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From Linear Optical Quantum Computing to Heisenberg-Limited Interferometry

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Abstract. The working principles of linear optical quantum computing are based on photodetection, namely, projective measurements. The use of photodetection can provide efficient nonlinear interactions between photons at the single-photon level, which is technically problematic otherwise. We report an application of such a technique to prepare quantum correlations as an important resource for Heisenberg-limited optical interferometry, where the sensitivity of phase measurements can be improved beyond the usual shot-noise limit. Furthermore, using such nonlinearities, optical quantum nondemolition measurements can now be carried out easily at the single-photon level.

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1. Effective nonlinearities from projective measurements

Looking back, scalable quantum computation with linear optics was considered to be impossible due to the lack of efficient two-qubit logic gates, despite the ease of implementation of one-qubit gates. Two-qubit gates necessarily need a nonlinear interaction between the two photons, and the efficiency of this nonlinear interaction is typically very tiny in bulk materials [1]. However, Knill, Laflamme, and Milburn recently showed that this barrier can be circumvented with effective nonlinearities produced by projective measurements [2], and with this work scalable linear optical quantum computation (LOQC) becomes a reality.

Let us consider the Kerr nonlinearity, which can be described by a Hamiltonian [3]

$$\mathcal{H}_{\text{Kerr}} = \hbar\kappa\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}, \quad (1)$$

where κ is a coupling constant depending on the third-order nonlinear susceptibility, and \hat{a}^\dagger , \hat{b}^\dagger and \hat{a} , \hat{b} are the creation and annihilation operators for two optical modes. One convenient choice of the logical qubit can then be represented by the two modes containing a single photon, denoted as

$$\begin{aligned} |0\rangle_L &= |0\rangle_l |1\rangle_k \\ |1\rangle_L &= |1\rangle_l |0\rangle_k, \end{aligned} \quad (2)$$

where l, k represent the relevant modes, and we have used the notation $|\cdot\rangle_L$ for a logical qubit, in order to distinguish it from the photon-number states $|\cdot\rangle_k$.

For a two-qubit gate, let us assign mode 1,2 for the control qubit, and 3,4 for the target qubit. Suppose now only the modes 2,4 are coupled under the interaction given by Eq.(1). For a given interaction time τ , the transformation can be written as

$$\begin{aligned} |0\rangle_L|0\rangle_L &\rightarrow |0\rangle_L|0\rangle_L \\ |0\rangle_L|1\rangle_L &\rightarrow |0\rangle_L|1\rangle_L \\ |1\rangle_L|0\rangle_L &\rightarrow |1\rangle_L|0\rangle_L \\ |1\rangle_L|1\rangle_L &\rightarrow e^{i\varphi}|1\rangle_L|1\rangle_L, \end{aligned} \quad (3)$$

where $\varphi \equiv \kappa n_a n_b \tau$ and $n_a = \langle \hat{a}^\dagger \hat{a} \rangle$, $n_b = \langle \hat{b}^\dagger \hat{b} \rangle$. This operation yields a conditional phase shift [4]. When $\varphi = \pi$, we have the two two-qubit gate called the conditional sign-flip gate. A typical two-qubit gate, controlled-NOT (CNOT), is then simply constructed by using the conditional sign flip and two one-qubit gates (e.g., Hadamard on the target, followed by the conditional sign flip and another Hadamard on the target). In order to have $\varphi \sim \pi$ at the single-photon level, however, a huge third-order nonlinear coupling is required [5]. Instead, Knill, Laflamme, and Milburn devised a nondeterministic conditional sign flip gate using nonlinear sign gate defined by

$$\alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle \longrightarrow \alpha|0\rangle + \beta|1\rangle - \gamma|2\rangle. \quad (4)$$

The nonlinear sign gate can be implemented non-deterministically by three beam splitters, two photo-detectors, and one ancilla photon [6] (see Fig. 1). The implementation of conditional sign flip gate is then made by the combination of the

