2012

Unit assessments for high school geometry

Damayanthi Srimathi Jegart
Louisiana State University and Agricultural and Mechanical College

UNIT ASSESSMENTS FOR HIGH SCHOOL GEOMETRY

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
In partial fulfillment of the
requirements for the degree of
Master of Natural Sciences

in

The Interdepartmental Program in Natural Sciences

by
Damayanthi Srimathi Jegart
B.A.,Nicholls State University 2000
August 2012

Acknowledgments

I would like to thank my family, especially my children, Natalie Cooney, Michael Jegart, Katelyn Jegart, Dylan Jegart, and my son-in-law, Tom Cooney for their encouragement, love and support during this process.

I would like to express my sincere appreciation to Dr. Madden, my professor, thesis advisor, and committee member. His knowledge, advice and guidance contributed to my growth as a professional. Words cannot express my gratitude for his time and help during this last year. I would also like to acknowledge and thank Dr. Neubrander and Dr. Sundar for being on my thesis committee.

Lastly I would like to thank my LaMSTI 2010 cohort for making this a pleasant and a great experience that I will remember and cherish for a long time. Your encouragement and support is greatly appreciated.

Table of Contents

Abstract

Eight unit tests closely aligned with the Louisiana Comprehensive Curriculum for high school geometry were developed. Five of these were administered, each to the same 115 students spanning all ability and attainment levels in a magnet school in a semi-rural Louisiana district. The results were analyzed to determine the quality of the questions as well as to glean information about student learning. The test scores were compared to the results of the state-administered end-of-course test for high-school geometry.

The main findings were as follows: **a)** most students do not communicate their reasoning or justification unless directed to do so, and even then only poorly, **b)** very basic skills are problematic for a very small (but troubling) number of students, **c)** pre-requisites from more recent grades are problematic for larger numbers, **d)** many students fail to read or understand directions, **e)** understanding the types of mistakes students make in these tests is likely to be useful in planning future lessons, **f)** only one of the unit tests was a good predictor of end-of-course results, suggesting that the end-of-course test might not represent all units in the course evenly.

Chapter I: Introduction

In the United States, geometry is the standard mathematics course for the 10^{th} grade. In Louisiana, the Louisiana Comprehensive Curriculum (LCC) for geometry controls the content of the geometry course. This document, which was developed and published by the Louisiana Department of Education, is aligned with state content standards as defined by the Louisiana Grade-Level Expectations (GLE'S).

The Louisiana Comprehensive Curriculum for Geometry is divided into eight units. Each unit includes a list of the GLE's to be addressed and sample activities by which to learn the GLE'S. Though some sample assessment problems are included, there are not enough to support proper assessment. Each school district is encouraged to create and administer unit tests for each unit. Teachers are expected to create additional assessments.

Our school district developed and administered six unit tests for the Geometry LCC. These were in use until 2011. The scores were typically very low. There was a consensus among teachers that these tests were of poor quality and inadequate for testing student knowledge of the subject. In particular, many questions were unrelated to the curriculum. With this in mind, I decided to make new unit tests for our parish based on the guidelines provided in each unit and on my experience teaching the course. I first expanded detailed plans for each unit, analyzing the content and the GLEs. After this, I chose assessment problems similar to those that I expected the students were likely to encounter on the End of Course Exam.

My student population for geometry included 115 students with various backgrounds and skill-levels in mathematics. I taught six full year geometry courses, the classes were on alternate block schedule. (Classes meet for 90 minutes on alternate days) Each class had between 17 and 24 students. All classes were called "honors" but in fact had mixture of ability and preparation.

I started working on designing the tests in the fall of 2011. I made eight unit tests for the eight units in the Louisiana Comprehensive Curriculum for geometry. When designing the tests, I first discussed the content and main ideas of the unit with my adviser, who is a mathematician. I composed the questions based on his advise on the content of each unit. Upon making each unit test, I reviewed the tests with my advisor to assure mathematical relevance of the test questions. I was able to administer five of the eight unit tests I designed.

After the tests were administered and corrected, I spend a long time studying the student work. I reviewed enough answers to get an idea of the range of the scores and then decided on procedures for awarding partial credit. Lastly I selected some test questions for deeper analysis.

The criterion for the test question I selected were:

- The questions had to have appreciable numbers for both correct and incorrect answers.
- I did not include any true/false question or multiple- choice questions.
- Area and volume questions, where students had to plug in values to a formula were also excluded.

I selected eight questions to be analyzed for my thesis; I included each question in the analyses. The question analyses included the following steps:

- Correct solution: I provided the solution to the test question.
- Hypothetical workflow: Describes a step-by-step process on how to solve the problem.
- Data: Describes the number of students who were successful at each stage of the hypothetical workflow.
- Analysis: Analyses the types of mistakes students made at each step, and types of common mistakes.
- Conclusions and recommendations: Describes possible interventions to improve student performance in the future as well as possible improvements for the test question.

By administering the unit tests I made for my students, I wanted to learn about student performance as well as the quality of the test questions.

Chapter II. Background

The purpose of this chapter is to learn about the history of geometry and developments that led to the current practices of teaching geometry in high schools. First the history of geometry beginning with Egypt, Greece and the influence of Euclid are examined. The Euclidean influence on geometry books from 1900s to present will be established by comparing the content of books from several periods. The influence of certain events in history will explain the changes and reforms that occurred in the mathematics education in the United States. National Council of Teachers of Mathematics and its influence on the development of state standards will show other changes in the high school mathematics curriculum. This chapter also includes an investigation into the assessment of mathematics, as well as explanation of the contents of each unit of the Louisiana Comprehensive Curriculum.

History of Geometry and the Influence of Euclid

The birthplace of geometry is widely considered to be Ancient Egypt. The science of earth measuring was developed there, and architects used geometric ideas to plan and construct buildings, including the pyramids.   The Egyptians applied geometry to measure lengths, find areas, and build solids. They also depicted some geometric designs in their mural decorations. (Walker Stamper, 1909). The Egyptians knew how to find the areas, and volumes of several geometric figures. The Moscow Mathematical Papyrus, an Ancient Egyptian papyrus, dating approximately to 1850 BC, and The Rhind Mathematical Papyrus dating to around 1650 BC, have calculations on finding the areas

of geometric objects. The second part of the Rhind papyrus is dedicated entirely to geometry. It contains problems on how to find the volume of cylindrical and rectangular based granaries, slopes of pyramids, and the area of a rectangular plot of land. Seven of the twenty-five problems in the Moscow Mathematical Papyrus are geometry problems that calculate the area of triangles and the volume of pyramids.

The Greeks extended the study of geometry to several new figures and surfaces, and they changed the approach to the subject from trial-and-error to the method of logical deductions. Thales of Miletus (635-543 BC) was the first mathematician to use deductive reasoning in proofs. Pythagoras of Ionia (582-496 BC), who is believed to be a student of Thales, was the first person to give a deductive proof for the relationship of the sides of a right triangle, now known as the Pythagorean Theorem.

Euclid (c. 325 B. C. – c. 270 B. C.) was a teacher who lived in Alexandria, Egypt. Many believe that he was a student at Plato's Academy. (Plato lived 428-348 B.C.) Through his work, all of geometry was systematized as a deductive system based on postulates that were perceived as obvious. His greatest contribution to geometry consisted in compiling the geometric knowledge of his day. In the book known as *The Elements*, Euclid systematized all the important works of geometers up to his time (Seidlin & Shuster, 1950). It begins with five postulates:

1. It is possible to draw a straight line joining any two points.
2. A straight- line segment may be extended without limit in either direction.
3. It is possible to draw a given circle with a given center through a given point.
4. All right angles are congruent to one another.

5. If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines intersect each other on that side if extended far enough.

Each of the 13 books of the *Elements* begins with additional definitions. Euclid derives theorems (true statements) from the postulates.

The Elements is considered a great tool in the development of logic and modern science. Book One treats triangles, parallels and area. Book two is on geometric algebra (a geometric treatment of some aspects of quadratic equations). Book three deals with circles and their properties. Book Four is on the constructions of inscribed figures. Book Five introduces ratio and proportion and Book Six treats similar triangles. Books Seven, Eight and Nine are on number theory. Books Eleven through Thirteen deal with spatial geometry.

Theon of Alexandria created an edition of Euclid's *Elements* in the 4th century AD. In 760, the Arabs received the *Elements* from the Byzantines, and it was translated into Arabic. Euclid was studied in the medieval universities. The Englishman, Adelhard of Bath, made the first translation of Euclid's *Elements* from the Arabic into Latin in the year 1120. Campano of Novara produced the first published version of the Elements in 1482. The text was merely an improved version of the first Latin translation. Sir Henry Billingsley introduced the first English translation in London in 1570. (Archibald, 1950)

In the 16th century, mathematicians recognized that algebra lacked the strong foundation found in geometry. The French mathematician Francois Viete was able to change this by creating the first symbolic algebra. With Viete's work began the modern paradigm in math. Rene Descartes' work *La Geometrie* appeared in 1637. This book

united algebra and geometry into a single subject, which today we call "analytical geometry." Descartes' writings greatly influenced the ideas of Leibniz and Newton, and were instrumental in the development of calculus (Boyer, 1959).

Teaching Geometry and Geometry Textbooks

Over two hundred geometry textbooks were published in the seventeenth century. Among these books, the Italian books are mostly devoted to the application aspect of geometry. Many included only those parts of geometry believed were of practical value, describing geometry as a tool to measure heights, depths and distances. Though geometry was one of the courses offered in Harvard College in 1642, no professorship in mathematics was established there until 1727. (Kokomoor, 1928)

Newton's work in the late 17th century marked the dawn of the Enlightenment. During this period, there was an explosion in the fields of math and science. Yet, until the 19th century, geometry was only taught in European and American universities, and was not yet included in the standard high school curriculum. Most universities, including Harvard and Yale used Euclid's *Elements* as the textbook for this course.

In the middle of the 19th century, some universities made geometry an entrance requirement. Now, there was motivation for high schools to teach geometry. Euclid's *Elements* set the curriculum. "Though little changed in the geometry curriculum, or in the assumed goals of geometry education, the university sanctioned transfer of college geometry to the high school setting, and established a precedent in the American

geometry curriculum that would prove to be very difficult to alter;" (Sinclaire, 2008, p. 19)

At the end of the 19^{th} century, at the Mathematics Conference of the Committee of Ten, it was argued, that gaining deductive reasoning skills through geometry was a valuable skill for high school students, since these skills once acquired could be transferred to other areas of reasoning. From 1900 to the 1950s, high school geometry textbooks in the USA followed the structure of the *Elements*. For example, <u>Plane and Solid Geometry</u> by G. A. Wentworth 1901 is mostly organized according to the Euclidean model. This book begins with general terms and axioms, then treats parallels and perpendiculars, congruent triangles, angle sums and proportions, parallelograms and trapezoids, and circles. In placing parallels before congruent triangles, this book alters the order of Book One of the *Elements*. <u>Plane Geometry & Its Reasoning</u> by Harry C. Barber and Gertrude Hendrix, 1937 is similar. Retaining the topics of Book One of the *Elements,* it begins with definitions, assumptions and then starts proving theorems on congruence of triangles. The table of contents of <u>Plane Geometry</u>, Seymour & Smith, 1949 shows the Euclidean model as well, beginning with basic concepts, then moving into formal geometry, parallels and perpendiculars, angle sums, parallelograms and trapezoids, circles, proportional line segments. All three high school geometry books, though published many years apart, clearly show the organization of content similar to Euclid's *Elements*.

The launch of Sputnik in 1957 made the U.S. recognize national shortcomings in the areas of technology. The United States wanted to be the leader in math and science and saw the improvement of K-12 education as the means. Thus began the "New Math

Movement". The School Mathematics Study Group (SMSG), which was founded in 1958, established a detailed curriculum and wrote textbooks for grades seven through twelve. "Over 60 texts were written as well as a variety of supplemental materials and reports. SMSG texts were to provide a model for commercial writers" (Herrera & Owens, 2001, p. 86). Many SMSG projects were funded by the National Science Foundation. Influenced by university mathematicians, SMSG authors viewed the geometry being taught in U.S. high schools as outmoded. SMSG tried to create a geometry course that was rigorous according to modern standards, hence even more rigorous than Euclid. The course was based on the SMSG Axioms (derived from work of Moise, and inspired by Moise's work in brining Euclidean geometry up to modern standards of rigor). "SMSG=New math emphasized deductive reasoning, set theory, rigorous, and abstraction, while the standards emphasize applications in real world context, especially experimentation and data analysis." (Herrera & Owens, 2009, p. 91)

Unfortunately, the SMSG math did not work well in schools, perhaps because it demanded too much of a reorientation in the culture of the math classroom. There were negative reactions to the New Math, and its influence declined in the 1970s. As this happened, the SMSG course was vulgarized, and isolated pieces made their way into new editions of more traditional texts.

The mathematics education in the United States during the 1970s has been viewed as a "back to the basics" period. The mythology was that the "New Math" had failed to produce any positive results and it was necessary to go back to the basics. Lesson objectives had to be clearly evident and measurable, teachers were to direct students

through a curriculum that was classified according to various criteria into successive levels.

In the early 80s, international studies showed that the U.S. students performed poorly compared to other nations. Results from the Second International Mathematics Study, and International Assessment of Educational Progress gave a sense of national crisis to the general public. The famous report, "A Nation at Risk," which was commissioned by the National Commission for Excellence in Education, was published in 1983. This report led the National Council of Teachers of Mathematics to take an active role in seeking to reform mathematics education in the United States. In 1989, NCTM produced its *Principles and Standards for School Mathematics*. In 1991, the NCTM *Professional Standards for Teaching Mathematics* were produced, and in 1995, the NCTM *Assessment Standards for School Mathematics* was published. The NCTM called for changes in content as well as pedagogy of math education. Some notable changes in content were math modeling and connection of math to the real world. Changes in pedagogy were, for the classroom to be student centered rather than teacher centered, by increasing student involvement and group work. Many states sought to align their curriculum with the standards developed by the NCTM. (Herrera & Owens, 2001)

The modern standard United States Geometry course is based on Euclidean geometry. If we compare the table of content of the most commonly used geometry books in high schools, we see a Euclidean based model. For example, the table of content for the *Glencoe Geometry* is as follows:

1. Introduction
2. Deductive reasoning

3. Lines and Angles
4. Congruence (SAS, SSS, ASA) (Isosceles triangles)
5. Inequalities in triangles
6. Parallel Lines
7. Trigonometry
8. Quadrilaterals
9. Transformations
10. Circles
11. Area
12. Surface area
13. Volume

High School Geometry in the Louisiana Comprehensive Curriculum

In the early 1990s, Louisiana began a process to raise academic standards in the public school system. The first set of general standards for math in the state appeared in the mid-1990s, and were called the “Frameworks”. Detailed and specific content standards in the form of Grade-Level Expectations (GLEs) were adopted in 2004. The GLE’s determine the appropriate content for each grade level. The Louisiana Comprehensive Curriculum (LCC), based on Grade-Level Expectations, was adopted in 2005. This document contains activities to guide instruction according to the GLEs. Each LCC course consists of several units that are organized according to the content that appears in state tests.

The Louisiana comprehensive curriculum for geometry is divided into eight units.

Unit 1: Geometric pattern and Reasoning. This unit is to be taught in approximately three weeks, and contains eight activities. The first two activities are devoted to inductive and deductive reasoning. The third and fourth activities are on linear “patternology.” The fifth is a very long activity on square and rectangular numbers. The sixth and seventh activities involve counting principals, and the eighth’s activity is on permutations and combinations. The first unit of the Louisiana Comprehensive Curriculum Geometry Course contains no geometry.

Unit 2: Reasoning and Proof. This unit is to be taught in approximately four weeks, and contains nine activities. Similar to the standard geometry textbook, unit two of this course introduces the vocabulary of points, lines, planes, etc. The first second and third activities address inductive and deductive reasoning. Activity four asks students to solve simple linear equations after they have been told that the terms of the equations refer to geometric objects. In activity five, students are asked to find the truth- values of conditional statements and to make truth tables. In activity six, students have to create simple statements using the law of syllogism. In activity seven, students are asked to give reasons for algebraic manipulations. In activity eight, students are to write proofs on segment and angle addition as well as more algebra proofs. Finally, in the ninth activity, students are introduced to some geometry with parallel lines and transversals.

Unit 3: Parallel and Perpendicular relationships. In this unit, students finally meet some Euclidean topics. The first three activities are devoted to the slopes of parallel and perpendicular lines. In activities three and four, students have to find the distance between two points and two lines. Activity five involves the relationship of angles

formed by two parallel lines and a transversal. In activity six, students are instructed to go to a website to complete an on-line activity. In activity seven, students show that the sum of the measures of the angles in a triangle is 180 degrees, and they review facts from earlier in the unit.

Unit 4: Triangles and quadrilaterals. Unit 4 is the longest unit in the LACC Geometry course. It contains 18 activities that are to be completed in 5 weeks. Students are introduced to some postulates and theorems connected with congruence and similarity. In activity one, students are instructed to use the definition of an isosceles triangle to find missing sides and angles. In activity two, students have to complete an online assignment involving congruent triangles. Activity three is a group activity identifying congruent parts of congruent triangles. In activities four and five, students use of the SSS, ASA, SAS and AAS criteria in numerous trivial examples. In activity six, students write a proof on triangle congruence. In activity seven, students have to draw different types of triangles and practice drawing angle bisectors, medians, altitudes and perpendicular bisectors. Activity eight does not exist. In activity nine, students use geometry software to draw more angle bisectors, medians, altitudes and perpendicular bisectors. In activity 10, students toy around with right triangles. Activity eleven instructs students to draw scalene, acute, and right triangles and measures their sides to understand the triangle inequality. In activity twelve, students solve simple equations having being told the terms of equations refer to the angles of a triangle. In activity 13, students make triangles using pieces of straws to understand the relationship of the sides of a triangle. Activity 14 is a discussion of similarity of geometric objects. Activity 15 instructs students to use geometry software (such as Geometer's Sketchpad) to

investigate convex quadrilaterals. In activity 16, students play around with quadrilaterals in the coordinate plane using the distance formula. Activity 17 deals with vocabulary for classifying quadrilaterals. In activity 18, students fiddle with the median of a trapezoid.

