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A Critique of a Student-Centered Learning Approach Used in a Geometry Classroom

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A CRITIQUE OF A STUDENT-CENTERED LEARNING APPROACH USED IN A
GEOMETRY CLASSROOM

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Masters of Natural Science

in

The Interdepartmental Program in Natural Sciences

by
Alana Blackwell Day
B.S., Louisiana State University, 2010
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Abstract

This thesis offers a framework for identifying effective classroom materials to support student-centered learning. Based on a review of published studies on effective classroom activities, as well as theses by Louisiana Math and Science Teachers Institute (LaMSTI) graduates, we identify promising characteristics. We employed these in five lessons, refined them into questions, and offer in final form for use.

Chapter 1: Introduction

The purpose of this thesis is to identify specific characteristics needed in effective activities and to determine if those qualities exist in certain activities that occur in a widely-used textbook. Many companies have created materials that claim to facilitate student-centered learning, but it can be difficult for teachers to determine which activities will be most productive in their classrooms. There is no reason to trust something will be effective just because it claims to implement a certain method.

I have used the research of other teachers and professionals in the education field to identify these important characteristics. Then I will look specifically at activities used from the Springboard© (SB) Geometry text, as they were implemented in my classroom, to see if these characteristics are present. For each of the selected lessons, I will do the following:

1. outline the lesson as it is presented in the text;
2. describe the implementation in my classroom;
3. discuss the students' and teachers' reactions throughout the activity;
4. present student results from our department created assessments, and
5. critique each activity in light of the data in the first four items.

Chapter 2 includes a review of the literature found on student-centered learning and effective classroom strategies. These studies were used to create the framework for evaluating each lesson. Chapter 3 discusses the implementation of each lesson in my classroom following the outline mentioned above. Chapter 4 contains the conclusions from this study and suggestions for future studies. The critiquing tool created for future use is found in Appendix A.

Chapter 2: Literature Review

This chapter includes an overview of research that identifies important features of a student-centered classroom. These features are used to develop a guiding framework with six questions that can be used to critique activities, tasks, or lessons used in mathematics classrooms. Each question will be discussed individually and clearly defined in reference to the purpose of the study. A final section will discuss the features of the textbook used in this specific study.

During the last several decades, researchers have introduced many different instructional approaches for teaching mathematics. A major debate exists between “student-centered” and “teacher-centered” methods. In the current Compass Classroom Evaluation system used in Louisiana Public Schools, teacher-directed classrooms are discouraged, and there is a major push toward “student-centered” curriculums. According to Gningue, Peach, and Schroder (2013), characteristics of the student-centered approach calls for teachers to engage students in critical, higher-order thinking through the use of manipulatives, technology, cooperative learning, and other pedagogical approaches that enable students to construct mathematics concepts on their own. Students do this through verifying, comparing, interpreting, investigating or solving problems, making connections, and constructing arguments (Sikula, Buttery, & Guyton, 1996).

Based on this research we can identify student-centered as an important feature of classroom instruction, but the term itself does not imply a single method. The following researchers, Stigler and Hiebert (1997), suggest that many teachers who think they are implementing this approach may not actually be doing so (as cited in Walters, et. al, 2014). Student-centered learning consists of a range of complementary approaches to teaching and learning that draws from multiple theories and trends in the field of education.

School districts have adopted different programs or textbooks and adapted their curriculums in hopes of achieving a “student-centered” classroom. However, little research can be found on what specifically makes an activity student-centered or effective in the mathematics classroom. Many professional development courses and training sessions offered by school boards and educational service companies claim to have identified the characteristics of quality mathematics lessons, tasks, and activities, but fail to provide research-based results. Also, a solid curriculum or structure alone does not ensure student success. There is increasing evidence that effective instruction is as important as well designed curriculum (Larson, 2002).

After looking closely at six classrooms in New England and one in New York, The American Institutes for Research concluded that there are multiple ways to create a student-centered environment. The researches (Walters, Smith, Leinwand, Surr, Stein, & Bailey, 2014) generally describe activities to be effective if they engage as many students as possible in reasoning about mathematics, communicating mathematical thinking, and persevering in problem solving. According to Larson (2002), “There are five essential characteristics of effective mathematics lessons: the introduction, development of the concept of skill, guided practice, summary, and independent practice.” The Mathematics Assessment Project associated with the Connecticut Common Core of Learning identified nine important qualities to look for in mathematical tasks: essential (standards-based), authentic (real-life), equitable (not-biased), rich (potential for extension and connections), feasible (developmentally appropriate), clear, scorable, active (visible student interaction), and accessible to differing levels of ability.¹

Many studies can be found on specific learning strategies that have been implemented with positive results. Studies done previously in the Louisiana Math and Science Teachers Institute (LaMSTI) examine the implementation of activities, tasks, and specific classroom

¹ <http://www.learner.org/workshops/missinglink/pdf/tools3.pdf>

strategies and how each effected student learning. The following theses submitted for the degree of Master of Natural Sciences (MNS) were referenced:

- “Implementing and Managing Self-Assessment Procedures,” Terry Armstrong (May 2013)
- “The Effects of Modeling Instruction in a High School Physics Classroom,” Mark Arseneault (August 2013)
- “The Effects of Implementing the Cooperative Learning Structures, Numbered Heads Together, In Chemistry Classes at a Rural, Low Performing High School,” Daniel Baker (August 2013)
- “Metacognition and Its Effect on Learning High School Calculus,” Bonnie Bergstresser (August 2013)
- “Teaching High School Geometry with Tasks and Activities,” Margaret Fazekas (August 2011)
- “The Effects of Self-Assessment on Student Learning of Mathematics,” Daniel Hotard (August 2010)
- “Problem Solving Strategies and Metacognitive Skills for Gifted Students in Middle School,” Lorena Java (August 2014)
- “Integrating Tasks, Technology, and the Common Core Standards in the Algebra II Classroom,” Beth McInnis (August 2012)
- “Evaluation of the Effectiveness of Cooperative Learning Structures in Improving Students’ Performance,” Jonah Njenga (December 2010)
- “Mathematical Modeling in the High School Classroom,” Selena Oswalt (August 2012)

- “Project Explorations and Student Learning in Geometry,” Verna Richard (August 2010)
- “Project-Based High School Geometry,” Danica Robinson (August 2009)

These theses and other research have been used to identify specific elements of effective classroom design. The characteristics found in the above research were used to formulate six questions that can be used by teachers to guide their evaluations of specific activities, tasks, or lessons that are used in their classrooms. The questions are:

1. Does the lesson state clear, standards-based objectives?
2. Does the lesson support teachers in providing guidance and feedback to students?
3. Does the lesson incorporate cooperative learning?
4. Does the lesson engage students in higher-order thinking?
5. Does the lesson include time for students to self-assess or reflect on their learning?
6. Can the lesson be completed in a reasonable amount of time?

2.1 Does the lesson state clear, standards-based objectives?

It is widely believed that students are more engaged when they can relate to and understand what they are learning. However, we are not free to teach any topic we choose. We are guided by national, state, and district standards that specify what students should know and be able to do. Educational objectives are often the criteria by which materials are selected, content is outlined, instructional procedures are developed, and tests are prepared. The purpose of stating the objectives is to indicate the kinds of changes the student can expect to experience. Beginning a lesson with the end goal in mind gives the activity meaning to both the student and teacher (Wiggins & McTighe, 1998). Students need to know there is a mathematical point to what they are being asked to complete if we are going to expect them to engage. Teachers need

to know the objectives covered by a lesson in order to establish if it meets specific curricular priorities for their subject.

2.2 Does the lesson support teachers in providing guidance and feedback to students?

According to research done concerning human cognitive architecture, Kirschner, Sweller, and Clark (2006) suggest minimally guided instruction is ineffective by itself. They suggest, “The aim of all instruction is to alter long-term memory (Kirschner, Sweller, & Clark 2006).” Pure discovery methods and minimal feedback often cause students to become “lost and frustrated”, and can lead to misconceptions (Brown & Campione, 1994). Guidance within a lesson offers an opportunity for students to use their working memory to develop problem-solving skills needed to accomplish the lesson’s goals. Examples of teacher guidance seen in studies include worked examples (Sweller & Cooper 1985) and process worksheets² (Van Merriënboer, 1997). Research shows that “free exploration” alone is not enough and guidance with meaningful feedback is needed.

2.3 Does the lesson incorporate cooperative learning?

Cooperative learning strategies have been widely adopted as an effective instructional tool at all levels of education (Baker, 2013). Njenga (2010) defines cooperative learning as “an instruction method in which students at various performance levels work together in small groups toward a common goal.” But this type of learning is not just simply achieved by placing students in groups. These strategies have elements of positive interdependence (students work together to achieve a learning goal), individual accountability (students are required to participate and are depended upon for group success), promotive interaction (students encourage each other to work toward their common goal(s)), appropriate use of group social skills

² Process worksheets provide a description of the phases one should go through when solving problems related to the learning tasks (Kirschner, Sweller, & Clark, 2006).

(members trust each other, communicate well, support each other, and resolve conflicts constructively), and group processing (each member is offered time to reflect on the strengths and weaknesses of their groups processes) (Johnson & Johnson, 1988, 2009; Slavin, 1988; Bowen, 2000).

A community approach enhances learning by helping students “make sense” and establish meaning to mathematics learning together (Stepanek, 2000). Creating this type of learning community involves establishing classroom standards of behavior, developing relationships that facilitate learning, learning group process skills, and sharing classroom authority. Stepanek (2000) explains that a community environment is distinguished by an emphasis on collaboration, where students do not formulate ideas in isolation, but through social interaction. Evidence shows that students working in cooperative learning environments achieve at higher levels of thought and retain information longer than students who work individually (Njenga, 2010). Some recognizable traits of cooperative learning seen in lessons are “think-pair-share,” “numbered heads together,” group discussion, “round robin,” and “jigsaw.”

2.4 Does the lesson engage students in higher-order thinking?

Many textbooks are comprised of problems that are merely iteration, but teachers are being asked to develop more than the basic skills in their subject areas (Lewis & Smith, 1993). There is a general agreement that a difference between lower order and higher order thinking can be distinguished, but higher-order thinking is difficult to define. According to Lewis and Smith (1993), experimental psychologist Norman Maier (1933, 1937) used the terms “learned behavior or reproductive thinking” to describe lower order and “reasoning or productive behavior” to describe higher order. Higher-order thinking challenges the student to interpret, analyze, or manipulate information (Lewis & Smith, 1993). Lewis and Smith (1993) combine research done

on the distinction between lower and high orders to offer the definition, “Higher order thinking occurs when a person takes new information and information stored in memory and interrelates and/or rearranges and extends this information to achieve a purpose or find possible answers in perplexing situations.” They say that in order to evaluate this type of thinking the answer cannot be found through simple recall. Students must examine the given information and make judgements regarding the logic needed to arrive at a conclusion.