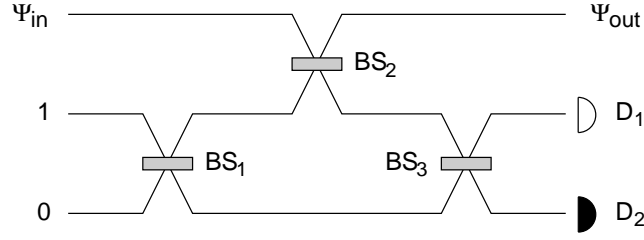


Figure 1. A diagram for the nonlinear sign gate. Conditioned upon a specific detector outcome, the desired output state can be obtained by choosing appropriate transmission coefficients of the beam splitters. The success probability of the gate operation is $1/4$, but we always know when it succeeds.

nonlinear sign gate and the physics of Hong-Ou-Mandel (HOM) interferometer [7]. For two arbitrary qubits

$$\begin{aligned} |Q_1\rangle &= \alpha_0|0\rangle_L + \alpha_1|1\rangle_L = \alpha_0|0\rangle_1|1\rangle_2 + \alpha_1|1\rangle_1|0\rangle_2, \\ |Q_2\rangle &= \alpha'_0|0\rangle_L + \alpha'_1|1\rangle_L = \alpha'_0|0\rangle_3|0\rangle_4 + \alpha'_1|1\rangle_3|0\rangle_4, \end{aligned} \quad (5)$$

the transformation of applying a condition sign flip gate can be written as

$$\begin{aligned} |Q_1\rangle|Q_2\rangle &\Rightarrow \alpha_0\alpha'_0|0\rangle_L|0\rangle_L + \alpha_0\alpha'_1|0\rangle_L|1\rangle_L + \alpha_1\alpha'_0|1\rangle_L|0\rangle_L - \alpha_1\alpha'_1|1\rangle_L|1\rangle_L \\ &= \alpha_0\alpha'_0|0, 1, 0, 1\rangle + \alpha_0\alpha'_1|0, 1, 1, 0\rangle + \alpha_1\alpha'_0|1, 0, 0, 1\rangle - \\ &\quad \alpha_1\alpha'_1|1, 0, 1, 0\rangle. \end{aligned} \quad (6)$$

where the modes 1 and 2 are designated for the control qubit, and 3 and 4 are for the target qubit. A sign change happens only when there is one photon in mode 1 and one photon in mode 3.

The implementation of the desired operation is achieved by two 50:50 beam splitters and two nonlinear sign gates (see Fig. 2), with probability of success $1/16$. Effectively then, a Kerr nonlinearity can be generated by linear optics and projective measurements. The probability of success then can be boosted by using gate-teleportation technique

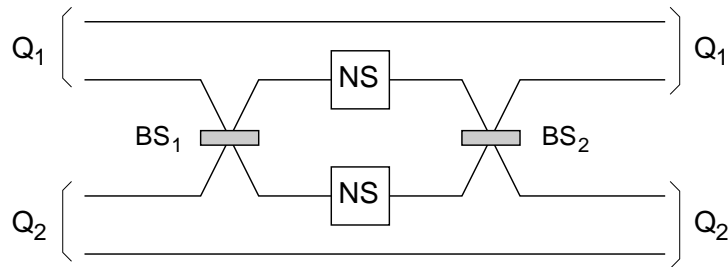


Figure 2. Nondeterministic conditional sign-flip gate. The relevant optical modes are assigned as $\{2,1,3,4\}$ from the top. When the modes 1, and 3 contain one photon each, $|1, 1\rangle_{1,3}$ ($|1\rangle_L|1\rangle_L$), it becomes $|2, 0\rangle_{1,3} - |0, 2\rangle_{1,3}$ after the first beam splitter (BS_1). Passing through the nonlinear sign gates, it yields $-|2, 0\rangle_{1,3} + |0, 2\rangle_{1,3}$. The second beam splitter (BS_2 —conjugate to beam splitter 1) then puts this into $-|1, 1\rangle_{1,3}$. Obviously, all other input states, $|0\rangle_L|1\rangle_L$, $|0\rangle_L|1\rangle_L$, $|1\rangle_L|0\rangle_L$, are not changed.