Unit 5: Similarity and Trigonometry. Unit five has 14 activities that are to be completed in approximately four weeks. In activity one, students draw similar triangles and formulate a definition for similar figures. Activity two asks students to use equilateral triangles, pattern blocks and cubes to find the ratios of sides, area, and volumes of similar figures. Activity three is an investigation of scale drawings. Activity four asks questions about similar triangles that are formed by light beams. In activity five, students find missing dimensions in similar figures. Activity 6 continues with variants of finding the ratios of corresponding segments in similar figures (now using special segments, such as medians, in triangles). In activity seven, students draw and investigate the midsegments of a triangle. In Activity eight, students review the material of the unit and call one another "Math Masters". In Activity nine, students talk about the ratios of sides of similar triangles; somehow, Pythagorean triples are supposed to be relevant, but the description of the activity does not explain exactly how. Activity ten is on the converse of the Pythagorean Theorem. Activity 11 is missing. In activity 12, students have to visit a website that explain the uses of trigonometry. In activity 13, students investigate 30-60-90 and 45-45-90 triangles. Activity 14 is on trigonometric ratios.

Unit 6: Area, Polyhedra, Surface Area, and Volume. Activity one provides a rationale for the area formula for rectangles. Activity two is titled "Area of Regular Polygons". It begins with an investigation of the angles of a regular polygon, and by the

end it develops the area formula for regular polygons. This is very complicated in comparison to most of the other activities. Activity three is a writing activity in which students make guesses and conjectures about the volumes of cylinders. In activity four, students investigate the surface areas and volumes of cubes of various sizes. In activity five, students build a cylinder out of paper and use this to understand the surface area formula. In activity 6, students build a pyramid and discuss its volume. In activity 7, students find the total surface area of the pyramid they constructed in activity 6. In activities 8 and 9, students compare the volume of a pyramid and a prism. Activity 10 involves the volume of irregular objects. Activity 11 is on geometric probability.

Unit 7: Circles and Spheres. Activity one is on vocabulary. Activity two develops the formula for the area of a disk. Activity three is on probability. Activity four is on central angles and arcs of circles—mostly notation. Activity five is on concentric circles; students discover that in larger circles, arcs subtended by the same angles have greater length. Activity 6 is on using circle graphs to represent data. Activity 7 is on geometric probability. Activity 8 is on arcs and chords of a circle. Activity 9 challenges students to find the center of an unmarked circle using the perpendicular bisectors of chords. Activity 10 is on inscribed angles. Activities 11 & 12 are on tangents and secants of a circle. Activities 13 & 14 are on the surface area and volume of spheres.

Unit 8: Transformations. Activity one involves understanding congruence, similarity and symmetry using transformations. Activities 2 & 3 concern reflections over x- and y-axis. In Activity 4 students draw the rotations of figures by 90, 180 and 270 degrees, using the coordinate plane. Activity 5 is on translation and activity 6 is on

dilation. In both cases, the transformations are represented in the coordinate plane. Activity 7 is on using geometry software to investigate transformation. (La Doe, 2010b)

The intention of the educators who wrote the Louisiana Comprehensive Curriculum was for the students to learn through activities. By comparing the table of content of the text- book to the Comprehensive Curriculum, we are able to see that the writers of the curriculum made an effort to align the units to the textbook. After completion of each unit, a unit assessment must be administered to measure learner outcome. Some sample assessment problems are included in the document. School districts are encouraged to make uniform unit tests and administer them after each unit.

When using the LACC, I found many activities that consumed a whole class period with very little learning as a result, even though I followed the guidelines closely. While some activities are engaging and are student centered, many are repetitious, obscure and shallow.

Assessment

Knowing What Students Know: The Science and Design of Educational Assessment is an important and influential book about assessment. According to the *Knowing What Students Know* model, also known as the assessment triangle, assessment is a process of creating evidence and drawing conclusions. All assessments, no matter what purpose they serve, involve three components:

- A set of goals and hypotheses about student learning (Cognitive Model),
- An opportunity for students to demonstrate learning (Observation Task), and

- A way to draw conclusions from the student performance (Interpretive Framework).

In large- scale assessments, a statistical model might be used to design assessments and interpret data. In the classroom the design and interpretation is often based on the teacher's professional judgment.

Assessment has many purposes. NCTM suggests that the main purposes from the point of view of the classroom teacher are three: it demonstrates what students have learned, it gives an opportunity for the teacher to communicate what the students are expected to learn and it provides opportunities for instruction.

The Educational Testing Service study, Cognitively Based Assessments *of*, *for*, and *as* learning states:

- Assessment *of* learning is done at the end of a task; its purpose is to provide evidence of learning.
- Assessment *for* learning happens during learning, students understand exactly what they are to learn.
- Assessment as learning is when students become aware of how they learn. (ETS)

Assessment Tasks: NCTM identifies four types of assessment tools: closed tasks, open-middled tasks, open-ended tasks, and projects. There is only one correct answer for closed assessment tasks. These types of questions include, fill in the blank, multiple choice, or true-false questions. Open-middled tasks are similar to closed tasks in that they also have only one correct solution, however, students can use different approaches and reasoning to solve them. Open-ended tasks allow the students to take many different approaches to the problem. These tasks contain many correct answers and

strategies. Students are asked to explain, make conjectures and justify their solutions. Projects then are basically extended open-ended tasks that may require students to use mathematics to solve a real-life problem. (NCTM, 2005)

Classroom assessment should mirror the activities and tools used during instruction. The type of questions the teacher asks when presenting the lesson, such as, “Why is that true?” Or “Can you explain?” need to be included during assessment.

My district uses the Louisiana Comprehensive Curriculum for geometry. Though there are some sample assessment problems included in the LCC, it does not contain any unit tests. According to the LCC, every district must administer a unit test after the completion of the unit. Our district wanted to administer unit tests for geometry based on the LCC to determine how well students will perform on the End of Course Exam. The skills tested on the End of Course Exam are aligned with the eight units to be taught in the LCC for geometry.

After closely examining the LCC and the activities contained in the document, I picked certain standards from each unit, using the models provided. The questions in my tests were motivated by the GLE’s the students need to have mastered after each unit. Based on the GLE’s, I made assumptions to the types of questions that will be asked on the End of Course Exam for Geometry. I wanted to determine whether my tests are good indicators of how well the students will perform on the End of Course Exam. My intention was to create test questions similar to the questions that will be on the End of Course Exam. The test questions in my tests include the types of assessments identified by the NCTM. For example, I have included closed task assessments in the form of filling in the blanks, true/false, and multiple-choice questions. There are also open-

middled assessments where students can arrive at the answer using different approaches and reasoning methods, and some open-ended tasks with only one right answer but students can take different paths to solve it. After the tests were administered and graded, I analyzed the results using certain guidelines.

Standard based reform has increased the amount of assessments given to students. in every subject in k-12 schools. Assessment is an important and necessary part of standard based reform. It is the principal method of determining whether students and teachers have achieved the standards and holding them accountable. However, assessment alone cannot improve learning. It will only be useful if the instruction is altered as a result of the information acquired through the assessment.

Chapter III. Test Design

This chapter will provide a description of how the tests for each unit were designed. The eight tests themselves are reproduced in the Appendix.

Before designing the unit tests, I carefully studied each unit and activities that accompany each unit. I also looked at the few sample assessment questions provided in each unit of the Louisiana Comprehensive Curriculum. I then made a list of the GLEs and the skills to be taught and assessed for each of the units in the LCC.

When designing the tests, I first discussed the content and main ideas of the units with my advisor, who is a mathematician. I composed the questions based on the content of the unit and his advice. When creating test questions, I wished to create questions that I hoped would be similar to the questions my students might encounter on the End of Course Test in geometry. I looked at several geometry textbooks to get ideas on how to write test questions. My advisor also provided me with ideas on how to create test questions. I used the free software program Geogebra to create graphs and geometric figures for the tests. After I created each unit test, I met with my advisor again to decide on the relevance of each test question.

The questions in my tests vary in difficulty and skill level. Some merely ask for definitions, some require students to use background knowledge, and different questions require different levels of reasoning. For example, I have included closed task assessments in the form of filling in the blanks, true/false, and multiple-choice questions. There are also "open-middled" assessments where students can arrive at the answer using different approaches and reasoning methods, and some open-ended tasks with only one right answer but students can take different paths to solve it.

Chapter IV. Test Item Analysis

From the eight tests that I developed, tests II through VI were administered to the students. From the Unit tests that I administered, I selected eight questions for deeper analysis. In order to be selected, the item had to have appreciable numbers of both correct and incorrect answers. I did not include true/false question or multiple-choice questions. I also excluded question where students had to plug values into a formula to arrive at the solution.

By administering the unit tests I made for my students, I wanted to learn about student performance as well as the quality of the test questions. The method of analysis was developed through experimentation, guided by these goals. After several attempts, I settled on a procedure that seemed objective and informative and was consistent with the assessment model from *Knowing What Students Know*. It consists of the following parts:

- Correct solution: I provided the solution to the test question.
- Hypothetical workflow: Describes a step-by-step process on how to solve the problem.
- Data: Describes the number of students who were successful at each stage of the hypothetical workflow.
- Analysis: What types of mistakes did students make at each step, and types of common mistakes.
- Conclusions and recommendations: Summarize what was learned about the test and about student-learning and suggest possible interventions to improve

student performance in the future as well as possible improvements for the test question.

The following tables show the overall grade distribution obtained by the 115 students who took the tests, as well as the grade distribution on the End of Course Exam. All rows total 115.

	Grade Distribution				
Test	A	B	C	D	F
Unit II	12	28	46	11	18
Unit III	26	17	23	21	28
Unit IV	13	29	40	20	13
Unit V	14	19	23	20	39
Unit Vi	56	28	19	5	7

The End of Course results are as follows:

Score	Exellent	Good	Fair	Needs Impr.
Frequency	12	56	31	16

Analysis of Unit III Test, Problem 22.

The problem, as it appeared on the test, is reproduced below.

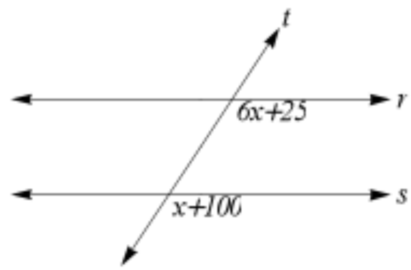


Figure 8.

22. In Figure 8 (above), line *r* is parallel to line *s*. Two angles are labeled with their degree measure. Find the value of *x*.

Correct answer

Since the angles are corresponding angles of a transversal of parallels, they are equal in measure. Therefore, $6x + 25 = x + 100$. Subtracting x + 25 from both sides, we get $5x = 75$. Diving both sides by 5, we get $x = 15$.

Hypothetical workflow

The "hypothetical work flow" was inferred from the answers that were observed and from my own approach to the problem. We can view the problem as a four-step task:

1. Recognize that a geometric fact can be used to transform the given problem into the problem of solving an equation;
2. Write the equation;
3. Select and execute a strategy for solving the equation;

4. Deal correctly with the arithmetic that is required.

However, the student work does not enable us to distinguish between students who completed the first step only and those who completed both the first and the second. So, we have to view this as a three-stage process. A student succeeds at the first stage if she (or he) writes the equation. She completes the second stage if she executes a correct strategy for solving it. She completes the last stage if he/she finds the correct solution.

Data

The number of students tested was 115. Fourteen (14) students had no answer. Of those who did answer:

- 101 (88%) completed the first stage. 93 of these wrote "$6x + 25 = x + 100$". The remaining 8 wrote: "$x + 100 = 6x + 25$".
- 97 (79%) completed the first and second stage. Typical answer at this stage was, $5x + 25 = 100$.
- 83 (72%) completed all three stages. The 14 students who completed two but not three stages all made basic errors with arithmetic.

The most common wrong answer was "$x = 75$". This occurred six times. The students who arrived at this answer didn't divide by five. They had $5x = 75$, yet ignored the fact that both sides need to be divided by 5 and wrote "$x = 75$".

Analysis

The most outstanding fact about the answers was that no student provided any reason or explanation. Certain habits of mind appear to be absent. The Common Core expects

students to “reason abstractly and quantitatively” and to “construct viable arguments and critique the reasoning of others.” To do this, students need to recognize the need for justifications and be in the habit of providing them. There is no evidence in the solutions to this problem that students had this habit.

Algebra and simple arithmetic seem to be the main problems for the students who were able to set up the equations correctly. 9% of the students don’t appear to be able to organize their work in the process of solving an equation of the form $Ax + B = Cx + D$. 7% of the students failed to perform basic arithmetic steps correctly. From this, we see that using algebra in the context of this problem is problematic for a small number of students.

Conclusions and Recommendations

The teacher can encourage the habits that the Common Core expects by constructing tests so that justifications are required. A test policy that always demands reasons might be useful in forming good habits.

Those students who have a problem with algebra or arithmetic need to be provided with some extra instruction. It might be carelessness, or there might be a deeper problem. At the beginning of the year, it would be useful to give some work to establish the general level of algebra skill, so that deeper problems are not encountered for the first time on a test like this.

Analysis of Unit III Test, Problem 25.

The problem, as it appeared on the test, is reproduced below.

25. A line passes through (3, 11) and has slope 4/5. Write the equation of this line.

Correct answer

The equation is found using the given slope 4/5 and point (3,11). There are two approaches:

1. Use point-slope form: $(y - y_0) = m(x - x_0)$. In this method, the equations is gotten in one step: $(y - 11) = (4/5)(x - 3)$.
2. Use the slope-intercept form. We know the equation is of the form $y = (4/5)x + b$, but we don't know b. We find it by substituting (3, 11) into the equation to get $11 = (4/5)(3) + b$, or .$b = 55/5 - 12/5 = 43/5$. This gives the final solution: $y=(4/5)x + 43/5$.

Hypothetical workflow

The "hypothetical work flow" was inferred from the answers that were observed and from my own approach to the problem. Students must first decide which of the two approaches to use.

<u>Approach 1:</u> The first approach produces an answer in one step. Students simply need to be able to recall the template for the point-slope form and fill in the given data appropriately. The problem statement does not ask for an equation in a specific form, but

students might decide to provide an answer in slope-intercept form—and actually some did. This would involve an additional step.

Approach 2: The second approach requires numerous steps:

1. Decide to use the slope-intercept form;
2. Refer to the given slope, and write "$y = (4/5)\ x + b$";
3. Recognize that the value of b can be determined by substituting the coordinates of the given point for (x, y) in this equation; perform the substitution.
4. Solve for b, which involves finding a common denominator and simplifying fractions;
5. Write the equation using the y-intercept found in step III.

The first step is completed without any writing. A student completes the second step if she writes

$$y = (4/5)\ x + b.$$

She completes the third step if she writes

$$11 = (4/5)(3) + b,$$

Or something equivalent She completes the fourth step if he/she correctly solves this equations for b; we would expect to see "$b = 43/5$" written somewhere. She completes the last step if he/she writes the correct equation.

Data

The number of students tested was 115. Eight (8) students used the first approach. Eighty-nine (89) students used the second approach. Eighteen (18) students had no response.

Approach 1: Three of the students had the correct equation. Five students tried to change the equation from point-slope form to slope-intercept form. All five of them had problems with operations on fractions, and consequently none of them succeeded.

Approach 2:

- All 89 students who chose the second approach completed the first and second stage, and wrote $y = mx + b$.
- Sixty-seven students (67) of these completed the third step and correctly substituted the coordinates of the given point for (x, y). They wrote $11 = 4/5(3) + b$
- Twenty-seven students (30%) completed the fourth stage and solved for b.
- Twenty-six students, (29%) completed all five stages.

Analysis

Only 8 out of 115 students (7%) used the easy method, and of these, five attempted to convert the equation into slope-intercept form, though this was not required. This suggests that students do not understand the meaning of "equation for a line" except in a very superficial way that refers to the format of the equation.

Seventeen (17) students, who completed the first two stages, mistook the *y*-coordinate of the given point for the *y*-intercept. Their answer was $y = 4/5x + 11$. It's surprising at first that 17 students assumed that the *y*-coordinate of the given point was the *y*-intercept. But this is the kind of error that would be expected if students did not have a clear understanding of how an equation works as a "point-tester", i.e., as a criterion (which may be written in many equivalent forms) for picking out the points that lie on a line.

Fifteen of the students, who completed the third stage correctly, did not attempt to solve the equation for *b*. The 25 students who attempted, didn't succeed in solving for *b* in an equation of the form $A = (B/C)(D) + b$, with A, B, C, D integers. They all failed to perform basic operations with fractions correctly.

Conclusions and Recommendations

It seems likely that most students do not have a clear understanding of what an equation for a line is. Instead, they have a procedural orientation. One would hypothesize that if asked, the students in this class would not be able to explain <u>why</u> an equation represents a line, but would only be able to tell how to produce an equation from given data.

Algebra and operations on fractions were a problem for 25 out of the 89 students who were able to set up the equations correctly. This is a rate of error in basic algebra or arithmetic that is somewhat greater than that seen in the previous problem. The 18 students who didn't respond probably are weak in basic algebra and arithmetic. This is still a minority, but approaches half the class. From these results, it is clear that some

students may be seriously behind, some need continued practice in basic operations with fractions and review of algebra I skills, yet many have the foundation to go on.

Analysis of Unit III Test, Problem 29.

