MEMS accelerometer: proof of concept for geotechnical engineering testing

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MEMS ACCELEROMETER: PROOF OF CONCEPT FOR GEOTECHNICAL ENGINEERING TESTING

A Thesis

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College In partial fulfillment of the Requirements for the degree of Master of Science in Civil Engineering

In

The Department of Civil and Environmental Engineering

by

Keith N. Hoffman
B.S. Louisiana State University, 2002
August 2004
DEDICATION

To Katherine, for her love and support
ACKNOWLEDGEMENTS

In light of the fact that there are many people who I might inadvertently omit from this section of the study, I would like to first thank you. The unnamed that have helped me throughout my graduate experience from day to day. They have done so much that I cannot possibly grasp what my experience would have been like without their help.

I would like to thank my future wife Katherine. She has continued to give her support throughout the completion of this degree. Her company has been a blessing at all times during this journey. This includes the late nights of work and the rewards for such exertion. She has been there to push me further when I began to waver in my diligence. She has been there to congratulate and celebrate. She has been there for everything.

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ABSTRACT

Geotechnical engineering materials are inherently variable, which leads to many simplifications when trying to model their behavior. The materials must always be characterized prior to any design work so that the engineer knows which direction he must progress to have a reliable design. Although subsurface characterization techniques and geotechnical design steadily improve, they are by no means infallible.

The combination of geotechnical subsurface characterization along with geophysical techniques for improved design and construction monitoring has begun to surface as a viable alternative to the standard techniques in geotechnical engineering. This is important because there is a lack of Quality Control/Quality Assurance during the construction stage of a project, which further compounds the problems inherent from the complexity of the subsurface. Geophysical techniques based on elastic wave propagation provide an excellent combination of characterization and monitoring for the observation of geotechnical engineering projects. Elastic wave propagation provides coverage between traditional boreholes and it helps infer changes in the state of stresses.

Unfortunately, sensors for this type of monitoring have typically been expensive, and the use of elastic wave propagation for characterization and monitoring has just begun to become to be implemented. The application of elastic wave tomography needs an inexpensive set of sensors to help justify its inclusion in the broad area of construction monitoring and characterization systems. This set of inexpensive sensors has arrived on the market developed from Miniature Electro-Mechanical Systems (MEMs) technology.

This research developed the Analog Devices’ ADXL250 MEMS accelerometer to determine its limitations and its range of applications. In addition, a packaging system developed to allow for a broader range of applications in geotechnical engineering. Once the sensor was fully calibrated, a long-term goal for the research was to utilize the instrument in a laboratory experiment to obtain a tomographic image of the state of stress within a model. While the sensor was utilized in a model in this study, the final reasoning for its use within the model was simply to show its capabilities and areas of application. Simple velocity distributions are given as well as inferences made about the driving factors for these behaviors.
CHAPTER 1
INTRODUCTION

1.1 Background

Geotechnical engineering projects are, for the most part, concerned with large scale operations that can range from the mundane to the truly extraordinary. Examples span from lackluster earthwork jobs such as cut and fill to glamorous levee and/or dam projects that test the limits of design. Because geotechnical engineering is a field in which construction is conducted with a wide variety of non-homogeneous materials the properties beneath a site are seldom truly known. This fact makes geotechnical materials extremely difficult to model. There are vast amounts of physical and chemical phenomenon that occur during any geotechnical process. Due to this, geotechnical engineers have created a variety of theories to model the behavior of geomaterials. Unfortunately there has been no way to accurately verify these theories because to do so would require a matrix of data acquisition instrumentation so dense that the cost would be unimaginable. A step toward this goal is to develop sensor technologies that are extremely cheap and very small for easy incorporation into the soil mass. This would allow for more accurate experimental calibration of the governing philosophies of geotechnical engineering. In the current state of practice a range of possible values is given and a decision is made as to what values seem most applicable (Dunnicliff, 1993). This is why monitoring has become a mainstay in the field. The use of monitoring systems is conducted in order to determine if the finished product is operating correctly both for safety concerns and efficiency matters.

In most cases, the vast size of these systems makes them both difficult and expensive to monitor. One of the major drawbacks to today’s monitoring systems is that the sensors are too expensive to purchase and install in the quantities that would be needed to create the kind of dense instrumentation array which would allow very intricate knowledge of the behavior of a geostructure. To address this problem in practice critical locations are determined and monitored by specialized equipment, which can either relay any pertinent information via communication link to a data acquisition system or require a work crew to come to the location and take readings. While there are systems in place that utilize real time response (i.e. automatic update monitoring), they are typically expensive and rather limited in size. Either way, the problem is a difficult obstacle to overcome.

Another option to ensure safe economic site performance is to start with more accurate knowledge of the subsurface. To do this a much larger scale initial subsurface investigation could be utilized to create a more accurate knowledge of the engineering properties of the materials. This requires a technique that is more widely applicable than tradition boring investigations. The inherent variability in the materials that are utilized in the field of geotechnical engineering raises concerns about the accuracy of material knowledge and subsequent design. The traditional techniques for soil investigation are adequate, but not infallible. Another alternative to these existing techniques is geophysical exploration and characterization. Geophysics provides an alternative solution to this problem, but there are issues with the cost of a geophysical exploration. The system components used in these techniques are very expensive, and therefore make them more costly and less accepted in the
industry. With the increasing application of seismic wave propagation techniques in geotechnical and more specifically geophysical engineering a more inexpensive sensor is needed to help further develop and promote their application. A very inexpensive set of system components would allow for increased application of these techniques, and eventual full acceptance. To do this all of the system components need to be cheaper and more accessible. Borrowing from the field of nano technologies, the arrival of inexpensive MEMS (Micro Electro Mechanical System) technologies has given the field of geotechnical engineering an option that could eventually lead to instrument arrays dense enough to fully understand the behavior of geomaterials.

1.2 Objectives

This study will attempt to evaluate the MEMS based ADXL250 accelerometer by Analog Devices Incorporated for applications within geotechnical engineering. This research will attempt to determine whether an inexpensive nano sensor (the Analog devices MEMS based ADXL250 accelerometer) that is readily available on the market coupled with wave propagation techniques can be applied in the field of geotechnical process monitoring. The objectives of this study are two fold. The first objective is the calibration and adaptation of the ADXL250 accelerometers and the second is conducting tests for actual engineering applications. While the manufacturer calibrated the instruments prior to making them commercially available their intended application was not for the field of geotechnical engineering and more specifically the very harsh geoenvironment. The aspects of the accelerometer that are most important to its possible applications within the field are its sensitivity and frequency range. Both of these properties will help to specify what type of applications the MEMS accelerometer is best suited for in the field. A statistical analysis of the sensitivity will be conducted versus frequency to determine the influence of frequency on the sensitivity if the instrument. Once the calibration is determined, a technique will be developed to adapt the MEMS to the geoenvironment. After the instrument is successfully sealed, it will be tested in corrosive environments in order to determine the capability of the sealing technique applied. The improved and geotechnically sound version of the MEMS accelerometer will then be recalibrated to observe what, if any, affect the sealing technique had on the sensitivity of the instrument.

Once the MEMS is prepared for application in the geoenvironment, two specific applications will be introduced and their experimental results will be compared to theoretic background on like situations. The two systems were the following setups:

- **Wave propagation in a plate**: A non destructive evaluation of the MEMS response in a simple homogeneous model.
- **Wave propagation 1-g braced excavation model**: An evaluation of the velocity field of a model braced excavation will be conducted utilizing the MEMS accelerometers.

While the first system will require very little in the way of design and evaluation, the second will be considerably more difficult to test and analyze. The specific design and results will be reviewed and compared to theoretically based models.
1.3 Outline

Chapter 2 of this thesis presents a review of MEMS technology and their various applications. This also includes various types of MEMS sensors and actuators and some limited information on their operation. Following this is a specific and detailed explanation of the MEMS based ADXL250 accelerometer by Analog Devices and its operation. Chapter 3 covers the calibration and recalibration in the post seal state for the MEMS accelerometer. This will include detailed information on the analysis of the response data and its interpretation. Chapter 4 is a review of the many elastic wave propagation concepts that will be used throughout the analysis of the two physical models that are presented Chapters 5 and 6. Chapter 5 presents detailed descriptions of the testing setup and results for the wave propagation in a homogeneous plate. Chapter 6 describes the braced excavation model and the analysis of wave propagation in the soil. Finally, Chapter 7 presents the conclusions of the research as well as the recommendations for future work and suggested corrections to the processes undertaken in the research conducted herein.
CHAPTER 2
NOVEL MINIATURE INSTRUMENTATION – MEMS

2.1 General Overview

MEMS is an acronym for Micro-Electro-Mechanical Systems and refers to a collection of microsensors and actuators which can sense and manipulate their environment and have the ability to react to changes in that environment with the use of microcircuit control (Varadan et al., 2003). MEMS have been in industry for some time now, but they were mostly overlooked in Geotechnical Engineering applications due to the major advances that have been made in the area of processing chips. Although the processing chip has received most of the attention, MEMS have been steadily developed over the past 25 years to create more intricate and advanced microelectronic sensors and micromechanical systems. They truly began to emerge in the 1990s with the development of the integrated circuit (Varadan et al., 2003). MEMS are developed at very small scales in much the same way as a computer chip. The process of fabrication is usually based on either a surface micromachining or bulk micromachining technique, which are the most popular, but there are other techniques on the market that are less commonly used. These manufacturing techniques as well as the low power requirements have allowed MEMS manufacturers to create smaller and smaller sensors and actuation systems. These systems have begun to replace the macro equivalents in industry due to their difference in size, durability, accuracy, speed, and cost. The size of MEMS has also produced a variety of new possibilities in the world of science that were deemed unfeasible prior to their development.

2.2 MEMS Fabrication

Silicon micromachining is the process of fashioning small mechanical parts out of a silicon substrate or on top of one. This process is used to develop a variety of micro parts or structures including, but not limited too, gears, beams, diaphragms, springs, and orifices. Using these micro parts a wide range of microsensors and actuators have been developed which have begun to revolutionize different industries (Varadan et al., 2003). There are two main techniques in the fabrication of MEMS, bulk silicon micromachining and surface silicon micromachining. Bulk silicon micromachining is the more popular of the two types of silicon based fabrication techniques. The reason for the popularity of the bulk micromachining technique is the ease with which the technique can incorporate actuators and viable sensors. That being said, surface micromachining has its advantages as well, which are based primarily on the fact that the 2 ½ dimensional structures (the thickness of the thin film structure is what is represented by the half) are very easily incorporated into integrated circuit technologies. Of course this small size has also limited the ability to incorporate the usually larger mechanical elements, actuators, into the MEMS (Varadan et al., 2003).

Bulk micromachining implies exactly what the technique embodies, that is the fact that the main micromechanical structure of the device is derived from within the bulk of a single-crystal silicon (SCS) by selectively removing the unwanted wafer material. Some of the typically seen size ranges in this technique are from submicrons to the depth of the full wafer, 200 to 500 µm, with lateral sizes in the range of microns to the full diameter of the wafer, 75 to 200 mm.
The key step in the bulk machining process is etching, which is the removal of the unwanted material. This is done in a number of ways, based primarily on the material and need of the manufacturer. Some etching techniques are the following: wet isotropic, wet anisotropic, Plasma isotropic, reactive ion, and etch-stop techniques. Knowing the intricacies of the techniques is not important for the scope of this report, but it should be know that they are used to remove material in order to obtain a final structure. An example of this process is shown in Figure 2.1.

![Figure 2.1: Process of creating a bulk micromachined structure using an etching technique (Gardner et al., 2001).](image)

The example shown in Figure 2.1 is the cross section of a cantilever beam structure during different aspects of its fabrication. On the oxidation and patterning and diffusion of phosphorus techniques, the addition of materials is done using diffusion or growth techniques, and the primary etching technique is shown in Figure 2.1d. The key is that the machining is all done within the wafer structure. This allows for larger and more robust structures such as actuators to be constructed.

Surface micromachining techniques produce structures that are mainly located on the surface of the silicon wafer and result in a thin film. The dimensions of these micromachined structures are an order of magnitude smaller than their bulk machined counterparts. Due to this surface mounting technique the structures can be easily incorporated into an integrated circuit as was stated earlier. There are several different techniques to fabricating MEMS using the surface machining processes. One common example is the sacrificial layer method shown in Figure 2.2, which is one of the simpler processes. The basic steps are the following:

1. Deposit and pattern the sacrificial SiO$_2$ layer on the silicon surface.
2. Deposit and pattern the permanent poly-Si film on the substrate.
3. Remove the required amount of sacrificial material in order to obtain the final structure, usually achieved through etching the unwanted material.
The sacrificial layer method is one of the simpler techniques for the fabrication of a surface micromachined structure. Some other methods in surface micromachining, which are covered in much more detail in *Microsensors, MEMS, and Smart Devices* by Gardner et al. are the following: Material systems in sacrificial layer technology, surface micromachining using plasma etching, and combined IC technology and anisotropic wet etching.

**2.3 Microsensors**

This section presents an overview of how these MEMS parts are used in different sensor technologies. There are a multitude of MEMS based sensors, which use the various parts constructed in the micromachining processes. A sensor is simply a device that converts a physical input into a quantifiable value. In the case of a MEMS, that quantifiable value is an electronic signal, which also has to be converted into the actual reading. According to Middelhoek, a sensor can be classified according to the energy domain of its primary input (Gardner et al., 2001). There are six primary energy domains for this classification:

- Electrical  E
- Thermal  T
- Radiation  R
- Mechanical  Me
- Magnetic  M
- Bio(Chemical)  C

Based on these domains a sensor or an actuator can be classified. This is helpful because of the wide variety of measurements and actions that can be associated with the MEMS industry. However, in some cases the MEMS based technology has created devices that fall under multiple areas of the primary energy domains. An example of this is the multi sensor platforms now available on the market. Figure 2.3 shows the relationship between the sensor or actuator and its energy domain classification.
Figure 2.3: Vectorial representation of sensors and actuators in the energy domain space. A processor would have a vector from E back to itself (Gardner et al., 2001).

Figure 2.4: Classification of microsensors based on the physical phenomenon which is measured (Gardner et al., 2001).
All six of these primary energy domains have associated MEMS sensors. The microsensors come in various shapes and sizes, and use different mechanisms for the measurement of their primary energy source. The book *Microsensors, MEMS, and Smart Devices* by Gardner *et al.* covers various types of sensors and their fundamental mechanism of operation. This chapter concentrates on the mechanical (Me) energy sensors because that is the type of sensor that is primarily used in this study. The mechanical sensor is perhaps the most important class of microsensor because of its wide acceptance among mass markets such as the automotive industry. There is a wide multitude of mechanical measurands, from acceleration to force to orientation. All of these can be quantified by some mechanism or combination of mechanisms in a microsensor device. The classification scheme for mechanical microsensors is shown in Figure 2.4, with the corresponding measurand.

For different device types there are different micromechanical components used for the measurement of the physical phenomenon. In their simplest form the devices consist of the following structures:

- A cantilever beam
- A bridge
- A diaphragm or membrane

If these micro parts are assumed to be a homogenous, uniform, and elastic material, we can use a simple mechanics of materials theory to describe how they deform when acted upon by a physical phenomenon. This is shown in equations 2.1 and 2.2 for a simple cantilever structure.

\[
\Delta x = \frac{l^3}{3 \cdot E_m \cdot I_m} F_x
\]  

(2.1)

The deflection, \( \Delta x \), is based on the material properties (Elastic modulus, \( E_m \), and Moment of Inertia, \( I_m \)) and the applied force, \( F_x \). The beam length is given as the variable \( l \). The Moment of Inertia is given by the following

\[
I_m = \frac{w \cdot d^3}{12}
\]  

(2.2)

Based on these simple relationships, it is clear that a cantilever can convert a mechanical force into displacement, and this is used in many different ways in MEMS. The particular use of a cantilevered structure that this report will concentrate on is the comb type structures used in acceleration sensors, which use multiple sets of cantilevered fingers that act as differential capacitors in the MEMS. One of the two sets is stationary, acting as the barrier for the center capacitor. The center set of fingers is allowed to move along one axis when an external force is applied parallel to the axis of motion. This in turn causes the capacitance to increase on one side of the differential capacitor, which can then be correlated to acceleration. A simple schematic of this structure is shown in section 2.5 of this chapter in Figure 2.6. This is just an example of how a micromachined part would be incorporated into MEMS.
2.4 MEMS Applications

MEMS cover a range of instruments and sensors that range from the mundane to the truly magnificent. At one end of the spectrum there are nanomachined nozzles used in inkjet printers to control the size of ink drop that is used in printing. The smaller the ink drops the better the resolution and quality of the printer. The ink jet nozzles really only fit the definition of a MEMS based on the process on which they are manufactured, outside of this they are simply a nanosized hole in a piece of silicon. The other end of the spectrum includes equipment to attempt to help the blind see, wireless multi sensor devices called smart dust (Warneke, 2002), and a gas turbine engine at MIT that is smaller than 1 cm$^3$ with a full set of fuel plenums, pressure ports, fuel injectors, igniters, fluid interconnects, and compressor airfoils (Singer, 2002). These are just some general examples of what can be done with MEMS.

The two distinct areas within MEMS:

- Microelectronic Sensors: A variety of physical, thermal, electrical, and chemical sensors. This area of MEMS has historically been the first due to their ease of fabrication.
- Micro Mechanical Systems: This are includes micro engines, transmissions, structures, and actuators. These are the more complex MEMS based systems.

Of the two areas of MEMS, the microsensors get the most attention due to their good track record. One of the foremost applications of microsensors is in the automotive industry. As a frontrunner in the use and development of MEMS the automotive industry has incorporated MEMS into their products is as airbag sensors. The MEMS are incorporated into the car easily due to their small size and low power requirements. The concept on which they work is simple. The sensor picks up the gross acceleration of the vehicle, and if that acceleration exceeds a preset level the airbag(s) deploys. There are other uses of MEMS in the automotive industry, but this is by far the most common between the different makers (Analog Devices, 2002).

Another very promising area of application for MEMS is smart dust applications. Smart dust is just what it sounds like, a system that is completely aware of its surrounding environment with multiple sensor types on board as well as a power source and communications abilities. This specific applications for this type of system are limitless, but they could simply be grouped into either monitoring or inquiry applications for different types of systems. For the most part, the real limit of this technology lies in the power source size because the rest of the components are already within targeted size ranges at this time. Eventually, assuming the power issue is solved, these sensors will become as small and numerous as dust revolutionizing the way in which people interact with the environment. One solution to the power problem is a passive MEMS system, which use an excitation from the communications to run the sensors. The problem with this type of system is it cannot be used for real time monitoring purposes because it must be turned on in a sense (Warneke, 2002).

MEMS have effectively given the electronic brain (i.e., the logic chip) eyes, ears, a nose, and the ability to manipulate their environment. There incorporation into most industries has the capability to yield tremendous advancements in the corresponding field. The downfall is that most companies do not have the money to fund the development of MEMS for there specific
needs. There are a few, but they are mostly automobile manufactures and very large corporations. Many automobile manufactures use MEMS for airbag deployment sensors. This in turn requires most other industries that cannot afford the venture of developing their own MEMS to adapt products already on the market to their needs. This may or may not be simple to do, but for the most part it requires more time and money than smaller corporations are willing to spend.

2.4.1 MEMS Applications in the Field of Civil Engineering

The more easily adapted type of MEMS is the nano sensor, which will usually offer many advantages over its macro counterparts in industry. First, and most obviously, is the size of MEMS sensors. They will usually be relatively small in comparison to their macro counterparts. This offers advantages in incorporating a MEMS sensor into a system that might lack room or need multiple points of data acquisition and/or monitoring. They have an inherent durability due to their small size and minimal external instrumentation following a no assembly required type of apparatus which calls for little additional fine-tuning other than to turn it on. MEMS also require very little in the way of power making them attractive to projects requiring nonstop monitoring. The final advantage that MEMS have over their counterparts is their cost. In most cases MEMS are very cheap when compared to the equivalent piece of macro instrumentation. Table 2.1 shows a summary of the characteristics of the Analog Devices MEMS accelerometer and macro sensor equivalents on the market.

