Performance of disc brushless DC motor applied as gearless drive for wheelchair

Deepti Rao Chikkam
Louisiana State University and Agricultural and Mechanical College, dchikk1@lsu.edu

Follow this and additional works at: https://digitalcommons.lsu.edu/gradschool_theses
Part of the Electrical and Computer Engineering Commons

https://digitalcommons.lsu.edu/gradschool_theses/3284
PERFORMANCE OF DISC BRUSHLESS DC MOTOR APPLIED AS GEARLESS DRIVE FOR WHEELCHAIR

A Thesis
Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College
in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering

in

The Department of Electrical & Computer Engineering

by

Deepti R Chikkam
B.Tech., J.N.T.U;2002
December, 2005
ACKNOWLEDGEMENTS

I would like to thank my father, Nageswara Rao Chikkam, my mother, Vijayalakshmi Chikkam, and my brother Bhaskar Srinivas for their encouragement and enduring patience as well as love and support during the course of my graduate studies.

I would like to express my deepest gratitude to my advisor and teacher, Dr. Ernest Mendrela for the tremendous amount of guidance and support that he provided during the preparation of this dissertation and throughout my entire graduate study.

In addition, I am very grateful to Dr. Leszek S. Czarnecki and Dr. Jin Woo Choi for being members of my committee. I would also like to thank Dr. Kenneth Paxton.
# TABLE OF CONTENTS

ACKNOWLEDGEMENTS..................................................................................................................ii

LIST OF TABLES............................................................................................................................v

LIST OF FIGURES..........................................................................................................................vi

ABSTRACT.........................................................................................................................................ix

CHAPTER 1: INTRODUCTION.............................................................................................................1

CHAPTER 2: GEARLESS DRIVE WITH DISC BRUSHLESS DC MOTOR .......4
  2.1 Currently Used Technology .................................................................................................4
  2.2 Gearless Drive ......................................................................................................................6
  2.3 Brushless DC Motors ..........................................................................................................8
  2.4 Disc Brushless DC Motors ..................................................................................................16
    2.4.1 Motor with Axial Flux in the Stator .............................................................................16
    2.4.2 Torus Type Motor .......................................................................................................19

CHAPTER 3: PROTOTYPE OF TORUS BRUSHLESS DC MOTOR...........24
  3.1 Motor Description and Design Parameters .................................................................24
  3.2 Motor Controller Parameters .........................................................................................27

CHAPTER 4: DETERMINATION OF MOTOR EQUIVALENT MODEL PARAMETERS.................................................................30
  4.1 Equivalent Circuit Parameters .......................................................................................30
  4.2 Parameters of the Mechanical System ..........................................................................31
    4.2.1 Friction Coefficient D ...............................................................................................31
    4.2.2 Moment of Inertia J .................................................................................................33
    4.2.3 Constant $K_e$ for Induced EMF ...........................................................................34

CHAPTER 5: MOTOR PERFORMANCE IN STEADY-STATE CONDITIONS..........................................................................................36
  5.1 Motor Model for Steady-State Operation ......................................................................36
  5.2 Performance Characteristics of the Motor ....................................................................37
  5.3 Setup of the Motor - Load System ................................................................................39

CHAPTER 6: SIMULATION OF MOTOR DYNAMICS.................................................42
  6.1 Mathematical Model of the Supply – Inverter – Motor System ....................................42
  6.2 Simulation of the Motor Operation under Constant Supply Voltage and Constant Load Torque..................................................................................................................48
    6.2.1 Data of the Drive System ..........................................................................................48
    6.2.2 A SIMULINK Block Diagram ..................................................................................49
    6.2.3 Simulation of a Wheelchair Drive under Dynamic Conditions .........................49
6.2.3.1 Starting Up Operation ...............................................................51
6.2.3.2 Driving a Wheelchair under Variable Road Conditions ..........55
6.2.4 An Influence of Switching Angle on Motor Performance ..........59
6.3 Comparison of Simulation Results with the Results Obtained from the Tests Conducted .................................................................67
6.3.1 Full – Load Conditions ...............................................................67
6.3.2 No – Load Conditions ...............................................................69

CHAPTER 7: CONCLUSION ...............................................................72

REFERENCES ..................................................................................75

APPENDIX-A: M-FILES FOR START AND PLOT BUTTONS ...............77

APPENDIX-B: M-FILES FOR THE ELECTROMECHANICAL CHARACTERISTICS OF THE MOTOR UNDER DYNAMIC CONDITIONS ....78

APPENDIX-C: M-FILES FOR PERFORMANCE CHARACTERISTICS OF DISC BRUSHLESS DC MOTOR UNDER STEADY STATE CONDITIONS ....81

APPENDIX-D: SUBSYSTEM MODELS .................................................82

VITA ...............................................................................................85
# LIST OF TABLES

3.1 Main design data of the motor designed as an application for a wheelchair ........27

3.2 Design specifications of the compact drive system SSC024D16 ..................29

4.1 Measured results from short-circuit and open-circuit test ......................31

4.2 Readings from friction coefficient measurements ...............................32

4.3 Readings from the moment of inertia measurements ............................34

4.4 Readings from the induced EMF measurements ..................................35

6.1 Wheelchair drive (single motor) performance at steady-state ...............59

6.2 Electromechanical parameters measured under full – load ..................67

6.3 Electromechanical parameters measured under no – load ....................69
LIST OF FIGURES

2.1 Mechanical system for a wheelchair drive. Motor, gears and axle together constitute the transmission system which runs the wheelchair........................................4

2.2 Block diagram of an electric drive system.............................................................5

2.3 Configurations of drives of electric vehicle..............................................................7

2.4 Cylindrical motor attached directly to a wheel.........................................................8

2.5 Permanent magnet brushless DC motor.................................................................10

2.6 Diagram of DC commutator motor, which explains its operation.........................12

2.7 Diagram of a DC motor with 3 coils (phases) in the armature..............................12

2.8 Scheme of the DC motor with 3-phase winding connected in Y.................................13

2.9 Rotor positions at two subsequent instants.............................................................14

2.10 The waveforms of torque (T) and electromotive force (E)....................................15

2.11 Block diagram of brushless DC motor.................................................................16

2.12 Disc motor with axial magnetic flux in the stator core........................................17

2.13 Distribution of permanent magnet on the rotor disc.............................................18

2.14 Three-phase stator winding with hall sensors......................................................18

2.15 Three-phase inverter (electronic commutator) for brushless DC motor...............19

2.16 View of the stator and the rotor of the disc motor................................................19

2.17 Scheme of the torus-type permanent magnet motor...........................................20

2.18 Magnetic flux in torus motor................................................................................20

2.19 Stator coils connected in a three-phase system....................................................21

2.20 Scheme of torus type permanent magnet motor with two rotor discs...............22

2.21 Scheme of the torus motor embedded in the wheel rim......................................22
2.22 Brushless DC torus motor ................................................................. 23
3.1 Scheme of the disc motor ................................................................. 24
3.2 Stator core with Gramme’s winding .................................................. 25
3.3 Part of stator core with teeth ............................................................ 26
3.4 Three-phase winding of the disc motor .............................................. 26
3.5 Dimensions of permanent magnet ..................................................... 26
3.6 Brushless DC motor ........................................................................ 27
3.7 SSC024D16 Drive System ................................................................. 28
4.1 Circuit to measure the inductance of the DC brushless motor ............. 30
4.2 Setup to measure the friction coefficient ........................................... 33
4.3 Graph to determine the moment of inertia ......................................... 34
4.4 Waveform of EMF’s induced across the stator windings ..................... 35
5.1 Equivalent circuit of the motor in the steady state conditions ............. 36
5.2 Electromechanical characteristics of torus motor supplied with 24 V voltage ................. 38
5.3 Motor - load setup ......................................................................... 40
6.1 Circuit diagram of supply-inverter-motor system ................................. 42
6.2 Scheme to the equation 4 ................................................................. 44
6.3 Position of the rotor with respect to the phase A .................................. 45
6.4 SIMULINK Model of brushless DC motor ....................................... 50
6.5 Waveform of rotary speed (\(\omega_r\)) and source current (\(i_s\)) .................. 51
6.6 Waveform of EMF (\(e_a\)) and armature voltage (\(V_a\)) ................. 52
6.7 Waveform of armature current (\(i_a\)) and armature voltage (\(V_a\)) .... 53
6.8 Waveform of electromagnetic torque ............................................... 53
6.9 Waveforms of EMF in the 3 phases \((e_a), (e_b), (e_c)\) and supply phase voltage \((V_a)\)…..54
6.10 Wheelchair under variable road conditions.................................................55
6.11 Forces acting on the wheelchair ...............................................................55
6.12 Waveform of torque component for the different road sections \((T_T)\)........57
6.13 Waveforms of the electromechanical parameters........................................57
6.14 Waveform showing the switching ON and OFF angles \(\Psi_{ON}\) and \(\Psi_{OFF}\).............60
6.15 Model showing a part of the pulse generator to change the switching angle \(\Psi_{ON}\).................................................................................................60
6.16 Input current \((I_s)\) vs load torque \((T_L)\) ......................................................62
6.17 Mechanical power output \((P_{em})\) vs load torque \((T_L)\)............................62
6.18 Efficiency \((Eff\%)\) vs load torque \((T_L)\).....................................................63
6.19 Electromechanical characteristics of the motor.........................................63
6.20 Input current \((I_s)\) vs load torque \((T_L)\)......................................................65
6.21 Mechanical power output \((P_{em})\) vs load torque \((T_L)\)............................65
6.22 Efficiency \((Eff\%)\) vs load torque \((T_L)\).....................................................66
6.23 Electromechanical characteristics of the motor.........................................66
6.24 Waveform of armature current \((I_a)\).........................................................68
6.25 Waveform of armature current \((I_a)\).........................................................69
6.26 Waveform of induced EMF .....................................................................70
ABSTRACT

The currently used electric drive for a wheelchair consists of brush permanent magnet DC motor and mechanical transmission that drives the wheels. The overall efficiency of this kind of drive usually does not exceed 60%.

At present there is under study much more effective drive that consists of brushless DC motor which embedded into the wheel rim directly drives the wheelchair. This type of high efficiency gearless drive is the object of this thesis project. The particular brushless DC motor is a torus – type motor with a high energy rare – earth magnets and ferromagnetic teeth that fill in the space between the coils of Gramme’s winding.

The objectives of the project were to determine by computer simulation and laboratory test the electromechanical characteristics of the motor prototype in steady – state and dynamic conditions. For this purpose the measurement stand has been built and measurements were carried out in variable load conditions.

To analyze the motor characteristics theoretically the mathematical models of the motor were developed one for steady – state and one for dynamic operation. The simulation of the motor drive was done using MATLAB/ SIMULINK software package.

The results obtained from simulation confirm the requirements put on the gearless wheelchair drive and the determined motor ratings are as follows: supply voltage – 24 V, input current – 8.7 A, torque – 9.7 Nm, speed – 161 rpm, efficiency, 78 %. The cogging torque is practically unnoticeable.