The problem was as follows:

> 29. Determine whether lines AB and MN are parallel, perpendicular, or neither. A is the point at (-1, 3), B is at (4, 4), M is at (3, 1), N is at (-2, 2)

Correct answer

Using the point-point slope formula, the slope of AB is $(4 - 3)/(4 - (-1)) = 1/5$. Slope of MN is $(2 - 1)/(-2-3) = -1/5$. Since these slopes are different and the product of the slopes is not -1, the lines are neither parallel nor perpendicular.

Hypothetical workflow

The "hypothetical work flow" was inferred from the answers that were observed and from my own approach to the problem. The steps of the problem are as follows:

1. Recall that whether lines are parallel or perpendicular can be deduced from slopes, and therefore begin by finding slopes.
2. Recall the point-point slope formula:

$$m = (y_2 - y_1)/(x_2 - x_1).$$

3. Select appropriate points and use the coordinates in the correct manner in order to deduce the slopes of the lines. If this is done, then the following is obtained:

   Slope of AB = (4 – 3)/(4 – (-1)) = 1/5.

   Slope of MN = (2 – 1)/(-2-3) = -1/5.

4. Use the knowledge that lines are parallel if they have the same slope and are perpendicular if the product of the slopes is -1. In this problem, the slopes are different, and their product is -1/25.

5. Conclude that the two lines are neither parallel nor perpendicular.

The student work does not enable us to distinguish between students who completed the first step only and those who completed the first and the second. It also doesn't allow us to distinguish between students who completed the fourth step and fifth step. So, we have to view this as a three- stage process:

- A student succeeds at the first stage if she (or he) recalls the point-point formula.
- She completes the second stage if she finds the two slopes.
- She completes the third stage, if he/she finds the correct answer.

Data

The number of students tested was 115. Three (3) did not respond. Eleven students wrote perpendicular, five students wrote parallel, however, they had no work to show how they arrived at this answer.

- 96 students completed the first stage.
- 89 students completed the second stage. In most cases, they wrote

  4-3/4-(-1) = 1/5

2-1/-2-3 = -1/5

- 71 students completed the third stage and wrote "neither".

Of the 96 who completed the first stage, 92 students plugged in the correct points into the formula, but only 89 arrived at the two correct slopes. Five students made computing errors, four arrived at 1/5, and 1/5 for both slopes and knew that parallel lines have the same slope. Their answer was "parallel." Two students plugged in the wrong values into the formula.

Eight students had the correct slopes, 1/5 and -1/5, yet they incorrectly identified them as parallel. Another 10 students who had the correct slopes identified them as perpendicular. One student who had the correct slopes didn't write an answer.

The most common wrong answer was "perpendicular". This answer appeared 22 times, 10 students who had this answer had the correct slopes.

Analysis

Only a small number of students, (5) made computing errors. The majority of the errors involved identifying the two slopes correctly. The knowledge that lines are parallel if they have the same slope, and are perpendicular if the product of their slopes is negative one is lacking in 18 of the students who arrived at the correct slope. 16% of the students appeared to be able to perform the correct procedures, but are not able to incorporate those procedures into strategy. Students, who identified the two slopes 1/5 and – 1/5 as parallel and perpendicular, clearly didn't remember the rule for parallel and perpendicular lines.

Conclusions and recommendations

As for the students who had the correct slopes, yet identified the lines as parallel, it is not clear whether they thought -1/5 equals 1/5, or whether they felt they made a mistake computing. It would be useful to give those students some similar problems where the answers are different numbers, such as 2 and ½. This would help identify and correct any misconceptions. Students, who identified them as perpendicular, clearly didn't remember the rule for perpendicularity. Students should have received more thorough instruction and practice in identifying the different types of slopes. A short quiz on just identifying parallel and perpendicular slopes prior to the test may have helped students memorize the rules.

This problem is not very informative about testing student knowledge. It leaves a lot of room for guessing and doesn't really tell us much about what students know about the slopes of parallel, perpendicular and skew lines. A better way to test student knowledge would be to ask students to explain why the two lines are neither perpendicular nor parallel.

Analysis of Unit III Test, Problem 23

The problem, as it appeared on the test, is reproduced below.

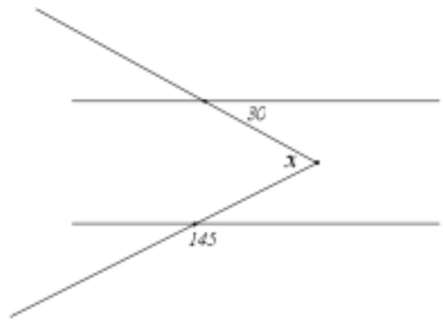


Figure 9.

23. In Figure 9 (above), the two horizontal lines are parallel. The angles are marked with their degree measure. Find *x*.

Correct answer

Draw a horizontal line through the vertex of the angle labeled "*x*". This divides that angle into two smaller angles. The upper one of these measures 30 degrees, because it is one of a pair of alternate interior angles in which the other angle is labeled "30". The lower one is in an alternate interior pair, where the second in the pair is supplementary to the angle labeled "145". So, its measure is 35. Thus, *x* is the sum of these two: 30 + 35 = 65 degrees.

Problem structure and hypothetical workflow

1. Note that a third line can be drawn through the vertex of angle *x* to produce two smaller angles.

2. Use the knowledge that alternate interior angles are congruent to conclude that the upper angle is congruent to the 30-degree angle.
3. Recognize the other angle is one of the pair of an alternate interior pair supplementary to the angle whose measure is 145 degrees.
4. Recall, supplementary angles measure 180, therefore, the supplement measures 180-145= 35.
5. Conclude that angle x measures 30 + 35 = 65 degrees.

Stages 2, 3 and 4 could be performed in numerous different orders. It's also possible that some of conclusions from these stages could be recognized before stage 1 is performed, and in fact, stage 1 might even be motivated by recognizing this information. Stage 1, on the other hand, is a "gimmick" that is often used in "missing angle" problems, and students who have seen a few problems of this kind may be primed to think of it. Rather than viewing this as a problem that requires sequential stages, we can view this as a problem that requires noticing several things and then putting those things together.

A. Foundation pieces

1) Recognizing vertical angles: Students were able to recognize and label vertical angles.
2) Recognizing supplementary angles: Students were able to recognize and label supplementary angles.
3) Recognizing transversals and alternate interior angles: students were able to recognize and label alternate interior angles.

B. Elaborating the figure. Students were able to draw the expected line

C. Coordinating the information. Students were able to combine information.

D. Final conclusion

Table 1. Scoring Rubric for III.23.

		Score and criterion:	
		0	**1**
Item	A1	No evidence of skill	Evidence of skill
	A2	No evidence of skill	Evidence of skill
	A3	No evidence of skill	Evidence of skill
	B	Did not draw line	Drew line
	C	No evidence of using facts together	Evidence of combining
	D	No correct final answer	Wrote “65” as final answer

Table 2. Distribution of Answer Types.

Student Response Types								Count	Comment
Item Part						Score (Part A)	Score		
A 1	A 2	A 3	B	C	D				
1	1	1	**1**	1	1	3	6	19	Performed well.
0	1	1	**1**	1	1	2	5	31	
1	1	1	**1**	1	0	3	5	1	
1	1	1	**1**	0	0	3	4	2	Had background (Part A), but did not complete.
0	1	1	**1**	1	0	2	4	2	
1	1	0	**1**	1	0	2	4	1	
0	1	1	**1**	0	0	2	3	4	
1	0	1	**1**	0	0	2	3	3	
0	0	1	**1**	0	0	1	2	12	Need assistance.
0	1	0	**1**	0	0	1	2	1	
1	0	0	**1**	0	0	1	2	1	
0	0	0	**1**	0	0	0	1	10	
1	1	0	**0**	0	0	2	2	1	Understanding or motivation lacking.
0	1	0	**0**	0	0	1	1	2	
1	0	0	**0**	0	0	1	1	1	
0	0	0	**0**	0	0	0	0	23	
							Total	**115**	

Data

A total of 115 students were tested. Of these, 23 students wrote nothing.

In Part A, 27 students recognized vertical angles and labeled them correctly. 64 students recognized supplementary angles and labeled them correctly. 73 students recognized alternate interior angles.

In Part B, 54 students made steps towards combining the supplement and the alternate interior angle to get the measure of the angle labeled x. 50 students arrived at the correct answer.

The most common wrong answer was $x = 75$. This answer appeared 12 times. Six of the students that arrived at this answer made computing errors when finding the measure of the supplement to the angle labeled 145, and the other six appeared to have guessed the measure to be 75.

Analysis

None of the respondents provided an explanation or reason for why the angles were congruent or supplementary. In all cases, only the angle measures were given. Most students gave evidence of at least partial understanding and background. Many were able to find the angle congruent to the 30-degree angle; however, they were unable to make the connection between the supplement of the 145-degree angle to the angle produced by

drawing the third parallel line. Some students seemed unable to deal with a problem that required several steps to arrive at the solution.

Conclusions and Recommendations

Many students did not connect information taken from the labeled 145-degree angle to information related to the third parallel line. If students had repeated opportunities in class to really analyze angle relationships like those created by the third line, they might have brought this experience to the test. In the future, I will be sure that there are more problems in class and in homework where students need to use pairs of alternate interior angles that occur when auxiliary lines are drawn. There is a question if such practice done independently without angle measures would transfer to cases where angle measures are involved. Students need two things: understanding of the angle relationships as well as ability to use angle measure in the context of these relations. Since this question has several steps, students may need more practice with these types of problems in order to remember to do all the necessary parts to solve it. Repetition and practice of similar problems would have been beneficial. Students should also be reminded to check their work once a problem is solved to avoid careless errors.

Analysis of Unit III Test, Problem 18.

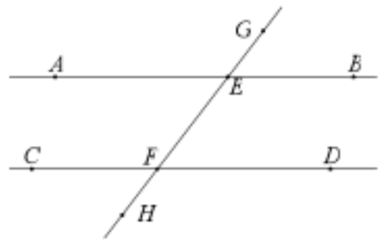


Figure 7.

18. In Figure 7 (above), line AB is parallel to line DC. Using point notation for angles, list all the angles that are congruent to angle AEG.

Correct answer

There are two different ways to solve this problem. One method would be to observe that angle CFE and AEG are corresponding angles of a transversal, and are congruent. Angle BEF is one of pair of alternate interior angle in which the second is CFE. Angle DFH is pair of alternate exterior angle in which the second is AEG.

The simplest way to solve this problem would be to observe that angle DFH is congruent to angle AEG since these two angles are alternate exterior angles of a transversal. Angle CFE is congruent to angle AEG since these two angles are corresponding angles of two parallel lines cut by a transversal. Angle BEF is congruent to angle AEG since these two angles are vertical angles.

Problem structure and hypothetical workflow

The problem requires the knowledge that three points can be used to name an angle. The three angles congruent to angle AEG can be recognized independently. This involves

using the knowledge that vertical angles are congruent and some knowledge of transversals. Here are some possible paths of reasoning:

1. Recall that when two parallel lines are cut by a transversal, the alternate exterior angles are congruent. Angle AEG and angle DFH are alternate exterior angles; therefore angle DFH is congruent to angle AEG.
2. Recognize that angle CFE is congruent to angle AEG since these two angles are corresponding angles of two parallel lines cut by a transversal.
3. Use the knowledge of vertical angles to see that angle BEF is congruent to angle AEG.

We can view answering as a three-part process. A student succeeds at the first part, if she (he) correctly uses three points to name an angle. He/she succeeds at the second part if she identifies either part of number two above. He/she succeeds at the last part, if she has correctly identified all three angles.

Data

Altogether, 115 students were tested. Of these:

- 5 (4%) students did not name any angles using three points. Five wrote nothing.
- 105 students (85%) completed the first stage and correctly used three points to name an angle.

Of the 105 who completed the first stage:

- 86 students (75%) correctly identified the alternate exterior angle;
- 70 students (61%) correctly identified the vertical angle;
- 58 students correctly identified the corresponding angle;

Of the 105 who completed the first stage:

- 99 (85%) completed the second stage. 99 students had at least one correct angle;
- 18 had only two correct angles;
- 32 had only one correct angle.

The most common answer was the correct answer, which was given by 49 students. The next most common answer, was naming the alternate exterior angle and no other. This occurred 16 times. Ten (10) students gave no answer. No other answer occurred more than 5 times.

The 115 responses can be classified as follows. Eighty (80) answers included at least one correct angle and no incorrect angles. Twenty-five (25) answers included an incorrect angle. Ten (10) students gave no answer.

Of the 86 who named the alternate exterior angle, only 12 named an incorrect angle. Of the 29 who did not name the alternate exterior angle, 13 named an incorrect angle.

Analyses

The most commonly named angle was the alternate exterior angle, which appeared in 86 answers. The following table shows that those who named this angle were far more likely to name other correct angles and far less likely to include incorrect angles in their answers.

Named AEA	N	P(named other correct)	P(named an incorrect)
Yes	86	63/86 = 0.73	12/86 = 0.14
No	29	13/29 = 0.45	13/29 = 0.45

Conclusions and Recommendations

The reason that most students identified the alternate exterior angle may be due to instruction. I did not intentionally emphasize this, but I may have gone to this angle first out of habit. The students who were paying attention may have picked up on this habit.

When two parallel lines are cut by a transversal, students need to understand that there are two pairs of congruent angles. Students should have had exposure to numerous situations where recognizing and using this fact is required. The “missing angle” problems in the Singapore curriculum could provide this.

Even before the fact is used, students need to understand what it means. A good strategy for this might be to have students draw two parallel lines cut by a transversal and then measure all of the angles.

In reviewing the answers, I found that some students (7) named the same angle using the letters in reverse order. These students may not have a clear understanding of how angle names work. It would be useful to have a sheet that explained this that could be given to them.

Analysis of Unit IV Test, problem 11

Find the measure of each side of an isosceles triangle ABC with AB = BC, if AB = 14, BC = $3x + 2$ and AC = $3x - 2$.

Correct answer

The problem is solved by observing that an isosceles triangle has two congruent sides, therefore, if AB = BC, and AB = 14, then:

$$3x + 2 = 14$$

$$3x = 12$$

$$x = 4$$

AB = BC = 14 and by substituting 4 for x, AC = 10.

Problem structure and hypothetical workflow

1. Recall that an isosceles triangle has two congruent sides.
2. Recognize that a geometric fact can be used to transform the given problem into a problem of solving an equation. Write the equation.
3. Select and execute a strategy for solving the equation.
4. Substitute the value from step three for x to find the measure of side AC.

The student work does not enable us to distinguish between the students who completed the first stage and those who completed both. So, we have to view this as a three-stage process. A student succeeds at the first stage if he/she writes the equation. She completes the second stage if she appears to be executing a correct strategy for solving it. He/she completes the last stage if he/she finds the correct solution.

Data

Altogether, 115 students were tested. Of these:

- 22 students wrote nothing.
- 80 students (70%) completed the first stage. Of these, 41 students wrote $14 = 3x + 2$, and 39 students wrote $3x + 2 = 14$.
- 13 students wrote the wrong equation, they wrote $3x + 2 = 3x - 2$.

Of the 80 who completed the first stage:

- 78 students (68% of the 115) completed the second stage and correctly solved the equation, they wrote $x = 4$.
- Two students wrote the correct equation, but were unable to solve the equation for x.

Of the 78 who completed the second stage:

- 60 students (52% of the 115) completed all three stages and wrote AB = 14, BC = 14 and AC = 10.
- 18 students (16%) correctly solved the equation, however did not substitute the value of x to find the measure of side AC. There answer was $x = 4$. This was also the most common mistake among the students.

Analyses

My hypothesis is that students did not read or understand the directions. Independently of this test, I know this to be a problem. 16% of the students failed to realize that solving the equation is only one step in solving the problem. Although they were able to complete the steps that require logical steps, such as recognizing that an isosceles triangle

has two congruent sides and using this fact to set up an equation, and solving the equation correctly, they didn't get credit for the problem because they didn't find the perimeter of the triangle. As for the students who wrote the equation incorrectly, they were clearly unable to recall the meaning of isosceles.

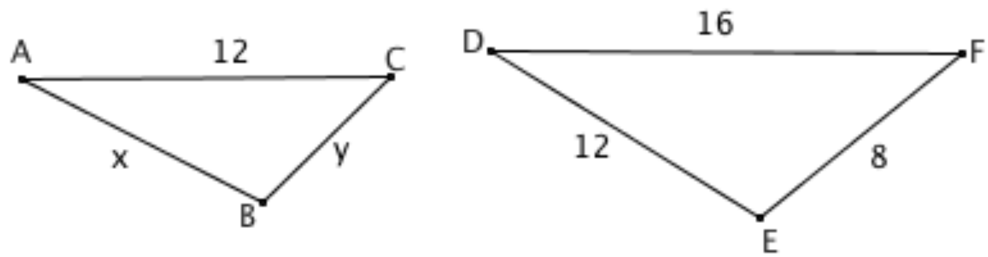
Conclusions and recommendations

Students need to be taught strategies to correctly read and understand the directions in a math problem. A helpful strategy might be: "Circle what the question is asking you to find; when finished solving, look at what you circled, did you answer that question?" Students need additional practice with problems that require multiple steps in finding the answer. Students should be given multiple opportunities in the classroom to practice such problems. Students need to understand that the language of geometry is extremely precise and they need to pay attention to details. It is important for students to recognize that solving an equation correctly doesn't always produce the right answer to a math problem it may be just one step towards finding the solution.

Analysis of Unit V Test, Problem 13

Each pair of triangles are similar. Find the perimeter of the indicated triangle.

13. Triangle ABC ______________________



Correct answer

The problem is solved by observing, that a proportion can be set up to find the missing sides of similar triangles.

$$12/x = 16/12 \qquad 12/y = 16/8$$

$$16x = 144 \qquad 16y = 96$$

$$x = 9 \qquad y = 6$$

Perimeter of ABC = 12 + 9 + 6 = 27

Problem structure and hypothetical workflow

1. Recall meaning of perimeter. Recognize that values for x and y are needed in order to compute it.
2. Recognize that the two triangles are similar and that this implies that there are proportions that give x and y.
3. Select and execute a strategy for solving the proportion.