<table>
<thead>
<tr>
<th>Instrumentation</th>
<th>Cost ($/unit)</th>
<th>Installation</th>
<th>Accuracy</th>
<th>Ease of Use</th>
<th>Capabilities</th>
<th>Durability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force Balance Unit</td>
<td>$300-$600</td>
<td>Casing must be within 3° of vertical</td>
<td>±2 mm per 25m up to 20°</td>
<td>Very user friendly, and can have repeated use</td>
<td>Rotation and Settlement</td>
<td>High, because electronics are separate/portable</td>
</tr>
<tr>
<td>Accelerometer based Tiltmeter</td>
<td>$300 - $500</td>
<td>Measurement plates must be rigidly attached</td>
<td>±0.001 arc degrees or 0.36 arc seconds</td>
<td>Very user friendly, and can have repeated use</td>
<td>Rotation</td>
<td>High, because electronics are separate/portable</td>
</tr>
<tr>
<td>Surveyor’s Electronic Total Station</td>
<td>$1,000 - $10,000+; surveyor can be hired</td>
<td>None required</td>
<td>±0.0348 cm - 0.914 cm of deformation</td>
<td>Requires someone with experience to use equipment</td>
<td>Vertical Settlement</td>
<td>High</td>
</tr>
<tr>
<td>Settlement Plates</td>
<td>$25 - $80</td>
<td>Relatively simple</td>
<td>±0.254 cm - 2.54 cm of deformation</td>
<td>Simple, but increased accuracy requires surveying</td>
<td>Vertical Settlement</td>
<td>High</td>
</tr>
<tr>
<td>Buried Plates</td>
<td>$10 - $25</td>
<td>Simple, but reading can be careful</td>
<td>Very Low</td>
<td>Requires someone with a drill to take readings</td>
<td>Vertical Settlement</td>
<td>High</td>
</tr>
<tr>
<td>Rupture Stake</td>
<td>$3 - $5</td>
<td>Simple, but must be through slip plane</td>
<td>Ver. Low</td>
<td>Problems with removal, but relatively simple</td>
<td>Identification of slip plane</td>
<td>Low</td>
</tr>
<tr>
<td>Shear Probe</td>
<td>$45-$65</td>
<td>Initial position must be vertical and through slip plane</td>
<td>Low</td>
<td>Very simple</td>
<td>Identification of slip plane and rotation</td>
<td>Medium</td>
</tr>
<tr>
<td>Inverted Pendulum</td>
<td>$200-$400</td>
<td>Initial position must be vertical otherwise accuracy will suffer</td>
<td>±0.03-0.5 mm of displacement</td>
<td>Simple</td>
<td>Horizontal deformation</td>
<td>High</td>
</tr>
<tr>
<td>Accelerometer based MEMs (ADXL202)</td>
<td>$15 - $25</td>
<td>Initial position must be recorded</td>
<td>5 mg of acceleration, or 0.33º, settlement variable</td>
<td>Wiring can be cumbersome, but fast data collection</td>
<td>Vertical Settlement</td>
<td>High, if sealed correctly from elements</td>
</tr>
</tbody>
</table>

Additionally, there are wireless MEMS developing on the market that would require no external power or hookups for taking data. This would allow for real time monitoring of engineered systems in order to find immediate updates of response. Some areas of measurement that MEMS can be used for are acceleration, temperature, chemical contaminant movement, and strain quantity. There seems to be a wide range of applications for MEMS, but there are very few that have started the research and development needed to promote their mainstream use within the field of civil engineering.

One individual that has pursued the use of MEMS within the field of civil engineering is professor Steven Glaser at the University of California. Professor Glaser’s interests reside in the
applications of advanced MEMS sensors for civil engineering and more specifically structural and geotechnical engineering. While Glaser has not done an in depth study on a particular sensor within the field, he has addressed the evolution of the civil engineering field with the application of these technologies. The following scenario was given as an example for Glaser’s real time natural hazards monitoring research:

- “Tiny self-contained wireless sensors are installed near critical structural points in a large commercial building. On board intelligence discerns normal structural deterioration and meaningful damage. Sensors report the location and kinematics of damage during and after an earthquake, allowing rapid, accurate, structural health determination (Glaser, 2000).”

Glaser’s research in real time natural hazard monitoring is centered around one particular piece of equipment called a Macro-Mote. The Macro-Motes are large-scale models for smart dust. They are fully self-contained interactive wireless units, which contain advanced MEMS sensors. Incorporated into the Macro-Mote package are communication abilities, processing capabilities, sensors of various types, and a power supply. The sensors within the Macro-Mote at present include: magnetic field measurement, humidity, light, temperature, and air pressure. All of this instrumentation fits neatly within a package about a cubic inch in total volume. This type of instrumentation could revolutionize real time monitoring in engineering systems.

The Colorado School of Mines (CSM) has also pursued research using MEMS technologies. In 1999 they entered into several feasibility discussions with MEMS manufactures. In addition to the feasibility studies, the Colorado School of Mines also considered the use of MEMS for monitoring landslide areas. Other areas of interest included assessing marine settlements and chemical monitoring of contaminant movements. The key points that were reached throughout the discussions and investigations were the following:

- Smaller lighter equipment can be installed and retrieved with little disturbance to the in-situ materials.
- The small size of the sensors allows for the ability to configure instrumentation with a wide variety of sensors on board measure large ranges of physical, chemical, and electrical properties of earth materials.
- The ever-decreasing cost of MEMS allows for the use of a much greater number of instruments that can be placed in spatial arrays to better evaluate the engineering system under scrutiny.
- The all in one MEMS layout, which incorporates both sensing and control capabilities on one platform, diminishes the need for complex instrumentation systems.
- The evolution of MEMS wireless applications will lead to the ability to place sensors in very unusual places or even those which hard wiring an instrument to would be impossible.

Both CSM and Glaser are pioneers in the applications of MEMS within the field of civil engineering. Traditionally considered a macro industry, the field of civil engineering is being revolutionized through the use of micro machinery, which allows for closer control and more precise monitoring of the health and security of essential areas within the nations infrastructure.
From the large structures that fill the skylines of the city nations to the levee systems that protect countless homes and workplaces, MEMS will be utilized. The technology is available, but the research on the implementation of this technology has only just begun with the work of people like those at CSM and professor Glaser.

2.5 Analog Devices ADXL250 Accelerometer

The ADXL250 accelerometer is a surface micro machined accelerometer which uses the concept of differential capacitance to sense acceleration. Some of the general characteristics of the ADXL250 are its ability to measure up to fifty times the earth’s gravity while still being able to discern down to ten micro gs. It also can measure acceleration in two orthogonal directions, which allows for more measurement capabilities. Table 2.2 shows a table of the ADXL250 with the different characteristics compared against other accelerometer types.

Table 2.2: Comparison of various accelerometers on the market. All information taken from company websites (Analog Devices, 2003; Entran, 2004; Sensotech, 2004).

<table>
<thead>
<tr>
<th></th>
<th>Analog Devices ADXL250 Accelerometer</th>
<th>Sensotec MA341 Accelerometer</th>
<th>Entran EGA Accelerometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity</td>
<td>38 mV/g</td>
<td>100 mV/g</td>
<td>4 mV/g</td>
</tr>
<tr>
<td>Frequency Response (Hz)</td>
<td>0 to 1000</td>
<td>0 to 15000</td>
<td>0 to 600</td>
</tr>
<tr>
<td>Temperature Range (°C)</td>
<td>-40 to 70</td>
<td>-12.22 to 143.33</td>
<td>-40 to 120</td>
</tr>
<tr>
<td>g range (g's)</td>
<td>±50</td>
<td>80</td>
<td>±50</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>15</td>
<td>301</td>
<td>530</td>
</tr>
</tbody>
</table>

While the ADXL250 has many possible applications, the concern of this section of this document is to review what it does and how it accomplishes that particular operation. The best way to work on this equipment is to start internally, so the first mechanism that will be tackled is the micro sensing element.

The sensing elements that the ADXL250 employs are multiple differential capacitors. The way it uses this particular physical mechanism is by orienting the capacitors in a way that allows them to react to accelerations caused by mechanical movement. The obvious question that arises from this explanation is why would the capacitors be affected by acceleration due to the earth’s gravity? The capacitors utilized by the MEMS accelerometer are, for lack of a better word, tiny. The entire ADXL250 system is manufactured from a single monolithic integrated circuit using a surface micro machining process and an etching technique. The whole process yields nano size elements for the differential capacitance, which in turn allows the specific elements to be easily shifted out of equilibrium by the pull of earth’s gravity not to mention multiple times that acceleration. The layout of the interior of the ADXL250 sensing elements might look something like two combs lying side by side with their multiple extensions extending toward and in
between each other without actually touching. The center beam has multiple fingers, which extend to the left and right side. These fingers fall between the two plates of a capacitor, and therefore create a differential capacitor. The central beam is in suspension and will move when an external force such as the earth’s gravity acts upon the ADXL250. This in turn causes the nano size elements to shift in one direction or the other causing an increase of capacitance on the side of the capacitor the finger moves toward. Figure 2.6 shows a three dimensional schematic representation of this comb type microstructure, which helps explain the phenomenon that occurs.

![Figure 2.6: Three-dimensional representation of MEM interior (Glaser, 2000)](image)

Once the sensor receives an input acceleration, it will then send an output voltage signal. The signal is sent through the internal circuitry of the ADXL250. This circuitry includes an amplifier to increase the signal strength, a demodulator to recreate the signal, and a buffer amplifier to help reduce noise from the internal components of the ADXL250. The Block diagram in Figure 2.7 below shows a schematic of the internal circuitry.

![Figure 2.7: Block diagram of the internal circuitry of the ADXL250 accelerometer (Analog Digital, 2002).](image)

The ADXL250 must be powered externally, as well as requiring the use of an external bypass capacitor for operation. Any output manipulation is generally done with the use of external elements that adjust the signal to the operator’s specific needs. This is one of the inherent weaknesses of the MEMS system. Any signal manipulation requires the use of additional external elements that the operator must know how to use and in some cases wire in order to attain the desired result. This will slowly be removed from the MEMS systems when the wireless instrumentation becomes more inexpensive and readily available. The signal in a wireless system
would simply be acquired through the use of a laptop, which would be able to manipulate the signal though the use of software algorithms and wireless excitation as opposed to additional hardware. The interior of the ADXL250 is packaged in a hermetic 14-lead surface mount cerpack package. This package serves two purposes for the MEMS accelerometers. One is to create a clean outward finish that is easily incorporated into its surroundings, while the other is to reduce external noise from entering the accelerometer. Figure 2.8 shows the external look of the ADXL250 with the leads labeled.

![Figure 2.8: External diagram of ADXL250 (Analog Digital, 2002).](image)

A real weakness for the accelerometer is brought to light by this particular diagram. The ADXL250 requires multiple connections to external circuitry to both power the instrument and record the signal. This is troublesome because these connections are not likely to be as robust as the ADXL250 itself causing the equipment to breakdown although the instrument is undamaged. This is specifically troubling within civil engineering where the environment in the field can be very harsh, particularly in times of disaster which is when the information from monitoring systems would be the most essential. This is an issue that will be tackled within this study. The goal is to find an inexpensive way to seal and protect the connections and electronics from the harsh external conditions they may encounter. Now that the ADXL250 has been overviewed, the manipulation of the signal will be considered in order to help facilitate the understanding and use of this instrumentation.

### 2.5.1 ADXL250 Signal

The signal that the ADXL250 produces is in an analog form, i.e. the signal is a continuous form, which needs to be converted to a digital signal in order to manipulate the data. The output signal from the ADXL250 is Ratiometric to the source voltage and is interrelated by the following relationship:

\[
V_{\text{OUT}} = \frac{V_{\text{source}}}{2} + \left( \text{sensitivity} \times \frac{V_{\text{source}}}{5V} \times a \right)
\]  

(2.3)

The different terms represent output voltage \( V_{\text{OUT}} \), source voltage \( V_{\text{source}} \), sensitivity, and input acceleration \( a \). The standard sensitivity is around 38mV per g and the functional voltage range is four too six volts. The ADXL250 can be used to measure accelerations due to the earth’s gravitational pull and due to time varying phenomena (e.g., vibrations).
If the acceleration of interest is of substantial magnitude, the ADXL250 output signal requires very little external manipulation to record acceleration data. The system can simply be wired and attached to the area of interest with little effort. The required external elements are a power supply and a something to record and view the output signal (oscilloscope suggested). Filtering may be needed if there is a large amount of external noise, but you may be able to achieve this with the instrument that reads the output signal. If, however, the input acceleration or differential acceleration is too small to discern with the equipment that was just mentioned the output signal will have to be externally amplified in order to obtain a better response.

If a true gravitational response is needed for the measurement of tilt for instance, the operator may want to adjust the ADXL250 to read very small amounts of the earth’s gravitational pull. The standard sensitivity (38 mV/g) can be doubled by connecting the voltage output \( V_{\text{OUT}} \) pin to the offset null pin (i.e. zero g adjust lead), but this may not be enough to achieve the resolution required to measure tilt accurately. To have more control over the sensitivity the signal will have to be dc coupled to an external amplifier because gravity will act constantly on the ADXL250 and it will be hard to discern any changes in the signal due too such a small input. With the dc-coupled connection, any deviation in the 2.5-volt zero g offset will be amplified. The circuitry for the external amplification of the signal is shown in Figure 2.9.

![Figure 2.9: Block diagram of the internal circuitry of the ADXL250 accelerometer connected to an external amplifier (Analog Digital, 2002).](attachment:figure2_9.png)

This is the set up for a “quick offset calibration” of the ADXL150 or ADXL250 accelerometer, and it is used to determine the value of the different components required to obtain the scale factor desired. The OP196 is an amplifier from Analog Devices that must be added to the MEMS in order to allow the external instrumentation to pick up the MEMS signal at such small changes in acceleration. The diagram gives the user a good idea of how to wire the system to obtain a desired output while maintaining a large enough range. One issue that might arise with this
system is problems due to multiple connections in the system. Between the MEMS and the external amplifier, as well as between the MEMS and the power source and data acquisition equipment, there are several places that could break down under field conditions.

2.6 Summary

MEMS are the next step in the development of the sensor field within civil engineering. This is due to their small size and inherent durability, as well as the low cost and accurate data acquisition. Their macro sensor counterparts in the field of civil engineering are slowly going to be phased out because the performance of the MEMS is equivalent if not better at a lower cost. The next step is to find way to ensure a durable package in the harsh environments that may be encountered in the field. In addition, the instruments must be tested to ensure their capabilities are compatible to the typical data that must be acquired. Once the sensors are fully tested and can be incorporated into a project, they may very well revolutionize the industry.
CHAPTER 3
EVALUATION OF MEMS ACCELEROMETERS RESPONSE

3.1 Introduction

This master thesis presents the development of the Analog Devices ADXL250 MEMS accelerometer for its application into geotechnical engineering measurements. To be able to use MEMS accelerometers in the geoenvironment, the response of the transducers must be evaluated and the proper mechanical and electrical protection must be provided. The problem arises in that Analog Devices and other MEMS manufacturers do not fabricate the packaging needed for geotechnical engineering applications and the development of the packaging for a small industry would be too costly (see for example xbow.com. Their TG Series are 3-axes accelerometers and are sold for $1,295.00). Therefore, this thesis presents the calibration and the design of a packaging for low-cost, use-and-lose transducers that will help in the wide implementation of these small-size, innovative transducers in both practice and research within geotechnical engineering.

This chapter discusses the methodology for the calibration of the MEMS for both dynamic acceleration measurement and for static (inclination) measurements. Then, a low-cost packaging methodology is proposed and the system is tested under an aggressive salt solution. The calibration and testing methodology is presented. The limitations of the sensors are determined to ensure adequate performance in the field. The first step was to check the calibration of the instrumentation. This was required for this project to ensure that the converted data was giving us the correct acceleration values. Then a similar testing was performed with the MEMS accelerometer under water with corrosive agents trying to simulate rigorous field conditions.

3.2 Calibration of the Sensor

In order to correctly calibrate the instrument the manufacturers calibration was reviewed to obtain a better understanding of how they acquired their scaling factor and frequency range (Analog Devices technical note, 1998). Through review of the data sheets offered by the manufacturer on the ADXL250 it was found that the MEMS were calibrated using a calibrated shaker. While this is a very accurate way to ensure a known input for the sensor, it is also quite expensive and outside the budget for this study.

3.2.1 Sensor Frequency Range and Sensitivity

To check the sensitivity and frequency range of the MEMS various tests were run using the ADXL250 accelerometer and a more expensive and calibrated miniature, piezocrystal accelerometer by PCB electronics (model number: U352C22, SN: 25554). Using the PCB accelerometer as a benchmark, the two accelerometers were tested simultaneously to obtain an idea of what the difference in the two readings was for a given input. Before any systems could be designed for testing the sensitivity, the frequency range of the MEMS had to be determined. Knowing this information would reduce the amount of work later in the study.
3.2.1.1 Evaluation of the Frequency Range of the MEMS Accelerometers: Coherence Test

In order to get a true frequency range for the MEMS a coherence function was utilized. This function indicates the magnitude of the energy in the output that is caused by energy in the input (Santamarina and Fratta, 1998). The function is defined as:

\[
\gamma^2 = \frac{\left| CC_{u}^{(x,z)} \right|_{avg}^2}{\left( AC_{u}^{(x,x)} \right)_{avg} \cdot \left( AC_{u}^{(z,z)} \right)_{avg}} = \frac{(Z_u \cdot X_u) \cdot (Z_u \cdot X_u)}{(X_u \cdot X_u) \cdot (Z_u \cdot Z_u)}
\]  

(3.1)

Where \( CC_{u}^{(x,z)} \) is defined as the cross correlation of the signals X and Z. \( AC_{u}^{(x,x)} \) and \( AC_{u}^{(z,z)} \) are the auto correlation of the signals X and Z respectively. The signals X and Z are the discrete Fourier transform of the time signals x and z, which are obtained from the original response data from the PCB and the MEMS respectively. The Fourier transform is defined as:

\[
X_u = \sum_{i=0}^{N-1} x_i \cdot e^{-j \left( \frac{2\pi i}{N} \right)}
\]  

(3.2)

Where the \( X_u \) is the frequency response of the time domain signal \( x_i \). N is the number of points in the vector x which is indexed by i. The mathematical software, MathCAD, used for the analysis has some algorithms to help facilitate the computation of the Fourier transform called the fast Fourier transform of fft and the complex fast Fourier transform or cfft. The cfft is used when the number of points in the time vector is not a power of two (i.e. \( 2^n \)).

The coherence function yields a value that ranges from zero to one for each of the frequencies in the spectrum. When the magnitude of the coherence function yields a values equal or near to one, it indicates that the sensor is responding to energy in the input. If the coherence function yields values close to zero, it indicates that the sensor is responding to just noise in the system. Therefore the frequency range that yields coherence function values close to one represents the frequency range of the transducer when the sensor is still within its operational frequency range.

The test set up consisted of a MEMS accelerometer, a PCB miniature piezocrystal accelerometer, and an impact excitation source. All of these transducers were placed on a tabletop and both the MEMS and PCB accelerometers were placed in the same location in order to respond to the same excitation. The sensor was excited by a seismic pulse, which sends a wide range of frequencies through the tabletop. The seismic excitation was applied at a distance of 12.7 cm (5 in) from the receivers. This was maintained to try and achieve a repetitive signal throughout the testing. The final frequency range for the MEMS was determined to be above the 900 Hz range specified by the manufacturer. However, the results from the sensitivity tests would show large deviations in sensitivity once beyond the manufacture’s specified range of 900 Hz, which would limit the capability of the MEMS in areas where frequencies would exceed this upper limit. Figure 3.1 shows the testing set up for the coherence testing as well as the results.
3.2.1.2 Evaluation of the Sensitivity of the MEMS Accelerometers: Rod Test

Once the frequency range of the accelerometer was determined, the analysis of the MEMS sensitivity could begin. To evaluate the sensitivity a signal with a high signal-to-noise ratio was desired. To create this signal a cantilever beam setup was designed that allowed the length, and therefore the frequency of excitation, to be controlled.

