Transport Phenomena in a Bay-Marsh System (Volumes I and II).

Hasitkumar Kantilal Trivedi
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ABSTRACT

A two-dimensional, time-dependent material transport model was developed that predicts distribution of different species of nitrogen, detritus, phytoplankton and other organisms contributing to primary production as a function of time for a coastal bay-salt marsh system when environmental conditions, tidal variations at the passes and fresh water flows into the system are specified. The material transport equation was solved on a digital computer for the Barataria Bay region of coastal Louisiana.

The biogeochemical cycling of nitrogen as it seems to occur in the Barataria Bay region is delineated. Nitrogen cycling is quantitatively represented by a series of rate equations with all rate constants in the rate equations evaluated from available information.

The material transport model was found to accurately predict the time-varying distribution of different species of nitrogen, live and dead standing crops of *Spartina alterniflora*, detritus, phytoplankton, and animal biomass in the marsh areas as well as open water of the bay. Verification of the analysis was made using comparison of computed daily average concentrations with available data in the literature; when these data were not available, qualitative behavior was consistent with how biologists understand the system to
function.

Analysis of the effects of planting marsh grass was made by increasing the marsh area. Results showed increase in total productivity of the system. This in turn increased the total fisheries catch. The total revenue increase due to this increase in catch was calculated at about two dollars per acre. It was concluded, therefore, that planting marsh grass would be economically feasible only if the planting cost can be covered by the annual increase of two dollars per acre if the justification is based on increased fisheries production.

Analysis of the effects of dredging and other activities that can permanently change marsh grass area to water area was made by decreasing the marsh area. Results showed decrease in total productivity of the system. This in turn decreased the total fisheries catch. The resultant total decrease in annual revenue was calculated at about $1.80 per acre. Hence it was concluded that the disadvantage of annual income loss from the fisheries sector would have to be balanced with gain from the activity which caused the loss of marsh grass area.

Analysis of the effect of increase in the amount of nitrogen in the marsh area was studied with the model. Results showed increase in total productivity of the bay-salt marsh system. The annual revenue increase derived by addition of nutrients was about $2.85 per acre. Hence it was concluded that addition of nutrients will be economically
feasible only when the cost per acre is less than $2.85 if the justification is based on increased fisheries production.

The material transport model was time averaged to obtain an equation that can be solved using a larger time step. New terms appeared in the time-averaged form of the material transport model, which were approximated by diffusion type terms.

The hydrodynamic model, developed by Hacker, was used to obtain the velocity data needed to solve the material transport model. A set of twelve quasi-steady state solutions was obtained representing twelve monthly average values of velocities. These data were interpolated to give velocities at intermediate points.

Computer programs of the hydrodynamic model and the material transport model are given in a form that can be readily used by engineers and scientists for studies of ecological design, e.g., for coastal planning and evaluating methods to improve the marine resources of the region. Users' manuals are included with the program for ease in applying them.
CHAPTER I

INTRODUCTION AND BACKGROUND

Introduction

The purpose of this dissertation is to evaluate the distribution of detrital food, nutrients, phytoplankton, and other organisms that contribute to the primary production in the bay-marsh system. Particular emphasis will be given to the application of this analysis to the Barataria Bay System. This chapter will serve as a general introduction to the subject of estuarine system analysis and will establish the appropriate groundwork for further development in subsequent chapters. The chapter will consist of three parts: the first part will be a discussion on the importance of modeling bay-marsh systems with particular emphasis on modeling bay-marsh systems in the state of Louisiana; the second part will be a discussion on the importance of modeling natural systems; and the conclusion will consist of a statement on the objectives of this present research.
Importance of Modeling Bay-Marsh Systems
with Emphasis on Coastal Louisiana Systems

Importance of Modeling Bay-Marsh Systems: Due to enormous pressures generated by our limited resources and population explosion, there is, today, an ever increasing need for the use of estuarine resources for economic growth, food and recreational centers. Estuaries are, collectively, of singular importance. In contra-distinction to the nutrient-rich, water-poor land and the water-rich, nutrient-poor ocean, estuaries are typically both a nutrient-rich and water-rich environment. They are highly productive and constitute the prime habitat for a myriad of species and the nursery and spawning area for many more (Ref. 1.1). Shallow coastal waters and semi-enclosed areas of the sea can be always characterized as more viable in productivity than the waters of open ocean at the same latitudes. Estuaries are nutrient traps and thus provide a surplus of usable fuel to the life they support. However, just as they accumulate nutrients, they can and do accumulate pollutants (Ref. 1.2).

Coastal waters and estuaries are of great importance to the world population that uses these waters in a variety of ways, some of which are conflicting. Besides their biological significance, however, estuaries are of specific importance to man. They have great propensity for stimulating urban development. Estuaries offer an accessibility to the sea, often with excellent shipping and harbor
facilities, while the feeding river provides a source of freshwater for domestic and industrial use (Ref. 1.1). In the United States, more than half the population lives in the coastal states, including those bordering the Great Lakes. A major share of the world's marine fisheries is obtained from coastal waters, and estuaries are essential as breeding grounds for many species of coastal fishes (Ref. 1.3). Unfortunately, these waters are also used for the disposal of municipal and industrial wastes. They often receive excessive volumes of wastes both from peripheral discharges and from upstream discharges carried into the estuary by the inflows, and are often adversely influenced by upstream fresh water consumption. The pollution of many estuaries is so intense that some species have been locally eliminated while others are unfit for human consumption (Ref. 1.4).

Besides these very practical reasons for a concern with estuaries, the estuary poses modeling problems of extreme scope and complexity. Its hydrodynamics, for instance, are determined by the interaction of tides from the ocean and the influx of fresh water from rivers. This is modified by complicated boundary stresses due to the semi-enclosed physiography. This is further altered by circulation arising due to Coriolis force and density gradients existing due to concentration differences. These factors along with the diffusion effects are responsible for the transport of various species such as detritus, nutrients,
phytoplankton, biochemical oxygen demand, salinity, etc., in the estuary. In addition, there are various reactions and interactions among the biological and chemical species. These range from simple first-order kinetics to reactions which depend nonlinearly upon additional variables and are possibly linked with the biota.

**Importance of Modeling Bay-Marsh Systems of Coastal Louisiana:** The state of Louisiana is a good example to show the importance of estuaries to the economy of coastal states. Approximately 45 percent of the state consists of coastal and floodplain wetlands-areas subject to intermittent flooding or characterized by near-surface water tables. These areas contain more than 75 percent of the state's population and 80 percent of its manufacturing capability. Water transportation played a major role in the development of the state, and waterborne commerce remains vital to its economic health. In recent years, annual statewide oil and gas production has been at $3.5 billion; sulphur, $140 million; and salt $50 million. Most of this production comes from offshore, coastal and floodplain areas. These extractive industries provide approximately 50 percent of the total state tax revenues. Most informed sources agree, however, that revenues Louisiana can derive from oil and gas production have reached a peak, and leveling off or decline is probable for future years. The need to evaluate the complete spectrum of marine and coastal resources and to
develop a plan for their most productive and effective development is thus increasingly obvious and important (Ref. 1.5).

Louisiana contains more than five million acres of coastal marshes, swamps, and estuaries. As more than two million of these acres are considered to be important habitat areas for fish and wildlife, Louisiana ranks first among all states in area of important estuarine habitat. Shrimp utilize the estuaries as nursery grounds, and Louisiana ranks first or second in shrimp production. In 1969 the state was first, with a production of more than 52,000,000 pounds of headless shrimp having a dockside value in excess of $33,400,000. Louisiana, the only state where oysters are harvested the year around, supplies 20 percent of the total U.S. market. Ten to fifteen million pounds of oysters are produced annually. The total annual value of all fishery operations is in the $100 to $150 million range, and total production of all species often exceeds one billion pounds annually. Fur and meat products provided by animals of the estuarine habitat is a several-million-dollar per year business (Ref. 1.5).

Louisiana's enormous fisheries catch requires a sizeable processing industry. In 1966 Louisiana had more ocean-related industries than any other state and ranked third in total employment of individuals in such industries. The value of canned fishery production in Louisiana in 1970 was $26.5 million. The value of Louisiana's industrial
fishery production was $30.1 million--30 percent of the national total in 1970. Louisiana's large-scale industrial fishery activity is primarily attributable to the menhaden catch, which is the chief industrial fish of the nation. Louisiana is by far the largest in the nation, supplying 55 percent of the total domestic supply of these useful and valuable fish (Ref. 1.6).

The basic problems for the future of the Louisiana fishery are from the ecological erosion of the estuarine and coastal environment. Certainly, one of the major considerations in future economic development of the Louisiana coastal region will be to maintain the biological environment (Ref. 1.6). The need to estimate the "spillover effects" of such activities as non-replenishable resource extraction, canal building, hurricanes, road construction, and other activities which may impinge on the rich estuarine spawning grounds is an absolute must if adequate management of the fishery resources is to be achieved.

The area selected for the study, the Barataria Bay region of coastal Louisiana, is a vast shallow water estuarine system bounded on the east by the Mississippi River and on the west by Bayou Lafourche. Barataria Bay is one of the most productive areas in the state. From 1963 to 1967, this area supplied over 44 percent of Louisiana's annual commercial fishery production (Ref. 1.7).

The dominant vegetation in the marsh area of Barataria
Bay region is *Spartina alterniflora* Loisel, which is the primary contributor of organic matter to the detrital food chain. At the base of this food web, and directly related to the input of solar energy into the system, is primary production from the *Spartina alterniflora*. The primary productivity of an ecological system, community or any part thereof, is defined as the rate at which energy is stored by photosynthetic and chemosynthetic activity of producer organisms (chiefly green plants) in the form of organic substances which can be used as food materials (Ref. 1.8).

The Barataria Bay is an extremely complex system, including marshes, brackish lakes, bayous, canals and streams. The primary producers in the region cannot be considered as a homogeneous group as has often been done in large deep lakes or in the ocean, because of differences in habitat, environment and growth habitats. The primary production in the Barataria Bay area can be divided into three different categories: 1) production by marsh grasses, 2) production by the epiphytic algae on the marsh grasses, and 3) production by the phytoplankton and benthic organisms.

Kirby (Ref. 1.9) has reported that the highest primary production is from *Spartina alterniflora* for this region. This high production of *Spartina alterniflora* is complemented by two other sources listed above to give a very high overall primary production. This is because of high fertility of the Barataria Bay region. Fertility of any area is proportional to the amounts of nutrients
available there. To occur and grow in a given situation, an organism must have essential materials—nutrients—which are necessary for growth and reproduction. All living organisms are made up of carbohydrates, fats and proteins. If we subject these three classes of organic molecules to elemental analysis, we can come up with the list of commonly involved elements in organic materials. Russell-Hunter (Ref. 1.10) states that there are about forty elements which are commonly involved in the organic materials and that the majority of even the invariably occurring, or "essential" elements make up only a tiny fraction of the mass of organisms. On a dry-weight basis, only five elements are present in the organic tissues of the majority of living organisms at levels greater than one percent. These are carbon, oxygen, hydrogen, nitrogen, and phosphorus.

The idea that an organism is no stronger than the weakest link in its ecological chain of requirements was first expressed by Justus Liebig, an agricultural chemist, in 1840 (Ref. 1.11). The concept of the "limiting factor" says that it is the basic kind of energy or matter most closely approaching the critical minimum required that will normally be the one whose quantitative availability determines and limits the extent of the productivity of the organism. Given the ready availability of carbon dioxide and of water, it is obvious why the usual limiting nutrients of aquatic and estuarine productivity are nitrogen and
phosphorus.

Pomeroy et al. (Ref. 1.12) have stated that there is no evidence that phosphorus ever is limiting to the productivity of the estuaries of the southeastern United States. Similar observations have been made by Patrick et al. (Ref. 1.13) for the Barataria Bay region also. Hence nitrogen may be the limiting nutrient for the ecology of the Barataria Bay. Often times a good way to determine the factors that improve the productivity of certain areas is to study the distribution of the limiting factor. Thus one of the aims of this dissertation research is to estimate and predict the distribution of nitrogen and its assimilable salts.

Nitrogen tends to circulate in the biosphere in definite patterns from environment to organisms and back to the environment. This circulation path is known as the biogeochemical cycle of nitrogen. The biogeochemical cycle of nitrogen is an immensely complex phenomenon. Additionally, the nature of this cycle varies from place to place. Hence, in attempting to find the distribution of nitrogen and its assimilable salts, it is of great importance to delineate the biogeochemical cycle of nitrogen as occurring in the Barataria Bay region. Once the biogeochemical cycle is accurately known, rate equations can be written for different steps in the sequence of reactions in the cycle. The rate equations obtained could be used along with a material transport model to evaluate the distribution of different
species. This analysis can be used as a tool for coastal planning and to evaluate methods to improve the marine resources of the region.

The Importance of Modeling Natural Systems

In our so-called "laissez-faire" society, industry has always tried to maximize its profit. The processes which they try to optimize are very complicated. In order to apply the optimization techniques the complicated process should be presented in a form that is amenable to the technique used. One of the procedures is to represent the process by a mathematical model. A mathematical model is a set or sets of equations in which the important variables of the system to be studied are included. Once a system is described by a suitable mathematical model, it can be studied by performing manipulations on the model equations rather than by an actual experimental work. For example, if a change in independent variable is known, it is possible to predict the behavior of the system being modeled by performing necessary manipulations.

As mentioned above, workers in industry use mathematical models to predict optimum conditions which will maximize the profit. This same idea can be applied to the management of natural environments. People use the natural resources in many different ways and many of them are in conflict with each other (for example, use of estuaries as stated in the previous section). To resolve the conflicts, a management
program of wide scope is extremely necessary. The development of such a program requires a thorough knowledge of the effect of the different demands put on a natural system. Techniques of mathematical modeling could be used to describe the relationships between different demands. Once the natural system is represented by a mathematical model, responses to different demands can be predicted using the mathematical model.