and sufficient number of ancilla photons. It has been also demonstrated that such a nondeterministic two-qubit gate can be made for qubits defined by the polarization degree of freedom [8, 9]. A general formalism for the effective photon nonlinearities generated by conditional measurement schemes in linear optics has been developed in some of our recent work [10]. Naturally, we emphasized that the ability to discriminate the number of incoming photons plays an essential role in the realization of such nonlinear quantum gates in LOQC [11, 12, 13].

2. Optical lithography beyond diffraction limit

Since the projective measurement can produce an effective photon-photon interaction, it can be a useful tool to manipulate quantum correlations between photons. A particularly interesting type of quantum state of light is the maximally entangled photon-number state. In our recent work, it has been shown that the Rayleigh diffraction limit in optical lithography can be overcome [14] by using a quantum state of light of the following form:

$$\frac{1}{\sqrt{2}} (|N, 0\rangle_{ab} + |0, N\rangle_{ab}), \quad (7)$$

where a, b denote two different paths. It is well known that the $N = 2$ path-entangled state of Eq. (7) can be generated using a Hong-Ou-Mandel interferometer and two single-photon input states. A 50:50 beam splitter, however, is not sufficient for producing path-entangled states with a photon number larger than two [15]. On the other hand, the generation of these states with $N > 2$ seems to involve a large Kerr nonlinearity, which makes their physical implementation very difficult [16].

Using the technique of projective measurement, we have shown that by conditioning on single-photon-detection, the generation of path-entangled photon-number states is possible for more than two photons [17, 18]. Figure 3 depicts a simple Mach-Zehnder type interferometric scheme for producing such a state with $N = 4$, using dual Fock-state inputs, $|N\rangle_a|N\rangle_b$.

Suppose that we have the $|3, 3\rangle$ state as the input entering into the modes a and b . Then, the first beam splitter transforms $|3, 3\rangle$ into a linear superposition of $|6, 0\rangle$, $|4, 2\rangle$,

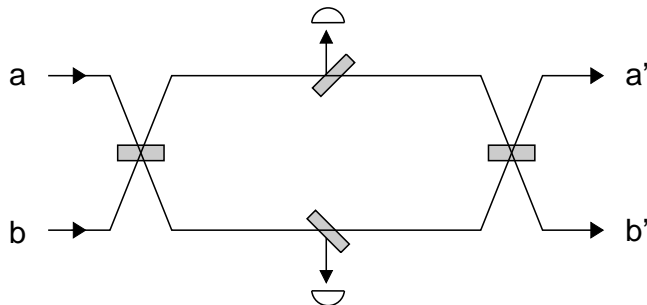


Figure 3. Mach-Zehnder interferometer with two additional beam splitters, which direct the reflected beams to photodetectors. Conditioned on a specific outcome of photodetection, a desired output state can be prepared in the mode a' and b' .