For the purpose of this study, higher-order thinking problems will require several steps without mimicking or just recalling steps. Many look to Bloom’s Taxonomy to further explain what symptoms of critical thinking should be seen in classroom instruction (Richard, 1985). Students will be asked to analyze problems by comparing or deconstructing their own reasoning or the reasoning of another. They will need to critique, judge, and check the information given or the solutions found. Students that design and construct a plan of action are also using these higher-order skills (Richard, 1985).

2.5 Does the lesson include time for students to self-assess or reflect on their learning?

According to Schoenfeld (1992), students expect their teachers to provide them with answers and are discouraged from discovering methods and answers on their own. A study by Bergstresser (2013) discussed an observation that students are often left playing a passive role in the classroom. Her study implemented metacognitive training showing a correlation between the skills learned and the students’ ability to retain content. Rickey and Stacy (2000) describe metacognition as, “thinking about one’s own thinking”. Other authors describe metacognition as one regulating their own cognitive process (Brown, 1987; Schraw, 2001) through planning, monitoring, and evaluating (Cooper and Sandi-Urena, 2009). A study done by Java (2014) identifies metacognitive skills used by “people who master problem solving”.

These skills include: (a) identifying of the problem's goal, (b) comprehending the problem before solving it, (c) recalling and relating to past knowledge, (d)attaining a higher grasp of their conceptual understanding, (e) cutting down the problem into several steps, (f) exercising flexibility by modifying techniques to attain the identified goal, and (g) employing self-evaluation of the solution made. (pages 8-9)

One of the steps indicated as a metacognitive skill is the process of self-evaluation.

Bransford (1999) says this approach of self-evaluation or self-assessment “is used when students can draw conclusions about their own work, set goals, keep records, and use aids or cuing devices to check for understanding.” Armstrong (2013) researched the implementation of self-assessment procedures in a classroom and established that his experimental group performed significantly better than the control group. The aid used in this study was a rubric.

A rubric is a tool students can use to score and identify the quality of their work based on a standard. Studies showed that the use of a rubric could improve students' understanding of objectives (Armstrong, 2013). However, rubrics can be ineffective if the scoring criteria are too general. Armstrong also concluded that the presence of feedback along with the use of the rubric was necessary for his students to fully assess their learning. Three steps identified to enhance student-evaluation are a) set clear expectations, b) circle key phrases on the assignment, c) revise work by identifying and correcting mistakes. Sadler and Good (2006) conclude that self-assessment improves student learning, but recommend that more study be done to determine if one technique is better than the other. These conclusions show the benefits of including elements of self-evaluation in classroom lessons.

2.6 Can the lesson be completed in a reasonable amount of time?

A main concern for teachers, as seen in a study done by Fazekas (2011), is the amount of time needed to complete a task or activity. Generally there are one hundred eighty instructional days given to teach all standards for a given course in public education. Time is very important.

Fazekas also pointed out that the amount of time spent of “the math” is often overtaken by the amount of time spent explaining the goals of a lesson or superficial aspects of creating the final product.

Stallings (1980) explained that student engagement time is positively correlated with student achievement on tests, but it does depend on how the time is used. This can be hard to judge without actually completing the activity or task with students. Once an activity is complete the teacher can reflect on the time needed to complete all aspects of the lesson. Teachers should look for the time spent on gaining understanding of the mathematical concepts to outweigh the time spent off-task (Karweit, 1984).

2.7 SpringBoard

The textbook used in Livingston Parish Geometry classrooms is SpringBoard© (SB) Geometry by College Board. According to the SB website and SB staff members sent to train the teachers in our district, SB is based on the “understanding by design” model. It is built around embedded assessments linked to specific standards and learning targets. The program emphasizes a methodical approach to learning new content using a balance of investigative, guided, and directed activities. The goal is to build content knowledge, encourage exploration, modeling, collaboration, practice, and application. The SB site (springboardprogram.collegeboard.org/mathematics/) identifies the following features as being evident in each lesson:

1. “Learning Targets” identify the relevant standards in student-friendly language.
2. “Suggested Learning Strategies” promote student ownership of learning.
3. “Meaningful Problems” provide real-world contexts.
4. “Cross-Curricular Connections” call out academic applications.

5. “Mathematical Practice Standards” are integrated into every lesson.

6. “Check Your Understanding” sections formatively assess student knowledge at the point of instruction.

Each lesson is also “chunked” to promote understanding. Chunking is the word used throughout the SB program that means the questions are grouped together in a specific way that is supposed to best help move the lesson along and bring about the most effective results. I followed the chunking recommended in each lesson and debriefed student results and answers after each section of questions was complete.

The features identified by the textbook align with the characteristics being evaluated. “Learning targets” are the objectives of the lesson. The learning strategies are provided in the margins and focus on collaboration among students and groups. Mathematical Practice Standards involve metacognitive skills in which students must go beyond recalling information to answer questions. The book includes “Check Your Understanding” to implement peer and self-assessment in their lessons. Elements not addressed by the text are teacher guidance and feedback and the effective use of time for each lesson.

Chapter 3: The Activities

In this chapter I will look specifically at activities used in the Springboard© (SB)

Geometry text. For each of the selected lessons, I will do the following:

1. outline the lesson as it is presented in the text;
2. describe the implementation in my classroom;
3. discuss the students' and teachers' reactions throughout the lessons;
4. present student results from our department created assessments, and
5. critique each activity using the framework created from the literature.

3.1 “Patios by Madeline”

This activity is contained in Unit 1 (spanning the first 6 weeks) of the Livingston Parish Geometry curriculum. It is “Activity 7” in Unit 1 of the SB book. It consists of three lessons, which we refer to as Lesson 7.1, Lesson 7.2, etc. According to the textbook, the focus of Activity 7 is to “develop the concept of proof in the context of parallel and perpendicular lines.” SB identifies this activity as “investigative,” (in contrast to “guided” and “directed”), meaning that students are expected to work with minimal input from the teacher.

The parish curriculum allotted two and a half ninety-minute periods (or nearly four hours) to the activity. In my implementation, the entire activity, including extra practice parts at the end of each lesson, was completed over the course of four ninety-minute class periods.

3.1.1. Lesson 1. Parallel Lines and Angle Relationships

Lesson 7.1 begins with a bell-ringer that asks the students to draw an oblique line on a sheet of lined paper and then measure the angles formed at the intersections using a protractor. The goal of this bell-ringer is not stated explicitly, but one can infer that it is intended to remind

students about what it means for lines to be parallel and to activate any prior knowledge they have about a parallel relationship.

After the bell-ringer, the textbook informs students that the learning targets of Lesson 7.1 are to:

1. Make conjectures about the angles formed by a pair of parallel lines and a transversal.
2. Prove theorems about these angles.

The activity in Lesson 7.1 is introduced with a scenario in which a customer asked for a patio and walkway to be designed, with the specification that the rows of paving bricks must be parallel to the walkway. The students are told that two parallel strings are tied to stakes and will be used to align the rows of bricks. Paint is used to identify the underground gas line to avoid accidents during construction. The string and gas lines intersect to form eight angles. This setting is referenced throughout all three lessons of the activity. The following diagram was given as a reference for the students

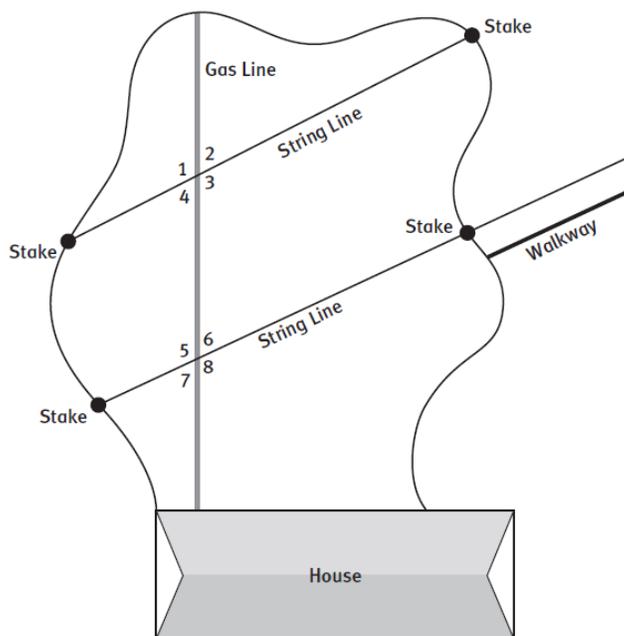


Figure 1. Diagram for Lesson 7.1, “Patios by Madeline” (SpringBoard Geometry, 2015).

The text asks students to use a protractor to measure all of the angles in the diagram. Then key terminology is introduced. The terms given are “transversal,” “same-side interior angles,” “alternate interior angles,” and “corresponding angles.” Students are asked to refer to the measurements they made, and then to use the terms to state conjectures about the relationships between angles. The text asks them to confirm their conjectures using another example of a transversal intersecting two parallel lines.

The next section of Lesson 7.1 introduces the name “Same-Side Interior Angles Postulate” and without stating it, asks students to write this postulate in if-then form based on the conjectures they made previously. They are also asked to write other statements about corresponding and alternate interior angle pairs formed by a transversal and two parallel lines. The lesson then inserts a Check Your Understanding (CYU) section in which the students assess their understanding of the terms defined previously and of the relationships between the angle pairs in a transversal of a pair of parallel lines.

After this, the lesson asks students to complete a proof of the so-called “Corresponding Angle Theorem,” which is not stated in the book. Actually, the students are given a labeled diagram of a transversal of a pair of parallels, and are asked to prove that the two angles in a named pair of corresponding angles are congruent. Prior to the proof, the book introduces the symbol “ \parallel ” for parallel lines. The book advises students, “It is sufficient to prove that one pair of corresponding angles formed by a pair of parallel lines and a transversal are congruent.” (Since the theorem has not been stated, it is unclear what this might mean to students.) The proof is in two-column format and all of the statements are already provided for the students, who need only supply two missing justifications. This requires a reference to the definition of supplementary angles and a reference to the Same-Side Interior Angles Postulate. The book justifies one of the

steps by reference to something called “The Congruent Supplements Theorem.” Apparently, this is new to the students, because the book puts a statement of the theorem in a side-bar.

Following this, the lesson asks students to provide all of the missing justifications in a proof of the “Alternate Interior Angles Theorem,” which is also never stated. At the end of the lesson, practice problems are included. These check vocabulary with four true-false questions, a contrived algebraic equation (presented by labeling a pair of same-side interior angles with linear expressions in the variable x), and a final question asking students to deduce the measures of supplementary angles (in a diagram with numerous pairs of supplements).

3.1.2. Lesson 2. Proving Lines are Parallel

Lesson 7.2 begins with a bell-ringer that asks students to write several if-then statements that are true and then swap the condition and conclusion to create a new if-then statement. The book calls this new statement the “converse.” Students are then asked to discuss the converses with their partners, determine which of them are true and provide evidence to support their conclusions.