There are living and nonliving factors in a natural environment. An ecosystem is a combination of these factors. In other words, an ecosystem is a complex of organisms and environment forming a functioning whole in nature. Since there are a great number of varying parameters and relationships existing in a given ecological system, development of an exact mathematical model of even the smallest and the simplest ecosystem is a big task. However, a simplified mathematical model using only important parameters can be developed which will adequately describe the system for the purpose under investigation.

In the mathematical modeling of natural systems, the modeling of the abiotic component (the nonliving environment) is important. In case of estuarine ecosystems, abiotic factor plays a more important role than in its terrestrial counterpart. The physical model (describing abiotic component) is generally combined with the biological model to form the complete mathematical model of the environmental system. The modeling is performed in the framework of a
systems analysis and one example of this is the system analysis for the estuarine environment of Barataria Bay (see Figure 1.1 [Ref. 1.14]).

In Figure 1.1 various relationships are represented that exist in the system. The first step in developing a mathematical model is to collect historical, biological and economic data that are available. Following this, transport phenomena models are developed. The hydrodynamic model is used to obtain the behavior of currents. Output from the hydrodynamic model is used as input to the energy transport model and the material transport model. The energy transport model describes the temperature distribution in the system. The material transport model describes the distribution of various substances such as salinity, dissolved gases, and microorganisms. Also, it can be visualized that constituents are transformed during the transport by chemical or biological actions, and for this process a reaction model must be incorporated with the material transport model. Transport processes have significant effect on biological communities in the system and the interaction between the two should be properly modeled. At the same time, economic models can be developed which describe regional activities and recreational and commercial values of the area. Since there is continuous interaction between transport phenomena models, biological models, and economic models, they all should be interconnected to give a regional model for the area. Regional analysis
Figure 1.1. System analysis diagram for Barataria Bay, Louisiana
represents interrelation between ecological and economic systems. Finally the regional model is combined with management objectives, management alternatives, and optimization techniques to give proper management decisions.

Statement of Objectives

The overall objective of this research program is to develop mathematical models of the ecology of the Barataria Bay region of coastal Louisiana. These models will be used as tools for coastal planning and to evaluate methods to improve the marine resources of the region. Specific aims of this research are to:

1) Delineate the biogeochemical cycle of nitrogen as occurring in the Barataria Bay and model its cyclic behavior.

2) Solve species continuity equation to evaluate distribution of nutrients, detritus, phytoplankton and other organisms that contribute to the primary and secondary production in the system.

3) Simulate typical and environmental stress conditions to obtain guidelines to serve as the basis for coastal management decisions.
REFERENCES


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1.13 Patrick, W. H., Jr., personal communication, June 1972, Baton Rouge.

CHAPTER II

A REVIEW OF PREVIOUS ANALYSES IN ESTUARINE SYSTEMS

Introduction

It is the purpose of this chapter to review the previous works on estuarine analysis that have significant relationships with the current work. The first part of this chapter will give a brief description of the historic development of the equations used to model two-dimensional estuarine transport phenomena. The second part will consist of a detailed discussion on the existing and related work on transport phenomena models of estuarine systems. The chapter will be concluded with a statement of the contributions that this work will make to the existing knowledge on estuarine analysis.

Historic Development of the Two-Dimensional Estuarine Systems Transport Phenomena Equations

The general equations that describe transport phenomena have long been known and are found in the literature (Ref. 2.1). These equations are three-dimensional time dependent partial differential equations. The representation of different parameters in three spatial dimensions and with instantaneous temporal variation is inaccessible analytically and is well beyond the capacities of present
computers. For this reason, the dimensionality of the equations is reduced by assuming that the phenomena under study is characterized by two-dimensional behavior.

In the case of shallow estuaries, this assumption is justifiable. The vast expanses of brackish waters that cover several hundred square miles of estuarine system found commonly in the East and Gulf Coasts of the United States are seldom deeper than twenty feet, and depths greater than these are found only in the proximity of the connection between the estuary and the ocean.

The modeling of the momentum transport of shallow waters with a two-dimensional hydrodynamic model was first proposed by Hansen in 1938 (Ref. 2.2). However, the computer hardware and software necessary to effectively obtain a solution were not available at the time. With the advance of technology, the necessary computers became available and the solutions of the models first proposed in 1938 came into being; the predecessor of today's estuarine models was presented by Hansen in 1956 (Ref. 2.3). All of the existing hydrodynamic models for shallow waters are essentially the same as Hansen's model. The number of hydrodynamic models presented in the literature progressed with time. Models were presented by Platzman in 1958 (Ref. 2.4), Miyazaki (Ref. 2.5) and Unoki and Iozaki (Ref. 2.6), both in 1963, a further refinement by Hansen (Ref. 2.7) in 1966, Leendertse (Ref. 2.8) in 1967, Reid and Bodine (Ref. 2.9) in 1968 and Hacker (Ref. 10) in 1973.
As the modeling of natural systems increased due to ecological concern, hydrodynamic models which were only used previously to model storm surges were used as a basis for an analysis of the overall transport phenomena in estuaries. One of the first suggestions of this idea in the literature was a proposed Systems Analysis of Galveston Bay in a report by TRACOR (Ref. 2.11) in 1968. Later, two-dimensional species and energy transport model derivations were presented by Leendertse (Ref. 2.12) in 1970, and Hacker, Pike and Wilkins (Ref. 2.13) in 1971, respectively. A comprehensive, two-dimensional model describing transport phenomena in a shallow estuary was first reported by Leendertse (Ref. 2.12) also in 1970. Other studies in which hydrodynamics is applied with energy and species models are TRACOR (Ref. 2.14) and Masch (Ref. 2.15), both in 1971, and Hacker (Ref. 2.10) in 1973.

Transport Phenomena Models of Estuarine Systems

An aim of this dissertation research is to solve the species continuity equations for distributions of nutrients (nitrogen, phosphorus), detritus, phytoplankton, and other microorganisms that contribute to the primary production in the Barataria Bay region of coastal Louisiana. Hence a detailed review connected with the developments of material transport models is desired in order to put the objectives of this research into proper perspective. It will be explained in this chapter that the solutions of hydrodynamic
models and energy transport models are prerequisites to solve the material transport model. A brief review of hydrodynamic models and energy transport models, therefore, is very much within the scope of this work.

The study of existing estuarine transport phenomena models is divided into three parts in this section. In the first part previous research on material transport models is presented. In the second part previous research on two-dimensional hydrodynamic models and the energy transport models is described. In the third and last part solution techniques applied to the model equations are reviewed.

Material Transport Models

In Bird, Stewart, and Lightfoot (Ref. 2.1) the general turbulent species continuity equation is given for a binary system:

$$\frac{\partial \rho_A}{\partial t} + (\vec{v} \cdot \rho_A \vec{v}) + (\vec{v} \cdot \vec{J}_A) - r_A = 0$$  \hspace{1cm} (2.1)

where the turbulent diffusion term is modeled by the following equation:

$$\vec{J}_A' = - D^*_{AB} \rho \vec{v}_A$$  \hspace{1cm} (2.2)

where

$$D^*_{AB} = D^*_{AB} \text{ laminar} + D^*_{AB} \text{ eddy}$$  \hspace{1cm} (2.3)

Assuming that constant density and diffusivity applies, Eq. (2.2) transforms to:
Writing Eq. (2.1) in terms of constant density and diffusivity by substituting Eq. (2.4), the result is:

\[ J_A = - B_{AB}^* \mathbf{v} \rho_A \]  

(2.4)

\[ \frac{\partial \rho_A}{\partial t} + (\mathbf{\nabla} \rho_A \mathbf{v}) + (\mathbf{\nabla} \cdot J_A) - r_A = 0 \]  

(2.5)

Eq. (2.5) is the general, binary diffusion species continuity equation. The concentration of a species in the estuarine system can be described by such binary diffusion. The reason for this is that the concentration of most of the species found in estuarine water is relatively low and each species diffuses independently from the others. Consequently, a binary diffusion coefficient can be used, where water is one component and the species in question is the other.

The above Eq. (2.5) expresses the variation of the species concentration under study, as resulting from flux of mass due to advection by the fluid containing the mass, dispersion of species in different directions, and reaction rates. In general, the computation of the concentration of a constituent may be conveniently divided according to the processes which transport the constituent through the system and the processes within the system which generate or degrade the constituent. The former are essentially hydrodynamic in origin, while the latter, the reaction rate terms, may be physical (breakdown of detritus), chemical
(transformation of nitrogen species), or biological (production of *Spartina alterniflora*, phytoplankton) in nature. Transposing terms, Eq. (2.5) can be written as:

\[
\frac{\partial \rho_A}{\partial t} + (\nabla \cdot \rho_A \mathbf{v}) = - (\nabla \cdot \mathbf{J}_A) + r_A \tag{2.6}
\]

In the above equation, all the terms except the first one represent the transport term, the last being the reaction rate term. The term \(\nabla \cdot \rho_A \mathbf{v}\) in Eq. (2.6) is the flux due to advection. The advective flux is determined by the hydrodynamics of the system which is related to the hydrology, meteorology and geomorphology of the areas. The term \(\nabla \cdot \mathbf{J}_A\) is the flux ascribed to diffusion in the different coordinate directions. The flux due to diffusion is proportional to the concentration gradient of the species under study. The constituent is transferred by this mechanism from a zone of high concentration to one of low concentration. The last term is the reaction rate term which accounts for production or degradation of any species as the result of various reactions of physical, chemical or biological nature.

As mentioned in the previous section, the solution of Eq. (2.6) is beyond the means of currently available solution techniques. Hence Eq. (2.6) is generally manipulated so as to reduce its effective dimensionality and its temporal variability. This is accomplished by integrating the Eq. (2.6) along one or more of the x, y, z, t axes. It must
be emphasized that dimensions are reduced by integration; it is not in general valid to merely neglect variation along a particular axis.

For the purpose of this dissertation a two-dimensional material transport model derived by Hacker et al. (Ref. 2.13) for the estuarine systems will be used. Hacker et al. obtained the material transport model by vertical integration of the species continuity equation, Eq. (2.6). The detailed derivation of the equation is given in Chapter IV. The important steps of the derivation are shown below:

Expanding Eq. (2.6):

\[
\frac{\partial \rho_A}{\partial t} + \frac{\partial}{\partial x} (\rho_A u) + \frac{\partial}{\partial y} (\rho_A v) + \frac{\partial}{\partial z} (\rho_A w) = \frac{\partial}{\partial x} \left( B_{AB}^* \frac{\partial \rho_A}{\partial x} \right) + \frac{\partial}{\partial y} \left( B_{AB}^* \frac{\partial \rho_A}{\partial y} \right) + \frac{\partial}{\partial z} \left( B_{AB}^* \frac{\partial \rho_A}{\partial z} \right) + r_A
\]  

(2.7)

In order to obtain a two-dimensional equation, Eq. (2.7) is integrated in the vertical direction as indicated below:

\[
\int_{z_b}^{z_s} \left[ \frac{\partial \rho_A}{\partial t} + \frac{\partial}{\partial x} (\rho_A u) + \frac{\partial}{\partial y} (\rho_A v) + \frac{\partial}{\partial z} (\rho_A w) \right] \, dz = \int_{z_b}^{z_s} \left[ \frac{\partial}{\partial x} \left( B_{AB}^* \frac{\partial \rho_A}{\partial x} \right) + \frac{\partial}{\partial y} \left( B_{AB}^* \frac{\partial \rho_A}{\partial y} \right) + \frac{\partial}{\partial z} \left( B_{AB}^* \frac{\partial \rho_A}{\partial z} \right) \right] \, dz + \int_{z_b}^{z_s} r_A \, dz
\]  

(2.8)
Define:
\[ S_A = \frac{1}{D} \int_{z_b}^{z_s} \rho_A \, dz \]  
\[ U = \frac{1}{D} \int_{z_b}^{z_s} u \, dz \]  
\[ V = \frac{1}{D} \int_{z_b}^{z_s} v \, dz \]  
\[ \overline{r_A} = \frac{1}{D} \int_{z_b}^{z_s} r_A \, dz \]

Applying Leibnitz's rule to each term in Eq. (2.8), rearranging, and neglecting the higher order diffusive terms, we obtain:

\[ \frac{\partial (DS_A)}{\partial t} + \frac{\partial (UDS_A)}{\partial x} + \frac{\partial (VDS_A)}{\partial y} = \frac{\partial}{\partial x} \left[ D \frac{\partial S_A}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D \frac{\partial S_A}{\partial y} \right] + \left[ \frac{\partial}{D_{AB}} \rho_A (z_s) - \frac{\partial}{D_{AB}} \rho_A (z_b) - \rho_A w (z_s) + \rho_A w (z_b) \right] + \overline{r_A} D \]

The terms in the square bracket in Eq. (2.13) represent the sink and source terms for species A. These sink and source terms take into account the convection and diffusion through the surface and bottom of the system and reaction rate of species A in the system. Eq. (2.13) can be written as:
\[
\frac{\partial (DS_A)}{\partial t} + \frac{\partial (UDS_A)}{\partial x} + \frac{\partial (VDS_A)}{\partial y} = \frac{\partial}{\partial x} \left(D \frac{\partial S_A}{\partial x}\right) + \frac{\partial}{\partial y} \left(D \frac{\partial S_A}{\partial y}\right) + P_A
\]  \tag{2.14}

Eq. (2.14) is the vertically averaged material transport equation which will be used for the estuarine analysis in this research. Comparing Eq. (2.7), the three-dimensional species continuity equation with the vertically integrated form, Eq. (2.14) the diffusion coefficient \((D_{AB}^* \frac{\partial}{\partial x})\) is integrated as a dispersion coefficient \((D_{AB}^* \frac{\partial}{\partial y})\). The dispersion coefficient includes not only the diffusion associated with turbulent mixing but also the dispersion due to velocity gradients and density differences (Ref. 2.16).

