2004

Joint optimization of process improvement investments for supplier-buyer cooperative commerce

Erick Comeaux

Louisiana State University and Agricultural and Mechanical College

Follow this and additional works at: https://digitalcommons.lsu.edu/gradschool_theses

Part of the [Operations Research, Systems Engineering and Industrial Engineering Commons](#)

Recommended Citation

Comeaux, Erick, "Joint optimization of process improvement investments for supplier-buyer cooperative commerce" (2004). *LSU Master's Theses*. 3161.

https://digitalcommons.lsu.edu/gradschool_theses/3161

This Thesis is brought to you for free and open access by the Graduate School at LSU Digital Commons. It has been accepted for inclusion in LSU Master's Theses by an authorized graduate school editor of LSU Digital Commons. For more information, please contact gradetd@lsu.edu.

**JOINT OPTIMIZATION OF PROCESS IMPROVEMENT
INVESTMENTS FOR SUPPLIER-BUYER
COOPERATIVE COMMERCE**

A Thesis

**Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Master of Science in Industrial Engineering**

in

The Department of Industrial & Manufacturing Systems Engineering

**by
Erick J. Comeaux
B.S. in Chemical Engineering, Louisiana State University, 1997
May 2004**

ACKNOWLEDGEMENTS

I would like to thank God for gently guiding me toward point in life that I have reached today and for protecting me along the way. I would also like to thank my wife, Courtney, for her encouragement and support, as well as the sacrifices that she has made with me along the way. I also thank my son Brennan, for making all of this worthwhile.

My sincere gratitude and appreciation to Crompton Corporation (Uniroyal Chemical Company) for supporting and funding my graduate work. This professional achievement would not have been possible without their cooperation and assistance.

My special thanks goes to Dr. Bhaba R. Sarker, Dr. Lawrence Mann, Jr. and Dr. Dennis B. Webster for reviewing my work and for serving on my committee. I sincerely appreciate all of the time and effort that they committed to this work.

Finally I would like to thank all my friends and other family members for their continuous encouragement and support.

TABLE OF CONTENTS

ACKNOWLEDGMENTS.....	ii
LIST OF TABLES.....	v
LIST OF FIGURES.....	vi
ABSTRACT.....	vii
CHAPTER 1 INTRODUCTION.....	1
1.1 Background.....	1
1.2 Problem Definition.....	3
1.3 Applications and Scope.....	3
1.4 Shortcomings.....	4
1.5 Research Goals.....	5
CHAPTER 2 REVIEW OF THE LITERATURE.....	7
2.1 Literature Survey.....	7
2.2 Deficiencies.....	11
2.3 Research Objectives.....	11
2.4 Methodology.....	12
CHAPTER 3 MATHEMATICAL MODEL.....	14
3.1 System Configuration.....	14
3.2 Notation.....	16
3.3 Basic Model.....	18
3.4 Quality Adjusted Model.....	21
3.4.1 Incorporation of Product Quality Inspection Policies.....	21
3.4.2 Division of Non-Conforming Product.....	22
3.4.3 Effect of Quality Control Errors.....	24
3.5 Quality Adjusted, Quality Improvement Model.....	28
CHAPTER 4 AN ILLUSTRATIVE EXAMPLE.....	35
CHAPTER 5 RESULTS AND DISCUSSION.....	39
5.1 Joint Economic Lot Size Model Effectiveness.....	39
5.2 Effect of Inspection Policy.....	41
5.3 Effect Scrap Generation.....	45
5.4 Effect of Type I Errors.....	51
5.5 Analysis of Critical Parameters.....	52
CHAPTER 6 CONCLUSION AND FUTURE RESEARCH.....	54
6.1 Future Research.....	55

REFERENCES.....	56
APPENDIX	
A QUALITY ADJUSTED JOINT ECONOMIC LOT SIZE MODEL.....	60
B QUALITY ADJUSTED QUALITY IMPROVED JOINT ECONOMIC LOT SIZE MODEL	61
VITA.....	62

LIST OF TABLES

Table 4.1 Summary of case study model parameters.....	36
Table 4.2 Investment exponential growth function parameters	38
Table 5.1 Case study optimization results from Affisco, <i>et al.</i> (2002)	40
Table 5.2 Effect of fraction inspected on optimal fraction conforming	42
Table 5.3 Effect of fraction inspected on optimization results	45
Table 5.4 Effect of non-conforming split on optimal fraction conforming	46
Table 5.5 Effect of non-conforming split on scrap cost and repair & replacement cost .	48
Table 5.6 Effect of non-conforming split on investment cost and joint relevant total cost.....	50

LIST OF FIGURES

Figure 3.1 Manufacturer-purchaser business relationship	15
Figure 3.2 Finite input rate, no backlogging inventory model.....	19
Figure 3.3 Non-conforming product routes	23
Figure 3.4 Opportunities for quality control errors in cooperative commerce	25
Figure 3.5 General shape of the process improvement investment function	29
Figure 4.1 Case study flow diagram	37
Figure 4.2 General form of the investment function for case study	38
Figure 5.1 Effect of joint economic lot size correction	41
Figure 5.2 Effect of fraction inspected on optimal fraction conforming	42
Figure 5.3 Effect of fraction inspected on optimization results	44
Figure 5.4 Effect of non-conforming split on optimal fraction conforming	46
Figure 5.5 Effect of non-conforming split on scrap cost and repair & replacement cost.....	48
Figure 5.6 Effect of non-conforming split on investment cost and joint relevant total cost	50
Figure 5.7 Type I error cost contribution to total annual cost.....	51

ABSTRACT

This research focuses on supporting the formation of strategic alliances through the concept of cooperative commerce, where suppliers and buyers work together to jointly optimize their businesses. The general goal of this research is to examine existing cooperative commerce models for obstacles that would hinder their successful implementation into modern industrial applications and to address those shortcomings.

Total annual cost equations are formulated to capture the joint total relevant cost of cooperative commerce business relationships. These total joint relevant cost models will include terms that capture the ordering cost, holding cost, and cost of quality, as well as any applicable investment cost for process improvements, consistent with traditional economic order quantity and economic production quantity theory.

This research corrects a modeling error of Affisco, *et al.* (2002) that led to underestimating the effectiveness of process improvements in joint economic lot size models. In addition, the models are expanded to accommodate a full range of product quality inspection policies, from zero to one hundred percent product inspections. Furthermore, the models are modified to account for the cost of scrap generation, as well as the effects of accepting non-conforming product and rejecting conforming product during quality inspections. Once the total cost models are expanded to account for these neglected costs, the joint total relevant cost equations are minimized to find the optimal batch sizes, and the effects of each model extension on the model solution are studied. Results indicate that these extensions do have a significant impact on the model results, such as reduced optimal batch sizes and increased optimal fraction conforming product.

CHAPTER 1

INTRODUCTION

Competition is rapidly intensifying in today's marketplace as a record number of companies struggle to keep their doors open for business. The recent global recession, coupled with rising natural gas prices and petrochemical feed stocks, have posed tremendous challenges to today's industry, and have led to an unsurpassed number of company closures. Surviving companies are responding with substantial spending curtailments, as well as renewed interest in fresh ideas for reducing costs. In all cases, emphasis is placed on cost reduction means that require minimal capital investments, and opportunities are sought to take advantage of existing synergies. Joint economic lot size models can offer an ideal way to exploit potential synergies between purchasers and manufacturers and provide substantial cost reductions on both sides, with minimal capital investment.

1.1 Background

Often, when companies seek potential synergies, they only consider possible internal consolidations among their facilities. Cooperative commerce, on the other hand, challenges companies to change their paradigms and look beyond their own firms by seeking to exploit potential opportunities for synergies between suppliers and buyers. Cooperative commerce, or the co-maker concept, focuses on integrating the economic lot size models for manufacturers and purchasers, hence, the term "joint economic lot size model."

The most fundamental premise behind cooperative commerce, or the co-maker, concept is that cooperative business relationships are more lucrative than adversarial ones along the supply chain. Deming first introduced this concept on a much broader scope in number four

of this “Fourteen Points” when he stated, “The aim is to minimize the total cost, not merely initial cost, by minimizing variation. This may be achievable by moving toward a single supplier for any one item, on a long-term relationship of loyalty and trust.” (Neave, 1990, pp. 307-319). Therefore, a joint optimal economic lot size policy can only be adopted between manufacturers and purchasers who demonstrate a committed, open spirit of true cooperation between their companies.

Essentially, implementing a joint optimal economic lot size policy requires a manufacturer and purchaser to cooperatively determine a joint economic lot size that will be used jointly by both the manufacturer, as the economic production quantity, and by the purchaser, as the economic order quantity. Consequently, joint economic lot size models are formulated with cost parameters that are pertinent to both the manufacturer and the purchaser, and the total cost equation is referred to as the “joint relevant total cost.” For instance, the set up cost term from the economic production quantity model, and the ordering cost term from the economic order quantity model, are combined to form the ordering and setup cost term in the joint relevant total cost equation. As with individual optimization, the joint economic lot size is simply determined by performing classical optimization to minimize the joint relevant total cost equation.

The importance of cooperation between the manufacture and purchaser is made evident through the requirement of combining cost terms in the mathematical model, but there is also another significant, although less apparent, reason for this cooperation. In general, the joint economic lot size is not the optimal lot size that would be found for either the manufacturer or purchaser through individual optimization. Even worse, either the manufacturer or purchaser will

typically be at a disadvantage if the joint economic lot size is adopted. In most circumstances, the purchaser will be at the disadvantage because setup cost reductions and quality improvements typically only improve the manufacturer's production cost. Therefore, to encourage participation in a joint economic lot size program, the manufacturer must offer some sort of price concession to the purchaser so that the overall cost savings are spread more equitably between both companies. Of course, this is often the most substantial hurdle to overcome when negotiating a joint economic lot size policy between manufacturer and purchaser because each company usually has its own idea of what should be considered equitable.

1.2 Problem Definition

The specific problem under consideration here is the formulation of a joint economic lot size model that will prescribe a single lot size, to be used as both the order and production quantity, that will minimize the overall total cost incurred by both the manufacturer and purchaser. Since both the manufacturer and purchaser will use the model jointly, the total cost function is comprised of the various costs incurred on both sides of the business relationship. In addition, the model must be able to determine if process improvement investments to enhance quality or reduce setup time are beneficial to the overall cooperative business relationship.

1.3 Applications and Scope

This research is applicable to any situation in which a buyer purchases a product from a manufacturer on a lot-for-lot basis, and at least one party is interested in achieving cost savings. Although retailers purchasing product from distributors, who are not manufacturers, may use the results of this work to their benefit, the financial rewards will be somewhat limited, since that

particular situation lies outside the scope of this work. To realize the full benefits of this research, at least one of the business parties must be able to significantly impact either the product quality or setup cost. Since product quality is controlled primarily by the manufacturer, excluding damage incurred during transportation, handling and storage, the distributor's ability to impact product quality would be greatly restricted, and the model would have limited effectiveness.

