2012

Writing in Geometry with the Common Core State Standards: developing mathematical thinkers

Yvonne Mariki Chimwaza
Louisiana State University and Agricultural and Mechanical College

WRITING IN GEOMETRY WITH THE COMMON CORE STATE STANDARDS: DEVELOPING MATHEMATICAL THINKERS

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Master of Natural Sciences

in

The Interdepartmental Program in Natural Sciences

by
Yvonne Mariki Chimwaza
B.S., Louisiana State University, 2009
August 2012

Acknowledgments

I would like to start by thanking God for this and every opportunity and privilege He has given me. I am nothing without Him.

I would like to thank my family, especially my husband, Tinashe Chimwaza; my mother Evetta Pitman; and my sister, Desiree Mariki; for their encouragement, love, prayers and support during this process and always. I am honored to have you in my life.

I would like to acknowledge and thank Dr. Frank Neubrander, my professor, thesis advisor, and committee member for his countless advice and help throughout this degree program. I am grateful to you and cannot express the level of my gratitude. I would also like to acknowledge and thank Dr. James Madden and Professor Tom Ricks for being members of my thesis committee. In addition, I must thank my principal, Debbie Bain for her support and everything she does.

I would like to also express my thanks to my friends for their time, encouragement and support.

Lastly, I would like to thank my LaMSTI 2010 cohort for every word of kindness, encouragement, support, inspiration and love. This process would have been a lot harder without you and I thank you for being there.

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Abstract

The newly released Common Core State Standards (CCSS) were adopted with the goal in mind that in the future our students will leave high school ready and better prepared for college and careers. In particular, the CCSS insists that a faithful implementation of the eight Standard of Mathematical Practices will lead to a generation of mathematical thinkers who have learned how to read, write, model, reason, and solve problems in mathematical terms. Unfortunately, at present, my students and others do not know how to write and reason mathematically. By way of this thesis, I searched for ways to help forty-five students in my Geometry classes improve their mathematical writing and reading skills by adding structured journaling. In this thesis, the work of three of the forty-five students was analyzed on the basis of three journal entries. Students A, B, and C's work showed that there was obvious change and growth in writing abilities, how they explained their reasoning, and how correct it was. All forty-five students completed a survey about their experiences with the journals and Geometry as a whole. The students, as seen in the survey responses, understood what the journals were designed to do and many of them saw the benefit of having a writing template. Within the same year I implemented the journals, our school's score on the Geometry End-of-Course test increased by fourteen percent from the previous year. Overall, though it cannot be said this is strong enough to stand alone and defend the template, it does show that three students, who represent a class of forty-five, with varying levels of understanding have all improved their mathematical writing and reasoning abilities. I do believe that this template should be tested to further solidify its effectiveness and that the success I had with my class on the End-of-Course test, due to the structured emphasis on writing and reasoning, can be replicated with ease.

Chapter 1: Setting the Standards

1.1 Introduction

It has become quite evident over the past decade that the American school system is in need of either reform or reconstruction. According to the Program for International Student Assessment's (PISA) 2009 "assessment of 15-year olds, the United States performs around average in reading (rank 14) and science (rank 17) and below the average in mathematics (rank 25) among the 34 OECD countries" (OECD, 2010). PISA has released data that shows the United States at a ranking as low as 30 in math overall and 23 in science overall.

Programme for International Student Assessment (2009)			
Rank	**Maths**	**Sciences**	**Reading**
1	Shanghai, China	Shanghai, China	Shanghai, China
2	Singapore	Finland	South Korea
3	Hong Kong	Hong Kong	Finland
4	South Korea	Singapore	Hong Kong
5	Taiwan	Japan	Singapore
6	Finland	South Korea	Canada
7	Liechtenstein	New Zealand	New Zealand
8	Switzerland	Canada	Japan
9	Japan	Estonia	Australia
10	Canada	Australia	Netherlands
11	Netherlands	Netherlands	Belgium
12	Macau	Liechtenstein	Norway
13	New Zealand	Germany	Estonia
14	Belgium	Taiwan	Switzerland
15	Australia	Switzerland	Poland
16	Germany	United Kindom	Iceland
17	Estonia	Slovenia	United States
18	Iceland	Macau	Liechtenstein
19	Denmark	Poland	Sweden
20	Slovenia	Ireland	Germany
21	Norway	Belgium	Ireland
22	France	Hungary	France
23	Slovakia	United States	Taiwan
24	Austria	Norway	Denmark
25	Poland	Czech Republic	United Kingdom
26	Sweden	Denmark	Hungary
27	Czech Republic	France	Portugal
28	United Kingdom	Iceland	Macau
29	Hungary	Sweden	Italy
30	United States	Latvia	Latvia

Figure 1.1 PISA countries rankings (1)

Maths			Sciences			Reading		
1.	Shanghai, China	600	1.	Shanghai, China	575	1.	Shanghai, China	556
2.	Singapore	562	2.	Finland	554	2.	South Korea	539
3.	Hong Kong, China	555	3.	Hong Kong, China	549	3.	Finland	536
4.	South Korea	546	4.	Singapore	542	4.	Hong Kong, China	533
5.	Taiwan	543	5.	Japan	539	5.	Singapore	526
6.	Finland	541	6.	South Korea	538	6.	Canada	524
7.	Liechtenstein	536	7.	New Zealand	532	7.	New Zealand	521
8.	Switzerland	534	8.	Canada	529	8.	Japan	520
9.	Japan	529	9.	Estonia	528	9.	Australia	515
10.	Canada	527	10.	Australia	527	10.	Netherlands	508
11.	Netherlands	526	11.	Netherlands	522	11.	Belgium	506
12.	Macau, China	525	12.	Liechtenstein	520	12.	Norway	503
13.	New Zealand	519	13.	Germany	520	13.	Estonia	501
14.	Belgium	515	14.	Taiwan	520	14.	Switzerland	501
15.	Australia	514	15.	Switzerland	517	15.	Poland	500
16.	Germany	513	16.	United Kingdom	514	16.	Iceland	500
17.	Estonia	512	17.	Slovenia	512	17.	United States	500
18.	Iceland	507	18.	Macau, China	511	18.	Liechtenstein	499
19.	Denmark	503	19.	Poland	508	19.	Sweden	497
20.	Slovenia	501	20.	Ireland	508	20.	Germany	497
21.	Norway	498	21.	Belgium	507	21.	Ireland	496
22.	France	497	22.	Hungary	503	22.	France	496
23.	Slovakia	497	23.	United States	502	23.	Taiwan	495
24.	Austria	496	24.	Norway	500	24.	Denmark	495
25.	Poland	495	25.	Czech Republic	500	25.	United Kingdom	494
26.	Sweden	494	26.	Denmark	499	26.	Hungary	494
27.	Czech Republic	493	27.	France	498	27.	Portugal	489
28.	United Kingdom	492	28.	Iceland	496	28.	Macau, China	487
29.	Hungary	490	29.	Sweden	495	29.	Italy	486
30.	United States	487	30.	Latvia	494	30.	Latvia	484

Figure 1.2 PISA countries ranking (2)

It must be said that this data averages the performance of American schools and is not a reflection of every school; some are much higher, while others are much lower (McCabe, 2010). Even so, this of course is not an ideal ranking for a country whose predecessors are known for their innovation. The results call for changes and

improvements the American school system especially in the areas of lowest ranking.

"Science, technology, engineering, and mathematics (STEM) education is a crucial issue in current educational trends" (Becker, 2011). One of the obvious reasons being, that in order for our students to later contribute productively to the post-industrial society they must receive a firm foundation in the STEM disciplines. A focus on the STEM disciplines will certainly prepare students for a collegiate career and will help develop reasoning skills for students who intend on going into the workforce after high school. If we want to prepare these students we must acknowledge that in many, if not all, socio-economically challenged regions in America, there is a breakdown in the educational effectiveness of the school system and something must be done especially in our weakest area, mathematics (McCabe, 2010).

1.2 Current Louisiana State Standards

In the current system, education standards vary from state to state; however for the purpose of this thesis we will focus on the Louisiana standards. The current standards for Louisiana are referred to as the Grade Level Expectations (GLEs). The GLEs "identify what all students should know or be able to do by the end of each grade from prekindergarten through grade 12 in Math, English, Science and Social Studies" (LaDoe, 2010a). A closer study of the GLEs will reveal that some of the exact topics covered in Geometry are also covered in eighth and ninth grade briefly. This brief teaching of many topics leaves little room for mastery. Along with the GLEs, the Louisiana Comprehensive Curriculum is a curriculum based on the GLEs and divided by subject and class (LaDoe, 2010b). It includes activities and pacing guides for the presentation of the GLEs to students.

Louisiana’s current assessments of students understanding of the GLEs are the LEAP (Louisiana Education Assessment Program), ILEAP (Integrated Louisiana Education Assessment Program), EOC (End Of Course), and GEE test (LaDoe, 2010b). The EOC is untimed and administered via computer; while the others are a series of timed and untimed multiple choice and constructed response questions delivered by paper test (LaDoe, 2010b). LEAP is taken in the fourth and eighth grade and the students’ promotion to the next grade can be hindered if a student does not score sufficiently. The ILEAP is administered grades three, five, six, and seven; however students’ promotion to the next grade level does not depend on the student’s score. The EOC is given to high schools students in six different core classes: Algebra I, Geometry, English II, English III, Biology, and American History. Students must pass one math, one English, and either Biology or American History to graduate from high school. Though in the past students were required to pass the GEE to graduate, the EOC and its requirements have since been enforced as of 2010-2011 for all Louisiana school districts (LaDoe, 2010b).

1.3 Common Core State Standards and PARCC Assessments

In an attempt to reform our school systems to compete more adequately on an international level, in 2009 the Common Core State Standards (CCSS) were released. These standards were authored to reverse the Unites States’ international rankings (National Governors Association Center for Best Practices, 2010a). The Common Core State Standards will be implemented in 45 states, 2 territories, and the District of Columbia ("About the Standards: Process," 2010). The goal and focus of the standards is to ensure that at the end of students’ primary and secondary educational careers they

will have developed reasoning skills that will have prepared them for a collegiate or workforce career of the twenty-first century (National Governors Association Center for Best Practices, 2010b). It is for this reason that the CCSS main focus is to develop independent thinkers and problem solvers in both the STEM and English Language Arts related disciplines. Independent thinkers have the ability to reason for themselves and explain that reasoning to others; this is a huge part of CCSS's plan for mathematics.

The CCSS for Mathematics contains Standards for Mathematical Practice and Standards for Mathematical Content, implying that the correct content delivery does not always produce the correct practice or ability to apply the content. "The standards stress not only procedural skill but also conceptual understanding, to make sure students are learning and absorbing the critical information they need to succeed at higher levels"("Key Points in Mathematics," 2010). This focus is to deter students from the practice of memorizing information for assessment purposes, but not retaining the information ("Key Points in Mathematics," 2010). The Standards for Mathematical Practice are to be applied daily to each grade level, while the Standards for Mathematical Content are grade specific.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

7. Look for and make use of structure.
8. Look for and express regularity in repeating reasoning

The Standards for Mathematical Practice focus on the long-term goals that teachers should have for their students. These standards were developed by combining the National Council of Teachers of Mathematics' (NCTM) process standards with the National Research Council's report *Adding It Up* (National Governors Association Center for Best Practices, 2010c). Each standard listed is geared towards a student's ability to retain, communicate and explain mathematically.

In addition to the eight practices, "the Standards for Mathematical Content are a balanced combination of procedure and understanding" (National Governors Association Center for Best Practices, 2010c). Students should not only be able to perform a procedure, but develop the understanding as to why the procedure is done and how/why it works. The CCSS for High School Geometry begins with an overview of six topic and ten mathematical practices which of course include the eight Standards for Mathematical Practice. A subject such as Geometry with its focus on reasoning and concrete or visual application has the rigor that CCSS is designed to produce in every strand (González & Herbst, 2006; National Governors Association Center for Best Practices, 2010c). Each of the six topics are then broken down into specific procedures that students are required to master.

The mastery of these skills will be tested by the Partnership for Assessment of Readiness for College and Careers (PARCC) assessments that will replace the LEAP and ILEAP assessments by the 2014-2015 school year. The standards and PARCC assessments focus heavily on student comprehension and retention as many programs

have planned to do in the past. The thing that makes the CCSS and PARCC different from the rest are their focus on writing mathematics to test and increase retention. The goal and plan of the PARCC assessments is to test how well students comprehend and retain mathematics by assessing how well they write mathematics through “innovative constructed response, extended performance tasks, and selected response (all of which will be computer based)” (Partnership for Assessment of Readiness for College & Careers, 2011).

The PARCC assessments focus on writing mathematics, which is not the same thing as writing about mathematics. There is a subtle difference between simply writing about mathematics and actually writing mathematics and “many have failed to distinguish reading and writing **about** mathematics [from] reading and writing **in** mathematics” (Bossé & Faulconer, 2008). When one is required to write and/or verbally explain a topic or concept it requires an understanding of the subject matter to do so effectively. The NCTM agreed that, “Students who have opportunities, encouragement, and support for speaking, writing, reading, and listening in mathematics classes reap dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically.” (National Council of Teachers of Mathematics, 2000). It is the ability to communicate mathematical understanding that the PARCC assessments will test.

With the PARCC assessments’ emphasis on writing in mathematics it is also important to look at the CCSS for English Language Arts. One of the key points in the standards for English Language Arts is that students have, “The ability to write logical arguments based on substantive claims, sound reasoning, and relevant evidence” (National Governors Association Center for Best Practices, 2010d). Though these skills

are necessary for English Language Arts, they are also a necessity for writing in a subject such as Geometry, which emphasizes the development of logical thinking and explanations. The ability of a student to give a clear logical explanation about abstract concepts, in which much of mathematics contains, is the whole goal and focus of the CCSS.

Chapter 2: Writing in Geometry

2.1 Understanding through Writing and Geometry

Since it is an essential part of communication, many would agree that reasoning and writing are essential parts of life. In addition, "researchers claim that writing sustains students' development of reasoning, communication and connections and consequently deepens mathematical knowledge and extends thinking" (Adu-Gyamfi, 2010). A student is able to retain information when he develops reasoning, communicates that reasoning, and makes connections to prior knowledge. Writing in itself is a powerful tool that allows students to connect former knowledge with current material to produce new knowledge (Cross, 2009). This is true of any discipline and not exclusive to writing in mathematics. However, writing in mathematics forces one to express the knowledge, ideas, and information they have attained about the abstract concepts in mathematics (Cross, 2009). Having students that write and reason about mathematics allows the teacher to gain insight into a student's rationale, which then gives way to the correction of misconceptions.

The original reason for teaching Geometry was to further develop students' thinking capacities. In actuality, "authors in 1877 indicated that the principal objective for the study of demonstrative geometry was the discipline of the mental faculties and memorization of a certain body of facts"(Brown, 1950). The educating of the mind is the only reason anyone teaches anything. A subject such as Geometry trains the mind to reason. In 1940 it was said by the NCTM that, "our goal aim in tenth year is to teach the material of deductive proof and to furnish pupils with a model for all their life thinking" (The Fifth Yearbook of National Council of Teachers of Mathematics, 1940). The

strength and finality of that statement implies that Geometry can aid the mind in understanding and making sense of the world around it because it requires written deductive proofs.