$|2, 4\rangle$, and $|0, 6\rangle$. After passing through the two intermediate beam splitters, and if one and only one photon is counted at each detector, the state is then projected onto an equal superposition of $|3, 1\rangle$ and $|1, 3\rangle$. Simply, the states $|6, 0\rangle$ or $|0, 6\rangle$ are discarded by this feedback from the photodetectors, since they cannot yield a click at both detectors. The $|4, 2\rangle$ and $|2, 4\rangle$ states, on the other hand, lose one photon in each arm of the interferometer and are projected to $|3, 1\rangle$ and $|1, 3\rangle$, respectively. Thus, just before the last beam splitter, we have a superposition of $|3, 1\rangle$ and $|1, 3\rangle$ with a known phase. We use an appropriate phase shifter in one of the two arms of interferometer so that the state after the projective measurement is reduced to $|3, 1\rangle - |1, 3\rangle$. Consequently after the last beam splitter, we get the desired state $|4, 0\rangle - |0, 4\rangle$. We have further shown that it is possible to produce any two-mode, entangled, photon-number state with only linear optical devices conditioned on photodetection [18]. Although the probability of success generally decreases exponentially as N increases [18, 19, 20, 21], it was shown that the scaling can be sub-exponential in N by using quantum memory [22]. For some applications, however, it can already be useful to have four-photon entanglement. Quantum interferometric lithography is such an example. Our approach has been used in a recent experiment to produce maximally entangled three-photon polarization states [23].

3. Phase-noise reduction beyond shot-noise limit

In a typical optical interferometer, in which ordinary coherent laser light enters via one input port, the phase sensitivity in the shot-noise limit scales as $\Delta\varphi = 1/\sqrt{\bar{N}}$ where \bar{N} is the mean number of photons. Over the last two decades, a lot of effort was devoted to overcoming this limit, due to the obvious practical applications. In the early 1980's, Caves first demonstrated that squeezing the vacuum noise in the unused input port of an interferometer causes the phase sensitivity to beat the standard shot-noise limit by $1/\sqrt{\bar{N}} \rightarrow 1/\bar{N}$ in the limit of infinite squeezing [24]. Bondurant and Shapiro proposed a multifrequency squeezed state interferometer for this same purpose [25]. Hermann Haus pioneered in the generation of squeezed light in optical fibers [26] as well as in Mach-Zehnder interferometers [27], towards achieving the goal of Heisenberg-limited interferometry.

On the other hand, in 1986 it had been suggested by Yurke and by Yuen that the phase-noise reduction can also be achieved using inputs with number eigenstates incident upon both input ports of a Mach-Zehnder interferometer [28, 29]. In particular, Yurke and collaborators showed that if the photons entered into each input port of the interferometer in nearly equal numbers with a certain type of correlation, then, it was possible to obtain an asymptotic phase sensitivity of $1/N$, the Heisenberg limit [30]. The so-called Yurke state is of the form:

$$\frac{1}{\sqrt{2}} [|N, N-1\rangle_{ab} + |N-1, N\rangle_{ab}], \quad (8)$$

where a, b denote the two input modes.

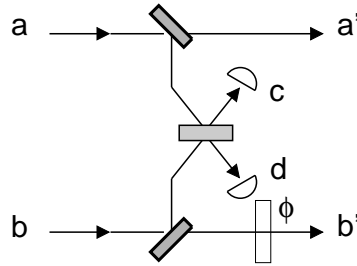


Figure 4. A simple path-entanglement generator. A Yurke-type quantum correlation between the two modes can be produced with a dual Fock-state. Suppose we post-select the outcome, conditioned upon only one photon detection by either one of the two detectors. Due to the 50:50 beam splitter in the midway, it is not possible to tell whether mode a or b lost one photon. The fundamental lack of which-path information provides the entanglement between the two output modes. For two-fold coincidence detection, the two detected photons are from either mode a or mode b , which eliminates the possibility of peeling off one photon from each mode.

Then, in the early 1990's, Holland and Burnett proposed Heisenberg-limited interferometry by the use of so-called *dual Fock states* of the form $|N, N\rangle_{ab}$ [31]. Such a state can be approximately generated by degenerate parametric down conversion or by optical parametric oscillation. In a conventional Mach-Zehnder interferometer, only the difference of the number of photons at the output is measured. However, to obtain increased sensitivity with dual Fock states, some special detection scheme is required, for which Hall and co-workers proposed a combination of a direct measurement of the variance of the difference current as well as a data-processing method based on Bayesian analysis [32]. Other types of special input states have been proposed for achieving the Heisenberg-limited phase sensitivity [33, 34, 35].

