2007

Optimal control of production and distribution in a supply chain system operating under a JIT delivery policy

Pablo Biswas

Louisiana State University and Agricultural and Mechanical College

OPTIMAL CONTROL OF PRODUCTION AND DISTRIBUTION IN A SUPPLY CHAIN SYSTEM OPERATING UNDER A JIT DELIVERY POLICY

A Dissertation

**Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy**

In

**The Interdepartmental Program
in Engineering Science**

By

**Pablo Biswas
B.S. M.E., Bangladesh University of Engineering and Technology, 1998
M. S. I. E., Louisiana State University and A and M College, 2003**

May, 2007

ACKNOWLEDGEMENTS

I would like to thank the Almighty for what I have become today. I would like to express my gratitude to Professor Bhaba R. Sarker for his time, support and guidance throughout the course of this work. His supervision and encouragement in pursuing this research were of paramount importance to me in completing this work. It was a pleasure working with him and learning the different facets of research. I really appreciate his generous help in this regard.

My special thanks to Professors Lawrence Mann, Jr., Ralph W. Pike, Luis A. Escobar, Gouli Ding, and Mehmet T. Tumay for serving on my committee. Their input during the course of this research has helped me see important aspects that were not addressed earlier.

I am also thankful to Professor Thomas G. Ray, the Chairman of the CMIE Department, and Professor Magd E. Zohdi for their encouragement and support to complete the work. I am grateful to the IE Department which has funded my study at LSU.

I really appreciate for the generous helps of my office mates Dr. Ahmed Diponegoro and Mohammad Anwar Ashek Rahman. I like to thank Joann Landry and Becky Labatut for their help in different ways.

Finally, I like to thank my father Pradhish Ranjan Biswas and my mother Supti Biswas for their sacrifices and support throughout my life. Last but definitely not least, I thank my cousin Lopa Sarker and my wife Sumi Biswas for their love, continuous encouragement, sacrifices, and constant moral support to achieve my goals.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS ii

LIST OF TABLES vi

LIST OF FIGURES viii

ABSTRACT… x

CHAPTER 1: INTRODUCTION 1
1.1 SUPPLY CHAIN SYSTEMS IN MANUFACTURING ENVIRONMENT 2
1.1.1 Supply Chain in Real Life 3
1.1.2 Just-in-Time (JIT) Delivery 5
1.1.3 Inventory of Raw Materials and Finished Goods 5
1.1.4 Idle Time of the Production Facilities 6
1.1.5 Single Facility Lot-Sizing Models 6
1.2 THE PROBLEM, RESEARCH GOALS AND OBJECTIVES 6
1.2.1 The Problem 6
1.2.2 Research Goals 7
1.2.3 Research Objectives 8
1.2.3 Overview 10

CHAPTER 2: LITERATURE REVIEW 11
2.1 MODELS WITH PERIODIC REVIEW 11
2.2 JOINT REPLENISHMENT POLICY 12
2.3 JUST-IN-TIME (JIT) MODELS 13
2.4 INVENTORY MODELS WITH SINGLE FACILITY LOT-SIZING 15
2.5 SHORTCOMINGS IN PREVIOUS RESEARCH 15
2.6 PROBLEM DEVELOPMENT FOR CURRENT RESEARCH 17

CHAPTER 3: PERFECT MATCHING SYSTEM WITH JUST-IN-TIME DELIVERY 21
3.1 PROBLEM DESCRIPTION AND NOTATION 21
3.2 ASSUMPTION 24
3.3 AVERAGE INVENTORY AND TOTAL COST FUNCTION 24
3.4 PROBLEM FORMULATION 29
3.5 SOLUTION METHODOLOGY 29
3.6 COMPUTATIONAL RESULTS 35
3.7 SPECIAL CASE 36
3.8 SINGLE FACILITY LOT-SIZING MODEL WITH PERFECT MATCHING 38
3.9 SOLUTION OF SINGLE FACILITY LOT-SIZING MODEL IN PERFECT MATCHING 42
3.10 NUMERICAL COMPUTATION OF OPTIMUM ROTATION CYCLE 46

CHAPTER 4: IMPERFECT MATCHING WITH JUST-IN-TIME DELIVERY 48
4.1 NOTATION 48
4.2 PROBLEM IDENTIFICATION 49
4.3 ASSUMPTION 49

4.4 FORMULATION OF COST FUNCTION BASED ON AVERAGE INVENTORY 50
4.5 SOLUTION TECHNIQUE 55
4.6 COMPUTATIONAL RESULTS 58
4.7 SPECIAL CASE FOR IMPERFECT MATCHING INVENTORY 59
4.8 SINGLE FACILITY LOT-SIZING MODELS FOR IMPERFECT MATCHING 61
4.9 SOLUTION TECHNIQUE OF ROTATION CYCLE 63
4.10 NUMERICAL COMPUTATION OF OPTIMUM ROTATION CYCLE 67

CHAPTER 5: SUPPLY CHAIN OF MULTIPLE SUPPLIERS AND BUYERS 69
5.1 PROBLEM DESCRIPTION FOR SINGLE PRODUCT, MULTI-SUPPLIER, AND MULTI-BUYER 71
5.2 NOTATION 73
5.3 TOTAL COST FUNCTION FOR RAW MATERIALS 75
5.4 TOTAL COST FUNCTION FOR FINISHED PRODUCTS 76
5.5 SOLUTION TECHNIQUES 79
5.5.1 Solution Procedure of Problem MB 80
5.5.2 Solution Procedure of Problem MS 81
5.6 NUMERICAL ANALYSIS 83
5.7 ALTERNATE SHIPMENTS SCHEDULE 84
5.7.1 Total Cost Function for Alternate Delivery Schedule 86
5.7.2 Numerical Tests for Alternate Delivery Schedule 89

CHAPTER 6: MULTIPLE PRODUCTS, SUPPLIERS AND BUYERS SUPPLY CHAIN 91
6.1 NOTATION USED TO DEVELOP THE MODEL 92
6.2 PROBLEM DESCRIPTION FOR MULTIPLE PRODUCTS SUPPLY CHAIN MODEL 93
6.2.1 Total Cost Function for Raw Material Supply 94
6.2.2 Total Cost Function for Finished Products 96
6.3 SOLUTION METHODS FOR ROTATION CYCLE OF MULTI-SUPPLIER-AND-BUYER 98
6.4 NUMERICAL ANALYSIS FOR MULTI-SUPPLIER-AND-BUYER ROTATION CYCLE 100

CHAPTER 7: SENSITIVITY ANALYSIS 105
7.1 EFFECT OF SHIPMENT SIZE (x) ON TOTAL COST FUNCTIONS 105
7.2 EFFECT OF RAW MATERIAL CONVERSION FACTOR (f) ON TOTAL COST FUNCTIONS 108
7.3 EFFECT OF ORDERING (K_0) AND SETUP (K_S) COSTS ON TOTAL COST FUNCTIONS 110
7.4 EFFECT OF RAW MATERIAL (H_R) AND FINISHED GOODS (H_F) CARRYING COSTS 111

CHAPTER 8: CONCLUDING REMARKS 113
8.1 SUMMARY 113
8.2 RESULTS 114
8.3 SIGNIFICANCE OF RESEARCH 115
8.4 FUTURE RESEARCH 116

REFERENCES 118

APPENDIX A: PROOF OF CONVEXITY 125
A.1 CONVEXITY TEST OF COST FUNCTION TC_{PM} 125
A.2 CONVEXITY TEST OF COST FUNCTION TC_{IM} 126

APPENDIX B: INVENTORY COMPUTATION 129
B.1 INVENTORY COMPUTATION FOR PERFECT MATCHING 129
B.2 INVENTORY COMPUTATION FOR IMPERFECT MATCHING 131

VITA........ 135

LIST OF TABLES

Table 2.1 Problem characteristics of few selected research and proposed model..................19

Table 3.1 Data set for numerical computation for perfect matching problem 36

Table 3.2 Results using given data set..36

Table 3.3 Inventory produced in accelerated and deferred production 38

Table 3.4 Data set for single facility lot-sizing model 47

Table 3.5 Optimum results for rotation cycle...47

Table 4.1 Data set for computation of Problem IM...59

Table 4.2 Results for imperfect matching conditions..59

Table 4.3 Comparison of quantity produced in accelerated and deferred production............61

Table 4.4 Data set for single facility lot-sizing model 67

Table 4.5 Optimum results for raw materials of imperfect matching case 68

Table 5.1 Buyers' demands and shipment sizes...83

Table 5.2 Solution for finished products based on buyers' demand 83

Table 5.3 Solution for raw materials based on buyers' demand 84

Table 5.4 Optimum system cost for fixed shipment periods..89

Table 6.1 Parametric values for five (5) different products 102

Table 6.2 Demand and shipment sizes for all five (5) products...102

Table 6.3 Results of optimal total costs for multiple suppliers, products, and buyers..........102

Table 6.4 Detail optimal results for multiple suppliers, products, and buyers......................104

Table 7.1 Effect of shipment size x on the total costs for perfect and imperfect matching ..107

Table 7.2 Effect of conversion factor f on the total system costs...109

Table 7.3 Effect of K_0/K_S on the total costs for perfect and imperfect matching 111

Table 7.4 Effect of H_R/H_F on the total costs for perfect and imperfect matching 112

LIST OF FIGURES

Figure 1.1. A typical supply chain system 1

Figure 2.1. Inventory models with just-in-time (JIT) delivery with idle time 16

Figure 2.2. Inventory models with just-in-time (JIT) delivery and without idle time........ 16

Figure 2.3. Problem development flow chart 20

Figure 3.1. Supply chain system with just-in-time (JIT) mechanism........ 21

Figure 3.2. (a) Inventory of finished goods; (b) Inventory of raw materials........ 25

Figure 3.3. (a) Finished goods inventory with idle time, (b) raw material inventory 37

Figure 3.4. Inventory for single facility with multiple products 40

Figure 4.1. (a) Finished goods inventory; (b) Raw material inventory 51

Figure 4.2. (a) Finished products inventory with idle time; (b) raw material inventory........ 60

Figure 5.1. Multi-supplier and multi-buyer supply chain system 71

Figure 5.2. Inventory diagrams for a multi-supplier-and-buyer system........ 72

Figure 5.3. Shipment duration according to buyers' schedule 84

Figure 5.4. Shipment duration according to manufacturer's schedule........ 85

Figure 5.5. Inventory diagrams of alternate delivery schedule 85

Figure 5.6. Variation in fractional savings based on alternate delivery schedule 90

Figure 6.1. Multiple suppliers, products, and buyers supply chain system........ 91

Figure 6.2. Inventory of raw material i for K finished products........ 94

Figure 6.3. Finished goods inventory of rotation cycle for buyer j 96

Figure 6.4. Supply chain results for multiple suppliers, products, and buyers......................103

Figure 7.1. Effect of shipment size *x* on the total system costs..106

Figure 7.2. Variation of the total costs for shipment size *x*..107

Figure 7.3. Effect of conversion factor *f* on the total system costs 108

Figure 7.4. Variation of the total costs for conversion factor *f*...109

Figure 7.5. Effect of K_0/K_S on total system costs 110

Figure 7.6. Effect of H_R/H_F on the total system costs 112

Figure B.1. Step-by-step inventory formation for perfect matching case 129

Figure B.2. Inventory behavior for imperfect matching in different stages..........................132

ABSTRACT

This research deals with a supply chain system where the production or manufacturing facility operates under a *just-in-time* (JIT) environment, and the facility consists of raw material suppliers, manufacturers, and retailers where inventory of raw materials, work-in-process, and finished goods are involved, respectively. This work considers that the production of finished goods in one cycle starts just after the production or uptime in preceding cycle to minimize the idle time of the facility. Considering this scenario, inventory models are developed for different delivery situations: (a) perfect matching condition where no finished good remains after the shipments and (b) imperfect matching condition where some finished goods remain after the shipments.

In this research, the problems are categorized as integer and mixed integer non-linear programming problems which are solved to find optimum number of orders and shipments, optimum production quantity, and minimum system cost. Moreover, multi-supplier and multi-buyer operations, where raw materials are ordered from different suppliers and finished goods are delivered to different customers, are considered. In addition to these problems, a single facility lot-sizing model is applied in perfect and imperfect matching cases, and, multi-supplier and multi-buyer case, to concentrate on more practical supply chain environments.

All the problems described in this research are non-convex functions for which the closed form solutions are cumbersome. Therefore, the heuristic solutions are developed to find the optimal lot-sizing techniques. Additionally, the multi-supplier and multi-buyer problem is solved with the help of integer approximation and the divide and conquer rule. The solutions are tested through numerical examples. Furthermore, the sensitivity analyses are performed to observe the variations of the different cost functions. Also, this research proposes an alternate

delivery schedule of finished product supply, for which both manufacturers and buyers will be benefited economically.

The production and supply chain management play a significant role for the necessary amounts of materials and parts arrive at the proper time and place. With the models obtained in this research, managers can quickly respond to consumers' demand by determining the right policies to order raw materials, to manage their production schedule efficiently and to deliver finished goods just-in-time.

CHAPTER 1

INTRODUCTION

Production and supply chain management play an important role on the current economy. The oscillating demand of various products and increasing expectations influence the social economy as well as the business enterprises in focusing their attention on the appropriate control of their supply chain. The continuous development of business environment has made it necessary to improve the knowledge and techniques of supply chain management.

A supply chain system consists of suppliers, manufacturers and retailers or customers including raw material inventory, work-in-process inventory, and the inventory of the finished goods. The raw materials are ordered from the suppliers, and manufacturers process the raw material as finished product and deliver to the customers or retailers. Supply chain philosophy enables an individual business organization to achieve superior productivity and minimizes its system cost by satisfying the service level requirements. A typical supply chain system is represented in Figure 1.1.

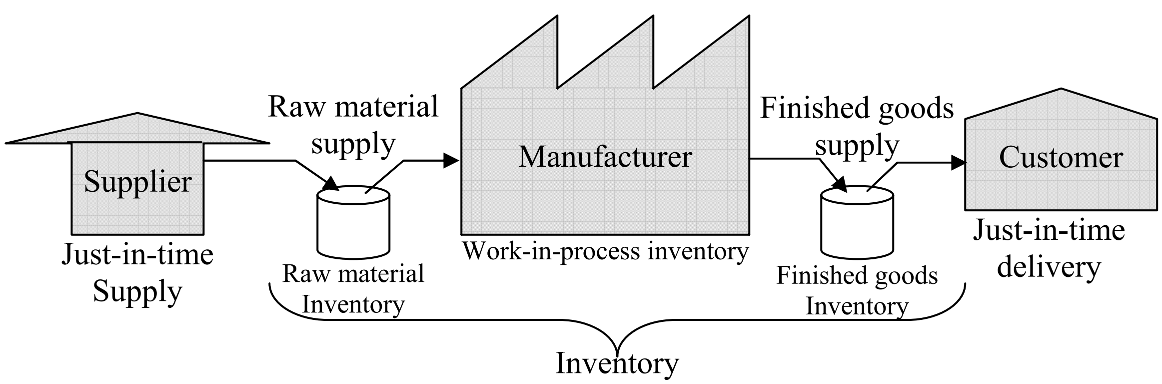


Figure 1.1. A typical supply chain system

For last few decades, *Just-in-Time* (JIT) philosophy has played an important role in supply chain systems such as the manufacturing sectors. The successful implementations of just-in-time (JIT) phenomena are frequent shipment of high quality parts to the customers and

ordering raw materials in small batches whenever required to process finished products. Applying just-in-time (JIT) philosophy, the researchers are searching for an economic quantity model of a production system which follows the just-in-time (JIT) philosophy for ordering raw materials and shipping process. Also, the long-term purchase agreements with the suppliers will minimize inventory-holding costs for the customers under just-in-time (JIT) manufacturing environment.

Suppose a manufacturing system which procures the raw materials from an outside supplier, and processes them to convert them into finished goods for customers. To minimize the retailer's holding cost, the manufacturer must deliver products in small quantities. Also, the manufacturer should accept the supply of raw materials in small quantities to minimize his holding cost. The manufacturing lot size depends on the retailer's sales volume, unit product cost, inventory-carrying cost, and setup cost. Conversely, the raw material purchasing lot size depends on raw material requirement in the production system, unit raw material cost, ordering cost, and inventory carrying cost. Therefore, the optimum quantity of finished goods may not be equal to the raw material requirement for an optimal manufacturing batch size. For an optimal operation of a just-in-time (JIT) environment, it is required to optimize both the raw material and production lot size simultaneously by taking all parameters in consideration. This research deals with such a system which operates under a just-in-time (JIT) environment.

1.1 SUPPLY CHAIN SYSTEMS IN MANUFACTURING ENVIRONMENT

The principal operational policies of a supply chain system include: (a) order, purchase and inventory of raw materials, (b) process of finished goods, inventory control, and (c) delivery of finished product to the warehouse, retailers or customers. Based on the business environment, a production system may have some or all of these operational activities. The main focus of this research is to study the different parts involved in a production system.

In this research, a single stage production facility is considered which purchases raw materials from outside suppliers and processes them to deliver a fixed quantity of finished products to retailers (or customers) at a fixed-interval of time. Also, the raw materials are replenished instantaneously to the manufacturing system to meet the just-in-time (JIT) operation.

1.1.1 Supply Chain in Real Life

The current research is based on the facts encountered in supply chain systems of different manufacturing systems, such as computer/electronics industries, sheet-metal industries, refineries, and paper industries. For example, to manufacture filing cabinet, metal sheets and L-angles are obtained from steel industries. The delivery of the finished cabinets depends on the downstream market demand. Similarly, computer and automobile industries procure various items and maintain supply chain both upstream and downstream to sustain uniform flow of products.

The refinery supply chain spans from the major crude lifting points around the world to the end of fuel consumers, with all the refining, transportation, storage, trading, exchanging, and swapping that goes on in between. For example, Navajo Refinery Artesia, New Mexico, purchases crude oil from producers in nearby southeastern New Mexico, West Texas, and from other major oil companies. After refining the crude oil into finished petroleum products, Navajo Refinery supplies the finished product to other refiners, convenience store chains, independent marketers, an affiliate of PEMEX (the government-owned energy company of Mexico), and retailers. The Company’s gasoline is marketed in the southwestern United States, including the metropolitan areas of El Paso, Phoenix, Albuquerque, Bloomfield, and Tucson, and in portions of northern Mexico. Therefore, it is important to optimize the inventory capacity for raw material (crude oil) and finished products (petroleum), production capacity,

and to generate an optimum supply chain management for the suppliers, refinery, and different retailers or distributors.

In electronics industries, a supply chain problem was addressed by Sarker and Parija (1994, 1996) in association with IBM. A silicon wafer vendor supplies wafers to Motorola Company for the production of Power PC chips, which are delivered to several customers such as Apple, IBM and Motorola itself. To satisfy customers' demand at different time-intervals, the manufacturer (Motorola) has to continue its production at an expected rate by procuring the silicon wafer at regular time-intervals and maintaining the inventory of finished Power PC chips. As a result, the wafer suppliers, the Power PC chips manufacturers, and customers' need to synchronize their logistics. The supply chain logistics should be well incorporated in order to keep the raw materials (wafer) and finished goods (chips) inventory system operative at minimum cost.

Georgia-Pacific Corporation in Atlanta, GA, manufactures and sells pulp, communication papers, containerboard, packaging and tissue, plywood, oriented strand board and industrial panels, lumber, gypsum products, and chemicals. The raw materials (mainly wood) for pulps supplied from Georgia-Pacific timberlands to the pulp and paper mill, Old Town, Maine, and the finished products distributed to the different customers and agents. The on-time deliveries of all the components from its subsidiary companies or other suppliers are important. A little deviation from schedule deliveries can cost millions of dollars to both the manufacturing and consuming industries. In another instance, however not the same field but equally responsive in just-in-time (JIT) operations, is a retail store (Wal-Mart, Safe-Way, Albertson's, Winn-Dixie, etc.) that supplies hundreds of items to local vendors. For economic survival and success of a company, the just-in-time (JIT) delivery and uninterrupted stream of products are of prime importance.

1.1.2 Just-in-Time (JIT) Delivery

The primary concept of a just-in-time (JIT) production system can be stated as 'producing and/or supplying the products only the right items in the right quantities at the right time.' Several manufacturing facilities, which previously carried large finished goods inventories to meet customers' demand, implemented the just-in-time (JIT) delivery system. Due to the implementation, the lot sizes are reduced and the frequency of delivery is scheduled for customers' demand. The direct impact of the just-in-time (JIT) system is to reduce inventory carrying costs by the customers. Conversely, the manufacturer should acquire accurate data of the customers' demand and maintain an optimum schedule to coordinate the distribution system. In order to coordinate the production with customers' demand in fixed time intervals, and to synchronize the ordering of raw materials with production schedules, both raw materials and finished items inventories should be maintained at an economic level to reduce the total cost of the system.

1.1.3 Inventory of Raw Materials and Finished Goods

In a supply chain system, the inventory control and the requirement for coordination of inventory decisions are important factors. One of the causes for maintaining inventory is to protect the company from unexpected customers' demands that are difficult to predict. Most consumers and industrial finished goods progress through different sorts of multi-stage inventory (especially in multi-plant operating facilities where inventories vary from one plant to another). Typically, the manufacturers order the raw materials form the suppliers to produce the finished products. Therefore, the types of inventory can be categorized into raw material inventory, work-in-process inventory and finished product inventory. It has been a challenge to determine the control mechanism for these inventories for efficient production, distribution, and control tactics to reduce the system cost of the supply chain system.

1.1.4 Idle Time of the Production Facilities

In large production companies, such as refineries, paper mills, etc., the production of finished goods follows a continuous production pattern, because of high maintenance cost and system restarting cost. Therefore, the system shutdown takes place only during the scheduled yearly maintenance. Considering these situations, it is important to manage the inventories of raw materials and finished goods as well as the supply chain network of those facilities, so that the system can run without interruption and at minimum cost.

1.1.5 Single Facility Lot-Sizing Models

The inventory control of physical objects has been the topic of interest for some time. Several extensions of basic *Economic Ordering Quantity* (*EOQ*) have been investigated considering single stage, single item, multi-stage, multi-item, finite production rates, quantity discounts, and backordering. It is important to give some attention to the rotation cycle (in each cycle there is exactly one setup for each product, and products are produced in same sequence in each production cycle) of production of multiple items in a single facility to ensure customers' demand, because most of the industries produced more than one item. Therefore, research should be conducted to optimize the rotation cycle of production of different products.

1.2 THE PROBLEM, RESEARCH GOALS AND OBJECTIVES

The previous sections indicate the problems, applications and other related issues involved in supply chain system in some selected manufacturing environments. This section discusses the precise problem, the motivation, and the objectives of this research.

1.2.1 The Problem

This research focuses on a manufacturing system, which receives raw materials from suppliers, converts them into finished products, and sells/delivers them to the customers or retailers based on their demand. The research also focuses on reducing the downtime or idle

time of the production facilities. To reduce the idle time of production, it is assumed that the production of succeeding cycle starts *immediately* after the production of preceding cycle. Moreover, the facility operates under just-in-time (JIT) mechanism for both supplier and manufacturer. The problem can be addressed in two ways: (a) reducing the downtime with perfect matching, and (b) reducing the downtime with imperfect matching, which are described as follows:

(a) Perfect matching

The *perfect matching* is the situation when there is no left-over finished goods at the end of the last delivery or shipment in a production cycle. In this research, it is considered that, to reduce idle time, the production of the subsequent cycle starts immediately after the end of production run of the previous cycle.

(b) Imperfect matching

The *imperfect matching* occurs when there are some finished goods remaining at the end of the previous delivery or shipment and this remaining inventory of finished goods are insufficient for the next delivery. It is considered that each cycle carries some finished goods which remain after all required shipments to customers are made. This remaining amount is carried over to the next production run and the production of this following cycle will start after the time required to consume this carried-over amount.

1.2.2 Research Goals

The goal of this research is to minimize the total system operation cost of a supply chain system that consists of suppliers, manufacturers and retailers. Moreover, these will result a systematic relationship which will lead to minimize the inventory costs. The effects of the total cost minimization can be expected to have large impact on batch sizes of raw material ordering, manufacturing, scheduling of shipments, and utilization of transportation.

1.2.3 Research Objectives

According to the supply chain principle, it can be stated that a production facility procures raw material from outside suppliers, processes them as finished goods and delivers with different methods of shipment quantities and time intervals. Therefore, a number of issues involved in supply chain system may be studied in different points of view. Considering those reasons, the specific objectives of this research can be itemized as follows:

(a) Determining the optimal policy for ordering raw materials

Raw materials are required at the beginning of a production cycle. If the necessary raw materials are ordered once in a cycle, it may incur a higher inventory carrying cost during the earlier part of the cycle. Therefore, a multi-ordering policy for raw material procurement may lower the carrying cost as well as encourage the appropriate use of raw materials. Hence, *finding the optimal number of orders, time intervals of orders and ordering quantity* are the objectives regarding raw material ordering policy.

(b) Economic production batch size

Generally, production rate is higher than the demand rate to satisfy customers' demand. Conversely, the inventory is projected to buildup during the continuation of production. The behavior of a just-in-time (JIT) oriented manufacturing system is different from typical economic batch quantity model. During production, a fixed quantity of finished goods is shipped to the customer after a fixed time interval, which results in a reduction in on-hand inventory which forces the inventory buildup in the traditional saw-tooth pattern. Also, during the downtime the inventory forms a staircase pattern due to the end of production. Moreover, for minimum idle time, the inventory must never reach the zero level, even if production stops. Hence, the primary objectives for finished products are *to determine the production batch quantity, production cycle length, time of production start and optimum*

number of shipments of finished goods in one cycle for both perfect and imperfect matching condition (discussed in the problem section) so that warehouse space can economically planned.

(c) Rotation cycle for multiple items with just-in-time (JIT) supply chain

Generally, a production facility produces multiple items according to the customers' demand. The production of a product starts immediately after the completion of the production of the preceding product, and so forth. Therefore, it is important to study the rotation cycle of the first product for multiple item production in a just-in-time (JIT) based supply chain facility.

(d) Optimal rotation cycle for just-in-time (JIT) supply chain system

Implementation of rotation cycle in a supply chain system needs to be optimized, so that the coordination between suppliers, producers and buyers operate in harmony. Hence, the objective of this research is *to find the optimal rotation cycle* applied to a minimal downtime based supply chain system.

(e) Multi-supplier and multi-buyer supply chain system

The supply chain of modern manufacturing facilities is coupled with multi-supplier and multi-buyer. Most of the time manufacturing orders its raw materials from different suppliers, and after production of finished products, they are delivered to multiple buyers. Therefore, the objective is to analyze the multi-supplier-and-buyer supply chain system, so that *the optimal ordering and shipment policy* can be determined for a large scale supply chain system.

(f) Multi-product, multi-supplier and multi-buyer supply chain system

Most of the manufacturing systems produce multiple products. Also, they deliver the products to multiple customers or retailers. To produce multiple products, the

manufacturers require various raw materials from different suppliers. Hence, the final objective of this research is to model the multi-product-supplier-and-buyer supply chain model with respect to the rotation cycle policy to confirm multiple items delivery over the rotation cycle time period.

1.2.3 Overview

The remainder of the chapters is presented in the following way. Chapter 2 presents the literature surveys on the supply chain systems and single facility lot-sizing system. The model formulation of supply chain inventory with perfect matching conditions and the solution procedure are described in Chapter 3. Also, Chapter 3 discusses the rotation cycle of multiple items production with perfect matching inventory condition with solution. Chapter 4 represents the imperfect matching with single-and multi-item production including the solutions for both problems. Chapter 5 deals with the multi-supplier and multi-buyer supply chain systems. Chapter 6 represents the multi-product, multi-supplier and multi-buyer supply chain system. Chapter 7 represents the sensitivity analysis of the perfect and imperfect matching system. Finally, Chapter 8 includes the concluding remarks of this research.

CHAPTER 2

LITERATURE REVIEW

The previous research in supply chain system deals with traditional inventory systems, warehousing, hierarchical productions, and logistics distribution with single-and multi-stage production systems. Earlier, researchers developed the optimum order and production quantity models for single and multi stage production system (Wang *et al*., 2004; Anderson and Marklund, 2000; Axsäter and Zhang, 1999; and Axsäter, 1997). Many researchers dealt with either centralized or decentralized systems. The concept of a *centralized* system is one which is owned by a single entity. This centralized concept allows for global optimization. Conversely, the *decentralized* system does not allow global optimization due to multi-ownership. However, the decentralized system can benefit with the help of a centralized system. This research deals with decentralized supply chain manufacturing systems composed of production with a single supplier and a single buyer operating under just-in-time (JIT) delivery. Several production policies, controlling the multi-stage supply chain system, are reviewed here.

2.1 MODELS WITH PERIODIC REVIEW

In *periodic review* models, the status of the stock of products in a facility is reviewed at a regular interval—the system defined by these models is referred to as the fixed replenishment interval system. This system allows a reasonable prediction of the level of labor involved. Conversely, a decision in the *continuous review* model can be made at any point in time. Therefore, continuous review models involve unpredictable level of labor. This characteristic shows that periodic review is less expensive than the continuous review in terms of cost and error of reviewing. Over the last three decades, there has been great progress in developing multi-stage supply chain theories with periodic review. Axsäter (1997) proposed a problem with replenishment policies for a one warehouse multi-retailer system. The model used a

recursive solution approach to determine the order up to inventory at the warehouse. This model also suggested that each retailer with stochastic demand can minimize the long-run system cost. Ouyang and Chuang (2000) studied a mixture of periodic review inventory models in which both the lead time and the review period were considered as decision variables. Instead of having a stock-out term in the objective function, a service level constraint was added to the model. They developed an algorithm to decide the optimal review period and lead time. McGavin *et al*. (1993) dealt with the case of one warehouse and *N*-identical retailer system. They studied a two interval policy of removal from the warehouse to the customers where the first removal occurs immediately after the replenishment from outside supplier and the second removal ships the remaining stock. The problem Parlar *et al*. (1995) studied contains occasional unavailability of materials and products which has an impact on inventory decisions of utility companies, manufacturers, wholesalers and retailers. They developed a model to address a periodic review, with setup costs, where the probability that an order placed now is filled in full, as opposed to whether supply was available in the previous period. Chen *et al*. (2001) combined pricing and replenishment strategies for profit maximization by means of optimizing the prices given by each retailer.

2.2 JOINT REPLENISHMENT POLICY

Researchers addressed the joint replenishment policy (JRP) to reduce the costs in two-stage supply chain systems owned by two parties. Goyal and Satir (1989) developed a joint vendor-buyer replenishment policy based on economic order quantity model and the related heuristics for deterministic and stochastic systems. Fung and Ma (2001) considered joint replenishment problems (JRP) of *n* items under deterministic and constant demand. They developed two algorithms for JRP based on tighter bounds for the optimal cycle time. Frenk *et al*. (1999) developed deterministic multi-item inventory problems, with general cost rate

functions and possibly service level constraints, of which the joint replenishment problem is a special case. Hill (1999) presented a model to minimize the mean total cost per unit time of manufacturing setup, stock transfer and holding for a system where a manufacturer supplies a product to a buyer. The vendor manufactures the product in batches at a finite rate and ships them to a buyer.

2.3 Just-in-Time (JIT) Models

Just-in-time (JIT) production systems have zero inventory systems and no buffer. The processing time variation and machine breakdowns cause disturbance in the production line. Therefore, the system efficiency drops. Intermediate buffers increase the efficiency of line. Sarker and Fitzsimmons (1989) studied the effects of above variability on the performance of push and pull systems. Golhar and Sarker (1992) addressed the perception that participation in just-in-time (JIT) delivery system is economically disadvantageous for suppliers. In just-in-time (JIT) system, the supplier has to coordinate his production with the buyer's demand so as to maintain zero inventory, but, in reality the supplier ends up with carrying large inventories to deliver limited shipments. An iterative solution is proposed. Generalized total cost inventory model is a piecewise convex function. Jamal and Sarker (1993) extended this problem and estimated the batch size from the lower bound concept of the just-in-time (JIT) delivery amount. They developed an efficient algorithm to calculate the optimal or near optimal batch sizes for both manufacturing and raw material ordering policies. Sarker and Parija (1994) developed a general cost model based on Golhar and Sarker's (1992) problem which considered both supplier and buyer to determine optimal ordering policy for the raw material and manufacturing batch size to minimize the total cost. They also considered that, at the end of the delivery, a few finished goods are left over which is less than the shipment amount. They solved the problem in semi-closed form and found that total cost function is piecewise convex.

Hill (1995) submitted a viewpoint on Sarker and Parija's (1994) work considering integral number of shipments. He modified the expression for average finished goods stock. Sarker and Parija (1996) further developed their research and proposed optimal multiple ordering policies from a single supplier for procurement of raw materials for single product manufacturing batch to minimize the total cost of production. Hill (1996) modified the ordering policy of the raw material by allowing a single order for multiple production cycles when the inventory cost for the raw material is much lower as compared to the ordering costs in each production cycle. In the same year, Nori and Sarker (1996) developed a model for evaluating the optimal batch size for a single-product manufacturing system operating under a fixed-quantity, periodic delivery policy. In this research, the authors added a multi-product situation and a single-facility scheduling scheme for the system considering two situations: (1) fixed setup cost, and, (2) variable setup cost. A few years later, based on previous research, Parija and Sarker (1999) provided an optimal ordering policy for procurement of raw material and optimal manufacturing batch size for fixed interval deliveries to multiple customers. The model gives a closed form solution for minimal total cost and also considers the use of carried over inventory to next cycle for determining the optimal starting time for each batch production cycle.

Sarker and Khan (1999) proposed an ordering policy for raw materials to meet the requirements of a production facility that must deliver finished goods according to customers' demand at a fixed point of time. They considered: (a) a finite production rate environment using raw materials from outside supplier, (b) only the product lot sizing and their associated raw material supply quantities, (c) a supply policy where the product was delivered in equal quantities at a fixed interval, and, (d) the products were supplied after processing the entire lot and quality certification of the products. They evaluated relationships between production

batch size, raw material quantity and delivery patterns. Few years later, Khan and Sarker (2002) developed another model for a manufacturing system which procures raw material from the suppliers in a lot and processes them as finished products. They estimated production batch sizes for a just-in-time (JIT) delivery system and incorporated a just-in-time (JIT) raw material supply system. Also, they assumed that the production system must deliver finished products demanded by outside buyers at fixed interval points in time. In view of all previous research, Diponegoro (2003) studied an operational policy for a lean supply chain system consisting of a manufacturer, multiple suppliers and multiple buyers. He dealt with three interrelated problems in supply chain. They are (a) single supplier and single buyer with fixed delivery size, (b) multiple suppliers and multiple buyers with individual delivery schedule and (c) time dependent delivery quantity with trend demand. He formulated these problems as mixed-integer, nonlinear programming problems with discrete, non-convex objective functions and constraints. Diponegoro (2003) also developed a closed-form heuristic which provided near optimal solutions and tight lower bounds.

2.4 INVENTORY MODELS WITH SINGLE FACILITY LOT-SIZING

Johnson and Montgomery (1974) explained a model for a multi-product, single-stage production inventory system in continuous time where they proposed a new policy in which all products require a stationary interval of time between successive productions of different products in a single facility. Nahmias (1997) also illustrated the same problem of rotation cycle policy for general case. The researchers did not give much attention to this rotation cycle policy for different production and delivery situation.

2.5 SHORTCOMINGS IN PREVIOUS RESEARCH

The supply chain system has received much attention by researchers and practitioners. In this current research, a few aspects considered are decentralized planning, just-in-time (JIT)

delivery and ordering system, infinite planning horizon and production inventory minimizing the down time.

During the model development, most of the researchers (Diponegoro, 2003; Sarker and Parija, 1994, 1999; Khan and Sarker, 2002) considered that the system remains idle until the shipments are made. Figure 2.1 shows the production with just-in-time (JIT) delivery.

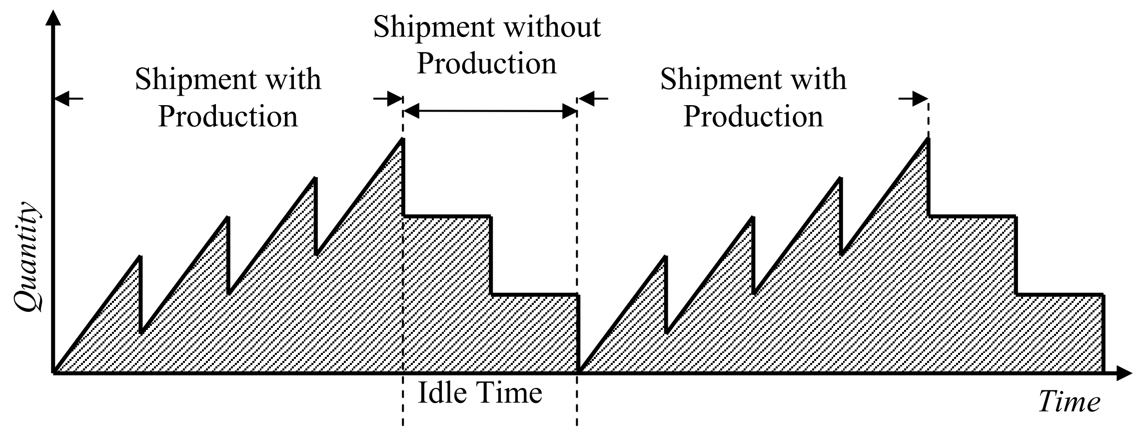


Figure 2.1. Inventory models with just-in-time (JIT) delivery with idle time

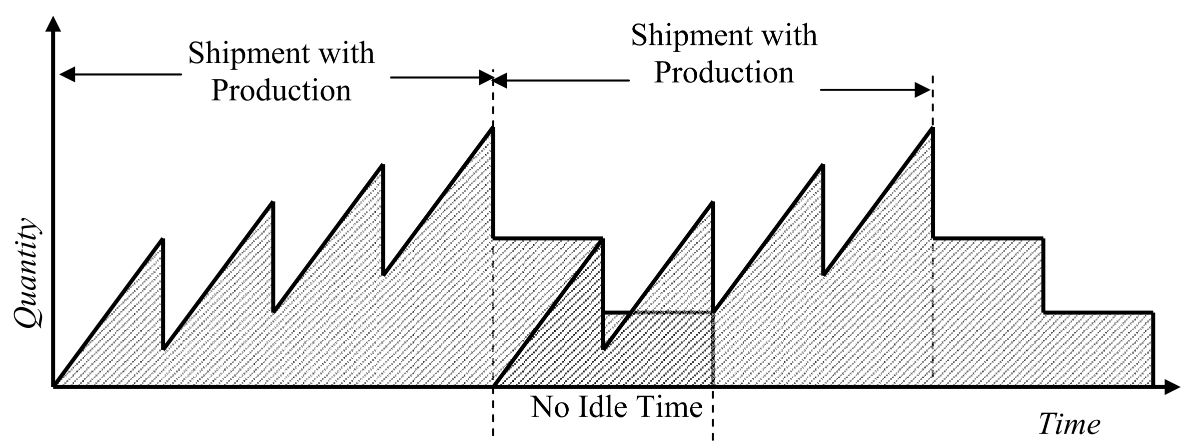


Figure 2.2. Inventory models with just-in-time (JIT) delivery and without idle time

In reality, large production industries (refineries, paper mills, sugar mills, etc.) do not let their production system be idle, because it incurs a high cost to restart their equipment. The production of these industries can be categorized as continuous production where the

subsequent production cycle (uptime) starts just after completion of prior production uptime. During production, the manufacturer also delivers the finished goods following the just-in-time (JIT) methods to lower finished goods handling cost. Therefore, these industries Keep their facility idle only during yearly maintenance or break down. Figure 2.2 represents the inventory built up without idle time for a continuous production facility that follows just-in-time (JIT) mechanism. Previous researchers ignored this type of models due to complexity of the problem.