4. Substitute the values from step 3 for the sides to find the perimeter.

The student work does not enable us to distinguish between the students who completed the first stage and who completed the first and the second stage. So we have to view this as a three-stage process. A student completed the first stage, if he/she set us a proportion to solve the problem. A student completed the second stage, if she correctly finds the values of x and y. She completes the third stage, if he/she finds the perimeter of triangle ABC.

Data

- Number of students tested: 115

  11 students wrote nothing

  20 students guessed a random number. The most common guess was 24, this answer appeared seven times.

- 84 (73%) students completed the first stage.

  29 students set up the proportions as: x/12 = 12/16 and y/12 = 8/16

  27 students set up the proportion as: 12/x = 16/12 and 12/y = 16/12

- 81 (70%) students completed the second stage.

  Three students who set up the proportion correctly made arithmetic errors.

  15 students who solved the proportion correctly didn't add the three sides to find the perimeter. Their answer was x = 9, y = 6. This was the most common wrong answer.

- 66 (57%) students completed the third stage and wrote perimeter of ABC = 27.

Analyses

Similar to the last problem, the most common error was not responding. 27% were lost at the beginning. After this, not following through to respond to what the question is asking was the next most common error. 15 students correctly set up a proportion and found the measure of the two missing sides but failed to find the perimeter. These students just did not follow through to present the solution. They might not have read the problem, or might have forgotten the goal. The good news is that 57% of the students arrived at the correct answer.

Conclusions and recommendations

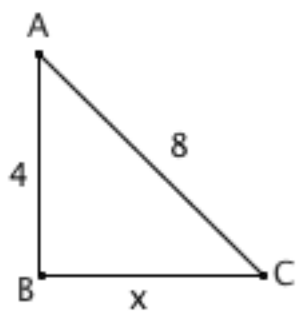
27% of the students didn't make any attempt to solve the problem. Since most of these students wrote nothing or guessed a random number, it is difficult to establish what they know about solving these types of problems. In the future, I plan on encouraging students to write down 'something' about the problem, even if they don't know how to set up the problem, they could write a few sentences about what they know about such problems. If one fourth of the class didn't make an attempt to solve a problem, this lesson may need to be re-taught. Assigning peer tutors may be one form of intervention to help these students. These students clearly need additional practice and instruction.

As for the 13% of the students who didn't follow through to arrive at the final answer, I wonder if it would have made a difference if I put "perimeter = _____" instead of "Triangle ABC ______". If they saw the word perimeter, some of the students might have realized that they hadn't found what was requested. I think sometimes students are so elated that they actually knew how to set up the problem and solve it, that they don't

pay attention to the detail of supplying the final answer. However, they need to be aware of the fact that paying attention to details is a very important habit to form in the subject of mathematics. Carelessness, not the lack of knowledge is often the reason students fail to perform well on tests.

Analysis of Unit V Test, Problem 16

Find x in each right triangle



Correct answer

The problem is solved by observing that the Pythagorean Theorem can be used to find the missing side of a right triangle given two sides.

$a^2 + b^2 = c^2$

$4^2 + x^2 = 8^2$

$x^2 = 48$

$x = 4\sqrt{3}$

Problem structure and hypothetical workflow

1. Recognize that the Pythagorean Theorem can be used to find the missing side measure of a right triangle.

2. Set up an equation using the correct values of the three sides.
3. Use and execute a strategy to solve the equation for the missing side.
4. Simplify the radical expression found in part 3.

We can view this as a four-step task. First, recognize that the Pythagorean Theorem can be used to find the missing side of a right triangle. Second, write the equation using the correct values. Third, select and execute a strategy for solving the equation and fourth, deal correctly with the radical expression. The student work does not enable us to distinguish between the students who completed the first stage and those who completed the first and the second stage. So we have to view this as a three-stage process. He/she completes the first stage if he/she writes the equation using the correct values. He/she completes the second stage, if he/she appears to be executing a correct strategy for solving it. He/she completes the last stage, if he/she simplifies the radical expression.

Data

- Altogether, 115 students were tested. Of these:
- Eight students wrote nothing and 9 students wrote a random number for the answer.
- 18 students did not correctly apply the Pythagorean Theorem. They wrote: $4^2 + 8^2 = C^2$, and their answer was $x = 4\sqrt{5}$. Eight students had this answer. This was also the most common wrong answer. An additional 4 students had $\sqrt{80}$.
- 21 students who set up the Pythagorean theorem correctly couldn't simplify radical expressions. They wrote $2\sqrt{3}$ or $16\sqrt{3}$.

- 80 (70%) students completed the first stage and wrote either, $x^2 + 4^2 = 8^{2,}$ or $4^2 + x^2 = 8^2$.
- 76 (66%) students completed the second stage and wrote: $x^2 = 48$
- 55 (48%) students completed all three stages correctly and wrote $x = 4\sqrt{3}$

Analyses

The two most common errors involved, simplifying radical expressions and understanding the Pythagorean Theorem. These results indicate that the students were not prepared for this test. Thirty-nine of the students tested didn't either know how to set up the Pythagorean Theorem correctly or simplify radical expressions. Eighteen students didn't know how to apply the knowledge that the hypotenuse of a right triangle is equal to the two shorter sides. Twenty-one students were not able to simplify radical expressions. If this many students have difficulty with these two skills, they needed a lot more practice and instruction to learn these skills before taking a test.

Conclusions and Recommendations

Since many students were unable to correctly use the Pythagorean Theorem, an activity that clarifies the relationship between the two smaller sides and the hypotenuse may have been helpful to some students. An activity such as drawing squares on the three sides of a right triangle on graph paper, and counting the squares to see that the number of squares on the two shorter sides add up to make the number of squares on the longest side, would have helped them visualize that $a^2 + b^2 = c^2$. Students also need more practice with

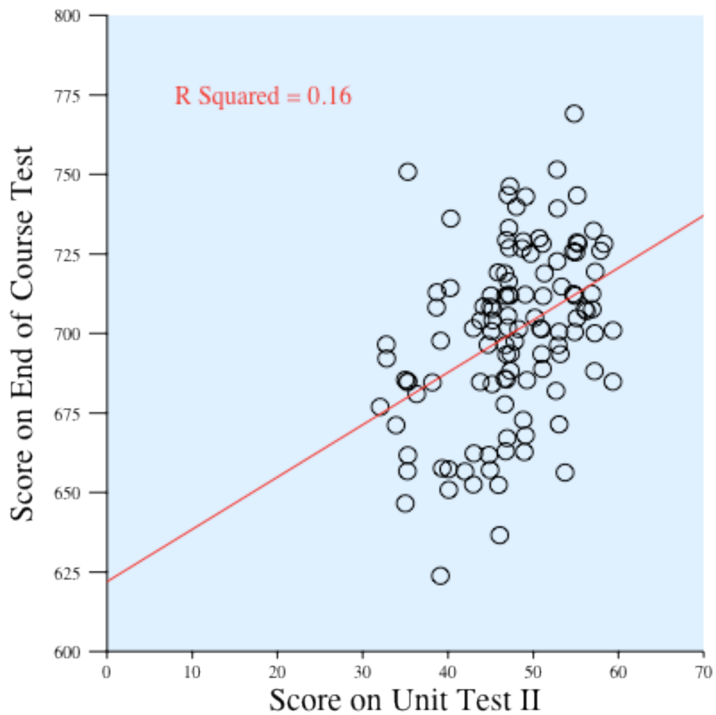
simplifying radical expressions. Since this is a skill that they will need in future math classes, more time should be devoted to practicing this skill.

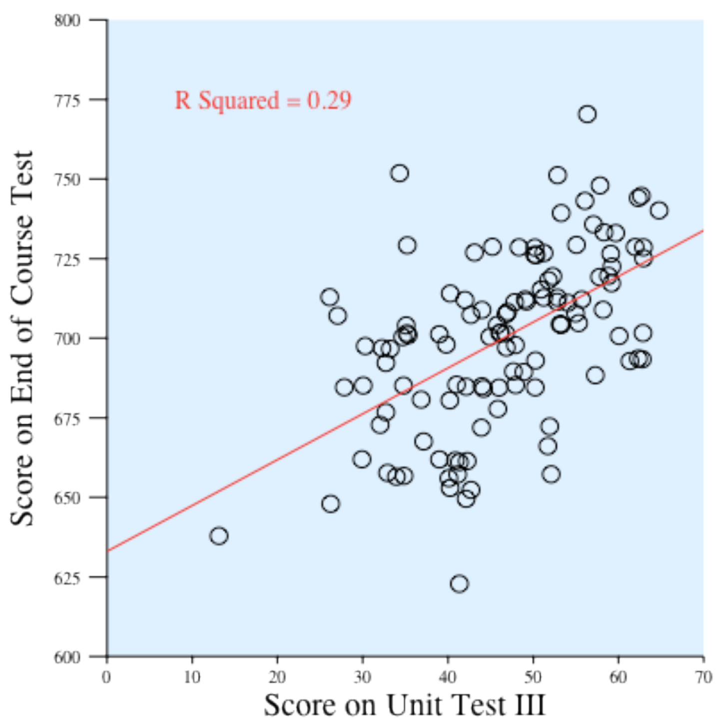
Chapter V. Comparison to End-of-Course Test

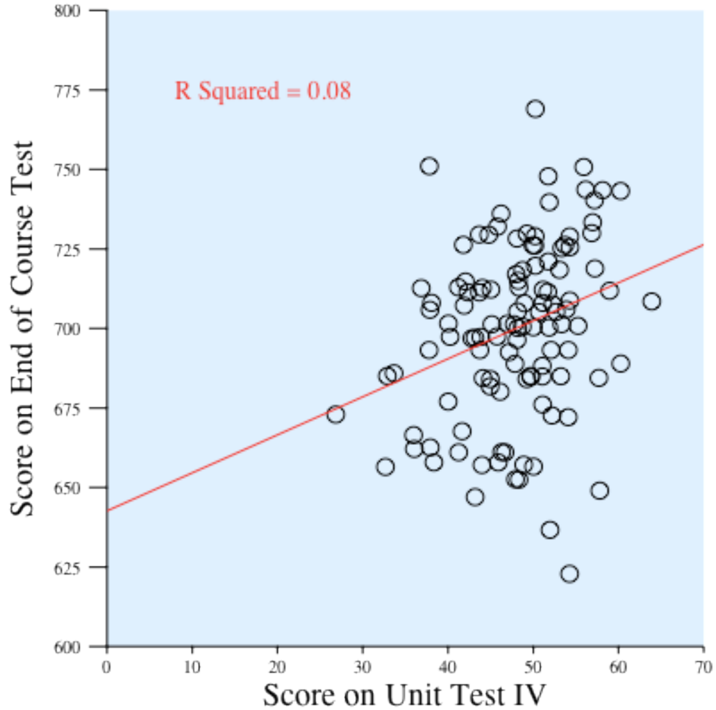
In order to estimate how informative the unit tests might be as predictors of performance on the state-administered Geometry End-of-Course Test, I consulted with statisticians regarding the use of regression models as a possible approach. In this chapter, I will present the findings.

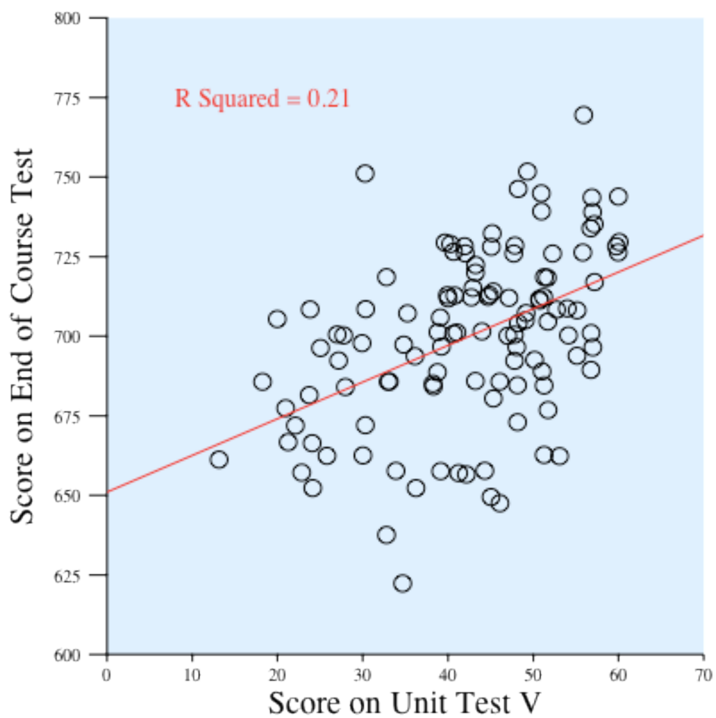
According to the "Operational Geometry End-of-Course Test Technical Report" submitted to Louisiana Department of Education by Pacific Metrics Corporation in February, 2011, the Geometry End-of-Course Test, was a test of the Grade 10 Louisiana GLEs. Only 50% of the test was directly about geometry, with the rest divided among the other strands (Number and Number Relations 12%, Algebra 6%, Measurement 12%, Data Analysis, Probability, and Discrete Math 12% and Patterns, Relations, and Functions 8%). Therefore, we should not expect the Unit Tests to be an especially good predictor. Knowledge of other mathematics will influence the scores in ways that we have no information about. By fitting linear models to the test data we have—the Unit Test scores and the End-of-Course score of 115 students—we may get a sense of how relevant what we tested was to End-of Course performance. In looking at the results, we conclude that some unit tests have very little correlation to the End-of-Course Test.

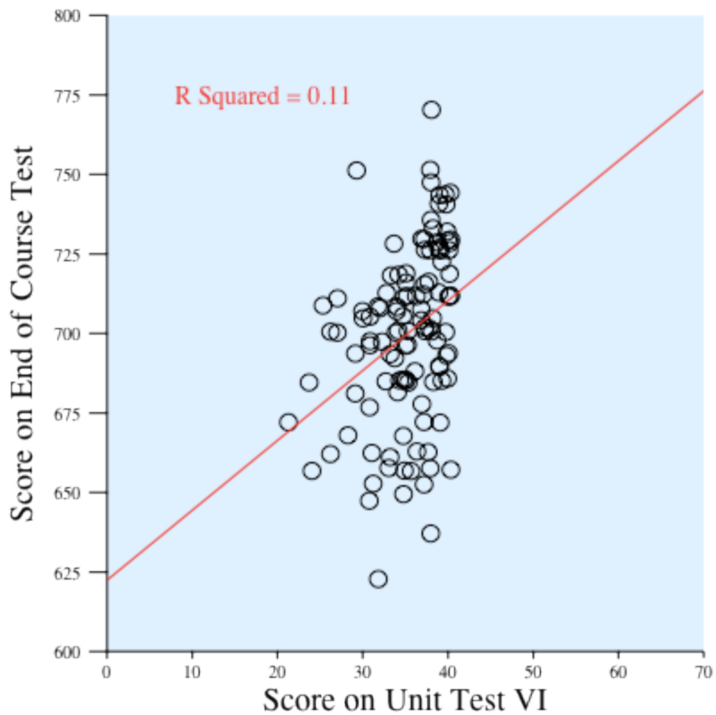
The following graphs show the End-of-Course scores of the 115 students plotted against the Unit Test scores. Each circle represents one student. Using *Mathematica*, we found the line of best fit by linear regression. The R^2 values for each data set are shone in the graphs. Conventionally, R^2 may be thought of as the portion of the variation in the End-of-Course scores that is accounted for by the variation in the unit test.


R Squared = 0.16
Score on End of Course Test
Score on Unit Test II
800
775
750
725
700
675
650
625
600
0
10
20
30
40
50
60
70


R Squared = 0.29
Score on End of Course Test
Score on Unit Test III
800
775
750
725
700
675
650
625
600
0
10
20
30
40
50
60
70


R Squared = 0.08
800
775
750
725
700
675
650
625
600
Score on End of Course Test
0
10
20
30
40
50
60
70
Score on Unit Test IV


R Squared = 0.21
800
775
750
725
700
675
650
625
600
Score on End of Course Test
0
10
20
30
40
50
60
70
Score on Unit Test V



We also fit a linear model to the entire data set, to obtain coefficients for all five tests simultaneously. This showed that tests IV and VI contained essentially no information about the End-of-Course score beyond what was in tests II, III and V, and that V was not significant at the level commonly asked for. For the full model, R^2 was about .34. Tests II and III were significant ($p < 10^{-5}$). The model with tests II and III only as predictor variables had an R^2 of about .32. As shown above the model with Test III as the only predictor variable had an R^2 of about .29. Unit tests II and III show some correlation to the End-of –Course Test in Geometry. From the five unit tests administered, unit III test shows the most correlation to the End-of-Course Test.

Chapter VI. Conclusions

This thesis has attempted to develop and analyze assessments of student performance in the Louisiana Comprehensive Curriculum for Geometry. I composed eight unit tests for this curriculum, all closely aligned with the GLEs and activities described in the LCC. I administered the tests to the 115 geometry students I teach at my school. I analyzed the test results to determine the quality of my test questions to make inferences about student learning.

Scores on the unit tests that I developed tended to be higher than scores on the tests previously used (though no students were tested on both).

The test questions that I analyzed varied greatly in terms of difficulty. On some questions, as few as 29% of the students had the correct answer while on others 72% of the students had the correct answer. The lowest scores were on questions that had operations on fractions or involved simplifying radicals. On average for the questions I analyzed, more than 50% of the students had the correct answers.

The test results suggest that very basic skills are problematic for a small (but troubling) number of students but that pre-requisites from more recent grades are problematic for larger numbers.

Many students fail to read or understand the directions for the test problems. In numerous answers, students performed correct operations and produced the needed information but did not follow through to state the requested answer. Though they had the skills necessary to solve the problem, they did not follow through to provide what was

called for. The same students often performed poorly on problems requiring multiple steps.

