Passive alignment of micro-fluidic chips using the principle of elastic averaging

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PASSIVE ALIGNMENT OF MICRO-FLUIDIC CHIPS USING THE PRINCIPLE OF ELASTIC AVERAGING

A Thesis

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College
In partial fulfillment of the Requirements for the degree of Master of Science in Mechanical Engineering

in

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by

Sitanshu Gurung
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ABSTRACT

This thesis presents a novel design for a passive alignment feature for vertically stacking chips in a micro fluidic device. Each chip is the size of a standard microscopic slide (75mm×25mm) and is a highly specialized functional unit designed and fabricated to process fluidic samples. The fluidic samples in the device flows from one chip to another using interconnecting holes 500μm-100μm in diameter. Hence an alignment of better than 100μm between chips is necessary to allow for the interconnecting holes to overlap so as to provide with a path for the fluid to flow. The feature to be used for alignment consists of v-grooves and dowel pins. Five v-grooves are hot embossed to each longer side walls of the chip such that the groove runs along the thickness of the chip. The dowel pins are fixed to a base and equal the number of v-grooves. An elastic fit in between the groove and the pin provide the necessary alignment. A mathematical model has been developed and is used to predict the alignment of the method. The results from the model have been further verified experimentally.
CHAPTER 1
GENERAL INTRODUCTION

1.1. Background

The idea of miniaturizing devices to mechanically manipulate matter at a molecular level was proposed by Dr Richard P. Feynman in his famous 1959 lecture at Stanford “There is plenty of room at the bottom”. One of his ideas was to be able to print volumes of the encyclopedia on a single head of a pin using a micro machine. The first real micro devices were developed in the field of electronics during the 1960s using miniaturized and compact electrical components such as transistors, capacitors, and resistors to obtain increased performance. As the individual electrical components got smaller in size, a lot of these components could now fit into a much smaller footprint on a common substrate keeping the cost, space requirement and energy consumption low. Since then micro electronics fabrication concepts have been expanded to fields other than electronic devices. During the 1980s, mechanical components like gears, pumps, channels etc were miniaturized and incorporated with electrical components such as sensors and actuators on silicon substrates using micro-fabrication technology. That led to the advent of MEMS (Micro electro-mechanical systems) devices. These MEMS devices were smaller, faster, inexpensive, more sensitive and accurate. They found application in all areas from life sciences to military and space applications but most commonly in consumer goods in the form of air bags in automobiles [1].

In principle MEMS devices are able to interact with the samples at a molecular level which is particularly advantageous in fields of biology and biochemistry as it greatly enhances the performance of the device. These devices are commonly known as Bio-MEMS devices and find use in processing, detection and analysis of biological samples [2]. One advantage of
miniaturization is that as the features on the device are on the scale of the samples, the interaction between them is much more efficient. Other advantages of using miniaturized features are minimal reagent use, quick response time and a compact and portable system. A miniaturized PCR (polymerase chain reaction) device is an example of a Bio-MEMS device used for the multiplication of DNA. The DNA sample is subjected to cyclic heating and cooling using micro heaters. Chemical reagents are supplied in micro wells. Reagents and the sample are mixed in micro mixers and special micro channels allow for a defined flow path [3]. Here we see an example of a complete Bio-MEMS device where the biological sample is acted upon by chemical reagents with the aid of specialized yet miniaturized mechanical and electrical features.

A Bio-MEMS device in most cases is a specialized micro fluidic device as most biological samples are either fluids or at-least suspended in a fluid. The processing of fluids requires micro channels, valves, pumps and other fluidic components. All these components are integrated along with electrical, optical and mechanical components to form a functional micro fluidic device. Each micro fluidic device is a combination of different components working together to obtain the desired result in an easily readable data format. The functions of individual components in the device could be sample preparation, processing of the sample, analysis of the sample and the generating the output or any one of the intermediate steps (Figure 1.1). In case of a Bio-MEMS device the input could be a drop of blood, a few strands of DNA, an enzyme, chemical reagents etc. The process could involve amplification of the sample, breaking down the sample, mixing of the sample and the output could be in the form of an electrical readout or maybe a certain spectrum of light. A blood glucose monitor for diabetics (Accu-check ®) (www.accu-check.com) is an example of a commercially available specialized Bio-MEMS device. The sample in this case would be a drop of blood and the output would be a numerical reading of the glucose level in the sample. Another example of an experimental Bio-MEMS
device is an array of thermo electric coolers used to cool individual cells for long term storage [4].

![Diagram of vertically stacked micro fluidic device]

Figure 1.1 Basic architecture of a vertically stacked micro fluidic device showing the different components.

Material selection for the components is an important factor when developing a Bio-MEMS device. The choice of material is largely dependent on the type of application and also on the “micro machining” processes available to work with that material. Historically silicon has been a natural choice of material for MEMS devices using well established micro fabrication techniques from the semi-conductor industry. Newer fabrication techniques have also led to the use of glass and polymers. Polymers are well suited for a lot many fluidic applications because of their compatibility with their samples and ease in fabrication. Polymer components are also cheap to fabricate, so disposable components can be fabricated, especially important for biological samples to prevent any cross contamination. The components under study in this thesis are fabricated out of polymers using hot embossing. Details of the fabrication process will be discussed in Chapter 2.
The next important step in the development of a device is the assembly and packaging of the components to form the device. One option is to vertically stack the different components (Figure 1.1) where the sample flows from one level to the next level through well aligned through holes. An alternative scheme to arranging the different components has been introduced by ThinXXS® (www.thinxxs.com) where the components are placed side-by-side and connected using interconnecting pipes. However the drawbacks of a horizontally arranged device are that they have a comparatively large fluidic dead volume as the adjacent chips have to be connected with tubes. A horizontal arrangement also leads to a larger device which reduces portability. Relatively a vertically stacked device has minimal dead volume and is much more portable and compact in size [5].

1.2. Objective

The objective of this thesis is to present a method for alignment for a vertically stacked micro fluidic device. The importance of alignment is shown in figure 1.2.

![Diagram of vertically stacked micro fluidic device with alignment examples]

Figure 1.2. Importance for alignment in a vertically stacked micro fluidic device.
A well aligned stack is necessary to provide a continuous path for the fluids to flow from one level to another. A poorly aligned stack would cause discontinuity in the flow of the fluid and the device would not function as desired. The alignment between the different levels is obtained by specialized and precisely placed alignment features. This thesis deals with one specific aspect in the development of a vertically stacked micro fluidic device: alignment of the different modules in a vertical stack. The alignment features are simple, inexpensive, and are fabricated in the same process step together with the micro structures. They utilize the principle of elastic averaging providing alignment performance comparable to that obtained from similar deterministic features while having higher load carrying capacity and ability to accommodate for fabrication tolerances of individual alignment features.

The focus of the thesis is the fabrication of the chips with the alignment features and optimizing the alignment performance. A mathematical model was developed and used to predict the alignment accuracy obtained by those features. Next an experimental verification of the mathematical model is done and the results are summarized with suggestions for possible future design improvements. Chapter 2 describes the fabrication process of the individual components and the post processing steps required to obtain the components in ready to use form. Chapter 3 starts with the description of the actual alignment features, the theory involved and a mathematical model based on the actual features to predict the alignment accuracy. The result of the simulation from the model is presented at the end of Chapter 3. The experimental setup and the alignment measurement methodology are discussed in Chapter 4. The final chapter summarizes the findings and presents a design for a newer feature that could improve the alignment and overcome some of the shortcomings of the present design.
CHAPTER 2

FABRICATION

Building a micro fluidic device begins with the design and fabrication of the individual components. The design of each component “chip” is very specific and highly dependent on its function. The fabrication process of components can be generic as standard micro fabrication procedures can be applied. In this thesis, the chips have been fabricated by hot embossing of polymers followed by some post processing, making each chip ready for use. This chapter describes the hot embossing process and the different post processing steps used for the fabrication of chips and the alignment features present on them.

2.1. Hot Embossing

Thermoplastic polymers can be formed into desired shapes using micro fabrication techniques such as injection molding [30] and hot embossing [31]. Hot embossing is the process of transferring a pattern from a metal template onto a polymer piece under elevated conditions of temperature and pressure (Figure 2.1). Hot embossing is a rapid prototyping technique that may be used to replicate high quality components in a short period of time while keeping the fabrication costs at a minimum.

![Figure 2.1 Schematic of hot embossing](image)

Molding of the components starts with the fabrication of the metal template (mold insert). Micro milling and LiGA (German acronym for lithography, electroplating and molding) are
some of the processes that can be used to fabricate the mold insert depending on the size of the structures. High quality nickel and nickel-iron mold inserts with structures having dimensions ranging from 10μm to 1mm with aspect ratio of up to 20 can be fabricated by LiGA process. However, the high fabrication cost and long fabrication time is only suitable for final designs and critical structures. Micro milling on the other hand can be used to machine brass, steel and aluminum mold inserts at a lower cost even though the minimum dimension of the feature is much larger and the quality and aspect ratio are much lower when compared to a LiGA mold insert but typically meet the demand of micro fluidic applications [5]. All the mold inserts for the fabrication of micro fluidic chips in our case were fabricated using micro milling. Brass blanks were milled on the Kern ® Micromilling machine to obtain the mold inserts [33] (Figure 2.2).