The test involved the use of a steel rod approximately .5 cm (0.197 in) in diameter and 71 cm (28 in) in length to serve as the observed or monitored system. Figure 3.2 shows a schematic of the system. A circular cross section was selected for the rod because the moment of inertia is the same throughout the cross section of the rod, which allowed for less variability in the system. The rod was attached to the table with a rubber clamp, which helped to damp out the vibrations of the input signal. The two accelerometers (MEMS and PCB accelerometers) were attached directly to the end of the rod at the tip in order to keep them at the same location in space so the signal being recorded would be equivalent. Testing was done on both axes separately for this system. The length of the rod was varied throughout the test to yield various frequencies of excitation. This system obtained signal frequencies that ranged from approximately 8 Hz to 950 Hz. These frequency values falls within the frequency range observed with the coherence function. This ensured a response for the MEMS accelerometers.
PCB Accelerometer- Model Number: U352C22, SN: 25554

Figure 3.2: Steel rod testing system (a) schematic, (b) actual testing setup with rubber clamp, and (c) close up two accelerometers attached to the rod.

Figure 3.3: Voltage response of the rod system at 64 cm long (~18Hz).

Once the system was set up at a predetermined length the voltage response was taken on an oscilloscope. The length of rod used in testing ranged from 4 cm (i.e., 950 Hz) to 64 cm (i.e., 8 Hz). Shown in figure 3.3 below is a typical result for discrete time signal from the testing conducted with the steel rod.
This was eventually done for eight different sensors on each axis of the sensor at five different rod lengths in order to yield enough data to determine a mean sensitivity value and standard deviation for different frequencies of excitation. As was shown in figure 3.3, the two accelerometers yield almost identical signals for the steel rod testing system. Although the accelerometers respond at the same frequency the magnitude of the response is quite different because the sensitivity of the two sensors it much different. Figure 3.4 shows the calibration curve for the piezoelectric accelerometers.

![Calibration curve for piezoelectric accelerometers](image)

**Figure 3.4:** PCB calibration data reconstructed from PCB model U352C22, SN 25554 calibration certificate.

Using these signals, an analysis was done in order to check the calibration result for the ADXL250 given by Analog Devices. The Fourier transform was again used to convert the time domain data to frequency domain to obtain a more accurate response frequency for the system. The Fourier transform was explained in detail in the previous section. Figure 3.5 shows the response in the frequency domain for the signals presented in Figure 3.3. As was stated previously, the peak in the frequency plot is 18 Hz and corresponds to the excitation frequency of the cantilever beam with a 64 cm length.

![Frequency response of rod system](image)

**Figure 3.5:** Frequency response of the rod system at 64 cm length (~18Hz).
The signals are identical in the frequency domain over the system’s natural frequency response, but they diverge as the input signal dies out and the noise to signal ratio increases. The next step was to identify the scaling factor for the ADXL250. Using the same data from the steel rod system, the analytical signal was obtained and the instantaneous amplitude determined. This was done using a methodology that was outlined in the text by Santamarina and Fratta, 1998. This method is shown below:

1. Compute the Discrete Fourier Transform, DFT, of the signal, x, obtaining the frequency domain equivalent, $X_u$ (Equation 3.2).
2. Set all of the values above the Nyquist frequency to zero, which basically means that the second half of the signal must be set to zero. For $u = N/2$ to $N$, $X_u = 0$, resulting in $X_{\text{modified}}$.
3. Compute the inverse discrete Fourier transform, IDFT, of the frequency domain signal, $X_{\text{modified}}$, and multiply the result, $x_{\text{A}}$, by two. This is the analytical signal.

After obtaining the analytical signal, the instantaneous amplitude can be obtained through equation 3.3 shown below.

$$\text{amp}_i = \sqrt{\text{Re}\left(x_{i}^{(A)}\right)^2 + \text{Im}\left(x_{i}^{(A)}\right)^2}$$ (3.3)

Using the relationship in Equation 3.3, the instantaneous amplitude of the signals from both the ADXL250 and the PCB accelerometer were obtained and compared in order to determine what sort of scaling factor was being employed by the ADXL250. An example of the result of the instantaneous amplitude measurement determined through the use of an analytical signal is shown in Figure 3.6. This result is taken form the signal shown earlier in both the time and frequency domain for a 64 cm cantilevered rod length.

![Figure 3.6](image.png)

**Figure 3.6:** Instantaneous amplitude of MEMS accelerometer response for 64 cm cantilevered length (please see also Figure 3.3).

Using the ratio of the two instantaneous amplitudes (ADXL250/PCB) the true value of the sensitivity of the MEMS could be determined. This was possible because the ratio between the two signals will always be the same for any given input. Given that the PCB piezocrystal accelerometer (model number: U352C22, Serial Number: 25554) is 8.29 mV/g and using the portion of the instantaneous amplitude ratio over which the value stays constant (i.e. the input
signal is strong), the ADXL250 scaling factor is obtained by multiplying the inverse of the PCB calibration factor by the ratio value. Typical results for this analysis are given in Figure 3.7, which shows the ratio of the two signal amplitudes multiplied by the scaling factor for the PCB accelerometer yielding the calibration value for the ADXL250 accelerometer. This result is also from the time and frequency domain information shown in Figures 3.3 and 3.4 for a cantilevered length of 64 cm.

![Figure 3.7](image_url)

**Figure 3.7:** Calibration factor graph for 64 cm length cantilevered steel rod. The dashed red line indicated the calibration value for the MEMS accelerometer that best fit the data.

The dashed red line in the above plot is a marker representing the mode of the calibration value. In this particular graph the tail begins to vary excessively because the signal began to die out toward the end of the data acquisition, which can be seen in Figure 3.3. This analysis yields the best results when the response amplitude is higher which usually occurs in the beginning of a signal. The response for each of the different beam lengths was measured for 8 different MEMS accelerometers. Using all of the compiled data, a statistical analysis was conducted for each of the input frequencies (i.e. different cantilevered lengths), that is, the mean sensitivity and standard deviation was determined for each frequency. Figure 3.8 shows the results for each of the frequencies tested.

![Figure 3.8](image_url)

**Figure 3.8:** (a) Mean sensitivity ($\mu$) and standard deviation ($\sigma$) for the MEMS versus various frequencies of excitation. (b) Percent error at a given frequency of excitation.
The mean sensitivity ($\mu$) for each frequency was within the range of 33 to 43 mV/g given by the manufacturer (Analog Devices, 2002). However, there was a difference between the frequency range listed by the manufacture (900 Hz) and what was found during testing. The range determined through the coherence test data indicated a frequency range greater than 900 Hz, but secondary testing showed the true capabilities of the sensor began to deviate once the input frequency was beyond the upper limit given by Analog Devices. This can be seen in Figure 3.8 where the standard deviation and percent error jump dramatically at the high frequency value. Due to this result it was felt that it was best to limit the application of the sensor to systems in which the input frequency values would remain beneath the 900 Hz limit that the manufacturer specifies.

### 3.2.1.3 Evaluation of the Source Voltage Influence on Sensitivity

One additional test was run to determine the affect on the sensitivity when varying the source voltage excitation. While the manufacturer specification recommends an excitation voltage range of 4-6 Volts, there is no mention of what affect a change in the voltage has on the sensitivity of the sensor.

In order to check this concept the steel rod test was again employed to create a system that allows the control of the input frequency excitation. In addition to the setup used for the previous testing, a multimeter was added to allow for fine adjustment of the power supply voltage. This allowed accurate discernment of the excitation voltage down to tenths of a millivolt. The sensitivity test that was described in section 3.2.1.2 was then run again at 21 different excitation voltages. Figure 3.9 shows the results for the testing.

![Figure 3.9: Source voltage versus the sensitivity of the MEMS accelerometer.](image)

It can clearly be seen that there is an affect on the sensitivity as the excitation voltage is increased. A higher density of points was taken near the median value because that is the excitation that was used throughout the calibration of the MEMS accelerometer. The median value tends toward the 39 mV/g sensitivity specified by the manufacture, but deviating from that value by plus or minus half a volt will affect the sensitivity quite a bit. The best-fit linear relationship depicted in Figure 3.9 is given by the following relationship:
\[ S = 9.55 \cdot 10^{-3} \left[ \frac{V}{gV} \right] V_{source} \left[ V \right] - 8.792 \cdot 10^{-3} \left[ \frac{V}{g} \right] \] (3.4)

where \( S \) is the sensitivity in mV/g and \( V_{source} \) is the source voltage in Volts used to power the MEMS accelerometer. This is an unfortunate trend, but a relatively inexpensive power supply can address this issue. The only key is deciding what excitation to power the instruments with and using the corresponding sensitivity.

### 3.2.1.4 Evaluation of the 0 Hz MEMS Accelerometers Response: Rotation Test

One additional system was used to check the MEMS capabilities as a rotational sensor. It was felt that the value given for sensitivity in rotation might be somewhat different than that obtained due to a vibration. A simple system was devised which would allow for controlled angular inputs. The ADXL250 was rotated through 180º of rotation taking the differential voltage reading every 5º of the test. Figure 3.10 shows the testing setup and Figure 3.11 shows the results for the tilt capability test.

\[
\theta = \left[ \sin^{-1} \left( \frac{V_{axis}}{S} \right) \right] \cdot \frac{180}{\pi}
\]

**Figure 3.10:** The schematic of the testing apparatus and the relationship to determine rotation from the sensor.

It can be seen in Figure 3.11 that the sensitivity throughout this test remained at or near 39 mV/g. The rotation data was extrapolated using symmetry in order to obtain a full 360º of rotation. This additional testing proved to further verify the value of the sensitivity of the MEMS. It should be noted that the sensitivity of the MEMS can be doubled by simply altering the wiring slightly which would give a more accurate resolution over one g of acceleration. However, this increase in sensitivity reduces the applicable total range by one half to ± 25 gs.
Figure 3.11: 0 Hz MEMS accelerometers response: (a) MEMS response as the testing apparatus rotates 180 degrees and (b) MEMS response converted into the portion of 1-g of acceleration discerned by the sensor.

3.3 Design of a Low Cost Packaging Solution for the MEMS Accelerometers

To address the problem of packaging for the MEMS based sensor a simple concept was developed. The sensor would first be fully wired and checked to determine proper working order. Once this was confirmed the entire sensor and the connection points would be covered in a material that would withstand the elements that might be encountered within the geoenvironment. In addition the material needed to be electronically neutral to reduce possible electrostatic discharge to the device that could cause irreversible damage.
As was stated previously, the packaging design of the sensor was originally intended to reduce external noise and withstand a range of temperatures within its working environment. While these traits are helpful to its application in the geoenvironment, by no means would the original packaging be adequate for submersion or possible corrosive attacks. Figure 3.12 shows the MEMS sensor both unwired and fully wired for use in the field in its standard packaging. It can be seen that the packaging for the MEMS accelerometer would be inadequate for the geoenvironment. The external connections are likely to short if submerged, and they would definitely rust under corrosive attack.

![Image](image.png)

**Figure 3.12:** ADXL250 MEMS technology based accelerometer

It was also felt that the soldered connections could be an area of concern if the sensor needed to be moved an excessive amount during its lifetime because it would create a very weak and brittle link. Therefore the additional packaging would also need to immobilize the wiring coming into the external connectors to reduce the risk of breakage during use in the field. One final requirement of the new packaging material was that it must still convey the geometry of the MEMS so that correct estimation of the orientation could be made in the field without knowledge of how the sensor was packaged.

There were a variety of options on the market for the additional packaging material. Initially, it was felt that a simple application of two or three coats of a covering agent such as an acrylic or polyurethane would suffice to keep the MEMS safe against attack and submersion. In addition, a few coats would still allow the geometry of the ADXL250 to be discerned through the coating. Though this concept seemed simple enough, it had some pitfalls. The most prominent of which was the lack of ability to determine weather or not the external connections had been fully covered after the application of the coating. Additionally, the external connections were difficult to keep covered and did not manage to stay immobilized after the application of either material. Because of these problems a simple coating system was abandoned in favor of a casting concept.
Again the MEMS would need to be fully wired and operational before the application of the protective material, but now the entire sensor would be cast into a mold slightly larger than its original nominal dimensions. The shape of the mold was an issue, because the ability to use both axes of the sensor would require a prismatic shape. However, this issue was avoided by using a clear casting material which allowed the orientation to be seen at all times once hardened. After a few unsuccessful trials, a final shape and material were selected. Figure 3.13 shows the final product for the MEMS external geoenvironment protection. The figure clearly shows the MEMS through the cast cylinder, which is created from a urethane-casting agent called Crystal Clear 204 by Smooth-On. The Nominal dimensions of the final cylindrical element were 13.6 mm (0.54”) in diameter by 14.2 mm (0.56”) in height. This solution worked nicely since it allowed the sensor and connections to be totally covered while maintaining a relatively small overall final size to maintain its ability to be retrofitted into existing projects.

![Figure 3.13: ADXL250 MEMS geoenvironment protection system](image)

### 3.3.1 Testing of a Low Cost Packaging Solution

To test the packaging technique the same tests that where run previously were run again on the cast MEMS accelerometer, but now the accelerometer was placed in a corrosive environment in order to test the ability of the packaging. This was challenging for a couple of reasons. The primary concern was that the benchmark sensor, the piezocrystal accelerometer by PCB electronics (model number: U352C22, SN: 25554), was not packaged in a manner that could cope with the corrosive environment. In addition, the PCB sensor cost approximately $400, which is by no means a use and loose transducer. Another concern was how to attain the same frequency levels with the MEMS submerged in a corrosive fluid.

#### 3.3.1.1 Reevaluation of the Frequency Range for Packaging influences

The frequency range was evaluated with the same methodology as was conducted for the unprotected sensor. Using the coherence function given in equation 3.1 the time domain data obtained from testing would be used to determine the applicable frequency range in the post packaging state for the MEMS accelerometer. The major concern was how the benchmark accelerometer would be located so that the sensor would receive the same signal. This was addressed by simply adhering the PCB accelerometer to the same external location on the test cell that the MEMS accelerometer was placed in during its submersion testing.
The testing apparatus consisted of a cylindrical Plexiglas cell that was 100 mm (4”) in diameter by 150 mm (6”) in height. The cylinder would also serve as the wave propagation medium, which would help solve the problem of locating the benchmark and the MEMS accelerometer at the same location. The cast MEMS accelerometer was attached to the interior of the cylinder at a height that would maintain submersion during the testing, and the cell was then filled with a dissolved one mole NaCl per liter solution that completely submerged the cast MEMS accelerometer. Once submerged the MEMS was checked to verify it was in working order. The piezocrystal accelerometer by PCB electronics (model number: U352C22, SN: 25554) was then attached to the external side of the cell at the same geometric location as the MEMS accelerometer. This was facilitated by a wax putty provided with the PCB accelerometer that allows easy attachment and removal during testing. Figure 3.14 depicts the submersion system.

![Figure 3.14: Submersion apparatus to facilitate testing the packaging system developed for the MEMS accelerometer. (a) Full view of the cell with sensor in place and (b) close of sensor location both inside and outside of the cell.](image)

Using an instrumented hammer, the opposite side of the cell was struck to induce wave propagation. The signals were recorded in the time domain on an oscilloscope and the same coherence algorithm that was used as was shown earlier in section 3.2.1.1. The results of the algorithm are shown in Figure 3.15.

![Figure 3.15: Frequency response of the Analog Devices ADXL250 accelerometer corresponding to the vertical axis of sensitivity.](image)
Although the packaging results indicate there may have been some influence on the frequency range due to the casting technique it seems that it can be assumed to be negligible. If Figure 3.1, which shows the coherence prior to the packaging being applied, is compared to Figure 3.15 there is a difference in the frequency response of the system. The prepackaging accelerometer reaches a frequency of 2000 Hz before any large deviation is seen. The response shown in Figure 3.15 shows a range that begins to fade around 1100 Hz. While there is a possibility that this could be due to the packaging, it is most likely due to the testing technique, which had a smaller range of input frequencies due to the physical constraints of the setup utilized. This matter very little because the sensitivity turns out to be the controlling factor in the operation of the MEMS accelerometers because it begins to loose accuracy at 900 Hz. This was the pre-established limit on the operation of the MEMS accelerometers for application in wave propagation detection prior to any packaging. Additional testing was also conducted to determine the influence of the casting material on the sensitivity of the MEMS accelerometer.

3.3.1.2 Revaluation of the Sensitivity for Packaging influences

Another set of data was also taken to revaluate the sensitivity analysis. Using the same setup as was shown in Figure 3.2 previously the MEMS were excited at a frequency within the range of operation for the instrument according to the coherence analysis. Although a rigorous statistical analysis was not conducted again, it was believed that the results of the analysis were indicative of the performance characteristics of the MEMS accelerometer.

After conducting the reduced analysis, the final values of sensitivity were again seen to approach the previous value. Figure 3.16 (following page) depicts an analysis of the response of the MEMS accelerometer tested against the piezocrystal accelerometer by PCB electronics (model number: U352C22, SN: 25554). The same PCB accelerometer was used throughout all of the testing to ensure consistency and reduce variability of the results.

Based on the results from the cast accelerometer testing the low cost packaging solution seems to have very little effect on the performance of the MEMS accelerometer. The calibration factor for the signal shown in Figure 3.16 was somewhat low in comparison to typical data that was seen for this experiment. The final calibration or sensitivity factor selected for the operation of the MEMS accelerometers was 38 mV/g. This matches almost exactly to manufacturer specification of 39 mV/g.

3.3.2 Low Cost Packaging Solution Evaluation

The final solution settled on for the packaging technique used on the MEMS accelerometers was very robust under the different testing that was conducted. The first (and most important) finding was that the selected packaging system had no influence on the operating characteristics of the MEMS accelerometer. This was shown through both the coherence and sensitivity tests results for the packaged accelerometer. Additionally, the packaging system was submersible and resistant to corrosion attack, which will allow the sensors to be used later in the field. Other positive characteristics were the clear finish for ease in establishing orientation and isolation of the brittle wiring points needed to operate the MEMS accelerometer. While the system has many positive points, it does have on very strong negative point, which is its permanence. This
unfortunate characteristic that leads to many of the positive characteristics of the system is also one of the downfalls of the technique. If there is any reason to have to adjust the wiring it is very difficult to remove the packaging material in order to reach the sensor. While this might be an issue with some sensors, when using the very inexpensive MEMS accelerometer the possibility of breaking a sensor during extraction from the packaging is less of a concern.

![Figure 3.16](image)

**Figure 3.16:** Results taken from the cast MEMS accelerometer for sensitivity analysis for a cantilevered length of approximately 64 cm (~18 Hz). (a) Raw data in the time domain, (b) data after conversion to the frequency domain, (c) instantaneous amplitude response of the data, and (d) the calibration factor determined from the instantaneous amplitude results.

### 3.4 Summary

The testing and analysis of the Analog Devices ADXL250 MEMS accelerometer was conducted in a fashion so that the instrument could be fully characterized before testing could be conducted utilizing the device as part of another geotechnical system. The results of the testing on the MEMS sufficiently characterized the accelerometers for further use in geotechnical testing. In addition, a packaging system was developed to facilitate possible field implementation once areas of geotechnical testing are identified for its application.
CHAPTER 4
REVIEW OF WAVE PROPAGATION CONCEPTS

4.1 Introduction – Wave Propagation

Chapters 2 and 3 present a review of MEMS technology and the calibration of the MEMS accelerometers and inclinometers for their using in geotechnical engineering. The next three chapters are dedicated to the use of MEMS to the measurement of engineering parameters and processes. Chapter 4 reviews wave propagation concepts that will be used in the analysis of the data in the later chapters. Then, Chapters 5 and 6 present the application of dynamic data for the evaluation of an impact source response and for the monitoring of changes of state of effective stresses behind a retaining wall.

