10-14-2014

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Recommended Citation

Mirhosseini, M., Magaña-Loaiza, O., Hashemi Rafsanjani, S., & Boyd, R. (2014). Compressive direct measurement of the transverse photonic wavefunction. *Frontiers in Optics, FiO 2014* https://doi.org/10.1364/fio.2014.fm4e.5

Compressive Direct Measurement of the Transverse Photonic Wavefunction

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Abstract: We generalize the method of direct measurement and combine it with compressive sensing. Using our method, we measure a 19200-dimensional state using only 20% of the total required measurements.

OCIS codes: 270.0270, 270.5585, 100.5070.

Determining an unknown wavefunction is of fundamental importance in quantum mechanics. Despite many seminal contributions, this task remains challenging for high-dimensional states. The direct measurement (DM) approach, introduced by Lundeen *et. al*, has provided a ground for meeting the high-dimensionality challenge [1]. Contrary to state tomography, this methods does not require a time-consuming post-processing. Nevertheless, the number of measurements required by the direct measurement protocol grows linearly with the dimensionality of the measured state. Here we combine a novel computational method known as compressive sensing with the direct measurement technique. Utilizing our approach, the wavefunction of a high-dimensional state can be estimated with a high fidelity using much fewer number of measurements compared to the standard direct measurement.

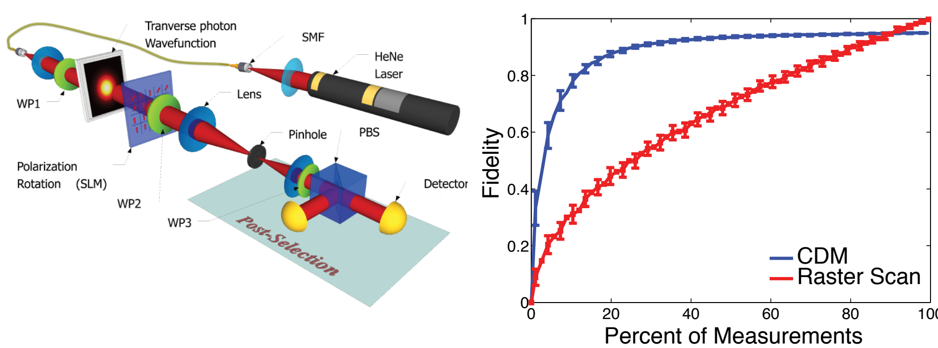

Fig. 1. A schematic illustration of the experimental setup (left). The fidelity of a reconstructed Gaussian state with the target wavefunction, shown in blue, as a function of the percentage of the total measurements (right). The fidelity of the state reconstructed from a partial pixel-by-pixel scan with the same number of measurements is shown in red for comparison.

A weak value is the expectation value of a weak measurement that is followed by a post-selection [2]. Consider a weak measurement of the position projector $\hat{\pi}_j = |x_j\rangle\langle x_j|$ at point x_j followed by a post-selection on the zeroth component of the Fourier transform of the spatial wavefunction, which we denote by $|o\rangle$. The complex wavefunction of a photon can be calculated at each point by measuring the real and imaginary part of the weak value as

$$\pi_w = \frac{\langle o|x_j\rangle\langle x_j|\psi\rangle}{\langle o|\psi\rangle} = \frac{\psi(x_j)}{\phi_0\sqrt{N}}. \tag{1}$$

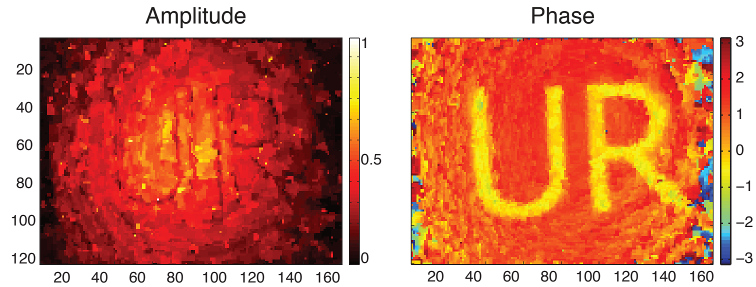

Fig. 2. The amplitude and phase of a Gaussian mode illuminating a custom phase mask (the initials of the University of Rochester). The data is reconstructed by the CDM method with N=19200, and $M/N = 20\%$ of total measurements.

Here we have used the Fourier transform property $\langle o|x_j\rangle = 1/\sqrt{N}$ where N is the dimension of the Hilbert space and $\phi_0 = \langle o|\psi\rangle$.

We generalize the DM to a form suitable for compressive sensing. Let the initial system-pointer state be $|\Omega\rangle = |\psi\rangle \otimes |V\rangle = \sum_{i=1}^{N} \psi_i |x_i\rangle \otimes |V\rangle$, where we have assumed to have a discrete Hilbert space for the spatial degree of freedom $|\psi\rangle$ and a two-level system such as the polarization of a single photon for the pointer state $|V\rangle$. We consider a situation where instead of a measuring a projector $\hat{\pi}_j$ we perform a weak measurement of the operator $\hat{Q}_m = \sum_j Q_{m,j}\hat{\pi}_j$ where the coefficients $Q_{m,j} \in \mathbb{R}$. In this situation the imaginary and the real part of ψ_j, $\Im[\psi_j]$ and $\Re[\psi_j]$, can be related to the expectation values of the polarization of the post-selected state $\bar{\sigma}_{x,m}$ and $\bar{\sigma}_{y,m}$ via a linear set of equations $\phi = \mathbf{Q}\ \psi$.

Here, $\phi_m = \frac{1}{\kappa}[\bar{\sigma}_{x,m} + i\bar{\sigma}_{y,m}]$ and $\kappa = \frac{2\alpha}{\phi_0\sqrt{N}}$. The numbers $m \in \{1:M\}$ and $n \in \{1:N\}$, where M is the total number of sensing operators and N is the dimension of the Hilbert state of the unknown wavefunction. To find the wavefunction ψ we need to (approximately) solve this linear system of equations in the case where $M \ll N$. A nonlinear strategy can be used to recover ψ with a high quality using the idea of compressive sensing (CS). If the wavefunction under the experiment ψ is known to have very few non-zero coefficients under a linear transformation $\mathbf{T}$, it can be reconstructed with a high probability by solving the convex optimization problem [3]

$$\min_{\psi'} ||\mathbf{T}\psi'||_{\ell_1}\ \text{, subject to } \mathbf{Q}\psi' = \phi. \tag{2}$$

Fig. 1 shows the schematics of the experiment. A vertically polarized Gaussian mode is prepared by spatially filtering a He-Ne laser beam with a single mode fiber and passing it through a polarizer. A random polarization rotation at each point is performed using a spatial light modulator (SLM) in combination with two quarter wave plates (QWP) [4]. To provide a quantitive comparison of the two methods we calculate the fidelity between a retrieved Gaussian state $|\psi'\rangle$ and the state $|\psi\rangle$ from a full pixel-by-pixel scan (See Fig. 1). We prepare a custom target state by illuminating phase mask depicting letters U and R with a phase jump of $\pi/2$ with a Gaussian beam. Figure 2 shows the amplitude and the phase of the reconstructed state with $M/N = 20\%$ of the total measurements. Notice that while the amplitude is relatively uniform, the phase shows the letters U and R with a remarkable accuracy.

To conclude, we have demonstrated high fidelity reconstruction of spatial states using the compressive direct measurement (CDM) method. This technique can be used for measurement of high-dimensional quantum states as well as classical applications such as wavefront sensing.

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