Latest work by researchers such as Hacker (Ref. 2.10), Espey et al. (Ref. 2.14), Leendertse (Ref. 2.12), and Shankar and Masch (Ref. 2.17) has been directed toward solution of the vertically integrated Eq. (2.14). Hacker (Ref. 2.10), in 1973, solved the time dependent, two-dimensional material transport model for the distribution of salinity in the Barataria Bay region of coastal Louisiana. Hacker's equation is summarized in Table 2.1. The convection terms required to solve the material transport model were computed from the equations of motion and continuity. He has used explicit numerical techniques to solve the partial differential equations. From his study Hacker has concluded that his material transport model accurately predicts salinity distributions in the marsh and in the open bay.


TABLE 2.1 TWO-DIMENSIONAL MATERIAL TRANSPORT MODELS

<table>
<thead>
<tr>
<th>Investigator</th>
<th>( \frac{\partial (DS_A)}{\partial t} )</th>
<th>( \frac{\partial (UDS_A)}{\partial x} )</th>
<th>( \frac{\partial (VDS_A)}{\partial y} )</th>
<th>( \frac{\partial}{\partial x} [D \frac{\partial S_A}{\partial x}] )</th>
<th>( \frac{\partial}{\partial y} [D \frac{\partial S_A}{\partial y}] )</th>
<th>( P_A = 0 )</th>
<th>System Modeled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hacker Ref. 2.10</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>Salinity distribution in estuary</td>
</tr>
<tr>
<td>Espey et al. Ref. 2.14</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>BOD, DO distribution in estuary</td>
</tr>
<tr>
<td>Leendertse Ref. 2.12</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>BOD, DO coliforms distribution in estuary</td>
</tr>
<tr>
<td>Shankar &amp; Masch Ref. 2.17</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>Salinity distribution in estuary</td>
</tr>
</tbody>
</table>

The equation used in the works referenced is formed by adding the terms that have been checked.
Espey et al., who were first to suggest the idea of using an overall transport phenomena model for estuarine analysis in 1968 (Ref. 2.11), reported solutions to the time dependent, two-dimensional material transport model in 1971 (Ref. 2.14). The equation used is summarized in Table 2.1. The model was used to describe the distribution of biochemical oxygen demand (BOD) and dissolved oxygen (DO) in the Morgan Point area of Galveston Bay. Since BOD and DO concentrations depend on each other, the first order kinetic rate expressions used are coupled, and the solution necessitates simultaneous solution of material transport models for different species. The explicit numerical solution technique was used. It was reported that the results obtained by this simulation study were in general agreement with the field data.

Leendertse (Ref. 2.12) in 1971, presented his solution to the time dependent two-dimensional model. His equation is also summarized in Table 2.1. Leendertse has solved the material transport model for the distribution of salinity, BOD, DO and coliform organisms. The numerical solution technique used was the alternating directions implicit (ADI) technique. With the help of charts, Leendertse has shown that computed values of salinity are smaller than the observed average values measured through the bay system. The probable reason for this was given as too large tidal circulation. He has also shown that for BOD, DO and coliform organisms the agreement between the
conditions simulated and averages of a large number of samples taken at different stations in the bay was quite good.

Shankar and Masch (Ref. 2.17), in 1970, solved the time dependent, two-dimensional material transport model for the salinity distribution in the Galveston Bay. The equation used is summarized in Table 2.1. The explicit numerical technique was used for solution of the model. It was reported that this model reproduced the concentration distributions throughout the bay system within reasonable limits of accuracy.

There are numerous references available in the literature discussed below which use one-dimensional material transport models for the analysis. In 1971, Espey et al. (Ref. 2.11) presented a one-dimensional, mass-balance model describing both the steady-state and dynamic transport within the Houston Ship Channel. The model has been developed to include variable cross-sectional area, flows and loadings. The equation presented is summarized in Table 2.2. The model was used to predict chloride, BOD, and DO distributions. The purpose of this work was to evaluate different parameters such as dispersion coefficient and reaction rate constants for the system under study. The results obtained were in general agreement with the data available.

Thomann et al. (Ref. 2.18), in 1970 presented one-dimensional steady-state material transport models
TABLE 2.2 ONE-DIMENSIONAL MATERIAL TRANSPORT MODELS

<table>
<thead>
<tr>
<th>Investigator</th>
<th>( \frac{\partial (DS_A)}{\partial t} + \frac{\partial (DUS_A)}{\partial x} - \frac{\partial }{\partial x} \left( DB_{AB} \frac{\partial S_A}{\partial x} \right) - P_A = 0 )</th>
<th>System Modeled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Espey et al.</td>
<td>X X X X</td>
<td>BOD, DO distribution in channel</td>
</tr>
<tr>
<td>Ref. 2.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thomann</td>
<td>X X X</td>
<td>Nitrogen, DO distribution in estuary</td>
</tr>
<tr>
<td>Ref. 2.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Callaway</td>
<td>X X X</td>
<td>Salinity, BOD distribution in river</td>
</tr>
<tr>
<td>Ref. 2.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bella</td>
<td>X X X</td>
<td>DO distribution in river or estuary</td>
</tr>
<tr>
<td>Ref. 2.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Harleman</td>
<td>X X X</td>
<td>Dye distribution in estuary</td>
</tr>
<tr>
<td>Ref. 2.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shubinski</td>
<td>X X X</td>
<td>Salinity, BOD distribution in estuary</td>
</tr>
<tr>
<td>Ref. 2.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>O'Connor</td>
<td>X X X</td>
<td>BOD, DO distribution in estuary</td>
</tr>
<tr>
<td>Ref. 2.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thomann</td>
<td>X X X</td>
<td>BOD, DO distribution in estuary</td>
</tr>
<tr>
<td>Ref. 2.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stommel</td>
<td>X X X</td>
<td>BOD distribution in estuary</td>
</tr>
<tr>
<td>Ref. 2.26</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 The equation used in the works referenced is formed by adding the terms that have been checked.
describing distribution of different species of nitrogen coupled with DO distribution in the Potomac and Delaware estuary. Their equation is summarized in Table 2.2. The objective of the analysis was to obtain greater insight into the effect of feedback of nutrients and to determine the usefulness of the approach in sequential reactions such as nitrification. They have obtained the values of reaction rate constants for their system. Using the constants obtained, they have evaluated concentrations of different species of nitrogen. With the help of graphs, they have shown that there is good agreement between calculated values and observed values.

In 1969, Callaway et al. (Ref. 2.19) presented a one-dimensional time dependent material transport model which is summarized in Table 2.2. The model was intended to be used to find salinity distribution in the Columbia River. No results are reported.

In 1968, Bella and Dobbins (Ref. 2.20) presented a general one-dimensional, time dependent material transport model that could be applied to river or estuary flows. The model is summarized in Table 2.2. They have used an explicit numerical technique as well as an analytical approach to evaluate DO profile. The analysis was used to compare the capabilities of finite difference scheme and analytical methods in solving water quality problems in estuaries. It was concluded that analytical solutions were of limited value in the investigation of the effects of BOD and DO
sources and sinks on the BOD and DO profiles.

Also in 1968, Harleman et al. (Ref. 2.21) used a one-dimensional material transport model to determine the longitudinal dispersion coefficient in the Potomac estuary using the dye test. The equation used is summarized in Table 2.2. The explicit finite-difference technique was used to solve the model. The results were compared with the field measurements of dye dispersion. Since available tidal input data were insufficient, the agreement between observed values and calculated values was not good.

In 1965, Shubinski et al. (Ref. 2.22) solved a one-dimensional branched network of interconnecting channels. Equations of motion and continuity were used to calculate velocities. Dispersion was computed from the scale (depth) of each channel according to relationships suggested for estuaries by Orlob (Ref. 2.23). This essentially one-dimensional system has been applied to approximate the two-dimensional behavior of San Francisco Bay, San Diego Bay and other similar systems. It is reported that the concentrations calculated were much lower than actually recorded in field observations. It is reasoned that the calculated net flows were always seaward in spite of the fact that strong current reversals actually occur.

Also in 1965, O'Connor (Ref. 2.24) presented the solution of a one-dimensional material transport model that could be used in general to evaluate the distribution of nonconservative substances (BOD, DO) in estuaries. The
equation is summarized in Table 2.2. The equation was solved analytically for the constant area estuary and for an estuary whose area has simple functional dependence on length. Data from the model tests and field surveys were compared. The agreement is reasonable.

In 1963, Thomann (Ref. 2.25) presented a one-dimensional, time dependent material transport model which is summarized in Table 2.2. The model was used to find BOD and DO concentration profiles in the Delaware River estuary. The concentration profiles have not been verified.

Apparently the first to employ the material transport model in estuaries was Stommel (Ref. 2.26) in 1953. He employed a steady-state assumption for a well-mixed system, i.e., one exhibiting negligible variations in the vertical and lateral directions as compared with those along the axis of the estuary. The equation used is summarized in Table 2.2. The velocity term was estimated from river runoff and estuary cross-sectional areas.

**Hydrodynamic Models**

Up to 1968, all of the modeling done on shallow estuaries were studies on the transport of momentum. All these hydrodynamic models were based on the derivation done by Hansen (Ref. 2.3). Hansen obtained the model equations by vertically integrating the general equations of motion and continuity. These vertically integrated equations are:
\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} - FV + g \frac{\partial L}{\partial x} = \frac{1}{\rho} \left( \tau_x - \tau_b \right) \tag{2.15}
\]

\[
\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + FU + g \frac{\partial L}{\partial y} = \frac{1}{\rho} \left( \tau_y - \tau_b \right) \tag{2.16}
\]

\[
\frac{\partial L}{\partial t} + \frac{\partial (DU)}{\partial x} + \frac{\partial (DV)}{\partial y} = (R - E_v) \tag{2.17}
\]

The above equations are summarized in Tables 2.3 and 2.4.

The latest in the line of hydrodynamic models is the one presented by Hacker et al. (Ref. 2.10). This model uses the powerful ADI technique similar to that used by Leendertse (Ref. 2.8). The equations used in the model are summarized in Tables 2.3 and 2.4. This model is used to predict the current distribution in the Barataria Bay region of coastal Louisiana. Hacker has solved the model for typical conditions that are observed for the system. It is concluded that the model accurately predicts the tidal variations in marsh and bay areas.

Leendertse (Ref. 2.8) presented his hydrodynamic model in 1967. The advantage of this model is due to the advanced ADI technique used for the numerical solution. Because of its inherent stability and rapid convergence, this technique is superior to the explicit techniques used on all the other hydrodynamic models that have the same equations as a basis. The equations used by Leendertse are summarized in Tables 2.3 and 2.4. Leendertse used his hydrodynamic model in conjunction with a material transport
TABLE 2.3 HYDRODYNAMIC MODELS: EQUATION OF CONTINUITY

<table>
<thead>
<tr>
<th>Investigator</th>
<th>$\frac{\partial L}{\partial t}$</th>
<th>$\frac{\partial (DU)}{\partial x}$</th>
<th>$\frac{\partial (DV)}{\partial y}$</th>
<th>$R - Ev$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hacker (Ref. 2.10)</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Leendertse (Ref. 2.9)</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Reid and Bodine (Ref. 2.11)</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Masch et al. (Ref. 2.17)</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Miyazaki, Ueno and Unoki (Ref. 2.18)</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Unoki and Isozaki (Ref. 2.7)</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Miyazaki (Ref. 2.6)</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Platzman (Ref. 2.5)</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Hansen (Ref. 2.4)</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

1 The equations used in the works referenced are formed by adding the terms that have been checked.
### TABLE 2.4 HYDRODYNAMIC MODELS: EQUATION OF MOTION\(^1\)

(X-Component)

<table>
<thead>
<tr>
<th>Investigator</th>
<th>(\frac{\partial U}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} - FV + \frac{\partial L}{\partial x} \cdot \tau_x^s/\rho - \tau_x^b/\rho)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hacker (Ref. 2.10)</td>
<td>X X X X X (Kw^2 \cos(\theta)) (gU(U^2 + V^2)^{1/2}/C^2)</td>
</tr>
<tr>
<td>Leendertse (Ref. 2.9 and 2.13)</td>
<td>X X X X X (Kw^2 \cos(\theta)) (gU(U^2 + V^2)^{1/2}/C^2)</td>
</tr>
<tr>
<td>Reid &amp; Bodine (Ref. 2.11)</td>
<td>X X (Kw^2 \cos(\theta)) (fU(U^2 + V^2)^{1/2}/D^2)</td>
</tr>
<tr>
<td>Masch et al. (Ref. 2.17)</td>
<td>X X X (Kw^2 \cos(\theta)) (fU(U^2 + V^2)^{1/2}/D^2)</td>
</tr>
<tr>
<td>Miyazaki, Ueno &amp; Unoki (Ref. 2.18)</td>
<td>X X (p_a \gamma^2 W</td>
</tr>
<tr>
<td>Unoki &amp; Isozaki (Ref. 2.7)</td>
<td>X X X (p_a \gamma^2 W</td>
</tr>
<tr>
<td>Miyazaki (Ref. 2.6)</td>
<td>X X X (p_a \gamma^2 W</td>
</tr>
<tr>
<td>Platzman (Ref. 2.5)</td>
<td>X X (K^* \rho dB^{-1} \frac{rU</td>
</tr>
<tr>
<td>Hansen (Ref. 2.4)</td>
<td>X X X X X (\eta*W</td>
</tr>
</tbody>
</table>

\((*) K=K_1 \ W \leq W_c \) critical wind speed = 14 knots  
\((*) K=K_1 + K_2(1 - \frac{W}{W_c})^2 \ W > W_c \)  
\(K_1 = 1.1 \times 10^{-6} \)  
\(K_2 = 2.5 \times 10^{-6} \)

\((*) \ y^2 = 2.6 \times 10^{-3} \)  
\((***) \ \alpha = 2.6 \times 10^{-3} = 0.25 \sim 0.50 \)

\((***) \ \mu = \text{coefficient of eddy viscosity (numerical value not reported)}\)

\((o) \ K^* > 1, M \equiv v dz, R \equiv T(h)/T(h) = \text{surface stress by wind (not reported)}\)

\((oo) \ \eta = 3.2 \times 10^{-6} \)  
\((ooo) \ r = 2.6 \times 10^{-3} \)

---

\(^1\) The equation used in the works referenced is formed by adding the terms that have been checked.
model to predict water quality in Jamaica Bay, New York. It is reported that excellent agreement between computed and observed tidal data has been obtained.