There were also a surprisingly large number of students who did not respond at all to certain test questions, it is not clear whether they chose not to respond or they didn't know how to set up the problem.

In the following table, I pull together the leading themes in the item analyses. After the table, I state overall conclusions.

Item	Summary Conclusions
III.22	Students did not include justifications. Basic skills were problematic for only about 10% of students.
III.25	Most students missed the easy solution method. There was evidence of superficial understanding of the equation of a line. 30% had problems with basic skills.
III.29	This problem did not provided unambiguous information about student knowledge.
III.23	Students did not provide explanations. Student work raised concern about students' ability to make connections and deal with multi-step problems.
III.18	Student answers suggest that students may be influenced by details of instruction that the teacher is not aware of. 43% of the students got a perfect score, and there was a lot of variety in the other answers. This problem was informative about the distribution of the knowledge in class.
IV.11	The analysis left us wondering whether the students had read the directions.
V.13	The analysis caused us to wonder whether students were reading the problem. Student answers did not provide a lot of information, raising concerns about item design.
V.16	More than 1/3 of the students were lacking skills from 8th grade.

Making the tests prior to teaching the skills was beneficial in planning the lessons. I was able to spend more time on the skills students had difficulty during instruction as well as include similar problems in their homework assignments to reinforce these skills.

Seeing the types of mistakes students make in these tests was very useful for future lessons. I will be able to address the types of common errors and possibly find a different method to teach these skills. I saw specific examples that show how background knowledge can influence performance on new skills. Students may be able to learn a new skill, but may be unable to use it if they cannot recall background. They may fail to complete problems for this reason.

The results of the unit tests that I developed and administered show major differences in student knowledge. While some students were fluent in certain math skills, such as simplifying radicals, and operations with fractions, others were not. I believe the students should be given a pretest at the beginning of the year that would provide a general idea of their ability level. This information then needs to be utilized when grouping these students into classes. In the six geometry classes I taught last year, I had students with various background knowledge. Had the students been grouped according to skill-levels as revealed by a well-designed pre-test, it would have been far easier for me to address their individual needs.

In geometry, many problems require reasoning skills. More students may retain the skills they learn, if they were given the opportunity to communicate and make connections to the skills they have learned previously. Seeing how students communicate the skills they have learned can also be an important tool for the teacher.

This will allow the teacher to provide meaningful feedback as well as clear any misconceptions.

Test designers need to provide opportunities for students to communicate their thought processes in solving the problem, and this may be difficult since students don't automatically do this. I now think it would have been beneficial if I had included more problems with instructions such as "Explain why?" or "Write an explanation for each step." In some of the problems that I composed, it would have been possible for mere guesses to get credit. Problems requiring explanations are not central in the present state-designed end-of-course test, but are likely to become much more prevalent as the Common Core State Standards are implemented.

References

Archibald, R. C. (1950). The First Translation of Euclid's Elements into English and its Source. *The American Mathematical Monthly, 57*(7), 443-452.

Barber, H. C., & Hendrix, G. (1937). Plane Geometry & Its Reasoning. New York, NY: Harcourt, Brace and Company.

Bock, R. D., Mislevy, R., & Woodson, C. (1982). The Next Stage in Educational Assessment. *Educational Researcher, 11*(3), 4-16.

Boyer, C. B. (1959). Descartes and the Geometrization of Algebra. The American Mathematical Monthly, 66(5), 390-393.

Bush, W. S., & Greer, A. S. (Eds.). (2005). Mathematical Assessment: A Practical Handbook. United States: NCTM.

Euclid's Elements, from http://en.wikipedia.org/wiki/Euclid%27s_Elements

Boyd, C. J. (2005). *Glencoe Geometry Textbook*. Columbus, OH: Glencoe McGraw-Hill.

Herrera, T. A., & Owens, D. T. (2001). The "New New Math"?: Two Reform Movements in Mathematics Education. *Theory into Practice, 40*(2), 84-92.

Kokomoor, F. W. (1928). The Teaching of Elementary Geometry in the Seventeenth Century. *Isis, 11*(1), 85-110.

Johnson, G. P. (1965). [untitled]. *The American Mathematical Monthly, 72*(10), 1152.

Joyce, D. E., (1998). Euclid's Elements. Retrieved from http://aleph0.clarku.edu/~djoyce/java/elements/elements.html

La Doe. (2010b). Louisiana Comprehensive Curriculum. Louisiana Department of Education, from http://www.doe.state.la.us/topics/comprehensive_curriculum.html

Moise, E. E. (1973). Elementary Geometry From an Advanced Standpoint. New York, NY: Addison-Wesley Publishing Company.

Pacific Metrics corporation. (2011). Operational Geometry End-of-Course Test Technical Report. Submitted to Louisiana Department of Education.

Pellegrino, J. W. Chudowsky, N., & Glaser, R. (Eds.). (1997). Knowing What Students Know: The Science and Design of Educational Assessment. Washington, DC: National Academy Press.

Seidlin, J. (1950). High Standards: Sacred and Profane. *Mathematics Magazine, 23*(4), 189-192.

Seymour, F. E., & Smith, P. J. (1949). Plane Geometry. New York, NY: The MacMillian Company.

Sinclair, N. (2008). The History of the Geometry Curriculum in the United States. Charlotte, NC: Information Age Publishing, Inc.

Stamper, A. W. (1909). Educational Aims in the Teaching of Elementary Geometry, Historically Considered. *Science, 30*(777), 701-703.

Wentworth, G. A. (1901). Plane and Solid Geometry. Boston, MA: The Athenaeum Press.

Western and Northern Canadian Protocol for Collaboration. (2006). Rethinking Classroom Assessment With Purpose in Mind (ISBN 0-7711-3499-1). Winnipeg, Manitoba, from http://www.edu.gov.mb.ca/ks4/assess/index.html

Appendix: Unit Tests

Unit 1 Test, LCC: Patterns and Reasoning.

Name: ______________________________ Date: ______________ Period: _______

For each of the following, write the next two terms and describe the pattern.

1. -2, 0, 2, 4, 6, …

2. 1, 3, 9, 27, 81, …

3. 1/2, 1/4, 1/8, 1/16, 1/32, …

Determine whether the following patterns are linear or non-linear.

4. 1, 3, 5, 7, 9, … ____________________

5. 1, 4, 9, 25, 36, … ____________________

6. $2n + 1$ ____________________

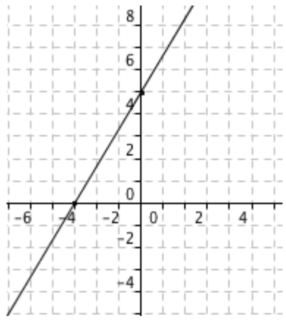
7. $n^2 - 2$ ____________________

8.

x	1	2	3	4	5
y	30	25	20	15	10

____

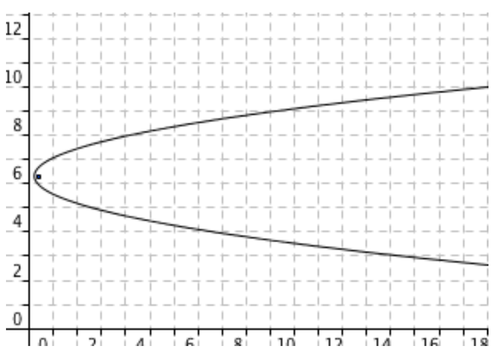
9. Is this graph linear or non-linear.



____

10. If the above graph is linear, write an equation in slope-intercept form. If it is not linear, explain why.

11. Is this graph linear or non-linear.



____

12. If the above graph is linear, write an equation in slope-intercept form. If it is not linear, explain why.

Write an equation for the n^{th} term of each linear sequence:

13. 1^{st} term = 5; 2^{nd} term = 10; etc.______________

14. 1^{st} term = 10; 2^{nd} term = 17; etc.______________

15. 0^{th} term = 4; 1^{st} term = -2; etc.______________

16. a) Draw a line to fit the data.
    b) Draw a circle around the *y*-intercept of your line.
    c) Draw two points on the line you made, and draw a right triangle with the segment between those points as its hypotenuse.
    d) What is the rise between your two points? (Label the triangle; answer with a number.)

    Answer to d):____

    e) What is the run between your two points? (Label the triangle; answer with a number.)

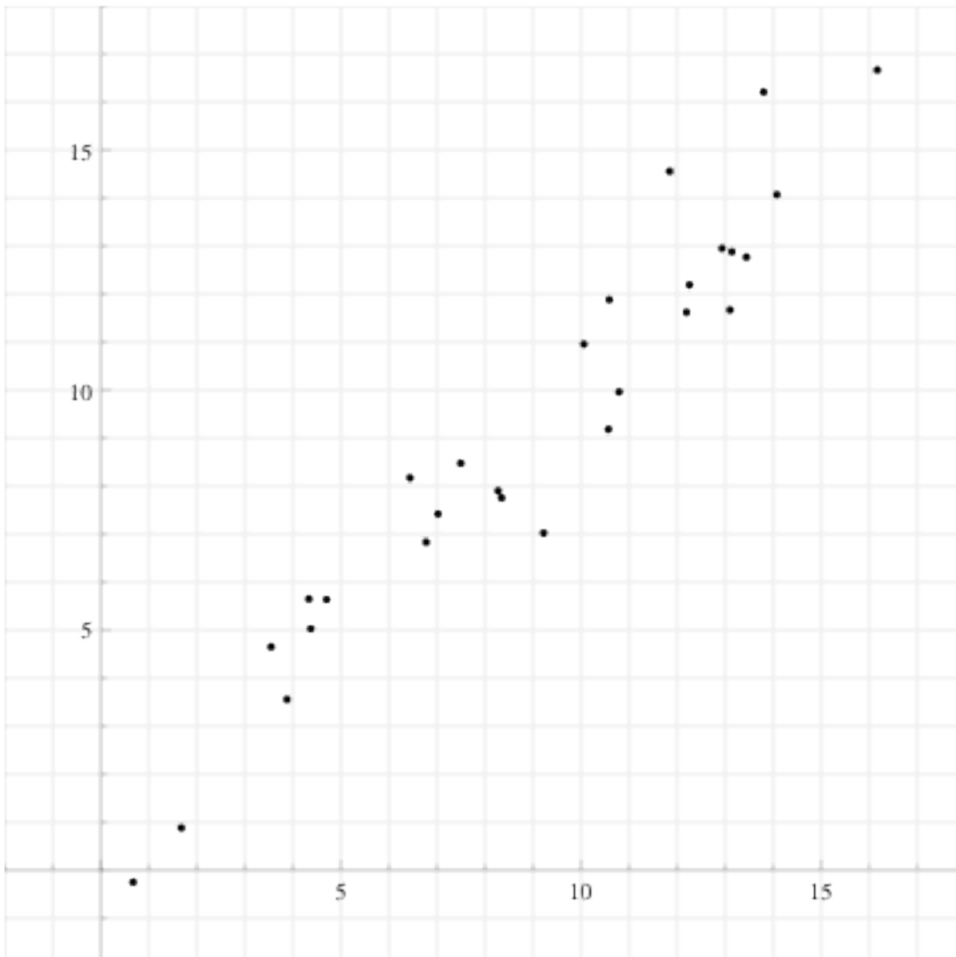
    Answer to e):____

    f) What is the "rise divided by the run" of your line? (Answer with a number.)

    Answer to f):____

    g) Write an equation in slope-intercept form for the line that you drew.

Answer to g):____



17. a) Draw a line to fit the data.
    b) Draw a circle around the y-intercept of your line.
    c) Draw two points on the line you made, and draw a right triangle with the segment between those points as its hypotenuse.
    d) What is the rise between your two points? (Label the triangle; answer with a number.)

Answer to d):____

e) What is the run between your two points? (Label the triangle; answer with a number.)

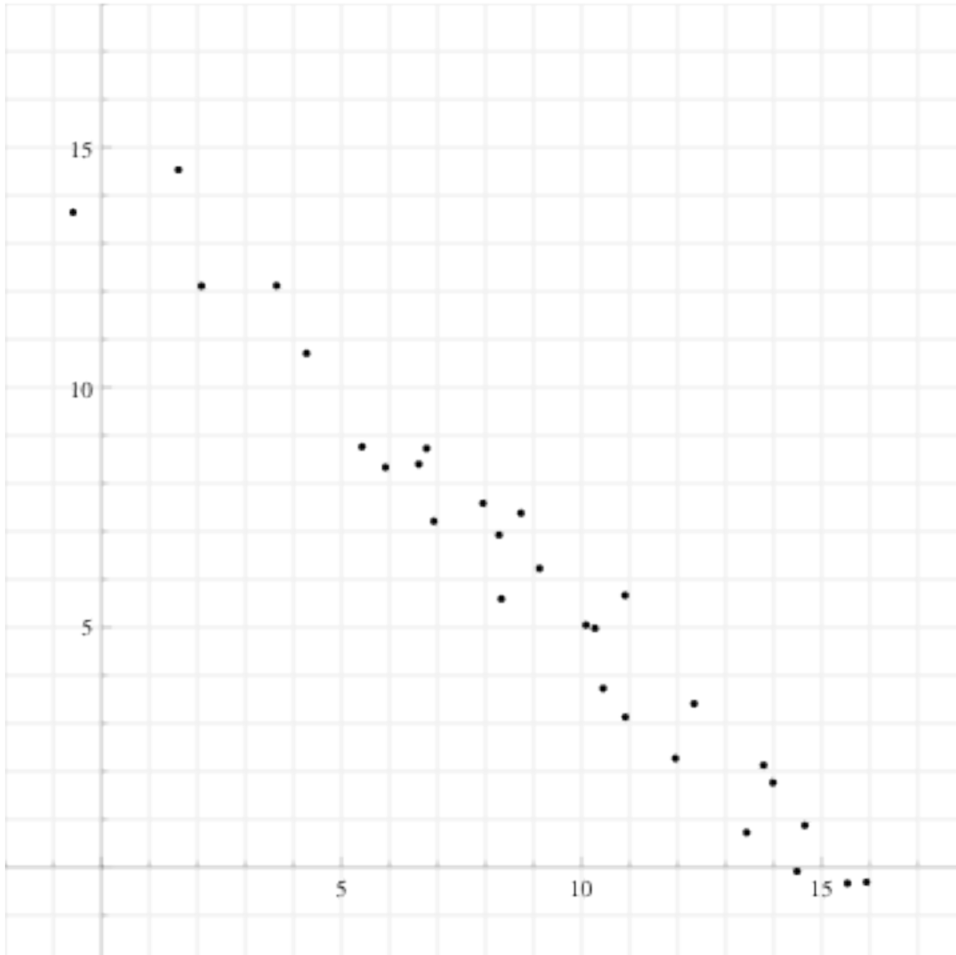
Answer to e):____

f) What is the "rise divided by the run" of your line? (Answer with a number.)

Answer to f):____

g) Write an equation in slope-intercept form for the line that you drew.

Answer to g):____



You may choose three pizza topping out of eight possible toppings: (A) Anchovies, (B) Bacon, (G) Green Pepper, (H) Ham, (M) Mushroom, (O) Onion, (P) Pepperomi, (S) Sausage. Here are some possibilities:

- ABG (you order anchovies, bacon and green pepper),
- ABH (you order anchovies, bacon and ham).
- Other possibilities are: ABM, ABO, …BGH, BGM, …,OPS.

18. How many possibilities are there if you may use a topping more than once, as for example SSS (triple sausage)?

19. How many possibilities are there if no topping can be used more that once? (Things like AAG or SSS are not allowed).

20. Kelsey has 3 blouses, 4 pairs of pants, and 6 pairs of shoes. How many different outfits can she choose?

Unit II Test, LCC: Reasoning and Proof

Name: ______________________________ Date: ______________________ Period: _________

Logical Compound Statements

True or false?

1. T F 1 + 1 = 2 AND 1 + 1 = 3

2. T F 1 + 1 = 2 OR 1 + 1 = 3

3. T F IF 1 + 1 = 3, THEN Brittney Spears is the President of the United States.

4. T F IF Jean lives in Baton Rouge, THEN Jean lives in Louisiana.

5. T F IF Jean does not live in Baton Rouge, THEN Jean does not live in Louisiana.

6. T F IF Jean does not live in Louisiana, THEN Jean does not live in Baton Rouge.

7. Suppose the following is true:

If A, then B.

Which of the following must be true.

If B, then A.

If not B, then not A.

If not A, then not B.

8. Suppose the following is true:

If Allen is late, then the bus is late.

Which of the following must be true.

If the bus is late, then Allen is late.

If the bus is not late, then Allen is not late.

If Allen is not late, then the bus is not late.

Which of these syllogisms is valid? Circle "YES" or "NO".

9. All Mary's pets are furry.

Wolfman is furry.

THEREFORE, Wolfman is one of Mary's pets. YES NO

10 All of Mary's pets like milk.

Goldfish is one of Mary's pets.

THEREFORE, Goldfish likes milk. YES NO

11. Some cats have no tails.

All cats are mammals.

THEREFORE, some mammals have no tails. YES NO

True or False?