After the bell-ringer, the textbook informs students that the learning targets of Lesson 7.2 are to:

1. Develop theorems to show that lines are parallel.
2. Determine whether lines are parallel.

This lesson begins by asking students to write the converse of the Same-Side Interior Angles Postulate, Alternate Interior Angles Theorem, and Corresponding Angles Theorem. The book informs students that “The converse of the statement “If p , then q ” is “If q , then p .” It also advises that the converse of the statement is not necessarily true, even if the conditional statement is true.

Then, the text asks students to recall the patio scenario from the previous lesson and informs them that the plans are being translated to blueprints, pictured on the next page (see Figure 2).

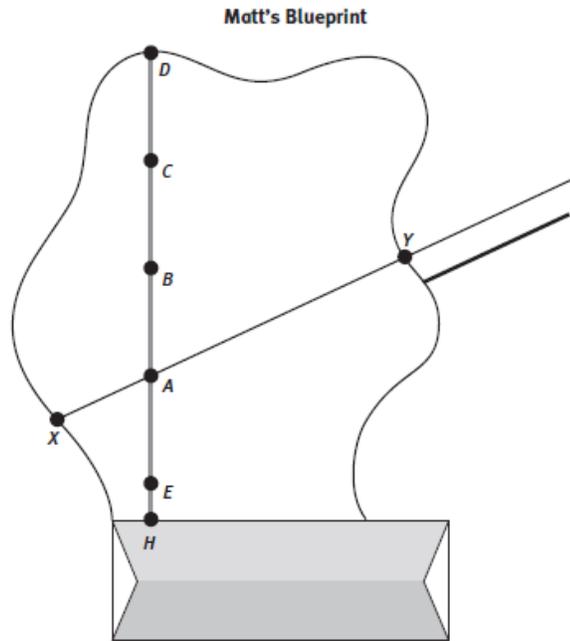


Figure 2. Diagram for Lesson 7.2, “Patios by Madeline” (SpringBoard Geometry, 2015).

The text then asks students to measure angle HAX and to draw lines through B, C and E that make the same angle with line HD in the corresponding position. (The instructions do not use this wording, but include enough detail to ensure that this is what is drawn). They are then asked to show how the drawing provides evidence in support of the converses to the Alternate Interior Angles Theorem and the Same Side Interior Angles Theorem. The text has not explicitly stated the converses at this point, so students are working off of what they wrote previously. Next, students are asked to explain why the given corresponding angle measures would not create parallel lines and will need to explain how to adjust the string lines creating these angles to ensure the lines will be parallel.

Lesson 7.2 then presents three Check-Your-Understanding questions. Students are asked to complete a proof of the Converse of the Corresponding Angles Theorem. This proof makes reference to the definition of congruent angles, the Linear Pair Postulate, the substitution property, and the definition of supplementary angles, and it uses the Converse of the Same Side Interior Angles Postulate. The lesson ends with five practice problems. The first three concern a line crossing three other lines. They are given the measures of some angles in the figure and are asked to determine lines are parallel. The fourth question asks about a pair of alternate interior angles that are both right. The last question asks how someone with a protractor, string and stakes could determine if two yard lines on a football field are parallel.

3.1.3. Lesson 3. Perpendicular Lines

Lesson 7.3 recommends a bell-ringer that asks students to draw a line perpendicular to one of the lines on a piece of lined notebook paper and then determine the relationship that line has with the other lines on the paper. After the bell-ringer, the textbook informs students that the learning targets of the lesson are to:

1. Develop theorems to show that lines are perpendicular.
2. Determine whether lines are perpendicular.

Lesson 7.3 begins with a similar scenario to Lesson 7.1, except the customer wants a patio with paving brick rows perpendicular to a walkway instead of parallel. In Problem 1, students are asked to draw a line through the point W perpendicular to the line XY using a protractor (see Figure 3). Then, in Problem 2, they are asked to draw a line parallel to the newly drawn line. The answer given by the textbook for Problems 1 and 2 is shown in Figure 3.

Meanwhile, in the margin, the text introduces the “Perpendicular Postulate,” and the “Parallel

Postulate.”³ These drawings lead into Problem 3, where students are asked to describe the relationship between the line drawn in Problem 2 and the line XY.

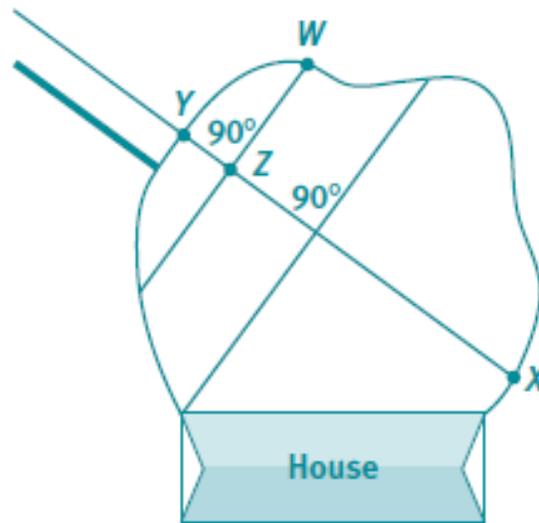


Figure 3. Lesson 7.3 Answer 2, "Patios by Madeline" (SpringBoard Geometry, 2015)

On the next page, the “Perpendicular Transversal Theorem” is stated in the margin but not discussed in the context of a problem.⁴ Problem 4, which is identified as a “Critique the Reasoning of Others” question, asks students to prove “Matt’s conjecture,” which happens to be none other than the Perpendicular Transversal Theorem. Problem 5 is formulated using the new vocabulary term “perpendicular bisector.” The problem asks students to restate the meaning of “midpoint” and of “perpendicular.” Problem 6 asks students to write an “instructional guide” that describes the process for creating rows of bricks parallel to a patio walkway and perpendicular to a walkway.

The lesson’s CYU section includes Problem 7, which essentially asks if perpendicularity is a transitive relationship between lines. Problem 8 checks whether the definition of “bisector”

³ The “X Postulate” states that given a line and a point not on it, there is exactly one line through the point having relation X to the given line.

⁴ “If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.”

is understood. The lesson's Practice section includes Problems 9--13. Problems 9-11 work from the assumption that two given lines are parallel and two given angles measure ninety degrees and students are asked to use this information to explain why lines are perpendicular, other angles are right, and show other lines are parallel. Problem 12 asks students to use the definition of perpendicular bisector to find the length of a segment. Problem 13 asks students to explain why a transversal of two parallels including obtuse angles could not be described as perpendicular lines.

3.1.4 Student and Teacher Reactions

3.1.4.1. Lesson 1

The time spent on the bell-ringer was mostly used to teach students how to use a protractor. "I've never learned how to use one of these things!" was heard multiple times throughout the bell-ringer even though we used them in an activity two weeks before. This process took longer than expected, but it definitely opened up my eyes to possible issues that could occur throughout the lesson. The students were divided into groups of three or four to complete the activity. I attempted to have at least one student more familiar with using a protractor in each group. The reason for this is that I noticed most of the students who could not use their protractor had a weaker understanding of angle measure and congruent angles which was necessary to be successful in this activity.

When the key terms were introduced and students were asked to match pairs of angles from the picture to the terms that match each description, the only definition they had trouble with was corresponding angles. When asked about why this occurred, many students explained they did not know what nonadjacent angles meant. Adjacent angles was defined in Activity 4, but the students failed to relate that definition to understand what nonadjacent would look like.

Many groups struggled to draw the desired conclusions about the relationship between pairs of angles because their measurements were not accurate in the first question. I was not expecting this to be an issue when planning the lesson, but was not surprised at this struggle after completing the bell-ringer and gaining a greater understanding of my students' previous knowledge and actual ability. I had to stop all groups and confirm their measures were correct before allowing them to move on even though this was not suggested by SB. The book did not anticipate that the majority of my students would be using the protractors incorrectly instead of just a few.

Students were then given two parallel lines and asked to draw their own transversal, label, and measure each angle formed. The students were then asked to explain why their conjectures were confirmed or not. However, the same protractor issues hindered some of the students from drawing clear conclusions. The main difficulty in this section was found when the students were told to draw their own transversal through the parallel lines which made all of the angle measures obtained vary within each group. They were unable to compare answers and were doubtful about their own findings. These inaccuracies meant that I had to address each group separately because of the varying locations of their transversal lines, taking time away from the overall goal of the section which was to confirm findings from the previous section. It also was not made clear by the activity that confirming using measurements is not the same as proving something to be true.

Next the students had to write the postulates and theorems in question in if-then form. Writing the postulates and theorems in this way was very confusing to the students. Many failed to see the connection between the previous questions and were already overwhelmed with the corrections they had to make thus far in the lesson. They just thought of this section as a separate

task. “Why are they asking this random question after all of that measuring?” was the main concern heard from ten of the fifteen groups. However, most were able to write the if-then form as asked when given the condition by the teacher even though they did not quite understand the purpose.

This led into the next section entitled Check Your Understanding (CYU). I noticed that the CYU section allowed the students time to reflect on what they had learned at this point in the activity and if they could apply it to a new diagram. Many students struggled to explain why they knew how to identify the names of given angle pairs even though most could label using the terms correctly. The struggle seemed to occur because the students only needed to simply “parrot” the definitions back and not provide any more reasoning. The last question involving algebraic expressions caused the most problems because of the student’s lack of adequate Algebra 1 foundation. They could explain that they knew corresponding angles were congruent but could not represent that with an equation.

To complete this lesson the last question only included one item in which the students are asked to complete a proof of the Corresponding Angles Theorem. I had to take on a guided instruction approach during this question. Many students struggle with writing proofs and were not making progress in their groups. Even though the statements were given, the students could not connect the postulates and theorems discussed previously in the course to the present problem. One proof was not enough practice before moving on to the second CYU section and lesson practice.

After completing the proof the students began another CYU section. They are first asked to reflect on their answers from the previous section. They needed to explain how they know the reasons used are accurate which led to the same problems experienced previously when they just

need to repeat the definitions. The students were not able to complete this CYU on their own or with their groups. I think this was due to their lack of understanding of the proof process in general and the lack of connection made between the theorems written in previous sections and activities that can be used in their proofs. This was the second and last proof of the activity. Due to a lack of time left in the class period the practice problems were assigned for homework.

3.1.4.2. Lesson 2

As students entered the classroom, they were instructed to turn in the lesson practice from the previous day and then begin the bell-ringer. Before discussing their bell-ringer results, I made note of the number of students that correctly answered their homework assignment and returned them to the students. We then reviewed the correct answers and addressed any misunderstandings from the previous day's lesson. The main misconception came from the problem involving writing and solving an algebraic equation based off of angle relationships. As mentioned previously, this misunderstanding was based on a lack of retention from their Algebra 1 course and was corrected easily. However, the angle relationship was understood by the majority of my students.