The results obtained from the test practically do not differ from those obtained from simulation. It means the calculation model used in simulation of steady – state and dynamic conditions has been verified positively.
CHAPTER 1: INTRODUCTION

People with both upper and lower extremity impairments due to cerebral palsy, high level spinal cord injury, or muscular dystrophy use electric wheelchairs. 93,000 electric wheelchair users are present in United States. The Medicare expenditures in 1997 for electric wheelchairs were found to be $166 million [1].

Brushed, direct current, internally rotating, permanent magnet motor is the industry standard in the case of electric drives used for wheelchairs. The overall efficiency under light loading of the electric wheelchair is found to be 60% to 70% but under loads typical to that of the electric wheel chair the efficiency drops to about 45%. Motor and drive train efficiency impacts battery performance i.e. capacity, peak current, life span, and time between recharge and overall performance i.e. range and speed of the wheelchair system. The motors, drive trains and batteries size and configuration constrain the physical dimensions, which are the weight, width and height of the wheelchair. The electric wheelchairs maintenance costs are estimated to be in excess of $1,000 over a 5 year period. The gears chatter and swipe and the friction associated with the motor and bearings are the potential sources of vibration and noise [1]. Brushes wear out and cause noise and need regular maintenance and replacement when required [1].

The above mentioned deficiency of the conventional solution can be overcome by the new type of DC drive based on brushless DC motors operating without mechanical transmission.

The brushless DC motors are permanent magnet motors with electronic commutator. The permanent magnet motors used in this case are single phase or poly
phase motors. When operating with single phase or poly phase motors, the inverter plays the role of electronic commutator.

The brushless DC motors are distinguished not only by the high efficiency but also by their practically no maintenance. No maintenance is due to lack of brushes. They are also capable of delivering much greater torque from the same mass of active material mainly due to the high energy of permanent magnets that are applied.

Another means to improve the electric drive efficiency is to eliminate the mechanical transmission by embedding the motor directly in the wheel. This solution will improve not only the efficiency but also improve the reliability of the drive and lower the cost of the whole system. Among a few geometrical structures, the motor with disc geometry is most suitable. Within the disc motors, a few structures can be distinguished:

- Motor with single-sided stator.
- Torus motor.
- Motor with axial flux through the stator.

The **object** of this project is a brushless DC torus motor.

The **objectives** of this project are:

- To determine the performance of the laboratory motor model at steady-state conditions.
- To analyze the motor operation under variable supply and load conditions.

The **tasks** to be accomplished in this project are:

- Literature study about the brushless DC torus motor.
- Preparation of a setup to test the brushless DC motor.
• Determination by measurements the parameters of the equivalent circuit and mechanical system.
• Testing the motor in the steady state and dynamic conditions.
• Formulation of the mathematical motor models for steady state and dynamic operation.
• Writing the program on PC for calculation of motor performance in steady-state conditions.
• Development of block diagram of the motor in MATLAB/SIMULINK software for simulation of the motor operation in variable load conditions.
• Comparison of the simulation results with those obtained from the test.
CHAPTER 2: GEARLESS DRIVE WITH DISC BRUSHLESS DC MOTOR

2.1 Currently Used Technology

The propulsion system of a currently used electric wheelchair consists of a pair of DC motors, one for each drive wheel and a drive train consisting of gears or belts or other mechanical elements that couple the motor’s shaft to the drive wheel shaft (Fig 2.1) [1].

A DC–DC converter drives each motor with a high frequency, square-wave pulse-train that rapidly turns each motor on and off (Fig 2.2). A microprocessor based control unit controls the speed and the torque generated by each motor by independently modulating the pulse-width into each motor. Solid state relays are generally used to switch supply voltage polarity to change the running direction of PM motor.

1 - driving rear wheel
2 - motor that drives the wheel
3 - gears for transmission
4 – semi-axle

Fig. 2.1 Mechanical system for a wheelchair drive. Motor, gears and axle together constitute the transmission system which runs the wheelchair
The control module of the wheelchair converts the positional information from the joystick into power signals to the motors. The control modules use feedback to check whether the motor is responding properly to the joystick position. These control modules adjust motor torque to maintain near constant speed while the load varies in response to changes in the terrain i.e. incline, bumps, grass, concrete, etc. and these controllers automatically limit the current to the motors when the wheelchairs get overloaded [1].

Permanent magnet motors have a linear torque-speed characteristic that makes them easy to control. The most commonly used DC motors in wheelchair drives are permanent magnet motors. The motor and drivetrain specifications for an electric wheelchair are very unique compared to the motors used in other industries. In the case of electric wheelchairs, two motor designs are preferred over one motor design. The motor must have an average efficiency of 75% and should have a high start up torque [2, 3]. It must
have performance characteristics of a true electrical transmission with a continuously variable gear ratio and should be independently controllable. The drive system should incorporate sensors that provide information to compensate for motor imbalance, diagnostics, steering, acceleration and wear status and must have good heat dissipation characteristics [1].

Recently proposed improvements in the electric wheelchair industry include the use of rare-earth magnets and brushless, gearless and direct-drive motors. Motors that use rare earth magnets can be more powerful than motors with iron magnets and much smaller in size. The brushless motors have better heat dissipation capability because the windings are on outside and there is no power loss through the brushes. Gearing and belts in the drivetrain are a source of noise and power loss. In order to reduce all these problems, gearless, direct-drive, brushless and rare earth magnet motors are proposed [1, 4].

As an example, the requirements for the motor that drives the wheelchair produced by Permobil, a Swedish company is as follows [5]:

- Torque developed by the motor at rotary speed $n = 150 \text{ rpm}$ - 8 $Nm$
- Maximum torque at starting - 60 $Nm$
- Supply source battery - 24 $V$
- Driving wheels of rim diameter - 240 $mm$

2.2 Gearless Drive

- In gearless drives, the motors are incorporated within the drive wheels. Fig 2.3 (a, b and c) shows drives with gears and without gears. Fig 2.3(c) shows a drive without gears. The motors incorporated within the wheels of the vehicle are called hub
motors. In the case of hub motors, the torque transmission elements from motor to wheel are eliminated [6]. Fig 2.4 shows an example in which the cylindrical motor is attached directly to the wheel rim.

(a)

(b)

(c)

Fig. 2.3 Configurations of drives of electric vehicle: (a) – central sprung motor with mechanical transmission, (b) – two sprung motors, (c) – two unsprung hub motors

The advantages of gearless drives are as follows:
• It eliminates the use of transmission elements like chains and belts. It improves system reliability and system performance, lowers installation costs and reduces system components all for approximately the same initial equipment investment as today’s geared drive system [7].

• The system prevents single wheel spins; it fits the wheel units for a variety of vehicles.

An increase of efficiency allows increasing the driving distance under the single charge, which in turn reduces the cost of driving [6, 8].

![Diagram of a cylindrical motor attached directly to a wheel](image)

**Fig. 2.4** Cylindrical motor attached directly to a wheel: 1-motor, 2-wheel rim, 3-rotor, 4-stator, 5-stator winding, 6-rotor magnets, 7-hollow shaft, 8-supply wire, and 9-tyre

### 2.3. Brushless DC Motors

The first earliest evidence of a brushless DC motor was in 1962 when T.G. Wilson and P.H. Trickey wrote about a DC brushless motor in a paper. It was subsequently
developed as a high torque, high response drive for special applications such as tape and disk drives for computers, robotics and positioning systems, and in aircraft, where brush wear was intolerable due to low humidity. This motor could not be used for industrial purposes for a power requirement greater than 5 hp. Over the years with the advent of high energy magnetic materials and high power and high voltage transistors e.g. thyristors and power MOSFETS, these motors came into existence. In 1980’s, the first DC brushless motor with thyristors was designed [3].

The first large brushless DC motors with a power capacity of 50 hp or more were designed by Robert E. Lordo at POWERTEC Industrial Corporation in the late 1980s [3]. At present all the major motor manufacturing industries make brushless DC motors. DC brushless motors have had a substantial impact in some industry market areas, primarily plastics and fibers, wire drawers, winders, cranes, and conveyors. Most recently a mining company has put several of these drives at 300 hp ratings operating coal conveyors in underground mines [3].

The increasing popularity of brushless permanent magnet motors in recent years is due to the drop in prices of the high energy magnets and electronic devices. The brushless permanent magnet motors perform better and have higher efficiency than the machines with electromagnetic excitation. Fig. 2.5 shows a permanent magnet brushless DC motor. All the parts are shown in Fig. 2.5. The rotor is a permanent magnet; the sensors used are hall elements [9, 10].

There are essential differences between the brush DC motor and brushless DC motor. The brush DC motor is equipped with the commutator, whereas an electronic commutator is used in a brushless DC motor. The use of electronic commutator implies
the armature winding to be on stator and the rotor is equipped with permanent magnets. In brush DC motor, the windings or the armature is always on the rotor.

**Fig. 2.5 Permanent magnet brushless DC motor** [9, 10]

The advantages of the brushless DC motor over the DC motors and the AC induction motors are [2, 4, 11]:

- **Performance**: The dynamic accuracy of the DC brushless motor is very high. Dynamic accuracy means the machine performs consistently, with the same efficiency.

- **Size**: The DC brushless motor is the smallest of the motors available with a given power rating. Thus the machine occupies lesser floor space, weighs lighter and hence it makes handling of the machine easier.

- **Efficiency**: The brushless DC motor is the most efficient motor available in the present industry.
• **Bearing stress**: In large AC motors, heat current flows from the rotor through the bearings, damaging the motor bearings. The rotor heating in the DC brushless motor is the least because there is no winding in the rotor since it has a permanent magnet. The rotor heat in brush motors is transferred to the stator through the bearings and the shaft before being removed by the ambient air. In brushless DC motor, the rotor heat produced is low thus it reduces the bearing stress.

Despite the differences between brush DC and brushless DC motors there are some basic similarities in these two motors. These similarities are discussed further.

The brush DC motor is shown schematically in Fig. 2.6. The motor is excited either by field winding or by permanent magnets. In both the cases, they are placed on the stator. The armature winding, which is placed on the rotor, consists of a number of coils. When the rotor turns, the current of the subsequent coils that are approaching stationary brushes, are commutated. Due to the commutator, the resultant magnetic flux $\Phi_a$ produced by the coils is always perpendicular to the field flux despite the current changes in the rotor coils.

The commutator can be regarded as a mechanical rectifier (in case of DC generators) and as a mechanical inverter (in case of a DC motor). This is seen particularly clear when the DC motor with three coils or phases (connected in delta) is considered as shown in Fig. 2.7. At particular time instant $t_1$, coils A, B and C are supplied generating the resultant flux $\Phi_a$ perpendicular to the field flux Fig. 2.8.a.