The ADXL250 MEMS accelerometer may be used to monitor the wave propagation from a source to a receiver over a medium (i.e. accelerations in solid materials). Therefore to be able to interpret the data that are presented in Chapters 5 and 6, the physics that govern elastic wave propagation should be presented. While the concept of wave propagation is intuitive, some of the elements involved with its explanation and theoretical background are less obvious. In order to provide a general and/or basic understanding of the propagation of waves through a material some basic examples as well as terminology will be presented in this chapter. If the reader is interested in a more thorough understanding of some of the concepts in this chapter much of the information within was taken from Kolsky (1963), Kramer (1996), and Santamarina et al (2001).

4.1.1 The Governing Equations

Consider an unbounded continuum (i.e. one that extends infinitely or indefinitely in the direction of wave propagation) that is linear-elastic, isotropic, and homogeneous. There are three sets of equations that govern the mechanical behavior of this medium. The equilibrium equations state that for any body to be in equilibrium the summation of forces and moments must be equal to zero. In terms of shear stress and normal stress the equilibrium equation in the x-direction is given as the following (taken from figure 4.1)

\[
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} + \frac{\partial \sigma_{yx}}{\partial y} + X = 0
\]  

(4.1)

where X is a body force in the corresponding direction. The x direction will be used as a representative direction for this chapter when the equations for the y and z directions can be obtained simply by substituting the required subscripts. Equilibrium equations, while a start, are not sufficient to solve the stress or strain field in a continuum. The second set of equations that govern the mechanical behavior of a continuum are compatibility equations. These equations provide the conditions required for the continuum to avoid cracking (Santamarina et al. 2001). This is important because breaks in the continuum would create anomalies, which would have vastly different properties from the continuum.
Figure 4.1: Stress components acting on an infinitesimal parallelepiped element (modified after Kolsky 1963).

For this example, the strains are gradients of deformations, which are given by $u_x$, $u_y$, and $u_z$. Therefore the definition of strain is

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}$$

(Similarly for $y$ and $z$ directions) \hspace{1cm} (4.2)

and

$$\gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

(Similarly for $y-z$ and $z-x$) \hspace{1cm} (4.3)

where $\varepsilon$ is given as the strain normal to a given plane and $\gamma$ is the shear strain. Also known is that for small strain, the volumetric strain is

$$\varepsilon_v = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \nabla \cdot \mathbf{u}$$

(4.4)

where $\nabla$ is the gradient and $\nabla^2$ is the Laplacian operator which is indicated by the following:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

(4.5)

While the compatibility and equilibrium equations provide an adequate number of equations to work within the continuum, they are in incompatible terms. The equilibrium equations are given in terms of stress and compatibility is in terms of displacement. In order to bring these two sets of equations together a third set, constitutive equations, is needed. The equations can be found in the form of Hooke’s law for a linear-elastic continuum.

$$\varepsilon_{xx} = \frac{1}{E} \left[ \sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz}) \right]$$

(Similarly for $y$ and $z$ directions) \hspace{1cm} (4.6)
Hooke’s law relates stress to strain through the use of elastic constants. While the most commonly used form of Hooke’s law uses the Young’s modulus, $E$, and Poisson’s ratio, $\nu$, it can also be written using other elastic constants. For its use in the wave equation, Hooke’s law will be in the following form

$$\sigma_{xx} = M \varepsilon_x + 2G \varepsilon_{xx}$$

(Similarly for y and z directions) \hspace{1cm} (4.8)

and

$$\sigma_{xy} = G \gamma_{xy}$$

(Similarly for y-z and z-x planes) \hspace{1cm} (4.9)

where $M$ is the constrained modulus defined as $\sigma_x/\varepsilon_x$ when $\varepsilon_x = \varepsilon_z = 0$ (Based on the assumption that this is an isotropic medium the other directions would yield the same value for $M$ with the same constraints). $G$ is the shear modulus, which is a material constant. It should be noted that only materials that can resist shear have a shear modulus. Therefore no fluid has a value for the shear modulus.

### 4.1.2 Equations of Motion

To obtain the equation of motion for an elastic continuum the variation of stress across an infinitesimal element much like that shown in Figure 4.1 is considered. While earlier there were no forces actually acting on the element in equilibrium, it will now be acted upon by some stress gradient $\partial \sigma_{xx} / \partial x$ over the length of $dx$ (substitute where required for $y$ and $z$). In order to obtain the external force acting on the element for each face the stress will be multiplied by the area of the corresponding face for each stress. In addition, the initial stress will be subtracted out of the formulation. Shown as the following for the x-direction:

$$\left( \sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} \right) dy dz + \left( \sigma_{xy} + \frac{\partial \sigma_{xy}}{\partial y} \right) dx dz + \left( \sigma_{xz} + \frac{\partial \sigma_{xz}}{\partial z} \right) dx dy$$

(4.10)

Applying Newton’s second law of motion, $F=ma$, and neglecting body forces such as gravity equation 4.10 must equal:

$$\rho dxdydz \frac{\partial^2 u_x}{\partial t^2}$$

(4.11)

where $\rho$ is the density of the continuum and $dxdydz$ is the infinitesimal volume (Kolsky 1963). Together these terms yield the mass while the second term in equation 4.11 is acceleration. Setting equation 4.10 and 4.11 equal and simplifying yields the following:
Using the constitutive equations, the stresses are redefined in terms of strain

\[
\rho \frac{\partial^2 u_x}{\partial t^2} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z}
\]

(Similarly for y and z directions) \hspace{1cm} (4.12)

and then the strain are further simplified to give the deformation using compatibility equations. Further simplifying the equation with the Laplacian operator yields the following result:

\[
\rho \frac{\partial^2 u_x}{\partial t^2} = (M + G) \frac{\partial \varepsilon_{vol}}{\partial x} + GV^2 u_x
\]

(Similarly for y and z directions) \hspace{1cm} (4.13)

Equation 4.14 is the equation of motion in one direction for an isotropic elastic solid in which body forces are absent, also known as the wave equation (Santamarina 2001). It describes a displacement field that varies in both time and space. This equation predicts only two modes of propagation through a medium (Santamarina 2001).

4.1.3 Compression Waves

The first mode of propagation (or solution) for the wave equation is obtained by differentiating each direction of equation 4.14 (x, y, and z) with respect to its direction and adding:

\[
\frac{\partial^2 u_x}{\partial t^2} = \left( \frac{\lambda + 2G}{\rho} \right) \frac{\partial^2 u_x}{\partial x^2} = v_p^2 \frac{\partial^2 u_x}{\partial x^2}
\]

(Similarly for y and z directions, plane wave) \hspace{1cm} (4.15)

This equation defines a waveform that propagates through a medium with dilation only (i.e. no rotation), which is defined as a P-wave or compression wave. This type of wave creates particle displacements that are parallel to the direction of propagation of the wave (Kramer 1996). Equation 4.15 also defines the p-wave velocity as \( V_p = [\frac{\lambda + 2G}{\rho}]^{0.5} \) or \( [\frac{M}{\rho}]^{0.5} \).

4.1.4 Shear Waves

The second mode of propagation for the wave equation corresponds to a wave that propagates in one direction with particle motion perpendicular to that direction. To obtain this solution to the wave equation the y-direction is differentiated with respect to z and the z-directions with respect to y. The two results are subtracted and dilation \( \varepsilon_{vol} \) is assumed to be zero yielding

\[
\frac{\partial^2 u_y}{\partial t^2} = \frac{G}{\rho} \frac{\partial^2 u_y}{\partial x^2}
\]

(Similarly for y and z directions, plane wave) \hspace{1cm} (4.16)
This defines an S-wave or shear wave, and the velocity (i.e. shear wave velocity) at which it propagates through the continuum is \( V_s = (G/\rho)^{0.5} \). It should be noted that this wave propagates only in a two dimensional plane. Because of this a given S-wave can be decomposed into the vector sum of its vertical (SV) and horizontal (SH) components. Generally, S-waves are always resolved into these two forms. Figure 4.2 shows an example of each type of wave propagation through a continuum.

![Figure 4.2](image_url)

**Figure 4.2**: Example of body waves as they move through a continuum. The dots are considered to be particle centers, which are connected by springs. (a) Positions before propagation. (b) Displacement during the propagation of a P-wave. (c) Displacement during the propagation of an S-wave (Santamarina et al. 2001).

While within an unbounded linear elastic solid, only two types of waves, body waves, can propagate (Kramer 1996). It should be noted that fluids, even if they satisfy the assumptions, can only support compression waves because they have no shear strength and therefore no shear modulus, G. Although they have little to no bearing to this research, there are other types of waves that exist.

### 4.2 Waves in Layered Media

The homogenous elastic half space model is very convenient for the explanation of body waves, but in reality the conditions are much more complex. There are many different material types that have vastly different properties, which waves travel through as they propagate from a source to a receiver. In order to gain a better understanding of wave propagation a thorough knowledge of the behavior of waves as they reach the interface between two materials is needed. Although the reality of wave propagation is that there are many different materials with varying geometry and material properties, simplification will be made to facilitate the explanation of the behavior at an interface between two materials. Some examples of wave propagation along the boundaries are Rayleigh waves and Love Waves. These two wave types are surface waves. The first, Rayleigh waves, can be shown to exist in a homogenous elastic half space near the surface. The
second, *Love waves*, requires a surficial layer of lower S-wave velocity than the underlying half-space (Kolsky 1963; Richart et al. 1970; Achenbach 1975). This description, however, will concentrate on the behavior of body waves because this is the subject that is most related to this study.

### 4.2.1 Incidence Wave Normal to Interface

Consider a wave propagating through medium 1, the *incidence wave*, approaching the interface between medium 1 and medium 2 at an angle normal to the interface. Also note each material has its own unique properties (i.e. impedance, $I_1$ and $I_2$). Compatibility of deformation states that the particle motion at the interface of the two materials must be the same. Effectively, this means the Amplitude of the incidence wave ($A_I$) plus the amplitude of the *reflected wave* ($A_R$) in medium 1 must be equal to the amplitude of the *transmitted wave* ($A_T$) in medium 2, $A_I + A_R = A_T$ (Kramer 1996). Figure 4.3 depicts the described scenario.

![Figure 4.3: Transmission and reflection in a two medium system.](image-url)

The amount of energy that is transmitted or reflected at the interface of two different materials is dependent on the impedance ratio, $\alpha_z$, of the two materials. The impedance, $I$, of a medium is defined as the product of the velocity, $V$, (dependent on wave type) and the mass density, $\rho$. The impedance ratio, $\alpha_z$, is defined as $I_2/I_1$. In order to determine the partitioning of the energy into the reflected and transmitted waves equation of equilibrium are used at the interface, $\Sigma F_1 = \Sigma F_2$. Combining the compatibility and equilibrium equations at the interface leads to the expressions for the reflection and transmission displacement amplitudes. These relationships are the following (Kramer 1996):

$$A_R = \frac{1 - \alpha_z}{1 + \alpha_z} A_I \quad (4.17)$$

and

$$A_T = \frac{2}{1 + \alpha_z} A_I \quad (4.18)$$

In addition to the determination of the partitioning of the displacement amplitude at the interface one can also determine the redistribution of stress at the interface. Also related through the impedance ratio, the relationships for the reflected a transmitted stress values are the following (Kramer 1996):
The importance of the impedance ratio in determining the nature of transmission and reflection is clearly seen by simply assigning values and obtaining the numbers. When the impedance ratio is less than one (i.e. rock to soil), the amplitude of the reflected wave is negative, which indicates a phase reversal from the incident wave to the reflected wave. A value equal to one indicated that there is no interface. Zero indicates a free end, which can support no transmitted stress. However, the displacement of the free end will be twice that of the incident value. Another interesting value is \( \infty \), which indicates a fixed end. While it cannot displace, the stress value is twice that of the incident value. Figure 4.4 shows a range of impedance ratios as well as the calculated reflection and transmission.

\[
\sigma_R = -\frac{1 - \alpha_z}{1 + \alpha_z} \sigma_i \tag{4.19}
\]

and

\[
\sigma_T = \frac{2\alpha_z}{1 + \alpha_z} \sigma_i \tag{4.20}
\]

**Figure 4.4:** Partitioning of reflection and refraction values based on the impedance ratio for displacement and stress for normal incidence.

### 4.2.2 Incidence Wave Oblique to Interface

While incident waves can approach an interface at a 90° angle, this is generally not the case. The orientation of an inclined body wave has a great effect on the manner in which the energy of the wave is partitioned into reflected and transmitted waves. Fermat’s principle states that the propagation of a seismic wave between two arbitrary points A and B is always the path of minimum travel time along any continuous path between A and B. The path that produces the minimum travel time is called the *ray path*, and it is usually represented by a vector called a *ray*. A ray is always perpendicular (in an isotropic medium) to the *wavefront*, which is defined as the surface of equal travel time. Snell considered the change of directions of ray paths at the interface between two materials. Utilizing Fermat’s principle of minimum travel time, Snell showed the following:
where \( i \) is the angle between the incident ray path and a path normal to the interface (Kramer 1996). \( C \) is a constant, and \( v \) is the wave velocity (velocity of interest) of the medium. This relationship is valid for both reflected and transmitted waves, and it indicates that all transmitted waves will be refracted unless the angle is zero (i.e. \( i = 0 \) and the wave is normal to the interface) or the propagation velocities of the two materials are equal.

Consider an example where two elastic half spaces of different material are in contact with each other. Much like previous examples, the conditions of equilibrium and compatibility along with the theory of elasticity can be used to determine the redistribution of energy across the interface for an incident P-wave, SH-wave, and SV-wave. Unlike the previous example, the oblique incidence of some of the wave types alters the particle motion across the interface, which is called mode conversion.

The mode conversion of S-waves depends on the direction of particle motion with respect to the interface. If the S-wave is polarized horizontally (i.e. SH-wave, there is no particle motion transverse to the interface), then only SH-waves will be reflected and transmitted and there is no mode conversion. Because the particle motion is parallel to the interface, the interface has no mode conversion effect. On the other hand, incident SV-waves do have particle motion transverse to the interface and therefore mode conversion does occur. In the case of an oblique SV-wave, both P- and SV-waves are reflected and refracted. The same is true for an incident P-wave, which also produces both P- and SV-waves in reflection and refraction. Figure 4.5 depicts the three types of body waves as they incur an interface at an oblique angle.

**Figure 4.5:** Reflected and refracted rays due to an incident (a) P-wave, (b) SV-wave, and (c) SH-wave (Kramer 1996).

The angle of incidence is uniquely related to the angle of refraction by the ratio of the wave velocities of the two materials on either side of the interface. This is seen in the generalized form of Snell’s law:

\[
\frac{\sin(i_{p1})}{v_{p1}} = \frac{\sin(i_{p2})}{v_{p2}} = \frac{\sin(i_{s1})}{v_{s1}} = \frac{\sin(i_{s2})}{v_{s2}}
\]
This law indicates that as a wave propagates from higher velocity material to lower velocity materials the angle of refraction will successively move closer and closer to normal to the interface (Kramer 1996). This sort of model reflects what is typically seen in the earth’s surface as a wave propagates upward through increasingly lower velocity materials coming nearer and nearer to a vertical path. This phenomenon is relied upon for a variety of ground response analysis methods. Another phenomenon that is important in seismic refraction tests is the critical angle of incidence, \( i_c \), which is defined as the incident angle that produces a refracted wave that travels parallel to the interface.

It is quite obvious that the interaction of stress waves with interfaces or boundaries can create quite complex wave paths. It is because of this phenomenon that many laboratory experiments are designed to simplify the travel path of a wave for being used to characterize a soil. When dealing with body waves, it is common that the researcher is interested in only the arrival of a specific waveform. Using the information about how waves interact with boundaries, researchers design their experiments to obtain either S- or P- waves as the first arrival.

There is a great deal of debate devoted to where the first arrival actually occurs, and in some cases it can be relatively obscured by noise. Generally, the arrival time can be taken as the first peak, trough, or deviation from zero (Arulnathan et al. 1998), but doing this assumes a plane wave front (i.e. direct travel path) with no reflected or refracted signals. While this is not necessarily incorrect, it can be seen why it is imperative to analyze source and receiver locations to ensure a correct first arrival. Figure 4.6 depicts data taken from a bender element test where the first arrival was actually somewhat obscured due to electromagnetic interference, P and S-wave arrivals, and noise.

![Figure 4.6: Data from a bender element test where the first arrival of the S-wave was relatively obscured due to both noise and possible interference problems (Tanner 2003).](image)

This bender element test data is taken from an experiment performed with an oedometric test cell. The bender elements were mounted in the side of the cell with a horizontal polarization for this particular data. With this type of physical setup, there are two types of waves that can be transmitted through the cell as well as additional noise from the bender elements themselves if improperly grounded. The P-waves generated from the compressive motion of the bender element out of plane are reflected off of the interior wall of the cell and can reach the receiver prior to the S-wave arrival. P-waves are also generated from the anchoring point of the bender
element during excitation. The movement causes an addition waveform to be transmitted into the cell wall at the anchoring point. This signal has a much larger velocity than the soil in the soil, and therefore would be very likely to arrive at the receiving bender element prior to the S-wave arrival. As was stated previously, the noise in these types of sensors is usually due to improper grounding or ambient background noise. The occurrence of these different types of wave interference for the oedometric soil-testing cell is depicted in Figure 4.7.

![Wave propagation in horizontally polarized bender elements within an Oedometric test cell (modified after Santamarina et al. 2001).](image)

**Figure 4.7:** Wave propagation in horizontally polarized bender elements within an Oedometric test cell (modified after Santamarina et al. 2001).

### 4.3 Wave Attenuation

Throughout this chapter, it has been assumed that the body waves will be propagating through homogeneous linear elastic half space. Waves can travel indefinitely in such an idealized material, but in reality this behavior cannot and does not occur. The amplitudes of waves in a realistic medium attenuate with distance due to two different sources.

#### 4.3.1 Material Damping Parameters

Real materials always partially dissipate the elastic energy of an elastic waveform. This dissipation is always accompanied by a decrease in the wave amplitude. Viscous damping is often used to represent this phenomenon because the energy that is lost in soils are actually due to a variety of mechanisms that are not fully understood (Kramer 1996). Generally, in viscoelastic wave propagation, soils are modeled as a Kelvin-Voigt solid. The stress strain relationship for the model is the following:

\[
\tau = G\gamma + \eta \frac{\partial \gamma}{\partial t}
\]  \hspace{1cm} (4.23)

Where \(\gamma\) is the shear strain, \(G\) is the shear modulus as was stated previously, and \(\eta\) is the material viscosity. Figure 4.8 illustrates a Kelvin-Voigt element represented by a spring and a dashpot in parallel.

As it is shown, the model is the sum of an elastic portion (proportional to strain) and a viscous portion (related to strain rate). For harmonic shear strain in the form \(\gamma_o \sin(\omega t)\) the shear stress becomes:

\[
\tau = G\gamma_o \sin(\omega t) + \omega \eta \gamma_o \cos(\omega t)
\]  \hspace{1cm} (4.24)
Figure 4.8: Kelvin-Voigt solid under applied horizontal shearing. The elements shearing resistance is represented by a spring (elastic) and dashpot (viscous) component (Kramer 1996).