1.	**T F**	Every line contains at least two different points.
2.	**T F**	If a line contains a point in segment A and B, then A and B are on that line.
3.	**T F**	If two points lie in a plane, then the entire line containing those point lies that plane.
4.	**T F**	If two different lines intersect, then their intersection is exactly one point.
5.	**T F**	If lines L and M have two different points in common, then L and M are the same Line.
6.	**T F**	Given any three points that do not all lie on a line, there is exactly one plane that Contains all three of them.
7.	**T F**	An angle is two rays with a common endpoint.
8.	**T F**	Some angles are straight angles.
9.	**T F**	If two angles are adjacent, they are complementary.
10.	**T F**	If two angles have the same vertex, they are said to be adjacent.
11.	**T F**	Two different lines that are perpendicular form four right angles.
12.	**T F**	Suppose AB is a line and C is a point between A and B. Suppose D is a point not

		on line AB. Then angle ACD and angle DCB form a linear pair.
13.	**T** **F**	If a ray is drawn from the vertex of a right angle and through a point in the interior of the angle, two complementary angles are formed.
14.	**T F**	Two perpendicular lines do not have any points in common.
15.	**T F**	If a line L has one point on one of a pair of parallel lines, and another point on the other line in the parallel pair, then L is called a Transylvanian (like Dracula).
16.	**T F**	If a line L has one point on one of a pair of parallel lines, and another point on the other line in the parallel pair, then L is called a transversal.
17.	**T F**	Supplementary angles form a linear pair.
18.	**T F**	An angle bisector divides and angle into two congruent angles.
19.	**T F**	Complementary angles are always equal.
20.	**T F**	Supplementary angles are always complementary.

Unit III Test: Parallel and Perpendicular Relationships

Name: ______________________________________ Date: ______________________

Each problem is worth 2 points.

Part A. Background knowledge.

1. Draw and label a picture of a pair of vertical angles. What do you know about vertical angles?

2. Draw and label a picture of a pair of supplementary angles. What do you know about supplementary angles?

__Part B. Parallels and Transversals: Vocabulary and Basic Facts.__

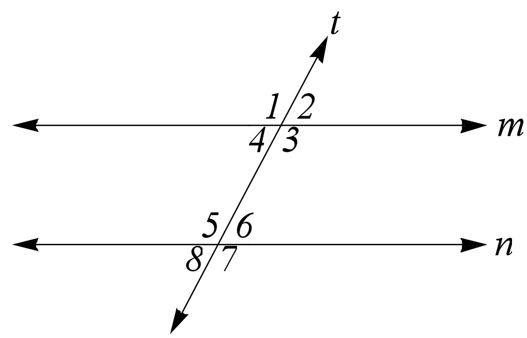


Figure 1.

In Figure 1, line *m* is parallel to line *n*. Problems 3 through 6 refer to Figure 1.

3. Two of the following pairs are supplementary angles. Put x's in the boxes next to them.

   $\angle 1$ and $\angle 7$ $\angle 1$ and $\angle 8$

   $\angle 2$ and $\angle 6$ $\angle 2$ and $\angle 5$

4. Two of the following pairs are alternate interior angles. Put x's in the boxes next to them.

   $\angle 4$ and $\angle 6$ $\angle 3$ and $\angle 6$

   $\angle 4$ and $\angle 5$ $\angle 3$ and $\angle 5$

5. Circle the angles that are congruent to $\angle 1$.

$\angle 1$ $\angle 2$ $\angle 3$ $\angle 4$ $\angle 5$ $\angle 6$ $\angle 7$ $\angle 8$

6. Circle the angles that are congruent to $\angle 2$.

$\angle 1$ $\angle 2$ $\angle 3$ $\angle 4$ $\angle 5$ $\angle 6$ $\angle 7$ $\angle 8$

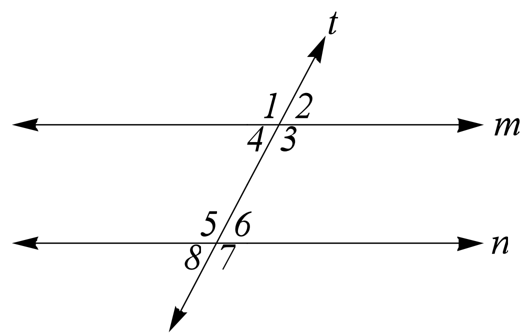


Figure 2.

7. In Figure 2 (above), consider the pair, $\angle 1$ and $\angle 7$. What term applies to this pair?
   a. These angles are supplementary.
   b. These angles are alternate exterior.
   c. These angles are corresponding exterior angles.

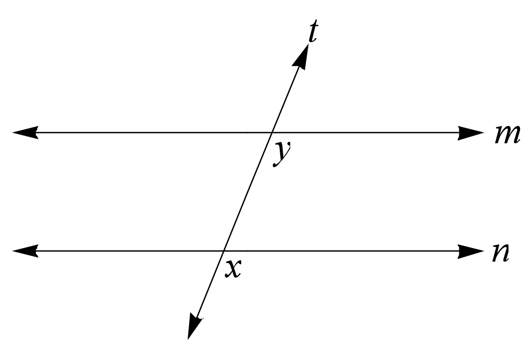


Figure 3.

8. In Figure 3 (above), angles x and y are:
   a) corresponding angles
   b) alternate interior angles
   c) alternate exterior angles

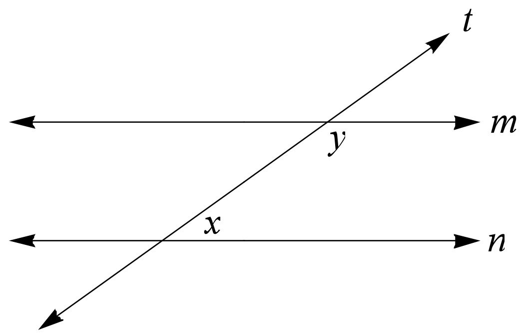


Figure 4.

9. In Figure 4 (above), angles *x* and *y* are:
    a) corresponding angles
    b) consecutive interior angles
    c) alternate interior angles

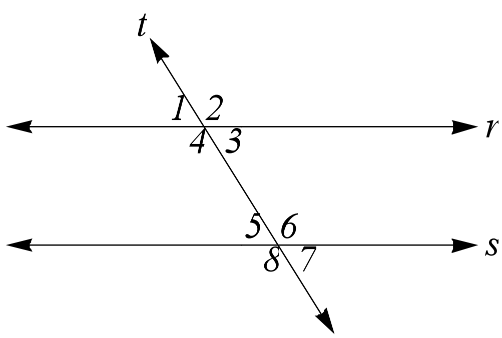


Figure 5.

In Figure 5, lines *r* and *s* are parallel. Decide whether each statement is true or false:

10. T F $\angle 2$ and $\angle 7$ are supplementary

11. T F $\angle 1$ is congruent to $\angle 5$

12. T F $\angle 2$ is congruent to $\angle 5$

Circle true or false for each statement:

13. T F If two parallel lines are cut by a transversal, then alternate exterior angles are congruent.

14. T F If two parallel lines are cut by a transversal, then corresponding angles are supplementary.

15. T F If two parallel lines are cut by a transversal, then consecutive interior angles are supplementary.

16. T F In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.

Part C. Parallels and Transversals: Applications.

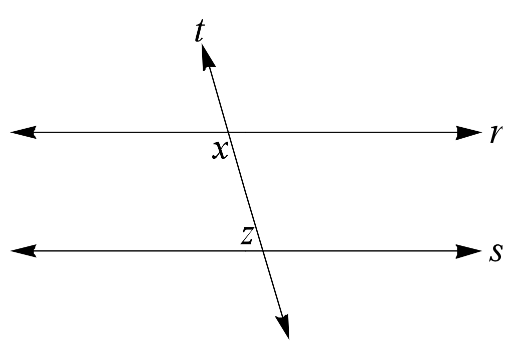


Figure 6.

17. In Figure 6, lines r and s are parallel, and angle x is 100 degrees. What is the measure of angle z?

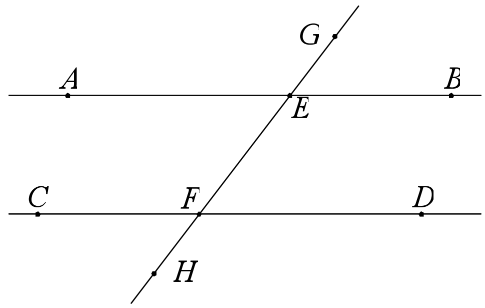


Figure 7.

18. In Figure 7 (above), line AB is parallel to line DC. Using point notation for angles, list all the angles that are congruent to < AEG.

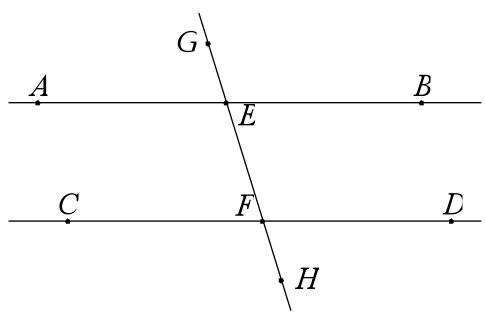


Figure 8.

19. In Figure 8 (above), angle ÐAEF is supplementary to ÐEFC. What can you say about lines AB and CD?

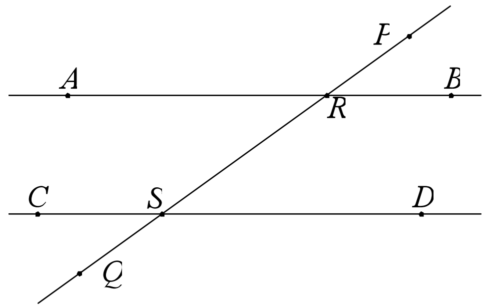


Figure 9.

20. This problem refers to Figure 9 (above). Mary thinks lines AB and CD are parallel, but she is not sure. What angles could she measure to check her guess, and what should she do?

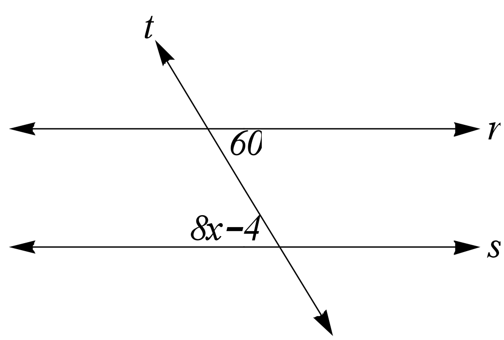


Figure 10.

21. In Figure 10 (above), line r is parallel to line s. Two angles are labeled with their degree measures. Find the value of x.

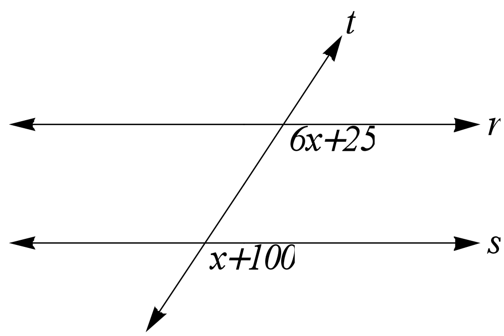


Figure 8.

22. In Figure 8 (above), line r is parallel to line s. Two angles are labeled with their degree measure. Find the value of x.

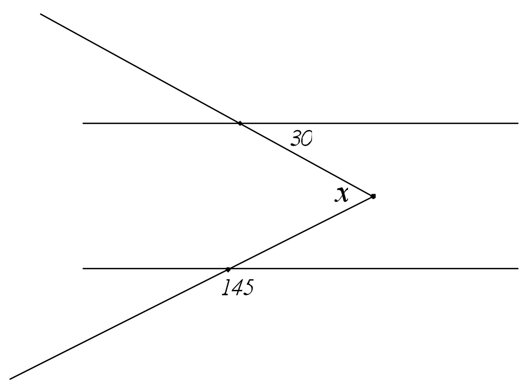


Figure 9.

23. In Figure 9 (above), the two horizontal lines are parallel. The angles are marked with their degree measure. Find x.

Part D. Parallels and Perpendicular Lines in Coordinate Geometry: Background Knowledge

24. What is the slope of the line through (2, 7) and (-1, 5)?

25. A line passes through (3, 11) and has slope 4/5. Write the equation of this line.

26. A line passes through (6, 5) and (x, 7). Its slope is 6. What is x?

Part E. Parallels and Perpendicular Lines in Coordinate Geometry: Vocabulary and Basic Fact

27. Suppose l_1 and l_2 are two lines with slopes m_1 and m_2, respectively.

a) l_1 and l_2 are parallel if and only if _____________________.

b) l_1 and l_2 are perpendicular if and only if _____________________.

Determine whether lines AB and MN are parallel, perpendicular, or neither.

28. A is the point at (0, 3), B is at (5, -7), M is at (-6, 7), N is at (-2, -1).

29. A is the point at (-1, 3), B is at (0, 5), M is at (2, 1), N is at (6, -1).

30. A is the point at (-1, 3), B is at (4, 4), M is at (3, 1), N is at (-2, 2).

Unit IV Test, LCC: Congruent triangles and quadrilaterals

Name: ______________________________Date: _________________ Period: _____

Each problem is worth 2 points.

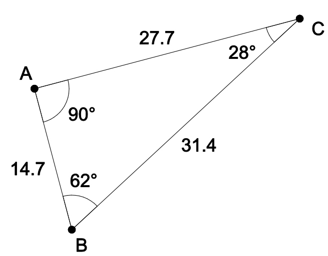
PART A. Vocabulary.

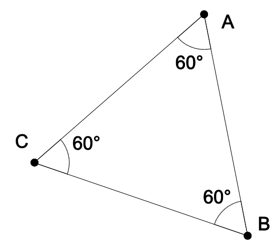
1. If all three sides of a triangle are congruent, then the triangle is ______________________________.

2. If at least two sides of a triangle are congruent, the triangle is _______________________________.

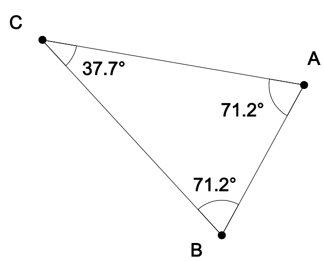
3. A triangle with all sides of different length is called _____________________________.

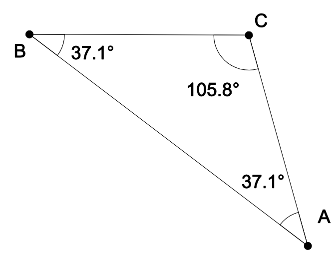
4. Classify the triangles (acute, obtuse, equiangular, right) by writing the appropriate descriptor under each.





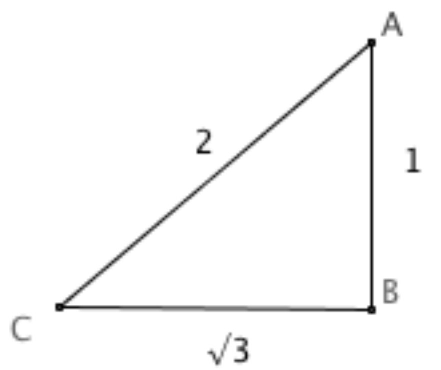
________________________ ________________________

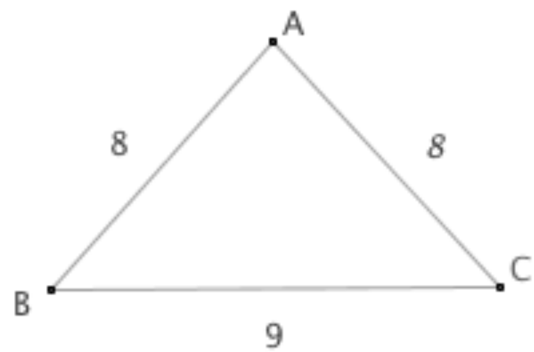




________________________ ________________________

5. Which properties (equilateral, isosceles, right or scalene) do the following triangles have? Write the appropriate descriptor or descriptors under each.





_________________________ _________________________

6. The sum of the measures of the angles of any triangle is ________________________

7. The sum of the measures of the acute angles of a right triangle is

____

8. If the measures of two angles of a triangle are 62 and 93, then the measure of the third angle is _______________

9. If triangle RST is congruent to triangle UWV, complete each pair of congruent parts.

a) < R = ________ b) < S = ________ c) < T = ________

d) RT = ________ e) ________ = UW f) ________ = WV

10. Find the measure of each side of equilateral triangle RST, if RS = 2x + 2, ST = 3x, and TR = 5x - 4.

11. Find the measure of each side of an isosceles triangle ABC with AB=BC, if AB = 14, BC = 3x+2 and AC = 3x-2.

True or False

12. _______The opposite sides of a parallelogram are congruent.

13. _______The opposite angles of a parallelogram are congruent.

14. _______If a parallelogram has one right angle, then it has four right angles.

15. _______The diagonals of a parallelogram are congruent.

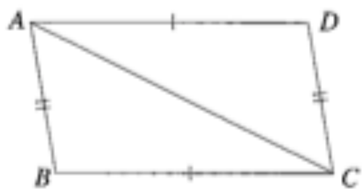
16. _______The diagonals of a rectangle bisect each other.

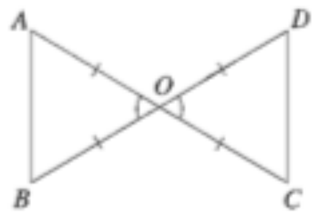
17. _______Consecutive angles of a parallelogram are complementary.

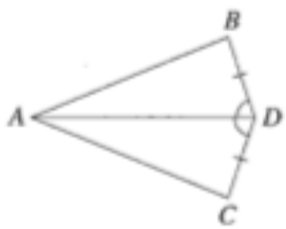
18. _______A diagonal of a parallelogram divides it into two congruent right triangles.

19. _______The diagonals of a parallelogram are congruent.

