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# *Ab initio* folding potentials for proton-nucleus scattering with NCSM nonlocal one-body densities

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**Abstract.** Based on the spectator expansion of the multiple scattering series we employ a nonlocal translationally invariant nuclear density derived from a chiral next-to-next-to-leading order (NNLO) and the very same interaction for consistent full-folding calculations of the effective (optical) potential for nucleon-nucleus scattering for light nuclei.

**Keywords:** Nuclear reactions, elastic scattering, no-core-shell model

The theoretical approach to the elastic scattering of a nucleon from a nucleus, pioneered by Watson [1], made familiar by Kerman, McManus, and Thaler (KMT) [2] and further developed as the spectator expansion [3,4] is now being applied together with *ab initio* structure calculations to obtain effective folding nucleon-nucleus (NA) potentials. The spectator expansion is predicated upon the idea that at projectile energies higher than about 100 MeV the two-body interactions between the projectile and the nucleons in the target dominate elastic scattering, for which a transition operator can be defined as

$$T_{el} \equiv PTP = PUP + PUPG_0(E)T_{el}. \tag{1}$$

The projector $P = \frac{|\Phi_A\rangle\langle\Phi_A|}{\langle\Phi_A|\Phi_A\rangle}$ is conventionally taken to project on the elastic channel so that $[G_0, P] = 0$. Here $|\Phi_A\rangle$ stands for the ground state of the target so that $H_A|\Phi_A\rangle = E_A|\Phi_A\rangle$, and $G_0(E) = (E - H_0 + i\varepsilon)^{-1}$, where $H_0 = h_0 + H_A$, being the free propagator for the projectile+target system. The effective (optical) potential is given by

$$U = V + VG_0(E)QU, \tag{2}$$

where the operator $Q$ is defined via the relation $P + Q = 1$. The first-order term involves two-body interactions between the projectile and one of the target nucleons, *i.e.* $U = \sum_{i=1}^{A} \tau_i$, where the operator $\tau_i$ is derived to be

$$\begin{aligned} \tau_i &= v_{0i} + v_{0i}G_0(E)Q\tau_i \\ &= v_{0i} + v_{0i}G_0(E)\tau_i - v_{0i}G_0(E)P\tau_i \\ &= \hat{\tau}_i - \hat{\tau}_i G_0(E)P\tau_i. \end{aligned} \tag{3}$$

Here $\hat{\tau}_i$ is the NN t-matrix and is defined as the solution of $\hat{\tau}_i = v_{0i} + v_{0i} G_0(E) \hat{\tau}_i$. In lowest order $\hat{\tau}_i \approx t_{0i}$, which corresponds to the conventional impulse approximation. Here the operator $t_{0i}$ stands for the standard solution of a Lippmann-Schwinger equation with the NN interaction as driving term.

For elastic scattering only $P\tau_i P$ needs to be considered,

$$\langle \Phi_A | \tau_i | \Phi_A \rangle = \langle \Phi_A | \hat{\tau}_i | \Phi_A \rangle - \langle \Phi_A | \hat{\tau}_i | \Phi_A \rangle \frac{1}{(E - E_A) - h_0 + i\varepsilon} \langle \Phi_A | \tau_i | \Phi_A \rangle, \quad (4)$$

and this matrix element determines the full-folding effective (optical) potential when summing over all target nuclei,

$$\langle \mathbf{k}' | U | \mathbf{k} \rangle = \langle \mathbf{k}' \Phi_A | \sum_i \tau_i | \mathbf{k} \Phi_A \rangle. \quad (5)$$

Since $\langle \mathbf{k}' | U | \mathbf{k} \rangle$ is the solution of the sum of one-body integral equations represented by Eq. (4), it is sufficient to consider the driving term

$$\langle \mathbf{k}' | \hat{U} | \mathbf{k} \rangle = \langle \mathbf{k}' \Phi_A | \sum_i \hat{\tau}_i | \mathbf{k} \Phi_A \rangle, \quad (6)$$

where $\hat{\tau}_i \approx t_{0i}$ when considering the first order single scattering term. Inserting