![Fabrication Process of Micro Fluidic Chip](image)

**Figure 2.2** The fabrication process of a micro fluidic chip

### 2.1.1. Details of Hot Embossing

Hot embossing of all the polymer chips for the micro fluidic device was carried out on a Jenoptik Mikrotechnik (http://www.jo-mt.de) HEX02 machine. The motion of the machine can
be controlled with a precision of 1 μm, the temperature is uniform to within ±2°C over a 100 mm diameter molding area and the machine uses a patented active demolding technology to separate the mold and the part after forming. Molding is done inside a vacuum chamber to keep air from being trapped in the micro-structures during the process. The molding process is controlled by a computer program interfaced to the molding machine.

The molding time, temperature and force are dependent on the type of polymer and the design of the mold. One important parameter for molding polymers is the glass transition temperature (Tg). The glass transition temperature is where the polymer becomes a viscous liquid able to flow and fill the insert cavities. This is the ideal temperature for molding. Details of the effect of other parameters on the different type of polymers can be obtained in the PhD dissertation of P. Datta [5]. A sample code for molding polymer is shown in Figure 2.3. Explanation of the code is given below.

<table>
<thead>
<tr>
<th>Code</th>
<th>Line Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initialize force Control(True/False)--------------</td>
<td>(1)</td>
</tr>
<tr>
<td>Heating (Top = 20°C, Bottom = 20°C)----------------</td>
<td>(2)</td>
</tr>
<tr>
<td>Close Chamber()----------------------------------</td>
<td>(3)</td>
</tr>
<tr>
<td>Evacuate Chamber()-------------------------------</td>
<td>(4)</td>
</tr>
<tr>
<td>Show chart window(Show/hide=11/0)----------------</td>
<td>(5)</td>
</tr>
<tr>
<td>Position relative(Position=14.000mm, Velocity=15.000/min)</td>
<td>(6)</td>
</tr>
<tr>
<td>Touch Force(Force=200N)----------------------------</td>
<td>(7)</td>
</tr>
<tr>
<td>Temperature&gt;(Temperature=18deg, channel=4)-------</td>
<td>(8)</td>
</tr>
<tr>
<td>Heating (Top = 21°C, Bottom = 19°C)---------------</td>
<td>(9)</td>
</tr>
<tr>
<td>Force-Force Controlled(force=15N,velocity=0.3mm/mm)</td>
<td>(10)</td>
</tr>
<tr>
<td>Wait Time(Time=675.0sec)--------------------------</td>
<td>(11)</td>
</tr>
<tr>
<td>Cooling (Top = -130°C, Bottom = -130°C)-----------</td>
<td>(12)</td>
</tr>
<tr>
<td>Temperature&lt;=(Temperature=45deg, channel=4)------</td>
<td>(13)</td>
</tr>
<tr>
<td>Temper(Top = 12deg, Bottom = 11deg)---------------</td>
<td>(14)</td>
</tr>
<tr>
<td>Open Chamber fast()-------------------------------</td>
<td>(15)</td>
</tr>
<tr>
<td>Unlock door()-------------------------------------</td>
<td>(16)</td>
</tr>
</tbody>
</table>

Figure 2.3 Sample code for Hot-embossing © CAMD, Jefferson Hwy (Details such as temperature, force have been changed)
• The first step in molding is to heat the mold insert and the polymer. While heating occurs the molding chamber slowly closes and the air is evacuated to form a vacuum (code lines 1-4).

• The mold moves slowly towards the polymer till they are in contact. No pressure is applied till the molding temperature is reached. The molding temperature is around or greater than the glass transition temperature of the polymer (code lines 6-7).

• After the molding temperature is reached a force is applied. This is the actual molding step (code lines 8-10).

• After the maximum applied force is reached, the mold and polymer are held at that pressure for a certain time to achieve good mold fill (code line 11).

• The mold temperature is reduced below the glass transition temperature and the mold and polymer are separated. Demolding starts after the temperature reaches that in line 13 (code lines 12-13).

• The molding chamber opens and returns to the original position so that the molded part can be removed and a new polymer substrate can be placed with machine for molding (code lines 15-16). Typical time for the entire process is usually between 10 minutes to 20 minutes.

The molded polymer part is not yet ready for use and has to go through post processing in order to obtain the ready to use chips.

2.2. Post Processing

Post processing in our case involves cutting the chip to the desired finish, cleaning the cut chip and then sealing the chip to obtain the desired finish. The molded chip at this point is formed as part of a much larger sheet of polymer (Figure 2.2). Excess polymer is present both on
the sides of the chip as well as in the thickness. The polymer on the sides needs to be removed to obtain the desired shape of the chip and the thickness needs to be reduced so as to open up the through holes which are fabricated in the chip to allow the sample to flow from one level to another. The thickness required can be obtained by fly cutting. Depth of cut can be controlled precisely to within 5μm by fly cutting. Fly cutting is done by fixing the sample to be cut onto a vacuum chuck while a rotating diamond tool cuts off the excess polymer until the desired thickness is reached.

The fly cut samples are then cleaned by placing them in an ultra sound bath with soap water for 2 minutes. The sample is then rinsed with Isopropyl alcohol and deionized water and is finally blown dry. The individual chips are not yet ready to be used as the micro channels present
in the chip are still open. Depending on the design of the chip, a second micro fluidic chip or a polymer cover sheet is bonded to the open chip face to seal the micro fluidic channels. For thermal sealing of microscopic slide format chip, a cover slip is placed on top of the chip. The two chips are then sandwiched between two glass pieces and then between copper plates. A force of about 800-1000 N is applied using six springs for a period of about 20 minutes at the glass transition temperature. Typically thermal sealing for PMMA chips was done at 105°C (glass transition temperature of PMMA).

Figure 2.5 Sonicator for ultra sound cleaning  
Figure 2.6 Fixture for thermal sealing of polymers

After post processing the chips are now ready to be aligned and used for the desired micro fluidic application. Post processing steps such as UV treatment or surface modification for specific biological applications may be performed according to requirement. The goal of all these post processing steps is to make the chip ready to use as a standalone device or as a component in more complicated micro-fluidic device [6, 20, 26]
CHAPTER 3

ALIGNMENT

The objective of this thesis is to design and fabricate features which can be used to align a multi-level micro fluidic stack. Each level in the stack is a polymeric chip the size of a standard microscopic slide (75mm × 25mm) which has been designed and fabricated to perform a specific function. Thickness of the chips in each level ranges from 1mm-3mm. The different levels are interconnected using through holes which allow for fluid to flow from one level to another (Figure 1.2). This flow is only possible if the chips (components) and hence the interconnecting hole in one level is aligned to the chip and the interconnecting hole in the next level. The size of these interconnects ranges from 500μm to 100μm. Hence the alignment between two chips in our case has to be better than 100μm for fluid to flow from one level to the next. Any alignment feature that we design and incorporate into the vertically stacked micro fluidic device has to achieve this degree of alignment.

3.1. Alignment Methodologies

There are basically two different kinds of methodologies available for the purpose of passive alignment. Passive alignment features are generally mechanical structures which provide the necessary alignment by means of coupling. The types of contact in between the coupled interfaces can be used to distinguish between the different types of couplings. The basic types of contact are surface, line and point contact, generally in increasing order of accuracy and repeatability [28].

3.1.1. Kinematic Couplings

The contacting members in case of a kinematic coupling make a point contact and hence provide the highest degree of accuracy and repeatability. A kinematic coupling makes use of
only the minimum number of constraints to restrict motion of a body in specified directions while keeping it free to move or rotate along all other degrees of freedom. The numbers of these constraints are usually equal to or less than the number of degrees of freedom to be constrained. A 3 ball 3 groove coupling is an example of a kinematic coupling. The number of degrees of freedom to be constrained is three, X-axis, Y-axis and rotation about the z-axis (where the z-axis is perpendicular to the surface), and the number of constraints is equal to three as well.

![Diagram of 3 ball 3 groove coupling](image)

Figure 3.1 Two circular plates being coupled using a 3 ball 3 groove coupling.

However, a kinematic coupling does have some drawbacks. To obtain high repeatability from a kinematic coupling the contacting points must have a high surface finish. Any manufacturing tolerance will adversely affect the surface finish and hence the repeatability of a kinematic coupling. The point contacts must also bear all the loads and are prone to undergo deformation. This limits the amount of load that can be applied. The load capacity of this type of coupling is hence limited by Hertzian stresses [16] or the stress produced by the deformation of the contacts. Friction and shear stress can cause wear of the surface and this would limit the repeatability as well as the accuracy of the contacting members.
3.1.2. Elastic Averaging

The type of contact in elastic averaging is a mixture of a line contact and a surface contact. Elastically averaged coupling is based on the principle that alignment can be obtained by using a large number of fairly compliant contacting members. A coupling using elastic averaging is basically an over constrained system with each constraint being fairly elastic [16]. The optical fiber connector shown in Figure 3.2 uses the principle of elastic averaging. Details of the working of the connector is presented in section 3.4.

