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## **An operational policy for a single vendor multi buyer integrated inventory supply chain system considering shipping time**

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**AN OPERATIONAL POLICY FOR A SINGLE VENDOR MULTI BUYER  
INTEGRATED INVENTORY SUPPLY CHAIN SYSTEM  
CONSIDERING SHIPPING TIME**

A Thesis

Submitted to the Graduate Faculty of the  
Louisiana State University and  
Agriculture and Mechanical College  
in partial fulfillment of the requirement for the degree of  
Master of Science in Industrial Engineering

in

The Department of Construction Management and Industrial Engineering

By  
Chiranjit Saha  
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## **ABSTRACT**

Since its introduction, the concept of integrated inventory supply chain has received a considerable amount of attention. The majority of studies in the last three decades revealed an increase in holding cost as product moves further down the chain or up the chain. A recent study Hoque (2008) considered vendor's setup cost and inventory holding cost. Some research also considered fixed transportation cost, which is unrealistic. This study focuses on a single-vendor, multi-buyer scenario and presents three models. First, two models illustrate the transferring of equally-sized batches. Then, a third model considers the transferring of unequally-sized batches in a lot. This study relaxes the assumption that vendor's holding cost must be greater than or less than all buyer's holding costs in the system. Also, this research facilitates unequal transportation time and cost for different buyers for greater flexibility. The total system cost is calculated by summing the annual operational cost for all the parties in the system. Optimum values of the decision variables are determined using a direct search method. As presented by the third model, a numerical example demonstrates that the total system cost is less when compared with other two models presented. This study also presents the following: solution procedures to solve each model, many numerical examples to support mathematical findings, and performance comparisons among three findings. In order to justify the lot-splitting approach for solving the integrated inventory problem, alternative models with no lot splitting are devised and tested under the same circumstances. Alternative models with no lot splitting produce similar or better results. Under the same circumstances, the alternate third model is observed to be offering the least total cost for the system. This study also presents a sensitivity analysis to check the robustness of the three models. The future extension of this research may involve considering storage capacity constraint and random demand.



# **CHAPTER 1**

## **INTRODUCTION**

Consider a single vendor supplying an item to multiple buyers. The vendor produces the item in batches and at a finite rate. The vendor then sends the finished items to multiple buyers. In this process, the vendor incurs batch set-up and transportation cost, and the vendor and buyer both carry the item holding cost proportional to time. Meanwhile, each buyer has his own deterministic yearly demand. In this scenario, the buyer has a problem with determining the ordering quantity, and the vendor has a problem with determining optimum production quantity and shipping schedule, which minimizes the operating cost. During the last three decades, researchers have been searching for the solution to these problems. Researchers have shown that by viewing the vendor and buyers as a system (also known as integrated supply chain) rather than as separate individuals, total system cost can be reduced significantly. The basis of the integrated supply chain concept is that each buyer has clear knowledge about their yearly demand, and buyers are ready to share this information with the supplier to enjoy the benefits of coordination. Today, great improvements in electronic information exchange have made this concept feasible.

In supply chain, transportation cost is a major part of operational cost. Transportation time, cost, and capacity constraint play a role in making decisions. In today's world, short life cycle and countless specialties of similar products have made the global market highly competitive. In order to survive market pressure, every company has to be highly competitive in terms of product quality, price and product supply. The flow diagram of a single vendor multi buyer supply chain system is shown in Figure 1.1.

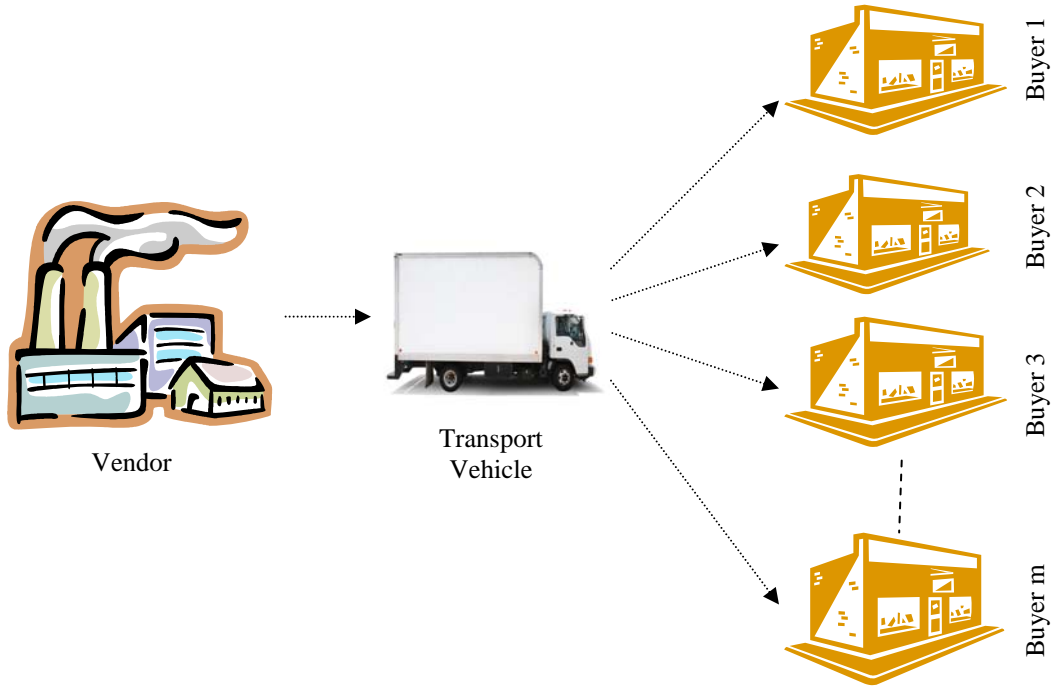


Figure 1.1: Flow diagram of a single vendor multi buyer integrated supply chain

### 1.1 The Problem

For decades, the primary objectives among research have been determining the optimal batch size, numbers of batch sizes in a lot, number of lots per year, and shipping schedule of integrated inventory models. The majority of this research is done with ideal assumptions. In both single and multistage supply chains, decision parameters, transportation time, cost and yearly demand vary with time. Demand of an individual buyer can be different, and holding cost and shipping cost may also vary. An individual buyer's economic order quantity and shipping schedule are also likely to differ. Unwise choices of these variables can lead to excessive product costs, which, in turn, can lead to customer dissatisfaction and lost sales.

The present research focuses on determining economic ordering quantity (EOQ) and shipping strategy of an inventory system integrated with a single-product, single-vendor,

and multi-buyer. In a single-vendor-multi-buyer system, a vendor produces and delivers an item to multiple buyers. Depending on the demand of all buyers, production and shipping policies are determined. If, for the sake of simplicity, we set a universal ordering quantity for all buyers, then we may fail to optimize everyone's cost. Some buyers may receive more than they need in a particular time span while others may receive less than needed. Again, imagine buyers are spread across the country where shipping cost and time varies significantly among buyers. For example, buyers who are far from the vendor need to ship earlier than those buyers who are relatively close to vendors. But maintaining different schedules for all buyers is difficult to accommodate, so we must find a cost-effective policy that better controls the complexity of operation. Also, we must determine how to regulate batch size in such a way that not only reduces ordering cost, setup cost and holding cost, but also avoids shortage.

## **1.2 Applications**

Automobiles are an example of such an item. All the showrooms around the country receive shipment of cars from the vendor. Suppose a manufacturing facility is in Michigan, and some of the showrooms are in Indiana, Louisiana or Alaska. Shipping cost and time is different for each case, and each showroom requires space for display, which requires high maintenance and surveillance cost. It is obvious that cost in California is more than that in Oklahoma or Missouri. No dealer likes to keep an excess of inventory because such inventory increases the holding cost. Again, if we look around we note, multiple showrooms with same automobiles which results in competition. No dealer can afford to not having cars in demand because the customer has a choice to get another next door. It is important that the dealer gets the shipment on time.

Another problem is every showroom has its own demand; based on ordering cost and holding cost, the showrooms' economic order quantities differ from one another. Again, considering setup cost and holding cost, vendor's economic manufacturing quantity in a batch can be different from the EOQ of all the showrooms. Now, if we arbitrarily assign each showroom the same shipping quantity and schedule, the system is likely to fail. The vendor interest is to produce as many items and ship them to the retailer as soon as possible to avoid holding cost. The interest of the retailer is to get the right amount of product within the right timeframe to avoid holding cost and other miscellaneous expenses. Again, transferring items in smaller lots results in lower inventory cost but higher ordering, setup, and transportation cost. On the other hand, transferring items in larger lots leads to higher inventory cost but lower ordering, setup, transportation cost, and scheduling interferences due to scarce storage capacity for both the vendor and the buyer.

Some examples of such industries are BMW, Ford, GMC, Mercedes, and Toyota, which produce cars, trucks, and other motorized vehicles. The proposed research will improve the inventory management of the supply chain system, which will also significantly reduce the system's operating cost and increase its profitability.

### **1.3 Research Goals**

The objective of this research is to study and model a single-vendor-multi-buyer inventory system, which constrains the transport capacity and maintains various transportation times between different buyers. In realistic situations, inventory holding costs are different for vendors and buyers; these transportation costs affect ordering policy, production policy, and total system cost, and transportation costs vary among all buyers. This research presents an operational policy to produce and deliver items in the right quantity at the right time while reducing the total system cost.

## **1.4 Research Objectives**

This problem addresses production and shipping policies of a product that flows from a vendor to multiple buyers. Yearly demands for all the buyers are recorded and are deterministic. Vendor and buyers maintain a close relationship to reduce overall system cost. In this research, three models are presented to address the problem under different operational policies. The vendor sends the product to buyers in multiple batches. Batches could be equally or unequally sized. Carrying costs for the vendor and each buyer may vary depending on their geographic location. Transportation time and cost for each buyer differs. Due to the above reasons, the nature of inventory of this supply chain system differs from traditional systems. Hence, the primary objectives of this research are:

- (i) To study the behavior of the inventories under three operational policies.
- (ii) To find optimal batch size.
- (iii) To find optimal batch number in a lot.
- (iv) To find best policy to reduce total system cost among three operational policies.

## **CHAPTER 2**

### **LITERATURE REVIEW**

This study has only considered the deterministic demand. The literature review organizes previous research in chronological order starting with the simplest, single-product, single-vendor, single-buyer scenario, unrestricted production rate and lot-for-lot policy (Goyal 1976). The review then shifts to the most recent single-vendor, multi-buyers scenario, which also involves setup and inventory cost for the vendor with finite production rate (Hoque 2008). Among the previous works, differences are mainly assumptions of production rate and replenishment period. Recently, other aspects of this problem have also been discussed in the literature. Other aspects refer to multiple buyers, transportation capacity and cost, lead time, variable production cost, quality and process failure, set up cost reduction and more realistic demand rates.

#### **2.1 Single Vendor Single Buyer Supply Chain System**

Goyal (1976) proposed his first model, addressing an integrated supply chain, assuming an infinite production rate, a uniform deterministic demand over time. He ignored lead time and restricted stock outs. In his research, the vendor produces in lots and sends the entire lot to the retailer. This process implies that the entire lot must be produced before shipment. Banerjee (1986) kept that lot-for-lot policy, but relaxed the assumption of infinite production rate. Banerjee (1986) also “coined” the term, “JELS” (joint Economic Lot Sizing) and argued about economic benefits of both vendor and retailer through JELS. Banerjee (1986) considered purchase transmit time, setup time and delivery time in accordance with actual production time. Goyal (1988) introduced a more generalized JELS model, which relaxed the assumption of lot-for-lot model and proposed to produce in a lot which can be supplied in “n” integer number of orders after the entire lot is produced. Goyal

(1988) showed that his joint total relevant cost is less than or equal to that of the JELS model. Goyal (1995) used a different approach than equally-sized shipment to come up with an idea of geometric shipment size. This means that the successive shipment size is the product of the prior shipment size in relation to the ratio of production and demand rate. Viswanathan (1998) presented two models: one in which the shipment sizes are similar and another in which the shipment size is equal to whatever inventory is available at that point. In this study Viswanathan named Lu's (1995) equal shipment size policy as "Identical delivery quantity" (IDE) and Goyal's (1995) an unequal shipment size model as "deliver what is produced" (DWP). Viswanathan showed that neither policy is better than the other for all type of problems, and he concluded that the best policy depends on the problem's parameters.

Goyal and Nebebe (2000) pointed out the difficulties faced by the vendor and retailer while applying the policy proposed by Goyal (1995) and Hill (1997), who pointed out the difficulty of determining batch size for the vendor, optimal number of shipments, and each shipment size for the buyer. Goyal and Nebebe (2000) proposed an alternate solution which suggests that among " $n$ " shipments, the first shipment is smaller and followed by  $(n-1)$  equal- sized, which is equal to the product of the first shipment size as well as its rate of production over rate of demand. Goyal (2000) extended the policy proposed by Hill (1997). He proposed that the following shipment sizes will be determined by first shipment size. The following shipment sizes may be increased by a factor of production rate over demand rate until it is impossible to do so. A likely drawback of this study is that the intended application of the model is unclear. Hill and Omar (2006) presented derivation of optimal manufacturing batch size and shipment policy when product-holding cost increases as the product moves down to the buyer under non-required

equal shipping size. Zhou and Young (2007) relaxed the assumptions that vendor's stock holding is always greater than the retailer's. They showed that their model performs equally well in reducing average total cost regardless of the vendor's or retailer's stockholding costs, which are never equal to each other. In this study they also allowed shortages but only for buyers. They also presented a production inventory policy for deteriorating items. In this study they proved that the optimal policy for vendors whose holding costs are greater than the retailers' holding cost is that all unequally-sized shipments increase the following shipments by ratio of production rate to the demand rate. Pan and Young (2002) explored how to reduce lead-time in an integrated system involving cost. They argued that better customer satisfaction levels and reduced safety stock levels can be achieved through improving lead-time; however, these changes occur at the expense of lead-time crashing cost. This study developed a model, which yields lower total cost and reduced lead time than that presented by Banerjee (1986) and Goyal (1988). Ultimately, this research is an extension of Goyal (1988), and it relaxed the infinite production capacity assumption. Hoque and Goyal (2000) considered transport capacity limitation by presenting an optimal policy for the single-vendor and single-buyer integrated supply chain, which considers equal and unequal shipment size and transportation capacity.

## **2.2 Single Vendor Multi Buyer Supply Chain System**

Joglekar and Tharthare (1990) considered another area of integrated supply chain, i.e., single vendor and multi buyer. In that study they presented an alternate solution of the same problem considered by Banerjee (1986) and named it as the 'Individual Responsible and Rational Decision' (IRRD). In IRRD, they refined JELS by breaking set-up cost into vendors' order processing and handling cost per production run setup cost. Based on the changes, the authors claimed IRRD's consistency in a free enterprise scenario and



superiority over Banerjee's (1986) JELS model in dealing with problems like single-vendor and non-identical buyers. A single-vendor, multi-retailer problem is also addressed by Affisco et al. (1988, 1991, and 1993). In these studies, researchers addressed the single vendor and many identical retailers with an objective of reducing production setup cost and retailer's ordering cost. They showed that substantial improvement can be achieved, under this model, through the independent cost optimization technique; thus, in a cooperative environment, an integrated inventory approach is suggested over independent cost optimization. Lu (1995) proposed his model in context of a single-vendor or multi-buyers scenario. Lu allowed shipments during production and ignored Goyal's assumption of producing an entire lot before shipment. Many other JELS-based models, e.g. Banerjee and Kim (1995) and Kim and Ha (2003), considered equally-sized shipment policy. Yau and Chiao (2004) presented a model in which the vendor produces and supplies to all the buyers; this minimizes the vendor's total annual cost based on the maximum cost buyers are willing to incur. They came up with an efficient algorithm to search an optimal cost curve. Siajedi et al. (2005) presented a single-vendor-multi-buyer scenario in which the vendor is the sole supplier for a specific item to all buyers. Supplies are delivered in equal sizes, but the shipment size may differ from one buyer to another based on their demand. Supplies are delivered in sequence; e.g., the first buyer will get first supply followed by second and third and so on, assuming production-cycle time and buyer's-ordering cycle time are the same. Also, the time between one delivery and the next is fixed for each buyer. Hoque (2008) presented single-vendor multi-buyer system considering and considered vendor's setup and inventory holding cost. In this research, he argued in favor of transferring of smaller lots over larger lots when storage capacity is scarce for both the vendor and the buyer. Viswanathan and Piplani (2001) addressed the same problem and tried to solve it by using a

game theoretic approach. They proposed that vendors will specify the common replenishment period for accepting the proposal, and in turn, buyers will receive a price discount from the vendor. The price discount will be sufficient to compensate the alleviated product carrying cost, if any. Viswanathan and Piplani (2001) derived optimal replenishment period and price discount quantity by solving Stackelberg game.