The same position of $\Phi_a$ can be achieved if the coils are supplied from DC source through the 3-phase inverter shown in Fig. 2.7.b.
Fig. 2.6 Diagram of DC commutator motor, which explains its operation [12]

Fig. 2.7 Diagram of a dc motor with 3 coils (phases) in the armature: (a) coils commutated by the mechanical commutator, (b) coils commutated by the electronic converter (electronic commutator) [12]

In case of star connected winding shown in Fig. 2.7.a two coils are energized at any time. This can be achieved using a 3-phase inverter shown in Fig. 2.7.b. The Fig. 2.7 shows the position of the motor and coils energized by commutator and inverter at three different time instants. No changes are observed in the resulting flux $\Phi_a$ position with respect to the stationary field. There are only some small changes in the position of flux $\Phi_a$ between two subsequent winding commutations which are shown in Fig. 2.8.
Fig. 2.8 Scheme of the DC motor with 3-phase winding connected in Y: (a) with mechanical commutator (winding placed on rotor), (b) with electronic commutator (inverter) (winding is on the stator and magnetic poles rotate) [12]
Due to the change in the position $\Delta \theta = 60^\circ$ (Fig. 2.9.c) the electromagnetic torque $T_{em}$, (the result of interaction of stationary flux $\Phi_f$ and flux $\Phi_a$) changes with time producing some torque ripple as shown in Fig. 2.9.b. The more the number of phases the smoother the torque waveform (Fig. 2.9.a) is obtained. With these similarities between conventional commutator DC motors and brushless DC motors, there is only one difference between the two. In a DC commutator motor, the armature winding which is mechanically commutated is placed on the rotor and the field winding (or permanent magnets) is on the stator, while in the case of brushless DC motor, both the parts are reversed.

**Fig. 2.9** Rotor positions at two subsequent instants: (a) $t_1$ and (b) $t_2$, (c) mutual positions of armature flux with respect to field flux position at instants $t_1$ and $t_2$[12 ]
This is because the armature winding in DC motor is self-commutated winding caused by rotating commutator while the brushless DC motor winding can be commutated by stationary electronic inverter. The moment of commutation in conventional DC motor is determined by the position of the coil with respect to the stationary brushes. In brushless DC motors, the moment of commutation is determined by the position of the sensor signal. It means that these motors cannot operate without the position sensors. Fig. 2.10 shows the schematic diagram of the brushless DC motor drive.

Hall sensors, optical sensors or induction sensors are used to sense the position of the rotor. The controller checks the position information and determines through simple logic which phase winding should be switched ON and switched OFF. The controller is built in a very similar way to the controller used in an AC variable frequency drive or in an AC vector drive. All three types use a PWM type for variable voltage control to their respective motors.

Fig. 2.10 The waveforms of torque (T) and electromotive force (E) : (a) with more phases (b) for 3-phase motor [12]
Inverter

Supply (DC Source)

Triggering signal to thyristors

Micro-controller

Position/speed signal

Hall sensor

Motor

PMDC Motor

2.4 Disc Brushless DC Motors

The disc-type permanent magnet DC brushless motors are the ones which are most suitable for a gearless drive in electric vehicles [13, 14].

Among several types of disc type permanent magnet DC brushless motors, two constructions have been most frequently proposed:

- Motor with axial flux in the stator.
- Torus type motor.

2.4.1 Motor with Axial Flux in the Stator

Axial flux PM motors can be designed as double-sided or single sided machines, with or without armature slots, with internal or external PM rotors and with surface mounted or interior type PM’s. Low power axial flux PM machines are usually machines with slotless windings and surface PMs. Rotors are embedded in power-transmission components to optimize the volume, mass, power transfer and assembly time.
Double-sided motor with internal PM disk rotor has the armature windings located on the two stator cores. The disk with the PM rotates between the two stators. PMs are embedded or glued in a nonferromagnetic rotor skeleton. When the stators are connected in parallel the motor can operate even when one stator windings break down. The stator cores are wound from electrotechnical steel strips and the slots are machined by shaping or planning [15].

Double-sided motor with one stator. The internal stator is more compact than the internal rotor. The double-sided rotor with PMs is located at two sides of the stator [15, 16, 17, 18]. The scheme of the motor is shown in Fig. 2.12.

**Fig. 2.12 Disc motor with axial magnetic flux in the stator core [13, 12]**

The stator consists of electromagnetic elements made of ferromagnetic cores and coils wound on them. These elements are placed axially and uniformly distributed on the stator circumference and glued together by means of synthetic resin. On both sides of the
The coils of the stator elements can be connected in different systems. Fig. 2.14 shows the connection of coils in 3-phase system. The position sensors are placed between the coils in the intervals of 60° of electrical angle. If a three-phase connection is considered as in Fig. 2.14, then a three-phase inverter (Fig. 2.15) is applied.
The motor of this type of construction was proposed for the in-wheel-drive of the light electric car as in Fig. 2.15 [12].

![Diagram of three-phase inverter (electronic commutator) for brushless DC motor](image)

**Fig. 2.15 Three-phase inverter (electronic commutator) for brushless DC motor [12]**

![Image of stator and rotor of disc motor](image)

**Fig. 2.16 View of the stator and the rotor of the disc motor [12]**

### 2.4.2 Torus Type Motor

Torus type motor seems to be the most suitable gearless drive. It is schematically shown in Fig. 2.17. The stator is placed between two rotor discs. It consists of a slotless core and the Gramme’s type winding. The stator core is made of laminated iron. The rotor disc is made of solid iron contains the high energy permanent magnets glued to their surfaces [19, 25]. Fig. 2.18 shows the magnetic flux in the motor. The magnetic flux is directed axially in the air-gap and in the stator winding zone. It turns its direction in the stator and rotor core.
Fig. 2.17 Scheme of the torus-type permanent magnet motor: 1-stator core, 2-stator winding, 3-rotor disc, 4-magnets [12, 19, 20]

Fig. 2.18 Magnetic flux in torus motor [12, 17]

The stator coils can be connected in different ways as single-phase winding or polyphase winding. Fig. 2.19 shows the distributed coils. They are distributed around the
stator connected in three phase system. The position sensors (usually Hall sensors) are also shown. Here, they are displaced at 120° of electrical angle.

Fig. 2.19 Stator coils connected in a three-phase system

To increase the electromagnetic torque the motor can contain a number of discs. In Fig. 2.20, two motor discs are coupled together. The torus motors were proposed for gearless drive of electric vehicles [17]. Fig. 2.21 shows schematically the motor embedded in the wheel rim.
Fig. 2.20 Scheme of torus type permanent magnet motor with two rotor discs: 1-stator core, 2-stator winding, 3-rotor, 4- magnets [12]

Fig. 2.21 Scheme of the torus motor embedded in the wheel rim: 1-motor, 2-wheel rim, 3-rotor disc, 4-stator core, 5-rotor magnets, 6-stator winding, 7-hollow shaft, 8-supply wire, 9-tyre
The stator is attached firmly to the wheel axle while the rotor is connected directly within the wheel. A three phase bridge of transistors is usually used as a converter for operating the permanent magnet brushless DC motor. To detect the rotor position optical sensors or hall sensors are used. They are distributed on the stator disc in electrical 120° intervals. The sensors sense the position of the rotor and they trigger the transistors so that they switch ON the respective stator winding [21].

Fig. 2.22.a shows the motor model with two rotor discs and stator built for driving the wheelchair [13, 20]. The same motor mounted in the wheel is shown in Fig. 2.22.b.

Fig. 2.22 Brushless DC torus motor (a) Motor with two rotor discs (b) Torus brushless DC motor in wheel rim on the test stand [13, 20]
CHAPTER 3: PROTOTYPE OF TORUS BRUSHLESS DC MOTOR

3.1 Motor Description and Design Parameters

The motor prototype, the subject of this thesis was designed and manufactured with the purpose to drive a wheelchair [22].

The motor is a disc type torus permanent magnet brushless DC motor. The scheme of the motor is shown in Fig. 3.1. The stator is placed between the two rotor discs. The rotor disc is made of soft iron with permanent magnets of rectangular shape glued to its surface. The stator coils are connected in a three phase winding system with the position sensors (hall sensors) placed at 120° electrical angle.

Fig. 3.1 Scheme of the disc motor
The stator shaft is firmly attached to the wheel suspension, while the rotor is embedded into the wheel rim. The wires that supply the stator winding are lead through the hollow shaft.

The scheme of the stator is shown in Fig. 3.2. The stator has the toothed core with the teeth placed between the coils Fig. 3.3. The coils are connected in a 3-phase star system as shown in Fig. 3.4. One of the requirements for this type of drives is to minimize the torque ripple as much as possible, which in case of direct wheel drive could deteriorate the riding comfort. The reduction of torque ripple has been achieved by the application of square shape magnets (instead of trapezoidal) of the particular dimensions related to the dimensions of the stator core. The magnet dimensions are shown in Fig. 3.5. The main design data of the motor are in Table 3.1.

![Stator Core with Gramme’s Winding](image)

**Fig. 3.2 Stator core with Gramme’s winding**
Fig. 3.3 Part of stator core with teeth

Fig. 3.4 Three-phase winding of the disc motor

Fig. 3.5 Dimensions of permanent magnet
Table 3.1 Main design data of the motor designed as an application for a wheelchair

<table>
<thead>
<tr>
<th>Component</th>
<th>Specification</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stator:</strong></td>
<td>Laminated core: outer diameter × inner diameter × thickness</td>
<td>170×110×12 mm,</td>
</tr>
<tr>
<td></td>
<td>Teeth made of composite of magnetic permeability $\mu=5\mu_0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gramme’s type 3-phase winding:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number of magnetic poles</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>Number of coils</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>Number of turns per coil</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>Air-gap length</td>
<td>1 mm</td>
</tr>
<tr>
<td><strong>Rotor:</strong></td>
<td>Soft iron discs: outer diameter × inner diameter × thickness</td>
<td>186×94×7 mm</td>
</tr>
<tr>
<td></td>
<td>Permanent magnets of rectangular shape:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>length×width×width</td>
<td>30×25×4 mm</td>
</tr>
</tbody>
</table>

The motor prototype is shown in Fig. 3.6.

![Disc brushless DC motor](image)

**Fig. 3.6 Brushless DC motor**

### 3.2 Motor Controller Parameters

The control drive system for the DC brushless motor applied here is SSC024D16 Compact Drive System designed by the SL-MTI as shown in Fig. 3.7.b. It allows controlling the speed and the direction of rotation of the motor. The control drive used is a standalone unit in an aluminium case enclosure Fig. 3.7.a. The drive can be field mounted in a remote location, or factory mounted or it can be connected directly to the
motor. The drive is with open loop control. In this case, the speed will remain constant when the load is constant and moderately vary when the load varies.