Based on the harmonic excitation and equations 4.23 and 4.24 it can be shown that the stress-strain loop of the Kelvin-Voigt is elliptical. The elastic energy dissipated in one cycle can be taken from the area of the ellipse from the stress strain loop,

$$\Delta W = \int_{t_o+2\pi/\omega}^{t_o} \tau \frac{d\gamma}{dt} \, dt = \pi \eta \omega \gamma^2$$

which indicates that the energy that is lost is proportional to the frequency of loading (Kramer 1996). A damping ratio, $\xi$, can be defined as the following

$$\xi = \frac{c}{2\sqrt{km}} = \frac{\Delta W}{4\pi W}$$

where $c$ is the system damping coefficient and the lower term is the critical damping coefficient (also $c_c$), which is defined as two times the square root of the stiffness times the mass (Santamarina et al. 2001). This is also the dissipation of energy per cycle, $\Delta W$, over the peak energy in the cycle, $W$, times a constant, $4\pi$ (the damping ratio is also sometimes denoted by D). However, this improperly models the true behavior of real soils because they exhibit a hysteretic effect during energy dissipation. This phenomenon occurs because energy is lost in soils through the slippage of soil grains with respect to one another, and this means that the energy dissipation of real soil is unaffected by the frequency of loading. To eliminate the dependence on frequency and maintain the visco-elastic model, the damping is placed in a formulation for an equivalent viscosity that is inversely proportional to frequency (Kramer 1996).

$$\eta = \frac{2G}{\omega \xi}$$

4.3.2 Intrinsic and Geometric Attenuation

Material damping absorbs some of the elastic energy in a body wave during attenuation; therefore, the energy dissipates as the wave travels through a material. The loss of that specific energy, energy per unit volume, causes a reduction in the amplitude of the wave as distance from the source increases. This behavior is typically modeled as an exponential decay in which the amplitude, $A$, at one location is related to the amplitude at a prior location by the following

$$A_z = A_t e^{-\alpha(t-t_z)}$$
where $\alpha$ is the attenuation coefficient, $r_1$ is the distance from the source at location one, and $r_2$ is the distance from the source at location two (Santamarina et al. 2001). The attenuation coefficient, $\alpha$, is related to the damping ratio, $\xi$ or $D$, for low loss wave propagation through the following

$$\alpha = \frac{2\pi D}{\lambda}$$  \hspace{1cm} (4.29)

The reduction of energy is also caused by another mechanism common to most wave propagation systems. When a source is created, it is usually done in a manner in which a large amount of energy is created over a small volume. In some cases this can be a charge in the ground or a hammer in the laboratory. In both cases the energy input at the source is very high. If the system is a uniform cross section rod, this energy should be maintained relatively well except for intrinsic attenuation. However, if the system is not uniform in cross section in the direction of wave propagation (i.e. a cone hit on its point) the same energy will be spread out over a larger and larger volume. This phenomenon is often referred to as radiation damping or geometric spreading. If the source of the stress wave is modeled as a point with a spherical wave front, it can be shown that the rate of amplitude decay for body waves is $1/r^2$ (Richart et al. 1970). The general formulation of the decay of the amplitude of particle motion is the following

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^\zeta$$  \hspace{1cm} (4.30)

where the exponent $\zeta$ is defined by the type of geometric attenuation. Typical values for the exponent are: $\zeta = 0$, corresponding to a plane wave in infinite media and in rods, $\zeta = 0.5$, corresponding to cylindrical wave front and, $\zeta = 1$, corresponding to spherical wave fronts (Richart et al. 1970).

### 4.4 Propagation of Waves through a Continuum

In order for a wave to traverse a medium as if it were a continuum its wavelength ($\lambda$) must be significantly larger than the characteristic dimension ($a_c$) of the medium. Wavelength, $\lambda$, is the distance in space between two points on a wave that exhibit the same amplitude and slope (Elmore 1969). Figure 4.9 depicts the definition of wavelength to better explain the concept.

![](Figure 4.9: Definition of wavelength depicted for explanation of concept.)
This concept is especially important when referring to geologic materials because it is well known that soils are actually made up of small particles. If a wavelength were small enough, it would see each particle as an individual continuum with specific properties (Fratta and Santamarina, 2002), which would make the problem of wave propagation orders of magnitudes greater in difficulty. (This is the same reason that the physical properties of soils are modeled on a macro rather than micro scale. The internal geometry alone would be very complex to model for just a handful of particles.) With the correct wavelength the soil mass will be traversed as if is an equivalent continuum that has the average properties of the particulate media. Therefore, it is easier to step back and describe wave propagation from the point of view of a uniform medium with average properties.

### 4.4.1 Elastic Waves and Geomaterials

The propagation of elastic waves through geomaterial due to small strain perturbations can be used to characterize the material without altering the soil structure. Due to this fact, velocity and attenuation are constant properties of granular materials at a given state, and they can be utilized in the characterization of processes within a soil mass without altering the internal effects of the process. Based on the previous sections in this chapter, there are four material parameters that control wave propagation: bulk stiffness, $B$, shear stiffness, $G$, mass density, and intrinsic attenuation, $\alpha$ (Santamarina et al. 2001). These four characteristics control the following wave properties:

\[
V_p = \sqrt{\frac{M}{\rho}} = \sqrt{\frac{B + \frac{4}{3}G}{\rho}} \quad \text{P-wave velocity} \quad (4.31)
\]

\[
V_s = \sqrt{\frac{G}{\rho}} \quad \text{S-wave velocity} \quad (4.32)
\]

\[
\frac{A_s}{A_i} = e^{-\alpha x} \quad \text{Intrinsic attenuation for plane wave} \quad (4.33)
\]

Using the above wave propagation properties, it is possible to characterize various properties of a geomaterial. Emphasis is typically placed on S-wave propagation because waveforms of this type yield more information about the material due to the way they propagate (i.e. particle displacement perpendicular to wave propagation direction).

This study will utilize the properties of the shear wave to determine velocity fields, and therefore a specific review of what effects their behavior as well as some example quantitative values for S-waves in relation to geomaterial will be demonstrated in this section. The shear wave velocity in soils is controlled by the stiffness of the granular skeleton, and the mass density of the geomaterial. Table 4.1 presents some example values of shear wave velocity for geomaterials for a range of densities.
**Table 4.1:** Material S-wave velocity and corresponding density based on assumed Poisson’s ratio, $\nu = 0.1$ (Santamarina et al, 2001, Telford et al, 1990, and Mavko et al, 1998).

<table>
<thead>
<tr>
<th>Materials</th>
<th>Velocity - $V_s$ [m/s]</th>
<th>Density $\rho$ [kg/m$^3$]</th>
<th>Damping ratio $[10^{-3}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluids (20 degrees C, 1 atm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pure Water</td>
<td>0</td>
<td>998</td>
<td>0.892</td>
</tr>
<tr>
<td>Air (100% relative humidity)</td>
<td>0</td>
<td>1.204</td>
<td>0.122</td>
</tr>
<tr>
<td>Metals</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aluminium</td>
<td>4290</td>
<td>2700</td>
<td></td>
</tr>
<tr>
<td>Copper</td>
<td>3350</td>
<td>8930</td>
<td>Pwave - 2.5 kHz: 0.227</td>
</tr>
<tr>
<td>Steel</td>
<td>3955</td>
<td>7900</td>
<td>Pwave at 2-8 Hz: 0.270</td>
</tr>
<tr>
<td>Rocks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sandstone</td>
<td>1340 - 3685</td>
<td>1950 - 2500</td>
<td>2.500</td>
</tr>
<tr>
<td>Shale</td>
<td>1475</td>
<td>1900</td>
<td>16.667 - 71.429</td>
</tr>
<tr>
<td>Limestone (dry)</td>
<td>2680 - 4690</td>
<td>2200 - 2750</td>
<td>2.632 - 4.545</td>
</tr>
<tr>
<td>Soils (100 kPa confining)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Granular Salt (dry)</td>
<td>155</td>
<td>1300 - 1500</td>
<td></td>
</tr>
<tr>
<td>Sand (dry)</td>
<td>79 - 115</td>
<td>1499 - 1900</td>
<td>3.33 - 6.24</td>
</tr>
<tr>
<td>Clay (saturated)</td>
<td>60 - 150</td>
<td>1800 - 2200</td>
<td>10.0 - 33.3</td>
</tr>
</tbody>
</table>

The state of effective stresses in a geomaterial may be inferred using the speed that a shear wave travels through a medium. The velocity-stress power relationship between the state of stress in the directions of wave propagation and shear wave velocity is the following

$$V_s = \alpha \left( \frac{\sigma_x}{1 \text{ kPa}} \right)^{\beta_1} \left( \frac{\sigma_y}{1 \text{ kPa}} \right)^{\beta_2}$$

(4.34)

where $V_s$ is the shear wave velocity and $\sigma_x$ and $\sigma_y$ are the perpendicular and parallel effective stresses in each direction of the plane of wave propagation respectively. Once the wave velocity is determined through the use of geometry and an obtained travel time, it will be combined with the velocity-stress power relationship to determine the state of stress in both the direction parallel and perpendicular to wave propagation. While this relationship is applied to determine stress values at a given state, the velocity-stress relationships are determined for wave measurements at multiple stress states, and therefore the relationship can also indirectly evaluate fabric changes in a soil mass.

The $\alpha$ and $\beta$ coefficients that relate shear wave velocity and stress states within a geomaterial must be experimentally obtained. These two values are based on a variety of physical parameters, and are therefore cannot be theoretically determined. Wave propagation is a small strain phenomenon, which maintains a level that is below the strain threshold. Due to this mechanical response is controlled by the contact effects, and the theoretically predicted values of $\beta$ are the following (Santamarina et al. 2001)

- $0$ for an idealized solid
- $\approx 0$ for a cemented soil
• 1/6 for Hertzian contacts (elastic spheres)
• 1/4 for cone-to-plane contacts (angular particles)
• 1/4 for spherical particles with contact yield
• 3/4 for contacts governed by Coulombian forces

The physical interpretation of $\alpha$ is related to packing, particle material properties, contact behavior, and soil structure. It is because these properties are very difficult to quantify and control that the values of $\alpha$ and $\beta$ must be obtained experimentally. There are published results that show a relationship between the two values of $\alpha$ and $\beta$ by Santamarina et al. (2001) after Fernandez (2000). Using these results, the values of the velocity-stress power relationship coefficients can be adequately determined for use in geomaterial investigation.

4.5 Summary

A general overview of the fundamental concepts that govern the propagation of body waves through a medium was presented in order to achieve a better understanding of the various phenomenons that may be observed during the conduction of elastic wave propagation testing techniques. Utilizing these concepts the results of the analyses conducted with the MEMS accelerometers were analyzed in order to evaluate their possible application as inexpensive (i.e. use-and-lose transducers) sensors for use in the field of geotechnical engineering.
CHAPTER 5
APPLICATION OF MEMS ACCELEROMETERS IN NON-DESTRUCTIVE EVALUATION: WAVE PROPAGATION IN A PLATE

5.1 Introduction

Non-destructive evaluation is a technique that utilizes the properties of wave propagation to determine material properties. This is especially interesting within geotechnical engineering because the field is traditionally dominated by destructive evaluation techniques. Also important is the fact that knowing the in-situ material properties is important to geotechnical design, and in every case that destructive evaluation occurs (i.e. borings) these properties are altered before adequate testing can be accomplished. There are some other techniques that have been developed to test in-situ properties of soil such as the Standard Penetration Test (SPT) and Cone Penetration Test (CPT), but they have their limitations. However, these penetration tests are not truly non-destructive as they change the state of stress of the material as it is pushed to the side modifying the material parameter as it is being measured.

The elastic wave propagation-based non-destructive evaluation requires a set of sources and receivers. While the source can be anything from a mundane knock on the surface with a blunt object to a sophisticated instrumented hammer or laser source, the receivers require somewhat greater detail. It is to this end that the MEMS accelerometer can be utilized. Geophysical investigations have traditionally used geophones or accelerometers to monitor wave propagation (Reynolds, 1997), but a more inexpensive instrument would allow for larger instrument clusters and more detailed data arrays.

To test this application, a simple system was used in order to reduce the amount of variables in the system. To this end a 2-D model was selected for the evaluation of an impact source behavior with MEMS accelerometers. In this test setup, the MEMS accelerometers were the receivers in a non-destructive test.

5.2 2-D Testing Model

The model used for the analysis of the MEMS accelerometers application as a non-destructive evaluation receiver was a wooden plate. While this system does little to approximate the reality of attempting to use MEMS in the geotechnical environment, it serves as a well-controlled system with homogeneous material properties throughout. In addition, the model is not 2-D because it has a finite thickness, but based on the manner in which it was tested the 2-D approximation will be adequate for the analysis of the collected data.

5.2.1 Model Material and Geometry

The wood plate had approximate dimensions of 60 cm x 60 cm x 1.3 cm (24 in x 24 in x 0.5 in). The wood type was a particleboard laminate with an apparent average density of 1157 kg/m$^3$ and approximate P-wave velocity $V_p=2050$ m/s. The MEMS were placed on the large face of the
plate in different geometric arrays in order to capture certain aspects of the wave propagation. To accomplish this, the model needed to stand vertically on its smallest dimension, and therefore a base was built which would allow this capability. Figure 5.1 depicts the 2-D model as well as a schematic of how the model was operated.

![Diagram](image1.png)

**Figure 5.1**: Picture showing the 2D plate test setup instrumented with the different layouts of MEMS accelerometers: (a) circular layout for constant travel times at each receiver, (b) square layout for travel times which vary consistently with travel length, and (c) picture of the actual instrumented test setup.
It can be seen in Figure 5.1 that there were two different geometric arrangements of MEMS on the face of the 2-D model. The first geometric setup was presented in Figure 5.1a (also see Figure 5.1c), it shows a semi-circular arrangement of sensors at three different distances from a centrally located impact source. This arrangement type was developed with the intent of evaluating the directivity of the impact source and monitoring the travel times at equal distances from the source with radii of 15 cm, 23 cm, and 30 cm (6 in, 9 in, and 12 in). The second testing setup was presented in Figure 5.1b. It was a square arrangement of sensors that was intended to capture different travel times at different distances and for different source positions. The collected data was then used to evaluate the homogeneity of the medium and to further interpret the directivity of the impact sources. For the square geometry, source excitations are spaced evenly across the top of the plate at 10 cm (4 in) intervals.

The impact source for each of the wave propagation experiments was a modally tuned impulse hammer by PCB electronics (Model Number: 086C01, Serial Number: 14833). The hammer was knocked against the surface perpendicular to the top of the plate at predetermined locations (Figure 5.1b). In addition, the hammer weight and coefficient of restitution was changed to achieve two different frequencies of excitation.

5.2.2 Sensor Geometry

The orientation of the sensors was also important for the wave propagation testing. Therefore, each sensor had to be oriented so that the signal response could be interpreted for a given accelerometer location. This was more difficult in the circular geometry where each of the accelerometers was placed perpendicular to the radial vector from the center of the half circle formed by the sensors (Figure 5.1c). The sensors in the circular geometry were spaced every 30°. In order to maintain this geometry the MEMS accelerometers were attached to the face of the 2-D model with hot glue. In the case of the square configuration, the sensors were simply arranged in the horizontal and vertical directions (x and y coordinates in Figures 5.1a and b).

Once in place, signals were recorded for each location of the MEMS accelerometers for a given excitation. In addition, this was done for both axes for comparison reasons later on in the study. After the traces were obtained from the oscilloscope, a data analysis was conducted to obtain displacement values from the acceleration traces.

5.3 Results of the 2-D Model

After obtaining all of the acceleration traces from the MEMS accelerometers, analysis was conducted to determine the maximum displacement caused by the arrival of the wave, which was later compared to published results. To obtain the displacements the acceleration was integrated twice with respect to time.

Each of the signals was extracted and placed into the mathematical analysis software MathCAD for display and analysis. Using this software the raw voltage vectors were converted into acceleration vector and then displacement vectors. The signals were first reduced to their differential voltage value by removing the DC offset component from the signal. Once this was
achieved the voltage values were converted to acceleration through the calibration factor (i.e., 38 mV/g) obtained in chapter three of this thesis.

Given the acceleration vector at a point, the displacement can be achieved through the second integration of the acceleration vector. In order to do this integration a numerical approximation was utilized. The formulation to determine the velocity (i.e. the first integral of displacement) was the following:

\[ v_i = \sum_{i=0}^{n} a_i \cdot \Delta t \]  

(5.1)

where \( a_i \) was the acceleration vector indexed by \( i \), \( \Delta t \) was the sampling interval (\( \Delta t = [125 \text{ kHz}]^{-1} \) in this study), and \( v_i \) was the velocity vector indexed by \( i \). Once the velocity vector was obtained, a second numerical integration would yield the displacement vector. The formulation for the displacement was the following:

\[ d_i = \sum_{n=0}^{i} v_n \cdot \Delta t \]  

(5.2)

where \( d_i \) was the particle displacement vector indexed by \( i \). After obtaining the numerous displacement vectors for each of the MEMS accelerometers, the maximum value was extracted. This was then compiled with the other maximum displacement values for the various geometries and excitation locations.

Figure 5.2 shows the results for the 15.2 cm (6 in) radius circular geometry (The locations shown in the figure corresponds to those shown on Figure 5.1a. The insert in the graph shows the positive response for each of the axes of the MEMS accelerometers in both in the radial and circumferential directions. Trends seen in the data are the initial response symmetry or anti-symmetry for symmetrically opposed pairs. The response symmetry was seen, for example, in the following pairs: there was symmetry between receiver pairs 1-7, 2-6, and 3-5 for the response parallel to the direction of propagation and anti-symmetry between receiver pairs 1-7, 2-6, and 3-5 for the response perpendicular to the direction of wave propagation. The same response trend was observed in both acceleration and particle motion. It was important to note that the response of the central accelerometer (MEMS accelerometer 4) was maximum in the case of the propagation in the direction of wave propagation and minimum in the direction perpendicular to the direction of wave propagation.

Figure 5.3 presents the raw data collected in the square accelerometer configuration. These data were collected using the instrumented hammer at 30 cm (12 in) from the edge. The accelerometers were aligned in the vertical direction as shown in the inserts. The horizontal acceleration data show that as the accelerometers were placed closer to the line of the source (i.e., \( \theta=0^\circ \)), the response became smaller and smaller; then it increased again as the accelerometers moved further from the line of the source.
Figure 5.2: Wave propagation data measured at different location 15 cm from the source in two perpendicular directions: (a) Measured acceleration versus time and (b) Calculated particle motion versus time. (The positive response of the MEMS is presented in the insert.)

The vertical acceleration data showed the change in the phase of the acceleration with the angle $\theta$ of the propagation path both for the horizontal and vertical accelerometers. In the case of vertical accelerometer responses, as the angle $\theta$ increased, the compressive wave seemed to change into a tension wave as shown, omitting accelerometers 1 and 13. The observations presented in Figures 5.2 and 5.3 were typical of source response and will be analyzed in next section using approximate theoretical solutions.
Figure 5.3: Accelerometer responses for an impulse excitation at 30 cm (12 in) from the edge: (a) horizontal acceleration and (b) vertical acceleration. (The positive response of the MEMS accelerometer is presented in the insert.)

5.4 Point Sources on a Free Surface

The idealization of a point source acting on a free surface is quite common. An understanding of the behavior caused by these forces in terms of wave propagation was essential to the evaluation of the capability of the MEMS accelerometers as receivers for non-destructive evaluation techniques. This section concentrates on the use of a vertical force acting on a free surface, which was a problem specifically addressed by Miller and Pursey (White, 1983). Miller and Pursey showed that the solutions to the wave equation could be combined in the cylindrical coordinate system to achieve uniform normal stress over a disk with a sinusoidally varying load. The cylindrical coordinate system is defined in Figure 5.4 for this solution.
Figure 5.4: Cylindrical coordinate solution as defined by Miller and Pursey for solution to point sources acting perpendicular to a free surface.