In particular, the Yurke state approach has the same measurement scheme as the conventional Mach-Zehnder interferometer; a direct detection of the difference current [36]. It is, however, not easy to generate the desired correlation in the input state. On the other hand, the dual Fock-state approach finds a rather simple input state, but requires a complicated data processing methods. However, by a simple utilization of the projective measurements with linear optical devices, it is possible to generate a desired correlation in the Yurke state directly from the dual Fock state.

Let us consider a linear optical setup depicted in Fig. 4. For a given dual Fock-state input $|N, N\rangle_{ab}$, the output state conditioned on, for example, a two-fold coincident count is given by

$$\frac{1}{\sqrt{2}} [|N, N-2\rangle + |N-2, N\rangle]. \quad (9)$$

It is not difficult to see that the condition of the coincident detection yields either one of the two modes before the beam splitter must contain two photons while the other modes contains no photon. This is an inverse-HOM situation where one photon at each mode cannot contribute to the coincident detection. Consequently, the coincident detection results in a situation where the main modes a and b can only lose two photons or not at

all. Here the probability success of this event can be optimized by choosing the reflection coefficient of the first beam splitters. For the reflection coefficient of $|r|^2 = 1/N$, its asymptotic value is found as $1/(2e^2)$, independent of N [37]. Furthermore, using a stack of such devices with appropriate phase shifters, we have developed a method for the generation of maximally path-entangled states of the form Eq. (7) with an arbitrary number of photons [18].

4. Single-photon QND measurement devices

In quantum optics the quantum nondemolition (QND) devices are usually considered in the context of photon-number measurements [38]. In 1985 Imoto, Haus, and Yamamoto developed the basic idea of QND in quantum optics, which consists of coupling the signal beam to the ‘meter’ beam in a nonlinear medium and the detection of the phase shift of the meter beam measures the number of photons in the signal beam [39]. The readouts of the number of photons in the signal beam are performed by phase-sensitive homodyne detection of the meter beam in interferometer arrangements.

By the same token, as discussed in Section 1, QND measurements at the single-photon level becomes extremely difficult due to the tiny strength of the nonlinear interaction between photons. In a recent experiment, a single-photon QND has been demonstrated by using a resonant coupling between a cavity field and the meter atoms [40]. Such a QND device at the single photon level can provide a key tool for optical quantum information processing, perhaps most importantly in quantum error correction. In contrast to the cavity approach, we have proposed a probabilistic device that signals the presence of a single photon without destroying it using the technique of projective measurement [41].

A simple way to perform a single-photon QND measurement is to use quantum teleportation. For example, a maximally polarization-entangled photon pair produced by a parametric down-converter can serve as a quantum channel. If the input state is in a arbitrary superposition of zero and one photon with a fixed polarization, the detector coincidence in Bell state measurement, signals the present of a single photon in the input and also the output states. Simply, a vacuum input can never yield a two-fold detector coincidence.

This teleportation-based QND scheme works only if the input states are restricted to one or zero photons. However, it breaks down if there are more than two photons in the input. For example, if the input state is of the form:

$$|\psi\rangle_{in} = c_0|0\rangle + c_1|1\rangle + c_2|2\rangle, \quad (10)$$

the two-photon term will contribute to the two-fold coincidence even when the output of the down-converter is vacuum, yielding a false identification of a single photon in the output state, conditioned on a detector coincidence.

If we restrict the number of photons in the input up to two, we can eliminate such a false identification by using an interferometric setup depicted in Fig.5. In Fig.