![Figure 3.2 An optical fiber connector utilizing the principle of elastic averaging [16]](image)

The alignment occurs in elastic averaging as multiple elastic members compete to locate the final position of the body. This method of alignment is useful where the individual members may not be located precisely but the many contacting members would be able to average out the positioning error or fabrication tolerances and provide the necessary alignment. The type of contact also allows an elastically averaged coupling to bear higher loads while developing much lower contact stresses when compared to a coupling which utilizes point contact.
3.2. Alignment Feature

The basic design of an alignment feature for most applications involves a projection and a recess [28]. As the projection fits into the recess the components are coupled and hence aligned to one another. The 3 ball 3 groove coupling or a derivative of this basic design is one of the most commonly used kinds of alignment features. The quality of the alignment required and the degrees of freedom to be constrained dictate the number, design and placement of the features.

Elastically averaged fit is commonly seen in day to day objects made out of polymers. One of the most common examples of an elastically averaged fit is seen in LEGO® blocks. The LEGO blocks work by the mating of projections and recesses on two different blocks. Inherent fabrication tolerance in the block gets averaged out among the contacting members providing with a cheap and easy method of coupling two bodies to one another. The same principle can be applied for coupling two chips in our case too. The chips in the micro fluidic stack are made of polymer and are fabricated by hot embossing. Dimensional variations of the alignment features are observed between different chip designs and even within the same chip [Table 4.1]. Hence an elastically averaged fit is a method of choice of working for the alignment features in our case. Another advantage of using elastic averaging is its higher load bearing capacity and lower stresses at the contacts as the chips will be subjected to considerable thermal stresses during post processing for example thermal sealing (section 2.2). The alignment features that we use must also address the following issues.

- Ease of Fabrication
- Ease of Assembly
- Minimize Space Usage
• Compatibility with post processing

3.2.1. Ease of Fabrication

Integrating alignment features onto the components for a vertically stacked micro fluidic device poses a problem for fabrication using hot embossing. If alignment features are required on both (top and bottom) surfaces of the chip then this would require that the polymer be embossed simultaneously from both sides using two inserts. This results in an inherent loss of accuracy between the top and bottom surface as only limited alignment can be obtained between the two mold inserts. (An alignment accuracy of about 20-50μm can be obtained between the two molds in the Hex02 Jenoptik machine)

![Diagram showing different layers of chips in a vertical stack with alignment features](image)

Figure 3.3 The different layers of chips in a vertical stack with the alignment features

One solution to this problem is to place the alignment features such that they can be formed using a single mold insert. This can be achieved by using alignment features on the side walls instead of the top and bottom surface.

3.2.2. Ease of Assembly

The use of any alignment feature on the top and bottom surface would cause a gap in between the surfaces. This is undesirable especially in a vertically stacked micro fluidic device because it would cause a discontinuity in the path of the flow of the liquid where the two levels
of chips are interconnected by simple alignment of the through holes and no interconnecting tubes are present.

![Diagram showing the gap between surfaces and interconnecting holes]

Figure 3.4 Gap between the surfaces when a conventional feature is used.

### 3.2.3. Minimize Space Usage

Each micro fluidic chip is the size of a standard microscopic slide (75mm × 25mm). We have very limited space for the various features to be “machined” onto the face of each chip let alone try and fit the alignment features on to it. Any alignment feature placed on the top and bottom surface will decrease the available space for other micro-fluidic features. Hence the design for the features should either not use the top and bottom surface or use only minimal space on them.

### 3.2.4. Compatibility with Post Processing

A fabricated chip with all the components will also need to be subjected to post processing such as fly-cutting and sealing. Fly-cutting is done to obtain the required thickness of the chip and to open the interconnecting through holes. Any alignment feature on the top and bottom surface hinders with the process of fly cutting. Thermal sealing is done to enclose the channel which is necessary to provide a path for the liquid to flow (section 2.2). Placing the features on the top and bottom surface would make it harder to seal the channels as they would hinder with the placement of the cover on top of the chip.
3.3. Design

Based on the constraints defined above a design for the alignment feature has been proposed. The alignment features to be used consists of v-grooves cut out on each longer side wall of the chip and a separate base with dowel pins (Figure 3.5). The chips with the v-grooves slide into the base with the dowel pins which provides the necessary alignment. The v-grooves are on the longer side walls of the chip cut at an angle of 90°. The height of each groove is equal to the thickness of the chip. There are five grooves on each longer side wall hence each chip has a total of ten grooves. Each dowel pin is 1mm in diameter and there is a total of ten dowel pins in the base.

![Figure 3.5. The alignment features (a) The concept (b) The geometry (c) The actual features](image)

The alignment feature discussed above is based on the principle of elastic averaging. This allows the use of alignment features with higher manufacturing tolerances able to withstand higher loads and contact stresses. The basic working of the alignment method is shown in Figure 3.6. The chip shown in dotted lines is being aligned to the chip shown in the continuous line.
Initially there exists a force and moment at the center of the dotted chip. After being aligned with respect to the chip in the continuous line the total force and moment at the center becomes equal to zero and the chip comes to a position of equilibrium.

Figure 3.6 Working of the alignment method.

In Figure 3.6 initially as the chip is placed into the base a resultant force and moment acts in the center. Elastic averaging occurs and the chip moves from its initial position (shown in dotted lines) to the final position where no force or moment acts on the center of the chip. The chip hence comes to a position of equilibrium.

3.4. Literature Review

There has been a varied use of elastic averaging described in the literature. One of the earliest examples of elastic averaging has been in the production of diffraction grating using the Merton nut designed by Sir Thomas Merton [9, 12]. Elastic averaging has also been used in the assembly of micro mechanical parts. Using a multi-beam parallelogram flexure model, the influence of geometric imperfection on the primary stiffness of the model has been given by Awatar [27]. Backlash is one problem faced by designer working with mating gears. Backlash causes a gap in time before the driver gear actually mates with the driven gear and starts rotating it. Mahadevan [11] has proposed the use of self interlocking tapers which utilize elastically averaged teeth to prevent backlash.
The use of elastic averaging in the alignment of chips and wafers is a new concept and Alexander Slocum [13] has been one of the forerunners in this area. He has developed a set of elastically averaged features for precision passive alignment of multiple wafers. The alignment features in his case are a row of concave v-shaped trenches which are built on one wafer. The other wafer has a set of pyramid shaped structures. These pyramid shaped structures are formed on the ends of cantilever beams. The other end of the cantilever is joined to the wafers. As the two wafers are brought into contact the pyramid structures engage into the v-shaped trenches (Figure 3.7). Any error in the structures is overcome by the bending of the beam. Elastic averaging here occurs by the bending and deformation of the beams as the two wafers get aligned to each other.

![Figure 3.7 Schematic of the alignment of the wafers using elastic averaging](image)

Patrick Willoughby [16] has proposed a new design for a connector for joining optical fibers which uses the principle of elastic averaging. Two trapezoidal halves with v-grooves are put together to form a hexagon (Figure 3.8). The optical fibers are placed in the v-grooves and then the two trapezoidal halves are put together to form a hexagonal ferrule. The elastic deformation of the groove-cylinder interface when averaged over a large number of grooves aligns the first trapezoid to the second hence forming the required hexagonal ferrule. Two such ferrules are aligned to each other using a thin circular piece of metal extrusion which makes
contact at 12 points. (The circular metal is not shown in the figure; only the two contacts on each wall of the hexagon are shown) The circular piece of metal also known as sleeve aligns two ferrules together as they deform elastically in the radial direction.

Figure 3.8 Showing an optical fiber connector which utilizes the principle of elastic averaging.

3.5. Mathematical Model

A mathematical model based on the design of the alignment features (Section 3.3) and the principle of elastic averaging will be presented in this section. The formulated model describes the translation of the center of one chip with respect to another. The basic idea behind alignment of two chips is that smaller the translations of the center of one chip with respect to another, better the alignment and vice-versa. The detailed mathematics is presented in the Appendix A.

3.5.1. Theory

The theory states that one body can be aligned to another body by over constraining provided that the constraining members are fairly elastic. The reasoning behind using our method for alignment is that both the dowel pins and v-grooves are fairly elastic yet have inherent fabrication tolerances. Formulation of the mathematical model of the problem involves representing each of the contacting members as springs and then applying randomly generated
tolerances to it. These randomly applied tolerances in real life would represent the unavoidable fabrication tolerances present in each of the contacting members. The range for this randomly generated tolerance is derived from measurements of individual chips [Table 4.1]. The springs on the other hand represent elasticity of each of the contacting members. Over constraining a body using these springs with randomly generated tolerances based on actual measurements would be a good approximation of a body which has been constrained using a large number of flexible contacting members (with inherent fabrication errors). Using these tolerances and the stiffness matrix (obtained from the springs) the local contact forces can be calculated. Force equations, moment equations and constraining equations can then be used to find the final locations of the contact points and hence the corresponding center points, respectively.

The mathematical model can be broken down into the following steps [16].

- Represent each of contacting members as springs and calculate the individual stiffness.
- Force calculation at each of the contacting members.
- Merge the local stiffness matrices.
- Find the forces and moment in the global co-ordinates.
- Combine the force, moment and constraint equations.
- Find the deflection of each of the contact points and the final deflection of the center.