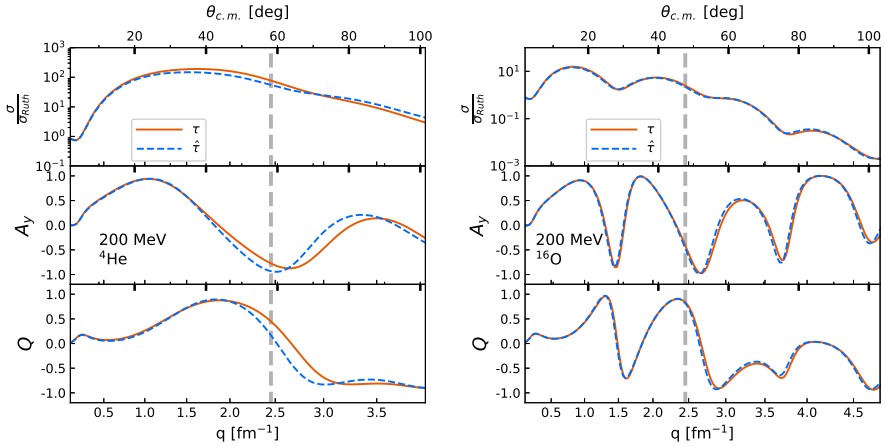


**Fig. 1.** Angular distribution of the differential cross section divided by the Rutherford cross section, analyzing power $A_y$ and spin rotation function $Q$ for elastic proton scattering from $^4$He (left) and $^{16}$O (right) as function of the momentum transfer and the c.m. angle calculated with the NNLO$_{\rm opt}$ chiral interaction [5]. The solid line represents the calculation based on $\tau_i$ (Eq. (5)), the dashed line the one based on $\hat{\tau}_i$ (Eq. (6)).

a complete set of momenta for the struck target nucleon before and after the collision, representing the sum over target protons and neutrons by $\alpha$, evaluating

the momentum conserving delta function and changing variables to $\mathbf{q} = \mathbf{k}' - \mathbf{k}$, $\mathbf{K} = \frac{1}{2}(\mathbf{k} + \mathbf{k}')$ and $\mathbf{P} = \frac{1}{2}(\mathbf{p}' + \mathbf{p}) + \frac{\mathbf{K}}{A}$, the final expression for the full-folding effective potential is given by

$$\hat{U}(\mathbf{q}, \mathbf{K}) = \sum_{\alpha=p,n} \int d^3\mathbf{P}\, \eta(\mathbf{P}, \mathbf{q}, \mathbf{K})\, \hat{\tau}_\alpha \left( \mathbf{q}, \frac{1}{2} \left( \frac{A+1}{A}\mathbf{K} - \mathbf{P} \right); \epsilon \right) \times \rho_\alpha \left( \mathbf{P} - \frac{A-1}{A}\frac{\mathbf{q}}{2}, \mathbf{P} + \frac{A-1}{A}\frac{\mathbf{q}}{2} \right). \quad (7)$$

Here $\eta(\mathbf{P}, \mathbf{q}, \mathbf{K})$ is the Møller factor for the frame transformation [6] relating the NN zero-momentum frame to the NA zero-momentum frame. Further details can be found in [7]. The quantity $\rho_\alpha$, with $\alpha = p(n)$, represents a nonlocal one-body density matrix (OBD) for the proton (neutron) distribution, and must be given in a translationally invariant fashion [8]. An important product of this work is