Chen et al. (2009) discussed delivery and shared transportation cost in a multiple-vendors integrated inventory system, and Comeaux et al. (2005) discussed the product quality inspection policy in an integrated inventory system.

### **2.3 Shortcomings of Previous Literature**

This literature review briefly recalls the development of the integrated supply chain management systems starting from the simplistic single-vendor, single-buyer system and moving toward more advanced single-vendor, multiple-buyers systems. The review points out that each study has its own shortcomings. Realistically, most of the problems are constrained. A vendor's holding cost could be higher or lower than other buyers' in the same system; transportation time to one buyer could be different from another buyer, and even transportation cost may differ among buyers. Although many researchers considered constraints mentioned above in their models one at a time, so far, they have given little attention to building a single-vendor-multiple-buyer integrated model, which considers all the above mentioned constraints. Here, we present a single-vendor-multi-buyer integrated supply chain model with equal and unequal batch sizes; this model considers vendor's set-up cost, transportation time, cost, capacity constrained and unequal holding cost for buyers. Three models are presented: the first two, which consider equal batch size and the third model, which considers unequal batch size.

## CHAPTER 3

### MODEL DEVELOPMENT

In this section the formulation of the three operational models are presented to illustrate different production and shipping policies. These models are based on some previously described assumptions and notations and are followed by average inventory and total system cost derivations.

#### 3.1 Assumptions

The following assumptions are made to construct models:

- (a) Demand and production rate are fixed and deterministic.
- (b) Every buyer estimates their own ordering and holding cost under various cost factor and lets it be known to the vendor.
- (c) The concerned parties share the benefits of coordination based on a costless way of sharing.
- (d) No backlogging or shortages are allowed, *i.e.*  $P \geq D$ .
- (e) Lot and batch sizes are integers.
- (f) For both vendor and the buyer, storage capacity is unconstrained.
- (g) All shipping vehicles are identical, and availability of any number of shipping media is unconstrained.
- (h) Transportation times are significant and can vary from buyer to buyer depending on buyer's distance from the vendor.
- (i) Set-up time and cost are significant.
- (j) Minimum batch size has to be greater than or equal to number of buyer in the system.
- (k) For the purpose of simplicity, inventory carrying cost during transportation is neglected.

### 3.2 Notations

The following notations will be required to formulate the model:

#### $i^{\text{th}}$ buyer parameter

- $a_i$  Ordering cost (\$/order)
- $C_{t_i}$  Cost of one vehicle for  $i^{\text{th}}$  buyer (\$/vehicle)
- $D_i$  Annual demand (unit/year)
- $d_i$  Daily demand of  $i^{\text{th}}$  buyer (unit/day)
- $h_i$  Holding cost per item per year (\$/unit/year)
- $q_t$  Vehicle capacity (units/vehicle)
- $t_i$  Shipping time to  $i^{\text{th}}$  buyer (days)

#### Vendor Parameter

- $D$  Annual rate of demand (unit/year)
- $g$  Smallest batch size
- $h$  Holding cost per item per year (\$/unit/year)
- $P$  Production rate per year (  $P > D, P / D = k$  ), (unit/year)
- $S$  Setup cost (\$/setup)

#### Variables

- $g_i$  Original shipping size for  $i^{\text{th}}$  buyer
- $G_i$  Shipping size for  $i^{\text{th}}$  buyer which includes transportation time demand
- $g$  Smallest batch size
- $G$  Batch size which includes transportation time demand for  $m$  buyers
- $n$  Number of equal or unequal batches in a lot

### 3.3 Model Formulation

In this research, the inventory models are developed for three production and shipping policies. The first model aims to produce items in  $n$  equal sized batches in a lot. Items are shipped to the buyers as soon as the batch production is finished. The second model also assumes that items are produced in  $n$  equal-sized batches in a lot and that the vendor holds the item until buyers place an order. The third model attempts to produce items in  $n$  unequal-sized batches in a lot, and the vendor holds the item until order is received.

Assuming the vendor produces items in lots, a lot consists of  $n$  batches, and batches are produced in size of  $g$ , buyers receive the item in proportion to the ratio of their demand to the total demand of batch size  $g$ . We can write:

$$g_i = g \frac{D_i}{D}. \quad (3.1)$$

Thus,

$$g = \sum_{i=1}^m g_i. \quad (3.2)$$

In this research, we are considering transportation time, and we acknowledge that transportation times vary between buyers. Now, we want to ship products to all buyers at the same time from the vendor's end, but since transportation times vary between buyers, some buyers might have to receive the product before or after the previous batch is exhausted. To minimize holding cost, we want to assure that each buyer receives a new shipment when the previous batch is nearing depletion. To consider this scenario, we propose to add individual transportation time demand with the buyer's corresponding shipment. Assume transportation time to buyer  $i$  is  $t_i$ , so the new shipment size for  $i^{th}$  buyer becomes

$$G_i = g_i + t_i d_i, \quad (3.3)$$

where  $d_i$  is the daily demand for  $i^{th}$  buyer. The summation of original batch size

$g$  and transportation time demand for all the buyers leads to

$$G = g + \sum_{i=1}^m t_i d_i . \quad (3.4)$$

### 3.4 Model I: Item Shipped Right After Production

As described above, a vendor ships items to buyers as soon as the batch is produced. In the first model, we are assuming all batch sizes in a lot are equal and that there are  $n$  batches in a lot. Figure 3.1 shows the production and inventory flow for the first model.

In Figure 3.1,  $g/P$ ,  $g_i/D_i$ , and  $t_i$  are the time segments, which represent the following: production time for a batch, consumption time for  $i^{th}$  buyer and transportation time for  $i^{th}$  buyer, respectively. In Figure 3.1, triangle (ABC) represents the inventory for the vendor. The triangle (MNO) represents the ideal inventory for the  $i^{th}$  buyer when an item is received at the end of previous inventory. The area (MOPQ) with a dashed line represents inventory in the buyers' warehouse while the previous batch is being consumed.

In the beginning of the cycle, the vendor begins production at a finite rate, and inventory begins to accumulate at a constant rate. In Figure 3.1, the slope AC is the rate of build inventory during production. As soon as production of the batch is complete, items are shipped to each buyer, and inventory of vendor reduces to zero. At this point, the vendor produces the next batch, repeating the process until the entire lot is produced. Although items to all buyers are shipped at the same time, they will receive them after  $t_i$  time, since shipping time differs among the buyers. Items are available for consumption as soon as the buyer receives them; and, the buyer consumes the item at a constant rate. In Figure 3.1, slope MN is the consumption rate. Since a buyer consumes items at a fixed rate, on-hand inventory starts decreasing at a constant rate. Again, as we assumed  $P > D$  and  $g/P < g/D$

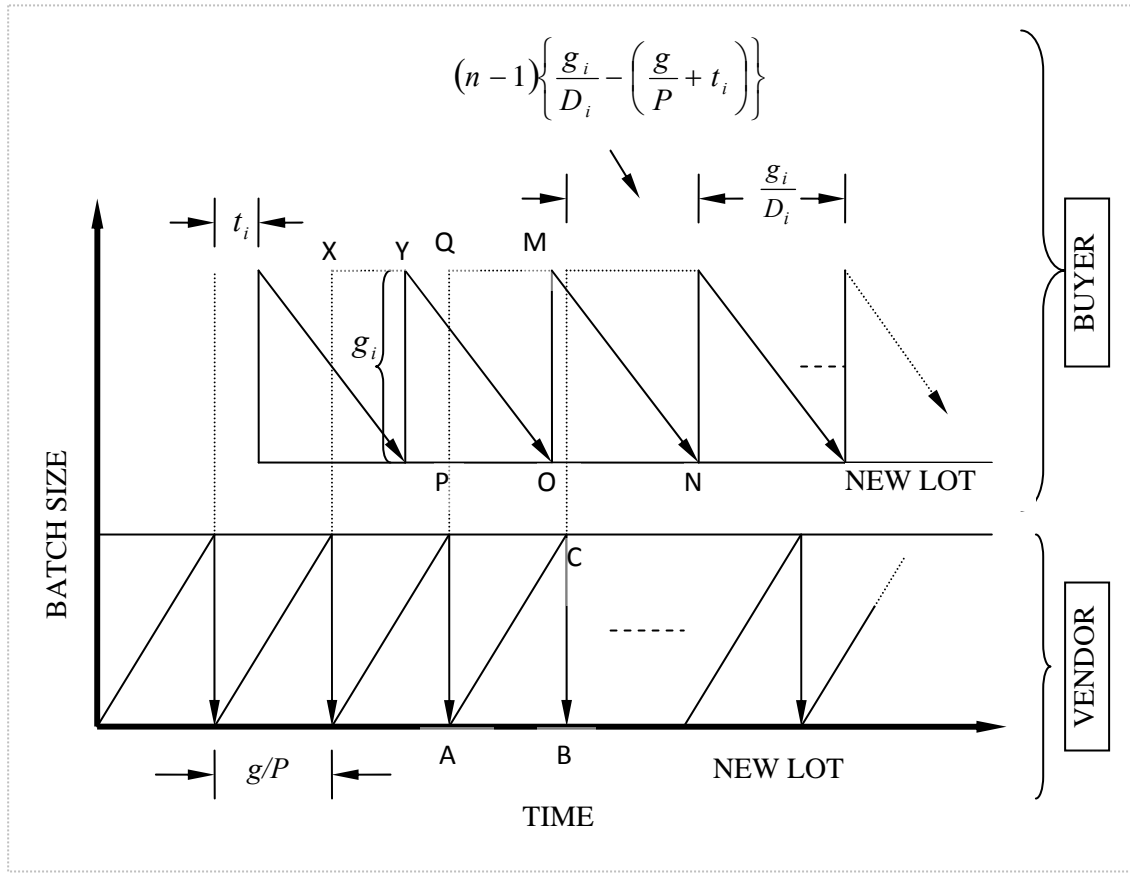


Figure 3.1: Inventory flow in Model I

to avoid shortage, a buyer will receive the next shipment before their on-hand inventory is exhausted. This new shipment remains in the warehouse until the previous batch is consumed. Notice, as the cycle continues, the time new shipment remains idle in the warehouse multiplies, as shown as MQ in the Figure 3.1, where  $(n-1)\left\{\frac{g_i}{D_i} - \left(\frac{g}{P} + t_i\right)\right\}$  the time new batch remains in the buyer's inventory. In contrast to this research, Hoque (2008) did not consider shipping time, and the buyer receives the item as soon as it is shipped. So, in the present research, the buyer does not have to hold inventory in its warehouse for  $t_i$  amount of time.

### 3.4.1 Average Inventory Calculation

This section derives the average inventory in Model I for the entire supply chain system containing a single vendor and multiple buyers. Since the demand is deterministic

and prohibits backlog or shortage, the production rate must be greater than the demand rate.

The total system inventory consists of buyer's inventory,  $I_b$  and vendor's inventory,  $I_m$ .

#### (a) Buyer's Average Inventory Calculation

Each buyer receives  $g_i$  amount of an item in each shipment and takes  $g_i/D_i$  amount of time to consume it. So, each buyer's average inventory for the first batch is  $(g_i^2/2D_i)$ . Since the second shipment arrives before the first shipment is finished, the buyer must hold the second shipment for  $\{g_i/D_i - (g/P + t_i)\}$  additional amount of time until previous shipment is consumed. Therefore, average inventory during the second shipment is  $[g_i^2/2D_i + \{g_i/D_i - (g/P + t_i)\}g_i]$ . The third shipment arrives before the second shipment is finished and actually remains in the warehouse twice the time of the second shipment. In Figure 3.1, we can see QM is double of XY, which are the current times in which shipping remains idle in the warehouse. This continues until the entire lot is supplied, if there are  $n$  batches in a lot, then the  $n^{th}$  shipment must remain in the warehouse for  $(n-1)\{g_i/D_i - (g/P + t_i)\}$  amount of time. Therefore, the total average inventory for  $i^{th}$  buyer per cycle can be expressed as follows:

$$Inv_{cycle_i} = \left[ \left( \frac{n g_i^2}{2 D_i} \right) + \left\{ \frac{g_i}{D_i} - \left( \frac{g}{P} + t_i \right) \right\} g_i (1 + 2 + 3 + \dots + (n-1)) \right]. \quad (3.5)$$

If each batch size is  $g$  and there are  $n$  batches in a lot, then in a complete cycle,  $ng$  amount of items are produced. So, the total number of cycles in a year is  $(D/ng)$ .

Therefore, average inventory for  $i^{th}$  buyer per year can be expressed as follows:

$$Inv_{year_i} = \left[ \left( \frac{n g_i^2}{2 D_i} \right) + \left\{ \frac{g_i}{D_i} - \left( \frac{g}{P} + t_i \right) \right\} g_i (1 + 2 + 3 + \dots + (n-1)) \right] \frac{D}{ng}. \quad (3.6a)$$



By substituting  $\left(g_i = g \frac{D_i}{D}\right)$  in equation (3.6a) and upon simplification, yields

$$Inv_{year_i} = \frac{gD_i}{2D} + g\left(\frac{n-1}{2}\right)\left(\frac{1}{D} - \frac{1}{P}\right)D_i - \left(\frac{n-1}{2}\right)D_i t_i. \quad (3.6)$$

Therefore, the total average inventory for all buyers per year is

$$Inv_b = \frac{g}{2D} \sum_{i=1}^m D_i + g \frac{(n-1)}{2} \left(\frac{1}{D} - \frac{1}{P}\right) \sum_{i=1}^m D_i - \left(\frac{n-1}{2}\right) \sum_{i=1}^m D_i t_i. \quad (3.7)$$

### (b) Vendor's Average Inventory Calculation

The vendor produces items in batches at a finite rate and holds them until production of the first batch is finished. If each batch size is  $g$  and requires  $g/P$  period to produce, the average inventory during the first cycle is  $(g^2/2P)$ . Since there are  $n$  batches in a cycle, the average inventory per cycle can be expressed as

$$Inv_{cycle_v} = \frac{n g^2}{2 P}. \quad (3.8a)$$

Since there are  $D/ng$  number of cycles per year, average inventory per year can be expressed as follows:

$$Inv_{year_v} = \frac{D}{ng} \frac{n g^2}{2 P} = \frac{Dg}{2P}. \quad (3.8)$$

### 3.4.2 Total Average Inventory Calculation

In an integrated supply chain system, the total average inventory is calculated by summing average yearly inventories of the vendor, equation (3.8), and all buyers, equation (3.7). Therefore, the yearly average inventory of the system can be written as follows:

$$Inv_{year_{system}} = \frac{Dg}{2P} + \frac{g}{2D} \sum_{i=1}^m D_i + g \frac{(n-1)}{2} \left(\frac{1}{D} - \frac{1}{P}\right) \sum_{i=1}^m D_i - \left(\frac{n-1}{2}\right) \sum_{i=1}^m D_i t_i. \quad (3.9)$$

### 3.5 Total Cost For Model I

In the current and previous sections, inventories for the vendor and all buyers are calculated under the assumptions of first model. Generally, the total cost of the system consists of major costs such as (a) ordering cost, (b) setup cost, (c) inventory holding cost, and (d) transportation cost. The total system cost, consisting of ordering cost, setup cost, inventory holding cost, and transportation cost, can be calculated.

