallel lines are cut by a transversal, then any pair of angles are either **congruent** or **supplementary**.

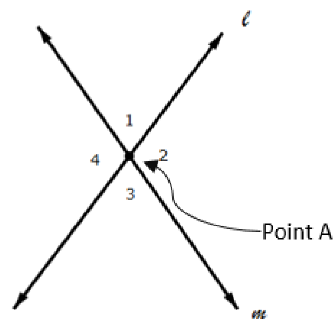
f. If two lines are parallel, and are cut by a transversal, then their **corresponding angles** are congruent. (**Corresponding Angles Theorem**: If two lines are cut by a transversal and the two lines are parallel, then their corresponding angles are congruent).

g. If two lines are parallel, then interior angles on the same side of the transversal are supplementary (called **same side interior angles** also called **consecutive interior angles. Consecutive Interior Angles Theorem:** If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary).

h. If two parallel lines are cut by a transversal, then each pair of **alternate interior angles** is congruent. (**Alternate Interior Angles Theorem:** If two lines are cut by a transversal and the two lines are parallel, then the alternate interior angles are congruent).
i. If two parallel lines are cut by a transversal, then each pair of **alternate exterior angles** is congruent. (**Alternate Exterior Angles Theorem**: If two lines are cut by a transversal and the two lines are parallel, then the alternate exterior angles are congruent).

Practice

1. Lines ℓ and m intersect at Point A. Present a formal argument proving vertical angles are congruent. Use the diagram below and the definitions in Practice Lesson 6, if necessary.

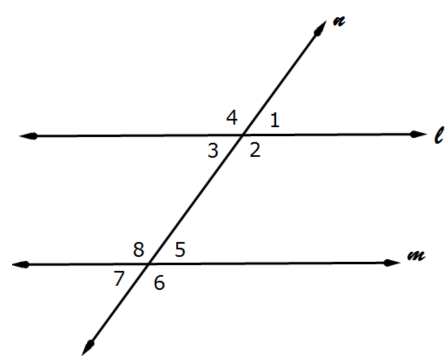


Solution: Since the lines intersect at one point, Point A, then the lines form adjacent pairs of angles (i.e.: $\angle 1$ is adjacent to $\angle 4$, $\angle 3$ is adjacent to $\angle 4$). If the exterior sides of a pair of adjacent angles form a straight line, then the angles are supplementary. Therefore, $\angle 1$ is supplementary to $\angle 4$ and $\angle 3$ is supplementary to $\angle 4$. If two angles are supplementary to the same angle, then they are congruent. Therefore, $\angle 1$ and $\angle 3$, which are vertical angles, are congruent because they are supplementary to the same angle.

2. Line ℓ and line m are parallel. Line n is a transversal that intersects line ℓ and line m.

For a - d, identify the angles in in the diagram at right.

a. all corresponding angles
b. all vertical angles
c. all alternate interior angles
d. all alternate exterior angles



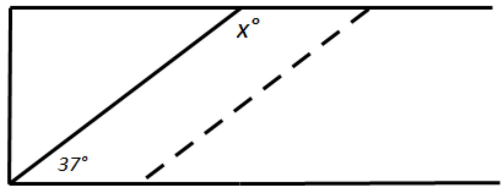
Solution:

a. corresponding angles: $\angle 1$ & $\angle 5$, $\angle 2$ & $\angle 6$, $\angle 4$ & $\angle 8$, $\angle 3$ & $\angle 7$
b. vertical angles: $\angle 1$ & $\angle 3$, $\angle 4$ & $\angle 2$, $\angle 5$ & $\angle 7$, $\angle 8$ & $\angle 6$
c. alternate interior angles: $\angle 2$ & $\angle 8$, $\angle 3$ & $\angle 5$
d. alternate exterior angles: $\angle 4$ & $\angle 6$, $\angle 1$ & $\angle 7$

3. If $m\angle 1$ is $54°$ and $m\angle 6$ is $126°$, what can you say about all of the acute angles from the problem above (#2). Explain your reasoning.

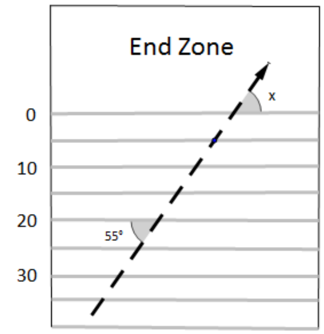
Solution: All of the acute angles measure $54°$. All of the acute angles are vertical angles. $\angle 1$ and $\angle 3$ are vertical angles so $\angle 1 \cong \angle 3$ therefore, $m\angle 3 = 54°$.
Also, if $m\angle 6 = 126°$, then $m\angle 5 = 54°$ since $\angle 6$ and $\angle 5$ form a linear pair (180 - 126 = 54).
Since $\angle 5$ and $\angle 7$ are vertical angles and vertical angles are congruent, then $m\angle 7 = 54°$.

4. Refer to the diagram below. A landscape architect is designing a stone walkway. One triangular shaped stone has been put into the design already. He wants the next stone to be a diagonal strip. The stone needs to be cut with a 37° angle on one end and an x° angle on the other end. What should the measure of angle x be in order to fit into the design?



Solution: The 37° angle and angle x are same side interior angles (they are on the same side of the transversal -- the hypotenuse of the triangle), therefore they are supplementary (180 - 37 = 143). The stone needs to be cut at a 143° angle.

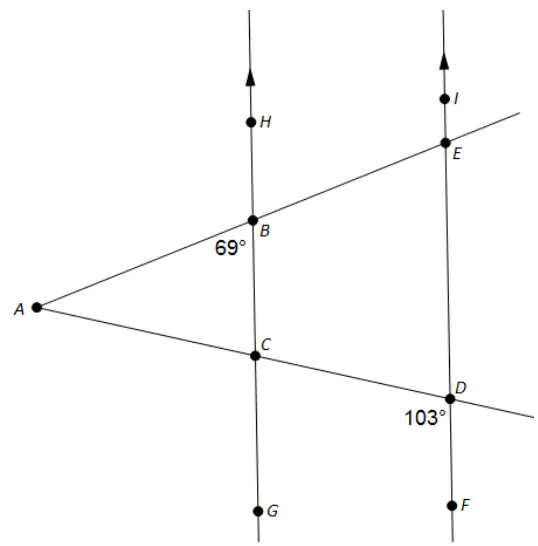
5. In the diagram at right, the dotted line indicates the path of a football player crossing the field toward the end zone. The path makes a 55° angle with the 20-yard line. What is the measure of angle *x* where the player crosses into the end zone? Support your reasoning with definitions, postulates, and/or theorems.



Solution: The path that crosses the parallel lines of the football field is a transversal. Angle x is an alternate interior angle to the angle that is created with the transversal and the 20-yard line. By the Alternate Interior Angles Theorem (7.5), alternate interior angles are congruent, so the m∠x = 55°.

6. Refer to the diagram at right. $\overleftrightarrow{BC} \parallel \overleftrightarrow{ED}$; m∠ABC = 69° and m∠CDF = 103°. Using the properties of angles created by a transversal crossing parallel lines, find the following angle measures. Support your reasoning with definitions and properties from Practice Lesson 6 and Practice Lesson 7.

a. m∠BCD
b. m∠BCA
c. m∠EDC
d. m∠EBC
e. m∠ACG

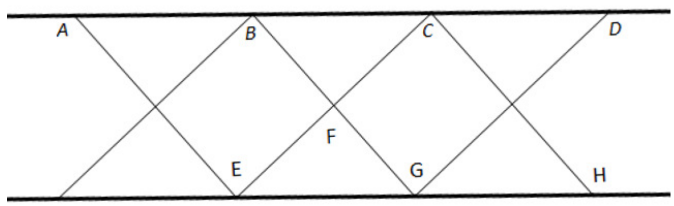


Solution: Since $\overleftrightarrow{BC} \parallel \overleftrightarrow{ED}$, and $\overline{AE}$ is a transversal and $\overline{AD}$ is a transversal to parallel lines $\overleftrightarrow{BC}$ and $\overline{ED}$, then by the properties of parallel lines cut by a transversal:

a. m∠BCD = 103°; ∠CDF ≅ ∠DCB; Alternate angles are congruent

b. m$\angle$BCA = 77°; $\angle$BCD and $\angle$BCA are a linear pair, so 180 - 103 = 77
c. m$\angle$EDC = 77°; $\angle$FDC and $\angle$EDC are a linear pair, so 180 - 103 = 77
d. m$\angle$EBC = 111°; $\angle$ABC and $\angle$EBC are a linear pair, so 180 - 69 = 111
e. m$\angle$ACG = 103°; $\angle$CDF and $\angle$ACG are corresponding angles, therefore congruent

7. The support beams on a bridge intersect in the pattern shown below. $\overline{AD} \parallel \overline{EH}$. If $\overline{BG}$ and $\overline{EC}$ intersect at point *F*, and m$\angle$BFC = 3x + 30 and m $\angle$EFG = 7x - 10, find the value of *x*.



Solution: Since intersecting lines create vertical angles and vertical angles are congruent, then $\angle$BFC $\cong$ $\angle$EFG. Set the equations equal to each other and solve for x:

$$3x + 30 = 7x - 10$$
$$40 = 4x$$
$$x = 10$$

Practice Lesson 8 Angles in Triangles

Objectives

- Use the properties of triangles to find unknown angles
- Use the properties of parallel lines to prove the Triangle Sum Theorem
- Use the Isosceles Triangle Theorem to find unknown angles

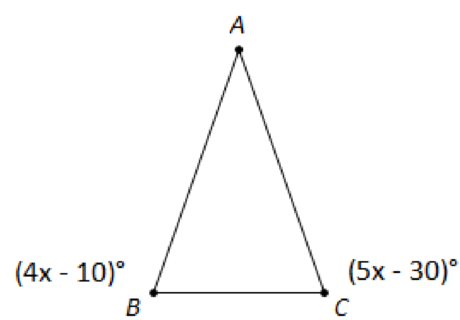
New Vocabulary: Isosceles triangle, Triangle Sum Theorem, converse, exterior angle of a triangle, Isosceles Triangle Theorem

Introduction

In this lesson, students will expand their knowledge about lines and angles to the study of triangles. Students learn the **Triangle Sum Theorem** in middle school (the sum of the measures of the angles in a triangle equal 180°). Now they will look at angle-sum relationships in a triangle and use these relationships to find unknown angles inside and outside a triangle. Students begin with a look at isosceles triangles and their properties and extend this knowledge to more complex configurations involving angles and triangles. The **converse** of a theorem will be introduced (the "if" part and the "then" part of the theorem are exchanged). Students will use the **Isosceles Triangle Theorem** (if two sides of a triangle are congruent, then the angles opposite those sides are congruent) and its **converse** (if two angles of a triangle are congruent, then the sides opposite those angles are congruent) to find unknown angle measures. To find unknown angles, students will also use the **Exterior Angle Theorem** which states, the measure of an exterior angle of a triangle is equal to the sum of the measures of the two non-adjacent interior angles. Students will also put their algebra skills to use in this lesson.

Practice

1. ΔABC is an isosceles triangle. $m\angle ABC = (4x - 10)°$ and $m\angle ACB = (5x - 30)$ Use the properties of isosceles triangles to find the measures of each angle of ΔABC.

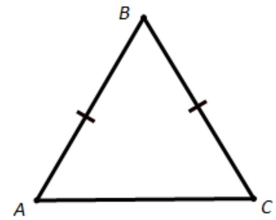


Solution: Since ΔABC is isosceles, then the $m\angle ABC \cong m\angle ACB$. Solving for x,

$$4x - 10 = 5x - 30$$
$$20 = x$$

$m\angle ABC = 70°$, $m\angle ACB = 70°$, $m\angle BAC = 40°$

2. ΔABC is an isosceles triangle with $\overline{AC}$ as the base. If $\overline{AB} = 18x$ inches, $\overline{CB} = 15x+3$ inches, find the length of the base.

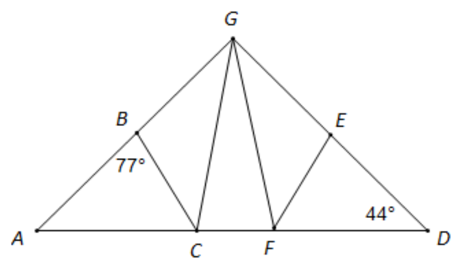


Solution: Since ΔABC is isosceles then, $\overline{AB} \cong \overline{CB}$ therefore,

$$18x = 15x + 3$$
$$3x = 3$$
$$x = 1$$

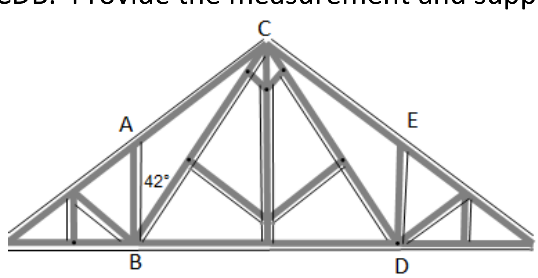
Since 18 x 1 = 18, the congruent legs of the isosceles triangle measure 18 inches.

3. ΔABC $\cong$ ΔEDF. Find the measure of $\angle$GCF. State your reasoning in precise language and support your conclusions with known facts and/or theorems



Solution: Since ΔABC $\cong$ ΔEDF, then $\angle$BAC $\cong$ $\angle$EDF. Therefore, m$\angle$BAC = 44°. By the Triangle Sum Theorem, m$\angle$BCA = 59°. Since $\angle$GCF is an exterior angle of ΔABC, then by the Exterior Angle Theorem, m$\angle$GCF = 77° + 44° = 121°.

4. One part of the support structure of the Alexandria River Bridge is shown below. The structure is composed of varying sized triangles and polygons. The following information is given: AB $\perp$ BD, ED $\perp$ BD, ΔABC $\cong$ ΔEDC, and m$\angle$ABC is 42°. Determine the measurement of $\angle$CDB. Provide the measurement and support your reasoning.



Solution: The exterior angle of ΔEDC is $\angle$EDB. Since the supports AB and ED are perpendicular to the same line, BD, and ΔABC $\cong$ ΔEDC, then $\angle$ABD must be congruent to $\angle$EDB.

Since m$\angle$ABC = 42°, then m$\angle$EDC = 42°. Since ED $\perp$ BD, then m$\angle$EDB = 90°. $\angle$CDB and $\angle$EDC are complementary, therefore m$\angle$CDB is 90- 42 = 48. m$\angle$CDB is 48°.

Practice Lesson 9 Writing Proofs Involving Angle Measures

Objectives

- use deductive reasoning to prove angle relationships

New Vocabulary: proof assumption
deductive reasoning

Introduction

Students develop their reasoning skills by making assumptions then proving the statements based on known facts. An **assumption** is that which is accepted as true without proof. A **proof** is a mathematical argument whose aim is to reach a conclusion that is true. Students will develop a logical structure in which angle relationships are proved as well as applied. When writing a proof involving angle measures, students must support their reasoning for each step [of the problem solving process] with a known fact or theorem. This is known as **deductive reasoning**. Students will be asked to write formal proofs using the two column method where statements are listed in the left hand column of a table and the reasons for supporting each statement are listed in the right hand column of the table. The two column table is shown below:

Statement	Reasons

Students will need the properties listed in Practice Lesson 7 in addition to the properties listed below to prove angle relationships.

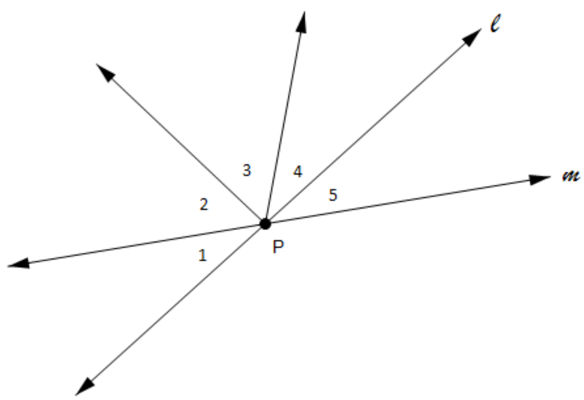
Reflexive Property of Equality a = a.
Symmetric Property of Equality if $a = b$, then $b = a$.
Symmetric Property of Congruency if $\angle a \cong \angle b$, then $\angle b \cong \angle a$.
Transitive Property of Equality if $a = b$ and $b = c$, then $a = c$.
Transitive Property of Congruency if $\angle a \cong \angle b$ *and* $\angle b \cong \angle c$, then $\angle a \cong \angle c$.
Addition Property of Equality if a = b, then a + c = b + c.
Subtraction Property of Equality if a = b, then a - c = b - c.
Substitution Property of Equality if a = b then a can be substituted for b. This means that if a = b then we can change any b to a or any a to b.
Angle Bisector if a ray divides an angle into two congruent parts (angles), then the angle is said to be bisected
Right Angles Theorem all right angles are congruent
Supplementary Angles Theorem If two angles are supplementary to the same angle (or to congruent angles), then the angles are congruent
Complementary Angles Theorem If two angles are complementary to the same angle (or to congruent angles) then these angles are congruent

The following abbreviations are permitted in the proofs of this section:

vert. ∠'s	vertical angles	corr. ∠'s	corresponding angles
alt. int. ∠'s	alternate interior angles	alt. ext. ∠'s	alternate exterior angles
supp. ∠'s	supplementary angles	comp. ∠'s	complementary angles
||	parallel to (or parallel lines)	Def.	Definition
∠'s on a line	angles on one side of a straight line add to 180°		

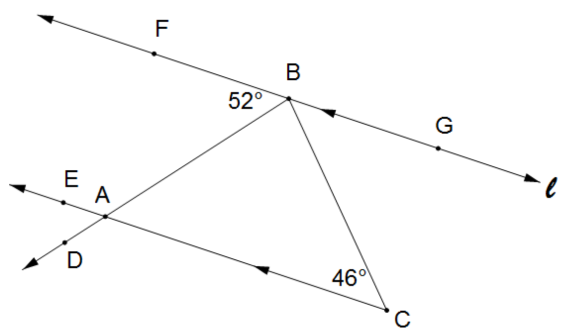
Practice

1. In the diagram shown below, lines ℓ and m intersect at point P. ∠3 and ∠5 are complementary. Justify the statement that ∠1 and ∠3 are complementary.



Solution: Since ∠1 and ∠5 are formed by intersecting lines, they are vertical angles and therefore congruent. Complementary angles are two angles whose measures sum to 90°. Since ∠3 and ∠5 are complementary, then m∠3 + m∠5 = 90°. Since ∠5 ≅ ∠1, then by substitution, m∠3 + m∠1 = 90°. Therefore, ∠1 and ∠3 are complementary.

2. It is given that line ℓ is parallel to $\overrightarrow{AC}$, m∠FBA = 52°, and m∠BCA = 46°. Show that m∠EAB is the sum of m∠ABC and m∠BCA.

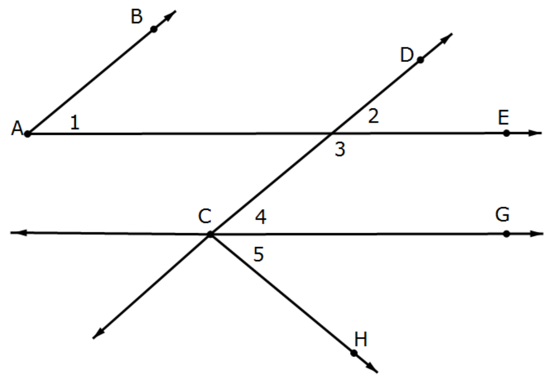


Solution: Since line ℓ is parallel to $\overrightarrow{AC}$, then ∠BAC & ∠FBA are alternate interior angles. By the Alternate Interior Angles Theorem, m∠BAC = 52° since alternate interior angles are congruent. The sum of the angles of any triangle is 180°, therefore, m∠ABC = 82° (180 - 52 -46 = 82). ∠EAB & ∠BAC form a linear pair so m∠EAB =128° (180 - 52 = 128). Since 82 + 46 = 128, then m∠EAB is the sum of m∠ABC and m∠BCA.

3. Write a two column proof for the following:

Given: $\overline{AB} \parallel \overline{CD}$, $\overline{AE} \parallel \overline{CG}$, $\angle 1 \cong \angle 5$

Prove: $\overline{CG}$ bisects $\angle$DCH



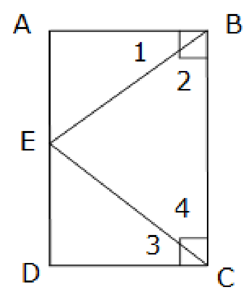
Solution:

Statements	Reasons
1. $\overline{AB} \parallel \overline{CD}$, $\overline{AE} \parallel \overline{CG}$, $\angle 1 \cong \angle 5$	1. Given
2. $\angle 1 \cong \angle 2$	2. Corr. $\angle$'s
3. $\angle 2 \cong \angle 4$	3. Corr. $\angle$'s
4. $\angle 1 \cong \angle 4$	4. Transitive Property
5. $\angle 4 \cong \angle 1$	5. Symmetric Property
6. $\angle 4 \cong \angle 5$	6. Transitive Property
7. $\overline{CG}$ bisects $\angle$DCH	7. Angle Bisector

4. Write a two column proof for the following:

Given: $m\angle 2 = m\angle 4$
$m\angle ABC = 90°$, $m\angle DCB = 90°$

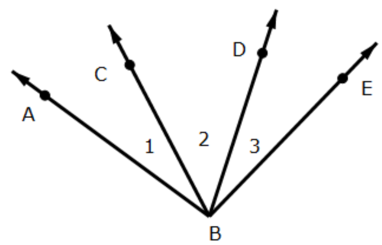
Prove: $m\angle 1 = m\angle 3$



Solution:

Statements	Reasons
1. $m\angle ABC = 90°$ $m\angle DCB = 90°$	1. Given
2. $m\angle 2 = m\angle 4$	2. Given
3. $m\angle 1 + m\angle 2 = m\angle ABC$ $m\angle 3 + m\angle 4 = m\angle DCB$	3. Angle Addition Postulate
4. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	4. Substitution Property
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	5. Substitution Property
6. $m\angle 2 = m\angle 2$	6. Reflexive Property
7. $m\angle 1 = m\angle 3$	7. Subtraction Property

5. Write a two column proof for the following:

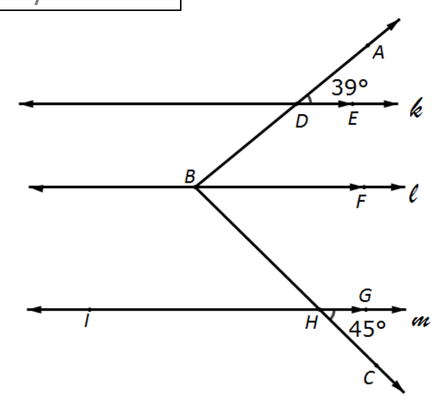


Given: ∠ABD ≅ ∠CBE
Prove: ∠1 ≅ ∠3

Solution:

Statements	Reasons
1. ∠ABD ≅ ∠CBE	1. Given
2. m∠1 + m∠2 = m∠ABD m∠3 + m∠2 = m∠CBE	2. Angle Addition Postulate
3. m∠1 + m∠2 = m∠2 + m∠3	3. Substitution Property
4. m∠2 = m∠2	4. Reflexive Property
5. m∠1 = m∠3	5. Subtraction Property

6. Write a two column proof for the following:

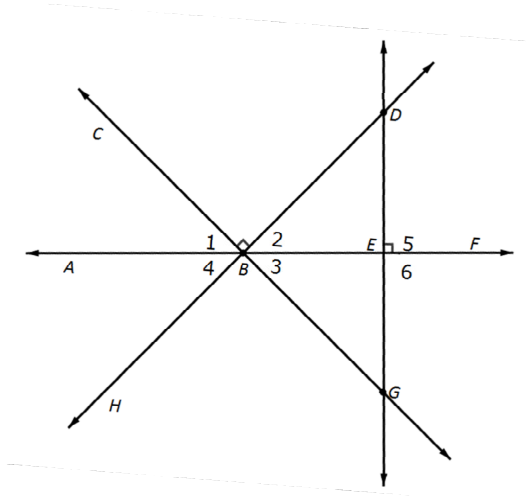


Given: $Line\ k \parallel Line\ l \parallel Line\ m$
m∠ADE = 39°
m∠GHC = 45°
Prove: m∠ABC = 84°

Solution:

Statements	Reasons
1. $Line\ k \parallel Linel \parallel Line\ m$	1. Given
2. m∠ADE = 39°	2. Given
3. ∠ADE ≅ ∠ABF	3. Corr. ∠'s
4. m∠ADE = m∠ABF	4. Def. of congruent ∠'s
5. m∠ABF = 39°	5. Substitution
6. m∠GHC = 45°	6. Given
7. ∠GHC ≅ ∠BHI	7. Vert. ∠'s
8. m∠GHC = m∠BHI	8. Def. of congruent ∠'s
9. m∠BHI = 45°	9. Substitution
10. ∠BHI ≅ ∠FBH	10. Alt. Int. ∠'s
11. m∠BHI = m∠FBH	11. Def. of congruent ∠'s
12. m∠FBH = 45°	12. Substitution
13. m∠ABC = m∠ABF + m∠FBH	13. Angle Addition Postulate
14. m∠ABC = 39° + 45°	14. Substitution
15. m∠ABC = 84°	15. Simplify

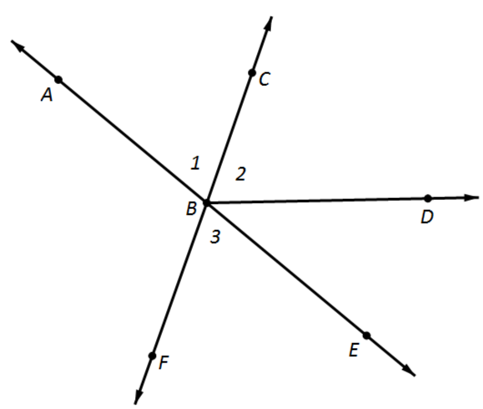
7. Write a two column proof for the following:
   Given: $\overline{AE}$ bisects ∠CBH and ∠DBG
   m∠CBD = 90°
   m∠DEF = 90°
   Prove: ∠DBG and ∠DEB are supplementary angles



Solution:

Statements	Reasons
1. $\overline{AE}$ bisects ∠CBH and ∠DBG	1. Given
2. ∠CBH = ∠1 + ∠4; ∠DBG = ∠2 + ∠3	2. Angle Bisector Theorem
3. ∠1 ≅ ∠4; ∠2 ≅ ∠3	3. Def. Angle Bisector
4. ∠CBD + ∠2 + ∠3 = 180°	4. ∠'s on a line
5. m∠CBD = 90°	5. Given
6. 90° + m∠2 + m∠3 = 180°	6. Substitution
7. m∠2 + m∠3 = 90°	7. Subtraction Property
8. m∠DBG = 90°	8. Substitution
9. m∠DEB + m∠DEF = 180°	9. ∠'s on a line
10. m∠DEF = 90°	10. Given
11. m∠DEB + 90° = 180°	11. Substitution
12. m∠DEB = 90°	12. Subtraction Property
13. m∠DEB + m∠DBG = 180°	13. Addition Property
14. ∠DEB and ∠DBG are supplementary angles	14. Def. of supplementary angles

8. $\overrightarrow{BC}$ bisects ∠ABD. Show that ∠2 ≅ ∠3.



Solution:

$\overrightarrow{BC}$ bisects ∠ABD	Given
∠1 ≅ ∠2	Def. of angle bisector
∠1 ≅ ∠3	Vert. ∠'s
∠3 ≅ ∠2	Transitive Property of Congruence
∠2 ≅ ∠3	Symmetric Property of Congruence

Practice Lesson 10 Proof by Construction

Objectives

- Use construction to prove angle relationships
- Construct auxiliary lines on diagrams given in proofs

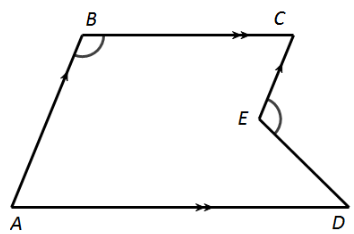
New Vocabulary: auxiliary line

Introduction

Students will sometimes find that a given diagram for a proof is insufficient or lacks a necessary element in order to reach a conclusion for the proof. The **auxiliary line** is a construction the student may add to the given diagram to help demonstrate the relationship between steps in a proof and the reason for the steps. In this lesson, angle relationships are proven with the aid of this construction. It is noted that students may find it useful to add more than one auxiliary line to aid in the process of the given proof. Also noted, the student should never alter or delete the original elements of the diagram.

Practice

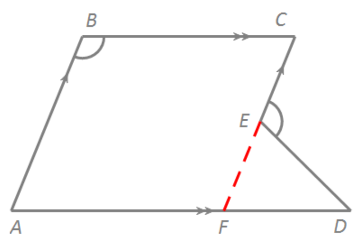
1. $\overline{AB} \parallel \overline{EC}$ and $\overline{AD} \parallel \overline{BC}$.
$\angle$ABC ≅ $\angle$CED.



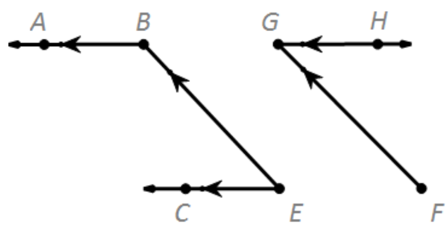
Prove $\angle$CED & $\angle$DAB are supplementary angles

Solution:
Extend $\overline{CE}$ by drawing auxiliary line $\overline{EF}$.

$\angle$ABC ≅ $\angle$EFA	alt. int. $\angle$'s
$\angle$CED is adjacent to $\angle$DEF	Def. of adjacent angles
$\angle$CED & $\angle$DEF are a linear pair	Def. of linear pair
m$\angle$CED + m$\angle$ DEF = 180°	Def. of linear pair
$\angle$CED & $\angle$ DEF are supp. $\angle$'s	Def. of supplementary angles
$\angle$ABC ≅ $\angle$CED	Given
$\angle$ABC & $\angle$DAB are supp. $\angle$'s	Consecutive Interior Angles Theorem
$\angle$CED & $\angle$DAB are supp. $\angle$'s	Substitution

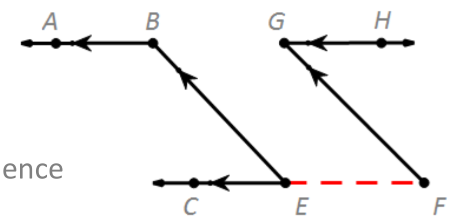


2. $\overline{BE} \parallel \overline{GF}$, $\overline{CE} \parallel \overline{AB}$, $\overline{CE} \parallel \overline{GH}$
Show that ∠CEB ≅ ∠HGF.



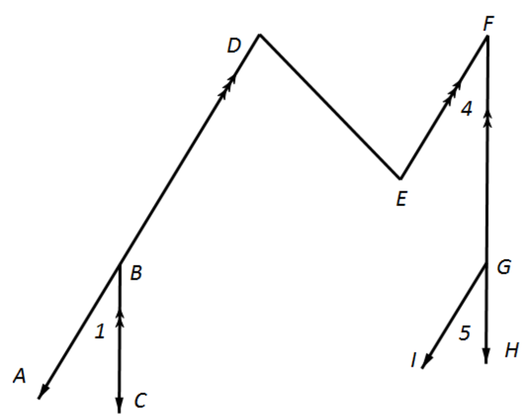
Solution:

Extend $\overline{CE}$ by drawing auxiliary line $\overline{EF}$ such that $\overline{CF}$ is parallel to $\overline{AB}$



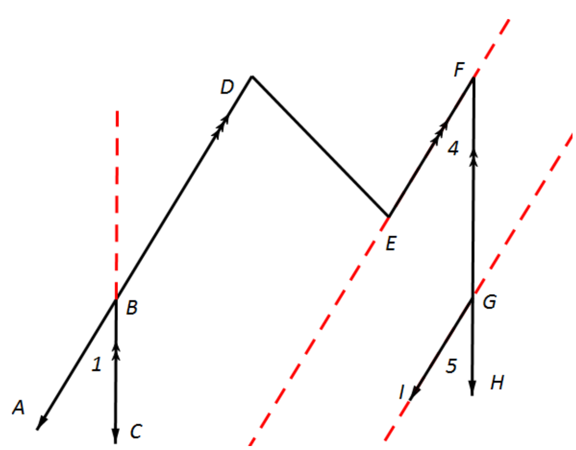
∠HGF ≅ ∠EFG	Alt. int. ∠'s
∠CEB ≅ ∠EFG	Corr. ∠'s
∠CEB ≅ ∠HGF	Transitive Property of Congruence

3. $\overline{AD} \parallel \overline{EF} \parallel \overline{IG}$ and $\overline{BC} \parallel \overline{FH}$.
Prove ∠1 ≅ ∠5



Solution:

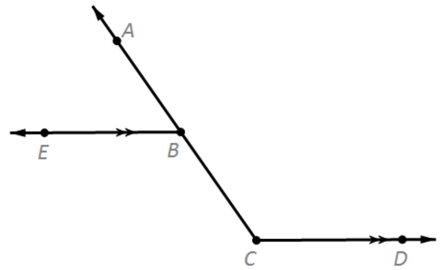
Draw auxiliary lines by extending $\overline{BC}$, $\overline{EF}$, and $\overline{IG}$.



$\overline{AD} \parallel \overline{EF} \parallel \overline{IG}$	Given
$\overline{BC} \parallel \overline{FH}$	Given
∠1 ≅ ∠4	Alt. int. ∠'s
∠4 ≅ ∠5	Corr. ∠'s
∠1 ≅ ∠5	Transitive Property of Congruence

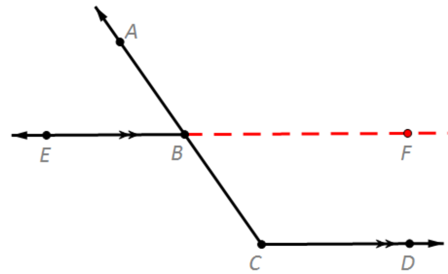
4. $\overline{EB} \parallel \overline{CD}$

Prove EBA is supplementary to DCB.