In addition, although supply chains with multiple manufacturers or purchasers could adapt this research to their particular systems, this situation is also outside the scope of this work. The most critical component of cooperative commerce is the spirit and degree of cooperation between the manufacturer and purchaser. As with any coordination effort, as the number of parties involved grows, effective coordination becomes more complex and both the spirit and degree of cooperation often weakens as each company attempts to further its own particular agenda. Therefore, the actual scope of this work is limited to individual manufacturer-purchaser relationships between two parties.

1.4 Shortcomings

Existing joint economic lot size models contain a number of weaknesses that would impede their successful utilization in practical, industrial applications. For instance, existing models are limited to handling only situations where one-hundred percent quality inspections are employed, and these models do not account for the cost of scrap generation. These existing models also assume that all non-conforming product is repairable or reworkable and that no non-conforming product will require disposal. In reality, most chemical industry quality control departments practice anywhere from zero to one-hundred percent product inspections, with many inspecting from twenty-five to fifty percent of the product. In addition, the models assume

that the quality inspection process is perfect in that all conforming product is accepted, while all non-conforming product is rejected. Unfortunately, although quality control technologies have advanced substantially, most product quality inspection processes are still not perfect, and product is mistakenly rejected or accepted. Finally, the literature contains a modeling error of not squaring the fraction conforming in the denominator of the quality adjusted optimal batch size, which leads to underestimating the effectiveness of quality improvements and setup cost reductions in joint economic lot size models. The modeling errors and simplifying assumptions of one hundred percent product quality inspections, zero scrap generation, and no quality control errors place severe restrictions on the usefulness of joint economic lot size models because they force the analyst to overlook significant components of the total annual cost. Therefore, this research will focus on relieving the models' limitations by addressing each of the problems mentioned in this section.

1.5 Research Goals

The general goal of this research is to examine existing joint economic lot size models for practical weaknesses that would hinder their successful implementation into industrial applications and to address those shortcomings. The motivation for employing this technology is strengthened by correcting an error in the optimal batch size for the quality adjusted joint economic lot sizes model, which also corrects the previously reported effectiveness of quality improvements. Specifically, the fraction conforming in the quality adjusted optimal batch size model denominator needs to be squared so that the basic optimal batch size is correct when comparing the optimal batch sizes for the quality improvement and setup cost reduction models. The models are also expanded to accommodate a full range of inspection policies of zero to one

hundred percent product inspections by multiplying the inspection cost by the fraction of product inspected. Furthermore, the models will be modified to account for the cost of scrap generation by separating non-conforming product into scrap and repairable, or reworkable, portions and multiplying the scrap portion by the disposal cost. The effects of Type I (rejecting conforming product) and Type II (accepting non-conforming product) errors in the quality inspection processes are also addressed by multiplying the frequency of inspection errors by the cost of inspection errors and incorporating these expenses into the total cost models.

The model enhancements from this research makes joint economic lot size models applicable to a wider range of real-world situations and offers the flexibility to still be valid if the simplifying assumptions from past research are accurate to any particular situation.

CHAPTER 2

REVIEW OF THE LITERATURE

Joint economic lot size models have shown considerable growth in the literature since they were introduced by Goyal (1977). Although a number of researchers have contributed to the body of knowledge available on these models, most of the work has been submitted by a limited group of authors, as indicated by Affisco, *et al.* (1993). Therefore, most of the research in this area has continued along a consistent track, which has strengthened the fundamental concepts and theories considerably, but has also preserved the same deficiencies from one article to the next.

2.1 Literature Survey

One of the first joint economic lot size models was formulated by Goyal (1977), and that model has initiated the propagation of an ever-growing web of joint economic lot size models, each adapted to a particular situation. Goyal (1977) launched the study of joint economic lot size models by using trial and error techniques to arrive at a jointly-optimized lot size, where the supplier's lot size was an integral multiple of the customer's order quantity.

While these joint economic lot size models were still in their infant stages, Porteus (1985) began to tinker with the idea that basic EOQ models should be able to provide a basis for analyzing process improvement investment options. Along with a number of subsequent researchers, Porteus (1985) believed that reducing lot sizes and increasing supplier flexibility by reducing setup costs was a fundamental premise of just-in-time (JIT) inventory systems. His initial efforts led to the development of a substantial body of research on incorporating the

effects of process improvement investments into EOQ-type models, which developed simultaneously along side joint economic lot size research. Within a short three years, these concepts were integrated into the developing joint economic lot size models.

Likewise, as researchers were pioneering the development of joint economic lot size models and investigating the effects of process improvement investments on EOQ models, other researchers were starting to study the effects of product quality on EPQ models. Rosenblatt and Lee (1986) instigated this area of research by investigating the effects of product quality on the optimal production cycle time in the traditional EPQ model. Their work basically concluded that incorporating product quality into the EPQ models resulted in shorter optimal cycle times. However, this concept did not make its way into the academic arena of joint economic lot size models for another sixteen years.

Banerjee (1986) employed classical optimization techniques to formulate his joint economic lot size model, where the jointly-optimized lot size was a function of demand, the annual inventory carrying charge, the manufacturer's annual production rate, setup cost, unit production cost, order cost and unit purchase cost. Banerjee's (1986) model was based on a situation with deterministic demand, where a single purchaser orders from a sole supplier, or manufacturer, who produces on a lot-for-lot basis. He also investigated the cost-tradeoffs involving the adoption of a joint economic lot size model, since the joint economic lot size is not the optimal lot size for either the purchaser or manufacturer, operating independently.

Building upon Porteus's (1985) earlier work, Billington (1987) investigated cases where the setup cost could be expressed as either an exponential or linear function of capital investment, and incorporated this concept into the traditional EPQ model. Around the same

time, Paknejad and Affisco (1987) published a similar paper where the setup cost was expressed as a logarithmic function of capital investment. Then, Affisco, *et al.* (1988) finally became the first to incorporate process improvement capital investments into the joint economic lot size model.

Goyal (1988) then relaxed the lot-for-lot assumption and designed a joint economic lot size model where the manufacturer's lot size is an integer multiple of the purchaser's order size. Shortly thereafter, Joglekar and Tharthare (1990) also modified the joint economic lot size model by relaxing the lot-for-lot assumption and took the co-maker concept a couple of steps further. Their model divided the setup cost term into the manufacturer's setup cost and the purchaser's ordering cost. These authors also presented their concept of the individually responsible and rational decision (IRRD) model in the same paper, which they proposed would require minimal coordination between the manufacturer and purchaser. In this paper, Joglekar and Tharthare (1990) claimed that the individually responsible and rational decision model actually resulted in lower setup costs than the existing joint economic lot size models.

Dumond and Newman (1990) then gave a boost to the cooperative commerce concept by publishing their "six activities for closing the gap between buyer and vendor." In their paper, Dumond and Newman (1990) encouraged manufacturers and purchasers to integrate their production planning and ordering functions and to implement cost-reducing technologies that would strengthen the relationships between the two companies.

Lu (1995) then modified Goyal's (1988) work to allow for lot splitting, where the manufacturer may supply the purchaser before completing the entire lot. Lu (1995) also modified Goyal's (1988) model to allow for multiple purchasers and a single manufacturer.

Later, Affisco, *et al.* (1988) integrated setup cost reductions into the joint economic lot size model, and Affisco, *et al.* (1991) extended that model to include the case of multiple purchasers with a single manufacturer. Nasri, *et al.* (1991) and Affisco, *et al.* (1993) then took the additional step of investigating the effects of simultaneous investments in setup cost and order cost reductions. In both cases, the authors concluded that superior results are achieved over traditional Economic Order Quantity/Economic Production Quantity (EOQ/EPQ) models when purchasers and manufacturers cooperate in the formulation of joint economic lot size policies. These results were not surprising since Goyal, *et al.* (1993) referenced a number of works that identified these three, basic foundations of Just-In-Time (JIT) purchasing, among others: reducing suppliers to a few or only one, establish long-term relationships with these suppliers, and decrease lot sizes to improve manufacturing performance. In addition, Affisco, *et al.* (1993) revisited Joglekar and Tharthare's (1990) work with the individually responsible and rational decision (IRRD) model and offered a wealth of evidence as to why joint economic lot size models are actually superior to the IRRD model. Affisco, *et al.*'s (1993) arguments were apparently well accepted, as the IRRD model has gone virtually unmentioned in the literature ever since.

Although Rosenblatt and Lee (1986) began investigating the impact of quality on economic lot size models, most of the research was focused on traditional EOQ/EPQ models until Affisco, *et al.* (2002) incorporated quality into joint economic lot size models. Affisco, *et al.* (2002) began with Banerjee's (1986) basic model and incorporated the effect of quality into the basic model, as well as the models for quality improvement, setup cost reduction, and simultaneous quality improvement and setup cost reduction.

2.2 Deficiencies

Past researchers have developed a number of models to address the problem of formulating joint economic lot size models for cooperative commerce. However, these models still contain a number of vulnerabilities that can be improved. For instance, Affisco, *et al.*'s (2002) work contained a modeling error that underestimated the power of joint economic lot size models by five to twelve percent by leading to an inaccurate optimal batch size for the quality adjusted model. The fraction conforming in the quality adjusted optimal batch size model denominator should have been squared so that the optimal batch sizes from the quality improved and setup cost reduction models would have been compared to an accurate reference point for the optimal batch size. In addition, past researchers assumed that all companies employ a one hundred percent product inspection policy, which is often not the case in industry. Furthermore, existing models assume that all non-conforming product is repairable. In reality, most manufacturers yield a portion of non-repairable, non-conforming product, typically referred to as "scrap." Moreover, the effects of accepting non-conforming product and rejecting conforming product in the quality inspection process are not addressed in existing joint economic lot size research. Therefore, the current research addresses the deficiencies identified in this section by correcting the quality adjusted optimal batch size model and modifying the models to incorporate zero to one-hundred percent product quality inspections, scrap cost, and quality inspection error costs.

2.3 Research Objectives

The objective of this research is to address the shortcomings within existing models that would result in implementation problems for practical, industrial applications. Specifically, the

fraction conforming in the quality adjusted optimal batch size model denominator is squared to accurately reflect the quality improved and setup cost reduction models' effectiveness. The joint economic lot size models are also expanded to address the full range of zero to one hundred percent product quality inspections by multiplying the fraction inspected by the inspection cost. In addition, the models are modified to account for the cost of scrap generation by separating the proportion of non-conforming product that requires disposal and multiplying that quantity by the disposal cost. Moreover, the effects of rejecting conforming product, as well as accepting non-conforming product, in the quality inspection processes are addressed by multiplying the frequency of quality control errors by the cost of each error, resulting in the cost of quality control errors, and including this value in the total cost models.