In the real world, the industry can not produce exactly what the buyers' require. Some of the finished product may remain after fulfilling the customers' demand. A small illustration of this problem can be found in big retail stores (Wal-Mart, Albertson's, and Winn-Dixie) where the items on the shelf are their inventories. Most of the time, these stores order the items which are still available on the shelf. Usually they place the newly ordered items behind the previous items. This is an important issue which researchers (Hill, 1995; Sarker and Parija, 1996; Khan and Sarker, 1999; Diponegoro, 2003) ignored while forming their model.

The application of rotation cycle policy was involved when multiple items are produced in a single facility. The researchers (Johnson and Montgomery, 1974, Nahmias, 1997) developed the models for general inventory models. In Table 2.1, a comparison between previous research and proposed research is represented as follows:

2.6 Problem Development for Current Research

The problem for this research is established by researching the shortcomings of the previous researchers. First, the researchers did not consider the minimization of idle time of the production systems, but Sarker and Khan (1999) proposed a model that happened to be minimizing inactive time of the production facility. In their model they did not consider the delivery during the production or up time. Therefore, this case is considered in the present research.

Deponegoro (2003) studied a just-in-time (JIT) oriented production system in finite planning horizon keeping inactive production during downtime. Sarker and Parija (1994) developed a model for just-in-time (JIT) based production system with some leftover inventory after the end of required shipments. They also did not incorporate the concept of minimizing idle time nor was the rotation cycle policy considered. In view of all these omissions, this research considered a single facility production system operating under just-in-time (JIT) environment with minimal idle time. The mathematical models are developed based on these omissions. Moreover, the solution procedures are discussed with numerical examples and a rotation cycle policy is developed for just-in-time (JIT) operated production system with different inventory situations. Figure 2.3 shows the flow diagram of the problem development here.

Table 2.1 Problem characteristics of few selected research and proposed model

Authors	Characteristics							
	Production Rate	Demand Rate	Planning Horizon	Model Configuration	Idle during Downtime	Shortage Considered	Inventory Pattern	Solution Methods
Banerjee and Burton (1990)	Finite	Constant	Unconstrained	Single-and Multi-stage	Yes	No	Perfect	Optimal
Banerjee (1992)	Finite	Periodic	Unconstrained	Single stage	Yes	No	Perfect	Optimal
Hill (1996)	Finite	Constant	Unconstrained	1-supplier 1-buyer	Yes	No	Perfect	Heuristic
Sarker and Parija (1994)	Finite	Constant	Unconstrained	1-supplier Multi-buyer	Yes	No	Imperfect	Heuristic
Sarker and Parija (1996)	Finite	Constant	Unconstrained	1-supplier Multi-buyer	Yes	No	Perfect	Optimal
Parija and Sarker (1999)	Finite	Constant	Unconstrained	1-supplier Multi-buyer	Yes	No	Perfect	Heuristic
Sarker and Khan (1999)	Finite	Constant	Unconstrained	1-supplier 1-buyer	Yes	No	Perfect	Heuristic
Khan and Sarker (2002)	Finite	Constant	Unconstrained	1-supplier 1-buyer	Yes	No	Perfect	Heuristic
Diponegoro (2003)	Finite	Time Varying	Constrained	Multi-supplier Multi-buyer	Yes	Yes	Perfect	Optimal
Proposed Models	Finite	Constant	Constrained	Single-supplier Single-buyer Single-product Multi-supplier Multi-buyer Multi-product	Minimal	No	Perfect and Imperfect	Heuristic

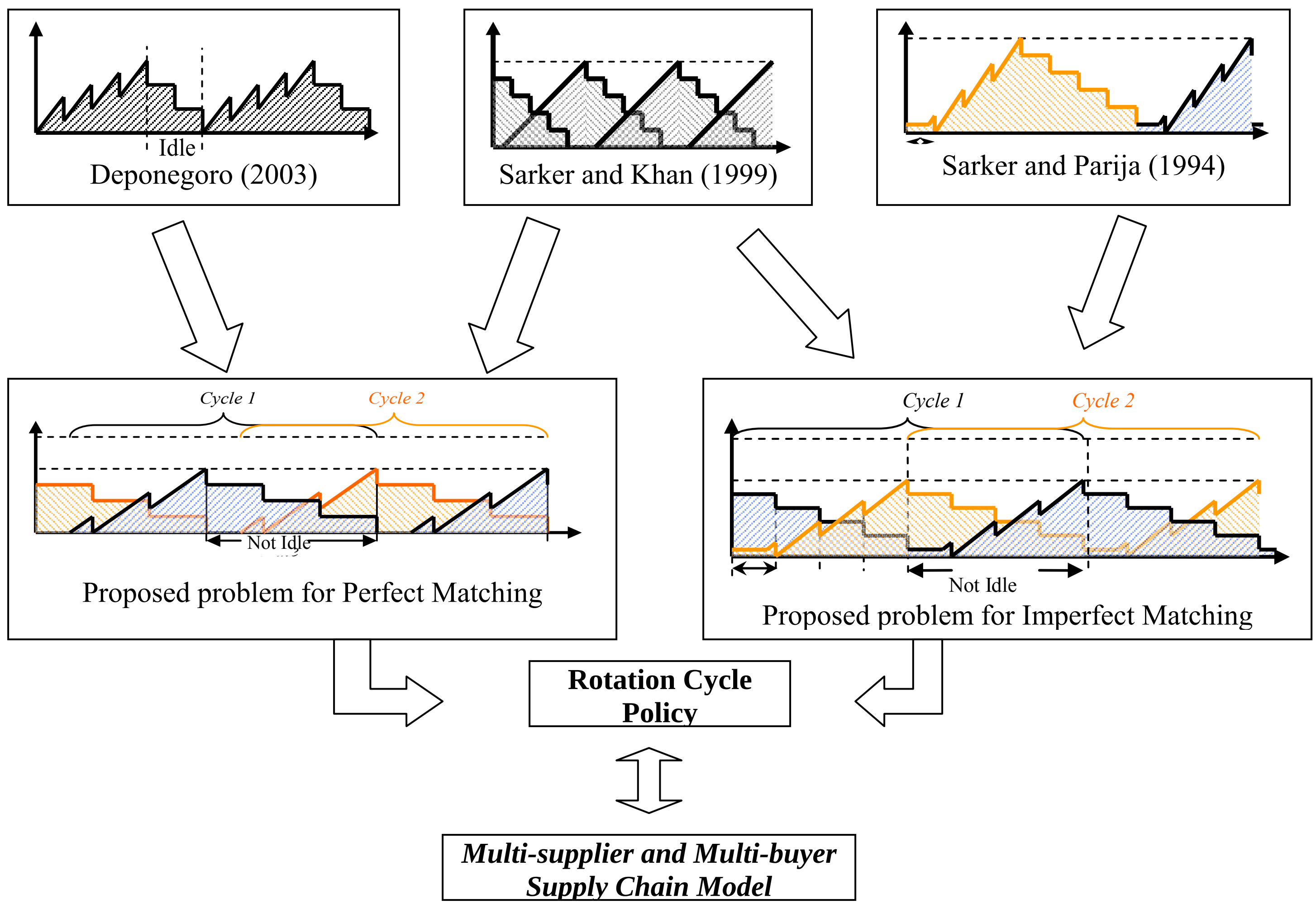


Figure 2.3. Problem development flow chart

CHAPTER 3

PERFECT MATCHING SYSTEM WITH JUST-IN-TIME DELIVERY

A typical supply chain system contains raw materials supplier, manufacturer of finished products and customers. The raw materials are procured from the suppliers and stored in inventory storage area at production centers. The finished goods are manufactured in the production centers and stored in warehouses, which, in turn, are shipped to the buyers or retailers. To improve productivity and reduce manufacturing costs, the just-in-time (JIT) technology has often been adopted by many production systems. In supply chain system with just-in-time (JIT) mechanism, the finished product output rate is controlled by the demand of the customers. Figure 3.1 represents the supply chain system with just-in-time (JIT) mechanism.

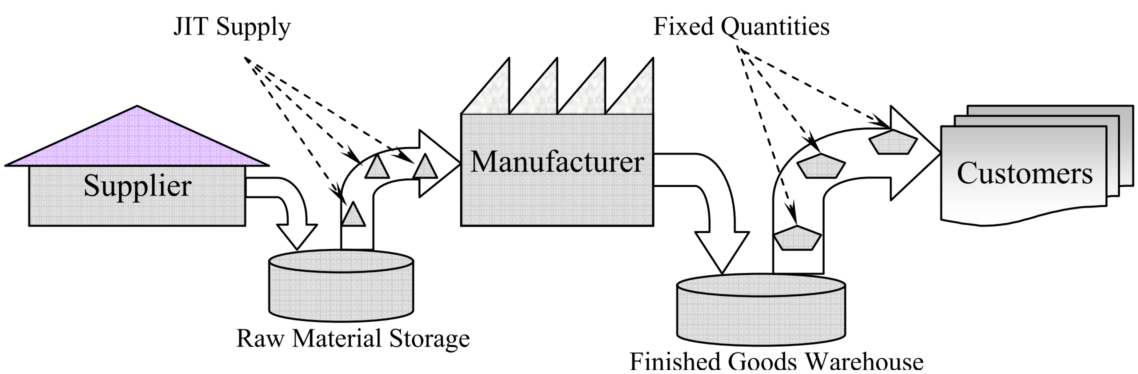


Figure 3.1. Supply chain system with just-in-time (JIT) mechanism

3.1 PROBLEM DESCRIPTION AND NOTATION

This part of the research deals with the supply chain system considering just-in-time (JIT) technique and considers a manufacturing system which has minimum idle time between successive production cycles. In a supply chain system, when the production quantity exactly matches the demand of a cycle time, it is called *perfect matching*. Therefore, the perfect

matching is the situation when there are no finished goods remaining after the shipments are completed to the customers at the end of a production cycle. To find an economic order quantity (*EOQ*) for the raw materials and an economic manufacturing quantity (*EMQ*) for the production facility with *prefect matching*, the following costs are considered: raw material ordering cost, raw material inventory cost, manufacturing setup cost and finished goods inventory carrying cost. In this section, an expression for the generalized cost function is developed that may be used to determine an optimal batch quantity for the production run with reduced idle time.

To develop the model for determining the interactions between raw materials and finished goods demand, following definitions and notation are used:

D_F : Demand for finished goods, units/year.

D_R : Demand for raw materials, units/year.

D_{Fk} : Demand for raw materials to produce k product, units/year.

D_{Rk} : Demand for k finished goods, units/year.

f : Conversion factor of the raw materials; $f = D_F / D_R = Q_F / Q_R$.

H_F : Holding cost of finished goods, \$/units/year.

H_R : Holding cost of raw materials, \$/units/year.

I_{PF} : Total finished goods inventory, units.

$\overline{I}_{PT}$: Average finished goods inventory, units.

I_{PR} : Total raw materials inventory, units.

$\overline{I}_{PR}$: Average raw materials inventory, units.

I_{PS} : Total finished goods inventory with idle time, units.

$\overline{I}_{PS}$: Average finished goods inventory with idle time, units/year.

K : Number of items to be produced.

k : Index for different finished products, $k = 1, \ldots, K$.

K_0 : Ordering cost of raw material, \$/order.

K_S : Manufacturing setup cost, \$/batch.

L : Time between successive shipments of finished goods, years, $L = x/D_F$.

m : Number of orders for raw materials; $n \geq m \geq 1$.

n : Number of *full* shipment of finished goods per cycle time.

P : Production rate, units/year.

Q_F : Quantity of finished goods manufactures per setup, units/batch.

Q_R : Quantity of raw materials required for each batch; $Q_R = Q_F / f$.

T_P : Production time (uptime), years; $T_1 = Q_F / P = nx/P$.

T_D : Pure consumption time, years (downtime);

T : Total cycle time, years; $T = nL$.

T_r : Rotation cycle length for multi-product production.

T_S : Setup time, years; $T_S < L$.

TC_{PF} : Total cost of finished goods, \$.

TC_{PR} : Total cost of raw materials, \$.

$TC_P(m, n)$: Total cost function, \$/year.

$TC_{PS}(m, n)$: Total cost function with idle time, \$/year.

$\overline{I}_{IS}$: Average finished goods inventory for with idle time, units/year.

x : Fixed quantity of finished goods per shipment at a fixed interval of time, units/shipment; $x = Q_F/n = LD_F$.

C_P : Objective function of rotation cycle.

$C_{PF}(Tr)$: Objective function for rotation cycle of finished product.

$C_{PR}(m_k)$: Objective function for rotation cycle of raw materials.

3.2 ASSUMPTIONS

To develop the mathematical model and to simplify the solution methodology, some assumptions considered are as follows:

1. Production rate is constant and finite.
2. Production rate is greater than the demand rate, $P > D_F$.
3. Production facility considers as just-in-time (JIT) delivery and supply of finished products and raw materials, respectively.
4. Production facility operates under the condition, where succeeding production cycles start immediately after the production period of preceding cycles.
5. There is only one manufacturer and one raw material supplier.
6. Only one type of product is produced in each cycle.
7. Finished goods delivery is in a fixed quantity at a regular interval.

3.3 AVERAGE INVENTORY AND TOTAL COST FUNCTION

In this part of the research, the production rate, P is assumed to be greater than the demand rate, D_F, so that there should be an inventory build-up during production. Figure 3.2 shows the inventory build-up due to processing of finished goods from raw material, where lower part of the Figure 3.2 represents the inventory of the raw material supply and the upper part represents the on-hand finished goods inventory. It is assumed that, the production of cycle 1 starts (at a finite rate of P) T_S time units after the end of the uptime of the previous cycle (i.e., at time A) and the first delivery of $x/2$ units for cycle 1 is made at L time units after the previous delivery. Since, the cycles overlap (uptime of succeeding cycle with downtime of preceding cycle), at the same time (every L time units) $x/2$ units are delivered both from the downtime of previous cycle and uptime of following cycle. Therefore, at fixed time period L, total x units are

being delivered which satisfies customers' demand. As the produced item during L-T_S time units is exactly $x/2$ amount for cycle 1, so there is no inventory after the delivery made at the end of L time units. After L time units, production starts again and for every L time units, shipments of x/2 units from each cycle are made.

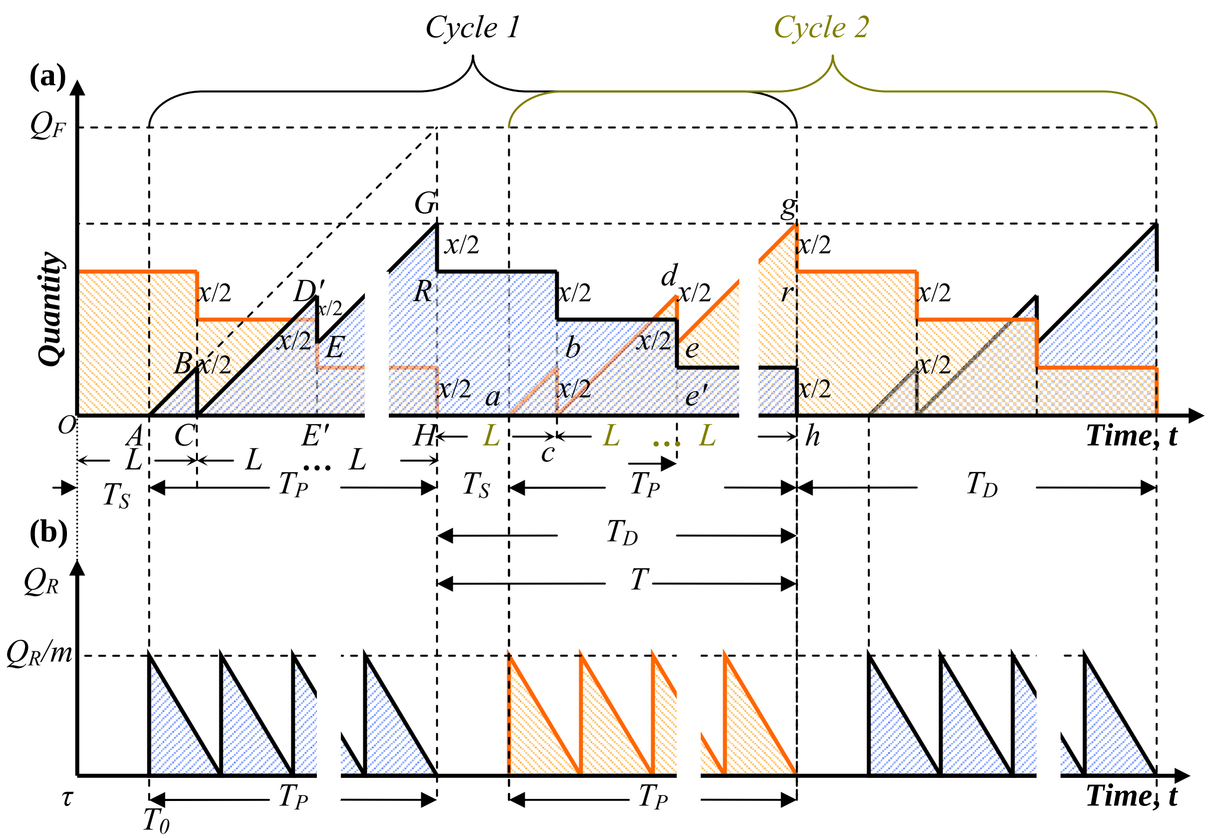


Figure 3.2. (a) Inventory of finished goods; (b) Inventory of raw materials

During L time period, Y amount of finished goods are produced and after shipping $x/2$ units, the remaining items are $Y - x/2 \geq 0$. Thus, the finished goods inventory build-up forms a saw-tooth pattern during the production uptime T_P. Clearly, $Q_F = PT_P$ units are produced in a cycle. After the end of the production, shipments of $x/2$ units at every L time units are made to the customers during the downtime, T_D, which is followed by *the new cycle*. During the down time of a cycle, finished goods are not produced, so that the on-hand inventory depletes at regular intervals (every L time units) from the end point of the production period to the end of

cycle. Thus, the later part of the inventory cycle (T_D period) forms a staircase pattern (under curve *Hg*). The finished products are delivered in *n* shipments (where $n \geq 1$) of *x* equal quantities at each *T* time period. Since, the uptime of all cycles and the downtime of their previous cycles coincide, the total delivery, in *L* time period, becomes *x* amount.

As shown in Figure 3.2, the production of cycle 2 begins at point *a*, which is T_S time unit after the end of *production period* (uptime) of cycle 1. Hence, the machines or the production cycle will not remain idle till the end of the shipments of the finished products after previous cycle. That is why, the overlapping part $\text{Area } GgH$ of $\text{Area } abcdegh$ is combined with the inventory during *T* time period denoted by $\text{Area}\, GghH$. The delivery follows the just-in-time (JIT) system and so does the raw material supply. Figure 3.2(a) shows that the on-hand inventory does not grow after production stops at the end of T_P in cycle 1. The quantity produced in T_P time units should meet the customer's demand for period *T* such that $Q_F = nx$, where *n* is the number of full shipments to customers per cycle and is assumed to be an integer for an infinite planning horizon. The raw materials for production are ordered during the time T_P time period.

If I_{PT}, I_{PP}, and I_{PD} are denoted as total inventory for perfect matching case, inventory produced at time T_P and inventory shipped at time T_D, respectively, then the total inventory during *T* time period can be written as

$$I_{PT} = I_{PP} - I_{PD}. \tag{3.1}$$

Also, from Figure 3.2, it is found that the total cycle time *T* is

$$T = T_P + T_S = T_D = nx / D_F, \tag{3.2}$$

where $L = x / D_F$.

Now, from Figure 3.2 (a), I_{PP} and I_{PD} can be calculated as [see Appendix B.1]

$$I_{PP} = \frac{nx}{2}T_P + \frac{nx}{2}T \text{ , and,} \tag{3.3}$$

$$I_{PD} = \frac{n(n-1)x^2}{2D_F}. \tag{3.4}$$

Therefore, using Equations (3.1), (3.3), and (3.4), the total inventory is found to be

$$I_{PT} = \frac{n^2x^2}{2D_F} + nx\left(\frac{x}{2D_F} - \frac{T_S}{2}\right) \tag{3.5}$$

(see Appendix B.1 for detailed calculation).

Therefore, the average inventory of the entire cycle can be found as

$$\bar{I}_{PT} = \frac{1}{T}\left[\frac{n^2x^2}{2D_F} + nx\left(\frac{x}{2D_F} - \frac{T_S}{2}\right)\right] = \frac{D_F}{nx}\left[\frac{n^2x^2}{2D_F} + nx\left(\frac{x}{2D_F} - \frac{T_S}{2}\right)\right]$$

$$= \frac{nx}{2} + D_F\left(\frac{x}{2D_F} - \frac{T_S}{2}\right). \tag{3.6}$$

Hence, the total cost function for the finished goods inventory can be written as

$$TC_{PF} = \frac{D_F}{Q_F}K_S + \bar{I}_{PF}H_F = \frac{D_F}{nx}K_S + \frac{H_F}{2}\left[nx + D_F\left(\frac{x}{D_F} - T_S\right)\right]. \tag{3.7}$$

The pattern of raw material inventory is shown in Figure 3.2(b) where Q_R is the raw materials required and these Q_R units are ordered in m instantaneous replenishments of Q_R/m units. It is assumed that each unit of finished good produced requires f units of raw material, so that $Q_F = fQ_R$. Again, in this research the raw materials are ordered and converted to finished goods during the production time or uptime, T_P. Thus, the time weighted inventory of raw material held in a cycle is given by

$$\bar{I}_{PR} = \left(\frac{Q_RT_P}{2m}\right) = \left(\frac{nx}{2mf}\right)\left(\frac{nx}{P}\right) = \frac{n^2x^2}{2mfP}. \tag{3.8}$$

where $Q_R = Q_F / f = nx / f, T = nx / D_F$ and $T_P = nx / P$.

Hence, the total cost for the raw material can be expressed as

$$TC_{PR} = \frac{D_R}{Q_R / m} K_0 + \bar{I}_{PR} H_R = \frac{mD_F}{nx} K_0 + \frac{n^2 x^2 H_R}{2mfP}. \tag{3.9}$$

where $\frac{Q_F}{Q_R} = \frac{D_F}{D_R} = f$.

Therefore, the total cost function for this problem can be written as

$$TC_P(m,n) = TC_{PR} + TC_{PF} = \frac{n^2 x^2 H_R}{2mfP} + \frac{mD_F K_0}{nx}$$

$$+ \frac{D_F}{nx} K_S + \frac{H_F}{2}\left[nx + D_F \left(\frac{x}{D_F} - T_S \right) \right]. \tag{3.10}$$

Upon simplification which yields

$$TC_P(m,n) = \frac{n^2}{m}\left(\frac{x^2 H_R}{2fP} \right) + \frac{m}{n}\left(\frac{D_F K_0}{x} \right) + \frac{1}{n}\left(\frac{D_F K_S}{x} \right)$$

$$+ \frac{nxH_F}{2} + \frac{D_F H_F}{2}\left(\frac{x}{D_F} - T_S \right). \tag{3.11}$$

Substituting the constant terms with different notation and simplifying, Equation (3.11) can be written as

$$TC_P(m,n) = A_1\left(n^2/m\right) + A_2\,(m/n) + A_3/n + A_4 n + A_5, \tag{3.12}$$

where $A_1 = \frac{x^2 H_R}{2fP}$,

$A_2 = \frac{D_F K_0}{x}$,

$A_3 = \frac{D_F K_S}{x}$,

$A_4 = \frac{xH_F}{2}$, and ,

$$A_5 = D_F H_F \left(\frac{x}{2D_F} - \frac{T_S}{2} \right).$$

3.4 PROBLEM FORMULATION

The total cost function for this part of research is a non-linear integer programming problem with has integer variables m and n. If the problem is defined as PM (perfect matching), then it can be expressed as follows:

***Problem PM*:** Find m, and n so as to

Minimize: $$TC_{PM} = A_1 \left(n^2/m \right) + A_2\,(m/n) + A_3/n + A_4 n + A_5, \tag{3.13}$$

Subject to: $$n \geq m \geq 1 \tag{3.13a}$$

m and n are integer. (3.13b)

3.5 SOLUTION METHODOLOGY

The total cost function developed for the perfect matching problem is a nonlinear integer programming (NLIP) problem, which is cumbersome to optimize (see Appendix A). Generally, the branch-and-bound (*B&B*) method is effective to find the optimum solution with integer variable. In this case the *B&B* method is not that effective, because to find the starting basic solution, variables have roots of a 4th degree polynomial equation and are inter-depended. Therefore, using the *Divide and Conquer* rule [Roundy (1989), Fürnkranz (1999)], the objective function is divided in two different parts; (1) supplier's side and (2) buyer's side.

Buyer's side deals with the finished product quantity, from where the manufacturer decides the optimum production quantity. Based on this decision, the manufacturer will order the required raw materials. Thus, the raw materials orders are dependent on the optimum finished product production. Let Equation (3.7) represents the buyer's side ($T_{BS} = T_{PF}$), and supplier's side ($T_{SS} = T_{PR}$) can be represented with Equation (3.9). According to the Equations

(3.7) and (3.9), it can be observed that T_{BS} is a function of m and T_{SS} is a function of both m and n, respectively. Here, n is dependent on m, and they both are integer variables. If m can be solved optimizing Equation (3.7), then this will be a static value for Equation (3.9) as the manufacturer has to decide the optimal order quantity of raw materials based on his production requirements of finished products, which will also satisfy the buyers' demand. To optimize both the buyer's side (T_{BS}), and supplier's side, (T_{SS}), a proposition is developed which is stated as follows:

Proposition 3.1: Optimum number of shipments (n^*) of finished products, optimizes the number of orders (m^*) of raw materials.

***Proof*:** $nx = Q_F = fQ_R$

If the optimum number of shipments is, n^*, then

$$n^* x = Q_F^* = fQ_R^* . \tag{3.14}$$

As the raw materials are ordered in m small lots from the suppliers, Q_R is ordered in lots sizes Q_R/m per order and the evaluated n^* provides the single decision variable for supplier side, which is convex for m. Also, the optimum Q_R^* can be obtained from Equation (3.13) and the supplier side can be readily solved. Therefore, n^* will provide the optimum number of orders m^* required for raw material supply.□

According to the Proposition 3.1, the first objective is to minimize the $T_{PF} = T_{BS}$, to find the value of m. Therefore, the objective function can be written as

Minimize:

$$T_{BS}(n) = \frac{D_F}{nx} K_S + \frac{H_F}{2}\left[nx + D_F \left(\frac{x}{D_F} - T_S \right) \right], \tag{3.15}$$

Subject to: $n \geq 1$ and is an integer. (3.15a)

It can readily be shown that T_{BS} is a convex function in n, but T_{BS} is a discrete optimization problem. Therefore, the objective function presented in Equation (3.15) can not be solved using the conventional techniques such as differentiation. A minimum value for T_{BS} can be obtained by substituting the value of n = 1, 2, …, and so on, until three successive values are obtained which presents a unimodal pattern for T_{BS} with a minimum point.

However, the search procedure is very tedious as the boundary of m is not well-defined. To avoid this situation, the incremental methods [Taha, (1992), Nori and Sarker (1998)] are used to establish a boundary for n, so that the interval over n^* may be searched and the optimal number of shipments can be obtained.

Let n^* be the value of n which minimizes the total cost T_{BS} where $n^* \geq 1$. Hence, $T_{BS}(n^*)$ is the minimum possible value for $T_{BS}(n)$ and in the neighborhood of n^*, the values like n^*-1 and n^*+1 obtain the values for the objective function such that

$$T_{BS}(n^* - 1) - T_{BS}(n^*) \geq 0 \text{, and} \tag{3.16}$$

$$T_{BS}(n^* + 1) - T_{BS}(n^*) \geq 0 . \tag{3.17}$$

Replacing the value of n with n^*-1, n^*, and n^*+1 in Equation (3.15), it can be found as

$$TC_P(n^* - 1) = \frac{D_F}{(n^* - 1)x} K_S + \frac{H_F}{2}\left[(n^* - 1)x + D_F\left(\frac{x}{D_F} - T_S\right)\right], \tag{3.18}$$

$$TC_P(n^*) = \frac{D_F}{n^* x} K_S + \frac{H_F}{2}\left[n^* x + D_F\left(\frac{x}{D_F} - T_S\right)\right] \text{, and} \tag{3.19}$$

$$TC_P(n^* + 1) = \frac{D_F}{(n^* + 1)x} K_S + \frac{H_F}{2}\left[(n^* + 1)x + D_F\left(\frac{x}{D_F} - T_S\right)\right] \text{, respectively.} \tag{3.20}$$

Substituting, Equation (3.16), by Equations (3.18) and (3.19) it can be found that

$$\frac{D_F K_S}{x}\left(\frac{1}{n^*} - \frac{1}{n^* - 1}\right) + \frac{x H_F (n^* - n^* + 1)}{2} \leq 0 \tag{3.21}$$

which yields

$$0 \le n^{*2} - n^{*} - \frac{2D_F K_S}{x^2 H_F}. \tag{3.22}$$

Therefore, considering only the positive root and simplifying, the upper bound of n^* can be found as

$$n^* \le \frac{1}{2}\left(\sqrt{1+4\Omega}+1\right), \tag{3.23}$$

where $\Omega = \frac{2D_F K_S}{x^2 H_F}$.

Again, substituting, Equation (3.17), by Equations (3.19) and (3.20) it can be found upon simplification as

$$\frac{D_F K_S}{x}\left(\frac{1}{n^*+1} - \frac{1}{n^*}\right) + \frac{x H_F (n^* + 1 - n^*)}{2} \ge 0, \tag{3.24}$$

which yields

$$0 \ge n^{*2} + n^* - \frac{2D_F K_S}{x^2 H_F} = n^{*2} + n^* - \Omega \tag{3.25}$$

as $\Omega = \frac{2D_F K_S}{x^2 H_F}$.

Solving Equation (3.25) and considering the positive value the lower bound of n^* can be found as

$$n^* \ge \frac{1}{2}\left(\sqrt{1+4\Omega}-1\right). \tag{3.26}$$

Therefore, the boundary condition of n^* can be expressed as

$$\left\lceil \frac{1}{2}\left(\sqrt{1+4\Omega}-1\right) \right\rceil \le n^* \le \left\lfloor \frac{1}{2}\left(\sqrt{1+4\Omega}+1\right) \right\rfloor. \tag{3.27}$$

From Equation (3.27) two values can be found as n_1^*, and n_2^*, respectively. Therefore, using Equation (3.15) and the following argument, optimum n^* can be evaluated as

$$n^* = \text{ arg min } \{TC_{BS}(n_1^*), TC_{BS}(n_2^*)\}\,. \tag{3.28}$$

After evaluation of the optimum value of n^*, is used as the fixed parameter of the supplier's side of the problem. Therefore, the objective function for the supplier's side T_{SS} can be written as

Minimize: $$T_{SS}(m) = TC_{PR} = \frac{mD_F}{n^* x} K_0 + \frac{n^{*2} x^2 H_R}{2mfP} \tag{3.29}$$

Subject to: $n^* \geq m \geq 1$ and is an integer. (3.29a)

This problem is also a discrete optimization problem and it is convex for m. Therefore, the induction method is used to set the boundary for m^*, and a search is applied around the boundary to optimize the objective function.

Let m^* be the value that minimizes T_{SS}, where m^*-1 and m^*+1 are in the neighborhood of m^* that obtains values for the objective function such that

$$T_{SS}(m^* - 1) - T_{SS}(m^*) \geq 0 \text{, and} \tag{3.30}$$

$$T_{SS}(m^* + 1) - T_{SS}(m^*) \geq 0\,. \tag{3.31}$$

Applying the values of m^*-1, m^*, and m^*+1 in Equation (3.29) it can be shown that

$$T_{SS}(m^* - 1) = \frac{(m^* - 1)D_F}{n^* x} K_0 + \frac{n^{*2} x^2 H_R}{2(m^* - 1) fP}, \tag{3.32}$$

$$T_{SS}(m^*) = \frac{m^* D_F}{n^* x} K_0 + \frac{n^{*2} x^2 H_R}{2m^* fP} \text{, and} \tag{3.33}$$

$$T_{SS}(m^* + 1) = \frac{(m^* + 1)D_F}{n^* x} K_0 + \frac{n^{*2} x^2 H_R}{2(m^* + 1) fP} \text{, respectively.} \tag{3.34}$$

Replacing Equations (3.30) and (3.31) with Equations (3.32), (3.33) and (3.34) it can be shown that

$$\frac{(m^*-1)D_F}{n^* x}K_0 + \frac{n^{*2}x^2 H_R}{2(m^*-1)fP} - \frac{m^* D_F}{n^* x}K_0 - \frac{n^{*2}x^2 H_R}{2m^* fP} \geq 0 \qquad (3.35)$$

$$\frac{(m^*+1)D_F}{n^* x}K_0 + \frac{n^{*2}x^2 H_R}{2(m^*+1)fP} - \frac{m^* D_F}{n^* x}K_0 - \frac{n^{*2}x^2 H_R}{2m^* fP} \geq 0 \qquad (3.36)$$

Solving Equation (3.35) and (3.36) for m^* it can be shown that

$$m^* \leq \frac{1}{2}\left(\sqrt{1+4\Psi}+1\right), \text{ and} \qquad (3.37)$$

$$m^* \geq \frac{1}{2}\left(\sqrt{1+4\Psi}-1\right), \text{ respectively,} \qquad (3.38)$$

where $\Psi = \dfrac{n^{*3}x^3 H_R}{2fPD_F K_0}$.

Hence, the boundary for m^* can be expressed as

$$\left\lceil \frac{1}{2}\left(\sqrt{1+4\Psi}-1\right) \right\rceil \leq m^* \leq \left\lfloor \frac{1}{2}\left(\sqrt{1+4\Psi}+1\right) \right\rfloor. \qquad (3.39)$$

Equation (3.39) will also provide two values for optimum number of orders as m_1^*, and m_1^*, respectively. Hence, the argument used in Equation (3.29) to solve optimum m^* as follows:

$$m^* = \arg\min \{TC_{SS}(m_1^*), TC_{SS}(m_2^*)\}. \qquad (3.40)$$

According to this solution it can be stated that both m^* and n^* are local optimum. Therefore, to find the global optimality for both the variables, a forward search is conducted [starting from the constraints presented in Equations (3.15a) and (3.29a)] to optimize both n^* and m^* simultaneously, by using integer step length 1 and Equation (3.12). The optimum solution will be

$$TC(m^{opt}, n^{opt}) = \arg \min_{a=1,..} \{TC(m^a, n^a)\}. \qquad (3.41)$$

Example 3.1: Total Cost Computation

A sample computation is presented using data set of Problem 1 from Table 3.1 and the solution technique discussed in Section 3.5 as follows:

The optimum boundary condition for the number of shipments can be evaluated by using Equation (3.27) as $\lceil 2.9 \rceil \le n^* \le \lfloor 4.1 \rfloor$ or $3 \le n^* \le 4$. From Equation (3.28), the optimum number of shipment can be evaluated as $n^* =$ arg min $\{TC_{BS}(3), TC_{BS}(4)\}$ $=$ arg min $\{\$799.45, \$797.60\} = 4$. Using this value in Equation (3.39), the boundary condition for number of raw materials orders can be found as $0 \le m^* \le 1$. Satisfying the constraint given in Equation (3.29a), the optimum $m^* = 1$ and $TC_{SS}(1) = \$911.11$. Now, using Equation (3.41) the optimum solution can be evaluated as, $TC_{BS}(1,7) = TC_{PF}(1,7) = \$969.03$, and $TC_{SS}(1,7) = TC_{PR}(1,7) = \$548.31$, from where the optimum system cost can be found as $TC_P^*(m^*, n^*) = TC_P^*(1, 7) = \$1{,}517.34$.

Appling the above results, the optimum results for perfect matching supply chain system are evaluated with some numerical data found in existing literature in the next section.

3.6 Computational Results

In this section, the numerical tests are presented using the solution procedures for the perfect matching supply chain system and six sets of data, which have been chosen from different hypothetical scenarios from industrial experience. The sets of problems are denoted as Problem 1, Problem 2, Problem 3, Problem 4, Problem 5, and Problem 6 and presented in Table 3.1. These data for Problem 1, Problem 2, Problem 3, Problem 4, Problem 5, and Problem 6 are found in Parija and Sarker (1999), Diponegoro (2003), and Biswas and Sarker (2005), respectively. The optimal results for all six problems are computed as Example 3.1 and are presented in Table 3.2.

Table 3.1 Data set for numerical computation for perfect matching problem

Parameters	*Problem 1*	*Problem 2*	*Problem 3*	*Problem 4*	*Problem 5*	*Problem 6*
P (units/year)	3,600	3,600	6,000	7,000	8,000	11,000
D_F (units/year)	2,400	2,400	3,000	5,200	5,200	7,200
K_0 ($/order)	150	100	150	200	200	300
K_S ($/setup)	50	100	60	70	200	250
H_R ($/unit/year)	1	10	3.5	4	4	10.5
H_F ($/unit/year)	2	10	5	15	25	45
F	2	3	3	2.5	3	4
x(units/shipment)	100	100	150	200	300	350
T_s (years)	0.001	0.002	0.002	0.003	0.005	0.006

Note: Problem 1 [Parija and Sarker (1999)], Problem 2 [Diponegoro (2003)], Problem 3-6 [Biswas and Sarker (2005)].