Solution:

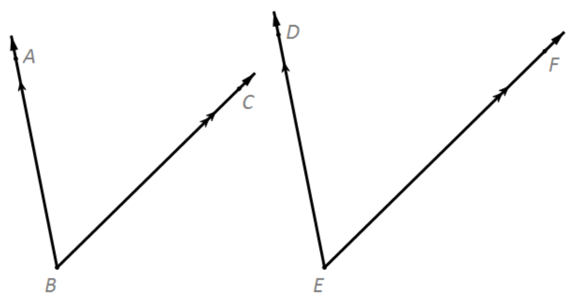
Draw an auxiliary line by extending $\overrightarrow{EB}$ to F.



$\overline{EB} \parallel \overline{CD}$	Given
∠ABF ≅ ∠DCB	Corr. ∠'s
∠EBA is adjacent to ∠ABF	Def. of adj ∠'s
∠EBA & ∠ABF are a linear pair	Def. of linear pair
m∠EBA + m∠ABF = 180°	Def. of linear pair
∠EBA & ∠ABF are supp. ∠'s	Def. of supplementary
∠EBA & ∠DCB are supp. ∠'s	Substitution

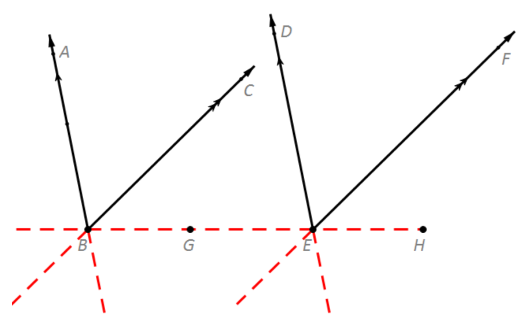
5. $\overline{AB} \parallel \overline{DE}$ and $\overline{CB} \parallel \overline{FE}$.

Prove ∠ABC ≅ ∠DEF.



Solution:

Draw auxiliary lines: extend $\overrightarrow{AB}, \overrightarrow{CB}, \overrightarrow{DE}, and\ \overrightarrow{FE}$ and then draw a transversal through the points B and E. Add points G and H on the transversal.



$\overline{AB} \parallel \overline{DE}$	Given
∠ABG ≅ ∠DEH	Corr. ∠'s
$\overline{CB} \parallel \overline{FE}$	Given
∠CBG ≅ ∠FEH	Corr. ∠'s
m∠ABC + m∠CBG = m∠ABG	Angle Addition Postulate
m∠DEF + m∠FEH = m∠DEH	Angle Addition Postulate
m∠ABC + m∠CBG = m∠DEF + m∠FEH	Substitution
m∠ABC + m∠CBG = m∠DEF + m∠CBG	Substitution
m∠ABC = m∠DEF	Subtraction

Practice Lesson 11 Proofs Involving Known Facts

Objectives

- Prove theorems about angles and triangles using known facts as reasons
- Prove the converse of theorems about angles and triangles using known facts as reasons

New Vocabulary: converse Segment Addition Postulate

The converse of previously stated theorems about parallel lines are:

a. **Converse of Corresponding Angles Theorem**: If two lines are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.
b. **Converse of Consecutive Interior Angles Theorem**: If two lines are cut by a transversal so that consecutive interior angles are supplementary, then the lines are parallel.
c. **Converse of Alternate Interior Angles Theorem**: If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.
d. **Converse of Alternate Exterior Angles Theorem**: If two lines are cut by a transversal so that alternate exterior angles are congruent, then the lines are parallel.

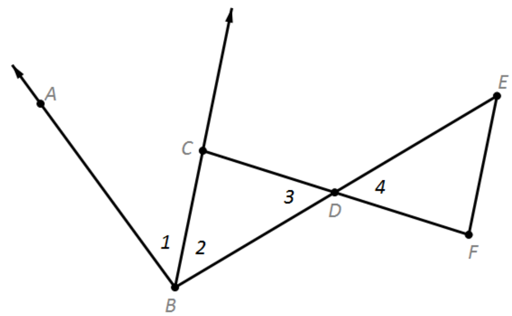
Introduction

Students expand their proof writing abilities with additional practice involving angle relationships and angles in triangles. Students will refer to the terms, properties, definitions and vocabulary learned in previous lessons. Refer to Practice Lessons 6-9 as needed to review these terms.
Students will construct the proofs in this lesson by utilizing definitions, theorems and known facts.
They will apply inductive and deductive reasoning skills to help them construct these proofs.
Students will also use logical reasoning to analyze an argument and to determine whether conclusions are valid.
This lesson will use the **converse** of a theorem to prove angle relationships. The **converse** of a theorem is a theorem obtained by reversing the roles of the premise and conclusion of the initial theorem. For example, the Alternate Interior Angles Theorem states: If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent. The **converse** of this theorem states: If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.
Students will also use a variety of theorems and postulates in this lesson. They have already been introduced to the Angle Addition Postulate, in this lesson the Segment Addition Postulate will be applied. The **Segment Addition Postulate** states that if point B lies on the line segment $\overline{AC}$ and B is between the endpoints A and C then $\overline{AB} + \overline{BC} = \overline{AC}$.

Practice

1. $\overrightarrow{BC}$ bisects ∠ABD.
   m∠2 = m∠3

   Prove m∠1 = m∠4

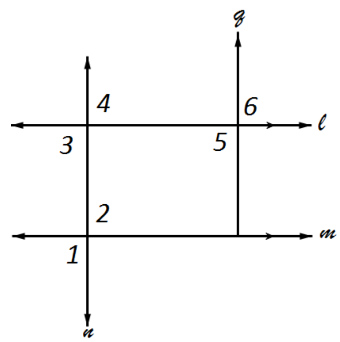


Solution:

$\overrightarrow{BC}$ bisects ∠ABD	Given
m∠2 = m∠3	Given
m∠1 = m∠2	Def. of angle bisector
m∠1 = m∠3	Transitive Property of Equality
∠3 ≅ ∠4	Vert. ∠'s
m∠3 = m∠4	Def. of congruent angles
m∠1 = m∠4	Transitive Property of Equality

2. $l \parallel m$; ∠2 ≅ ∠5

   Prove $n \parallel q$

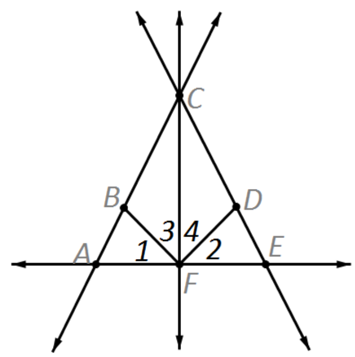


Solution:

∠2 ≅ ∠5	Given
∠3 ≅ ∠2	Alt. int. ∠'s
∠3 ≅ ∠5	Transitive Property
$n \parallel q$	Corr. ∠'s Converse

3. ∠BFD is bisected by $\overline{CF}$.
   m∠CFA = 90°, m∠CFE = 90°. ∠3 ≅ ∠4

   Prove: m∠1 = m∠2

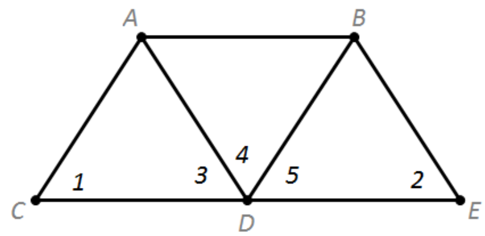


Solution:

∠BFD is bisected by $\overline{CF}$	Given
m∠CFA = 90°, m∠CFE = 90°	Given
∠3 ≅ ∠4	Given
m∠3 = m∠4	Def. of congruent
m∠1 + m∠3 = m∠CFA	Angle Addition Postulate
m∠1 + m∠3 = 90°	Substitution
m∠2 + m∠4 = m∠CFE	Angle Addition Postulate
m∠2 + m∠4 = 90°	Substitution
m∠1 + m∠3 = m∠2 + m∠4	Substitution
m∠1 + m∠3 = m∠2 + m∠3	Substitution
m∠1 = m∠2	Subtraction

4. $\angle CDB \cong \angle EDA$; $\angle 1 \cong \angle 5$

Prove: $\angle 1 \cong \angle 3$



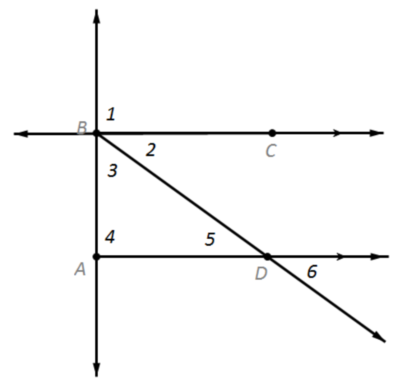
Solution:

$\angle CDB \cong \angle EDA$	Given
$\angle 1 \cong \angle 5$	Given
$\angle 4 \cong \angle 4$	Reflexive Property
$\angle 3 \cong \angle 5$	Vert. $\angle$'s
$\angle 3 \cong \angle 1$	Substitution
$\angle 1 \cong \angle 3$	Symmetric Property

5. Write a two column proof for the following:

$\overline{BC} \parallel \overline{AC}$, $\overline{AB} \perp \overline{BC}$, $\angle 2 \cong \angle 6$

Prove: $m\angle 4 = m\angle 5 + m\angle 3$



Solution:

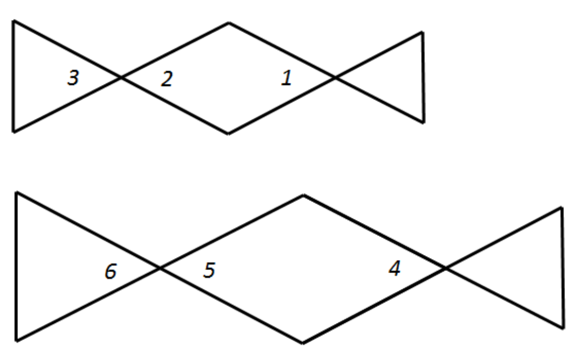
Statements	Reasons
1. $\overline{BC} \parallel \overline{AC}$	1. Given
2. $\angle 2 \cong \angle 6$	2. Given
3. $\angle 6 \cong \angle 5$	3. Vert. $\angle$'s
4. $\angle 2 \cong \angle 5$	4. Transitive Property
5. $\overline{AB} \perp \overline{BC}$	5. Given
6. $\angle 1$ is a right angle	6. Def. of perpendicular
7. $m\angle 1 = 90°$	7. Def. of right angle
8. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	8. $\angle$'s on a line
9. $m\angle 2 + m\angle 3 = 90°$	9. Subtraction

6. $\angle 1 \cong \angle 3$, $\angle 4 \cong \angle 6$, $\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 5$

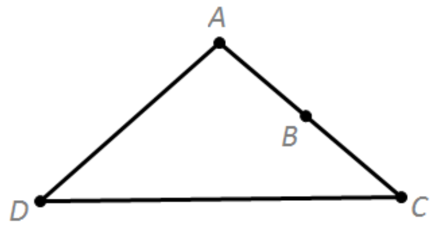
Solution:

$\angle 1 \cong \angle 3$	Given
$\angle 2 \cong \angle 3$	Vert. $\angle$'s
$\angle 2 \cong \angle 1$	Transitive Property
$\angle 4 \cong \angle 6$	Given
$\angle 5 \cong \angle 6$	Vert. $\angle$'s
$\angle 5 \cong \angle 4$	Transitive Property
$\angle 1 \cong \angle 4$	Given
$\angle 2 \cong \angle 5$	Substitution



7. $\overline{AC} \cong \overline{AD}$

Prove: $\overline{AB} + \overline{BC} = \overline{AD}$

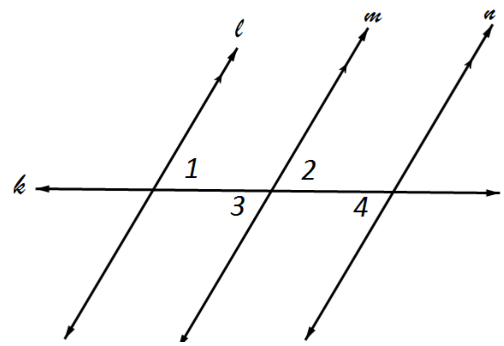


Solution:

$\overline{AC} \cong \overline{AD}$	Given
$\overline{AC} = \overline{AD}$	Def. of congruent segments
$\overline{AB} + \overline{BC} = \overline{AC}$	Segment Addition Postulate
$\overline{AB} + \overline{BC} = \overline{AD}$	Substitution

8. $\ell \parallel m$ and $m \parallel n$

Prove: $\ell \parallel n$

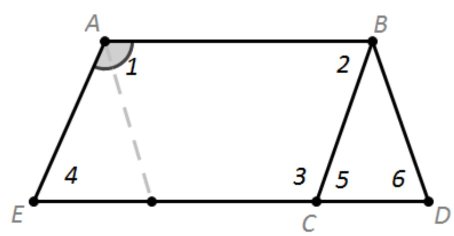


Solution:

line $\ell \parallel m$ and $m \parallel n$	Given
$\angle 1 \cong \angle 3$	Alt. int. $\angle$'s
$\angle 3 \cong \angle 2$	Vert. $\angle$'s
$\angle 1 \cong \angle 2$	Transitive Property
$\angle 2 \cong \angle 4$	Alt. int. $\angle$'s
$\angle 1 \cong \angle 4$	Transitive Property
$\ell \parallel m$	Alt. int. $\angle$'s converse

9. $\angle 2 \cong \angle 5$

Prove: $\angle 1$ and $\angle 4$ are supplementary



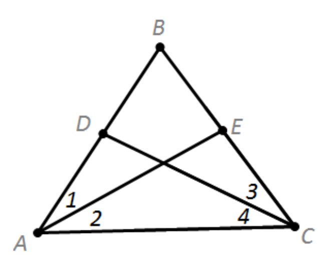
Solution:

$\angle 2 \cong \angle 5$	Given
$\overline{AB} \parallel \overline{EC}$	Alt. int. $\angle$'s converse
$\angle 1$ and $\angle 4$ are supp.	Consecutive Interior Angles Theorem

10. $\overline{AE}$ bisects $\angle$BAC and $\overline{CD}$ bisects $\angle$BCA.

$\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 3$



Solution:

$\overline{AE}$ bisects $\angle$BAC; $\overline{CD}$ bisects $\angle$BCA	Given
$\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$	Angle Bisector Theorem
$\angle 1 \cong \angle 4$	Given
$\angle 1 \cong \angle 3$	Transitive Property
$\angle 2 \cong \angle 3$	Transitive Property

Practice Lesson 12 Rigid Motion Transformations

Objectives
- Identify rigid motion transformations
- Identify angles of rotation
- Construct a line of reflection

New Vocabulary:	Rigid Motion Transformation	image
	isometry	pre-image
	reflection	line of reflection
	rotation	angle of rotation
	center of rotation	distance preserving
	translation	angle preserving
	glide reflection	

Introduction

With this lesson, students shift their focus to isometries. An **isometry** is a transformation that preserves length, angle measures, parallel lines, and distances between points. Transformations that are isometries are called **rigid motion transformations**. **Rigid motion transformations** are functions that map a set of input points from an original image (the **pre-image**) to a set of output points (which is the new location of the **image**) after applying a function rule (such as a reflection, rotation, etc.). Rigid motion transformations do not change the size or shape (or even the location!) of the original image; in other words, rigid motion transformations are **distance preserving** (meaning the distance between the points of the pre-image is equal to the distance between the corresponding points of the image) and **angle preserving** (meaning the angle size is not altered).

Another word on **distance preserving**. This term may confuse some students, so it is best to begin by saying that an object is composed of a set of points. These points may resemble a letter, a triangle or other polygon, or even a tree. Anything at all. Distance preserving in simple terms means that the distance between any two points of the pre-image is equal to the distance between the image of these two points therefore, the size of the shape is not altered under a rigid motion transformation. For example, the length of each side of a triangle remains the same after the triangle has been rotated, reflected, or translated. Students are already familiar with the terms **rotation, reflection, translation,** and **glide reflection**. These four transformations describe the rigid motion function rules. These terms were introduced in the elementary grades and students in 8^{th} grade are well skilled in these transformations in the coordinate plane. In this lesson students will examine transformations of objects and shapes without the benefit of coordinate points.

In this lesson, students will also identify angles of rotation and construct a line of reflection. These rudimentary concepts relating to rotations and reflections will be developed further in later lessons.

Another point to clarify about rigid motion transformations is the fact that it is the plane that contains the figure that is being transformed, not the figure itself. To see this, take a

transparency, piece of tracing paper, or piece of patty paper and draw a figure on it. Then make the reflection, rotation, or translation. You will see that as you move the paper (the plane) the figure on the plane also is reflected, rotated, or translated.

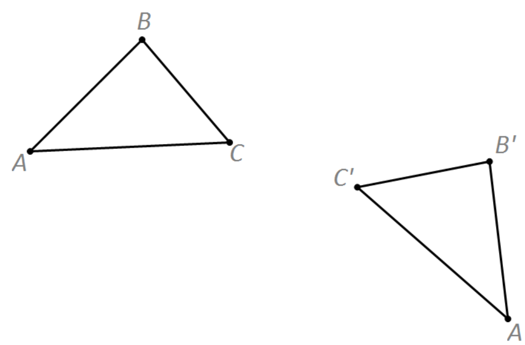
A **reflection** is defined: "if ℓ is a line and if P is a point not on ℓ, then the reflection of P across ℓ is the point P' such that

a) the distance from P' to ℓ is equal to the distance from P to ℓ, and
b) the line joining P to P' is perpendicular to ℓ

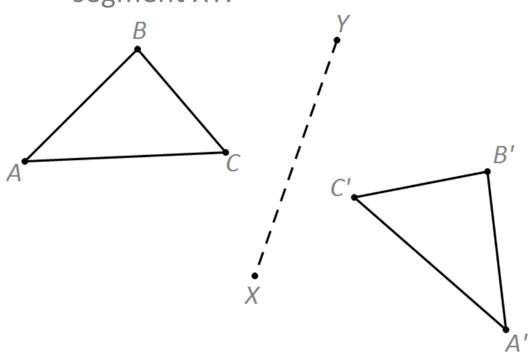
A **rotation** is a rigid transformation that turns all points of the plane about a fixed point called the **center of rotation**. The **angle of rotation** is the number of degrees through which points rotate around the center of rotation. Rotations can turn in either a clockwise direction, or counterclockwise direction.
A **translation** is a movement of the plane in which a figure is translated a specific direction for a specific distance.
A **glide reflection** is a composition of a reflection and a translation.

Practice

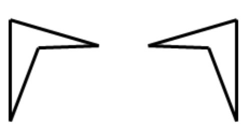
1. Identify whether the transformation of △ABC is a reflection, rotation, or translation.

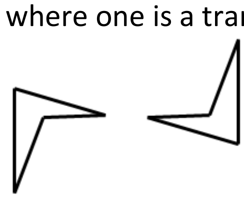


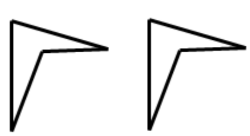
Solution:
The transformation is a reflection about the segment XY.



2. Which of the following shows a pair of figures where one is a translation of the other?

a) 

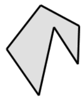
b) 

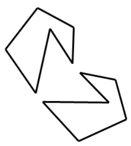
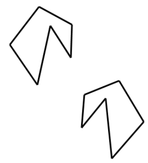
c) 

d) 

Solution: c is correct, it is a translation. a) reflection, b) rotation, d) dilation

3. Which of following pairs of figures shows a rotation of the shaded figure on the left?



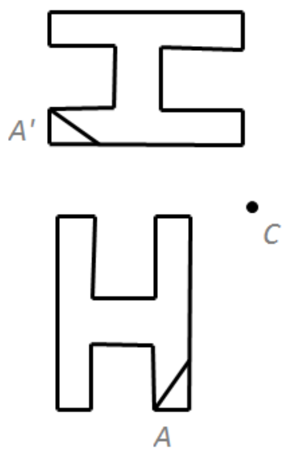
a)  b)  c)  d) 

Solution: b is correct, it is a rotation. others: a) reflection, c) glide reflection, d) dilation

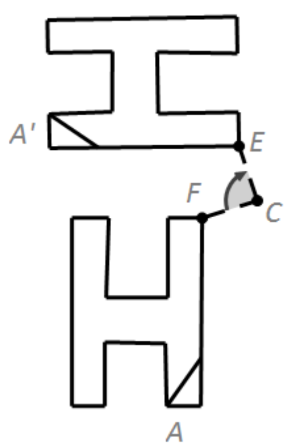
4. The **angle of rotation** is the amount of rotation a figure makes when rotated about a fixed point called the **center of rotation**. In the diagram below, the letter H is rotated about point C. Identify the angle of rotation as

a) 90° b) 180°

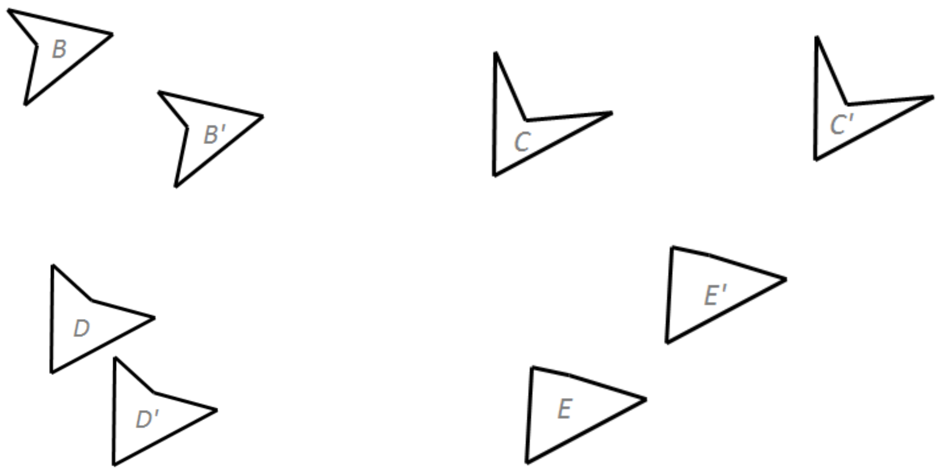
c) -90° d) -180°



Solution: c) is correct.
A positive rotation is movement in a counterclockwise (left) direction. Negative rotations turn in a clockwise (right) direction. The H has rotated in clockwise direction , therefore the angle of rotation is -90°.

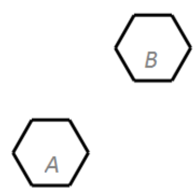


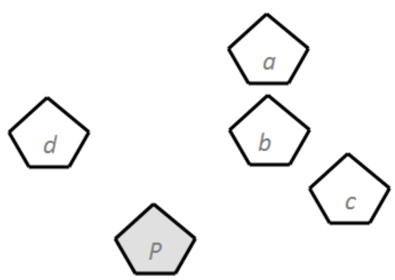
5. In the diagram at right, figure A has been translated so that the image is A'. From the figures below, which pair represents the same translation as A to A'?



Solution: C to C' is correct.

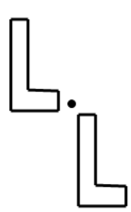
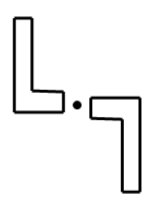
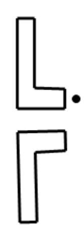
6. In the diagram at right, the same translation that translates hexagon A to hexagon B would translate pentagon P to which pentagon (below)?





Solution: pentagon b shows the correct translation from pentagon P.

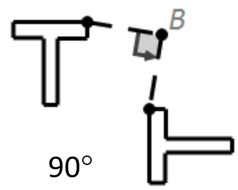
7. The four figures below all represent a rigid motion transformation. Which one represents a rotation?

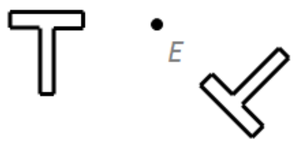
a)  b)  c)  d) 

Solution: c) is correct

8. The figure below shows a 90° rotation about the center point B. Using this figure as a guide, match the remaining figures with the degree of rotation from the list at left.

-90°
180°
135°
60°





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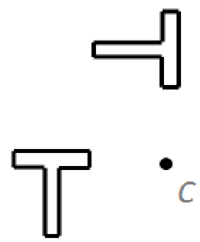

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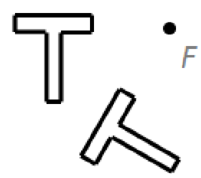

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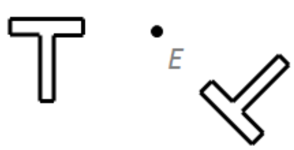
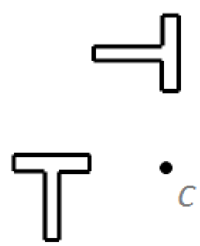
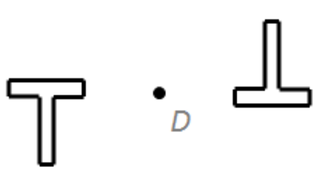
Solution:



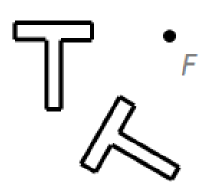
135°



-90°

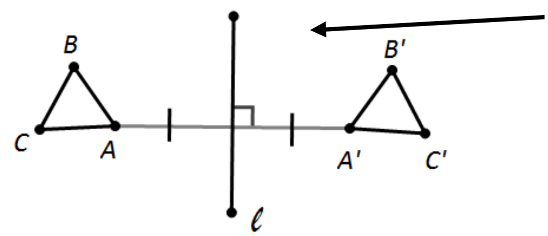


180°



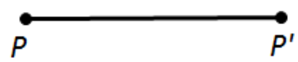
60°

9. A reflection is a rigid motion constructed across a **line of reflection**. The line of reflection is the **perpendicular bisector** of the segment joining every point of the pre-image and its image. The diagram below demonstrates this definition visually.

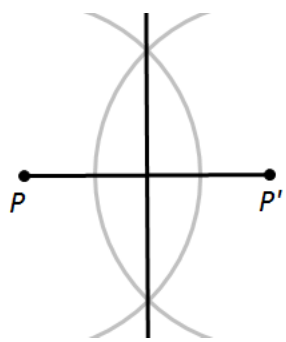


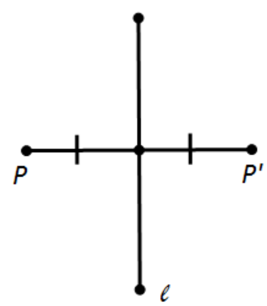
Given the informal definition of line of reflection above, find the line of reflection, between P and P' (notation: r_ℓ P = P').

Solution: Since the line of reflection is the perpendicular bisector of the line that connects P to P', then first construct the line that connects the two points.

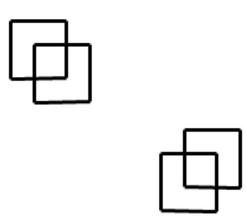


Next, construct the perpendicular bisector of $\overline{PP'}$. The result is a mapping of P to P' over the line of reflection, ℓ. (Notation: r_ℓ P = P')





10. What type of rigid motion does the figure below appear to be?



Solution: the rigid motion is a glide reflection.

Practice Lesson 13 Rigid Motion Transformations Rotations

Objectives
- Rotate figures about a center of rotation
- Find the angle of rotation
- Find the center of rotation
- Use function notation to write rigid motion functions

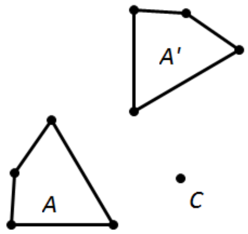
New Vocabulary: clockwise
counterclockwise

Introduction

Students concentrate on the rigid motion transformation: rotations. Function notation for transformation is introduced in this lesson, as well as precise language regarding the direction of rotations. First, the direction of the angle of rotation. Students have learned in elementary grades the notion of clockwise. When the hands on a clock move (to the right) they move in a **clockwise** direction. However, a movement of an angle in a clockwise direction (to the right) results in a negative measurement for that angle. The **counterclockwise** movement of an angle results in a positive measurement for that angle. So when denoting angle measurement, positive measurements represent an angle that has moved in a counterclockwise direction and negative measurements represent an angle that has moved in a clockwise direction.
Now we can discuss function notation. Function notation for rigid motion transformation of the plane have specific parts that gives you information about the transformation. It is a precise language and certain terms must be defined in order to understand it. Let θ represent any angle. Let $\mathbf{R}$ represent the transformation function rotation. Let $\mathbf{C}$ be the center of rotation. Then, $\mathbf{R_{C,\theta^\circ}\ (A) = A'}$ says that the image of point A after a rotation of θ° about point C is A'. For example, if the pre-image is point B and the angle of rotation is 30° and the center of rotation is point D, then the function notation we write is: $\mathbf{R_{D,30^\circ}\ (B) = B'}$. A clockwise rotation of 75° with the same parameters as the 30° angle would be written: $\mathbf{R_{D,-75^\circ}\ (B) = B'}$.
In addition to writing function notation, students will practice rotating figures about a center of rotation, find the angle of rotation, and locate the center of rotation.

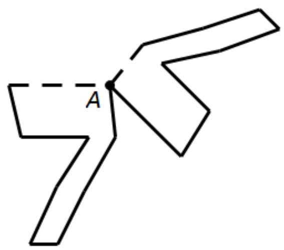
Practice

1. Identify the following as a clockwise rotation or counterclockwise rotation of figure A about center C.



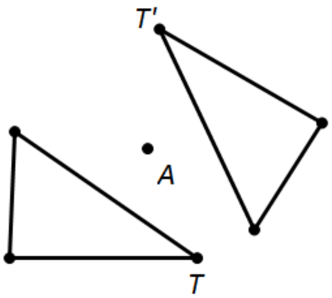
Solution: The rotation of figure A is a counterclockwise rotation about C.

2. The figure below has been rotated about the center of rotation point A. What is the angle of rotation? Describe the rigid transformation in function notation.

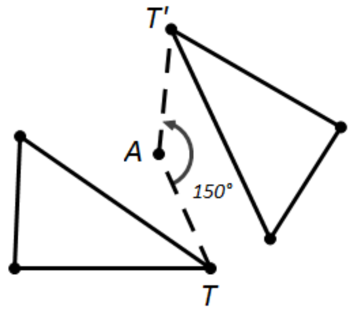


Solution: Using a protractor, measure the angle indicated by the dashed lines. You will find the angle of rotation about the center of rotation A is 135°.
In function notation: $R_{A,\,135°}(A) = A$

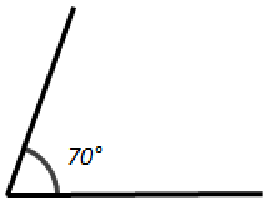
3. The figures below show a rotation of point T about the center of rotation point A. What is the angle of rotation of point T? Write your answer in function notation.

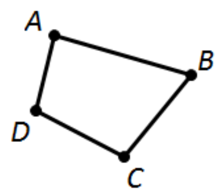


Solution: Draw a line from T to A and another from A to T'. Using a protractor, measure the angle made by the newly formed lines (the dashed lines in the diagram, right). You will find the angle of rotation of point T about the center of rotation A is 150°. In function notation, $R_{A,\,150°}(T) = T'$

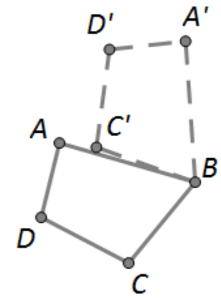


4. Rotate the figure ABCD -70° about the center of rotation point B. A 70° angle is provided for tracing if a protractor is not available.

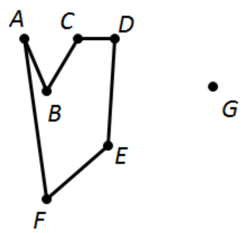




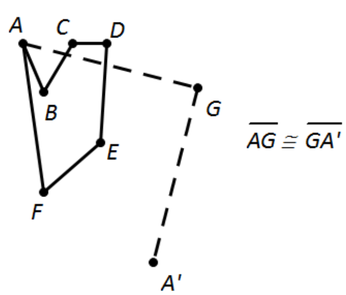
Solution:

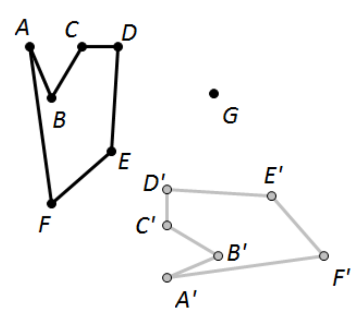


5. Rotate the figure below 90° counterclockwise about the center of rotation G. Use a protractor or use the angle provided as your guide for making the angle of rotation.

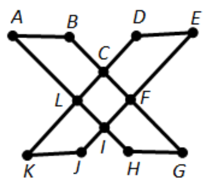


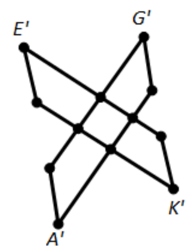
Solution: Trace and cut out the angle provided or use patty paper to trace it and place the vertex on point G and one ray intersecting point A. Mark the distance from the vertex to point A on the ray. Fold along the vertex so that the rays lie on top of each other and mark the other ray with point A'. Place the vertex on point G with the original mark lined up with point A and transfer point A' to the page . Continue this process for each point. When your are done connect each point with a line segment. You now have the image A'B'C'D'E'F' as shown in below.
(You may use a protractor for this same process, taking care that points of the image are the exact distance from G as the pre-image; demonstrated by the congruency statement $\overline{AG} \cong \overline{GA'}$).



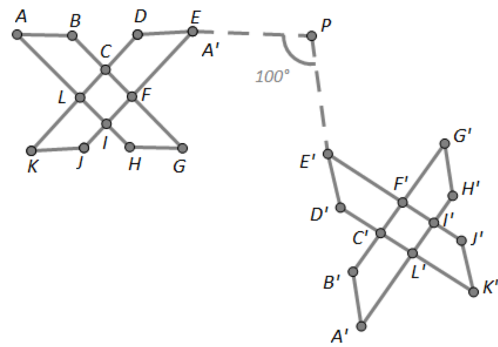


6. Find the center of rotation and angle of rotation of the pair figures below. Finish labeling the points on the image, then write the transformation of point E in function notation.