(a)   (b)

Fig. 3.7 SSC024D16 Drive System (a) top view (b) view of the internal circuit [23]

The unit is simple to install and easy to operate. The unit has high power density; it can operate up to 1500 watts of output power, 20 Amperes of continuous current, and 35 Amperes of peak current. The internal components and construction of the circuit is rugged which provides high reliability under adverse conditions. The commutation frequency is high - 3.33 kHz, 100,000 rpm is achievable with a 4 pole motor. Hall sensor feedback is used. Three-phase Hall sensor feedback signals are used for motor commutation. A compact protective enclosure, durable anodized aluminium case shields electronic components from harmful environmental elements. It is thermally designed to maximize heat dissipation. Mounted to a metal structure, this enclosure becomes a convenient heat sink. Dimensions: 2.9 X 2.9 X 1.8 inches (74 X 74 X 46 mm)
The design specifications of the Compact Drive System are in Table 3.2. The signal sources for the SSC024D16 are hall sensors. The rotor position is sensed by the hall sensors and the signals are sent to the SSC024D16 [23].

Table 3.2 Design specifications of the compact drive system SSC024D16

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input voltage</td>
<td>$V-DC$</td>
<td>16-48</td>
</tr>
<tr>
<td>Continuous output current</td>
<td>$A$</td>
<td>20</td>
</tr>
<tr>
<td>Peak current</td>
<td>$A$</td>
<td>35</td>
</tr>
<tr>
<td>PWM Frequency</td>
<td>$kHz$</td>
<td>14</td>
</tr>
<tr>
<td>Command voltage</td>
<td>$V-DC$</td>
<td>0-6</td>
</tr>
<tr>
<td>Operating temperature</td>
<td>$^\circ C$</td>
<td>-40 to 45</td>
</tr>
<tr>
<td>Output power</td>
<td>$W$</td>
<td>900</td>
</tr>
</tbody>
</table>
CHAPTER 4: DETERMINATION OF MOTOR EQUIVALENT MODEL PARAMETERS

4.1 Equivalent Circuit Parameters

The motor equivalent circuit is shown in Fig. 4.1.a. The phase resistance of the motor was measured using DC current, its value \( R_{ph} = 0.25 \). 

![Motor equivalent circuit diagram](image)

**Fig. 4.1 Circuit to measure the inductance of the DC brushless motor: (a) Motor equivalent circuit (b) Open circuit (c) Short circuit**

The motor inductance was measured using the open circuit (Fig. 4.1.b) and the short circuit (Fig. 4.1.c) test. The rotor was driven by an external motor with a speed \( n \) and the stator windings were initially kept open. The open circuit line voltages across the windings were measured. During the short circuit test, the stator winding was short circuited and the phase currents flowing through the stator winding were measured. This is shown in Fig. 4.1.c. Additional 10Ω resistors were connected in series with each of the stator windings for protection. The mean values of the measured quantities are in Table 4.1. When the inductance was measured \( R_{ph} \) was equal to 0.3695Ω. This value is not the
same value obtained when the resistance was measured because of the change of winding temperature.

Table 4.1 Measured results from short-circuit and open-circuit test

<table>
<thead>
<tr>
<th>$n$ ( \text{rpm} )</th>
<th>$I_1$ ( \text{A} )</th>
<th>$V$ ( \text{V} )</th>
<th>$Z_{ph}$ ( \Omega )</th>
<th>$L_{ph}$ ( H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>137.6</td>
<td>0.62</td>
<td>11.14</td>
<td>10.37</td>
<td>1.009m</td>
</tr>
</tbody>
</table>

The inductance was calculated in the following way:

$$L_{ph} = \frac{X_{ph}}{2 \cdot \pi \cdot f}$$

where,

$$X_{ph} = \sqrt{(Z_{ph}^2 - (R_{ph} + 10)^2)}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}}$$

$$V_{ph} = \frac{V}{\sqrt{3}}$$

$$f = \frac{(p / 2) \cdot n}{60}$$

$p = 14$ number of magnetic poles

4.2 Parameters of the Mechanical System

The parameters of the mechanical system are the moment of inertia $J$, constant for induced EMF $K_e$ and the friction coefficient $D$.

4.2.1 Friction Coefficient D

To determine the friction coefficient used in simulation model of mechanical system, it was assumed that friction torque $T_f$ depends on rotary speed $\omega_m$ according to
the equation 4. During the no-load conditions, the electromagnetic torque produced by the motor equals the friction torque. There is no load torque, and inertia torque equals zero, because the inertia torque is proportional to the rate of change of speed.

\[ T_{fr} = D \cdot \omega_m \]  

(4)

To find D, the no-load test was carried out. The stator winding was supplied from the battery through the inverter. The stator attached firmly to the axle could freely rotate. The diagram of the test system is shown in Fig. 4.2. During the no-load test, the same torque that was driving the rotor with speed \( n \) was acting on the stator axle of the radius \( r \). The dynamometer that was attached to the axle showed the friction force \( F_{fr} \). The friction torque was then calculated as:

\[ T_{fr} = F_{fr} \cdot l \]  

(5)

The friction coefficient was found next from the following equation:

\[ D = \frac{T_{fr}}{\omega_m} \]  

(6)

where \( \omega_m = \frac{2 \cdot \pi \cdot n}{60} \) is the angular speed of the rotor.

The readings and calculated values are shown in Table 4.2.

**Table 4.2 Readings from friction coefficient measurements**

<table>
<thead>
<tr>
<th>( n ) ( \text{rpm} )</th>
<th>( F_{fr} ) ( \text{N} )</th>
<th>( l ) ( m )</th>
<th>( T_{fr} ) ( \text{Nm} )</th>
<th>( \omega_m ) ( \text{rad/s} )</th>
<th>( D ) ( \text{N/(rad/s)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>221.8</td>
<td>0.3</td>
<td>0.29</td>
<td>0.087</td>
<td>23.22</td>
<td>0.0037</td>
</tr>
</tbody>
</table>
4.2.2 Moment of Inertia J

A moment of inertia was determined from another no-load test according to the well-known procedure. During this no-load test, the motor running with speed $\omega_o$ is switched off. Its speed steadily goes down due to the friction power losses $\Delta P_m$ (the kinetic energy of the motor is dissipated as power loss) as shown in Fig. 4.3. The kinetic energy $W_{k,p}$ at a particular speed $\omega_p$ is equal to the energy $\Delta W_m$ dissipated due to friction losses. It means

$$W_{k,p} = \frac{1}{2} J \cdot \omega_p^2 = \Delta W_m = \frac{\Delta P_m \cdot \Delta t}{2}$$  \hspace{1cm} (7)

from the above equation

$$J = \frac{\Delta P_m \cdot \Delta t}{\omega_p^2}$$  \hspace{1cm} (8)

The readings obtained from the test and the calculated moment of inertia is in Table 4.3.

Fig. 4.2 Setup to measure the friction coefficient

4.2.2 Moment of Inertia J

A moment of inertia was determined from another no-load test according to the well-known procedure. During this no-load test, the motor running with speed $\omega_o$ is switched off. Its speed steadily goes down due to the friction power losses $\Delta P_m$ (the kinetic energy of the motor is dissipated as power loss) as shown in Fig. 4.3. The kinetic energy $W_{k,p}$ at a particular speed $\omega_p$ is equal to the energy $\Delta W_m$ dissipated due to friction losses. It means

$$W_{k,p} = \frac{1}{2} J \cdot \omega_p^2 = \Delta W_m = \frac{\Delta P_m \cdot \Delta t}{2}$$  \hspace{1cm} (7)

from the above equation

$$J = \frac{\Delta P_m \cdot \Delta t}{\omega_p^2}$$  \hspace{1cm} (8)

The readings obtained from the test and the calculated moment of inertia is in Table 4.3.
Table 4.3 Readings from the moment of inertia measurements

<table>
<thead>
<tr>
<th>$\omega_o$ (rpm)</th>
<th>$\omega_p$ (rpm)</th>
<th>$\Delta P_m$ (W)</th>
<th>$\Delta t$ (s)</th>
<th>$J$ (Kg-m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>224.5</td>
<td>210.3</td>
<td>1.794</td>
<td>2.6</td>
<td>0.0096</td>
</tr>
</tbody>
</table>

The graph used to obtain the value of the moment of inertia is shown in Fig. 4.3.

![Graph to determine the moment of inertia](image)

**Fig. 4.3 Graph to determine the moment of inertia**

4.2.3 Constant $K_e$ for Induced EMF

The EMF induced in the stator winding is proportional to the rotor speed $\omega_r$ and the flux $\Phi$ according to equation $E = K_e \cdot \Phi \cdot \omega_r$. Since the rotor flux in a brushless DC motor is constant, the EMF is $E = K_e \cdot \omega_r$. To determine EMF constant $K_e$, the rotor was driven by an external motor with a speed $n$. The phase-phase voltages across the windings of the stator are observed using an oscilloscope. The waveforms are shown in Fig. 4.4.
The constants are calculated using the following equations:

\[ \omega = \frac{n \cdot P}{60} \]  

(9)

\[ K_e = \frac{E_{ph\text{ (max)}}}{\omega_r} \]  

(10)

\[ E_{ph\text{ (max)}} = \frac{E \cdot \sqrt{2}}{\sqrt{3}} \]  

(11)

The readings obtained from the test and the calculated constants for induced EMF are in Table 4.4.

**Table 4.4 Readings from the induced EMF measurements**

<table>
<thead>
<tr>
<th>$E_V$</th>
<th>$E_{ph\text{ V}}$</th>
<th>$E_{max\text{ V}}$</th>
<th>$f$ Hz</th>
<th>$\omega$ rpm</th>
<th>$K_e$ V/(rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.88</td>
<td>3.97</td>
<td>5.61</td>
<td>11.7</td>
<td>100.28</td>
<td>0.5349</td>
</tr>
</tbody>
</table>
CHAPTER 5: MOTOR PERFORMANCE IN STEADY-STATE CONDITIONS

5.1 Motor Model for Steady-State Operation

The brushless DC motor with low winding inductance, which is usually for surface mounted permanent magnets, can be regarded at steady-state operation as a conventional separately excited DC motor. Such a motor can be analyzed applying circuit model shown in Fig. 5.1 The motor considered here has the phase inductance \( L_s = 1.009 \text{ mH} \). The motor operates at rated speed equal to 137.6 rpm. The relation between the speed and the frequency of the stator current is

\[
f = \frac{n \cdot p}{60} \tag{1}
\]

![Fig. 5.1 Equivalent circuit of the motor in the steady state conditions](image)

For the considered motor, the value of number of pole pairs \( p = 7 \). Using equation (1), \( f = 16.06 \text{ Hz} \). This means that the phase reactance \( X_s = 2 \cdot \pi \cdot f \cdot L_s = 0.102 \Omega \). Comparing with phase resistance equal to \( R_{ph} = 0.25 \Omega \), its value is smaller. It means the motor electromechanical characteristics will be little affected by the inductance.
They may be affected more at light loads when the rotor speed is higher and the frequency is higher too. Concluding the above reasoning it can be said that equivalent circuit shown in Fig. 5.1 without inductance may be used for analysis of the motor operating in steady-state conditions.

The equations, which describe the motor model, are as follows:

\[ T_{em} = K \cdot I_a \] (2)

\[ E_a = K_e \cdot \omega_r \] (3)

\[ V = E_a + I_a \cdot R_a \] (4)

where,

- \( T \) – electromagnetic torque,
- \( E_a \) – line-to-line electromotive force,
- \( \omega_r \) – rotor angular speed,
- \( K_e \) – constant,
- \( R_a = 2R_{ph} \) – armature resistance,
- \( I_a \) – average armature current
- \( V \) – source voltage.