Reid and Bodine (Ref. 2.9) vertically integrated the equations of motion and continuity to produce a model of Galveston Bay used to predict storm surges in 1968. The model ignores Coriolis forces, advection of momentum, and uses a quadratic bed resistance based on Manning's coefficient. The equations used are summarized in Tables 2.3 and 2.4. This model used empirical correlations to correct for flow conditions due to submerged barriers, weirs, and tidal inputs. Comparison was made between computed and observed values. A significant deviation between the two values was observed in the first 12 hours of the storm period. This variation is attributed to the approximation of a static system in the numerical model for beginning the problem. However, it is shown that the computed and observed hydrographs approximately converge after 12 hours of storm time. It is concluded that overall correlation obtained is adequate.

Masch et al. (Ref. 2.27) presented a hydrodynamic model identical to the Galveston Bay Model by Reid and Bodine except that he includes Coriolis force terms. Masch used this model as a basis for a salinity model of the San Antonio and Matagorda Bays in Texas (Ref. 2.15). It is shown with the help of graphs that tidal variations computed with the help of the model are in general agreement
with observed values.

Miyazaki, Ueno, and Unoki (Ref. 2.28), in 1962, developed a hydrodynamic model based on long wave equations in which advection of momentum was ignored. This model was used to investigate typhoon surges along the Japanese coast. Special care was given to analyzing wind generated currents in this study. Using a number of charts, it is shown that the results obtained by the model are in fair agreement with the observations at three stations out of a total of four investigated. At the fourth station, the computed pattern agrees with the observation but the amplitude is only one-half the latter. No explanation is given for such behavior.

Using this work as a base, Unoki and Isozaki (Ref. 2.6) studied the effects of storm surges caused by typhoons on a dike with openings in Tokyo Bay in 1963. Empirical equations, to calculate the flow through the opening of a dike, were developed in this work. It is reported that the computation coincides with observation as a whole, but the remarkable oscillations found in the record are not represented sufficiently in computation. This is attributed to the fact that the bottom friction expressed in the model was not adequate and the abrupt change of wind was not sufficiently accounted for in the model. It was later on that Reid and Bodine used this work to arrive at a series of empirical equations for flows for their Galveston Bay Model.

Miyazaki (Ref. 2.5) also used the same equations to
produce a model to study the effects of Hurricane Carla in 1961 in the Gulf of Mexico. The equations used by the past three works mentioned are summarized in Tables 2.3 and 2.4. The model was applied to the whole gulf area. A combination of coarse and fine grid size was used. Fine-mesh was used along the Texas-Louisiana coast where storm surges were computed at nine different stations. It is reported that the computed values were in good agreement with available data.

In 1958, Platzman (Ref.-2.4) developed a model to study the surge of June 26, 1954 on Lake Michigan. The main forcing function on this model was a drastic change in atmospheric pressure due to an intense and fast-moving squall line. This model describes the surge generated by this pressure gradient and is the only one found in the literature that uses atmospheric pressure gradients as a forcing function. All of the other hydrodynamic models have tidal variation as a forcing function, and most ignore atmospheric pressure effects. It was in this work that Platzman developed the explicit "leap frog" numerical technique used by most of the recent investigators. The equations used by Platzman in his model are summarized in Tables 2.3 and 2.4. The reported results are as follows:

a. The computed amplitude of the main surge is approximately one-half the observed amplitude, but the inclusion of the effect of wind stress very probably will remove the major part of this discrepancy.

b. The computed phases between significant events are in excellent agreement with the observations.
c. The structure of the tail of the main surge is not in agreement with the observed structure, probably because the resolving power of the grid is inadequate for this purpose.

In 1956, Hansen (Ref. 2.3) crystallized the ideas he first presented in 1938 (Ref. 2.2) of vertically integrating the equations of motion and continuity to produce a two-dimensional hydrodynamic model, or the long-wave equations. He used his model in conjunction with a rudimentary explicit numerical scheme, to predict hydrodynamic behavior in open shallow seas. Coriolis forces and advection of momentum were considered in this model. The equations used are summarized in Tables 2.3 and 2.4.

Energy Transport Models

The following part of this section will deal with the equation used for the two-dimensional energy transport model for the shallow estuaries. The first mathematical derivation of the vertically averaged two-dimensional energy equation was given by Hacker et al. (Ref. 2.13) in 1971. This equation is analogous to the vertically averaged species continuity equation and can be written as:

$$\rho C_p \left[ \frac{\partial (DT)}{\partial t} + \frac{\partial (UDT)}{\partial x} + \frac{\partial (VDT)}{\partial y} \right] = \frac{\partial}{\partial x} \left[ DK x \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[ DK y \frac{\partial T}{\partial y} \right] + PE$$  

(2.18)

Hacker et al. solved the above equation to find the temperature distribution in the Barataria Bay region of coastal Louisiana. The equation used is summarized in Table 2.5.
TABLE 2.5 ENERGY TRANSPORT MODELS

<table>
<thead>
<tr>
<th>Investigator</th>
<th>(\frac{\partial (TD)}{\partial t} + \frac{\partial (UDT)}{\partial x} + \frac{\partial (VDT)}{\partial y} - \frac{\partial^2}{\partial x^2} \left( \frac{DK}{x} \frac{\partial T}{\partial x} \right) - \frac{\partial^2}{\partial y^2} \left( \frac{DK}{y} \frac{\partial T}{\partial y} \right) - P_E )</th>
<th>System Modeled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hacker et al. (Ref. 2.13)</td>
<td>X  X  X  X  X  X  X</td>
<td>Temporary distribution in an estuary</td>
</tr>
<tr>
<td>Tracor (Ref. 2.14)</td>
<td>X  X  X  X  X  X  X</td>
<td>Temporary distribution in an estuary</td>
</tr>
</tbody>
</table>

1The equation used in the works referenced is formed by adding the terms that have been checked.
The equation is solved using an explicit, finite-difference technique. Hacker, from his study, concluded that the energy transport model accurately predicts the time-varying temperature distribution in the Barataria Bay system for marsh areas as well as open water of the bay.

TRACOR (Ref. 2.14) in 1971 presented a model for energy transfer in Galveston Bay. This work, combined with their hydrodynamic model and material transport model, produced a comprehensive transport model. The energy transport model used is summarized in Table 2.5. The model is solved using an explicit, finite-difference technique. It is reported that the agreement between computed and observed temperatures is quite satisfactory.

Numerical Techniques

Numerical techniques used for the solution of vertically averaged two-dimensional models for momentum, energy and material transfer can be broadly classified into two groups: explicit schemes and implicit schemes. Until 1967, only explicit schemes were used to solve the equations. While some differences exist among the techniques used by the different investigators, the explicit schemes used were basically the same. When attempting to solve equations by numerical techniques, the area under study is placed under a square grid system with a rough approximation of the irregular boundaries, as shown in Figure 2.1. The partial differential equations which form the models are transformed
Figure 2.1. Square grid over irregularly shaped bay
into difference equations with the substitution of finite difference approximations for the partial derivative terms. In the explicit technique, the resulting algebraic equations can be explicitly solved for the dependent variables.

Explicit schemes suffer from the disadvantage that they are conditionally stable. Platzman (Ref. 2.4) classified the long-wave equations as of the hyperbolic type, assuming that the two-dimensional long-wave equations behave as the one-dimensional system. Subsequent researchers seemed to go along with this result. With this, the stability criterion for the above scheme is obtained as:

For size step:

$$\Delta x = \Delta y \leq \min \left[ \frac{8 \theta_{\text{min}}}{|U_{\text{max}}|}, \frac{8 \theta_{\text{min}}}{|V_{\text{max}}|} \right]$$

(2.19)

For time step:

$$\Delta t \leq \min \left[ \frac{\Delta x}{2|U_{\text{max}}|}, \frac{\Delta y}{2|V_{\text{max}}|} \right]$$

(2.20)

and/or

$$\Delta t \leq \min \left[ \frac{(\Delta x)^2}{4 \theta_{\text{max}}}, \frac{(\Delta y)^2}{4 \theta_{\text{max}}} \right]$$

(2.21)

This condition raises a severe problem for long, real-time solutions because extensive time is required.

All hydrodynamic models used similar explicit techniques with the exceptions of Leendertse (Ref. 2.8) and Hacker (Ref. 2.10). All energy and species models presented also used explicit technique with the exception of
Leendertse (Ref. 2.12) and Masch (Ref. 2.15). Implicit schemes for vertically averaged equations of change are difficult to obtain. It was not until 1967 that Leendertse presented an implicit scheme for solving the hydrodynamic model. The advantage of this technique over the explicit one is its inherent stability and rapid convergence.

Leendertse (Ref. 2.8) has discussed the convergence of his alternating-direction-implicit scheme. Leendertse used an identical technique to solve the two-dimensional species continuity model for Jamaica Bay, New York (Ref. 2.12). The vertically averaged species continuity and energy equations are of the parabolic type and alternating directions implicit schemes are ideally suitable for their solution. Masch also used this technique in solving for the salinity distribution in the San Antonio and Matagorda Bays, Texas (Ref. 2.15).

The combination of the hydrodynamic model and the energy and species transport models results in a complete transport model of the area under study. The validity of these models has been established in the literature (Refs. 2.4, 2.7, 2.8, 2.9, 2.12, 2.14).

Contributions

The major objective of the current research in the coastal area of Louisiana by Louisiana State University is coastal zone planning and evaluation of methods to improve the marine resources of the region. An important
prerequisite to the determination of such a plan is the development and validation of a set of management tools for use in evaluating the various available alternatives. These management tools consist of mathematical models which describe the hydraulic behavior and distribution of various parameters that have significant effect on the ecology of the area. Such models have already been proposed by Hacker et al. (Ref. 2.10) for the Barataria Bay region of coastal Louisiana.

The specific aim of this research is to solve the material transport model to find the distribution of nutrients (specifically nitrogen), detritus, phytoplankton, and other organisms that contribute to the primary production. The sink and source terms required to solve the material transport model will be developed during this research. The biogeochemical cycle of nitrogen is inextricably related to the primary production. The nitrogen cycle is an immensely complex phenomenon. Moreover, various features of the cycle change from place to place. Hence, delineation of the complete nitrogen cycle in the area of interest is the first step toward developing sink and source terms.

Several nitrogen forms may serve as a nutrient source for algal growth, Spartina production, and bacterial growth. These species release nitrogen upon death, thereby completing the cycle. This behavior can be represented as a sequence of coupled reactions. The material transport model
should be solved for every different species and since various species are coupled, the solution should be carried out simultaneously. The literature survey shows that such complex analysis has never been attempted for estuarine systems. Moreover, Barataria Bay, the estuary under study in this work, is unique in the sense that there is continuous interaction between the marsh and the water. An attempt is made to analyze this complex and vast body of water. In summary, this work will:

1. Delineate the biogeochemical cycle of nitrogen as occurring in the Barataria Bay and model the cyclic behavior.

2. Solve material transport model to find distribution of nutrients, detritus, phytoplankton, and other organisms that contribute to the primary production.

3. Simulate different conditions to obtain guidelines to base coastal management decisions.
REFERENCES


2.5 Miyazaki, M., A Numerical Computation of the Storm Surge of Hurricane Carla in the Gulf of Mexico, Technical Report No. 10, Dept. of Geophysical Sciences, University of Chicago, Chicago, Ill.


CHAPTER III

ECOSYSTEM CONCEPTS AND NITROGEN CYCLING

Introduction

The purpose of this chapter is to delineate the framework of biogeochemical cycling of nitrogen and to develop the mathematical models for this cycling of nitrogen. The cycling of nitrogen is related to the production of organic matter in the water body. Hence, in developing this framework, involvement of biotic community should be clearly defined. In other words, we are looking at the interaction of biotic and abiotic factors which together form the ecosystem.

This chapter is divided into three parts. The first part describes the interactions in the ecosystem. The second part describes the biogeochemical cycle of nitrogen in the ecosystem. This part is further divided into two subsections. The first section describes the previous work on nitrogen cycling as applicable to this study. In the second section, nitrogen cycling and biomass dynamics as it seems to occur in the Barataria Bay region is given. In the third part, the mathematical expressions describing the biogeochemical cycling of nitrogen and biomass dynamics are developed. The procedures are given to evaluate the rate
coefficients appearing in the rate equations that are de­
veloped.

**Interactions in the Ecosystem**

As explained in Chapter I, abiotic factors play an important role in the life cycle of biotic components. Living organisms and the nonliving environment continuously interact and influence each other. The abiotic and the biotic components are so interactive that in the study of an ecosystem, they cannot be considered separately. In other words, an ecosystem is a complex of organisms and environment forming a functioning whole in nature. The interaction between living organisms and nonliving environment exchanges energy in various forms. The energy transfer in the ecosystem follows the biotic diversity in the ecosystem, and it also leads to the exchanges of chemical substances.