We then moved to the bell-ringer where they wrote their own true if-then statements and used that as an introduction to that day's objectives. One example given by a student was "If I make a field goal kick, then the Wildcats get three points." After hearing other examples, I instructed the students to swap the condition and conclusion to create a new if-then statement as suggested by SB. This led into the discussion of how to write the converse of a given statement. They were asked to discuss if the converse of their original statements were true with their partners and had to provide examples to support their conclusion. The student that mentioned the previous example explained that his converse was not necessarily true because our football team

has two kickers. This discussion led into the first question of the lesson where they had to write the converse of the postulate and theorems from Lesson 1.

The students were allowed to look back at the previous day's lesson if needed. The bell-ringer activity definitely helped make this question move smoothly. Next the students returned to their groups of either three or four to complete the rest of the activity. They were asked to recall the patio scenario from the previous lesson, but were told that the plans were now being translated to blueprints that can be seen on the next page. This created some confusion when the students got to Problem 2, because many groups did not turn the page to see the diagram mentioned. Once directed to the diagram, students were able to complete part a and measure the angle but struggled to extend the lines and draw their own angle congruent to the original. After teacher assistance most students were able to create a congruent angle and extend the line to see that it appeared parallel.

Problem 3 led into a discussion about the converse postulates and theorems written at the beginning of the activity. One student asked, "Is this (referring to the line drawn) what the converse of the corresponding angles one would look like in a picture?" His group member answered, "I don't see why not, since we know the corresponding angles are congruent. I mean we drew it so they would be congruent so the lines must be parallel." This student discussion led the class into being able to complete the rest of the section (Problems 4-7) in a similar manner. They drew multiple parallel lines by creating congruent alternate interior angles and supplementary same-side interior angles.

Students were able to show further understanding and explain when the assistant's angles would not create parallel lines but did not realize the scenario provided was not actually a converse relationship. Throughout the CYU section, the students did not have any trouble until

they arrived at the formal proof question. Even though this followed true SB fashion of fill in the blank, the students were still unable to provide the correct statements and reasons. I had to review the question with most of the groups and explain the reasoning. The two reasons twelve out of the fifteen groups missed were the use of the substitution property and the definition of supplementary angles. After reviewing the proof, we still had time to begin the lesson practice in class before the bell rang. Only seven of my students were unable to answer the practice questions correctly. These students claimed that the use of three parallel lines intersecting a transversal is what caused confusion. The practice did not involve any formal proofs.

3.1.4.3. Lesson 3

The third day of this activity fell on club schedule day which, at my school, means that students leave throughout the class block to attend club meetings. This did not change the structure of the beginning of the class and all students still had to complete their bell-ringer. Almost all students were able to complete the bell-ringer without any peer or teacher assistance. They were also able to see that the line drawn was perpendicular to the other lines on the paper pretty quickly and almost seemed perturbed that was not understood information. This bell-ringer led into the lesson well, but groups did need to be rearranged due to the number of students participating in club meetings.

The scenario of this lesson was similar to the previous lessons but now students had to draw a perpendicular line. Only three groups needed guidance to complete these first few tasks because they used many of the skills developed in the previous lesson and bell-ringer. Most groups were able to move on to critique the reasoning of the patio designer and create a diagram for justification, but struggled to use words to justify their reasoning. The main reason I observed used by many groups was, “Line t is perpendicular to line n because it is perpendicular to line

m.” They did not connect to the previous lessons using the Corresponding Angles Theorem until prompted.

Before moving into the introduction of perpendicular bisector, I stopped the groups to discuss the postulates introduced at the beginning of the lesson to make sure these were not overlooked. This led into the new term perpendicular bisector being introduced. Only one group was able to work from the given definition to write three conclusions, so I led a whole group discussion to move the activity along. Students had a difficult time breaking up the definition into terms of “midpoint” and “perpendicular”. Once asked to look at the two words separately, students were able to break down the definition. I also provided another diagram and asked them to make the conclusions if given that the transversal was a perpendicular bisector to the given segment. Almost all groups were successful after using my previous example.

The last question before the CYU section was for students to write an instruction guide for creating rows of bricks parallel to a patio walkway. I had them complete one guide per group to allow for more discussion and collaboration, but needed to remind students that we were writing for both parallel and perpendicular rows. I found it helpful to give feedback immediately as groups finished their guides to allow for edits while it was fresh on their minds. Only one group was able to move on without rewriting, almost all groups needed to include more details about which angles needed to be congruent to ensure the lines would be parallel. All groups finished their guides and the CYU section before class ended but not all were able to begin the lesson practice, but the time needed to complete the guides varied for each group. This variation caused this activity to go into the next day.

Before reviewing the answers to the CYU section the next day, I prompted the students to draw a picture to help them make conclusions, which enabled seven out of twelve groups to be

able to answer the section correctly instead of the original two groups upon entering the classroom. After reviewing the lesson practice, forty two of the fifty three students answered all questions correctly. Out of the eleven not answering correctly only three made attempts to complete the assignment and of those three the misunderstanding lied in labeling the wrong angles ninety degrees from the given measures.

3.1.5. Results

After completing this activity the students were given a department quiz (see Appendix B) created to assess the learning targets of Activity 7. Eighty nine percent (89%) of my students were able to correctly identify when angles were congruent when given two parallel lines. Seventy four percent (74%) of the students correctly classified supplementary angles. However, even though the majority of the students were able to show that they knew given angles were congruent, only seventy one percent (71%) could solve the algebraic equations correctly. The algebraic equations were the most missed questions outside of the proofs. The two proofs on the quiz both followed the two-column format. They were given parallel lines and were asked to prove angles congruent using the angle pair relationships and the transitive property. Thirty two percent (32%) of my students left these two questions blank. Only fifty three (53%) of the students answered the proofs correctly. The ones that made unsuccessful attempts at this proof struggled to identify the relationship between the angles correctly. For example, many confused alternate interior angles and corresponding angles leaving them unable to give the correct reason for two angles to be congruent. After asking the students about this confusion of angle relationships, many explained that the presence of two sets of intersecting parallel lines in the same diagram was the reason for their mistake.

3.1.6. Critique

3.1.6.1. Lesson 1

The questions mentioned previously guided the critique of Lesson 1.

1. Does the lesson state clear, standards-based objectives?

Yes, the learning targets are listed in terms that students can understand and are linked to a Common Core State Standard (CCSS). The CCSS is only visible in the teacher edition and includes examples of theorems that should be proven in Geometry courses. However, the standards focus of the activity described in the teacher edition (“develop the concept of proof in the context of parallel and perpendicular lines”) is very vague. It is unclear what is meant by “develop the concept of proof.”

2. Does the lesson support teachers in providing guidance and feedback to students?

No, it does not offer sufficient opportunity for this to occur. Even though tips to the teacher in the margin mention the need to “guide” students, it is not specific about what is expected. It often seemed like “guide” meant to go over the answers given by the text, but this is never stated or encouraged explicitly. The teacher must be very attentive to student conversation in order to provide meaningful guidance and feedback during group discussions. This is difficult when there are multiple groups discussing questions at the same time. The best opportunity to provide guidance and timely feedback was during the CYU and Lesson Practice section. However, the depth of the questions only left opportunity to simply check the students’ answers for correctness.

There are many times throughout this lesson where the students are expected to think about, prove and use statements that they are asked to formulate, but which are never stated in the book they have in front of them. That puts a tremendous burden on the teacher, in assuring

that all students agree on the meaning of certain statements. It's hard to imagine a better way of doing this than writing the statement out on the white board or on a hand-out. It makes sense to have students formulate the statement before proving it or using it, but at some point, the statement needs to be nailed down. When the students are left without guiding examples in this manner it creates a stand-still and no progress is made within the groups.

3. Does the lesson incorporate cooperative learning?

Yes, but not effectively. The cooperative learning aspect is not prominent throughout the first lesson of the activity. When cooperative learning was called for, too many of the students were unable to measure the angles correctly and were unable to compare answers and help their group members arrive at conclusions. The sharing of knowledge was missed multiple times throughout this lesson due to student struggles using the protractor. Many cooperative strategies (think-pair-share, look for a pattern, summarizing) are suggested in the margins, but the students did not engage in this type of learning because the lesson answers did not provide much discussion to understand. The lesson outline did not fit with the strategies listed.

4. Does the lesson engage students in higher-order thinking?

Attempts to have the students engage in higher-order thinking were only seen through the use of specific strategies and terminology such as explain, summarize, predict and confirm. However, this did not occur at a higher level. The confirmations were only made through examples, which the teacher was asked to explain was not enough evidence for proof.

Most questions involved students having to explain their reasoning or justify their answers, which would require students to process why they can answer each question. However, many of the attempts to include higher order thinking during this activity were only to explain the students understanding of simple definitions. It confused students to use the definition to

explain their reasoning when the question included the definition itself. The answers were almost included in the questions; which many would think helps the students formulate the correct conclusion, but it actually has a reverse effect and steers them away from the simple solutions. As mentioned earlier, if the student is only simply required to recall facts to obtain an answer that is not considered higher-order. This allows us to conclude that higher-order thinking is not present in this lesson.

5. Does the lesson include time for students to self-assess or reflect on their learning?

The CYU sections were intended to allow time for peer and self-evaluation. However, the students were unable to do this without the corrected answers first. There was not a rubric that specifically stated the expectations in these sections so the students had no easy way to tell if their attempts were going in the right direction. Even without the presence of a rubric, students did not have clear statements available to them to refer to in order to correct their reasoning. There was good discussion between some students who were engaged and I heard conversations where students critiqued the reasoning of group members, but this was not seen in all groups. The students who were most engaged did not have any trouble using protractors. There was a minimal amount of opportunity for students to self-evaluate their learning throughout this lesson.

6. Can the lesson be completed in a reasonable amount of time?

In the future I would include a longer review of measuring angles with protractors the day before completing this activity or use technology to measure the given angles, such as Geometry Sketchpad. The length of time spent on the mathematic concepts in this lesson was largely affected by the inaccuracies in angle measurement. The goal was not to learn how to use protractors correctly, even though that is where most of the class time was spent. Many students

missed learning about the relationships between angles on a transversal intersecting parallel lines because of the time spent correcting their use of a protractor.

Additional comments about this lesson are in response to its adaptability. It was not easily accessible to learners at different levels. If the student is unable to answer question 1 correctly, then they were be unable to participate in any of the discussions following that question. There were not many ways for the students on lower levels to enter into the lesson without feeling like they were incapable of completing the tasks in the lesson.

3.1.6.2. Lesson 2

The same questions used previously guided the critique of Lesson 2.

1. Does the lesson state clear, standards-based objectives?

Yes, the learning targets are listed in terms that students can understand and are linked to the same a CCSS as Lesson 7.1. To make sure there is not confusion for students, the text could clarify what it means to “determine whether lines are parallel” and what tools will be used to make that determination.

2. Does the lesson support teachers in providing guidance and feedback to students?

Similar to Lesson 7.1 students were only provided feedback when the teacher reviewed the answers to the group of questions from the CYU and lesson practice sections. In Problem 2 the steps guided students to create a congruent angle on a transversal. These steps were very clear and all but one group was able to complete the tasks involving this process. Since students were able to create the congruent angles, they were also able to see the relationships between the converse statements written and their drawings. In conclusion, this lesson offered more opportunities for guidance and feedback, but it did not play a large role.