In addition, Miller and Pursey’s solution assumed zero stresses elsewhere on the free surface. Using the resulting formulation, they applied the following boundary conditions to the obtained displacement integrals: small disk radius and a large radial distance from the source when compared to typical body wave wavelengths (i.e., far-field assumption). At this limit, the source can be considered a point source with the following formulation:

$$F(t) = F_0 \cdot e^{i\omega t} \quad (5.3)$$

where $F_0$ was the magnitude, $\omega$ was the frequency, and $t$ was the time. Symmetry around the vertical axis made the displacement out of plane zero (i.e. $u_\theta=0$). The remaining displacements (radial and transversal respectively) were found to be the following:

$$u_r = \frac{F_0 \cos(\theta) \left[ 1 - 2 \left( \frac{V_S}{V_P} \right)^2 \cdot \sin^2(\theta) \right] e^{-i\omega t}}{2\pi \rho V_P^2 r \left( 1 - 2 \left( \frac{V_S}{V_P} \right)^2 \cdot \sin^2(\theta) \right)^2 + 4 \left( \frac{V_S}{V_P} \right)^3 \sin^2(\theta) \cos(\theta) \left[ 1 - \left( \frac{V_S}{V_P} \right)^2 \cdot \sin^2(\theta) \right]^{0.5}} \quad (5.4)$$

$$u_\theta = \frac{-F_0 \sin(\theta) \cos(\theta) \left[ \left( \frac{V_S}{V_P} \right)^2 - \sin^2(\theta) \right]^{0.5} e^{-i\omega t}}{\pi \rho V_S^2 r \left( 1 - 2 \sin^2(\theta) \right)^2 + 4 \sin^2(\theta) \cos(\theta) \left[ \left( \frac{V_S}{V_P} \right)^2 - \sin^2(\theta) \right]^2} \quad (5.5)$$

where $V_P$ was the P-wave velocity, $V_S$ was the S-wave velocity, and $\rho$ was density of the medium. The equation for tangential displacement $u_\theta$ yielded positive values for $\sin(\theta)<V_S/V_P$. Outside of this range, the tangential displacement yielded negative values, which indicate a phase shift in the material response. Solving for the amplitude we obtain the following formulation for radial and tangential respectively:

$$U_r = \left( \frac{4\pi \rho V_P^2}{F_0} \right) |u_r| \quad (5.6)$$
Using these relationships an understanding of body waves propagation through a solid from a vertical force on a plane boundary was obtained. Figure 5.5 depicts the solution for a body wave propagating from a free surface with a velocity squared ratio of \((V_P/V_S)^2=3\). The exponential portion of the applied force was dropped to simplify the solution, and \(F_o\) was defined as one. Figure 5.5a presents the typical directivity of a source acting perpendicular; that is, the radial displacement is greatest in the direction parallel to the application of the sources and continuously decreases with increasing angle \(\theta\) till the radial displacement becomes zero at \(\theta=90^\circ\) and \(-90^\circ\). Figure 5.5b shows the transversal displacement caused by the perpendicular excitation. This plot shows the phase shifts in the displacement as the angle \(\theta\) increases. This transition angle depends on the ratio of S and P-wave velocities and it indicates the how the stresses in a medium change from compression to tension in a response to normal load as a function of the elastic properties.

\[
\theta = \left( \frac{4\pi V_P^2}{F_o} \right) u_o
\]  

Figure 5.5: Amplitude of particle displacement in the (a) radial and (b) transversal directions caused by a force acting at the free boundary of a solid, perpendicular to the surface (after White, 1983).

5.5 Experimental Versus Theoretical Results

In order to compare the MEMS results to those shown for the theoretical solution, the MEMS accelerometer values were employed in conjunction with their geometric location and plotted against the theoretical solution. The maximum value was extracted and combined with its geometric location in cylindrical coordinates (i.e. its normalized distance and rotation angle). Plotting this against the theoretical solution based on the model parameters will provide correct comparison to evaluate the MEMS response. However, it is important to note that the theoretical solution was derived for a semi-infinite medium (White 1981) and the experimental study corresponds to wave propagation in a plate. Therefore, the comparison with theoretical solution should only be considered for qualitative evaluation.
5.5.1. Monitoring the Directivity of Impulse Sources

The MEMS accelerometers were used to monitor elastic wave propagation and evaluate the impact source response. These data were gathered with the accelerometer arrangement presented in Figure 5.1a. The accelerometers were located at radial distances equal to 15 cm (6 in), 23 cm (9 in) and 30 cm (12 in) from the source. Two types of sources were used. One source was labeled low frequency (high mass and soft hammer tip) and the other source was labeled high frequency (low mass and hard hammer tip). The pictures of the two hammers used are presented in Figure 5.6

![Impulse hammer physical properties](image)
- Mass: 0.1 kg (0.23 lb)
- Length: 21.6 cm (8.5 in)
- Head diameter: 1.57 cm (0.62 in)
- Tip diameter: 0.63 cm (0.25 in)

**Additional mass (.050 kg) for low frequency signal**
- Softer plastic tip for low coefficient of restitution

**Harder steel tip for high coefficient of restitution**

![Figure 5.6: Picture of the PCB (Model Number: 086C01, Serial Number: 14833) Modually tuned impulse hammer: (a) Low-frequency hammer and (b) high-frequency hammer.](image)

- Harder steel tip for high coefficient of restitution

The acceleration data were integrated twice to obtain the particle displacement (Figure 5.2b) and this data were plotted in polar coordinates to show the effect of direction in the response of the source. These results are presented in Figures 5.7. The data were presented normalized with respect to source amplitude and the maximum displacement. The radial displacement closely followed the theoretical model however the transversal displacement deviated from the theoretical model at angles $\theta = 90^\circ$ and $-90^\circ$. This observation may be tentatively explained by the fact that at the impulse interface there are also surface waves propagating along the boundary. Surface wave particle motion has a large component in the direction perpendicular the horizontal boundary and a small component in the direction parallel to wave propagation along the surface (see also Figure 5.8a). Figures 5.8b and 5.8c show the acceleration responses of two sensors that were near the surface of the plate. The data in these figures shows the retrograde particle motion triggered by surface wave propagation. This last observation also explains the small radial displacement at angles $\theta = 90^\circ$ and $-90^\circ$. Data in Figure 5.7 clearly shows that along the vertical direction the radial displacement is maximum while the transversal displacement is minimum as shown in Figure 5.5. Therefore, both the radial and transversal motion capture with the accelerometer may be justified with theoretical models.
Figure 5.7: Radial and transversal particle displacement versus direction for both low and high frequency hammer. (a) Radial displacement at 15 cm (6 in) from the source, (b) Transversal displacement at 15 cm (6 in) from the source, (c) Radial displacement at 22 cm (9 in) from the source, (d) Transversal displacement at 22 cm (9 in) from the source, (e) Radial displacement at 30 cm (12 in) from the source, and (f) Transversal displacement at 30 cm (12 in) from the source.

The theoretical model is presented with a continuous line (Model parameters $V_p=2050$ m/s; $V_s=1000$ m/s; $\rho=1100$ kg/m$^3$).
Figure 5.8: (a) Horizontal and vertical particle motion of a surface waves (Richart et al. 1970), (b) and (c) vertical and horizontal accelerations for accelerometers 1 and 7 along the surface of the plate.

Furthermore, the analysis of the data in the rectangular sensor arrangement (Figure 5.1b) allows for the further evaluation of properties of the impact source. Figure 5.10 shows the analysis of the rectangular geometry for determination of the phase angle. Each signal’s first arrival was examined to determine if the initial value was opposite the expected polarity. If this was the case a zero was assigned to the receiver-source combination the signal corresponded to and its geometric information was recorded. After going through each of the source locations and analyzing the data it was plotted against its angular location in a polar plot. This allowed all of the data taken to be compared for different source locations, which allowed for an easier determination of the phase shift angle.
After identifying the phase shift in the acceleration responses and plotting the resulting data in the angular coordinate system the phase shift location was clearly identified at ±48°. The data was then superimposed over the solution by Miller and Pursey for tangential displacement (Figure 5.10) to better help calibrate the theoretical solution to the real model. This allowed the S-wave velocity to be more accurately determined for the solution. The S-wave velocity was approximately 1500 m/s while maintaining the P-wave velocity of 2050 m/s. The back calculated Poisson’s ratio from the velocity values was ν = −0.076. While this value is physically possible, it is unlikely that it is correct, which can be attributed to the fact that the theoretical model used was for a three dimensional solid and the reality was effectively a two dimensional plate. The reason there are a few points which fall beyond the phase shift angle cutoff is most likely due to inaccurately specified geometry at the source location. This inaccuracy was caused by a lack of impact location control when the hammer was used to excite the system.

5.6 Summary

The application of MEMS accelerometers as wave propagation sensors is the area where they would have the most impact in Geotechnical engineering and more specifically geophysics. While the simple system utilized does not accurately reflect the complexity of a soil mass, it does allow general capabilities of the sensor to be examined. The results of this testing have shown that the ADXL250 MEMS accelerometer is well suited for the analysis of elastic wave propagation in a solid. This was shown through the MEMS ability to yield an acceleration response at a location due to a seismic pulse on a free surface and through the reduced data’s accurate comparison to a theoretical solution.
CHAPTER 6
APPLICATION OF MEMS ACCELEROMETERS IN
GEOTECHNICAL ENGINEERING: MONITORING
INCLINATIONS AND EFFECTIVE STRESSES CHANGES

6.1 Introduction

Earth pressure problems are some of the oldest addressed by the field of civil engineering, and more specifically geotechnical engineering. Braced excavations fall into this category, but are somewhat neglected in research due to the fact that they are usually temporary in practice. It seems that in many cases the responsibility for the excavation is actually placed on the contractor because the excavation is only a temporary part of a larger project (Lambe and Turner, 1970). Under many situations these types of structures are necessary in order to develop a project when there is limited space on the site. This is especially common in urban areas that have been developed previous to the construction of the current project. Whatever the reason for the use of a braced excavation, a better understanding of the behavior of soils subjected to the loading cases seen within these systems is needed to help advance the technology of the field.

While earth pressure problems are traditionally important when dealing with excavations, the development of new technologies has helped form a way to redefine the knowledge we have about these types of systems. The area that is of primary importance to this study is the ability to now obtain an image of the velocity filed and in turn the state of stress in and around a geostructures through the use of tomography. Borrowing from areas such as nano-machining technologies the geotechnical engineer now has the capacity to perform tomographic study of projects to obtain a better understanding of the problem in question. While in the past these studies were expensive due to the costs of sensors and data acquisition systems, ever developing electronics technologies have created cheaper and cheaper instrumentation. Given the reduced cost for the system components, the next step is simply education and acceptance of the techniques. While this paper is not designed specifically to develop tomographic techniques, it will help promote its and MEMS capabilities through the use of the braced excavation problem.

6.2 Braced Excavations

While the behavior of braced excavations is not the primary focus of this study, it is an interesting system to analyze using the tomographic imaging process. Before proceeding into the analysis of the 1-g model developed for this study, a general review of the description of braced excavations and the theory that governs their design will be covered.

Braced excavations are generally used in situations where, as was stated previously, the space available for the project is limited requiring a cut with very steep to vertical side slopes. In the field, steel sheet piles are driven into the ground along the sides of the intended excavation location, and the soil is excavated from between the two sheet pile walls. While the cut is being excavated, intermediate members must be placed perpendicular to the direction of the sheet pile walls (i.e. across the excavation). These members are struts. The struts tie into another set of system elements called whales, which help redistribute the load at the level of the struts. Whales
can either be plates that distribute the load over a larger area or members that run along the length of the wall intercepting multiple struts and distributing the load over the wall length. In cases where the whales run parallel to the wall they are sometimes structural members such as H-piles. These bracing members (i.e. whales and struts) are used to restrict the movement of the sheet pile wall as the excavation is completed. As the excavation increases in depth, additional levels of struts may be needed to stabilize the structure. This process is continued until the excavation reaches the design depth. In most cases the sheet piles are driven beyond the depth of the excavation in order to stability against rotation and bottom heave. Bottom heave is a phenomenon by which the bottom of the excavation will actually heave upward because the depth of the excavation becomes so large that the surrounding soil weight overcomes the shear strength of the material. Figure 6.1 shows an example of a braced excavation.

![Figure 6.1: Example braced excavation with embedded sheet pile tips.](image)

### 6.2.1 Braced Excavation Design

This study will concentrate on only sand-based design methodology for braced excavations because only dry uniform sand will be used throughout the tomographic testing analysis. This material was selected to simplify both testing and analyses because the apparatus design and construction would require a substantial amount of effort before testing could even begin.

In dry sands (i.e. above the water table), failure can occur in a variety of manners such as failure of the struts, failure of inadequately stiffened whales, rotation failure of the sheet (rare), base heave, and piping failure. Generally the failure of the structural elements is very rare, and the stability of the geostructure is of primary concern. However, if the excavation is unstable it is likely that the loads will be non-uniformly distributed into the structural elements causing their progressive failure. To analyze the stability of a braced excavation requires an adequate knowledge of the earth pressure distribution against the sheet pile.
6.2.1.1 Lateral Earth Pressure Distribution

In order to determine the pressure distribution against the sheeting of a braced excavation a few assumptions are required. While these assumption help simplify the problem for analysis they are by no means infallible, and it is very likely that the problem does not represent the exact reality that it is used for in design. The assumptions and analysis are based on Figure 6.2.

\[ r = r_0 e^{\alpha \tan \phi} \quad (6.1) \]

where \( r \) equals the length of any radius incline at an angle \( \alpha \) to \( r_0 \), \( r_0 \) is the distance from the center of the spiral to the point \( d \) in Figure 6.2 and \( \phi \) is the angle of internal friction of the sand. The center of the spiral \( C \) is located at a point that is on the same line as \( d \) and inclined at an angle \( \phi \) from the horizontal. The forces acting on the wedge are as shown in 6.2:

I. The resultant wall pressure \( P_1 \) inclined at an angle \( \delta \) to the normal of the wall. This is equal and opposite of the pressure acting against the back of the sheet pile.
II. The self-weight \( W \) of the soil wedge and applied at the center of gravity of the wedge.

III. The soil reaction \( R_1 \) due to the shear strength of the material. This is inclined at the friction angle \( \phi \) from the normal to the failure surface.

The value of the resultant wall pressure \( P_1 \) can be determined through the conditions of equilibrium \( \Sigma F = 0 \) and \( \Sigma M = 0 \). Summing moments about the center of the failure surface \( C_1 \) the following relationship is obtained

\[
P_1 l_w - W l_w = 0
\]  \hspace{1cm} (6.2)

and

\[
P_1 = \frac{W l_w}{l_u}
\]  \hspace{1cm} (6.3)

While this seems simple enough, there are numerous alternative failure surfaces that must be drawn from different assumed position of point \( d \) on the surface. The maximum value of earth pressure obtained from these various surfaces is taken as the value of pressure on the sheeting.

The final value of the pressure acting on the sheeting is related to the value of \( n_a \), which is a proportionality factor which locates the height of application of the earth pressure \( P_1 \). In the previous analysis this was assumed. The distribution of the earth pressures can be approximated as was shown by Terzaghi (1936). Terzaghi’s analysis of the distribution of earth pressure was based on Figure 6.3 and the following assumptions:

![Figure 6.3](image)

**Figure 6.3:** Terzaghi’s analysis of the distribution of earth pressure: (a) Sheet pile system with infinitesimal element depicted, (b) Variation of lateral earth pressure coefficient with height, (c) Force break down on element, (d) Force polygon solution (modified after Prakash et al, 1979).
I. The depicted failure surface bc in Figure 6.3 (a) is a straight line.

II. The lateral earth pressure coefficient $K$ varies throughout the wall height and is only equal to $K_A$ at the bottom of the excavation.

III. Coefficient of lateral earth pressure $K$ is inversely proportional to the deflection of the sheeting at a given depth.

If the sheeting is assumed to deflect linearly from zero at the top of the excavation to an unknown value of $\rho$ at the bottom of the excavation the lateral earth pressure coefficient $K$ can be defined by the following relationship (Prakash et al, 1979)

$$K = K_A \left(1 + \lambda \frac{y}{H}\right)$$

(6.4)

where $H$ is the height of the excavation and $\lambda$ is a proportionality factor. As was shown in Figure 6.3 (b), $K$ varies from the active earth pressure value $K_A$ to $(1 + \lambda)K_A$ over the height of the excavation. If $\rho$ is assumed to be the minimum deflection that will reduce the value of $K$ to $K_A$ then $\lambda$ is simply a proportionality factor that relates the deflection to the lateral earth pressure.

The forces that act on element $dy$, which is defined by abcd, are shown in figure 6.3 (c). Using the conditions of equilibrium and summing the forces in the $y$ and $x$ directions respectively yields the following expression

$$-\gamma(dy \cdot \tan(\theta)) + (q + dq)y \cdot \tan(\theta) - q[(y + dy)\tan(\theta)] + \tan(\delta)dy + dF_x \cot(\theta') = 0$$

(6.4)

and

$$P_x dy - dF_x = 0$$

(6.5)

Combining equations 6.4 and 6.5 and substituting in the relationship $P_x = qK_A$ the following is obtained

$$y(dy - dq) + qdy = q\left(\frac{\tan(\delta) + \cot(\theta')}{\tan(\theta)}\right)Kdy$$

(6.6)

Based on Figure 6.3 (d) in which a force polygon obtains the solution to the vertical earth pressure, the value of $K_A$ can be determined. Knowing that $P_x$ is the sum of the two vertical components taken from the geometry and the equating this to the vertical pressure the following can be shown

63
\[ \frac{1}{2} \gamma H^2 \tan(\theta) = P_x (\tan(\delta) + \cot(\theta)) \] (6.7)

which then yields

\[ P_x = \frac{1}{2} \left( \frac{\tan \theta}{\tan \delta + \cot \theta} \right) \gamma H^2 = \frac{1}{2} K_A \gamma H^2 \] (6.8)

Using the previously determined value for the active lateral earth pressure coefficient \( K_A \) and the conditions of equilibrium demonstrated earlier in conjunction with the assumption made in reference to the actual value of the lateral earth pressure coefficient an approximation can be made for the vertical pressure. Substituting equation 6.4 and the value of \( K_A \) into equation 6.6 the following is obtained

\[ \frac{dq}{dy} + \frac{\lambda}{H} q - \gamma = 0 \] (6.9)

and integrating we obtain

\[ q = \frac{\gamma H}{\lambda} + Ce^{-\lambda y/H} \] (6.10)

where \( C \) is a constant of integration. If the boundary conditions of \( y = H \) and \( q = 0 \) are applied we obtain the following solution:

\[ q = \frac{\gamma H}{\lambda} \left( 1 - e^{-\lambda \beta} \right) \] (6.11)

and

\[ P_x = \frac{\gamma H}{\lambda} \left( 1 - e^{-\lambda \beta} \right) K_A \left( 1 + \frac{y}{H} \lambda \right) \] (6.12)

Where \( \beta = (1 - y/H) \). Using this solution a plot of lateral earth pressure versus vertical location in the excavation is shown in Figure 6.4 to help show the effect that \( \lambda \) has on the distribution of pressure behind the sheeting.

The four traces in the Figure 6.4 are for the \( \lambda \) values shown. The pressure values were normalized versus the value of earth pressure for the base of an excavation (i.e. \( K_A \gamma H \)). It is seen that the lower the \( \lambda \) value the more closely the solution to the general wedge method represents Rankine’s active condition.
Figure 6.4: Lateral earth pressure distribution determined through the general wedge method (modified after Prakash et al, 1979).

It is also apparent is that the active pressure values do not differ greatly from the generalized wedge method, and in fact may yield a higher total pressure value over the entire sheet. While the general wedge method provides some insight to the true lateral earth pressure behind an excavation it is still a simplified method, which relies on a handful of assumptions. The reality is that the lateral earth pressure behind an excavation is vastly different from a linear distribution (Prakash et al, 1979).

Terzaghi and Peck utilized field experimentation in order to determine the values of the apparent earth pressure against sheeting in a braced excavation. The results were obtained from sands above the water table. Figure 6.5 depicts the pressure against four struts in one vertical plane for four different locations in their Berlin Subway data.