3.5.2. Modeling Steps

- **Step 1- Represent each of the contacting members as springs.**

   Each of the contacting members (the v-grooves and the dowel pins) must be represented by springs to form an analytical solution to the problem. Each contact is represented by two springs; one representing the beam bending ($k_{\text{geom}}$) and the other the polymer deformation ($k_{\text{hertz}}$).
After denoting each of the contact as a pair of springs, the individual stiffness of each of the contact can be calculated as

\[
\frac{1}{k_{total}} = \frac{1}{k_{geom}} + \frac{1}{k_{hertz}} \quad (3.1)
\]

Where,

\[
k_{geom} = \frac{8EI}{l^3}
\]

and \(K_{hertz}\) represents the total stiffness of each of the contact. The derivations of \(k_{geom}\) and \(k_{hertz}\) are shown in Appendix A.

- **Step 2- Force calculation at each of the contacting members.**

Figure 3.9 Showing the actual chip as springs and a single contact with the corresponding stiffness

The next step involves arranging the stiffness of each of the contact points in matrix form. Each pair of v-groove and dowel pin will have two contacts. Each of these contacts will have stiffness as shown in equation 3.1 which is a sum of the Hertzian stiffness and the geometric stiffness. This means that each dowel pin and v-groove will have a pair of stiffness, one for each of the contacts it makes. (It has been assumed that the springs denoting these stiffness are perpendicular to each other)
The combined stiffness matrix for each of the contact elements is of the following form. Stiffness \( K_1 \) and \( K_2 \) represent the stiffness for contact 1 and 2. The other two elements in the stiffness matrix are zero as there are no springs in any other direction. The stiffness in this matrix can be used to find the forces at each of the contact using the randomly generated tolerances.

\[
\begin{bmatrix}
K_1 & 0 \\
0 & K_2
\end{bmatrix} - - - (3.2)
\]

\[
\begin{bmatrix}
f_1 \\
f_2
\end{bmatrix} =
\begin{bmatrix}
K_1 & 0 \\
0 & K_2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} - - - (3.3)
\]

However these forces and displacements are in local co-ordinates and need to be transformed into global co-ordinates. Transformation matrix is used to transform the local stiffness matrix from local into global co-ordinates.

\[
\text{Transformation matrix} =
\begin{bmatrix}
-sin\theta & cos\theta \\
cos\theta & sin\theta
\end{bmatrix} - - - (3.4)
\]

The transformed stiffness matrix is described in equation 3.5.

\[
\begin{bmatrix}
K_1 \sin^2 \theta + K_2 \cos^2 \theta & -K_1 \sin \theta \cos \theta + K_2 \sin \theta \cos \theta \\
-K_1 \sin \theta \cos \theta + K_2 \sin \theta \cos \theta & K_1 \cos^2 \theta + K_2 \sin^2 \theta
\end{bmatrix} - - - (3.5)
\]

- **Step 3- Merge the local stiffness matrices.**

Each stiffness matrix as shown in equation 3.5 denotes the overall stiffness of a pair of dowel pin and v-groove. Since each alignment will involve ten pins and ten grooves the mathematical model will have to account for ten matrices as shown in equation 3.5. Furthermore each of the stiffness for a pair of v-groove and dowel pin will have an effect on the adjacent pair. It has been assumed each pair will have a 5% cross coupling effect on the adjacent pair [16]. For a very similar geometry a FEA simulation in the PhD dissertation of P Willoughby shows 5%
cross coupling. This means that the deflection at one pair of v-groove and dowel pin will have an effect on the adjacent pair i.e. if the first pair were to deflect then the pair next to it would deflect by 5% that amount even if the second pair were not subjected to any external force.

- **Step 4- Find forces in global co-ordinates.**

The next step would involve finding the forces in global co-ordinates. After the stiffness of each individual pair was integrated into a matrix it can be multiplied by the randomly generated tolerances which would provide the forces being exerted on each of the element. We can see that the forces on the left hand side would not only depend on the stiffness and displacement of the individual elements but also on that of the adjacent elements. These are the forces that are being exerted on each of the individual elements due to the displacements of the elements and the cross coupling effect of the neighboring elements. A resultant force and a moment can be calculated at the center of the chip.

\[
\begin{bmatrix}
    k_1 & 5k_2 \\
    5k_1 & k_2 & 5k_3 \\
    5k_2 & k_3 & 5k_4 \\
    5k_3 & k_4 & 5k_5 \\
    \vdots & \ddots & \ddots
\end{bmatrix} \quad \text{--- (3.6)}
\]

Where \(k_1, k_2, k_3\) etc. represent the transformed matrices for the 1\(^{st}\), 2\(^{nd}\), 3\(^{rd}\), pair of dowel pins and v-grooves, respectively.

\[
\begin{bmatrix}
    \bar{F}_1 \\
    \bar{F}_2 \\
    \bar{F}_3 \\
    \bar{F}_4 \\
    \vdots
\end{bmatrix} =
\begin{bmatrix}
    k_1 & 5k_2 \\
    5k_1 & k_2 & 5k_3 \\
    5k_2 & k_3 & 5k_4 \\
    5k_3 & k_4 & 5k_5 \\
    \vdots & \ddots & \ddots
\end{bmatrix}
\begin{bmatrix}
    \bar{x}_1 \\
    \bar{x}_2 \\
    \bar{x}_3 \\
    \bar{x}_4 \\
    \vdots
\end{bmatrix} \quad \text{--- (3.7)}
\]
• **Step 5- The force, moment and geometric constraint equations are combined to find the vectors of displacements of the individual points.**

The global stiffness matrix can be used to calculate the force equilibrium equations and moment equilibrium equations (equation 3.8).

\[
\begin{bmatrix}
\text{force\_equilibrium\_equation} \\
\text{Moment\_equilibrium\_equation} \\
\text{geometric\_constraint}
\end{bmatrix}
\begin{bmatrix}
\text{vectors\_of\_displacement}
\end{bmatrix}
= \begin{bmatrix}
\text{force} \\
\text{moment} \\
\text{zeros}
\end{bmatrix} 
\tag{3.8}
\]

For calculating the force equilibrium equations each row of the global stiffness matrix is summed up. The sum of each row represents stiffness of each element and the effect of the stiffness of the adjacent element on it. This sum of stiffness forms the force equilibrium equation. The force equilibrium equation when multiplied by the respective distances from the center of the chip gives the moment equilibrium equation. For forming the geometric constrain equations we have assumed that each side of the chip is at a constant slope and that the horizontal distance between any two elements is constant. Constant slope refers to the fact that irrespective of the individual displacements at the contacts on the side walls, the two walls with contacts are parallel. The geometric constraints equations have been arranged so that the product of the geometric constraints and the vectors of displacements is zero. The reason for forming matrix 3.8 is to find the vectors of displacements.

• **Step 6- Finding the vectors of displacements and hence alignment of the chips.**

After writing all the equations and finding the force and moment in equation 3.8 the vectors of displacements can be found out. The vectors of displacements describe the final positions of the contacting members. The difference between the initial tolerance due to manufacturing of the contacting members and the vectors of displacements will give the actual
displacements of each of the members. Averaging out the displacements will give us final position of the center of the chip and hence the alignment.

3.6. Results of Simulation

Mathematical modeling begins with the generation of 20 random numbers between +13 microns and -13 microns. This range is based on table 4.1 (standard deviation). This number signifies how much each contact point moves about the initial position. The movement of the contact is due to manufacturing tolerances in the fabrication of the parts (mainly the polymer chip). These 20 numbers (in the range of ±13 microns) hence denote the manufacturing tolerance transferred at the points of contact. The model utilizes these tolerances to determine the position of the center. The origin (0, 0) is assumed to be the initial position of the center and any calculated center is measured in reference to the origin. Smaller the distance from the origin better the alignment and vice versa.

3.6.1 Distribution of the Center

A Matlab code was written according to the theory of the mathematical model. The code generates a set of 20 different random numbers [± 13 microns]. These random numbers are fed into the program as possible tolerances at each of the 20 contacts. The output from the program is in the form of simulated centers. The origin (0, 0) is considered to be the actual center. Any simulated center closer to the origin represents much better alignment than a simulated center farther away from the origin. Figure 3.10 shows the distribution of 100 simulated centers. The origin denotes the actual center of the chip. The alignment gets better with decreasing distance from the center. It can be observed that there are more simulated centers clustered around the origin. There are 74/100 centers within the 15 micron radius i.e. 74% of the simulated centers
will have an accuracy of better than 15 microns. There are 94 centers within the 20 micron radius which means 94% of the simulated centers will have an accuracy of 20 microns or better. A total of 99 centers are present within the 25 micron radius which means that 99% of the centers will have an accuracy of 25 microns or better.

To get a better quantitative idea for the distribution of the center two simulations of 1000 centers each were done. The first run simulates the distance from the origin as shown in figure 3.11. A peak in the number of centers at about 5-10 microns from the origin is seen. However a better approximation of the experiments would be measuring the distance between

---

**Figure 3.10 Distribution of the simulated center**

**Table 3.1 Distribution of simulated centers**

<table>
<thead>
<tr>
<th>Simulated centers</th>
<th>Number of centers</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;15 microns</td>
<td>74</td>
<td>74</td>
</tr>
<tr>
<td>&lt;20 microns</td>
<td>94</td>
<td>94</td>
</tr>
<tr>
<td>&lt;25 microns</td>
<td>99</td>
<td>99</td>
</tr>
</tbody>
</table>
the centers of two chips. For this run of simulation it was seen that the maximum number of centers were at a distance of 10-15 microns and is shown in figure 3.11.