20. Identify the congruent relationships and the rule (SAS, ASA, SSS, AAS) that tells them

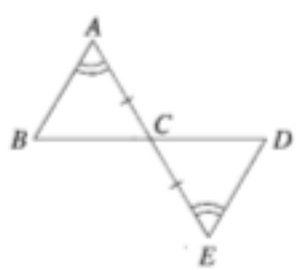
a) 

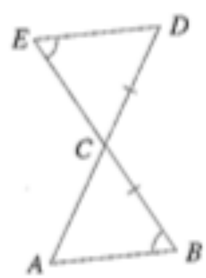
b) 

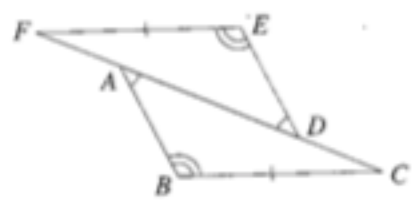
c) 

a)_______________ b)_______________

c)_______________

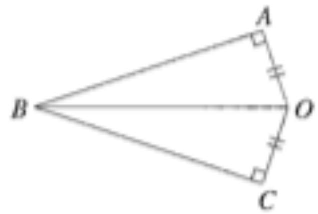
d) 

e) 

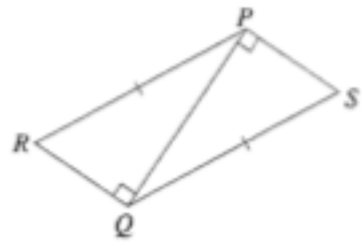
f) 

d)_______________ f)_______________

e)_______________



g)



h)

g)________________ h) ________________

Unit V Test, LCC: Similarity and Trigonometry.

Name: ________________________ Date: __________________ Period: ________

Fill in the blanks using the following words: **scale factor, quantities, proportion**.

1. A ratio is a comparison of two ____________________

2. The ratio of two corresponding quantities is called a ___________________

3. An equation stating that two ratios are equal is a _____________________

4. There are 175 girls in the sophomore class of 315 students. Find the ratio of the number of girls to the total number of students.

5. The length of a rectangle is 10 inches and its width is 5 inches. Find the ratio of the length to the width.

6. The length of a model train is 12 inches. It is a scale model of a train that is 36 feet long. Find the scale factor. (Caution! Pay attention to the units.)

7. The ratio of the sides of a triangle is 2:5:7 and its perimeter is 140 feet. Find the measures of the sides.

Determine whether the this is an example of congruent figures, or figures that are only similar. Write "similarity" or "congruence"

8. Two triangles that have exactly the same shape, but not the same size. ______________

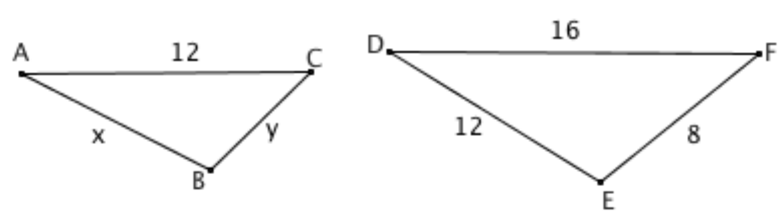
9. Two sides and the included angle of one triangle are congruent to two sides and the included angle of the other triangle. _________________

10. The measures of all three pairs of corresponding sides of two triangles are proportional. ____________________

11. The three angles of one triangle are congruent to the three angles of the other triangle. ______________________

12. The three sides of one triangle are congruent to the three side of the other triangle. ______________________

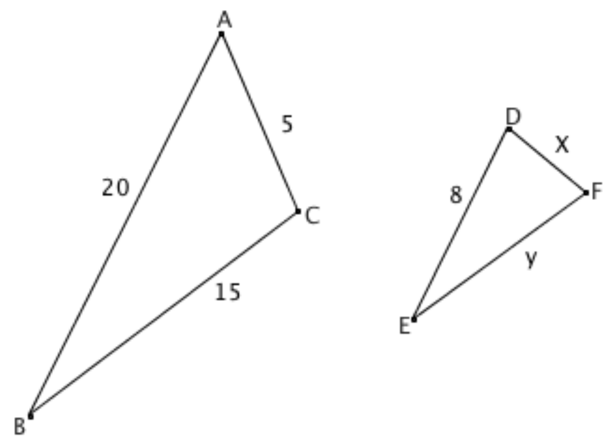
13. The triangles in each pair are similar. Find the perimeters of the triangles.



The perimeter of triangle DEF is ______________________

The perimeter of triangle ABC is ______________________

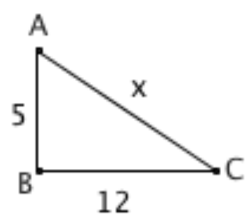
14.



The perimeter of triangle ABC is ____________________

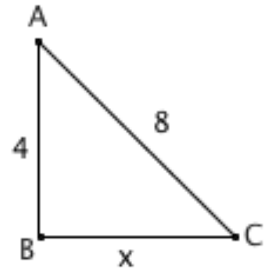
The perimeter of triangle DEF is ____________________

15.



In right triangle ABC, the side AC is marked with x. The lengths of the other sides are shown in the picture. What is the value of x ? __________

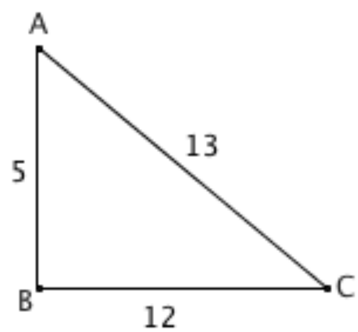
16.



In right triangle ABC, the side BC is marked with *x*. The lengths of the other sides are shown in the picture. What is the value of x ? __________

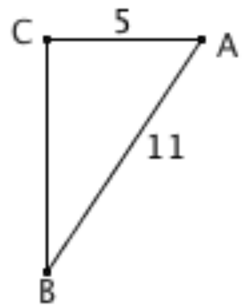
In 17 and 18, find the indicated trigonometric ratio as a fraction *and* as a decimal. Round to the nearest thousandth. All the triangles have a right angle.

17.



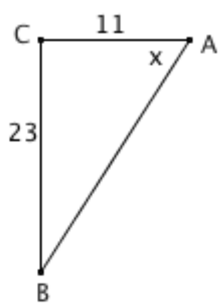
Sin A = ____________ Cos A = ____________ Tan A = ____________

18.



Sin A = _______________ Cos A = _______________ Tan A = _______________

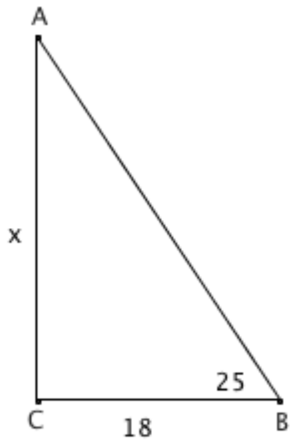
19.



x = _______________.

Using the inverse trig functions on your calculator, find the measure of the angle in the triangle above marked x. Round your answer to the nearest tenth.

20.



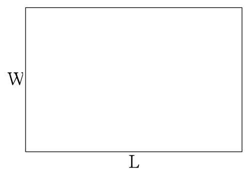
$x =$ ______________.

Using the appropriate trig functions on your calculator, find the length of the side marked x. Round your answer to the nearest tenth.

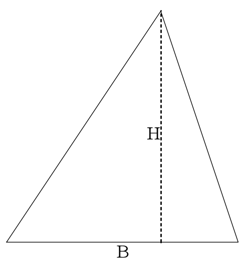
Unit VI Test, LCC: Area, Polyhedra, Surface Area, and Volume.

Name: _____________________________ **Date:** _______________ **Period:** _______

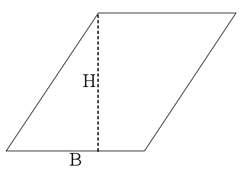
1. Write the formula for the area, using the letters in the figures

a. 

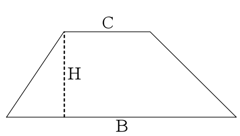
The area of the rectangle is _______________.

b. 

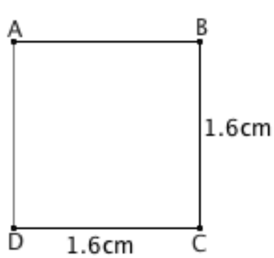
The area of the triangle is _______________.

c. 

The area of the parallelogram is _____________.

d. 

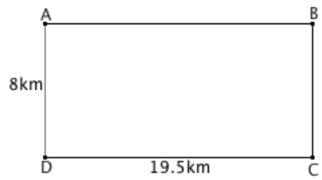
The area of the trapezoid is _____________.



2. Find the perimeter and area of the square. Round to the nearest tenth if necessary. Include units.

a. Perimeter = _______________

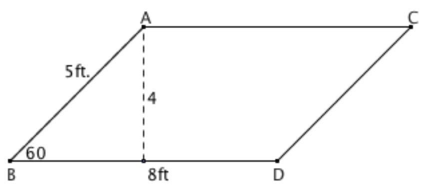
b. Area = ________________



3. Find the perimeter and area of the rectangle. Round to the nearest tenth if necessary. Include units.

a. Perimeter = _______________

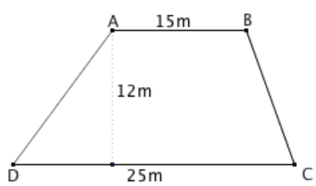
b. Area = __________________________



4. Find the perimeter and area of the parallelogram. Round to the nearest tenth if necessary. Include units.

a. Perimeter = _______________________

b. Area = __________________________

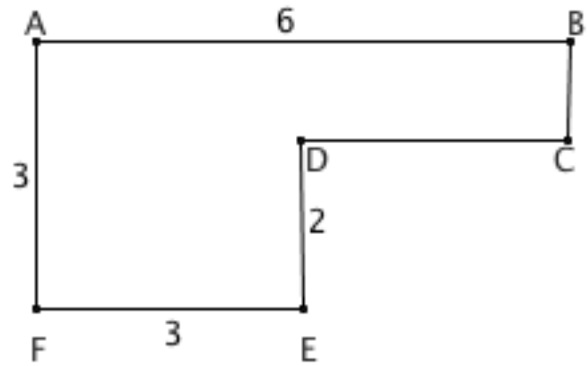


5. Find the perimeter and area of the trapezoid. Round to the nearest tenth if necessary. Include units.

a. Perimeter = _______________________

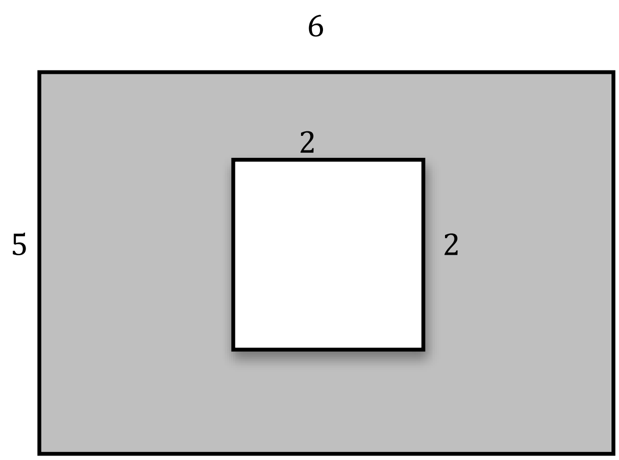
b. Area = __________________________

6. Find the area of the figure.

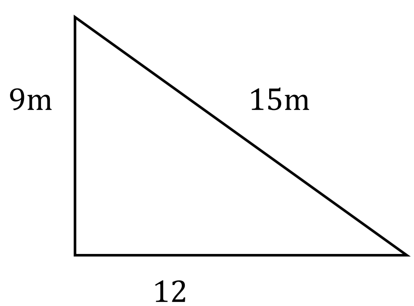


Area: _______________

7. Find the area of the shaded region.

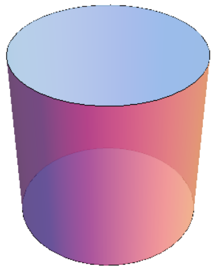
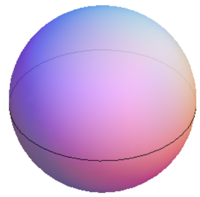
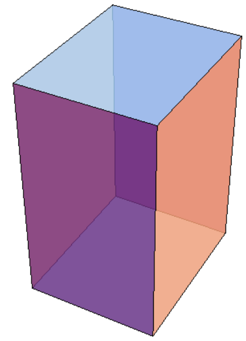


8. Find the area and perimeter of the right triangle below.



Area = _______________ Perimeter= _______________

9. Identify each solid:

a. ________________ b. ________________ c. ________________

True or False

10. _____ A base of a prism is a face of the prism.

11. _____ If a base of a prism has n vertices, then the prism has n lateral edges.

12. _____ In a right prism, the lateral edges are also altitudes.

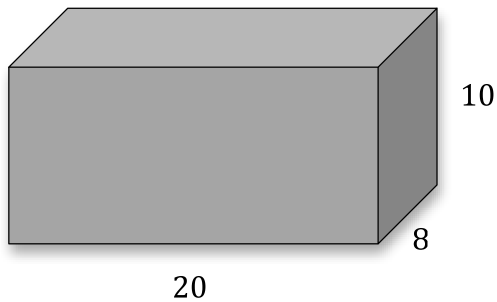
13. _____ Any two lateral edges of a prism are perpendicular to one another.

14. _____ In a rectangular prism, any pair of opposite faces can be called the bases.

15. _____ The bases of a cylinder are disks.

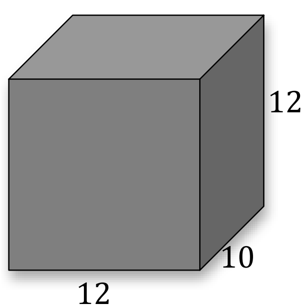
Find the total surface area of each prism.

16.



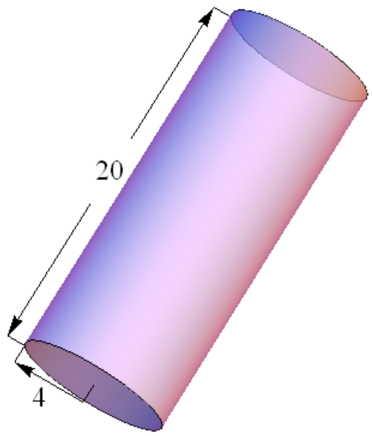
Total surface area = ________

17.



Total surface area = __________

18. Find the lateral area (= surface area NOT including bases) of of the cylinder below.

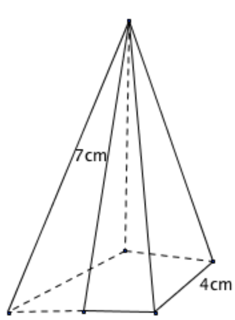


Lateral surface area = __________

19. Find the total surface area of the cylinder above.

Total surface area = ______________________________________

20. Find the lateral area of the pyramid below.

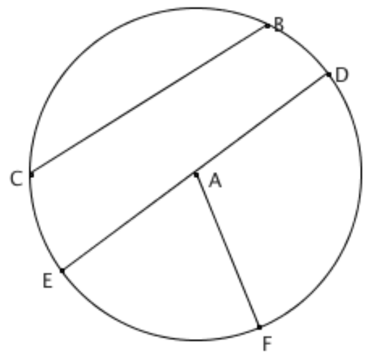


Lateral area = ______________________________

Unit VII Test, LCC: Circles and Spheres

Name: ____________________________ Date: _______________ Period: ________

1. Name each part of the circle.

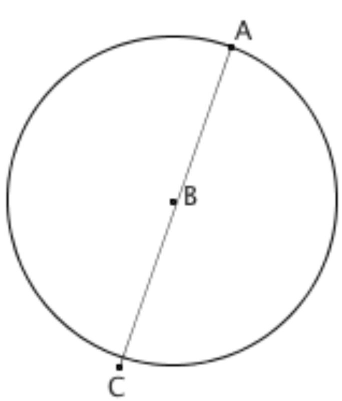


a. A radius ___________________

b. A diameter ___________________

c. A chord ____________________

2. In the circle below, if AC = 20, find AB



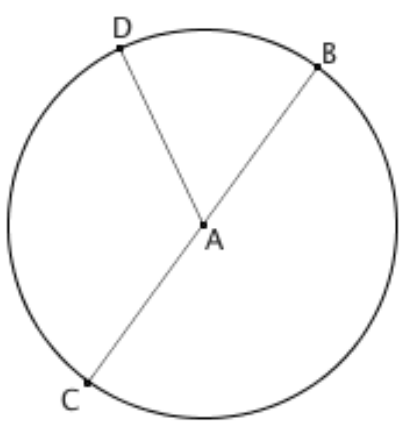
AB = ____________

The radius (r), diameter (d), or circumference (c) of a circle is given. Find the missing measures. Round your answer to the nearest hundredth.

3. $r = 6.5$ mm. d= ________________ c = _______________

4. $c = 220$ yd. d = _______________ r = ________________

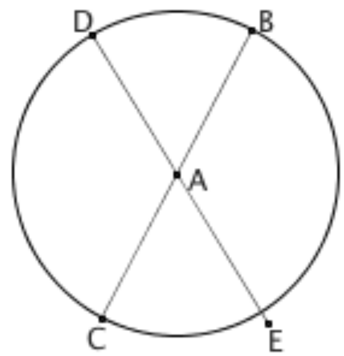
4. In the circle below, if the measure of angle BAD = 40 degrees, and BC is a diameter, find the measure of arc BD and the measure of arc DCB.



a. arc BD = ___________

b. arc DCB = _________

In the circle below, if the measure of angle BAD = 46, find each measure.