The energy transfer in the ecosystem can be understood from a biological energy flow diagram. The biological energy flow diagram represents flows of energy among various species in an ecosystem. The concepts of food chain and trophic level are used in formulating the energy flow dia­
gram.

The transfer of food energy from the source in plants takes place through a series of organisms. This repeated eating and being eaten is referred to as the food chain (Ref. 3.1). Ricker (Ref. 3.2) recognized the food chain as an important conceptual tool in describing an ecosystem.
he reported that production in any major ecosystem begins with photosynthesis. It continues by way of a maze of food chain links. The products of these links are successive stocks of organisms. At each transfer, a large proportion, 80 to 90 percent, of the potential energy is lost as heat (Ref. 3.3). Therefore, the number of steps or links in a sequence is limited, usually to four or five. The shorter the food chain or nearer the organism to the beginning of the chain, the greater the available energy. Thus food chains are not isolated sequences but are interconnected with one another.

The trophic level is the other important conceptual tool in an ecosystem. This concept is a classification of species by diet. As Odum (Ref. 3.3) has reported, the organisms whose food is obtained from plants by the same number of steps are said to belong to the same trophic level.

From the trophic standpoint, the biotic community of an ecosystem can be divided into two fundamental components, an autotrophic component or producers and a heterotrophic component or consumers (Ref. 3.4). An autotrophic component is self-feeding. By the photosynthetic process, these organisms absorb radiant light energy, and using chlorophyll and simple inorganic substances, they build up complex organic molecules. These molecules serve the nutritional requirements of the producer's own growth and metabolism.

A heterotrophic component depends on other organisms for its nourishment. Organisms in this group require
complex organic materials for their nutrition and rely on the availability of organic food of high energy content. For their energy requirements, they use the complex organic material synthesized elsewhere after it is broken down and rearranged into simple components.

Heterotrophs can be further divided as primary, secondary, or tertiary consumers. A primary consumer or a herbivore is a heterotroph that obtains its nutrition directly from plants; a secondary consumer or a primary carnivore is a heterotroph that obtains its nutrition from herbivore; and a tertiary consumer or a secondary carnivore is a heterotroph that obtains its energy from primary carnivore. Thus, from the above discussion, it is emphasized that this trophic classification is one of function and not of species as such (Ref. 3.3). Consequently, a given species population may occupy one, or more than one, trophic level according to the source of energy actually assimilated.

**Barataria Bay Ecosystem**

Having the general concepts of the biological energy flow diagram, the one for Barataria Bay is presented in Figure 3.1. This was from the study by Vora (Ref. 3.5) and was modified by deleting menhaden which is predominantly a near-shore rather than a marsh species. In the discussion to follow, it will also be shown how the various terms discussed above apply to the ecology of the Barataria Bay.

The biological energy flow diagram for Barataria Bay
Figure 3.1. Biological energy flow diagram, Barataria Bay, Louisiana (Ref. 3.5)
Figure 3.2. Explanation of symbols used in biological energy flow diagrams (Ref. 3.5)
is divided into three compartments, marsh, water and sediment. The primary production in Barataria Bay can be divided into four categories:

1. Production by marsh grasses (production on marsh)
2. Production by epiphytic algae on marsh grass and on marsh surface (production on marsh)
3. Production by phytoplankton (production in water)
4. Production by benthic organisms (production in sediment)

Kirby (Ref. 3.6) has reported that the most important source of primary production in the region is *Spartina alterniflora*. This is complemented by three other sources listed above to give a very high overall primary production. In analyzing the primary producers in the region in this study, epiphytic algae and marsh grasses are grouped together, and benthic organisms and phytoplankton are grouped together. Epiphytic algae grow on the stems of *Spartina alterniflora* and on the marsh surface; and compared to the production of *Spartina alterniflora*, the production of epiphytic algae is not significant enough (less than 3.5% of *Spartina alterniflora* production) to deserve a separate treatment. In the case of benthic organisms, not only is the production comparatively small (less than 8% of the phytoplankton production), but studies of benthic organisms have not been frequent enough to establish any seasonal trends. Thus the primary production can in fact be divided into two groups, namely: (1) production by marsh grass and epiphytic algae and (2) production by phytoplankton and benthic organisms.
Beginning with the two different above listed sources of primary production, there are two main food chains. In one group, the primary producers are marsh grasses. Little Spartina alterniflora is consumed alive, and most of the live standing crop of Spartina alterniflora is transformed to dead standing crop (Ref. 3.7). The dead standing crop is subjected to microbial activity and is decomposed to form detritus. Detritus is defined as all types of biogenic material in various stages of microbial decomposition which represent potential energy sources for consumer species (Ref. 3.8). Detritus furnishes food for much of the marsh fauna and herbivores and browsing animals of various sorts. These are in turn fed upon by predaceous animals which could be termed secondary consumer or primary carnivores. In turn these might be fed upon by larger predators.

Detritus plays an extremely important role in the estuarine ecosystem. Organic detritus is the chief link between primary and secondary productivity. As only a small portion of the net production of the marsh grass is grazed while it is alive, the major energy flow between autotrophic and heterotrophic levels is by the way of the "detrital food chain" (Ref. 3.9, 3.10, 3.11, 3.12). After conversion of the dead standing crop to detritus and further reduction in size, the material enters in the water body. Despite the recognition of its importance, there remains a great deal of ignorance as to the quantity of the material
and its fate after it leaves the marsh.

The detritus in the estuary can be partitioned into suspended detritus and detritus settled on the bottom. The partitioning will be continuously fluctuating depending on water turbulence and microbial activity. It is very difficult to differentiate the actual detrital particles from the microflora and fauna which live upon it and ingest it. These include bacteria, fungi, protozoans, algae, and rotifers.

Odum and De La Cruz (Ref. 3.13), from oxygen consumption measurements and metabolism measurements of detritus, concluded that the suspended particles are by no means "dead" or "inert" bodies, and the detritus particles and the attendant microorganisms make up a highly active heterotrophic microecosystem. They studied the decomposition of dead standing crop of Spartina alterniflora in Georgia salt marshes and reported that although the small suspended particles were 70-80 percent ash, the organic matter in them proved to be rich in protein, up to 24 percent as compared to ten percent in living grass. A buildup of microbial populations is presumed to account for the enrichment of the decomposing Spartina alterniflora. Since decomposing Spartina alterniflora is enriched because of microbial activity, detritus rich in bacteria may be a better food source for animals than the grass tissue that forms the base for most of the particulate matter. The detritus enriched with protein by microbial activity is eaten by animals at all
trophic levels either in coarse form or in suspended form.

Darnell (Ref. 3.14-3.18) considered detritus to be the particles with the associated organisms and studied the food habits of the large consumers by examination of their gut contents. In the headwater streams of the Rio Tamesi in Mexico, organic detritus made up over one-half of the ingested food material in seven of 17 species which were considered to be the important large consumers. In similar studies in Lake Pontchartrain (Ref. 3.16), nearly all of the major consumers had ingested some detritus. This material made up over one-half the observed diets in eight of the 35 species examined. The most abundant species in the community were the detritus feeders. In a study of the Neuse River estuary in North Carolina, organic detritus made up over one-half of the food of three of 12 species examined, although it was represented in the stomach contents of most species. Darnell states that not only benthic, but many planktonic species as well take in substantial amounts of organic detritus.

At this point it can be summarized that in one of the food chains, the base is *Spartina alterniflora*. This marsh grass is converted into dead plant and then by microbial activity into detritus. The detritus so formed is the food source for many animals at various trophic levels.

The second food chain begins with the microscopic green plants of phytoplankton. The primary producers in the plankton form a mixed population of photosynthesizing
microorganisms of which diatoms (single-celled algae) are usually the most important. These primary producers are fed upon by the zooplankton which are primary consumers. Zooplankton is the food supply for small fishes and a few large predaceous insect larvae, and these in turn form the prey of the larger fishes.

Thus in the second food chain, the base is phytoplankton. The phytoplankton is eaten by zooplankton at the next trophic level, which is subsequently eaten by animals at higher trophic levels.

Correspondingly in the biotic and abiotic components of an ecosystem, there is continuous interaction between autotrophic and heterotrophic components and between different levels of heterotrophic component. As mentioned earlier, the various components interact in a fundamentally energy-dependent fashion. The transfer of energy between different trophic levels follows the first and the second laws of thermodynamics. In an ecosystem, incoming energy to any species is always equal to the energy leaving the species plus that accumulated by the species. This concept of an energy balance satisfies the first law of thermodynamics, which states that energy may be transformed from one type to another, but it is never created or destroyed.

A portion of the energy coming to a species is dissipated as heat in metabolic activity and is measured as respiration. Odum (Ref. 3.3) described this concept stating that the energy transfer among species is always associated
with the progressive decrease in (but not destruction of) energy at each trophic level. Thus this concept is supported by the second law of thermodynamics which states that "no process involving an energy transformation will spontaneously occur unless there is a degradation of the energy from a concentrated form into a dispersed form" (Ref. 3.3). Consistent with the second law of thermodynamics, a large proportion of energy is lost as heat (for example, 82% for live Spartina alterniflora).

The Role of Nutrients

The dependency of life on energy is coexistent with a dependency on the availability of some 20 elements required in the dynamics of life processes (Ref. 3.19). Although carbohydrates can be photosynthesized from water and carbon dioxide of atmosphere, the more complex organic substances require additional components either in considerable abundance, as in the case of nitrogen and phosphorus, or in trace amounts, as in the case of zinc and molybdenum. Further, the very process of photosynthesis occurs in the presence of enzymes which themselves contain an array of elements. Thus it is important to consider the movement of these nutrients in the system.

Unlike the unidirectional flow of energy, nutrients circulate in the ecosystem. Although there is a progressive diminution of energy in trophic feeding chains, the nutrient components are not diminished. In fact, some may even become
concentrated in certain steps of the chain. In any event, nutrients are not lost in the same manner as energy. When nutrient-containing protoplasm is eventually subjected to decomposer activity, the nutrients are released to the environment, and here they are available for re-use, for recycling. Thus energy does not circulate but materials do.

Russell-Hunter (Ref. 3.20) states that from the elements which are commonly involved in the organic materials, the majority of the essential elements make up only a tiny fraction of the mass of organisms. On a dry-weight basis, only five elements are present in the organic tissues of the majority of living organisms at levels greater than one percent. These are carbon, oxygen, hydrogen, nitrogen, and phosphorus. The essential material available in amounts most closely approaching the critical minimum needed will tend to be the limiting one. Given the ready availability of carbon dioxide and of water, the usual limiting nutrients of estuarine productivity are nitrogen and phosphorus.

Nitrogen along with phosphorus have long been considered to be the limiting nutrients for photosynthetic production in estuarine systems. Many in the past felt that phosphorus was most likely the limiting nutrient, but recent determinations of the total phosphorus content of estuarine waters and of the natural populations of algae have indicated that, relative to its need, phosphorus is not often present in critically low concentrations (Ref. 3.21). Pomeroy et al. (Ref. 3.22) have stated that there is no evidence that
phosphorus ever is limiting the productivity of the estuaries of the southeastern United States. Similar observations have been made by Patrick et al. (Ref. 3.23) for the Barataria Bay region. It is a general opinion (Ref. 3.23, 3.24, 3.25, 3.26) that nitrogen may be the limiting nutrient for the ecology of the Barataria Bay. Thus at this stage, we establish the important point that nitrogen may be the limiting nutrient for the ecology of the Barataria Bay region and that the study of the distribution of the different species of nitrogen will help us to determine the factors that can improve the productivity of the area.

Although the biogeochemical cycling of most of the elements follows a similar general pattern, they circulate through the system in a variable and complex manner. Most cycles involve an element alternately incorporated in inorganic salts in the environmental medium and then in organic molecules as part of the biomass of the ecosystem. Normally, the inorganic form is first taken out of the abiotic environment by an autotrophic green plant, and it is returned after passage through the units of the food chain as a result of the activities of decomposing bacteria. In the following section, the biogeochemical cycle of nitrogen is discussed. This discussion explains the above mentioned cycling pattern.
Biogeochemical Cycle of Nitrogen

In this section the biogeochemical cycle of nitrogen is discussed in two parts. The first part describes the previous work on nitrogen cycling as applicable to this study. In the second part, an analysis of nitrogen cycling is proposed that specifically represents the phenomena occurring in the Barataria Bay system.

Nitrogen Cycling: This is an immensely complex phenomenon in ecosystems. The cycle involves transfer of material from environment to living organisms and back to the environment. The cycle is inextricably related to the organic production in the system. Biological involvement in the nitrogen cycle is by far the most extensive, and ordered. This involvement is highly specific in the sense that certain organisms are able to act only in certain phases of the cycle. The biomass dynamics play such an important role in the cycling of nitrogen in estuarine systems that a predictive model of nitrogen dynamics in such a system must include the capability to model biomass activity as well as the behavior of factors which regulate biomass dynamics. Consequently, the biomass dynamics of the Barataria Bay ecosystem will be included in the analysis.

Nitrogen exists in the biosphere in a number of diverse forms. It is present primarily as organic nitrogen (plant and cellular constituents and as particulate matter), ammonia, and nitrate. These various forms of nitrogen are
all interrelated by a series of reactions, which collectively represent the "nitrogen cycle." The important steps of the nitrogen cycle can be listed as follows: assimilation of ammonia and nitrate, ammonification, nitrification, denitrification, and nitrogen fixation. Each of these important steps will be discussed in the following paragraphs.

Assimilation of Ammonia and Nitrate: Basically, the nitrogen cycle follows the flow of nitrogen into and out of living matter. By far the greatest influx of inorganic nitrogen into organisms results from ammonia and nitrate assimilation. These reactions are carried out by most microorganisms and plants. In natural waters, the primary agents of nitrate and ammonia assimilation are algae, although bacteria and fungi also carry out the process.