#### 3.5.1 Ordering Cost

Each time a buyer places an order to the vendor incurs a cost, which may consist of paper work, telephone conversations, etc., then each buyer presumably places an order before the cycle starts. Hence, each buyer will place  $D/ng$  number of orders in a year. Therefore,  $(m)$  buyers in a year will place  $mD/ng$  number of orders. The cost of placing one order for  $i^{th}$  buyer is  $a_i$ . Therefore, the total ordering cost,  $A$ , for all buyers can be expressed as

$$A = \frac{D}{ng} \sum_{i=1}^m a_i, \quad (3.10)$$

where  $D$  is the demand rate (units/year),  $n$  is the number of batches in a cycle and  $g$  is batch size.

#### 3.5.2 Setup Cost

In each new production cycle that a vendor starts for a new lot, a setup cost, such as changing dye, putting raw materials etc is required. If the manufacturing process requires setup for every new lot, the total number of setups required is  $D/ng$ . Hence, the total setup cost ( $S_m$ ) can be calculated as,

$$S_m = \frac{SD}{ng}, \quad (3.11)$$

where  $D$  is the demand rate (units/year),  $n$  is the number of batches in a cycle,  $g$  is the batch size, and  $S$  the setup cost per lot.

### 3.5.3 Inventory Carrying Cost

While the batch is being produced, inventory builds up. Thus, the vendor incurs the inventory carrying cost until production is finished and the items are shipped. Similarly, each buyer receives items and holds them until all the items are consumed. Therefore, each buyer also incurs item carrying costs. The total system inventory carrying cost can be calculated.

#### (a) Inventory Carrying Cost for Buyers

From equation (3.7) we know the average yearly inventory for all buyers. If  $h_i$  is the carrying cost for  $i^{th}$  buyer, inventory holding cost for all buyers per year can be calculated as

$$Inv_b = \frac{g}{2D} \sum_{i=1}^m D_i h_i + g \frac{(n-1)}{2} \left( \frac{1}{D} - \frac{1}{P} \right) \sum_{i=1}^m D_i h_i - \left( \frac{n-1}{2} \right) \sum_{i=1}^m D_i h_i t_i, \quad (3.12)$$

where  $D_i$  is yearly demand of  $i^{th}$  buyer,  $g$  is batch size,  $D$  is demand rate (units/year),  $n$  is number of batches in a lot;  $P$  is production capacity (units/year) and  $t_i$  is transportation for  $i^{th}$  buyer.

#### (b) Inventory Carrying Cost for Vendor

From equation (3.8) we know average yearly inventory for the vendor. If  $h$  is the carrying cost for vendor, inventory holding cost ( $h_m$ ) for the vendor per year can be calculated as

$$h_m = \frac{Dhg}{2P}. \quad (3.13)$$

### (c) Total System Carrying Cost Calculation

Total system's carrying cost consists of the buyer's carrying cost per year and vendor's carrying cost per year. Equations (3.12) and (3.13) represent all buyers' carrying costs and the vendor's carrying cost, respectively. Hence, the total system's carrying cost ( $h_{sys}$ ) can be expressed as

$$h_{sys} = \frac{g}{2D} \sum_{i=1}^m D_i h_i + g \left( \frac{n-1}{2} \right) \left( \frac{1}{D} - \frac{1}{P} \right) \sum_{i=1}^m D_i h_i - \left( \frac{n-1}{2} \right) \sum_{i=1}^m D_i h_i t_i + \frac{Dhg}{2P}. \quad (3.14)$$

### 3.5.4 Transportation Cost

Every time the vendor sends a shipment to a buyer, the buyer incurs a transportation cost. Realistically, the capacity of a transportation vehicle is limited. Again, even if a conveyance is partially filled, the buyer has to pay the price of a full load. Another consideration is transportation cost for one shipment, which may differ among buyers depending on their distances from the vendor. If  $q_t$  is the carrying capacity of a conveyance and  $g_i$  is the shipment size, the receiving buyer has to pay for  $(g_i/q_t)$  number of loads. If the conveyance cost is  $C_{t_i}$  for  $i^{th}$  buyer and there are  $n$  number of batches in a cycle, then transportation cost per cycle ( $T_{cycle_i}$ ), paid by  $i^{th}$  buyer, can be calculate as

$$T_{cycle_i} = n \left\lceil \frac{g_i}{q_t} \right\rceil C_{t_i}. \quad (3.15)$$

Since there are  $(D/ng)$  number of cycles in a year, transportation cost to  $i^{th}$  buyer per year can be calculated as

$$T_{year_i} = \frac{D}{ng} n \left\lceil \frac{g_i}{q_t} \right\rceil C_{t_i} = \frac{D}{g} \left\lceil \frac{gD_i}{Dq_t} \right\rceil C_{t_i}. \quad (3.16)$$

By equating ( $g_i = \frac{gD_i}{D}$ ), where  $g$  is batch size (items/batch) and  $D$  is the yearly demand (items/year). Since there are  $m$  buyers in the system, total transportation cost ( $T_b$ ) for all buyers in a year can be expressed as

$$T_b = \frac{D}{g} \sum_{i=1}^m \left[ \frac{gD_i}{Dq_t} \right] C_{t_i} . \quad (3.17)$$

### 3.5.5 Total System Cost

The total system cost,  $TC_I$  consists of ordering cost, setup cost, inventory carrying cost, and transportation cost. Hence, the total system cost for Model I can be calculated by adding equations (3.10), (3.11), (3.14), and (3.17) as

$$TC_I = \frac{SD}{ng} + \frac{Dhg}{2P} + \frac{D}{ng} \sum_{i=1}^m a_i + \frac{D}{g} \sum_{i=1}^m \left[ \frac{gD_i}{Dq_t} \right] C_{t_i} + \frac{g}{2D} \sum_{i=1}^m D_i h_i + g \left( \frac{n-1}{2} \right) \left( \frac{1}{D} - \frac{1}{P} \right) \sum_{i=1}^m D_i h_i - \left( \frac{n-1}{2} \right) \sum_{i=1}^m D_i h_i t_i. \quad (3.18)$$

### 3.6 Solution Methodology

At this point, one must understand the nature of the total system cost ( $TC_I$ ) function for optimization purposes. If the total cost function is found to be convex, then  $\partial TC_I / \partial g = 0$  leads to optimality. By looking at  $D/g \sum_{i=1}^m [gD_i/Dq_t] C_{t_i}$  in equation (3.18), it is clear that  $TC_I$  is not a convex function in  $g$  given  $n$  constant. An optimal batch quantity  $g^*$  can be calculated by a direct search method within a boundary of  $n = (1,10)$ , and  $g = (m, D)$ . Similarly, an optimal number of batches  $n^*$  can also be calculated by a direct search method within a boundary of  $n = (1,10)$ , and  $g = (m, D)$  using equation (3.18).

**Example 3.1** Item shipped in equally-sized batches upon production completion

Assume,  $P = 1900$  units/year,  $D = \sum_{i=1}^m D_i = 1250$  units/year,  $S = 90$ ,  $h = 1.3$ ,

$q_i = 30$ . Data is given in Table 1. For integer  $n$ , the minimum total cost is presented in Table 2. Necessary unit conversion is performed prior to solution. For detailed results, see Appendix. Figure 3.2, a plot using MatLab 2009b; the minimum total cost is marked on the figure. In Figure 3.2, a sudden cut occurs in the plot because at that point the number of vehicle required changes for some buyers.

Table 1: Data for single vendor 9 buyers problem

| Buyer | $a_i$ | $D_i$ | $h_i$ | $t_i$ (days) | $C_{t_i}$ |
|-------|-------|-------|-------|--------------|-----------|
| 1     | 9     | 150   | 1.9   | 4.0          | 50        |
| 2     | 14    | 230   | 1.2   | 2.0          | 40        |
| 3     | 13    | 180   | 2.8   | 2.5          | 40        |
| 4     | 21    | 114   | 1.3   | 7.0          | 50        |
| 5     | 20    | 185   | 1.5   | 6.0          | 60        |
| 6     | 27    | 80    | 2.2   | 14.0         | 100       |
| 7     | 12    | 45    | 2.6   | 19.0         | 100       |
| 8     | 16    | 145   | 2.8   | 5.0          | 50        |
| 9     | 10    | 121   | 1.7   | 9.0          | 60        |
| SUM   | 142   | 1250  |       |              |           |

Table 2: Summary of results

| $N$      | $g$        | $TC_I$         |
|----------|------------|----------------|
| 1        | 608        | 3951.11        |
| <b>2</b> | <b>468</b> | <b>3870.08</b> |
| 3        | 468        | 3900.73        |
| 4        | 405        | 3981.31        |

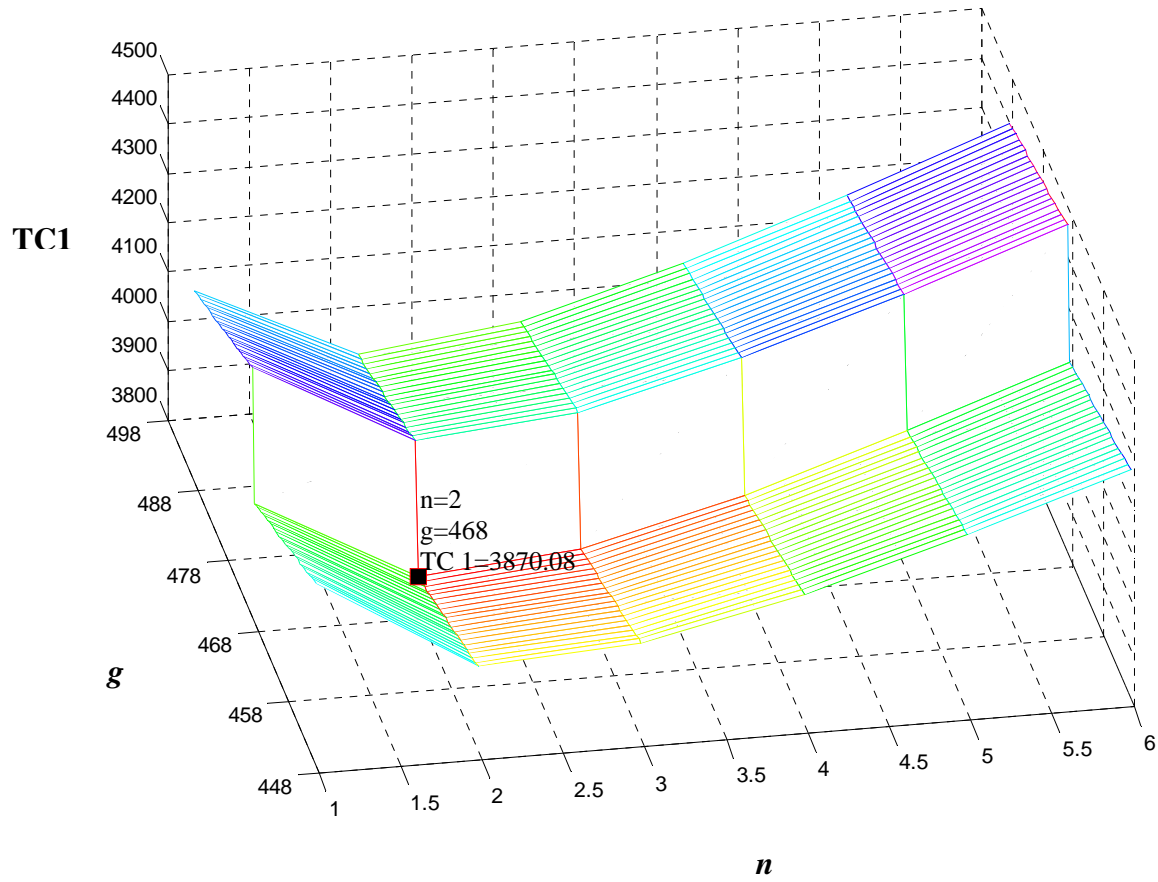


Figure 3.2: Graphical representation of  $TC_I$

### 3.7 Sensitivity Analysis for Model I

The total cost is a solution of the model where model parameters (vendor holding cost, setup cost, and production to total yearly demand ratio) are presumably fixed. It is useful to carry out sensitivity analysis of the model. For this purpose, effect of changes in system parameters must be verified to check if the current solution

1. Remains unchanged.
2. Becomes sub-optimal, etc.

### 3.7.1 Effect of $S$ on $TC_I$

The vendor setup cost  $S$  plays a major role in determining total cost of the system, and it also affects the optimal batch size and number of batches in a lot. The effect of  $S$  on total system cost,  $TC_I$  is expressed mathematically in equation (3.19),

$$\frac{\partial TC_I}{\partial S} = \frac{D}{ng}, \quad (3.19)$$

which is a constant term. The rate and direction of change of  $TC_I$  with respect to  $S$  depend on the values of parameters used in the numerical example. If we put values of  $D$ ,  $n$ , and  $g$  from Table 10 and Table 11 in equation (3.19), then we can observe that for unit change in  $S$ ,  $TC_I$  will increase by \$0.93.

### 3.7.2 Effect of $h$ on $TC_I$

The vendor's holding cost  $h$  is an important parameter in developing the model. The effect of  $h$  on total system cost  $TC_I$  is expressed mathematically in equation (3.20),

$$\frac{\partial TC_I}{\partial h} = \frac{Dg}{2P}, \quad (3.20)$$

which is also a constant term. The rate and direction of change of  $TC_I$  with respect to  $h$  depend on the values of parameters used in the numerical example. If we put values of  $D$ ,  $n$ , and  $g$  from Table 10 and Table 11 (presented later), then we can write, that for every unit increase (decrease) in  $h$ ,  $TC_I$  will increase (decrease) by \$232.00.

### 3.7.3 Effect of $P/D$ on $TC_I$

The ratio between production rate and total yearly demand  $P/D$  is not only an important parameter to determine the total cost; but also, in conjugation with the vendor holding cost  $h$ , the ratio determines which model to choose for a particular scenario among the three. The effect of  $P/D$  on total system cost,  $TC_I$  is expressed mathematically in equation (3.21),



$$\frac{\partial TC_I}{\partial A} = -\frac{hg}{2A^2} + \frac{I}{DA^2} g \left( \frac{n-1}{2} \right) \sum_{i=1}^m D_i h_i . \quad (3.21)$$

The effect of  $P/D$  over  $TC_I$  is represented by Figure 3.3 using equations (3.21) and  $P/D \in [0,5]$ . A study is performed with respect to  $P/D$  where the parameter values are the same except for the values of  $P/D$ , which changes from 1 to 5, and the results are presented in Table 3. Figure 3.3 illustrates that  $P/D$  ratio affects  $TC_I$  changes rapidly up to 2.1, and beyond this point,  $TC_I$  becomes less sensitive to a change in  $P/D$ . We can also write that  $TC_I$  increases as the ratio increases. Table 3 summarizes the change in  $TC_I$  for the change in  $P/D$  ratio.

### 3.8 Model II: Items Are Shipped In Every $g/D$ Period

As described above, a vendor holds items and ships them in every  $g/D$  period. In the second model we are assuming all batch sizes in a lot are equal and there are  $n$  batches in a lot. Figure 3.4 shows the production and inventory flow for the second model. In Figure 3.1,  $G/P$ ,  $G_i/D_i$ , and  $t_i$ , are the time segments, which represent the production time for a batch including transportation time demand, consumption time for  $i^{th}$  buyer and transportation time for  $i^{th}$  buyer, respectively. In Figure 3.4, triangle (MNO) represents the ideal inventory for the vendor. The triangle (ABC) represents the ideal inventory for the  $i^{th}$  buyer when the item is received at the end of previous inventory. The area (XYZ) represents the inventory built up in the vendor's warehouse due to constant production of items.