2.4 Methodology

The quality adjusted optimal batch size model found in Affisco, *et al.* (2002), is corrected by squaring the fraction conforming in the denominator. The optimal batch size obtained from the quality-adjusted-quality-improved model is then compared to quality adjusted optimal batch size before investment to determine the effectiveness of the process improvement. The optimal batch size obtained from the quality adjusted quality improved model is also compared to the quality adjusted optimal batch size reported by Affisco, *et al.* (2002), to demonstrate the difference in batch sizes between the corrected model found in this research and the error found in Affisco, *et al.* (2002).

The joint total relevant cost models are also modified by multiplying the inspection cost by the fraction of product inspected, in order to accommodate a full range of zero to one hundred percent product quality inspections. The proportion non-conforming product in the

joint total relevant cost models is then separated into a scrap portion and a portion that is suitable for repair or rework, to account for the cost of scrap generation. Finally, the joint total relevant cost models are expanded to account for the costs of rejecting conforming product, as well as accepting non-conforming product, by multiplying the frequency of the quality control errors by the average cost of each error.

Once the joint total relevant cost models are expanded to account for zero to one hundred percent product quality inspections, the cost of scrap generation, and the effects of quality control errors, the convexity of the total cost models is assessed by evaluating the Hessian matrices. After demonstrating the convexity of the joint total relevant cost models, the optimal batch size models are generated by classical optimization.

CHAPTER 3

MATHEMATICAL MODEL

The mathematical models under consideration in this work belong to a class of inventory models commonly referred to as joint economic lot size models. Joint economic lot size models were developed years ago to model the co-maker, or cooperative commerce, concept, where both manufacturers and purchasers work together to jointly optimize the overall business relationship. Therefore, the models will be formulated with system parameters from both the manufacturer's and purchaser's points of view.

3.1 System Configuration

Figure 3.1 illustrates the relationship between manufacturer and purchaser, as well as the relevant cost parameters for the mathematical model. The figure demonstrates a clear separation between the manufacturer and purchaser since the two entities are typically completely independent companies, connected only through a supplier-buyer relationship. The solid arrows indicate material flows; whereas, dotted arrows represent the flow of information. As depicted in the diagram, orders for a given lot size (Q) of items are sent from the purchaser to the manufacturer, and the purchaser incurs an ordering cost (A) for each order. After receiving the order, the manufacturer sets up his production process, incurring a setup cost (S), and brings in the raw materials required to fulfill the order. The manufacturer then produces the ordered items at a production rate (P), and incurs a unit production cost (C_v) per item. Non-conforming items from the production process are repaired or replaced by the manufacturer, at a cost (C_M) per item, and the conforming items are shipped to the purchaser at an annual demand rate (D). The purchaser must then pay a purchase cost per item (C_p), and must incur an inspection cost (C_N) per item. Non-conforming items are returned to the manufacturer, where they are repaired or replaced, at a cost (C_M), by the manufacturer. In certain cases, however, a portion of the non-conforming items cannot be repaired ($1-w$) and must be discarded as scrap. This material incurs a disposal cost of (d). The purchaser stores conforming items until they are sold to the open market through distributors, retailers, etc.

While the conforming items are waiting for shipment, the purchaser incurs an inventory carrying charge of (h), which is expressed as a fractional dollar value.

Figure 3.1 Manufacturer-purchaser business relationship

In typical Economic Order Quantity (EOQ) or Economic Production Quantity (EPQ) models, the mathematical model is formulated for the sole use of either the purchaser or the manufacturer, but not both. Often, this results in different batch sizes being used by the purchaser and manufacturer. Therefore, either the purchaser orders more items than needed, or the manufacturer produces more items than needed. In either situation, both companies waste valuable money by not operating in harmony with the other because either the purchaser

inspects and stores unneeded items, or the manufacturer produces and stores excess inventory, and passes a portion of this cost onto the purchaser through a higher purchase cost (C_p). The joint economic lot size model, however, will be formulated to generate a lot size to be used by both purchaser and manufacturer to minimize the overall cost incurred by both companies.

3.2 Notation

a	Dollar increase in a_p^- per percentage increase in $\bar{p}$
b	Simplifying term, equal to $(-LN p_0)/D$, in the quality improvement investment cost function
e_c	Cost of Type I errors (\$/error)
e_f	Frequency of Type I errors (fraction of errors/year)
D	Percentage increase $\bar{p}$ per dollar increase in a_p^-
I	Fraction inspected
w	Proportion reworkable, or repairable, non-conforming product
a_p^-	Amount of capital investment for quality-related process improvement (\$)
A	Ordering cost (\$/order)
C_H	Total annual Holding Cost (\$/year)
C_O	Total annual Ordering Cost (\$/year)
C_p	Purchasing cost (\$/unit)
C_M	Manufacturer's non-conforming item repair and replacement cost (\$/unit)
C_N	Purchaser's inspection cost (\$/unit)
C_Q	Annual Quality Cost (\$/year)
C_v	Production cost (\$/unit)

d	Scrap cost (\$/unit)
D	Annual demand (units/year)
G	Total procurement cost (manufacturer's production cost plus purchaser's purchasing cost) (\$)
h	Annual unit inventory carrying cost (\$/unit/year)
i	Cost of capital investment (\$)
P	Annual production rate (units/year)
p	Proportion of non-conforming items produced by manufacturer's process
$\bar{p}$	Proportion of conforming items produced by manufacturer's process ($1-p$)
p_0	Original proportion of non-conforming units produced by manufacturer's process before an investment is made
$\bar{p}_0$	Original proportion of conforming units produced by manufacturer's process before an investment is made ($1-p_0$)
Q	Order or production size (units/batch)
Q^*	Optimum order or production size of item (units/batch)
S	Manufacturer's setup cost (\$/setup)
S_0	Original manufacturer's setup cost before investment (\$/setup)
TC	Joint total relevant cost of manufacturer/purchaser commerce (\$/year)

3.3 Basic Model

For cooperative commerce to work effectively, the joint economic lot size model must be formulated for both the manufacturer and the purchaser, as opposed to traditional *EOQ*, or *EPQ*, models, which are only developed from a single vantage point, for either the purchaser or the manufacturer. Therefore, the terms in both the total cost and lot size equations are taken from both the manufacturer's costs and the purchaser's costs. For this reason, the total cost equations actually represent the joint total relevant cost of the supplier/buyer business relationship, as an overall entity. Likewise, the lot size equations are designed to yield a single lot size that will be used by both the purchaser, as the order quantity, and by the manufacturer, as the production quantity.

According to Banerjee (1986), the joint total relevant cost (*TC*) can be optimized by minimizing the joint relevant total cost, for a single manufacturer, producing on a lot-for-lot basis, in response to orders from a single purchaser. Assume that demand is deterministic, and the manufacturer is the sole supplier.

For the annual demand, D , the manufacturer's annual production rate, P , the ordering cost, A , the manufacturer's setup cost, S , the annual inventory carrying charge, h , the unit production cost, C_v , the unit purchase cost, C_p , and the order or production size in units, Q , the

ordering cost can be written as $\left(\frac{D}{Q}\right)(S + A)$, and the holding cost can be written as

$\left(\frac{Q}{2}\right)h\left[\left(\frac{D}{P}\right)C_v + C_p\right]$, from which the joint total relevant cost model can be written as

$$TC(Q) = \left(\frac{D}{Q}\right)(S + A) + \left(\frac{Q}{2}\right)h\left[\left(\frac{D}{P}\right)C_v + C_p\right] \quad (1)$$

In addition, Equation (1) assumes that the inventory position resembles a finite input rate model, with no backlogging, as illustrated in Figure 3.2.

Figure 3.2 Finite input rate, no backlogging inventory model

As shown in Figure 3.2, in this form of inventory model, the inventory builds at a finite, linear rate during the production period of the inventory cycle. Then, production ceases for a period of time, and the inventory decreases at a finite, linear rate, corresponding to the demand. The

cycle then repeats itself continuously, at regular, equally-spaced time intervals. Therefore, the average inventory can be estimated as $\frac{Q}{2}$.

The joint total relevant cost equation presented in Equation (1) was determined to be strictly convex by evaluating the one-dimensional Hessian, as shown in Equation (2).

$$f''(Q) = \frac{2D(S+A)}{Q^3} > 0 \quad (2)$$

Therefore, since Equation (1) is a convex, non-linear function, the basic joint total relevant cost model can be minimized by performing classical optimization and setting the partial derivative of the function, with respect to Q , equal to zero, as expressed by Equation (3). The value obtained by Equation (3) will give the joint economic lot size that will be used by both the purchaser, as the order quantity, and by the manufacturer, as the production quantity.

$$Q^* = \sqrt{\frac{2D(S+A)}{h \left[\left(\frac{D}{P} \right) C_v + C_p \right]}} \quad (3)$$

However, this basic model does not take the quality of the manufacturer's process or the effects of potential process improvements into account. Therefore, a quality adjusted model will be developed in the next section to address this missing element.

All joint total relevant cost equations presented here are convex, non-linear functions, similar to Equation (1) and minimizing the functions are achieved in the same manner, by performing classical optimization.

3.4 Quality-Adjusted Model

When the joint total relevant cost is adjusted for the quality factor, Equation (1) can be extended to include the cost of quality (inspections, non-conforming item repairs, and non-conforming item replacements), and the lot size must be adjusted for the quality factor ($\bar{p}$). Updating the equations for ordering cost and holding cost merely involve multiplying the lot size in the expressions by the quality factor, giving $\left(\frac{D}{\bar{p}Q}\right)(S + A)$ for the ordering cost, and $\left(\frac{Q}{2}\right)\bar{p}h\left[\left(\frac{D}{P}\right)C_v + C_p\right]$ for the holding cost. The cost of quality, on the other hand, is defined as the summation of the inspection cost and the non-conforming item repair and replacement cost.

$$C_Q = \left(\frac{D}{\bar{p}}\right)[(1 - \bar{p})C_M + C_N] \quad (4)$$

where $\bar{p} = 1 - p$ (the proportion of good items produced by the manufacturer's process), C_M is the manufacturer's cost of repair and replacement of non-conforming items, and C_N is the purchaser's inspection cost. Hence, according to Affisco, *et al.* (2002), the joint total relevant total cost can be written as

$$TC_p(Q, \bar{p}) = \frac{D(S + A)}{\bar{p}Q} + \left(\frac{Q}{2}\right)\bar{p}hG + \left(\frac{D}{\bar{p}}\right)[(1 - \bar{p})C_M + C_N] \quad (5)$$

where $G = \left(\frac{D}{P}\right)C_v + C_p$.