Table 3.2 Results using given data set

Parameters	*Problem 1*	*Problem 2*	*Problem 3*	*Problem 4*	*Problem 5*	*Problem 6*
n^*	7	3	3	2	1	1
m^*	1	2	2	1	1	1
Q^*	400	300	300	400	600	350
$T_{BS} = TC_{PF}(n^*)$	$969.03	$2,723.22	$1,885.00	$5,012.05	$10,641.67	$21,870.15
$T_{SS} = TC_{PR}(m^*)$	$548.31	$908.55	$1,019.69	$3,054.86	$3,474.17	$4,286.03
$TC_P^*(m^*, n^*)$	$1,517.34	$3,631.77	$2,904.69	$7,911.29	$14,115.83	$26,106.90

3.7 SPECIAL CASE

If there is no overlapping in between the cycles, which means during the downtime no production or uptime takes place, then the inventory diagram in Figure 3.2 becomes similar to Diponegoro's (2003) model shown in Figure 3.3. In this case the cycle time, T, becomes $T = T_S + T_P + T_D = nL$ and there is idle time during the pure shipment or downtime and each cycle delivers x units of finished goods every L time units. Therefore, the finished goods inventory, I_{PT}, from Equation (3.5) will convert to I_{PS} when

$$T_S \to T_S + T_D = T_S + T_S + T_P = 2T_S + nx/P\,. \tag{3.42}$$

Applying Equation (3.42) in Equation (3.5) it can be found that

$$I_{SP} = \frac{n^2x^2}{2D_F} + nx\left\{\frac{x}{2D_F} - \frac{1}{2}\left(2T_S + \frac{nx}{P}\right)\right\}$$

$$= \frac{n^2x^2}{2D_F} + nx\left\{\frac{x}{2D_F} - T_S - \frac{nx}{2P}\right\}$$

$$= \frac{n^2x^2}{2D_F} - \frac{n^2x^2}{2P} + nx\left(\frac{x}{2D_F} - T_S\right)$$

$$= \frac{n^2x^2}{2D_F}\left(1 - \frac{D_F}{P}\right) + nx\left(\frac{x}{2D_F} - T_S\right) \qquad (3.43)$$

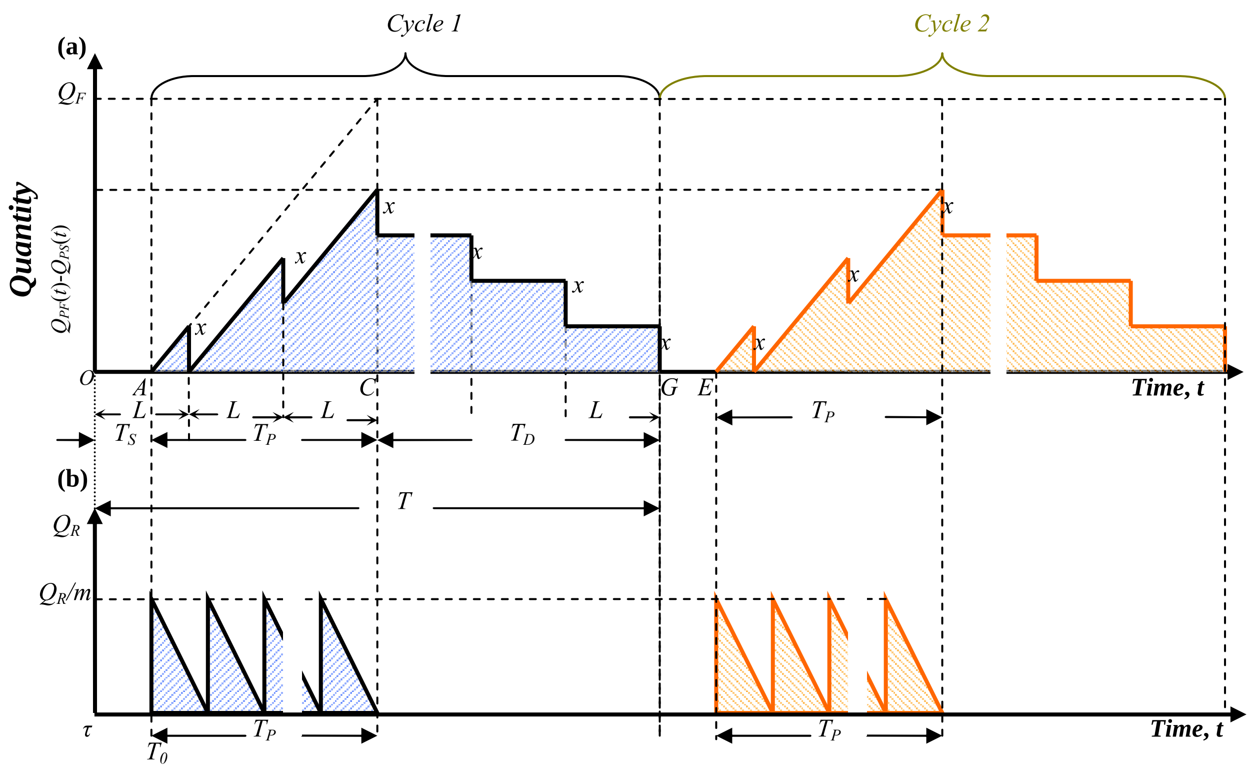


Figure 3.3. (a) Finished goods inventory with idle time, (b) raw material inventory

and the average inventory becomes

$$\bar{I}_{SP} = \left[\frac{nx}{2}\left(1 - \frac{D_F}{P}\right) + D_F\left(\frac{x}{2D_F} - T_S\right)\right]. \qquad (3.44)$$

Therefore, the total cost function given in Equation (3.10) converts to

$$TC_{PS}(m,n) = \frac{mD_F K_0}{nx} + \frac{n^2x^2H_R}{2mfP} + \frac{D_F K_S}{nx} + H_F\left[\frac{nx}{2}\left(1 - \frac{D_F}{P}\right) + D_F\left(\frac{x}{2D_F} - T_S\right)\right], \qquad (3.45)$$

which is Diponegoro's (2003) cost function model for infinite planning horizon.

Let Diponegoro's (2003) model be 'deferred production' (as there is a long idle time after the production stops at end of time period T_P), and the model described here for perfect matching case is 'accelerated production'. Applying the parametric values given in Table 3.1 in Equation (3.5), and using the operation schedule presented in Figure 3.2, the total inventory produced in accelerated production for all six problems are computed and presented in Table 3.3. Also, a similar computation is presented in Table 3.4 by using Figure 3.3 and Equation (3.45). From Table 3.3, it can be observed that, in accelerated system produces, more finished products result than the deferred production for perfect matching system.

Table 3.3 Inventory produced in accelerated and deferred production

Problems	Accelerated			Deferred			Accelerated	Deferred
	Cycle time (years)	Quantity produced (units)	Idle time (years)	Cycle time (years)	Quantity produced (units)	Idle time (years)	Quantity Produced (units/year)	Quantity Produced (units/year)
1	0.125	24.85	0.001	0.250	36.90	0.124	198.80	147.60
2	0.125	24.70	0.002	0.250	36.30	0.123	197.60	145.20
3	0.150	44.55	0.002	0.300	88.20	0.148	297.00	294.00
4	0.115	45.25	0.003	0.231	55.08	0.113	392.17	238.68
5	0.173	101.60	0.005	0.346	151.96	0.168	587.02	439.00
6	0.146	98.93	0.006	0.292	144.24	0.140	678.38	494.54

3.8 Single Facility Lot-Sizing Model with Perfect Matching

The models and examples, given in previous sections of this chapter, deal with a single product manufacturing system with minimal down time. This section considers a single production facility that produces K products, where product k has a constant demand of D_{Fk} units per year and produced at a constant rate of P_k units per year and delivered at a fixed amount of x_k after every L_k time units. It is assumed that $P_k > D_{Fk}$ so that the production of all K items must meet customers' demand and $\sum_{k=1}^{K}(D_{Fk} / P_k) \leq 1$. Without permitting any shortages, it is a problem to determine the time of production and optimum number of units to produce for

each item. However, the rotation cycle [as defined in Johnson and Montgomery (1974)] cannot be applied in production with minimal downtime model, because this policy cannot satisfy customers' demand due to the formation of the cost function. In the minimal down time model the uptime and downtime delivery combines x_k units at every L_k time units and only k product is being produced which satisfies the customers' demand for one product. When a single facility is producing multiple items in rotation, production of each item has to satisfy customers' demand. Therefore, the special case will be more appropriate for rotation cycle application where the downtime will be minimal as one product will be produced at the end of the production or uptime of another product. Hence, the rotation cycle is applied on special case described in Section 3.7.

If the problem is formulized as perfect matching case to determine the lot-sizes, it will be undoubtedly difficult to schedule the production of each item on a single facility layout without incurring shortages. Therefore, a rotation cycle policy is used to avoid shortages and to schedule the production cycle of each item. This policy will also minimize the idle time of the production facility. Figure 3.4 represents the inventory models (on-hand finished goods inventory and its corresponding raw materials supplies) for single facility lot sizing model.

According to the definition of rotation cycle, all products must have same cycle time, T_r, of production and during the time period of T_r, lot of each product is produced in the facility. Due to the rotation, the products are produced in a fixed order, which is repeated from cycle to cycle. Therefore, the lot size for product k must equal the demand during the production cycle without permitting shortages as

$$Q_{Fk} = n_k x_k = T_r D_{Fk} \,. \tag{3.46}$$

Substituting this constraint into the objective function given in Equation (3.46), the expression for the average cost per unit time for rotation cycle policy can be expressed as

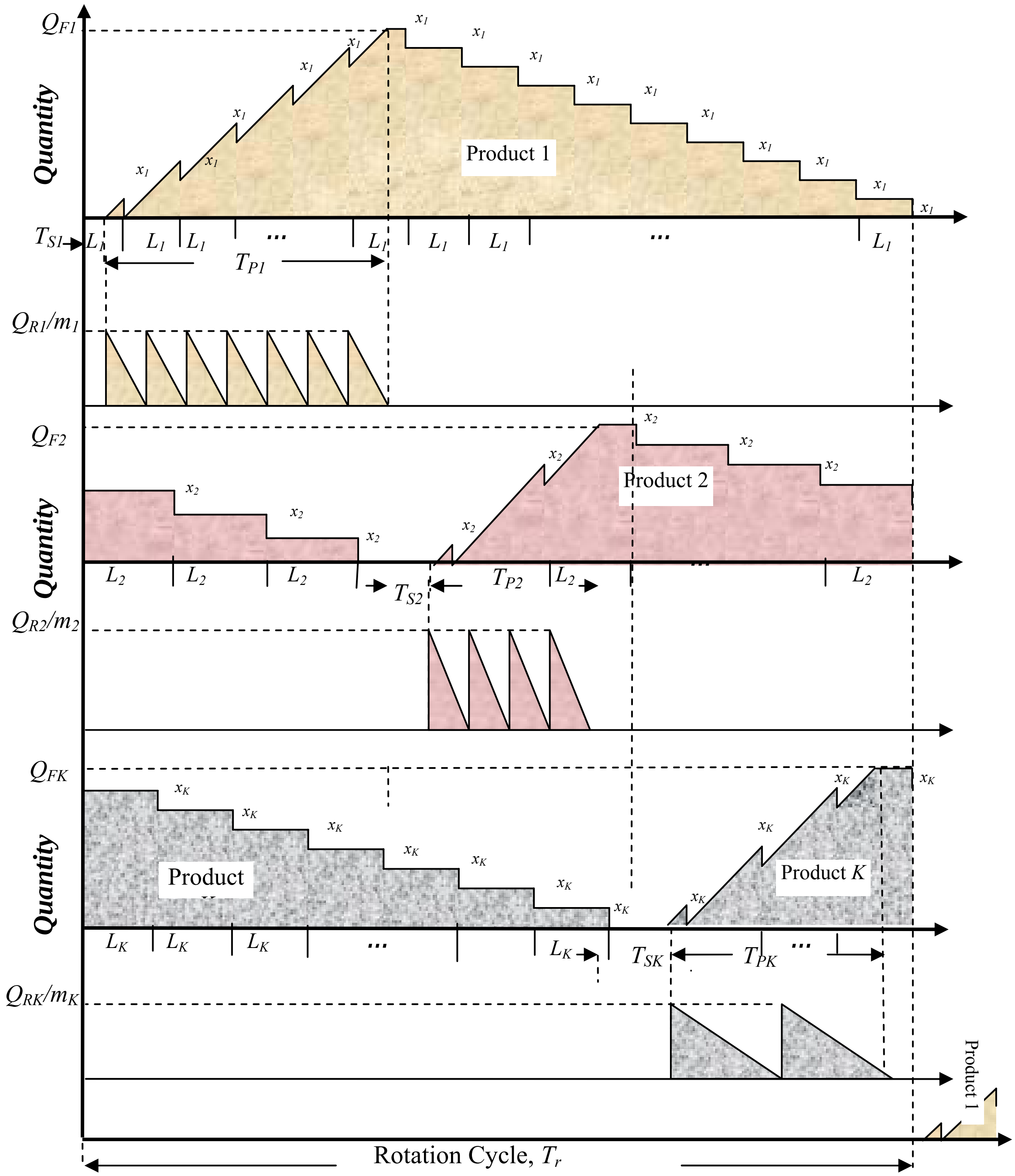


Figure 3.4. Inventory for single facility with multiple products

$$C_P(T_r, m_1, m_2, ..., m_K) = \sum_{k=1}^{K} \left[\frac{T_r^2 D_{Fk}^2}{m_k} \left(\frac{H_{Rk}}{2 f_k P_k} \right) + \frac{m_k K_{0k}}{T_r} + \frac{K_{Sk}}{T_r} + \frac{T_r D_{Fk} H_{Fk}}{2} \left(1 - \frac{D_{Fk}}{P_k} \right) + D_{Fk} H_{Fk} \left(\frac{x_k}{2 D_{Fk}} - T_{Sk} \right) \right]. \quad (3.47)$$

Substituting the constant value from Equation (3.48), it can be expressed as

$$C_P(T_r, m_1, ..., m_K) = \sum_{k=1}^{K} \left[A_{ak} T_r^2 / m_k + A_{bk} m_k / T_r + A_{ck} / T_r + A_{dk} T_r + A_{ek} \right], \qquad (3.48)$$

where $A_{ak} = D_{Fk}^2 \left(\dfrac{H_{Rk}}{2 f_k P_k} \right),$

$A_{bk} = K_{0k},$

$A_{ck} = K_{Sk},$

$A_{dk} = \dfrac{D_{Fk} H_{Fk}}{2} \left(1 - \dfrac{D_{Fk}}{P_k} \right),$ and,

$A_{ek} = D_{Fk} H_{Fk} \left(\dfrac{x_k}{2 D_{Fk}} - T_{Sk} \right).$

Before minimizing the problem, it is necessary to study the constraints related to the rotation cycle policy, such as the setup times the number of raw material deliveries for each product. If the setup time for product k is T_{Sk}, then the total setup time per cycle and the total production time per cycle must be smaller or equal to the rotation cycle length. Therefore, the following constraint on T_r will be

$$T_r \geq \sum_{k=1}^{K} \left[T_{Sk} + \frac{Q_{Fk}}{P_k} \right]. \qquad (3.49)$$

Replacing Q_{Fk}, by using Equation (3.46), it can be re-written as

$$T_r \geq \frac{\sum_{k=1}^{K} T_{Sk}}{1 - \sum_{k=1}^{K} \left[D_{Fk} / P_k \right]} \equiv T_{\min} \geq 0. \qquad (3.50)$$

Also, the number of raw material delivery, m_k for product k can not be less than 1 and should be an integer variable. Hence, the constraint on m_k is

$$m_k \geq 1 \text{ and is an integer,} \qquad \text{for } k = 1, 2, ..., K. \tag{3.51}$$

For these reasons, the rotation cycle minimization problem for single facility lot-sizing model C_{PR} can be formulated as

Minimize C_{PR}

$$C_P = \sum_{k=1}^{K} \left[A_{ak} T_r^2 / m_k + A_{bk} m_k / T_r + A_{ck} / T_r + A_{dk} T_r + A_{ek} \right], \tag{3.52}$$

Subject to:

$$T_r \geq T_{\min} \geq 0\,, \tag{3.52a}$$

$$m_k \geq 1 \text{ and is an integer,} \qquad \text{where } k = 1, 2, ..., K. \tag{3.52b}$$

The solution procedure of the rotation cycle policy problem is described in the following section.

3.9 SOLUTION OF SINGLE FACILITY LOT-SIZING MODEL IN PERFECT MATCHING

According to the formulation of the single facility lot-sizing problem, it can be categorized as a mixed-integer non linear programming problem where m_k's are integer and T_r is a real variable and the number of variables are (K + 1). Due to formulation of the problem, it cannot be solved using derivatives and a closed form solution cannot be determined. Therefore, to solve the problem in a simplified way, the objective function is divided in to two parts (a) finished product shipments, and (b) raw material orders. Basically the rotation cycle for the finished products (T_r) is the same for the raw material delivery, because the raw materials are delivered from the supplier by instantaneous replenishments. Again, the raw material for a product k is ordered when the finished product k goes for production. Figure 3.4 shows the finished different goods supply to the customers and the corresponding raw materials ordered. The next part of the research deals with the solution techniques of the entire problem by separating the total cost for finished products, and solving the new objective function.

(a) Finished product shipments

To solve the rotation cycle policy for the part finished product supply, the cost function from Equation (3.47) can be divided as

$$\text{Minimize: } C_{PF}(T_r) = \sum_{k=1}^{K}\left[\frac{K_{Sk}}{T_r} + \frac{T_r D_{Fk} H_{Fk}}{2}\left(1-\frac{D_{Fk}}{P_k}\right) + D_{Fk} H_{Fk}\left(\frac{x_k}{2D_{Fk}} - T_{Sk}\right)\right] \tag{3.53}$$

$$\text{Subject to:} \qquad T_r \geq T_{\min} \geq 0 \tag{3.53a}$$

It can be shown that the Equation (3.53) is a convex function for T_r. Therefore, it can be solved by differentiation with respect to T_r and equate it to zero as follows:

$$\frac{dC_{PF}(T_r)}{dT_r} = \sum_{k=1}^{K}\left[-\frac{K_{Sk}}{T_r^2} + \frac{D_{Fk} H_{Fk}}{2}\left(1-\frac{D_{Fk}}{P_k}\right)\right] = 0, \tag{3.54}$$

which yields

$$T_r^* = \sqrt{\frac{2\sum_{k=1}^{K} K_{Sk}}{\sum_{k=1}^{K} D_{Fk} H_{Fk}\left(1-D_{Fk}/P_k\right)}}. \tag{3.55}$$

Equation (3.55) has to satisfy the constraint given in Equation (3.53a). Using the optimal rotation cycle T_r^*, the number of shipments for different finished products can be obtained from Equation (3.46). After that the optimal rotation cycle, T_r^* is used to solve the optimal number of orders for raw materials.

(b) Raw material orders

As the raw materials order policy is instantaneous, the production rate for the raw material is ∞. Therefore, this also satisfies the condition for rotation cycle. Now, applying the value of T_r^* from Equation (3.55), the total cost/objective function for raw material k can be written as [from Equation (3.47)]

Minimize: $$C_{PR}(m_k) = \frac{m_k K_{0k}}{T_r^*} + \frac{T_r^{*2} D_{Fk}^2}{m_k}\left(\frac{H_{Rk}}{2 f_k P_k}\right) \qquad (3.56)$$

Subject to: $m_k \geq 1$ and is an integer, where $k = 1, ..., K.$ (3.56a)

This objective function [Equation (3.56)] is convex in m_k, but it is a discrete optimization, which cannot be solved by usual technique (differentiation). Therefore, the induction method is used to solve for m_k.

Let m_k^* be the value which minimizes $C_{PR}(m_k^*)$. In the neighborhood of m_k^*, points like $m_k^* - 1$ and $m_k^* + 1$ for which the values of the objective function provide the results as follows:

$$C_{PR}(m_k^* - 1) - C_{PR}(m_k^*) \geq 0 \text{, and} \qquad (3.57)$$

$$C_{PR}(m_k^* + 1) - C_{PR}(m_k^*) \geq 0 \,. \qquad (3.58)$$

Now, substituting the values of $C_{PR}(m_k^* - 1), C_{PR}(m_k^*)$ and $C_{PR}(m_k^* + 1)$ in Equation (3.56), the modified equations can be written as

$$C_{PR}(m_k^* - 1) = \frac{(m_k^* - 1) K_{0k}}{T_r^*} + \frac{T_r^{*2} D_{Fk}^2}{(m_k^* - 1)}\left(\frac{H_{Rk}}{2 f_k P_k}\right), \qquad (3.59)$$

$$C_{PR}(m_k^*) = \frac{m_k^* K_{0k}}{T_r^*} + \frac{T_r^{*2} D_{Fk}^2}{m_k^*}\left(\frac{H_{Rk}}{2 f_k P_k}\right) \text{, and} \qquad (3.60)$$

$$C_{PR}(m_k^* + 1) = \frac{(m_k^* + 1) K_{0k}}{T_r^*} + \frac{T_r^{*2} D_{Fk}^2}{(m_k^* + 1)}\left(\frac{H_{Rk}}{2 f_k P_k}\right) \text{, respectively.} \qquad (3.61)$$

Using Equations (3.57) to (3.61) the boundary for m_k^* can be evaluated as

$$\left\lceil \frac{1}{2}\left(\sqrt{1 + 4\Delta_k} - 1\right) \right\rceil \leq m_k^* \leq \left\lfloor \frac{1}{2}\left(\sqrt{1 + 4\Delta_k} + 1\right) \right\rfloor, \qquad (3.62)$$

where $\Delta_k = \dfrac{T_r^{*3} D_{Fk}^2 H_{Rk}}{2 f_k P_k K_{0k}}$, and $k = 1, ..., K.$

Moreover, Equation (3.62) has to satisfy the constraint given in Equation (3.56a). Hence, optimum total cost for all K raw materials can be expressed as

$$C_{PR}(m_k^*) = \frac{m_k^* K_{0k}}{T_r^*} + \frac{T_r^{*2} D_{Fk}^2}{m_k^*}\left(\frac{H_{Rk}}{2 f_k P_k}\right), \tag{3.63}$$

where $k = 1, 2, \ldots, K$.

Again, the optimum value of T_r^* and m_k^* are both local optimum. Hence, to find the global optimal solution for both T_r^* and m_k^*, a forward search is conducted using Equation (3.47) with starting form the value given by the constraints for T_r^* and m_k^* [presented in Equations (3.52a) and (3.53b), respectively]. As this problem is a non-linear mixed integer programming problem, the step sizes for T_r^* and m_k^* are considered as 0.01 and 1, respectively. The search will be ended when the minimum total cost will be reached, and thus, the optimum T_r^{opt} and m_k^{opt} can be evaluated. From the above computations the following procedure is presented to solve the rotation cycle policy.

Example 3.2: Rotation Cycle Estimation

Consider six products are being produced in a single facility manufacturing system. The respective parameters for all six products are presented in Table 3.4. Using these data and Equation (3.55), the T_r^* can be found as

$$T_r^* = \sqrt{\frac{2(50+100+60+70+120+150)}{(857.13+128.57+480.00+98.40+41.60+43.51)}} = 0.82\,. \tag{3.64}$$

Now using the value of T_r^* in Equation (3.62) the boundary conditions for m_k^* can be found as

$$\lceil 0.21 \rceil \le m_1^* \le \lfloor 1.21 \rfloor, \lceil 0.98 \rceil \le m_2^* \le \lfloor 1.98 \rfloor, \lceil 0.73 \rceil \le m_3^* \le \lfloor 1.73 \rfloor,$$

$$\lceil 0.47 \rceil \le m_4^* \le \lfloor 1.47 \rfloor, \lceil 0.23 \rceil \le m_5^* \le \lfloor 1.23 \rfloor, \text{ and } \lceil 0.40 \rceil \le m_6^* \le \lfloor 1.40 \rfloor. \tag{3.65}$$

From where it can be found that

$$m_1^* = 1, m_2^* = 1, m_3^* = 1, m_4^* = 1, m_5^* = 1, \text{ and } m_6^* = 1. \tag{3.66}$$

Using these values the total costs can be found as $C_P(T_r^*, m_1^*, m_2^*, m_3^*, m_4^*, m_5^*, m_6^*)$ $= (0.82,1,1,1,1,1,1)$ $= \$38,359.07$ per year, and this is local optimum solution. Therefore, a forward search is conducted starting from T_r^*= 0.21 (with step size 0.01), and m_k^* = 1 (with step size 1). Applying the search the optimum solution is found as $C_P(T_C^{opt}, m_1^{opt}, m_2^{opt}, m_3^{opt}, m_4^{opt}, m_5^{opt}, m_6^{opt})$ $= (0.21,1,1,1,1,1,1)$ $= \$38,408.87$ per year.

Next section describes the numerical example for rotation cycle policy using the data set for six products to be produced in a single facility.

3.10 NUMERICAL COMPUTATION OF OPTIMUM ROTATION CYCLE

In this section, an optimum rotation cycle and number of orders are determined using a set of numerical data for six products and assuming $\sum_{k=1}^{6}(D_{Fk}/P_k) \le 1$. The dataset is presented in Table 3.4. In this case, it is considered that the all six products are produced in a single facility in a sequence and they will be delivered using just-in-time (JIT) phenomena. Also, the raw materials for each product will be ordered following just-in-time (JIT) ordering policy. According to the constraint given in Equation (3.53a), it can be determined by using the data given in Table 3.3:

$$T_r \ge \frac{0.001+0.002+0.002+0.003+0.005+0.006}{\left[1-\left(\frac{2,000}{14,000}+\frac{1,500}{10,500}+\frac{3,000}{15,000}+\frac{1,800}{10,000}+\frac{1,200}{9,000}+\frac{2,200}{20,000}\right)\right]} = \frac{0.019}{0.091} = 0.21. \tag{3.67}$$

It is also observed that

$$\sum_{k=1}^{6}\left(D_{Fk}/P_k\right) = \frac{2,000}{14,000}+\frac{1,500}{10,500}+\frac{3,000}{15,000}+\frac{1,800}{10,000}+\frac{1,200}{9,000}+\frac{2,200}{20,000} = 0.91 \le 1 \tag{3.68}$$

which satisfies the rotation cycle assumption.

Table 3.4 Data set for single facility lot-sizing model

Parameters	*Product 1*	*Product 2*	*Product 3*	*Product 4*	*Product 5*	*Product 6*
P_k (units/year)	5,000	10,500	15,000	8,000	8,000	20,000
D_{Fk} (units/year)	2,400	2,000	3,000	2,000	1,200	2,200
K_{0k} ($/order)	150	100	150	200	200	300
K_{Sk} ($/setup)	50	100	60	70	200	250
H_{Rk} ($/unit/year)	1	10	3.5	4	4	10.5
H_{Fk} ($/unit/year)	2	10	5	15	25	45
f_k	2	3	3	2.5	3	4
x_k (units)	100	100	150	200	300	350
T_{Sk} (years)	0.001	0.002	0.002	0.003	0.005	0.006

Note: Data set is found in Johnson and Montgomery (1973).

Using solution procedure, the optimum rotation cycle results are presented in Example 3.2 and Table 3.5.

Table 3.5 Optimum results for rotation cycle

Parameters	Optimum results for rotation cycle policy					
	Product 1	*Product 2*	*Product 3*	*Product 4*	*Product 5*	*Product 6*
m_k^{opt}	1	1	1	1	1	1
T_r^{opt}	0.21					
n_k^*	4	3	4	2	1	3
Q_{Fk}^*	400	300	600	200	300	350
Q_{Rk}^*	200	100	200	80	100	88
$C_{PR}(T_r^*, m_1^*, ..., m_k^*)$	$38,408.87					

Thus, this chapter concludes the research for perfect matching situation with minimal system idle time. The next chapter deals with the imperfect matching situation with just-in-time (JIT) delivery policy and obviously the system idle time is minimal.

CHAPTER 4

IMPERFECT MATCHING WITH JUST-IN-TIME DELIVERY

In reality, the inventory of a supply chain system never becomes empty. A number of products are always left-over after the deliveries are made. These left-over amounts are added to the next shipment after the production of required amount to make-up a complete batch. An illustration may be observed in retail stores such as Albertson's, Target, Wal-Mart, etc. Usually these stores order their supplies before their stock runs out. Large industries also start their production before the finished product inventories fall to zero. Therefore, it is important to search for an optimal supply chain system for these types production facilities with left-over finished goods inventory. This part of the research focuses on these issues of a supply chain system.

4.1 NOTATION

Most of the notation used to develop the model for this part of research is defined in previous chapter. Some additional important notation is given below:

I_{IT} : Total finished goods inventory for imperfect matching, units.

$\overline{I}_{IT}$: Average finished goods inventory for imperfect matching, units/year.

I_{IR} : Total raw materials inventory, units.

$\overline{I}_{IR}$: Average raw materials inventory, units/year.

I_{IS} : Total finished goods inventory with idle time, units.

$\overline{I}_{IS}$: Average finished goods inventory with idle time, units/year.

I_0 : Quantity remained after the *m* number of shipments, or initial inventory, units.

Q_F' : Quantity of finished goods manufactures per setup, units/batch.

T' : Total cycle time, years.

T_C : Rotation cycle time, years.

T_1 : Production uptime, years.

T_2 : Downtime, years

TC_{IF} : Total cost of finished goods, \$/year.

TC_{IR} : Total cost of raw materials, \$/year.

$TC_I(Q'_F, m)$: Total cost function for imperfect matching, \$/year

$TC_{IS}(m, n)$: Total cost function with idle time, \$/year

C_{IR} : Objective function of rotation cycle for imperfect matching.

4.2 PROBLEM IDENTIFICATION

When the production run stops, the on-hand inventory stops building up. Finished goods may be adequate for shipments after the production run is over as represented in Figure 4.1. In reality, the last shipment size may be less than the required amount. This situation leads to an *imperfect matching*. In 1992, Golhar and Sarker explained the imperfect matching system in their research. They optimize the total cost function using search procedure. After that, Sarker and Parija (1994) also dealt with the imperfect matching system and evaluated the piecewise convexity of the total function. In this part of the research, the function is developed to find economic order quantity (*EOQ*) for the raw materials and an economic manufacturing quantity (*EMQ*) for the production facility with imperfect matching and minimal idle time. Here, raw material ordering cost, raw material inventory cost, manufacturing setup cost and finished goods inventory carrying cost are considered. In this section, an expression for the generalized cost function is developed that may be used to determine an optimal batch quantity for the production run.

4.3 ASSUMPTION

To develop the mathematical model for this part, two more assumptions are made:

1. A fixed quantity is left-over after required shipments and carried over to the succeeding cycle.
2. Production run of succeeding cycle starts immediately after the uptime or production run of previous cycle and setup time.

4.4 Formulation of Cost Function based on Average Inventory

In this case, the production rate, P, is assumed to be greater than the demand rate, D_F, so that there is no shortage during the production. The lower part of Figure 4.1 represents the inventory of the raw material and the upper part represents the on-hand finished goods inventory. For a better explanation of Figure 4.1, let the cycle that starts from point O be cycle 1 and the following cycle that starts from point C be cycle 2. It is assumed that the production of cycle 1 starts T_S time units after the end of the uptime (at point A') of the previous cycle of cycle 1. Hence, the downtime of the cycle preceding cycle 1 overlaps the uptime of cycle 1. It is also assumed that an initial inventory, I_0, remains at the end of all possible full shipments of previous cycles of cycle 1. Therefore, at the beginning of cycle 1, the item produced during L-T_S time units is exactly $x/2–I_0$ amount so that a delivery of $x/2$ units can be made at the end of L time units. Again, at point a', production starts and shipments of $x/2$ units from both cycles (uptime of cycle 1 and downtime of preceding cycle) are made at every L time units. Therefore, both cycles combined x units of shipment at every L time which satisfies the customers' requirements. Similarly, the initial inventory, I_0 is carried over from preceding cycle of cycle 1 to the start of uptime of cycle 2, and amount remaining at cycle 1 is carried over till the beginning of cycle 3, and so forth. As this is an infinite planning horizon and the downtime of preceding cycle and uptime of succeeding cycle overlaps throughout the planning horizon, the cycle time is considered for average inventory calculation is T', which is the overlapping time periods (Figure 4.1). According to the description above, production starts after T_S time

units and produces exactly Q'_F (= $nx + I_0$) amount to deliver $x/2$ after L time units. Hence, during time L–T_S time the quantity produced is $x/2$–I_0 at the rate of P, so that $I_0 + (x/2–I_0)P \geq x/2$. Figure 1 is used to calculate the average on-hand inventory of the finished goods. $\hat{I}_{IT}$, $\hat{I}_{IP}$, and $\hat{I}_{ID}$ are the total inventory, uptime inventory and downtime inventory for the imperfect matching, respectively. Therefore, the total inventory can be calculated as

$$\hat{I}_{IT} = \hat{I}_{IP} - \hat{I}_{ID}\,. \tag{4.1}$$

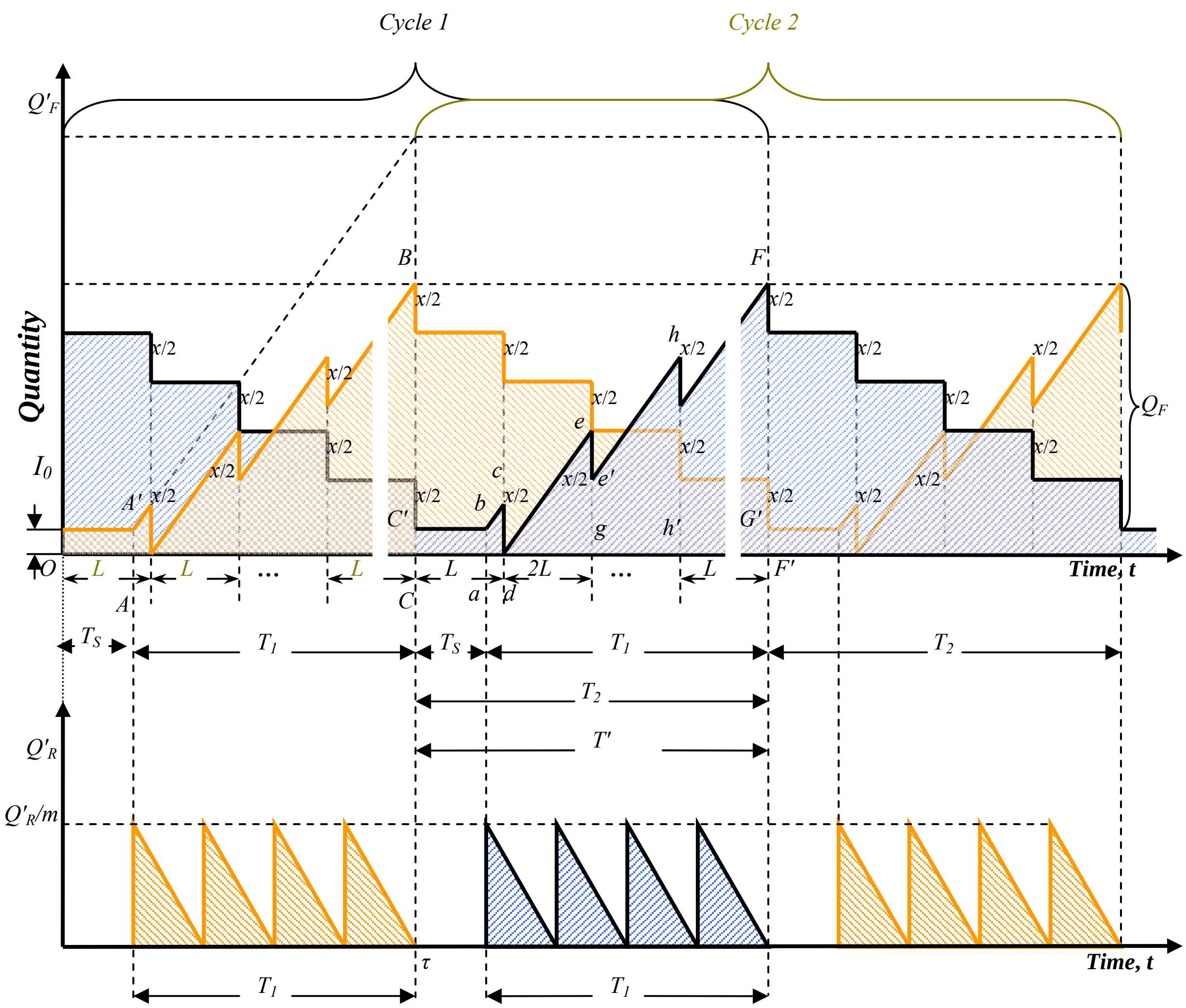


Figure 4.1. (a) Finished goods inventory; (b) Raw material inventory

From Figure 4.1, it can be found that

$$\hat{I}_{IP} = \text{area } C'bac + \text{area } abcd + \Delta deg + \text{area } ge'hFF'\,, \tag{4.2}$$

Therefore, the total produced inventory can be found as (see Appendix B.2 for detail computations)

$$\hat{I}_{IP} = \frac{nx}{2}T_1 + \frac{nx}{2}T' + 2I_0T' . \tag{4.3}$$

Again, the total inventory shipped can be calculated from Figure 4.1(a) as

$$\hat{I}_{ID} = L(x/2) + 2L(x/2) + ... + (n-1)L(x/2) + L(x/2) + 2L(x/2) +$$

$$... + (n-1)L(x/2) = 2\frac{n(n-1)}{2}\frac{Lx}{2} = \frac{nx^2}{2D_F}(n-1), \tag{4.4}$$

where $L = x/D_F$.

Hence, the total inventory for time period, T', can be calculated by combining Equations (4.3) and (4.4) as

$$\hat{I}_{IT} = \frac{Q_F'^2}{2D_F} + \frac{Q_F'}{2D_F}(4I_0 + x - D_FT_S) - \frac{I_0}{2D_F}(I_0 + x - D_FT_S), \tag{4.5}$$

where $T' = Q_F' / D_F$ and $Q_F' = nx + I_0$ (see Appendix B.2 for detail computations).