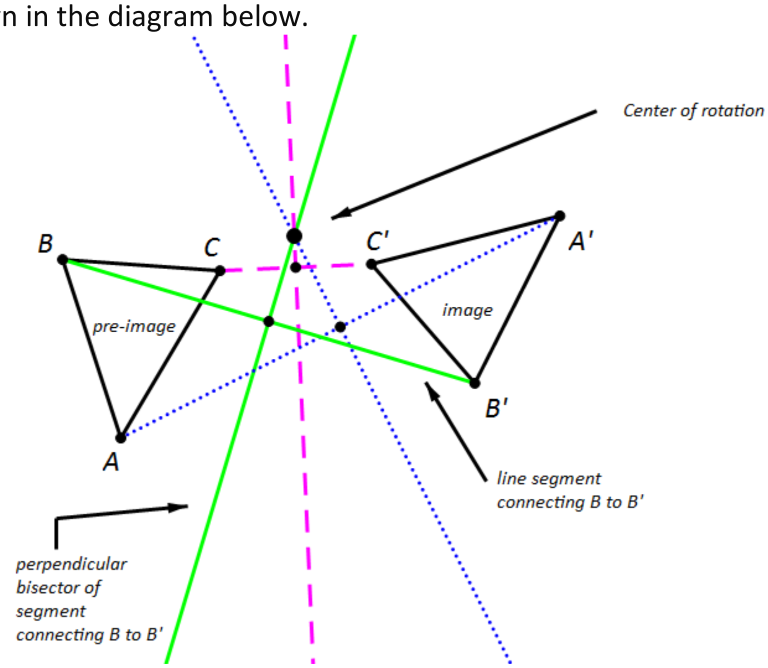




Solution: $R_{P,100°}(E)=E'$

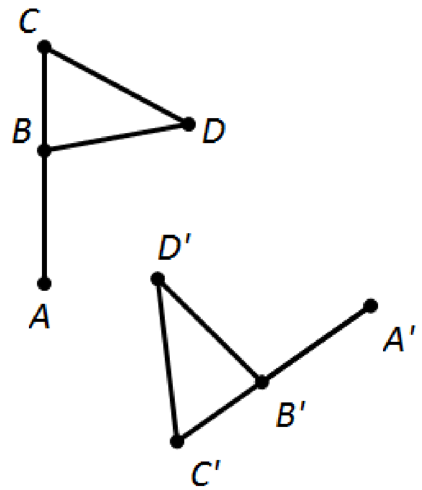


7. To find the center of rotation you locate the **intersection of the perpendicular bisectors** of the line segments connecting each point of the pre-image to each corresponding point of its image as shown in the diagram below.

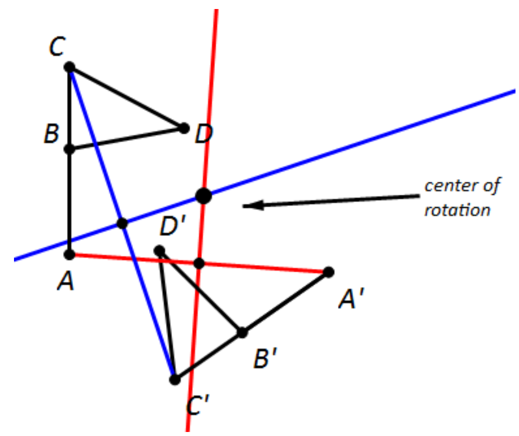


Go to next page for your task.

Your task: find the center of rotation for the figures below:



Solution: As evidenced from the above example, you need only one intersection of the $\perp$ bisectors to find the center of rotation as all of the $\perp$ bisectors will intersect at this point.



Practice Lesson 14 — Rigid Motion Transformations Reflections

Objectives

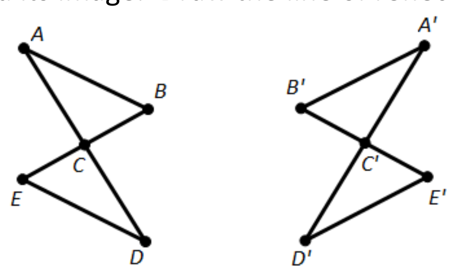
- Construct the line of reflection
- Identify reflections and line of reflections
- Construct reflections over a line of reflection

Introduction

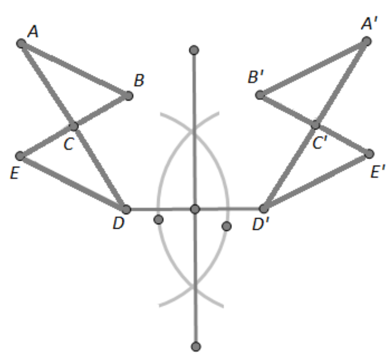
Students concentrate on the rigid motion transformation: reflection. Students will use previous knowledge of the perpendicular bisector and its construction to perform reflections without the benefit of a coordinate grid. Students will construct reflections and lines of reflection. They will also identify the correct reflection given a group of figures and identify the line of reflection for a given reflection.

Practice

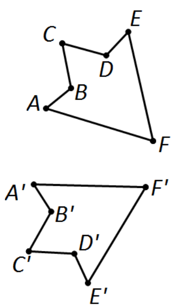
1. The line of reflection is the perpendicular bisector of the segment joining every point of the pre-image and its image. Draw the line of reflection for the figures below.



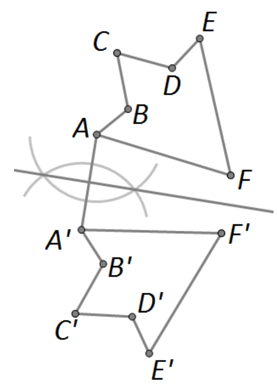
Solution: Draw a line segment connecting and point from the pre-image to its corresponding point of the image (D to D' in the figure below). Construct the perpendicular bisector of $\overline{DD'}$.



2. Construct the line of reflection of the figures below (left).

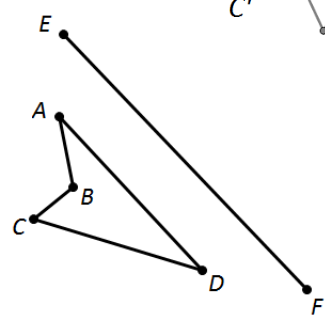


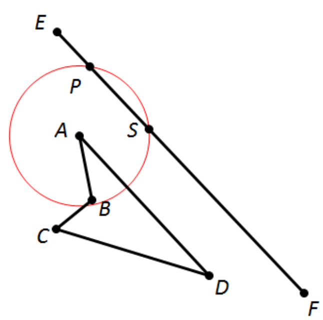
Solution: figure below (right).



3. Reflect quadrilateral ABCD over the line of reflection, EF.

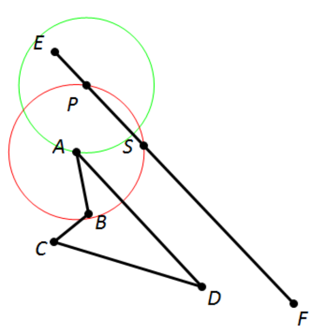
Solution: To construct a reflection of ABCD over line EF, the construction of three circles for each vertex is necessary. Follow the steps below to construct the reflection of ABCD over EF.

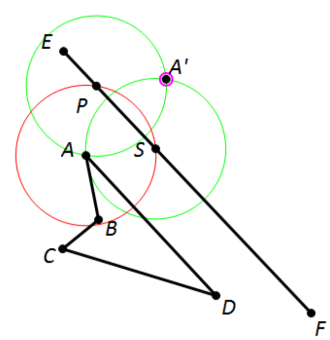




a. Construct a circle with center A and an intersection of the circle at two points with EF. Label the intersections P and S as shown at left.

b. Construct a 2nd circle, Center P, radius A (below left).

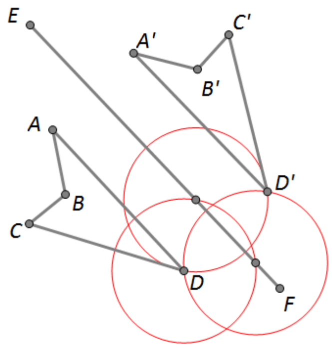




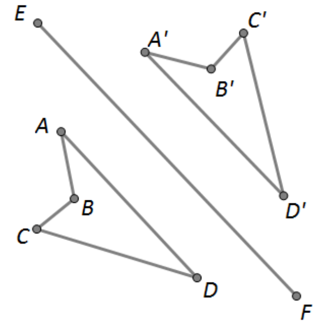
c. Construct a 3rd circle, Center S, radius A. The intersection of Circle P and Circle S is the image of A' (above right).

d. Repeat steps a - c for vertices B, C & D.

(Construction of B’ & C’ are not shown here).

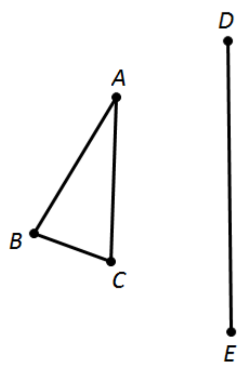


Construction of D’

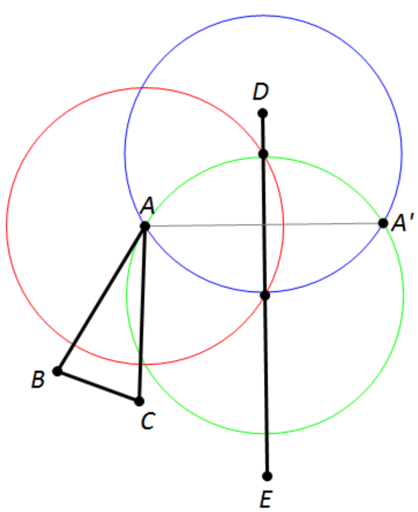


Completed reflection of ABCD

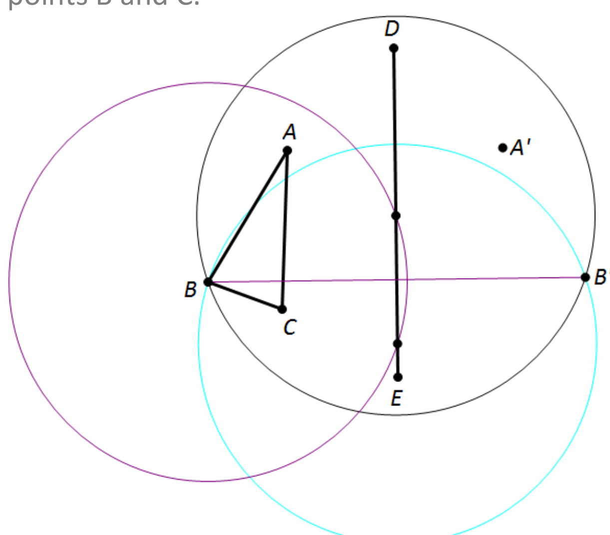
4. Reflect the triangle over the line of reflection DE



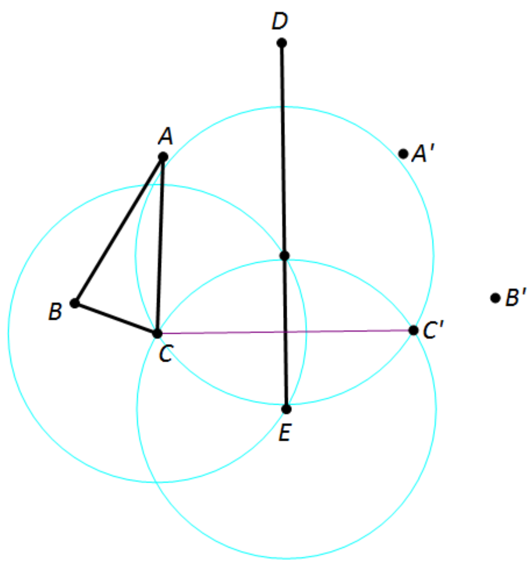
Solution: Construct a circle, Center A that intersects DE at two points. Label the points F and G, respectively. Construct Circle F with A on the circle. Construct Circle G with A on the circle. Label the intersection of Circles F & G as A’. Construct a line segment from A to A’. Repeat this process for points B and C.



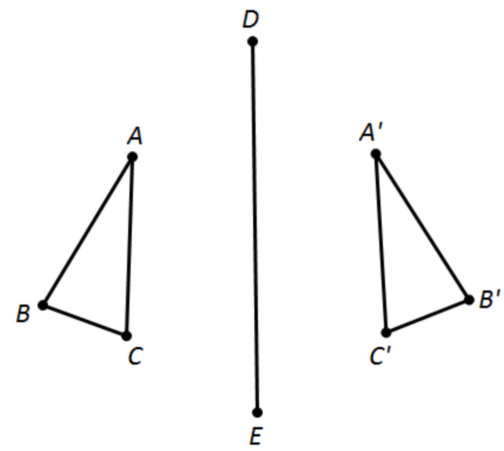
Reflection of Point A



Reflection of Point B

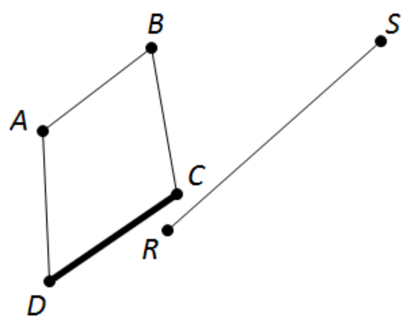


Reflection of Point C



Reflection of ΔABC

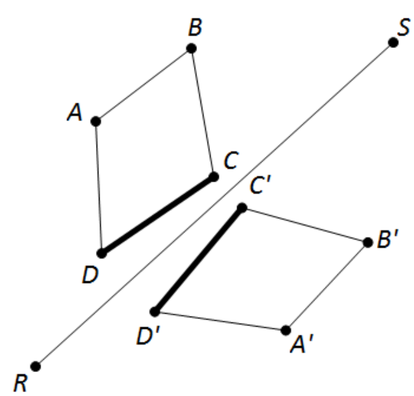
5. Reflect quadrilateral ABCD over the line RS.



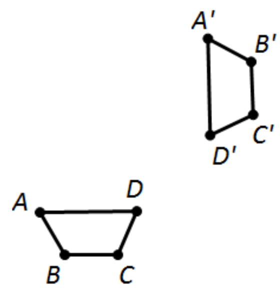
Solution:

Note: Extend the line past point D since the line of reflection is the perpendicular bisector of the segment connecting each point of the pre-image to each point of the image (you want to be able to construct the segment for point D as well).

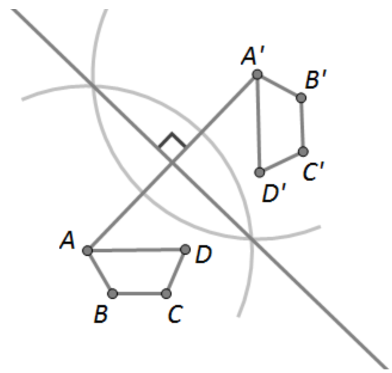
Continue the same process you followed in problems 3 & 4. The final result will be as shown below.



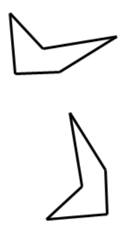
6. Construct the line of reflection for the following figure below:



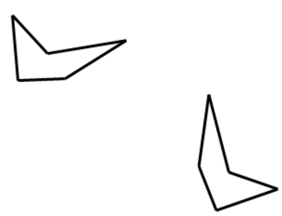
Solution: Connect point A to point A' with a line segment. Construct the perpendicular bisector of the connecting line segment connecting point A to point A'.



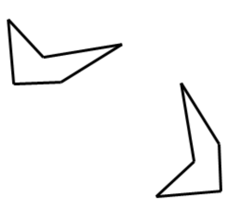
7. Identify the correct reflection about the line of reflection DE from the following:



A)



B)

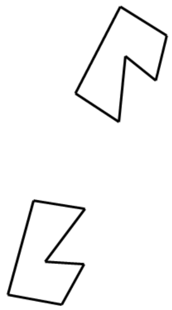


C)

D)

Solution: The correct choice is B).

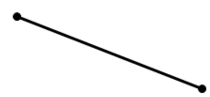
8. Identify the correct line of reflection of the following:



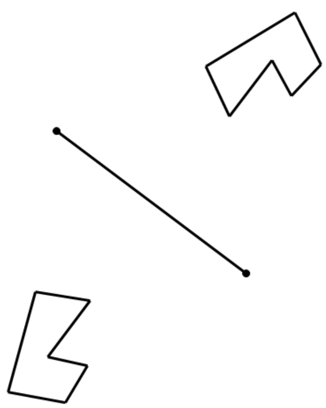
A) 

B) 

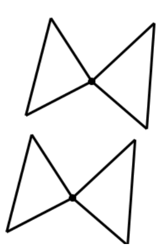
C) 

D) 

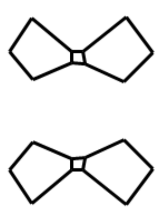
Solution: The correct choice is D).



9. Below are four examples of rigid motion. Identify the reflection:

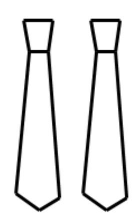


A)



B)

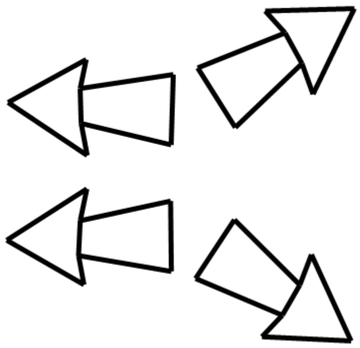
C)

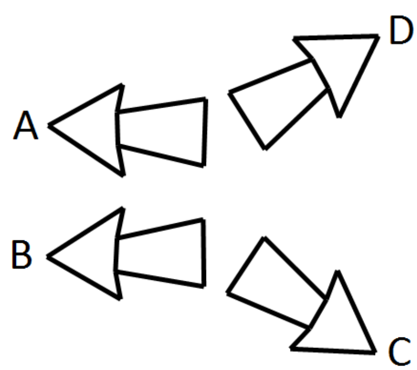


D)

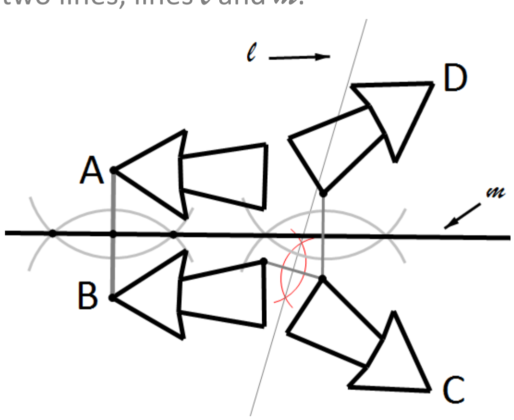
Solution: the correct choice is B)

10. State the number of and construct the line(s) of reflection of the following:

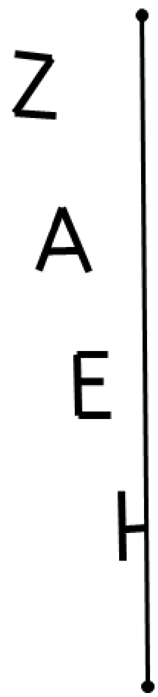




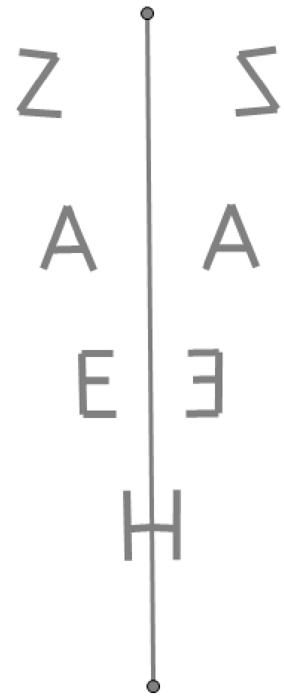
Solution: There are two lines of reflection.
Label the figures as shown below:
The reflections of arrow A are in the following order: A to B, B to C, C to D.
Draw a line segment from any point on A to the corresponding point on B (this example uses the tip of the arrow). Construct the perpendicular bisector of the line segment.
Next, draw a line segment from any point on B to the corresponding point on C (this example uses the top of the base).
Construct the perpendicular bisector of this line segment. Finally, draw a line segment from any point on C to the corresponding point on D (this example uses the tip of the arrow). Construct the perpendicular bisector of this segment. You will see that this perpendicular bisector coincides with the perpendicular bisector drawn between A and B, thus creating only two lines, lines ℓ and m.



11. Refer to the diagram at left. Reflect the following letters about the given line of reflection



Solution:



12. Describe your procedure in problem 11. Be sure to precise mathematical language so that anyone without knowledge of constructing a reflection could be successful in this construction.

Solution: since the line of reflection is the perpendicular bisector of the line connecting the pre-image to the image, then a construction of a perpendicular bisector of the line of reflection to each key point of the pre-image can be constructed.

Step 1: Extend the line of reflection in both directions as it is not long enough for a successful construction.
Step 2: Determine key points on the pre-image and mark them.
Step 3: With the compass point on the first key point, (labeled point A) extend the width of the compass out past the line of reflection and construct a circle with point A as the center.
Step 4: The circle intersects the line of reflection in two places. Label them point B and point C.
Step 5: With compass point at point B and compass width adjusted to point A, construct a circle with center B.
Step 6: With compass point to point C and compass width adjusted to point A, construct a circle with center C.
Step 7: Circle B and Circle C intersect at point D. Mark it.
Step 8: Repeat Steps 3 through 8 for each key point of the letter.
Step 9: Draw the lines connecting the points marked in Step 7.
Step 10: Repeat the entire process (Step 2 through Step 10) for each letter.

Practice Lesson 15 — Rigid Motion Transformations Rotations, Reflections, and Symmetry

Objectives
- Examine the relationship between reflections and rotations
- Observe the symmetry created by reflections and rotations

New Vocabulary:

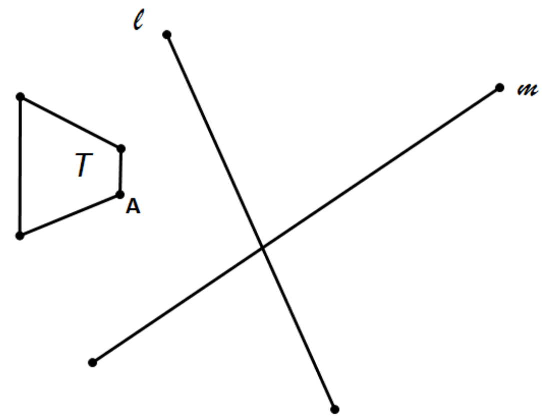
symmetry	line of symmetry
rotational symmetry	reflection symmetry
line symmetry	identity transformation
identity symmetry	regular polygon
order of rotational symmetry	magnitude of rotational symmetry

Introduction

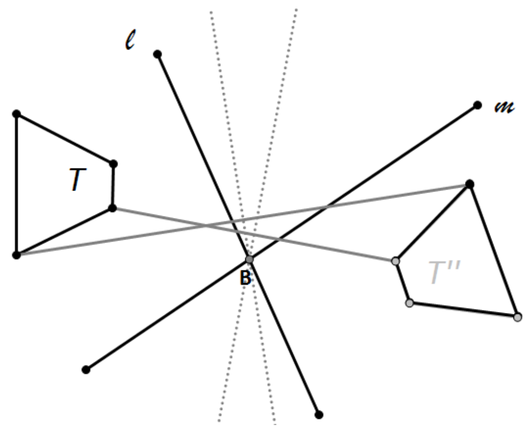
In this lesson, students will examine the relationship that exists between the reflection over intersecting lines and the rotation. The reflection of a figure over two intersecting lines produces the rigid motion transformation: a rotation. The point of intersection of the two lines is the center of rotation and the angle of rotation is the angle formed by the intersection of the lines and a point of the figure and its image. The angle created by the intersecting lines is one-half the measure of the angle of rotation. Before the study of this lesson begins, students must become familiar with the vocabulary listed above. We begin by defining the most basic rigid motion transformation: the **identity transformation**. This transformation maps each point in the plane back onto itself. A figure has **symmetry** if there is an isometry that maps the figure onto itself. The **identity symmetry** is one such isometry. In this lesson, students will investigate **rotational symmetry** and **reflection symmetry**. **Rotational symmetry** is the property by which figure, upon a certain rotation, is mapped onto itself. In other words, if you can rotate a figure around a center point by fewer than 360° and the figure appears unchanged, then the figure has **rotational symmetry**. A figure has **reflection symmetry** if you can reflect a figure over a line and the figure appears unchanged. The **line of symmetry** is a line that divides a figure into two congruent parts, each of which is the mirror image of the other. All points of the pre-image are exactly the same distance from the line of symmetry as the corresponding points of the image. The number of positions a figure can be rotated to, without bringing any changes to the way it looks originally, is called its **order of rotational symmetry**. The **magnitude of rotational symmetry** is the measure of the angle of each turn. Also in this lesson, students will examine the symmetrical properties of **regular polygons.** A **regular polygon** is a figure in which all interior angles have the same measure and all sides have the same length**.**

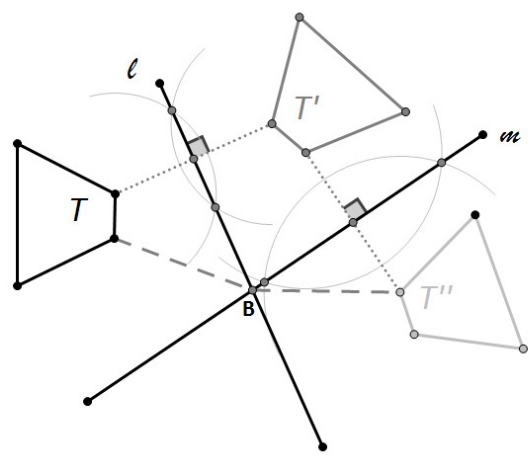
Practice

1. Refer to the diagram below, right. Reflect polygon T about line ℓ and then reflect polygon T' about the line m. Describe what you notice about polygon T'' as it relates to the pre-image T.



Solution: The image T'' appears to be a clockwise rotation of polygon T about the point B.
As you can see from the diagram below, left, the dotted lines are the perpendicular bisectors of the line segments connecting the corresponding points of polygon T and polygon T''. Their intersection is point B, the center of rotation, which is also the intersection of ℓ and m. The final result is shown in the diagram below, right.



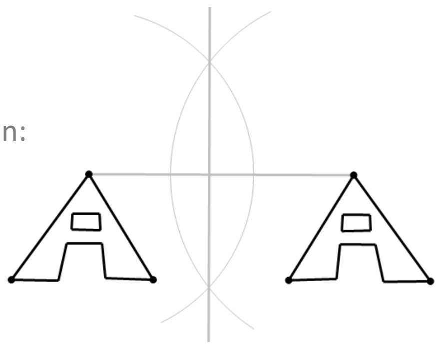


2. If the angle measure of ∠B in the previous problem is 80°, what is the angle of rotation of T to T''? Explain.

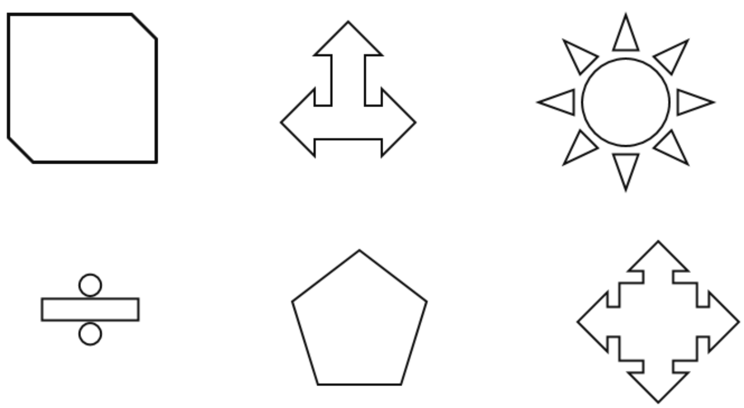
Solution: Since the angle of rotation is twice the size of the angle formed by intersecting lines, the angle of rotation is 160°.

3. Recall from Practice Lesson 14 that the line of reflection is the perpendicular bisector of the line segment connecting a point of the pre-image to its corresponding point of the image. This line of reflection is also known as the line of symmetry. Construct the line of symmetry of the following figures.

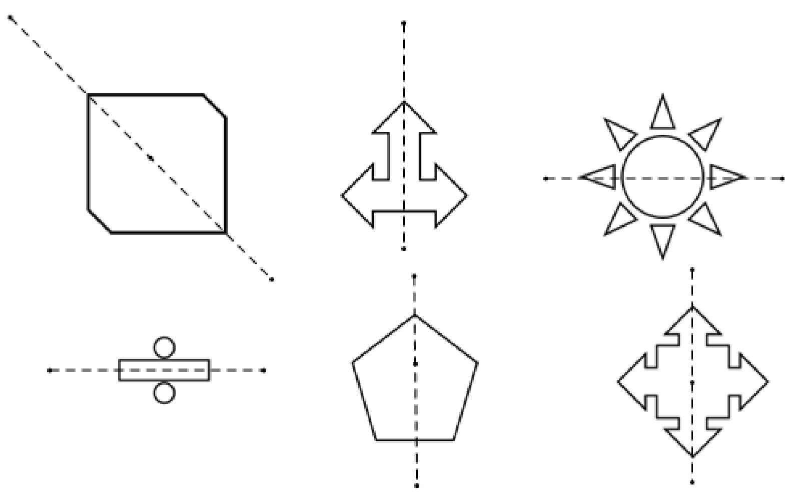
Solution:



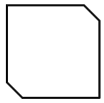
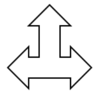
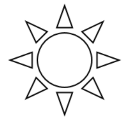
4. The figures below each have at least one line of symmetry. Show a line of symmetry for each figure.



Solution:



5. The figures below are the figures from problem 3. Give the number of lines of symmetry for each figure in the space provided.

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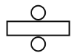 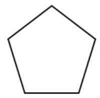 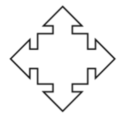

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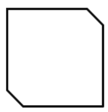
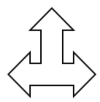
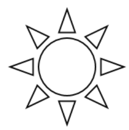
Solution:

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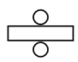
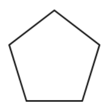
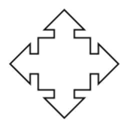
  

2 5 2

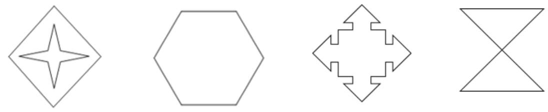
5. Regular polygons have a degree of rotational symmetry equal to 360 divided by the number of sides.
   a. What is the degree of rotational symmetry for a square?
   b. A pentagon?
   c. A hexagon?
   d. An octagon?

   Solution:
   a. A square has 90° rotational symmetry
   b. A pentagon has 72° rotational symmetry
   c. A hexagon has 60° rotational symmetry
   d. An octagon has 45° rotational symmetry

6. Based on your answers from number 5, can you make a conjecture about the lines of symmetry any regular pentagon will have? State the number of lines of symmetry for each polygon in problem number 5.

   Solution: a regular polygon will have the number of lines of symmetry equal to the number of sides of the polygon. A square has 4 lines of symmetry; a pentagon has 5 lines of symmetry; a hexagon has 6 lines of symmetry; an octagon has 8 lines of symmetry.

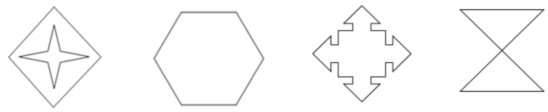
7. For the following figures state the order and magnitude of rotational symmetry.



order of rotational symmetry ________ ________ ________ ________

magnitude of rotational symmetry ________ ________ ________ ________

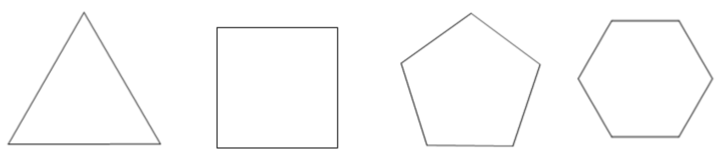
Solution:



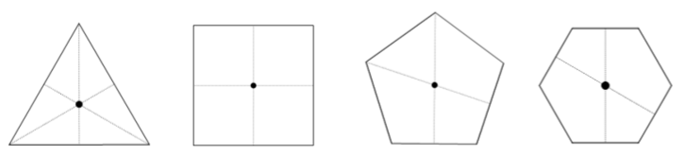
order of rotational symmetry: 2, 6, 4, 2

magnitude of rotational symmetry: 180°, 60°, 90°, 180°

8. Locate the center of rotation for the following regular polygons. Explain how you found them.



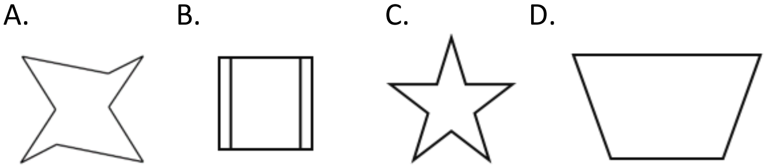
Solution:



The center of rotation is found by constructing the perpendicular bisectors of at least two sides of the regular polygon. (Note: the center of rotation can also be found by constructing the angle bisectors of at least two angles of the regular polygon).

9. Name all the types of symmetry for the following figures:

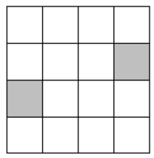
A. B. C. D.



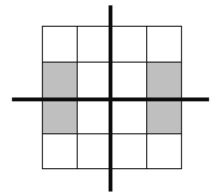
Solution:

A. rotational & identity symmetry
B. rotational, reflection and identity symmetry
C. rotational, reflection and identity symmetry
D. reflection & identity symmetry

10. Use the following figure to solve this problem: Shade two more boxes to make a diagram with two lines of symmetry. Show the lines of symmetry.