2.2 Writing in Mathematics

One of the biggest aspects of teaching mathematics is understanding that it constantly builds on itself. Having a tool such as writing emphasizes making connections, which when incorporated into the learning environment will increase student understanding and achievement (Cross, 2009). Writing to solve a problem in mathematics forces students to make sense of the question asked the method(s) of answering, and explaining it to an audience. This type of reasoning combines problem solving and investigation skills. Both skills are believed to be related by many teachers, but too few force students to connect them (YEO & YEAP, 2010). Students need to "investigate during problem solving" and adding a writing component, where students have a template for the investigation would aid in this process (YEO & YEAP, 2010).

There are many aspects associated with being able to write mathematics. As with any language, there is necessary vocabulary that one must know in order to communicate it. However, memorizing vocabulary is only the beginning; true mathematics begins when students can correctly use and explore the definitions (Hiebert & Stigler, 1999). Mathematical writing is slightly more challenging than many other subjects because it contains more than mere words but also numeric, symbolic, graphical, and verbal depictions that communicate its meanings (Freitag, 1997). To the trained eye a graph in mathematics can say as much and even more than two well

written paragraphs. It can tell us the equation of a function, its behavior, its intercepts, which, depending on the function, can lead to maximizing and minimizing values; the possibilities are numerous. Writing mathematics requires the writer to make sense of abstract ideas by relating them to concrete objects and situations. It allots the writer the opportunity to quantify the world around them by applying mathematics to it and concretely understanding mathematics by applying the world to it. For example, a problem may be given to a student such as, "Before lunch, Mark catches double the amount of fish as Ryan. After lunch Mark catches 3 more, but Ryan does not catch any and immediately flies to a remote area of Timbuktu. If Mark caught a total of 13 fish, how many fish did Ryan catch? Write an equation to solve." The previous problem is applying math to a natural occurring situation. The converse would then be "Write a real life situation that can be solved by the equation $2x + 3 = 13$ and explain why it works." As a classroom mathematics teacher I've observed that students who take the time to decompose the problem and attempt to explain it in natural terms tend to be better mathematical writers. These students are careful to identify four main components necessary to solve the given problem: isolate important given information, recognize the question being asked, look for clues about the necessary mathematical operation(s), and connect to prior knowledge. However, even these students will have to take their problem solving skills a step further when the new standards are in place. The CCSS will require that students be able to "justify" their answers (National Governors Association Center for Best Practices, 2010c). Students will have to explain their reasoning and argue a strong case as to why it is true. It is an accepted fact that a student truly understands mathematics when they can justify their answer in words,

herein being the importance of writing in mathematics (National Governors Association Center for Best Practices, 2010c). One must understand that writing in mathematics does not automatically produce or equate learning, but it produces questioning which leads to investigation that then leads to discovery and understanding.

Chapter 3: Proposal, Preparation and Process

3.1 Proposal

In order for students to prosper in the ever changing post-industrial, technology fused world, they must acquire the reasoning skills that the STEM disciplines offer. Though mathematics only makes up a fourth of the STEM disciplines it is closely intertwined with the others and absolutely necessary to be successful in the others. It is evident that the current American education system as a whole might not be working to its full potential and there is hope that the new CCSS and PARCC assessments just may be a step in the right direction. The new CCSS standards and PARCC assessments will require students to become far more independent thinkers, problem solvers, and thorough explainers in all disciplines. Mathematics is a powerful tool in developing independent thinkers and problem solvers, because it requires logic and reasoning. The study of writing in Geometry forces students to reason and describe their thinking process in a way that makes sense on paper. Geometry, by requiring students to not only reason logically, but also logically explain their reasoning through writing allows students to become the teacher. The goal of this thesis is to use the CCSS Standards for Mathematical Practice to infer the practice of using a template for journal writing in a Geometry class to improve and better understand student reasoning, while creating a template that can easily be replicated by other teachers.

3.2 Preparation

In designing this project I wanted to be sure that it was fair, relevant, and insightful. The fairness and relevance are in regards to the questions students will

answer. I asked myself “how can I maintain that questions are rigorous, but ‘doable’”? In order to ensure the questions were fair, all questions (with the exception of one) for the students’ journal entries were taken from the assigned Glencoe Geometry textbook for Avoyelles Parish School System and the other was taken from Pearson Education’s My Math Lab for Geometry. The questions were given to students after we covered the topic of the question. Also, students were progressively given more specific information on the expectations for the journals, which are later defined as phases.

The insightfulness then surfaced by creating a step-by-step template for students to follow. Using the Common Core State Standards, I designed a series of questions that students use to completely answer the given entries. The questions are designed in such a way that students actively use all of the Standards for Mathematical Practice from the CCSS every time they perform a journal entry. Students were also asked to address a specific audience which tells them how in depth they need to be in their explanation as shown in Figure 3.1; certain audiences will have more understanding than others.

Journal Questions (Math Standards)

Question:

Audience:

1. What is this problem asking you to do?
2. What steps/information will be needed to solve the problem?
3. Solve and explain your method of solving the problem.
4. How can you relate this to the real world?
5. What tools would have been helpful in solving this problem and why?
6. How can you be more precise in your explanation, if possible?
7. What patterns do you observe, if any?
8. What conjectures can you make about possible shortcuts to solving a problem like this?

Figure 3.1 Standards for Mathematical Practices questions

Students' assignments were then analyzed using a grading rubric (Figure 3.2). The rubric was designed to not only determine mathematical correctness, but also the ability to communicate the use of previous knowledge and diverse methods of solving the given entry. A student's correct usage of vocabulary, pictures/graphs/tables, and connection to real life, are all valuable pieces of information the journal offers the teacher. The rubric also addresses the issue of being grammatically correct.

Entry #	Use of vocabulary 1 2 No Yes	Correct vocabulary use 1 2 3 No Partial Yes	Grammatically correct 1 2 3 < 50% 50% >50%	Mathematically correct 1 2 3 < 50% 50% >50%	Graph, Picture, Table 0 1 2 N/A No Yes	Real world connection 1 2 No Yes
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

Figure 3.2 Geometry Journal Rubric

Finally, a survey (Appendix A) was created with three main questions in mind. The first is to ascertain what the student feels contributed to their learning in Geometry. The second was to allow students to rate what methods of assessment they found to be important in a Geometry course. Lastly, and most importantly, they described what they gathered from the journals and how well they understood the purpose of the journals.

3.3 Process

The thesis was conducted while I was teaching at Louisiana School for the Agricultural Sciences (LaSAS), a Type II charter school (meaning it is within the parish school system) in the small town of Bunkie, Louisiana—rural Avoyelles Parish School System. The school caters to students who are at risk of dropping out of high school. Many of the students plan to attend a vocational school after graduation, and therefore LaSAS prepares many of them with classes such as Agriculture, Welding, and Family and Consumer Sciences. Though there are several different learning tracks students can choose to take, all students are required to take Geometry. In Geometry class, journal assignments were given on Wednesdays and students would have three-fourths of the class time to work on it and the final fourth was to review and sometimes the review was continued the next day. There were a total of ten journal entries assigned during the school year (Appendix B). The question was given to students as well as the writing guidelines I designed based on the CCSS Standards for Mathematical Practices; these guidelines are referred to as SMPQs (Figure 3.1). The students were required to use the eight questions to give a detailed explanation of how to solve the entry. Students were asked to only write their response on the right side of their journals and the left side was used for reflection (Ramos, 2010). The model for this type of journaling (also known as an interactive notebook) can be seen in Figure 3.3. I decided to use the right side for the personal responses and the left side for student reflection.

Left Side of the Interactive Notebook

Left Side – student input/application	Right Side – teacher input
• **Reorganize new information in creative formats** • **Express opinions and feelings** • **Explore connections to what has been learned** • **Apply skills learned (diagrams, analogies, political cartoons)** • **see examples below**	• Title and Unit pages • Unit Assignment Calendars • Class, reading, and discussion notes • Informative Handouts • Essays • Personal Responses

Figure 3.3 Left Side/Right Side Journal Model

The journals were given in three phases. In phase one, which covered only the first two journals, students were given the question and audience along with the SMPQs without ever showing them an example of journal writing. The purpose of doing this was to observe what type of work the students would produce when given only the guidelines. The next phase was to give students a detailed example (Appendix C). The example included both the “student’s” initial approach and his/her reflection after the question was discussed as a class. This gave students a chance to see how in depth answers could and should be. The third phase was to give the students a list of the criteria their journals were judged on. Clearly, in order to validate the effectiveness of my writing approach, I could have ascertained a control group of students. However, I decided against it because I might have caused this group to have life-long deficiencies in logical processing skill that the other group might have gained.

All students received participation points for doing the journals, but using the rubric allowed me to understand my students’ level of understanding and how they

reasoned (Figure 3.2). I wanted to assess their ability to reason and think mathematically along with their ability to arrive at the correct answer. With the previously mentioned as my goal, my role in the classroom had to be clearly defined. My role was to only facilitate the students' work by only giving the initial problem and prompting them as they worked through it (Cross, 2009). The students knew that I would only answer their questions with more questions; this in turn led to more collaboration amongst my students. This was an added bonus to the journal's effect. "Peer collaboration embodies a reciprocal process where each member has opportunities to share his or her thoughts and explore the reasoning of others" (Cross, 2009). Students began to practice mathematics as the CCSS Standards for Mathematical Practice wanted them to. Students reasoned with each other, they justified their own answers, modeled for their classmates how to work the problem and persevered in their attempt to solve the problems. However, the hard issues were getting student to a point where they could write those things on paper in a logical manner and filling in the gaps for those having trouble making connections. In the next chapter we will look at three journals questions and three different students' responses to each question. Students A, B, and C were chosen because of their varying levels of understanding. These journals reflect the progression of the three phases, and I hope, will provide a glimpse at the potential power of my journaling method.

Chapter 4: Results

4.1 Question One

Question one, as seen in figure 4.1, focuses on a student's understanding of the vocabulary, properties, and postulates concerning parallel lines (Boyd, 2005). The question is asking students to reason about why two angles formed in part by the same parallel line and are consecutive, may not have the characteristic of being consecutive interior angles that are supplementary. A student had to have knowledge of the exact definition of "consecutive interior angles" but what is more important is having an understanding of the importance of a transversal line in determining whether other characteristics (such as alternate interior angles, etc) of parallel lines hold true.

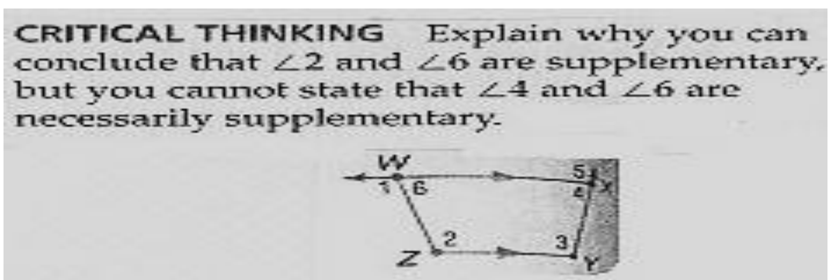


Figure 4.1 Question one

Journal Questions (Math Standards)

Question:

Audience:

1. What is this problem asking you to do?
2. What steps/information will be needed to solve the problem?
3. Solve and explain your method of solving the problem.
4. How can you relate this to the real world?
5. What tools would have been helpful in solving this problem and why?
6. How can you be more precise in your explanation, if possible?
7. What patterns do you observe, if any?
8. What conjectures can you make about possible shortcuts to solving a problem like this?

Figure 4.2 Standards for Mathematical Practice Questions (SMPQ)

Student A always attempted each problem and did his best to work through each SMPQ. For most journal entries Student A reflected on his work and made correction. However, Student A had a difficult time understanding the necessary mathematics and definitions to arrive at the correct answer. Figure 4.3 is Student A's attempt to solve question one.

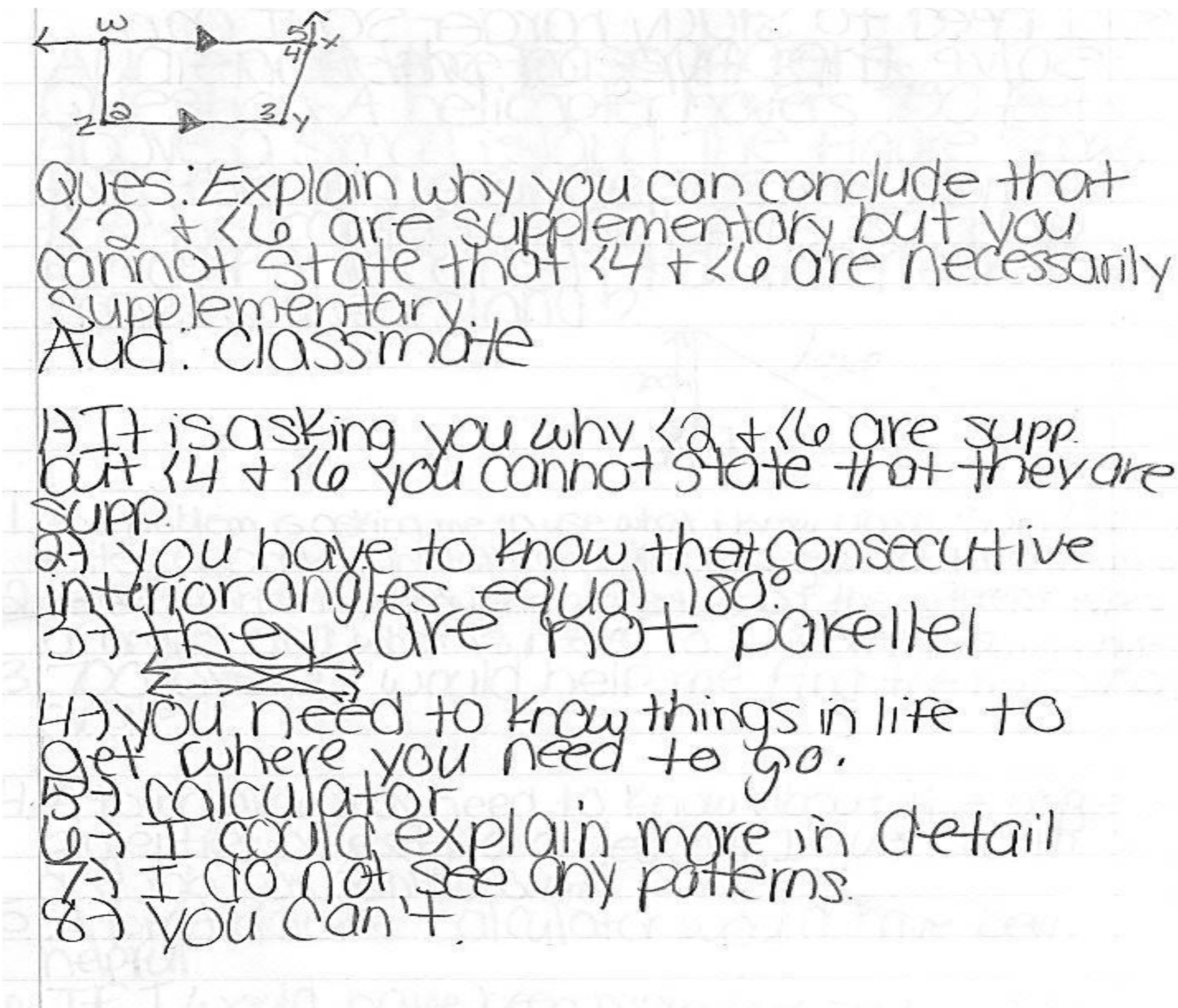
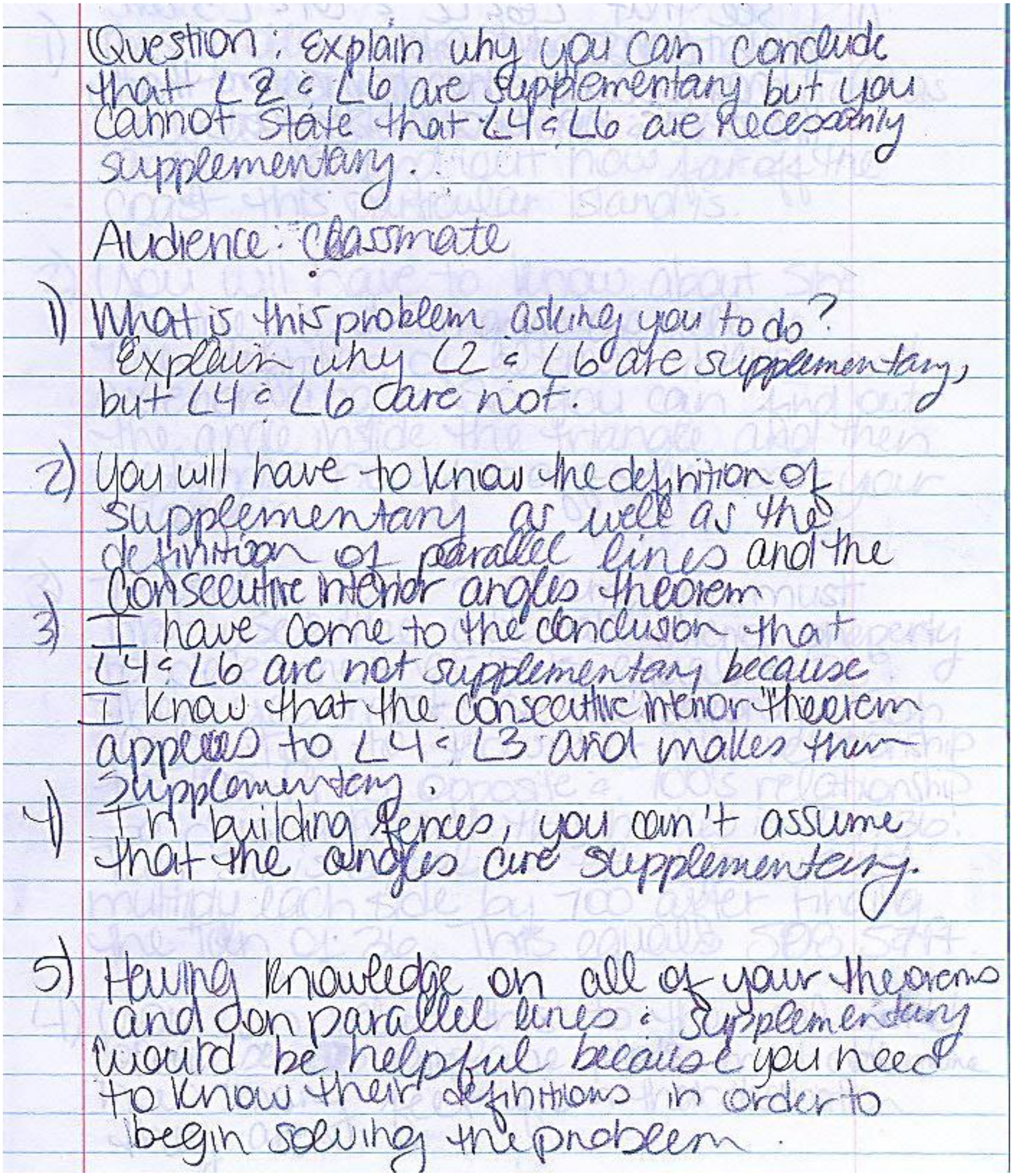