![Number of simulated centers vs. distance](image)

Figure 3.11 Number of centers vs. the distance

A decrease in the alignment accuracy was seen when the relative distance between two centers was measured. The physical significance of this result is that when aligned, centers for two unique chips will be much further away than the centers for two similar chips. Simulating the distance between the origin and center can be compared to experimental measurement of the centers between two similar chips because changing one chip with an exactly similar chip the center for the changed chip will be almost be at the same place as the one before (Relatively the center for the unchanged chip doesn’t move and can be visualized as being a stationary origin). However if we use two unique chips the probability of two centers being close is much less (Relative position of the centers will change with each run and there is no fixed point). This is closer to the experimental results and is also expected in the simulation because the probability
of two simulated centers lying close to each other is much less compared to lying farther away (Figure 3.10)

3.6.2. Repeatability and Accuracy

Repeatability and accuracy are a quantitative measure of the alignment of any system. Figure 3.12 is a graphical representation of the accuracy and repeatability of a system.

Figure 3.12 Graphical representation of accuracy and repeatability

- **Repeatability**: An alignment system is said to be repeatable if within certain limits of error each repeat is not as different from the previous one. In Figure 3.12 the red dots denote the actual distribution of actual centers and the red cross hair denotes the mean of this distribution. The red circle around the red dots denotes the repeatability of the alignment system.

- **Accuracy**: The accuracy is defined as the difference between the actually obtained points and the intended point. In Figure 3.12 the black cross hair represents the intended center. The distance between the red cross hair and the black cross hair denotes the accuracy of the system.
• **Accuracy and Repeatability of simulated centers**

Figure 3.13 shows the distribution of the simulated centers. A circle of 25 microns radius drawn with the mean as the center denotes the repeatability. The distance between the mean of the simulated centers and the origin denotes the accuracy of the simulation. The accuracy of the simulation is 2 microns.

![Distribution of simulated centers](image)

Figure 3.13 Repeatability and Accuracy of the simulated centers

### 3.6.3. Varying Numbers of Contacts

Varying the number of contacts will give us a difference in the degree of elastic fit. Using a lesser number of contacts will give a looser fit than compared to using a larger number of contacts which will give a much tighter fit. The number of dowel pins has been varied to see the effect the degree of elastic fit has on the alignment accuracy.
Figure 3.14 Varying the number of contact by varying the number of dowel pins.

The number of contacts is varied by adding or removing dowel pins. Each dowel pin adds 2 contacts; hence figure 3.14 (a) has two dowel pins or four contacts and so on. To mathematically model this only the stiffness of the dowel pin is taken into consideration and the rest are all set to zero. The cross coupling is also considered accordingly i.e. only the dowel pin with non-zero stiffness will have a cross coupling effect on the adjacent dowel pins stiffness. This is turn result in all the zero stiffness elements to have a corresponding zero force. Hence any contact element missing physically from the model will have no effect on the chip.

Each point on the graph represents the displacement of the center for a certain number of contacts. Each point is a mean of 5 simulations. It can be observed that as the number of contacts
increases from 4 to 20 the displacement of the center decreases. Hence, the alignment gets better as the number of contacts increases. The results obtained from running the simulation can be summarized as follows. The repeatability and accuracy for the simulation are both below the acceptable limit of 100 microns. The results from the simulation show that the alignment increases with increasing number of contacts.

![Displacement vs. number of contacts (Simulated)](image)

Figure 3.15 Displacement of the simulated center with varying number of contacts

3.7. Different Polymers

The alignment accuracy for chips fabricated using different materials could be easily simulated using this mathematical model. However the reader needs to take into consideration that while simulating for different materials only the material properties were changed (Young’s modulus and Poisons ratio). It is easily possible the chips while being fabricated from different
materials will form considerably different (different manufacturing tolerance) and hence caution should be taken to interpret the results.

![Distribution of center for different polymers](image)

Figure 3.16 Alignment accuracy for different polymers

The material used for manufacturing the chip was PMMA (Poly methyl meth acrylate). However other materials such as PC (Poly Carbonate), PP (Poly propylene) etc could also have been used for fabrication of the chip. This section shows in brief the alignment accuracy that can be expected for a chip made from a different material.

Table 3.2 Mechanical property of different polymers (Source: [www.goodfellow.com](http://www.goodfellow.com))

<table>
<thead>
<tr>
<th>Type of Polymer</th>
<th>Young’s Modulus of elasticity (GPa)</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMMA (Poly methyl meth acrylate)</td>
<td>3.2</td>
<td>0.35-0.4</td>
</tr>
<tr>
<td>PC (Poly carbonate)</td>
<td>2.6</td>
<td>0.37</td>
</tr>
<tr>
<td>PP (Poly propylene)</td>
<td>1.5-2</td>
<td>0.1-0.3</td>
</tr>
</tbody>
</table>

We see that the even when the Young’s modulus of elasticity changes significantly the alignment accuracy doesn’t change. This is because the value of the total stiffness (equation 3.1),
is a small number (in the order of 3-8 N/mm) and the Young’s modulus of elasticity and the Poisson’s ratio don’t have a significant effect (Alignment of the chip was measured with respect to the frame).

3.8 Different Manufacturing Tolerance

A manufacturing tolerance of about ±13 microns was measured for a typical hot embossed part (Table 4.1). Using the mathematical model the alignment accuracy for different manufacturing tolerance was predicted (Five different manufacturing tolerance ±50 microns, ±25 microns, ±16 microns, ±12.5 microns and ±10 microns were considered in the study and are shown in Figure 3.17). It was seen that as the manufacturing tolerance got better the number of center closer to the origin increase suggesting an increase in the alignment accuracy.

![Alignment accuracy with varying manufacturing tolerance](image)

Figure 3.17 Alignment accuracy with varying manufacturing tolerance

Figure 3.17 shows that for a smaller range of manufacturing tolerance the alignment accuracy is much higher compared to a larger range of tolerance. An important observation to
make would be that higher the manufacturing tolerances, flatter the curve suggesting that the alignment accuracy would be evenly distributed over a larger range. While a high manufacturing tolerance does produce a large number of chips which can be aligned accurately there is a sharp slope the number decreases rapidly.
EXPERIMENTS AND RESULTS

The results for the alignment obtained from the simulation have been verified using experiments. The experimental methodology and the results have been discussed here.

4.1. Alignment Verification Method

The feature on each of the chips which measures the alignment includes two square patterns on diagonally opposite corners (Figure 4.1). The relative position between the squares when two chips are placed one on top of each other describes the translation of one chip with respect to the other and hence the alignment between the two chips.

Figure 4.1 The squares on diagonally opposite ends to measure the alignment

When the two squares overlap there is a gap all around the squares. A measure of this gap between the sides of the squares will give us a measure of the alignment of one chip with respect to another. However the reading obtained may not be a true measure of the alignment as it may have either one or both of the following tolerances.

- Manufacturing error
- Systematic error
Manufacturing errors: The difference in dimension between the design value and that of the final product is the manufacturing error. A molded polymer structure (23.5 mm long) was measured repeatedly to determine this error.

Table 4.1 Table showing the manufacturing as well as systematic error

<table>
<thead>
<tr>
<th>Initial design</th>
<th>Final product</th>
<th>Mean</th>
<th>Standard Deviation of B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue print Dimensions (microns)</td>
<td>Blue print Dimensions (microns)</td>
<td>Mean of B (microns)</td>
<td>Standard Deviation of B (microns)</td>
</tr>
<tr>
<td>A 23500</td>
<td>23195.3</td>
<td>23191.92</td>
<td>13.42</td>
</tr>
<tr>
<td>B 23500</td>
<td>23195.5</td>
<td>23191.92</td>
<td>13.42</td>
</tr>
<tr>
<td>A 23500</td>
<td>23171.2</td>
<td>23191.92</td>
<td>13.42</td>
</tr>
<tr>
<td>B 23500</td>
<td>23189.5</td>
<td>23191.92</td>
<td>13.42</td>
</tr>
<tr>
<td>A 23500</td>
<td>23208.1</td>
<td>23191.92</td>
<td>13.42</td>
</tr>
</tbody>
</table>

Systematic error: This is inherent in the measurement methodology and is a cumulative error of both human error and the limitation of the measuring instrument. The systematic error is included in the total error in table 4.1

4.2 Experiment Methodology

The method of measurement of the alignment between the chips has been based on the doctoral thesis from P. Datta [5]. An explanation of the experimental measurement of the alignment of the chips is given below. Figure 4.2 shows the distance between the walls of the blue and the red square. X1 and Y1 denote the translation of the chip in the bottom left corner and X2 and Y2 represents the translation of the chips the top right corner. In case of perfect alignment, theoretically X1 must be equal to X2 and Y1 must be equal to Y2, i.e. the total translation along the X-axis and the Y-axis must be equal no matter where it is measured. However due to error in measurement and fabrication, the total distance translated may or may
not be the same. This difference in distances can be shown better if measurement is taken in different places instead of a single place. This is the reason the readings were taken in two squares placed on the two opposite corners of chips. While measurement error may suggest no translation if measured at a single point, measuring the translation at two different locations gives a better probability of finding the true translation.