### 5.2 Performance Characteristics of the Motor

Using the above mentioned equations in section 5.1, the program was written in MATLAB (see Appendix C file `steadystate.m`) to calculate the electromechanical characteristics. The characteristics obtained from simulation at supply voltage of 24 V are shown in Fig. 5.2.

The mechanical power was calculated as follows
\[ P_m = T_{em} \cdot \omega_m \]  \hspace{1cm} (5)

The efficiency of the motor is

\[ \text{Eff} \% = \frac{P_{out}}{P_{in}} \cdot 100\% \]  \hspace{1cm} (6)

where,

\[ P_{out} = P_m - \Delta P_m \]  - output power \hspace{1cm} (7)

\[ P_{in} = V \cdot I_a \]  - input power \hspace{1cm} (8)

In calculations, the mechanical power losses were expressed by the equation (9)

\[ \Delta P_m = \omega_m^2 \cdot D \]  \hspace{1cm} (9)

where D is the friction coefficient of 0.0037 (Nm/(rad/s))

---

Fig. 5.2 Electromechanical characteristics of torus motor supplied with 24 V voltage
From equations (3) and (4):

\[ \omega = \frac{V - R_a \cdot I_a}{K_e} \]  

(10)

or

\[ \omega_m = \frac{V}{K_e} - R_a \left( \frac{T_{em}}{K_e^2} \right) \]  

(11)

where,

\[ T_{em} = \frac{E_a \cdot I_a}{\omega_m} \]  

(12)

5.3 Setup of the Motor - Load System

To verify the mathematical model of the motor used in the analysis of the motor performance setup of motor – load has been built. It is shown schematically in Fig. 5.3. It consists of the following components:

- DC supply of 24 V
- SSC024D16 Compact drives system designed by SL-MTI, which is an inverter that supplies to the 3 – phase winding of the brushless DC motor.
- Disc permanent magnet brushless DC motor.
- Torque sensor TK20N manufactured by HBM, which generates signals for the torque and the speed of the motor.
- Data Acquisition Card *NI – DAQ 7.x* manufactured by National Instruments, which is used as a tool to capture the waveforms from the torque sensor and also the currents flowing in the circuit, on the computer using LABVIEW software.
- Oscilloscope manufactured by Tektronix.
- Permanent magnet DC generator applied as load for the brushless DC motor.
Fig. 5.3 Motor - load setup

The current, speed and torque waveforms are observed in the oscilloscope and the computer using the LABVIEW software.

A 24 V DC supply voltage is applied to SSC024D16 compact drive system, which generates a three phase voltage waveform. The voltages are displaced at 120° apart. The voltages are generated depending on the rotor position, which is sensed by the hall sensors and sent as feedback signals to the compact drive system. SSC024D16.

The torque sensor TK20N has to be supplied from a 12 V DC source. The output of the torque sensor is a voltage signal with the magnitude varying between ±10 V. The frequency output signal is proportional to \( n \). Its value can be obtained from equation below:

\[
n = \frac{f_{CRO} \cdot 60}{360} = \frac{f_{CRO}}{6}
\]  

(13)

The measurements of the motor characteristics were carried out on described above setup at constant supply voltage of 24 V for variable load torque. The test results are shown in Fig. 5.2. The differences between theoretical and test characteristics that occur at higher loadings are due to the constant resistance that was used in mathematical
model, while during the measurements the armature resistance was increasing, due to the increase of temperature.
CHAPTER 6: SIMULATION OF MOTOR DYNAMICS


The supply-inverter-motor circuit model is shown in Fig. 6.1. The model is proposed under the following assumptions:

- All elements of the motor are linear and no core losses are considered,
- Electromotive force $e_a$ varies sinusoidally with the rotational electric angle $\phi_e$,
- The cogging torque of the motor is negligible,
- Due to the surface mounted permanent magnets winding inductance is constant (does not change with the $\phi_e$ angle),
- Voltage drops across diodes and transistors and connecting wire inductance are ignore.

![Fig. 6.1 Circuit diagram of supply-inverter-motor system](image)

The equations that describe the model are as follows:

**Voltage equations**

- Voltage equation at the source side:
\[ E_b - i_s \cdot R_b - i_c \cdot R_c = 0 \]  
\[ (1.a) \]
\[ v_s = v_c + i_c \cdot R_c \]  
\[ (1.b) \]
\[ i_s = i_{sk} + i_c \]  
\[ (1.c) \]

where:

- \( E_b \) and \( R_b \) – voltage and resistance of the source
- \( R_c \) – capacitor resistance
- \( i_s \) – source circuit current
- \( i_{sk} \) – converter input current
- \( v_c \) – voltage across capacitor

\[ v_c = \frac{Q_c}{C} \]  
\[ (2) \]

- \( Q_c \) – charge in capacitor,
- \( C \) – capacitance,
- \( i_c \) – current flowing through the capacitor:

\[ i_c = \frac{dQ_c}{dt} \]  
\[ (3) \]

- Voltage equation at the motor side (Fig. 6.2) are:

\[ v_A = v_N + v_{sA} \]
\[ v_B = v_N + v_{sB} \]
\[ v_C = v_N + v_{sC} \]  
\[ (4) \]

where:

- \( v_{sA}, v_{sB}, v_{sC} \) are the inverter output voltages that supply the 3-phase winding.
- \( v_A, v_B, v_C \) are the voltages across the motor armature winding.
- \( v_N \) – voltage at the neutral point.
The equation of the voltages across the motor winding

\[
\begin{bmatrix}
v_A \\
v_B \\
v_C \\
\end{bmatrix} = \begin{bmatrix} R_A & 0 & 0 \\ 0 & R_B & 0 \\ 0 & 0 & R_C \\
\end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \\
\end{bmatrix} + \frac{d}{dt} \begin{bmatrix} L_A & L_{AB} & L_{AC} \\ L_{BA} & L_B & L_{BC} \\ L_{CA} & L_{CB} & L_C \\
\end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \\
\end{bmatrix} + \begin{bmatrix} e_A \\ e_B \\ e_C \\
\end{bmatrix}
\] (5)

or in shorten version:

\[
V_a = R_a \cdot I_a + \frac{d}{dt} L_a \cdot I_a + E_a
\] (6)

Since the resistances \( R_a \) of all phases are the same:

\[
R_a = \begin{bmatrix} R_a & 0 & 0 \\ 0 & R_a & 0 \\ 0 & 0 & R_a \\
\end{bmatrix}
\] (7)

Since the self- and mutual inductances are constant for surface mounted permanent magnets and the winding is symmetrical:

\[
L_A = L_B = L_C = L; \text{ and } L_{AB} = L_{BC} = L_{CA} = L_{BA} = L_{AC} = L_{CB} = M
\]

the inductance matrix takes the form:
\[ \mathbf{L}_s = \begin{bmatrix} L & M & M \\ M & L & M \\ M & M & L \end{bmatrix} \]  

(8)

For Y connected stator winding:

\[ i_a + i_b + i_c = 0 \]

Thus the voltage equation takes the form:

\[
\begin{bmatrix}
\nu_A \\
\nu_B \\
\nu_C
\end{bmatrix} =
\begin{bmatrix}
R_A & 0 & 0 \\
0 & R_B & 0 \\
0 & 0 & R_C
\end{bmatrix}
\begin{bmatrix}
i_A \\
i_B \\
i_C
\end{bmatrix} +
\begin{bmatrix}
L_s & 0 & 0 \\
0 & L & 0 \\
0 & 0 & L_s
\end{bmatrix}
\begin{bmatrix}
\frac{di_A}{dt} \\
\frac{di_B}{dt} \\
\frac{di_C}{dt}
\end{bmatrix} +
\begin{bmatrix}
e_A \\
e_B \\
e_C
\end{bmatrix}
\]

where \( L_s = L - M \) synchronous inductance

![Fig. 6.3 Position of the rotor with respect to the phase A](image)

The electromotive force induced in the phase \( A \) winding (see Fig. 6.3):

\[ e_a = K_E \omega_r \sin(\theta_e) \]  

(9)

where:

\[ K_E \] – constant,

\[ \omega_r \] – rotor angular speed:
\[
\omega_r = \frac{1}{p} \frac{d\theta_e}{dt} \quad (10)
\]

\(\theta_e\) – electrical angle (Fig.2),

\(p\) – number of pole pairs.

For three-phase winding the electromotive forces written in a form of matrix \(E_a\)

\[
E_a = \frac{K_e}{p} \begin{bmatrix}
\sin \theta_e \\
\sin(\theta_e - \frac{2}{3} \pi) \\
\sin(\theta_e - \frac{4}{3} \pi)
\end{bmatrix} \frac{d\theta_e}{dt} \quad (11)
\]

Equation that links the supply and motor sides:

\[
i_{sk} = \frac{1}{v_s} (i_A v_{sA} + i_B v_{sB} + i_C v_{sC}) \quad (12)
\]

results from the equality of the powers at input and output of the inverter.

Supply voltages for the phases (\(v_{sA}, v_{sB}\) and \(v_{sC}\)) results from the operation of converter.

**Motion equation:**

\[
T_j + T_D + T_S + T_L = T_{em} \quad (13)
\]

where:

- Inertia torque:

\[
T_j = J \frac{d\omega_r}{dt} \quad (14)
\]

\(J\) – moment of inertia,

- Viscous friction torque

\[
T_D = D \cdot \omega_r \quad (15)
\]
\[ D \] – friction coefficient,

- Coulomb friction torque

\[ T_s = \text{sign}(\omega_r)T_d \] (16)

\[ T_L \] - load torque

- Electromagnetic torque for 3-phase motor

\[ T_{em} = \frac{e_A i_A}{\omega_r} + \frac{e_B i_B}{\omega_r} + \frac{e_C i_C}{\omega_r} \] (17)

\[ T_{em} = \frac{e_A i_A}{\omega_r} + \frac{e_B i_B}{\omega_r} + \frac{e_C i_C}{\omega_r} = K_E (f_a(\phi_e) i_A + f_b(\phi_e) i_B + f_c(\phi_e) i_C) \] (18)

Where

\[ f_a(\phi_e) = \sin(\theta_e) \]

\[ f_b(\phi_e) = \sin(\theta_e - \frac{2\cdot\pi}{3}) \]

\[ f_c(\phi_e) = \sin(\theta_e - \frac{4\cdot\pi}{3}) \]