Figure 4.3 Student A Example 1

Student A in SMPQ 1 was able to restate what the problem asked him to do, but he did not do so in a way that would hint that he understood the question fully. He did use vocabulary for SMPQ 2, and it was a vital part to solving the problem. However, SMPQ 3 shows us that Student A did not know how to explain his reasoning and therefore could not solve the problem. Another thing to note is that Student A created a “new” math symbol to explain that two lines are not parallel; this hints to the idea that he does not have a clear understanding of vocabulary and symbolic representation of key elements in Geometry. This student has many of the pieces necessary to solve this problem, but was not able to argue them in a correct and logical sequence. There were also grammatical errors, mainly that complete sentences were not used. The answer given for SMPQ 4 was supposed to relate the problem to real life, but the connection here is unclear. This particular entry was done before a detailed example was given. However, what this entry says to me as a teacher is that there is not a clear understanding of terminology or concepts, but the willingness to persevere in solving is there.

Student B had a better understanding of the definitions and concepts. She was always on the right track with her reasoning, but did not always make the connection. Figures 4.4a and 4.4b show the work that student B completed on the same question. In SMPQ 1, Student B seems to show an understanding of what the question is asking. She continues in SMPQ 2 with the correct necessary vocabulary. In SMPQ 3 she demonstrates that she understands the meaning of the vocabulary she is using, but she does not define the vocabulary enough to make her argument concrete. As she points out in SMPQ 5, she does understand the importance of the vocabulary as a tool to solve

the problem. When asked if there are any patterns she identified that there are two pairs of consecutive interior angles.



Question: Explain why you can conclude that $\angle 2$ & $\angle 6$ are supplementary but you cannot state that $\angle 4$ & $\angle 6$ are necessarily supplementary.

Audience: Classmate

1) What is this problem asking you to do? Explain why $\angle 2$ & $\angle 6$ are supplementary, but $\angle 4$ & $\angle 6$ are not.

2) You will have to know the definition of supplementary as well as the definition of parallel lines and the consecutive interior angles theorem

3) I have come to the conclusion that $\angle 4$ & $\angle 6$ are not supplementary because I know that the consecutive interior theorem applies to $\angle 4$ & $\angle 3$ and makes them supplementary.

4) In building fences, you can't assume that the angles are supplementary.

5) Having knowledge on all of your theorems and on parallel lines & supplementary would be helpful because you need to know their definitions in order to begin solving the problem.

Figure 4.4a Student B Example 1

Her work would prompt me to ask her more questions about the definitions in order to determine how well she actually understands.

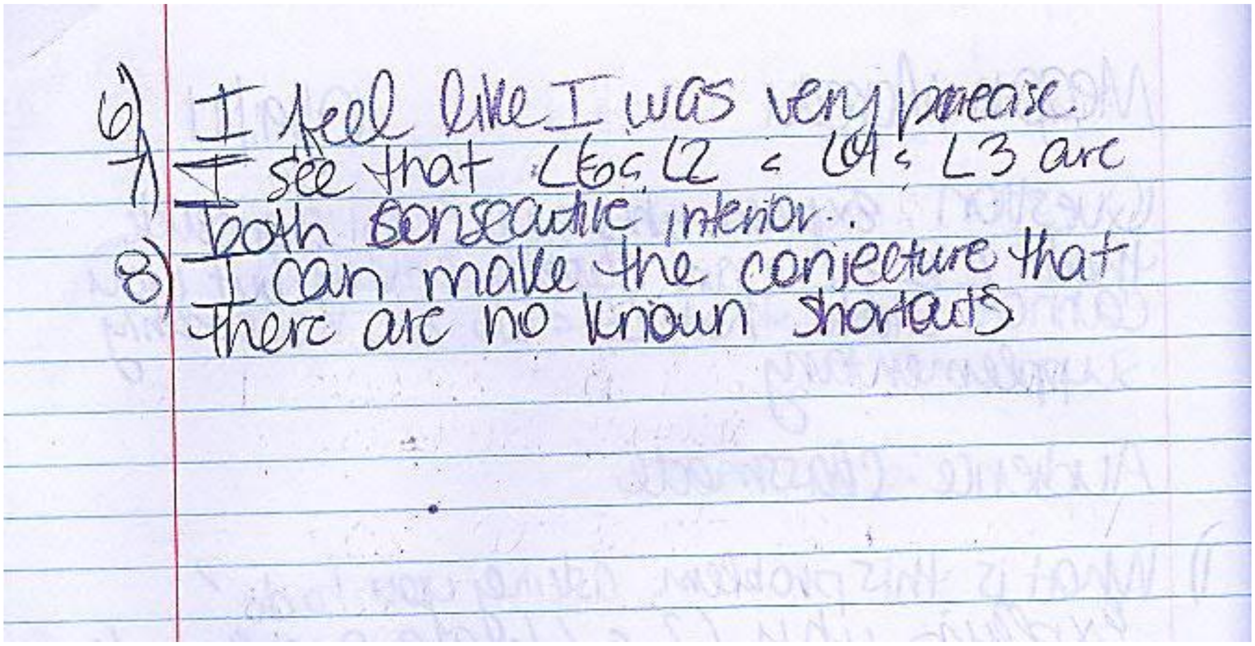
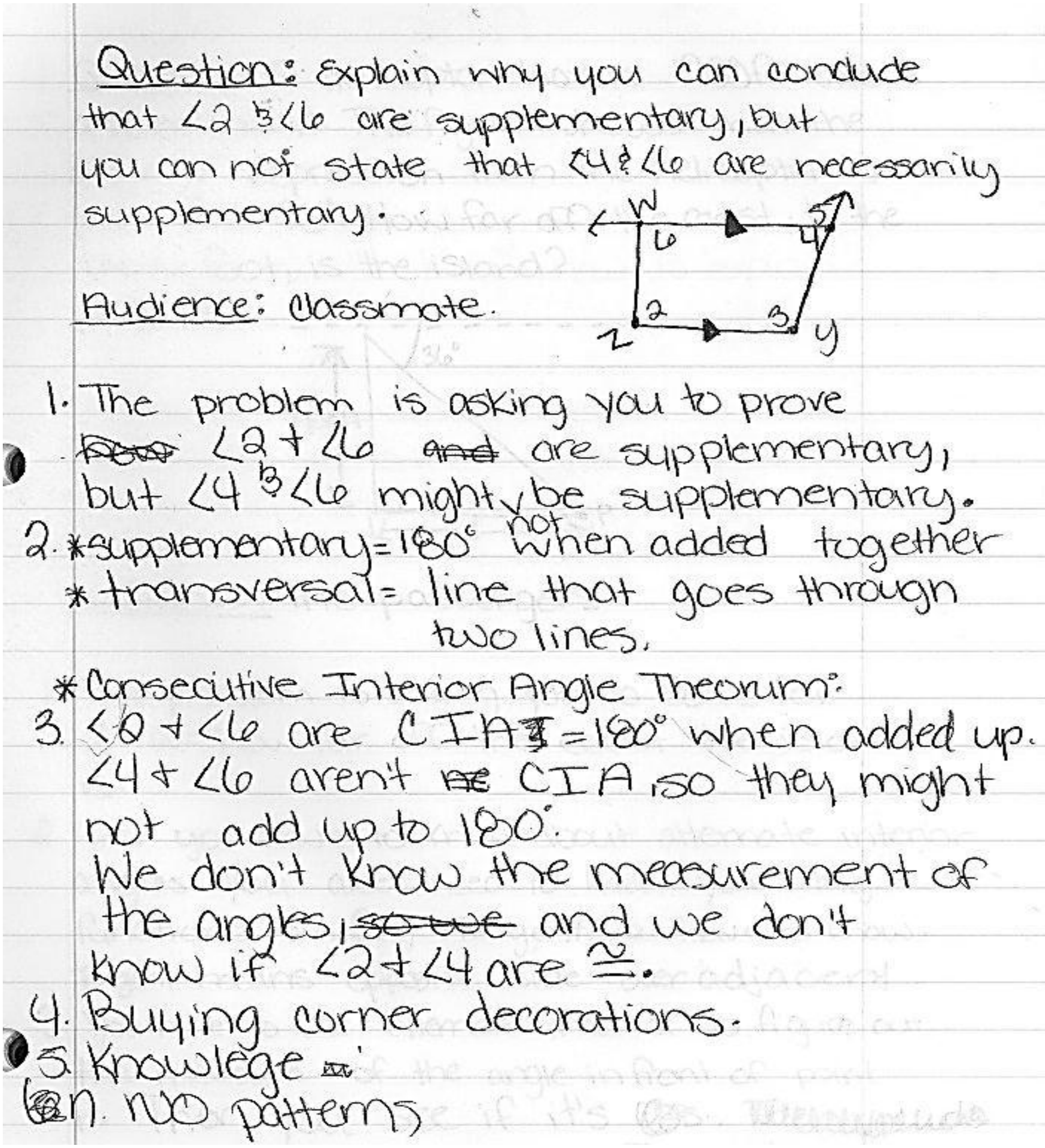

6) I feel like I was very precise.
7) I see that ∠6 & ∠2 & ∠4 & ∠3 are both consecutive interior.
8) I can make the conjecture that there are no known shortcuts

Figure 4.4b Student B Example 1 Continued

Student C also had trouble explaining the entry in its entirety; however she did realized the importance of the transversal line (figure 4.5). She explains that one pair of angles followed the definition of consecutive interior angles and the other pair did not. It is possible that Student C was not very detailed in her explanation because the audience was a classmate. Generally, when the audience was a classmate, students were allowed to make some assumptions about the audience's knowledge of definitions and such. Student C also paid attention to grammar as seen by her changing her word choice a few times. The connection to real life here is also unclear as seen with Students A and B. Student C ran out of time and was not able to complete this journal

entry. Student C's entry says to me the same thing student B's did, "review and question vocabulary," though she did show the most understanding of the three.



Question: Explain why you can conclude that $\angle 2$ & $\angle 6$ are supplementary, but you can not state that $\angle 4$ & $\angle 6$ are necessarily supplementary.

Audience: Classmate.

1. The problem is asking you to prove $\angle 2 + \angle 6$ ~~and~~ are supplementary, but $\angle 4$ & $\angle 6$ might not be supplementary.
2. *supplementary = 180° when added together
*transversal = line that goes through two lines.

*Consecutive Interior Angle Theorum:

3. $\angle 2 + \angle 6$ are CIA = 180° when added up. $\angle 4 + \angle 6$ aren't CIA, so they might not add up to 180°.
We don't know the measurement of the angles, ~~so we~~ and we don't know if $\angle 2 + \angle 4$ are $\cong$.
4. Buying corner decorations.
5. Knowlege
6. No patterns

Figure 4.5 Student C Example 1

Overall, each student persevered in solving even though there were different levels of understanding about question one. This entry was completed before the students received an example journal writing. I realized that I needed to review definitions with the students and it also gave me an idea of the detail needed in the example they would receive in the next phase.

4.2 Question Two

Question two, as seen in Figure 4.6, asks students to use their knowledge about ratios and right triangle to find the area of an inscribed shape (Boyd, 2005).

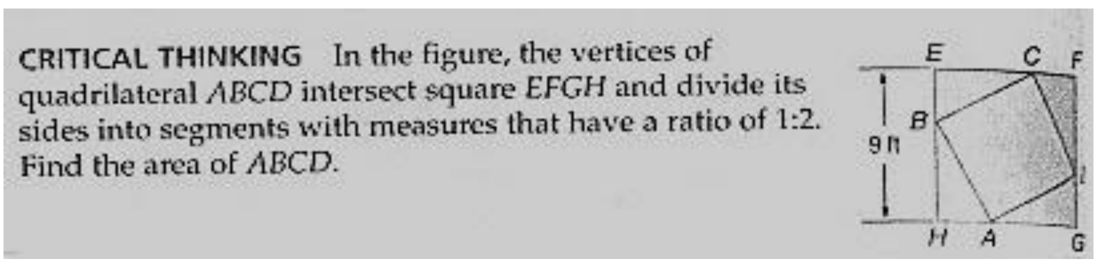

CRITICAL THINKING In the figure, the vertices of quadrilateral *ABCD* intersect square *EFGH* and divide its sides into segments with measures that have a ratio of 1:2. Find the area of *ABCD*.