3.4.1 Incorporation of Product Quality Inspection Policies

This quality adjusted joint economic lot size model found in Equation (5) assumes that the purchaser will conduct one hundred percent product quality inspections. In practice, many

companies cannot afford the resources required for one hundred percent product inspections, but insist on performing some degree of quality inspections. We must keep in mind that, in spite of Deming's work, noted in Neave (1990, pp. 297 - 307), proving that only zero or one hundred percent quality inspection policies are effective, many corporate executives and front-line supervisors are not familiar with, or do not subscribe to, Deming's thoughts on inspection policies. In reality, many purchasers have product quality inspection policies between zero and one-hundred percent. Therefore, a term should be added to the model that will accommodate quality inspection policies within the full range of zero to one hundred percent.

Since the cost of quality is directly related to the number of inspections taking place, Equation (4) must be modified to accommodate inspection policies of zero to one hundred percent product inspection. This modification is accomplished by multiplying the unit inspection cost C_N , by the fraction of products inspected, I .

$$C_Q = \left(\frac{D}{P} \right) [(1 - \bar{p})C_M + IC_N] \quad (6)$$

3.4.2 Division of Non-Conforming Product

In addition, Affisco, *et al.* (2002) assumed that the cost of quality was strictly composed of inspection costs and repair and replacement costs. In reality, a portion of all non-conforming product is usually not repairable, or capable of being reworked, and requires disposal. Affisco, *et al.*'s (2002) model would capture the supplier's cost of replacing defective products, but would not take into account the disposal cost of non-repairable, or non-reworkable, defective products. Therefore, the cost of quality term should be extended to account for this very common, significant expense.

However, the repair and replacement cost term, C_M , cannot simply be defined such that it includes disposal costs because two separate and distinct categories of non-conforming product actually exists, each with it's own cost of non-conformance, as illustrated in Figure 3.3.

Figure 3.3 Non-conforming product routes

Affisco, *et al.* (2002) addressed the portion of non-conforming product that passes along the repairable, or reworkable, route back to the customer (w). Along this route, the non-conformances are corrected, and the manufacturer incurs a repair and replacement cost of C_M dollars per unit. The repair and replacement cost, C_M , is typically equal to some incremental amount, less than the full original production cost (unless the disposal cost is overriding), to cover the repair or rework of the defective product. However, the portion of non-conforming product that passes along the scrap route ($1-w$) experiences a completely different situation. The scrap product is considered to be defective beyond repair, or rework, and requires disposal. Therefore, the manufacturer incurs a cost of d dollars per unit for non-conforming

product passing along this route. Unlike the repair and replacement cost, the manufacturer's disposal cost, d , is composed of the full production cost, C_v , since the entire value of the original product is lost, plus a disposal fee. Therefore, d will always be substantially greater than C_M . Equation (5) is then expanded to include both routes for non-conforming product, as follows:

$$C_Q = \left(\frac{D}{\bar{p}} \right) \left[w(1 - \bar{p})C_M + (1 - w)(1 - \bar{p})d + IC_N \right] \quad (7)$$

Inspection policies and scrap generation are only considered in quality-adjusted models because in non-quality-adjusted models all production is assumed to be conforming product, eliminating the need for inspection and disposal. Therefore, the equations of the basic joint economic lot size and basic joint total relevant cost models do not change with respect to the number of inspections taking place or the amount of scrap generated.

3.4.3 Effect of Quality Control Errors

The cost of quality term, however, still has not adequately addressed Type I and Type II errors. Type I errors would consist of rejecting conforming product. The manufacturer is typically the primary controller of product quality, and quality control automation on the part of manufacturers has greatly reduced the frequency of these errors to the point that they are almost non-existent. However, purchasers do not usually analyze in-coming product with the same level of sophistication as manufacturers, resulting in a higher frequency of Type I errors. Furthermore, since purchasers operate their inspection processes from a defensive perspective, the inspections are usually conducted to ensure that if an error is made, Type I errors will be

favorable over Type II errors. In the case of a Type I error, the purchaser returns a perfectly good product to the manufacturer and submits a defective product claim.

Figure 3.4 Opportunities for quality control errors in cooperative commerce

Type II errors, on the other hand, consists of accepting non-conforming product. Again, the current state of quality control technology has greatly reduced the frequency of these errors, but there is still a very small chance that they may occur. Figure 3.4 illustrates the flow of non-conforming product through the cooperative commerce system and identifies the points of possible quality control errors.

If the purchaser evaluates in-coming product, as this model assumes, the probability of the Type II error repeating itself will be extremely small. The reason is that, as described in Figure 3.4, the purchaser's inspection process serves as an independent validator of the

manufacturer's quality control process. Therefore, Type II errors were not addressed in this model because the error would have to evade both the manufacturer's and purchaser's quality control systems.

Type I errors, however, are a real problem that must be addressed in the model. In most situations, any time that the purchaser rejects a conforming product, that product is returned to the manufacturer, at the manufacturer's expense, without an additional screening step. Although good screening processes will minimize the frequency of this error, the model should account for the cost incurred by the manufacturer when these errors do occur. The average cost of Type I errors can be computed by multiplying the frequency of Type I errors, e_f , by the cost of each error, e_c , which further extends the cost of quality expression given in Equation (7) to

$$C_Q = \left(\frac{D}{\bar{p}} \right) \left[w(1 - \bar{p})C_M + (1 - w)(1 - \bar{p})d + IC_N + e_f e_c \right] \quad (8)$$

Although the effect of Type II errors was not included in this model, the average cost of each Type II error could be multiplied to the frequency of the errors and added to the end of Equation (8), exactly as the Type I errors were accommodated.

Hence, the joint total relevant cost expression, given in Equation (5) becomes:

$$TC_{\bar{p}}(Q, \bar{p}) = \frac{D(S + A)}{\bar{p}Q} + \left(\frac{Q}{2} \right) \bar{p}hG + \left(\frac{D}{\bar{p}} \right) \left[w(1 - \bar{p})C_M + (1 - w)(1 - \bar{p})d + IC_N + e_f e_c \right] \quad (9)$$

By taking the fraction of products inspected into account the models are able to accommodate inspection policies within the full range of zero to one hundred percent product inspections, rather than being limited to solely one hundred percent inspection. In addition, by incorporating both routes for non-conforming product into the equations, the model can now accurately analyze a manufacturer's true cost of quality. In the extremely rare case where a manufacturer is able to correct all defects on non-conforming products, and yields no scrap, w simply becomes 1, and Equation (7) reverts back to Equation (6), where non-conforming product only passes along the repair and replacement route back to the customer. Therefore, the improved model does not lose any of its original applicability; rather, the model simply becomes more suitable for a broader range of applications.

The quality adjusted joint total relevant cost model given in Equation (9) was confirmed to be strictly convex by evaluating the two-dimensional Hessian matrix, as demonstrated in Equation (10). The two dimensional Hessian matrix defined by Equation (10) was obtained by evaluating the second partial derivative of Equation (9), with respect to Q , and assigning the coefficients of that resulting expression to the first row, and by evaluating the second partial derivative of Equation (9), with respect to $\bar{p}$, and assigning the coefficients of that resulting expression to the second row.

$$H_f(Q, \bar{p}) = \begin{bmatrix} \frac{\partial^2 TC}{\partial Q^2} & \frac{\partial^2 TC}{\partial Q \partial \bar{p}} \\ \frac{\partial^2 TC}{\partial \bar{p} \partial Q} & \frac{\partial^2 TC}{\partial \bar{p}^2} \end{bmatrix} \geq 0 \quad (10)$$

Since the leading principle determinants of the Hessian matrix were determined to be non-negative, Equation (9) was verified to be strictly convex, with a unique minimum point. See Appendix A for a more detailed synopsis of the Hessian evaluation.

Therefore, since Equation (9) is a convex, non-linear function, the model can also be minimized by setting the partial derivative, with respect to Q , equal to zero, leading to the optimal joint economic lot size. Minimizing Equation (10), by setting $\frac{\partial TC}{\partial Q} = 0$, leads to the following joint economic lot size model:

$$Q_p^* = \sqrt{\frac{2D(S+A)}{hG\bar{p}^2}} \quad (11)$$

Although this model does account for the quality of the manufacturing process, the equations still do not consider the effects of potential process improvements, which will be addressed in the next section.

3.5 Quality-Adjusted, Quality-Improvement Model

When we consider the option of investing in process improvements to enhance the manufacturer's production process, we must further extend the joint total relevant cost model to account for the cost of the capital investment. For quality-related process improvements, the investment cost term will be represented as $ia_{\bar{p}}(\bar{p})$, and the remainder of the equation can be simply expressed as $TC_{\bar{p}}(Q, \bar{p})$, since the terms are identical to those in Equation (9). Therefore, we will seek to minimize the sum of the investment cost for increasing $\bar{p}$ and the quality-adjusted joint total relevant cost, which can be expressed as

$$TC_{\bar{p}}^Q(Q, \bar{p}) = ia_{\bar{p}}(\bar{p}) + TC_{\bar{p}}(Q, \bar{p}) \quad (12)$$

subject to $\bar{p}_0 < \bar{p} \leq 1$, where i is the unit cost of capital investment, and $\bar{p}_0 = 1 - p_0$ is the original proportion of good units produced by the manufacturer's process. According to Affisco, *et al.* (2002), the exponential growth function of the process improvement investment can be expressed as

$$\bar{p} = \bar{p}_0 e^{\Delta a_{\bar{p}}} \quad (13)$$

subject to $\bar{p}_0 < \bar{p} \leq 1$, where Δ is the percentage increase in $\bar{p}$ per dollar increase in $a_{\bar{p}}$.

Equation (13) is a strictly increasing exponential function of the proportion conforming units, representing the change of the proportion conforming units from an initial level ($\bar{p}_0$) to some improved level ($\bar{p}$), as a function of the amount of capital invested, as shown in Figure 3.5.

Figure 3.5 General shape of process improvement investment function

The law of diminishing returns governs the shape of this function, as the proportion conforming units cannot be improved above a certain level regardless of the capital invested.

Furthermore, as the proportion of conforming units continues to improve, each successive improvement requires a greater incremental amount of capital.

In addition, the amount of investment, $a_{\bar{p}}$, is restricted by the following limits:

$$0 \leq a_{\bar{p}} \leq \frac{\ln \left[\frac{1}{\bar{p}_0} \right]}{\Delta} \quad (14)$$

Taking the log of both sides of Equation (14) gives

$$a_{\bar{p}} = \frac{\ln(\bar{p})}{\Delta} - \frac{\ln(\bar{p}_0)}{\Delta} = \mathbf{a} \ln \bar{p} + \mathbf{b} \quad (15)$$

where $\mathbf{a} = \frac{1}{\Delta}$ and $\mathbf{b} = \frac{-\ln \bar{p}_0}{\Delta}$.