3. Does the lesson incorporate cooperative learning?

Collaboration was possible at the beginning because of the reference back to Lesson 7.1. Think-pair-share was an effective strategy in this lesson since students were able to create parallel lines using congruent angles. I heard meaningful connections to the converse statements written in Problem 1 being discussed among group members.

4. Does the lesson engage students in higher-order thinking?

The lesson's main attempt to have the students "attend to precision" was to explain the meaning of the Contrapositive of the Corresponding Angles Theorem. This by itself is not a difficult problem, but in the context of the lesson it is confusing to the students. The lesson focused on the difference between a statement and its converse. It claims to follow a logical approach in making this distinction, but implies that you can use the converse when needing to explain the contrapositive. Students were able to explain that the given corresponding angles would have to be congruent to make the lines parallel but did not see the difference between the statement made and the converse of Corresponding Angles Theorem.

This lesson did a better job about not including the answer in the question, but they still did not ask for student to come to their own conclusions. For example, Problems 12-14 gave the students two angle measures and asked if two lines were parallel. Instead they could have given the students the same angle measures and asked what they could conclude, if anything, from the given relationship.

5. Does the lesson include time for students to self-assess or reflect on their learning?

The CYU sections were intended to allow time for peer and self-evaluation. However, similar to Lesson 7.1 the students were unable to do this without the corrected answers first and without a rubric. Two of the three questions asked in the CYU section did not refer back to this

specific lesson which does not allow time for students to build on the current objectives. These two questions involved writing the inverse of the Alternate Interior Angles Theorem (AIAT) and writing a proof for the Converse of the Corresponding angles Theorem. The AIAT is from Lesson 7.1 and writing an inverse was discussed in Lesson 3.3, so students failed to see any connection to the current lesson. The proof addressed a theorem formulated in this particular lesson, but did not effectively check the students understanding of the theorem itself by asking students to prove it to be true.

6. Can the activity be completed in a reasonable amount of time?

This activity and the lesson practice were all able to be completed during one class period (90 minutes). Many of my students were able to move through this lesson easily in their groups.

3.1.6.3. Lesson 3

The previous questions also guided the critique of Lesson 3.

1. Does the lesson state clear, standards-based objectives?

The learning targets are listed in terms that students can understand, but upon completion of the lesson the students questioned if that was actually what they learned. The targets claim that students will show lines are perpendicular and determine if lines are perpendicular. However, the lesson bell-ringer works off of the assumption that students already know how to create perpendicular lines. There is no mention of using the definition of perpendicular bisector in the targets, but understanding this definition seems to be important to the text because there are three questions involving a perpendicular bisector. Based on this information I would say that the objectives are not clear due to the fact that they do not apply to the whole lesson. The only new theorem mentioned in this lesson was the Perpendicular Transversal Theorem, which was given in the margin. Students were only asked to affirm it was true, not “develop” it.

2. Does the lesson support teachers in providing guidance and feedback to students?

This lesson does not incorporate any feedback past reviewing correct answers. Teacher guidance is also minimal due to the abundance of “math tips” found in the margins. Each question is disconnected from the previous questions which also made it difficult to help students make any connections to the objectives or previous learning.

3. Does the lesson incorporate cooperative learning?

No, the cooperative learning aspect is not prominent throughout this lesson. The students were able to complete all of the questions independently. Many checked answers with their group members, but further discussion was not needed. The only reason students collaborated at all was because I made them answer Problem 6 as a group. I broke the group of four students into pairs and each pair had to write part of the “instruction guide.” Then, the group had to come together and correct each pair’s guides before compiling them into one set of instructions. The text did not encourage this, but I found it was necessary to have some time where the students worked together.

4. Does the lesson engage students in higher-order thinking?

As mentioned earlier, if the student is only simply required to recall facts to obtain an answer that is not considered higher-order. Postulates, theorems, and definitions are given in abundance during this lesson when compared to Lesson 1 and 2, but this is not beneficial to the students because they are not connected to specific questions. The students are simply asked to use the definitions to confirm conjectures are true. Only surface level knowledge is needed to complete the given tasks. Many of the conclusions asked to be made were already common knowledge to the students.

5. Does the lesson include time for students to self-assess or reflect on their learning?

The students do not reflect on their learning in Lesson 3. The six problems in this lesson did not rely heavily on each other. Students did not have to reflect on their previous answers to make conclusions in the latter portion of the lesson. The lack of reflection throughout left the students feeling like the lesson was scattered. They did have to recall their previous knowledge from middle school courses about perpendicular lines, but this was not difficult.

As mentioned previously, Problem 6 was adjusted to provide time for collaboration. This forced them to correct their group members' reasoning before turning in their final product. Many good discussions were heard during this time, but this would not have happened if I did not adjust the texts' instructions.

6. Can the lesson be completed in a reasonable amount of time?

This lesson can be completed in one class period, especially if writing the instruction guide is removed.

Additional comments about this lesson pertain to changes I would recommend based not only on my experiences, but others in my department and the adaptability of the lesson. The "scattered" nature of this lesson made it easy for students to jump around and answer questions they felt comfortable with, but this is not necessarily a good thing. Most students were able to answer every question correctly, but were annoyed at the simplistic nature of the lesson. I would recommend starting with Problem 5 and then moving to the CYU and Lesson practice sections. This means I would also delete the task of writing an instruction guide. This task took a lot of time for some groups and did not benefit their learning. These changes would still keep the lesson accessible to multiple students on multiple learning levels, but would not spend time focused away from the new objectives students need to learn.

3.2 “Is That Right?”

This activity was completed during Unit 3 of the Livingston Parish Geometry curriculum. It is “Activity 20” in Unit 3 of the SB book, consisting of two lessons, 20.1 and 20.2. According to the textbook, the activity focus is to enable “students to prove the Pythagorean Theorem using triangle similarity and to apply the relationships in the theorem, including Pythagorean triples, to solve problems.” SB identified this lesson as investigative, which implies little guidance is needed from the teacher. The entire activity and extra practice was completed over the course of three ninety minute class periods which was the same time allotted by the parish curriculum.

3.2.1. Lesson 1. Pythagorean Theorem

The SB-recommended bell-ringer asked for students to find the geometric mean of given pairs of numbers. The goal for using this bell-ringer is not stated explicitly, but one can infer that it is intended to review the objectives of the Lesson 19.3 entitled “Geometric Mean” and to anticipate relationships that appear in the proof later in the lesson. (However, the text never points out this connection explicitly). The bell-ringer also reviewed an algebraic skill needed to solve equations with exponents that will be used in the lesson.

After the bell-ringer, the textbook informs students that the learning targets of the lesson are to:

1. Use similar triangles to prove the Pythagorean Theorem.
2. Apply the Pythagorean Theorem to solve problems.

Lesson 20.1 begins with a scenario describing an online company that manufactures custom kites. Customers can go to this website, design a kite, and this company will build the kite. The owner of the company is creating a webpage to educate her customers about the parts of a kite.

The book tells students that many kites include right triangles which is why the Pythagorean Theorem is “useful in analyzing the dimensions of a kite.”

The Pythagorean Theorem is introduced in the margin and the students are asked to label the sides of a given triangle using the terms “hypotenuse” and “leg.” The lesson assumes that the students are familiar with this terminology and no reference to the definition of these terms is given. Students are then expected to find the length of the hypotenuse using the equation stated in the margin.

The lesson then provides a diagram (Figure 4) of a right triangle with an altitude drawn to the hypotenuse. The book tells students that the altitude drawn forms three right triangles that are similar. Problems 2-4 guide students through a proof of the Pythagorean Theorem using the similar triangles formed. Using this diagram the students are first asked to write a similarity statement involving the three triangles present. Then they are asked to fill in the blanks to create proportions from this statement using their knowledge of similar triangles. Students are asked to use the proportions found to prove the Pythagorean Theorem. It can be assumed that the students will use cross multiplication similar to previous activities discussing proportional relationships.

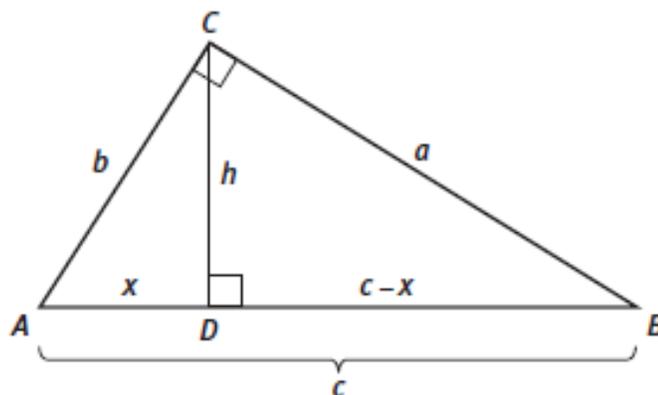


Figure 4. Diagram for Lesson 20.1, "Is it Right?" (SpringBoard Geometry, 2015)

This proof led into the first CYU section. Problem 5 introduced the definition of a Pythagorean Triple and students were asked to explain why given numbers do not form such a triple. The definition given in the text⁵ is misleading because it implies that numbers can be used to satisfy the Pythagorean Theorem, which is not precise. (The equation $a^2 + b^2 = c^2$ is not the Pythagorean Theorem, but only part of it). Problem 6 then asked students to explain why c must be greater than both a and b in order for $a^2 + b^2 = c^2$ to be true.

Problem 7 is introduced with a labeled diagram of a rhombus-shaped kite. Students are asked first to explain how to find the perimeter in words and then to actually find the perimeter of this kite using the Pythagorean Theorem if the length of the “spar” and “spine” are known. The properties of a rhombus were identified in Unit 2 if students needed to use them as a reference. This question led into the second CYU section. Problem 8 also needed to use properties learned in Unit 2 and students were asked to use the Pythagorean Theorem to show that the diagonals of a square are congruent. Problem 9 provided students with a scenario in which a ladder was leaning against a wall and they were asked find the height reached on the wall by a ladder.

Lesson 20.1 practice included Problems 10-15. Problems 10, 11, 13, and 14 required students to find the missing side of a right triangle using the Pythagorean Theorem. This side length could then be used to find the perimeter of the figure in Problems 13 and 14. Problem 12 asked students to identify a Pythagorean triple from 4 given sets of numbers. Problem 15 provided students with the measurements of a cube-shaped box. Students were then asked if a paintbrush of a specified length could fit in the box. Students needed to be able to explain how they knew if the paintbrush could fit.

⁵ “A Pythagorean triple is a set of three nonzero whole numbers that satisfy the Pythagorean Theorem.”