Figure 4.6 Question 2

Journal Questions (Math Standards)

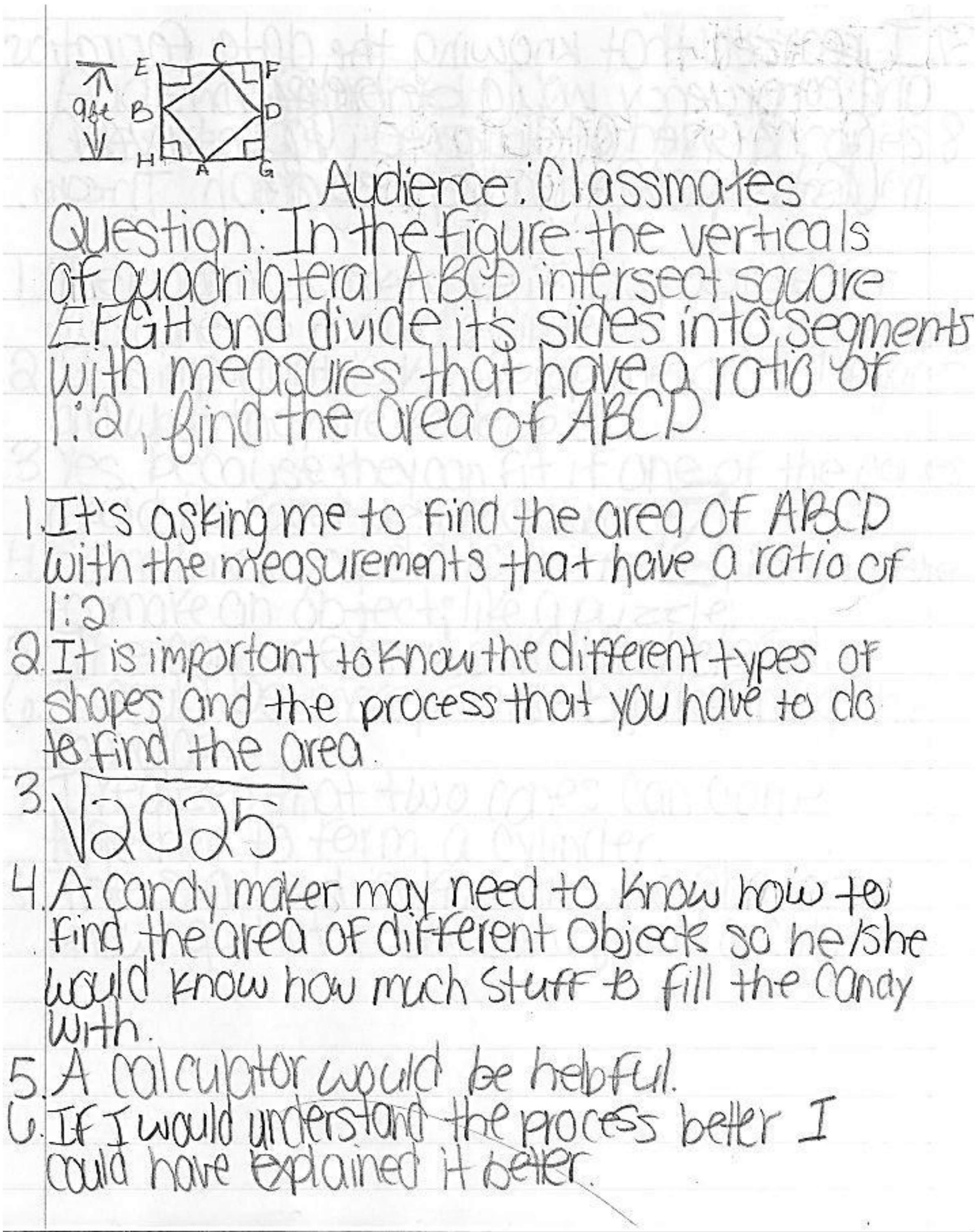
Question:

Audience:

1. What is this problem asking you to do?
2. What steps/information will be needed to solve the problem?
3. Solve and explain your method of solving the problem.
4. How can you relate this to the real world?
5. What tools would have been helpful in solving this problem and why?
6. How can you be more precise in your explanation, if possible?
7. What patterns do you observe, if any?
8. What conjectures can you make about possible shortcuts to solving a problem like this?

Figure 4.7 Standards for Mathematical Practice Questions (SMPQ)

Here students who did not immediately know what the measures of the legs of the triangles were would have wanted to set up an equation $x + 2x = 9$, where x is shortest leg and 2x is the other leg. Students would have then found $3x = 9$, therefore $x = 3$ and if $x = 3$ then $2x = 6$. Keeping in mind that EFGH is a square, so all sides are congruent and the angles of the square equal ninety degrees; students would have then gone to use the Pythagorean Theorem ($a^2 + b^2 = c^2$ where "a" and "b" are legs of the triangle and c is the hypotenuse) to find the hypotenuse of the triangles formed by the square and inscribed quadrilateral, which was found to be $\sqrt{45}$. Students would then realize that all the sides of quadrilateral ABCD are equal and therefore a rhombus. Using the formula for area of a rhombus (area = height x side) and after verifying that the height and side were congruent, students would find the area to be 45 ft^2. Another approach to solving this problem is again using $x + 2x = 9$ to find the triangles' lengths, but using the lengths to find the area of the triangle. Area = ½bh (that is half the base times the height), so we have Area = ½ x 6 x 3 = 9, then multiply the area of one triangle by four since there are four triangles, 9 x 4 = 36. From there find the area of the square by multiplying side times side or $side^2$ which equals 81. Finally, subtract the two areas 81 – 36 = 45. Students worked diligently on this entry and when time was up they did not ask for the answer, but a hint to find the answer. Though all of them received the verbal hint only a few wrote it down and used it and this will be clearly illustrated in the students' work.



Audience: Classmates

Question: In the figure the verticals of quadrilgteral ABCD intersect sqaure EFGH and divide it's sides into segments with measures that have a ratio of 1:2, find the area of ABCD

1. It's asking me to find the area of ABCD with the measurements that have a ratio of 1:2
2. It is important to know the different types of shapes and the process that you have to do to find the area.
3. $\sqrt{2025}$
4. A candy maker may need to know how to find the area of different objeck so he/she would know how much stuff to fill the candy with.
5. A calculator would be helpful.
6. If I would understand the process better I could have explained it better.

Figure 4.8a Student A Example 2

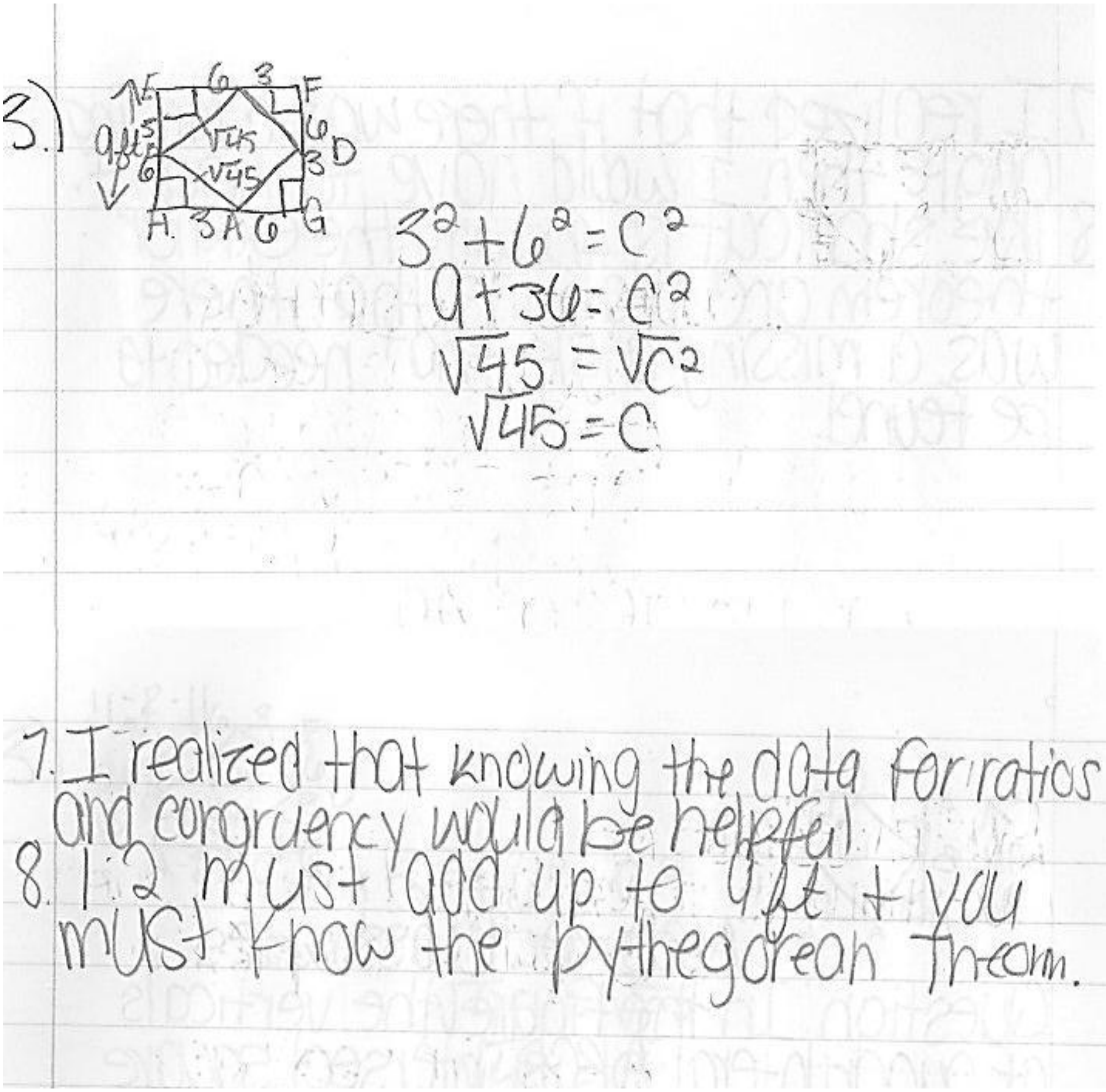


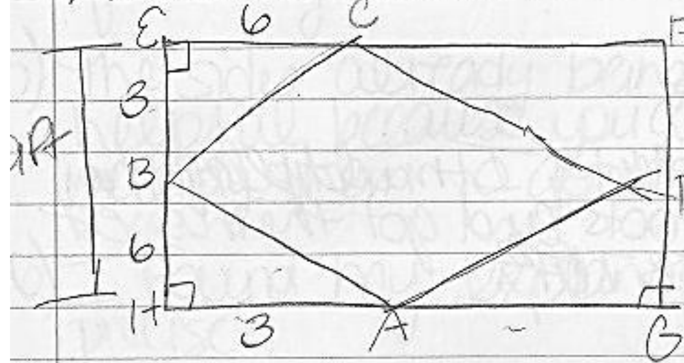
Figure 4.8b Student A Example 2 Reflection

Student A did improve in grammar and in formatting his journal, but at first glance it appears he did not understand how to answer. As seen in Figure 4.8a and SMPQ 1, student A did not understand the question being asked. In Student A's SMPQ 2 he mentions the importance of "[knowing] the different shapes and the process that you have to do to find area," this statement shows that he understands he needs to use an

area formula, but does not explicitly say which formula to use. No work is shown to support his answer and he could not make a clear connection to real life, but his answer is correct. If we take the square root of 2025 we find that it equals 45, the correct answer. As we look at Figure 4.8b, his reflection shows us that he is beginning to see the importance of showing his work. In his reflection of SMPQ 8, which he originally did not attempt, he infers that knowing the correct way to solve the problem is a shortcut, since SMPQ 8 asks for conjectures about possible shortcuts.

For Question two, Student B showed a great deal of understanding (Figure 4.9a and 4.9b). She explained both SMPQ 1 and 2. On SMPQ 3 she did not show us how she found the measures 3ft and 6ft, but the audience happens to again be a classmate, which may account for her lack of detail. She also made a calculation error when finding the area, but she addresses it in her reflection, Figure 4.9c. Her connection to real life here makes sense and is applicable to the problem—someone trying to put carpeting in a specific shape in a square room. She was confident about the precision of her answer and the triangles lengths being same was a pattern she notices. She has shown improvement in her writing and explaining. She addresses her audience and is explaining to them how to work the problem instead of only explaining how she works the problem. I believe that this subtlety changes how the student will communicate her ideas. The focus shifts from explaining what you have learned to teaching and helping someone else to learn.

Audience: Classmate



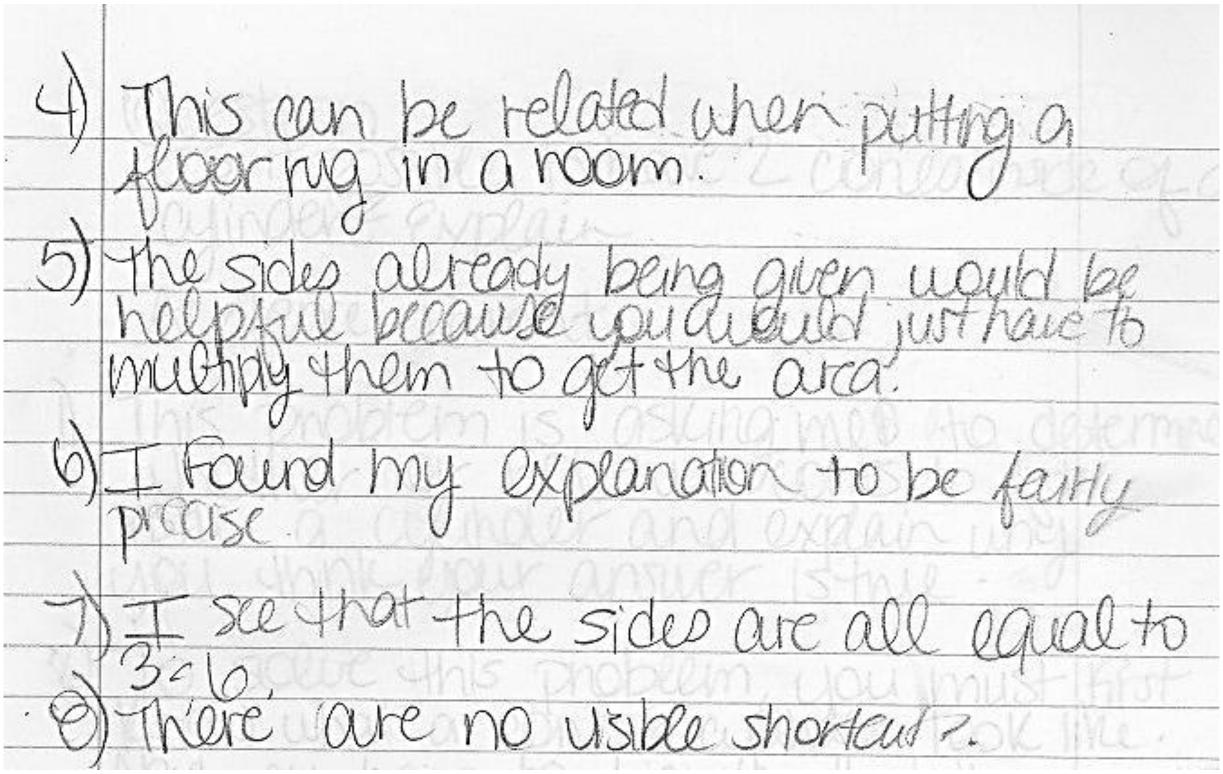
In the figure, the vertices of quadrilateral ABCD intersect square EFGH & divide its sides into segments with measures that have a ratio of 1:2. Find the area of ABCD.