Combining all the above equations, the system in steady-space form is [24]

\[ \dot{x} = Ax + Bu \] (19)

\[ x = [i_A \ i_B \ i_C \ \omega_r \ \theta_e]^T \] (20)

\[ A = \begin{bmatrix}
-\frac{R_s}{L_s} & 0 & 0 & 0 & -\frac{K_E(f_a(\phi_e))}{L_s} & 0 \\
0 & -\frac{R_s}{L_s} & 0 & 0 & -\frac{K_E(f_b(\phi_e))}{L_s} & 0 \\
0 & 0 & -\frac{R_s}{L_s} & 0 & -\frac{K_E(f_c(\phi_e))}{L_s} & 0 \\
0 & 0 & 0 & 0 & \frac{K_E(f_a(\phi_e))}{J} & \frac{K_E(f_b(\phi_e))}{J} \\
0 & 0 & 0 & 0 & \frac{K_E(f_c(\phi_e))}{J} & \frac{D}{J} \\
0 & 0 & 0 & 0 & 0 & \frac{P}{2}
\end{bmatrix} \] (21)
6.2 Simulation of the Motor Operation under Constant Supply Voltage and Constant Load Torque

6.2.1 Data of the Drive System

The inverter – motor system is supplied from the battery of 24 V with the capacitor connected in parallel as shown in Fig. 6.1 (circuit diagram shown in 6.1 section). The data of these elements are as follows:

- \( E_b = 24 \, V \) - EMF of the battery
- \( R_s = 0.005 \, \Omega \) - source resistance
- \( R_c = 10 \, \Omega \) - resistance in series with capacitor
- \( C = 0.00001 \, F \) - capacitance

The inverter is assumed to be ideal without any power losses. The parameters of the motor circuit are as follows:

- \( R_a = 0.25 \, \Omega \) - phase resistance of the brushless DC motor
- \( L_a = 0.00101 \, H \) - phase inductance of the DC brushless motor
- \( K_e = 0.53 \, V/(rad/s) \) - EMF constant

The parameters of the mechanical system are:
\[ J = 0.0096 \text{ Kg/m}^2 \]  - moment of inertia

\[ D = 0.0037 \text{ Nm/(rad/s)} \]  - friction coefficient

\[ T_{\text{load}} = 10\text{Nm} \]  - load torque

### 6.2.2 A SIMULINK Block Diagram

The simulation of the motor operation was done using software package MATLAB/SIMULINK®. The block diagram of the drive system which is shown in Fig. 6.4 was developed using the mathematical model derived in section 6.1.

The main block diagram consists of 3 parts; supply source, inverter + motor winding and mechanical system of the drive + pulse generator. The subsystem related to inverter – motor circuit is shown in Fig 6.4.b.

In this diagram the electromotive forces \( e_A, e_B, e_C \) are generated by rotor position signal \( \theta_e \) and appropriate functions: \( f_a(\phi_e), f_b(\phi_e), f_c(\phi_e) \). The phase voltages \( v_{sA}, v_{sB} \) and \( v_{sC} \) are generated by position signal \( \theta_e \) and blocks are shown in the appendix D (pulsegenerator.mdl, inverter.mdl). More about the other START subsystems and and PLOT buttons m-files are shown in the appendix A (initial.m, speed.m).

### 6.2.3 Simulation of a Wheelchair Drive under Dynamic Conditions

Two cases of a dynamic drive operation were considered:

- Starting up operation
- Driving of wheelchair on variable road conditions
Fig. 6.4.a SIMULINK Model of brushless DC motor
6.2.3.1 Starting Up Operation

To simulate this operation it was assumed that:

- The drive system is supplied from constant voltage of 24 V
• The system is loaded by rated torque of 10 Nm.

The simulation results are shown in Figs. 6.5, 6.6, 6.7 and 6.8. The rotary speed \((\omega_r)\) and source current \((i_s)\) are shown in Fig. 6.5. The ripple in the speed waveform is due to the electronic commutation (switching of the transistors).

The waveform of EMF \((e_a)\) and armature voltage \((V_a)\) and waveform of armature current \((i_a)\) and armature voltage \((V_a)\) are shown in Figs. 6.7 and 6.8. The shape of induced EMF is sinusoidal and this is due to the square magnets on the rotor. The induced EMF and the voltage applied to the motor are in phase which shows that the windings are ON when the absolute value of induced EMF is maximum. The armature current waveform is also a square waveform but has ripples and this is due to the commutation of the current phases.

![Waveform of EMF \((e_a)\) and armature voltage \((V_a)\)](image)

**Fig. 6.6 Waveform of EMF \((e_a)\) and armature voltage \((V_a)\)**
Fig. 6.7 Waveform of armature current ($i_a$) and armature voltage ($V_a$)

The waveform of electromagnetic torque of the motor and torque share in phase $A$ ($Tem_a$) are shown in Fig. 6.8. The shape of the torque is important. The torque ripple is only due to the switching of the transistors. The motor is a torus motor with rectangular magnets therefore the cogging torque is practically reduced to zero.

Fig. 6.8 Waveform of electromagnetic torque (a) 3-phases $Tem$ (b) phase $A$ ($Tem_a$)
Fig. 6.9 Waveforms of EMF in the 3 phases ($e_a$), ($e_b$), ($e_c$) and supply phase voltage ($V_a$)
To illustrate the switching conditions Fig. 6.9 shows the EMFs of the three phases and supply voltage $v_{xa}$ of phase $A$ at steady state conditions.

This phase as well as other phases remains always switched ON when the absolute values of their EMFs are greater than those of other phases.

6.2.3.2 Driving a Wheelchair under Variable Road Conditions

In order to simulate the operation of wheelchair drive under variable road condition it was assumed that the road consists of 5 parts shown in Fig. 6.10.

![Fig. 6.10 Wheelchair under variable road conditions](image)

Forces which act on the wheelchair are shown in Fig. 6.11

![Fig. 6.11 Forces acting on the wheelchair](image)
The horizontal component of force $F_{TM} = F_M \cdot \sin(\alpha)$, where $F_M = M \cdot g$, here $M = 150$ Kg.

The torque equation can be written as $T_J + T_D + T_L = T_{em}$, described earlier in section 6.1. The load torque can be obtained using the equation $T_L = T_{LOAD} + T_T$, where the torque for one motor is $T_T = \frac{F_{TM} \cdot D_w}{N_M \cdot 2}$. $D_w = 8''$, is the diameter of the wheel used in the wheelchair and $N_M = 2$ is the number of motors that drive the wheelchair. There is an additional steady load torque $T_{LOAD} = 8Nm$. The mathematical model used is derived in section 6.1. The road consists of 5 parts and the wheelchair drive operates as a motor in the 1st, 2nd, 3rd and 5th parts. It operates as a generator in the 4th part. The load torque varies in the 2nd, 3rd and the 4th parts. The torque equations are:

$$T_T = 19Nm, T_L = 19 + 8 = 27Nm$$ - 2nd part (for $\alpha = 15^\circ$)

$$T_T = 0Nm, T_L = 8Nm$$ - 3rd part (for $\alpha = 0^\circ$)

$$T_T = -13Nm, T_L = -13 + 8 = -5Nm$$ - 4th part (for $\alpha = -10^\circ$)

The power output of the machine when it is behaving as a motor is $P_{out} = T_L \cdot \omega$ and power input is $P_{in} = E_b \cdot i_s$. When the machine operates as a generator in the 4th part the power output is $P_{out} = E_b \cdot i_s$. The efficiency of the motor is $Eff = \frac{P_{out}}{P_{in}} \cdot 100\%$.

The road conditions are simulated by the torque component $T_T$ that varies along the road according to the waveform shown in Fig. 6.12. The input current, torque and speed waveforms are shown in Fig. 6.13.
Fig. 6.12 Waveform of torque component for the different road sections ($T_r$)

Fig. 6.13 Waveforms of the electromechanical parameters (a) source current ($i_s$), electromagnetic torque ($Tem$) and speed ($\omega$)
(b)

(c)

58
The average electromechanical parameters of single motor calculated for 3 road sections: 2, 3 and 4 at steady state operation are in Table 6.1.

Table 6.1 Wheelchair drive (single motor) performance at steady-state

<table>
<thead>
<tr>
<th>Road section</th>
<th>$I_s$ [A]</th>
<th>$T_{em}$ [Nm]</th>
<th>$\omega$ [rad/sec]</th>
<th>$N$ [Km/hr]</th>
<th>$P_{out}$ [W]</th>
<th>$P_{in}$ [W]</th>
<th>Eff [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part 2</td>
<td>30</td>
<td>26.9</td>
<td>7.9</td>
<td>0.24</td>
<td>213.3</td>
<td>720</td>
<td>30</td>
</tr>
<tr>
<td>Part 3</td>
<td>9.1</td>
<td>8.02</td>
<td>20.2</td>
<td>0.61</td>
<td>161.6</td>
<td>217.2</td>
<td>75</td>
</tr>
<tr>
<td>Part 4</td>
<td>-5.5</td>
<td>-4.85</td>
<td>31.9</td>
<td>0.96</td>
<td>131.5</td>
<td>-159.5</td>
<td>82</td>
</tr>
</tbody>
</table>

- After passing the point $A$ and in the part 1 the machine operates as a motor.
- After passing the point $B$ machine operates as a brake and in part 2 as a motor.
- After passing the point $C$ machine operates as a brake and in the part 3 it operates as a generator, charging the DC source.
- After passing the point $D$ machine operates as a generator and in part 5 it operates as a motor.

The ripple is very high when the machine is in the 2nd part because the motor is becoming unstable. The efficiency of the motor is low in the 2nd part when it is going up the incline. The machine operates at maximum efficiency in the 3rd and 4th part, when it is operating as a generator.

6.2.4 An Influence of Switching Angle on Motor Performance

One of the many factors that have an influence on the motor performance is the switching ON angle shown in Fig. 6.14.
Fig. 6.14 Waveform showing the switching ON and OFF angles $\Psi_{ON}$ and $\Psi_{OFF}$

A change of the switching angle was achieved in simulation block diagram by changing the parameters in the pulse generator (see Fig. 6.15). Practically the switching angles $\Psi_{ON}$ and $\Psi_{OFF}$ may change as well as duty ratio $d$.

Fig. 6.15 Model showing a part of the pulse generator to change the switching angle $\Psi_{ON}$.

Two cases are considered:

a) $\Psi_{ON} = \text{variable}, \quad \Psi_{OFF} = \text{variable}, \quad d = \text{constant}$

b) $\Psi_{ON} = \text{variable}, \quad \Psi_{OFF} = \text{constant}, \quad d = \text{variable}$
Case a:

The simulation was done for the following switching angles $\Psi_{ON} = 20^\circ, 10^\circ, 0^\circ, -10^\circ$ and $-20^\circ$. The duty ratio $d = \frac{T_{ON}}{T} = 0.67$, which is kept constant. It means the switching OFF angle $\Psi_{OFF}$ was also changed accordingly. The results of simulation were plotted in form of characteristics of average values of input current ($I_s$), mechanical power output ($P_{em}$) and efficiency ($Eff_{\eta}$) shown in Figs. 6.16, 6.17 and 6.18. The characteristics were drawn using file plot_pem_idc.m and plot_efficiency.m in MATLAB (see Appendix C plot_pem_idc.m and plot_efficiency.m).