Again, the total cycle time for imperfect matching case can be calculated as

$$T' = Q_F' / D_F . \tag{4.6}$$

Hence, the average inventory for imperfect matching case is

$$\bar{\hat{I}}_{IT} = \frac{\hat{I}_{IP} + \hat{I}_{ID}}{T'} = \frac{D_F}{Q_F'}\left[\frac{Q_F'^2}{2D_F} + \frac{Q_F'}{2D_F}(4I_0 + x - D_FT_S) - \frac{I_0}{2D_F}(I_0 + x - D_FT_S)\right]$$

$$= \frac{Q_F'}{2} - \frac{I_0}{2Q_F'}(I_0 + x - D_FT_S) + \frac{1}{2}(4I_0 + x - D_FT_S) . \tag{4.7}$$

Therefore, the total cost function for the finished products can be written as

$$TC_{IF} = \frac{D_F}{Q_F'}K_S + \bar{\hat{I}}_{IF}H_F$$

$$= \frac{D_F}{Q'_F} K_S + H_F \left[\frac{Q'_F}{2} - \frac{I_0}{2Q'_F}\left(I_0 + x - D_F T_S\right) + \frac{1}{2}\left(4I_0 + x - D_F T_S\right) \right]. \quad (4.8)$$

During the production time or uptime, T_1, the raw materials are ordered to produce finished products, which require Q'_R units of raw materials to produce Q'_F units of finished goods and they are instantaneously replenished by the outside supplier in m batches. Also, f units of raw materials required to produce one unit of finished product, i.e., $fQ_R = Q_F$. As a result, *raw materials* inventory of entire cycles can be expressed as

$$\overline{I}_{IR} = \left(\frac{Q'_R T_1}{2m} \right) = \left(\frac{Q'_F T_1}{2mf} \right) = \frac{Q'^2_F}{2mfP}, \quad (4.9)$$

where $Q'_R = Q'_F / f$, and $T_1 = Q'_F / P$.

Therefore, the total cost for the raw material can be expressed as

$$TC_{IR} = \frac{D_R}{Q'_R / m} K_0 + \hat{\overline{I}}_{IR} H_R = \frac{mD_F}{Q'_F} K_0 + \frac{Q'^2_F H_R}{2mfP}, \quad (4.10)$$

where $\frac{Q'_F}{Q'_R} = \frac{D_F}{D_R} = f$.

Therefore, the total cost for this imperfect matching case can be found as

$$TC_I(Q'_F, m) = TC_{IR} + TC_{IF} = \frac{mD_F}{Q'_F} K_0 + \frac{Q'^2_F H_R}{2mfP} + \frac{D_F}{Q'_F} K_S$$

$$+ H_F \left[\frac{Q'_F}{2} - \frac{I_0}{2Q'_F}\left(I_0 + x - D_F T_S\right) + \frac{1}{2}\left(4I_0 + x - D_F T_S\right) \right], \quad (4.11)$$

which yields

$$TC_I(Q'_F, m) = \frac{Q'^2_F H_R}{2mfP} + \frac{mD_F K_0}{Q'_F} + \frac{1}{2} Q'_F H_F$$

$$+ \frac{1}{Q'_F} \left\{ D_F K_S - \frac{I_0 H_F}{2}\left(I_0 + x - D_F T_S\right) \right\} + \frac{H_F}{2}\left(4I_0 + x - D_F T_S\right). \quad (4.12)$$

Also, by applying $Q'_F = (nx + I_0)$ in Equation (4.12), the cost function can be expressed as

$$TC_I(m,n) = \frac{(nx+I_0)^2 H_R}{2mfP} + \frac{mD_F K_0}{(nx+I_0)} + \frac{1}{2}(nx+I_0)H_F$$

$$+\frac{1}{(nx+I_0)}\left\{D_F K_S - \frac{I_0 H_F}{2}(I_0 + x - D_F T_S)\right\} + \frac{H_F}{2}(4I_0 + x - D_F T_S). \quad (4.13)$$

Now, Replacing the constant term with appropriate notation and simplifying Equation (4.12), it can be re-written as

$$TC_I(Q'_F, m) = B_1(Q'^2_F / m) + B_2(m / Q'_F) + B_3 Q'_F + B_4 / Q'_F + B_5, \quad (4.14)$$

where $B_1 = \dfrac{H_R}{2fP}$,

$B_2 = D_F K_0$,

$B_3 = \dfrac{1}{2} H_F$,

$B_4 = D_F K_S - \dfrac{I_0 H_F}{2}\left(I_0 + x - D_F T_S\right)$,

and, $B_5 = \dfrac{H_F}{2}(4I_0 + x - D_F T_S)$.

The total cost function for this part of research is a non-linear integer programming problem with two integer variables Q'_F and m. Let the problem be defined as IM (imperfect matching). Again, the production quantity Q'_F and the number of raw material shipment cannot be less than or equal to 1. Hence, the Problem IM can be expressed with two constraints as

***Problem IM*:** Find Q'_F and m so as to

Minimize:

$$TC_{IM} = B_1(Q'^2_F / m) + B_2(m / Q'_F) + B_3 Q'_F + B_4 / Q'_F + B_5, \quad (4.15)$$

Subject to: $Q'_F \geq 1$, (4.15a)

$$m \geq 1, \text{ and is an integer.} \tag{4.15b}$$

In the following section, the solution procedures for Problem IM are described in details.

4.5 SOLUTION TECHNIQUE

The total cost function developed for imperfect matching problem is also a nonlinear integer programming (NLIP) problem and a non-convex function (see Appendix A.2). To find the starting basic solution it is observed that one of the variables have roots of 4th degree polynomial equation and also the variables are inter dependent. Considering these situations, the objective function is separated (using *Divide and Conquer* rule) in to two different parts as Section 3.5. They are (a) finished product shipments and (b) raw material orders. To solve these objective functions a proposition is developed as follows:

Proposition 4.1: Optimum production quantity of finished product, $Q_F'^*$, provides the solution for optimum number of orders m^*.

Proof: It can be stated that $fQ_R' = Q_F'$.

If optimum production quantity of finished products is $Q_F'^*$, then

$$fQ_R'^* = Q_F'^* \tag{4.16}$$

Again, according to the ordering policy of raw material, the total required raw materials, Q_R' are ordered in m small batches as Q_R' / m. Thus, the objective function for raw material inventory becomes a single variable objective function which is convex in *m*. Hence, the objective function can easily be solved by induction technique. Therefore, optimum production quantity found from Equation (4.16) will provide the optimum number of orders m^*.□

Based on Proposition 4.1, the objective function for the entire system is divided in to two different parts and solved each of the parts using following approach. This solution process generates the local optimal solution. Therefore, finding the global optimal is also explained.

(a) Finished product shipments

From Equation (4.12) the objective function for finished product production and shipments can be written as

Minimize: $$TC_{IF}(Q'_F) = \frac{D_F}{Q'_F} K_S + H_F \left[\frac{Q'_F}{2} - \frac{I_0}{2Q'_F} \left(I_0 + x - D_F T_S \right) \right.$$

$$\left. + \frac{1}{2} \left(4I_0 + x - D_F T_S \right) \right], \tag{4.17}$$

Subject to: $Q'_F \geq 1.$ (4.17a)

It can be shown that Equation (4.17) is convex in Q'_F. Hence, the optimum production quantity of finished products can be evaluated by differentiating Equation (4.17) with respect to Q'_F and equate to zero as

$$\frac{dTC(Q'_F)}{dQ'_F} = -\frac{D_F}{Q'^2_F} K_S + \left[\frac{H_F}{2} + \frac{I_0 H_F}{2Q'^2_F} \left(I_0 + x - D_F T_S \right) \right] = 0, \tag{4.18}$$

which yields

$$Q'^*_F = \sqrt{\left[\frac{2D_F K_S}{H_F} - I_0 \left(I_0 + x - D_F T_S \right) \right]}. \tag{4.19}$$

As Q'^*_F is an integer variable, so that the optimum finished products can be evaluated as $\lfloor Q'^*_F \rfloor$ or $\lceil Q'^*_F \rceil$, whichever provides the minimum cost for total cost for finished products. After that the optimum number of shipments n^* can be evaluated as

$$n^* = \left\lfloor \frac{Q'^*_F - I_0}{x} \right\rfloor, \tag{4.20}$$

this value has to satisfy $n^* \geq 1$ and is an integer. The optimum Q'^*_F is used for solving the raw material orders.

(b) Raw material orders

The objective function for the raw material order can be written as

Minimize: $$TC_{IR}(m) = \frac{mD_F}{Q_F'^*} K_0 + \frac{Q_F'^{*2} H_R}{2mfP}, \tag{4.21}$$

Subject to: $m \geq 1$ and is an integer. (4.21a)

Equation (4.21) is a discrete optimization problem, which is convex in *m*. Therefore, the induction method is used to solve for optimum number of orders.

To solve the problem of raw material orders, the induction method is used as Chapter 3. As a result, the boundary condition for m^* is evaluated from Equation (4.21) as

$$\left\lceil \frac{1}{2}\left(\sqrt{1+4\Delta} - 1\right) \right\rceil \leq m^* \leq \left\lfloor \frac{1}{2}\left(\sqrt{1+4\Delta} + 1\right) \right\rfloor, \tag{4.22}$$

where $\Delta = \dfrac{Q_F'^{*3} H_R}{2fD_F K_0 P}$.

From the above equation, two values of number of orders can be found as m_1^*, and m_2^*, respectively. Applying the boundary values in Equation (4.21), and the argument, the optimum raw materials orders m^* can be evaluated as

$$m^* = \text{arg min } \{TC_{IR}(m_1^*), TC_{IR}(m_2^*)\}. \tag{4.23}$$

Also, the boundary condition should satisfy the constraint given in Equation (4.21a), otherwise optimum m^* will be replaced by 1. In this case m^* and $Q_F'^*$ both are local optimal solution. Therefore, a search forward search is conducted starting from the constraints presented in Equations (4.15a) and (4.15b), and using Equation (4.12) and a integer step size of 1, with respect to $Q_F'^*$ and m^* for which $TC_I(Q_F'^{opt}, m^{opt})$ is minimum. The following example is presented with some numerical values to the optimum production quantity, $Q_F'^{opt}$ and number

of orders for raw materials m^{opt}. This following example shows the detail computation for imperfect matching inventory situation.

Example 4.1: Total Cost for Imperfect Matching

Consider P = 3,600 units/year, D_F = 2,400 units per year, K_0 = \$150/order, K_S = \$50/setup, H_R = \$1/unit/year, H_F = \$2/unit/year, f = 2, x = 100 units/shipment, I_0 = 25 units, and T_S = 0.001 years. Applying these values in Equation (4.19) the $Q_F'^*$ can be found as

$$Q_F'^* = \sqrt{\left[\frac{2\times 2,400\times 150}{2} - 25(25+100-2,400\times 0.001)\right]} = 342 \text{ units/year.}$$

Now, using this values in Equation (4.22) the boundary condition for m^* can be evaluated as $\lceil 0.0076 \rceil \le m^* \le \lfloor 1.0076 \rfloor$, which yields $m^* = 1$. Using these values the optimum total cost can be evaluated as $TC_I(Q_F'^*, m^*) = TC_I(342,1) = \$1,708.71$ per year. This solution is also local optimal solution. Hence, the forward search is conducted to find the global optimal solution starting from $TC_I(1,1)$, and the optimality is reached at $TC_I(Q_F'^{opt}, m^{opt}) = TC_I(661,1) = \$1,610.48$ per year, where $Q_F'^{opt} = 661, n^{opt} = 6$, and $m^{opt} = 1$.

Next section shows the results for the total system cost of imperfect matching system with the numerical values found in the literature.

4.6 COMPUTATIONAL RESULTS

In this section the numerical values are used to solve six sets of problems with imperfect matching inventory condition. The parametric values for six different problems are given in Table 4.1. It can be observed that the values presented in Table 4.1 are as same as the values presented in Table 3.1, except the initial inventory I_0. Using the solution process, a sample computation for Problem 1 is shown in Example 4.1. Applying the same procedure, the results for all six problems are presented in Table 4.2.

Next section deals with a special case of the imperfect matching problem when system is idle during the downtime.

Table 4.1 Data set for computation of Problem IM

Parameters	*Problem 1*	*Problem 2*	*Problem 3*	*Problem 4*	*Problem 5*	*Problem 6*
P (units/year)	3,600	3,600	6,000	7,000	8,000	11,000
D_F (units/year)	2,400	2,400	3,000	5,200	5,200	7,200
K_0 ($/order)	150	100	150	200	200	300
K_S ($/setup)	50	100	60	70	200	250
H_R ($/unit/year)	1	10	3.5	4	4	10.5
H_F ($/unit/year)	2	10	5	15	25	45
f	2	3	3	2.5	3	4
x(units/shipment)	100	100	150	150	200	150
I_0	25	30	50	40	60	55
T_S (years)	0.001	0.002	0.002	0.003	0.005	0.006

Table 4.2 Results for imperfect matching conditions

Parameters	*Problem 1*	*Problem 2*	*Problem 3*	*Problem 4*	*Problem 5*	*Problem 6*
Q'^*	661	296	483	405	365	367
n^*	6	2	3	2	1	1
m^*	2	1	2	1	1	1
$TC_I^*(Q'^*, m^*)$	$1,610.48	$4,154.74	$3,344.32	$8,601.73	$14,966.41	$25,871.18

4.7 SPECIAL CASE FOR IMPERFECT MATCHING INVENTORY

As discussed in Section 3.7, when plant is idle during the downtime period only (Figure 4.2), the inventory model transforms to the imperfect matching model with idle time. Therefore, if $T_S \to T_S + T_D = T_S + T_S + T_P = 2T_S + nx/P = 2T_S + (Q'_F - I_0)/P$, the finished goods inventory for this special case becomes

$$\hat{I}_{IT} = \frac{Q_F'^2}{2D_F} + \frac{Q'_F}{2D_F}(4I_0 + x - 2D_F T_S - D_F(Q'_F - I_0)/P)$$

$$-\frac{I_0}{2D_F}(I_0 + x - 2D_F T_S - D_F(Q'_F - I_0)/P). \qquad (4.24)$$

Similarly, the total cost function expressed in Equation (4.12) transforms to

$$TC_{IS}(Q'_F, m) = \frac{Q_F'^2 H_R}{2mfP} + \frac{mD_F K_0}{Q'_F} + \frac{1}{Q'_F}\left\{D_F K_S - \frac{I_0 H_F}{2}\left(I_0 + x - 2D_F T_S\right)\right\}$$

$$+\frac{Q'_F H_F}{2}\left(1 - \frac{D_F}{P}\right) + \frac{H_F}{2}\left\{4I_0 + x + D_F\left(\frac{I_0}{P} - 2T_S\right)\right\}, \tag{4.25}$$

which is the modified Parija and Sarker (1994) model (deferred production).

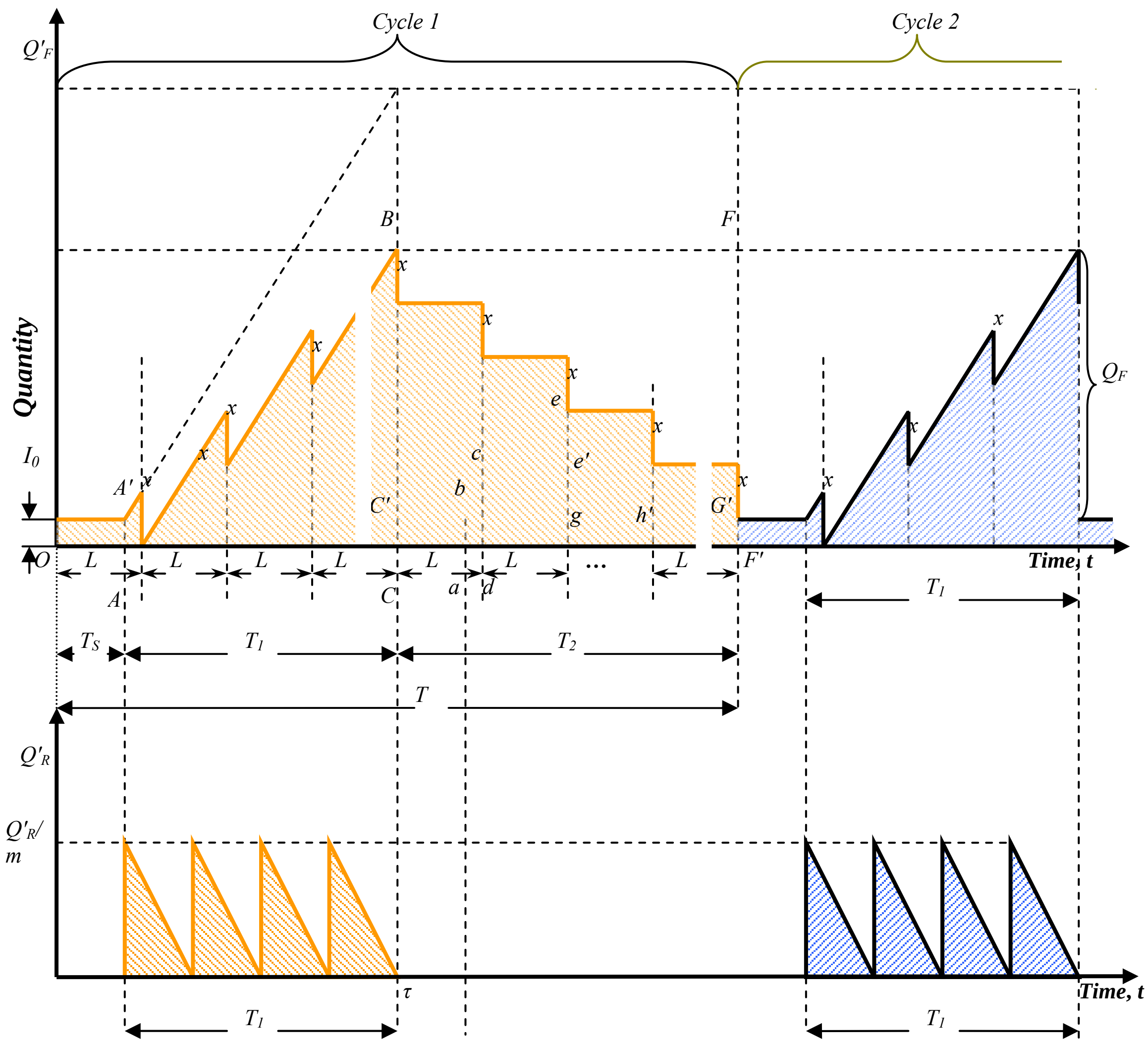


Figure 4.2. (a) Finished products inventory with idle time; (b) raw material inventory

Again, replacing $Q'_F = nx + I_0$, Equation (4.5) can be rewritten as

$$\hat{I}_{IT} = \frac{(nx + I_0)^2}{2D_F} + \frac{(nx + I_0)}{2D_F}(4I_0 + x - 2D_F T_S - nxD_F / P)$$

$$-\frac{I_0}{2D_F}(I_0 + x - 2D_F T_S - nxD_F / P)\,. \qquad (4.26)$$

Let Figure 4.1 represents the 'accelerated production' and Figure 4.2 presents the 'deferred production.' If the numerical data from Table 4.1 are applied to the Equations (4.24), and (4.26), a comparison between 'deferred production' and 'accelerated production,' respectively, can be presented in Table 4.3, based on the idle time and number of items produced during the cycle time.

Table 4.3 Comparison of quantity produced in accelerated and deferred production

Problems	Accelerated			Deferred			Accelerated	Deferred
	Cycle time (years)	Quantity produced (units)	Idle time (years)	Cycle time (years)	Quantity produced (units)	Idle time (years)	Quantity Produced (units/year)	Quantity Produced (units/year)
1	0.167	54.49	0.001	0.333	85.83	0.165	326	258
2	0.167	55.58	0.002	0.333	91.06	0.164	333	273
3	0.200	114.14	0.002	0.400	200.88	0.198	571	502
4	0.154	196.92	0.003	0.308	374.84	0.151	1279	1217
5	0.231	399.15	0.005	0.462	724.35	0.226	1728	1568
6	0.194	529.86	0.006	0.389	958.31	0.189	2731	2464

4.8 Single Facility Lot-Sizing Models for Imperfect Matching

In this section, the rotation cycle policy for single facility lot-sizing model for the imperfect matching situation is discussed. Also, the rotation cycle is applied on the special case of imperfect matching system described in the Section 4.7 to satisfy customers' demand as well as minimize the idle time of the system. As described in Section 3.8, this section also considers a single production facility that produces K products with a constant demand of D_{Fk} units per year for product k (where k = 1, 2, …,K), and k product is produced at a constant rate of P_k units per year to satisfy the demand D_{Fk}. All products are delivered at a fixed amount of x_k units after every L_k time units. According to the assumption, production of all k items must meet customers' demand and $\sum_{k=1}^{K}(D_{Fk} / P_k) \le 1$. Also, due to rotation cycle policy, all products with

the same production cycle time, T_C, and a lot of each product is produced during this time period. Due to the rotation, the products are produced in a fixed order, which is repeated from cycle to cycle. Now, the lot size for product j must equal the demand during the production cycle without permitting shortages as

$$Q'_{Fk} = (m_k x_k + I_0) = T_C D_{Fk} . \tag{4.27}$$

Applying Equation (4.27) in Equation (4.25), it can be written as

$$\begin{aligned} C_{IR}(T_C, m_1, ..., m_K) = \sum_{k=1}^{K} \Bigg[& \frac{T_C^2 D_{Fk}^2}{m_k} \left(\frac{H_{Rk}}{2 f_k P_k} \right) + \frac{m_k K_{0k}}{T_C} + \frac{T_C D_{Fk} H_{Fk}}{2} \left(1 - \frac{D_{Fk}}{P_k} \right) \\ & + \frac{1}{T_C} \left\{ K_{Sk} - \frac{I_{0k} H_{Fk}}{2 D_{Fk}} \left(I_{0k} + x_k - 2 D_{Fk} T_{Sk} \right) \right\} \\ & + \frac{H_{Fk}}{2} \left\{ 4 I_{0k} + x_k + D_{Fk} \left(\frac{I_{0k}}{P_k} - 2 T_{Sk} \right) \right\} \Bigg] . \end{aligned} \tag{4.28}$$

Equation (4.28) can be rewritten by replacing the constant terms as follows:

$$C_{IR}(T_C, m_1, ..., m_K) = \sum_{m=1}^{K} \left[B_{ak} T_C^2 / m_k + B_{bk} m_k / T_C + B_{ck} T_C + B_{dk} / T_C + B_{ek} \right], \tag{4.29}$$

where $B_{ak} = D_{Fk}^2 \left(\dfrac{H_{Rk}}{2 f_k P_k} \right)$,

$$B_{bk} = K_{0k},$$

$$B_{ck} = \frac{D_{Fk} H_{Fk}}{2} \left(1 - \frac{D_{Fk}}{P_k} \right),$$

$$B_{dk} = K_{Sk} - \frac{I_{0k} H_{Fk}}{2 D_{Fk}} \left(I_{0k} + x_k - 2 D_{Fk} T_{Sk} \right), \text{ and,}$$

$$B_{ek} = \frac{H_{Fk}}{2} \left\{ 4 I_{0k} + x_k + D_{Fk} \left(\frac{I_{0k}}{P_k} - 2 T_S \right) \right\}.$$

This rotation cycle problem is restricted by two constraints, (1) Rotation Cycle time, T_C, must be greater than or equal to the cumulative setup times and production time for all products which are produced in the facility, and (2) the number of orders of raw material must be greater than or equal to 1. Using these two constraints, the problem can be formulated as

$$C_{IR} = \sum_{k=1}^{K} \left[B_{ak} T_C^2 / m_k + B_{bk} m_k / T_C + B_{ck} T_C + B_{dk} / T_C + B_{ek} \right], \tag{4.30}$$

Subject to:

$$T_C \geq \frac{\sum_{k=1}^{K} T_{Sj}}{1 - \sum_{k=1}^{K} \left[D_{Fk} / P_k \right]} \equiv T_{\min} \geq 0, \tag{4.30a}$$

$$m_k \geq 1 \text{ and is an integer,} \qquad \text{for } k = 1, 2, ..., K. \tag{4.30b}$$

Therefore, the problem becomes a mixed integer non-linear programming problem and the solution procedure to this problem is discussed in the next section.

4.9 Solution Technique of Rotation Cycle

The formulation of the single facility lot-sizing problem for imperfect matching system can be categorized as a mixed-integer non linear programming problem where m_k's are integer and T_C is a real variable and the number of variables are (K + 1). Due to formulation of the problem, it cannot be solved using derivatives and a closed form solution cannot be determined. As discussed in Section 3.9, the objective function is divided in to two parts (a) rotation cycle for finished products, and (b) number of raw material orders. The rotation cycle for the finished products (T_C) is the same for the raw material delivery, because the raw materials are delivered from the supplier by instantaneous replenishments. Again, the raw material for a product k is ordered when the finished product k goes in production. The solution procedures are shown as follows:

(a) Rotation cycle for finished products

To solve the rotation cycle policy for the part finished product supply, the cost function from Equation (4.28) can be divided as

$$\text{Minimize: } C_{IRF}(T_C) = \sum_{k=1}^{K}\left[\frac{T_C D_{Fk} H_{Fk}}{2}\left(1-\frac{D_{Fk}}{P_k}\right)\right.$$

$$+\frac{1}{T_C}\left\{K_{Sk} - \frac{I_{0k} H_{Fk}}{2D_{Fk}}\left(I_{0k} + x_k - 2D_{Fk}T_{Sk}\right)\right\}$$

$$\left.+\frac{H_{Fk}}{2}\left\{4I_{0k} + x_k + D_{Fk}\left(\frac{I_{0k}}{P_k} - 2T_{Sk}\right)\right\}\right], \qquad (4.31)$$

Subject to: $T_C \geq T_{\min} \geq 0$ (4.31a)

It can be shown that the Equation (4.31) is a convex function for T_C; therefore, it can be solved by differentiation with respect to T_C and equate it to zero as follows:

$$\frac{dC_{IRF}(T_C)}{dT_C} = \sum_{k=1}^{K}\left[\frac{D_{Fk} H_{Fk}}{2}\left(1-\frac{D_{Fk}}{P_k}\right)\right.$$

$$\left.-\frac{1}{T_C^2}\left\{K_{Sk} - \frac{I_{0k} H_{Fk}}{2D_{Fk}}\left(I_{0k} + x_k - 2D_{Fk}T_{Sk}\right)\right\}\right] = 0, \qquad (4.32)$$

which yields

$$T_C^* = \sqrt{\frac{2\sum_{k=1}^{K}\left[K_{Sk} - \frac{I_{0k} H_{Fk}}{2D_{Fk}}\left(I_{0k} + x_k - 2D_{Fk}T_{Sk}\right)\right]}{\sum_{k=1}^{K} D_{Fk} H_{Fk}\left(1 - D_{Fk}/P_k\right)}}, \qquad (4.33)$$

where $k = 1, \ldots, K$.

Equation (4.33) has to satisfy the constraint given in Equation (4.31a). Using the optimal rotation cycle T_C^*, the number of shipments for different finished product can be obtained from

Equation (4.27). The optimal rotation cycle, T_C^* is used to solve the optimal number of orders for raw materials in following section.

(b) Number of raw material orders

As the raw materials order policy is instantaneous, the production rate for the raw material is ∞; therefore, this also satisfies the condition for rotation cycle. Now, applying the value of T_C^* from Equation (4.33), the total cost/objective function for raw material k can be written as [from Equation (4.28)]

Minimize: $$C_{IRR}(m_k) = \frac{m_k K_{0k}}{T_C^*} + \frac{T_C^{*2} D_{Fk}^2}{m_k}\left(\frac{H_{Rk}}{2 f_k P_k}\right) \tag{4.34}$$

Subject to: $m_k \geq 1$ and integer, $\forall k = 1,\ldots,K.$ (4.34a)

This objective function [Equation (4.34)] is convex in m_k and the objective function is a discrete function, which cannot be solved using differentiation. Hence, the induction method is used to solve m_k. Using the induction method in Equation (4.34), the boundary condition for m_k^* is can be evaluated as

$$\left\lceil \frac{1}{2}\left(\sqrt{1+4\Delta_k} - 1\right) \right\rceil \leq m_k^* \leq \left\lfloor \frac{1}{2}\left(\sqrt{1+4\Delta_k} + 1\right) \right\rfloor, \tag{4.35}$$

where $\Delta_k = \dfrac{T_C^{*3} D_{Fk}^2 H_{Rk}}{2 f_k P_k K_{0k}}$, and $k = 1,2,\ldots,K.$. In addition, Equation (4.35) has to satisfy the constraint given in Equation (4.34a). Applying the boundary condition in Equation (4.34) the optimal objective function can be evaluated as well as the optimum number of orders m_k^* for raw material k, where $k = 1, \ldots, K$.

Hence, optimum total cost for *all* raw materials can be expressed as

$$C_{IRR}(m_1^*,\ldots,m_K^*) = \sum_{k=1}^{K} \frac{m_k^* K_{0k}}{T_C^*} + \frac{T_C^{*2} D_{Fk}^2}{m_k^*}\left(\frac{H_{Rk}}{2 f_k P_k}\right). \tag{4.36}$$

As discussed before, both m_k^* and T_C^* is not globally optimal. Therefore, another forward search is conducted using Equation (4.28), starting from the constraints for T_C^* and m_k^* [given in Equations (4.30a) and (4.30b)] and with step sizes 0.01 and 1, respectively, to evaluate the optimal T_C^{opt} and m_k^{opt} that will minimize the $C_{IR}(T_C^{opt}, m_1^{opt}, ..., m_K^{opt})$.

Example 4.2: Rotation Cycle and Total Cost

Consider six products are being produced in a single facility manufacturing system. The respective parameters for all six products are presented in Table 4.4. Using these data and Equation (4.33), the T_C^* can be found as

$$T_C^* = \sqrt{\frac{2 \times 259.59}{1694.23}} = 0.56 \text{ years.} \tag{4.37}$$

Now using the value of T_C^* in Equation (4.35) the boundary conditions for m_k^* can be found as

$$\lceil 0.04 \rceil \le m_1^* \le \lfloor 1.04 \rfloor, \lceil 0.05 \rceil \le m_2^* \le \lfloor 1.05 \rfloor, \lceil 0.03 \rceil \le m_3^* \le \lfloor 1.03 \rfloor,$$

$$\lceil 0.07 \rceil \le m_4^* \le \lfloor 1.07 \rfloor, \lceil 0.07 \rceil \le m_5^* \le \lfloor 1.07 \rfloor, \text{ and } \lceil 0.11 \rceil \le m_6^* \le \lfloor 1.11 \rfloor, \tag{4.38}$$

that yields

$$m_1^* = 1, m_2^* = 1, m_3^* = 1, m_4^* = 1, m_5^* = 1, \text{ and } m_6^* = 1. \tag{4.39}$$

Using these values the total costs can be found as $C_{IR}(T_C^*, m_1^*, m_2^*, m_3^*, m_4^*, m_5^*, m_6^*)$ $= (0.56, 1, 1, 1, 1, 1, 1)$ $= \$33,928.26$ per year, and this is local optimum solution. Therefore, a forward search is conducted starting from T_C^*= 0.21 (with step size 0.01), and m_k^* = 1 (with step size 1) and the optimum solution is obtained in $C_{IR}(T_C^{opt}, m_1^{opt}, m_2^{opt}, m_3^{opt}, m_4^{opt}, m_5^{opt}, m_6^{opt})$ $= (0.32, 1, 1, 1, 1, 1, 1)$ $= \$32,373.85$ per year.

The detailed results of rotation cycle policy are presented with numerical values in the following section.

4.10 NUMERICAL COMPUTATION OF OPTIMUM ROTATION CYCLE

In this section, an optimum rotation cycle and number of orders are determined using a set of numerical data for six products and assuming $\sum_{k=1}^{6}(D_{Fk}/P_k) \le 1$. The dataset is presented in Table 4.4. In this case, it is considered that all six products are produced in a single facility in a sequence and they will be delivered using just-in-time (JIT) policy. Also, the raw materials for each product will be ordered following multiple ordering policies.

Table 4.4 Data set for single facility lot-sizing model

Parameters	*Product 1*	*Product 2*	*Product 3*	*Product 4*	*Product 5*	*Product 6*
P (units/year)	14,000	10,500	15,000	10,000	9,000	20,000
D_F (units/year)	2,000	1,500	3,000	1,800	1,200	2,200
K_0 (\$/order)	150	100	150	200	200	300
K_S (\$/setup)	50	100	120	130	200	150
H_R (\$/unit/year)	1	10	3.5	4	4	10.5
H_F (\$/unit/year)	2	10	5	15	25	45
f	2	3	3	2.5	3	4
x (units)	100	100	150	200	300	350
I_0 (units)	25	30	50	40	60	55
T_s (years)	0.001	0.002	0.002	0.003	0.005	0.006

According to the constraint given in Equations (4.30a) or (4.31a), it can be determined by using the data given in Table 4.3 that

$$T_C \ge \frac{0.001+0.002+0.002+0.003+0.005+0.006}{\left[1-\left(\frac{2,000}{14,000}+\frac{1,500}{10,500}+\frac{3,000}{15,000}+\frac{1,800}{10,000}+\frac{1,200}{9,000}+\frac{2,200}{20,000}\right)\right]}=\frac{0.019}{0.09}=0.21. \quad (4.40)$$

It is also observed that

$$\sum_{k=1}^{6}\left(D_{Fk}/P_k\right)=\left(\frac{2,000}{14,000}+\frac{1,500}{10,500}+\frac{3,000}{15,000}+\frac{1,800}{10,000}+\frac{1,200}{9,000}+\frac{2,200}{20,000}\right)=0.91\le 1 \quad (4.41)$$

which satisfies the assumption for rotation cycle policy.

The detailed results for the single facility lot sizing models for imperfect matching case are presented in Example 4.2 and Table 4.5.

Table 4.5 Optimum results for raw materials of imperfect matching case

Parameters	**Results**					
	Product 1	***Product 2***	***Product 3***	***Product 4***	***Product 5***	***Product 6***
T_C^*, years	0.32					
m_k^*	1	2	2	1	1	1
n^*	6	4	6	2	1	1
$Q_{Fk}'^*$, units/year	600	400	900	400	300	350
$Q_{Rk}'^*$, units/year	300	133	300	160	100	88
$TC_P^*(T_r^*, m_k^*)$	\$32,373.85					

Thus, this chapter concludes the research for imperfect matching situation when the system idle time is negligible. The next chapter deals with multi-supplier and multi-buyer problem under just-in-time (JIT) delivery policy and the system idle time is minimal as before.

CHAPTER 5

SUPPLY CHAIN OF MULTIPLE SUPPLIERS AND BUYERS

This chapter deals with a supply chain system where a manufacturer procures raw materials from multiple vendors, converts them to finished products at a finite production rate, and finally ships the finished products to multiple customers or buyers. Each supplier supplies a different set of items to the manufacturer so that there is no competition between the suppliers. The buyers are dispersed geographically and each has individual demands of finished products. The quantity and interval of shipments to each buyer are fixed for an infinite planning horizon. Also, the raw materials ordering and holding costs from each supplier are unique and different from each other. The manufacturing system operates under the minimal downtime policy (which has been discussed in previous chapters), which effectively means that the production of succeeding cycle starts immediately after the production of preceding cycle and setup time.

Generally, the manufacturer carries two types of inventories; (a) raw material inventory, and (b) finished goods inventory. The work-in-process inventories are negligible as compared to other inventories. The system cost consists of ordering cost and holding cost of raw materials, production setup cost and holding cost of finished products. The main problems in this chapter are to determine the manufacturing batch size and the number of orders of raw materials by minimizing of the system cost, when the production of the system remains inoperative only during the setup time.

A supply chain that consists of suppliers and retailers has been addressed in the literature as a class of two-echelon, warehouse-retailer distribution system with infinite replenishment rate [Askin and Goldberg (2002)]. In 1990, Axsäter studied the replenishment policies for the case of a one-warehouse, *N*-retailer system. Axsäter's model determined a

recursive solution procedure for the order-up-to inventory position at the warehouse and each retailer with stochastic demand to minimize the long-run system cost. McGavin *et al*. (1993) dealt with the case of a one-warehouse and *N*-identical retailer system. They examined a two-interval policy of withdrawals from warehouse to retailers where the first withdrawal occurs immediately after replenishment from outside suppliers and second withdrawal on the second interval ships the remaining stock at the warehouse. Schwarz *et al*. (1985) aimed at maximizing the fill rate by determining safety stock at the warehouse and retailers. The model studied by Chen *et al*. (2001) combined pricing and replenishment strategies to maximize profits by means of optimizing the prices given in each retailer.

This part of the research deals with a supply chain system with two-echelon inventory having finite production rate and multiple replenishments of raw materials. A few researchers have studied this class of problem. In 1994, Sarker and Parija dealt with the stated problem with a single-buyer and single-supplier system. Later Parija and Sarker (1999) extended this model to a multi-retailer system. They introduced the problem of determining the production start time and proposed a method that determines the cycle length and raw material order frequency for a long-range planning horizon.

The cycle length is restricted to be an integer-multiple of all shipment intervals to the buyers, which may be sub-optimal. Lu (1995) developed a one-vendor, multi-buyer, integrated inventory model while Goyal (1995), Goyal and Gupta (1989), and Aderohunmu *et al*. (1995) developed models for joint vendor-buyer policy in a just-in-time (JIT) manufacturing environment without considering the raw material related costs.

Since periodic demands create the total inventory cost to be a piecewise convex function in manufactured quantity, Park and Yun (1984) proposed a stepwise partial enumeration algorithm for solving such economic lot scheduling problems. Pan and Liao

(1989) and Ramasesh (1990) developed optimal orders and production quantity models for a single-echelon production system. Related research on supplying finished goods to a customer at fixed-time intervals with carried over products to the next cycle was first raised by Hill (1995, 1996). In 2003, Diponegoro studied an exact analytical method to obtain an optimal policy for a more general class of problem with multiple suppliers, non-identical buyers, finite production rate and finite planning horizon. In the current research, the system idle time is minimized so that the system can produce more with a finite production rate. Also, an analytical method is presented to determine the optimum number of shipments from different suppliers and optimum number of deliveries to non-identical multiple customers or retailers for an infinite planning horizon.