1) This problem is asking me to find the area of ABCD.

2) You must use information about ratios & Pythagorean theorem to find out the Area.

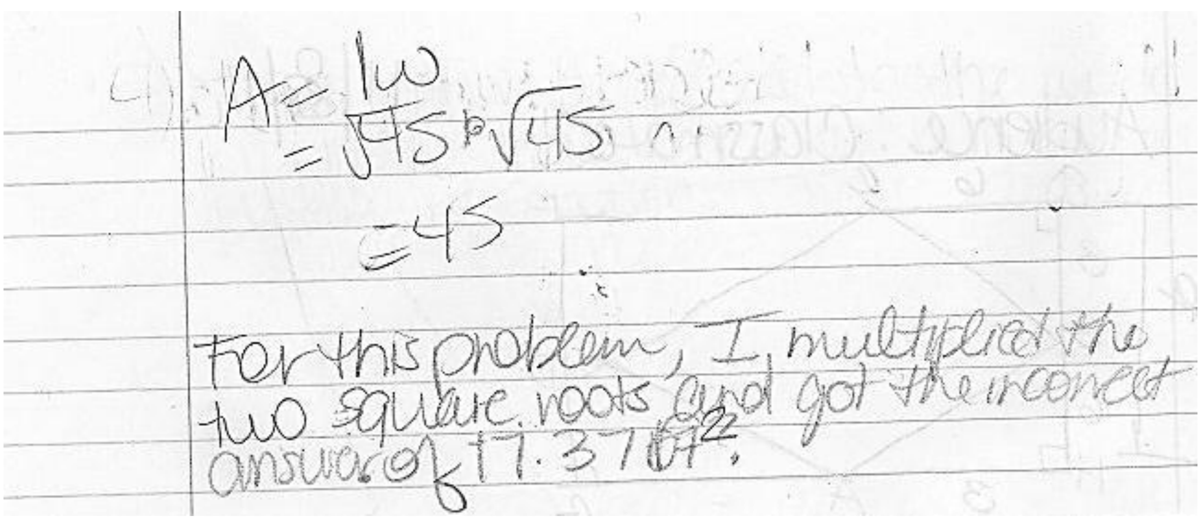
3) You must first understand that the 1:2 ratio has to equal to 9ft, which makes it a 3:6 ratio. You have to find the hypotenuse using the Pythagorean theorem which is $\sqrt{45}$ for both triangles. To find out the area of the rectangle, use the formula l·w which makes the area to be 13.37ft².

Figure 4.9a Student B Example 2



4) This can be related when putting a flooring rug in a room.
5) The sides already being given would be helpful because you would just have to multiply them to get the area.
6) I found my explanation to be fairly precise
7) I see that the sides are all equal to $3\sqrt{6}$,
8) There are no visible shortcuts.

Figure 4.9b Student B Example 2 continued



$A = lw$
$= \sqrt{45} \cdot \sqrt{45}$
$= 45$

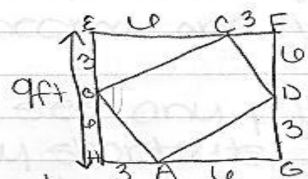
For this problem, I multiplied the two square roots and got the incorrect answer of 17.3712.

Figure 4.9c Student B Example 2 Reflection

For question two, student C uses the information from the hint to solve the entry (Figures 4.10a and 4.10b). Though, she misspells Pythagorean and does not

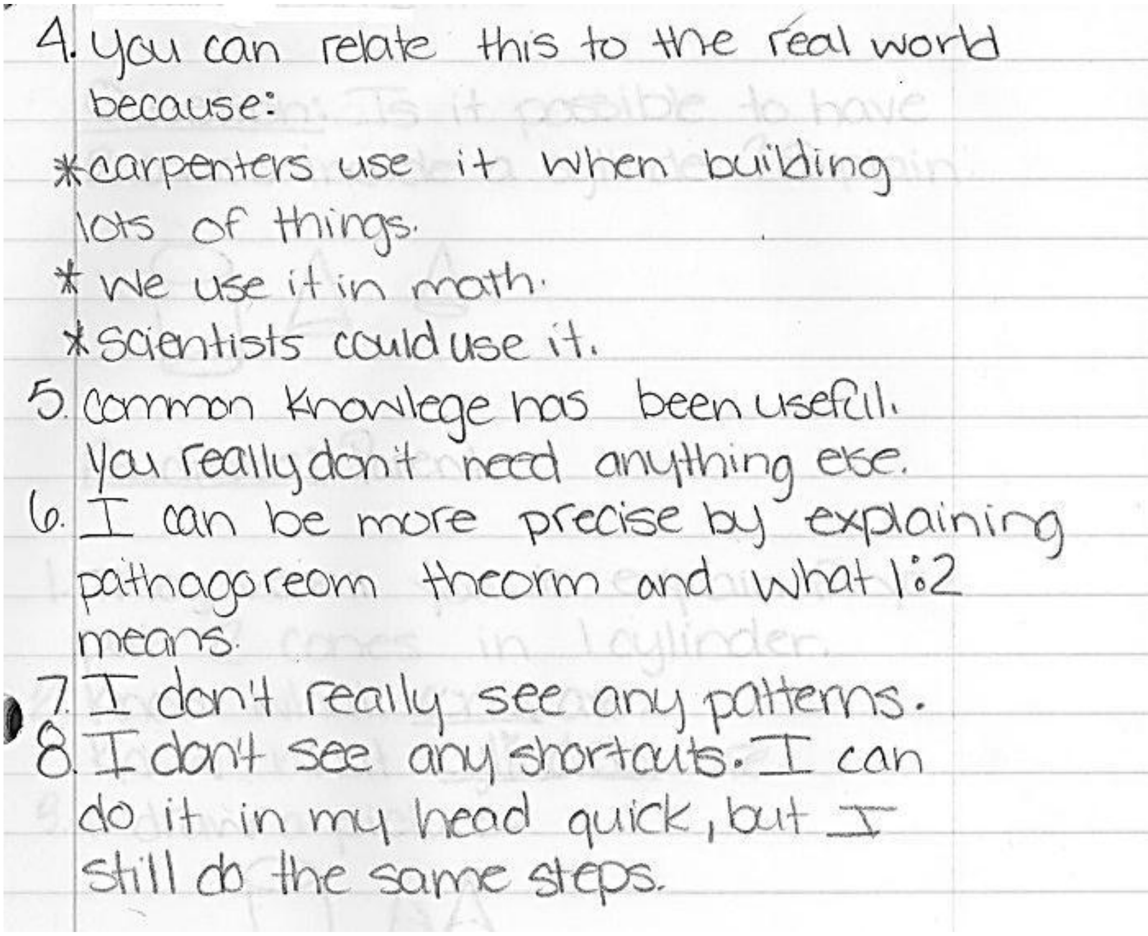
completely spell the word proportion on SMPQ 2, there are no errors in her solution. In SMPQ 3 she could have demonstrated how to use the ratio to find the lengths of the legs, but mentions this in SMPQ 6. She understood that she could have been more precise with her explanation. She makes connections to real life and is confident about her answers and her knowledge. This is the confidence students need to have in their explanations.

Question:
In the figure, the vertices of quadrilateral ABCD intersect square EFGH and divides its sides into segments with measures that have a ratio of 1:2. Find the area of ABCD.
Hint: ① 1:2 must add up to be 9ft.
② Pathagoreum Theorum
picture:



Audience: classmates
1. The problem is asking us to find the area of □ABCD.
2. steps taken:
① ratios + pro
② pathagoreum theorm
③ multiply l×w = Area of □
3. First use proportion to find the 2 sides of a Δ. Then use pathagoreum theorm. $6^2+3^2=6.7^2$ Then square, 6.7^2 ft because that will give you the area of a square (all sides are even on a square). You should get 45′ ft²

Figure 4.10a Student C Example 2



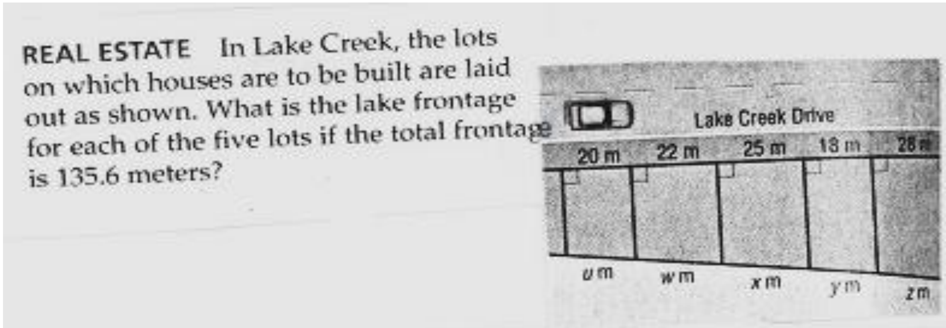
4. You can relate this to the real world because:
*carpenters use it when building lots of things.
* We use it in math.
*scientists could use it.
5. common Knowlege has been usefull. You really don't need anything else.
6. I can be more precise by explaining pathagoreom theorm and what 1:2 means.
7. I don't really see any patterns.
8. I don't see any shortcuts. I can do it in my head quick, but I still do the same steps.

Figure 4.10b Student C Example 2 continued

Question two showed improvements in understanding and grammar, in some way for all three students. The students were more detailed with their explanations and supported their work better, with some exception.

4.3 Question Three

The final question that we will explore is question three (Boyd, 2005). In question three, as seen in Figure 4.11, students are being asked to use their knowledge of proportions. In this problem the total frontage length is given and the five individual lots' street lengths are given. Students are asked to find the frontage length for the five lots.



REAL ESTATE In Lake Creek, the lots on which houses are to be built are laid out as shown. What is the lake frontage for each of the five lots if the total frontage is 135.6 meters?

Figure 4.11 Question 3

Journal Questions (Math Standards)

Question:

Audience:

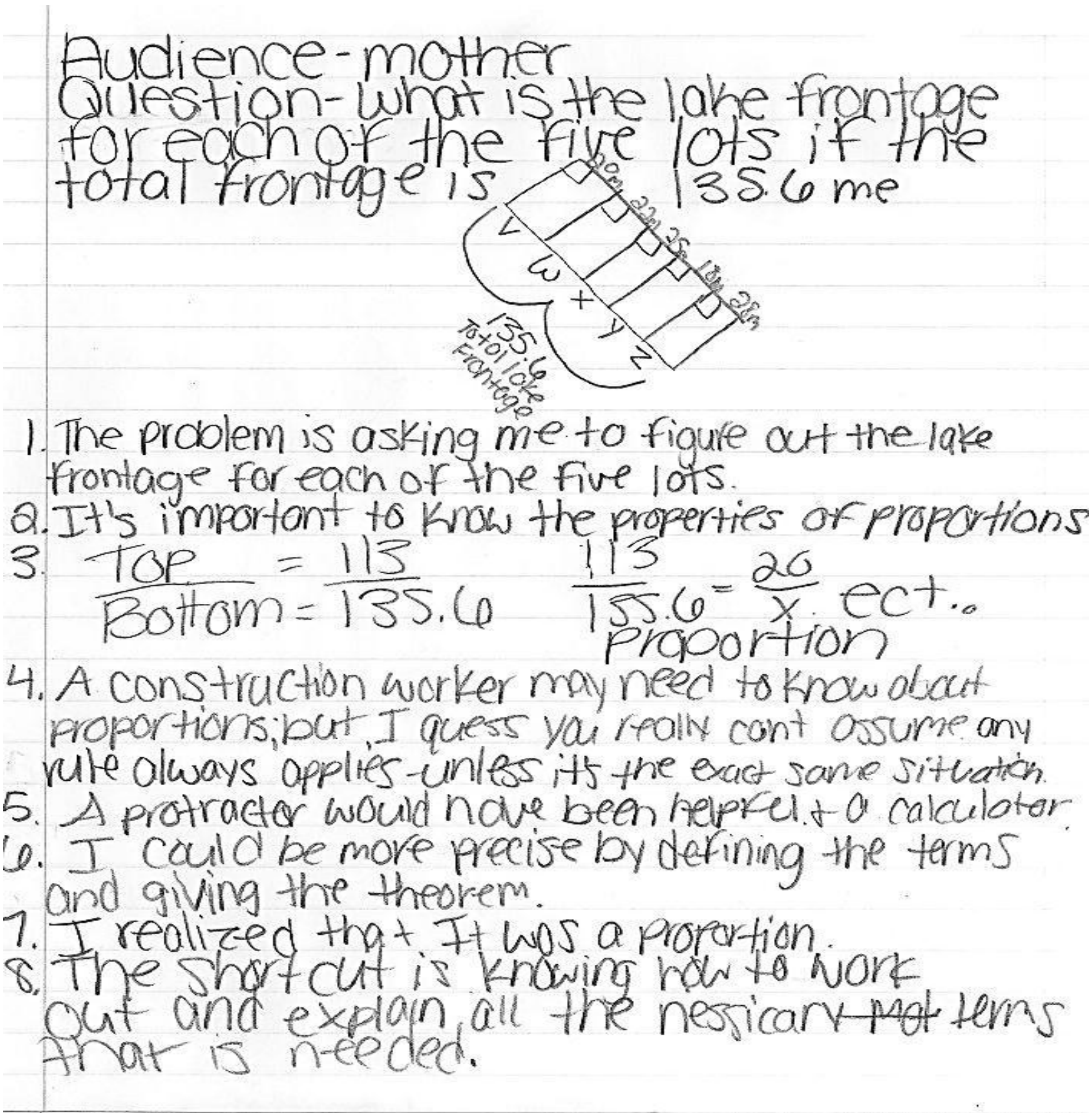
1. What is this problem asking you to do?
2. What steps/information will be needed to solve the problem?
3. Solve and explain your method of solving the problem.
4. How can you relate this to the real world?
5. What tools would have been helpful in solving this problem and why?
6. How can you be more precise in your explanation, if possible?
7. What patterns do you observe, if any?
8. What conjectures can you make about possible shortcuts to solving a problem like this?

Figure 4.12 Standards for Mathematical Practice Questions (SMPQ)

To solve this entry, students would start by adding all of the street lengths together. They can then create a ratio of $\frac{total\ street\ length}{total\ frontage\ length} = \frac{113\ meters}{135.6\ meters}$. Students would then proceed to use this ratio to set up proportions for the five lots with solutions being u = 24m, w = 26.4m, x = 30m, y = 21.6m, and z = 33.6m. Some students may have wanted to prove proportionality before ever solving. In this case students can use similar triangles to prove that the leg of the triangle is the same length as the street side of the lots, which then would make the hypotenuse of the triangles proportional to the street side of the lots, an example of this solution can be seen in appendix D. In question three, the audience here is the student's mother; a person who should have

some knowledge of proportions, but is far removed from them and needs a detailed explanation. This problem was given to students after they knew what the rubric analyzed. The answers for question three should be more thorough and have correct vocabulary usage, seeing as how, students knew what to expect.