Solution:



Practice Lesson 16 Rigid Motion Transformations Translations

Objectives

- Translate figures according to a vector length and direction
- Translate figures in the coordinate plane
- Explore relationship between vectors and parallel lines
- Identify and describe vectors

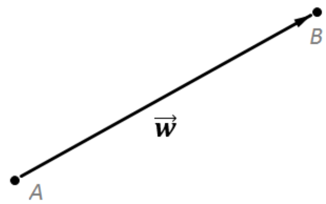
New Vocabulary: translational symmetry, vector, initial point, terminal point, magnitude

Introduction

In this lesson, students will examine the third type of symmetry known as **translational symmetry**. A translation is a function that takes points in the plane as inputs and returns other points as outputs. The translation is a movement of the plane in which a figure is translated a specific direction for a specific distance. A basic translation is a reflection about a pair of parallel lines. A translation is an isometry and a figure has **translational symmetry** if after a translation with the specified distance and direction, the figure appears to be unchanged. One way to achieve this specified movement is the application of a **vector**. A vector (in very simple terms) is an arrow! A vector is a quantity that has a fixed length (its **magnitude**) and a fixed direction. The **initial point** (where it starts) of a vector is represented by the "tail" of the arrow and the **terminal point** (where it ends) is represented by the "head" of the arrow. Vectors describe translations by helping to specify a direction and an amount of length to move every point of the pre-image. The movement of the pre-image along a vector is a construction of a parallel line through each point of the pre-image you want to translate parallel to the vector and translated the length and direction of the vector. Students will use translation and vector notation as figures are translated in the coordinate plane. Students will also tell the difference between the ordered pair notation and vector notation of $\langle a, b \rangle$ also known as component form. Naming a vector using component form specifies the horizontal change a and the vertical change b from the initial point to the terminal point of the vector.
In this lesson, students will identify and describe vectors, translate figures, and explore the relationship of translated figures, reflections, rotations, and vectors.

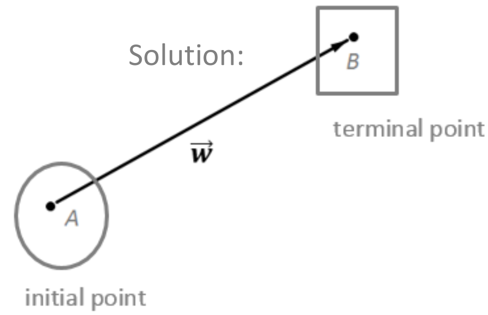
Practice

1. The vector shown below left describes the distance from point A to point B for vector $\vec{w}$.



Draw a circle around the initial point and a box around the terminal point of the vector.

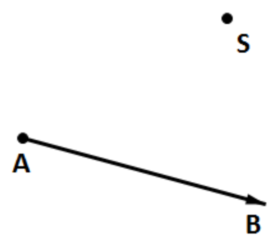
Solution:



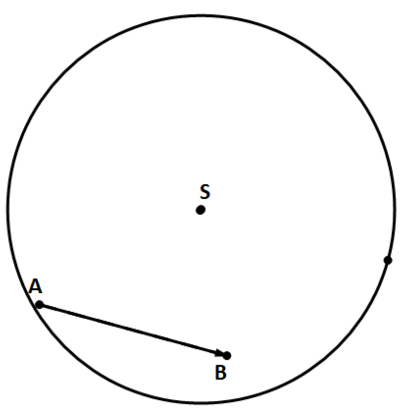
2. Tell the difference between a line segment and a vector.

Solution: a vector has direction; a line segment does not have direction.

3. Given the vector $\overrightarrow{AB}$, translate the point S using a compass and straightedge. Proper function notation for this translation is apply $T_{\overrightarrow{AB}}$ to point S (read: "apply the translation the length and direction of vector AB to point S").

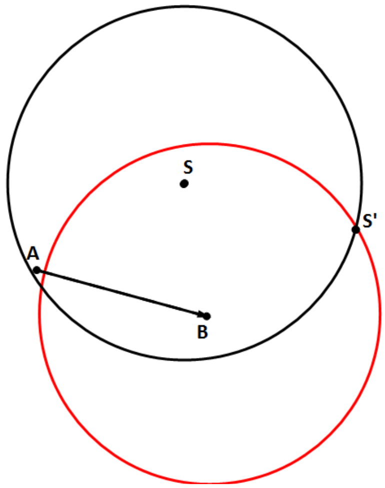


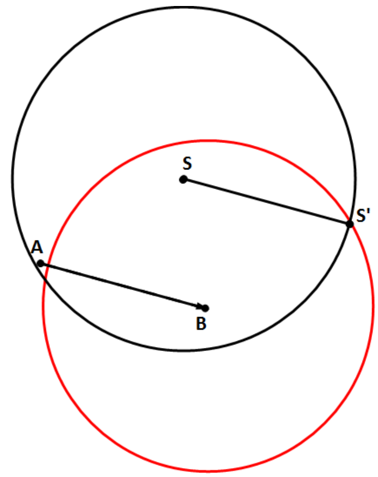
Solution: To translate the point S we must construct a parallel line to $\overrightarrow{AB}$ and the exact length of $\overrightarrow{AB}$. To do this draw a circle with center S and radius $\overrightarrow{AB}$.



Next adjust the compass so that the radius is AS and draw another circle with center B.

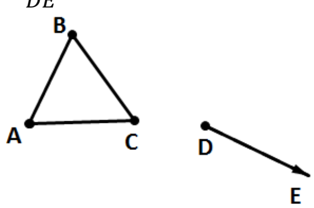
Label the intersection of the circles S'. Draw a line connecting S and S'.





Point S has been translated the length (magnitude) of $\overrightarrow{\boldsymbol{AB}}$ and in the direction of $\overrightarrow{\boldsymbol{AB}}$ **.** Use this construction technique to translate plane figures in the direction of a vector and the exact length of a vector.

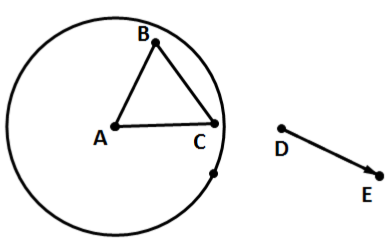
4. Apply $T_{\overrightarrow{DE}}$ to ΔABC.

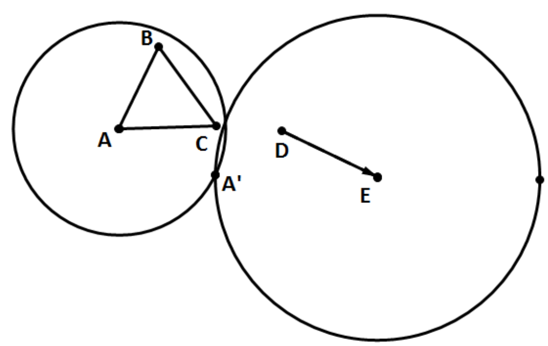


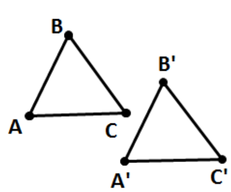
Solution: Draw a circle with center A and radius $\overrightarrow{DE}$.

Next, adjust the compass to width to AD and draw a circle with center E. Label the intersection of Circle A and Circle E as A'.

Repeat these two steps until all vertices of ΔABC have been translated. Draw the ΔA'B'C'.







The translated image

5. An image has **Translational Symmetry** if it can be divided by straight lines into a sequence of identical figures. Translational symmetry results from moving a figure a certain distance in a certain direction by a vector. The figures below possess translational symmetry.

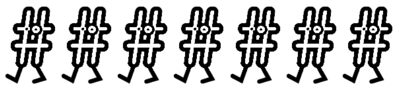
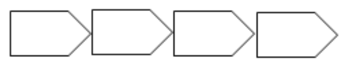


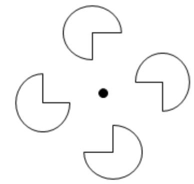
image 18

Determine if the following figures have translational symmetry.

A.



B.



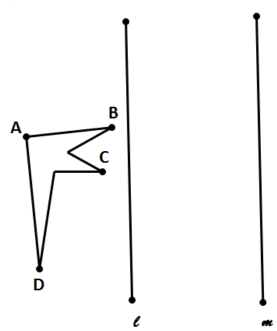
C.

Solution: A. No, does not have translational symmetry

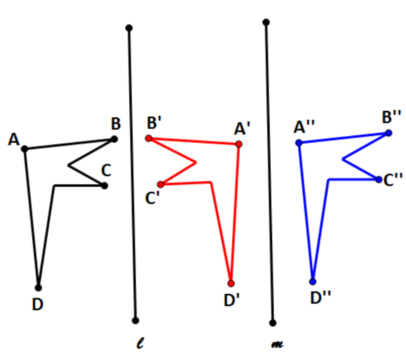
B. Yes, has translational symmetry

C. No, does not have translational symmetry

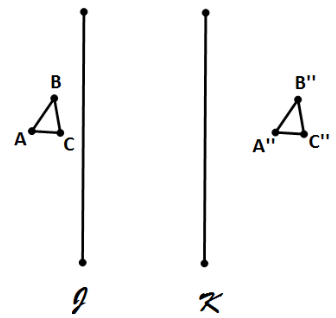
6. The line segments ℓ and m are parallel. Reflect ABCD across each line and compare your results to a translation.



Solution: The relationship between the pre-image and two parallel lines results in a translation of the of ABCD to A"B"C"D" that is equivalent to a reflection across each of the parallel lines. (Note: the distance of D to D" is exactly twice the distance of ℓ to m).

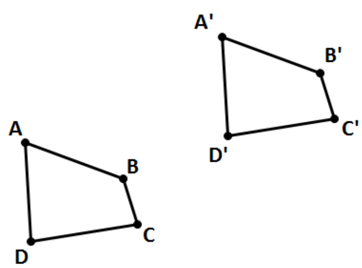


7. The distance between lines $\mathcal{J}$ and $\mathcal{K}$ is 2 cm. State the magnitude of the vector $\overrightarrow{CC''}$.

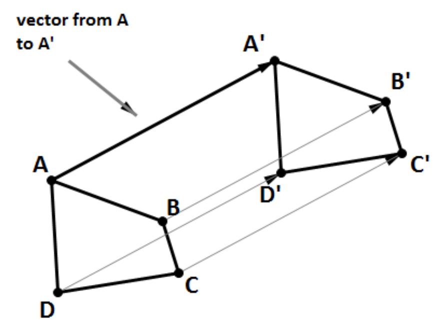


Solution: Since a translation is directly related to a sequence of two reflections across parallel lines, and the distance between the lines is one half the distance between the corresponding points of the translated figure, then the magnitude of the vector $\overrightarrow{CC''}$ is 1 cm.

8. Draw the vector that shows the distance and direction of the following translated figure:



Solution: Construct a line segment from A to A'. Point the arrow in the direction of A'. If you create a line segment (vector) from each point of the pre-image ABCD to each point of the image A'B'C'D', you will see that each vector is exactly the same length and each vector is parallel to each other.

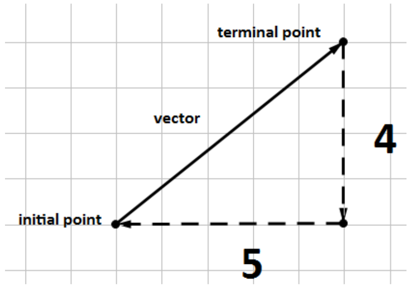


9. Translations can be made in the coordinate plane. Directions in the coordinate plane are given in units and movement can be vertical positive (up), vertical negative (down), horizontal positive (right), horizontal negative (left). Vectors can also describe translations in the coordinate plane and use a specific notation, for example a vector that moves a point left 3 units and down 4 units in the coordinate plane is notated: $\langle -3, -4 \rangle$.

What is the difference between $\langle -2{,}5 \rangle$ and (-2,5)?

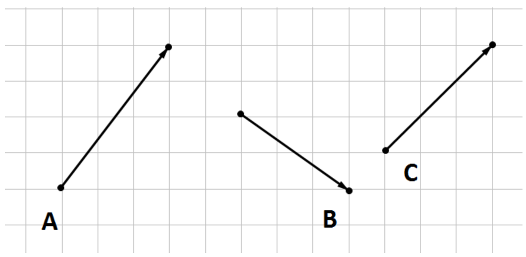
Solution: $\langle -2{,}5 \rangle$ is the component form for a vector which represents the horizontal change of -2 (a movement left two units) and the vertical change of 5 (a movement of 5 units up); (-2,5) is the coordinate position of a point in the second quadrant.

10. The magnitude of a vector in the coordinate plane can be found by using the distance formula or the Pythagorean Theorem (after the creation of a right triangle formed by joining the terminal and initial points of the vector (see diagram). Note: magnitude is always positive.



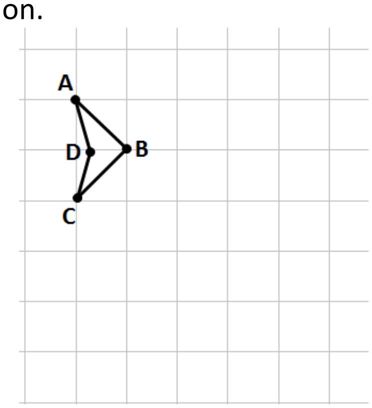
As you can see from the example, you can create a right triangle and use the Pythagorean Theorem to find the magnitude (length) of the vector when translating objects in the coordinate plane. The magnitude of the vector shown is $\sqrt{41}$.

Find the magnitude of the vectors shown below:

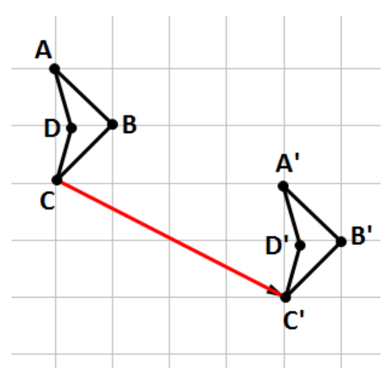


Solution: A) 5 B) $\sqrt{13}$ C) $3\sqrt{2}$

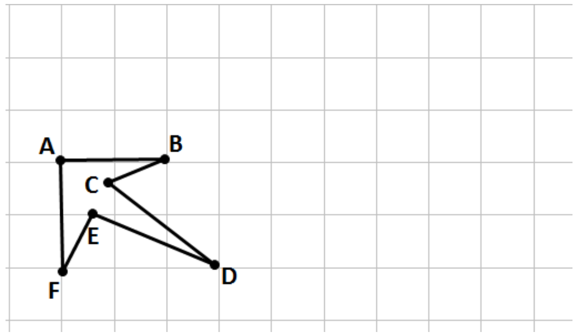
11. Translate the figure below four units to the right and two units down. State the magnitude, write the vector in vector component form, and draw the vector that defines the translation.



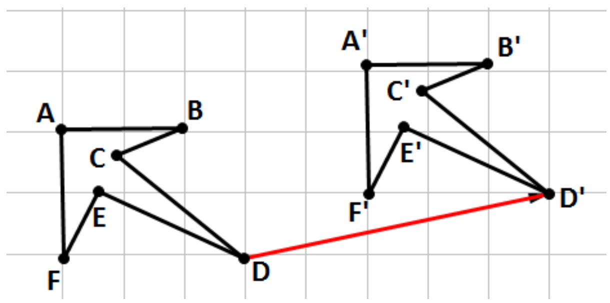
Solution: the magnitude of the vector $\langle 4, -2 \rangle$ is $2\sqrt{5}$. The translation is shown at right:



12. Translate the figure below 5 units right and 1 unit up. State the magnitude, write the vector in vector notation, and draw the vector that defines the translation.



Solution: the magnitude of the vector $\langle 5,1\rangle$ is $\sqrt{26}$. The translation is shown below:



Practice Lesson 17 Rigid Motion and the Perpendicular Bisector

Objectives

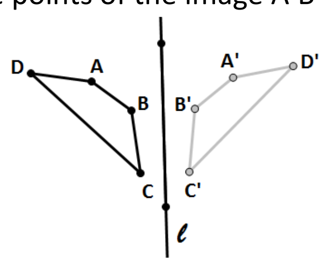
- Explore the relationship between rigid motion transformations and the perpendicular bisector

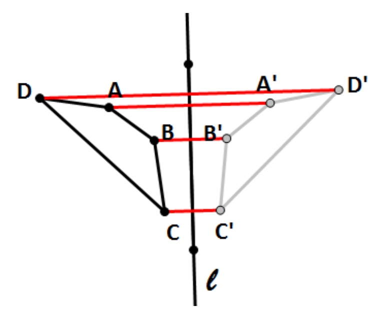
Introduction

This lesson expands the study of rigid motion transformations as they relate to the perpendicular bisector. As previously discovered, a line of reflection is the perpendicular bisector of the line segment connecting a point of the pre-image to its corresponding point of the image. Recall that the **perpendicular bisector** of a segment is a line, ray or segment that intersects a given segment at a 90° angle and passes through the midpoint of the given segment. An important property of the perpendicular bisector is that each point on the bisector is equidistant to the endpoints of the given segment it bisects. Students will employ this property when exploring transformations specifically reflections and rotations.

Practice

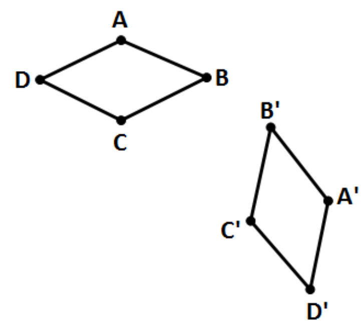
1. Quadrilateral ABCD has been reflected across line ℓ. Discuss the relationship between the distance between line ℓ and the points of the pre-image ABCD and the distance between line ℓ and the points of the image A'B'C'D'.



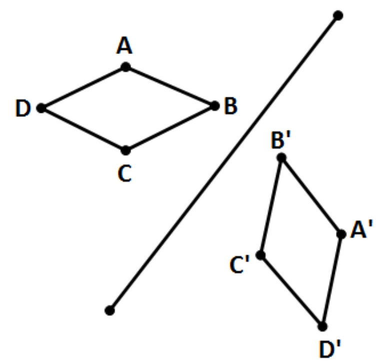


Solution: Connect each point of the pre-image to its corresponding point of the image with a line segment. Measure the distance between each point and line ℓ. The distance between each point of the pre-image and line ℓ is the same as the distance between each point of the image and line ℓ. Measure the angle of intersection of each segment and line ℓ. The angle of intersection of each segment and line ℓ is 90°. Because each point and its corresponding point are the same distance from line ℓ, and each angle formed by these segments and line ℓ are 90°, then by the definition of perpendicular bisector, line ℓ bisects each of these segments, thus line ℓ is the perpendicular bisector of each of the segments.

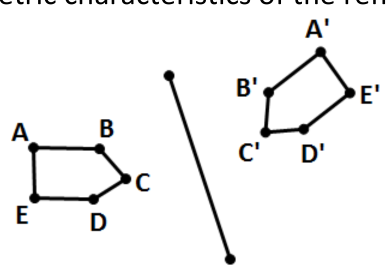
2. This next problem asks you to find the line of reflection of the following figures. The pre-image ABCD is reflected over a line of reflection resulting in the reflected image, A'B'C'D'. Discuss the necessity of using the perpendicular bisector to accomplish this task.



Solution: Since the line of reflection IS the perpendicular bisector of the line segments connecting each point of the pre-image to its corresponding point of the image, it would be nearly impossible to find the line of reflection without employing this construction. One could draw each segment and construct the midpoints of each segment and connect them with a line, but this method would be more arduous and would not be as precise as constructing the perpendicular bisector, as there is more room for error with this method.

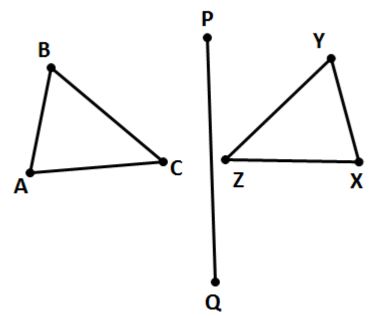


3. Discuss the isometric characteristics of the reflected figure, A'B'C'D', shown below.

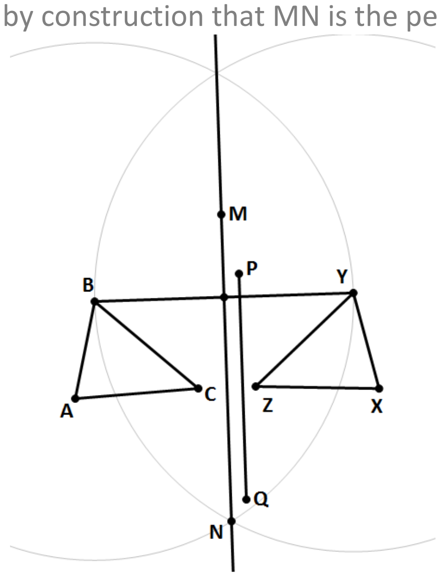


Solution: An isometry is a transformation in which the pre-image and its image are congruent. A reflection is an isometry that preserves angle measure and distance between points. This means that the distance between any two points of the pre-image is the same as the distance between their corresponding points of the image. This characteristic makes the reflected figure congruent to the pre-image.

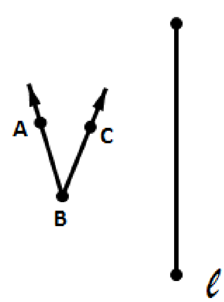
4. In the figure below, explain why PQ could not be the line of reflection of pre-image ΔABC and ΔXYZ.



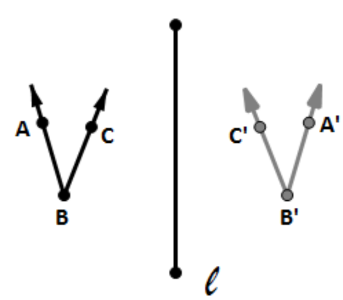
Solution: Since the line of reflection is the perpendicular bisector of the line segments connecting each point of the pre-image ΔABC to its corresponding point of image ΔXYZ you can see by construction that MN is the perpendicular bisector of BY, not PQ.



5. Reflect ∠ABC over line segment ℓ. Based on your construction, what can you say about the reflection of ∠ABC?



Solution: C is the same distance from ℓ as C' is to ℓ, A is the same distance from ℓ as A' is to ℓ, B is the same distance from ℓ as B' is to ℓ; therefore, ∠ABC ≅ ∠A'B'C' so they must have the same angle measure.



Practice Lesson 18 Rigid Motion and the Parallel Postulate

Objectives
- Construct parallel lines by various methods
- Prove lines are parallel by the indirect proof method

New Vocabulary: parallel postulate, indirect proof/proof by contraction, direct proof

Introduction

In this lesson, student's will apply Euclid's Fifth Postulate, commonly called the **parallel postulate**.

Euclid's Fifth Postulate states:

> *If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, will meet on that side on which are the angles less than two right angles.*
>
> ~ History of the Parallel Postulate, Florence P. Lewis

The modern version of this postulate which has become widely accepted as a restatement of the parallel postulate is:

> *Given any straight line and a point not on it, there exists one and only one straight line which passes through that point and never intersects the first line, no matter how far they are extended.*
>
> ~ Euclid, restated by Godel, Escher, Bach : An Eternal Golden Braid, Douglas R. Hofstadter

In order to be successful in this lesson, students must understand what is meant by the parallel postulate and how its meaning can be applied to establish commonly known principles such as corresponding angles and alternate interior angles. Therefore, this lesson will also revisit terms and concepts previously introduced in Practice Lesson 7, Parallel Lines. These terms are **parallel, corresponding angles, alternate interior angles** and **transversal.** In this lesson, students will construct lines that are parallel, give proof by construction, and use theorems as well as construction to prove that two lines are parallel using **direct proof** (a proof that starts with a given statement and uses logical reasoning to arrive at a conclusion or the statement to be proven) or **indirect proof** (sometimes called **proof by contradiction** - which is a proof that starts with a negation or opposite of the statement to be proven and uses logical reasoning to arrive at a conclusion that is a contradiction to the given statement proving the statement is false). It is helpful to follow these strategies when writing an indirect proof:

1. Identify the conclusion (the statement you want to prove is true)
2. Assume the statement is false
3. This false assumption will be the first statement of your indirect proof
4. Use logical reasoning based on your assumption
5. If the conclusion you reach is a contradiction, your original statement must be true

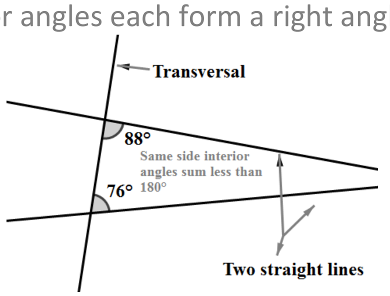
Practice

1. The following is Euclid's Fifth Postulate (The Elements, circa 300 BC).

> *If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, will meet on that side on which are the angles less than two right angles.*
>
> ~ History of the Parallel Postulate, Florence P. Lewis

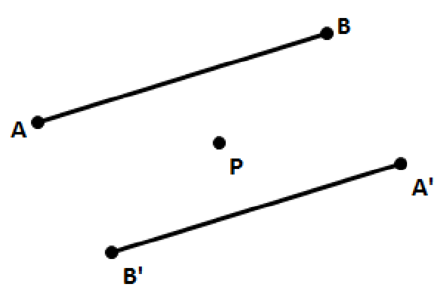
Given your previous experience (particularly Lesson 7), in your own words using previously learned vocabulary, restate Euclid's Fifth Postulate in a statement you and your peers can understand. Draw a diagram to demonstrate your understanding.

Solution: Answers will vary but should be similar to: Euclid is referring to the situation that two lines (when crossed by a transversal) cannot be parallel if the sum of same side interior angles is less than 180°; i.e., two lines crossed by a transversal ***are*** parallel if the same side interior angles each form a right angle.

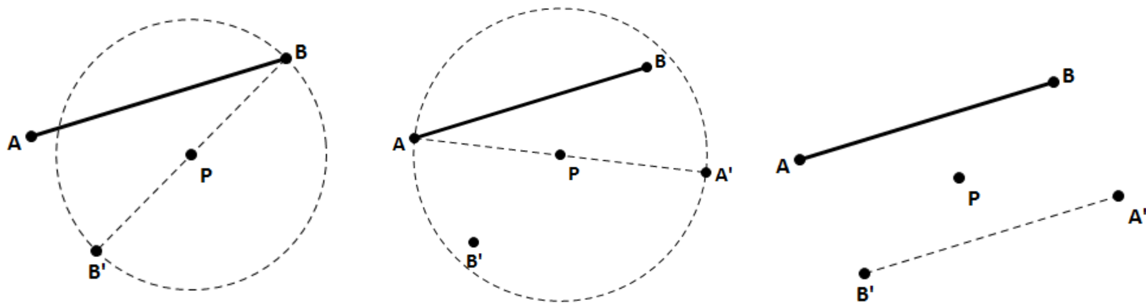


2. Rotate the line segment $\overline{AB}$ 180° counterclockwise about point P ($\mathbf{R_{P,180^\circ}}\ (\overline{AB}) = \overline{AB'}$).
Make sure you label the image correctly. Describe your process.

Solution: The rotation of line segment $\overline{AB}$ 180° about point P results in the diagram at right.

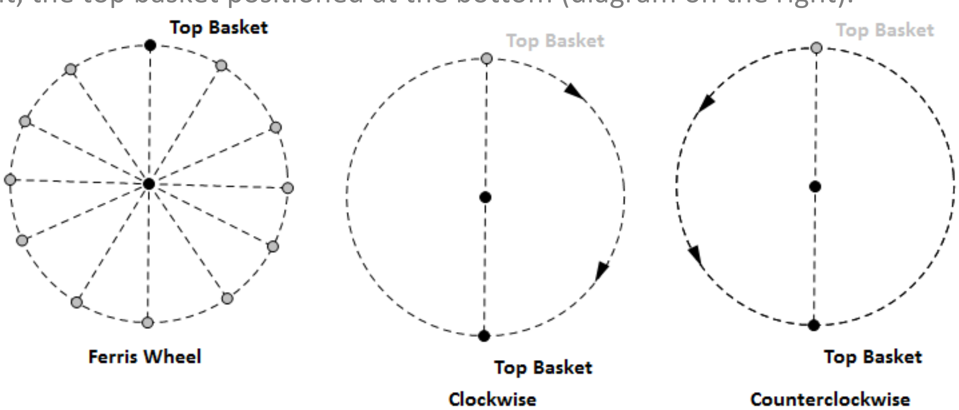


Since the 180° is the angle measure of a straight line (and is one-half turn of a circle) I need a straight line connecting the endpoints of the line segment $\overline{AB}$ that passes through Point P (the center of rotation). Since a rotation is a rigid motion that preserves distance, the image under the rotation will be the same length as the pre-image. Since the radius of a circle is one half the diameter, P will be the midpoint of the diameter for each straight line. First, I constructed a circle with P as the center and BP the radius. Then I drew a straight line from B through P until it intersected the circle and labeled the intersection B'. I then constructed a circle with P as the center and AP the radius. I drew a straight line from A through P until it intersected the circle and labeled the intersection A'. I connected B' to A' with a straight line segment.

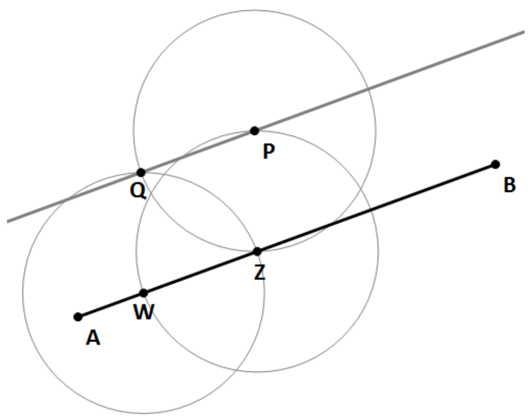


3. Refer to your rotation in problem 2. Compare a rotation of 180° clockwise to your original rotation (counterclockwise). Will the result be the same or different. Discuss your reasoning and give an example to support your reasoning.

Solution: A rotation of 180° in either direction will produce the same result. Fact: 180° is one-half a complete rotation of a circle (360°). Consider a Ferris Wheel with a basket stopped directly at the top. A forward movement (clockwise) of one-half of a complete rotation results in the top basket being positioned at the bottom (the middle diagram). A reverse movement (counterclockwise) of one-half a complete rotation produces the same result, the top basket positioned at the bottom (diagram on the right).

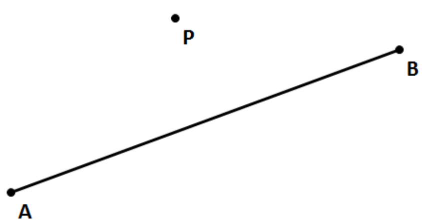


4. Recall from the introduction to this lesson, *"Given any straight line and a point not on it, there "exists one and only one straight line which passes" through that point and never intersects the first line, no matter how far they are extended."* This problem asks you to think about what makes two lines parallel and how you would construct the parallel lines with a compass and straightedge. There are actually a few ways parallel lines can be constructed using a compass and straight edge. One method is to construct circles shown here:

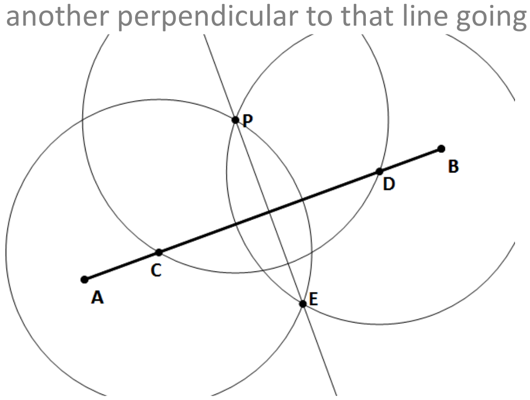


Construct three circles one with center P that intersects $\overline{AB}$ (call this Point Z), another circle with center Z and radius PZ that intersects $\overline{AB}$ (call this intersection Point W), and a final circle with center W and radius WZ that intersects the first circle with center P (call this intersection Point Q). Draw a line through Q and P. This line is parallel to $\overline{AB}$.

Can you describe and construct two additional constructions that will produce a line parallel to $\overline{AB}$ and passes through Point P?

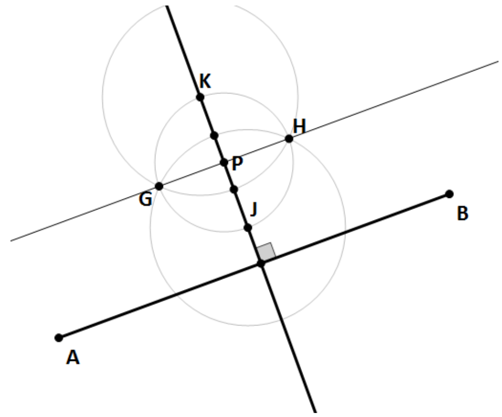


Solution One: construct perpendicular lines (one through P and perpendicular to $\overline{AB}$, and another perpendicular to that line going through P.

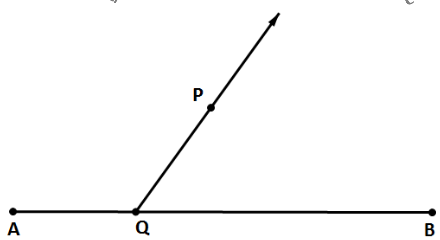


Part I: construct a circle with center P, so that it intersects $\overline{AB}$. Label the intersections C, D respectively. With center C construct a circle with radius CP. Construct a circle with center D and radius DP. Where these two circles intersect, label E. Draw a line through PE. This is perpendicular to $\overline{AB}$ (image above left).

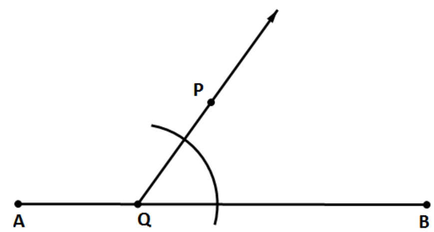
Part II: construct a circle with center P that intersects PE. Label the intersections J, K respectively. Construct circle with center J and extend the radius a little past P. Keeping the same radius, construct circle with center K. Where these two circles intersect, label G and H respectively. Draw a line through GH. It should pass through P and is parallel to $\overline{AB}$.



Solution Two: Draw a line from $\overline{AB}$ through P forming and angle with vertex Q as shown in (1). Copy the angle with P as the new vertex of the copied angle. To Copy the angle: With compass at Q, draw an arc across $\overline{PQ}$ through $\overline{AB}$ as shown in (2).