3.2.2. Lesson 2. Converse of the Pythagorean Theorem

Building off of the previous day's lesson the bell-ringer asked for students to find the missing side length of a triangle when given two side lengths. Two values were given and were stated in terms of a , b , and c . The directions asked students to solve the equation $a^2 + b^2 = c^2$. After the bell-ringer, the textbook informs students that the learning targets of the lesson are to:

1. Use the converse of the Pythagorean Theorem to solve problems.
2. Develop and apply Pythagorean inequalities.

We used the bell-ringer to review the Pythagorean Theorem and discuss how to describe the equation in words, leading into the first question of the lesson. In Problem 1 students were asked to write the Pythagorean Theorem and its converse in if-then form. The students were then asked to use the converse to determine if the given triangle (see Figure 5) was a right triangle.

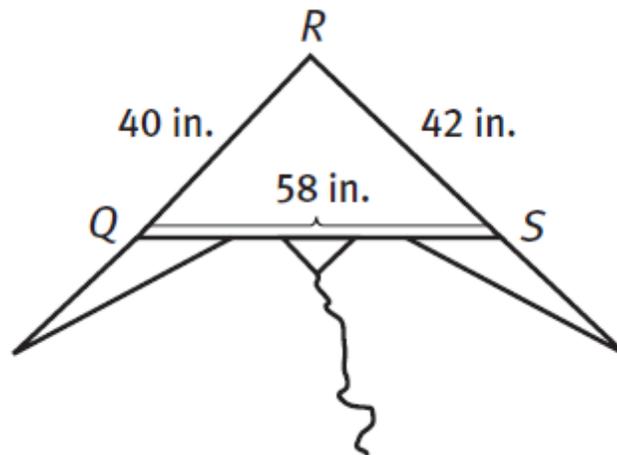


Figure 5. Diagram for Lesson 20.2, "Is it Right?" (SpringBoard Geometry, 2015).

Problem 2 asked students to write an algebraic proof to show that if " a , b , and c form a Pythagorean triple, than any positive whole-number multiple of the numbers is also a Pythagorean triple." This question provided further explanation and explicitly stated the equation

that was given ($a^2 + b^2 = c^2$) and the equation that need to be proved for any positive whole number x , $(xa)^2 + (xb)^2 = (xc)^2$.

Problem 3 was labeled a “hands-on exploration” by the text. It involved using straws as manipulatives. The groups cut the straws into specific lengths and were asked to build the triangles listed in the table and identify the types of triangles, if any, created as obtuse, acute, or right. To complete the rest of the table they had to square the longest side separately from summing the square of the two shortest sides. Figure 6 shows the answers given in the teacher edition for this problem. This table would be used to answer six of the remaining eight questions which seems to imply the importance of ensuring students have correctly filled in their charts before moving on.

Triangle Side Lengths	Type of Triangle	Square of Longest Segment	Sum of the Squares of the Two Shorter Sides
5, 12, 13	right	169	169
6, 6, 12	not a triangle	144	72
5, 6, 12	not a triangle	144	61
5, 12, 15	obtuse	225	169
5, 12, 12	acute	144	169
6, 12, 13	acute	169	180
6, 12, 15	obtuse	225	180

Figure 6. Lesson 20.2 Answer 3, "Is it Right?" (SpringBoard Geometry, 2015).

Problem 4 labeled “express regularity in repeated reasoning” asked student to determine if there was a relationship between $a^2 + b^2$ and c^2 for the different types of triangles. Based on

the answers in the teacher edition, students were expected to write equations and inequalities from the comparison made between these quantities.

Problems 5 and 6 ask students to classify triangles as acute, obtuse, or right using the given side lengths and relationships identified in Problem 4. The CYU section had two questions that shared the same reasoning as problems seen previously in the activity. Problem 7 was similar to problems 5 and 6 and students were asked to determine what type of triangle was given. Problem 8 asked students to find other Pythagorean triples when given the values of one known triple.

Lastly the students were asked to complete the Lesson Practice. Two of the three problems asked students to determine if a triangle could be formed using the given side lengths and if so, they were asked to classify what type of triangle would be formed. Problem 11 did not provide the students with side measures, but did provide a diagram representing a picture frame. Students were asked to explain how they could use a ruler to determine if the sides of the frame met to form right angles.

3.2.3. Student and Teacher Reactions

3.2.3.1. Lesson 1

As students entered the classroom, they were instructed to sit in assigned groups of three or four. They completed the bell-ringer in groups that involved finding the geometric mean. Only five of the fourteen groups did not need prompting in how to solve equations involving a variable that is “squared.” Once reminded of the square root function either by a classmate or the teacher, students were successful in completing this bell ringer.

To complete Problem 1, students had to be reminded of the definition of hypotenuse and leg. It was difficult for students to know they needed to use the given equation to find the

missing side length because the text introduced it in the margin without making reference to its importance in the specific problem. Most groups needed help making these connections, but did not need help performing the specific operation.

Given the diagram in Figure 4, the students had to write a similarity statement involving the three triangles present, which did not stump many groups. Creating the proportions from the similarity statement also did not cause any trouble except for comments of discontent with “no numbers” being present. I only needed to step in when it came time to use the equations found by the proportions to prove $a^2 + b^2 = c^2$ using substitution. Even after the whole group discussion and example given by the teacher, many groups still asked “how did you know you could do that?” referencing substituting one equation into another. This misunderstanding goes deeper into not understanding equality which needed to be addressed outside of this specific lesson on the Pythagorean Theorem.

This proof led into the CYU section which was different than most activities. The first question involved the definition of a Pythagorean Triple and students were asked to prove why given numbers do not form such a triple. After this, they were asked to prove why c must be greater than both a and b in order for $a^2 + b^2 = c^2$ to be true. Both of these questions required assistance from the teacher for all groups to complete. However, after discussing the results with all groups most students were able to understand the reasoning behind the explanations. Perhaps the problem lay in the introduction of new concepts in a section designed to check their understanding of previously discussed questions.

Next the students were given the diagram of a rhombus-shaped kite in which the perimeter needed to be found. Ten of the fourteen groups were able to answer this question without any assistance, the other groups only needed to be reminded that the diagonals of a

rhombus bisect each other in order to answer this question correctly moving everyone into the second CYU section. The first question required a whole group discussion because all but one group labeled their diagonal “s” instead of the side lengths as directed. I drew a diagram to represent the square with side lengths s on the board and isolated one triangle formed by the diagonal off to the side of the square. This enabled the groups to draw the correct conclusions and show that the diagonals of the square were congruent. All groups were able to find how high up the wall a ladder of specific length would reach if the length between the base of the ladder and wall. Only five students needed to be prompted to draw a triangle to represent the word problem.

The only trouble the groups had with the lesson practice was rounding to the tenths place correctly. In general all groups answered each question correctly without need for much teacher assistance. The only prompting given was recommending that the groups should draw the figures before attempting to make conclusions. I also needed to review simplifying radicals during this practice because many students were able to get the correct decimal answer but could not simplify the radicals as the text asked.

3.2.3.2. Lesson 2

Students were instructed to sit in assigned groups of three or four from the previous day. They completed the bell-ringer in groups. The bell-ringer asked them to find the missing value using the Pythagorean Theorem when given the value of two variables. All students that attempted the bell-ringer were able to get the correct answer without peer or teacher assistance.

The bell-ringer led to a discussion about how to describe the equation of the Pythagorean Theorem in words. This enabled the students in groups to write the theorem and its converse easily. However, when asked to determine if the given triangle was “right” they did not initially

think about using the equation $a^2 + b^2 = c^2$ and just reread the statement written in if-then form. Eventually one student from a group asked “Don’t we just test the numbers like we did with the triples in this part?” This question spurred on the other groups who seemed to be stuck and most students were able to show that the kite created a right triangle. The students who did not come to this conclusion noticed that they put they value of the hypotenuse in for a leg and vice versa. One student asked, “Does it matter where you put the numbers?” His classmate answered with an emphatic, “Yes!” This discussion caused another student in a different group to ask me, “How do you know which one is the hypotenuse if there isn’t a right angle labeled?” I used this as an opportunity to pose the same question to the class and gave them time to discuss this in groups. Multiple groups concluded that, “if the triangle was right, then the longest side would have to be the hypotenuse.” This discussion set them up nicely for the exploration activity they would complete in Problem 3.

The next question, Problem 2, involving proof using only variables needed to be teacher led because none of the groups were able to complete this question. Even though the text explicitly stated the equations the students should use, they lacked the Algebra foundation needed.

Problem 3 involved using straws as manipulatives. The groups cut the straws into specific lengths and were asked to build the triangles listed in the table and identify the types of triangles, if any, created as obtuse, acute, or right. The grid paper was very helpful in ensuring the appropriate lengths were cut. As mentioned previously, to complete the rest of the table students had to square the longest side separately from summing the squares of the two shortest sides. This process required some prompting from me and nine of the fourteen groups needed to either be shown examples of how to obtain the squares in question or reminded how to determine

which number is the hypotenuse. Many groups also did not understand that “not a triangle” was an option for some of the side lengths so that was explained to the class as a whole after addressing the same concern in multiple groups. As a class we were able to agree on the correct answers for the table and then used the results to express our findings with general equations and inequalities for right, obtuse, and acute triangles. The students could express their findings in words, but were unable to write the equations and inequalities without teacher guidance. These general statements were used to complete the next questions and CYU section. All groups except for two were able to answer these questions correctly. Those groups needed to be reminded about the proportionality of similar triangles to find other Pythagorean Triples using one that was given.

The lesson practice had to be completed for homework due to a lack of class time remaining. We reviewed the answers the next day and the only question the majority of the students missed involved a picture frame without actual measures. Students had to reason abstractly and were unable to put their process into words. When I asked students what they needed to show that the corners created right angles, all knew stated they needed to show $a^2 + b^2 = c^2$. They just could not understand how to replace a, b, and c with the segments in the actual frame “without numbers.”

3.2.4. Results

This activity was assessed formally at my school using the Mid-Unit 3 Test (see Appendix C). Seven of the twenty five questions specifically correlated to the learning targets of this activity. Three of those questions involved finding the missing side length of a right triangle, but the right triangle could only be drawn using the given word problem. Ninety seven percent (97%) of my students were able to draw and correctly label the triangle and ninety one percent

(91%) were able to find the correct side length. The students that could draw the correct triangles without solving correctly did not always substitute the measure of the hypotenuse into the appropriate part of the equation. Most students just used both given numbers as a and b even though that did not match the picture drawn. A few of the students did not use the square root function correctly.

The remaining four questions gave the students three potential side lengths of a triangle. The students had to first identify if the given lengths could form a triangle, if so, they had to show what kind of triangle could be formed. Only seventy one (71%) of my students were able to identify the side lengths that did not form a triangle. Eighty nine (89%) correctly identified the right triangle and forty two percent (42%) switched the inequalities for acute and obtuse triangles. Overall this section showed a poor understanding of how to identify triangles that were not right.

3.2.5. Critique

3.2.5.1. Lesson 1

The questions provided in chapter 2 also guided the critique of this lesson.

1. Does the lesson state clear, standards-based objectives?

Yes, the learning targets are listed in terms that students can understand and are linked to a Common Core State Standard (CCSS). The CCSS is only visible in the teacher edition and includes examples of theorems that should be proven in Geometry courses.