The comparative value of ammonium and nitrate ions as a source of nitrogen for plants has been the subject of a number of investigations (Ref. 3.27). Considerable controversy exists in the literature (Ref. 3.28) concerning the relative merits of ammonia and nitrate as algal nitrogen sources. In the past many workers concluded that nitrate is preferred on the basis of laboratory culture experiments. However, the results are difficult to interpret because the nitrogen concentrations were much higher than those found in natural waters. Hutchinson (Ref. 3.29) refers to earlier literature which concluded that ammonia is assimilated as undissociated NH₃ rather than as ammonium ion, but the
currently accepted mechanism is just the opposite. Evidence for the latter mechanism includes growth of certain bacteria in culture media having low pH values with ammonia as the sole nitrogen source. Undissociated ammonia is toxic to certain organisms at high concentrations and high pH values, and because of this some workers may have concluded that ammonia is not a satisfactory nitrogen source.

If both ammonia and nitrate are present, ammonia is generally preferred. Ammonium ion has a valence of minus three, which is the same as the valence of nitrogen in amino acids. The valence of the nitrate ion is plus five. In plants, the nitrogen is present in amino acids. On this basis plants must expend energy to reduce nitrogen from a valence of plus five to minus three.

In view of the necessity of reduction in the process for assimilation of nitrate, it is likely that ammonia would be as good or better a source of nitrogen than is nitrate. In natural waters nitrate has often been assumed to be the nitrogen nutrient source. This is probably because of its normally greater abundance than ammonia. However, recent studies (Ref. 3.30, 3.31) using \(^{15}\text{N}\) have shown higher rates of ammonia assimilation than nitrate assimilation in several North American lakes and the Sargasso Sea. Many higher terrestrial plants can use both ammonia and nitrate, but it is known that the pH of the nutrient medium plays a part in the availability of ammonia. Tiedjens and Robbins (stated by Hutchinson [Ref. 3.32]) found that
several crop plants assimilated ammonia preferentially above pH seven and nitrate preferentially below pH seven. At a pH of four, no ammonia was used. The same behavior was observed for nitrogen assimilation of *Spartina alterniflora* in the Barataria Bay. Ho (Ref. 3.33) obtained the value of pH between 7.0 and 7.8 for the sediments in southern Barataria Bay. Brannon (Ref. 3.34), from his studies, has concluded that *Spartina alterniflora* roots assimilate ammonia from the sediment.

Some form of nitrogen is required by phytoplankton for the synthesis of their cellular amino acids. They satisfy most of their needs by utilizing the ammonia and nitrate present in the water.

Considerable disagreement is seen in the literature as to the form of nitrogen phytoplankton can assimilate. Conover (Ref. 3.35) concludes from his study that the phytoplankton production in Long Island Sound is controlled by the supply of nitrate nitrogen. Riley and Chester (Ref. 3.36) state that deficiency of nutrients, especially nitrate, is probably the main factor limiting marine primary production of phytoplankton. Elsewhere in their book, Riley and Chester (Ref. 3.36) also state that phytoplankton satisfy their nitrogen needs by utilizing the ammonia, nitrate and nitrite present in the water. Harvey (Ref. 3.37) states that although all three sources of nitrogen can be absorbed by most species of phytoplankton, ammonia is usually used preferentially. Vaccaro (Ref. 3.38) has found that
ammonia is probably more important than nitrate as a nutrient in the coastal waters off New England; and it is likely that this is the case in many other areas. For example, Domogella and Fred (stated by Hutchinson [Ref. 3.32]) have observed a very sudden decrease in the concentration of ammonia, accompanied by a great increase in phytoplankton. At the same time, they observed that there was not much change in the concentration of nitrate. This can most easily be explained as direct assimilation of ammonia by phytoplankton. Schreiber (as stated by Riley and Chester [Ref. 3.36]) has found that a few classes of phytoplankton, e.g. phytoflagellates, appear to be able to satisfy their nitrogen requirements by utilization of dissolved organic nitrogen compounds, such as amino acids, and some others, e.g. diatoms, can do so after attached bacteria have deaminated these compounds (Ref. 3.39).

Thus we can see that claims have been made that phytoplankton can use almost any type of nitrogen available. Williams (Ref. 3.40) has found from his studies that many species of phytoplankton can thrive with the inorganic nitrogen (ammonia, nitrite, nitrate) commonly present in estuaries. He further states that it is likely that the actively photosynthesizing species are adapted to existing nutrient conditions. For our system, i.e., in Barataria Bay, nitrate is almost absent (Ref. 3.23, 3.24, 3.26) and it is likely that phytoplankton obtain all their nitrogen requirement by ammonia assimilation.
In summary, the greatest influx of inorganic nitrogen into organisms results from ammonia and nitrate assimilation. Claims have been made that either is suitable as the source of nitrogen for plants as well as microorganisms. Available information supports the fact that ammonia seems to be the most likely source of nitrogen for *Spartina alterniflora* and phytoplankton in the Barataria Bay.

**Ammonification:** Following the incorporation of inorganic nitrogen into an organic form in protein, it is metabolized and returned as waste products of that metabolism or as organized protoplasm in dead organics. Many heterotrophic bacteria, actinomycetes, and fungi occurring in both the soil and water utilize this organic nitrogen-rich substrate; in their metabolism of it, they convert and release it in inorganic form, ammonia (Ref. 3.41). This process is referred to as ammonification or mineralization. Thus this process is the reverse of ammonia assimilation, and organic nitrogen is returned to the inorganic nitrogen pool as ammonia.

The breakdown of organic nitrogen is a complex process. In microorganisms organic nitrogen is usually broken down into amino acids which are deaminated, and the resulting ammonia is excreted. Most higher animals excrete their nitrogen as organic compounds such as urea and uric acid. These organic nitrogen compounds are further degraded to ammonia by chemical reactions or by microorganisms. Under conditions
of low pH, amino acids are sometimes decarboxylated to form amines which are subsequently deaminated to yield ammonia.

Organic nitrogen decomposition proceeds typically from complex high weight polymers (e.g., proteins) into the monomeric units (e.g., amino acids) and then to ammonia. This can be represented by the following sequence:

\[
\text{Particulate organic nitrogen} \xrightarrow{\text{bacteria}} \text{Soluble organic nitrogen} \xrightarrow{\text{bacteria}} \text{Ammonia}
\]

Von Brand et al. (as stated by Brezonik [Ref. 3.42]) have proposed that ammonia is liberated from planktonic organisms by direct bacterial action, without the formation of soluble organic intermediates. Most studies on ammonification have not dealt with this aspect directly, however, and have followed the process by determining concentrations of end-product, i.e., ammonia. It is reported that zooplankton excrete large amounts of amino acids (Ref. 3.43) and that phytoplankton also excrete significant amounts of amino acids (Ref. 3.44). These compounds may then be reassimilated by algae (thus short circuiting the ammonification process) or may be further degraded to ammonia by bacteria. In any event it would seem that soluble organic nitrogen is a significant intermediate in the overall recycling of nitrogen.

The agents of ammonification in natural waters are bacteria, algae and zooplankton. The relative importance of
these agents is not well understood and depends on the location of the water mass under consideration (Ref. 3.45). Bacteria are almost solely responsible for ammonification in the anoxic hypolimnia of lakes. They also apparently play a significant role in the upper layers of the water column, but in recent years it has become increasingly apparent that other agents, especially zooplankton, also are prominent in the process (Ref. 3.46, 3.47). Zooplankton feed on phytoplankton and detrital particles; they excrete (largely as ammonia) the organic nitrogen in these particles in excess of their own requirements. This process is important in that it is a rapid alternative pathway to bacterial degradation. Algae are themselves agents of ammonification, primarily in the process of autolysis after death. Hutchinson (Ref. 3.48) has reported that apart from the production of ammonia by the direct or indirect decomposition of proteins and other nitrogenous organic matter, production of ammonia by bacterial reduction of nitrate is also of considerable importance.

Whatever the exact biochemical pathways of ammonification are, the process is obviously of great importance in renewing a normally quite limited supply of nitrogen for synthesis by primary producers in natural waters. Unfortunately, there is no specific information about any aspect of ammonification for Barataria Bay.
Nitrification: In natural waters nitrate is obtained from ammonia in the nitrogen cycle through the process of nitrification. Nitrification is a biochemical process consisting of two distinct steps. In the first step, ammonia is oxidized to nitrite and in the second step, nitrite is oxidized to nitrate. These reactions are carried out by chemolithotrophic bacteria (autotrophs), who obtain their energy by ammonia and nitrite oxidation and their cellular carbon by reducing carbon dioxide (Ref. 3.28, 3.49, 3.50, 3.51, 3.52). Nitrification is an aerobic process, and nitrifiers require a high oxygen partial pressure for optimum growth.

The nitrification reactions may be represented by the following equations (Ref. 3.53, 3.54, 3.55, 3.56, 3.57):

\[
\text{NH}_4^+ + \text{OH}^- + \frac{1}{2} \text{O}_2 \xrightarrow{\text{bacteria}} \text{H}^+ + \text{NO}_2^- + 2\text{H}_2\text{O} \quad \Delta G^o = -59.4 \text{ K. Cal. (3.1)}
\]

\[
\text{NO}_2^- + \frac{1}{2} \text{O}_2 \xrightarrow{\text{bacteria}} \text{NO}_3^- \quad \Delta G^o = -18.0 \text{ K. Cal. (3.2)}
\]

where \(\Delta G^o\) represents change in internal energy for the reactions.

A few microorganisms represented by the genus Nitrosomonas, which are autotrophs, employ nitrification of the ammonium ion as their sole source of energy. In the presence of oxygen, ammonia is converted to nitrite ion (\(\text{NO}_2^-\)) plus water, in an exothermic reaction, with a change in internal energy of about -59.4 kilocalories per mole. This is quite sufficient for their comfortable existence.
There is another specialized group of microorganisms, represented by *Nitrobacter*, that are capable of extracting additional energy from the nitrite generated by *Nitrosomonas*. The result is the oxidation of a nitrite ion to a nitrate ion in an exothermic reaction with a change in internal energy of about -18 kilocalories per mole.

No specific information is available for Barataria Bay except that nitrification is known to occur. Also, no information exists about the presence of *Nitrosomonas* and *Nitrobacter* in Barataria Bay.

**Denitrification**: The process of denitrification balances nitrogen fixation by reducing fixed nitrogen (in the form of nitrate or nitrite) to molecular nitrogen. Denitrification is a process whereby certain bacteria can use nitrate as their terminal electron acceptor in the absence of oxygen (Ref. 3.58). As with most of the other nitrogen cycle reactions, the biochemical pathway for denitrification is complex and poorly understood. It is known that nitrite is the first intermediate in the process and is formed by a molybdenum containing enzyme, nitrite reductase. Nitrous oxide can sometimes be formed along with molecular nitrogen. Nitrous oxide can be further reduced to molecular nitrogen by bacterial action.

Denitrification is a bacterial phenomenon carried out by anaerobic bacteria. Delwiche (Ref. 3.59) has discussed the agents of denitrification in greater detail. The
Denitrifying bacteria, if allowed to exist in the absence of oxygen, use the nitrate or nitrite ion as electron acceptors for the oxidation of organic compounds. In these reactions the change in internal energy is nearly as large as it would be if pure oxygen were the oxidizing agent.

A controversy existed in the literature for a long time concerning the possibility of denitrification occurring in the presence of oxygen. Brezonik (Ref. 3.60) has reviewed some of this controversy, as has Delwiche (Ref. 3.59). The works of Skerman and Macrae (Ref. 3.61) and Jannasch (as stated by Brezonik [Ref. 3.60]) have demonstrated that denitrification is essentially an anaerobic process. It will occur in the presence of small amounts of dissolved oxygen, especially if suspended material is present. In this case it is felt that anoxic microzones are developed within the suspended material enabling denitrification to occur.

Denitrification is known to occur widely in soils (Ref. 3.62). Also, denitrification is known to occur in anoxic trenches and fjords of the oceans (Ref. 3.63). There is evidence for its occurrence in wide areas of the Pacific Ocean where high photosynthetic production in the surface layers and poor subsurface circulation leads to very low dissolved oxygen concentrations in the deep water. The status of denitrification in lakes is not well known, but Hutchinson (Ref. 3.64) feels that it is probably of greater significance in lakes than in the oceans.
In Barataria Bay, nitrate is almost absent (Ref. 3.24, 3.26). It is therefore believed that as soon as nitrate is formed, it is rapidly denitrified and lost from the system. As far as the marsh surface is concerned, there is an absence of an oxidized zone (Ref. 3.65), and hence the nitrate appears to be immediately reduced and lost from the surface.

**Nitrogen Fixation:** Although men and other animals live in an ocean of air that is 79 percent nitrogen, their supply of food is limited more by availability of fixed nitrogen than by that of any other plant nutrient (Ref. 3.28). By "fixed" it means nitrogen incorporated in a chemical compound that can be utilized by plants and animals. The process of nitrogen fixation is important in maintaining a nitrogen balance in the biosphere, which would otherwise be depleted of nitrogen within a few million years (Ref. 3.63). The process is of importance in the Barataria Bay ecosystem since it allows organic production to go on by continually furnishing a supply of fixed nitrogen.