In both SMPQ 1 and 2 Student A shows that he understands the problem, though he could have further explained the steps in SMPQ 2 (Figure 4.13a). He demonstrates in SMPQ 3 that he knows how to set up the problem and even found the first lot frontage length, but feels that there is no need to complete the other four. Not completing the problem, of course, is not ideal, but if we take audience into account it may be the he reason he did not finish. His connection to real life if much better than it has been previously, but a protractor is not a necessary tool for this problem as he states in SMPQ 5. This shows that there are still some missing pieces in either his understanding of this problem or his understanding of the purpose of a protractor. He also states that he could have been more precise by actually “defining the terms” which I believe to mean solving the other four proportions. The pattern that he discovers is that using a proportion to solve is repeated several times. There are some misspellings and subject-verb agreement problems, but he is confident that the short cut is: knowing how to work the problem. He understood this problem, whereas he had issues with some of the other journal entries. His work on this problem is not thorough, because he did not completely solve. However, his work has greatly improved and he was able to demonstrate how to solve.


Audience-mother
Question- What is the lake frontage for each of the five lots if the total frontage is 135.6 me

1. The problem is asking me to figure out the lake frontage for each of the five lots.
2. It's important to know the properties of proportions
3. $\frac{\text{Top}}{\text{Bottom}} = \frac{113}{135.6}$ $\frac{113}{135.6} = \frac{26}{x}$ ect.. proportion
4. A construction worker may need to know about proportions; but I guess you really can't assume any rule always applies-unless it's the exact same situation.
5. A protractor would have been helpful + a calculator.
6. I could be more precise by defining the terms and giving the theorem.
7. I realized that It was a proportion.
8. The shortcut is knowing how to work out and explain all the nessicary ~~mat~~ terms that is needed.

Figure 4.13a Student A Example 3

In his reflection he duplicates his work, but still does not show the work for the other four lots, Figure 4.13b.

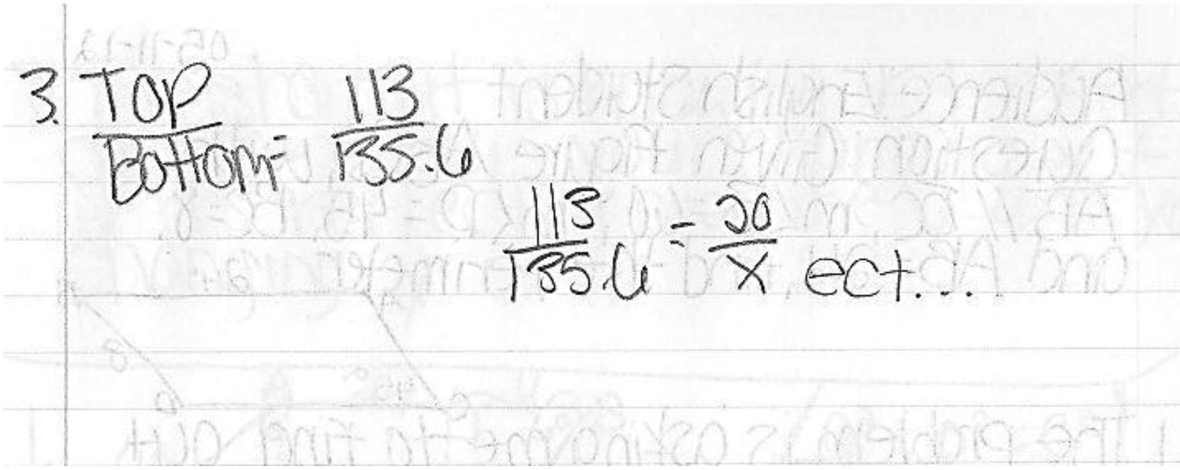
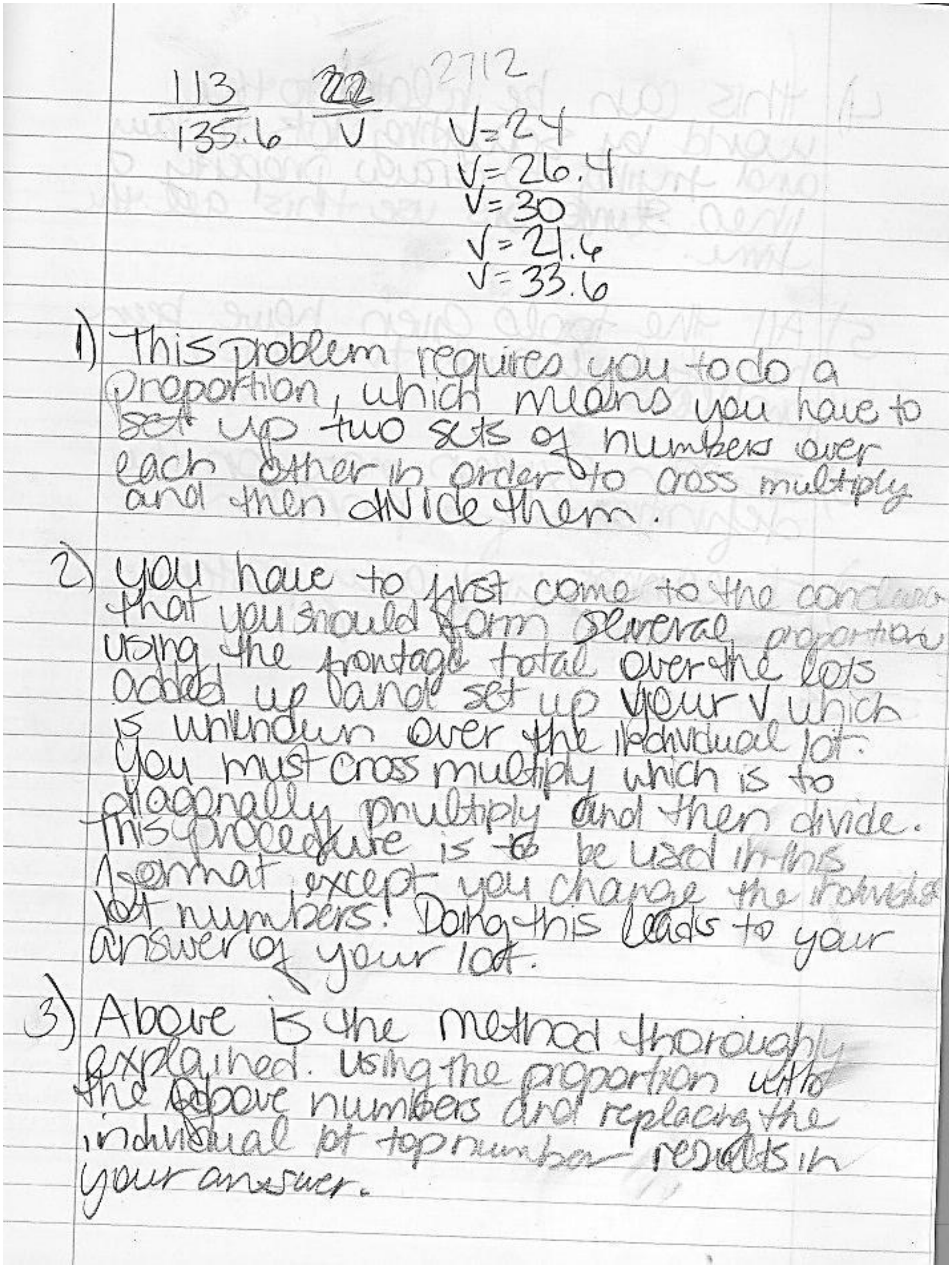


Figure 4.13b Student A Example 3 reflection

Student B also shows that she understand the problem and seen in Figures 4.14a and 4.14b. Like Student A, Student B does not show all of her, but she does at least give the other solutions. Student B explains the process of finding a missing part of a proportion in both SMPQ 1 and 2, then refers to her explanation in SMPQ 3. In SMPQ 2 there is a mistake in her wording, but the work at the top illustrates that what she meant to write was “the total lot over the frontage total.” The question already relates to real life and she realized this and stated it as a connection, but she also relates it surveyors. Unlike Student A, she does not see a pattern; I believe that this is due to the students’ differing ideas on what constitutes as a pattern. When differences in students’ work and ideas occurred it usually led to class discussion. Student B does not answer SMPQ 8; it is possible that she ran out of time or simply did not notice any short cuts to this particular question. Student B maintains her focus on the audience and writes her explanation as if the person has limited understanding. An example of this is seen on SMPQ 2, “you must cross multiply which is to diagonally multiply and then divide.” She knows her audience may not understand her terminology.



$\frac{113}{135.6}$ $\frac{22}{V}$ 2712

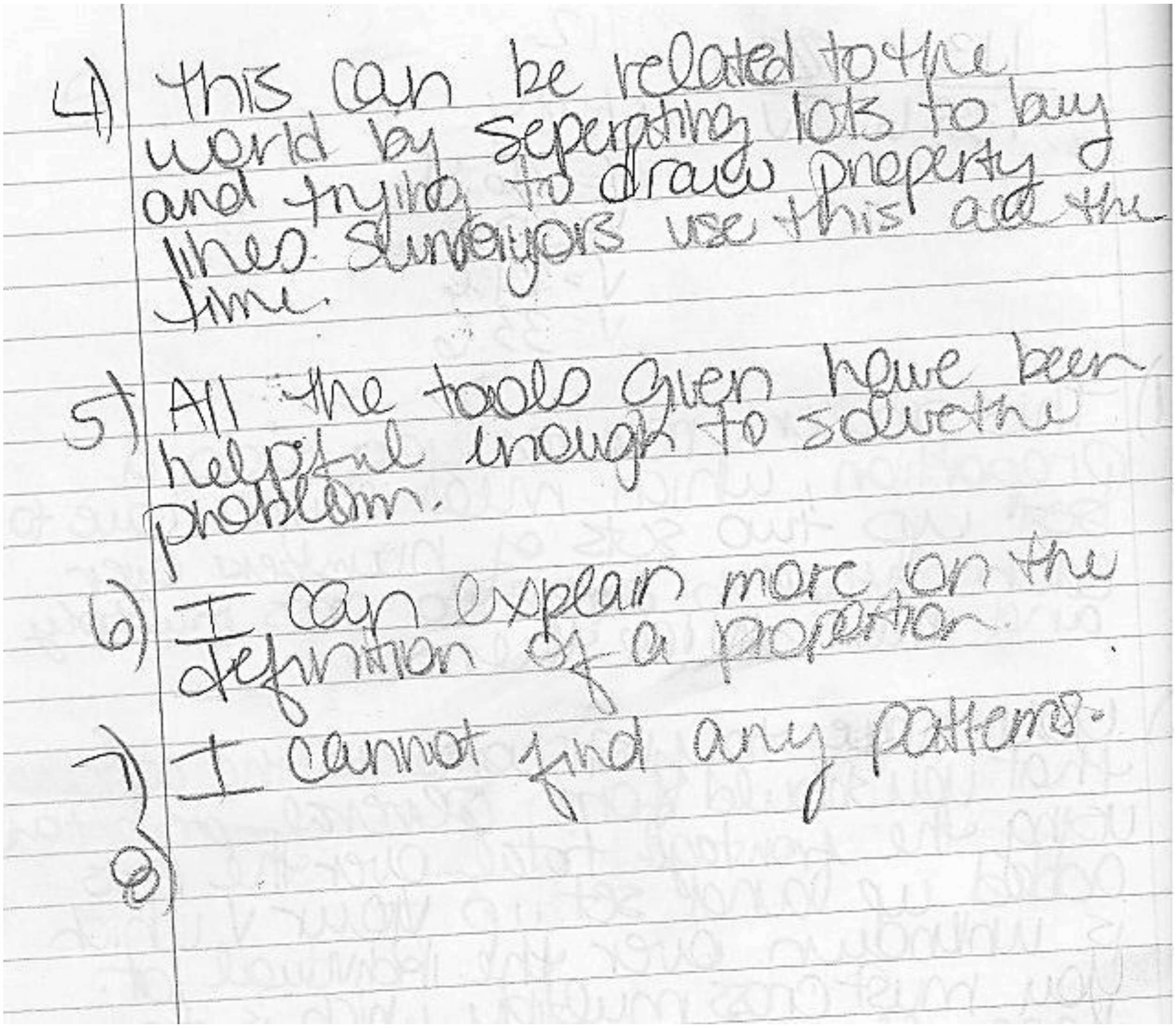
V=24
V=26.4
V=30
V=21.6
V=33.6

1) This problem requires you to do a proportion, which means you have to set up two sets of numbers over each other in order to cross multiply and then divide them.

2) you have to first come to the conclusion that you should form several proportion using the frontage total over the lots added up and set up your V which is unknown over the individual lot. You must cross multiply which is to diagonally multiply and then divide. This procedure is to be used in this format except you change the individual lot numbers! Doing this leads to your answer of your lot.

3) Above is the method thoroughly explained. Using the proportion with the above numbers and replacing the individual lot top number results in your answer.

Figure 4.14a Student B Example 3


4) this can be related to the world by seperating lots to buy and trying to draw property lines. Surveyors use this all the time.

5) All the tools given have been helpful enough to solve the problem.

6) I can explain more on the definition of a proportion.

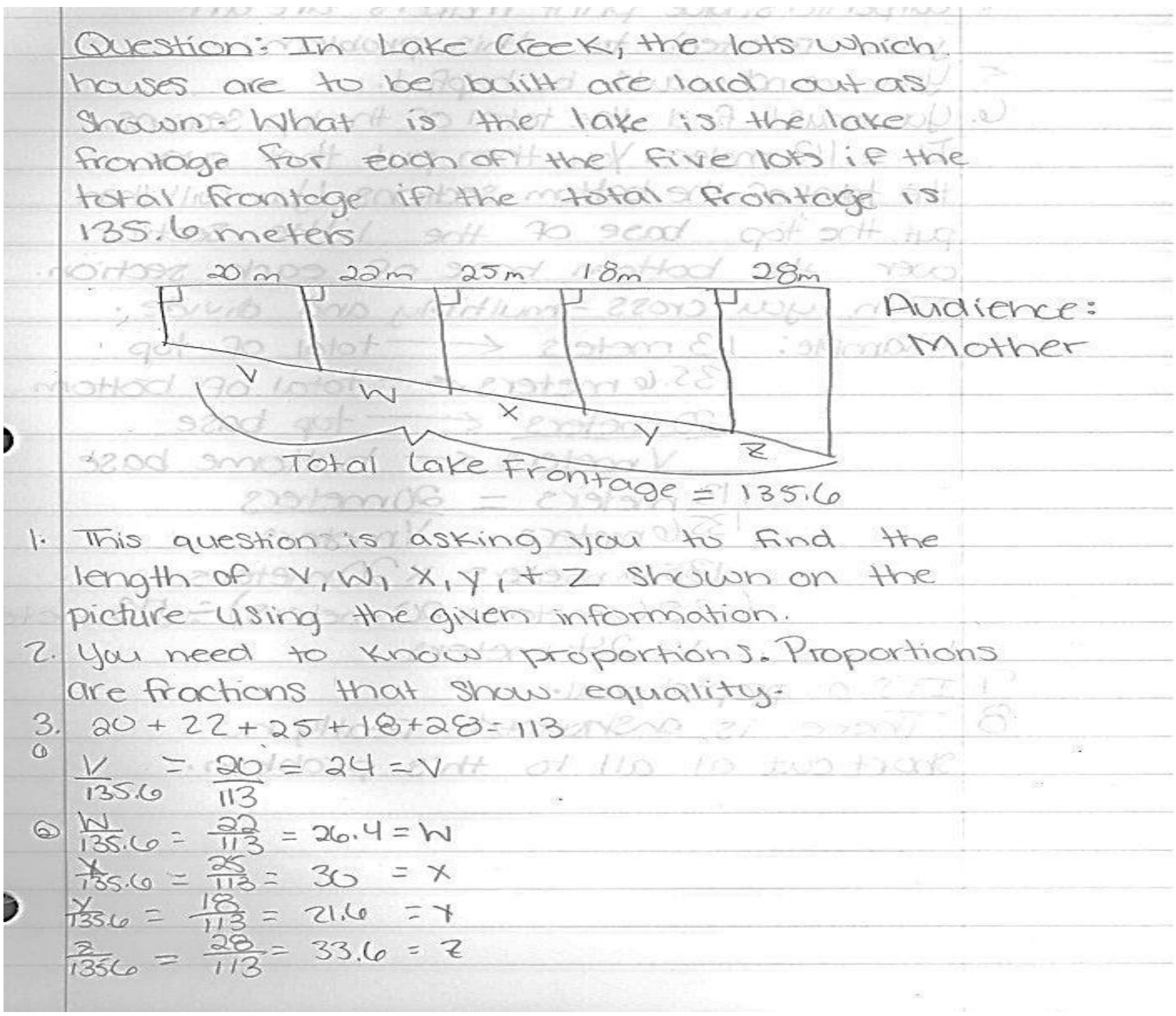
7) I cannot find any patterns.