Figure 6.5: Apparent earth pressure distribution in braced excavations in Berlin Subway (Terzaghi and Peck 1967 after Prakash et al., 1979).
While it can be seen from these apparent pressure diagrams that there is an increase in lateral earth pressure, there is very little trend in how that pressure seems to increase. The data does seem to have some similitude with the generalized wedge method because both results tend to a distribution that is somewhat parabolic. This is most obvious from the location of center of pressure, which in both cases is near the middle of the excavation height. This is in opposition of a linear increase with depth, which would yield a center of pressure at approximately a third of the excavation height. In design an equivalent rectangular pressure distribution is used to simplify the computation of the earth pressure and to remove some of the uncertainties about the system. Figure 6.6 depicts the distribution as well as the value of the pressure envelope \( p_a = 0.65\gamma K_A H \).

![Figure 6.6: Apparent lateral pressure distribution for dry sand.](image)

This apparent pressure envelope method was employed for the design of the 1g model developed for this tomographic study. In addition, the design of the excavation accounted for other failure mechanisms such as base heave and structural member failure using standard methodologies. While there was limited freedom in the design of the excavation, these areas were used to pre size the various elements that would later be required to run the test.

### 6.3 Model Braced Excavation

The braced excavation utilized in this tomographic study was developed for this study, therefore a large amount of system development was conducted prior to any testing to construct the apparatus. The development of the system was two fold involving both a geometric design and a sensor design. It is important to note that the physical dimensions of the apparatus were restricted due to both laboratory space and available materials.

#### 6.3.1 Geometric Design

The first step in the design of the model braced excavation was to determine in what size container the system would be housed. This was an important step because a variety of problems could occur within the system during testing based on this decision. However, the space allotted to this experiment was limited, and therefore the final container was selected based on both its size and maneuverability. A prismatic wood container with internal dimensions equal to 0.61m x 0.61m x 1.219m (2’ x 2’ x 4’) was selected. This helped reduce the amount of material weight required to fill the container and only took limited space within the laboratory. In addition, the container utilized was already constructed for a previous system in another laboratory, which
reduced cost and material requirements to build the container. Furthermore, the container had been sanded and finished on the interior leaving a relatively smooth surface. The main argument against the use of this type of container is the nominal dimensions and weather or not scaling effects would be an issue. While this was considered, there were other problems that influenced the decision.

The other options for a braced excavation model were to design and test a centrifuge model or analyze a full-scale test section. A full-scale test section was ruled out primarily for cost reasons. On the other hand, the centrifuge model was a viable option and it would have reduced the scaling effects on the system. Outside of the fact that access to a centrifuge is not available at Louisiana State University, problems arose due to the type of system that would be required for the analysis. The sensors used to monitor the 1-g model require power sources and data acquisition system interfaces. The use of the centrifuge becomes much more complicated when the entire monitoring system requires wireless remote capabilities. Therefore, a 1-g model was selected to allow effective user control of the monitoring system during testing.

With the container selected, the next step was to size the wall to fit within the container. The internal dimensions of the container lead to a wall design that would utilize the entire width in order to have more control over the system. While this concept would achieve a three-section system, there were concerns raised about the boundary effects due to the internal dimensions of the container. To address these concerns, a sheet of Teflon was attached to the internal surface of the container to reduce friction between the wall and the container and between the surfaces and the sand. Since the wall was going to traverse the container across the short dimension, this dimension was controlled and set to just short of 0.61m. The reasoning behind the sheeting being just short of the long dimension was to allow rotation under loading. However, the short dimension would allow sand to fall back into the excavation and change the physical characteristics of the problem. Therefore a flexible rubber gasket was placed along the height of the wall to keep the sand on the exterior of the excavation out of the interior. This is shown in Figure 6.7.

![Figure 6.7: Sheet pile interface mechanism and Teflon surface sheeting for the reduction of friction between the surfaces of the container and the elements of the braced excavation.](image)
6.3.1.1 Reaction Frame

Another addition to the container design was the incorporation of a reaction frame to load the soil. This concept surfaced in the design of the walls because the load of the soil was extremely small and to create any failure mechanism the soil would require a surcharge load. In order to add this to the system the container was reinforced with steel plates across the bottom and four 183 cm (6ft) all thread columns (i.e. two sets) were passed along the side of the container. This allowed the area of the excavation to be bridged by a piece of 5.08 cm x 7.62 cm x 0.317 cm (2 in x 3 in x 0.125 in) steel box tubing on either side of the excavation. This horizontal member would provide the reaction point for the soil to be loaded against. In addition the all thread members allowed for full adjustment of the system height. The loading mechanism selected was manual hydraulic jacks. In order to control the application of the load, in-line load cells were utilized. The load cells were rigidly attached to the horizontal cross member and also allowed for height adjustment. In order to transfer the load to the jack without eccentricities a loading sleeve was machined which rigidly attached to the load cell. This sleeve slid over the loading piston of the jack and was seated using a 1.905 cm (0.75 in) diameter steel ball bearing. The jack load was then applied to the soil through a 31 cm x 61 cm x 0.317 cm (24 in x 48 in x 0.125 in) steel plate, which distributes the load uniformly over the exterior surface of the excavation. The size of the bearing plate was selected so that the failure wedge would fall within the loaded area. Figure 6.8 shows the loading system for the model.

![Figure 6.8: Reaction frame and load transfer mechanism. (a) Reaction Frame, (b) in-line load cell and loading sleeve, and (c) hydraulic jack and bearing plate.](image-url)
6.3.1.2 Sheet Pile Design

The design of the vertical dimension of the sheeting would prove to be difficult because the anticipated load from the soil was extremely low due to the amount of material being such a small volume of soil. Using standard design techniques for braced excavations, an analysis of the excavation was executed at various vertical dimensions for the sheeting. There were a few factors that would control the height of the sheeting such as the possibility of heave failure and the proximity of the bottom surface of the container to the bottom of the sheet. In addition to these issues, the sheeting material would have to be lighter and thinner than typical sheeting material to help promote the type of behavior desired for the system. Therefore, Aluminum was pre selected for the sheeting material and the section modulus would have to be calculated to determine if the design was satisfactory.

The first portion of the analysis that would be checked is what size excavation could be utilized in order to avoid heave failure and promote desired behavior. The methodology followed was taken from the *Pile Buck Steel Sheet Piling Design Manual*. As was stated previously the lateral earth pressure distribution utilized for design would follow standard practice.

\[
p_A = 0.65\gamma HK_A
\]  

(6.13)

Due to our very limited height, this number was extremely low. This led us to the incorporation of a reaction frame on the braced excavation, which would allow for the application of a uniform surcharge load on the system to develop the desired behavior. Using the anticipated ability of applying this surcharge load, the design of the wall height wall conducted with these additional loads. Starting with a surcharge load, the system was analyzed to determine wall height. Using the typical design techniques the surcharge load was applied to the sheeting with the following methodology

\[
\sigma_h(q) = K \cdot q
\]  

(6.14)

where K is the lateral earth pressure coefficient which is either the active or passive earth pressure coefficient based on weather of not the wall is moving toward or away from the soil. The value q is the surcharge load on the soil. Additional considerations were the point loads from the struts, which were spaced laterally to achieve an equal contributing distance of 15.24 cm (6 in) on either side of the strut. Furthermore, only one level of struts at the height of the exterior ground level was used for space reasons. While this seldom happens in practice, an interest in the development of pressures under this set of conditions was brought into view. In many cases the use of equipment is restricted within the excavations due to multiple levels of struts. The selected excavation depths were 7.62 and 15.24 cm (3 and 6 in). After obtaining the lateral pressure distribution acting on the sheet pile, equilibrium conditions were applied and the maximum moment in the sheet was determined. Using the calculated moment, the required section modulus of the sheet was determined. After a variety of analyses varying both sheet height and surcharge load the final required section modulus was found to be 0.159 cm\(^3\) (0.0095 in\(^3\)) with a maximum surcharge of approximately 29 kPa (600 psf) and sheet height of 45.7 cm (1.5 ft). Figure 6.9 shows a picture of the sheeting as well as the whales and struts utilized in the model.
Figure 6.9: Sheet pile used in the braced excavation model as well as the whale and strut locations along the sheet.

Figure 6.9 shows an intermediate load cell in each of the struts. These were 1.334 kN (300 lb) capacity load cells, which were incorporated to determine what type of load was being transferred through the struts. The struts were 0.952 cm (0.375 in) diameter aluminum rods threaded to match the load cell and the whale on respective ends. The whales were 1.27 cm (0.5 in) square by 55.9 cm (22 in) long bar stock that were machined to rigidly attach to the sheeting. Another addition to the testing setup was the angle brackets used to geometrically locate the sheets before filling the excavation with sand. Once filled enough to hold the sheeting in position, the brackets are removed and the soil is raised to the pre excavation level. Using this geometric layout the excavation was then wired with a monitoring system, which utilized a variety of sensors determine the behavior of the system. Figure 6.10 depicts the final physical setup of the braced excavation model.

Figure 6.10: Physical setup for 1-g model braced excavation utilized in this study.
6.3.2 Soil Characterization

The selected soil for the system was uniform clean dry sand. This was done for a variety of reasons, the most prominent of which was to initially simplify the system in order to determine the capabilities before attempting something as complex as a fully or partially saturated clay material. A variety of tests were run to determine different characteristics of the material.

A specific gravity test was conducted as per ASTM D 854-92 for the sand in order to facilitate the calculation of soil pressures in the model. The final value of specific gravity, \( G_s \), was found to be 2.65. The unit weight of the material was estimated to be 16 kN/m\(^3\) (100 lb/ft\(^3\)). In addition to the weight volume testing a grain size distribution was conducted using a sieve analysis as per ASTM D 422-63. The results of the grain size distribution are shown in Figure 6.11.

![Grain size distribution results for clean sand utilized in model braced excavation](image)

**Figure 6.11:** Grain size distribution results for clean sand utilized in model braced excavation

The Friction angle was first estimated based on the angle of repose and then later verified through a triaxial test. It was felt that the critical state (i.e. residual) friction angle, \( \alpha \), should be used in the analysis of the final design. Based on the analysis of the triaxial results, \( \alpha = 33^\circ \). Figure 6.12 shows the results for a triaxial test set conducted on the sand sample.

![Results for triaxial test conducted on clean sand](image)

**Figure 6.12:** Results for triaxial test conducted on clean sand
6.3.3 Monitoring System Design

After designing the model a methodology had to be developed so that the behavior could be effectively monitored during operation utilizing the MEMS based ADXL250 accelerometers. The incorporation of the MEMS accelerometers into the model was found to be more difficult than anticipated because of the stringent geometric restraints on their placement due to the subsequent data analysis. However, a methodology was devised that would allow the MEMS location to be controlled during the different sequences of testing.

6.3.3.1 MEMS Accelerometers Wave Propagation System

The placement of the MEMS was crucial because the geometric layout of the sources and receivers dictates how well the wave propagation data will can be interpreted. Before a spatial array could be determined the system of sources and receiver had to selected. While the receivers were previously determined (i.e. the MEMS), the sources were later selected based on both the receiver capabilities and the layout of the braced excavation model. Leading into the design, the source selected was a metallic rod which would be placed at a certain location and plucked in order to propagate waves through the system, but the lack of control for source location was felt to be to great. Due to the control required in this technique the type of source element selected were piezoelectric transducers or bender elements. The bender elements allowed the polarization of the wave to be better controlled during the test and we could create S-waves in two orthogonal directions with the correct anchoring system.

Bender elements are flexurally responsive piezoelectric elements, which when excited by a voltage yield a displacement response if properly anchored (hence the name bender element). The elements utilized consist of two very thin length expander plates that are joined to a metal shim. The plates are polarized to produce a displacement response under a voltage and a voltage response when displaced (Morgan Electro Ceramics: TP -245, 2003). There are two types of polarization for bender elements, series or parallel. The parallel type was selected for this study because they would be utilized only as a source, which meant additional displacement and reduced electromagnetic noise. The reasoning for the selection is based on the signal created. The parallel elements create twice the tip displacement and have very reduced electromagnetic cross-talk in comparison to the series type transducer (Morgan Electro Ceramics: TP -245, 2003). The main disadvantage of a parallel type bender element is a much lower sensitivity than its series counterpart. This has no bearing on its use as a source in the MEMS wave propagation system. Figure 6.13 shows a parallel type bender element.

![Figure 6.13: Parallel operation bender element (Modified after Morgan Electro Ceramics: TP -245, 2003).](image-url)
Utilizing the bender elements was an essential step toward the final wave propagation techniques that would be employed on the project. It was felt that S-waves should be created in two orthogonal directions in order to determine velocity field in two planes. To do this and anchoring system was developed that rigidly held one end of the bender elements cantilevering the element. The modules that were finally created had benders in orthogonal directions that would create S-waves in the vertical and horizontal planes. Figure 6.14 depicts the anchoring system used as well as a picture of the final product.

**Figure 6.14:** Bender element module for anchoring elements in specific orthogonal orientation for the horizontal and vertical polarization of S-waves.

Using the sources and receivers a spatial array was developed for wave propagation analysis of the model braced excavation. There were multiple competing effects that directed the placement and spacing of the array. It was felt that the centerline of the short dimension of the excavation was the primary location for the MEMS accelerometers, and to get more information from the setup the wave propagation techniques would take place on the exterior of the excavation. The primary reason for this was the lack of space on the interior of the excavation. Placing the MEMS accelerometers in the center of the short dimension of the model and spacing them vertically would yield results for the velocity distribution behind the sheeting. While the MEMS needed to be close to the wall, if they were to close the waves might travel through the wall and arrive earlier at certain sensors causing inaccuracies in the data. Based on this, the final location of the MEMS was selected to be 5.08 cm (2 in) from the wall on the exterior of the excavation on the centerline of short dimension of the container. The eight receivers were spaced vertically at 5.08 cm (2 in) intervals except between the depths of 25.4 cm (10 in) and 30.5 cm (12 in) where they were spaced at 2.54 cm (1 in) intervals. The reasoning behind this was to create a denser information array at depths more information was expected. The four source modules were then placed on the same line 17.1 cm (6.75 in) away so they could maintain a
position outside of the failure wedge. The vertical spacing was 7.62 cm (3 in) between modules with a larger 12.7 cm (5 in) space above the first source. There were two concerns for the positioning of the sources. The first was that the load from the bearing plate might cause the vertically polarized bender element to lose displacement amplitude and therefore cause the signal to be more difficult to observe and the second was simply concern about damaging the sources.

Figure 6.14: Elevation view of the spatial array for the wave propagation setup. The position in the perpendicular direction is centered in the excavation 30.5 cm (1 ft) from either side.

Once the final spatial array was determined a directivity analysis was conducted to determine if there were any problems with the final arrangement. Figure 6.14 depicts the final arrangement of the sources and receivers. The directivity analysis for the seismic wave propagation setup was conducted according to the specification given by Stokoe and Santamarina (2000). The concern was that the arrival of the reflected P-wave would precede that of the direct S-wave for some of the travel paths from source to receiver. In order to determine whether or not there was cause for concern for the given setup, the shortest and longest travel (i.e. outer most limits) paths were examined to determine the ratio of arrival times, $T_R$, for a range of Poisson’s Ratios, $\nu$, from 0.1 to 0.2. This is shown in Figure 6.15. This range is typical for clean dry sands (Stokoe and Santamarina, 2000).

The equation for $T_R$ is the following

$$T_R = \frac{t_s}{t_p} = \sqrt{\frac{(\nu - 1)}{(1 - 2\cdot\nu)}} \frac{d_s}{d_p}$$

(6.15)

where $t_s$ and $t_p$ are the travel times for the S-waves and P-wave respectively, and $d$ is the length of the travel path following the same format. Using a range of Poisson’s ratios from 0.1 to 0.2 the arrival time ratio, $T_R$, was plotted to determine weather or not the value ever was above one. The results are shown in Figure 6.16.
Figure 6.15: Directivity analysis wave paths on the spatial array depicted for the wave propagation analysis in both the vertical and horizontal planes of polarization. (a) Vertically polarized bender elements paths, (b) Shortest horizontally polarized S-wave path, and (c) longest horizontally polarized S-wave path.
Figure 6.16: Result of the directivity analysis on the spatial array depicted for the wave propagation analysis in both the vertical and horizontal planes of polarization.

The travel time ratio for all of the scenarios was found to be acceptable other than the short vertical path. This particular sensor-receiver set was the combination of the upper most sensor and receiver. The reflected P-wave is due to the steel bearing plate used to distribute the load into the system uniformly. Because of the impedance mismatch at the soil plate interface the reflected P-wave will arrive before the direct S-wave. The data set from this combination was analyzed keeping this concern in mind.

6.3.3.2 Additional Instrumentation

In addition to the wave propagation study there was additional instrumentation placed on the excavation to monitor what sort of behavior had occurred during testing. The braced excavation model was also equipped with load cells and MEMS accelerometers on the sheeting for rotation sensing. The four MEMS rotation sensors were placed on the exterior of the sheet between the centerline of the sheeting and the strain gauges. The use of the MEMS was designated to allow the deflected shape of the sheeting to be more accurately determined. The vertical spacing employed was based on the number of sensors available and was simply devised to cover the height of the wall below the whales and struts. In addition to the vertical location the initial orientation of the MEMS accelerometers was very strictly controlled because a certain sensor orientation would yield a better response for a given rotation. This phenomenon occurs because the tilt sensing capabilities of the MEMS accelerometer is based on earth’s gravity, and therefore the response is higher at an initial orientation orthogonal to the direction of earth’s gravitation pull. This was shown in the calibration chapter on the MEMS accelerometer. The MEMS used
for rotation sensing were also rewired in order to obtain a doubled sensitivity of approximately 79 mV/g. This was done to yield a better final resolution. Figure 6.17 shows the location of the MEMS accelerometers on the excavation sheeting.

Figure 6.17: Rotational MEMS locations on the sheeting on the exterior of the excavation

**6.3.3.3 Instrumentation Instillation**

The installation of the instrumentation was relatively simple for the most part because the instruments could be rigidly attached to a certain location. This was the case for the strain gauges, rotation sensors, and load cells. The wave propagation system on the other presented a rather interesting challenge. In order to maintain the geometric location of the sensors during the pre test filling and densification of the sand all of the sensors (i.e. sources and receivers) had to be located in a way that would allow little to no movement. In addition to this, the sensors then had to be independent of each other once the test was in progress. This meant that whatever methodology was employed to hold them in place during the filling and densification process had to be easily removed once testing was ready to begin.

To accomplish this for the accelerometers a tensioned string was used to maintain a location throughout the pre test procedures. The string was carefully placed prior to attaching any of the accelerometers and tension so that it would maintain its location during the pre test processes. Once located and tensioned the sensors were rigidly attached with a strong adhesive in the predetermined vertical locations. The sand was then filled in around them and compacted. The methodology used for compaction was vibratory compaction. Once the soil was compacted, the
string was released to remove the tension and allow the accelerometers to be independent elements during the wave propagation studies.

The sources accomplished this in much the same way, but a thin walled plastic cylinder was placed around the tensioned string in order to give the system more rigidity. The additional rigidity of the anchoring system was needed due to the additional weight of the source modules. The sources were then taped to the cylinder with adhesive strips, which would maintain the vertical location. Prior to running the test the tension would also be released in the source string. While there was some concern that the waves could travel through the stiffer plastic cylinder, the adhesive strips employed do not allow shear waves to propagate and therefore they cannot reach the cylinder. Figure 6.18 shows the anchoring system prior to filling and compaction.

![Anchoring system for elastic wave propagation system utilized in the braced excavation model. (a) Entire system prior to fill and compaction and (b) close up of source and receiver attachments (note that the MEMS were not coated).](image)

### 6.4 Pretest Compaction and Excavation

Prior to conducting the test the excavation had to be both filled and compacted. The compaction effort would help make the soil more uniform prior to applying the surcharge loads during the test. It was also felt that the denser the soil the less attenuation would occur in the elastic wave propagation techniques. This was very important to help attain a better signal to noise ratio from the accelerometers.