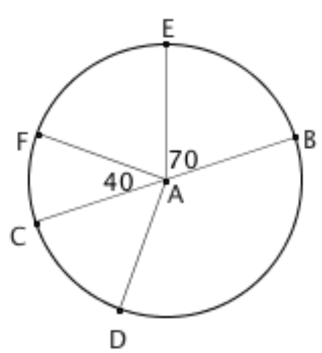
5. measure of arc BD = _______________

6. measure of arc DC = _______________

7. measure of arc CE = _______________

8. measure of arc BECD = ______________

9. In the circle below, find the measure of each angle.

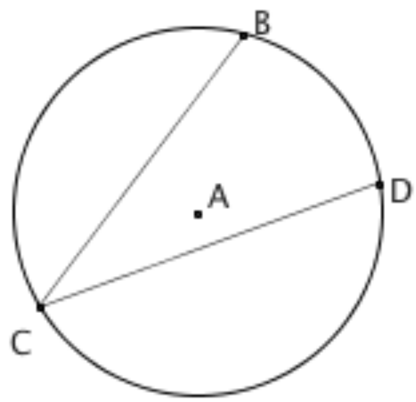


a. angle FAE = _______________ b. angle FAB = _______________

10. Angle FAD in the picture above is a right angle. What is the measure of angle CAD?

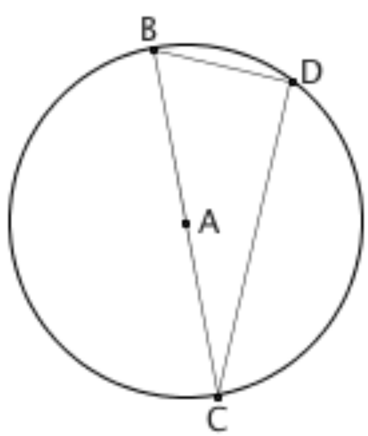
____

11. In the circle below, if the measure of arc BD = 80. Find measure of angle BCD.



angle BCD = _____________

12. In the circle below, BD = 6 and BC = 10

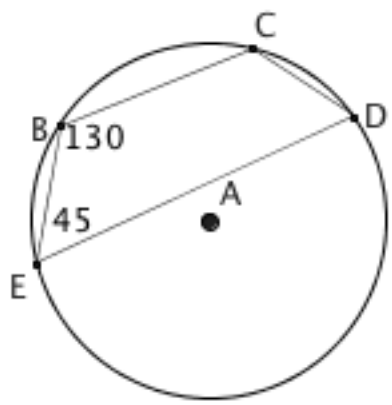


a. What is the measure of angle D? _______

b. How do you know the measure of angle D?____________________________

c. Find DC ___________________

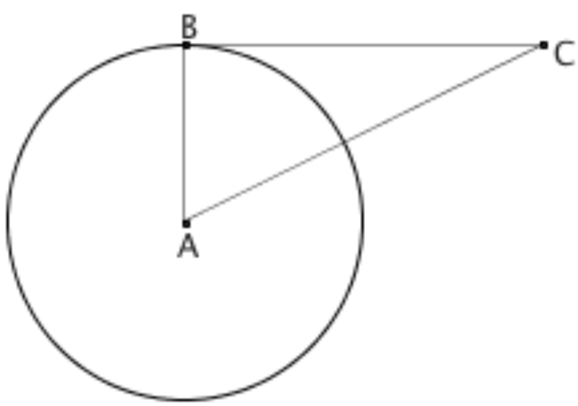
13. Find the measure of angle C and angle D.



a. angle C = ____________

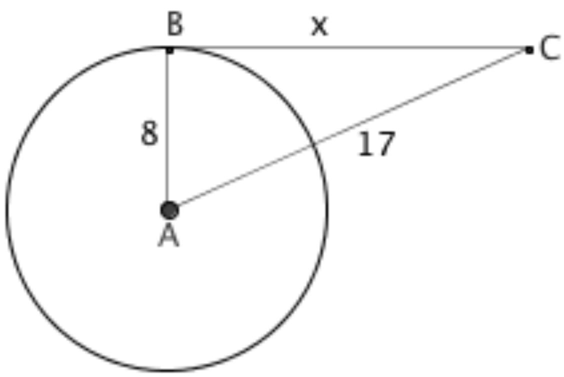
b. angle D = ____________

14. Which segment is tangent to the given circle? A is the center, and angle ABC is right.



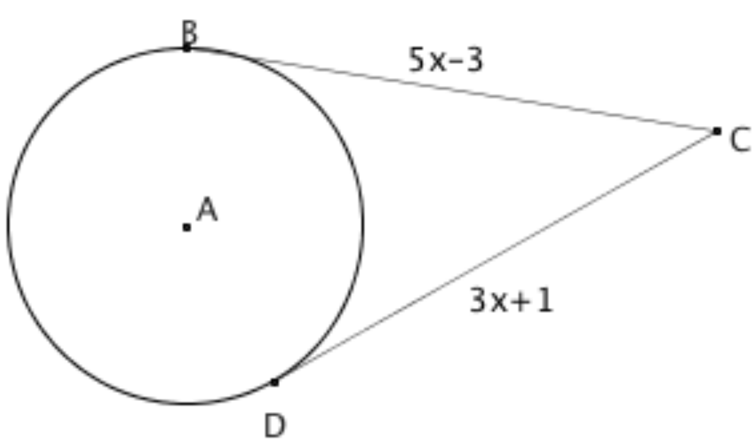
Answer: ________________

15. Find x. Assume that segments that appear to be tangent are tangent.



x =________________

16. Find x. Assume that segments that appear to be tangent are tangent.

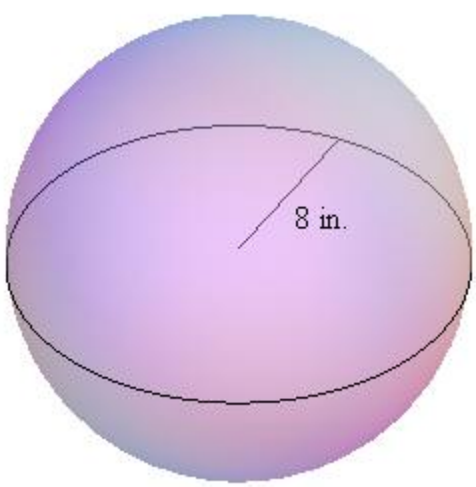


x = ________________

17. Find the surface area (to the nearest tenth) of a sphere of radius 6 cm.

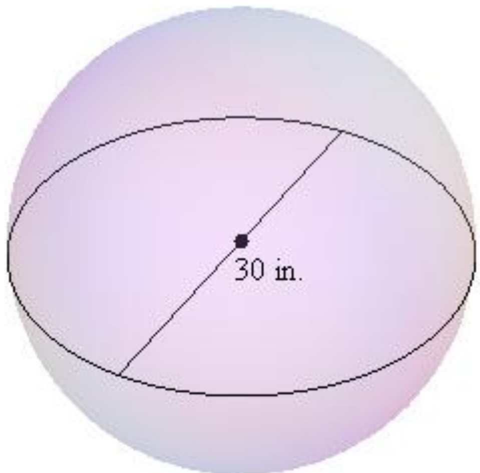
18. Find the volume (to the nearest tenth) of a sphere of diameter 6 cm.

19. Find the surface area:



Answer: ______________________________

20. Find the surface area:



Answer: ______________________________

Unit VIII Test, LCC: Transformation.

Name: ____________________________ Date: ________________ Period: ______

Identify each transformation as a reflection, translation, dilation, or rotation.

1. A figure has been turned around a point. _______________

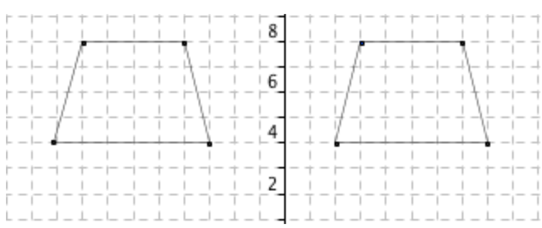
2. A figure has been increased in size. _______________

3. A figure has been flipped over a line. _______________

4. A figure has been shifted horizontally to the right. _______________

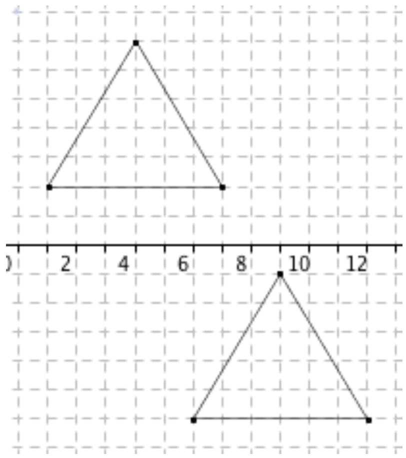
Identify each picture as a reflection, translation, dilation or rotation.

5.



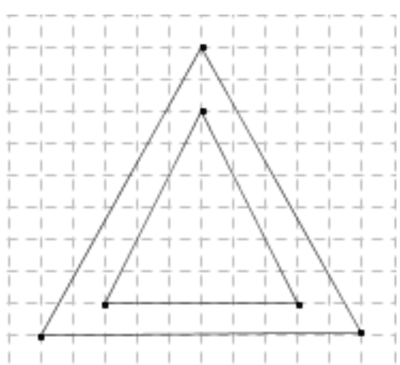
____

6.



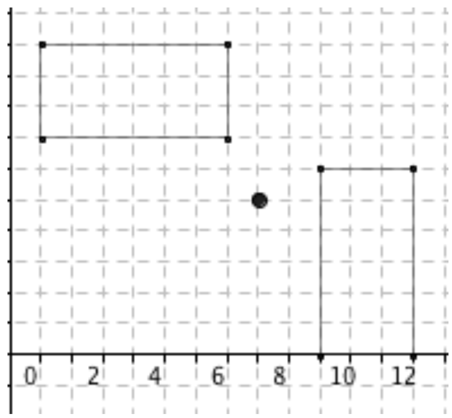
____

7.



____

8.



____

True or False

9. ______ To reflect a point over the *x*-axis, multiply the *y*-coordinate by -1.

10. ______ To translate a point by an ordered pair (*a*, *b*), add a to the *x*-coordinate and b to the *y*-coordinate.

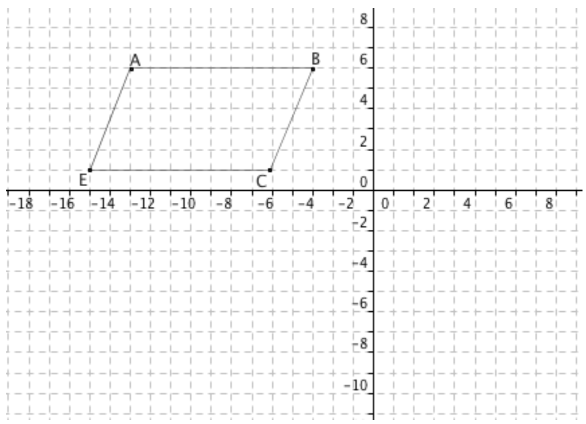
11. ______To dilate a figure by a scale factor *k*, multiply both coordinates by *k*.

12. ______When a figure is dilated by a scale factor of *k*, if *k*< 0, the figure is enlarged.

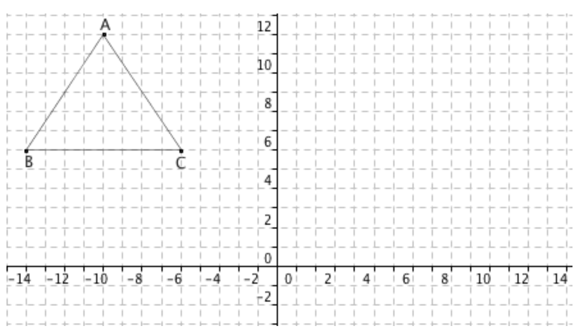
13. ______To rotate a figure 90 degrees counterclockwise, about the origin, switch the coordinates of each point and then multiply the new first coordinate by -1.

14. ______A reflection is a congruence transformation.

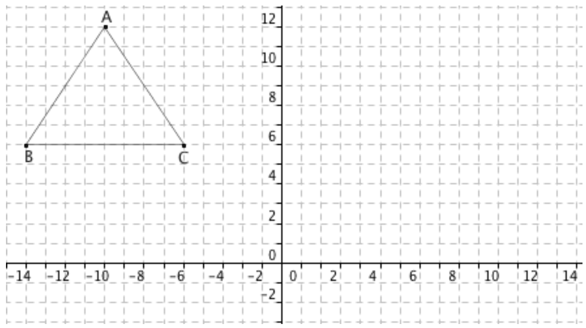
15. Reflect the parallelogram ABCD over the *x*-axis.



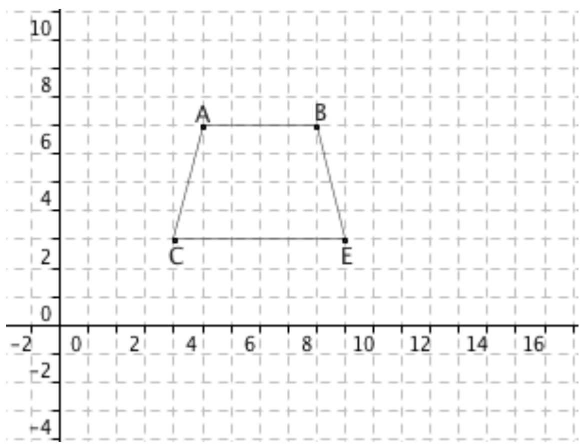
16. Reflect the triangle ABC over the *y*-axis.



17. Translate the triangle *ABC* 2 units to the right and 3 units down.



18. Dilate the trapezoid ABCD about the origin by a scale factor of two.



Find the coordinates of the vertices of each figure after the given transformation is performed.

19. Triangle RST with R(1, 3), S(5, -1), T(4, 5), translated by (3, -2).

Coordinates of the image: ______________________________

20. Parallelogram E(0,0)F(5,1)G(7,6)H(2,5) reflected over the x-axis.

Coordinates of the image: ____

Appendix B: IRB Approval

Application for Exemption from Institutional Oversight

LSU
Institutional Review Board
Dr. Robert Mathews, Chair
131 David Boyd Hall
Baton Rouge, LA 70803
P: 225.578.8692
F: 225.578.6792
irb@lsu.edu
lsu.edu/irb

Unless qualified as meeting the specific criteria for exemption from Institutional Review Board (IRB) oversight, ALL LSU research/ projects using living humans as subjects, or samples, or data obtained from humans, directly or indirectly, with or without their consent, must be approved or exempted in advance by the LSU IRB. This Form helps the PI determine if a project may be exempted, and is used to request an exemption.

-- Applicant,Please fill out the application in its entirety and include the completed application as well as parts A-E, listed below, when submitting to the IRB. Once the application is completed, please submit two copies of the completed application to the IRB Office or to a member of the Human Subjects Screening Committee. Members of this committee can be found at http://research.lsu.edu/CompliancePoliciesProcedures/InstitutionalReviewBoard%28IRB%29/item24737.html

-- A Complete Application Includes All of the Following:
(A) Two copies of this completed form and two copies of part B thru E.
(B) A brief project description (adequate to evaluate risks to subjects and to explain your responses to Parts 1&2)
(C) Copies of all instruments to be used.
*If this proposal is part of a grant proposal, include a copy of the proposal and all recruitment material.
(D) The consent form that you will use in the study (see part 3 for more information.)
(E) Certificate of Completion of Human Subjects Protection Training for all personnel involved in the project, including students who are involved with testing or handling data, unless already on file with the IRB. Training link: (http://phrp.nihtraining.com/users/login.php)
(F) IRB Security of Data Agreement: (http://research.lsu.edu/files/item26774.pdf)

1) Principal Investigator: Damayanthi Jegart **Rank:**

Dept: Natural Science **Ph:** 985-713-9528 **E-mail:** djegart@yahoo.com

2) Co Investigator(s): please include department, rank, phone and e-mail for each
*If student, please identify and name supervising professor in this space

IRB# E6014 LSU Proposal #
Complete Application
Human Subjects Training

3) Project Title: Teaching and Assessment of Geometry in Secondary Schools

Study Exempted By:
Dr. Robert C. Mathews, Chairman
Institutional Review Board
Louisiana State University
203 B-1 David Boyd Hall
225-578-8692 | www.lsu.edu/irb
Exemption Expires: 7/8/2015

4) Proposal? (yes or no) No **If Yes, LSU Proposal Number**

Also, if YES, either
◯ **This application completely matches the scope of work in the grant**
OR
◯ **More IRB Applications will be filed later**

5) Subject pool (e.g. Psychology students)

*Circle any **"vulnerable populations" to be used**: (children <18; the mentally impaired, pregnant women, the ages, other). Projects with incarcerated persons cannot be exempted.

6) PI Signature **Date** (no per signatures)

**** I certify my responses are accurate and complete.** If the project scope or design is later changes, I will resubmit for review. I will obtain written approval from the Authorized Representative of all non-LSU institutions in which the study is conducted. I also understand that it is my responsibility to maintain copies of all consent forms at LSU for three years after completion of the study. If I leave LSU before that time the consent forms should be preserved in the Departmental Office.

Screening Committee Action: Exempted √ 4 Not Exempted ____ Category/Paragraph 4

Reviewer Mathews **Signature** [signature] **Date** 7/9/12

Figure 1: IRB Approval

(Figure 1 continued)

Study Exempted By:
Dr. Robert C. Mathews, Chairman
Institutional Review Board
Louisiana State University
203 B-1 David Boyd Hall
225-578-8692 | www.lsu.edu/irb
Exemption Expires: 7/8/2015

Dear Parents,

As your child's math teacher and in order to meet my responsibilities as a teacher, I have developed unit tests for the Louisiana Comprehensive Curriculum that I am responsible for administering.

I am currently enrolled the Louisiana Math and Science Teacher Institute (LaMSTI) Master of Natural Science degree program. As part of this I am conducting research on student learning. I would like to be able to use the student unit test results from the past semester as data. No names or identifying information will be included in any public work. Student identity will remain confidential.

I am requesting permission to use the data from the tests that your child has taken. No names will be associated with the work. No one will be able to tell if the work is your child's.

If you have any questions please contact me at damijegart@ipsb.net or 985-526-6584.

Sincerely,
Dami Jegart

Vita

Damayanthi Srimathi Jegart was born in Sri Lanka. She is the mother of four children. She graduated Magma Cum Laude from Nicholls State University in 2000. After graduating she received an alternate certification to teach secondary math. She taught algebra I for two years at Assumption High School. She transferred to Saint James Parish in 2004 and taught at the Science and Math Academy for four years. She is currently teaching geometry and advanced math at the Math and Science Academy West in Iberville Parish. This will be her fifth year teaching at this school.