8)

Figure 4.14b Student B Example 3 continued

Student C's approach to this problem was different than the other two students as seen in Figures 4.15a and 4.15b. Her answer to SMPQ 1 shows that she completely understands the question and in SMPQ 2 she describes what is necessary to solve and gives a definition. In SMPQ 3, student C sets up her proportions to be $\frac{lot\ frontage\ length}{total\ frontage\ length} = \frac{lot\ street\ length}{total\ street\ length}$, this approach then led to questions from other classmates about how she was able to get the same answer. In SMPQ 5 she

explains that the only tool you need is your mind, because she feels that it is common sense for their particular grade level. When asked about being precise, Student C explains another method of solving the problem, which is the same method that Students A and B used. When a student is able to demonstrate multiple methods of reaching a solution then the student fully understands the big concept or idea. Student C, like student A, believed that the repeated use of proportions or the fact that all of the lots were proportional to the totals were patterns in the question.


Question: In Lake Creek, the lots which houses are to be built are laid out as shown. What is the lake is the lake frontage for each of the five lots if the total frontage if the total frontage is 135.6 meters

1. This question is asking you to find the length of V, W, X, Y, + Z shown on the picture using the given information.
2. You need to know proportions. Proportions are fractions that show equality.
3. 20 + 22 + 25 + 18 + 28 = 113

① $\frac{V}{135.6} = \frac{20}{113} = 24 = V$

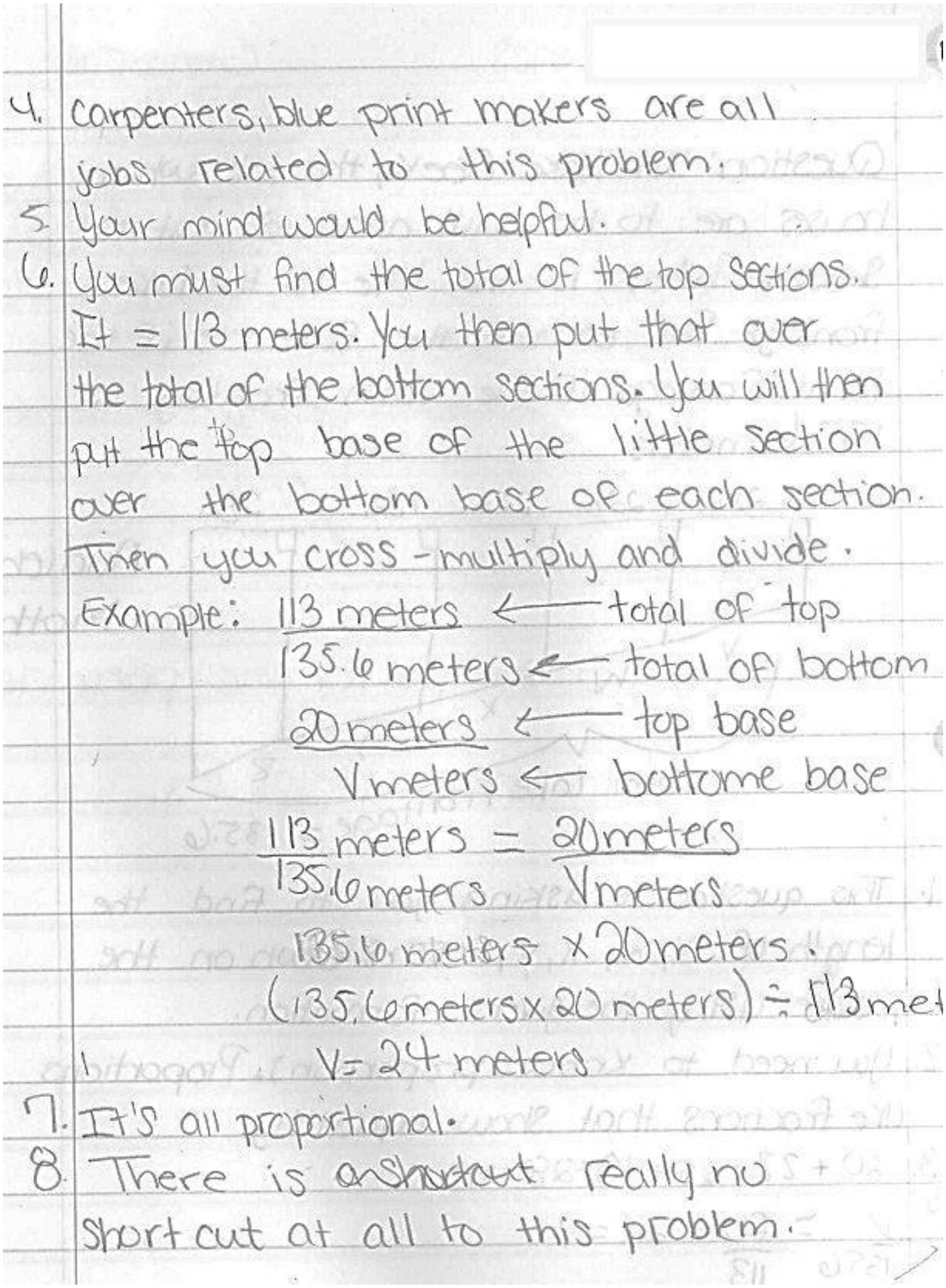
② $\frac{W}{135.6} = \frac{22}{113} = 26.4 = W$

$\frac{X}{135.6} = \frac{25}{113} = 30 = X$

$\frac{Y}{135.6} = \frac{18}{113} = 21.6 = Y$

$\frac{Z}{135.6} = \frac{28}{113} = 33.6 = Z$

Figure 4.15a Student C Example 3



4. carpenters, blue print makers are all jobs related to this problem.
5. Your mind would be helpful.
6. You must find the total of the top sections. It = 113 meters. You then put that over the total of the bottom sections. You will then put the top base of the little section over the bottom base of each section. Then you cross-multiply and divide.

Example: $\frac{113 \text{ meters}}{135.6 \text{ meters}}$ ← total of top / ← total of bottom

$\frac{20 \text{ meters}}{V \text{ meters}}$ ← top base / ← bottome base

$$\frac{113 \text{ meters}}{135.6 \text{ meters}} = \frac{20 \text{ meters}}{V \text{ meters}}$$

135.6 meters × 20 meters

(135.6 meters × 20 meters) ÷ 113 met

V = 24 meters

7. It's all proportional.
8. There is ~~a shortcut~~ really no short cut at all to this problem.

Figure 4.15b Student C Example 3 continued

Overall, question three showed great progress in all three students. The students all wrote enough information to make it possible for another person with some knowledge of proportions to follow it. The students were all able to make good connections to real life and all of them recognized areas they could have been more precise in. One even demonstrated multiple methods for solving, which shows a true understanding since, “mathematical problems can have one right answer, but usually there are many ways to find the answer” (Hiebert & Stigler, 1999).

4.4 Geometry Survey

The Geometry Student Survey, as seen in appendix A, allowed the students to rate their experience in Geometry as a whole and more specifically, the journals. There were ten areas the students were asked to rate with five being the highest and one being the lowest. In the section dealing with effects on student learning, when asked how much the *teacher’s enthusiasm* affected their learning thirty-nine out of forty-five students rated it as five. This means that students feel that they learn more from a teacher that enjoys the subject and communicates that enjoyment of the subject to students. Of the six that did not rate it as a five, four rated it either a four or three; leaving only two student who found it not important to his/her learning. Another interesting finding is that only six out of forty-five students ranked *previous knowledge* as a five. The other ranking vary greatly from one to four. Students do not feel that their previous knowledge aided them much in Geometry, however in order for them to make many of the connections there had to be some previous knowledge at work. This part of the survey served mostly for my own knowledge; however I felt that this information was something that other teachers would appreciate. The data from the

Geometry journals section of the surveys can be found in appendix A. It depicts the mean and standard deviation of students answers in this section; the data is given in two tables: by student and by question

The students' thoughts about the journals tended to be a love/hate feeling. Students understood that the journals required them to give a more detailed response, but they did not like the rigor of the problems. A student writes, "Before journals, I couldn't explain math problems into words. Now I can," when asked *how did the journals help you write a logical response to the questions asked.* To the same question, a second student answers, "It helped me write a logical response because they related to the real world, which is easier to explain." This statement shows the importance of CCSS math standard four, model with mathematics. According to the CCSS, when further describing standard four, "Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (National Governors Association Center for Best Practices, 2010c). Student B replied, when asked *what aspect of the journals aided you in writing a logical response,* that it was "being able to have a format." Having a template for students to follow as they answer helps them realize what things are necessary in completely explaining an answer. To the question, *how did reflecting (left-side of notebook) help you better understand the problem and complete your journal,* a student writes, "It helped me understand where I went wrong and how to fix it." It is extremely important for students to not only know their answer is wrong, but to also know what makes it wrong and how to make it right. I explained these journals to my students as puzzles they needed to solve. With this in mind, many of them developed a competitive nature

with the entries. The students did not want to entry to “win,” so they were anxious to know the way to the right answer, not simply the answer itself. When asked for *other comments*, many of the students made references to summer break and me teaching them next year along with other nice things, but one student writes, “I don’t like it (the journals)!!! I just don’t like to write!!!” Herein lies the challenge of the CCSS; students, like it or not, will be required to write logical response and will need help doing it. To the same question, another student writes, “the journals are confusing, but the point is to make your mind wonder.” I feel that this comment sums up the feelings of most of the students. They did not enjoy having to struggle through the problems, but they understood the purpose, which was to “make [their minds] wonder” or think deeper about the problems, the steps it takes to solve them, and how to explain their work to others.

Chapter 5: Concluding Thoughts

5.1 Research

It is evident that there must be change in the United States education system. The Common Core State Standards are an attempt to bring about the necessary change to make our students career and college ready. The CCSS requires that students be able to explain and justify their work. By focusing on the CCSS Standards for Mathematical Practices it is possible to create a tool that will help to develop mathematical thinkers, that is, students who have the ability to logically reason. The purpose of this research was to design a template using the CCSS Standards for Mathematical Practice to aid both teachers and students in developing mathematical thinkers. When the work of students A, B, and C was analyzed there was obvious change and growth both in how they explained their reasoning and how correct it was. The students, as seen in the survey responses, understood what the journals were designed to do and many of them saw the benefit of having a writing template. Within the same year I implemented the journals, our school's score on the Geometry End-of-Course test increased by fourteen percent from the previous year. Overall, though it cannot be said that the research is strong enough to stand alone and defend the template, it does show that three students, who represent a class of forty-five, with varying levels of understanding have all improved their mathematical writing and reasoning abilities. I do believe that this template should be tested to further solidify its effectiveness and I do believe that the success I had with my class on the End-of-Course test, due to the structured emphasis on writing and reasoning, can be replicated with ease.

5.2 Future Research

For future researchers testing the effectiveness of this mathematics writing template I suggest that some alterations be made. Instead of the three different phases I used (i.e. journal without an example, with an example, and with rubric details), give students an entry to answer without the SMPQs as a guide and then give them the same question with the SMPQs. This will give you a better baseline or control item to refer back to. Also, do this for more than one journal entry and assign more entries. Another idea is to have students compile their answers to the eight SMPQs and write one paragraph explanation. You can then have students critique each other's work based on the rubric to decide how thorough their peers were and how they, themselves, can become more thorough. My final suggestion is to be as consistent as possible with the level of difficulty each entry presents. This is why I chose to use questions from the book; however I do feel that some of them did not have the same rigor that others had. Be sure to give student challenging, but doable problems so they do not become overwhelmed. I would still suggest only comparing the work of low, medium, and high performing students to gage were the class is as a whole.

5.3 Closing Remarks

Our goal in all that we do as educators is to prepare this generation to become the leaders of the next generation. We must do all that we can to accomplish that goal, which includes research such as this. As earlier stated, anyone who teaches anything is mainly teaching reasoning. Developing methods that aid in producing independent,

mathematical thinkers, is another step in the right direction if we hope to prepare our students for the world.

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Appendix A: Geometry Student Survey and Results

Geometry Student Survey

Please answer all survey questions to the best of your ability.

Effects on Student Learning

On a scale from 1 to 5 with 5 being the highest rate how much the following has affected your education in Geometry this year:

Teacher's enthusiasm: 1 2 3 4 5

Learning material (textbook, etc): 1 2 3 4 5

Previous knowledge: 1 2 3 4 5

Classmates behavior: 1 2 3 4 5

Geometry As a Course

On a scale from 1 to 5 with 5 being the highest rate how important to you view the following in Geometry:

Working skill problems: 1 2 3 4 5

Word problems: 1 2 3 4 5

Writing/Constructive Responses: 1 2 3 4 5

Geometry Journals

On a scale from 1 to 5 with 5 being the highest rate how well you understood the purpose of the following:

Writing to a specific audience: 1 2 3 4 5

Explaining what the question was asking: 1 2 3 4 5

Relating questions to real life: 1 2 3 4 5

Please answer the following short answer questions.

How did the journals help you write a logical response to the questions asked?

What aspect of the journals aided you in writing logical response?

How did reflecting (left-side of notebook) help you better understand the problem and complete your journal?