The biochemistry of nitrogen fixation is rather complex and incompletely understood. The fixation process is endothermic and requires a plentiful supply of organic material as an energy source (Ref. 3.66). Before nitrogen can be fixed it must be "activated," which means that molecular nitrogen must be split into two atoms of free nitrogen. This step requires at least 160 kilocalories for each mole of nitrogen. The actual fixation step, in which two atoms
of nitrogen combine with three molecules of hydrogen to form two molecules of ammonia, releases about 13 kilocalories. Thus the two steps together require a net input of at least 147 kilocalories. Whether nitrogen-fixing organisms actually invest this much energy, however, is not known. Reactions catalyzed by enzymes involve the penetration of activation barriers, and there is not a simple change in internal energy between a set of initial reactants and their end products. One would like to know how the enzyme (nitrogenase) used by nitrogen-fixing bacteria can accomplish at ordinary temperatures and pressures what takes hundreds of degrees and thousands of pounds per square inch of pressure in a synthetic ammonia reactor. Molybdenum seems to be universally necessary for biochemical nitrogen fixation and cobalt is also needed by *Azotobacter* when it is fixing nitrogen.

The nitrogen fixation process is limited to a comparatively small group of organisms, although the list is continually increasing as research discovers more species capable of fixation. For a complete list of the known nitrogen fixing organisms, reference is made to Fogg (Ref. 3.67), Hutchinson (Ref. 3.64), and Stewart (Ref. 3.68).

As noted by Brenner (stated by Brannon [Ref. 3.69]), it is an established fact that bacteria in soils have the capacity to fix nitrogen. In the case of Barataria Bay, *Spartina alterniflora* serves in effect as a nutrient pump. Nitrogen fixed by bacteria in marsh soils is moved by
Spartina alterniflora from deep sediments into the water via bacterial degradation of the grass. This mechanism of transport is an important source of nitrogen in the marshes (Hall et al., as stated by Brannon [Ref. 3.69]).

Nitrogen Cycling in Barataria Bay

It was pointed out in the previous section that the biological involvement in the nitrogen cycle is the most extensive. Biomass dynamics play a very important role in the estuarine system, and hence a predictive model of nitrogen dynamics in such a system must include the biomass dynamics as well as the behavior of factors which regulate the biomass dynamics. This section will present the analysis of the biomass dynamics and the biogeochemical cycle of nitrogen. In Figure 3.3 an energy flow diagram is given which is a simplification of the previous one, but groups species based on the best understanding of the flow of nitrogen in the system. Figure 3.4, the biological mass flow diagram, shows the biological involvement in the cycling of nitrogen. In Figure 3.5, the nitrogen flow diagram for Barataria Bay is given.

In Figure 3.3 the biological energy flow diagram is given, which is a simplification of Figure 3.1. Spartina alterniflora and phytoplankton are the two species at the base of the food chain in the Barataria Bay. The live standing crop of Spartina alterniflora is converted to the dead standing crop of Spartina alterniflora. The dead
Figure 3.3. Simplified biological energy flow diagram for Barataria Bay for nitrogen cycling analysis
**TABLE 3.1 Key to Figure 3.3**

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<th>Description</th>
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<td>BM2</td>
<td>Dead Standing Crop of Marsh Grass</td>
</tr>
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<td>Coarse Detritus</td>
</tr>
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<td>BM4</td>
<td>Fine Detritus</td>
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<td>BM5</td>
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<td>Natural Death</td>
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Figure 3.4. Biological mass flow diagram for Barataria Bay
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Figure 3.5. Nitrogen flow diagram for Barataria Bay
### TABLE 3.3 Key to Figure 3.5

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<td>CC</td>
<td>Catch</td>
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<tr>
<td>DN</td>
<td>Denitrification</td>
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<td>E</td>
<td>Export</td>
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<td>Feces and Excretion</td>
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<tr>
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<td>Grazing</td>
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<tr>
<td>MD</td>
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<tr>
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<td>Natural Death</td>
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<tr>
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<td>Nitrification</td>
</tr>
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<td>N1</td>
<td>Organic Nitrogen in Live Standing Crop of Marsh Grass</td>
</tr>
<tr>
<td>N2</td>
<td>Organic Nitrogen in Dead Standing Crop of Marsh Grass</td>
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<tr>
<td>N3</td>
<td>Organic Nitrogen in Coarse Detritus</td>
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<tr>
<td>N4</td>
<td>Organic Nitrogen in Fine Detritus</td>
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<td>N5</td>
<td>Dissolved Organic Nitrogen</td>
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<tr>
<td>N6</td>
<td>Dissolved Ammonia Nitrogen</td>
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<tr>
<td>N7</td>
<td>Dissolved Nitrite-Nitrate Nitrogen</td>
</tr>
<tr>
<td>N8</td>
<td>Organic Nitrogen in Animal Biomass</td>
</tr>
<tr>
<td>N9</td>
<td>Organic Nitrogen in Phytoplankton</td>
</tr>
<tr>
<td>PC</td>
<td>Phytoplankton Consumption</td>
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standing crop is transformed into coarse detritus. Some of the coarse detritus is eaten by animals and the rest is converted to fine detritus by the microbial decomposition. Animals on the marsh, such as crabs and snails, also ingest fine detritus. It is believed that coarse as well as fine detritus enters the bay due to tidal flushing. A part of the coarse detritus in the bay is eaten by animals in the water. A second part is converted to fine detritus, and the rest is transported from the bay to the Gulf. A portion of fine detritus is consumed by animals in the water. The remaining fine detritus is transported to the Gulf. A portion of the animal biomass is removed from the bay by fishing. Another portion of the biomass is transported to the Gulf, and the rest is converted to the detritus which is distributed between coarse and fine detritus.

The origin of a second food chain in the bay is phytoplankton. A portion of the phytoplankton biomass is converted to detritus after natural death, and another portion is eaten by animals at higher trophic levels.

Comparing Figure 3.3, a simplified form of biological energy flow diagram, with that of Figure 3.1, which is the detailed form of biological flow diagram, we can see that both the diagrams are closely related. There are two major differences between the simplified and the detailed diagrams. In the simplified diagram, the primary production is divided into two groups instead of four as in the case of the detailed diagram. The production by marsh grasses
and the production by epiphytic algae, which exist on both the marsh grasses and marsh surface, were combined to form one group. The growth of phytoplankton and benthic organisms were combined to form another group in the water. The justification for such grouping was discussed in the previous section. Also, the simplified diagram (Figure 3.3) shows that the production from all animals is combined. This is consistent with the aim of this research which is to evaluate the distribution of primary producers, detritus, and the species that affect primary production in the Barataria Bay region. Hence, it is not necessary to consider animals separately in different trophic levels. They do affect the distribution of primary production, detritus, and nitrogen and hence must be included in the analysis.

It was pointed out earlier that biomass dynamics play a very important role in the estuarine system, and hence a predictive model of nitrogen dynamics in such a system must include the biomass dynamics as well as the behavior of factors which regulate the biomass dynamics. In Figure 3.4, the biological mass flow diagram, the biological involvement in the cycling of nitrogen is shown. This diagram directly follows from the previously discussed simplified biological energy flow diagram (Figure 3.3).

The nitrogen flow diagram for Barataria Bay is given in Figure 3.5 which follows from the biological mass flow diagram (Figure 3.4). In this diagram each species is represented by its nitrogen content only. In the biological
mass flow diagram, there is a loss of biomass as respiration from all species except dissolved organic material, dissolved ammonia, and dissolved nitrite and nitrate. In addition, live standing crops of *Spartina alterniflora* and phytoplankton have input from solar radiation. Nitrogen is not lost from the system because of respiration and it is not added to the system from solar radiation input. Hence, in the nitrogen flow diagram, input from solar radiation and loss by respiration are eliminated.

The live standing crop of marsh grass is converted to the dead standing crop of marsh grass, and nitrogen is also transferred as part of the dead standing crop. The dead standing crop is transformed into detritus, and with this transformation there is the associated organic nitrogen. The detritus and associated organic nitrogen are partitioned into coarse (settled) detritus and fine (suspended) detritus. The loss of biomass and associated nitrogen in coarse as well as fine detritus due to the microbial activity is shown. Both the detritus species have input from animal biomass (because of death) and output to animal biomass (grazing). In addition, the fine detritus has an input from phytoplankton which results from the natural death of this species.

As mentioned previously, decomposing marsh grass becomes enriched in nitrogen because of microbial buildup. The additional nitrogen required is assimilated from dissolved ammonia in the water. Since nitrate is in very low
concentrations in the Barataria Bay ecosystem it appears that ammonia is the only source of inorganic nitrogen available for this buildup. In addition, the increase in concentration is also due to the fact that organic nitrogen is not lost in the microbial respiration of detritus as other materials are (i.e., carbon and hydrogen).

Dissolved organic nitrogen in the water has input from two sources, namely animal biomass and fine detritus as shown in Figure 3.5. Certain animals like zooplankton excrete organic nitrogen. This is shown by a flow from animal biomass to organic nitrogen. Animals which are converted to detritus have organic nitrogen bound in them. A portion of this is converted to dissolved organic nitrogen. This is shown by a flow from fine detritus to organic nitrogen.

By the process of ammonification, this dissolved organic nitrogen is converted to ammonia nitrogen. This is shown by a flow from organic nitrogen to ammonia nitrogen.

Ammonia-nitrogen in the water has input from two sources. One is from organic nitrogen and the other is from the animal biomass. This second flow represents the ammonia excreted in feces by different animals (Ref. 3.26).

There are four output flows from ammonia nitrogen in the water. One flow represents the ammonia uptake by phytoplankton. Two more flows represent uptake of ammonia by bacteria on the fine and coarse detritus and the fourth flow represents the nitrification of ammonia nitrogen to nitrate
Nitrate nitrogen is essentially absent on the marsh surface as well as in the water body (Ref. 3.23, 3.24, 3.26). Nitrate formed by the nitrification process is immediately denitrified and is lost from the system. Consequently the concentration is very low, but it does appear as part of the cycle as shown on the figure.

Animal biomass and its associated organic nitrogen receive nitrogen from coarse detritus and fine detritus, and transfer nitrogen to coarse and fine detritus, organic nitrogen and ammonia nitrogen.

Phytoplankton is the other main primary producer and its production is also limited by available solar energy and nitrogen. It was pointed out previously that phytoplankton assimilates ammonia, and this is represented by the flow from ammonia nitrogen to phytoplankton. Nitrogen is transferred from phytoplankton by natural death and by grazing animals.

The following section describes how the previous information can be represented quantitatively by rate equations. The rate equations will be used along with the material transport model to evaluate the distribution of different species in Barataria Bay and how they are affected by the nitrogen available in the system.
Rate Equations Describing the Biomass Dynamics

In order to evaluate the distribution of the species in the system quantitatively, the information on Figure 3.4, the biological mass flow diagram, will be represented as reaction or growth rate equations. Once these relationships are developed in the mathematical form, it is necessary that the rate coefficients that appear in these equations be evaluated. When this is done the mathematical expressions become the sink and source terms in the material transport model equation (2.14). The material transport model is then solved for the distribution of the various species over the bay-marsh system. This part of the chapter discusses the development of the rate equations and the procedures to evaluate the rate coefficients that appear in these rate equations.

Development of Rate Equations: As a strategy to develop the reaction or growth rate equations for the transformation of different species which contain nitrogen, the following procedure was selected. If the required rate form was available in the literature, the same rate form was used in this analysis. If not, then the required rate form was developed using the available data.

In estimating the parameters of the rate expressions, the following technique was used. If the model parameters were available from the literature, then these values were used. If the parameters were not available but sufficient
data existed, a least squares estimate of the parameters was obtained. If the values were not available in the literature and if sufficient data were not reported, the parameters were then calculated using yearly averaged data. If it was impossible to arrive at the yearly averaged value of a species, then the model parameters were adjusted such that the peaks of the concentrations reasonably matched with the observed values and the peaks reached the maximum and minimum values at observed times during the year.

In the following part of this section the rate expressions for different species are described. The values of the rate coefficients that appear in the rate expressions are given. The methods used to evaluate the coefficients are also given.

**Live Standing Crop of Spartina alterniflora:** In Table 3.4, the rate expression for live standing crop of *Spartina alterniflora* and the values of the rate coefficients that appear in the equation are given. No rate expression was available in the literature to represent the growth rate of *Spartina alterniflora*, and this expression was developed from the information available in the literature. The first two terms on the right hand side of the rate expression express the gross photosynthesis. The rate of gross photosynthesis by plants is proportional to the intensity of solar radiation and is influenced by the amount of available nutrients and by self shading. The effect of
TABLE 3.4 Rate Expression for Live Standing Crop of *Spartina alterniflora*

The Rate Expression:

\[ \bar{r}_1 = k_1 \cdot SS \cdot S_1 - k_2 S_1^2 - k_{1R} S_1 - k_{12} S_1 \]  

(3.3)

The Model Parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Method of Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_1)</td>
<td>0.045 (day/year•langley)</td>
<td>Least Squares</td>
</tr>
<tr>
<td>(k_2)</td>
<td>0.0112 (1/year•gm•organic/m² of marsh area)</td>
<td>Least Squares</td>
</tr>
<tr>
<td>(k_{12})</td>
<td>(10.5 \left( \frac{58.33 - T}{58.33} \right)) (1/year)</td>
<td>Least Squares</td>
</tr>
<tr>
<td>(k_{1R})</td>
<td>0.4693 (x) (3^{T/10}) (1/year)</td>
<td>Literature</td>
</tr>
</tbody>
</table>

where \(SS\) = daily averaged solar radiation (langley/day)—a function of time given in Table 3.13.

\(T\) = temperature in degrees Centigrade—given in Table 3.13.

\(S_1\) = concentration of live standing crop of *Spartina alterniflora* in gram organic matter/m² of marsh area.
solar radiation is often modeled in a manner similar to second-order chemical kinetics (Ref. 3.70, 3.71, 3.72), and in this case is given by the first term in the equation. The effects of limited amounts of nutrients, and of self-shading were accounted for by a logistic type of relationship proposed by Williams (Ref. 3.73) for the lake system which is given as the second term in the equation. Unfortunately there was not sufficient information to explicitly include the effect of available nitrogen on the growth rate.