2. Does the lesson support teachers in providing guidance and feedback to students?

No, it does not offer many opportunities for guidance or feedback. In the first problem, students are asked to find the missing side of a triangle, but the only thing given to them was the equation in the margin. A worked example would have been more beneficial because most

students were using this formula for the first time due to gaps in CCSS. The guidance through the proof was helpful, but many students missed the purpose of the process because they claimed the variables used in the diagram “threw them off.”

3. Does the lesson incorporate cooperative learning?

As in other lessons, cooperative learning strategies were mentioned in the margins but were not applicable to most of the questions they were recommended for. Discussion groups could not be used in the first CYU section because new definitions were being introduced leaving no opportunity to draw conclusions from the previous problems. Most questions could be solved by the students individually without needing any discussion between group members.

4. Does the lesson engage students in higher-order thinking?

Most questions only required students to understand how to “plug numbers” into the equation in the Pythagorean Theorem to find the missing values. However, there was an opportunity for students to extend their learning and use the solutions to find out more about specific figures. Discussion about what was needed before groups could find perimeters, congruent diagonals, and figure restrictions were heard from the majority of group members. Students were able to identify what they knew, what was needed, and a process for finding the correct solution. Because of these conversations, I would say that high-order thinking was present in this lesson.

5. Does the lesson include time for students to self-assess or reflect on their learning?

The best opportunity for students to reflect on their learning is normally during the CYU section. However, The CYU sections did not provide the same opportunity for self-critique that in other lessons had. The questions introduced new terms and concepts seemingly unrelated to the previous sections not leaving room for students to determine if they understood the previous

sections. The teacher had to direct most of the groups through this section. Teacher direction often takes away from meaningful reflection time. However, the lesson practice provided some opportunity for the students to assess their learning. This was only accomplished because time in class could be allotted to complete and review the answers to these sections. Even though there was not a rubric that specifically stated the expectations in these sections, students could tell if their attempts were going in the right direction based on their solutions to the multiple computation problems. These helped them feel more confident when applying what was learned to the word problems.

“Self-Revision/Peer-Revision” was a given strategy for Problems 3 and 4. These problems involved filling in the blanks for two proportional relationships and finding cross products. “What did you get?” was the main question used to make “revisions.” Checking your answers is not the desired collaboration in this type of strategy, but again the problem did not lend itself to more.

6. Can the lesson be completed in a reasonable amount of time?

Yes, the lesson can be completed in a reasonable amount of time. This lesson was completed in the amount of time suggested to stay on track with the parish recommended time frame. The majority of the time was spent on the learning targets mentioned at the beginning of the lesson.

3.2.5.2. Lesson 2

The questions used in Lesson 1 also guided the critique of this lesson.

1. Does the lesson state clear, standards-based objectives?

Learning targets are listed specifically and are in terms easy to understand. Students may need to be reminded of what the converse of a statement is, but this was not necessary for the majority of the groups in my class.

2. Does the lesson support teachers in providing guidance and feedback to students?

As in other lessons, there was a problem asking students to write equations or inequalities without being told that explicitly. The question asked students to identify what was suggested by comparing two quantities. It is not clear to students that they are supposed to express their comparison algebraically. Reviewing the answers to the previous questions and guiding students through the exploration in Problem 3 was beneficial and allowed time to help students see the relationship between the side lengths of a triangle instead of simply giving them the answers. This problem showed support for necessary guidance and feedback from the teacher and the answers obtained by students were used throughout the rest of the lesson.

3. Does the lesson incorporate cooperative learning?

The cooperative learning aspect was prominent throughout the activity. The hands-on exploration using straws cut to specific dimensions to create triangles promoted meaningful group discussion. Multiple groups offered to share their results with the class even before being prompted. Students were discussing answers and the process they used to obtain those answers with their classmates. Multiple students were able to show their group members where mistakes had been made and needed to be corrected.

4. Does the lesson engage students in higher-order thinking?

Higher-order thinking is examined during Problems 3 and 4 of this lesson where students “express regularity in repeated reasoning.” After completing the hands-on exploration the students compared quantities and were asked to formulate conclusions from the comparison. The book intended for them to write equations and inequalities, but this was not explicitly asked for so most groups just used complete sentences. Most students were also able to clearly justify their reasoning for classifying triangles as right, acute, or obtuse even though they could not express that reasoning algebraically.

5. Does the lesson include time for students to self-assess or reflect on their learning?

There were multiple opportunities available for student to reflect on their learning after completing the task in Problem 3. Repeated reasoning was used to complete the lesson, CYU section, and Lesson Practice. This allowed students to continue to attempt to classify triangles correctly. Students also had examples to refer to when evaluating if they met all of the proper criteria needed.

6. Can the lesson be completed in a reasonable amount of time?

The activity will be difficult to complete in one class period. Even when I had the materials needed (straws, scissors, centimeter grid paper, and rulers) organized prior to the lesson, the exploration still took up a lot of time. The CYU and lesson practice sections had to be completed for homework. However, the exploration and questions following gave students enough practice to be able to complete these on their own at home. Even though the exploration took a lot of time, the students did spend most of the time in the lesson focused on developing and understanding the objectives stated at the beginning of the lesson.

Additional comments on this activity can be made about the importance of including hands-on explorations when applicable. The text used the appropriate amount of examples to provide evidence that students could build upon. Lesson 20.2 also provided many opportunities for students to answer questions based off of their conjectures. However, there was still a need for the text to state the equations and inequalities explicitly to clear up any doubts the students might still have after finishing the exploration.

Chapter 4: Conclusions

From the literature we defined six preliminary questions that would be helpful in evaluating the effectiveness of classroom activities, but did not provide many details. After reviewing the lessons we added more details to expand on each question. Appendix A includes a document with these revisions. I will share it with my department and other teachers to help guide them in their evaluations of classroom materials.

Too often I assume the resources given to use by our district or through educational services have been critiqued and are without flaw. Looking at each lesson through the lens of this structure allowed me to identify weaknesses and strengths. Knowing these features opens up the opportunity to supplement extra materials and provide appropriate guidance when needed. However, if the lesson is not dissected in such a way, the weakness would not be exposed to give opportunity to elevate student learning.

Critiquing such a widely used resource also opened up my eyes to the proper mathematics training needed for teachers. The text often misused definitions, presupposed certain background knowledge, and left students without guiding examples. This can be easily overlooked if the teacher does not have the proper understanding of mathematics concepts.

I will continue to use these questions to critique future activities, tasks, and lessons being considered for use in my classroom. If I were to complete a similar study again, I would want to use different assessments to determine if significant changes can be seen upon completion of specific activities. The department created tests used did not ask students to implement higher-order thinking. They simply needed to recall definitions on the tests. There were no questions that asked for students to extend learning. I will recommend to my department that we create

new assessments that require students to do more than just repeat a list of steps learned or show that they have memorized a definition.

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Appendix A: Critique Tool for Lessons Featuring Active Learning

1. Does the lesson include a clear statement of standards-based learning goals?
 - Are the objectives written so that students and teachers can understand them? (If not, what needs to be cleared up?)
 - Is there evidence that the lesson will accomplish the stated objectives?
 - Do the objectives line up with the standards required by my district?
2. Does the lesson support the teacher in providing guidance and feedback to students?
 - Are the students given sufficient background information?
 - Are the important ideas (e.g., theorems, postulates, and definitions) stated explicitly?
 - Is there opportunity for the teacher to direct students toward the correct thinking without giving the answer directly? Is the teacher able to let students know if they are working in the right direction?
 - Are sufficient examples incorporated?
3. Does the lesson incorporate cooperative learning?
 - What tasks are students asked to complete that require collaboration?
 - Is there opportunity for students in a group to have different roles to help accomplish a single task?
 - Will students be able to complete the tasks and share their knowledge with the members of their group? With the class?
4. Does the lesson engage students in higher-order thinking?
 - Will students only need to recall basic information to complete tasks?
 - Will mimicking steps allow students to answer questions?
 - Are students required to provide written explanation for their results?
 - What evidence is required from the students to justify their results?
5. Does the lesson include time for students to self-assess or reflect on their learning?
 - Are students asked to draw conclusions from their work?
 - Is there a rubric or scoring tool available for students to check for understanding?
 - Do students have opportunity to critique their peers learning at some point in the lesson?
 - Are students able to make revisions to their answers after self-assessment?
6. Can the lesson be completed in a reasonable amount of time?
 - What materials are needed to complete this lesson?
 - Will the majority of the time used be spent on mathematics?

Appendix B: Department Created Unit 1 Quiz 4 Summative Assessment

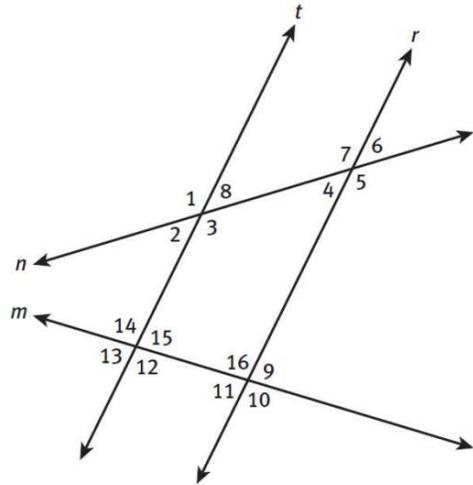
Name: _____ Date: _____ Block: _____

Geometry Quiz 4: Lessons 7-1, 7-2 and 7-3

Use the diagram below to answer #1 – 14. In the diagram, $t \parallel r$ and line m is not parallel to line n .

Complete the statement with alternate interior, alternate exterior, corresponding, same-side interior, or vertical. (2 points each)

1. $\angle 2$ and $\angle 15$ are _____ angles.
2. $\angle 6$ and $\angle 9$ are _____ angles.
3. $\angle 3$ and $\angle 4$ are _____ angles.
4. $\angle 13$ and $\angle 15$ are _____ angles.
5. $\angle 1$ and $\angle 5$ are _____ angles.



If $m\angle 15 = 85^\circ$ and $m\angle 2 = 45^\circ$, determine the measure of each of the following angles. (2 points each)

6. $m\angle 4 =$ _____
7. $m\angle 13 =$ _____
8. $m\angle 16 =$ _____
9. $m\angle 7 =$ _____
10. $m\angle 11 =$ _____

Find the value of x and the missing angle using the given information. (5 points each)

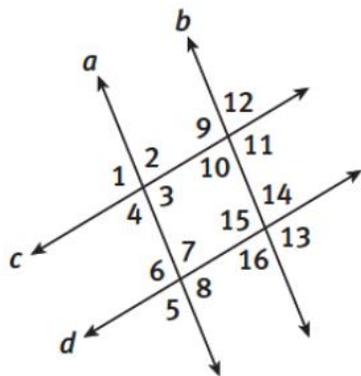
11. If $m\angle 8 = (5x - 2)^\circ$ and $m\angle 4 = (3x + 24)^\circ$, then $x =$ _____ and $m\angle 7 =$ _____.