The densification of the sand was accomplished with a hand held vibratory compaction probe. Filling the excavation in 10.16 cm (4 in) to 15.24 cm (6 in) lifts and then compacting the sand with two passes of the vibratory probe was the basic technique. All three of the excavation sections were compacted with a square compaction point layout until the predetermined ground height was reached. Care had to be taken not to directly touch the sensors with the probe during compaction in fear of damaging the instrumentation. The compaction technique method is shown in Figure 6.19 as well as the probing pattern of the compaction technique.
After the compacted sand had reached the predetermine ground level, it was preloaded so that the excavation material already felt the weight of the surcharge load prior to excavation. Once the jack loads stabilized the interior section of the model (i.e., excavation area) was excavated to a depth of 7.62 cm (3 in) and the wave propagation readings were conducted. The system was then excavated again to a depth of 15.24 cm (6 in) and readings were taken again. The excavation was accomplished with the assistance of a wet/dry vacuum. The sand was simply removed until the correct height was reached at which time readings are taken from all instruments. Figure 6.20 shows a picture of the model pre and post excavation.

**Figure 6.19:** (a) Example of the square pattern used for compaction point layout of the excavation and (b) the affected area due to a single probe location.

**Figure 6.20:** Braced excavation system (a) prior to excavation and (b) after excavating to down 15.24 cm (6 in).
6.5 Data Collection

The data acquisition for the braced excavation was conducted on a variety of equipment that ranged from notebooks to oscilloscopes. Depending on what was required of the data that was being collected, a specific data acquisition system was employed. Most of the instrumentation only required data to be collected once, but the elastic wave propagation system required a more sophisticated collection.

6.5.1 State Instrumentation

The instrumentation that falls into this category is basically everything other than the wave propagation system. While there was a variety of instrumentation used, most gave a voltage response, which could be acquired through a multimeter. The only trick to this was that unhooking and reconnecting equipment constantly throughout the test was not an option; therefore a ten-point switch was utilized to allow speedy change over for the various instruments being read off of the multimeter. Once a steady voltage was reached for the instrument in question, the reading was manually recorded into a spreadsheet. The two system just described are shown in Figure 6.21.

![Figure 6.21: Data acquisition for the state sensors employed on the model excavation.](image)

6.5.2 Wave Propagation Instrumentation

An oscilloscope was used for wave propagation data acquisition. This allowed the signals to be cleaned up prior to the analysis of the data. This was an important part of the system because the typical signal was very noisy. In order to obtain a clear image of the shear wave arrival the signal was averaged approximately 1024 times. This canceled out the random noise in the signal. This was all accomplished prior to the recording of the signal with the oscilloscope. Of note if the fact that the signals had no gain applied. There were eight receivers and eight sources, which meant that a signal had to be recorded from each of the sources to all eight receivers, yielding 64 signals per loading and excavation case. The oscilloscope allowed for fast acquisition because
four signal could be taken and averaged at once without degrading the quality of the signals. Once a nice signal was obtained the data was saved directly to the hard drive of a computer.

In order to excite the system a signal generator was employed. This allowed user control of the amplitude and frequency of the signal used to excite the bender element sources. Each bender element was wired to a switch box for ease in triggering. They could be controlled individually or as a group for plane wave type propagation. The frequency control also gave the user an idea of how long it would take to receive enough signals to reach the required signal number for averaging on the oscilloscope. Figure 6.22 shows the oscilloscope and signal generator used for testing.

![Figure 6.22: (a) Signal generator and (b) oscilloscope used for triggering and data acquisition respectively.](image)

### 6.6 Analysis and Results

The results for the analysis of this model are presented within this section. All of the instrumentation was recording for both surcharge load and excavation depth in order to determine behavior changes due to either variable. As was stated previously, the load was brought to the predetermined amount prior to excavation of the braced excavation system. Once the load stabilized the system was excavated and the load was restabilized if required. At this point readings were taken for the various state instrumentation equipment and the wave propagation system. This will be reported versus the load case and the excavation height. The load cases applied to the excavation were 9.5 kPa (200 psf), 19.1 kPa (400 psf), and 28.7 kPa (600 psf). Each load increment had data taken at 7.62 cm (3 in) and 15.24 cm (6 in).

#### 6.6.1 State Instrumentation Results

While the load cells used for the vertical surcharge load were set to the pre design amount, the rest of the system was based on the soil reaction to the surcharge load applied. The load from the surcharge was applied with a ±0.24 kPa (±5 psf) error due to the relaxing of the load from the hydraulic jacks. This phenomenon was possibly due to a combination of soil movement and slow release of the jack load from slack in the system. The loads were observed throughout the testing and kept within this range of ±0.24 kPa (±5 psf).
6.6.1.1 Strut Loads

The strut loads yield a very nice behavior during testing. The basis seemed to show that the material within the excavation provides support before it is removed at which point the struts begin to take the load. While the 7.62 cm (3 in) excavations show very little variation load to load, the 15.24 cm (6 in) does begin to show the strut taking the load after the second lift of soil is removed. Figure 6.23 below shows the trend for the strut versus load for each excavation depth.

![Strut Load vs Surcharge Load](image)

**Figure 6.23:** Trend for strut load versus the surcharge and the excavation depth

It seems that before the second lift of soil was removed very little of the load was transferred into the actual struts. Also shown is that the trend increases with increases surcharge load. There are only three points given for the following because the data has been reduced. Each of the strut loads was averaged for a given surcharge and excavation depth to more accurately show the trend.

6.6.1.2 Rotation Sensors

Based on the previous results it was expected that the rotation sensors would yield a larger change under the higher loadings, however very little changed throughout the testing for the rotation sensors. The reason for this minimal change in sheet orientation was felt to be a result of sheets that were stiffer than what was needed. Because the final design for the sheets turned out to be to stiff to yield results that showed any sort of trend, the deflected shape remained almost constant for the different load cases. This anticipated trend was reinforced after an analysis of the results for each of the load cases and excavation depths. Figure 6.24 below shows the results of the data analysis for the MEMS rotation sensors on the excavation.

Unfortunately the upper most tilt sensor (i.e., MEMS 1) gave physically impossible results during testing so the final results had to be based on the MEMS below the cut only. While there was very little change from test to test in the deflected shape the final results were considered
reasonable. The lack of a trend for increasing loads was attributed to the fact that the active condition was mobilized under very little displacement at the lower loading conditions.

![Graph showing rotational MEMS readings and deflected shape]

**Figure 6.24:** (a) Readings for the rotational MEMS during the different tests conducted on the braced excavation model. (b) Typical deflected shape which all of the tests yielded.

### 6.6.2 Wave Propagation Results and Discussion

The wave propagation analysis using the MEMS was conducted for each of the loading cases at both the 7.62 cm (3 in) and 15.24 cm (6 in) excavation. This allowed trends to be shown for the position, load case, and excavation height. The results for the velocity profiles from each of the sources will be shown and then discussed on the following sections.

#### 6.6.2.1 Wave Propagation Results: Velocity Profiles

Each test was run in the same manner in order to allow for an easy comparison. While it is important to shown the change in velocities for the various conditions, the presentation of this data was developed to demonstrate the MEMS capability for use in wave propagation studies. All of the results shown are based on the estimated travel time determined from the time domain analysis of the voltage response. An example of the raw voltage response signals with which the velocities were obtained can be seen in Figure 6.25. Figures 6.26, 6.27, 6.28, and 6.29 show the results for all of the wave propagation analysis conducted at 0.5 kPa (12.5 psf), 9.5 kPa (200 psf), 19.1 kPa (400 psf), and 28.7 kPa (600 psf). Each load increment had data taken at 7.62 cm (3 in) and 15.24 cm (6 in) with the exception of the base load. The results shown are a report the results for the velocity profiles seen from each source for a given load case and excavation depth.
Figure 6.25: response at each receiver (1 to 7 from top to bottom) for Source 3.
Figure 6.26: Velocity profile (horizontally polarized case) for all load cases and excavation depths of (a) 7.62 cm (3 in) and (b) 15.24 cm (6 in) for Source 1 in the braced excavation model.
Figure 6.27: Velocity profile (horizontally polarized case) for all load cases and excavation depths of (a) 7.62 cm (3 in) and (b) 15.24 cm (6 in) for Source 2 in the braced excavation model.
Figure 6.28: Velocity profile (horizontally polarized case) for all load cases and excavation depths of (a) 7.62 cm (3 in) and (b) 15.24 cm (6 in) for Source 3 in the braced excavation model.
Figure 6.29: Velocity profile (horizontally polarized case) for all load cases and excavation depths of (a) 7.62 cm (3 in) and (b) 15.24 cm (6 in) for Source 4 in the braced excavation model.
6.6.2.2 Wave Propagation Discussion: Velocity Profiles

The results from the velocity profile were very accurately recorded, and the magnitudes of velocity are very reasonable. There are a variety of trends seen in the data, which demonstrate the expected behavior in the elastic wave propagation. The most obvious information obtained from the velocity profiles is the trend shown due to the increased surcharge load.

In each of the velocity profiles (i.e. from each of the sources) the increased load yielded increased velocities for the most part. In particular, there are very nice trends closer to the bearing plate where the additional load is applied. This trend can be attributed to the increased soil density and shear modulus $G$ under the increased surcharge loading (see for example Roessler, 1979). Also seen due to in the loading sequences is the shape of each of the velocity profiles.

At the higher load cases the shape of the curve consistently increases yielding the highest velocity values at the upper most receivers. This is expected as well because the induced stress from the surcharge load has the most effect on the upper most soil. This induced stress value slowly decreases as the depth increases making the velocity values slowly approach unity. While this was not seen in this example, probable reasons for the lack of uniformity at the lower receivers could be due to the small size of the excavation system or the displacement of the sheet pile. This size would not allow enough depth for the in-situ stresses to become the controlling factor for the shear wave velocity.

Comparing the different curves from different sources it is seen that the results from Source 1 and Source 4 show the most scatter. What you see in Source 1 is a general lack of approached uniformity. In fact, the velocity profiles for Source 1 show very little increase from the top to bottom receivers. This is because the velocity of the material slowly decreases with increased distance, which means the velocity value changes very little at each receiver location. Another way to look at the phenomenon is that the time and distance increase as the depth increases, and if both values increase the velocity changes very little. One the other hand, the velocity profiles from Source 4 show the exact opposite trend. These velocities show the highest values for the same pressures seen throughout the test. This can be attributed to and increasing velocity as the signal travels from Source 4 to the upper receivers. In other words the time decreases as the distance increases which means an overall increase in velocity. This particular behavior can actually be seen in all of the plots as the source depth is increased, but it is most obvious in Sources 1 and 4. This type of trend was also seen by Tanner (2004) for test conducted on clean sand beneath a circular footing. At the lower source elevations the apparent velocities would increase in comparison to the higher source elevations, much like what has occurred in the tests conducted for this study. Figure 6.30 (following page) shows the results of Tanner’s testing for specific source-receiver combinations.

Unfortunately, there was an oversight that causes the vertical data to be lost. The size of the bender elements used for the vertically polarized S-wave were too large, and therefore the pressure due to the surcharge load caused the benders movement to be greatly reduced. This made the signal much too small and the first arrival from the raw voltage response signals unable to be evaluated. The sensitivity of the ADXL250 MEMS accelerometers should be doubled prior

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to their use as a field sensor to achieve a better signal to noise ratio and possibly remove this sort of problem. Additionally, the stress changes due to the excavation depth change were too small to be discerned for the most part. Although there was a slight trend seen which seemed to show the velocities reducing due to the deeper excavation height, it was too small to distinguish. Lastly, the seventh receiver also shorted during testing and no data could be recorded for the remainder of testing. Therefore none of its results were reported in the study.

**Figure 6.30:** Velocity profiles from research by Tanner (2004) showing the effect of shear wave path on the velocity distribution from each source. (a) Source 2 shows effect of traveling downward to decreasing stress levels while (b) Source 6 shows the effect of traveling upward through increasing stress levels (Tanner, 2004).
6.7 Summary

While the design of the excavation was relatively simple the data obtained from both the state instrumentation and the wave propagation instrumentation seem to indicate a relatively nice set of trends. Comparing the velocity profile with the approximate deflected shape yields an interesting observation. The deflection increased with depth, which basically infers that the velocity should decrease with depth, which is what was seen in each of the profiles. This can be attributed to the reduced stress due to the sheet movement away from the soil behind the excavation. The change in velocity profile was indistinguishable due to a change in excavation depth, but an increased load in the struts was seen for this same event. This was a nice confirmation of the model behavior.

Although the data inversion for the state of stress could not be accomplished, the trends in the velocity do depict a picture of the stress trends behind the sheet. Regrettably, all that can be inferred from the velocity profiles is an idea of what the stress trends are behind the sheet. To obtain the full data inversion, a much larger quantity of data would be required. The MEMS accelerometers have shown adequate capabilities for use as receivers in wave propagation experiments conducted in particulate material.
CHAPTER 7
CONCLUSIONS AND FURTHER RECOMMENDATIONS

7.1 Conclusions

In order to determine possible uses in the field of geotechnical engineering for MEMS based technologies one needed to discover an application that was readily available and presented many advantages to the field. This opportunity presented itself in the form of the ADXL250 MEMS based accelerometer by Analog Devices. Its cheap price and range of possible applications made the MEMs based accelerometer an excellent candidate to merge the two fields.

This thesis presents the results of an in depth calibration of MEMS technologies and an investigation into their applications in the field of geotechnical engineering. The following is a summary of the findings as well as limitations in the final results.

Sensor development for use within a specific field requires a variety of sequences, which includes: a calibration check of the sensor, improvement to the sensor where needed for application purposes, recalibration based on the improvement effects, and testing in specific applications to evaluate performance.

The calibration of the MEMS accelerometers was accomplished with the use of a simple cantilever beam system and seismic pulse test (Chapter 3). The cantilevered beam allowed moderate control over the input frequency, and allowed testing of all frequencies that were within the applicable range of the sensor. In addition, the seismic pulse test also yielded adequate control of the testing parameters. After a large compilation of data for each of the parameters, the final sensitivity and frequency range were determined which were later employed for the evaluation of the MEMS response in elastic wave propagation testing. The results of the analyses matched the manufacturer specifications adequately enough to proceed into the development of an improved packaging technique.

Improvement of the MEMS accelerometers packaging was an essential step into the possible application of the sensors into the field of geotechnical engineering. This is attributed to the fact that the packaging on the MEMS was originally developed to deal with elevated temperatures, electrostatic discharge (i.e. shock), and possible excessive stress values due to massive acceleration changes (i.e. something such as being dropped on a floor). These parameters are not necessarily the major concerns for applications in geotechnical engineering. The area that was focused on specifically was submersion into corrosive agents. This hurdle was cleared by utilizing a casting technique that completely isolated the accelerometer and its connection points from the outside environment (Chapter 3). The packaging system was tested to determine if the apparent seal had in fact corrected the previous deficiencies. After a successful evaluation of the packaging the sensor was recalibrated in the same manner as previously used to determine what influences the packing had on its performance. No performance affects were found (Chapter 3). While the packaging system was successful, it should be noted that the cost of this process would increase the overall price of the sensor because the packaging technique is an addition to the processes already required to prep the instrumentation for field application. A very ruff estimate of the cost is approximately $30 to reach the final geotechnically sound product. This cost
includes fully wiring the sensor and packaging it for use in the field. This was estimated to be worth about an hour of time at a rate of $10 an hour. The additional materials were estimated to be about $5, which includes the packaging and the wiring materials.

After a review of some the fundamental elastic wave propagation concepts (Chapter 4), the MEMS were employed as acceleration sensors in a simple 2-D model (Chapter 5) to evaluate the response trends and measurement capabilities of the sensors. Utilizing a simple particleboard laminate wood plate the MEMS were run through a variety of simple test applications to determine if the response was visible and accurate. While the testing was relatively simple, it gave a relatively large amount of insight into the capabilities of the sensors. Not only did the MEMS perform well, but their comparison to a theoretical model (originally by Miller and Pursey) for a similar type of excitation was good. They matched trends very well for the analysis of wave propagation in a solid from a seismic pulse perpendicular to a free surface. Additional analysis showed that where the theoretical model and the experimental results differed were due additional wave types not considered in the model. Additional analyses conducted on the plate data demonstrated the correct symmetry trends throughout, and also compared well to the same model. This meant the results from the MEMS had captured the wave propagation phenomenon exceptionally well.

While the simple plate test demonstrated the MEMS capabilities for use in wave propagation techniques, the model was relatively simple and by no means represented the complexity of wave propagation in particulate media. The 1g-braced excavation model testing (Chapter 6) conducted with the use of the MEMS was created to evaluate their capabilities within a geotechnical state of circumstances. Use as a seismic sensor in wave propagation studies was felt to be the most sensible application for the accelerometers. Additionally the sensors were used as tilt sensors for the sheeting in the experiment. The tilt sensing capabilities of the MEMS (Chapter 3) helped to evaluate the deflected shape of the sheeting. The MEMS performed well and allowed a relatively accurate representation of the velocity field to be determined. Utilizing bender element sources, the soil mass was impregnated with the accelerometers and a few different surcharge loads were applied to the model to show trends in the velocity field. Most of the trends seen in the data were relatively easily explained by the coupling of stress theory in soils and elastic wave propagation theory. While an in depth explanation of soil behavior was not achieved the MEMS performed well. However, a problem did occur at low surcharge loading. Because the sources used apply such a small deformation, the signal energy is relatively low. This is somewhat unfortunate because the MEMS are basically inertial sensors, and if the already low wave energy attenuates to quickly before reaching the sensor it will not be seen above the noise. Fortunately, with increased stresses the waveforms became increasingly clearer reducing the time required to determine the first arrival. After analyzing and reducing the data for the model including the state instrumentation on the model, a simple explanation of its behavior was established. It was felt that the MEMS’ velocity trends could allow inferences to be made about the stress state in the model as well as the pressures on the exterior face of the sheeting.

7.2 Recommendations and Future Work

Although this study has identified the MEMS accelerometer as a definite candidate for wave propagation applications, improvements can be found easily. First and foremost, the MEMS
must be recalibrated after doubling the sensitivity. Although this reduces the applicable range to ±25 gs, the maximum expected accelerations for application within elastic wave propagation in soil are well below this limit. The doubled sensitivity could improve all aspects of the MEMS performance given in this thesis.

Testing should be conducted on the packaging technique to determine more physical properties, such as melting point, insulation capabilities, and evaluation of its long term performance under long corrosive attack. Further evaluation of a molding technique should be investigated to possible create a better geometry for MEMS use in the field. Additionally, while the packaging is a permanent type of technique, a quick and gentle removal technique should be established for trouble shooting sensor problems.

A full tomographic imaging test should be conducted with the MEMS in order to create a more practical use for the accelerometers. If a model is used, it is suggested that the model undergoes a full dimensionless analysis prior to testing to ensure an adequate representation of reality. While this test can be conducted on a model, a full-scale test is the preferred and recommended choice. This leads to the next application for the inexpensive accelerometers: field evaluation. The use of these sensors in the field is essential to their possible application as inexpensive sensors for the industry. An analysis of their application as slope stability sensors should be conducted because the accelerometers are already inclined to use as a tilt meter. Furthermore, other applications should be developed in order to give the field more applications for this type of sensor due to its accuracy and ease of implementation. Some areas in which the accelerometer could possibly be applied are the following:

- Application as centrifuge calibration sensors for geotechnical testing.
- Field application for the analysis of deep foundation systems to determine if loss of structural element occurred during installation using wave propagation techniques.
- Slope stability applications.
- Settlement analysis sensors using wave propagation techniques.

With these applications and the others that this study focused on this sensor is quite suited for application within the field of geotechnical engineering. However, in order to achieve overall acceptance the sensor needs further simplification to ease its application in the field.
REFERENCES


VITA

Keith Hoffman was born on June 8th, 1979, in New Orleans, Louisiana. He received his bachelor’s degree from Louisiana State University in August of 2002, following which he immediately enrolled in the Louisiana State University graduate school program. Keith will graduate from the graduate program at Louisiana State University in August of 2004. He will marry Katherine Marino on June 12th, 2004, in Mandeville, Louisiana, which is their hometown. Keith has stayed in Baton Rouge, Louisiana, upon accepting a job with CDM’s local branch.