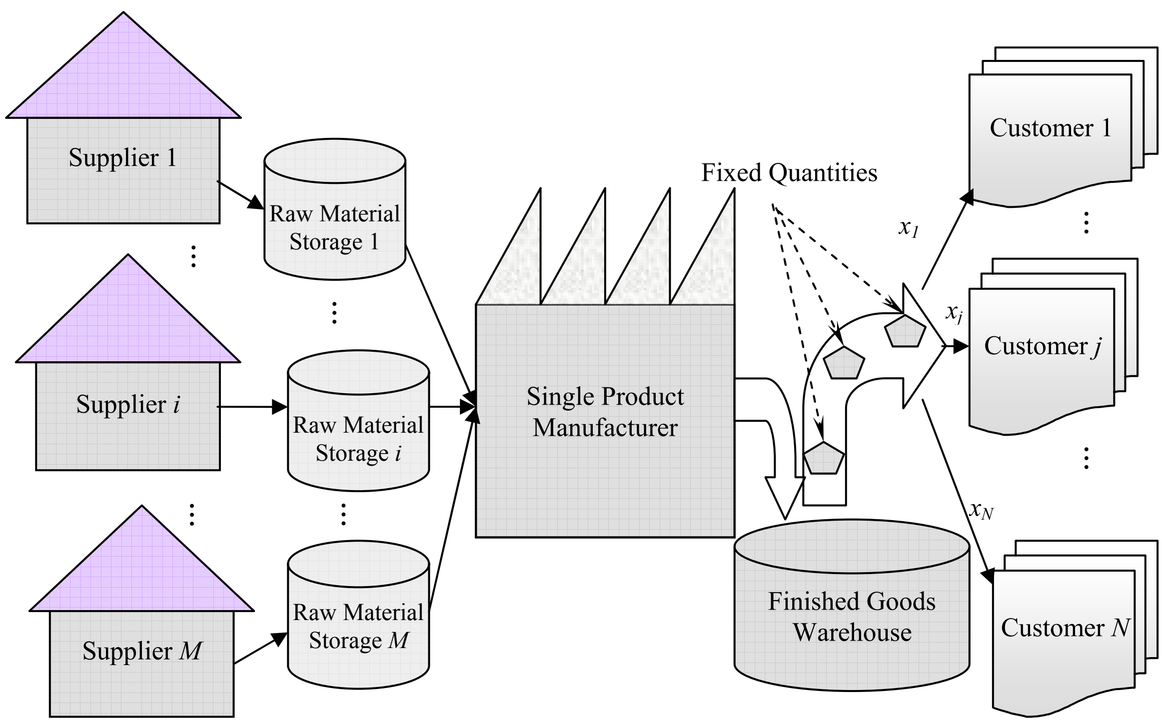
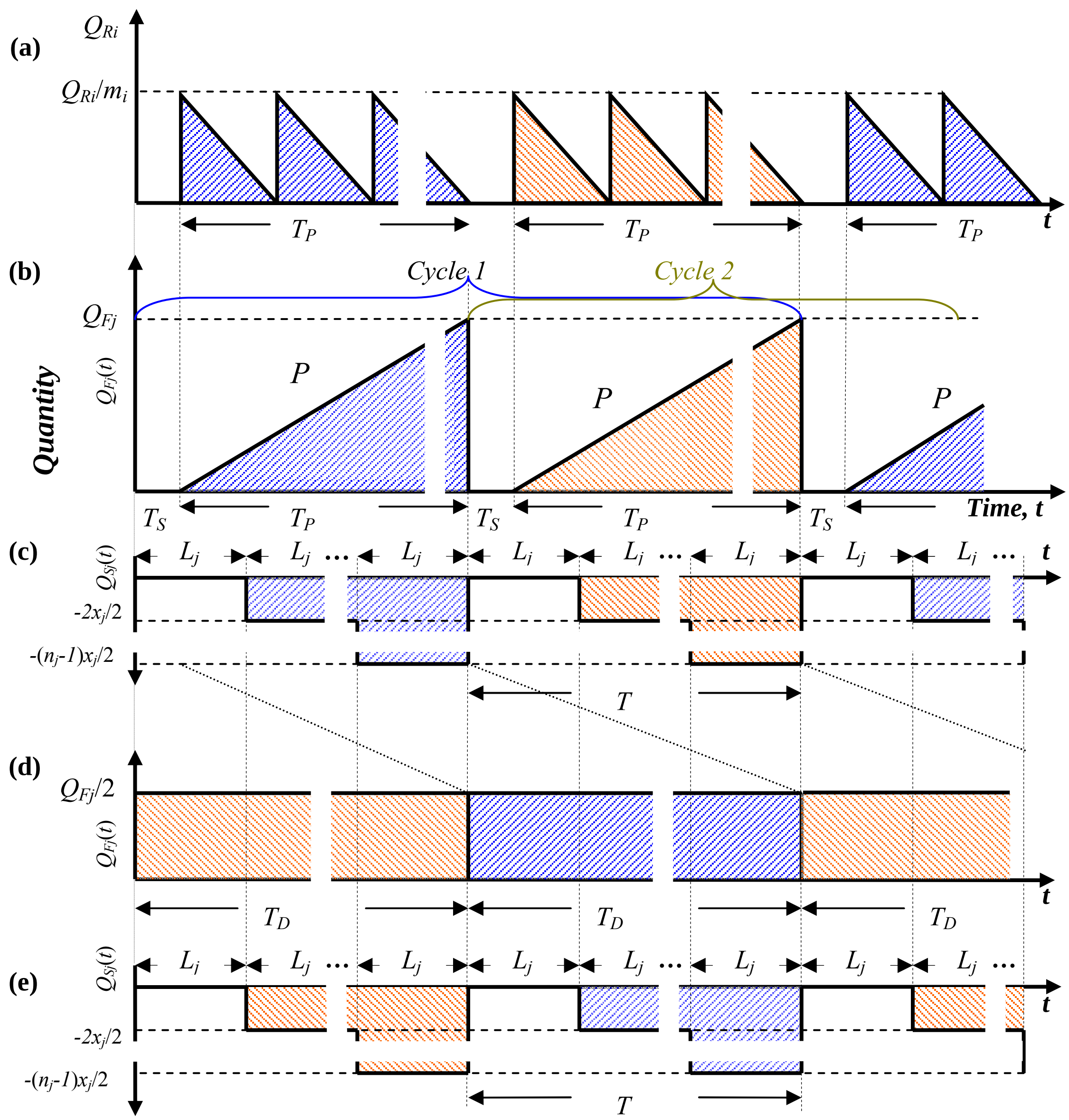


Figure 5.1. Multi-supplier and multi-buyer supply chain system

5.1 Problem Description for Single Product, Multi-Supplier and Multi-Buyer

A manufacturing facility procures raw materials from suppliers, converts them into finished goods and delivers the finish products to the buyers according to their demand (Figure

5.1). Also, the production cycle of the system starts immediately after the production of the previous cycle which reduces the system idle time.



(a) Raw material inventory from supplier i, (b) Finished goods produced; (c) Inventory shipped during uptime to buyer j; (d) Inventory during downtime; (e) Inventory shipped during downtime to buyer j.

Figure 5.2. Inventory diagrams for a multi-supplier-and-buyer system

The supply chain system (Figure 5.1) consists of M non-competing suppliers $\{i = 1, 2, \ldots, M\}$, a manufacturer and N non-identical buyers $\{j = 1, 2, \ldots, N\}$, over an infinite planning

horizon. The products are manufactured at a finite and constant rate P(units/year), and shipped to j^{th} buyer $\{j = 1, 2, \ldots, N\}$ every L_j time interval with a fixed shipment size of x_j. The total number of shipments made during the time period T to the buyer j is n_j, such that $n_j x_j = D_{Fj}$, where D_{Fj} is the demand of finished product for j^{th} buyer.

In this case, the finished products are delivered at regular intervals and these intervals may vary from customer to customer. The inventory build-up is presented in Figure 5.2, where it can be observed that the production of the system starts T_S time period after the end of the production of the previous cycle. Figures 5.2 (a), (b), (c), (d) and (e) represent the ordered quantity from supplier i during uptime T_P in multiple installments, the uptime (T_P) inventory, quantity shipped during uptime (T_P) to buyer j, remaining inventory during down time (T_D), and quantity shipped during uptime (T_D) to buyer j, respectively. As discussed in previous chapters, it is observed from the Figure 5.2 that the downtime of cycle 1 overlaps the uptime of cycle 2, and in both uptime and downtime situations $n_j x_j/2$ units are delivered to buyer j. Therefore, the total units are shipped to the customer j is $n_j x_j$ (= D_{Fj}) during the time period T and that satisfies the buyer j.

5.2 Notation

The notation used to construct the cost function for the multi-supplier and multi-buyer supply chain system are as follows:

D_F : Cumulative demand of finished goods from all N buyers, units/year.

D_{Ri} : Demand for raw materials i, units/year.

f_i : Quantities of raw materials required from supplier i per unit of finished products.

H_F : Holding cost of finished goods, \$/units/year.

H_{Ri} : Holding cost of raw materials from i^{th} supplier, \$/units/year.

I_F : Total finished goods inventory, units.

$\overline{I}_F$: Average finished goods inventory, units.

I_{Ri} : Total raw materials inventory from i^{th} supplier, units.

$\overline{I}_{Ri}$: Average raw materials inventory i^{th} supplier, units.

K_{0i} : Ordering cost of raw material from supplier i, \$/order.

K_S : Manufacturing setup cost, \$/batch.

L_j : Time between successive shipments of finished goods to j^{th} buyer, years

m_i : Number of orders for raw materials from supplier i.

M : Number of suppliers.

n_j : Number of *full* shipment of finished goods per cycle time to j^{th} buyer.

N : Number of buyers.

P : Production rate, units/year.

Q_F : Quantity of finished goods manufactures per setup, units/batch, $Q_F = \sum_{j=1}^{N} n_j x_j$.

Q_{Fj} : Quantity of finished goods delivered to buyer j, units/batch, $Q_{Fj} = n_j x_j$.

Q_{Ri} : Quantity of raw materials required for each batch from supplier i.

T_P : Production time (uptime), years; $T_P = Q_F / P = n_j x_j / P$.

T_D : Consumption time, years (downtime);

T : Total cycle time, years; $T = n_j L_j$.

T_S : Setup time, years; $T_S < L_j$.

$TC_{MF}(n_j)$: Total cost function of finished goods for buyer j, \$/year, where $j = 1, \ldots, N$.

$TC_{MR}(m_i)$: Total cost of raw materials, from suppliers i, \$/year, where $i = 1, \ldots, M$.

$TC_M(m_i, n_j)$: Total system cost, \$/year, where $i = 1, \ldots, M$, and $j = 1, \ldots, N$.

x_j : Fixed quantity of finished goods per shipment at a fixed interval of time L_j to j^{th} buyer.

5.3 Total Cost Function for Raw Materials

In this part of the research, it is considered that the raw materials are delivered in multiple installments from M different suppliers [Figures 5.1 and 5.2 (a)]. It is assumed that the each supplier supplies a unique raw material and they are non-competing suppliers. There are M buffers for the raw material storage from M suppliers. Also, if f_i units of raw material i are required to produce one finished good; then the total raw materials required to produce one finished product are $\sum_{i=1}^{M} f_i Q_{Ri} = Q_F = \sum_{j=1}^{N} Q_{Fj}$. Again, the produced finished goods Q_F are supplied to N customers as Q_{F1}, …, Q_{FN} units over the time period. Recalling that f_iQ_{Ri} are procured in m_i replenishments of equal quantities at equal interval of time T_P, the time-weighted inventory of i^{th} types of raw materials (I_{Ri}) is

$$I_{Ri} = f_i Q_{Ri} T_P / 2m_i = f_i Q_{Ri} Q_F / 2m_i P. \tag{5.1}$$

Considering that the raw material i is supplied from the supplier i, so that the total cost function for raw material i is

$$TC_{MRi}(m_i) = \frac{m_i D_{Ri} K_{0i}}{Q_{Ri}} + \frac{f_i H_{Ri} Q_{Ri} Q_F}{2m_i P}. \tag{5.2}$$

Again, from Equation (5.1), the total time-weighted raw material inventory (I_{Ri}) required for Q_F finished goods are

$$\sum_{i=1}^{M} I_{Ri} = \sum_{i=1}^{M} f_i Q_{Ri} Q_F / 2m_i P = \frac{1}{2P} \sum_{i=1}^{M} \frac{f_i Q_{Ri}}{m_i} \sum_{i=1}^{M} f_i Q_{Ri}. \tag{5.3}$$

where $\sum_{i=1}^{M} f_i Q_{Ri} = Q_F$.

Therefore, the total cost function for the raw materials (TC_{MR}) is

$$TC_{MR}(m_1,...,m_M) = \sum_{i=1}^{M} \frac{m_i D_{Ri} K_{0i}}{Q_{Ri}} + \frac{1}{2P} \sum_{i=1}^{M} \frac{f_i Q_{Ri}}{m_i} \sum_{i=1}^{M} f_i Q_{Ri}, \tag{5.4}$$

where i = 1, 2, …, M.

From Equation (5.4) it can be observed that the total cost function for raw material is the function of m_i (i = 1, 2, …, M), and Q_{Ri}. As the Q_{Ri} is strictly dependent on Q_F, and Q_F^* can be evaluated by solving the total cost function of the finished products. Therefore, the objective function for multi-supplier (MS) for different raw materials supply is

Problem MS: Find $m_1,...,m_M$, by

$$\text{Minimize } TC_{MR}(m_1,...,m_M) = \sum_{i=1}^{M} \frac{m_i D_{Ri} K_{0i}}{Q_{Ri}} + \frac{1}{2P} \sum_{i=1}^{M} \frac{f_i Q_{Ri}}{m_i} \sum_{i=1}^{M} f_i Q_{Ri} \tag{5.5}$$

Subject to: $m_1,...,m_M \geq 1$ and are integer. (5.5a)

Therefore, it is required to find the total cost function for finished product delivery for multi-buyer with just-in-time (JIT) policy.

5.4 Total Cost Function for Finished Products

It is considered that there are N (j =1, 2, …, N) number of buyers and each of them has their own demand (D_{Fj}) for the finished products. Therefore, the total demand for the finished product for all N buyers is $D_F = \sum_{j=1}^{N} D_{Fj}$. As discussed earlier, that the fixed shipment time for j^{th} buyer is L_j where j =1, 2, …, N. The step-by-step finished goods inventory of the production system for buyer j is represented in Figures 5.2 (b), (c), (d), and (e). The finished goods production of a cycle begins at a finite rate P after the end of production of the previous cycle and setup time T_S [Figure 5.2 (b)]. The production continues untill T_P time period and the inventory builds up. During this T_P time period buyer j has a demand of x_j at the end of every L_j time period. According to the Figure 5.2 (c), from the uptime inventory of cycle 1 $x_j/2$ units are delivered every L_j time period in n_j number of shipments. From the downtime T_D inventory [Figures 5.2 (d) and (e)] the other $x_j/2$ units are delivered during the same L_j time period in n_j

number of shipments. Therefore, during the cycle time T, the total delivery of the finished products is $n_j x_j$ units, which satisfies the demand D_{Fj} for j^{th} buyer. It can be observed that the uptime of a cycle overlaps the downtime of previous cycle throughout the planning horizon, the average inventory is the same in every T time period. Again, $n_j x_j = Q_{Fj}$ units are produced in each cycle for buyer j. Hence, the total finished products are produced for all N buyers are $Q_F = \sum_{j=1}^{N} n_j x_j$ units.

Now, from Figures (b), (c), (d) and (e) the finished goods inventory for buyer j can be computed as

$$I_{Fj} = \frac{n_j^2 x_j^2}{2D_{Fj}} + n_j x_j \left(\frac{x_j}{2D_{Fj}} - \frac{T_S}{2} \right). \tag{5.6}$$

Again, the cycle time T can be computed as

$$T = Q_{Fj} / D_{Fj} = n_j x_j / D_{Fj} = n_j L_j \quad \forall\ i = 1, 2, ..., N. \tag{5.7}$$

Using Equations (5.6) and (5.7) the average inventory for buyer j can be evaluated as

$$\begin{aligned} \overline{I}_{Fj} = I_{Fj} / T \quad &= \frac{D_{Fj}}{n_j x_j} \left[\frac{n_j^2 x_j^2}{2D_{Fj}} + n_j x_j \left(\frac{x_j}{2D_{Fj}} - \frac{T_S}{2} \right) \right] \\ &= \frac{1}{2} \left[n_j x_j + x_j - D_{Fj} T_S \right]. \end{aligned} \tag{5.8}$$

Hence, the total cost function for buyer j can be found as

$$\begin{aligned} TC_{MFj}(n_j) &= \frac{D_{Fj}}{Q_{Fj}} K_S + H_F \overline{I}_{Fj} \\ &= \frac{D_{Fj}}{n_j x_j} K_S + \frac{H_F}{2} \left[n_j x_j + x_j - D_{Fj} T_S \right], \end{aligned} \tag{5.9}$$

where $Q_{Fj} = n_j x_j$.

As a result, the total finished goods inventory for all N buyers can be found as

$$TC_{MF}(n_1,...,n_N)=\sum_{j=1}^{N}TC_{MFj}$$

$$=\sum_{j=1}^{N}\left[\frac{D_{Fj}}{n_j x_j}K_S+\frac{H_F}{2}(n_j x_j+x_j-D_{Fj}T_S)\right], \quad (5.10)$$

which is a function of n_j and $j = 1, \ldots, N$.

The manufacturer needs to decide the number of orders for different raw materials required to produce the amount of finished product $Q_F=\sum_{j=1}^{N}Q_{Fj}=\sum_{j=1}^{N}n_j x_j$ by solving the Equation (5.10), which has the decision variables $n_1,n_2,...,n_N$ for all N buyers.

From Equation (5.10) the total cost function for multi-buyer supply chain delivery problem (MB) is defined as integer non-linear programming (INP) problem:

Problem MB: Find $n_1,n_2,...,n_N$ so as to

Minimize $$TC_{MF}(n_1,..,n_N)=\sum_{j=1}^{N}\left[\frac{D_{Fj}}{n_j x_j}K_S+\frac{H_F}{2}(n_j x_j+x_j-D_{Fj}T_S)\right], \quad (5.11)$$

Subject to: $n_1,n_2,...,n_N\geq 1$ and are integer. (5.11a)

Hence, the total system cost for multiple suppliers and buyers supply chain system can be evaluated by adding Equations (5.5) and (5.11):

$$TC_M(m_i,n_j)=\sum_{i=1}^{M}\frac{m_i D_{Ri}K_{0i}}{Q_{Ri}}+\frac{1}{2P}\sum_{i=1}^{M}\frac{f_i Q_{Ri}}{m_i}\sum_{i=1}^{M}f_i Q_{Ri}$$

$$+\sum_{j=1}^{N}\left[\frac{D_{Fj}}{n_j x_j}K_S+\frac{H_F}{2}(n_j x_j+x_j-D_{Fj}T_S)\right], \quad (5.12)$$

Subject to: $n_j\geq m_i\geq 1,$ and is an integer, for $i=1,...,M$, and $j=1,...,N$. (5.12a)

The next section discusses the solution procedure for the multi-supplier and multi-buyer supply chain system with just-in-time (JIT) delivery.

5.5 SOLUTION TECHNIQUES

In previous sections, the total cost functions are developed for multi-buyer and multi-supplier supply chain problem with just-in-time (JIT) delivery. The total cost functions are separated from each other because the problem solution is complicated if they are combined. Therefore, the solution is separated as both cost functions are convex according to their decision variables.

The solution of the cost functions will begin by solving the total cost function of the finished product, because a manufacturer needs to know the amount of finished goods to produce to satisfy the customer/buyers' demand. Whenever the manufacturer can estimate the required amount of finished goods, he can estimate the number of orders.

Proposition 5.1: Optimum number of shipments n_j^* for N buyers solves the optimum number of raw material orders m^*_i for all M suppliers.

***Proof*:** $\sum_{j=1}^{N} n_j x_j = Q_F$.

By solving *Problem MB* for optimum number of shipments for buyer j, n_j^* (j = 1, …, N), the optimum number of finished products for all N buyers can be evaluated as

$$\sum_{j=1}^{N} n_j^* x_j = Q_F^* . \tag{5.13}$$

Again, $\sum_{i=1}^{N} f_i Q_{Ri} = Q_F$ if $f_i Q_{Ri}$ units of raw materials are required to produce Q_F finished products. Therefore, $Q_F^* = \sum_{i=1}^{k} f_i Q_{Ri}^*$. From this result the amount of raw materials required are fixed for Problem MS which has as decision variables the number of orders for raw materials, m_i. So the variable for Problem MS is m_i, {where i = 1, …, M}. Hence, solving Problem MS, the optimum number of shipment can be obtained.□

5.5.1 Solution Procedure of Problem MB

Proposition 5.1 required to solve the Problem MB, which is a convex function of n_j. Also, Problem MB is a discrete optimization problem (as discussed in previous chapters). Hence, differentiating Equation (5.9) with respect to n_j cannot solve the problem. Considering this situation, the induction method is used to solve Problem MB.

Suppose, n_j^* is the optimum number of shipments for buyer j, which minimizes the total cost $TC_{MF}(n_j^*)$ where $n_j^* \geq 1$. In the neighborhood of n_j^*, the values like $n_j^* - 1$ and $n_j^* + 1$ for the objective function are

$$TC_{MF}(n_j^* - 1) - TC_{MF}(n_j^*) \geq 0 \text{, and} \tag{5.14}$$

$$TC_{MF}(n_j^* + 1) - TC_{MF}(n_j^*) \geq 0 \,. \tag{5.15}$$

Substituting the values of $TC_{MF}(n_j^* - 1), TC_{MF}(n_j^*)$ and $TC_{MF}(n_j^* + 1)$ in Equation (5.11), it can be found that

$$TC_{MF}(n_j^* - 1) = \frac{D_{Fj}}{(n_j^* - 1)x_j} K_S + \frac{H_F}{2}\left[(n_j^* - 1)x_j + x_j - D_{Fj}T_S\right], \tag{5.16}$$

$$TC_{MF}(n_j^*) = \frac{D_{Fj}}{n_j^* x_J} K_S + \frac{H_F}{2}\left[n_j^* x_j + x_j - D_{Fj}T_S\right] \text{, and} \tag{5.17}$$

$$TC_{MF}(n_j^* + 1) = \frac{D_{Fj}}{(n_j^* + 1)x_j} K_S + \frac{H_F}{2}\left[(n_j^* + 1)x_j + x_j - D_{Fj}T_S\right], \tag{5.18}$$

respectively.

Solving Equations (5.14) to (5.18) and considering only the positive roots, the boundary for n_j^* is

$$\left\lceil \frac{1}{2}\left(\sqrt{1 + 4\Omega_j} - 1\right) \right\rceil \leq n_j^* \leq \left\lceil \frac{1}{2}\left(\sqrt{1 + 4\Omega_j} + 1\right) \right\rceil \tag{5.19}$$

where $\Omega_j = \dfrac{2D_{Fj}K_S}{x_j^2 H_F}$, and $j = 1, \ldots, N$.

A neighborhood search has to be performed using the above boundary conditions for all n_j^* which have two values, n_{j1}^*, and n_{j2}^*, to find the minimum costs $TC_{MF}(n_j^*)$ for all *j*. Therefore, the optimum n_j^* can be evaluated as

$$n_j^* = \arg\ \min\{TC_{MF}(n_{j1}^*), TC_{MF}(n_{j2}^*)\}, \tag{5.20}$$

where, $j = 1, \ldots, N$.

Example 5.1: Total Cost for Buyer 1

Consider D_{F1} = 400 units/year, P = 7,000 units/year, K_S = \$50/setup, H_F = \$5/unit/year, x_1 = 50 units per shipment, and T_S = 0.001 year. Applying these values in Equation (5.19), the boundary condition for buyer 1 is $0.97 \le n_1^* \le 2.12$ or $1 \le n_1^* \le 2$. Total cost can be computed using Equation (5.9) as $TC_{MF}(n_{j1}^*) = \$651.00$ and $TC_{MF}(n_{j2}^*) = \$576.00$. Hence, the optimal number of shipments for buyer 1 [from Equation (5.20)] are $n_1^* = \arg\ \min\ \{TC_{MF}(1), TC_{MF}(2)\}$ =2, from where Q_{F1}^* = 100 units/year. All computational results for different buyers are shown in Section 5.6.

5.5.2 Solution Procedure of Problem MS

In Section 5.2.1, the optimal production quantity for finished products Q_F^* is determined using the optimum n_j^* evaluated from Equations (5.19) and (5.20). This value of Q_F^* provides the optimum ordered quantity for the raw material as

$$Q_F^* = \sum_{i=1}^{M} f_i Q_{Ri}^*. \tag{5.21}$$

From Equation (5.21), Q_{Ri}^* (where $i = 1,...,M$) is fixed for the objective function or *Problem MS*. Also, the variables for this problem are $m_i, \forall i$, which have to be solved by induction technique as before. Therefore, by applying induction method in Equation (5.5), the boundary condition for m_i^* is

$$\left\lceil \frac{1}{2}\left(\sqrt{1+4\Psi_i}-1\right)\right\rceil \le m_i^* \le \left\lfloor \frac{1}{2}\left(\sqrt{1+4\Psi_i}+1\right)\right\rfloor, \qquad (5.22)$$

where $\Psi_i = \dfrac{f_i Q_{Ri}^{*2} Q_F^* H_{Ri}}{2PD_{Ri}K_{0i}}$, and $i = 1,...,M$.

Using the boundary for all m_i^* [all have two values $(m_{i1}^*,$ and $m_{i2}^*)$ for all *i*], the optimum m_i^* can be obtained by

$$m_i^* = \arg\ \min\{TC_{MR}(m_{i1}^*), TC_{MR}(m_{i2}^*)\}, \qquad (5.23)$$

where, $i = 1, \ldots, M$.

It is discussed in previous chapters that $m_1^*,...,m_M^*$ and $n_1^*,...,n_N^*$ are local optimal. Therefore, a search is conducted with $TC_M(m_i^*, n_j^*)$ presented in Equation (5.12) with respect to $m_1^*,...,m_M^*$ and $n_1^*,...,n_N^*$, to solve the optimum $m_1^{opt},...,m_M^{opt}$ and $n_1^{opt},...,n_N^{opt}$ and so as the optimum system cost $TC_M(m_i^{opt}, n_j^{opt})$.

Example 5.2: Total Cost for Supplier 1

Let P = 7000 units/year, f_1 = 3, H_{R1} = \$1/unit/year, K_{01} = \$60/order, and T_S = 0.001 year. Also, the values found solving for all buyers are $Q_{R1}^* = 367$ units/year [from Table 5.3], and $D_{R1} = 2000$ [from Table 5.3], for raw material 1. Using these values in Equation (5.21) the optimum number of orders for raw material 1 is evaluated as $\lceil 0.21 \rceil \le m_1^* \le \lceil 1.21 \rceil$ or $m_1^* = 1$.

Satisfying the constraints given in Equation (5.5a), the optimum number of orders and total cost for raw material 1 is found as $m_1^* = 1$ and $TC_{MR}(m_1^*) = \$413.70$ per year.

In the next section, the solution procedures are illustrated and the detailed results are presented with numerical values.

5.6 NUMERICAL ANALYSIS

This part of the research deals with the numerical test for the multi-supplier and multi-buyer supply chain problem. The following parametric values are used to compute the multi-buyer and multi-supplier supply chain system.

Suppose, there are two types of raw materials i = {1, 2} that supplied from 2 non-competing suppliers to produce a finished product. There are 10 buyers and each buyer demands are presented in Table 5.1 as follows:

Table 5.1 Buyers' demands and shipment sizes

Buyers, j	1	2	3	4	5	6	7	8	9	10
Demand, D_{Fj}, units/year	400	550	600	700	200	400	650	720	820	900
x_j, units/ shipment	50	50	60	70	20	40	50	80	60	60
L_j, years	0.13	0.09	0.10	0.10	0.10	0.10	0.08	0.11	0.07	0.06
Total Demand	6000 units/year									

P = 7000 units/year, f_1 = 3, f_2 = 4, H_{R1} = \$1/unit/year, H_{R2} = \$2/unit/year, H_F = \$5/unit/year, K_{01} = \$60/order, K_{02} = \$70 per setup, K_S = \$50 per setup, and T_S = 0.001 year.

Table 5.2 Solution for finished products based on buyers' demand

Buyers, j	n^*_j	Q^*_{Fj} units/year	TC_{MFj}, \$/year	Total Cost, TC_{MF}
1	2	100	\$576.00	\$6,815.00/year
2	2	100	\$651.38	
3	2	120	\$701.50	
4	2	140	\$776.75	
5	3	60	\$367.17	
6	2	80	\$551.00	
7	3	100	\$718.29	
8	2	160	\$826.80	
9	2	120	\$793.72	
10	2	120	\$852.40	

Applying the above values in the solution techniques discussed in Section 5.5, and computing in MS Excel, the optimum number of shipments, the total cost for each buyer, and optimum number of raw materials orders and total cost from different suppliers are presented in Tables 5.2 and 5.3, respectively. Therefore, the total system cost can be evaluated as $TC_M(m_i^*, n_j^{*opt})$ = \$7,604.13 per year.

Table 5.3 Solution for raw materials based on buyers' demand

Supplier, i	m^*_i	D_{Ri} units/year	Q^*_{Ri} units/year	TC_{MRi}, \$/year	Total Cost, TC_{MR}
1	1	2000	367	\$413.70	\$805.80/year
2	2	1500	275	\$392.09	

5.7 Alternate Shipments Schedule

In previous sections, the multi-supplier and multi-buyer supply chain system is discussed where the supply chain system operated under just-in-time (JIT) delivery condition. Also, it was assumed that the finished goods are delivered to customer j {= 1, …, N}at fixed amounts x_j after every L_j time period [Figure 5.3]. The optimum solution is evaluated and illustrated with some numerical example based on the customers/buyers requirements.

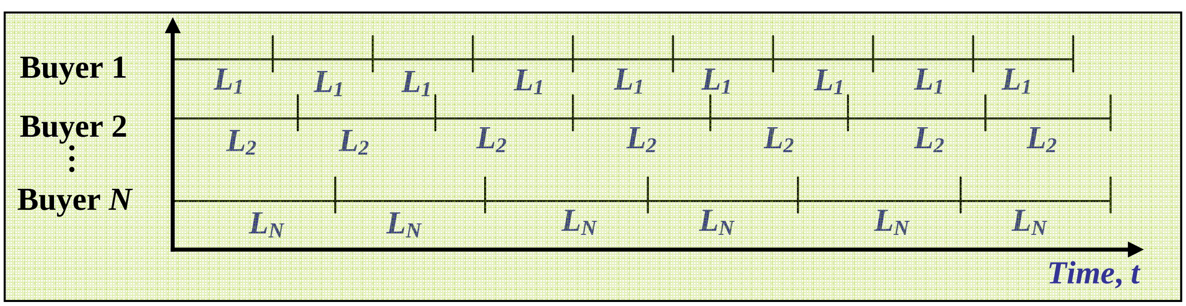


Figure 5.3. Shipment duration according to buyers' schedule

Occasionally, the total numbers of such intervals for all the N buyers may be numerous which may result in many deliveries. Therefore, for satisfying the buyers' need, the manufacturer and the buyers must agree on a fixed time period for different shipments which is

considered in this research as the average of L_1, L_2, L_3, …, and $L_N = L_F$ for all N buyers. Figure 5.4 represents the alternate delivery schedule for buyer j at every L_F time period. Based on the alternate delivery schedule the inventory diagrams for buyer j are illustrated in Figure 5.5.

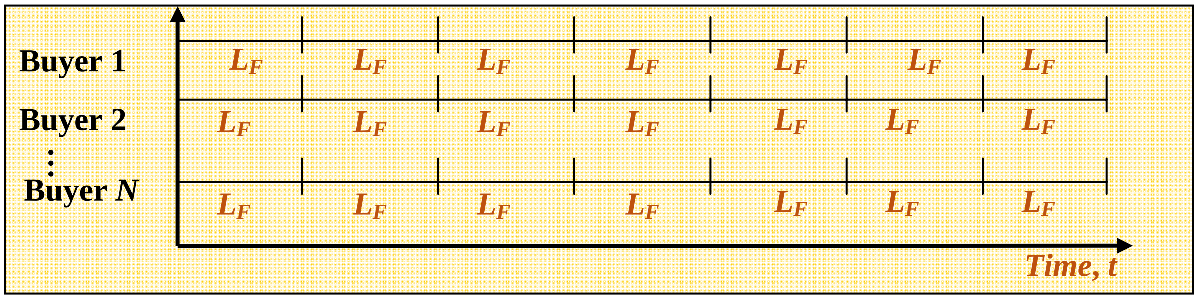


Figure 5.4. Shipment duration according to manufacturer's schedule

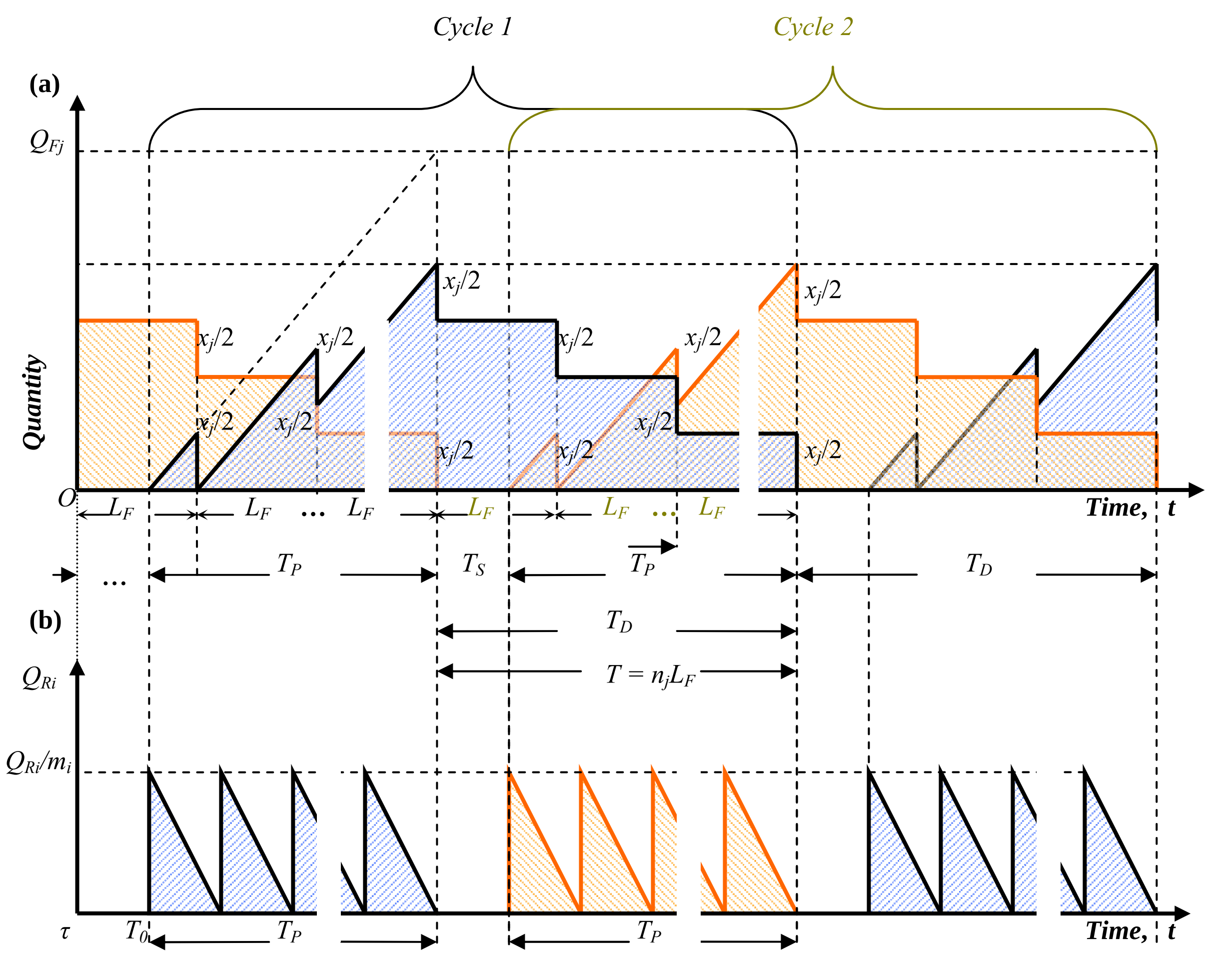


Figure 5.5. Inventory diagrams of alternate delivery schedule

5.7.1 Total Cost Function for Alternate Delivery Schedule

In this section, the total cost function is formed by using the alternate delivery schedule L_F [Figure 5.5]. Rearranging Equation (5.8) and using L_F and $T = n_j L_F$, the average inventory can be evaluated as

$$\bar{I}_{Fj} = I_{Fj} / T = \frac{1}{n_j L_F} \frac{n_j x_j}{2} \left[(n_j + 1) L_F - T_S \right]$$

$$= \frac{x_j}{2L_F} \left[(n_j + 1) L_F - T_S \right]. \tag{5.24}$$

Note that Equation (5.23) will be the same as Equation (5.8) if $L_F = L_j = x_j / D_{Fj}$. Hence, the cost function, $TC_{MFj}^{L_F}$, for buyer j at L_F fixed delivery can be written using Equation (5.24) as

$$TC_{MFj}^{L_F} = \frac{D_{Fj}}{n_j x_j} K_S + \frac{x_j H_F}{2L_F} \left[(n_j + 1) L_F - T_S \right], \tag{5.25}$$

from where the total cost function, $TC_{MF}^{L_F}$, for all N buyers using the alternate delivery schedule can be written as

$$TC_{MF}^{L_F}(n_1, \ldots, n_N) = \sum_{j=1}^{N} \left[\frac{D_{Fj}}{n_j x_j} K_S + \frac{x_j H_F}{2L_F} \left\{ (n_j + 1) L_F - T_S \right\} \right], \tag{5.26}$$

where the total cost for raw material supply remains the same as Equation (5.2) as

$$TC_{MRi}^{L_F}(m_i) = \frac{m_i D_{Ri} K_{0i}}{Q_{Ri}} + \frac{f_i H_{Ri} Q_{Ri} Q_F}{2 m_i P}. \tag{5.27}$$

Therefore, the total cost function for the raw materials supply ($TC_{MR}^{L_F}$) is

$$TC_{MR}^{L_F}(m_1, \ldots, m_M) = \sum_{i=1}^{M} \frac{m_i D_{Ri} K_{0i}}{Q_{Ri}} + \frac{1}{2P} \sum_{i=1}^{M} \frac{f_i Q_{Ri}}{m_i} \sum_{i=1}^{M} f_i Q_{Ri}. \tag{5.28}$$

Therefore, the total system cost for the multiple suppliers and buyers system with alternate delivery schedule is

$$TC_M^{L_F}(m_i, n_j) = \sum_{i=1}^{M} \frac{m_i D_{Ri} K_{0i}}{Q_{Ri}} + \frac{1}{2P} \sum_{i=1}^{M} \frac{f_i Q_{Ri}}{m_i} \sum_{i=1}^{M} f_i Q_{Ri}$$

$$+ \sum_{j=1}^{N} \left[\frac{D_{Fj}}{n_j x_j} K_S + \frac{x_j H_F}{2L_F} \left\{ (n_j + 1) L_F - T_S \right\} \right], \tag{5.29}$$

subject to: $n_j \geq m_i \geq 1,$ and are integer, for $i = 1, ..., M,$ and $j = 1, ..., N.$ (5.29a)

The same solution techniques [using induction method in Equations (5.25) and (5.27)] are used to solve the alternate delivery schedule policy and the number of shipments (n_j^*) and number of orders (m_i^*) are

$$\left\lceil \frac{1}{2}\left(\sqrt{1+4\Omega_j} - 1\right) \right\rceil \leq n_j^* \leq \left\lfloor \frac{1}{2}\left(\sqrt{1+4\Omega_j} + 1\right) \right\rfloor, \text{ and} \tag{5.30}$$

$$\left\lceil \frac{1}{2}\left(\sqrt{1+4\Psi_i} - 1\right) \right\rceil \leq m_i^* \leq \left\lceil \frac{1}{2}\left(\sqrt{1+4\Psi_i} + 1\right) \right\rceil, \text{respectively,} \tag{5.31}$$

where $\Omega_j = \dfrac{2D_{Fj} K_S}{x_j^2 H_F}$, and $j = 1, ..., N,$ and $\Psi_i = \dfrac{f_i Q_{Ri}^{*2} Q_F^* H_{Ri}}{2PD_{Ri} K_{0i}}$, and $i = 1, ..., M.$

From the above results it can be stated that there is no affect of L_F on the number of shipments (n_j^*) and the number of orders (m_i^*), but L_F does affect the total cost function. Let the optimal total costs for regular delivery schedule be $TC_M(m_i^*, n_j^*)$ and the alternate delivery schedule be $TC_M^{L_F}(m_i^*, n_j^*)$, respectively. Therefore, the fractional savings due to this change in total cost can be given by

$$\text{Fractional Savings} = \frac{TC_M(m_i^*, n_j^*) - TC_M^{L_F}(m_i^*, n_j^*)}{TC_M(m_i^*, n_j^*)} \tag{5.32}$$

$$= 1 - \frac{TC_M^{L_F}(m_i^*, n_j^*)}{TC_M(m_i^*, n_j^*)}. \tag{5.33}$$

Now applying the values of $TC_M(m_i^*, n_j^*)$ and $TC_M^{L_F}(m_i^*, n_j^*)$, from Equations (5.12) and (5.29)

$$= 1 - \frac{\Phi - \rho \sum_{j=1}^{N} \left(\frac{x_j}{L_F} \right)}{\Phi - \rho \sum_{j=1}^{N} D_{Fj}}, \tag{5.34}$$

where $\Phi = \sum_{i=1}^{M} \frac{m_i D_{Ri} K_{0i}}{Q_{Ri}} + \frac{1}{2P} \sum_{i=1}^{M} \frac{f_i Q_{Ri}}{m_i} \sum_{i=1}^{M} f_i Q_{Ri} + \sum_{j=1}^{N} \left(\frac{D_{Fj}}{n_j x_j} K_S \right) + \sum_{j=1}^{N} \left[\frac{x_j H_F}{2} (n_j + 1) \right],$

$\rho = \frac{H_F T_S}{2}$, $j = 1, \ldots, N$, and $i = 1, \ldots, M$.