The efficiency was calculated as

$$Eff_{\eta} = \frac{P_{out}}{P_{in}} \times 100\%$$

where

$$P_{in} = \frac{1}{T} \int_0^T (E_p \cdot i_s)dt \quad \text{and} \quad P_{out} = \frac{1}{T} \int_0^T (T_{em} \cdot \omega)dt$$

The characteristics are similar to that of a shunt DC motor with no armature reaction or a separately excited DC motor. The magnetic flux is constant, predominated by the magnetic field of the permanent magnets. The current – torque characteristic is a straight line which is shown in Fig. 6.16. The speed decreases as the current increases showing that the speed is directly proportional to the induced EMF. The electromechanical power is directly proportional to the input current, since the magnetic flux is constant. The electromagnetic power depends only on the armature current since the magnetic flux is constant. The motor efficiency is maximum when the switching angle is $\Psi_{ON} = -10^\circ$, which means that the transistors are switched earlier. The motor
efficiency is minimum when the $\psi_{ON} = 20^\circ$, which means the transistors are switched on with a delay. This is shown in Fig. 6.18

Fig. 6.16 Input current ($I_s$) vs load torque ($T_L$)

Fig. 6.17 Mechanical power output ($P_{em}$) vs load torque ($T_L$)
Fig. 6.18 Efficiency (Eff %) vs load torque ($T_L$)

On the basis of this simulation, calculation of performance characteristics were carried out for the switching angle $\Psi_{ON} = -10^\circ$, where the efficiency has reached maximum. The calculation results are shown in Fig. 6.19.

Fig. 6.19 Electromechanical characteristics of the motor
Under heavy load conditions the speed – torque characteristics is almost a straight line. The winding inductance can be neglected at heavy loads. The performance characteristics obtained considering the motor to behave as a separately excited motor in section 5.2, Fig. 5.2 and the performance characteristics shown in Fig. 6.19 are similar. This proves that the motor behaves as a separately excited DC motor. The mechanical power output depends on the electromagnetic torque and speed. The electromagnetic torque is directly proportional to the armature current.

**Case b:**

Switching was done for the following switching angles and the corresponding duty ratios: $\Psi_{ON} = 5^\circ$, $d = 0.69$; $\Psi_{ON} = 0^\circ$, $d = 0.67$; $\Psi_{ON} = 5^\circ$ and $d = 0.64$. It means the switching OFF angle $\Psi_{OFF}$ is kept constant. The results of simulation were plotted in Figs. 6.20, 6.21 and 6.22 in form of characteristics of average values of input current ($I_s$), mechanical power output ($P_{em}$) and efficiency ($Eff_{\psi}$). The characteristics were drawn using file `plot_pem_idc_duty.m` and `plot_efficiency_duty.m` in MATLAB (see Appendix C `plot_pem_idc_duty.m` and `plot_efficiency_duty.m`).

The current – torque characteristic is a straight line which is shown in Fig. 6.20. The mechanical power output depends on electromagnetic torque, which depends only on the armature current since the magnetic flux is constant. The motor efficiency is maximum when the duty ratio is $d = 0.69$, which means that the transistors are switched on for a longer time. The motor efficiency is minimum when the $d = 0.64$ which means the transistors are switched on for a shorter time. This is shown in Fig. 6.22.
Fig. 6.20 Input current ($I_s$) vs load torque ($T_L$)

Fig. 6.21 Mechanical power output ($P_{em}$) vs load torque ($T_L$)
On the basis of this simulation, calculation of performance characteristics were carried out for the duty ratio $d = 0.69$ where the efficiency has reached maximum. The calculation results are shown in Fig. 6.23.
6.3 Comparison of Simulation Results with the Results Obtained from the Tests Conducted.

To verify the mathematical dynamic model of the brushless DC motor drive the measurements were carried out on the set up shown in Fig. 5.3. During the measurements at the steady – state conditions the waveforms of currents and EMFs were observed at no – load and full – load conditions.

6.3.1 Full – Load Conditions

During the test, the motor electromechanical parameters were measured and their values are in Table 6.2. The current waveform of phase A measured by means of oscilloscope is shown in Fig. 5.18.a.

Table 6.2 Electromechanical parameters measured under full - load

<table>
<thead>
<tr>
<th>$V_{DC}$ V</th>
<th>$I_{DC}$ A</th>
<th>$n$ rpm</th>
<th>$T_L$ N-m</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.44</td>
<td>10</td>
<td>144.4</td>
<td>9.85</td>
</tr>
</tbody>
</table>

For similar conditions a simulation was carried out. Since the winding warms up during the test its phase resistance was increased to the value $R_a = 0.37 \, \Omega$. This value was used for simulations. The switching angle of $\Psi_{ON} = 9^\circ$ is used after trial and error method to obtain the right waveform. The current waveform of phase A was calculated and is shown in Fig. 6.24.b.

The shape of the current waveform obtained from simulation of the motor model and the experimental results are the same. The speed of the motor from the simulation results is 146.1 $rpm$ and the speed obtained from the experimental results is 144 $rpm$. The dip in the current waveform is due to the commutation of the current phases.
Fig. 6.24 Waveform of armature current ($I_a$): (a) experimental result (b) simulation result
6.3.2 No – Load Conditions

The motor during the no – load test was loaded only by the friction torque. The rest of the parameters values are shown in Table 6.3.

Table 6.3 Electromechanical parameters measured under no - load

<table>
<thead>
<tr>
<th>$V_{DC}$</th>
<th>$I_{DC}$</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>$A$</td>
<td>$rpm$</td>
</tr>
<tr>
<td>24</td>
<td>0.5</td>
<td>210.4</td>
</tr>
</tbody>
</table>

Fig. 6.25 Waveform of armature current ($I_a$): (a) experimental result (b) simulation result
The current waveforms measured during the steady – state by an oscilloscope is shown in Fig. 6.25.a. For similar operation conditions simulation was done. The current waveform of phase A obtained from simulation is shown in Fig. 6.25.b.

Fig. 6.26 Waveform of induced EMF ($E_a$): (a) experimental result (b) simulation result
The currents waveforms obtained from the simulation of the motor model and the experimental results are similar.

To measure the EMF induced in the winding the motor was driven with speed $n = 100.28$ rpm, by a DC motor. The measured waveform of line – to – line voltage is shown in Fig. 6.26.a. It can be observed that the voltage has a sinusoidal shape. Such a sinusoidal shape has been used in the simulation of the motor.
CHAPTER 7: CONCLUSION

A disc brushless DC motor that is applied as a gearless wheelchair drive was analysed. The disc motor is a torus – type motor with the space between the stator coils filled with teeth. These teeth increase the flux density in the air-gap that improved the motor performance. Unfortunately this results also in an increase in the torque ripple of the motor. Due to the rectangular – shaped permanent magnets applied in the rotor disc the torque – ripple was reduced, becoming practically unnoticed. This improved the riding comfort of the wheelchair.

This type of motor was the object of intensive research in this project.

Two major types of analysis of the motor performance were carried out and are presented in this thesis. These are:

- Analysis in steady – state operation
- Analysis in dynamic conditions.

In both cases a theoretical and experimental study was done for the disc motor whose ratings determined in this research are as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>supply voltage</td>
<td>$24 , V$</td>
</tr>
<tr>
<td>output power</td>
<td>$163 , W$</td>
</tr>
<tr>
<td>torque</td>
<td>$9.7 , Nm$</td>
</tr>
<tr>
<td>input current</td>
<td>$8.7 , A$</td>
</tr>
<tr>
<td>speed</td>
<td>$161 , rpm$</td>
</tr>
<tr>
<td>efficiency</td>
<td>$78 , %$</td>
</tr>
</tbody>
</table>

Conclusions from the study of the motor in steady – state operating conditions are as follows:
• The electromechanical characteristics measured on the experimental setup show that the motor behavior is similar to the behavior of conventional separately excited DC motor.

• Since the speed-torque and the armature current-torque characteristics are nearly linear it indicates that the armature reaction is negligible. It means that the magnetic flux is produced by the applied high-energy magnets are not diminished by the magnetic flux produced by the armature current. These results confirm the expectation for surface mounted permanent magnet motors in which the large non magnetic gap (air gap + thickness of the permanent magnet) does not allow the armature reaction flux to be strong.

• The analyzed motor with performance similar to separately excited DC motor is very good object for control of gearless wheelchair drive.

• A theoretical analysis has been done applying mathematical model used for steady state operation of separately excited DC motor. The results obtained from this study practically do not differ from the results obtained during the measurement. This verifies the formulated mathematical model.

Conclusions from the motor study in the dynamic conditions are as follows:

• The research on winding commutation shows that a change of switching angle influences the motor performance. The maximum efficiency of the motor is achieved when the three phases are switched ON with the angle \( -10^\circ \) at a constant duty ratio \( d = 0.67 \).
• A study on influence of duty ratio on motor performance shows that maximum efficiency is when \( d = 0.69 \). It means if the winding is ON longer than at natural commutation \( (d = 0.67) \) the motor performs better.

• A study on influence of road conditions on motor operation shows that the efficiency greatly changes when the slope of the road the wheelchair is driving on varies. When the wheelchair is driving downhill the motor operates as generator preventing it from accelerating to too high speed.

• The waveforms of currents obtained from computer simulation and test does not differ much what proves that the mathematical model sufficiently accurate describe the motor in dynamic conditions.

In general brushless DC motor studied in this project satisfies the requirements put on the electric drive wheelchair. The speed and torque of the wheelchair drive can be controlled easily by varying the input current.
REFERENCES


[22] Work supported by *NSF under grant ECS – 0217308* and M. Jagiela was beneficiary of the foundation for Polish science.