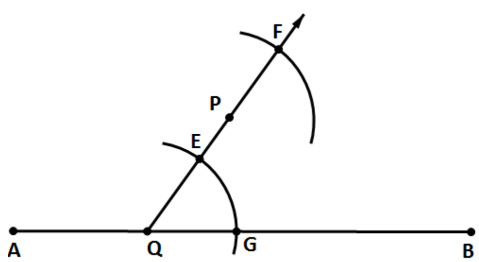


(1)

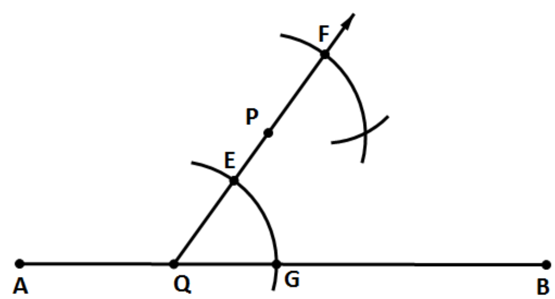


(2)

Without adjusting compass, place point on P and draw an arc across $\overline{PQ}$ as shown in (3). Label the intersections of the arcs with both line segments. With the point of the compass at point E, adjust width to point G. Without adjusting the compass, place on point F and draw an arc across the top arc as shown in (4).

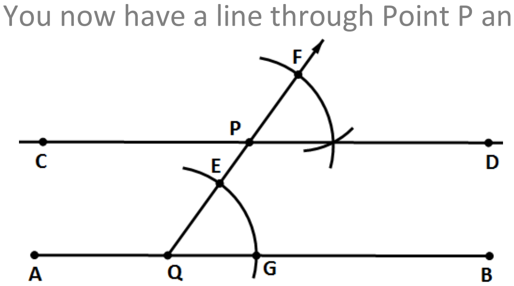


(3)



(4)

With a straightedge, draw a line through P and the intersection of the two arcs, as shown in (5). You now have a line through Point P and parallel to $\overline{AB}$.



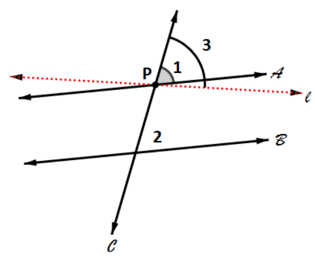
(5)

5. Given the final construction in problem 4, how can you be sure the lines are parallel?

Solution (this is not a proof): $\overline{QF}$ is a transversal to $\overline{AB}$ and $\overline{CD}$. Since ∠EQG ≅ ∠FPD by construction (copied angle). By the corresponding angles theorem converse, if a transversal intersects two lines such that the corresponding angles are congruent, then the lines are parallel.

6. From Practice Lesson 7 we learned the **Corresponding Angles Theorem**: If two lines are crossed by a transversal and the two lines are parallel, then their corresponding angles are congruent. In this proof we will use the **converse** of the **Corresponding Angles Theorem**: If the corresponding angles formed by two lines and a transversal are congruent, then the lines are parallel. The following is a **indirect proof** proving the Corresponding Angles Theorem.

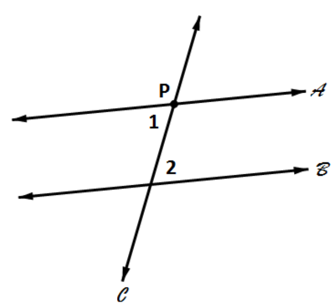
Given: line $\mathcal{A}$ // line $\mathcal{B}$; line $\mathcal{C}$ is a transversal of lines $\mathcal{A}$ & $\mathcal{B}$.
Prove: ∠1 ≅ ∠2



Assume ∠1 & ∠2 are **not** congruent. Then there must be another line through P with a corresponding angle congruent to ∠2. Let that line be line ℓ. Let the angle line ℓ makes with the transversal be ∠3. Now ∠2 ≅ ∠3. By the Corresponding Angles Theorem Converse, line ℓ // line $\mathcal{B}$. But now we have a contradiction since line $\mathcal{A}$ // line $\mathcal{B}$. By the Parallel Postulate, there can be only one line passing through P parallel to line $\mathcal{B}$. Therefore the assumption is false and ∠1 ≅ ∠2 must be true.

Your Task: prove the following:

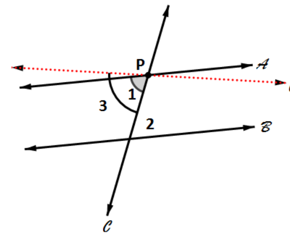
Given: line $\mathcal{A}$ // line $\mathcal{B}$; line $\mathcal{C}$ is a transversal of lines $\mathcal{A}$ & $\mathcal{B}$.
Prove: ∠1 ≅ ∠2



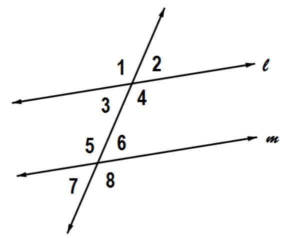
Solution: ∠1 & ∠2 are alternate interior angles. The proof will essentially be the same as in the example except the theorems will be replaced by the alternate interior angles theorem and its converse. Let's begin.
State the two relevant theorems for reference.
The **Alternate Angles Theorem:** If two lines are cut by a transversal and the two lines are parallel, then the alternate interior angles are congruent. **Alternate Angles Theorem Converse:** If the alternate angles formed by two lines and a transversal are congruent, then the lines are parallel.

Begin the proof:

Construct a line ℓ passing through P and let the angle it creates with transversal $\mathcal{C}$ be $\angle 3$. Now $\angle 2 \cong \angle 3$. By the Alternate Angles Theorem Converse, line ℓ // line $\mathcal{B}$. But now we have a contradiction since line $\mathcal{A}$ // line $\mathcal{B}$. By the Parallel Postulate, there can be only one line passing through P parallel to line $\mathcal{B}$. Therefore the assumption is false and $\angle 1 \cong \angle 2$ must be true.



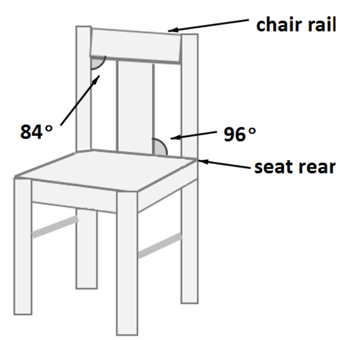
7. Use a two-column format and the diagram at right to prove the following:
   Given: $\angle 1 \cong \angle 8$
   Prove: $\ell // m$



Solution:

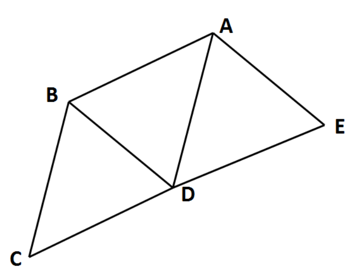
Statements	Reasons
$\angle 1 \cong \angle 8$	Given
$\angle 8 \cong \angle 5$	Vert. $\angle$'s
$\angle 1 \cong \angle 5$	Transitive Property
$\ell // m$	Corresponding Angles Theorem Converse

8. Is it possible to prove that the chair rail and the seat rear of the chair at right are parallel? Support your answer with postulates or theorems.



Solution: Yes. The angle made by the chair rail and chair back is 84° and the angle made by the seat rear and the chair splat is 96° which make these angle supplementary and same side interior angles (or consecutive angles). By the Consecutive Angles Theorem Converse, the chair rail and the seat rear are parallel.

9. Refer to the diagram at right.
   Given $\angle ABC \cong \angle ADC$
   $m\angle DAB + m\angle ABC = 180$
   Prove: $\overline{BC} // \overline{AD}$



Solution:

$\angle ABC \cong \angle ADC$	Given
$m\angle DAB + m\angle ABC = 180$	Given
$m\angle ABC = m\angle ADC$	Def. $\cong \angle$'s
$m\angle DAB + m\angle ADC = 180$	Substitution
$\angle DAB$ and $\angle ABC$ are supp. $\angle$'s	Def. of supplementary
$\overline{BC} // \overline{AD}$	Consecutive Angles Theorem Converse

Practice Lesson 19 Constructing Sequences of Rigid Motions

Objectives

- Use function notation to describe sequences of rigid motions
- Determine whether figures are congruent by analyzing rigid motion
- Explain why figures are congruent or not congruent

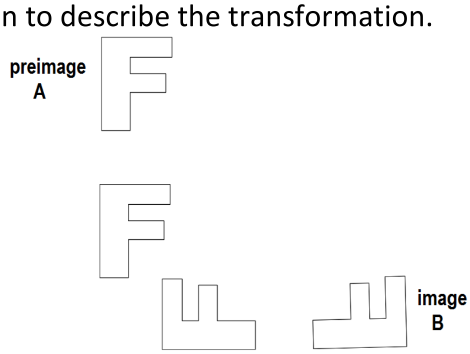
New Vocabulary: congruence correspondence

Introduction

In this lesson, students will revisit rigid motion transformations and study them in terms of congruence and correspondence. Students have an basic understanding of both congruence and correspondence but this lesson will go deeper bringing their understanding further by being more comprehensive. **Congruence** is defined as the existence of a finite composition of rigid motions that maps one figure onto another. A **Correspondence** is a function, and with respect to plane figures, pairs vertices to vertices, angles to angles, and sides to sides of one figure to the another, but not necessarily congruent figures or congruent parts. There is no requirement of a rigid motion in terms of correspondence like there is with congruence (i.e., you don't need to have a rigid motion to produce a correspondence between the parts of two figures). In this lesson, students will analyze and construct sequences of rigid motions and describe them with function notation. For example, a sequence of rigid motions composed of a reflection about BC, a translation along vector ω, and 90° rotation about point B for ΔABC that produces Δ EFG would be written: $(r_{\overline{BC}}\,(T_{\vec{\omega}}(R_{C,90^\circ}(\Delta ABC)))) = \Delta EFG$ (Remember to write the function notation in reverse order).

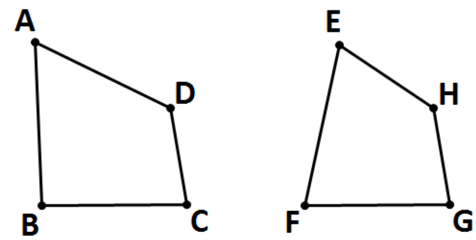
Practice

1. Below is a sequence of rigid motions of pre-image A that produces image B. Use function notation to describe the transformation.



Solution: there is a translation, rotation, and then a reflection. Label the vector ω, label the center of rotation point A and label the line of reflection $\overline{CD}$. The notation therefore, would be $(r_{\overline{CD}}\,(R_{A,90^\circ}(T_{\vec{\omega}}(\text{pre-image A})))) = \text{image B}$.

2. Explain why quadrilateral EFGH is not congruent to quadrilateral ABCD.



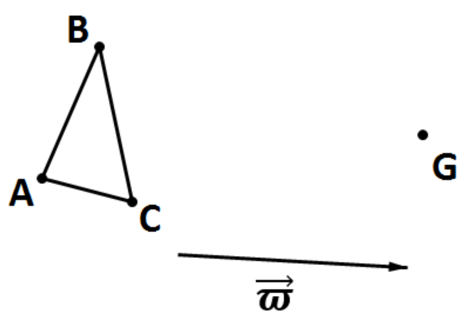
Solution: the definition of congruence states that there is a series of rigid motion transformations that maps one figure onto another. This means that a figure's parts are mapped: line segments to line segments of equal length, angles to angles of equal measure, points to corresponding points. The quadrilateral ABCD is not congruent to quadrilateral EFGH because there is no rigid motion that can produce EFGH from ABCD because AD ≠ EH.

3. Give the proper sequence of rigid motions performed on ΔABC that produces ΔJKL by analyzing the function composition.

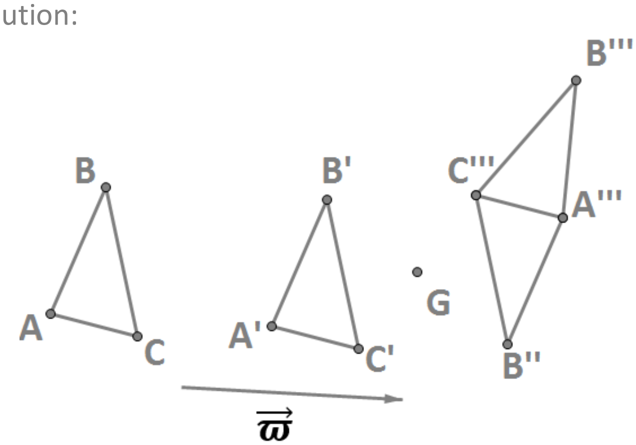
$$(R_{P,35^\circ}\ (r_m(T_{\vec{v}}\ (R_{Q,220^\circ}\ (\Delta ABC))))) = \Delta JKL$$

Solution: ΔABC is rotated 220° about point Q, then translated along vector υ, reflected across line m and then rotated about point P 35° to produce ΔJKL.

4. With a compass and straightedge, perform the following composition of rigid motions. Using the diagram and vector given. $(r_{\overline{A''C''}}\ (R_{G,180^\circ}\ (T_{\vec{\varpi}}\ (\Delta ABC)))) = \Delta A'''B'''C'''$ Show each step.



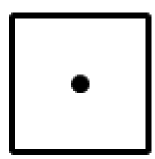
Solution:

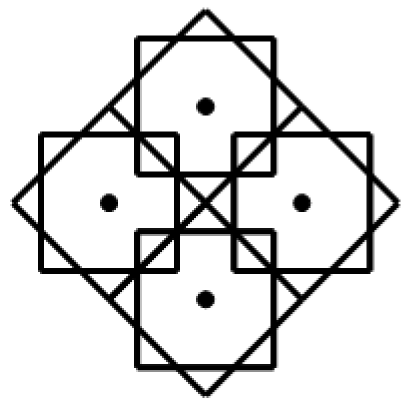


5. Explain how $\Delta A'''B'''C'''$ can be congruent to ΔABC in problem 4.

   Solution: $\Delta A'''B'''C'''$ is congruent to ΔABC because there is a series of rigid motions that takes ΔABC to $\Delta A'''B'''C'''$ which means angle is mapped to angle of equal measure and side is mapped to side of equal length.

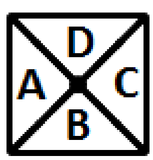
6. The design below right is the result of three rigid motions of the square and dot below left. Can you name them?





   Solution: a rotation about the center dot then two reflections.

7. The square below consists of four congruent triangles, ΔA, ΔB, ΔC, ΔD. Name two different sequences of two rigid motions that will produce ΔD from ΔA.



   Solution: A 90° rotation of ΔA results in ΔB. Then a reflection of ΔB about the center will produce ΔD; or, a reflection of ΔA will produce ΔC then a 90° of ΔC will produce ΔD.

8. Explain why the triangles in problem 7 are congruent.

   Solution: Because a sequence of rigid motion transformations maps each triangle onto the other.

Practice Lesson 20 Congruence and Rigid Motion

Objectives

- describe a correspondence in terms of congruence
- identify corresponding parts of congruent figures
- Use function notation to describe rigid motion transformations

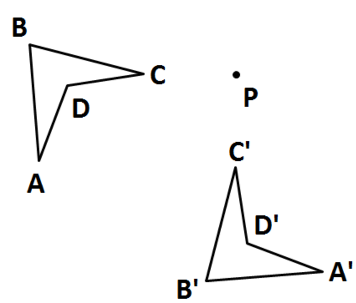
New Vocabulary: CPCTC

Introduction

A correspondence is a comparing of parts. In terms of a congruence however, the comparison goes much deeper. In fact, a rigid motion or sequence of rigid motions produces a one-to-one correspondence of all parts of one figure to another. This means that two or more figures' corresponding parts are also congruent. When referring to "parts" of a figure we usually mean the vertices, angles, and sides. NOTE: in this lesson, when identifying correspondences, if the vertex is labeled, there is no need to list the correspondence for angles. Students may use an arrow (→) to represent the "correspondence" rather than writing out the word many times. The phrase **Corresponding Parts of Congruent Triangles are Congruent (CPCTC)** is commonly used when proving triangles congruent. Since a composition of rigid motions maps corresponding parts to corresponding parts of equal measure, CPCTC is basically a restatement of the definition of congruence. In this lesson, students will identify correspondences of congruent and non-congruent figures.

Practice

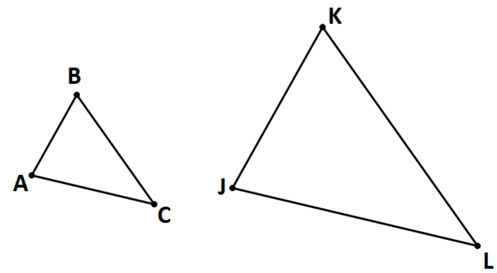
1. In the following diagram, quadrilateral ABCD has been rotated about point P to produce quadrilateral A'B'C'D'. Tell how many and list all of the correspondences between the parts of the quadrilaterals. . You may also refer to the vertex by its letter label (for example, A for angle A).



Solution: There are 8 correspondences with these two quadrilaterals.

vertices	**sides**
A→A'	**AB→A'B'**
B→B'	**BC→B'C'**
C→C'	**CD→C'D'**
D→D'	**DA→D'A'**

2. State the correspondences of the following figures and state whether they are congruent or not. If not congruent, explain why not.

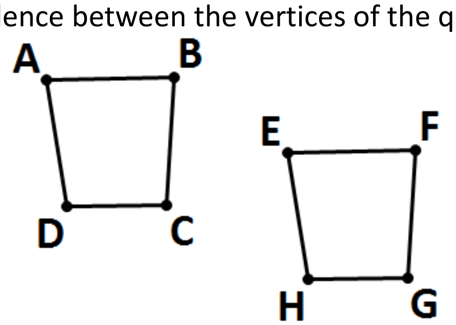


Solution:

vertices	sides
A→J	AB→JK
B→K	BC→KL
C→L	AC→JL

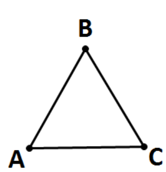
The triangles are not congruent. There is not a sequence of rigid motions that will produce ΔJKL from ΔABC.

3. The quadrilateral ABCD is congruent to quadrilateral EFGH.
   a. What type of correspondence exits for these quadrilaterals and explain what it means.
   b. List the correspondence between the vertices of the quadrilaterals.



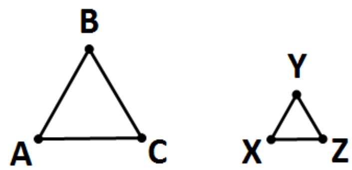
Solution:
a. there is a one-to-one correspondence between these quadrilaterals - this means that a sequence of rigid motions has produced quadrilateral EFGH from quadrilateral ABCD.
b. A→E, B→F, C→G, D→H

4. ΔABC is an equilateral triangle. How many one-to-one correspondences are there for this triangle?



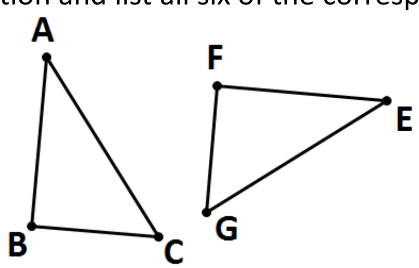
Solution: there are six one-to-one correspondences for equilateral triangles

5. In the diagram below, both triangles are equilateral triangles. How many one-to-one correspondences are there between the two triangles? List the one-to-one correspondences.



Solution: the angles are congruent in similar equilateral triangles but the sides are not therefore, there are three one-to-one correspondences between ΔABC and ΔXYZ
The one-to-one correspondences are:
A→X, B→Y, C→Z

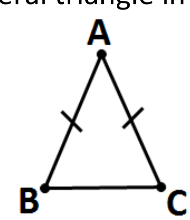
6. ΔABC has been rotated 270° about point Q to produce ΔEGF. Write the transformation in function notation and list all six of the correspondences.



Solution: $R_{Q,270°}(\Delta ABC) = \Delta EFG$.

vertices	sides
A→E	AB→EF
B→F	BC→FG
C→G	AC→EG

7. The triangle shown below is an isosceles triangle. How does the correspondences of this triangle differ from the equilateral triangle in problem 4?



Solution: Since only two angles and two sides are congruent in an isosceles triangle, there will only be four one-to-one correspondences for an isosceles triangle as opposed to the six one-to-one correspondences of an equilateral triangle.

Practice Lesson 21 Correspondence and Transformations

Objectives

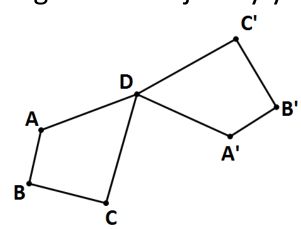
- Identify correspondences resulting from rigid motion transformations
- Write transformations and composition of transformations in function notation

Introduction

This lesson extends Practice Lesson 20. Students continue to identify correspondences and determine congruencies. Students will identify rigid motion transformation and write them in function notation.

Practice

1. Identify the transformation for quadrilateral ABCD that has produced quadrilateral A'B'C'D, write the transformation in function notation, and list the corresponding vertices and sides. State whether the figures are congruent and justify your answer.

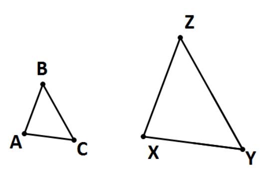


Solution: The quadrilateral has been rotated about the vertex D: ($R_{D,\,135^\circ}$(ABCD)) = A'B'C'D

vertices	sides
A→A'	AB→A'B'
B→B'	BC→B'C'
C→C'	CD→C'D
D→D	DA→DA'

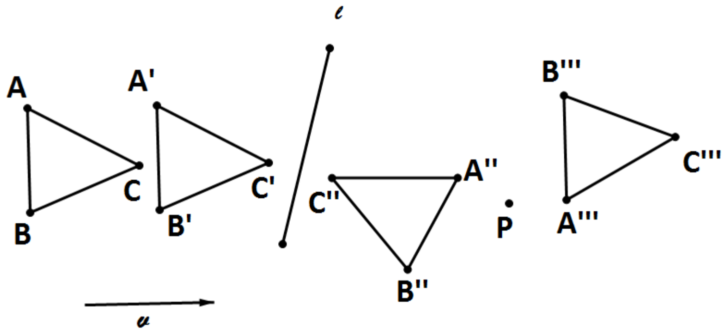
The quadrilaterals are congruent because a rigid motion maps a figure onto itself where all the sides and angles of the quadrilateral will map onto their corresponding side or corresponding angle which proves they are congruent.

2. ΔXYZ is twice the size of ΔABC. The angles are the same size. Is there a congruence correspondence between the two triangles. Explain your reasoning.



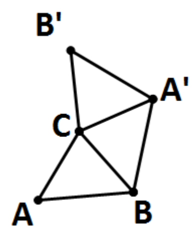
Solution: There is not a congruence correspondence between the two triangles because there is not a rigid motion that maps ΔABC onto ΔXYZ where the sides are the same length.

3. Refer to the figure below.
   a. Write the transformations in function notation.
   b. Is ΔABC congruent to ΔA'''B'''C'''? If so, state the correspondences.



a. $(R_{P,210^\circ}(r_\ell(T_{\vec{v}}(\Delta ABC))) = \Delta A'''B'''C'''$
b. Yes ΔABC≅ΔA'''B'''C'''. A→A''', B→B''', C→C''', AB→A'''B''', AC→A'''C''', BC→B'''C'''.

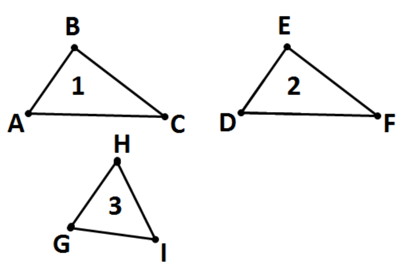
4. $(r_{CB}(r_{CB'}(\Delta ABC)$ describes the transformation of ΔABC. State the correspondence and congruence of the triangles.



Solution:

A→A'	AB→A'B→A'B'	ΔABC ≅ A'BC ≅ A'B'C
B→B'	AC→A'C	
C→C	CB→CB'	

5. Refer to the three triangles below. Δ1 ≅ Δ2; AB ≅ DE ≅ GH. List all of the congruent correspondences.



Solution:

A→D AB→ DE→GH
B→E BC→ EF
C→F AC→DF

Practice Lesson 22 Congruence Criteria for Triangles SAS

Objectives
- Show how congruence criteria for triangles follow from rigid motion transformations
- Prove two triangles are congruent using the SAS Criteria for proving triangles congruent

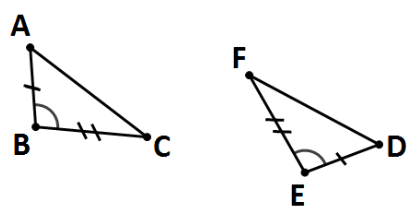
New Vocabulary: SAS - Side - Angle - Side SAS criterion

Introduction

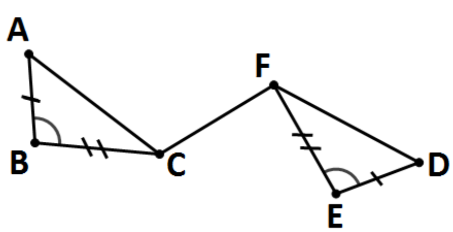
In this lesson, students will show that the criteria for proving triangles congruent follows from the definition of congruence in terms of rigid motion. In this lesson, students are introduced to the first of four congruence criteria, **SAS** (Side Angle Side), in proving triangle congruency. Students will use the **SAS criterion** to prove that two triangles are congruent if two sides and the angle in between them are congruent. Recall, for triangles to be congruent, a sequence of rigid motions maps corresponding part to corresponding parts of equal measure (i.e., after a sequence of rigid motions, vertices must coincide, angles of equal measure must coincide, sides of equal length must coincide).

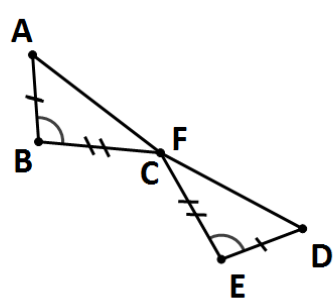
Practice

1. Is it possible that the two triangles below are congruent? If so, demonstrate how they could be congruent.

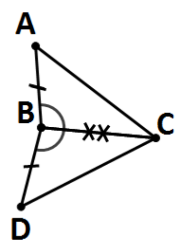


Solution: The triangles are congruent by the congruence criterion SAS. There is a composition of rigid motions that maps congruent parts onto one another. Rigid motions preserve side length and angle measure. To show the triangles are congruent, $\triangle DEF$ has been translated, reflected and rotated as shown below:
Step 1: Mark the translation vector connecting point F to point C and translate. Point F coincides with point C.

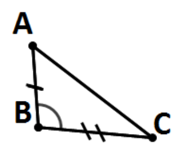




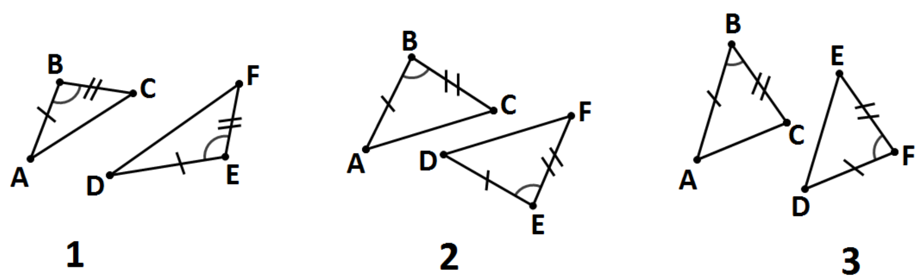
Step 2: Rotate ΔDEF about F so that FE coincides with BC. The sides are congruent since rotations preserve side lengths.



Step 3: Reflect ΔDEF over EF. The triangles coincide and are therefore congruent since reflections preserve angle measures and side lengths. ΔABC≅ ΔDEF.

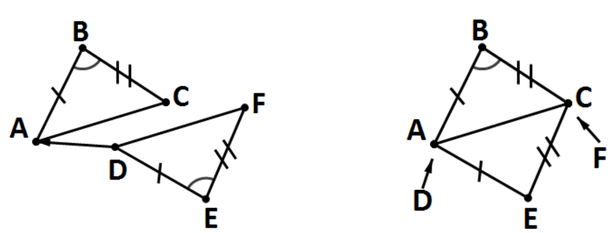


2. The diagram below shows three pair of triangles, all marked with SAS congruence criterion. Only one pair is congruent. Determine which pair is congruent by finding the composition of rigid motion transformations that makes it so. Demonstrate the congruency by performing the rigid motion composition.

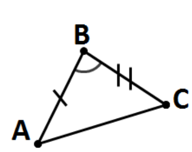


Solution: The triangles in set 2 are congruent. The composition of rigid motion to make SAS true is a translation and a reflection.

Step 1: Draw the translation vector connecting point A to point D. Translate ΔDEF along the translation vector so that point D coincides with point A. (Consequently, F coincides with point C).

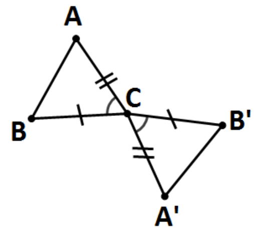


Step 2: Reflect ΔDEF over the line DF so that it coincides with ΔABC proving SAS is true.



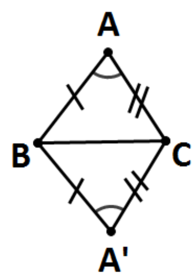
3. What rigid motion transformation would prove the SAS criteria true for the diagrams below?

   a. shared vertex



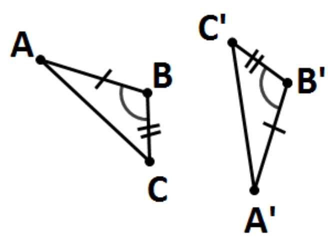
   Solution a.: A rotation would map side B’C to BC, angle C to angle C and side A’C to AC proving SAS to be true.

   b. shared side



   Solution b.: A reflection would map side A’B to AB, angle A’ to angle A and side A’C to AC proving SAS to be true.

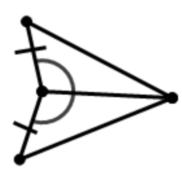
   c. two distinct triangles (no shared parts)



   Solution b.: A reflection, rotation, or translation (or any combination of the three) would map side A’B’ to AB, angle B’ to angle B and side A’C’ to AC proving SAS to be true.

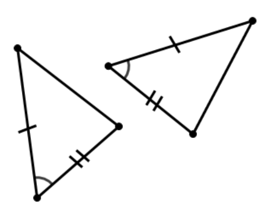
4. Tell whether the following triangles make the SAS criterion true. State a reason for your response.

   a.



   Solution a: Yes, the SAS criterion is true. There is a shared side as a result of a reflection.

   b.



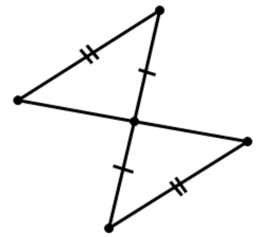
   Solution b: Yes, the SAS criterion is true. A composition of rigid motions (a rotation and translation) maps the triangles’ corresponding parts onto each other.

c.

Solution c: No, the angle is not between the two congruent sides, therefore, SAS cannot be proved true.

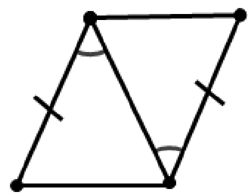
d.

Solution d: No, although vertical angles are congruent here, it is not between the congruent sides, therefore SAS criterion cannot be proved true.



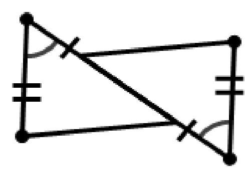
e.

Solution e: Yes, the SAS criterion holds true. There is a composition of rigid motions (two reflections, then rotation) that maps one triangle onto the other, therefore, the SAS criteria can be proved true.



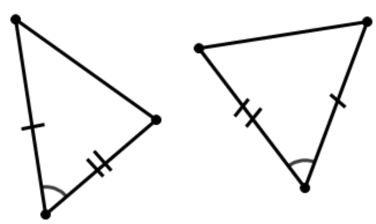
f.

Solution f: Yes, the SAS criterion holds true. There is a composition of rigid motions (rotation, translation) that maps one triangle onto the other proving the SAS criterion true.



g.

Solution g: No, there is not a composition of rigid motions that could map the corresponding parts of one triangle onto the corresponding parts of the other triangle.



Practice Lesson 23 Base Angles of Isosceles Triangles

Objectives
- Prove two isosceles triangles are congruent using rigid motion
- Prove two isosceles triangles are congruent using the SAS Criterion

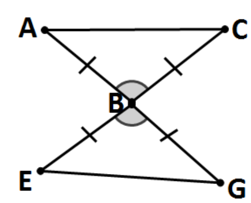
New Vocabulary: base angles of an isosceles triangle

Introduction

In this lesson, students will show how the SAS Criterion follows from rigid motion transformation. Students will widen their experience with proof, this time with isosceles triangles. The terms **midpoint**, **isosceles triangle**, and **SAS** will be revisited. Students must be familiar with the properties of the isosceles triangle learned in middle school such as an isosceles triangle has two congruent sides and two congruent angles. The **base angles of an isosceles triangle** are the two congruent angles of the isosceles triangle. Key terms and theorems used previously will also be used in the proofs of this lesson. To be successful in this lesson, students should review CPCTC, SAS, vertical angles theorem, alternate interior angles theorem, definition of isosceles, definition of perpendicular and the properties of congruence learned in Practice Lesson 9.

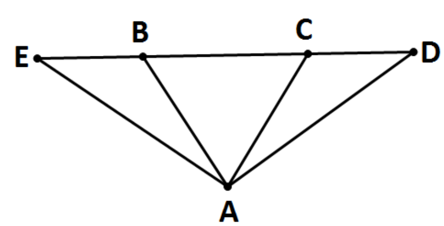
Practice

1. ΔABC has been reflected over point B to produce ΔEBG. Using what you know about rigid motion transformations, prove ΔEBG $\cong$ ΔABC.