Substituting the investment cost function given in Equation (15) into the general expression given in Equation (12), we obtain the joint total relevant cost after investment,

$$\begin{aligned} TC_p^Q(Q_p^-, \bar{p}_I) = & i[\mathbf{a} \ln \bar{p}_I + \mathbf{b}] + \frac{D(S+A)}{\bar{p}_I Q_p^-} + \frac{Q_p^- \bar{p} h G}{2} \\ & + \left(\frac{D}{\bar{p}} \right) [\mathbf{w}(1-\bar{p})C_M + (1-\mathbf{w})(1-\bar{p})d + \mathbf{I}C_N + \mathbf{e}_f \mathbf{e}_c] \quad (16) \end{aligned}$$

Therefore, the optimal joint relevant total cost is determined by comparing the joint relevant total costs with and without process improvement investments, where the joint relevant total cost before investment was given by Equation (9). Therefore, the optimal joint relevant total cost is selected according to Equation (17):

$$TC^* = \min \{ TC_p^-(Q_p^*), TC_p^Q(Q_p^*, \bar{p}_I) \} \quad (17)$$

The quality adjusted quality improved joint total relevant cost model given in Equation (16) was confirmed to be convex by evaluating the two dimensional Hessian matrix, shown in Equation (10). A more detailed explanation of the Hessian evaluation is shown in Appendix B. The second partial derivative of Equation (16), with respect to Q , was evaluated, and the coefficients of that resulting expression were assigned to the first row of the Hessian matrix, and the second partial derivative of Equation (16), with respect to $\bar{p}$, was evaluated, and the coefficients of that resulting expression were assigned to the second row.

In addition, since Equation (16) is a convex, non-linear function, the model can be minimized by setting $\frac{\partial TC}{\partial Q} = \frac{\partial TC}{\partial \bar{p}} = 0$, and solving the two simultaneous equations, leading to the optimal joint economic lot size and optimal fraction conforming. Equation (18) gives the optimal joint economic lot size after investment, and Equation (20) give the optimal proportion of conforming parts after investment.

$$Q_p^{Q*} = \sqrt{\frac{2D(S+A)}{hG}} \left[\frac{ia}{(\mathbf{w}C_M + (1-\mathbf{w})d + \mathbf{l}C_N + \mathbf{e}_f\mathbf{e}_c)D} \right] \quad (18)$$

The optimal joint economic lot size is found by comparing the original joint economic lot size, given in Equation (11), with the joint economic lot size after investment, given in Equation (18).

Therefore, the optimal joint economic lot size can be expressed mathematically as:

$$Q^* = \min\{ Q_p^*, Q_p^{Q*} \} \quad (19)$$

Likewise, the optimal fraction of conforming items is found by comparing the original proportion of good parts before investment, $\bar{p}_0$, and the proportion of good parts after the investment, which is found to be:

$$\bar{p}_I = \frac{(\mathbf{w}C_M + (1-\mathbf{w})d + \mathbf{I}C_N + \mathbf{e}_f\mathbf{e}_c)D}{i\mathbf{a}} \quad (20)$$

by once again solving the two simultaneous equations, $\frac{\partial TC}{\partial Q} = \frac{\partial TC}{\partial \bar{p}} = 0$.

Therefore, the optimal fraction of conforming items is the maximum of the fraction conforming items before the investment and the fraction conforming after the investment. This decision can be expressed mathematically as,

$$\bar{p}^* = \max\{ \bar{p}_0, \bar{p}_I \} \quad (21)$$

As evidenced by the preceding paragraphs, the development of joint economic lot size models with process improvements can consume a considerable amount of time and engineering resources. To make matters worse, all of this time and energy can be spent to find that the investment isn't even worthwhile. The derivation of critical points for the quality-improvement investment model can help to establish benchmarks to quickly determine whether or not the investment is worthwhile. An investment cannot be justified unless the proportion of good units produced by the manufacturer's process after investment is greater than the original proportion of good units. In other words, $\bar{p} > \bar{p}_0$, or

$$\bar{p}_I = \frac{(\mathbf{w}C_M + (1-\mathbf{w})d + \mathbf{I}C_N + \mathbf{e}_f\mathbf{e}_c)D}{i\mathbf{a}} > \bar{p}_0 \quad (22)$$

This inequality can be solved for the cost of capital, i , demand, D , or any other parameter, providing equations for the critical values that determine whether or not the investment is worthwhile. For example, the critical value for the inventory carrying charge, found in Equation (23), defines the minimum carrying charge for which an investment is worthwhile. For any carrying charge below this value, an investment will not prove to be worthwhile.

$$i < \frac{(\mathbf{w}C_M + (1 - \mathbf{w})d + \mathbf{I}C_N + \mathbf{e}_f\mathbf{e}_c)D}{i\bar{a}\bar{p}_0} \quad (23)$$

Likewise, the critical demand can be found using Equation (24).

$$D > \frac{i\bar{a}\bar{p}_0}{(\mathbf{w}C_M + (1 - \mathbf{w})d + \mathbf{I}C_N + \mathbf{e}_f\mathbf{e}_c)} \quad (24)$$

Similarly, using Equation 22, we can determine the critical value for the technology coefficients, Δ , which is found to be

$$\Delta > \frac{i\bar{p}_0}{(\mathbf{w}C_M + (1 - \mathbf{w})d + \mathbf{I}C_N + \mathbf{e}_f\mathbf{e}_c)D} \quad (25)$$

Following the same logic, we can define equations for the critical scrap cost, d , proportion reworkable, or repairable non-conforming product, $\mathbf{w}$, repair and replacement cost, C_M , or inspection cost, C_N .

However, it is important to remember that the sole purpose of these critical points is to provide a quick, simple means of deciding whether or not to proceed with a process improvement investment feasibility study. Therefore, unnecessarily complicating that decision with a large number of critical points will only defeat the original intent of those useful tools. In

most cases, it is only necessary to evaluate the critical inventory carrying charge, demand, and technology coefficient.

CHAPTER 4

AN ILLUSTRATIVE EXAMPLE

The specific example used to illustrate this study focuses on a single purchaser that periodically orders a batch of an item from a manufacturer who is the sole source for this item. The numerical values are based on an example used by Affisco, *et al.* (2002). The manufacturer produces this item on a lot-for-lot basis, and both the manufacturer and purchaser have agreed to cooperate according to the results of a joint economic lot size model. For the specific example under consideration in this proposal, the demand is 1,000 units/year, and the production rate is 3,200 units/year. The production cost is \$20/unit, and the purchase cost is \$25/unit. The setup cost is \$400/setup, and the ordering cost is \$100/order. The inventory carrying cost is 20%, and the original proportion of conforming units is 75%. The inspection cost is \$5/unit, and the repair or replacement cost for non-conforming items is \$12/unit. The purchaser commits Type I errors at a rate of about 5 errors per year and at a cost of \$5/error. Figure 4.1 illustrates the context of these model parameters on a flow diagram of the manufacturer-purchaser cooperative commerce relationship, and Table 4.1 summarizes their values.

The relevance of this illustrative example to any particular situation can be assessed by considering the relative magnitude of the various cost parameters. Highly specialized technical industries, such as computers and electronics, will have relatively high production, purchase, repair and disposal costs. Mass-produced, discrete commodities, such as plastic pipe fittings, on the other hand, will have relatively low production, purchase, repair and disposal costs. This specific illustrative example falls between those two extremes.

Table 4.1 Summary of case study model parameters

Model Parameter	Value
Demand (D)	1,000 units/year
Production rate (P)	3,200 units/year
Production cost (C_v)	\$20/unit
Purchase cost (C_p)	\$25/unit
Setup cost (S)	\$400/setup
Ordering cost (A)	\$100/order
Inventory carrying cost (h)	\$0.20/unit/year
Original proportion conforming units ($\bar{p}$)	5%
Inspection cost (C_N)	\$5/unit
Repair and replacement cost (C_M)	\$12/unit
Scrap generation (w)	10%
Disposal cost (d)	\$21/unit
Type I error frequency (e_f)	5 errors/year
Type I error cost (e_c)	\$5/error

Figure 4.1 illustrates the relationship between manufacturer and purchaser, as well as the relevant cost parameters for the mathematical model. The solid arrows indicate material flows, and dotted arrows represent the information and data flows. As depicted in the diagram, orders for a given lot size (Q) of items are sent from the purchaser to the manufacturer, and the purchaser incurs an ordering cost. After receiving the order, the manufacturer incurs a setup cost, brings in the raw materials required to fulfill the order, then produces the ordered items. Non-conforming items from the production process are repaired or replaced by the manufacturer, and the conforming items are shipped to the purchaser. The purchaser must then pay the purchase cost and must incur an inspection cost. Non-conforming items are returned to the manufacturer, where they are repaired or replaced by the manufacturer. In certain cases, however, a portion of the non-conforming items cannot be repaired and must be discarded as scrap. The purchaser stores conforming items until they are sold to the open market.

Figure 4.1 Case study flow diagram

In addition, the manufacturer is considering capital investments that will improve the quality of the manufacturing process. The cost of the quality improvement capital investment can be expressed as a strictly increasing exponential function of the fraction conforming product, representing the cost of changing the fraction conforming from an initial position to some improved level, as depicted in Figure 4.2.

Figure 4.2 General form of investment function for case study

As explained previously in the mathematical model, the investment function can be specifically expressed as $a_{\bar{p}} = \frac{\ln(\bar{p})}{\Delta} - \frac{\ln(\bar{p}_0)}{\Delta}$ for the quality-related process improvement.

The coefficients of the logarithmic investment function, used by Affisco, *et al.* (2002), are as follows.

Table 4.2 Investment exponential growth function parameters

Coefficients of Investment Function	Quality Improvement Investment
<i>i</i>	0.10
<i>D</i>	5.75 x 10 ⁻⁶

CHAPTER 5

RESULTS AND DISCUSSION

An illustrative example was necessary to bring the parameters and equations of the joint economic lot size models into a practical perspective, and to provide a specific example of how this technology can be used. However, simply solving the joint economic lot size model for a single example would be of limited academic value, other than providing another explanation of how to apply the models. Therefore, the effects of each model extension from this research are illustrated in this section by examining the effects of each new parameter over as wide a range as possible. Hence, although the actual numerical figures may not be applicable beyond this example, the general trends will be pertinent to most industrial situations. Preliminary results indicate that this research will make a useful contribution to the existing body of knowledge on joint economic lot size models. In this chapter, the preliminary results are presented in a general form, but the observed trends will be studied and explained in more mathematical detail.

5.1 Joint Economic Lot Size Model Effectiveness

The optimization results of the case study from Affisco *et al.* (2002) are summarized in Table 5.1.