Table 3: Effect of  $P/D (= A)$  ratio on  $TC_I$

| $A$ | $\frac{\partial TC_I}{\partial A}$ |
|-----|------------------------------------|
| 1.0 | 404.75                             |
| 1.5 | 179.89                             |
| 2.0 | 101.19                             |
| 2.5 | 64.76                              |
| 3.0 | 44.97                              |
| 3.5 | 33.04                              |
| 4.0 | 25.30                              |
| 4.5 | 19.99                              |
| 5.0 | 16.19                              |

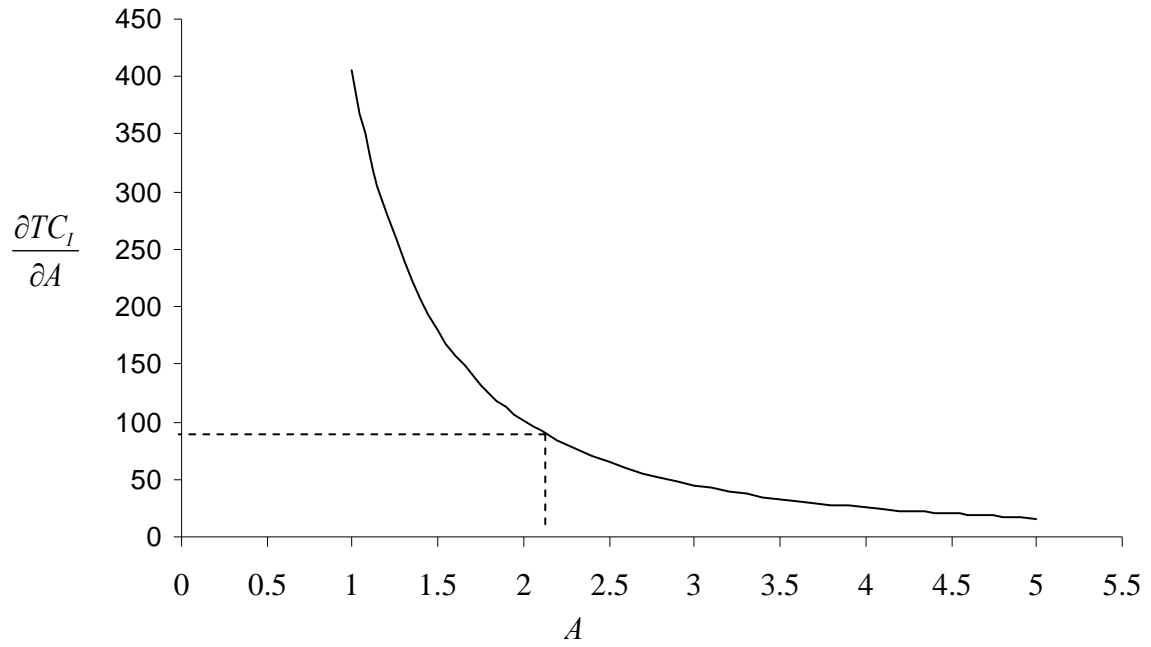


Figure 3.3: Effect of  $P/D (= A)$  ratio on  $TC_I$

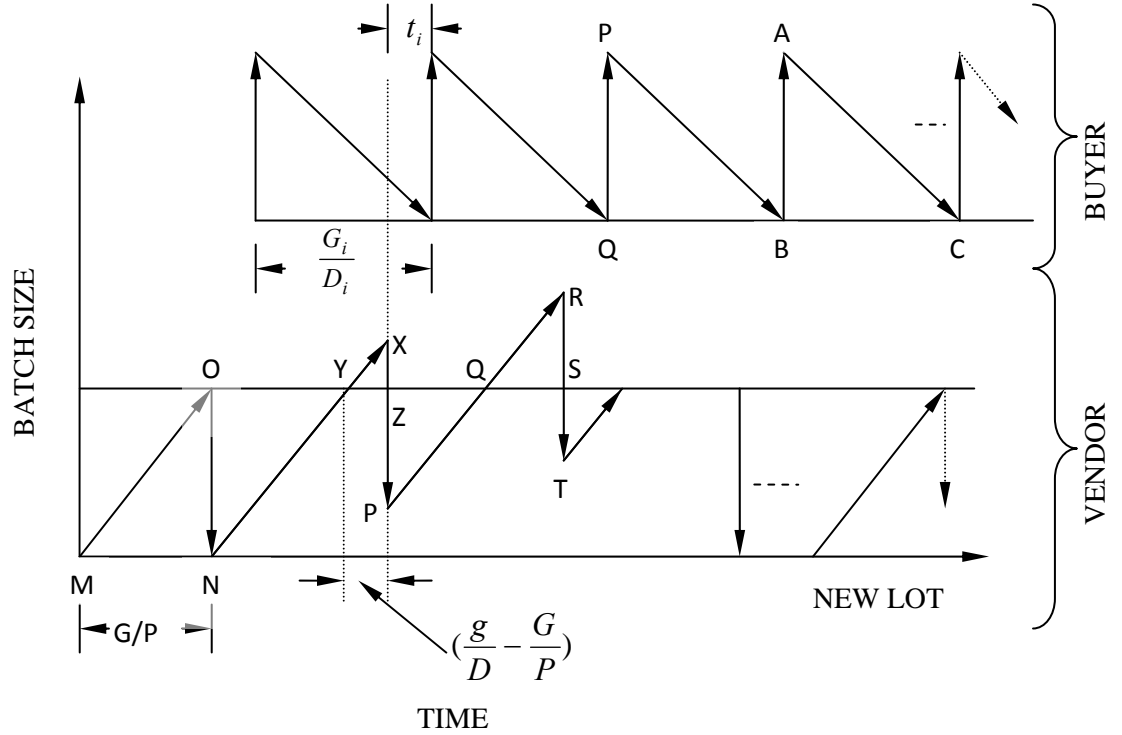


Figure 3.4: Inventory flow in Model II

In the beginning of the cycle, the vendor starts production at a finite rate and inventory builds up at a constant rate. In Figure 3.4, the slope MO is the rate of accumulating inventory during production. As soon as production of the first batch is complete, items are shipped to each buyer, and inventory of vendor is depleted. At this point, the vendor begins producing the next batch. Since we assumed  $P > D$  to avoid shortage, inventory begins building up for the vendor beyond  $G$  as  $G/P < G/D$ , as shown in Figure 3.4. When a buyer's inventory drops to transportation time demand, new items are shipped, and the vendor's inventory is reduced by  $G$ . This process is repeated until the entire lot is produced. Items are shipped to all buyers at the same time, but buyers will receive items after  $t_i$  time since shipping time for each buyer varies. Items are readily available to consume as soon as the buyer receives them, and the buyer consumes the items at a constant rate. In Figure 3.4, slope AC is the consumption rate. Since buyer consumes items in a

constant rate, on-hand inventory begins decreasing constantly. When the new items arrive, inventory drops to zero. Hence, the buyer does not have to hold new items for extra time, unlike the first model.

Notice that the cycle propels the time so that new items in the vendor's warehouse multiply. Since the production is continuous until the entire lot is produced, the inventory level also rises with time. Figure 3.4 proves this notion with the area of triangle (QRS), which is double that of triangle (XYZ).

### 3.8.1 Average Inventory Calculation

This section derives the average inventory in Model II for the entire supply chain system containing a single vendor and multiple buyers. Again, the demand is deterministic and prohibits backlog and shortage; the production rate is presumably greater than the demand rate to avoid shortage. The total system inventory consists of buyer's inventory, and vendor's inventory,  $I_m$ .

#### (a) Buyer's Average Inventory Calculation

Each buyer receives  $G_i$  amount of item in each shipment and takes  $G_i/D_i$  amount of time to consume. So each buyer's average inventory for the first batch is  $(G_i^2/2D_i)$ . If there are  $n$  batches in a lot, average inventory,  $I_{b_i}$ , for  $i^{th}$  buyer per cycle can be expressed as,

$$I_{b_i} = \frac{n G_i^2}{2 D_i}. \quad (3.22)$$

We determine that  $G_i = g D_i / D + t_i d_i$ , and upon substituting in Equation (3.22) and simplifying, we obtain

$$I_{b_i} = \frac{n}{2} \left( g^2 \frac{D_i}{D^2} + 2g \frac{t_i d_i}{D} + \frac{t_i^2 d_i^2}{D_i} \right). \quad (3.23)$$

Since there are  $D/ng$  number of cycles per year, then the average inventory,  $I_{i_{year}}$  per year can be expressed as

$$I_{i_{year}} = \frac{D}{nG} \frac{n}{2} \left( g^2 \frac{D_i}{D^2} + 2g \frac{t_i d_i}{D} + \frac{t_i^2 d_i^2}{D_i} \right). \quad (3.24)$$

We determine  $g = (G - t_i d_i)$ , so substituting  $g$  in Equation (3.24), we get,

$$I_{i_{year}} = \frac{D}{2G} \left[ (G - t_i d_i)^2 \frac{D_i}{D^2} + 2(G - t_i d_i) \frac{t_i d_i}{D} + \frac{t_i^2 d_i^2}{D_i} \right]. \quad (3.25)$$

Upon simplification this results in,

$$I_{i_{year}} = \frac{D_i}{2D} G + t_i d_i \left\{ 1 - \frac{D_i}{D} \right\} + t_i^2 d_i^2 \left\{ \frac{D_i}{2DG} - \frac{1}{G} + \frac{D}{2GD_i} \right\}, \quad (3.26)$$

Therefore, average inventory,  $I_b$  for all ( $m$  buyers) buyers per year can be expressed as

$$I_b = \frac{G}{2D} \sum_{i=1}^m D_i + \sum_{i=1}^m t_i d_i \left\{ 1 - \frac{D_i}{D} \right\} + \frac{1}{2G} \sum_{i=1}^m t_i^2 d_i^2 \left\{ \frac{D_i}{D} - 2 + \frac{D}{D_i} \right\}. \quad (3.27)$$

### (b) Vendor's Average Inventory Calculation

The vendor produces items in batches at a finite rate and holds them until the production of the first batch is finished, and then, he ships in every  $g/D$  period. Remember, to save on ordering cost, buyers only place order in the beginning of the lot and every batch is automatically shipped after a  $g/D$  period. If each batch size is  $G$  and requires  $G/P$  periods to produce, the average inventory during the first batch is  $G^2/2P$ . Since the second batch is shipped after it was produced, the vendor holds this batch for  $(g/D - G/P)$  period of time. Therefore, the average inventory during the second batch is  $[G^2/2P + G(g/D - G/P)]$ . The third shipment also remains for a while before it is shipped and actually remains relatively

longer (twice) than the second batch. In Figure 3.4, we can see that QS is double YZ, which represents the times that previous batches are remaining in the warehouse. This continues to occur until the entire lot is exhausted. If there are  $n$  batches in a lot, the  $n^{th}$  batch will have to remain in warehouse for  $(n-1)(g/D - G/P)$  amount of time. Therefore, the total average inventory for the vendor, per cycle, can be expressed as

$$I_{v_{cycle}} = \frac{1}{2} \frac{nG^2}{P} + \left( \frac{g}{D} - \frac{G}{P} \right) G \{1 + 2 + 3 + \dots + (n-1)\}. \quad (3.28)$$

If each batch size is  $G$  and there are  $n$  batches in a lot, then in a complete cycle,  $nG$  amount of items are produced. So, the total number of cycles in a year is  $D/ng$ . Therefore, average inventory for the vendor per year can be expressed as

$$I_v = \left[ \frac{1}{2} \frac{nG^2}{P} + \left( \frac{g}{D} - \frac{G}{P} \right) G \{1 + 2 + 3 + \dots + (n-1)\} \right] \frac{D}{Q}, \quad (3.29)$$

where the lot size  $Q = nG$ . We define  $G = g + \sum_{i=1}^m t_i d_i$ , by substitution expression  $g$  in terms of  $G$ , so

$$I_v = \frac{D}{nG} \left[ \frac{n}{2} \frac{G^2}{P} + \left( \frac{G - \sum_{i=1}^m t_i d_i}{D} - \frac{G}{P} \right) G \{1 + 2 + 3 + \dots + (n-1)\} \right], \quad (3.30)$$

this, after simplification, yields

$$I_v = \frac{DG}{2P} + \frac{(n-1)G}{2} - \frac{(n-1)}{2} \sum_{i=1}^m t_i d_i - \frac{(n-1)DG}{2P}. \quad (3.31)$$

### 3.8.2 Total Average Inventory Calculation

In an integrated supply chain system the total average inventory is calculated by adding average yearly inventories for all buyers using equation (3.27) and vendor equation (3.31). Therefore, the yearly average inventory of the system can be written as follows:

$$\begin{aligned}
I = & \frac{G}{2D} \sum_{i=1}^m D_i + \sum_{i=1}^m t_i d_i \left\{ 1 - \frac{D_i}{D} \right\} + \frac{1}{2G} \sum_{i=1}^m t_i^2 d_i^2 \left\{ \frac{D_i}{D} - 2 + \frac{D}{D_i} \right\} + \frac{DG}{2P} \\
& + \frac{(n-1)}{2} \left\{ G - \sum_{i=1}^m t_i d_i - \frac{DG}{P} \right\}.
\end{aligned} \tag{3.32}$$

### 3.9 Total Cost for Model II

In the current and previous sections, inventories for the vendor and all of the buyers are calculated under assumptions of the second model. Usually, the total cost of the system consists of the major costs, such as (a) ordering cost, (b) setup cost, (c) inventory holding cost, and (d) transportation cost. The total system cost consisting of these four elements can be calculated.

#### 3.9.1 Ordering Cost

Each time a buyer places an order with the vendor, he incurs a cost, which may be related to paperwork, telephone calls, etc. We assume each buyer places an order before the cycle starts. Hence, each buyer will place  $D/nG$  number of orders in a year. Therefore,  $m$  buyers in a year will place  $mD/nG$  number of orders. The cost of placing one order for the  $i^{th}$  buyer is  $a_i$ . Therefore, total ordering cost  $A$  for all buyers can be expressed as

$$A = \frac{D}{nG} \sum_{i=1}^m a_i, \tag{3.33}$$

where  $D$  is the demand rate (units/year),  $n$  is the number of batches in a lot and  $G$  is batch size.

#### 3.9.2 Setup Cost

Each time the vendor starts a new lot, production requires a setup such as changing die, setting raw materials, etc. If the manufacturing process requires setup for every new lot, then the total number of setup required is  $D/nG$ . Thus, the total setup cost ( $S_m$ ) can be calculated as

$$S_m = \frac{SD}{nG}, \quad (3.34)$$

where  $D$  is the demand rate (units/year),  $n$  is the number of batches in a cycle,  $G$  is batch size, and  $S$  is setup cost per lot.

### 3.9.3 Inventory Carrying Cost

While the batch is being produced, inventory builds up. Thus, the vendor incurs the inventory carrying cost until production is complete and items are shipped. Similarly, each buyer receives items and holds them until all of the items are consumed. Therefore, each buyer also incurs the item carrying cost, and the total system inventory carrying cost can be calculated.

#### (a) Inventory Carrying Cost for Buyers

From equation (3.27) we know the average yearly inventory for all buyers. If  $h_i$  is the carrying cost for  $i^{th}$  buyer, inventory holding cost for all buyers per year can be calculated as

$$H_b = \frac{G}{2D} \sum_{i=1}^m D_i h_i + \sum_{i=1}^m t_i d_i h_i \left\{ 1 - \frac{D_i}{D} \right\} + \frac{1}{2G} \sum_{i=1}^m t_i^2 d_i^2 h_i \left\{ \frac{D_i}{D} - 2 + \frac{D}{D_i} \right\}, \quad (3.35)$$

where  $D_i$  is the yearly demand of  $i^{th}$  buyer,  $G$  is batch size,  $D$  is demand rate (units/year),  $n$  is number of batches in a lot, and  $P$  is production capacity (units/year).