12. If $m\angle 10 = (8x + 7)^\circ$ and $m\angle 12 = (4x + 55)^\circ$, then $x = \underline{\hspace{2cm}}$ and $m\angle 16 = \underline{\hspace{2cm}}$.

13. If $m\angle 3 = (5x - 8)^\circ$ and $m\angle 4 = (2x + 20)^\circ$, then $x = \underline{\hspace{2cm}}$ and $m\angle 5 = \underline{\hspace{2cm}}$.

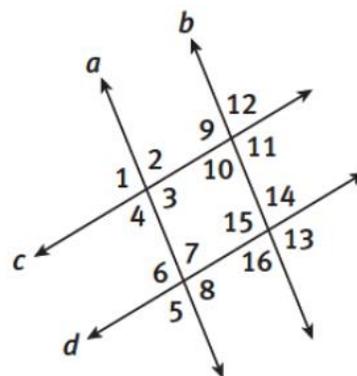
14. If $m\angle 14 = (4x)^\circ$ and $m\angle 11 = (2x + 36)^\circ$, then $x = \underline{\hspace{2cm}}$ and $m\angle 10 = \underline{\hspace{2cm}}$.

Use the diagram above each proof to supply the missing reasons. (1 point per blank)



15. **Given:** $a \parallel b, c \parallel d$
Prove: $\angle 8 \cong \angle 9$

Statements	Reasons
1. $c \parallel d$	1.
2. $\angle 8 \cong \angle 3$	2.
3. $a \parallel b$	3.
4. $\angle 3 \cong \angle 9$	4.
5. $\angle 8 \cong \angle 9$	5.



16. **Given:** $a \parallel b, c \parallel d$
Prove: $\angle 1 \cong \angle 13$

Statements	Reasons
1. $c \parallel d$	1.
2. $\angle 1 \cong \angle 8$	2.
3. $a \parallel b$	3.
4. $\angle 8 \cong \angle 13$	4.
5. $\angle 1 \cong \angle 13$	5.

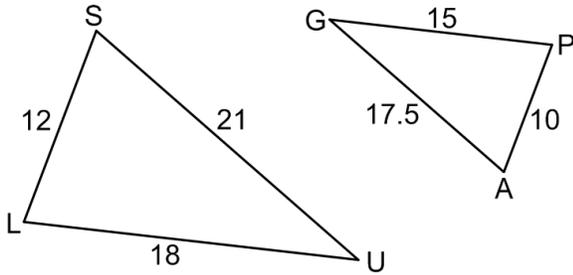
Appendix C: Department Created Mid-Unit 3 Summative Assessment

Name: _____ Date: _____ Block: _____

Geometry Mid-Unit Test 3: Lessons 18, 20, and 21

Determine if the figures are similar. If so, state the scale factor and write a similarity statement.

(3 points)



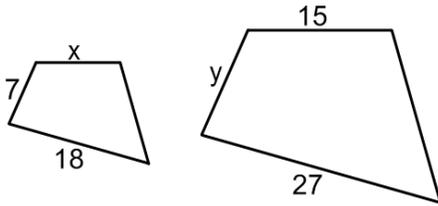
Similar: Yes or No (Circle One)

Scale Factor: _____

Similarity Statement: _____

The figures below are similar. Find the value of each variable. (6 points each)

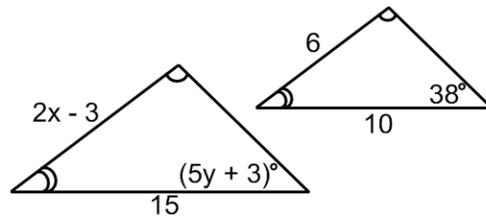
2.



$x =$ _____

$y =$ _____

3.



$x =$ _____

$y =$ _____

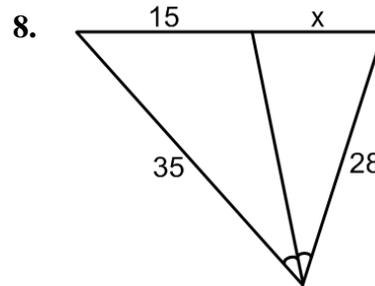
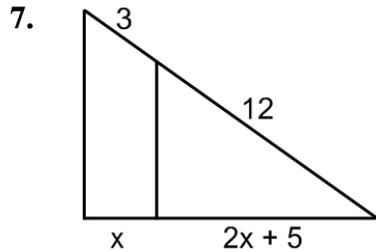
Use ratio and proportion to solve the following word problems. (3 points each)

4. Scott has typed 4 pages of a term paper in 15 minutes. At the same rate, how long should it take him to type the remaining 24 pages?

5. A statue that is 12 feet tall casts a shadow that is 15 feet long. Find the length of the shadow that an 8 foot cardboard box casts.

6. On a map scale, 2 centimeters represents 5 kilometers. If two towns on the map are 20 kilometers apart, how long would the line segment be between the two towns on the map?

Find the value of x . (3 points each)

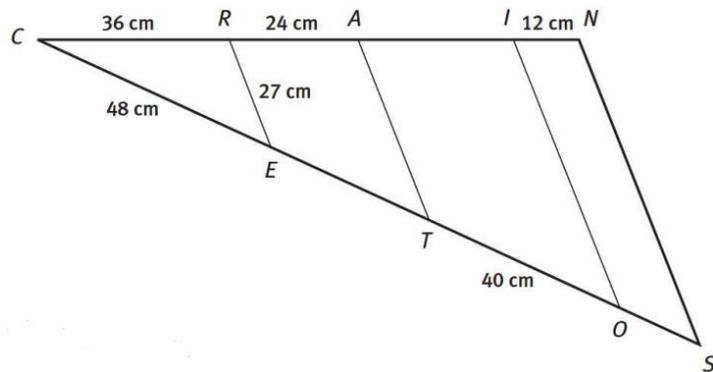


Use the diagram to find the following lengths. (3 points each)

9. $ET =$ _____

10. $IO =$ _____

11. $OS =$ _____



Use Pythagorean Theorem to solve the following. (3 points each)

12. A ship leaves port and sails 9 kilometers west and then 12 kilometers north. How far is the ship from the port?

13. The diagonal of a television screen is 37 inches. The television has a width of 32 inches, find the height.

14. A 10 foot ladder is leaning against a wall. The foot the ladder is 6 feet from the wall. How high up does the ladder hit the wall?

Tell whether a triangle can be formed having the following side lengths. If a triangle can be formed, tell whether it is right, acute, or obtuse. (3 points each)

15. 8, 10, 11

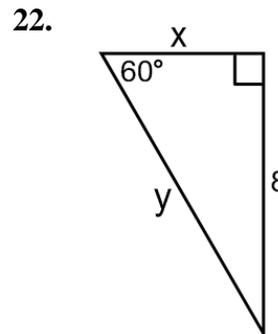
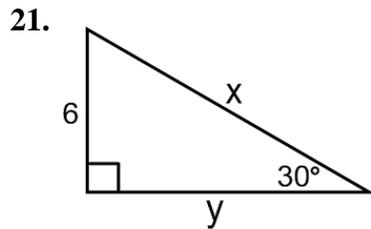
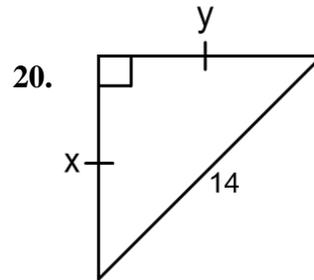
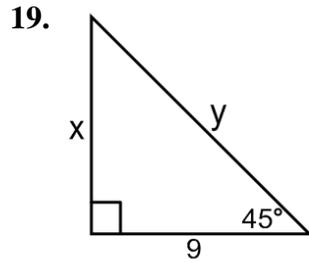
16. $\sqrt{31}$, 7, 14

17. 5, 15, $5\sqrt{10}$

18. 12, 12, 12

Use Special Right Triangles to find the value of each variable. Write answer in simplest radical form.

(4 points each)



Solve the following. Write answer in simplest radical form. (3 points each)

23. The perimeter of a square is 28 inches. Find the length of a diagonal.

24. The hypotenuse of a $30^\circ - 60^\circ - 90^\circ$ triangle is 14 inches. What is the length of the longer leg?

25. The perimeter of an equilateral triangle is 24 centimeters. Find the length of an altitude.

Appendix D: IRB Approval



ACTION ON EXEMPTION APPROVAL REQUEST

TO: Alana Day
Natural Sciences

FROM: Dennis Landin
Chair, Institutional Review Board

DATE: May 25, 2015

RE: IRB# E9350

TITLE: Critique of the Effectiveness of an Activity-based Learning Approach in a Geometry Classroom

Institutional Review Board
Dr. Dennis Landin, Chair
130 David Boyd Hall
Baton Rouge, LA 70803
P: 225.578.8692
F: 225.578.5983
irb@lsu.edu | lsu.edu/irb

New Protocol/Modification/Continuation: New Protocol

Review Date: 5/13/2015

Approved X **Disapproved** _____

Approval Date: 5/22/2015 **Approval Expiration Date:** 5/21/2018

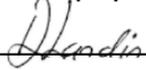
Exemption Category/Paragraph: 1

Signed Consent Waived?: No, parental consent and child assent forms need to be signed.

Re-review frequency: (three years unless otherwise stated)

LSU Proposal Number (if applicable):

Protocol Matches Scope of Work in Grant proposal: (if applicable)

By: Dennis Landin, Chairman _____ 

PRINCIPAL INVESTIGATOR: PLEASE READ THE FOLLOWING –
Continuing approval is CONDITIONAL on:

1. Adherence to the approved protocol, familiarity with, and adherence to the ethical standards of the Belmont Report, and LSU's Assurance of Compliance with DHHS regulations for the protection of human subjects*
2. Prior approval of a change in protocol, including revision of the consent documents or an increase in the number of subjects over that approved.
3. Obtaining renewed approval (or submittal of a termination report), prior to the approval expiration date, upon request by the IRB office (irrespective of when the project actually begins); notification of project termination.
4. Retention of documentation of informed consent and study records for at least 3 years after the study ends.
5. Continuing attention to the physical and psychological well-being and informed consent of the individual participants, including notification of new information that might affect consent.
6. A prompt report to the IRB of any adverse event affecting a participant potentially arising from the study.
7. Notification of the IRB of a serious compliance failure.
8. **SPECIAL NOTE:**

**All investigators and support staff have access to copies of the Belmont Report, LSU's Assurance with DHHS, DHHS (45 CFR 46) and FDA regulations governing use of human subjects, and other relevant documents in print in this office or on our World Wide Web site at <http://www.lsu.edu/irb>*

Vita

Alana Blackwell Day is a native of Baton Rouge, Louisiana. She attended Louisiana State University and received a Bachelor of Science in Mathematics in 2010. Alana has been teaching high school mathematics in the Livingston Parish School District since August, 2010.