Table 5.1 Case study optimization results from Affisco *et al.* (2002)

Variable	Quality Adjusted	Quality Improved	Setup Cost Improved	Simultaneous Investment
Q (units)	461.88	409.16	249.41	191.35
S (\$)	400	400	9.35	9.35
$\bar{p}$	0.75	0.9776	0.75	0.9776
Repair/ Replacement Cost (\$)	4,000.00	274.96	4,000.00	274.96
Purchaser Inspection Cost (\$)	6,667.00	5,114.57	6,667.00	51,114.57
Quality Improvement Cost (\$)	-	4,606.00	-	4,606.00
Setup Reduction Cost (\$)	-	-	187.70	187.70
Joint Total Relevant Cost (\$)	13,192.58	12,295.56	12,023.36	11,352.49

Careful review of Affisco, *et al.*'s (2002) solution unveiled an error in the basic, quality-adjusted joint economic lot size model. However, Affisco, *et al.*'s (2002) error appeared to be either a typographical error or editing oversight since the error did not carry into further models. In addition, the error was not observed in previous models that formed the foundation for Affisco, *et al.*'s (2002) work. The accurate joint economic lot size, Q , for the quality adjusted model was calculated to be 533.33 units. It was found that Affisco, *et al.*'s (2002) value of 461.88 units is obtained if the $\bar{p}$ term in the denominator of the quality adjusted optimal joint economic lot size model is not squared, which is in error. Correcting this modeling error results in an observed 23.3% to 64.1% improvement in the process improvement investment models, compared to Affisco, *et al.*'s (2002) reported 11.4% to 58.6%. Figure 5.1 demonstrates this increase in the improvement observed from the process improvement models, which strengthens the motivation for studying this field of production and inventory control.

Figure 5.1 Effect of joint economic lot size correction

5.2 Effect of Inspection Policy

Furthermore, incorporating a full-range of zero to one hundred percent product quality inspections opened the models to a larger number of practical applications. Specifically, companies that employ less than one hundred percent inspection policies can now accurately utilize the mathematical models to generate joint economic lot sizes, as well as to evaluate the feasibility of proposed process improvement investments. The effects of the new product quality inspection policy term on the quality-adjusted quality improved model are described below, in Figures 5.2 and 5.3, and in Table 5.2.

Figure 5.2 Effect of fraction inspected on optimal fraction conforming

Table 5.2 Effect of fraction inspected on optimal fraction conforming

Fraction Inspected	0	25%	50%	75%	100%
Optimal Fraction Conforming	69%	76.2%	83.4%	90.6%	97.8%

As Figure 5.2 and Table 5.2 indicate, more rigorous inspection policies result in increased values for the optimal fraction conforming. This conclusion makes logical sense because as more items are inspected, more non-conforming items will be identified, requiring repair and replacement, and thus, directly increasing the total cost in a linear fashion. Algebraically manipulating Equation (20), for the optimal fraction conforming after the process improvement

investment, finds that the slope of the optimal fraction conforming versus fraction inspected curve is equal to $\frac{C_N D}{i a}$, and the intercept is equal to $\frac{D(w C_M + (1-w)d + e_f e_c)}{i a}$. Hence, increasing the annual demand (D) or inspection cost (C_N) makes the optimal fraction conforming more sensitive to the fraction inspected. On the other hand, increasing the cost of capital investment (i) or the dollar increase in capital investment per percentage increase in fraction conforming (a) makes the optimal fraction conforming less sensitive to the fraction inspected.

Although lax inspection policies appear to offer less severe financial penalties for non-conforming items, because many go unidentified and do not require immediate repair or replacement, this would be an unwise assumption to make. In the long run, non-conforming items always result in increased costs and reduced sales and have led to the demise of many corporations.

As Figure 5.2 indicates, the relationship between the fraction inspected and the optimal fraction conforming is linear. The observed linearity is a direct result of the non-conforming product cost being defined as a constant, average cost per unit. Hence, as more units are inspected and found to be defective, the total cost of non-conforming product rises, resulting in a higher value for the optimal fraction conforming to minimize the amount of defective product requiring repair or disposal.

In addition, a base optimal fraction conforming was observed at zero percent inspected because even if the purchaser does not conduct any quality inspections, the manufacturer still encounters a cost of quality, which encourages the production of good product. On the other hand, at one hundred percent product quality inspections, the optimal fraction conforming is still

not one hundred percent because the non-conforming product costs are not high enough to force an optimal fraction conforming of one hundred percent. In other words, the non-conforming product costs are arranged in such a fashion that it is financially more advantageous to produce a small amount of non-conforming product than it would be to assure that no defective product is manufactured. Often, this is not the case, which is why six sigma programs have become so popular. In some situations, however, the cost of non-conforming product could be high enough to force an optimal fraction conforming of one hundred percent, even though that objective would be very difficult to achieve.

Figure 5.3 and Table 5.3 demonstrate the effect of the fraction inspected on the optimal joint economic lot size, optimal joint total relevant cost and inspection cost.

Figure 5.3 Effect of fraction inspected on optimization results

Table 5.3 Effect of fraction inspected on optimization results

Fraction Inspected	0%	25%	50%	75%	100%
Optimal Joint Economic Lot Size	533	525	480	442	409
Optimal Joint Total Relevant Cost	\$6,608.70	\$8,415.03	\$9,606.79	\$10,955.67	\$12,207.78
Inspection Cost	\$0.00	\$1,666.67	\$3,333.33	\$5,000.00	\$6,666.67

According to Figure 5.3 and Table 5.3, more rigorous inspection policies result in smaller batch sizes, increased inspection costs, and higher joint total relevant costs. All of these results were anticipated because more rigorous inspection policies led to a higher proportion of conforming parts, creating a more dependable process, which does not require larger batch sizes to satisfy demand, while accommodating non-conforming production. On the other hand, increased inspection frequencies do increase both the inspection cost and total cost in an almost linear fashion because more effort is required to perform the inspections. The slope of the relationship between the fraction inspected and the inspection cost is DC_N , with an intercept of zero.

5.3 Effect of Scrap Generation

Dividing non-conforming product into a repairable, or reworkable, portion and a scrap portion also had a significant effect on the model's solution. Since the supplier absorbs the full value of the original defective product for the scrap portion of non-conforming product, as well as a disposal fee, the disposal cost is always higher than the repair and replacement cost. Recognizing the impact of this significant expense on the total cost equation forces the optimal

fraction non-conforming to a higher value than was obtained when disposal costs were ignored. Figures 5.4 through 5.5 illustrate the effects of the non-conforming product split on the optimal fraction conforming, scrap cost, repair and replacement cost, investment cost and joint relevant total cost.

Figure 5.4 and Table 5.4 illustrate the influence that the scrap portion of non-conforming product has on the optimal fraction conforming.

Figure 5.4 Effect of non-conforming product split on optimal fraction conforming

Table 5.4 Effect of non-conforming product split on optimal fraction conforming

Proportion Repairable/Reworkable	0%	25%	50%	75%	100%
Optimal Fraction Conforming	100%	100%	100%	89%	76%

Together, the disposal cost and the repair and replacement cost provide the financial penalty for non-conforming product. As Figure 5.4 illustrates, if less than 50% of the non-conforming product is suitable for repair or rework in the illustrative example, the financial

penalty for non-conforming product is so prohibitively high that 100% conforming product is considered to be optimal. In another situation, however, with different cost parameters, the repairable or reworkable proportion that forces an optimal fraction conforming of 100% or more will be different. This point could be estimated by recognizing that the slope of the optimal fraction conforming versus proportion repairable or reworkable correlation is $\frac{D(C_M - d)}{ia}$, and the intercept of the line is $\frac{D(d + IC_N + e_f e_c)}{ia}$. Hence, increasing the annual demand (D) or inspection cost (C_N) makes the optimal fraction conforming more sensitive to the proportion repaired or reworked. On the other hand, increasing the cost of capital investment (i) or the dollar increase in capital investment per percentage increase in fraction conforming (a) makes the optimal fraction conforming less sensitive to the proportion repaired or reworked.

However, as a larger portion of the non-conforming product can be repaired or reworked, the overall cost of non-conforming product decreases, allowing the supplier to tolerate lower levels of conforming product. In highly specialized industries, however, where practically all non-conforming product is considered scrap, off-grade levels must be maintained extremely close to zero. This is the case in many segments of the electronics industry.

A sharp decline was observed in Figure 5.4 around 50% scrap generation even though the mathematical model did not appear to predict such a sharp transition. The reason for this sharp transition is the fraction conforming has a practical ceiling of one hundred percent. Purely applying the model results in an optimal fraction conforming of approximately 130% at zero percent product inspections and 120% at twenty five percent product inspections. However,

since 130% and 120% conforming product is not feasible, the optimal fraction conforming was set at one hundred percent for those points.

Figure 5.5 and Table 5.5 verify the logic used to explain Figure 5.4 by demonstrating the competing influences of the scrap cost and the repair and replacement cost.

Figure 5.5 Effect of non-conforming split on scrap cost and repair & replacement cost

Table 5.5 Effect of non-conforming split on scrap cost and repair & replacement cost

Proportion Repairable/Reworkable	0%	25%	50%	75%	100%
Disposal Cost	\$7,000.00	\$5,250.00	\$3,500.00	\$1,750.00	\$0.00
Repair/Replacement Cost	\$0.00	\$1000.00	\$2,000.00	\$3,000.00	\$4,000.00
Total Non-Conforming Cost	\$7,000.00	\$6,250.00	\$5,500.00	\$4,750.00	\$4,000.00

As mentioned earlier, the summation of the disposal cost and the repair and replacement cost form the basic financial penalty for non-conforming product. When the repairable fraction

is between 0% and roughly 65%, the disposal cost is the primary contributor to the cost of non-conforming product. When the repairable fraction rises above 65%, however, a smaller amount of non-conforming product is discarded, and the repair and replacement cost becomes the dominant factor. In general, by setting the disposal cost from Equation (8) equal to the repair and replacement cost from Equation (8), the proportion repairable/reworkable where these two non-conforming costs are equal can be defined as $w^* = (1 + C_M / d)^{-1}$.

This trend observed in Figure 5.5 can be explained by examining the cost of quality parameters given in Equation (8). When none of the non-conforming product is suitable for repair or rework, the disposal cost (d) is the total non-conforming product cost because no non-conforming product is being repaired or reworked. On the other hand, when all of the non-conforming product is suitable for repair or rework, the repair cost (C_M) is the total non-conforming product cost being no product requires disposal. Furthermore, since the repair cost is typically less than the disposal cost, the total cost of non-conforming product has an inverse relationship to the proportion repairable/reworkable.

Figure 5.6 Effect of non-conforming split on investment cost and joint relevant total cost

Table 5.6 Effect of non-conforming split on investment cost and joint relevant total cost

Proportion Repairable/Reworkable	0%	25%	50%	75%	100%
Investment Cost	\$9,287.89	\$7,433.81	\$5,358.21	\$3,000.90	\$273.21
Joint Total Relevant Cost	\$7,654.09	\$8,133.76	\$8,413.13	\$8,441.27	\$8,145.03

Continuing along the same line of logic, as the percent scrap increases, the financial penalty for non-conforming product rises, necessitating major quality-related process improvements, and therefore, the investment cost also rises. As the scrap portion of non-conforming product decreases, however, the investment cost also decreases.