#### (b) Inventory Carrying Cost for Vendor

From equation (3.31) we know the average yearly inventory for the vendor. If  $h$  is the carrying cost for vendor, inventory holding cost ( $h_m$ ) for the vendor per year can be calculated as

$$I_v = \frac{DGh}{2P} + \frac{(n-1)Gh}{2} - \frac{(n-1)h}{2} \sum_{i=1}^m t_i d_i - \frac{(n-1)DGh}{2P}. \quad (3.36)$$



### (c) Total System Carrying Cost Calculation

Total system's carrying cost consists of the buyer's carrying cost per year and vendor's carrying cost. Equations (3.35) and (3.36) represent all buyers' carrying cost and vendor's carrying cost, respectively. Hence, the total system's carrying cost  $H$  can be expressed as

$$H = \frac{G}{2D} \sum_{i=1}^m D_i h_i + \sum_{i=1}^m t_i d_i h_i \left\{ 1 - \frac{D_i}{D} \right\} + \frac{1}{2G} \sum_{i=1}^m t_i^2 d_i^2 h_i \left\{ \frac{D_i}{D} - 2 + \frac{D}{D_i} \right\} + \frac{D G h}{2P} + \frac{(n-1)h}{2} \left\{ G - \sum_{i=1}^m t_i d_i - \frac{D G}{P} \right\}. \quad (3.37)$$

### 3.9.4 Transportation Cost

Every time the vendor sends a shipment to a buyer, the buyer incurs transportation cost. In reality, the capacity of a transportation vehicle is limited. Again, even if a vehicle is partially filled, the buyer has to pay the entire price of full load. Another factor is transportation cost for one shipment can be different among buyers depending on their distances from the vendor. If  $q_t$  is the carrying capacity of a vehicle and  $g_i$  is the shipment size, the receiving buyer has to pay for  $G_i/q_t$  number of vehicles. If the cost/vehicle is  $C_{t_i}$  for  $i^{th}$  buyer, and there are  $n$  number of batches in a cycle, then, transportation cost per cycle ( $T_{cycle_i}$ ) to the  $i^{th}$  buyer can be calculate as,

$$T_{cycle_i} = n \left\lceil \frac{G_i}{q_t} \right\rceil C_{t_i}. \quad (3.38)$$

Since there are  $D/ng$  number of cycles in a year, transportation cost to  $i^{th}$  buyer per year can be calculated as,

$$T_{year_i} = \frac{D}{nG} n \left\lceil \frac{G_i}{q_t} \right\rceil C_{t_i} = \frac{D}{G} \left\lceil \frac{G D_i}{D q_t} \right\rceil C_{t_i}. \quad (3.39)$$

Equating  $G_i = \frac{GD_i}{D}$ , where  $G$  is batch size (items/batch) and  $D$  is yearly demand (items/year). Since there are  $m$  buyers in the system, total transportation cost  $T$  for all buyers in a year can be expressed as

$$T = \frac{D}{G} \sum_{i=1}^m \left[ \frac{GD_i}{Dq_t} \right] C_{t_i} . \quad (3.40)$$

### 3.9.5 Total System Cost

The total system cost  $TC_{II}$ , consists of ordering cost, setup cost, inventory carrying cost, and transportation cost. Hence, the total system cost for Model I can be calculated by adding equations (3.33), (3.34), (3.37), and (3.40) as

$$\begin{aligned} TC_{II} = & \frac{D}{nG} \sum_{i=1}^m a_i + \frac{SD}{nG} + \frac{D}{G} \sum_{i=1}^m \left[ \frac{GD_i}{Dq_t} \right] C_{t_i} + \frac{G}{2D} \sum_{i=1}^m D_i h_i + \sum_{i=1}^m t_i d_i h_i \left\{ 1 - \frac{D_i}{D} \right\} \\ & + \frac{1}{2G} \sum_{i=1}^m t_i^2 d_i^2 h_i \left\{ \frac{D_i}{D} - 2 + \frac{D}{D_i} \right\} + \frac{DGH}{2P} + \frac{(n-1)h}{2} \left\{ G - \sum_{i=1}^m t_i d_i - \frac{DG}{P} \right\}. \end{aligned} \quad (3.41)$$

### 3.10 Solution Methodology

At this point, it is necessary to understand the nature of the total system cost  $TC_{II}$  function for optimization purposes. If the total function  $TC_{II}$  is convex, then,  $\partial TC_{II} / \partial g = 0$ , which allows for optimality and optimal batch quantity  $g^*$  evaluation.

By looking at  $D/G \sum_{i=1}^m [GD_i/Dq_t] C_{t_i}$  in equation (3.41), it is clear that  $TC_{II}$  is not a convex function in  $g$  given  $n$  remains constant. An optimal batch quantity  $g^*$  can be calculated by a direct search method within a boundary of  $n = (1,10)$  and  $g = (m,D)$ . Similarly, an optimal number of batches  $n^*$  can also be calculated by a direct search method within a boundary of  $n = (1,10)$  and  $g = (m,D)$  using equation (3.41).

**Example 3.2** Order shipped after every  $g/D$  period

Assume,  $P = 2300$  units/year,  $D = \sum_{i=1}^m D_i = 1300$  units/year,  $S = \$150$ ,  $h = 2.1$ , and  $q_i = 40$ . Data is given in Table 4. For integer  $n$ , the minimum total cost is \$3727.11 (for  $n = 2$ , and  $g = 506$ ), presented in Table 4. Necessary unit conversion is performed prior in to solution. For detailed result see Appendix. In Figure 3.5, a sudden cut occurs in the plot because at that point the number of vehicle required changes for some buyers.

Table 4: Data for single vendor 9 buyers problem

| Buyer | $a_i$ | $D_i$ | $h_i$ | $t_i$ (days) | $C_{t_i}$ |
|-------|-------|-------|-------|--------------|-----------|
| 1     | 15    | 170   | 1.9   | 6.0          | 50        |
| 2     | 11    | 250   | 1.1   | 2.0          | 40        |
| 3     | 18    | 175   | 2.9   | 2.8          | 40        |
| 4     | 19    | 115   | 1.3   | 5.7          | 50        |
| 5     | 23    | 190   | 1.7   | 7.2          | 60        |
| 6     | 24    | 70    | 2.2   | 16.3         | 100       |
| 7     | 13.5  | 50    | 2.9   | 17.1         | 100       |
| 8     | 17.3  | 150   | 2.6   | 6.2          | 50        |
| 9     | 11.6  | 130   | 1.7   | 7.0          | 60        |
|       | 152.4 | 1300  |       |              |           |

Table 5: Summary of results

| $n$      | $g$        | $TC_{II}$      |
|----------|------------|----------------|
| 1        | 647        | 3841.33        |
| <b>2</b> | <b>506</b> | <b>3727.11</b> |
| 3        | 506        | 3806.98        |
| 4        | 506        | 3948.97        |
| 5        | 233        | 4075.77        |
| 6        | 233        | 4111.09        |

**3.11 Sensitivity Analysis for Model II**

Sensitivity analysis has been performed in this section to test the robustness of the proposed model, which is subject to change in given parameters, setup cost, vendor holding cost, and production to demand ratio.

### 3.11.1 Effect of $S$ on $TC_{II}$

The vendor setup cost plays a major role in determining total cost of the system, and it also affects the optimal batch size. So, it is important to understand the effect of change in  $S$  on  $TC_{II}$ . The effect of  $S$  on total system cost  $TC_{II}$  is expressed in equation (3.42),

$$\frac{\partial TC_{II}}{\partial S} = \frac{D}{nG}, \quad (3.42)$$

which is a positive constant term. It can be written that  $TC_{II}$  increases (decreases) with an increase (decrease) in setup cost  $S$ . If we implement values of  $D$ ,  $n$  and  $g$  from Table 10 and Table 11 (shown later) for a unit change in  $S$ , then  $TC_{II}$  will increase by \$1.11.

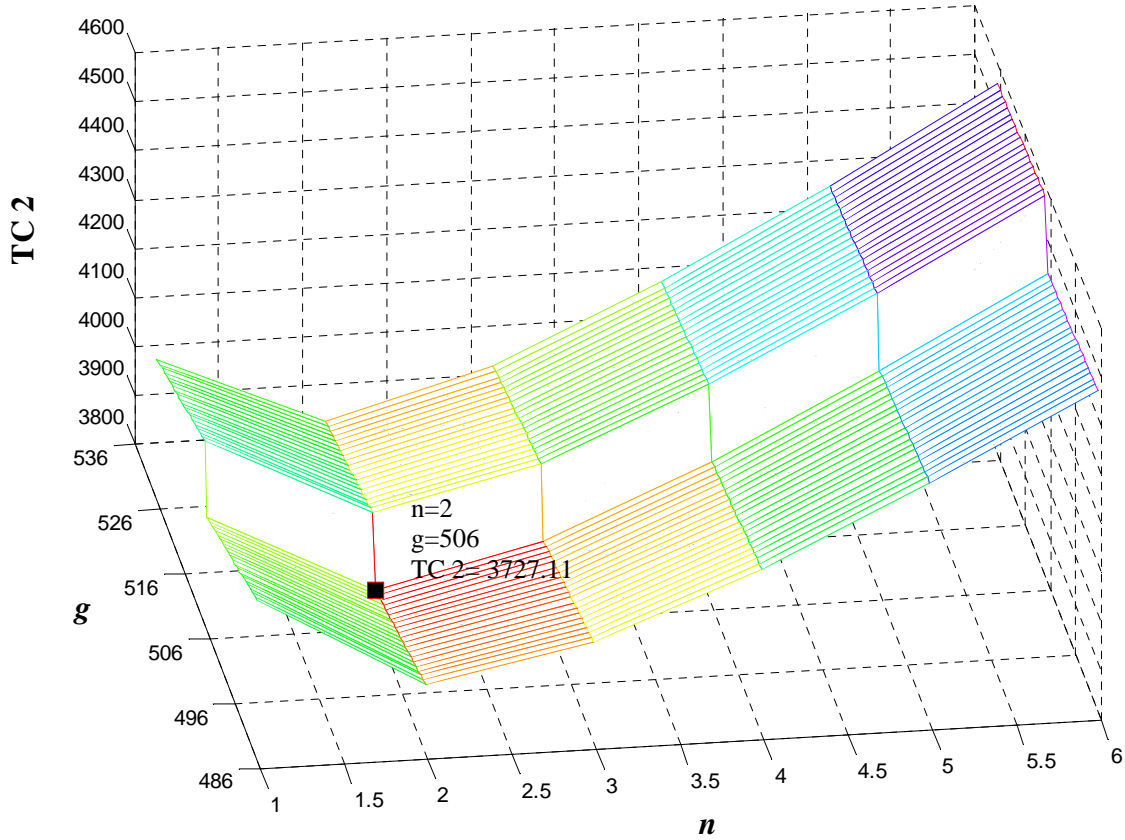


Figure 3.5: Graphical representation of  $TC_{II}$

### 3.11.2 Effect of $h$ on $TC_{II}$

The vendor's holding cost is an important parameter in determining the total cost. Also, to an extent, the vendor's holding cost governs which model should be chosen among the three presented.

It is important to understand how vendor holding cost  $h$  affects  $TC_{II}$ . The effect of  $h$  on total system cost  $TC_{II}$  is expressed in equation (3.43),

$$\frac{\partial TC_{II}}{\partial h} = \frac{DG}{2P} + \frac{(n-1)}{2} \left\{ G - \sum_{i=1}^m t_i d_i - \frac{DG}{P} \right\}, \quad (3.43)$$

which is also a positive, constant term. It can be written that  $TC_{II}$  increases (decreases) with an increase (decrease) in holding cost  $h$ . By inserting values of  $D$ ,  $n$ ,  $P$ , and  $G$  from Table 10 and Table 11 (shown later), there will be a unit change in  $h$  and  $TC_{II}$  will increase by \$257.

### 3.11.3 Effect of $P/D$ on $TC_{II}$

The ratio between production rate and total yearly demand not only determines the total cost, but also, it determines, in conjugation with vendor holding cost, which model to choose for a particular scenario among the three presented. Hence, it is crucial to understand the effect of  $P/D$  on total system cost  $TC_{II}$ . The effect of  $P/D$  on total system cost  $TC_{II}$ , is expressed in equation (3.44).

$$\frac{\partial TC_{II}}{\partial A} = -\frac{Gh}{2A^2} + \frac{(n-1)h}{2} \frac{G}{A^2}. \quad (3.44)$$

The effect of  $P/D$  over  $TC_{II}$  is represented by Figure 3.6 using equations (3.44) and  $P/D \in [1,5]$ . A study is performed with respect to  $P/D$  where the parameter values are the same except for the values of  $P/D$ , which shifts from 1 to 5; the results are presented in Table 6. Figure 3.6 reveals that  $P/D$  ratio affects  $TC_{II}$  and changes rapidly up to 2.0.

Beyond this point,  $TC_{II}$  becomes less sensitive to change in  $P/D$ . We can also write that  $TC_{II}$  increases as the ratio decreases. Table 6 summarizes the change in  $TC_{II}$  for the change in  $P/D$  ratio.

### 3.12 Model III: Items Shipped in Unequal Batches

As described above, the vendor holds items until the order is placed and then ships items to buyers. In the third model, we assume that batch sizes in a lot are unequal, that they increase by a factor  $k$  (where  $k = P/D$ ) and that there are  $n$  batches in a lot. Figure 3.7 shows the production and inventory for the third model. A lot is produced with batch sizes of  $\left(g + \sum_{i=1}^m t_i d_i\right)$ ,  $\left(kg + \sum_{i=1}^m t_i d_i\right)$ ,  $\left(k^2 g + \sum_{i=1}^m t_i d_i\right), \dots, \left(k^{n-1} g + \sum_{i=1}^m t_i d_i\right)$ . In Figure 3.7,  $\left(g + \sum_{i=1}^m t_i d_i\right)/P$ ,  $(g_i + t_i d_i)/D_i$ , and  $t_i$ , are the time segments, which represent the batch production time, including transportation time demand, consumption time for  $i^{th}$  buyer and transportation time for  $i^{th}$  buyer, respectively. In Figure 3.7, triangles MNO, OPQ and QRS represent inventories for the vendor for subsequent batches. Triangles ABC, CDE and EFH represent inventories for the  $i^{th}$  buyer when the item is received immediately at the end of previous inventory. In the beginning of the cycle, the vendor begins production at a finite rate and inventory builds up at a constant rate. In Figure 3.7 the slope MN is the rate of inventory build-up during production. As soon as production of the first batch  $\left(g + \sum_{i=1}^m t_i d_i\right)$  is complete, items are shipped to each buyer, and vendor inventory reduces to zero. At this point, the vendor begins producing the next batch of size  $\left(kg + \sum_{i=1}^m t_i d_i\right)$ . Remember, we assumed  $P > D$  to avoid shortage. Unlike the first two models, the third model's batch size increases with a multiplication factor of  $k$ . Hence, to avoid shortage,