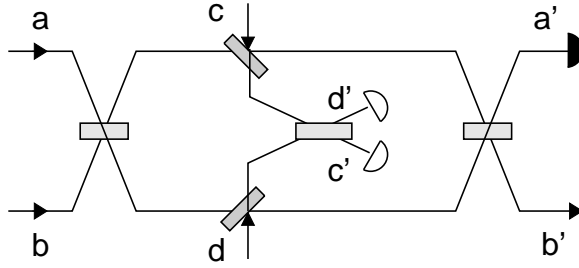


Figure 5. QND measurement device for single-photon detection. The input state, of an arbitrary superposition of $|0\rangle$, $|1\rangle$, and $|2\rangle$, enters into mode a , and an auxiliary single photon is prepared for both modes c and d . Conditioned upon a detector coincidence in modes c' and d' , and no count in mode a' , the outgoing mode b' is a single-photon state.

5, we assume that the input state of the form Eq.(10) enters into in mode a , and we further prepare single photons for mode c and d . Assuming beam splitters are 50:50, the transformation of the probe photons in the mode c and d can then be written as

$$\hat{c}^\dagger \hat{d}^\dagger \rightarrow \frac{1}{4}(\hat{b}'^{\dagger 2} - \hat{a}'^{\dagger 2} + \hat{d}'^{\dagger 2} - \hat{c}'^{\dagger 2} - 2\hat{a}'^\dagger \hat{c}'^\dagger + 2\hat{b}'^\dagger \hat{d}'^\dagger). \quad (11)$$

Then we post select the photodetection outcome for one and only one photon counted at each detector. This condition requires either two photons are in mode c or two photons are in mode d , which eliminates the contribution from $c_0|0\rangle$ of the input state. For one-photon and two-photon input states, we have

$$\hat{a}^\dagger \rightarrow (\hat{a}'^\dagger - \hat{c}'^\dagger)/\sqrt{2}, \quad \hat{a}^{\dagger 2} \rightarrow (\hat{a}'^{\dagger 2} - 2\hat{a}'^\dagger \hat{c}'^\dagger + \hat{c}'^{\dagger 2})/2. \quad (12)$$

Now the only two-fold coincidence in the mode c' and d' by a two-photon input is possible when the $2\hat{b}'^\dagger \hat{d}'^\dagger$ from Eq.(11) and $2\hat{a}'^\dagger \hat{c}'^\dagger$ from Eq.(12) combine, yielding $\hat{a}'^\dagger \hat{b}'^\dagger \hat{c}'^\dagger \hat{d}'^\dagger$. However, further postselecting on the vacuum in the mode a' eliminates this two-photon contribution to the two-fold coincidence in c' and d' .

A single photon in mode a yields a contribution $\hat{b}'^\dagger \hat{c}'^\dagger \hat{d}'^\dagger$, indicating that there is a two-fold coincidence in mode c' and d' , and a single photon in the output mode b' . As can be seen if Eqs.(11,12), the probability of success for this interferometric device is given by $1/8$. By adjusting the transmission coefficients of the beam splitters in modes c and d , the probability of success can be increased further. Obviously, this scheme does not work when the incoming state has an unknown polarization. However, it turns out that a more sophisticated interferometric setup with polarization beam splitters can do the job while preserving the unknown polarization [41]. Of course, such a scheme is not a full QND measurement of the photon-number observable, since it works for only zero, one, and two photons. It can, however, still play an important role in linear optical quantum computation, where up to only two photons are used in each logic gate [42, 43]. Furthermore, such a single-photon QND device can be used in various quantum communication protocols such as quantum repeaters [44, 45].

5. Summary

Linear optics with projective measurements, can be used to replace the use Kerr nonlinearities and provide a much higher efficiency. Using this technique, we have studied the generation of useful photonic quantum correlations. The maximally path-entangled photon-number states provide an essential way for optical lithography to proceed beyond the Rayleigh diffraction limit. The Yurke-type path-entanglement is of particular importance in Heisenberg-limited interferometry. Projective measurements also enable us to construct a device that signals the presence of a single photon without destroying it. Single-photon non-demolition measurement is of great importance in quantum information processing with photons, since most error-correction codes in the presence of qubit loss requires QND measurements [46].

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