From Equation (5.34), it can be stated that the lower value is the L_F, the higher value is the fractional savings. The relationship presented in Equation (5.34) is tested numerically in the following section.

Example 5.3: Fractional Savings for Alternate Delivery

Consider L_F = 0.06 years (maximum of all L), D_{F1} = 400 units/year, P = 7,000 units/year, K_S = \$50/setup, H_F = \$5/unit/year, x_1 = 50 units per shipment, f_1 = 3, H_{R1} = \$1/unit/year, K_{01} = \$60/order, T_S = 0.001 year, $Q_{R1}^* = 367$ units/year [from Table 5.3], and $D_{R1} = 2000$ [from Table 5.3], for raw material 1. and T_S = 0.001 year. Applying these values in Equations (5.30) and (5.31), the boundary conditions for buyer 1 is found as $\lceil 0.97 \rceil \le n_1^* \le \lfloor 2.12 \rfloor$ or $1 \le n_1^* \le 2$, and $\lceil 0.21 \rceil \le m_1^* \le \lceil 1.21 \rceil$ or $1 \le m_1^* \le 1$. Total cost can be computed using Equations (5.25) and (5.27) as $TC_{MF}(n_1^*) = \$572.92$, and $TC_{MR}(m_1^*)$. $= \$413.70$ per year, respectively. Hence, the optimum total cost for buyer 1 is $TC_M^{L_F}(m_1^*, n_1^*) =$ \$986.62 per year. Again, the optimum total cost for buyer 1 with scheduled delivery was found as $TC_M(m_1^*, n_1^*)$ = \$989.70 per year. Therefore, the fractional savings for buyer 1 can be computed using Equation (5.33) as

$$\text{Fractional Savings} = 1 - \frac{TC_M^{L_F}(m_i^*, n_j^*)}{TC_M(m_i^*, n_j^*)} = 1 - \frac{\$986.62}{\$989.70} = 0.0031.$$

Using the similar computations for all 5 buyers and 5 suppliers, the fractional savings total system

5.7.2 Numerical Tests for Alternate Delivery Schedule

In this numerical test, the identical parameters presented in Section 5.6, and the results of m_i^* and n_j^* given in Tables 5.2 and 5.3, are used. The fractional savings of between the total cost of alternate delivery schedule, $TC_M^{L_F}(m_i^*, n_j^*)$ and regular delivery schedule, $TC_M(m_i^*, n_j^*)$, respectively, can be evaluated by applying the numerical values in Equation (5.33) or (5.34) for different L_F, and are presented in Table 5.4.

Table 5.4 Optimum system cost for fixed shipment periods

Methods of Computation	**L_F, days**	**L_F, years**	**$TC_M^{L_F}(m_i^*, n_j^*)$**	**Fractional savings**
Minimum	23	0.06	\$7,566.63	0.0049
Average	34	0.09	\$7,574.76	0.0039
Mode	37	0.10	\$7,575.63	0.0037
Maximum	46	0.13	\$7,578.74	0.0033
Optimum total cost for assigned shipment period			\$7,604.13	

According to the results presented in Table 5.4 and Figure 5.6, it can be observed that the total system cost of multi-supplier and multi-buyer supply chain system with alternate delivery schedule is lower than the total system cost of multi-supplier and multi-buyer supply chain system with assigned delivery schedule. Also, the manufacturer can offer buyers a fractional saving from 0 to 0.0033, if all the buyers agree to receive the finished products at every 0.13 years.

The next chapter of this research deals with the supply chain system with multiple suppliers, products and buyers.

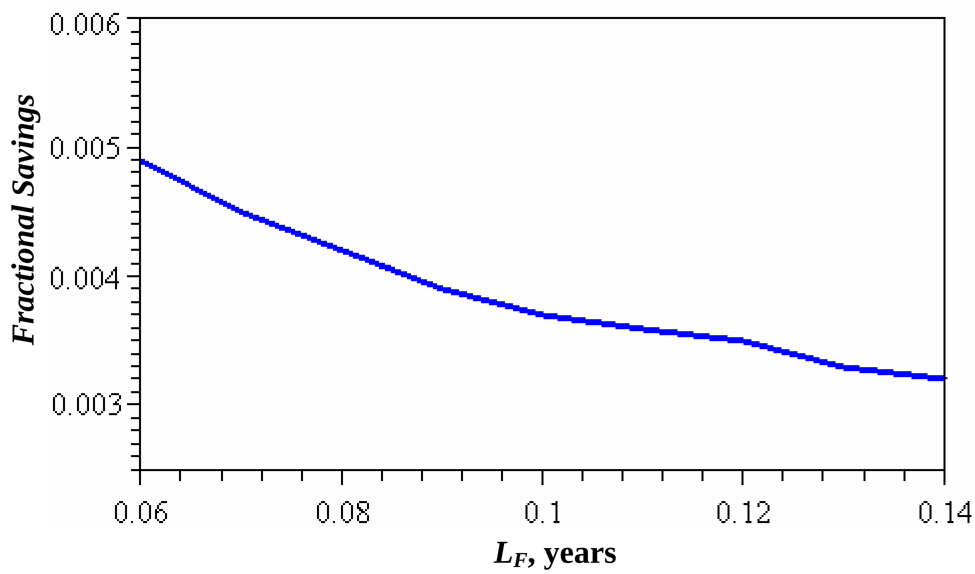


Figure 5.6. Variation in fractional savings based on alternate delivery schedule

CHAPTER 6

MULTIPLE PRODUCTS, SUPPLIERS AND BUYERS SUPPLY CHAIN

Generally, most of the manufacturing facilities deal with multiple products, which are supplied to multiple customers. To produce those multiple products, the manufacturer orders the raw materials from various suppliers. This chapter focuses on the supply chain system with multiple product production and delivery to multiple buyers, including multiple raw material supply. The main objective of this chapter is to find the optimum finished product delivery schedule and optimum number of raw material order by minimizing the rotation cycle for all products. Figure 6.1 represents the product flow diagram for multiple products manufacturing with multiple suppliers and multiple buyers.

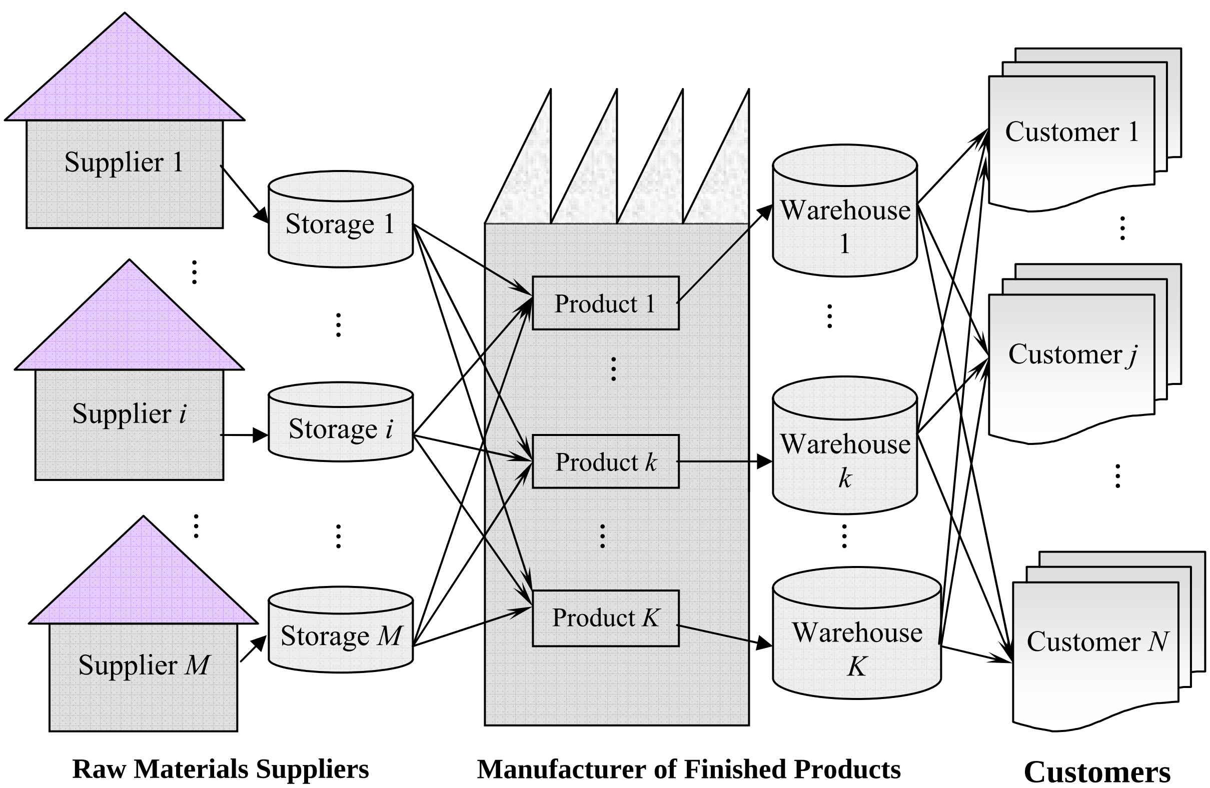


Figure 6.1. Multiple suppliers, products, and buyers supply chain system

6.1 NOTATION USED TO DEVELOP THE MODEL

The following notation is used to develop the lot sizing model:

D_{FkN} : Cumulative demand of finished goods k from all N buyers, units/year.

D_{Fkj} : Demand of finished goods k from j^{th} buyer, units/year.

D_{Rik} : Demand for raw materials i for finished product k, units/year.

f_{ik} : Quantities of i^{th} raw materials required from supplier i to produce one unit of finished product k.

H_{Fk} : Holding cost of finished goods k, \$/units/year.

H_{Ri} : Holding cost of i^{th} raw materials from i^{th} supplier, \$/units/year.

i : Index for raw materials and suppliers.

j : Index for buyers.

k : Index for finished products.

K : Number of items to be produced.

M : Number of suppliers.

K_{0i} : Ordering cost of raw material from supplier i for finished product k, \$/order.

K_{Sk} : Manufacturing setup cost for finished product k., \$/batch.

L_{kj} : Time between successive shipments of finished goods k to buyer j, years

n_{kj} : Number of *full* shipment of finished goods k per cycle time to j^{th} buyer.

m_{ik} : Number of orders for raw materials from supplier i for k^{th} finished product.

P_k : Production rate for k^{th} finished product, units/year.

Q_{FKN} : Quantity of K finished goods for N buyers per setup, units/batch,

$$Q_{FKN} = \sum_{k=1}^{K} \sum_{j=1}^{N} m_{kj} x_{kj} .$$

Q_{Fkj} : Quantity of finished goods k delivered to j^{th} buyer, units/batch, $Q_{Fkj} = n_{kj} x_{kj}$.

Q_{Rik} : Quantity of i^{th} raw materials required for k^{th} product in each batch from supplier i.

T_M : Rotation cycle length for multi-supplier-product-and-buyer production.

T_{Sk} : Setup time for finished goods l, years; $T_{Sk} < L_{kj}$.

C_M : Objective function for rotation cycle policy.

x_{kj} : Fixed quantity of k^{th} finished goods per shipment at a fixed interval of time L_{kj} to j^{th} buyer.

6.2 PROBLEM DESCRIPTION FOR MULTIPLE PRODUCTS SUPPLY CHAIN MODEL

As discussed in previous chapters (Chapter 3 and 4), this section deals with the rotation cycle policy for single facility lot-sizing model where the production facility produces K number of different products. The raw materials are delivered from M number of non-competing suppliers and supplier i {= 1, …, M} delivers i^{th} raw material to produce k^{th} {= 1, …, K} finished products in a single facility. Also, K finished products are delivered to N number of buyers according to their demand. Figure 6.1 represents the supply chain for the single facility lot-sizing models with a multi-supplier, multi-product, and multi-buyer system.

Here, it is considered that finished product k is delivered to j^{th} {= 1, …, N}customer according to his demand D_{Fkj}, (where D_{Fkj} is the demand of k^{th} product to j^{th} customer in n_{kj} multiple lots). In this facility, product k is produced at a finite rate of P_k (units per year) to satisfy the buyers' demand D_{Fkj}. All products are delivered at a fixed amount of x_{kj} units after every L_{kj} time units to follow the just-in-time (JIT) delivery policy. According to the assumption, production of all K items must meet customers' demand and $\sum_{k=1}^{K}\sum_{j=1}^{M}(D_{Fkj}/P_k) \le 1$. Due to rotation cycle policy, all products must have same production cycle time, T_M. Therefore,

a cycle lot of each product is produced during this time period. Also, because of the rotation, the products are produced in a fixed order, which is repeated from cycle to cycle.

6.2.1 Total Cost Function for Raw Material Supply

According to the lot-sizing problem, the demand for raw material i, (D_{Rik}) must satisfy the accumulated demand for all K products to produce. Also, f_{ik} units of raw material i are required to produce one of each k^{th} finished product. Now, if raw materials i are delivered for product k, are $\sum_{i=1}^{M} Q_{Rik}$ and $\sum_{j=1}^{N} Q_{Fkj}$ are the finished products k required to satisfy buyer j, then

$\sum_{i=1}^{M} f_{ik} Q_{Rik} = \sum_{j=1}^{N} Q_{Fkj}$ and $\sum_{i=1}^{M} f_{ik} D_{Rik} = \sum_{j=1}^{N} D_{Fkj}$.

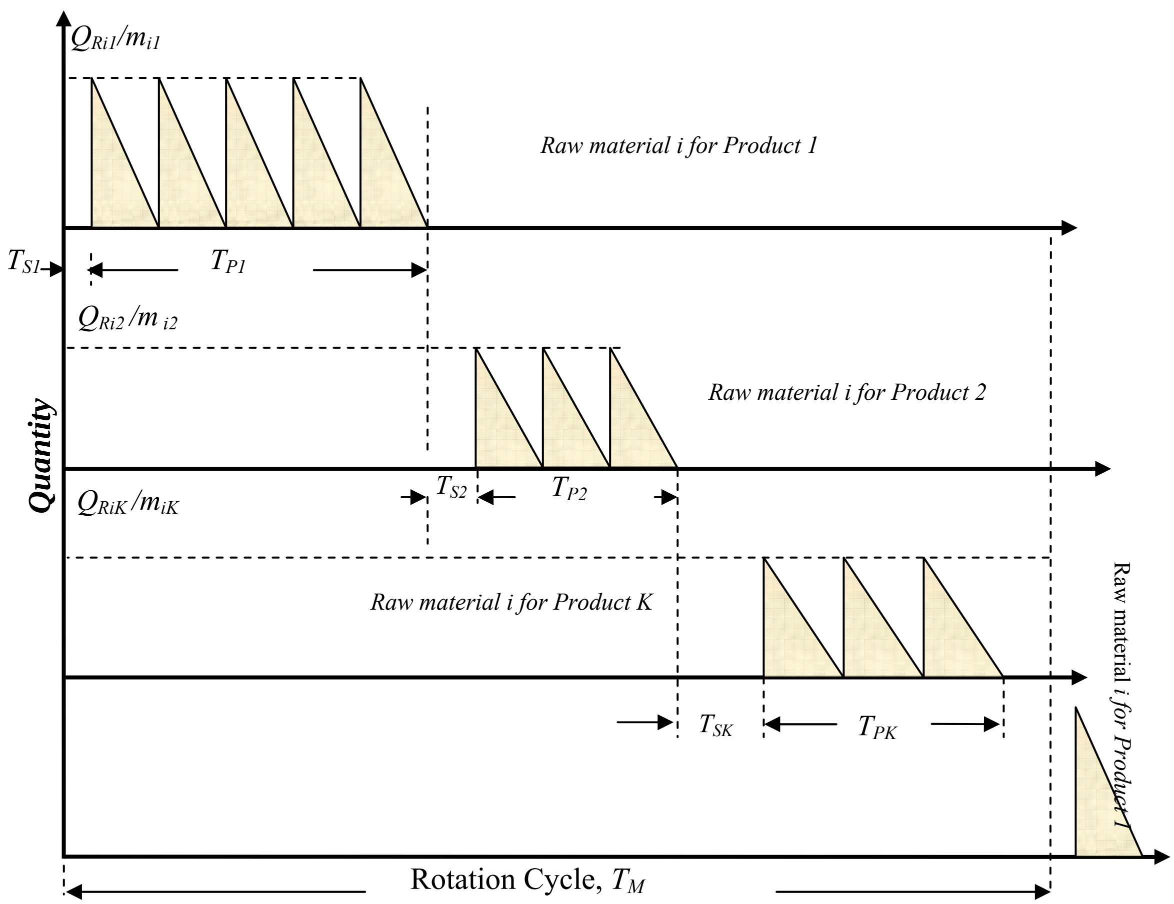


Figure 6.2. Inventory of raw material i for K finished products

Now, the lot size for product k for buyer j must equal the demand during the entire production cycle T_M for all products without permitting shortages as

$$Q_{Fkj} = T_M D_{Fkj}, \tag{6.1}$$

which is also true for raw material supply for K products as

$$Q_{Rik} = T_M D_{Rik}. \tag{6.2}$$

Therefore, the relationship between finished products and raw materials are

$$T_M = \frac{Q_{Fkj}}{D_{Fkj}} = \frac{Q_{Rik}}{D_{Rik}}. \tag{6.3}$$

Figure 6.2 represents the rotation cycle of raw material supply from supplier i for K different types of finished products. Therefore, the total cost function for i type raw material from supplier i for finished product k is (according to the previous chapters)

$$\begin{aligned} C_{Rk}(m_{ik}) &= \frac{m_{ik} D_{Rik}}{Q_{Rik}} K_{0i} + \frac{Q_{Rik} H_{Ri}}{2m_{ik}} \sum_{j=1}^{N} \frac{Q_{Fkj}}{P_k} \\ &= \frac{m_{ik} K_{0i}}{T_M} + \frac{T_M^2}{2m_{ik}} (D_{Rik} H_{Ri}) \sum_{j=1}^{N} \frac{D_{Fkj}}{P_k} \text{ [Equation (6.3)]}, \end{aligned} \tag{6.4}$$

Subject to: $m_{ik} \geq 1$ and is an integer. (6.4a)

From Equation (6.4) the total cost function for all M the raw materials for all K finished products is

$$C_{MR}(T_M, m_{ik}) = \sum_{i=1}^{M} \sum_{k=1}^{K} \left[\frac{m_{ik}}{T_M} K_{0i} + \frac{T_M^2}{2m_{ik}} (D_{Rik} H_{Rk}) \sum_{j=1}^{N} \frac{D_{Fkj}}{P_k} \right], \tag{6.5}$$

which can be written as

$$C_{MR}(T_M, m_{ik}) = \frac{1}{T_M} \sum_{i=1}^{M} \sum_{k=1}^{K} m_{ik} K_{0i} + \frac{T_M^2}{2} \sum_{i=1}^{M} \sum_{k=1}^{K} \left[\frac{(D_{Rik} H_{Ri})}{m_{ik}} \sum_{j=1}^{N} \frac{D_{Fkj}}{P_k} \right], \tag{6.6}$$

Subject to: $m_{ik} \geq 1$ and is an integer, for $i = 1, \ldots, M$, and $k = 1, \ldots, K$. (6.6a)

From Equation (6.6), it can be observed that the total cost function of i raw materials is a function of T_M, and m_{ik}.

6.2.2 Total Cost Function for Finished Products

The previous section represents the formulation of the total cost function of multiple raw materials supply from multiple suppliers to produce multiple finished products. This section illustrates the total cost function of multiple finished products for multiple buyers according to their demands. Figure 6.3 represents the inventory build-up for the rotation cycle policy of the k finished products, which are supplied to buyer j according to their demand of x_{kj} units (where k = 1, …,K and j = 1, …, N) at the end of L_{kj} time period.

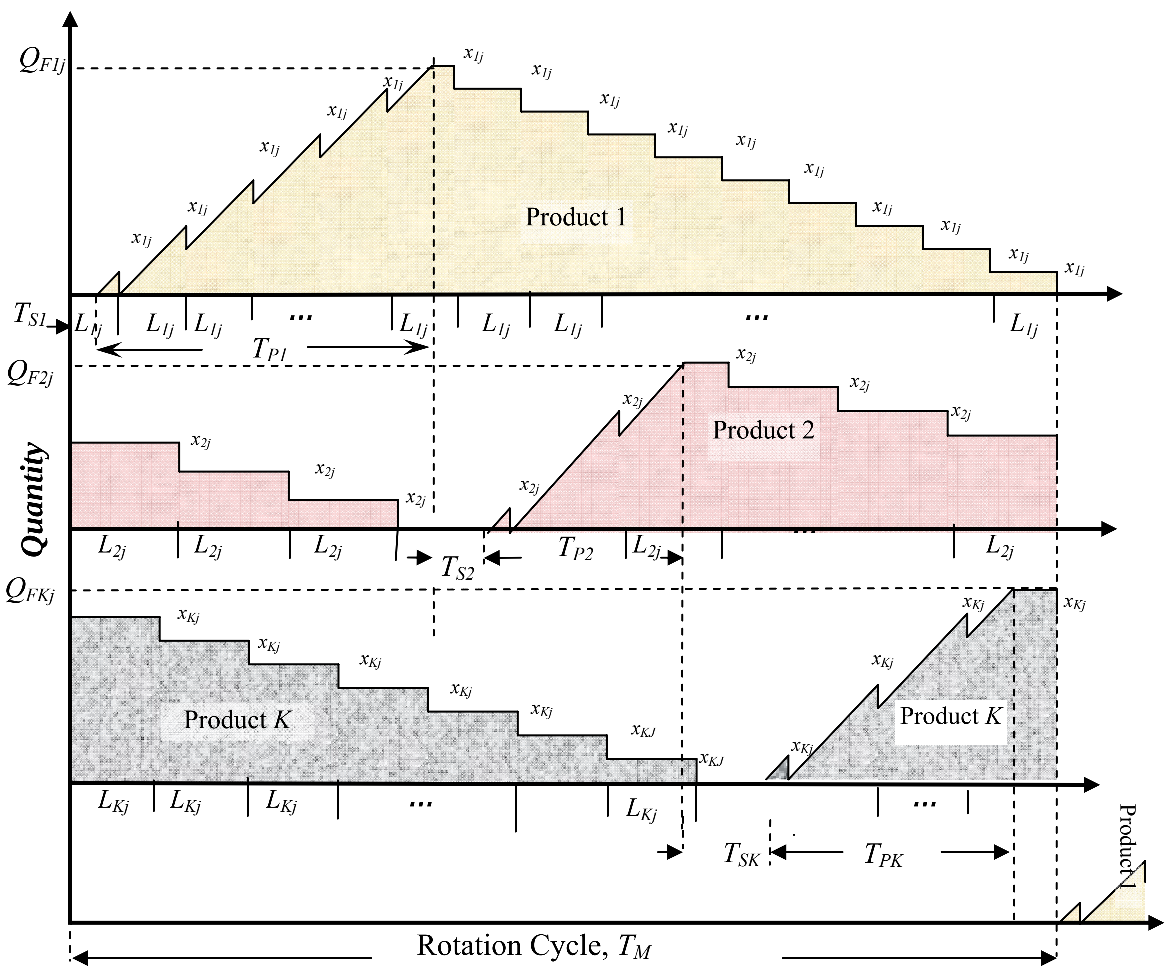


Figure 6.3. Finished goods inventory of rotation cycle for buyer j

If buyer j required a supply for Q_{Fkj} finished products in n_{kj} shipments of x_{kj} units in every L_{kj} time period, then the total cost function for product k for buyer j is

$$C_{Fkj} = \frac{D_{Fkj} K_{Sk}}{Q_{Fkj}} + H_{Fk}\left[\frac{Q_{Fkj}}{2}\left(1-\frac{D_{Fkj}}{P_k}\right)+\left(\frac{x_{kj}}{2}-D_{Fkj}T_{Sk}\right)\right]. \tag{6.7}$$

Applying the relationship described in Equation (6.3) in Equation (6.7) the total cost function of rotation cycle for finished product k and buyer j can be written as

$$C_{Fkj}(T_M) = \frac{K_{Sk}}{T_M} + H_{Fk}\left[\frac{T_M D_{Fkj}}{2}\left(1-\frac{D_{Fkj}}{P_k}\right)+\left(\frac{x_{kj}}{2}-D_{Fkj}T_{Sk}\right)\right]. \tag{6.8}$$

Using Equation (6.8), the total cost function for all K finished products and N buyers can be evaluated as

$$C_{MF}(T_M) = \frac{N}{T_M}\sum_{k=1}^{K} K_{Sk} + T_M \sum_{k=1}^{K}\sum_{j=1}^{N}\left[\frac{H_{Fk}D_{Fkj}}{2}\left(1-\frac{D_{Fkj}}{P_k}\right)\right]$$

$$+\sum_{k=1}^{K}\sum_{j=1}^{N} H_{Fk}\left(\frac{x_{kj}}{2}-D_{Fkj}T_{Sk}\right). \tag{6.9}$$

Hence, the combined total cost function for the entire rotation cycle policy for M multiple suppliers, K finished products and N multiple buyers [Equations (6.6) and (6.9)] can be found as

$$C_{MR}(T_M, m_k) = \frac{1}{T_M}\sum_{i=1}^{M}\sum_{k=1}^{K}\left[\frac{(D_{Rik}H_{Ri})}{m_{ik}}\sum_{j=1}^{N}\frac{D_{Fkj}}{P_k}\right] + \frac{N}{T_M}\sum_{k=1}^{K} K_{Sk}$$

$$+ T_M \sum_{k=1}^{K}\sum_{j=1}^{N}\left[\frac{(H_{Fk}D_{Fkj})}{m_{ik}}\left(1-\frac{D_{Fkj}}{P_k}\right)\right] + \sum_{k=1}^{K}\sum_{j=1}^{N}\left[H_{Fk}\left(\frac{x_{kj}}{2}D_{Fkj}T_{Sk}\right)\right]. \tag{6.10}$$

This problem is restricted with two constraints, (1) Rotation Cycle time, T_M, must be greater than or equal to the cumulative setup times and production time for all products which are produced in the facility, and (2) the number of orders of raw material must be greater than or

equal to 1. Using these two constraints, the objective functions for this problem can be formulated as

$$C_{MR}(T_M, m_{ik}) = \frac{1}{T_M}\sum_{i=1}^{M}\sum_{k=1}^{K} m_{ik}K_{0i} + \frac{T_M^2}{2}\sum_{i=1}^{M}\sum_{k=1}^{K}\left[\frac{(D_{Rik}H_{Ri})}{m_{ik}}\sum_{j=1}^{N}\frac{D_{Fkj}}{P_k}\right]$$

$$+\frac{N}{T_M}\sum_{k=1}^{K}K_{Sk} + T_M\sum_{k=1}^{K}\sum_{j=1}^{N}\left[\frac{H_{Fk}D_{Fkj}}{2}\left(1-\frac{D_{Fkj}}{P_k}\right)\right]$$

$$+\sum_{k=1}^{K}\sum_{j=1}^{N}\left[H_{Fk}\left(\frac{x_{kj}}{2}-D_{Fkj}T_{Sk}\right)\right]. \tag{6.11}$$

Subject to:

$$T_M \geq \frac{\sum_{k=1}^{K} T_{Sk}}{1-\sum_{k=1}^{K}\sum_{j=1}^{N}\left[D_{kj}/P_k\right]} \equiv T_{\min} > 0\,, \tag{6.11a}$$

$$m_{ik} \geq 1 \text{ and is an integer,} \qquad \text{for } i = 1,\ldots,M, \text{ and } k = 1,\ldots,K. \tag{6.11b}$$

The problem is a mixed integer non-linear programming problem and the solution procedure to this problem is discussed in the next section.

6.3 Solution Methods for Rotation Cycle of Multi-Supplier-and-Buyer

To solve the rotation cycle policy for multi-supplier and multi-buyer problem, Proposition 5.1 is used. In this case, the rotation cycle is the same for all the raw materials and finished products. Therefore, solving Equation (6.9) will provide the optimal rotation cycle for all finished products and raw materials.

As Equation (6.9) is a convex function for the decision variable T_M; it can be solved by differentiating Equation (6.9) with respect to T_M and equating the result to zero as

$$\frac{dC_{MF}(T_M)}{dT_M} = -\frac{1}{T_M^2}N\sum_{k=1}^{M}K_{Si} + \sum_{k=1}^{N}\sum_{j=1}^{J}\left[\frac{H_{Fk}D_{Fkj}}{2}\left(1-\frac{D_{Fkj}}{P_k}\right)\right] = 0\,, \tag{6.12}$$

upon simplification which yields

$$T_M^* = \sqrt{\frac{2N\sum_{k=1}^{K} K_{Sk}}{\sum_{k=1}^{K}\sum_{j=1}^{N}\left[H_{Fk}D_{Fkj}\left(1-\frac{D_{Fkj}}{P_k}\right)\right]}}\,. \tag{6.13}$$

Now, this $T_M{}^*$ has to satisfy the constraint presented in Equation (6.11a). Therefore, T_M^{opt} will be that value which one [between the Equation (6.13) and Equation (6.11a)] is the maximum, i.e.,

$$T_M^{opt} = \max\{T_{\min}, T_M^*\}\,. \tag{6.14}$$

Hence, using the value of T_M^{opt}, Equation (6. 4) can be solved by induction technique as this part of the problem is a discrete optimization problem.

Suppose, m_{ik}^* optimizes the objective function $C_{Rik}(m_{ik}^*)$ where $m_{ik}^* - 1$ and $m_{ik}^* + 1$ are the points in the neighborhood which obtain the boundary conditions for m_{ik}^* as

$$\left\lceil \frac{1}{2}(\sqrt{1+4\Delta_{ik}}-1)\right\rceil \le m_{ik}^* \le \left\lceil \frac{1}{2}(\sqrt{1+4\Delta_{ik}}+1)\right\rceil, \tag{6.15}$$

where $\Delta_{ik} = \dfrac{(T_M^{opt})^3}{2K_{0i}}(D_{Rik}H_{Ri})\sum_{j=1}^{N}\dfrac{D_{Fkj}}{P_k}, i = 1,...,M, k = 1,...,K,$ and $j = 1,...,N.$

After the limits are evaluated a search for minimum total cost can be made using the following equation:

$$C_{MR}(m_{ik}) = \frac{1}{T_M^{opt}}\sum_{i=1}^{M}\sum_{k=1}^{K} m_{ik}K_{0i} + \frac{(T_M^{opt})^2}{2}\sum_{i=1}^{M}\sum_{k=1}^{K}\left[\frac{(D_{Rik}H_{Rk})}{m_{ik}}\sum_{j=1}^{N}\frac{D_{Fkj}}{P_k}\right]. \tag{6.16}$$

It must be noted that $m_{ik}^* \ge 1$, otherwise the values of m_{ik}^* must be replaced by 1. In this case m_{ik}^* and $T_M{}^*$ are local optimal. As a result, a search is applied with respect to $T_M{}^*$ and m_{ik}^*, [starting from the values given by the constraints presented in Equations (6.11a) and (6.11b) and with the step sizes of 0.01 and 1, respectively] on the objective function presented in

Equation (6.11) to find the optimum total cost $C_M^*(T_M^{opt}, m_{ik}^{opt})$ as well as T_M^{opt} and m_{ik}^{opt}. A sample computation is presented below:

Example 6.1: Total Cost Estimation

Consider the raw material 1 is supplied to the manufacturer to produce product 1 from suppler 1. Also, the manufacturer will produce the amount of product 1 to satisfy the demand of buyer 1. The parameters for product 1 are P_1 = 35,000 units per year, D_{F11} = 580 units/year, x_{11} = 20 units per shipments, f_{11} = 3, H_{R1} = \$1/unit/year, K_{01} = \$60/order, H_{F1} = \$2/unit/year, K_{S1} = \$20/setup, and T_{S1} = 0.001 years. The additional parameters are presented in Table 6.1 and 6.2 in the next section. Applying these values in Equation (6.13), the rotation cycle T_M^* can be found as

$$T_M^* = \sqrt{\frac{2 \times 5 \times 210}{31232.48}} = 0.26\,.$$

Applying T_M^* = 0.26 in Equation (6.15) the boundary for m_{11}^* is found as

$\lceil 0.01 \rceil \le m_{11}^* \le \lceil 1.01 \rceil$ which yields $m_{11}^* = 1$. Using these values in Equation (6.10) the total cost for raw material 1, finished product 1, and buyer 1 can be found as $C_M(T_M^*, m_{11}^*) = \$475.36$ per year, which is local optimal. Therefore, using the forward search from $C_M(T_M^*, m_{11}^*) = C_M(0.02, 1)$ with the step sizes 0.01 for T_M^* and 1 for m_{11}^*, the optimum result is reached at $C_M(T_M^{opt}, m_{11}^{opt}) = C_{MR}(0.37, 1) = \$447.17$ per year (where T_M^{opt} = 0.37 years and $m_{11}^{opt} = 1$). All the detailed results for rotation cycle policy are presented in Tables 6.3 and 6.4 in next section.

6.4 Numerical Analysis for Multi-Supplier-and-Buyer Rotation Cycle

This section presents a numerical computation for the rotation cycle policy. Suppose, there are two types of raw materials $i = \{1, 2\}$ are supplied from 2 non-competing suppliers,

and supplier 1 supplies raw material 1 and supplier 2 supplies raw material 2 to produce a finished products. The other parametric values for raw material supply are $f_{11} = f_{12} = f_{13} = f_{14} = f_{15} = 3$, $f_{21} = f_{22} = f_{23} = f_{24} = f_{25} = 4$, H_{R1} = \$1/unit/year, H_{R2} = \$2/unit/year, K_{01} = \$60/order, and K_{02} = \$70 per setup. The parametric values for different components for 5 products and 5 buyers are presented in Tables 6.1 and 6.2.

To apply rotation cycle policy, the problem has to satisfy the constraint, which is

$$\sum_{k=1}^{5}\sum_{j=1}^{5}(D_{Fkj} / P_k) \leq 1. \tag{6.17}$$

Apply all the values from Table 6.4 in Equation (6.17) it can be found that

$$\sum_{k=1}^{5}\sum_{j=1}^{5}(D_{Fkj} / P_k) = 0.36 \leq 1. \tag{6.18}$$

Hence, the rotation cycle policy can be applied for this problem. Using solution procedures used in Example 6.1, the optimal solution for multiple suppliers, products and buyers can be found as presented in Tables 6.3 and 6.4 [all computations are evaluated using Maple 6.0 and MS Excel]. Also, a supply chain diagram for 2 suppliers, 5 products, and 5 buyers with results are presented in Figure 6.4. Hence, the system total cost can be evaluated by adding all the total costs presented in Table 6.4 as

$$\boldsymbol{C_M (T_M^{opt}, m_{ik}^*)} = \textbf{\$25,783.46} \text{ [where } i = 1,2, \text{ and } k = 1,\ldots,5], \text{per year.}$$

Next chapter deals with the sensitivity analysis which has been performed for both perfect and imperfect matching inventory policies.