Other comments:

Geometry Survey: Geometry Journals Section (by Student)					
Student	Mean	Standard Deviation	22	3	.82
1	4	0.82	23	3	.82
2	4	0	24	5	0
3	4.33	.47	25	2.33	.47
4	5	0	26	5	0
5	3	.82	27	3.67	.47
6	5	0	28	3.33	.94
7	3	1.41	29	5	0
8	4.33	.47	30	4	0
9	4.22	.94	31	4.33	.47
10	3	0	32	1.53	1.25
11	3	0	33	2.33	.94
12	3.33	1.25	34	3	.82
13	3	1.63	35	2.33	.47
14	1.67	.94	36	4	.82
15	2	0	37	2.67	1.70
16	3.33	1.70	38	4.33	.94
17	3	.82	39	3.66	.94
18	1	0	40	4.67	.47
19	2.67	.94	41	4	.47
20	4.67	.47	42	5	0
21	3.33	.47	43	5	0
			44	5	0
			45	5	0

Figure A.1 Geometry Journals Section (by Student)

Geometry Survey: Geometry Journals Section (by Question)		
Question	Mean	Standard Deviation
1	3.489	1.424
2	3.600	1.143
3	4.273	3.400

Figure A.2 Geometry Journals Section (by Question)

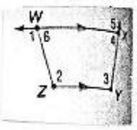
Appendix B: Journal Questions

Questions for Student Journals

1.

COORDINATE GEOMETRY Triangle ABC has been reflected in the x-axis, then the y-axis, then the origin. The result has coordinates $A'''(4, 7)$, $B'''(10, -3$ and $C'''(-6, -8)$. Find the coordinates of A, B, and C.

2. **CRITICAL THINKING** Explain why you can conclude that $\angle 2$ and $\angle 6$ are supplementary, but you cannot state that $\angle 4$ and $\angle 6$ are necessarily supplementary.



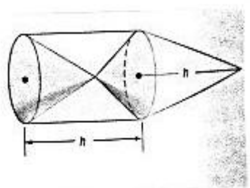
3.

CRITICAL THINKING The point-slope form of an equation of a line can be rewritten as $y = m(x - x_1) + y_1$. Describe how the graph of $y = m(x - x_1) + y_1$ is related to the graph of $y = mx$.

4.

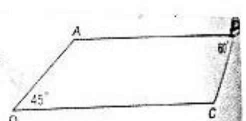
CRITICAL THINKING

Is it possible for the two cones inside the cylinder to be congruent? Explain.



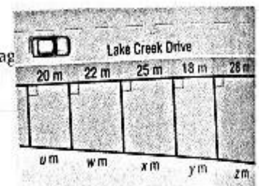
5.

CRITICAL THINKING Given figure $ABCD$, with $\overline{AB} \parallel \overline{DC}$, $m\angle B = 60$, $m\angle D = 45$, $BC = 8$, and $AB = 24$, find the perimeter.



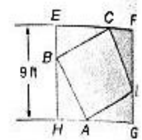
6.

REAL ESTATE In Lake Creek, the lots on which houses are to be built are laid out as shown. What is the lake frontage for each of the five lots if the total frontag is 135.6 meters?



7.

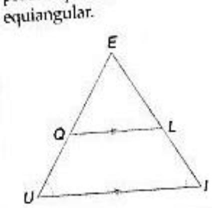
CRITICAL THINKING In the figure, the vertices of quadrilateral $ABCD$ intersect square $EFGH$ and divide its sides into segments with measures that have a ratio of 1:2. Find the area of $ABCD$.



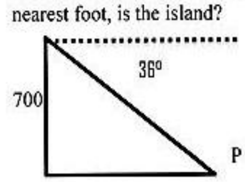
8.

CRITICAL THINKING $\overline{KL}$ is a segment representing one side of isosceles rig triangle KLM, with $K(2, 6)$, and $L(4, 2)$. $\angle KLM$ is a right angle, and $\overline{KL} \cong \overline{LM}$ Describe how to find the coordinates of vertex M and name these coordinat

9. **PROOF** Write a two-column proof to prove that $\triangle EQL$ is equiangular.



10. A helicopter hovers 700 feet above a small island. The figure shows that the angle of depression from the helicopter to point P is 36 degrees. How far off the coast to the nearest foot, is the island?



(These do not depict the order the entries were given in)

Appendix C: Journal Example for Students

Left-Side 10/20/11

Reflections:

1. I guess what problem was really asking me was about // lines and their properties.

2. It also important to say which properties/theorems I need.

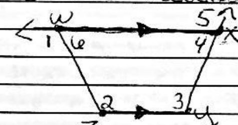
5. A Straight edge would have also been a good tool

4. What other jobs would use this? (Be sure to ask the teacher about this) ← Reminder*

Right side

10/19/11 Right Side

Question: Explain why you can conclude that ∠2 & ∠6 are supplementary, but you can't state that ∠4 & ∠6 are necessarily supplementary.



Audience: Classmate

1. The problem is asking me to use what I know about parallel lines to prove ∠2 & ∠6 are supplementary but that ∠4 & ∠6 may not be supplementary.
2. It's important to know properties of // lines and what it means for to angles to be supplementary.
3. Since $\overline{WX} \parallel \overline{ZY}$ the properties or theorems of alternate interior & exterior and consecutive interior angles apply. You can look and see that ∠2 & ∠6 are consecutive interior angles so they are supplementary. But if you didn't remember that then notice that ∠2 & ∠1 are alt interior so they are ≅ and ∠1 is supp to ∠6 because they make a straight line (180°). However, we can't tell if $\overline{WZ} \parallel \overline{XY}$ so we can't assume those ~~same~~ rules ~~apply~~ same rules apply.
4. A construction worker may need to know about the angle properties of // lines, but I guess with any job you can't assume any rule always applies unless it is the exact same situation.
5. A protractor would have been helpful, so I could measure the angles.
6. I could be more precise by defining the terms and giving the theorem, but my classmate should know them, that's why I didn't

Right Side

Left Side 10/20/11

7. Also since ∠6 and ∠4 don't share the same transversal the // theorems aren't necessarily true. ∠6's transversal is $\overline{WZ}$ and ∠4's transversal is $\overline{XY}$.

10/19/11 Cont... Right Side

7. I realized that if two angles (∠2 & ∠1) are congruent then it makes sense that if ∠1 is supp to ∠6 then ∠6 is also supp to ∠2.

8. The shortcut is knowing the consecutive interior theorem and realizing $\overline{WZ}$ would have to be paralle to $\overline{XY}$ for ∠6 to be supplementary to ∠4.

Appendix D: Problem Three's Proportionality

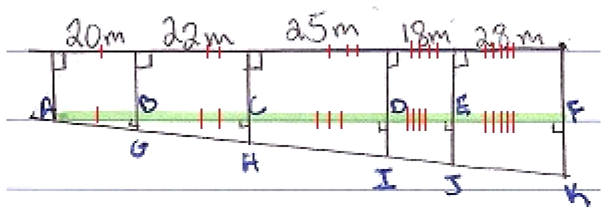
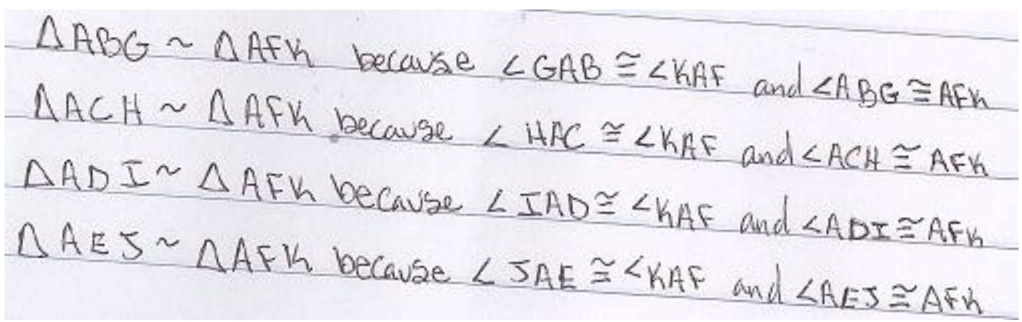


Figure D.1 Similar Triangles



ΔABG ~ ΔAFK because ∠GAB ≅ ∠KAF and ∠ABG ≅ AFK
ΔACH ~ ΔAFK because ∠HAC ≅ ∠KAF and ∠ACH ≅ AFK
ΔADI ~ ΔAFK because ∠IAD ≅ ∠KAF and ∠ADI ≅ AFK
ΔAEJ ~ ΔAFK because ∠JAE ≅ ∠KAF and ∠AEJ ≅ AFK

Figure D.2 Similar Triangles Argued

Appendix E: IRB Approval

Application for Exemption from Institutional Oversight

LSU

Institutional Review Board
Dr. Robert Mathews, Chair
131 David Boyd Hall
Baton Rouge, LA 70803
P: 225.578.8692
F: 225.578.6792
irb@lsu.edu
lsu.edu/irb

Unless qualified as meeting the specific criteria for exemption from Institutional Review Board (IRB) oversight, ALL LSU research/projects using living humans as subjects, or samples, or data obtained from humans, directly or indirectly, with or without their consent, must be approved or exempted in advance by the LSU IRB. This Form helps the PI determine if a project may be exempted, and is used to request an exemption.

- Applicant,Please fill out the application in its entirety and include the completed application as well as parts A-E, listed below, when submitting to the IRB. Once the application is completed, please submit two copies of the completed application to the IRB Office or to a member of the Human Subjects Screening Committee. Members of this committee can be found at http://research.lsu.edu/CompliancePoliciesProcedures/InstitutionalReviewBoard%28IRB%29/item24737.html

- A Complete Application Includes All of the Following:
  - **(A)** Two copies of this completed form and two copies of part B thru E.
  - **(B)** A brief project description (adequate to evaluate risks to subjects and to explain your responses to Parts 1&2)
  - **(C)** Copies of all instruments to be used.
    - *If this proposal is part of a grant proposal, include a copy of the proposal and all recruitment material.
  - **(D)** The consent form that you will use in the study (see part 3 for more information.)
  - **(E)** Certificate of Completion of Human Subjects Protection Training for all personnel involved in the project, including students who are involved with testing or handling data, unless already on file with the IRB. Training link: (http://phrp.nihtraining.com/users/login.php)
  - **(F)** IRB Security of Data Agreement: (http://research.lsu.edu/files/item26774.pdf)

1) Principal Investigator: Yvonne M. Chimwaza **Rank:** Graduate Student

Dept: Basic Science **Ph:** 504.301.5648 **E-mail:** ychimwaza4@gmail.com

2) Co Investigator(s): please include department, rank, phone and e-mail for each
*If student, please identify and name supervising professor in this space

IRB# E5990 LSU Proposal #

✓ Complete Application

✓ Human Subjects Training

3) Project Title: Writing in Geometry with The Common Core State Standards

Study Exempted By:
Dr. Robert C. Mathews, Chairman
Institutional Review Board
Louisiana State University
203 B-1 David Boyd Hall
225-578-8692 | www.lsu.edu/irb
Exemption Expires: 5/31/2015

4) Proposal? (yes or no) No **If Yes, LSU Proposal Number**

Also, if YES, either
○ **This application completely matches the scope of work in the grant**
OR
○ **More IRB Applications will be filed later**

5) Subject pool (e.g. Psychology students) High school students

*Circle any **"vulnerable populations" to be used**: (children <18; the mentally impaired, pregnant women, the ages, other). Projects with incarcerated persons cannot be exempted.

6) PI Signature Yvonne M. Chimwaza **Date** 5/14/12 (no per signatures)

**** I certify my responses are accurate and complete.** If the project scope or design is later changes, I will resubmit for review. I will obtain written approval from the Authorized Representative of all non-LSU institutions in which the study is conducted. I also understand that it is my responsibility to maintain copies of all consent forms at LSU for three years after completion of the study. If I leave LSU before that time the consent forms should be preserved in the Departmental Office.

Screening Committee Action: Exempted ✓ Not Exempted ____ Category/Paragraph 1

Reviewer Mathews **Signature** [signature] **Date** 6/1/12

Study Exempted By:
Dr. Robert C. Mathews, Chairman
Institutional Review Board
Louisiana State University
203 B-1 David Boyd Hall
225-578-8692 | www.lsu.edu/irb
Exemption Expires: 5/31/2015

Child Assent Form

I, ____________________________, agree to allow my regular classroom assignment be used in my teacher's research. I have been informed that my name will not be used, but that the journal entries I write will be used. I understand that the purpose of this is to help students understand Geometry better. I also understand that I will not receive any extra credit for participating and will be graded the same as those who are not participating. I can decide not to allow my journal entries to be used and will face no consequences for doing so.

Child's Signature:____________________ Age:_______ Date:__________________
Witness*__________________________________ Date:____________________
*(Witness must be present for the assent process, not just the signature by the minor.)

Study Exempted By:
Dr. Robert C. Mathews, Chairman
Institutional Review Board
Louisiana State University
203 B-1 David Boyd Hall
225-578-8692 | www.lsu.edu/irb
`xemption Expires: 5/31/2015

Parental Consent Form

Project Title: Writing in Geometry with The Common Core State Standards

Performance Site: Louisiana School of Agricultural Sciences (LaSAS)

Investigator: The following investigator is available for questions,
M-F, 7:30 a.m.-2:30 p.m.
Yvonne Chimwaza
Geometry Teacher, LaSAS
(318)-346-8029

Purpose of the Study: The purpose of this research project is to develop a writing template for other teachers to use in order to increase students understanding of math.

Exclusion Criteria: Student must be in one of Yvonne Chimwaza's three Geometry Classes.

Description of the Study: The students are to be given ten math problems that they will solve using the eight Standards for Mathematical Practice as provided by the Common Core State Standards. The students will then reflect on the questions and complete a survey about the project. The purpose of this project is to produce a math writing template that will aid in developing logical and deep thinking amongst math students. The project will be done in two phases. The first phase includes having students complete journal writing on problems taken directly from their textbook. The students work will be analyzed and graded based on a rubric. The other phase is to have students complete a survey about the journal writings. The goal is to test the effectiveness of the writing template and produce a working template that other mathematics educators can use to inspire deeper exploration into mathematic assignments.

Risks: There are no known risks.

Right to Refuse: Participation is voluntary, and a child will become part of the study only if both child and parent agree to the child's participation. At any time, either the subject may withdraw from the study or the subject's parent may withdraw the subject from the study without penalty or loss of any benefit to which they might otherwise be entitled.

Privacy: The school records of participants in this study may be reviewed by investigators. Results of the study may be published, but no names or identifying information will be included for publication. Subject identity will remain confidential unless disclosure is required by law.

Financial Information: There is no cost for participation in the study, nor is there any compensation to the subjects for participation.

Signatures:
The study has been discussed with me and all my questions have been answered. I may direct additional questions regarding study specifics to the investigator. If I have questions about subjects' rights or other concerns, I can contact Robert C. Mathews, Chairman, Institutional Review Board, (225) 578-8692, irb@lsu.edu, www.lsu.edu/irb. I will allow my child to participate in the study described above and acknowledge the investigator's obligation to provide me with a signed copy of this consent form.

Parent's Signature:______________________________

Date:____________________

The parent/guardian has indicated to me that he/she is unable to read. I certify that I have read this consent from to the parent/guardian and explained that by completing the signature line above he/she has given permission for the child to participate in the study.

Signature of Reader:______________________________

Date:____________________

Vita

Yvonne Mariki Chimwaza was born in Nassau Bay, TX, to Geoffrey Mariki and Evetta Pittman. She is the third of five siblings and the wife of Tinashe Eugene Chimwaza Esq. She graduated from Louisiana State University with a Bachelor of Science in mathematics, a concentration in secondary education in May 2009. While at Louisiana State University she completed research under the authorization on the Ronald E. McNair Program. After graduating, she taught Sixth Grade Mathematics for two years at Westdale Middle School in East Baton Rouge Parish. She then began teaching at Louisiana School for the Agricultural Sciences in Avoyelles Parish and continues there teaching Geometry, Algebra II, and Advanced Math. While teaching in Avoyelles Parish she was awarded 2012 High School Teacher of the Year for the parish.