As depicted in Figure 5.6, the joint relevant total cost curve passes through a maximum around 65% repairable, or reworkable, non-conforming product, which is the same point that the annual disposal and repair and replacement costs are equivalent. This point was previously determined to be $w^* = \left(1 + \frac{C_M}{d}\right)^{-1}$ in the analysis of Figure 5.5 and can be considered the least optimal proportion repairable or reworkable since it yields the highest joint total relevant cost.

5.4 Effect of Type I Errors

In this particular case, Type I errors had a very minor effect on the results of the model. Figure 5.7 illustrates the contribution of Type I errors on the total annual cost for each of the four models considered in this research, for the case study presented.

Figure 5.7 Type I error cost contribution to total annual cost

As indicated, the cost of Type I errors contributed only 0.19% to the total annual cost for the quality adjusted model of the case study presented in this paper. Likewise, Type I errors contributed 0.20% of the total annual cost for the quality adjusted, quality improved model,

0.21% for the quality adjusted setup cost reduction model, and 0.22% for the quality adjusted simultaneous improvement model.

Although the cost of Type I errors had minimal impact on the particular case study featured in this work, this cost parameter should still be included in the general model because of the broad applicability of this research. For instance, especially in the petrochemical industry, products are commonly sold and transported in large, bulk quantities, and in some situations, transportation may involve multiple means, such as truck, railcar and barge, with intermediate transfers from one mode to another. Hence, transportation costs can be particularly high, and a single Type I error can result in the return of a major portion of the annual demand. Therefore, including the Type I error cost in the mathematical models helps to insure that the effect of these errors is properly evaluated and a potentially significant impact on the total annual cost is not overlooked.

5.5 Analysis of Critical Parameters

The equations for critical points clearly provides managers with a highly useful tool for quickly and efficiently determining whether or not to proceed with a full-fledged process improvement investment study. Although a number of critical points can be defined for any of the model parameters, an extensive list of critical points provides little practical value outside of the academic arena, since the very purpose of these values is to provide rapid, uncomplicated decision making tools. Therefore, we must consider the two most common questions that managers ponder when selecting projects: “What is the opportunity for improvement?” and “How effective could this solution be?” These two questions can always be answered using the critical points for demand, inventory carrying charge, and technology coefficients. The other

critical points, such as for repair/replacement cost, inspection cost, and scrap cost, would only be truly useful in limited situations.

The critical point for demand is essential because it evaluates the economies of scale to determine whether or not the product volume itself justifies the possibility of a process improvement. The larger the demand is relative to the critical demand point, the more incentive is present to implement a process improvement to improve quality and/or reduce the setup cost. Likewise, the critical point for inventory carrying charge defines the point at which the inventory carrying charge rate is substantial enough to justify an attempt to improve quality or reduce setup cost. The critical points for the technology coefficients, on the other hand, help managers to evaluate the effectiveness of each process improvement investment dollar. Similar to the demand and inventory carrying charge rate, technology coefficients that lie well above their critical points indicate that the process improvements could be very lucrative and are well worth consideration.

CHAPTER 6

CONCLUSION AND FUTURE RESEARCH

New joint total relevant cost models were developed to address the fraction of product quality inspections conducted, scrap generation and Type II errors. In addition, these models were minimized to yield models for the optimal joint economic lot size and optimal fraction conforming.

Correcting the modeling error of not squaring the fraction conforming in the denominator of the quality adjusted joint economic lot size model in Affisco, *et al.*'s (2002) paper indicates that cooperative commerce models are actually five to twelve percent more effective than previously reported. In addition, introducing a term into the model that allows for less than one hundred percent product quality inspections makes the model more applicable to industrial situations where less than a one hundred percent product inspection policy is employed. More importantly, accounting for the percent of product quality inspections can change the joint relevant total cost by as much as 84.7% as the portion of product inspections varies from zero to one hundred percent, which verifies the critical significance of introducing this new model parameter. Furthermore, it was demonstrated that increasing the fraction of product inspected, increases the optimal fraction conforming, the inspection cost, and the total cost, while decreasing the optimal batch size.

Breaking the fraction non-conforming product down into scrap and repairable, or reworkable, portions has a number of implications on the joint economic lot size model. Specifically, as the proportion of repairable/reworkable non-conforming product increases, the optimal fraction conforming product decreases. Likewise, the total financial penalty for non-

conforming product also decreases. Finally, as the portion repairable/reworkable non-conforming product increases, the investment decreases, and the joint relevant total cost basically increases, after passing through a local maximum at the point where the annual disposal and repair and replacement costs are equivalent.

It was also demonstrated that although Type I errors are somewhat easier to commit than Type II errors, they both may have some effect and should be considered. Finally, the critical points were shown to be useful for setting benchmarks to determine whether or not to proceed with full investment analyses.

6.1 Future Research

1. Future research could investigate the effect of “intelligent” product quality inspection procedures on the joint total relevant cost models, where the frequency of product inspections could increase or decrease, based the number of defects observed.
2. The results of this research could also be applied to joint total relevant cost models with process improvement investments aimed at reducing set up cost, production cost, repair and replacement cost, or inspection cost.
3. Since discrete item repair costs often vary, depending on the defect, the repair and replacement cost could be treated as a matrix, with a number of possible values.
4. This research was applied to deterministic models to achieve closed form solutions. However, a similar approach could be applied to stochastic inventory control models.
5. The comparison between the joint optimization and individual optimization could be studied in more detail.

REFERENCES

- Affisco, J.F., Nasri, F. and M.J. Paknejad, 1988. "Investing in Setup Cost Reduction in the Joint Economic Lot Size Model." *Proceedings of the Decision Sciences Institute*: 1126-1129.
- Affisco, J.F., Nasri, F. and M.J. Paknejad, 1991. "Investing In Setup Cost Reduction In Setup Cost Reduction in the One-Vendor and Many-Nonidentical-Purchasers Joint Economic Lot Size Model." *Proceedings of the Decision Sciences Institute*: 1340-1343.
- Affisco, J.F. Nasri, F. and M.J. Paknejad, 1993. "Reducing Lot Sizes-Analytical Results and Techniques for Implementation," *Management of New Technologies for Global Competitiveness*, Quorum Books, Westport, Connecticut: 351-367.
- Affisco, J.F., Nasri, F. and M.J. Paknejad, 1993. "A Comparison of Alternative Joint Vendor-Purchaser Lot Sizing Models." *International Journal of Production Research*, **31**(11): 2661-2676.
- Affisco, J.F., Paknejad, M.J. and F. Nasri, 2002. "Quality Improvement and Setup Reduction in the Joint Economic Lot Size Model." *European Journal of Operational Research*, **142**(3): 497-508.
- Arinze, B., Kim, S.L., and A. Banerjee, 1995. "A Multicriteria Model for Supporting Setup Reduction Investment Decisions." *Production Planning and Control*, **6**(5): 413-420.
- Billington, P.J., 1987. "The Classical Economic Production Quantity Model with Setup Cost as a Function of Capital Expenditure." *Decision Sciences*, **18**(1): 25-42.
- Banerjee, A., 1986. "A Joint Economic-Lot-Size-Model for Purchaser and Manufacturer." *Decision Sciences*, **17**(3): 292-311.
- Cheng, T.C.E, 1989. "An Economic Production Quantity Model with Flexibility and Reliability Considerations." *European Journal of Operations Research*, **39**(1): 174-179.
- Cheng, T.C.E, 1991. "EPQ with Process Capability and Quality Assurance Conditions." *Journal of the Operational Research Society*, **42**(8): 713-720.
- Dumond, E.J., and W. Newman, 1990. "Closing the Gap Between Buyer and Vendor." *Production and Inventory Management Journal*, **31**(4): 13-17.
- Gitlow, H., Oppenheim, A., and R. Oppenheim, 1995. *Quality Management—Tools and Methods for Improvement, second edition*, Burr Ridge, Illinois.

- Goyal, S.K., 1977. "An Integrated Inventory Model for a Single Supplier-Single Customer Problem." *International Journal of Production Research*, **15**(2): 107-111.
- Goyal, S.K., 1988. "A Joint Economic-Lot-Size Model for Purchaser and Vendor: A Comment." *Decision Sciences*, **19**(1): 236-241.
- Goyal, S.K., and A. Gunasekaran, 1990. "Effect of Dynamic Process Quality Control on the Economics of Production." *International Journal of Operational Research*, **10**(1): 69-77.
- Goyal, S.K., et al., 1993. "Integrating Production and Quality Control Policies: A Survey." *European Journal of Operational Research*, **69**(1): 1 - 13.
- Goyal, S.K., et al., 1995. "A One-Vendor Multi-Buyer Integrated Inventory Model: A Comment." *European Journal of Operational Research*, **82**(1): 209-210.
- Hahn, C.K., Pinto, P.A., and D.J. Bragg, 1983. "Just In Time Production and Purchasing." *Journal of Purchasing and Materials Management*, Fall: 2-10.
- Hayek, P.A., and M.K. Salameh, 2001. "Production Lot Sizing with the Reworking of Imperfect Quality Items Produced." *Production Planning and Control*, **12**(6): 584-590.
- Hill, R.M., 1997. "The Single-Vendor Single-Buyer Integrated Production-Inventory Model with a Generalized Policy." *European Journal of Operational Research*, **97**(3): 493-499.
- Hoffman, C., 1998. "Investments in Modern Production Technology and the Cash Flow-Oriented EPQ-Model." *International Journal of Production Economics*, **54**(2): 193-206.
- Hong, J.D., and J.C. Hayya, 1995. "Joint Investment in Quality Improvement and Setup Reduction." *Computational Operations Research*, **22**(6): 567-574.
- Joglekar, P., and S. Tharthare, 1990. "The Individually Responsible and Rational Decision Approach to Economic Lot Sizes for One Vendor and Many Purchasers." *Decision Sciences*, **21**(3): 492-506.
- Johnson, L.A. and D.C. Montgomery, 1974. *Operations Research in Production Planning Scheduling and Control*. John Wiley and Sons, Inc., New York.
- Juran, J., and F. Gryna, 1980. *Quality Planing and Analysis*. McGraw-Hill Book Company, New York.