Table 6.1 Parametric values for five (5) different products

Parameters	*Product 1*	*Product 2*	*Product 3*	*Product 4*	*Product 5*
P_k (units/year)	35,000	20,000	50,000	30,000	40,000
D_{Fk} (units/year)	2,550	2,010	3,000	2,110	2,200
K_{Sk} ($/setup)	20	30	50	40	70
H_{Fk} ($/unit/year)	2	10	4	6	5
T_{Sk} (years)	0.001	0.002	0.002	0.003	0.004

Table 6.2 Demand and shipment sizes for all five (5) products

Products	Parameters	*Buyer 1*	*Buyer 2*	*Buyer 3*	*Buyer 4*	*Buyer 5*
1	D_{F1j} (units/year)	580	520	450	550	450
	x_{1j} ($/order)	20	40	30	50	50
2	D_{F2j} (units/year)	400	360	450	500	300
	x_{2j} ($/order)	20	45	30	50	50
3	D_{F3j} (units/year)	600	560	560	720	560
	x_{3j} ($/order)	50	35	40	40	40
4	D_{F4j} (units/year)	500	350	450	510	300
	x_{4j} ($/order)	20	25	30	30	50
5	D_{F5j} (units/year)	440	540	540	320	360
	x_{5j} ($/order)	40	60	30	40	45

Table 6.3 Results of optimal total costs for multiple suppliers, products, and buyers

Parameters	*Products*	*Buyer 1*	*Buyer 2*	*Buyer 3*	*Buyer 4*	*Buyer 5*	Total Cost
$C^*_M(T^*_M)$	1	$283.94	$1,207.67	$521.54	$847.52	$716.76	$3,577.42
	2	$219.56	$952.89	$521.54	$794.86	$583.61	$3,072.46
	3	$321.05	$1,251.87	$620.41	$995.17	$788.74	$3,977.24
	4	$255.41	$835.25	$521.54	$745.40	$583.61	$2,941.21
	5	$253.93	$1,342.31	$586.10	$573.76	$624.49	$3,380.59
Total System Cost for Finished Products							**$16,948.91**
$C^*_M(T^*_M, m^*_{1k})$	1	$163.23	$162.90	$163.13	$162.98	$162.88	$815.13
	2	$162.90	$162.68	$163.13	$162.91	$162.64	$814.26
	3	$163.27	$162.96	$163.37	$163.23	$163.06	$815.90
	4	$163.09	$162.66	$163.13	$162.92	$162.64	$814.45
	5	$162.98	$162.93	$163.32	$162.64	$162.74	$814.61
Total System Cost for Raw Material 1							**$4,074.35**
$C^*_M(T^*_M, m^*_{2k})$	1	$190.80	$190.30	$190.65	$190.42	$190.27	$952.45
	2	$190.30	$189.96	$190.65	$190.31	$189.91	$951.12
	3	$190.86	$190.39	$191.00	$190.80	$190.53	$953.57
	4	$190.58	$189.94	$190.65	$190.33	$189.91	$951.41
	5	$190.41	$190.35	$190.93	$189.90	$190.05	$951.65
Total System Cost for Raw Material 2							**$4,760.20**
Total Cost		**$1,333.89**	**$5,589.99**	**$2,771.13**	**$3,956.70**	**$3,297.20**	**$25,783.46**

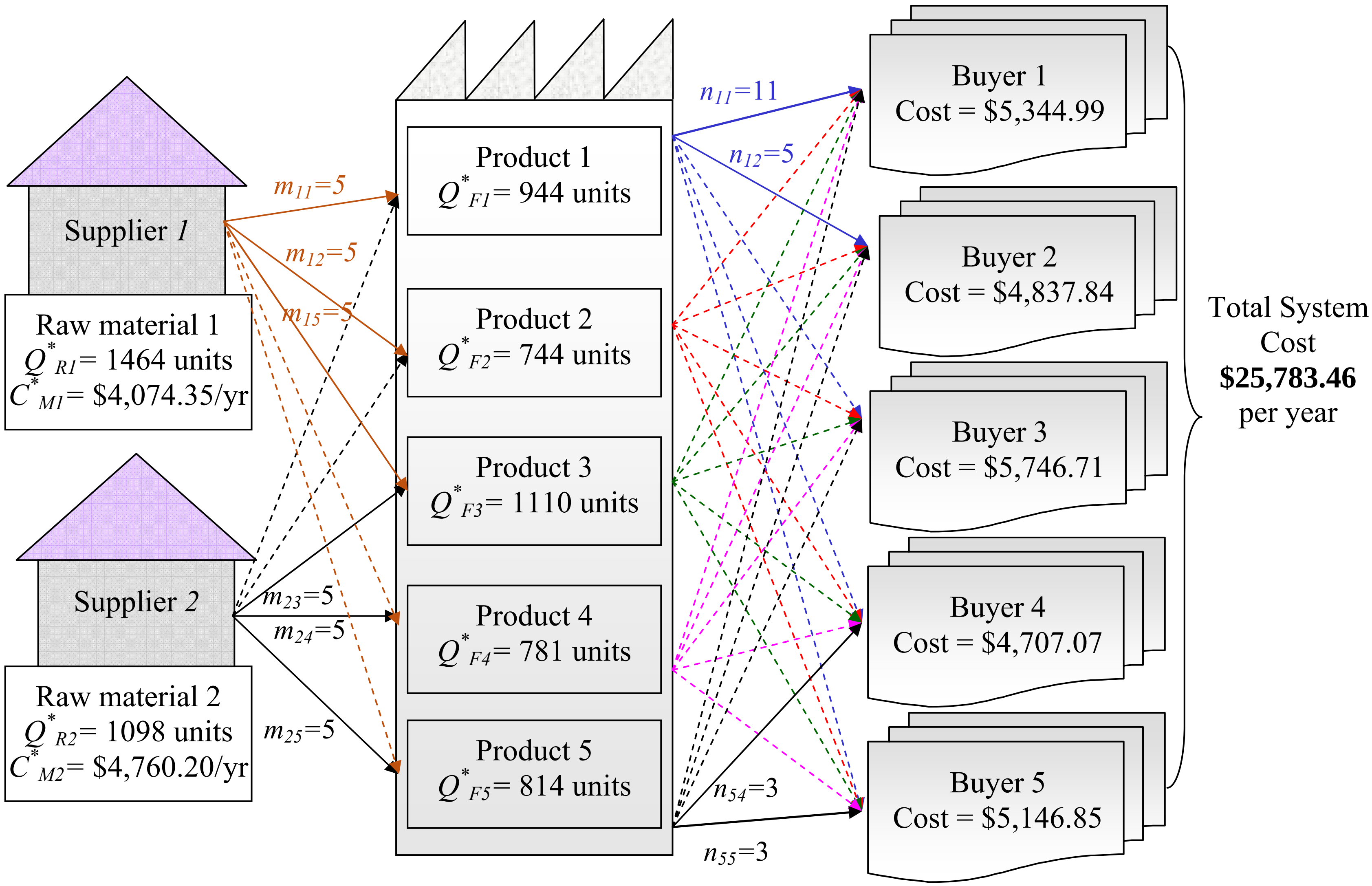


Figure 6.4. Supply chain results for multiple suppliers, products, and buyers

Table 6.4 Detail optimal results for multiple suppliers, products, and buyers

Itemized Optimum Results							
Products	**Parameters**	***Buyer 1***	***Buyer 2***	***Buyer 3***	***Buyer 4***	***Buyer 5***	**Total**
	T_M^{opt}	**0.37**					
1	m_{11}^*	1	1	1	1	1	**5**
	m_{21}^*	1	1	1	1	1	**5**
	n_{1j}^*	11	5	6	4	3	**29**
	Q_{F1j}^*, units/year	215	192	167	204	167	**945**
	Q_{R11}^*, units/year	72	64	56	68	56	**316**
	Q_{R21}^*, units/year	54	48	42	51	42	**237**
	$C_M^*(T_M^{opt}, m_{11}^{opt}, m_{21}^{opt})$	**$5,344.99**					
2	m_{12}^*	1	1	1	1	1	**5**
	m_{22}^*	1	1	1	1	1	**5**
	n_{2j}^*	7	3	6	4	2	**22**
	Q_{F1i}^*, units/year	148	133	167	185	111	**744**
	Q_{R12}^*, units/year	49	44	56	62	37	**248**
	Q_{R22}^*, units/year	37	33	42	46	28	**186**
	$C_M^*(T_M^{opt}, m_{12}^{opt}, m_{22}^{opt})$	**$4,837.84**					
3	m_{13}^*	1	1	1	1	1	**5**
	m_{23}^*	1	1	1	1	1	**5**
	n_{3j}^*	4	6	5	7	5	**27**
	Q_{F3i}^*, units/year	222	207	207	266	207	**1109**
	Q_{R13}^*, units/year	74	69	69	89	69	**370**
	Q_{R23}^*, units/year	56	52	52	67	52	**279**
	$C_M^*(T_M^{opt}, m_{13}^{opt}, m_{23}^{opt})$	**$5,746.71**					
4	m_{14}^*	1	1	1	1	1	**5**
	m_{24}^*	1	1	1	1	1	**5**
	n_{4j}^*	9	5	6	6	2	**28**
	Q_{F4i}^*, units/year	185	130	167	189	111	**782**
	Q_{R14}^*, units/year	62	43	56	63	37	**261**
	Q_{R24}^*, units/year	46	32	42	47	28	**195**
	$C_M^*(T_M^{opt}, m_{14}^{opt}, m_{24}^{opt})$	**$4,707.07**					
5	m_{15}^*	1	1	1	1	1	**5**
	m_{25}^*	1	1	1	1	1	**5**
	n_{5j}^*	4	3	7	3	3	**20**
	Q_{F5i}^*, units/year	163	200	200	118	133	**814**
	Q_{R15}^*, units/year	54	67	67	39	44	**271**
	Q_{R25}^*, units/year	41	50	50	30	33	**204**
	$C_M^*(T_M^{opt}, m_{15}^{opt}, m_{25}^{opt})$	**$5,146.85**					

CHAPTER 7

SENSITIVITY ANALYSIS

Sensitivity analysis is an important issue for conducting research. This analysis helps to view the characteristics of the mathematical formulation of the problem. The total cost functions are the snapshot of the real solution in which the model parameters (shipment quantity, raw material conversion factor, ordering cost, holding cost, etc.) are assumed to be static values. It is reasonable to study the sensitivity, i.e., the effect of making changes in the model parameters over a given optimum solution. It is important to find the effects on different system performance measures, such as cost function, inventory system, etc. For this purpose, sensitivity analyses of various system parameters for the proposed models are required to observe whether,

(a) The current solutions remain unchanged,

(b) The current solutions become sub-optimal,

(c) The current solutions become infeasible, etc.

This part of the research presents the sensitivity analyses of the total cost functions of perfect and imperfect matching supply chain systems which have been discussed in Chapters 3 and 4, respectively. These analyses are performed based on the static values involved in the cost function. They are shipment quantity, raw material conversion factor, and ordering cost.

7.1 EFFECT OF SHIPMENT SIZE (x) ON TOTAL COST FUNCTIONS

In a just-in-time (JIT) delivery based production system, the shipment size is an important factor. The total cost functions revolve around the shipment size, x. Also, x determines the on-hand inventory and its carrying costs. Therefore, it is necessary to perform a sensitivity analysis base on the variation of shipment size, x. To perform this analysis it is

necessary to evaluate the differentiations with respect to x of the Equations (3.11) and (4.12) as follows:

$$\frac{dTC(m^*,n^*)}{dx} = \frac{n^{*2}xH_R}{m^* fP} - \frac{D_F}{n^* x^2}(m^* K_0 + K_S) + \frac{1}{2}H_F(n^* + 1), \text{ and} \tag{7.1}$$

$$\frac{dTC(Q_F'^*,m^*)}{dx} = \frac{dTC(m^*,n^*)}{dx} = \frac{n^*(n^* x + I_0)^2 H_R}{m^* fP} - \frac{n^* D_F}{(n^* x + I_0)^2}(m^* K_0 + K_S)$$
$$+ \frac{H_F I_0 (I_0 + x - D_F T_S)}{2(n^* x + I_0)^2} + \frac{1}{2}H_F(n^* + 1), \tag{7.2}$$

where $Q_F'^* = n^* x + I_0$.

Applying the parametric values from Chapters 3 and 4 in Equations (7.1) and (7.2) and the values of x from 30 to 250 units/shipment, the graphical presentation is given in Figure 7.1.

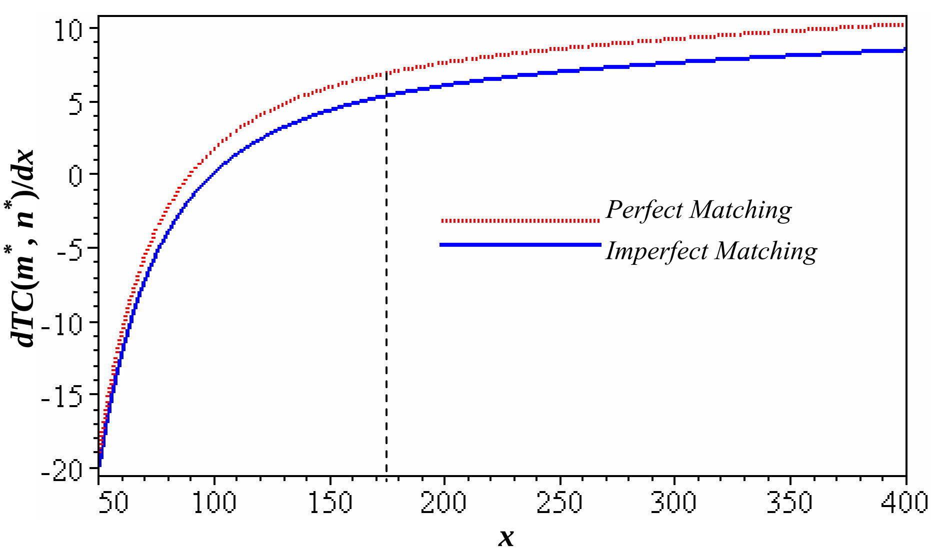


Figure 7.1. Effect of shipment size x on the total system costs

Again, using parametric values in Equations (3.11) and (4.13) and the values of x from 10 to 500 units per shipment the plots have been presented in Figure 7.1. According to Figure 7.2, it can be observed that as the total cost functions increases, x values varies from 1 to 160 after which the total cost increases in a linear fashion.

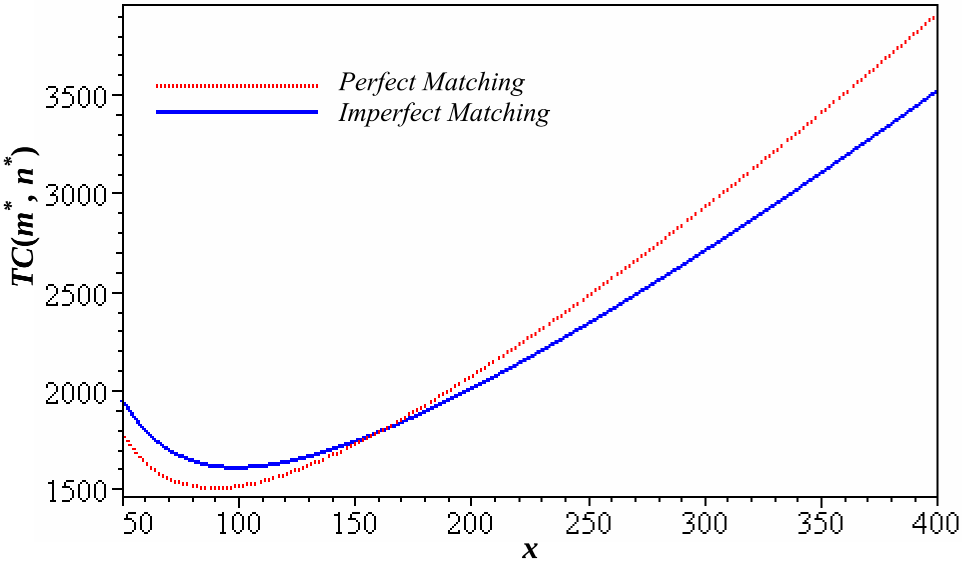


Figure 7.2. Variation of the total costs for shipment size x

Table 7.1 Effect of shipment size x on the total costs for perfect and imperfect matching

x units/shipments	$TC_P(m^*, n^*)$	$TC_I(m^*, n^*)$	$dTC_P(m^*, n^*)/dx$	$dTC_I(m^*, n^*)/dx$
1	-68563.42	-2987.98	68577.03	15594.51
10	-677.65	-391.16	6935.08	5830.57
30	-67.99	-61.29	2527.38	2670.57
60	-10.64	-12.09	1632.71	1794.28
90	0.15	-1.54	1507.07	1619.34
120	4.05	2.44	1578.03	1640.65
150	5.97	4.41	1731.31	1747.27
180	7.11	5.56	1928.80	1897.20
210	7.87	7.33	2154.19	2077.28
240	8.44	7.88	2399.31	2274.81
270	8.90	7.31	2659.63	2487.86
300	9.28	7.66	2932.42	2712.49
330	9.62	7.96	3215.95	2947.77
400	10.29	8.53	3913.47	3524.56
500	11.13	9.21	4985.44	4412.42

From Figure 7.5, it can be seen that the total cost functions are minimum when x value is 100 units/shipment, because the parametric values used for this plot are 100 units/shipment and m^* and n^* are minimized according to these values. All the values used for these graphical

representations are presented in Table 7.1. In Table 7.1 it is also observed that the change in total costs is minimal when the shipment size is 1. This is because whatever is produced is shipped to the customers. Therefore, there is no inventory holding costs for finished goods items.

7.2 EFFECT OF RAW MATERIAL CONVERSION FACTOR (*f*) ON TOTAL COST FUNCTIONS

Another important parameter here is raw material conversion factor, *f*, which is a determination factor of ordering required raw materials. In this section, the sensitivity analysis is performed for both perfect and imperfect matching system with respect to *f*. Differentiating Equations (3.11) and (4.13) with respect to *f* it can be found that

$$\frac{dTC(m^*,n^*)}{df} = -\frac{1}{2}\frac{n^{*2}x^2 H_R}{m^* f^2 P}\text{, and} \tag{7.3}$$

$$\frac{dTC(Q_F'^*,m^*)}{df} = \frac{dTC(m^*,n^*)}{df} = -\frac{1}{2}\frac{(n^* x + I_0)^2 H_R}{n^* f^2 P}, \tag{7.4}$$

where $Q_F'^* = n^* x + I_0$.

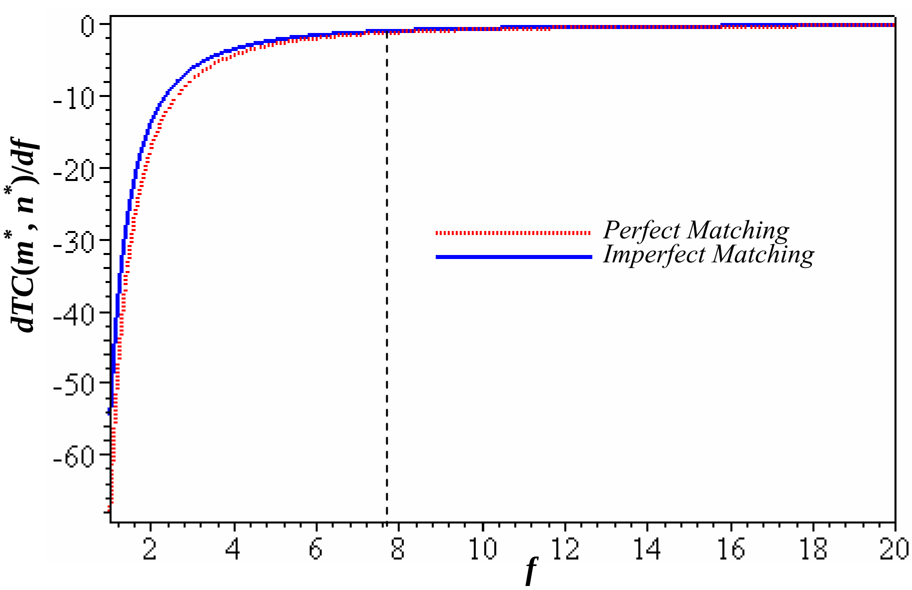


Figure 7.3. Effect of conversion factor *f* on the total system costs

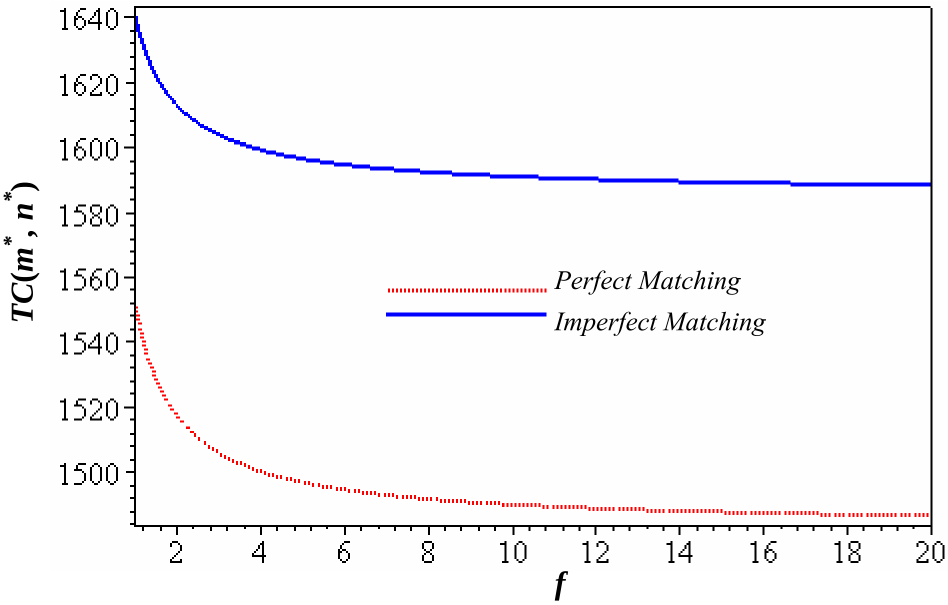


Figure 7.4. Variation of the total costs for conversion factor *f*

Table 7.2 Effect of conversion factor *f* on the total system costs

f	$TC_P(m^*, n^*)$	$TC_I(m^*, n^*)$	$dTC_P(m^*, n^*)/dx$	$dTC_I(m^*, n^*)/dx$
1	-68.06	-54.25	1551.37	1639.95
3	-7.56	-7.03	1507.00	1603.78
5	-2.72	-2.17	1497.93	1597.55
7	-1.39	-1.11	1493.04	1593.45
9	-0.84	-0.67	1490.88	1591.72
11	-0.56	-0.45	1489.50	1590.63
13	-0.40	-0.32	1488.55	1589.87
15	-0.30	-0.24	1487.85	1589.31
17	-0.24	-0.19	1487.32	1588.89
25	-0.11	-0.09	1487.04	1587.87
27	-0.09	-0.07	1485.83	1587.71
30	-0.08	-0.06	1485.58	1587.50

Applying the parametric values in Equations (7.3) and (7.4) as before and varying the values of f from 1 to 15, the illustration is shown in Figure 7.3. Figure 7.3 shows that the change in total cost for both the cases increases with the increase of f, as the quantity of raw material ordering increases. Figure 7.4 is drawn by using the parametric values in Equations

(3.11) and (4.13) and changing the values of f from 1 to 20. It can be observed for higher f value the total cost functions decreases linearly.

7.3 EFFECT OF ORDERING (K_0) AND SETUP COSTS (K_S) ON TOTAL COST FUNCTIONS

Raw material ordering (K_0) and setup (K_S) costs have significant impact on the total cost functions. According to the formation of the total cost functions of perfect and imperfect matching systems [Equations (3.11) and (4.13), respectively], it can be observed that the ordering (K_0), and setup (K_S) costs are a linear operator for the cost functions. Therefore, the total cost will increase with the increase of both the K_0 and K_S. Conversely, the ratio of K_0/K_S also plays an important role on the system cost. Therefore, a sensitivity analysis has been performed by increasing the ratio of K_0/K_S from 0.02 to 13, and the variation of the total cost function has been observed. The detailed results are presented in Table 7.3 and a graphical representation is shown in Figure 7.5. According to the Figure 7.5 and Table 7.3, it can be observed that both total costs for perfect and imperfect matching systems decrease with the increase of ordering and setup cost ratio.

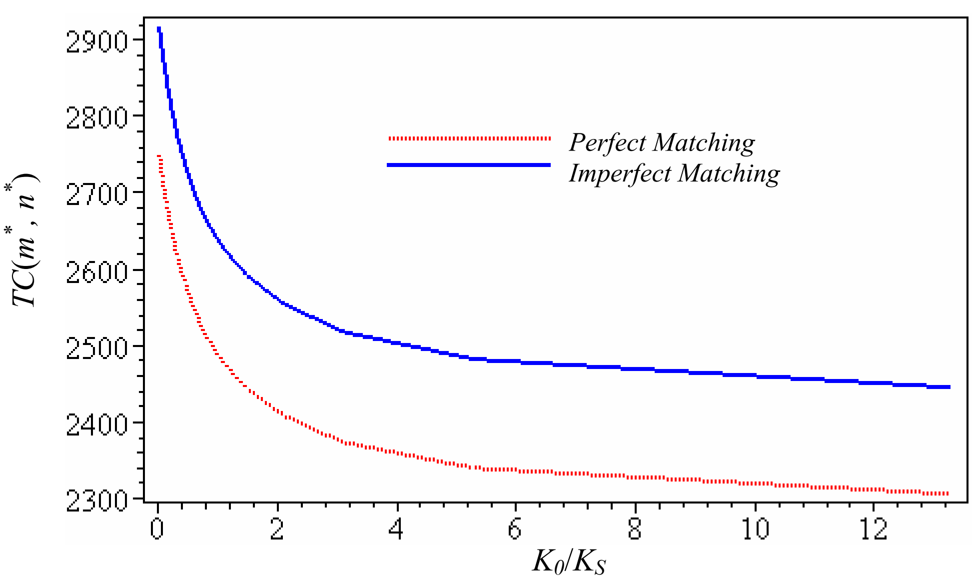


Figure 7.5. Effect of K_0/K_S on total system costs

Further, it is observed that the total cost function decreases rapidly when the ratio of K_0/K_S varies from 0.02 to 4.0, after which the total costs decrease in linear fashion. Moreover, in this analysis, the total cost for imperfect matching system, $TC_P(m^*, n^*)$, is found to be higher than the total cost for imperfect matching system, $TC_I(m^*, n^*)$.

Table 7.3 Effect of K_0/K_S on the total costs for perfect and imperfect matching

Ordering Cost (K_0)	Setup Cost (K_S)	Ratio	$TC_P(m^*, n^*)$	$TC_I(m^*, n^*)$
10	550	0.018	\$2,751.63	\$2,917.59
40	510	0.078	\$2,717.34	\$2,881.28
70	470	0.149	\$2,683.06	\$2,844.97
100	430	0.233	\$2,648.77	\$2,808.66
130	390	0.333	\$2,614.48	\$2,772.35
160	350	0.457	\$2,580.20	\$2,736.04
190	310	0.613	\$2,545.91	\$2,699.74
220	270	0.815	\$2,511.63	\$2,663.43
250	230	1.087	\$2,477.34	\$2,627.12
280	190	1.474	\$2,443.06	\$2,590.81
310	150	2.067	\$2,408.77	\$2,554.50
340	110	3.091	\$2,374.48	\$2,518.19
370	70	5.286	\$2,340.20	\$2,481.88
400	30	13.333	\$2,305.91	\$2,445.58

7.4 Effect of Raw Material (H_R) and Finished Goods (H_F) Carrying Costs

In the total cost function of a two echelon inventory system, the raw material (H_R) and finished goods (H_F) carrying costs, play an important role. So, it is essential to find the impact in the total cost functions for both perfect and imperfect matching case, with variations in both the raw material (H_R) and finished goods (H_F) carrying costs. As the total cost functions are linearly dependent on both the raw material (H_R) and finished goods (H_F), the total costs will increase with the increase of both H_R and H_F. Conversely, the ratio of raw material (H_R) and finished goods (H_F) carrying costs have different effects on the total cost $TC(m^*, n^*)$.

Therefore, varying the ratio of H_R/H_F from 0.03 to 9.0, and using the parametric values in Equations (3.11) and (4.13), the effects on both perfect and imperfect matching systems total costs, $TC_P(m^*, n^*)$, and $TC_I(m^*, n^*)$, are presented in Figure 7.6, respectively. Also, details

computational results are presented in Table 7.4. From both Figure 7.6 and Table 7.4 it is observed that the total cost functions decrease with the increase of the ratio of H_R/H_F.

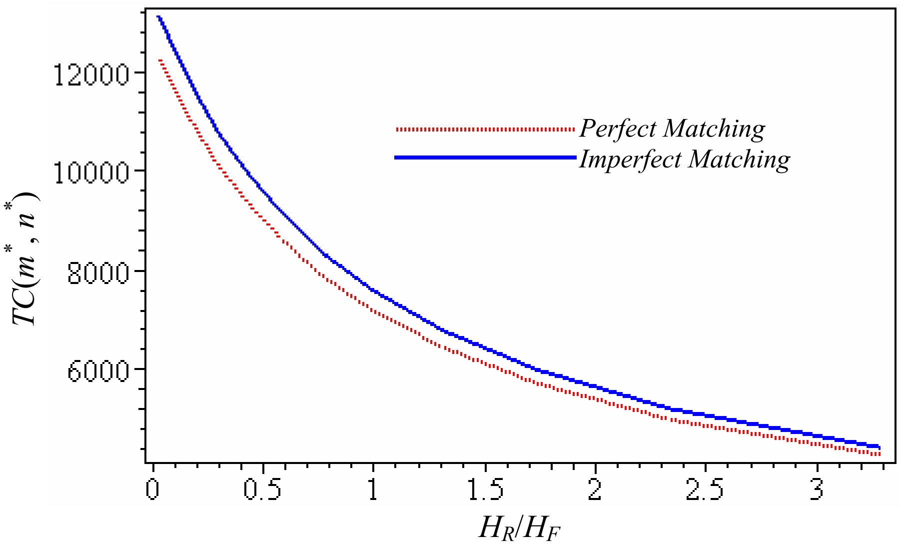


Figure 7.6. Effect of H_R/H_F on the total system costs

Table 7.4 Effect of H_R/H_F on the total costs for perfect and imperfect matching

Raw Material Carrying Cost, (H_R)	Finished Goods Carrying Cost, (H_F)	Ratio	$TC_P(m^*, n^*)$	$TC_I(m^*, n^*)$
1	29	0.03	\$12,284.94	\$13,138.98
3	27	0.11	\$11,555.40	\$12,345.70
5	25	0.20	\$10,825.85	\$11,552.42
7	23	0.30	\$10,096.31	\$10,759.14
9	21	0.43	\$9,366.76	\$9,965.86
11	19	0.58	\$8,637.22	\$9,172.58
13	17	0.76	\$7,907.68	\$8,379.30
15	15	1.00	\$7,178.13	\$7,586.02
17	13	1.31	\$6,448.59	\$6,792.74
19	11	1.73	\$5,719.04	\$5,999.46
21	9	2.33	\$4,989.50	\$5,206.18
23	7	3.29	\$4,259.95	\$4,412.90
25	5	5.00	\$3,530.41	\$3,619.62
27	3	9.00	\$2,800.86	\$2,826.34

From the above analysis it can be concluded that, in most cases, the total cost function for perfect matching case is always lower than the imperfect matching case. Therefore, it can be stated that whenever the system enters in real world the total cost increases.

CHAPTER 8

CONCLUDING REMARKS

The primary objective of this research was to determine the operational policy for a two stage supply chain system with reduced system idle time, to minimize both the inventory and system cost. This work studied three types of problems for a serial system with fixed demand quantity in (1) perfect matching situation, (2) imperfect matching situation, and (3) multiple suppliers and multiple buyers situations. This chapter assembles the conclusive remarks in the form of short summary, research results, important conclusions and future research issues.

8.1 SUMMARY

This study presents an operations policy of a supply chain system with just-in-time (JIT) deliveries. The supply chain system operates under a reduced idle time, meaning the production of a cycle starts immediately after the end of its preceding cycle. A set of problems are categorized as a serial system with a fixed quantity and a fixed delivery interval in a perfect matching condition, where the finished goods produced are the same as the finished goods delivered to the customers. Further, this study considered the serial system with a fixed quantity, fixed delivery interval, and imperfect matching condition, in which some produced finished goods remains in storage after the end of all possible shipments of finished goods to the customers. Based on this research, it was further extended for multi-supplier and multi-buyer supply chain system as well as multiple products manufacturing in a single production facility.

For all the parts of the research, the optimum number of orders, optimum batch sizes, and optimum number of shipments were evaluated to minimize the total system cost. A sensitivity analysis was performed for both perfect and imperfect matching cases to visualize

the characteristics of the cost functions with respect to their static parametric values (shipment sizes, raw material conversion factor, etc.).

The operation policies prescribe the number of orders and the ordered quantities of raw materials from suppliers, production quantities, and number of shipments to the customers for an infinite planning horizon. The heuristics for optimization based on the integer approximation search procedures, were described.

8.2 RESULTS

In this research, the problems perfect matching and imperfect matching are formed as non-linear integer (NILP) non-convex functions. The problems with rotation cycles are formed as non-convex mixed integer non-linear programming (MINLP) problems. The solution techniques were proposed using integer approximations, and divide and conquer rules. Based on these solution processes, this research used various numerical analysis based on numerical data found in previous research. The minimum costs obtained for perfect and imperfect matching inventory conditions using the proposed solution techniques are optimal, which are confirmed by using CONOPT and DICOPT solvers available in GAMS software. According to the results of perfect matching and imperfect matching inventory condition, it can be observed that whenever the realistic situations are considered in the supply chain models, the total cost increases. Moreover, the total cost for the accelerated production (current research) is higher than the cost for the deferred production (found in literature), because the current research produced more finished products as the facility has less idle time or down time. From the results for multi-supplier and multi-buyer supply chain system, it was found that instead of the finished goods delivery schedule assigned by the buyers, if manufacturer and the buyers can agree on a fixed time period for shipments to all buyers, the manufacture will be able minimize costs.

From the sensitivity analysis for the accelerated production, it was found that for large shipment sizes, the total costs do not increase rapidly. Therefore, this model can be applied to systems which consider large shipment sizes. Moreover, the total cost functions decreases rapidly when the ratio of ordering cost and setup cost increases from 0.02 to 4.0. The same type of variation can be observed for the ratio of raw material and finished goods holding costs with the increase of holding costs, ordering cost and setup costs. Therefore, it can be concluded that the increase in different cost parameters has huge impacts on the total cost functions up to a certain level after which the impacts are insignificant.

8.3 SIGNIFICANCE OF RESEARCH

In the past, researchers tried to develop the proper supply chain management with ideal conditions. This research focused more on real life situations where the production facility does not remain idle and does not wait until the end of shipments. In many industries, the production facilities stay idle only during the routine maintenance because of high costs of shutting down and restarting the production facilities, such as refineries, paper mills, etc.

The proposed models will allow the decision makers to quickly respond to the changes in demand and setup parameters by adjusting the cost parameters and the planning horizon. System performances such as work-in-process, inventory costs, and system cost can be reduced down to a significant level by implementing the prescribed policies and their solution techniques. This research has potential applications in industries for determining the operational policies such as production quantity, cycle length, order quantity, and number of orders for two-stage supply storage system. Specific applications can be found in supply chains for refinery, paper mills, microchips, electronic industries, and retailers industries.

This research will have a significant impact on the real life production facilities where the idle time of the facility is negligible. This will help to develop a better supply chain

management to any industry. According to the managerial perspective, the manager always faces the questions regarding the determination of production cycle, number of raw material order, the quantities of raw materials per order, batch size, number of shipment to the customers, etc. When the finished goods supply to buyers has same quantity and interval of shipments, the solution methodology for the problems perfect matching and imperfect matching can be applied by the manager. The solution method calls for an efficient search procedure and produces near optimum solutions that minimize the system costs. The methods can be implemented on MS Excel, which makes it easier for the managers to decide the course of action. In a manufacturing process, the managers and the upper level personnel are always aware of the production status and schedules for the delivery. Therefore, this research will help the managers take decisions regarding the issues addressed above.

8.4 FUTURE RESEARCH

Prospective research issues that can be pursued further concerning the supply chain system addressed in this research are as follows:

1. **Time Varying Demand:** In real world, the demand of a product is typically either increasing or decreasing, or it remains constant over a certain period during its life cycle. Usually, the electronic products industries such as computers, softwares, .etc, fall under this category of demand profiles, in most cases. Therefore, this type of demand profiles can be incorporated into the model to make the supply chain problem more realistic.
2. **Power-of-Two:** According to the research presented here, it can be observed that the solutions of these supply chain system are complicated. In 1986, Roundy proposed a near optimal solution technique named as *power-of-two* policy for two echelon inventory techniques. If this technique can be applied to the supply chain system, a

solution can be obtained which may help by adding more realistic constraints to the supply chain system.

3. **Variable Production Capacity:** The capacity of production in some manufacturing facilities can be adjusted for the demand of the products. The system considered in here operates under constant production capacity. The categories of problems addressed in this research may be extended to systems with a variable production capacity as a decision variable, which is a more general class of supply chain system.
4. **Transportation Cost:** In each supply chain system transportation of materials from one facility to other is an important decision variable. A significant amount of costs is involved in the supply chain system due to transportation of goods, (raw material, finished products, etc.). Hence, including the transportation cost as a decision variable will enhance the supply chain system analysis.
5. **Multi-stage System:** Finally, an extension of the current research can be addressed with multiple stages with network structured supply chain system, which might be of interest to many researchers. This system is more applicable with those industries who owned multiple facilities with different stages.

REFERENCES

Aderohunmu, R., Mobolurin, A. and Bryson, N., (1995) "Joint vendor-buyer policy in JIT manufacturing," *Journal of Operational research Society*, **46**(3), 1995, 375-385.

Anderson, J. and Marklund, J., (2000), "Decentralized inventory control in a two-level distribution system," *European Journal of Operational Research*, **127**(3), 483-508.

Anderson, J., Axsäter, S. and Marklund, J., (2000), "Decentralized multi-echelon inventory control," *Production and Operations Management*, **7**, 370-388.

Audet, C. and Dennis, J. E. Jr., (2003), "Analysis of generalized pattern searches," *Siam Journal of Optimization*, **13**(3), 889-903.

Axsäter, S., (1997), "Simple evaluation of echelon stock (R, Q)-policies for two level inventory systems," *IIE Transactions*, **29**(8), 661-669.

Axsäter, S., (2001), "A frame work for decentralized multi-echelon inventory control," *IIE Transactions*, **33**(1), 91-98.

Axsäter, S. and Zhang, W. F., (1999), "A joint replenishment policy for multi-echelon inventory control," *International Journal of Production Economics*, **59** (1-3), 243-250.

Banerjee, A. (1986), "A joint economic lot sizing model for purchaser and vendor," *Decision Science*, **17**(3), 292-311.

Banerjee, A. (1992), "Production lot sizing with work-in-process considerations in response to periodic demand," *International Journal of Production Economics,* **26**(2), 145-151.

Banerjee, A. and Burton, J. S., (1990), "Single and multi-stage lot sizing with work-in-process inventory consideration," *Engineering Cost and Production Economics*, **19**(3), 287-294.

Barbosa, L. C. and Friedman, M., (1978), "Deterministic inventory lot size models-a general root law," *Management Science*, **24**(8), 819-828.

Best, M. J. and Chakravarty, N., (1988), "Active set algorithms for isotonic regression when the underlying graph is an arborescence," *Research Report CORR*, **88-18**, Faculty of Mathematics, University of Waterloo, Waterloo, IA.