Solution: Since a reflection is a rigid motion and rigid motion preserve distance, then $\overline{AB} \cong \overline{BE}$.
Also, rigid motion preserves angle measure,
so $\angle$ABC $\cong$ $\angle$EBG. And finally, since rigid motion preserves distance, $\overline{CB} \cong \overline{GB}$. By the SAS criterion, ΔEBG $\cong$ ΔABC.

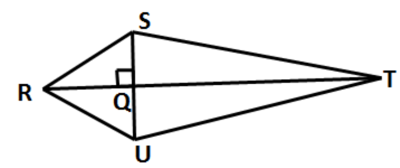
2. ΔABC is isosceles. $\overline{AE} \cong \overline{AD}$. $\angle$EAB $\cong$ $\angle$DAC.
Prove: ΔEAB $\cong$ ΔDAC.



Solution:

Statements	Reasons
1. ΔABC is isosceles	1. Given
2. $\overline{AB} \cong \overline{AC}$	2. Def. of isosceles
3. $\angle$EAB $\cong$ $\angle$DAC	3. Given
4. $\overline{AE} \cong \overline{AD}$	4. Given
5. ΔEAB $\cong$ ΔDAC	5. SAS Criterion

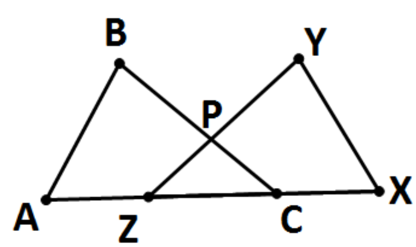
3. Q is the midpoint of $\overline{SU}$. $\overline{RT} \perp \overline{SU}$
Prove: ΔRST $\cong$ ΔRUT



Solution:

Statements	Reasons
1. Q is the midpoint of $\overline{SU}$. $\overline{RT} \perp \overline{SU}$	1. Given
2. $\overline{SQ} \cong \overline{UQ}$	2. Def. of midpoint
3. $\overline{RQ} \cong \overline{RQ}$	3. Reflexive Property
4. $\angle$SQR and $\angle$UQR	4. Def. of perpendicular
5. $\angle$SQR $\cong$ $\angle$UQR	5. All right angles are congruent
6. ΔSQR $\cong$ ΔUQR	6. SAS Criterion
7. $\overline{SR} \cong \overline{UR}$; $\angle SQR \cong \angle UQR$	7. CPCTC
8. $\overline{TR} \cong \overline{TR}$	8. Reflexive Property
9. ΔRST $\cong$ ΔRUT	9. SAS Criterion

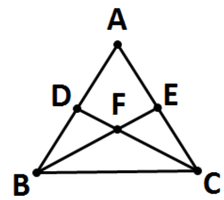
4. Given: $\overline{AZ} \cong \overline{XC}$, $\overline{AB} \cong \overline{XY}$, $\angle$A $\cong$ $\angle$X
Prove: ΔCPZ is isosceles



Solution:

Statements	Reasons
1. $\overline{AZ} \cong \overline{XC}$, $\overline{AB} \cong \overline{XY}$, $\angle$A $\cong$ $\angle$X	1. Given
2. $\overline{ZC} \cong \overline{ZC}$	2. Reflexive Property
3. $AC + \overline{ZC} = \overline{AC}$ $\overline{XC} + \overline{CZ} = \overline{XZ}$	3. Segment Addition Postulate
4. $\overline{AC} \cong \overline{XZ}$	4. Substitution
5. ΔABC $\cong$ ΔXYZ	5. SAS Criterion
6. $\angle ACB \cong \angle XZY$	6. CPCTC
7. ΔCPZ is isosceles	7. Def. of isosceles triangle

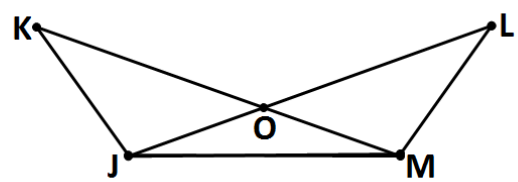
5. ΔBFC is isosceles. $\overline{DF} \cong \overline{EF}$
Prove ΔBDC $\cong$ ΔCEB



Solution:

Statements	Reasons
1. ΔBFC is isosceles. $\overline{DF} \cong \overline{EF}$	1. Given
2. $\overline{BC} \cong \overline{BC}$	2. Reflexive Property
3. $\angle$EBC $\cong$ $\angle$DCB	3. Isosceles Triangle Theorem
4. $\overline{CF} \cong \overline{BF}$	4. Def. of isosceles Δ
5. $\overline{DF} + \overline{CF} = \overline{DC}$ $\overline{EF} + \overline{BF} = \overline{EB}$	5. Segment Addition Postulate
6. $\overline{DC} \cong \overline{EB}$	6. Substitution
7. ΔBDC $\cong$ ΔCEB	7. SAS Criterion

6. Given: $\overline{KJ} \cong \overline{LM}$, $\angle$JKM $\cong$ $\angle$MLJ
Prove: ΔJKM $\cong$ ΔMLJ



Solution:

Statements	Reasons
1. $\overline{KJ} \cong \overline{LM}$	1. Given
2. $\angle$JKM $\cong$ $\angle$MLJ	2. Given
3. $\overline{JM} \cong \overline{JM}$	3. Reflexive Property
4. ΔJKM $\cong$ ΔMLJ	4. SAS Criterion

Practice Lesson 24 — Congruence Criteria for Triangles ASA and SSS

Objectives

- Use the ASA and SSS criteria for proving triangles congruent

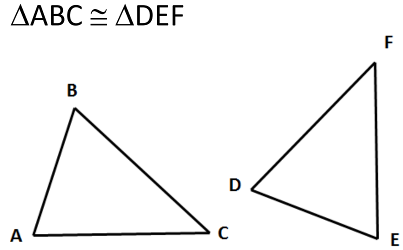
New Vocabulary: ASA - Angle - Side - Angle — ASA Criterion
SSS - Side - Side - Side — SSS Criterion

Introduction

In this lesson, students will investigate two more triangle congruency criteria: **ASA (Angle - Side - Angle)** and **SSS (Side - Side - Side)** criteria. Triangles are proved congruent using the **ASA criterion** when two angles and the side in between them are congruent. Triangles are proved congruent using the **SSS criterion** when all three sides of one triangle are congruent to all three sides of another triangle. Students will be asked to identify criterion for congruency and will also continue to expand their experience with proof using these two congruence criteria. Recall, for triangles to be congruent, a sequence of rigid motions maps corresponding part to corresponding parts of equal measure (i.e., after a sequence of rigid motions, vertices must coincide, angles of equal measure must coincide, sides of equal length must coincide).
Note: Students will be required to used their previous knowledge of unknown angles and SAS criterion to complete the following proofs.

Practice

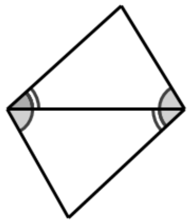
1. Given: $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{BC} \cong \overline{EF}$
   Prove: $\Delta ABC \cong \Delta DEF$

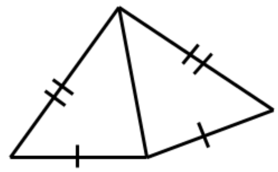


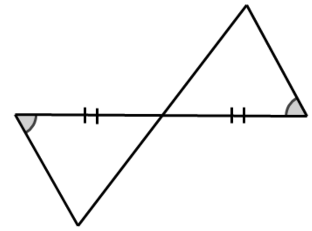
Solution:

Statements	Reasons
1. $\angle B \cong \angle E$	1. Given
2. $\overline{BC} \cong \overline{EF}$	2. Given
3. $\angle C \cong \angle F$	3. Given
4. $\Delta ABC \cong \Delta DEF$	4. ASA Criterion

2. Given the following diagrams, determine the congruence criterion for each set of triangles:

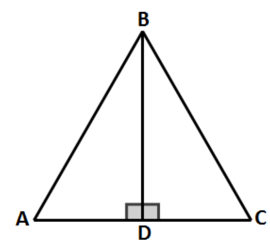
A) 

B) 

C) 

Solution: A) ASA Criterion B) SSS Criterion C) ASA Criterion

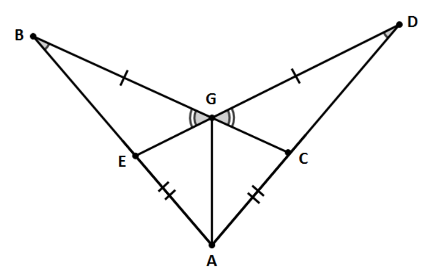
3. Given: $\overline{\text{BD}}$ bisects ∠ABC, $\overline{\text{BD}} \perp \overline{\text{AC}}$,
Prove: $\overline{AB} \cong \overline{BC}$



Solution:

Statements	Reasons
1. $\overline{\text{BD}}$ bisects ∠ABC	1. Given
2. ∠ABD ≅ CBD	2. Angle bisector divides an angle into two congruent angles
3. $\overline{BD} \cong \overline{BD}$	3. Reflexive Property
4. $\overline{\text{BD}} \perp \overline{\text{AC}}$	4. Given
5. ∠BDA & ∠BDC are right ∠'s	5. Def. of ⊥
6. ∠BDA ≅ ∠BDC	6. Right angles are congruent
7. ΔABD ≅ ΔCBD	7. ASA Criterion
8. $\overline{AB} \cong \overline{BC}$	8. CPCTC

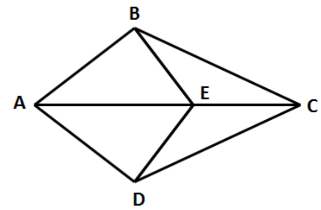
4. Given: ∠B ≅ ∠D, $\overline{BG} \cong \overline{GD}$, $\overline{EA} \cong \overline{CA}$
Prove: ∠EAG ≅ ∠CAG



Solution:

Statements	Reasons
1. ∠B ≅ ∠D	1. Given
2. $\overline{BG} \cong \overline{GD}$	2. Given
3. ∠BGE ≅ ∠DGC	3. Vertical ∠'s are ≅
4. ΔBGE ≅ ΔDGC	4. ASA Criterion
5. $\overline{EA} \cong \overline{CA}$	5. Given
6. $\overline{EG} \cong \overline{CG}$	6. CPCTC
7. $\overline{GA} \cong \overline{GA}$	7. Reflexive Property
8. ΔGEA ≅ ΔGCA	8. SSS Criterion
9. ∠EAG ≅ ∠CAG	9. CPCTC

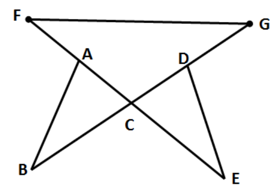
5. Given: $\Delta CEB \cong \Delta CED$, $\Delta ABC \cong \Delta ADC$
   Prove: $\angle EBA \cong \angle EDA$



Solution:

Statements	Reasons
1. $\Delta CEB \cong \Delta CED$	1. Given
2. $\Delta ABC \cong \Delta ADC$	2. Given
3. $\overline{BE} \cong \overline{DE}$	3. CPCTC
4. $\overline{EC} \cong \overline{EC}$	4. Reflexive Property
5. $\overline{AB} \cong \overline{AD}$	5. CPCTC
6. $\Delta EBA \cong \Delta EDA$	6. SSS
7. $\angle EBA \cong \angle EDA$	7. CPCTC

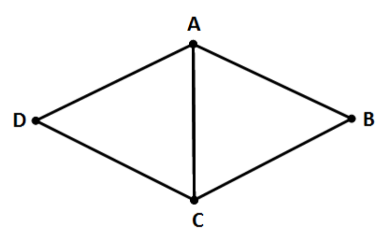
6. Given: $\angle FAB \cong \angle GDE$, $\overline{AC} \cong \overline{DC}$
   Prove: $\Delta ABC \cong \Delta DEC$



Solution:

Statements	Reasons
1. $\angle FAB \cong \angle GDE$	1. Given
2. $\angle FAB$ is supp. to $\angle CAB$; $\angle GDE$ is supp. to $\angle CDE$	2. If two angles form a straight line then they are supplementary
3. $\angle CAB \cong \angle CDE$	3. Supplementary $\angle$'s Theorem
4. $\overline{AC} \cong \overline{DC}$	4. Given
5. $\angle ACB \cong \angle DCE$	5. Vertical $\angle$'s are $\cong$
6. $\Delta ABC \cong \Delta DEC$	6. ASA Criterion

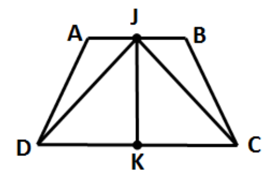
7. Given: $\overline{AC}$ *bisects* $\angle DAB$, $\overline{CA}$ *bisects* $\angle DCB$
   Prove: $\Delta DAC \cong \Delta ABC$



Solution:

Statements	Reasons
1. $\overline{AC}$ *bisects* $\angle DAB$	1. Given
2. $\angle DAC \cong \angle BAC$	2. Def. of angle bisector
3. $\overline{AC} \cong \overline{AC}$	3. Reflexive Property
4. $\overline{CA}$ *bisects* $\angle DCB$	4. Given
5. $\angle DCA \cong \angle BCA$	5. Def. of angle bisector
6. $\Delta DAC \cong \Delta BAC$	6. ASA Criterion

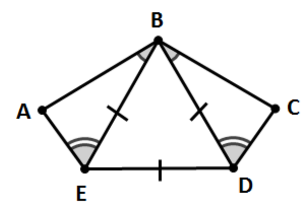
8. Given: $\overline{JK}$ bisects $\overline{AB}$, K is the midpoint of $\overline{DC}$
$\angle A \cong \angle B$, $\overline{AD} \cong \overline{BC}$
Prove: $\angle DKJ \cong \angle CKJ$



Solution:

Statements	Reasons
1. $\overline{JK}$ bisects $\overline{AB}$	1. Given
2. $\overline{AJ} \cong \overline{BJ}$	2. Def. of segment bisector
3. $\angle A \cong \angle B$	3. Given
4. $\overline{AD} \cong \overline{BC}$	4. Given
5. $\Delta ADJ \cong \Delta BCJ$	5. SAS Criterion
6. $\overline{DJ} \cong \overline{CJ}$	6. CPCTC
7. K is the midpoint of $\overline{DC}$	7. Given
8. $\overline{DK} \cong \overline{CK}$	8. Def. of midpoint
9. $\overline{JK} \cong \overline{JK}$	9. Reflexive Property
10. $\Delta DKJ \cong \Delta CKJ$	10. SSS Criterion
11. $\angle DKJ \cong \angle CKJ$	11. CPCTC

9. State whether ΔABE can be proved congruent to ΔCBD and explain your answer.



Solution:
Yes, the triangles are congruent by the ASA criterion.

Practice Lesson 25 Congruence Criteria for Triangles SAA and HL

Objectives

- Use the SAA and HL criteria for proving triangles congruent
- Prove the HL criterion
- Know why using AAA and SSA criteria will not prove triangles congruent

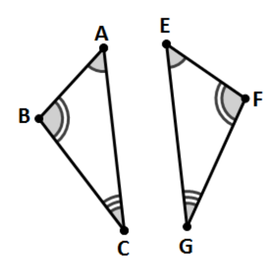
New Vocabulary:	SAA (Side-Angle-Angle)	SAA criterion
	HL (Hypotenuse-Leg)	HL criterion
	AAA (Angle-Angle-Angle)	AAA criterion
	SSA (Side-Side-Angle)	SSA criterion

Introduction

In this lesson, students will investigate two more triangle congruency criteria: **SAA (Side-Angle-Angle)** and **HL Hypotenuse-Leg)** criteria. Triangles are proved congruent using the **SAA criterion** if two angles and the non-included side of one triangle are congruent to the two angles and the non-included side of another triangle, then the two triangles are congruent (which ultimately leads to a variation of the **ASA congruence criterion**). Triangles are proved congruent using the **HL criterion** when the hypotenuse and one leg of a right triangle is congruent to the hypotenuse and one leg of another triangle. Students will also prove the **HL criterion** by using a sequence of rigid motions. Students will also tell why that given two sides and a non-included angle the **SSA criterion** is *not enough* to prove congruence. Additionally, when all three corresponding angles (**AAA criterion**) are equal does *not* prove congruence.

Practice

1. From the information given in the diagram at right, can you prove $\Delta ABC \cong \Delta EFG$? Explain.

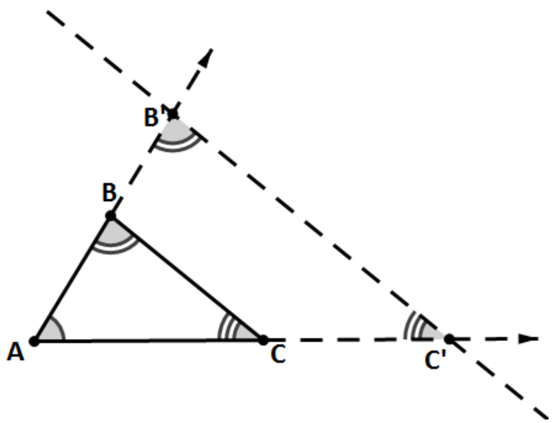


a) yes, by SAS b) yes, by AAA

c) yes, by ASA d) no

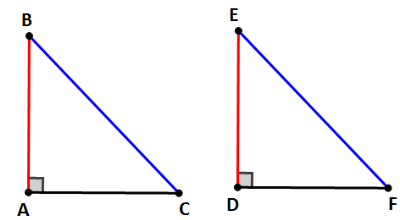
Solution:
d) no. The information marking the triangles shows an AAA criterion, which is not a valid criterion for proving triangle congruency.

2. Explain why AAA is not enough to prove two triangles congruent.

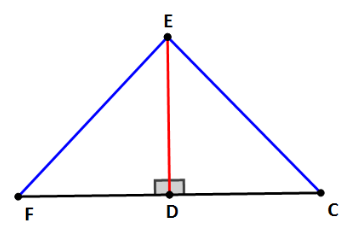


Solution: Consider ΔABC. By extending $\overline{AB}$ and $\overline{AC}$ and then constructing a line parallel to $\overline{BC}$, we can create a triangle with corresponding angles since $\overline{AB}$ and $\overline{AC}$ are transversals to parallel lines. Since corresponding angles are congruent and ∠A is congruent to itself, we have two triangles with AAA congruence but clearly the two triangles are not congruent because they have different side lengths.

3. Given: ΔABC & ΔDEF are right triangles.
Prove the HL criterion for triangle congruence.

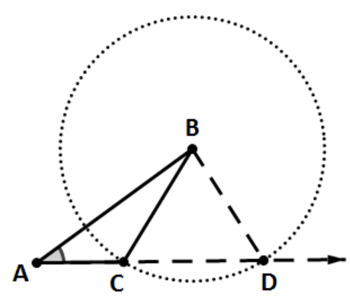


Solution: The HL criterion states: when the hypotenuse and one leg of a right triangle is congruent to the hypotenuse and one leg of another triangle.
Let $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong EF$. by using a sequence of rigid motions (reflection about $\overline{DE}$ and a translation left), we can map ΔDEF onto ΔABC such that an isosceles triangle is formed with the congruent legs ($now\ \overline{ED}$) as an altitude as in the figure below:



Both congruent hypotenuses form an angle with the same straight line ($\overline{FC}$), such that ∠F ≅ ∠ C. Also, the altitude of an isosceles triangle is the angle bisector of the vertex angle, therefore, ∠FED ≅ ∠CED. By SAS, ΔABC ≅ ΔDEF proving the HL congruence criterion for right triangles.

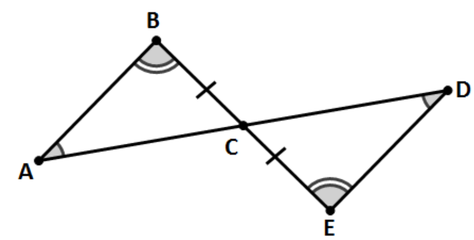
4. Show why SSA criterion is not enough to prove triangles congruence.



Solution: Consider ΔABC and ΔABD with fixed side $\overline{AB}$. $\overline{BC}$ is the radius to circle with center B. Construct another radius to circle B to point D. Since all radii of the same circle are the same length, $\overline{BC} \cong \overline{BD}$. Now we have ∠A ≅ ∠A (reflexive property), $\overline{AB} \cong \overline{AB}$ (reflexive property), $\overline{BC} \cong \overline{BD}$ (all radii to Circle B have the same length) presenting the SSA criterion. But ΔABC and

ΔABD are clearly not congruent since $\overline{AD} > \overline{AC}$. Therefore, the SSA criterion does not prove triangle congruence in this case. *Note: When the fixed angle is acute, SSA does not work for proving triangle congruency, but when the fixed angle is obtuse, SSA can prove triangle congruence since this presents a situation where the arbitrary side can only be one length.*

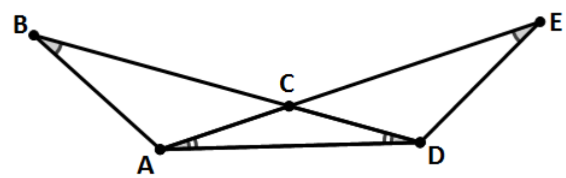
5. Given: C is the midpoint of $\overline{BE}$.
   $\overline{AB} \parallel \overline{DE}$
   Prove: $\Delta ABC \cong \Delta DEC$



Solution:

Statements	Reasons
1. $\angle BAC \cong \angle CDE$	1. Alt. int. $\angle$'s
2. C is the midpoint of $\overline{BE}$	2. Given
3. $\overline{BC} \parallel \overline{CE}$	3. Definition of midpoint
4. $\angle ABC \cong \angle DEC$	4. Alt. int. $\angle$'s
5. $\Delta ABC \cong \Delta DEC$	5. AAS criterion

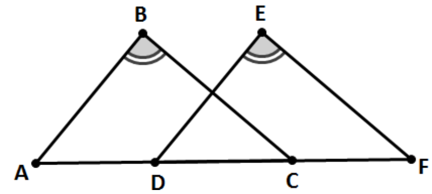
6. Given: $\angle B \cong \angle E$, $\angle A \cong \angle D$
   Prove $\overline{BC} \cong \overline{EC}$



Solution:

Statements	Reasons
1. $\angle B \cong \angle E$, $\angle A \cong \angle D$	1. Given
2. $\overline{AC} \cong \overline{DC}$	2. Definition of Isosceles Δ
3. $\angle BCA \cong \angle ECD$	3.Vert. $\angle$'s are congruent
4. Δ BCA $\cong \Delta$ECD	4. AAS criterion
5. $\overline{BC} \cong \overline{EC}$	5.CPCTC

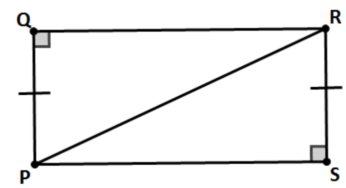
7. Given: $\overline{BC} \parallel \overline{EF}$, ∠B ≅ ∠E, $\overline{AD} \cong \overline{CF}$
Prove: ΔABC ≅ ΔDEF



Solution:

Statements	Reasons
1. $\overline{BC} \parallel \overline{EF}$	1. Given
2. ∠B ≅ ∠E	2. Given
3. ∠C ≅ ∠F	3. Corr. ∠'s
4. $\overline{AD} \cong \overline{CF}$	4. Given
5. $\overline{AD} + \overline{DC} = \overline{FC} + \overline{CD}$	5. Addition prop. of equality
6. $\overline{AC} = \overline{AD} + \overline{DC}$	6. Segment Addition Postulate
7. $\overline{FD} = \overline{FC} + \overline{CD}$	7. Segment Addition Postulate
8. $\overline{AC} = \overline{FD}$	8. Substitution
9. ΔABC ≅ ΔDEF	9. AAS Criterion

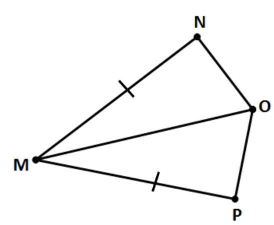
8. Given: $\overline{PQ} \cong \overline{SR}$
Prove: ΔPQR ≅ ΔRSP



Solution:

Statements	Reasons
1. $\overline{PQ} \cong \overline{SR}$	1. Given
2. $\overline{PR} \cong \overline{PR}$	2. Reflexive Property
3. ΔPQR ≅ ΔRSP	3. HL Criterion

9. Given: $\overline{NO} \perp \overline{MN}$, $\overline{PO} \perp \overline{MP}$, $\overline{MN} \cong \overline{MP}$
Prove: ΔMNO ≅ ΔMPO



Solution:

Statements	Reasons
1. $\overline{NO} \perp \overline{MN}$, $\overline{PO} \perp \overline{MP}$	1. Given
2. ∠ONM, ∠OPM are right angles	2. Definition of perpendicular lines
3. ΔMNO & ΔMPO are right triangles	3. Definition of right triangles
4. $\overline{MN} \cong \overline{MP}$	4. Given
5. $\overline{MO} \cong \overline{MO}$	5. Reflexive Property
6. ΔMNO ≅ ΔMPO	6. HL Criterion

Practice Lesson 26 Triangle Congruency Proofs

Objectives

- Prove triangles are congruent using SAS, ASA, SSS, SAA, & HL Criteria

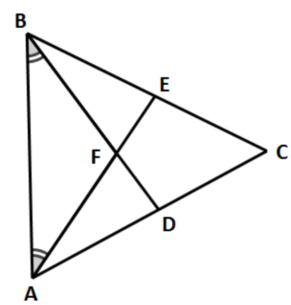
New Vocabulary: Partition Postulate

Introduction

Bringing it all together! Students will practice their skill in solving proofs involving triangle congruency. In this lesson, students will use the various congruency criteria learned in Lessons 22-25. In addition, a new postulate is introduced that is useful in proving triangles congruent: **Partition Postulate**. The **Partition Postulate** states that the whole is equal to the sum of its parts.

Practice

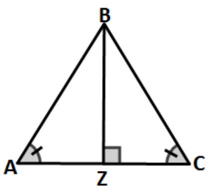
1. Given: $\overline{BC} \cong \overline{AC}$, ∠DBA ≅ ∠EAB
   Prove: $\overline{EF} \cong \overline{DF}$

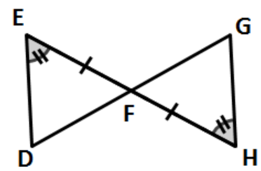


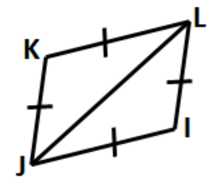
Solution:

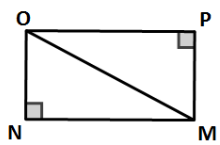
Statements	Reasons
1. $\overline{BC} \cong \overline{AC}$	1. Given
2. ∠ABC ≅ ∠BAC	2. Definition of Isosceles
3. $\overline{AB} \cong \overline{AB}$	3. Reflexive Property
4. ∠DBA ≅ ∠EAB	4. Given
5. ΔEBA ≅ ΔDAB	5. ASA Criterion
6. ∠BEF ≅ ∠ADF	6. CPCTC
7. ∠EFB ≅ ∠DFA	7. Vert. ∠'s are ≅
8. ∠DBA ≅ ∠EAB	8. Given
9. $\overline{BF} \cong \overline{AF}$	9. Definition of Isosceles
10. ΔEFB ≅ ΔDFA	10. AAS
11. $\overline{EF} \cong \overline{DF}$	11. CPCTC

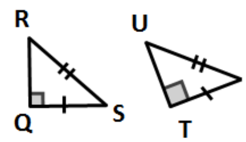
2. The following diagrams are pairs of triangles. Equivalent tick marks indicate a congruence within the pair. State whether each pair can be proven congruent. If congruent, state the criterion and reasons why the triangles are congruent. If you cannot prove congruent, state a reason why they are not congruent.

A) 

B) 

C) 

D) 

E) 

Solution:

A) Yes, congruent. $\overline{BZ} \cong \overline{BZ}$ by the reflexive property. ∠BZC is a right angle, so $\overline{BZ} \perp \overline{AC}$. ∠A ≅ ∠C (given). Therefore, ∠BZA is a right angle. ΔBZA ≅ ΔBZC by the SAA criterion.

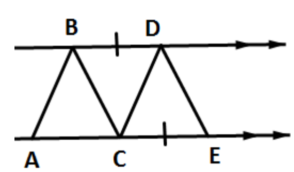
B) Yes, congruent. ∠F ≅ ∠H (given). $\overline{EF} \cong \overline{HF}$ (given). ∠EFD ≅ ∠HFG (vertical angles). ΔDEF ≅ ΔGHF by the ASA criterion.

C) $\overline{KL} \cong \overline{LI} \cong \overline{IJ} \cong \overline{JK}$ (given). $\overline{JL} \cong \overline{JL}$ by the reflexive property. ΔJKL ≅ ΔLIJ by the SSS criterion.

D) No, cannot prove congruent. This pair of right triangles do not have congruent legs marked congruent, so congruence cannot be proved by the HL criterion.

E) No, cannot prove congruent. The triangles are marked indicating the SSA criterion. But SSA is not a valid criterion for which triangles can be proved congruent.

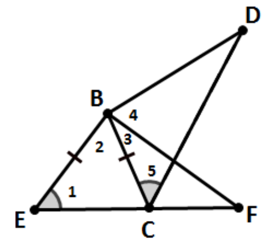
3. Consider the diagram at right.
Given: $\overline{BD} \parallel \overline{AE}$. $\overline{BD} \cong \overline{CE}$.
Prove ΔBCD ≅ ΔEDC.



Solution:

Statements	Reasons
1. $\overline{BD} \cong \overline{CE}$	1. Given
2. ∠BDC ≅ ∠ECD	2. Alternate interior angles
3. $\overline{CD} \cong \overline{CD}$	3. Reflexive Property
4. ΔBCD ≅ ΔEDC	4. SAS criterion

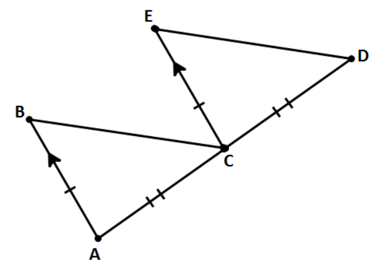
4. Consider the diagram at right.
Given: $\overline{EB} \cong \overline{BC}$, $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 4$
Prove: $\Delta EBF \cong \Delta CBD$



Solution:

Statements	Reasons
1. $\angle 1 \cong \angle 5$	1. Given
2. $\overline{EB} \cong \overline{BC}$	2. Given
3. $\angle 2 \cong \angle 4$	3. Given
4. $\angle 2 + \angle 3 \cong \angle 3 + \angle 4$	4. Angle Addition Postulate
5. $\angle EBF \cong \angle CBD$	5. Partition Postulate
6. $\Delta EBF \cong \Delta CBD$	6. ASA criterion

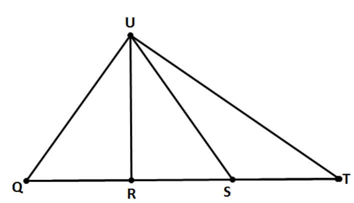
5. Consider the diagram at right.
Given: $\overline{AB} \cong \overline{CE}$, $\overline{AB} \parallel \overline{CE}$, $\overline{BC} \cong \overline{CE}$
Prove: $\Delta CAB \cong \Delta DCE$



Solution:

Statements	Reasons
1. $\overline{AB} \cong \overline{CE}$	1. Given
2. $\overline{AB} \parallel \overline{CE}$	2. Given
3. $\overline{AC} \cong \overline{CD}$	3. Given
4. $\angle CAB \cong \angle DCE$	4. Corresponding Angles
5. $\Delta CAB \cong \Delta DCE$	5. SAS criterion

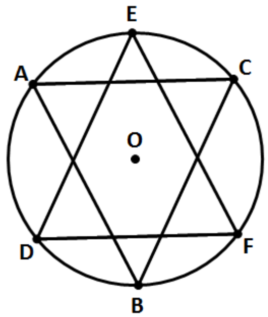
6. Consider ΔQUT shown at right. $\overline{UR} \perp \overline{QT}$. If $\overline{QR} \cong \overline{ST}$, which statement could always be proven? Explain how knowing this statement could help you prove $\Delta RQU \cong \Delta RUS$.



A) $\overline{QS} \cong \overline{TR}$ B) $\overline{QU} \cong \overline{UT}$ C) $\overline{QR} \cong \overline{RS}$ D) $\overline{UQ} \cong \overline{US}$

Solution: A) $\overline{QS} \cong \overline{TR}$.
$\overline{QR} + \overline{RS} = \overline{AC}$ (segment addition)
$\overline{ST} + \overline{RS} = \overline{AC}$ (substitution)
$\overline{QS} \cong \overline{TR}$ (Partition Postulate)
$\overline{UR} \cong \overline{UR}$ (Reflexive Property)
$\angle URQ$ & $\angle URS$ are right angles (Definition of perpendicular)
$\angle URQ \cong \angle URS$ (right angles are congruent)
$\Delta RQU \cong \Delta RUS$ (SAS criterion)

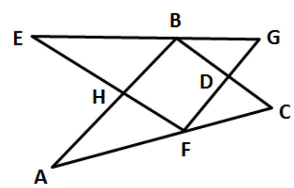
7. The diagram at right is one of three geometric designs a bicycling club, The Knight Riders, is considering for their logo. The club insists on congruent triangles inside the circle. Create a proof that shows the two triangles, $\triangle ABC$ and $\triangle$ DEF are congruent using the following information: Circle O with $\triangle ABC$ and $\triangle DEF$ that are isosceles triangles with vertex angles $\angle B$ and $\angle E$, respectively. $\overline{AB} \cong \overline{FE}$. $\angle CBA \cong \angle DEF$. Prove $\triangle ABC \cong \triangle FED$.