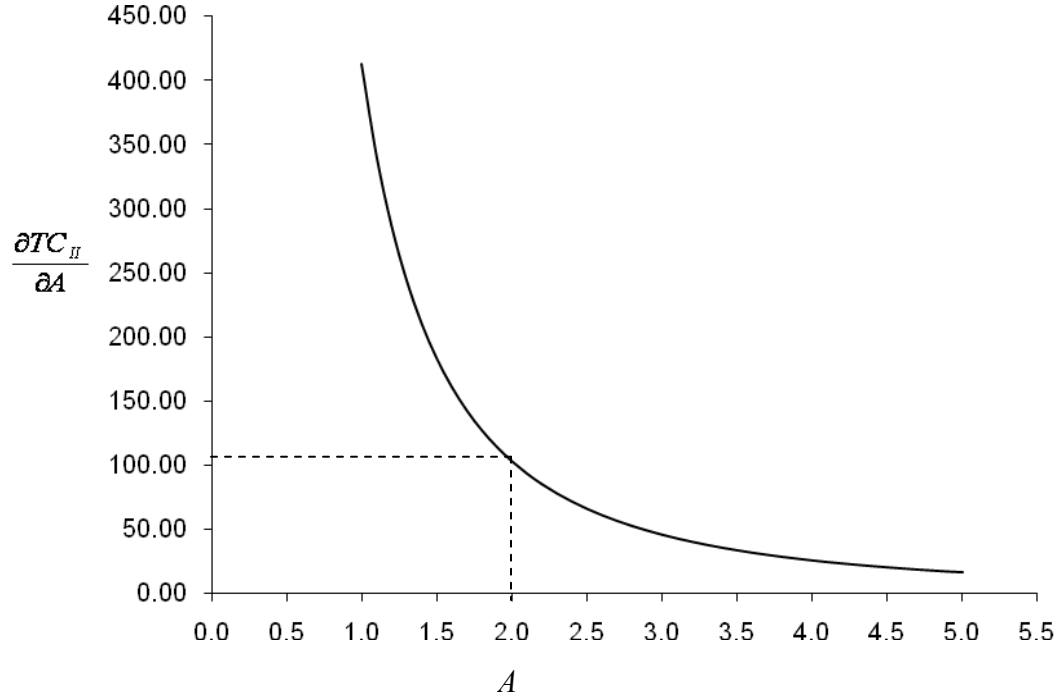


Figure 3.6: Effect of  $P/D (= A)$  ratio on  $TC_{II}$

Table 6: Effect of  $P/D (= A)$  ratio on  $TC_{II}$

| $A$ | $\frac{\partial TC_{II}}{\partial A}$ |
|-----|---------------------------------------|
| 1.0 | 412.33                                |
| 1.5 | 183.26                                |
| 2.0 | 103.08                                |
| 2.5 | 65.97                                 |
| 3.0 | 45.81                                 |
| 3.5 | 33.66                                 |
| 4.0 | 25.77                                 |
| 4.5 | 20.36                                 |
| 5.0 | 16.49                                 |

$\left(k^n g + \sum_{i=1}^m t_i d_i\right) / P = \left(k^{n-1} g + \sum_{i=1}^m t_i d_i\right) / D$ . When the buyer's inventory falls to the transportation time demand, new items are shipped, and the vendor's inventory is reduced to zero. This is repeated until the entire lot is produced. Items are shipped to all buyers at the same time, but buyers will receive items after  $t_i$  time since shipping times vary among

buyers. Items are readily available to consume as soon as the buyer receives them, and buyer consumes the items at a constant rate. In Figure 3.7, slope BC is the consumption rate. Since the buyer consumes items at a constant rate, on-hand inventory begins to decrease constantly. When the new items arrive, inventory drops to zero. Hence, the buyer is not required to hold new items for extra time, unlike the first model. Unlike the other two models, in this model, neither the vendor nor the buyers need to hold a new batch in its warehouse while the previous batch is being consumed.

### 3.12.1 Average Inventory Calculation

This section derives the average inventory in Model III for the entire supply chain system containing a single vendor and multiple buyers. Similar to the other two models, the demand is deterministic and no backlog or shortage is allowed, the production rate has to be greater than the demand rate. The total system inventory consists of buyer's inventory  $I_b$  and vendor's inventory  $I_m$ .

#### (a) Buyer's Average Inventory Calculation

Each buyer receives  $g_i + t_i d_i$  amount of items for the first shipment, followed by  $kg_i + t_i d_i$ ,  $k^2 g_i + t_i d_i$ , ...,  $k^{n-1} g_i + t_i d_i$  items. Each buyer takes  $(k^{n-1} g_i + t_i d_i)/D_i$  time to consume the batch fully so each buyer's average inventory for the first batch is  $(g_i + t_i d_i)^2 / 2D_i$ . For the second batch, the average inventory is  $(kg_i + t_i d_i)^2 / 2D_i$ . Similarly, for the  $n^{th}$  batch, the average inventory is  $(k^{n-1} g_i + t_i d_i)^2 / 2D_i$ . If there are  $n$  batches in a lot, the average inventory ( $I_{b_i}$ ) for  $i^{th}$  buyer per cycle can be expressed as

$$I_{b_i} = \frac{1}{2D_i} \left[ (g_i + t_i d_i)^2 + (kg_i + t_i d_i)^2 + (k^2 g_i + t_i d_i)^2 + \dots + (k^{n-1} g_i + t_i d_i)^2 \right]. \quad (3.45)$$



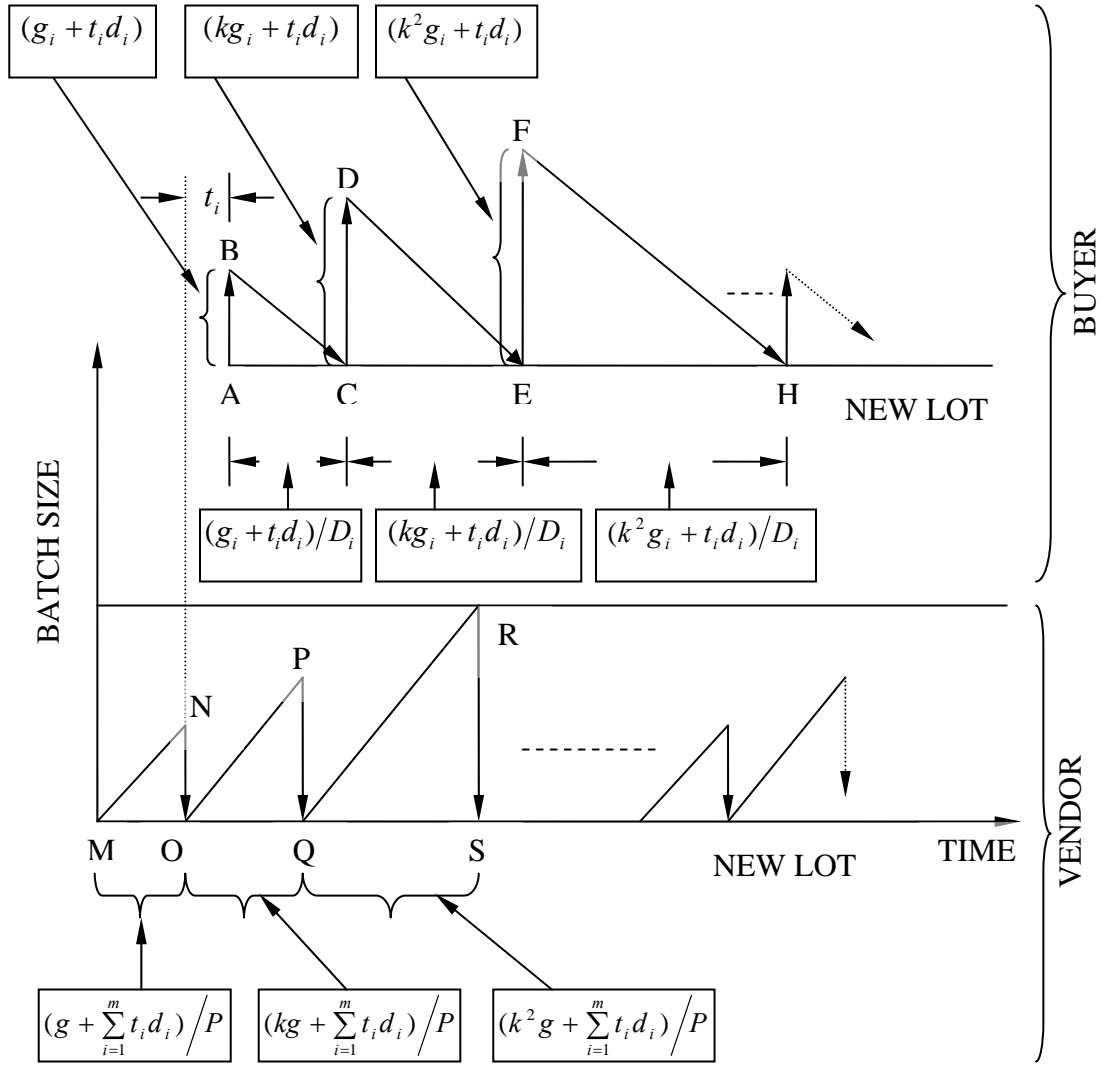


Figure 3.7: Inventory flow in Model III

Upon simplification this leads to

$$I_{b_i} = \frac{1}{2D_i} \left[ g_i^2 \frac{k^{2n} - 1}{k^2 - 1} + 2g_i t_i d_i \frac{k^n - 1}{k - 1} + n t_i^2 d_i^2 \right]. \quad (3.46)$$

We know  $g_i = g \frac{D_i}{D}$ , and substituting in equation (3.46), we get,

$$I_{b_i} = \frac{1}{2D_i} \left[ g^2 \frac{D_i^2}{D^2} \frac{k^{2n}-1}{k^2-1} + \frac{k^n-1}{k-1} 2g \frac{D_i}{D} t_i d_i + n t_i^2 d_i^2 \right]. \quad (3.47)$$

Assume the sum of all batches in a lot equals  $Q'$ , so the lot size is

$$Q' = \frac{k^n-1}{k-1} G. \quad (3.48)$$

Therefore, there will be  $D/Q'$  number of lots in a year. Hence, the average inventory  $I_{i_{year}}$

for  $i^{th}$  buyer per year can be expressed as

$$I_{i_{year}} = \frac{D}{Q'} \frac{1}{2D_i} \left[ g^2 \frac{D_i^2}{D^2} \frac{k^{2n}-1}{k^2-1} + \frac{k^n-1}{k-1} 2g \frac{D_i}{D} t_i d_i + n t_i^2 d_i^2 \right]. \quad (3.49)$$

If there are  $m$  buyers in the system, the average inventory for all buyers per year can be expressed as

$$I_b = \frac{D}{Q'} \frac{1}{2} \left[ \frac{g^2}{D^2} \frac{k^{2n}-1}{k^2-1} \sum_{i=1}^n D_i + \frac{k^n-1}{k-1} \frac{2g}{D} \sum_{i=1}^m t_i d_i + n \sum_{i=1}^m \frac{t_i^2 d_i^2}{D_i} \right]. \quad (3.50)$$

### (b) Vendor's Average Inventory Calculation

The vendor produces batch items at a finite rate with a batch of sizes  $\left( g + \sum_{i=1}^m t_i d_i \right)$ ,  $\left( kg + \sum_{i=1}^m t_i d_i \right)$ ,  $\left( k^2 g + \sum_{i=1}^m t_i d_i \right)$ , ...,  $\left( k^{n-1} g + \sum_{i=1}^m t_i d_i \right)$ . The vendor holds items until production is finished and ships as soon as the batch production is completed. Since batch sizes are unequal, we can calculate the work-in-process (WIP) inventory for the vendor. Hence, the WIP inventory per cycle can be expressed as:

$$WIP_{cycle} = \frac{\left( g + \sum_{i=1}^m t_i d_i \right)^2}{2P} + \frac{\left( kg + \sum_{i=1}^m t_i d_i \right)^2}{2P} + \frac{\left( k^2 g + \sum_{i=1}^m t_i d_i \right)^2}{2P} + \dots + \frac{\left( k^{(n-1)} g + \sum_{i=1}^m t_i d_i \right)^2}{2P}. \quad (3.51)$$

If the lot size is  $Q'$  and there are  $D/Q'$  number of cycles in a year, the average WIP inventory ( $WIP_{year}$ ) per year can be expressed as

$$WIP_{year} = \frac{D}{Q'} \frac{1}{2P} \left[ g^2 \frac{k^{2n} - 1}{k^2 - 1} + \frac{k^n - 1}{k - 1} 2g \sum_{i=1}^m t_i d_i + n \sum_{i=1}^m t_i^2 d_i^2 \right], \quad (3.52)$$

Where  $P/D = k$ ,  $D$  is yearly demand (item/year),  $g$  is the minimum batch size,  $n$  is number of batches in a lot,  $Q'$  is lot size (item/lot) and  $P$  is production rate (item/year).

### 3.12.2 Total Average Inventory Calculation

In an integrated supply chain system, the total average inventory is calculated by adding average yearly inventories of all buyers and vendors using equations (3.50) and (3.52). Therefore, yearly average inventory  $I$  of the system can be written as

$$I = \frac{D}{2Q'} \left[ \left( \frac{k^{2n} - 1}{k^2 - 1} \right) \left\{ \frac{g^2}{D^2} \sum_{i=1}^m D_i + 2g \frac{1}{D} \sum_{i=1}^m t_i d_i + \sum_{i=1}^m \frac{t_i^2 d_i^2}{D_i} \right\} + \frac{1}{P} \left\{ g^2 \frac{k^{2n} - 1}{k^2 - 1} + \frac{k^n - 1}{k - 1} 2g \sum_{i=1}^m t_i d_i + n \sum_{i=1}^m t_i^2 d_i^2 \right\} \right]. \quad (3.53)$$

### 3.13 Total Cost for Model III

In the current and previous sections, inventories for the vendor and all buyers are calculated as in the second model. Usually, the total cost of the system consists of major costs: (a) ordering cost, (b) setup cost, (c) inventory holding cost, and (d) transportation cost. The total system cost consisting of these costs can be calculated.

#### 3.13.1 Ordering Cost

Each time a buyer places an order to the vendor, the buyer incurs a cost, which may consist of paperwork, telephone calls, etc. Assume each buyer places an order before the cycle starts. Hence, each buyer will place  $D/Q'$  number of orders in a year. Therefore,  $m$  buyers in a year will place  $mD/Q'$  number of orders. The cost of placing one order for  $i^{th}$

buyer is  $a_i$ . Therefore, total ordering cost  $A$  for all buyers can be expressed as

$$A = \frac{D}{Q'} \sum_{i=1}^m a_i, \quad (3.54)$$

where  $D$  is the demand rate (units/year),  $n$  is the number of batches in a cycle, and  $Q'$  is lot size (items/lot).

### 3.13.2 Setup Cost

Each time the vendor begins production of a new lot, he incurs a setup cost for changing die, setting raw materials, etc. If the manufacturing process requires setup for every new lot, the total number of setup requires is  $D/Q'$ . Hence, the total setup cost  $S_m$  can be calculated as,

$$S_m = \frac{SD}{Q'}, \quad (3.55)$$

where  $D$  is the demand rate (units/year),  $n$  is the number of batches in a cycle,  $Q'$  is lot size (items/lot) and  $S$  setup cost per lot (\$/lot).

### 3.13.3 Inventory Carrying Cost

While the batch is being produced, inventory builds up. Thus, the vendor incurs an inventory carrying cost until production is completed, and he then ships the items. Similarly, each buyer receives items, holding them until all items are consumed. Therefore, each buyer also incurs a carrying cost. The total system inventory carrying cost can be calculated.

#### (a) Inventory Carrying Cost for Buyers

From equation (3.50), we know the average yearly inventory for all buyers. If  $h_i$  is the carrying cost for  $i^{th}$  buyer, then inventory holding cost  $h_b$  for all buyers per year can be calculated as

$$h_b = \frac{D}{Q} \frac{1}{2} \left[ \frac{g^2}{D^2} \frac{k^{2n} - 1}{k^2 - 1} \sum_{i=1}^n D_i h_i + \frac{k^n - 1}{k - 1} \frac{2g}{D} \sum_{i=1}^m t_i d_i h_i + n \sum_{i=1}^m \frac{t_i^2 d_i^2 h_i}{D_i} \right], \quad (3.56)$$

where  $D_i$  is the yearly demand of  $i^{th}$  buyer,  $g$  is batch size,  $D$  is demand rate (units/year),  $n$  is number of batches in a lot and  $P$  is production capacity (units/year).