Biswas, P. and Sarker, B. R., "Economic order quantity for a JIT production system with minimal downtime," *The Proceedings of the Industrial Engineering and Research Conference* (*CD-ROM)*, Atlanta, GA, May 14-18, 2005.

Biswas, P. and Sarker, B. R., "An Improved Production Plan to Minimize Machine Idle Time," Accepted for *The Proceedings of the Industrial Engineering and Research Conference* (*CD-ROM)*, Orlando, FL, May 20-24, 2006.

Cachon, G. P., (2001), "Exact evaluation of batch ordering inventory policies in two-echelon supply-chains with periodic review," *Operations Research*, **49**(5), 658-674.

Cao, Q. and Schniederjans, M. J., (2004), "A revised EMQ/JIT production-run model: An examination of inventory and production costs," *International Journal Of Production Economics*, **87** (1), 83-95.

Chen, F. R. and Zheng, Y. S., (1994), "Evaluating echelon stock (R, nQ) policies in serial production inventory systems with stochastic demand," *Management Science*, 40(10), 1262-1275.

Chen, F., Federgruen, A. and Zheng, Y. S., (2001), "Coordination mechanics for a distribution system with one supplier and multiple retailers," *Management Science*, **47**(6), 963-708.

Clark A. R. and Armentano V. A., (1995), "A heuristic for a resource-capacitated multi-stage lot-sizing problem with lead times," *Journal of Operations Research Society*, **46**(10), 1208-1222.

Coope, I. D. and Price, C. J., (2001), "On the convergence of grid-based method for unconstrained optimization," *Siam Journal of Optimization*, **11**(4), 859-869.

Diponegoro, A., (2003), "Finite-horizon operations planning for a lean supply chain system," Ph. D. Dissertation, Department of Industrial and Manufacturing Systems, Louisiana state University, Baton Rouge, LA, 2003.

Diponegoro, A. and Sarker, B. R., (2006), "Operations policy for a supply chain system with fixed interval delivery and linear demand," *Journal of the Operational Research Society*, (in press).

Fung, R. Y. K. and Ma, X., (2001), "A new method for joint replenishment problems," *Journal of The Operational Research Society*, **52**(3), 358-362.

Federgruen, A. and Zheng, Y-S., (1993), "Optimal power-of-two replenishment strategies in capacitized general production/distribution networks," *Management Science*, **39**(6), 710-728.

Federgruen, A., Queyranne, M. and Zheng, Y-S., (1992), "Simple power-of-two policies are close to optimal in a general class of production/distribution networks with general joint setup costs," *Mathematics of Operations Research*, **17**(4), 951-963.

Frenk, J. B. G., Kleijn, M. J. and Dekker, R., (1999), "An efficient algorithm for a generalized joint replenishment problem," *European Journal of Operational Research*, **118**(2), 413-428.

Fürnkranz, J., (1999), "Separate and Conquer rule learning," *Artificial Intelligence Review* **13** (1), 3–54.

Golhar, D. Y. and Sarker, B. R., (1992), “Economic manufacturing quantity in a just in time delivery system,” *International Journal of Production Research*, **30**(5), 961-972.

Goyal, S. K. (1976), “An integrated inventory model for a single supplier-single customer problem,” *International Journal of Production Research*, **15**(1), 107-111.

Goyal, S. K. and Nebebe, F., (2000), “Determination of economic production-shipment policy for a single-vendor-single-buyer system,” *European Journal of Operational Research*, **121**(1), 175-178.

Goyal, S. K. and Satir, A. T., (1989), “Joint replenishment inventory control - deterministic and stochastic-models,” *European Journal of Operational Research*, **38**(1), 2-13.

Goyal, S. K., (1978), “Economic batch quantity in a multi-stage production system,” *International Journal of Production Research*, **16**(4), 267-273.

Himmelblau, D. M., (1972), *Applied Nonlinear Programming*, McGraw-Hill Book Company, New York, USA.

Hill, R. M. (1996), “Optimizing a production system with a fixed delivery schedule,” *Journal of Operational Research Society,* **47**(7), 954-960.

Hill, R. M., (1995), “Viewpoints on: an optimal batch size for a production system operating under a fixed-quantity, periodic delivery policy,” *Journal of Operational Research Society*, **46**(2), 271-273.

Hill, R. M., (1999), “The optimum production and shipment policy for the single-vendor single-buyer integrated production-inventory problem,” *International Journal of Production Research*, **37**(11), 2463-2475.

Hook, R and Jeeves, T. A., (1961), “Direct search solution of numerical and statistical problems,” *Journal of the ACM*, **8**(2), 212-229.

Jackson, P. L., Maxwell, W. L. and Muckstadt, J. A., (1988), “The joint replenishment problem with power-of-two restriction,” *AIIE Transactions*, **17**, 25-32.

Jamal, A. M. M. and Sarker, B. R., (1993), “An optimal batch size for a production system operating under a just-in-time delivery system,” *International Journal of Production Economics*, **32**(2), 255-260.

Jamal, A.M.M, Sarker, B. R. and Mondal, S., (2004), “Optimal manufacturing batch size with rework process at a single-stage production system,” *Computers and Industrial Engineering*, **47**(1), 77-89.

Jamal, A. M. M., Sarker, B. R., and Wang, S., (1997), “An ordering policy for deteriorating items with allowable shortage, and permissible delay in payment,” *Journal of the Operational Research Society*, **48**(8), 826-833.

Jamal, A. M. M., Sarker, B. R. and Wang, S., (2000), "Optimal payment time for a retailer under permitted delay of payment by the wholesaler," *International Journal of Production Economics*, **66**(1), 59-66.

Johnson, L. A. and Montgomery, D. C., (1974), *Operations Research in Production Planning, Scheduling and Inventory Control*, First Edition, John Wiley and Sons, Inc., New York, U.S.A.

Khan, L. R. and Sarker, R. A., (2002), "An optimal Batch size for a JIT manufacturing system," *Computers and Industrial Engineering*, **42**(2-4), 127-138.

Lewis, R. M. and Torczon, V., (1999), "Pattern search algorithms for bound constrained minimization," *Siam Journal of Optimization*, **9**(4), 1082-1099.

McGavin, E. J., Schwartz, L. B. and Ward, J. E., (1993), "Two-interval inventory-allocation policies in a one-warehouse *N*-identical-retailer distribution system," *Management Science*, **39**(9), 1092-1108.

Mitra, S. and Chatterjee, A. K., (2004), "Echelon stock based continuous review (R, Q) policy for fast moving items," *OMEGA-International Journal of Management Science*, **32**(2), 161-168.

Muckstadt, J. A. and Roundy, R. O., (1993), "Analysis of multistage production system," *Handbook of Operations Research*, **4**, 59-131.

Nahmias, S., (1997), *Production and Operations Analysis*, Third Edition, Irwin/McGraw-Hill, New York, USA.

Nori, V. S. and Sarker, B. R., (1996), "Cyclic scheduling for a multi-product, single-facility production system operating under a just-in-time delivery policy," *Journal of Operational Research Society*, **47**(7), 930-935.

Nori, V. S. and Sarker, B. R., (1998), "Optimum number of kanbans between two adjacent stations," *Production Planning and Control*, **9**(1), 60-65.

Ojha, D., Sarker, B. R. and Biswas, P., "An optimal batch size for an imperfect production system with quality assurance and rework," *International Journal of Production Research* (in press).

Ouyang, L. Y. and Chuang, B. R., (2000) "A periodic review inventory model involving variable lead time with a service level constraint," *International Journal Of Systems Science*, **31**(10), 1209-1215.

Parlar, M., Wang, Y.Z. and Gerchak, Y., (1995), "A periodic review inventory model with Markovian supply availability," International Journal of Production Economics, **42**(2), 131-138.

Parija, G. R. and Sarker, B. R., (1999), "Operations planning in a supply chain systems with fixed-interval deliveries of finished goods to multiple customers," *IIE Transactions*, **31**(11), 1075-1082.

Powell, M. J. D., (1994), "A direct search optimization methods that models the objective and constrained functions by linear interpolation," *Advances in Optimization and Numerical Analysis*: *Proceedings of the Sixth Workshop on Optimization and Numerical Analysis*, Oaxaca, Mexico, **275**, 51-68.

Powell, M. J. D., (1998), "Direct search algorithm for optimization calculation," *Acta. Numer*., **7**, 287-338.

Rao, S. S., (1992), *Optimization*: *Theory and Applications*, Second Edition, Wiley Eastern Limited, Delhi, India.

Ravindran, A., Phillips, D. T. and Solberg, J. J., (1987), *Operations Research*: *Principal and Practice*, Second Edition, John Wiley and Sons, Inc, New York, U.S.A.

Roundy, R., (1986), "A 98%-effective lot sizing rule for a multi-product, multi-stage production/inventory system," *Mathematics of Operations Research*, **11**(4), 699-728.

Roundy, R., (1989), "Rounding off to powers of two in continuous relaxations of capacitated lot sizing problems," *Management Science*, **35**(12), 1433-1442.

Sarker, B. R. and Balan, C. V., (1998), "Operations planning for a single-cycle kanban system operating under linear demand," *International Journal of Production Research*, **36**(2), 357-375.

Sarker, B. R. and Balan, C. V., (1999), "Operations planning for a multi-cycle kanban system," *European Journal of Operations Research*, **112**(2), 284-303.

Sarker, B. R. and Diponegoro, A., (2006), "Finite horizon planning with fixed interval of deliveries and no shortage," *OPSEARCH* (in press).

Sarker, B. R. and Fitzsimmons, J. A., (1989), "The performance of push and pull systems: a simulation and comparative study," *International Journal of Production Research*, **27**(10), 1715-1731.

Sarker, B. R., Jamal, A. M. M., and Wang, S., (2000), "Supply chain models for perishable products under inflation and permissible delay in payment," *Computers and Operations Research*, **27**(1), 59-75.

Sarker, B. R., Jamal, A. M. M., and Wang, S., (2000), "Optimal payment time under permissible delay in payment for products with deterioration," *Production Planning and Control*, **11**(4), 380-390.

Sarker, B. R. and Parija, G. R., (1994), "An optimal batch size for a production system operating under a fixed-quantity, periodic delivery policy," *Journal of Operational Research Society*, **45**(8), 891-900.

Sarker, B. R. and Parija, G. R., (1995), "Reply on: an optimal batch size for a production system operating under a fixed-quantity, periodic delivery policy," *Journal of Operational Research Society*, **46**(2), 273-274.

Sarker, B. R. and Parija, G. R., (1996), "Optimal batch size and raw material ordering policy for a production system with a fixed-interval, lumpy demand delivery system," *European Journal of Operational Research*, **89**(3), 593-608.

Sarker, R. A. and Khan, L. R., (1999), "An optimal batch size for a production system operating under periodic delivery policy," *Computers and Industrial Engineering*, **37**(4), 711-8.

Sarker, R. A., Karim, A. N. M. and Azad, S., (1995), "Two cases of integrated inventory," *Journal of the Institution of Engineers*, *Bangladesh*, **21**(4), 45-52.

Sarker, R. A., Karim, A. N. M. and Haque, A. F. M. A., (1995), "An optimal batch size for a production system operating under a continuous supply/demand," *International Journal of Industrial Engineering*, **2**(3), 189-198.

Teunter, R., (2004), "Lot-sizing for inventory systems with product recovery Computers and Industrial Engineering, **46** (3), 431-441.

Taha, A. H, (1992), *Operations Research*: *An Introduction*, Macmillan Publishing Company, New York.

Venkateswaran, J. and Son, Y. J., (2004), "Impact of modeling approximations in supply chain analysis-an experimental study*," International Journal of Production Research* **42**(15), 2971-2992.

Wang, H. and Wang, H-P. B., (1991), "Optimum number of kanbans between two adjacent workstations in a JIT system," *International Journal of Production Economics*, **22**(2), 179-188.

Wang, H. W., Guo, M. and Efstathiou, J., (2004), "A game-theoretical cooperative mechanism design for a two-echelon decentralized supply chain," *European Journal of Operational Research*, **157**(2), 372-388.

Wang, S., (2001), "Control of supply chain systems by kanban mechanism," Ph. D. Dissertation, Department of Industrial and Manufacturing System Engineering, Louisiana State University, Baton Rouge, LA.

Wang, S. and Sarker, B. R., (2002), "Locating cells with bottleneck machines in cellular manufacturing systems," *International Journal of Production Research*, **40**(2), 403-424.

Wang, S. and Sarker, B. R., (2004), "A Single-stage Supply Chain System Controlled by Kanbans under Just-in-time Philosophy," *Journal of the Operational Research Society*, **55**(5), 485-494.

Wang, S. and Sarker, B. R., (2005), "An assembly-type supply-chain system controlled by kanbans under just-in-time philosophy, *European Journal of Operational Research*, **162**(1), 153-172.

Wang, S., Sarker, B. R., Triantaphyllou, E., and Mann, Jr., L., (2004), "Resource Planning for a Depot Location Model in Electric Power Restoration," *European Journal of Operational Research*, **155**(1), 22-43.

Wang, W., Fung, R. Y. K. and Chai Y. T., (2004), "Approach of just-in-time distribution requirements planning for supply chain management," *International Journal of Production Economics*, **91**(2), 101-108.

Zhou, Y. W., Lau, H. S. and Yang, S. L., (2004), "A finite horizon lot-sizing problem with time-varying deterministic demand and waiting-time-dependent partial backlogging," *International Journal of Production Economics*, **91**(2), 109-119.

APPENDIX A

PROOF OF CONVEXITY

A.1 Convexity Test of Cost Function TC_{PM}

Since TC_{PM} is a function of (m, n), it is sufficient to show that $TC(m, n)$ is convex for m, $n \geq 1$. Note that $P, D_F, K_0, K_S, H_R, H_F, x, f \geq 0$ and $m, n \geq 1$, Hence, it is required to prove that the principal minors of the Hessian matrix of Equation (3.12) are positive. The total cost function for the perfect matching case is

$$TC_{PM} = \frac{n^2}{m}\left(\frac{x^2 H_R}{2fP}\right) + \frac{m}{n}\left(\frac{D_F K_0}{x}\right) + \frac{1}{n}\left(\frac{D_F K_S}{x}\right)$$

$$+\frac{nxH_F}{2} + \frac{D_F H_F}{2}\left(\frac{x}{D_F} - T_S\right). \tag{A.1}$$

The Hessian of TC_{PM} can be found by partial differentiation with respect to m and n as follows:

$$H(m,n) = \begin{bmatrix} \mathbb{R} & \mathbb{Z} \\ \mathbb{Z} & \mathbb{C} \end{bmatrix}, \tag{A.2}$$

where $\mathbb{R} = \dfrac{\partial^2 TC_{PM}}{\partial n^2} = \dfrac{x^2 H_R}{mfP} + \dfrac{2D_F}{n^3 x}(mK_0 + K_S)$,

$$\mathbb{Z} = \frac{\partial^2 TC_{PM}}{\partial m \partial n} = -\left(\frac{D_F K_0}{n^2 x} + \frac{nx^2 H_R}{m^2 fP}\right), \text{ and}$$

$$\mathbb{C} = \frac{\partial^2 TC_{PM}}{\partial m^2} = \frac{n^2 x^2 H_R}{m^3 fP}$$

From Equation (A.2), the first principal minors of Hessian $H(m,n)$ is found as

$$H_1(m,n) = \mathbb{R} = \frac{x^2 H_R}{mfP} + \frac{2D_F}{n^3 x}(mK_0 + K_S). \tag{A.3}$$

Hence, $H_1(m,n) \geq 0$ as all parameters are positive.

Using Equation (A.2) the second principal minor for Problem PM can be found as

$$H_2(m,n) = \mathbb{RC} - \mathbb{Z}^2 = \frac{2xD_F K_S H_R}{m^3 nfP} - \frac{D_F^2 K_0^2}{n^4 x^2}. \tag{A.4}$$

Therefore, from Equation (A.4) it can be confirmed that $H_2(m,n) \geq 0$ if and only if

$$\frac{n^3 x^3 K_S H_R}{m^3 fPD_F K_0^2} \geq \frac{1}{2}, \tag{A.5}$$

which confirms that the total cost function is a quasi-convex function.

A.2 Convexity Test of Cost Function TC_{IM}

The cost function TC_{IM}, which is stated as Equation (4.12) in Chapter 4 can be rewritten as follows:

$$TC_I = \frac{Q_F'^2 H_R}{2mfP} + \frac{mD_F K_0}{Q_F'} + \frac{1}{2} Q_F' H_F$$
$$+\frac{1}{Q_F'}\left\{D_F K_S - \frac{I_0 H_F}{2}\left(I_0 + x - D_F T_S\right)\right\} + \frac{H_F}{2}\left(4I_0 + x - D_F T_S\right). \tag{A.6}$$

From Equation (A.6), the Hessian matrix is evaluated as follows:

$$H(Q_F', m) = \begin{bmatrix} \mathbb{R}' & \mathbb{Z}' \\ \mathbb{Z}' & \mathbb{C}' \end{bmatrix}, \tag{A.7}$$

where, $\mathbb{R} = \frac{\partial^2 TC_I}{\partial Q_F'^2} = \frac{H_R}{mfP} + \frac{2}{Q_F'^3}\left[mD_F K_0 + D_F K_S - \frac{I_0 H_F}{2}\left\{\left(I_0 + x - D_F T_S\right)\right\}\right],$

$$\mathbb{Z}' = \frac{\partial^2 TC_I}{\partial Q_F' \partial m} = -\left(\frac{Q_F' H_R}{m^2 fP} + \frac{D_F K_0}{Q_F'^2}\right), \text{ and}$$

$$\mathbb{C}' = \frac{\partial^2 TC_I}{\partial m^2} = \frac{Q_F'^2 H_R}{m^3 fP}$$

Now, from Equation (A.7) the first principal minor is

$$H_1(Q_F', m) = \mathbb{R} = \frac{H_R}{mfP} + \frac{2}{Q_F'^3}\left[mD_F K_0 + D_F K_S - \frac{I_0 H_F}{2}\left\{\left(I_0 + x - D_F T_S\right)\right\}\right]. \tag{A.8}$$

For convexity, $H_1(Q'_F, m)$ has to be greater than or equal to zero, so that from Equation (A.8) it can be written as

$$mD_F K_0 + D_F K_S - \frac{I_0 H_F}{2}\left\{\left(I_0 + x - D_F T_S\right)\right\} \geq 0\,. \tag{A.9}$$

According to the assumption, $T_S \leq \left(\frac{x}{D_F} - \frac{x/2 - I_0}{P}\right) = \frac{x}{D_F} - \frac{x}{2P} + \frac{I_0}{P}$. Replacing T_S, from Equation (A.10) it can be found that

$$mD_F K_0 + D_F K_S - \frac{I_0 H_F}{2}\left\{I_0 + x - D_F\left(\frac{x}{D_F} - \frac{x}{2P} + \frac{I_0}{P}\right)\right\} \geq 0$$

which yields $$mD_F K_0 + D_F K_S - \frac{I_0 H_F}{2}\left\{I_0 + \frac{xD_F}{2P} - \frac{I_0 D_F}{P}\right\} \geq 0$$

$$mD_F K_0 + D_F K_S - \frac{I_0 H_F}{2}\left\{I_0 + \frac{D_F}{P}\left(\frac{x}{2} - I_0\right)\right\} \geq 0$$

$$D_F\left(mK_0 + K_S\right) \geq \frac{I_0 H_F}{2}\left\{I_0 + \frac{D_F}{P}\left(\frac{x}{2} - I_0\right)\right\} \tag{A.10}$$

(according to the assumption $x/2 - I_0 \geq 0$).

Therefore, Equation (A.8) is positive if and only if Equation (A.10) holds.□

Using Equation (A.7) the second principal minor $H_2(Q'_F, m)$ can be evaluated as

$$H_2(Q'_F, m) = \mathbb{R}'\mathbb{C}' - \mathbb{Z}'^2$$

$$= \left[\frac{H_R}{mfP} + \frac{2}{Q_F'^3}\left\{mD_F K_0 + D_F K_S - \frac{I_0 H_F}{2}\left\{\left(I_0 + x - D_F T_S\right)\right\}\right\}\right]\left(\frac{Q_F'^2 H_R}{m^3 fP}\right)$$

$$-\left(\frac{Q'_F H_R}{m^2 fP} + \frac{K_0 D_F}{Q_F'^2}\right)^2. \tag{A.11}$$

which yields

$$H_2(Q_F', m) = \frac{2D_F K_S H_R}{Q_F' m^3 fP} - \frac{I_0^2 H_R H_F}{Q_F' m^3 fP} - \frac{xI_0 H_R H_F}{Q_F' m^3 fP} + \frac{I_0 H_R H_F D_F T_S}{Q_F' m^3 fP} - \frac{D_F^2 K_0^2}{Q_F'^4}$$

$$= \frac{H_R}{Q_F' m^3 fP}\left(2D_F K_S - I_0^2 H_F - xI_0 H_F + I_0 H_F D_F T_S\right) - \frac{D_F^2 K_0^2}{Q_F'^4}$$

$$= \frac{H_R}{Q_F' m^3 fP}\left\{2D_F K_S + I_0 H_F\left(D_F T_S - I_0 - x\right)\right\} - \frac{D_F^2 K_0^2}{Q_F'^4}. \qquad \text{(A.12)}$$

Equation (A.12) is positive if and only if

$$\frac{H_R}{Q_F' m^3 fP}\left\{2D_F K_S + I_0 H_F\left(D_F T_S - I_0 - x\right)\right\} \geq \frac{D_F^2 K_0^2}{Q_F'^4}. \qquad \text{(A.13)}$$

Hence, Equation (A.10) is only positive if and only if

$$\frac{Q_F'^3 H_R\left\{2D_F K_S + I_0 H_F\left(D_F T_S - I_0 - x\right)\right\}}{D_F^2 K_0^2 fPm^3} \geq 1, \qquad \text{(A.14)}$$

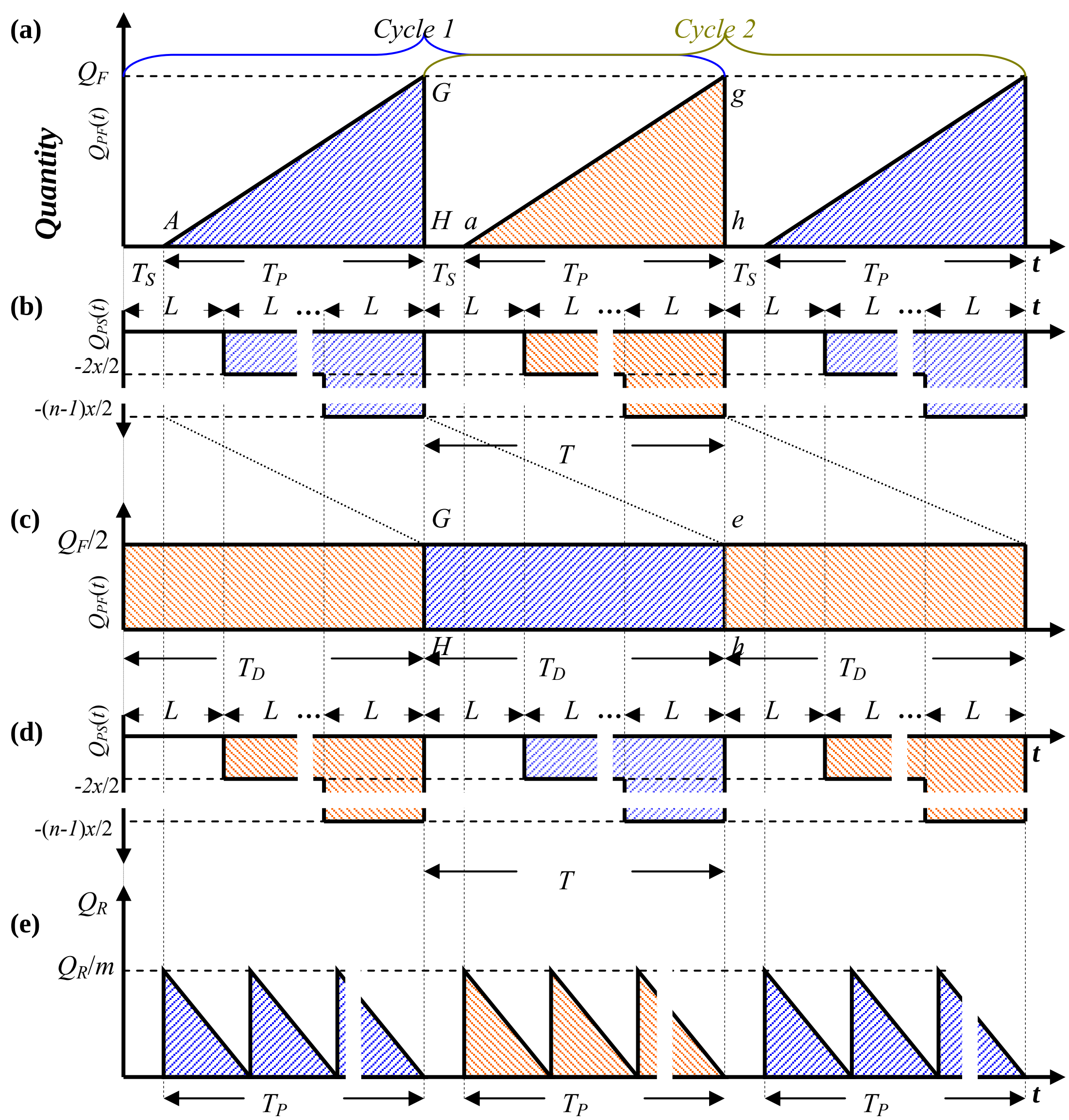
which indicates that the function is convex if and only if Equations (A.10) and (A.14) hold.□

APPENDIX B

INVENTORY COMPUTATION

B.1 Inventory Computation for Perfect Matching

To calculate the total inventory for perfect matching case, Figure 3.2 is broken down in different sections and represented in Figure B.1as follows:



(a) Inventory produced; (b) Inventory shipped during production; (c) Inventory during downtime; (b) Inventory shipped during downtime; and (e) Raw material inventory

Figure B.1. Step-by-step inventory formation for perfect matching case

The area under the curve *GghH* of Figure 3.2 is

$$I_{PT} = I_{PP} - I_{PD}. \tag{B.1}$$

The area I_{PP} and I_{PD} are shown individually on their own time scale and presented in Figure B.1. Figure B.1 (a) represents the inventory produced during the time period T_P. Figure B.1 (b) shows the inventory shipped during the time period T_P. Figure B.1(c) illustrates the inventory remained during the downtime and Figure B.1(d) presents the inventory shipped during time period T_D. Using Figure B.1 (a) and (c), the total inventory I_{PP} during time period T can be calculated as follows:

$$I_{PP} = \Delta agh + AreaGehH\ , \tag{B.2}$$

where $\Delta agh = \frac{Q_F}{2} T_P = \frac{nx}{2} T_P$, and

$$AreaGehH = \frac{Q_F}{2} T_D = \frac{nx}{2} T\ .$$

Again, from Figure B.1 (b) and (d), the total inventory shipped I_{PD} time period T can be calculated as

$$I_{PD} = \frac{Lx}{2} + 2\frac{Lx}{2} + ... + (n-1)\frac{Lx}{2} + \frac{Lx}{2} + 2\frac{Lx}{2} + ... + (n-1)\frac{Lx}{2}$$

$$= \frac{2Lx}{2}\left[1 + 2 + ... + (n-1)\right] = \frac{n(n-1)}{2} Lx = \frac{nx^2(n-1)}{2D_F}, \tag{B.3}$$

where $L = x/D_F$.

From Figure 3.2 and Figure B.1 the total cycle time T is

$$T = T_P + T_S = T_D = nx / D_F\ . \tag{B.4}$$

Adding Equations (B.2) and (B.3), the total inventory of one cycle can be found as

$$I_{PT} = \frac{nx}{2} T_P + \frac{nx}{2} T - \frac{nx^2(n-1)}{2D_F}$$

$$= \frac{nx}{2}(T - T_S) + \frac{nx}{2}T - \frac{nx^2(n-1)}{2D_F} \qquad \text{[using Equation (B.4)]}$$

$$= \frac{nx}{2}\frac{nx}{D_F} + \frac{nx}{2}\frac{nx}{D_F} - \frac{nx^2(n-1)}{2D_F} - \frac{nx}{2}T_S$$

$$= \frac{n^2x^2}{2D_F} + \frac{n^2x^2}{2D_F} - \frac{n^2x^2}{2D_F} + \frac{nx^2}{2D_F} - \frac{nx}{2}T_S$$

$$= \frac{n^2x^2}{2D_F} + \frac{nx^2}{2D_F} - \frac{nx}{2}T_S$$

$$= \frac{n^2x^2}{2D_F} + nx\left(\frac{x}{2D_F} - \frac{T_S}{2}\right), \qquad \text{(B.5)}$$

where $T = nx / D_F$.□

The above Equation is used to calculate the average inventory of perfect matching case in Chapter 3.

B.2 Inventory Computation for Imperfect Matching

To evaluate the total inventory for imperfect matching, the area of Figure 4.1 must be evaluated as

$$\hat{I}_{IT} = \hat{I}_{IP} - \hat{I}_{ID}, \qquad \text{(B.6)}$$

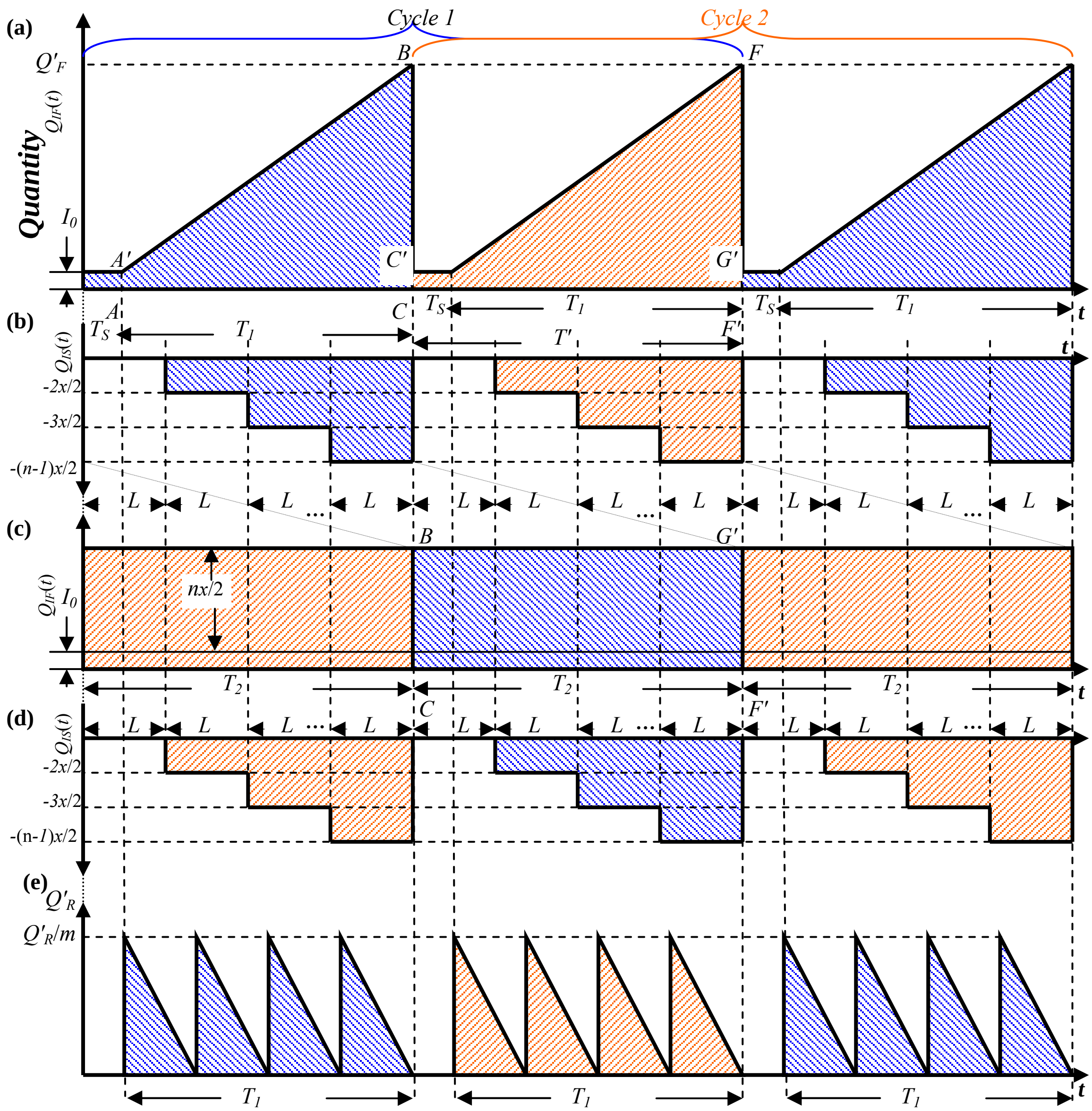
where $\hat{I}_{IT}$, $\hat{I}_{IP}$ and $\hat{I}_{ID}$ are the total inventory for imperfect matching, the total inventory produced and the total inventory shipped during time *T'*, respectively. To illustrate the behavior of the inventory diagram presented in Figure 4.1, the diagram is divided in different parts and represented in Figure B.2.

From Figures B.2 (a) and B2 (c), it can be found that

$$\hat{I}_{IP} = \text{area } CC'FF' + \text{area } CBGF', \qquad \text{(B.7)}$$

where $$\text{area } CC'FF' = \frac{nx}{2}T_1 + I_0T' \text{, and} \tag{B.8}$$

$$\text{area } CBGF' = \frac{nx}{2}T' + I_0T' . \tag{B.9}$$



(a) Inventory produced; (b) Inventory shipped during production; (c) Inventory during downtime; (b) Inventory shipped during downtime; and (e) Raw material inventory

Figure B.2. Inventory behavior for imperfect matching in different stages

Considering Equations (B.7), (B.8) and (B.9), the total produced inventory can be found as

$$\hat{I}_{IP} = \frac{nx}{2}T_1 + \frac{nx}{2}T' + 2I_0T'. \tag{B.10}$$

Again, the total inventory shipped can be calculated from Figure B.1 (b) and B.1 (d) as

$$\hat{I}_{ID} = L(x/2) + 2L(x/2) + \ldots + (n-1)L(x/2) + L(x/2) + 2L(x/2) +$$

$$\ldots + (n-1)L(x/2) = 2\frac{n(n-1)}{2}\frac{Lx}{2} = \frac{nx^2(n-1)}{2D_F}, \tag{B.11}$$

where $L = x/D_F$.

Hence, the total inventory for time period $T' = Q'_F / D_F$ is

$$\hat{I}_{IT} = \frac{nx}{2}T_1 + \frac{nx}{2}T' + 2I_0T' - \frac{nx^2(n-1)}{2D_F}$$

$$= \frac{nx}{2}(T' - T_S) + 2I_0T' + \frac{nx}{2}T' - \frac{n^2x^2}{2D_F} + \frac{nx^2}{2D_F}$$

$$= \frac{(Q'_F - I_0)}{2}\left(\frac{Q'_F}{D_F} - T_S\right) + 2I_0\frac{Q'_F}{D_F} + \frac{(Q'_F - I_0)}{2}\frac{Q'_F}{D_F} - \frac{(Q'_F - I_0)^2}{2D_F} + \frac{(Q'_F - I_0)x}{2D_F}$$

$\left[\text{as the total quantity produced is } Q'_F = nx + I_0\right]$

$$= \frac{(Q'_F - I_0)Q'_F}{D_F} - \frac{(Q'_FT_S - I_0T_S)}{2} + \frac{2I_0Q'_F}{D_F} - \frac{Q'^2_F - 2Q'_FI_0 + I_0^{\,2}}{2D_F} + \frac{xQ'_F - xI_0}{2D_F}$$

$$= \frac{Q'^2_F}{2D_F} + \frac{2I_0Q'_F}{D_F} - \frac{Q'_FT_S}{2} + \frac{xQ'_F}{2D_F} - \frac{I_0^{\,2}}{2D_F} - \frac{xI_0}{2D_F} + \frac{I_0T_S}{2}$$

$$= \frac{Q'^2_F}{2D_F} + \frac{2I_0Q'_F}{D_F} + \frac{xQ'_F}{2D_F} - \frac{Q'_FT_S}{2} - \frac{I_0^{\,2}}{2D_F} - \frac{xI_0}{2D_F} + \frac{I_0T_S}{2}$$

$$= \frac{Q'^2_F}{2D_F} + Q'_F\left(\frac{2I_0}{D_F} + \frac{x}{2D_F} - \frac{T_S}{2}\right) - \left(\frac{I_0^{\,2}}{2D_F} + \frac{xI_0}{2D_F} - \frac{I_0T_S}{2}\right)$$

$$= \frac{Q_F'^2}{2D_F} + \frac{Q_F'}{2D_F}(4I_0 + x - D_F T_S) - \frac{I_0}{2D_F}(I_0 + x - D_F T_S), \qquad \text{(B.12)}$$

where $T' = Q_F' / D_F$ and $Q_F' = nx + I_0$. □

The above Equation is used to calculate the total cost function of finished product for imperfect matching case in Chapter 4.

VITA

Pablo Biswas was born on January 1, 1974, in Jessore, Bangladesh. He received his Bachelor of Science in Mechanical Engineering degree from Bangladesh University of Engineering and Technology (BUET), Dhaka, in 1998. He received a Master of Science in Industrial Engineering degree from Louisiana State University, Baton Rouge, Louisiana, in May 2003. The Doctor of Philosophy degree will be awarded to him on May 2007.

Since 2000, he has been working as a Teaching Assistant for a number of courses (such as project management, engineering economics, engineering statistics, and manufacturing processes and methods) at the Department of Industrial Engineering, Louisiana State University. He has also assisted in the LSU Manufacturing Process Laboratory for six years. As a Research Assistant, he worked on three projects in the area of transportations, logistics, and simulation, funded by the US Army Corps of Engineering Research and Development Center (USAC-ERDC). He also worked on the project "Multi-product Flowline Design for a Flexible Manufacturing System" funded by the National Science Foundation (NSF) during 1999-2000.

He has published one paper in *International Journal of Production Research*, and three of others in refereed conference proceedings. Five of his recent articles are in different stages of review processes by *Journal of the Operational Research Society*, *OPSEARCH*, and *Journal of Transportation Engineering*. His research interests are in production and manufacturing systems with special interest in supply chain management, logistics, lean manufacturing system, production planning and control, cellular/flexible manufacturing systems, operations research, and simulation. He is a member of the Institute of Industrial Engineers (IIE), the Institute for Operations Research and Management Science (INFORMS), and the Decision Science Institute (DSI).