Solution:

Statements	Reasons
1. $\triangle ABC$ and $\triangle DEF$ are isosceles triangles with vertex angles, $\angle B$ and $\angle E$	1. Given
2. $\overline{AB} \cong \overline{CB}$. $\overline{FE} \cong \overline{DE}$	2. Definition of isosceles
3. $\overline{AB} \cong \overline{FE}$	3. Given
4. $\overline{CB} \cong \overline{DE}$	4. Transitive Property
5. $\angle CBA \cong \angle DEF$	5. Given
6. $\triangle ABC \cong \triangle FED$	6. SAS Criterion

8. Given: $\triangle HBE \cong \triangle HFA$. $\triangle GDB \cong \triangle CDF$
Prove: $\triangle ABC \cong \triangle EFG$



Solution:

Statements	Reasons
1. $\triangle HBE \cong \triangle HFA$	1. Given
2. $\overline{HB} \cong \overline{HF}$, $\overline{HE} \cong \overline{HA}$, $\overline{EB} \cong \overline{AF}$, $\angle HEB \cong \angle HAF$	2. CPCTC
3. $\triangle GDB \cong \triangle CDF$	3. Given
4. $\overline{BG} \cong \overline{FC}$	4. CPCTC
5. $AH + \overline{HB} = \overline{EH} + \overline{HF}$; $\overline{EB} + \overline{BG} = \overline{AF} + \overline{FC}$	5. Segment Addition Postulate
6. $\overline{AB} \cong \overline{EF}$, $\overline{AC} \cong \overline{EG}$	6. Substitution
7. $\triangle ABC \cong \triangle EFG$	7. SAS Criterion

Practice Lesson 27 Triangle Congruency Proofs Part II

Objectives

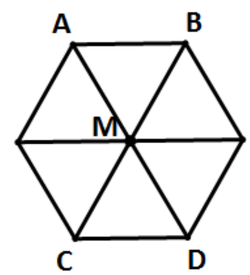
- Prove triangles are congruent using SAS, ASA, SSS, SAA, & HL Criteria

Introduction

A continuation of the previous lesson (Practice Lesson 26). Students are still applying their skill at writing proofs using triangle congruency criteria.

Practice

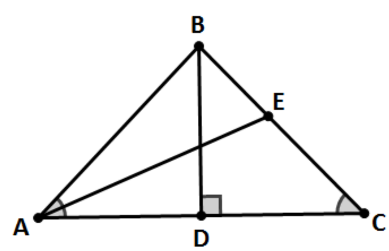
1. Consider the hexagon in the diagram at right.
   Given: M is the midpoint of $\overline{AD}\ and\ \overline{BC}$.
   Prove: ΔAMB ≅ ΔDMC



Solution:

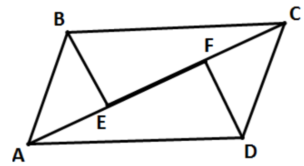
Statements	Reasons
1. M is the midpoint of $\overline{AD}$	1. Given
2. $\overline{AM} \cong \overline{MD}$	2. Definition of midpoint
3. M is the midpoint of $\overline{BC}$	3. Given
4. $\overline{CM} \cong \overline{MB}$	4. Definition of midpoint
5. M is the vertex of ∠AMB & ∠DMC	5. Definition of vertical angles
6. ∠AMB ≅ ∠DMC	6. Vertical angles are congruent
7. ΔABM ≅ ΔCDM	7. SAS criterion

2. Determine which of the triangles in the diagram at right are congruent by AAS.



Solution: ∠CDB ≅ ∠ADB by AAS

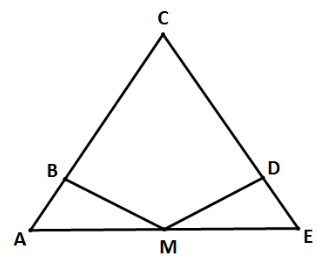
3. Consider the quadrilateral at right. All lines are straight lines.
Given: $\overline{AE} \cong \overline{CF}$, $\overline{FD} \cong BE$, ∠CFD ≅ ∠AEB
Prove: ∠FAD ≅ ∠ECB



Solution:

Statements	Reasons
1. $\overline{FD} \cong BE$	1. Given
2. ∠CFD ≅ ∠AEB	2. Given
3. ∠CFD is supplementary to ∠DFA	3. If two ∠'s form a straight line then they are supplementary.
4. ∠DFA ≅ ∠BEC	4. Supplementary Angles Theorem
5. $\overline{AE} \cong \overline{CF}$	5. Given
6. $\overline{EF} \cong \overline{EF}$	6. Reflexive Property
7. $\overline{AE} + \overline{EF} = \overline{AF}$	7. Segment Addition Postulate
8. $\overline{CF} + \overline{EF} = \overline{CE}$	8. Segment Addition Postulate
9. $\overline{AF} \cong \overline{CE}$	9. Partition Postulate
10. ΔFAD ≅ ΔECB	10. SAS criterion
11. ∠FAD ≅ ∠ECB	11. CPCTC

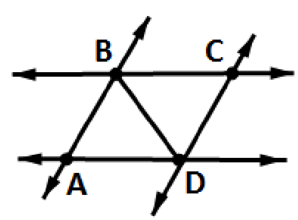
4. Consider the diagram at right.
Given: $\overline{BM} \cong \overline{MD}$. M is the midpoint of $\overline{AE}$.
∠BMA ≅ ∠DME, $\overline{AC} \cong \overline{EC}$.
Prove: $\overline{BC} \cong \overline{DC}$



Solution:

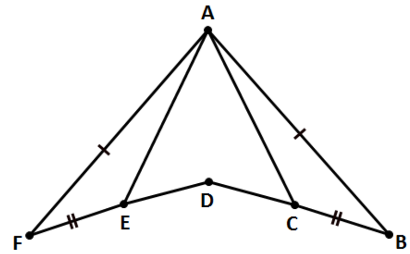
Statements	Reasons
1. $\overline{BM} \cong \overline{MD}$	1. Given
2. ∠BMA ≅ ∠DME	2. Given
3. M is the midpoint of $\overline{AE}$	3. Given
4. $\overline{AM} \cong \overline{ME}$	4. Definition of midpoint
5. ΔBMA ≅ ΔDME	5. SAS criterion
6. $\overline{BA} \cong \overline{DE}$	6. CPCTC
7. $\overline{AC} \cong \overline{EC}$	7. Given
8. $\overline{BC} \cong \overline{DC}$	8. Subtraction

5. In the diagram at right, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$
State the triangle congruence criterion that makes ΔABD ≅ ΔCDB.

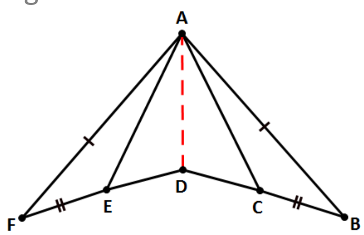


Solution: The triangles are congruent by the SAS criterion.

6. Consider the diagram at right.
Given: $\overline{AF} \cong \overline{AB}$, $\overline{FE} \cong \overline{BC}$, $\overline{DF} \cong \overline{DB}$.
Prove: ∠FAE ≅ ∠BAC.

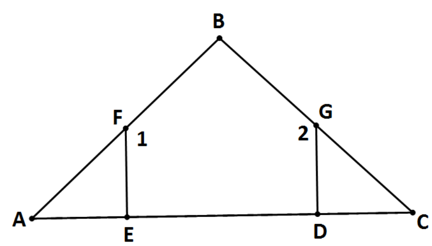


Solution: Construct the auxiliary line segment $\overline{AD}$ as shown below.



Statements	Reasons
1. $\overline{AD} \cong \overline{AD}$	1. Reflexive Property
2. $\overline{AF} \cong \overline{AB}$	2. Given
3. $\overline{DF} \cong \overline{DB}$	3. Given
4. $\overline{AM} \cong \overline{ME}$	4. Definition of midpoint
5. ΔADF ≅ ΔADB	5. SSS criterion
6. $\angle F \cong \angle B$	6. CPCTC
7. $\overline{FE} \cong \overline{BC}$	7. Given
8. ΔAFE ≅ ΔABC	8. SAS
9. ∠FAE ≅ ∠BAC	9. CPCTC

7. Consider the diagram at right.
Given: $\overline{AB}$ & $\overline{BC}$ are straight lines
∠1 ≅ ∠2, $\overline{FE} \cong \overline{GD}$, $\overline{FA} \cong \overline{GC}$
Prove: ΔABC is isosceles.



Solution:

Statements	Reasons
1. ∠1 ≅ ∠2	1. Given
2. $\overline{AB}$ & $\overline{BC}$ are straight lines	2. Given
3. ∠1 is supplementary to ∠AFE, ∠2 is supplementary to ∠CGD	3. If two angles form a straight line, then they are supplementary
4. ∠AFE ≅ ∠CGD	4. Supplementary Angles Theorem
5. $\overline{FE} \cong \overline{GD}$	5. Given
6. ΔAFE ≅ ΔCGD	6. SAS
7. ∠A ≅ ∠C	7. CPCTC
8. ΔABC is isosceles	8. If two angles of a triangle are congruent, then the triangle is isosceles

Practice Lesson 28 Parallelograms

Objectives

- Prove the properties of parallelograms

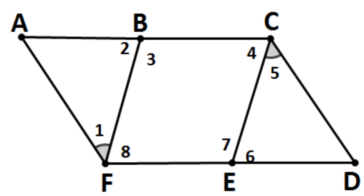
New Vocabulary: parallelogram

Introduction

Students will explore parallelograms in this lesson. Parallelograms are quadrilaterals that possess special properties. If a quadrilateral is a **parallelogram**, then its opposite sides are not only parallel but equal in measure and opposite angles are congruent. Additional properties of parallelograms are the diagonals bisect each other and consecutive angles are supplementary.

Practice

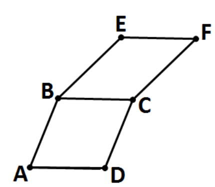
1. In the diagram at right, it is given that ACDF is a parallelogram. $\angle 1 \cong \angle 5$.
   Prove: $\overline{BF} \cong \overline{CE}$



Solution:

Statements	Reasons
1. ACDF is a parallelogram	1. Given
2. $\overline{AF} \cong \overline{CD}$	2. Opposite sides of a parallelogram are congruent.
3. $\angle$A $\cong$ $\angle$D	3. Opposite angles of a parallelogram are congruent.
4. $\angle 1 \cong \angle 5$	4. Given
5. ΔFAB $\cong$ ΔCDE	5. ASA criterion
6. $\overline{BF} \cong \overline{CE}$	6. CPCTC

2. Consider the diagram at right.
   Given: ABCD is a parallelogram; BEFC is a parallelogram.
   Prove: $\overline{AD} \cong \overline{EF}$

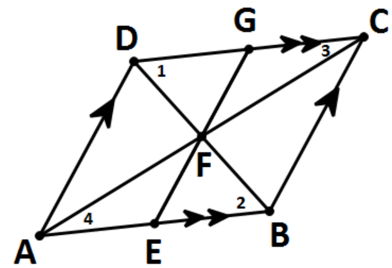


Solution:

Statements	Reasons
1. ABCD is a parallelogram BEFC is a parallelogram	1. Given
2. $\overline{AD} \cong \overline{BC}$	2. Opposite sides of a parallelogram are congruent
3. $\overline{BC} \cong \overline{EF}$	3. Opposite sides of a parallelogram are congruent

4. $\overline{AD} \cong \overline{EF}$	4. Transitive Property

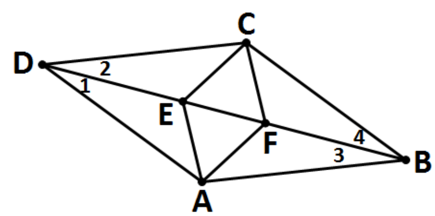
3. Given: ABCD is a parallelogram
Prove: $\overline{EF} \cong \overline{FG}$



Solution:

Statements	Reasons
1. ABCD is a parallelogram	1. Given
2. $\overline{DC} \parallel \overline{AB}$	2. Definition of a parallelogram
3. $\overline{DA} \parallel \overline{CB}$	3. Definition of a parallelogram
4. $\angle 1 \cong \angle 2$	4. Alternate interior angles are congruent
5. $\angle 3 \cong \angle 4$	5. Alternate interior angles are congruent
6. ΔDCF≅ ΔBFA	6. ASA criterion
7. $\overline{AF} \cong \overline{FC}$	7. CPCTC
8. $\angle$AFE ≅ $\angle$CFG	8. Vertical angles
9. ΔAFE ≅ Δ CFG	9. ASA criterion
10. $\overline{EF} \cong \overline{FG}$	10. CPCTC

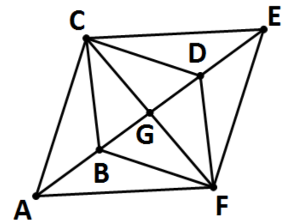
4. Consider the diagram at right.
Given: ABCD is a parallelogram
$\overline{DE} \cong \overline{BF}$
Prove: AFCE is a parallelogram



Solution:

Statements	Reasons
1. ABCD is a parallelogram	1. Given
2. $\overline{AB} \parallel \overline{CD}, \overline{AD} \parallel \overline{CB}$	2. Definition of a parallelogram
3. $\overline{DE} \cong \overline{BF}$	3. Given
4. $\overline{AB} \cong \overline{CD}$	4. Opposite sides of a parallelogram are congruent
5. $\angle 2 \cong \angle 3$	5. Alternate interior angles are congruent
6. ΔDEC≅ ΔBFA	6. SAS criterion
7. $\overline{EC} \cong \overline{FA}$	7. CPCTC
8. $\overline{AD} \cong \overline{CB}$	8. Opposite sides of a parallelogram are congruent
9. $\angle 1 \cong \angle 4$	9. Alternate interior angles are congruent
10. ΔDEA ≅ ΔBFC	10. SAS criterion
11. $\overline{AE} \cong \overline{CF}$	11. CPCTC
12. AFCE is a parallelogram	12. If opposite sides of a quadrilateral are congruent then it is a parallelogram

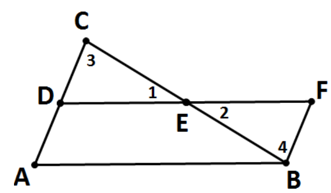
5. Consider the diagram at right.
Given: BCDF is a parallelogram, $\overline{AB} \cong \overline{ED}$
Prove: ACEF is a parallelogram



Solution:

Statements	Reasons
1. BCDF is a parallelogram	1. Given
2. $\overline{CG} \cong \overline{GF}$, $\overline{BG} \cong \overline{DG}$	2. Diagonals of parallelograms bisect each other
3. $\overline{AB} \cong \overline{ED}$	3. Given
4. $\overline{AB} + \overline{BG} = \overline{ED} + \overline{DG}$	4. Segment Addition Property of equality
5. $\overline{AG} \cong \overline{EG}$	5. Partition Postulate
6. ACEF is a parallelogram	6. if the diagonals of a quadrilateral bisect each other then it is a parallelogram

6. Consider the diagram at right.
Given: D is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{CB}$
$\overline{DE} \cong \overline{EF}$.
Prove ABFD is a parallelogram.



Solution:

Statements	Reasons
1. E is the midpoint of $\overline{CB}$	1. Given
2. $\overline{CE} \cong \overline{BE}$	2. Definition of midpoint
3. $\overline{DE} \cong \overline{EF}$.	3. Given
4. $\angle 1 \cong \angle 2$	4. Vertical angles
5. $\triangle$CED $\cong$ $\triangle$BEF	5 SAS criterion
6. $\angle 3 \cong \angle 4$	6. CPCTC
7. $\overline{FB} \parallel \overline{CA}$	7. Alternate Interior Angles Theorem Converse
8. $\overline{FB} \cong \overline{DC}$	8. CPCTC
9. D is the midpoint of $\overline{AC}$	9. Given
10. $\overline{DC} \cong \overline{DA}$	10. Definition of midpoint
11. $\overline{FB} \parallel \overline{DA}$	11. Transitive property
12. ABFD is a parallelogram	12. If a quadrilateral has one pair of opposite sides that is both congruent and parallel, then it is a parallelogram

Objectives

- Construct the mid-segment of triangles
- Prove the properties of the mid-segment of triangles
- Apply the properties of the mid-segment of triangles
- Describe the relationships between the mid-segment and other parts of the triangle

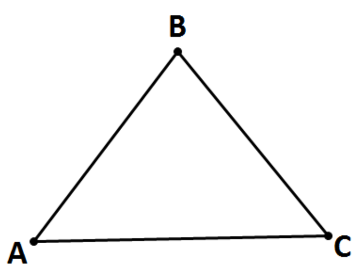
New Vocabulary: mid-segment Triangle Mid-segment Theorem

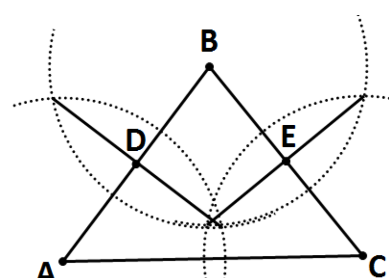
Introduction

The focus of this lesson is the **mid-segment** of a triangle. The **mid-segment** of a triangle is a line segment that connects the midpoints of two sides of a triangle. The mid-segment can connect any two sides of the triangle. Students will find the mid-segment of a triangle by constructing the perpendicular bisector of the side. Recall from Lesson 4, problem 4, the intersection of the perpendicular bisector and the side of the triangle is the midpoint of the side. By connecting the two midpoints with a line segment the mid-segment is created. The mid-segment has two distinct properties: 1) the mid-segment is parallel to the non-connected side of the triangle, 2) the mid-segment is one-half the length of the non-connected side of the triangle (the side it is parallel to). These two properties are collectively known as the **Triangle Mid-segment Theorem.** Students will use this theorem to find segment and side lengths of triangles.

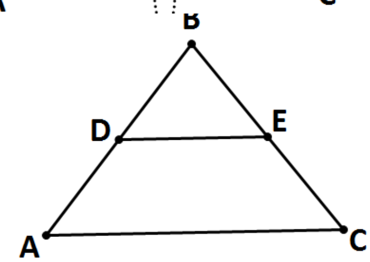
Practice

1. Given $\triangle$ABC below, construct the mid-segment parallel to $\overline{AC}$.

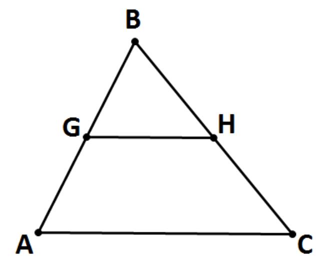




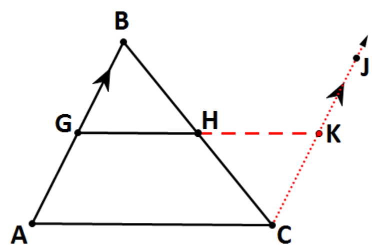
Solution: Construct the perpendicular bisector of $\overline{AB}$ and the perpendicular bisector of $\overline{BC}$. (Refer to Lesson 4, pages 21-22). Mark each intersection of the perpendicular bisector and the side with a point (D & E respectively in the figure at right). This is the midpoint of the side.
Draw line segment $\overline{DE}$. $\overline{DE}$ is the mid-segment of $\triangle$ABC.



2. You can prove the Triangle Mid-segment Theorem by using the properties of parallelograms. Consider the triangle at right.
Given: ΔABC with mid-segment $\overline{GH}$.
G is the midpoint of $\overline{AB}$; H is the midpoint of $\overline{BC}$.
Prove: $\overline{GH} \parallel \overline{AC}$; $\overline{GH} = \frac{1}{2}\overline{AC}$

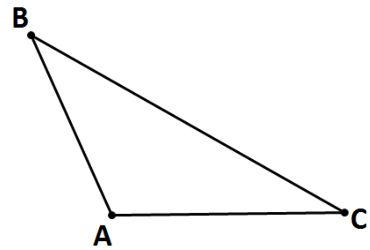


Solution: Construct auxiliary line $\overline{CJ}$ parallel to $\overline{AB}$.
Extend $\overline{GH}$ to intersect $\overline{CJ}$ at point K.

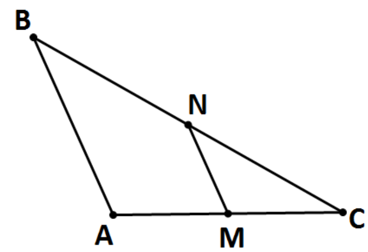


Statements	Reasons
1. ΔABC with mid-segment $\overline{GH}$	1. Given
2. G is the midpoint of $\overline{AB}$; H is the midpoint of $\overline{BC}$	2. Given
3. $\overline{GH}$ is a mid-segment of ΔABC	3. A mid-segment joins the midpoints of two sides of a triangle
4. $\overline{AB} \parallel \overline{CJ}$; $\overline{GH}$ extends to intersect $\overline{CJ}$ at point K	4. By construction
5. $\overline{AG} \cong \overline{GB}$; $\overline{CH} \cong \overline{HC}$	5. Midpoint of a segment divides a segment into two congruent segments
6. $\angle$ABC $\cong$ $\angle$BCK	6. Alternate interior angles are congruent
7. $\angle$BHG $\cong$ $\angle$KHC	7. Vertical angles are congruent
8. ΔBHG $\cong$ ΔKHC	8. ASA Criterion
9. $\overline{GB} \cong \overline{CK}$ and $\overline{GH} \cong \overline{KH}$	9. CPCTC
10. $\overline{AG} \cong \overline{CK}$	10. Transitive Property
11. AGKC is a parallelogram	11. If a quadrilateral has one pair of opposite sides that is both congruent and parallel, then it is a parallelogram
12. $\overline{GH} \parallel \overline{AC}$	12. A parallelogram is a quadrilateral with two pair of opposite sides parallel
13. $\overline{GK} \cong \overline{AC}$	13. Opposite sides of a parallelogram are congruent
14. $\overline{GK} = \overline{AC}$; $\overline{GH} \parallel \overline{KH}$	14. Congruent segments have equal measure
15. $\overline{GH} + \overline{KH} = \overline{GK}$	15. Segment Addition Postulate
16. $\overline{GH} + \overline{GH} = \overline{AC}$	16. Substitution
17. $2\overline{GH} = \overline{AC}$	17. Simplify
18. $\overline{GH} = \frac{1}{2}\overline{AC}$	18. Division Property of Equality

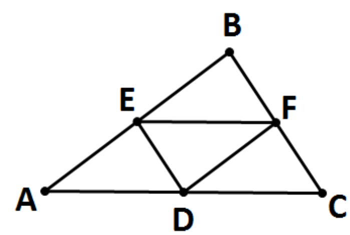
3. Construct a segment in $\triangle$ABC that is $\frac{1}{2}$ the length of $\overline{AB}$.



Solution: Since the mid-segment $\frac{1}{2}$ the length of the non-connected side of a triangle, then $\overline{AB}$ must be that side. Also, the mid-segment is parallel to $\overline{AB}$, so the mid-segment must connect the midpoints of $\overline{AC}$ and $\overline{BC}$. To find the midpoints of $\overline{AC}$ and $\overline{BC}$, construct the perpendicular bisectors of each side. The intersection of the perpendicular bisector and the side is the midpoint of the side. The figure at right shows the mid-segment $\overline{MN}$ connecting midpoints M and N respectively.



4. Given $\triangle$ABC at right, $\overline{AE} \cong \overline{EB}$, $\overline{BF} \cong \overline{FC}$, $\overline{AD} \cong DC$.
Find the following:



a) $\overline{ED} \parallel$ ______ d) $\overline{EF} \cong$______ $\cong$ ______

b) $\overline{AB} \parallel$ ______ e) $\overline{FD} \cong$______ $\cong$ ______

c) $\overline{EF} \parallel$ ______ f) $\overline{ED} \cong$______ $\cong$ ______

Solution:

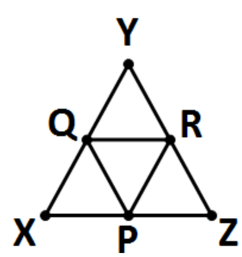
a) $\overline{ED} \parallel$ $\overline{BC}$ d) $\overline{EF} \cong$ $\overline{AD}$ $\cong$ $\overline{DC}$

b) $\overline{AB} \parallel$ $\overline{DF}$ e) $\overline{FD} \cong$ $\overline{AE}$ $\cong$ $\overline{EB}$

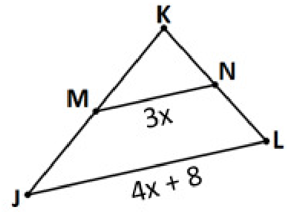
c) $\overline{EF} \parallel$ $\overline{AC}$ f) $\overline{ED} \cong$ $\overline{BF}$ $\cong$ $\overline{FC}$

5. Given $\triangle$XYZ, $\overline{QP}$ = 12 cm, $\overline{QR}$ = 14 cm, $\overline{RP}$ = 17 cm.
Find the perimeter of $\triangle$XYZ.

Solution: Since each mid-segment is $\frac{1}{2}$ the length of its parallel side, then the perimeter of $\triangle$XYZ will be twice the perimeter of $\triangle$QPR. The perimeter of $\triangle$QPR is 43 cm therefore, the perimeter of $\triangle$XYZ is 86 cm.

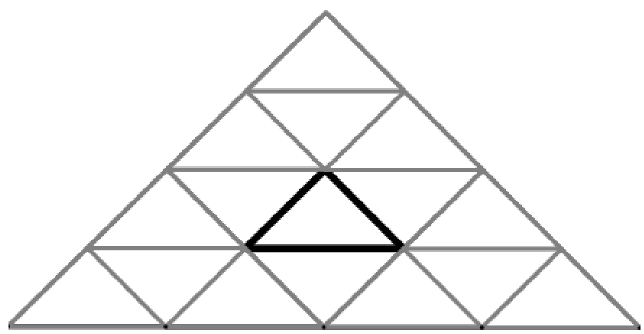


6. Given $\triangle$JKL, $\overline{MN}$ = 3x and $\overline{JL}$ = 4x + 8. Find the lengths of $\overline{MN}$ and $\overline{JL}$ in inches.

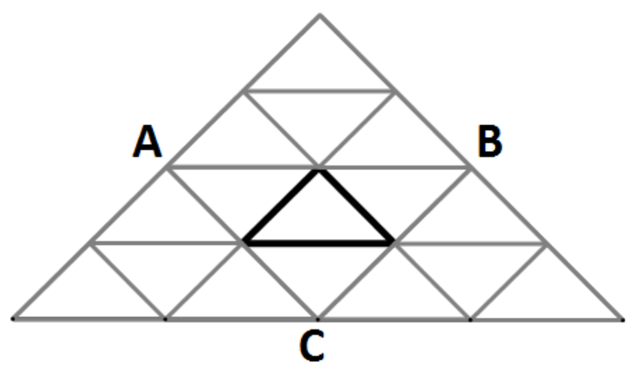


Solution: 2(3x) = 4x + 8
6x = 4x + 8
2x = 8
x = 4
$\overline{MN}$ = 12 inches and $\overline{JL}$ = 24 inches.

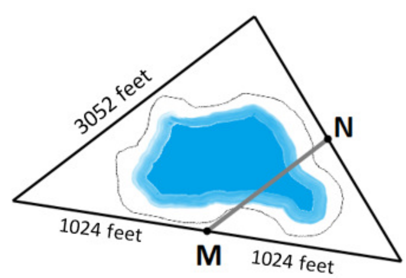
7. A sculpture near the Bronx Zoo in New York City is an isosceles triangle made up of smaller triangles based on mid-segments. The length of the base of the sculpture is 114 feet. What is the length of the base of the bold black triangle shown below?



Solution: 28.5 feet (28 feet, 6 inches). Label the triangle, $\triangle$ACB , whose vertex (C) is the midpoint of the base of the large triangle. $\overline{AB}$ is the mid-segment of the large triangle. Therefore, $\overline{AB}$ is 57 feet in length. The mid-segment of $\triangle$ACB is the base of the bold triangle. Since $\overline{AB}$ measures 57 feet, then the base of the smaller, bold triangle is 28.5 (one-half of 57 feet).



8. $\overline{MN}$ is the location of a new bridge being built across Lake Buena Vista. How long will the bridge be when it is finished?

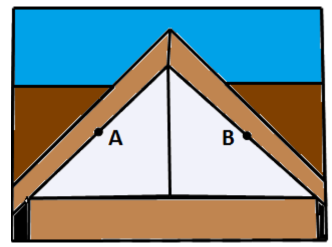


Solution: Since $\overline{MN}$ is the mid-segment of the triangle that surrounds the lake, the length is $\frac{1}{2}$(3052) which is 1526 feet.

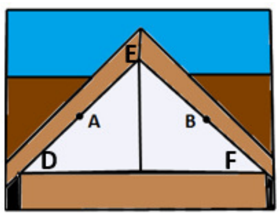
9. If the mid-segment of an isosceles triangle is 25 meters and each of the congruent sides of the triangle are 22 meters, what is the perimeter of the isosceles triangle?

Solution: The perimeter is 94 meters. (22 x 2) + (2 x 25) = 94 .

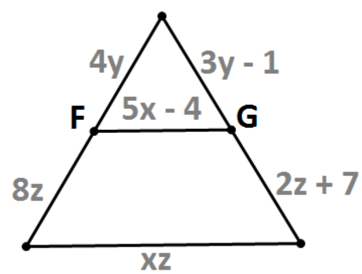
Consider the sketch of the dormer window at right. The purlins (labeled A and B, respectively) are set at the midpoints of the fascia board of the window. If each triangular window pane measures 5 feet at its base, how far apart are A and B?



Solution: Consider one triangle, ΔDEF (below), consisting of the two window panes. Since the total width of the two panes of glass ($\overline{DF}$) measures 10 feet (5 + 5), then the distance between A and B is 5 feet since extending a line between the two points would create a mid-segment (which is $\frac{1}{2}$ the length of the parallel side of ΔDEF).



11. Consider the figure at right.
Solve for x, y, & z.



Solution: Solve algebraically:

4y = 8z

$\boxed{y = 2z}$

3(2z) - 1 = 2z + 7

6z = 2z + 8

4z = 8

$\boxed{z = 2}$

2(5x - 4) = 2x

10x - 8 = 2x

8x = 8

$\boxed{x = 1}$

x = 1, y = 4, z = 2

Practice Lesson 30 Special Lines in Triangles: Medians

Objectives

- Construct the medians of triangles
- Apply the properties of the medians of triangles to solve problems

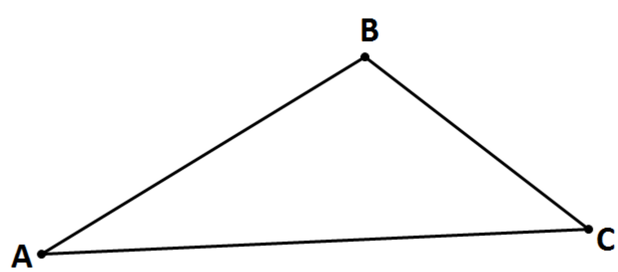
New Vocabulary: Concurrency of Medians of a Triangle Theorem

Introduction

In this lesson, students will revisit the medians of triangles first introduced in Lesson 5. Recall, a median is a line segment that connects any vertex of a triangle to the midpoint on the opposite side. There are three medians of a triangle. The intersection of the three medians is a point of concurrency (the centroid) formerly introduced in Lesson 5. A theorem about medians states: the medians of a triangle intersect at a point that is two thirds the distance from each vertex to the midpoint of the opposite side. This theorem is the **Concurrency of Medians of a Triangle Theorem**. We can also say that the distance from the vertex to the centroid is twice the distance from the centroid to the midpoint. This divides the median into a 2:1 ratio. Student will use this special property to solve problems. Students will also construct the medians of a triangle using a compass and straight edge (by revisiting the perpendicular bisector of a line segment construction).

Practice

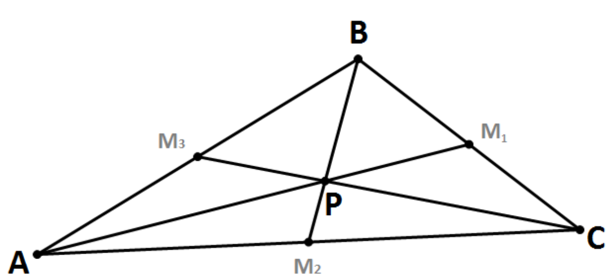
1. Construct the medians of ΔABC (below). Use your knowledge about constructing midpoints for this practice session.



Solution: **Step 1**: Construct the perpendicular bisectors of each side to locate the midpoints. Label them M_1, M_2, M_3.

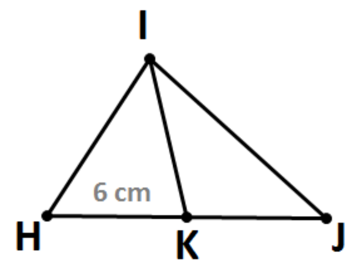
Step 2: Connect the midpoints of each side to the vertex of the opposite side

Step 3: Label the intersection of the medians, P (this is the centroid of the triangle).



2. The following triangle has one median, ($\overline{IK}$), showing. Find the length of ($\overline{HJ}$) if ($\overline{HK}$) = 6 cm.

   Solution: Since the median connects a <u>midpoint</u> to the opposite vertex, then $\overline{HJ}$ must be 12 cm since the midpoint (K) bisects $\overline{HJ}$.



3. $\overline{TW}, \overline{VY}, and\ \overline{XU}$ are medians of ΔTVX. Find $\overline{TW}$ if $\overline{TZ}$ = 3x + 2 and $\overline{TW}$ = 5x

   Solution: By the theorem stated in the introduction to this lesson, $\overline{TZ} = \frac{2}{3}\overline{TW}$

   $3x + 2 = \frac{2}{3}(5x)$

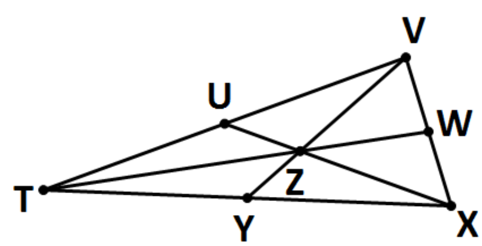
   $3x + 2 = \frac{10x}{3}$

   $3(3x + 2) = 10x$

   $9x + 6 = 10x$

   $x = 6$

   $\overline{TZ} = 20$ and $\overline{TW} = 30$



4. C is the centroid of ΔEFG shown at right. If $\overline{CM}$ = 5, find $\overline{FM}$ and $\overline{FC}$.

   Solution: Since C is the centroid of ΔEFG, $\overline{FC} = \frac{2}{3}\overline{FM}$

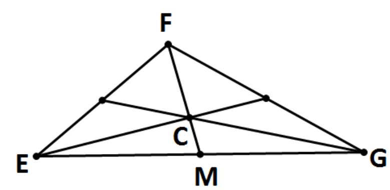
   $\overline{CM} = \overline{FM} - \overline{FC}$ $\quad\quad$ $\overline{FC} = \frac{2}{3}(15)$

   $\overline{CM} = \frac{1}{3}\overline{FM}$ $\quad\quad$ $\boldsymbol{\overline{FC} = 10}$

   $5 = \frac{1}{3}\overline{FM}$

   $\boldsymbol{15 = \overline{FM}}$

   $$\boldsymbol{\overline{FM} = 15,\ \overline{FC} = 10}$$



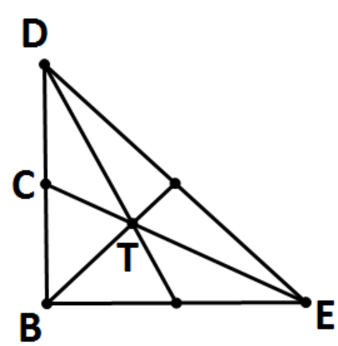
5. Given ΔBDE, if $\overline{CT}$ = 22, how long is $\overline{TE}$?