### (b) Inventory Carrying Cost for Vendor

From equation (3.52), we know the average yearly inventory for the vendor. If  $h$  is the carrying cost for the vendor, then inventory holding cost  $h_v$  for the vendor per year can be calculated as follows:

$$h_v = \frac{D}{Q} \frac{h}{2P} \left[ g^2 \frac{k^{2n} - 1}{k^2 - 1} + \frac{k^n - 1}{k - 1} 2g \sum_{i=1}^m t_i d_i + n \sum_{i=1}^m t_i^2 d_i^2 \right]. \quad (3.57)$$

### (c) Total System Carrying Cost Calculation

Total system's carrying cost consists of the buyer's carrying cost per year and the vendor's carrying cost. Equations (3.56) and (3.57) represent the carrying costs for all of the buyers as well as for the vendor, respectively. Hence, the total system's carrying cost  $H$  can be expressed as

$$H = \frac{D}{2Q} \left[ g^2 \frac{k^{2n} - 1}{k^2 - 1} \left\{ \frac{h}{P} + \frac{1}{D^2} \sum_{i=1}^n D_i h_i \right\} + 2g \frac{k^n - 1}{k - 1} \left\{ \frac{h}{P} \sum_{i=1}^m t_i d_i + \frac{1}{D} \sum_{i=1}^m t_i d_i h_i \right\} + n \left\{ \frac{h}{P} \sum_{i=1}^m t_i^2 d_i^2 + \sum_{i=1}^m \frac{t_i^2 d_i^2 h_i}{D_i} \right\} \right]. \quad (3.58)$$

### 3.13.4 Transportation Cost

Every time the vendor sends a shipment to a buyer, the buyer incurs transportation cost. In reality, capacity of a transportation vehicle is limited. Again, even if a vehicle is partially filled, the buyer has to pay according to the full-load price. Another thing to consider is the transportation cost for one shipment because it may vary among different buyers depending on their distances from the vendor. If  $q_t$  is carrying capacity of a vehicle

and  $(k^{n-1}g_i + t_id_i)$  is the shipment size, then the receiving buyer has to pay for  $(k^{n-1}g_i + t_id_i)/q_t$  number of vehicles. If the cost of a vehicle is  $C_{t_i}$  for  $i^{th}$  buyer, and there are  $n$  number of batches in a cycle, then the transportation cost per cycle  $T_{i_{cycle}}$  paid by  $i^{th}$  buyer, can be calculate as

$$T_{i_{cycle}} = \sum_{j=1}^n \left[ \frac{k^{j-1}g_i + t_id_i}{q_t} \right] C_{t_i}, \quad (3.59)$$

since there are  $(D/Q)$  number of cycles in a year, transportation cost  $T_{i_{year}}$  to  $i^{th}$  buyer per year can be calculated as

$$T_{i_{year}} = \frac{D}{Q} \sum_{j=1}^n \left[ \frac{k^{j-1}g_i + t_id_i}{q_t} \right] C_{t_i}, \text{ and} \quad (3.60)$$

Since there are  $m$  buyers in the system, the total transportation cost  $T_{year}$  for all buyers in a year can be expressed as follows

$$T_{year} = \frac{D}{Q} \sum_{i=1}^m \sum_{j=1}^n \left[ \frac{k^{j-1}g \frac{D_i}{D} + t_id_i}{q_t} \right] C_{t_i}. \quad (3.61)$$

### 3.13.5 Total System Cost

The total system cost  $TC_{III}$  consists of ordering cost, setup cost, inventory carrying cost, and transportation cost. Hence, the total system cost for Model I can be calculated by adding equations (3.54), (3.55), (3.58), and (3.61) as

$$\begin{aligned} TC_{III} = & \frac{D}{2Q} \left[ g^2 \frac{k^{2n}-1}{k^2-1} \left\{ \frac{h}{P} + \frac{1}{D^2} \sum_{i=1}^n D_i h_i \right\} + 2g \frac{k^n-1}{k-1} \left\{ \frac{h}{P} \sum_{i=1}^m t_i d_i + \frac{1}{D} \sum_{i=1}^m t_i d_i h_i \right\} + n \left\{ \frac{h}{P} \sum_{i=1}^m t_i^2 d_i^2 + \sum_{i=1}^m \frac{t_i^2 d_i^2 h_i}{D_i} \right\} \right] \\ & + \frac{D}{Q} \sum_{i=1}^m a_i + \frac{SD}{Q} + \frac{D}{Q} \sum_{i=1}^m \sum_{j=1}^n \left[ \frac{k^{j-1}g \frac{D_i}{D} + t_id_i}{q_t} \right] C_{t_i}. \end{aligned} \quad (3.62)$$

### 3.14 Solution Methodology

At this point, we need to understand the nature of the total system cost  $TC_{III}$  function for optimization purposes. If the total function  $TC_{III}$  is convex, then  $\partial TC_{III} / \partial g = 0$  leads to optimality, and the optimal batch quantity  $g^*$  can be evaluated.

By looking at  $D/Q \cdot \sum_{i=1}^m \sum_{j=1}^n \left[ (k^{j-1} g D_i / D + t_i d_i) / q_t \right] C_{t_i}$  in equation (3.62), it is clear that  $TC_{III}$  is not a convex function in  $g$  given  $n$  constant. An optimal batch quantity  $g^*$  can be calculated by a direct search method within a boundary of  $n = (1, 10)$ , and  $g = (m, D)$ . Similarly an optimal number of batches  $n^*$  can also be calculated by a direct search method within a boundary of  $n = (1, 10)$ , and  $g = (m, D)$  using equation (3.62).

**Example 3.3** Unequal size batches are shipped.

Assume,  $P = 2200$  units/year,  $D = \sum_{i=1}^m D_i = 1400$  units/year,  $S = 160$ ,  $h = 1.1$ , and  $q_t = 25$ . Data is given in Table 7. For integer  $n$ , the minimum total cost is \$2552.67 (for  $n = 7$ , and  $g = 137$ ), and a summary of results are presented in Table 8. Necessary unit conversion is performed prior to solution. For detailed results see Appendix. In Figure 3.2, a sudden cut occurs in the plot because at that point the number of vehicle required changes for some buyers.

### 3.15 Sensitivity Analysis for Model III

Sensitivity analysis has been performed in this section to test the robustness of the proposed model, which is subject to change in the following areas: given parameters, setup cost, vendor holding cost and production to demand ratio.

Table 7: Data for single vendor 9 buyers problem

| Buyer | $a_i$ | $D_i$ | $h_i$ | $t_i$ (days) | $C_{t_i}$ |
|-------|-------|-------|-------|--------------|-----------|
| 1     | 12    | 160   | 2.1   | 6.0          | 50        |
| 2     | 14    | 310   | 1.0   | 2.0          | 40        |
| 3     | 16    | 170   | 2.8   | 4.0          | 40        |
| 4     | 18    | 120   | 1.4   | 5.0          | 50        |
| 5     | 17    | 187   | 1.7   | 7.4          | 60        |
| 6     | 23    | 72    | 1.9   | 13.8         | 100       |
| 7     | 15    | 60    | 2.7   | 17.8         | 100       |
| 8     | 19    | 175   | 2.3   | 5.9          | 50        |
| 9     | 11    | 146   | 1.9   | 8.3          | 60        |
|       | 145   | 1400  |       |              |           |

Table 8: Summary of results

| $n$      | $g$        | $TC_{III}$     |
|----------|------------|----------------|
| 1        | 633        | 5032.51        |
| 2        | 414        | 4776.54        |
| 3        | 137        | 4331.17        |
| 4        | 137        | 3584.15        |
| 5        | 137        | 3057.53        |
| 6        | 137        | 2714.58        |
| <b>7</b> | <b>137</b> | <b>2552.67</b> |
| 8        | 127        | 2582.08        |
| 9        | 94         | 3495.29        |
| 10       | 63         | 4140.88        |

### 3.15.1 Effect of $S$ on $TC_{III}$

The vendor setup cost plays a major role in determining total cost of the system, and it also affects the optimal batch sizes. So, it is important to understand the effect of change in  $S$  on  $TC_{III}$ . The effect of  $S$  on total system cost  $TC_{III}$  is expressed in equation (3.63),

$$\frac{\partial TC_{III}}{\partial S} = \frac{D}{Q}, \quad (3.63)$$

which is a positive, constant term. It can be written that  $TC_{III}$  increases (decreases) with increase (decrease) in setup cost  $S$ . If we apply the values of  $D$ ,  $n$  and  $g$  from Table 10 and Table 11 (shown later), then, for unit change in  $S$ ,  $TC_{III}$  will increase by \$0.49.



### 3.15.2 Effect of $h$ on $TC_{III}$

The vendor's holding cost is an important parameter in determining total cost. Vendor's holding cost also governs to an extent which model should be chosen among three presented here. So, an understanding of how a change in vendor holding cost  $h$  affects  $TC_{III}$

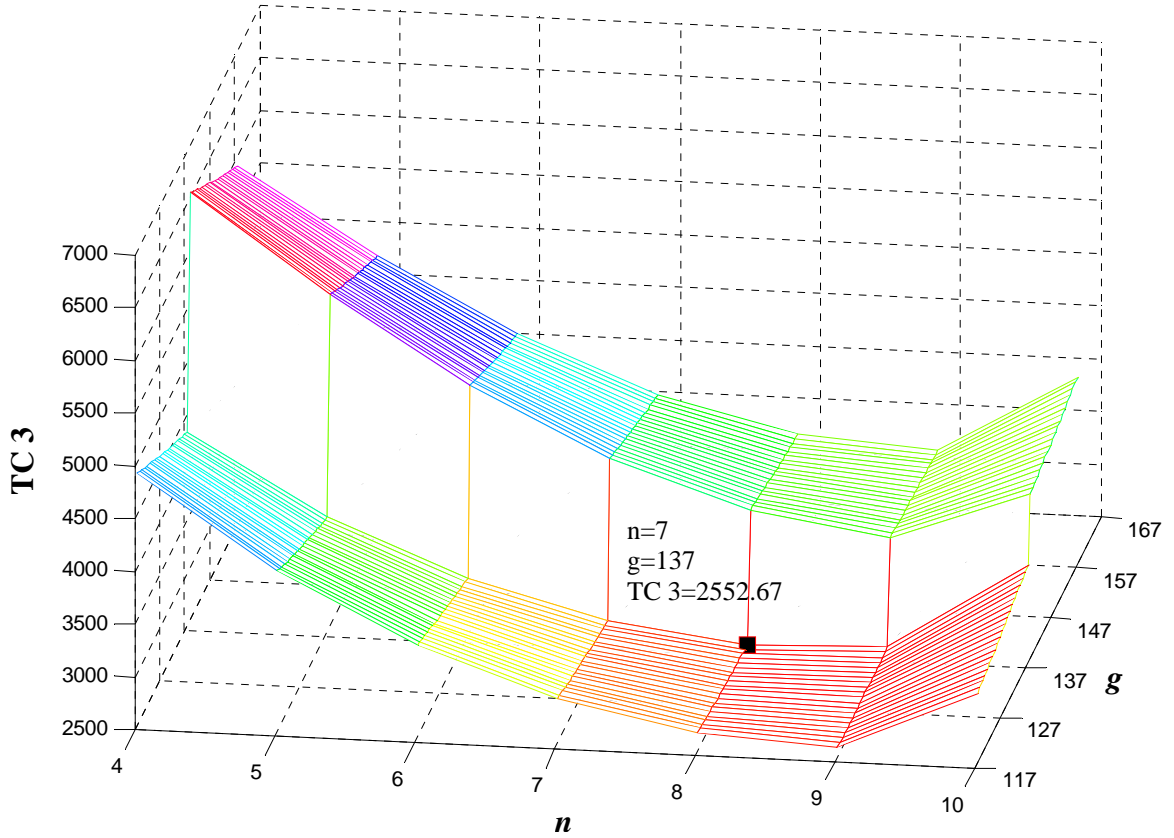


Figure 3.8: Graphical representation of  $TC_{III}$

is required. The effect of  $h$  on total system cost  $TC_{III}$  is expressed in equation (3.64),

$$\frac{\partial TC_{III}}{\partial h} = \frac{D}{2Q} g^2 \frac{k^{2n} - 1}{k^2 - 1} \frac{1}{P} + 2g \frac{k^n - 1}{k - 1} \frac{1}{P} \sum_{i=1}^m t_i d_i + \frac{n}{P} \sum_{i=1}^m t_i^2 d_i^2, \quad (3.64)$$

which is a positive, constant term. It can be written that  $TC_{III}$  increases (decreases) with increase (decrease) in setup cost  $h$ . If we use values of  $D$ ,  $n$  and  $g$  from Table 11 (shown

later), then for unit change in (increase or decrease)  $h$ ,  $TC_{III}$  will change (increase or decrease) by \$516.90.

### 3.15.3 Effect of $P/D$ on $TC_{III}$

The ratio between the production rate and total yearly demand  $P/D$  is an important parameter used in determining the total cost. The performance of the model also largely depends on  $P/D$  ratio. So, it is crucial to understand the effect of change in  $P/D$  ratio on total system cost  $TC_{III}$ . The effect of change in  $P/D$  ratio on total system cost,  $TC_{III}$  is expressed in equation (3.65),

$$\frac{\partial TC_{III}}{\partial A} = -\frac{1}{2Q} g^2 \frac{k^{2n}-1}{k^2-1} \frac{h}{A^2} - 2g \frac{1}{D} \frac{k^n-1}{k-1} \frac{h}{A^2} \sum_{i=1}^m t_i d_i - n \frac{1}{D} \frac{h}{A^2} \sum_{i=1}^m t_i^2 d_i^2. \quad (3.65)$$

The effect of  $P/D$  over  $TC_{III}$  is represented by Figure 3.9 using equations (3.65) and  $P/D \in [1,5]$ . A study is performed with respect to  $P/D$  where the parameter values are same except the values of  $P/D$ , which shifts from 1 to 5, and the results are presented in Table 9. It can be observed from the Figure 3.9 that  $TC_{III}$  changes rapidly with a change in  $P/D$  ratio up to 1.8. Beyond this point,  $TC_{III}$  becomes less sensitive to change in  $P/D$ . We can also write that  $TC_{III}$  reduces as the ratio increases; in other words, the bigger the ratio, and the lower the sensitivity. Table 9 summarizes the change in  $TC_{III}$  for the change in  $P/D$  ratio.

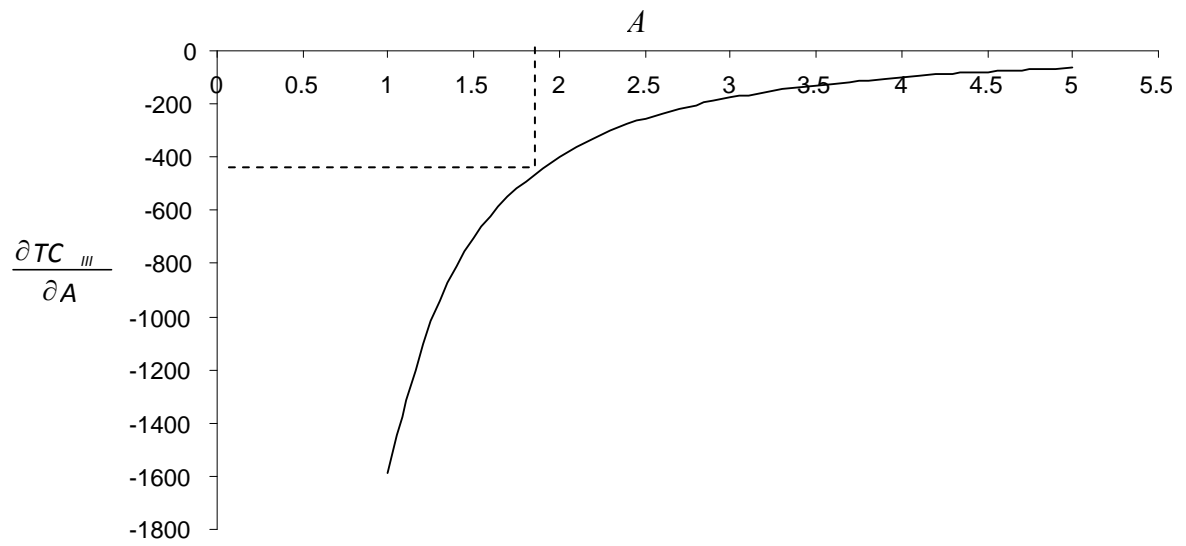


Figure 3.9: Effect of  $P/D (= A)$  ratio on  $TC_{III}$

Table 9: Effect of  $P/D (= A)$  ratio on  $TC_{III}$

| $A$ | $\frac{\partial TC_{III}}{\partial A}$ |
|-----|--|
| 1.0 | -1590.45                               |
| 1.5 | -706.87                                |
| 2.0 | -397.61                                |
| 2.5 | -254.47                                |
| 3.0 | -176.72                                |
| 3.5 | -129.83                                |
| 4.0 | -99.40                                 |
| 4.5 | -78.54                                 |
| 5.0 | -63.62                                 |

## CHAPTER 4

### RESULTS

This section summarizes the results of all the models presented in this research and compares their efficiency under similar circumstances. This section also presents a comparison of performance of three models with their alternatives under same scenario.