[23] SSC024D16 BLDC motor –


Initial.m

% Start file for the thesismodeltest
% Calculation of Initial Conditions
% Protection circuit parameters and input voltage
Rs=0.005;              %source resistance
Eb=24;                  %supply voltage
Rc=10;                  %resistance in series with capacitor
C=0.00001;                  %capacitance
J=0.00962;            %moment of inertia
D=0.0037;             %coefficient of friction
Ke=0.5349;                %Emf constant

%Motor parameters
Ra=0.25;                        %Phase resistance of the DC brushless motor
La=0.00101;                  %Phase inductance of the DC brushless motor

Speed.m

% thesismodeltest work
load speed w;                   %loading speed
load tload tl;                     %loading load torque
load tinput tem;                %loading electromagnetic torque
load inducedemf emfa;     %loading induced emf in phase A
load inputcurrent idc;       %loading input DC current
load position fi;                %loading position
w=w';tl=tl';tem=tem';
t=w(:,1);w=w(:,2);
tl=tem(:,2);tem=tem(:,2);
CLF;
figure(1);
subplot(1,1,1),plot(t,idc(2,:),'b'),xlabel('time [s]'),ylabel('i_s[A]'),gtext('i_s'),grid %plot for input current
figure(2);
subplot(1,1,1),plot(t,tem(:,1),'b'),xlabel('time [s]'),ylabel('T_em[Nm]'),gtext('Tem'),grid %plot for electromagnetic torque
figure(3);
subplot(1,1,1),plot(t,w(:,1),'b'),xlabel('time [s]'),ylabel('w[rad/sec]'),gtext('w'),grid %plot for speed
APPENDIX-B: M-FILES FOR THE
ELECTROMECHANICAL CHARACTERISTICS OF
THE MOTOR UNDER DYNAMIC CONDITIONS

Case A: Varying switching angle

Plot_pem_idc.m

%program to plot the electromagnetic power and input DC current for various switching angles

tl=[ 0 1 2 3 4 5 6 7 8 9 10 12 14 16 18]; %Vector for load torque
pem1=[2.1 30.1 56.1 101.7 121.5 140.2 156.9 171.9 185.2 197.3 217.4 230.8 239.7 243.1]; %Vector for electromagnetic power beta = -20
pem2=[2.4 28.8 53.4 76.1 97.1 116.6 134.4 150.8 165.5 179.0 191 210.6 225.1 236.4 241.2]; %Vector for electromagnetic power beta = -10
pem3=[2.8 28.7 52.8 74.8 95.3 114.6 132.2 148 162.6 175.8 187.7 207.6 221 231.7 239]; %Vector for electromagnetic power beta = 0
pem4=[3.1 28.9 53.2 75.3 95.9 114.8 132.2 148 162.5 175.3 186.9 205.2 219.8 228.9 234.5]; %Vector for electromagnetic power beta = 10
pem5=[3.7 30.2 55.2 77.6 98.5 117.6 135.2 150.8 164.9 177.4 188.5 206.4 218.4 225.8 230.3]; %Vector for electromagnetic power beta = 20
idc1=0.5*[0.31 2.64 4.99 7.35 9.65 12.01 14.34 16.63 18.99 21.35 23.67 28.6 33 37.8 42.4]; %Vector for input DC current beta = -20
idc2=0.5*[0.24 2.49 4.74 6.98 9.21 11.47 13.73 15.98 18.22 20.53 22.74 27.2 31.6 36.4 40.8]; %Vector for input DC current beta = -10
idc3=0.5*[0.24 2.48 4.71 6.93 9.17 11.39 13.64 15.89 18.13 20.32 22.61 27 31.4 36 40.4]; %Vector for input DC current beta = 0
idc4=0.5*[0.31 2.60 4.91 7.18 9.46 11.76 14.04 16.31 18.62 20.88 23.19 27.8 32.2 36.8 41.4]; %Vector for input DC current beta = 10
idc5=0.5*[0.46 2.85 5.31 7.74 10.14 12.55 15.04 17.39 19.82 22.25 24.66 29.6 34.4 39 43.8]; %Vector for input DC current beta = 20
plot(tl, idc1,'r*-', tl, idc2, 'b+-', tl, idc3,'go-', tl, idc4,'mx-', tl, idc5,'k^-'),grid
legend('beta1=-20 deg','beta2=-10 deg','beta3=0 deg','beta4=+10 deg','beta5=+20 deg');
xlabel('T_L[N-m]');
ylabel('I_s [A]')
figure(2);
plot(tl,pem1,'r*-',tl,pem2,'g+-',tl,pem3,'bx-',tl,pem4,'mo-',tl,pem5,'k^-'),grid
legend('beta1=-20 deg','beta2=-10 deg','beta3=0 deg','beta4=+10 deg','beta5=+20 deg');
xlabel('T_L [Nm]');
ylabel('Pem [W]')
Plot_efficiency.m

% program to plot the efficiency for various switching angles and the electromechanical characteristics for the optimum switching angle

tl=[ 0 1 2 3 4 5 6 7 8 9 10 12 14 16 18]; % Vector for load torque
n1=[0 88 90 89 86 84 80 77 74 70 65 60 53 49]; % Vector for efficiency beta= -20
n2=[0 89 91 86 83 80 77 75 73 70 65 60 53 49]; % Vector for efficiency beta= -10
n4=[0 84 85 84 83 80 77 72 71 68 61 58 51 47]; % Vector for efficiency beta= 0
n5=[0 79 81 80 79 76 73 70 69 67 64 57 53 47 44]; % Vector for efficiency beta= +10
n3=[0 87 88 87 84 84 80 77 73 72 70 63 59 56 48]; % Vector for efficiency beta= +20

plot(tl, n1, 'r+',tl,n2,'b*',tl,n3,'go',tl,n4,'y^',tl,n5,'kx'),grid
legend('beta1=-20 deg','beta2=-10 deg','beta3=0 deg','beta4=+10 deg','beta5=+20 deg');
figure(1);
xlabel('T_L [N-m]');
ylabel('efficiency [%]');
figure(2);
pem2=[2.4 28.8 53.4 76.1 116.6 134.4 150.8 165.5 179.0 191.0 210.6 225.1 236.4 241.2]; % Vector for electromagnetic power beta= -10
idc2=0.5*10*[0.24 2.49 4.74 6.98 9.21 11.47 13.73 15.98 18.22 20.53 22.74 27.2 31.6 36.4 40.8]; % Vector for input DC current beta= -10
w2=(60/(2*3.1415))*[28 27 26 25 24 23 22 21 21 20 19 18 16 15 13]; % Vector for speed in rpm beta= -10

plot(tl, pem2, 'r*',tl,idc2,'g+',tl,n2,'bo',tl,w2,'kx'),grid
xlabel('torque [Nm]'),gtext('speed [r.p.m.]'),gtext('efficiency [%]'),gtext('mechanical power [W]'),gtext('current/10 [A]')

Case B: Varying duty ratio \( d \)

Plot_pem_idc_duty.m

% program to plot the electromagnetic power and input DC current for various duty ratios

tl=[0 1 2 4 6 8 10 12 14 16 18]; % Vector for load torque
pem1=[2.9 27.2 50.7 94.5 130.9 161.6 186.9 206.4 222.5 232.7 237.3 ]; % Vector for electromagnetic power duty ratio = 0.69
pem2=[2.7 28.5 52.5 95.3 132.2 162.6 187.7 207.6 221 231.7 239]; % Vector for electromagnetic power duty ratio = 0.67
pem3=[4.6 28.1 48.3 83.0 111.1 133.2 149.2 162.7 171.5 177.2 180.1]; % Vector for electromagnetic power duty ratio = 0.64
idc1=[0.17 1.2 2.4 4.6 6.8 9 11.2 13.4 15.7 17.9 20.1]; % Vector for input DC current duty ratio = 0.69
idc2=[0.12 1.3 2.4 4.6 9.1 11.3 13.5 15.7 18 20.2]; % Vector for input DC current duty ratio = 0.67
idc3=[0.22 1.2 2.3 4.4 6.6 8.6 10.8 12.9 15 17.1 19.2 ]; % Vector for input DC current duty ratio = 0.64

79
figure(1);
plot(tl,idc1,'r*', tl, idc2, 'b^', tl,idc3,'ko'),grid
legend('d = 0.69','d = 0.67','d = 0.64');
xlabel('T_L [N-m]');
ylabel('I_s [A]')
figure(2);
plot(tl,pem1,'r*',tl,pem2,'go',tl,pem3,'bx'),grid
legend('d = 0.69','d = 0.67','d = 0.64');
xlabel('T_L [Nm]');
ylabel('Pem [W]')

\textbf{Plot\_efficiency\_duty.m}

%program to plot the efficiency for various duty ratios and the electromechanical
characteristics for the optimum duty ratio

tl=[0 1 2 4 6 8 10 12 14 16 18]; %Vector for load torque
n1=[0 91.7 88 85 70 65 60 55 50 ]; %Vector for efficiency duty ratio = 0.69
n3=[0 87 86 81 73 67 61 55 45 41]; %Vector for efficiency duty ratio = 0.67
n2=[0 87 88 84 80 73 70 63 59 56 48]; %Vector for efficiency duty ratio = 0.64
plot(tl, n1, 'r*',tl,n2,'go',tl,n3,'k+'),grid
legend('d = 0.69','d = 0.67','d = 0.64');
figure(1);
xlabel('T_L [N-m]');
ylabel('efficiency [%]');
figure(2);
pem1=[2.9 27.2 50.7 94.5 130.9 161.6 186.9 206.4 222.5 232.7 237.3 ]; %Vector for electromagnetic power duty ratio = 0.69
idc1=10*[0.17 1.2 2.4 4.6 8.9 11.2 13.4 15.7 17.9 20.1]; %Vector for input
w1=(60/(2*3.1415))*[27.3 25.9 25.1 23.4 21.8 20.3 18.8 17.4 16.0 14.7 13.4] %Vector for speed in rpm duty ratio = 0.69
plot(tl, pem1, 'r*',tl,idc1,'g+',tl,n1,'bx',tl,w1,'ko'),grid
xlabel('T_L [Nm]'),gtext('speed [r.p.m.]'),gtext('efficiency [%]'),
gtext('mechanical power [W]'),gtext('current/10 [A]')
APPENDIX-C: M-FILES FOR PERFORMANCE CHARACTERISTICS OF DISC BRUSHLESS DC MOTOR UNDER STEADY STATE CONDITIONS

Steady_statemodel.m

% load test
Ip=[1.6 3.4 5.2 6.7 7.2 8.7 9.5 12];
Up=[24 24 23.8 23.8 23.7 23.6 23.4];
Mp=[1.5 3.75 5.8 7.5 7.96 9.69 10.63 14.3];
OMEGAp=pi/30*[195 182 178 168 165 161 153 137.5];
Pp=24*Ip;
Pmp=Mp.*OMEGAp;
np=OMEGAp*30/pi;
Effp=Pmp./Pp*100;

%simulation
Is=1:1:13; Kms=1.15;
Us=24;Rs=0.5;
Ps=Us*Is;
Ms=Is.*Kms;
omgas=(Us-Rs*Is)./Kms;
ns=omgas*30/pi, Mst=Ms, nst=ns
Pms=Ms.*omgas-0.004*omgas;
Effs=Pms./Ps*100;
CLF;
figure(1),whitebg,
plot(Ms,Effs,'r',Ms,10*Is,'r',Ms,ns,'r',Ms,Pms/1,'r',Mp,Effp,'xr',Mp,10*Ip,'ro',Mp,ns,'rx',
Mp,Pmp/1,'r*'),grid,
xlabel('torque [Nm]'),gtext('speed [r.p.m.]'),gtext('efficiency [%]'),
gtext('mechanical power [W]'),gtext('current/10 [A]')
APPENDIX-D: SUBSYSTEM MODELS

Pulsegenerator model

Pulsegenerator subsystem model
Inverter model

Rampgenerator model

83
Motor subsystem model
Deepti R. Chikkam, daughter of Nageswara Rao Chikkam and Vijayalakshmi Chikkam was born on 15th day of April 1981, in Visakhapatnam, Andhra Pradesh, India. Having an interest in electrical phenomena she chose electrical engineering as her major and received her Bachelor of Technology degree from Jawaharlal Technological University in June of 2002. With a continuing interest in that subject, she joined the Graduate Program in the Electrical and Computer Engineering Department at Louisiana State University in August 2002.