   Solution: Since the distance from the vertex to the centroid is twice the distance from the centroid to the midpoint, we have:

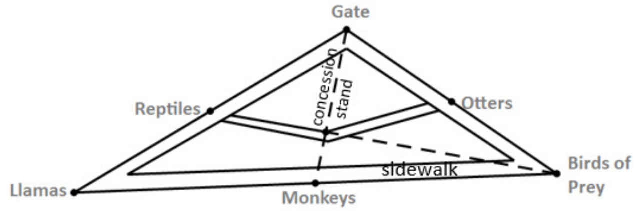
   $\overline{CT} = \frac{1}{2}\overline{TE}$

   $22 = \frac{1}{2}\overline{TE}$

   $44 = \overline{TE}$



6. Josh and Perry are going to the Zoo. They purchase a map (shown below, right) and notice the following information: The map is in a triangular shape. The exhibits are located at the midpoints and vertices of the triangle. The concession stand is the intersection of the midpoints and the vertices. The distance from the Gate to the concession stand is twice the distance than the concession stand from the Monkey exhibit. The Reptile exhibit is 60 meters away from the Birds of Prey exhibit.



a) How far are the Birds of Prey from the concession stand?

b) If the concession stand is located 10 meters from the Monkey exhibit, how far is the concession stand from the Gate?

Solution:

a) Since the Reptiles are 60 meters away from the Birds of Prey, then the Birds of Prey exhibit is $\frac{2}{3}(60) = 40$. **The Birds of Prey are 40 meters from the concession stand**.

b) The concession stand to the monkeys is $\frac{1}{2}$ the gate to the concession stand. Since the monkeys are 10 meters from the concession stand, then the gate to the concession stand is 20 meters away.

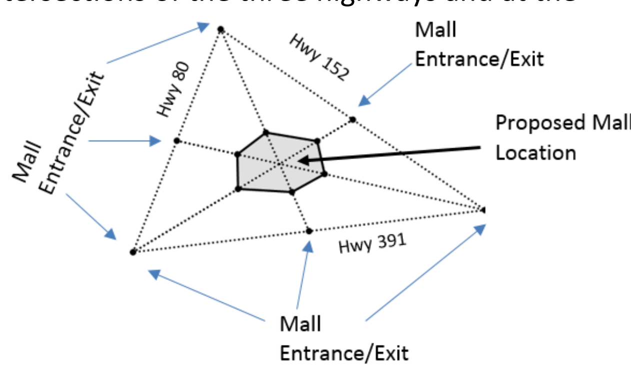
7. A developer has plans to build a shopping mall within easy access of three existing highways. The plans are shown below.

Entrances/Exits are placed at the three intersections of the three highways and at the midpoints of the stretch of road between each intersection. The distances from the midpoint entrances/exits to the mall parking are:

Hwy 152 to the mall: $\frac{3}{8}$ mile

Hwy 80 to the mall: $\frac{1}{2}$ mile

Hwy 391 to the mall: $\frac{5}{6}$ mile



a) Find the length of each access road to the mall (assume that they intersect at one point then subtract one mile total for the assumption).

b) Grading and paving $\frac{1}{2}$ mile of road costs \$35,000. How much will it cost to grade and pave all of the access roads?

Solution:

a) Mall to intersection of Hwy 391 & Hwy 80 is $(2)(\frac{3}{8}) = \frac{3}{4}$ mile. Total road: $1\frac{1}{8}$ miles.

Mall to intersection of Hwy 152 & Hwy 391 is $(2)(\frac{1}{2}) = 1$ mile. Total road: $1\frac{1}{2}$ miles.

Mall to intersection of Hwy 80 & Hwy 152 is $(2)(\frac{5}{6}) = 1\frac{2}{3}$ miles. Total road: $2\frac{1}{2}$ miles.

b) 70,000 x $5\frac{1}{8}$ miles = 358,750. Subtract the cost of 1 mile for the assumption: \$288,750 total cost to grade and pave the access roads to the mall.

Practice Lesson 31 Construct a Square

Objectives

- Use a compass and straight edge to construct squares of different sizes
- List the properties of a square

Introduction

Students will use a compass and straight edge to construct a square. Students define a square and will list the properties of a square and use them during the construction of the square.

Note: The drawings are not drawn to scale.

Practice

1. What is a square? Providing a precise definition of a square will prepare you for the construction of a square. Describe the types of lines that create the angles of a square.

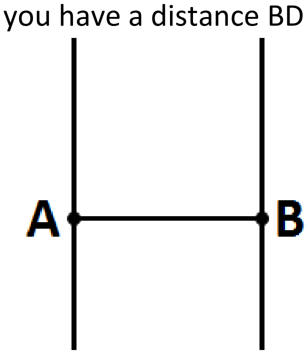
   Solution: A square is a quadrilateral with four congruent sides and four right angles. Right angles are formed by the intersection of perpendicular lines (lines intersect at 90° angles).

2. Use a compass and straight edge and the following instructions to construct a one inch square.
   a. Draw a line segment 1 inch long.
   b. Label each endpoint A, B respectively.
   c. Construct a perpendicular line through A, perpendicular to $\overline{AB}$
   d. Construct a perpendicular line through B, perpendicular to $\overline{AB}$
   e. Construct circle C_A (center A) so that $\overline{AB}$ is the radius
   f. Construct circle C_B (center B) so that $\overline{BA}$ is the radius
   g. Both circles intersect the perpendicular lines at four points (one point above and below each center of each circle). Choose either above or below the center and label the intersections D and E respectively so that you have a distance BD and AE.
   h. Draw line segments $\overline{BD}$, $\overline{DE}$, and $\overline{AE}$

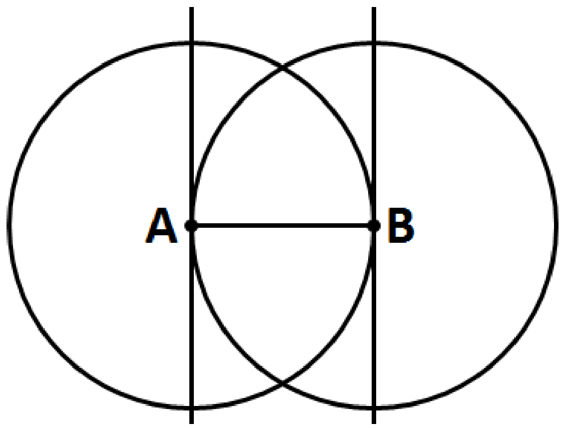
   Solution:

   a-b

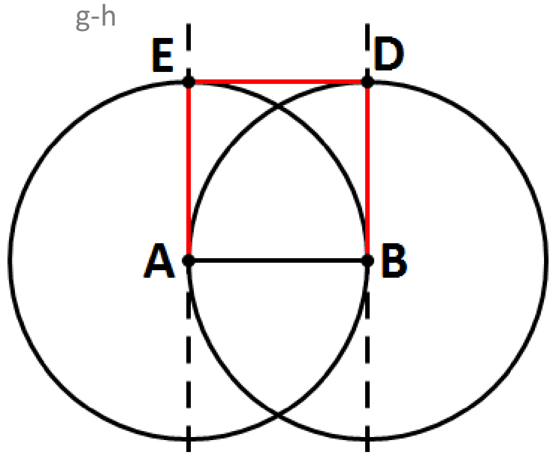
c-d



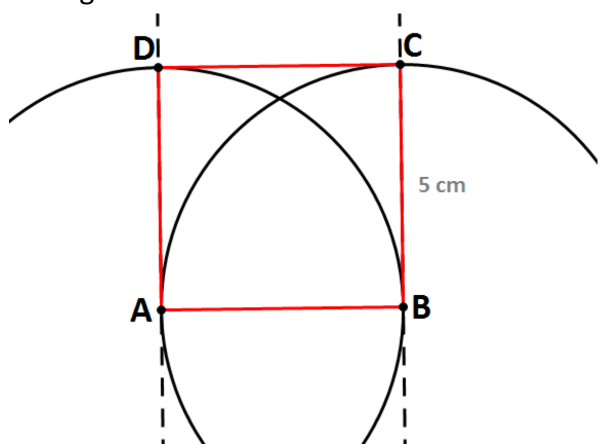
e-f



g-h



3. Construct a square with side lengths 5 cm.

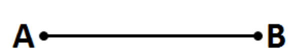


4. List the properties of a square.

Solution: Four congruent sides
Four congruent right angles
Diagonals bisect each other
Diagonals are perpendicular
Diagonals bisect the angles
Opposite sides are parallel and equal in length
Opposite angles are congruent

5. Examine the first step in constructing a square given a diagonal, $\overline{AB}$. What is the relationship between Step 1 given below and the properties of a square?

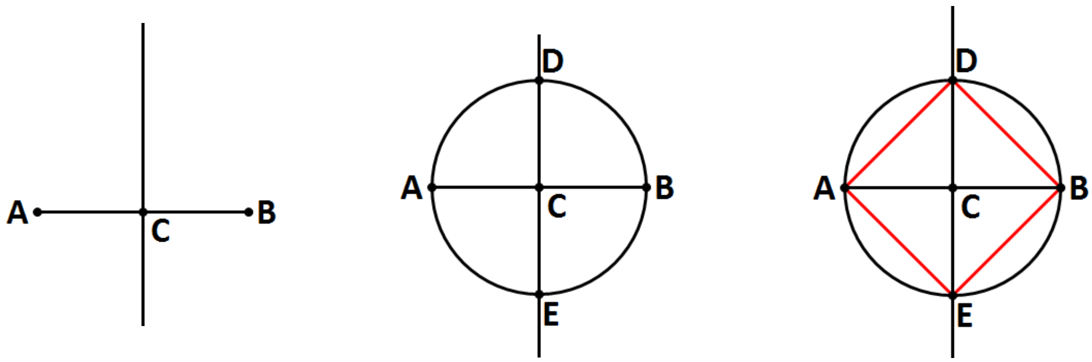
Step 1: Construct the perpendicular bisector of $\overline{AB}$



Solution: The diagonals bisect each other at 90° angles. Therefore, the construction of the perpendicular bisector of $\overline{AB}$ will create the other diagonal of the square.

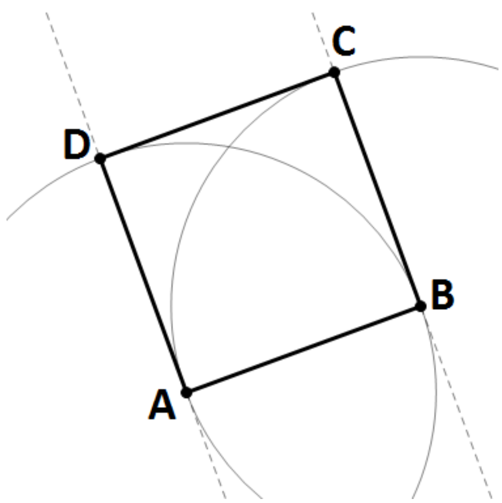
6. Complete the construction of the square given the diagonal in problem 5. Provide a set of step-by-step instructions for your construction.

Solution:
Step 1: Construct the perpendicular bisector of $\overline{AB}$. Label the intersection C.
Step 2: Construct a circle with center C and radius $\overline{CB}$. Label each intersection of the perpendicular line and the circle as D, E, respectively.
Step 3: Join A, E, B, & D with line segments.

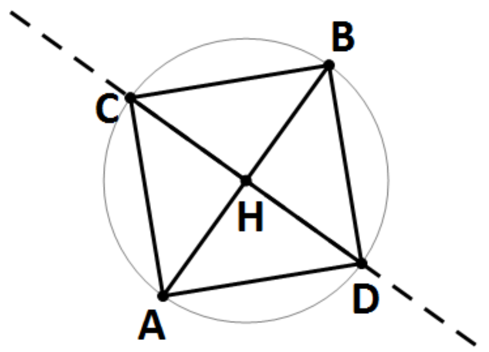


7. Use both techniques to construct two squares of different sizes.

Solution: Answers will vary. Sample squares are shown below.



Two Perpendicular lines through $\overline{AB}$



Diagonal Method

Practice Lesson 32 Construct a Nine Point Circle

Objectives

- Use the midpoints and altitudes of a triangle to construct a nine point circle

New Vocabulary: Nine-Point Circle

Introduction

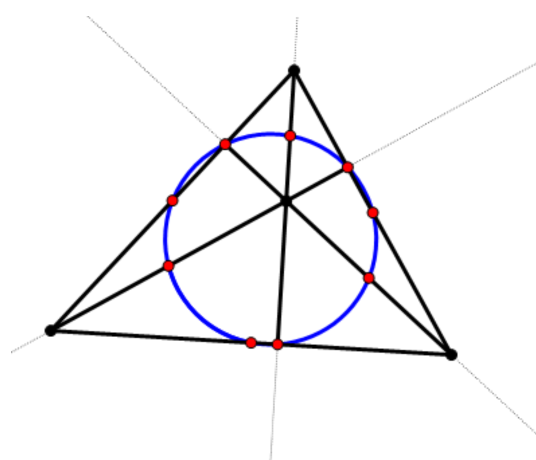
Students will use what they know about midpoints and altitudes of a triangle to construct a **Nine-Point Circle** (a circle defined by nine points). The nine points of the Nine-Point Circle are determined by midpoints and altitudes of a triangle. Recall that an altitude of a triangle is line segment that connects a vertex of the triangle to the opposite side at a 90° angle. and the intersection of the three altitudes is a point of concurrency called the orthocenter. Also recall that the midpoint of a line segment is created by the perpendicular bisector of that segment.

Practice

1. Every triangle has a nine point circle. To construct your first nine point circle it is best to start with an acute triangle (since the obtuse triangle has an altitude outside the triangle making the construction a bit awkward. Follow the steps below to construct a nine-point circle.

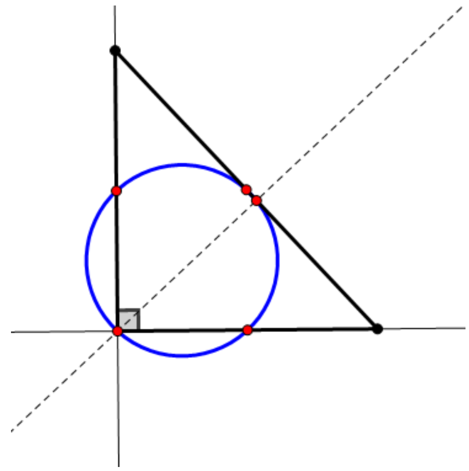
   Step 1: Create any acute triangle.
   Step 2: Construct the midpoints of each side (construct the perpendicular bisector of each side).
   Step 3: Construct the altitudes of the triangle. (To construct an altitude, construct a perpendicular line through a point (Lesson 4), with the vertex as your point. Repeat for all three vertices. Mark each intersection of the altitude and side with a point.
   Step 4: On each altitude, from the vertex to the orthocenter, construct a midpoint. Notice the circle made by the nine points just constructed. Connect the points and you have the Nine Point Circle!

   Solution: Sample triangle and nine point circle at right.

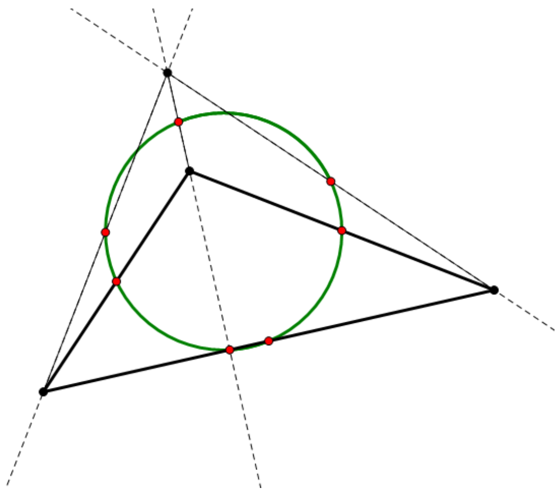


2. Construct a nine point circle using a right triangle. Explain any difficulties or abnormalities.

Solution: Since two of the altitudes of a right triangle are the sides of the triangle, the midpoints from Step 2 and Step 4 are the same, thus "reducing" the number of points by four (two midpoints on the sides and two midpoints on the vertex to orthocenter segments). So only five points are visible to draw the circle.



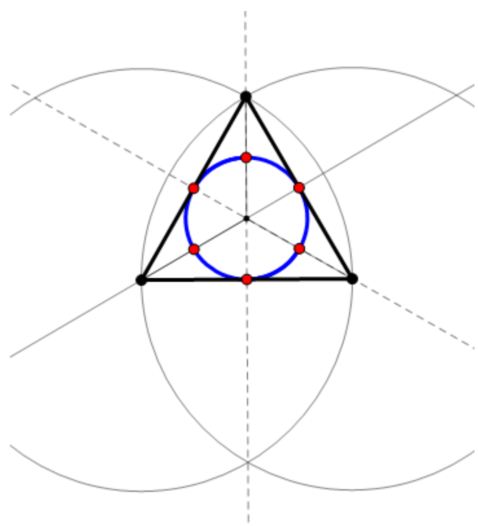
3. Construct a nine point circle using an obtuse triangle. Explain any difficulties or abnormalities.



Solution: Since two of the altitudes are outside the triangle, there is no intersection of them with the sides, thus reducing the number of points visible by two.

4. Construct a nine point circle using an equilateral triangle. Explain any difficulties or abnormalities.

Solution: Since the midpoints of the sides coincide with the intersection of the altitudes and sides, the number of points on the circle is reduced by three. (Note: this circle is inscribed in the triangle because the circle is completely inside the triangle and the circle's side touches each side at one point).



Practice Lesson 33 Review of Assumptions Part I

Objectives

- List, define and provide examples of important principles used in Lessons 1-32

Introduction

In this lesson students will review the properties, postulates, principles and theorems used in Lessons 1-32.

Practice

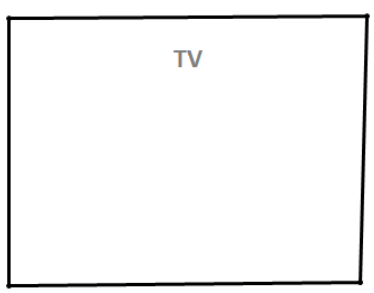
1. Match the meaning with the word.

1.	Assumption	a.	terms that cannot be precisely defined therefore can only be explained by examples and descriptions
2.	Definition	b.	a proven fact using undefined terms, postulates and other theorems
3.	Postulate	c.	that which is accepted as true without proof
4.	Theorem	d.	the accepted meaning of a term
5.	Undefined terms	e.	a basic statement accepted as true but cannot be proven

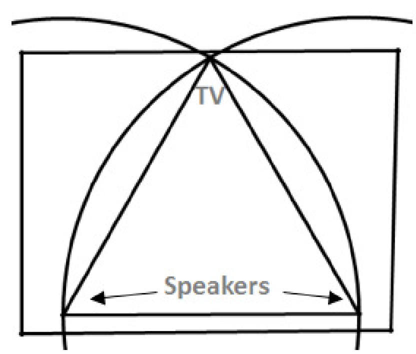
Solution:

1.	Assumption	c.	that which is accepted as true without proof
2.	Definition	d.	the accepted meaning of a term
3.	Postulate	e.	a basic statement accepted as true but cannot be proven
4.	Theorem	b.	a proven fact using undefined terms, postulates and other theorems
5.	Undefined terms	a.	terms that cannot be precisely defined therefore can only be explained by examples and descriptions

2. Kari wants to put her speakers for her surround sound equidistant from the TV in her living room. Describe how she could achieve this distance. Use the floor plan provided at left.



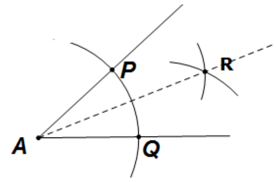
Solution: Kari could use the floor plan to construct an equilateral triangle with the TV at one of the vertices and the speakers at the other two vertices.



3. Describe the properties of the equilateral triangle.

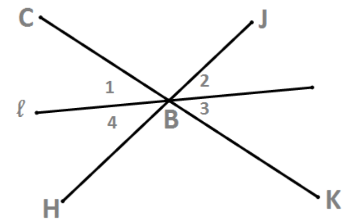
Solution: an equilateral triangle is a three sided polygon with equal sides and equal angles. Angles measure 60°. Equilateral triangles are constructed by the intersection of two circles.

4. Describe the construction at right:



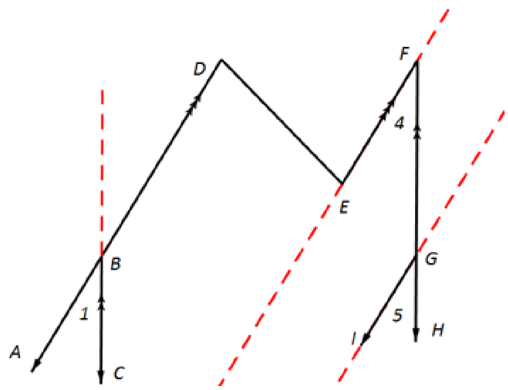
Solution: This is the construction of the angle bisector

5. Consider the diagram at right. If line ℓ bisects $\angle CBH$ and $\angle JBK$, what theorem would be used to prove $\angle CBH = \angle 1 + \angle 4$ and $\angle DBG = \angle 2 + \angle 3$



Solution: The Angle Bisector Theorem

6. Consider the diagram below, right. $\angle 4 \cong \angle 5$.
Explain why $\angle 4 \cong \angle 5$.



Solution: Because they are corresponding angles

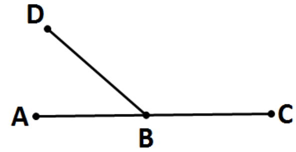
7. Describe the Perpendicular Bisector Theorem.

Solution: if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

8. Give a definition of collinear.

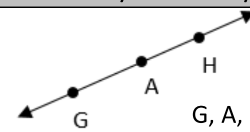
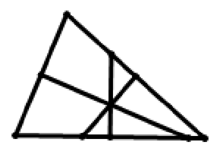
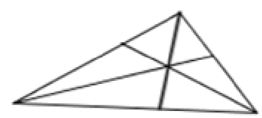
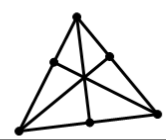
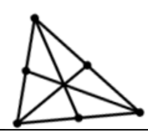
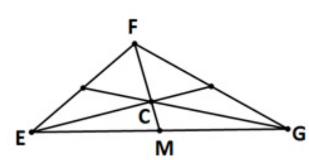
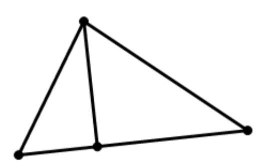
Solution: three or more points that lie on the same line

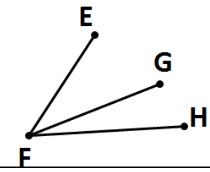
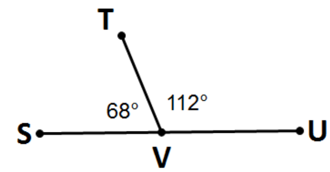
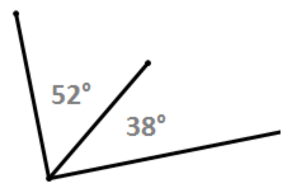
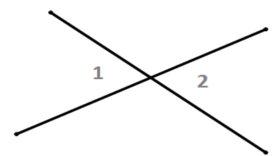
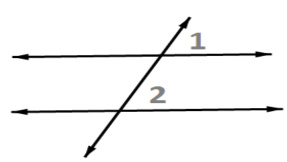
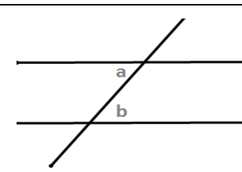
9. Describe the relationship between $\angle ABD$ & $\angle CBD$ if $\overline{AC}$ is a straight line.



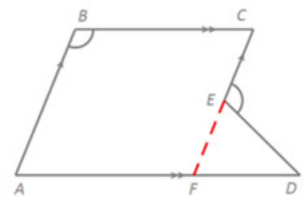
Solution: $\angle ABD$ & $\angle CBD$ are a linear pair

11. Complete the table. (Note: the solutions are shown in the original table to save space. Student copies will have the table with empty cells where the type is gray. In the last column some cells are left blank - indicated by shading).

Term/Postulate/Theorem	Define, describe or list it	Sketch/notation/example
collinear	three or more points that lie on the same line.	 G, A, H are collinear
perpendicular bisector	divides a line segment into two equal parts	⊥ bisector
concurrent	When three or more lines intersect at a single point	
points of concurrency	incenter, circumcenter, centroid, orthocenter	
circumcenter	the point where the three perpendicular bisectors meet	
incenter	the point where the three angle bisectors meet	
orthocenter	the point where the three altitudes meet	
centroid	the point where the three medians meet	
median	a segment connecting any vertex of a triangle to the midpoint of the opposite side	
altitude	a line segment connecting a vertex to the line containing the opposite side and perpendicular to that side	

adjacent angles	two angles that have the same vertex, share a common side and have no interior points in common.	 $\angle$EFG & GFH are adjacent angles
supplementary angles	The sum of the measures of two angles is 180°	
complementary angles	The sum of the measures of two angles is 90°	
vertical angles	angles formed by two intersecting lines	
Angle Addition Postulate	if B is in the interior of AOC, then $m\angle AOB + m\angle BOC = m\angle AOC$	
parallel	Two lines are parallel if they lie in the same plane and do not intersect	notation for parallel: // or ||
corresponding angles	two congruent angles, both lying on the same side of the transversal and situated the same way on two different parallel lines	
alternate interior angles	when two lines are crossed by a transversal, the pairs of angles on opposite sides of the transversal but inside the two parallel lines	
Triangle Sum Theorem	the sum of the measures of the angles in any triangle equal 180°	

Isosceles Triangle Theorem	(if two sides of a triangle are congruent, then the angles opposite those sides are congruent	
Reflexive Property of Equality	a = a	
Reflexive Property of Congruency	a ≅ a	
Symmetric Property of Equality	if $a = b$, then $b = a$	
Symmetric Property of Congruency	if $\angle a \cong \angle b$, then $\angle b \cong \angle a$.	
Transitive Property of Equality	if $a = b$ and $b = c$, then $a = c$.	
Transitive Property of Congruency	if $\angle a \cong \angle b$ *and* $\angle b \cong \angle c$, then $\angle a \cong \angle c$	
Addition Property of Equality	if a = b, then a + c = b + c.	
Subtraction Property of Equality	if a = b, then a - c = b - c	
Substitution Property of Equality	if a = b then a can be substituted for b. This means that if a = b then we can change any b to a or any a to b	
Angle Bisector Theorem	if a ray divides an angle into two congruent parts (angles), then the angle is said to be bisected	
Supplementary Angles Theorem	If two angles are supplementary to the same angle (or to congruent angles), then the angles are congruent	
Complementary Angles Theorem	If two angles are complementary to the same angle (or to congruent angles) then these angles are congruent	

auxiliary line	a construction added to the given diagram to help demonstrate the relationship between steps in a proof and the reason for the steps	
converse	a theorem obtained by reversing the roles of the premise and conclusion of the initial theorem	
Segment Addition Postulate	if point B lies on the line segment $\overline{AC}$ and B is between the endpoints A and C then $\overline{AB} + \overline{BC} = \overline{AC}$.	

Practice Lesson 34 Review of Assumptions Part II

Objectives

- List, define and provide examples of important principles used in Lessons 1-32

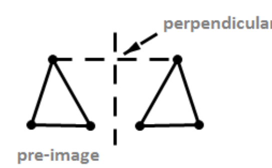
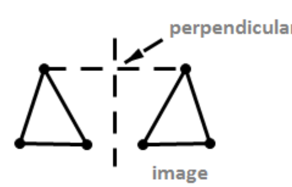
Introduction

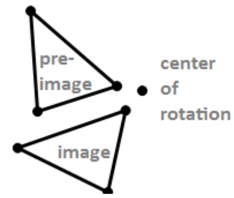
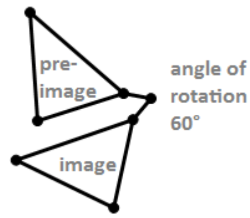
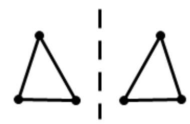
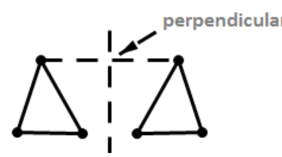
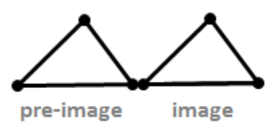
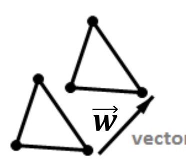
Students will continue listing, defining and providing examples of important principles learned in Lessons 1-32.

Practice

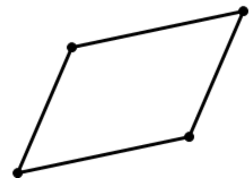
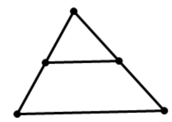
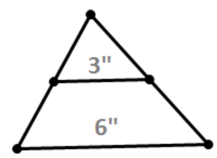
1. Complete the table. (read the note in the previous lesson)

Term/Postulate/Theorem	Define, describe or list it	Sketch/notation/example
proof	mathematical argument whose aim is to reach a conclusion that is true	
Corresponding Angles Theorem	If two lines are crossed by a transversal and the two lines are parallel, then their corresponding angles are congruent	
Converse Corresponding Angles Theorem	If two lines are crossed by a transversal so that corresponding angles are congruent, then the lines are parallel	
Consecutive Interior Angles Theorem	If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary	
Converse Consecutive Interior Angles Theorem	If two lines are crossed by a transversal so that consecutive interior angles are supplementary, then the lines are parallel.	
Alternate Interior Angles Theorem	If two lines are cut by a transversal and the two lines are parallel, then the alternate interior angles are congruent	

Converse Alternate Interior Angles Theorem	If two lines are crossed by a transversal so that alternate interior angles are congruent, then the lines are parallel.	
Alternate Exterior Angles Theorem	If two lines are crossed by a transversal and the two lines are parallel, then the alternate exterior angles are congruent	
Converse Alternate Exterior Angles Theorem	If two lines are crossed by a transversal so that alternate exterior angles are congruent, then the lines are parallel	
Exterior Angle Theorem	the measure of an exterior angle of a triangle is equal to the sum of the measures of the two non-adjacent interior angles	
rigid motion transformation	functions that map a set of input points from an original image to a set of output points (a new location) after applying a function rule. These functions are distance and angle preserving (i.e. from original image to transformed image, segments have the same length, angles have the same measure and parallel lines remain parallel	rotation, reflection, translation
pre-image	the image of points before a rigid motion transformation has been applied (the original image)	
image	the image of points after a rigid motion transformation has been applied	

rotation	a rigid motion transformation that moves around a center point in a clockwise or counterclockwise angle	
center of rotation	a point around which an image is rotated	
angle of rotation	number of degrees through which points rotate around the center of rotation	
line of reflection	the line over which reflections are made	 the dashed line is the line of reflection
reflection	if ℓ is a line and if P is a point not on ℓ, then the reflection of P across ℓ is the point P' such that a) the distance from P' to ℓ is equal to the distance from P to ℓ, and b) the line joining P to P' is perpendicular to ℓ	 Each point of each figure is exactly the same distance from the line of reflection
translation	a movement of the plane in which a figure is translated a specific direction for a specific distance	
vector	a quantity that has a fixed length (its magnitude) and a fixed direction. Images are translated along a vector. Vectors can be names as points and lines are named. In the diagram, $\overrightarrow{w}$ is the name of the vector.	

Correspondence	a function, and with respect to plane figures, pairs vertices to vertices, angles to angles, and sides to sides of one figure to the another, but not necessarily congruent figures or congruent parts	
Corresponding Parts of Congruent Triangles are Congruent	a restatement of the definition of congruence where congruence is defined as the existence of a finite composition of rigid motions that maps one figure onto another. Used when proving triangles congruent	CPCTC
Congruence Criterion	Used for proving triangles congruent. Triangles must meet certain criteria in order to be proved congruent.	
Side-Angle-Side Criterion	The criterion that says two triangles are congruent when two sides and the angle in between them are congruent	SAS
Side-Side-Side Criterion	The criterion that says two triangles are congruent when all three sides are congruent	SSS
Angle-Side-Angle Criterion	The criterion that says two triangles are congruent when two angles and the side in between them are congruent	ASA
Side-Angle-Angle Criterion	The criterion that says two triangles are congruent when two angles and the non-included side are congruent	SAA
Hypotenuse-Leg Criterion	The criterion that says two right triangles are congruent when the hypotenuse and one leg are congruent	HL

parallelogram	a quadrilateral where its opposite sides are parallel, opposite sides are equal in measure, and opposite angles are congruent. Also, these quadrilaterals have diagonals that bisect each other and their consecutive angles are supplementary	
mid-segment of a triangle	a line segment that connects the midpoints of two sides of a triangle.	
Triangle Mid-segment Theorem	1) the mid-segment is parallel to the non-connected side of the triangle, 2) the mid-segment is one-half the length of the non-connected side of the triangle (i.e. it is $\frac{1}{2}$ the side it is parallel to)	

VITA

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