#### 4.1 Performance Comparison

In order to understand the merit of each model, we performed a comparison among these three models by using same set of parameters. The data used are presented in the Table 10.

##### Example 4.1: Performance Comparison of Three Models

Assume,  $P = 2000$  units/year,  $D = \sum_{i=1}^m D_i = 1300$  units/year,  $S = 200$ ,  $h = 2$ ,

and  $q_i = 25$ .

Table 10: Data for single vendor 9 buyers problem

| Buyer | $a_i$ | $D_i$ | $h_i$ | $t_i$ (days) | $C_{t_i}$ |
|-------|-------|-------|-------|--------------|-----------|
| 1     | 10    | 170   | 2     | 5            | 50        |
| 2     | 13    | 250   | 1     | 2            | 40        |
| 3     | 15    | 175   | 3     | 3            | 40        |
| 4     | 20    | 115   | 1.2   | 6            | 50        |
| 5     | 21    | 190   | 1.5   | 7            | 60        |
| 6     | 25    | 70    | 2.1   | 15           | 100       |
| 7     | 14    | 50    | 2.8   | 18           | 100       |
| 8     | 17    | 150   | 2.5   | 6            | 50        |
| 9     | 13    | 130   | 1.8   | 8            | 60        |
|       | 148   | 1300  |       |              |           |

Table 11 summarizes the performance of three models under same scenario.

Table 11: Summary of results

| Model | $N$ | $g$ | Total Cost |
|-------|-----|-----|------------|
| I     | 3   | 464 | \$4628.76  |
| II    | 3   | 391 | \$4829.82  |
| III   | 6   | 112 | \$2789.46  |

It is clear from Table 8 that Model III provides lower total cost than Model I and Model II under same circumstances.

#### **4.2 Comparison with Alternative Approach**

In our original models, items are produced in batches noting that there are  $n$  batches in a lot and that there can be multiple lots in a year. In an alternative approach, we verified that when the items are transferred in lots instead of batches, we performed a comparative study between two approaches to understand which one is more efficient. We used the same data set (given in Table 10) as input and presented the summary of results in Table 12.

Table 12: Performance comparison with alternative models

|                | Model I   | Model II  | Model III |
|----------------|-----------|-----------|-----------|
| Original Model | \$4628.76 | \$4829.82 | \$2789.46 |
| Alternative    | \$4887.00 | \$5070.92 | \$2455.89 |

## **CHAPTER 5**

### **RESEARCH SUMMARY AND CONCLUSIONS**

This section summarizes the research and concludes the findings of the research. This section also discusses the significance of this work and applicability of the research. Finally, a few suggestions are made to provide direction for the future researcher.

#### **5.1 Conclusions**

This research describes three models to solve an integrated inventory problem where a single vendor supplies a single product to multiple buyers. In the first and second model, the vendor transfers items in equally-sized batches; in the third, however, the vendor transfers items in unequally-sized batches. Each transfer of batches incurs transportation cost. Optimum solution techniques for all three models are presented while sensitivity analysis has also been performed.

This study does not restrict the vendor to have either greater or less holding cost than everyone else in the system. The study is also flexible in accommodating unequal transportation time and cost for each buyer in the system.

A numerical study performed on the three models presented here reveals that under similar circumstances, the third model is always more efficient in solving the integrated inventory system.

Through a set of numerical problems, we found that splitting lots does not always produce better results. In some cases, alternate models, which do not split lots, produce better results than the models that participate in splitting lots. However, an extensive review is required to conclude whether or not models with no lot-splitting consistently produce better results.

## 5.2 Research Significance

In the recent past, multiple researchers have investigated the integrated inventory supply chain system and developed models for it. These studies are based on many ideal variables such as perfect transportation system and equal carrying costs for vendor and buyers in the system regardless of geographic location, which are unrealistic. This research, however, refines the work of Hoque (2008) into a more realistic model. This research presents an approach to obtain just-in-time delivery of products with an objective of reducing the overall system cost. Models presented in this research allow different transportation cost and time; also, they facilitate different ordering and holding costs for vendors and buyers. This research also allows vendors to ship batches of equal or unequal sizes and does not restrict shipping before the entire lot is produced. The results of this research is to find an optimal batch size; input only includes basic information such as holding cost, ordering cost, transportation capacity, transportation cost and yearly demand. However, the storage capacity constraint for both the vendor and the buyers was not been considered.

## 5.3 Possible Future Extensions

The inventory models presented here are subject to certain restrictions, which can be considered in future research. By relaxing some of those restrictions, the problem will become more complex but more realistic. However, in order to enhance the applicability and model performance, the following extensions are suggested:

(a) Nondeterministic demand: In this study, the yearly demand for the system is considered deterministic, but in reality, this may not always the case. However, if a non-deterministic demand, rather than a deterministic demand, is considered for an integrated supply chain system, the models presented here will be closer to reality.

(b) Storage capacity constraint: In this research, storage capacity for vendor and buyers are considered to be unconstrained. Again, realistically, that might not be the case especially because some buyers may be located where the demand is high but where a big storage capacity is not feasible. Thus, storage capacity constraints should be examined in future research.



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## APPENDIX

This section presents computational results of three models.

### 1. Computational results for Model I

Table 13: Computational results in detail

| <i>n</i> | <i>g</i> | <i>TC</i> | <i>n</i> | <i>g</i>   | <i>TC</i>      | <i>n</i> | <i>g</i> | <i>TC</i> |
|----------|----------|-----------|----------|------------|----------------|----------|----------|-----------|
| 1        | 101      | 9818.18   | 2        | 101        | 8396.16        | 3        | 101      | 7931.24   |
|          | 201      | 5390.48   |          | 201        | 4715.49        |          | 201      | 4521.43   |
|          | 301      | 4661.31   |          | 301        | 4258.76        |          | 301      | 4177.37   |
|          | 401      | 4209.07   |          | 401        | 3959.43        |          | 401      | 3950.86   |
|          | 501      | 4341.99   |          | <b>468</b> | <b>3870.08</b> |          | 468      | 3900.73   |
|          | 601      | 3977.62   |          | 501        | 4197.31        |          | 501      | 4245.57   |
|          | 608      | 3951.11   |          | 601        | 913.87         |          | 601      | 4010.97   |
|          | 701      | 4113.38   |          | 701        | 4116.83        |          | 701      | 4258.18   |
|          | 801      | 4015.76   |          | 801        | 4077.82        |          | 801      | 4260.55   |
|          | 901      | 4164.80   |          | 901        | 4279.73        |          | 901      | 4501.94   |
|          | 1001     | 4386.68   |          | 1001       | 4550.46        |          | 1001     | 4810.81   |
|          | 1101     | 4423.12   |          | 1101       | 4632.84        |          | 1101     | 4930.36   |
|          | 1201     | 4362.09   |          | 1201       | 4615.55        |          | 1201     | 4949.50   |
| <i>n</i> | <i>g</i> | <i>TC</i> | <i>n</i> | <i>g</i>   | <i>TC</i>      | <i>n</i> | <i>g</i> | <i>TC</i> |
| 4        | 101      | 7705.59   | 5        | 101        | 7575.65        | 6        | 101      | 7493.56   |
|          | 201      | 4447.60   |          | 201        | 4421.87        |          | 201      | 4420.18   |
|          | 301      | 4176.26   |          | 301        | 4207.27        |          | 301      | 4254.34   |
|          | 401      | 4002.56   |          | 401        | 4078.36        |          | 401      | 4166.22   |
|          | 405      | 3981.31   |          | 405        | 4058.78        |          | 405      | 4148.18   |
|          | 501      | 4342.08   |          | 501        | 4457.87        |          | 501      | 4583.32   |
|          | 601      | 4148.28   |          | 601        | 4301.67        |          | 601      | 4463.11   |
|          | 701      | 4434.01   |          | 701        | 4623.62        |          | 701      | 4820.13   |
|          | 801      | 4473.46   |          | 801        | 4698.44        |          | 801      | 4929.45   |
|          | 901      | 4750.98   |          | 901        | 5010.74        |          | 901      | 5275.87   |
|          | 1001     | 5095.30   |          | 1001       | 5389.45        |          | 1001     | 5688.43   |
|          | 1101     | 5249.82   |          | 1101       | 5578.07        |          | 1101     | 5910.70   |
|          | 1201     | 5303.57   |          | 1201       | 5665.69        |          | 1201     | 6031.84   |

## 2. Computational results for Model II

Table 14: Computational results in detail

| <i>n</i> | <i>g</i> | <i>TC</i> | <i>n</i> | <i>g</i>   | <i>TC</i>      | <i>n</i> | <i>g</i> | <i>TC</i> |
|----------|----------|-----------|----------|------------|----------------|----------|----------|-----------|
| 1        | 9        | 36595.18  | 2        | 9          | 30106.52       | 3        | 9        | 27937.39  |
|          | 101      | 9255.87   |          | 101        | 7679.31        |          | 101      | 7173.88   |
|          | 201      | 5571.28   |          | 201        | 4760.32        |          | 201      | 4538.73   |
|          | 301      | 4714.38   |          | 301        | 4220.63        |          | 301      | 4133.42   |
|          | 401      | 4491.87   |          | 401        | 4185.46        |          | 401      | 4189.34   |
|          | 501      | 3917.19   |          | 501        | 3742.84        |          | 501      | 3819.37   |
|          | 601      | 3974.78   |          | <b>506</b> | <b>3727.11</b> |          | 506      | 3806.98   |
|          | 647      | 3841.33   |          | 601        | 3903.85        |          | 601      | 4043.49   |
|          | 701      | 3988.10   |          | 701        | 4003.86        |          | 701      | 4201.03   |
|          | 801      | 4052.36   |          | 801        | 4144.17        |          | 801      | 4395.32   |
|          | 901      | 4108.70   |          | 901        | 4269.38        |          | 901      | 4572.12   |
|          | 1001     | 4197.60   |          | 1001       | 4422.07        |          | 1001     | 4774.72   |
|          | 1101     | 4240.84   |          | 1101       | 4525.40        |          | 1101     | 4926.71   |
|          | 1201     | 4356.00   |          | 1201       | 4697.84        |          | 1201     | 5146.88   |
| <i>n</i> | <i>g</i> | <i>TC</i> | <i>n</i> | <i>g</i>   | <i>TC</i>      | <i>n</i> | <i>g</i> | <i>TC</i> |
| 4        | 9        | 26848.14  | 5        | 9          | 26190.84       | 6        | 9        | 25749.51  |
|          | 101      | 6936.24   |          | 101        | 6805.71        |          | 101      | 6728.75   |
|          | 201      | 4464.49   |          | 201        | 4449.18        |          | 201      | 4463.34   |
|          | 301      | 4147.84   |          | 233        | 4075.77        |          | 233      | 4111.09   |
|          | 401      | 4270.78   |          | 301        | 4202.92        |          | 301      | 4278.32   |
|          | 501      | 3958.61   |          | 401        | 4383.25        |          | 401      | 4511.23   |
|          | 506      | 3948.97   |          | 501        | 4122.95        |          | 501      | 4299.82   |
|          | 601      | 4235.77   |          | 601        | 4449.10        |          | 601      | 4672.96   |
|          | 701      | 4443.55   |          | 701        | 4704.21        |          | 701      | 4973.94   |
|          | 801      | 4686.31   |          | 801        | 4993.23        |          | 801      | 5308.12   |
|          | 901      | 4910.38   |          | 901        | 5262.85        |          | 901      | 5622.43   |
|          | 1001     | 5159.41   |          | 1001       | 5556.92        |          | 1001     | 5960.84   |
|          | 1101     | 5357.21   |          | 1101       | 5799.39        |          | 1101     | 6247.41   |
|          | 1201     | 5622.73   |          | 1201       | 6109.29        |          | 1201     | 6601.22   |

### 3. Computational results for Model III

Table 15: Computational results in detail

| <i>n</i> | <i>g</i> | <i>TC</i> | <i>n</i> | <i>g</i>   | <i>TC</i>      | <i>n</i> | <i>g</i> | <i>TC</i> |
|----------|----------|-----------|----------|------------|----------------|----------|----------|-----------|
| 3        | 101      | 5492.03   | 4        | 101        | 4488.71        | 5        | 101      | 3738.08   |
|          | 137      | 4331.17   |          | 137        | 3584.15        |          | 137      | 3057.53   |
|          | 201      | 5666.82   |          | 201        | 4763.74        |          | 201      | 4112.97   |
|          | 301      | 5320.53   |          | 301        | 4708.49        |          | 301      | 4273.36   |
|          | 401      | 4414.86   |          | 401        | 4058.38        |          | 401      | 3889.31   |
|          | 501      | 4941.65   |          | 501        | 4581.68        |          | 501      | 4454.23   |
|          | 601      | 5086.68   |          | 601        | 4855.24        |          | 601      | 4847.76   |
|          | 701      | 5009.74   |          | 701        | 4837.95        |          | 701      | 4954.70   |
|          | 801      | 5214.26   |          | 801        | 5149.61        |          | 801      | 5374.19   |
| <i>n</i> | <i>g</i> | <i>TC</i> | <i>n</i> | <i>g</i>   | <i>TC</i>      | <i>n</i> | <i>g</i> | <i>TC</i> |
| 6        | 101      | 3187.07   | 7        | 101        | 2821.68        | 8        | 101      | 2646.56   |
|          | 137      | 2714.58   |          | <b>137</b> | <b>2552.67</b> |          | 127      | 2582.08   |
|          | 201      | 3696.18   |          | 201        | 3523.79        |          | 201      | 3627.32   |
|          | 301      | 4064.75   |          | 301        | 4133.54        |          | 301      | 4547.29   |
|          | 401      | 3965.57   |          | 401        | 4355.40        |          | 401      | 5153.00   |
|          | 501      | 4625.15   |          | 501        | 5177.67        |          | 501      | 6229.35   |
|          | 601      | 5164.00   |          | 601        | 5913.64        |          | 601      | 7243.46   |
|          | 701      | 5447.04   |          | 701        | 6432.42        |          | 701      | 8078.79   |
|          | 801      | 6004.21   |          | 801        | 7181.56        |          | 801      | 9102.48   |

## **VITA**

Chiranjit Saha was born to Mr. Ranjit Saha (father) and Mrs. Sipra Saha (mother) in Kolkata, India, in 1984. He earned a Bachelor of Science in Electronics and Instrumentation Engineering from West Bengal University of Technology, Siliguri, India in 2006. He joined the Department of Construction Management and Industrial Engineering at Louisiana State University at Baton Rouge, Louisiana, in fall 2007 for the degree of Master of Science in Industrial Engineering.