Figure 4.2 The method of measuring the alignment

The total translation of the center:

\[
\text{Translation of the center in X direction} = \frac{X_1 + X_2}{2}
\]

\[
\text{Translation of the center in Y direction} = \frac{Y_1 + Y_2}{2}
\]

It is possible that the distances between all the sidewalls are equal. If this is the case than the value of X1, X2, Y1, Y2 are all equal to zero. Hence the total translation in the X and Y direction is zero as well. This means that the centers of the two chips are exactly on top of one another. The two chips are hence perfectly aligned.
4.3 Experimental Setup

Two polymer chips of microscopic slide format with the red and blue squares have been used. Each chip is 75mm × 25mm in dimension. A base with ten dowel pins was used to align these two chips. Nikon MM-22U (measuring microscope) interfaced to the computer using the QC500 software was used to measure the lateral distance between two features on the substrate. The software has been interfaced with the microscope to measure either the direct movement of the table of the microscope (the table of the microscope can be moved with motors controlled via a joystick) or the movement of the cross-hair (Cross hair in the screen can be moved using the mouse). The measurements were done inside a class 100 clean room at CAMD.

![Figure 4.3 The schematic and the actual chips inserted on the base and the Nikon M-22U measuroscope](image)

The chips were placed onto the base as shown in figure 4.3. After that the base was fixed firmly to the table on the measuroscope using adhesive tape. Care was taken to make sure that the base would not move during measurements. The chips were then placed carefully onto the base while the dowel pins aligned the chips. After the chips were placed onto the dowel pins the measurements were taken.
4.4. Results

The following section of the chapter discusses the various experimental results and a comparison with the respective results from the theoretical model. For experimental purpose 3 different sets of chips have been used. The translation of center in each set is measured and compared with the theoretical value.

4.4.1. Distribution of Centers

Two chips were stacked and aligned using the base. One of the chips is considered to be the reference and the center of the other chip was measured with respect to that chip. In Figure 4.5 the origin is the reference center. The other points are the centers of the chip that we are measuring with respect to the origin. Three different combinations of chips were used to show the distribution of the centers. The mean for all centers for one combination was taken and a circle was drawn to encircle all the centers for that combination. Hence each circle represents the distribution for one combination of chips. The experimental centers are distributed so far off because each combination of chip was unique and that different fabrication tolerances are expected. This causes the different set of centers to be far off from each other; however centers
of same combinations of chips are close to each other proving that while unique chips have way
different tolerances, similar chips have tolerances much closer to each other.

![distribution of center (experimental)](image)

Figure 4.5 Distribution of the centers for 3 different combinations of chips

### 4.4.2. Accuracy and Repeatability

Repeatability of the alignment for each combination is radius of the circle with the mean
as the center and the circle is drawn so as to include all the distributed centers. Accuracy is
defined as the distance between the mean and the reference point, origin [Figure 4.5].

We see that the repeatability and accuracy obtained from the alignment method is better
than 100 microns in all the three combination. Combination 1 had the best alignment accuracy
and repeatability results as both the radius of the circle (repeatatebility) and the distance from the
origin was small.

**Table 4.2 Repeatability and Accuracy for the three combinations**

<table>
<thead>
<tr>
<th>Combination numbers</th>
<th>Repeatability (microns)</th>
<th>Accuracy (microns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55.0</td>
<td>29.6</td>
</tr>
<tr>
<td>2</td>
<td>55.6</td>
<td>64.9</td>
</tr>
<tr>
<td>3</td>
<td>78.5</td>
<td>37.1</td>
</tr>
</tbody>
</table>
4.4.3. Varying the Number of Contacts

The objective of varying the number of dowel pins is to determine the effect of the number of contacts on the alignment of the chips. Each dowel pin makes two contacts i.e. the 10 dowel pins make 20 contacts. To vary the number of contacts, two dowel pins are added in each set of experiment; hence the first set of experiments is done with 2 dowel pins i.e. 4 contacts, the second set of experiment is done with 4 dowel pins i.e. 8 contacts and so forth until the total number of dowel pins is 10. For every number of contacts 3 different sets of chips have been used, since there are five possible combinations of contacts (4, 8, 12, 16, 20), a total of 15 different sets of experiments have been performed. The dowel pins that have been used to provide the contacts are represented as blue dots next to the V-grooves on the chips (figure 3.14). For the 3 combinations with varying number of dowel pins the movement of the center is shown in the tables below.
For varying number of contacts we see that the mean distance from the center decreases with increasing number of contacts. This result has been further verified by using three different combinations of chips. For all the combinations we see that mean of the movement of center decreases with increasing number of contacts.

The mathematical model predicts alignment accuracy between two chips of being better than the required 100 microns and the experimental results verify this result. This proves the concept that chips in a vertically stacked micro fluidic device can be aligned to each other within 100 microns using this alignment method enabling the flow of sample from one level to another.
A novel method for passively aligning polymer micro fluidic chips in a vertical stack has been presented. The alignment methodology is based on the principle of elastic averaging which uses an over constrained system with multiple points of contacts to average out the manufacturing tolerances at the individual points. Alignment features have traditionally been placed on the top and bottom surfaces of components being assembled, however in order to avoid inaccuracies stemming from misaligned molds during double sided hot embossing, the alignment features were placed on the edges of the components. A third body, an alignment frame, was used to align the individual components.

A mathematical model was developed to predict the alignment of the components. The results from the model were then verified experimentally. The model predicted the accuracy of the alignment method to be around 2 microns and the whole system to have a repeatability of 25 microns. The experiments showed a mean accuracy of 40 microns and repeatability of 60 microns. The model and experimental results also showed corresponding trends for varying number of contacts; more contacts produced better alignment accuracy. The model also predicted that there was no significant difference in the alignment accuracy when using polymers such as Polycarbonate or Polypropylene instead of PMMA. For reduced manufacturing tolerance the model predicted an increased alignment accuracy and vice-versa. Decreasing the number of contacts also predicted a worsening in the alignment accuracy from both the model as well as the experiment. Thus it was seen that the final alignment accuracy was more sensitive to the number of contacts and comparatively less sensitive to the tolerances at each contact.
5.1. Future Works

Based on the results, it can be assumed that two components with a larger number of alignment members produce a better accuracy, hence future designs of the assembly mechanisms should aim to maximize the number of alignment features. Another shortcoming of the current design was the limited number of chips that can be stacked one on top of the other because the dowel pins used for alignment tend to bend after the first few are stacked (Figure 5.1). This leads to a loss in alignment accuracy. A new design has been proposed to overcome this drawback (Figure 5.2).

This newer design has both the v-grooves and pins built into the single chip. Each chip is aligned with respect to the adjacent chip and has no effect on any other chip whatsoever. The stack doesn’t suffer from the bending of the beam as it does in this case. This design improves on the original while retaining the simplicity of single sided molding. With this new design any number of chips may be stacked without losing alignment accuracy.
REFERENCES


APPENDIX A

DETAILS OF MATHEMATICAL MODEL

The mathematical model has been formulated by representing each contact as a pair of spring. There are 10 pairs of dowel pins and V-grooves; hence we have 20 contacts

**Step 1 (Representing each contact as springs)**

Figure A. Representing each contact as a pair of spring.

Approximation of a beam as a cantilever

Figure B. Schematic representation of a steel dowel pin in a PMMA base which can be modeled as a uniformly loaded cantilever beam. The figure on the left (a) is that of the dowel pin which has been approximated as a cantilever on the right (b).
\[ \delta = -\frac{wl^4}{8EI} \]

Where,
\( \delta \) = deflection at the end of the beam.
\( w \) = uniformly distributed load along the length.
\( l \) = length of the beam.
\( E \) = Young's modulus of material of the dowel pin.
\( I \) = Moment of inertia of the beam.

\[ \text{stiffness} = \frac{F}{\delta} = \frac{wl}{wl^4} = \frac{8EI}{l^3} \]

Therefore stiffness (due to geometry) of a uniformly loaded cantilever beam

\[ = \frac{8EI}{l^3} \]

CALCULATION OF STIFFNESS DUE TO HERTZIAN STRESS

Fig C. Showing the radius of the flat surface (\( r \): which is close to infinity) and the radius of the beam (\( R \): which is 0.5 mm). Hence the relative radius (\( R_c \)) is 2.

Relative radius :\( R_c \)

\[ \frac{1}{R_c} = \frac{1}{R} + \frac{1}{r} \]
Where,

\( R_c = \) contact radius

\( r = \) Radius of the flat surface

\( R = \) Radius of the beam.

- Load per unit length of the contacts (\( P \))

\[
f_1 = k_1 \times x_1
\]

\[
P = \frac{f_1}{\text{length of beam (incontact)}} : l
\]

- Normal displacement of the cylindrical surface and the flat PMMA at 1.

\[
\delta = \lambda [p^2 (\eta_1 + \eta_2)^2 (B + A)]^{1/3}
\]

Where,

\[
\eta_1 = \frac{1 - v_1^2}{E_1}, \eta_2 = \frac{1 - v_2^2}{E_2}
\]

\[
B + A = \frac{1}{2} \left[ \frac{1}{R_1} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_2} \right] = \frac{1}{2} \left[ \frac{1}{.5} + \frac{1}{.5} + 0 \right] = 2
\]

Figure D. Schematic of the polymer chip fitted and aligned using the dowel pins.
The value of $\lambda$ is obtained from fig 11.6 of book “Mechanical Analysis and Design 2nd edition”-Burr and Chetam. Pg 681

- Normal stiffness:

$$ K = \frac{F}{\delta} $$

$$ K_{hertz} = \frac{P \times \text{Height of beam}}{\lambda [P^2 (\eta_1 + \eta_2)^2 (B + A)]^{1/3}} $$

Total combined stiffness

$$ \frac{1}{K_{total}} = \frac{1}{K_{geom}} + \frac{1}{K_{hertz}} $$

**Step 2 (Force calculation at each of the contacts)**

The second step involves forming the local stiffness matrix for each of the contact points. The forces in the local co-ordinates are denoted by $f_1$ and $f_2$ respectively and $F_x$ and $F_y$ denote the forces in the global co-ordinates.