- Lam, S.M., and D.S. Wong, 1996. "A Fuzzy Mathematical Model for the Joint Economic Lot Size Problem with Multiple Price Breaks." *European Journal of Operational Research*, **95**(3): 611-622.
- Lee, H.H., Chandra, M.J., and V.J. Deleveau, 1997. "Optimal Batch Size and Investment in Multistage Production Systems with Scrap." *Production Planning and Control*, **8**(6): 586-596.
- Lu, L., 1995. "A One-Vendor Multi-Buyer Integrated Inventory Model." *European Journal of Operational Research*, **81**(3): 312 - 323.
- Nasri, F., Paknejad, M.J. and J.F. Affisco, 1991. "Simultaneous Investment in Setup Cost and Order Cost Reduction in the Joint Economic Lot Size Model." *Proceedings of the Northeast Decision Sciences Institute*, 263 - 267.
- Nasri, F., Paknejad, M.J. and J.F. Affisco, 1990. "Setup Cost Reduction in an Inventory Model with Finite-Range Stochastic Lead Times." *International Journal of Production Research*, **28**(1): 199-212.
- Neave, H.R., 1990. *The Deming Dimension*. SPC Press, Inc., Knoxville, Tennessee.
- Nye, T.J., Jewkes, E.M., and D.M. Dilts, 2001. "Optimal Investment in Setup Reduction in Manufacturing Systems with WIP Inventories." *European Journal of Operational Research*, **135**(1): 128-141.
- Paknejad, M.J., Nasri, F., and J.F. Affisco, 1995. "Defective Units in A Continuous Review (s,Q) System." *International Journal of Production Research*, **33**(10): 2767-2777.
- Paknejad, M.J., and J.F. Affisco, 1987. "The Effect of Investment in New Technology on Optimal Batch Quantity." *Proceedings of the Northeast Decision Sciences Institute*, 118-120.
- Papadakis, G.P., 1985. "The Deming Inspection Criteria for Choosing Zero or 100% Inspection." *Journal of Quality Technology*, **17**(2): 121-127.
- Porteus, E., 1985. "Optimal Lot Sizing, Process Quality Improvement and Setup Cost Reduction." *Operations Research*, **34**(1): 137-144.
- Porteus, E., 1986. "Investing in Reduced Setups in the EOQ Model." *Management Science*, **31**(8): 998-1010.

- Rosenblatt, M.J., and H.L. Lee, 1986. "Economic Production Cycles with Imperfect Production Processes." *IIE Transactions*, **18**(3): 48 - 55.
- Salameh, M.K., and M.Y. Jaber, 2000. "Economic Production Quantity Model for Items with Imperfect Quality." *International Journal of Production Research*, **64**(1): 59-64.
- Tayi, G.K., and D.P. Ballou, 1988. "An Integrated Production-Inventory Model with Reprocessing and Inspection." *International Journal of Production Research*, **26**(3): 1299-1315.
- Tersine, R.J., Barman, S., and R.A. Toelle, 1995. "Composite Lot-Sizing with Quantity and Freight Discounts." *Computational Industrial Engineering*, **28**(1): 107-122.
- Tripathy, P.K., Wee W.M., and P.R. Majhi, 2003. "An EOQ Model with Process Reliability Considerations." *Journal of the Operations Research Society*, **54**(5): 549-554.
- Voros, J., 1999. "Lot Sizing with Quality Improvement and Setup Time Reduction." *European Journal of Operational Research*, **113**(3): 568-574.
- Wu, S.Y., 2002. "Optimal Policy for Set-up Time Reduction in a Multistage Production-Inventory System." *International Journal of System Sciences*, **33**(7): 551-556.
- Yum, B.J., and E.D. McDowell, 1987. "Optimal Inspection Policies in a Serial Production System Including Scrap Rework and Repair: An MILP Approach." *International Journal of Production Research*, **25**(3): 1451-1460.

APPENDIX A

QUALITY ADJUSTED JOINT ECONOMIC LOT SIZE MODEL

Equation (8) Quality Adjusted Joint Economic Lot Size Model

>TC:=((D/(p*Q))*(S+A)+((Q/2)*p*h*G)+((D/P)*(w*(1-p)*Cm+((1-w)*(1-p)*d)+(lambda*Cn)+(Ef+Ec)));

$$TC := \frac{D(S+A)}{pQ} + \frac{1}{2}QphG + \frac{D(w(1-p)Cm + (1-w)(1-p)d + \mathbf{I}Cn + EfEc)}{p}$$

>diff(TC,Q,Q);

$$2 \frac{D(S+A)}{pQ^3}$$

>Diff(TC,Q,Q)=diff(TC,Q,Q);

$$\frac{\partial^2}{\partial Q^2} \left(\frac{D(S+A)}{pQ} + \frac{1}{2}QphG + \frac{D(w(1-p)Cm + (1-w)(1-p)d + \mathbf{I}Cn + EfEc)}{p} \right) =$$

$$2 \frac{D(S+A)}{pQ^3}$$

>diff(TC,Q,p);

$$\frac{D(S+A)}{p^2Q^2} + \frac{1}{2}hG$$

>Diff(TC,Q,p)=diff(TC,Q,p);

$$\frac{\partial^2}{\partial p \partial Q} \left(\frac{D(S+A)}{pQ} + \frac{1}{2}QphG + \frac{D(w(1-p)Cm + (1-w)(1-p)d + \mathbf{I}Cn + EfEc)}{p} \right) =$$

$$\frac{D(S+A)}{p^2Q^2} + \frac{1}{2}hG$$

>diff(TC,p,p);

$$2 \frac{D(S+A)}{p^3Q} + 2 \frac{D(w(1-p)Cm + (1-w)(1-p)d + \mathbf{I}Cn + EfEc)}{p^3} - 2 \frac{D(-wCm - (1-w)d)}{p^2}$$

>Diff(TC,p,p)=diff(TC,p,p);

$$\frac{\partial^2}{\partial p^2} \left(2 \frac{D(S+A)}{p^3Q} + 2 \frac{D(w(1-p)Cm + (1-w)(1-p)d + \mathbf{I}Cn + EfEc)}{p^3} - 2 \frac{D(-wCm - (1-w)d)}{p^2} \right) =$$

$$2 \frac{D(S+A)}{p^3Q} + 2 \frac{D(w(1-p)Cm + (1-w)(1-p)d + \mathbf{I}Cn + EfEc)}{p^3} - 2 \frac{D(-wCm - (1-w)d)}{p^2}$$

>with(linalg):

>Hessian(TC, [Q,p]);

$$\begin{bmatrix} 2 \frac{D(S+A)}{pQ^3} & \frac{D(S+A)}{p^2Q^2} + \frac{1}{2}hG \\ \frac{D(S+A)}{p^2Q^2} + \frac{1}{2}hG & 2 \frac{D(S+A)}{p^3Q} + 2 \frac{D(w(1-p)Cm + (1-w)(1-p)d + \mathbf{I}Cn + EfEc)}{p^3} - 2 \frac{D(-wCm - (1-w)d)}{p^2} \end{bmatrix} > 0$$

APPENDIX B
QUALITY ADJUSTED QUALITY IMPROVED
JOINT ECONOMIC LOT SIZE MODEL

Equation (14) Quality Adjusted Quality Improved Joint Economic Lot Size Model

$$>TC:=((i*(\alpha*\ln(p)+\beta))+(\frac{D}{p*Q})*(S+A))+(\frac{Q}{2}*p*h*G)+((\frac{D}{P})*(w*(1-p)*Cm+((1-w)*(1-p)*d)+(\lambda*Cn)+(Ef+Ec)));$$

$$TC := i(a \ln(p) + b) + \frac{D(S+A)}{pQ} + \frac{1}{2}QphG + \frac{D(w(1-p)Cm + (1-w)(1-p)d + ICn + EfEc)}{p}$$

>diff(TC,Q,Q);

$$2 \frac{D(S+A)}{pQ^3}$$

>Diff(TC,Q,Q)=diff(TC,Q,Q);

$$\frac{\partial^2}{\partial Q^2} \left(i(a \ln(p) + b) + \frac{D(S+A)}{pQ} + \frac{1}{2}QphG + \frac{D(w(1-p)Cm + (1-w)(1-p)d + ICn + EfEc)}{p} \right) =$$

$$2 \frac{D(S+A)}{pQ^3}$$

>diff(TC,Q,p);

$$\frac{D(S+A)}{p^2Q^2} + \frac{1}{2}hG$$

>Diff(TC,Q,p)=diff(TC,Q,p);

$$\frac{\partial^2}{\partial p \partial Q} \left(i(a \ln(p) + b) + \frac{D(S+A)}{pQ} + \frac{1}{2}QphG + \frac{D(w(1-p)Cm + (1-w)(1-p)d + ICn + EfEc)}{p} \right) =$$

$$\frac{D(S+A)}{p^2Q^2} + \frac{1}{2}hG$$

>diff(TC,p,p);

$$-\frac{ia}{p^2} + 2 \frac{D(S+A)}{p^3Q} + 2 \frac{D(w(1-p)Cm + (1-w)(1-p)d + ICn + EfEc)}{p^3} - 2 \frac{D(-wCm - (1-w)d)}{p^2}$$

>Diff(TC,p,p)=diff(TC,p,p);

$$\frac{\partial^2}{\partial p^2} \left(i(a \ln(p) + b) + \frac{D(S+A)}{pQ} + \frac{1}{2}QphG + \frac{D(w(1-p)Cm + (1-w)(1-p)d + ICn + EfEc)}{p} \right) =$$

$$-\frac{ia}{p^2} + 2 \frac{D(S+A)}{p^3Q} + 2 \frac{D(w(1-p)Cm + (1-w)(1-p)d + ICn + EfEc)}{p^3} - 2 \frac{D(-wCm - (1-w)d)}{p^2}$$

>with(linalg):

>Hessian(TC, [Q,p]);

$$\begin{bmatrix} 2 \frac{D(S+A)}{pQ^3} & \frac{D(S+A)}{p^2Q^2} + \frac{1}{2}hG \\ \frac{D(S+A)}{p^2Q^2} + \frac{1}{2}hG & -\frac{ia}{p^2} + 2 \frac{D(S+A)}{p^3Q} - 2 \frac{D(w(1-p)Cm + (1-w)(1-p)d + ICn + EfEc)}{p^3} - 2 \frac{D(-wCm - (1-w)d)}{p^2} \end{bmatrix} > 0$$

VITA

Erick Comeaux was born in Baton Rouge, Louisiana, on July 16, 1974. He grew up in New Roads, Louisiana, and graduated as valedictorian of Catholic High School of Point Coupée. He then received his Bachelor of Science degree in chemical engineering in 1997, from Louisiana State University's Honor's College.

His professional career started with Louisiana State University's Macromolecular Studies Group, as a Research Associate, before joining Georgia Gulf Corporation as a Research and Development Engineer. Currently, Erick is a Production Superintendent within the Polymer's Business Unit at Crompton Corporation's Geismar facility, and is registered with the Louisiana Board of Professional Engineers and Land Surveyors. Throughout his career, he has been involved with the development, synthesis, characterization, manufacturing, processing, and applications of a wide range of polymers.

Erick joined the Industrial and Manufacturing Systems Engineering Department of Louisiana State University part-time in August of 2000, and expects to receive his Master of Science degree in May of 2004. His primary research interests are in material science, polymer engineering, operations management, lean manufacturing systems, production planning and control, and supply chain management.