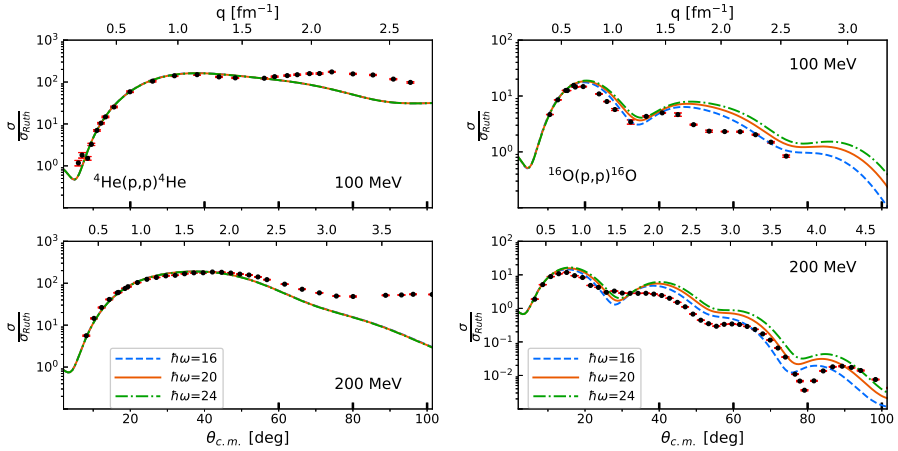


**Fig. 2.** Angular distribution of the differential cross section divided by the Rutherford cross section for elastic proton scattering from $^4$He (left) and $^{16}$O (right) as function of the c.m. angle calculated with the NNLO$_{\rm opt}$ chiral interaction [5]. The calculations for $^4$He are carried out with $N_{\rm max} = 18$, while the ones for $^{16}$O with $N_{\rm max} = 10$. The values of $\hbar\omega$ are indicated in the lower panels. The $^4$He data for 100 MeV are taken from [9] and for 200 MeV from [10]. The $^{16}$O data for 100 MeV are taken from [11] and for 200 MeV from [12].

that the NN t-matrix and OBD now use the same underlying NN interaction. For this we choose the optimized chiral NN interaction at next-to-next-to-leading order NNLO$_{\rm opt}$ from Ref. [5]. In the $A = 3$, 4 nucleon systems the contributions of the 3NFs are smaller than in most other parameterizations of chiral interactions. From this point of view, the NNLO$_{\rm opt}$ interaction is very well suited for our calculations, since the first-order folding potential does not contain any

explicit 3NF contributions. We calculated the full-folding integral for the first-order effective (optical) potential for NA scattering *ab initio*. i.e. they are based consistently on one single NN interaction, in our case the chiral $\mathrm{NNLO_{opt}}$ interaction from Ref. [5], which is fitted to NN data up to 125 MeV laboratory kinetic energy with $\chi^2 \approx 1$ per degree of freedom. Based on this interaction the one-body nonlocal, translationally invariant nuclear densities are calculated as laid out in Ref. [8]. Further details of the calculations of the effective potential and the NA scattering are described in Ref. [7].

First, we want to illustrate the difference in employing $U$ or $\hat{U}$ as effective NA potential in Fig. 1 for proton scattering from $^4$He and $^{16}$O at 200 MeV projectile laboratory kinetic energy. The figure shows that taking into account the effect of the operator $Q$ by solving Eq. (4) to obtain $U$ is clearly visible for the light nucleus $^4$He, while very small for a heavier nucleus like $^{16}$O.

As examples for elastic proton scattering based on an *ab initio* effective potential we show in Fig. 2 the angular distributions (divided by the Rutherford amplitude) for $^4$He and $^{16}$O for energies between 100 and 200 MeV laboratory kinetic energy. We find them in very good agreement with the data in the angle and momentum transfer regime where the first order term of the full-folding effective potential should be valid. We also want to point out that the first order term in the multiple scattering expansion does not explicitly contain any 3NF contributions, thus the choice of the $\mathrm{NNLO_{opt}}$ interaction works well with the theoretical content of the effective potential. Further in the future with different interactions will have to shed more light on the effect of including 3NFs in the one-body density for the first-order effective potential.

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