Figure E. Contact no 1 and 2 and the included angle $\theta$ between the local and global co-ordinates
Let us define forces $f_1$ and $f_2$.

\[
f_1 = K_1 \times x_1 \quad \text{------------------------ (1)}
\]

\[
f_2 = K_2 \times x_2 \quad \text{------------------------ (2)}
\]

Where $K_1$ and $K_2$ are the stiffness of contact 1 and 2 and $x_1$ and $x_2$ are the displacements along $f_1$ and $f_2$ respectively.

Representing equation (1) and (2) in matrix form.

\[
\begin{bmatrix}
    f_1 \\
    f_2
\end{bmatrix} =
\begin{bmatrix}
    K_1 & 0 \\
    0 & K_2
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} \quad \text{------------------------ (3)}
\]

We need to transform the forces $F_1$ and $F_2$ along the X direction and Y direction.

\[
F_{x1} = f_2 \cos \theta - f_1 \sin \theta \quad \text{------------------------ (4)}
\]

\[
F_{y1} = f_2 \sin \theta + f_1 \cos \theta \quad \text{------------------------ (5)}
\]

Representing equation (4) and (5) in matrix form.

\[
\begin{bmatrix}
    F_{x1} \\
    F_{y1}
\end{bmatrix} =
\begin{bmatrix}
    -\sin \theta & \cos \theta \\
    \cos \theta & \sin \theta
\end{bmatrix}
\begin{bmatrix}
    f_1 \\
    f_2
\end{bmatrix} \quad \text{------------------------ (6)}
\]

Where $F_{x1}$ and $F_{y1}$ represent the forces along X and Y

Let the displacements along X and Y are $\Delta X_1$ and $\Delta Y_1$ respectively.

We need to transform $x_1$ and $x_2$ to $\Delta X_1$ and $\Delta Y_1$.

\[
\Delta X_1 = x_2 \cos \theta - x_1 \sin \theta \quad \text{------------------------ (7)}
\]
\[ \Delta Y_1 = x_2 \sin \theta + x_1 \cos \theta \quad \text{--------- (8)} \]

Representing equation 7 and 8 in matrix form

\[
\begin{bmatrix} \Delta X_1 \\ \Delta Y_1 \end{bmatrix} = \begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \text{----- (9)}
\]

Using equation 6 and equation 9 in equation 3

\[
\begin{bmatrix} F_{x1} \\ F_{y1} \end{bmatrix} = \begin{bmatrix} \sin \theta & \cos \theta & 0 & K_1 \\ \cos \theta & \sin \theta & 0 & K_2 \end{bmatrix} \begin{bmatrix} \sin \theta & -\cos \theta \\ -\cos \theta & -\sin \theta \end{bmatrix}^{-1} \begin{bmatrix} \Delta X_1 \\ \Delta Y_1 \end{bmatrix} \quad \text{----- (10)}
\]

Equation 10 gives us the forces in the global co-ordinates.

\[
\begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}^{-1} = \frac{1}{-\left(\cos^2 \theta + \sin^2 \theta \right)} \begin{bmatrix} \sin \theta & -\cos \theta \\ -\cos \theta & -\sin \theta \end{bmatrix} = \begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{bmatrix} \quad \text{----- (11)}
\]

Therefore

\[
\begin{bmatrix} F_{x1} \\ F_{y1} \end{bmatrix} = \begin{bmatrix} -K_1 \sin \theta & K_2 \cos \theta \\ K_1 \cos \theta & K_2 \sin \theta \end{bmatrix} \begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta Y_1 \end{bmatrix} \quad \text{----- (12)}
\]

The \( F=K*X \) for dowel pin no 1.

\[
\begin{bmatrix} F_{x1} \\ F_{y1} \end{bmatrix} = \begin{bmatrix} K_1 \sin^2 \theta + K_2 \cos^2 \theta & -K_1 \sin \theta \cos \theta + K_2 \sin \theta \cos \theta \\ -K_1 \sin \theta \cos \theta + K_2 \sin \theta \cos \theta & K_1 \cos^2 \theta + K_2 \sin^2 \theta \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta Y_1 \end{bmatrix} \quad \text{----- (13)}
\]

**Stiffness for a pair of dowel pin and V-groove is**

\[
\begin{bmatrix} k_{total1} & 0 \\ 0 & k_{total2} \end{bmatrix} \quad \text{Where } k_{total1} \quad \text{and } k_{total2} \quad \text{are total stiffness for contacts no 1 and 2.}
Transformed stiffness matrix for each pair of dowel pin and V-groove

\[
\begin{bmatrix}
    K_1 \sin^2 \theta + K_2 \cos^2 \theta & -K_1 \sin \theta \cos \theta + K_2 \sin \theta \cos \theta \\
    -K_1 \sin \theta \cos \theta + K_2 \sin \theta \cos \theta & K_1 \cos^2 \theta + K_2 \sin^2 \theta
\end{bmatrix}
\]

The stiffness matrix for the rest of the dowel pins/V-grooves can be found out similarly.

**Step 3 (Merging the local stiffness matrix)**

Defining the $K_{\text{big}}$ matrix

The stiffness of all the contacts is incorporated into a bigger matrix.

\[
K_{\text{big}} =
\begin{bmatrix}
    K_{11} & K_{12} & 0.05K_{21} & 0.05K_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\
    K_{12} & K_{22} & 0.05K_{12} & 0.05K_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\
    0.05K_{11} & 0.05K_{12} & K_{21} & K_{22} & 0.05K_{31} & 0.05K_{32} & 0.05K_{31} & 0.05K_{32} & 0.05K_{31} & 0.05K_{32} \\
    0.05K_{12} & 0.05K_{22} & K_{21} & K_{22} & 0.05K_{32} & 0.05K_{32} & 0.05K_{32} & 0.05K_{32} & 0.05K_{32} & 0.05K_{32} \\
    0 & 0 & 0 & 0 & 0.05K_{91} & 0.05K_{92} & K_{101} & K_{102} & K_{101} & K_{102} \\
    0 & 0 & 0 & 0 & 0 & 0.05K_{92} & K_{102} & K_{102} & K_{102} & K_{102}
\end{bmatrix}
\]

Where $K_1, K_2, \ldots, K_{10}$ represent the stiffness matrix for each pair of contact.

Here (from equation 13),

\[
\begin{bmatrix}
    k_{11} & k_{12} \\
    k_{21} & k_{22}
\end{bmatrix} =
\begin{bmatrix}
    K_1 \sin^2 \theta + K_2 \cos^2 \theta & -K_1 \sin \theta \cos \theta + K_2 \sin \theta \cos \theta \\
    -K_1 \sin \theta \cos \theta + K_2 \sin \theta \cos \theta & K_1 \cos^2 \theta + K_2 \sin^2 \theta
\end{bmatrix}
\]

For simplification purpose,
Similarly,

\[
\begin{bmatrix}
K_1 \sin^2 \theta + K_2 \cos^2 \theta & -K_1 \sin \theta \cos \theta + K_2 \sin \theta \cos \theta \\
-K_1 \sin \theta \cos \theta + K_2 \sin \theta \cos \theta & K_1 \cos^2 \theta + K_2 \sin^2 \theta
\end{bmatrix}
= k_1
\]

\[
\begin{bmatrix}
K_3 \sin^2 \theta + K_4 \cos^2 \theta & -K_3 \sin \theta \cos \theta + K_4 \sin \theta \cos \theta \\
-K_3 \sin \theta \cos \theta + K_4 \sin \theta \cos \theta & K_3 \cos^2 \theta + K_4 \sin^2 \theta
\end{bmatrix}
= k_2
\]

\[
\begin{bmatrix}
K_5 \sin^2 \theta + K_6 \cos^2 \theta & -K_5 \sin \theta \cos \theta + K_6 \sin \theta \cos \theta \\
-K_5 \sin \theta \cos \theta + K_6 \sin \theta \cos \theta & K_5 \cos^2 \theta + K_6 \sin^2 \theta
\end{bmatrix}
= k_3
\]

Here, \(k_1, k_2, k_3\) represent the stiffness matrix for the 1\(^{st}\), 2\(^{nd}\) and 3\(^{rd}\) pair of dowel pin and V-groove respectively. We can similarly represent the stiffness for all the other pairs of dowel pins and V-grooves too.

**Step 4 (Force and moment calculation)**

Force calculation:

![Forces at each of the contact.](image)

Figure F. The total forces developed at each of the contacts
The force at each of the contact can be calculated as a product of the stiffness of that contact and the displacement.

\[
\begin{bmatrix}
F_1 \\
F_2 \\
F_3 \\
\vdots \\
F_8 \\
F_9 \\
F_{10}
\end{bmatrix} =
\begin{bmatrix}
k_1 & .5k_2 & 0 & 0 & 0 \\
.5k_1 & k_2 & .5k_3 & 0 & 0 \\
0 & .5k_2 & k_3 & .5k_4 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & .5k_5 & k_4 & .5k_5 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & .5k_6 & k_7 & .5k_8 \\
0 & 0 & .5k_7 & k_8 & .5k_9 \\
0 & 0 & .5k_8 & k_9 & .5k_{10} \\
0 & 0 & 0 & .5k_9 & k_{10}
\end{bmatrix}
\begin{bmatrix}
x'_1 \\
x'_2 \\
x'_3 \\
\vdots \\
x'_8 \\
x'_9 \\
x'_{10}
\end{bmatrix}
\]

\(k_1, k_2, \ldots, k_{10}\) represent the stiffness matrix for the 1st, 2nd and 10th pair of dowel pin and V-groove respectively and \(x'_1, x'_2, \ldots\) represent the displacements.

Moment calculation:

\[
\text{Moment } F_{x_1} \text{ about the center} = F_{x_1} \times \text{Height}_1
\]

\[
\text{Moment } F_{y_1} \text{ about the center} = F_{y_1} \times \text{Length}_1
\]
Step 5 (Combine force, moment and geometric constraint equation)

\[
\begin{bmatrix}
\text{Force equilibrium equation} \\
\text{Moment equilibrium equation} \\
\text{Geometric constraints}
\end{bmatrix}
\begin{bmatrix}
\text{Vectors of displacements}
\end{bmatrix}
= 
\begin{bmatrix}
\text{force} \\
\text{moment} \\
\text{zeros}
\end{bmatrix}
\]

Here, we know all the terms on the right hand side. The vectors of displacements are the final positions of the contact points. However to find the vectors of displacements we still need to define the force equilibrium equations, moment equilibrium equations and the geometric constraints. The force equilibrium equations would be the sum of individual stiffness and the effect of cross coupling. (Effect of the neighboring springs) The product of the force equilibrium equations with the final displacements gives the final force.

\[
\text{Force equilibrium equation} =
\begin{bmatrix}
K_1 + 5\% K_2 \\
5\% K_1 + K_2 + 5\% K_3 \\
5\% K_2 + K_3 + 5\% K_4 \\
\vdots
\end{bmatrix}
\]

The moment equilibrium equations would be the sum of the force equilibrium equations multiplied by the respective distances from the center. The product of the moment equilibrium equations with the final displacements gives the final moment.

\[
\text{Moment equilibrium equation} =
\begin{bmatrix}
(K_1 + 5\% K_2) \times L_1 \\
(5\% K_1 + K_2 + 5\% K_3) \times L_2 \\
(5\% K_2 + K_3 + 5\% K_4) \times L_3 \\
\vdots
\end{bmatrix}
\]

The geometric constraints equations are written such that when these equations are multiplied by the final displacements the product is zeros.
Figure G. The geometric constraints for the chips

For writing the geometric constraint equations we assume the body has a constant slope and the horizontal displacement is constant.

i.e. for all practical purposes

Constant Slope

\[
\begin{bmatrix}
0 & 1 & 1 \\
\frac{1}{\text{dist}(1 \& 2)\text{ pair}} & -\frac{1}{\text{dist}(1 \& 3)\text{ pair}} & \cdots & \frac{1}{\text{dist}(1 \& 3)\text{ pair}}
\end{bmatrix}
\begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3 \\
\vdots
\end{bmatrix}
= 0
\]

\[
\therefore \frac{Y_2}{\text{dist}(1 \& 2)\text{ pair}} = \frac{Y_3}{\text{dist}(1 \& 3)\text{ pair}} = 0
\]

Constant horizontal distance

\[
\begin{bmatrix}
1 & -1 & 0 & \cdots \\
X_1 \\
X_2 \\
X_3 \\
\vdots
\end{bmatrix}
= 0
\]
\[ X1 - X2 = 0 \]

Therefore

\[
\text{Geometric constraint equation: } \begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & 1 \\
\frac{1}{\text{dist}(1 \& 2)_{pair}} & \frac{1}{\text{dist}(1 \& 3)_{pair}} & \vdots
\end{bmatrix}
\]

The vectors of displacements can be easily found out. The difference in-between the vectors of displacements and the initially generated random numbers give us the final position of the dowel pins and V-grooves.
### APPENDIX B

#### EXPERIMENTAL AND SIMULATED DATA

- Displacement of center with varying no of contacts (Simulated)

<table>
<thead>
<tr>
<th>No of contacts</th>
<th>Displacement of center</th>
<th>Mean displacement</th>
</tr>
</thead>
</table>
| 4              | 0.1097  
0.1255  
0.0803  
0.1539  
0.0988 | 0.011364 |
| 8              | 0.0718  
0.0934  
0.0686  
0.0735  
0.0816 | 0.07778 |
| 12             | 0.0656  
0.1161  
0.0405  
0.0786  
0.0901 | 0.07818 |
| 16             | 0.0486  
0.0259  
0.0539  
0.0541  
0.0569 | 0.04788 |
| 20             | 0.0277  
0.0252  
0.0262  
0.055  
0.0262 | 0.03206 |
• Distribution of the center about the origin (Experimental)

<table>
<thead>
<tr>
<th>Combination</th>
<th>No of contacts</th>
<th>ΔX (mm)</th>
<th>ΔY (mm)</th>
<th>Distribution of center(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>-0.0891</td>
<td>-0.03725</td>
<td>0.09657315</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>-0.0566</td>
<td>-0.0354</td>
<td>0.06675867</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>0.06895</td>
<td>-0.01385</td>
<td>0.07032727</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>-0.0737</td>
<td>-0.0095</td>
<td>0.07430976</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>-0.0879</td>
<td>-0.00205</td>
<td>0.0879239</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>0.04125</td>
<td>0.0976</td>
<td>0.10595906</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>0.02005</td>
<td>0.09935</td>
<td>0.10135297</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>0.00105</td>
<td>0.0614</td>
<td>0.06140898</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>-0.0302</td>
<td>0.1026</td>
<td>0.10695233</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>-0.05088</td>
<td>0.02375</td>
<td>0.06341532</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>-0.06815</td>
<td>0.027</td>
<td>0.07330363</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>-0.1114</td>
<td>-0.03095</td>
<td>0.11561947</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>0.06315</td>
<td>-0.01275</td>
<td>0.06442426</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>-0.0609</td>
<td>0.00795</td>
<td>0.06141671</td>
</tr>
</tbody>
</table>

• Repeatability and Accuracy for combination 1 (Experimental)

<table>
<thead>
<tr>
<th>S. No</th>
<th>Co-ordinates of center</th>
<th>Co-ordinates of the mean</th>
<th>Repeatability (distance between the individual centers and mean)</th>
<th>Accuracy (distance between the mean and the origin)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X-coordinate</td>
<td>Y-coordinate</td>
<td>X-coordinate</td>
<td>Y-coordinate</td>
</tr>
<tr>
<td>1</td>
<td>-0.0891</td>
<td>-0.03725</td>
<td>-0.04767</td>
<td>-0.01961</td>
</tr>
<tr>
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<td>-0.0566</td>
<td>-0.0354</td>
<td>-0.04767</td>
<td>-0.01961</td>
</tr>
<tr>
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<td>0.06895</td>
<td>-0.01385</td>
<td>-0.04767</td>
<td>-0.01961</td>
</tr>
<tr>
<td>4</td>
<td>-0.0737</td>
<td>-0.0095</td>
<td>-0.04767</td>
<td>-0.01961</td>
</tr>
<tr>
<td>5</td>
<td>-0.0879</td>
<td>-0.00205</td>
<td>-0.04767</td>
<td>-0.01961</td>
</tr>
</tbody>
</table>

• Repeatability and Accuracy for Combination 2 (Experimental)

<table>
<thead>
<tr>
<th>S. No</th>
<th>Co-ordinates of center</th>
<th>Co-ordinates of the mean</th>
<th>Repeatability (distance between the individual centers and mean)</th>
<th>Accuracy (distance between the mean and the origin)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X-coordinate</td>
<td>Y-coordinate</td>
<td>X-coordinate</td>
<td>Y-coordinate</td>
</tr>
<tr>
<td>1</td>
<td>0.04125</td>
<td>0.0976</td>
<td>0.00685</td>
<td>0.08922</td>
</tr>
<tr>
<td>2</td>
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• Repeatability and Accuracy for Combination 3 (Experimental)

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<th>Co-ordinates of the mean</th>
<th>Repeatability (distance between the individual centers and mean)</th>
<th>Accuracy (distance between the mean and the origin)</th>
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• Movement of center for combination 1 (Experimental)

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• Movement of center for combination 2 (Experimental)

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• Movement of center for combination 3 (Experimental)

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</table>
VITA

Sitanshu Gurung was born in Kathmandu, Nepal. He graduated from Birla Institute of Technology, India with a bachelor in mechanical engineering in the May of 2003. He worked at Kantipur City College, as an Instructor from 2003 to 2004. He completed the requirement for and will receive the degree of master in Mechanical Engineering from Louisiana State University, Department of Mechanical Engineering, in December 2007. At present he is working as a design engineer for Steam and process repairs, Gonzales, Louisiana.