The rate of respiration by marsh grass and the rate of conversion of live standing crop to dead standing crop are represented as first order kinetic equations. The third term represents the rate of respiration by the marsh grass and the fourth term represents the rate of conversion of live standing crop of Spartina alterniflora to the dead standing crop of Spartina alterniflora.

The rate coefficient for respiration $k_{IR}$ was obtained from Teal and Kanshwan. They have extensively studied the respiration by marsh grasses, and have estimated the metabolic quotient and the respiration coefficient. Their values were used in this study. The other rate coefficients were evaluated by the least squares analysis.

Kirby (Ref. 3.6) has measured the biomass of marsh grass in the Barataria Bay region. Kirby has divided his monthly biomass measurements into two groups, namely the streamside biomass and the inland biomass. In this research, the weighted average based on area of these data was used.
Kirby concluded that the death rate of marsh grass is at a maximum in winter due to colder temperatures, and a temperature dependent death coefficient was formulated. The statistical information such as predicted and measured values and the overall correlation coefficient are given in Figure 3.6. The correlation coefficient obtained in this case is 0.7977. Compared to the correlation coefficient at 95 percent confidence limit (0.602) this fit is statistically acceptable, but does not appear to be a "good" fit compared to laboratory data. However, in this case of biological system, the statistical fit with such correlation coefficient for field data is considered very good. Riley (Ref. 3.74), when fitting the data for phytoplankton biomass, obtained the value 0.602 for the correlation coefficient, and he considered it to be a very good fit of the field data.

Dead Standing Crop of Spartina alterniflora: In Table 3.5, the rate expression for dead standing crop of Spartina alterniflora, and the values of the rate coefficients that appear in the equation are given. In the rate expression, the first term represents the rate of formation of dead standing crop of marsh grass from the live standing crop of marsh grass. It was mentioned previously that the rate coefficient $k_{12}$ was evaluated from the least squares analysis. The second term in the rate expression represents the rate of decomposition of dead standing crop of marsh
Correlation Coefficient = 0.7977
Correlation Coefficient at 95% confidence limit = 0.602

Figure 3.6. Comparison of least squares fit with measured values of live standing crop of *Spartina alterniflora*
TABLE 3.5 Rate Expression for Dead Standing Crop of *Spartina alterniflora*

The Rate Expression:

\[ \bar{r}_2 = k_{12}S_1 - k_{23}S_2 - k_{2R}S_2 \]  

(3.4)

The Model Parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Method of Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{12})</td>
<td>(10.5\left(\frac{58.33 - T}{58.33}\right)) (1/year)</td>
<td>Least Squares</td>
</tr>
<tr>
<td>(k_{23})</td>
<td>(0.4 \cdot 2.25^{T/10}) (1/year)</td>
<td>Least Squares</td>
</tr>
<tr>
<td>(k_{2R})</td>
<td>(0.025 \cdot 2.25^{T/10}) (1/year)</td>
<td>Least Squares</td>
</tr>
</tbody>
</table>

where \(T\) = temperature in degrees Centigrade--given in Table 3.13.

\(S_1\) = concentration of live standing crop of *Spartina alterniflora* in gram organic matter/m\(^2\) of marsh area.

\(S_2\) = concentration of dead standing crop of *Spartina alterniflora* in gram organic matter/m\(^2\) of marsh area.
grass into coarse detritus, and the third term represents the rate of respiration of the associated microbial population. The rate coefficient $k_{23}$ and $k_{2R}$ were also evaluated by a least squares analysis.

Kirby (Ref. 3.6) has measured the dead grass organic matter over the year and a first order rate coefficient for total output of dead grass. Here also his data are divided into streamside and inland measurements and for this study the data were averaged based on the area. Since there are only two outputs from the dead standing crop of marsh grass, and since Kirby has measured the rate coefficient for total output of dead grass over the year, the individual values can be calculated if one of them is known. Odum and De La Cruz (Ref. 3.11) have reported that the bacterial respiration loss from detritus on the marsh is five times as high as that on the dead standing crop. Day et al. (Ref. 3.75) have reported the values of yearly respiration losses for marsh fauna. This gave the respiration loss for a dead standing crop. Using this information and the average flows, the values of the two coefficients were separated. In Figure 3.7, the measured and predicted rate coefficients are compared for the decomposition coefficients ($k_{23} + k_{2R}$). The correlation coefficient obtained in this case is 0.8176. Compared to the correlation coefficient at 95 percent confidence limit (0.632) this fit is very good. In Figure 3.8, the measured and predicted values from the least squares fit of dead standing crop of marsh grass are compared. The
Correlation Coefficient = 0.8176
Correlation Coefficient at 95% confidence limit = 0.632

Figure 3.7. Comparison of least squares fit with measured values of decomposition coefficients $(k_{23} + k_{2R})$ for dead standing crop of *Spartina alterniflora*. 
Correlation Coefficient = 0.7496
Correlation Coefficient at 95% confidence limit = 0.553

Figure 3.8. Comparison of least squares fit with measured values of dead standing crop of *Spartina alterniflora*
correlation coefficient obtained in this case is 0.7496. Compared to the correlation coefficient at 95 percent confidence limit (0.553) this fit is very good.

Coarse Detritus: In Table 3.6, the rate expression for coarse detritus and the rate coefficients that appear in the rate equation are given. No rate expression was available in the literature for the coarse detritus. Consequently, a first order kinetic equation was selected to represent the dynamics of the coarse detritus. The terms in the rate expression follow from the flows in the biological mass flow diagram (Figure 3.4). The first three terms in the expression contribute to the formation of coarse detritus and the next three terms contribute to the reduction of coarse detritus. Unfortunately there was not sufficient information to explicitly include the effect of available nitrogen on the growth rate.

It was mentioned earlier that the rate coefficient $k_{23}$ was evaluated by the least squares analysis. The other coefficients, $k_{83}$, $k_{63}$, $k_{38}$, $k_{34}$ and $k_{3R}$, were estimated from the yearly averaged flows. Using the average biomass, respiration coefficient, net production and assimilation efficiency data given by Day (Ref. 3.75), the sum of the outputs and sum of the inputs were calculated. Thus for each species, the outputs and inputs were known. The first order rate coefficients were evaluated as:

\[
\text{Rate Coefficient} = \frac{\text{Yearly Averaged Biological Mass Flow}}{\text{Yearly Averaged Biomass (Standing Crop)}}
\]
TABLE 3.6 Rate Expression for Settled Coarse Detritus

The Rate Expression:

\[
\tilde{r}_3 = k_{23} \cdot \frac{1}{D} \cdot S_2 + k_{83} \cdot S_8 + k_{63} \cdot A_3 C_6 - k_{3R} S_3 \\
- k_{38} S_3 - k_{34} S_3
\]

The Model Parameters:

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<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Method of Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{23} )</td>
<td>( 0.4 \cdot 2.25^{T/10} ) (1/year)</td>
<td>Least Squares</td>
</tr>
<tr>
<td>( k_{83} )</td>
<td>9.087 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{63} )</td>
<td>0.09 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{38} )</td>
<td>9.68 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{34} )</td>
<td>50.25 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{3R} )</td>
<td>17.07 (1/year)</td>
<td>Average Flow</td>
</tr>
</tbody>
</table>

where \( T = \) temperature in degrees Centigrade--given in Table 3.13.

\( S_2 = \) concentration of dead standing crop of Spartina alterniflora in gram organic matter/m² of marsh area.

\( S_3 = \) concentration of coarse detritus in gram organic matter/m³.

\( S_8 = \) concentration of animal biomass in gram organic matter/m³.

\( C_6 = \) concentration of ammonia nitrogen in gram/m³.

\( A_3 = 100.0 \left( \frac{\text{gram organic matter in coarse detritus}}{\text{gram nitrogen in coarse detritus}} \right) \) (Ref. 3.25, 3.26)

\( D = \) Depth of water in meters
Fine Detritus: In Table 3.7, the rate expression for fine detritus and the rate coefficients that appear in the rate equation are given. No rate expression was available in the literature for fine detritus. Consequently a first order kinetic rate equation was selected to represent the dynamics of the fine detritus. The terms in the rate expression follow directly from the flows in biological mass flow diagram (Figure 3.4). The first four terms in the rate expression contribute to the formation of fine detritus and the next three terms contribute to the reduction of fine detritus. Unfortunately there was not sufficient information to explicitly include the effect of available nitrogen on the growth rate.

Values of different rate coefficients were not available in the literature. All the coefficients except $k_{45}$ representing formation of dissolved organic nitrogen from fine detritus were evaluated from the yearly averaged flows. For the case of dissolved organic nitrogen, yearly averaged estimates of flows were not available. The following section on dissolved organic nitrogen describes the method by which the constant $k_{45}$ was evaluated.

Dissolved Organic Nitrogen: In Table 3.8, the rate expression for dissolved organic nitrogen and the rate coefficients that appear in the rate equation are given. Thomann et al. (Ref. 3.51) have developed a steady-state, one-dimensional materials transport model for dissolved
TABLE 3.7 Rate Expression for Fine Suspended Detritus

The Rate Expression:

\[
\bar{r}_4 = k_{34} \cdot S_s + k_{94} \cdot S_9 + k_{84} \cdot S_8 = k_{64} \cdot A_4 \cdot C_6
\]

\[- k_{45} \cdot S_4 - k_{4R} \cdot S_4 - k_{48} \cdot S_4 \quad (3.6)
\]

The Model Parameters:

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
<th>Method of Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{34} )</td>
<td>50.25 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{94} )</td>
<td>2.82 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{84} )</td>
<td>20.41 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{45} )</td>
<td>0.41 (1/year)</td>
<td>Average Flow + Parametric Study</td>
</tr>
<tr>
<td>( k_{4R} )</td>
<td>21.26 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{48} )</td>
<td>19.50 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{64} )</td>
<td>0.21 (1/year)</td>
<td>Average Flow</td>
</tr>
</tbody>
</table>

where \( S_s \) = concentration of coarse detritus in gram organic matter/m\(^3\).

\( S_4 \) = concentration of fine detritus in gram organic matter/m\(^3\).

\( S_8 \) = concentration of animal biomass in gram organic matter/m\(^3\).

\( S_9 \) = concentration of phytoplankton biomass in gram organic matter/m\(^3\).

\( C_6 \) = concentration of ammonia nitrogen in grams/m\(^3\).

\( A_4 \) = \( \frac{gram \ organic \ matter \ in \ fine \ detritus}{gram \ nitrogen \ in \ fine \ detritus} \) (Ref. 3.13)
TABLE 3.8 Rate Expression for Dissolved Organic Nitrogen

The Rate Expression:

\[ \bar{F}_5 = k_{85} \frac{1}{A_8} S_8 + k_{45} \frac{1}{A_4} S_4 - k_{56} C_5 \]  \hspace{1cm} (3.7)

The Model Parameters:

<table>
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<tr>
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<td>( k_{85} )</td>
<td>0.14 (1/year)</td>
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</tr>
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<td>( k_{45} )</td>
<td>0.41 (1/year)</td>
<td>Average Flow + Parametric Study</td>
</tr>
<tr>
<td>( k_{56} )</td>
<td>2.0 (1/year)</td>
<td>Average Flow + Parametric Study</td>
</tr>
</tbody>
</table>

where \( S_4 \) = concentration of fine detritus in gram organic matter/m\(^3\).

\( S_8 \) = concentration of animal biomass in gram organic matter/m\(^3\).

\( C_5 \) = concentration of organic nitrogen in gram/m\(^3\).

\[ A_4 = \frac{58.8 \text{ (gram organic matter in fine detritus)}}{\text{gram nitrogen in fine detritus}} \] (Ref. 3.13)

\[ A_8 = 33.3 \frac{\text{gram organic matter in animal biomass}}{\text{gram nitrogen in animal biomass}} \] (Ref. 3.25, 3.26)
organic nitrogen. For the rate equation, they have successfully used a first order kinetic equation. Here also, a first order kinetic equation was selected. Different terms in the rate expression directly follow from the biological mass flow diagram (Figure 3.4). The first two terms in the rate equation contribute to the formation of dissolved organic nitrogen and the last term contributes to the loss of the species by microbial action.

Values of rate coefficients were not available in the literature. In addition, the measured data were not sufficient to analyze by statistical techniques. Hence the rate coefficients were obtained from the qualitative information available (Ref. 3.24) for dissolved organic nitrogen.

**Dissolved Ammonia Nitrogen:** In Table 3.9, the rate expression for dissolved ammonia nitrogen and the rate coefficients that appear in the rate equation are given. Thomann et al. (Ref. 3.51) have also developed a steady-state, one dimensional material transport model for dissolved ammonia nitrogen. For the rate expression, they have successfully used a first order kinetic equation. Here also, a first order kinetic equation was selected. The different terms in the rate expression directly follow from the biological mass flow diagram (Figure 3.4). The first two terms represent the formation of dissolved ammonia nitrogen from animal biomass and dissolved organic nitrogen. The next three terms represent loss of dissolved ammonia.
TABLE 3.9 Rate Expression for Dissolved Ammonia Nitrogen

The Rate Expression:

\[
\bar{T}_6 = k_{86} \cdot \frac{1}{A_8} \cdot k_{56} \cdot C_5 - k_{67} \cdot C_6 - k_{63} \cdot C_6 - k_{64} \cdot C_6 \\
- A_{ph} \cdot \frac{1}{A_9} \cdot S_9
\]  (3.8)

The Model Parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Method of Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{86})</td>
<td>0.08 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>(k_{56})</td>
<td>2.0 (1/year)</td>
<td>Average Flow + Parametric Study</td>
</tr>
<tr>
<td>(k_{67})</td>
<td>4.0 (1/year)</td>
<td>Average Flow + Parametric Study</td>
</tr>
<tr>
<td>(k_{63})</td>
<td>0.09 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>(k_{64})</td>
<td>0.21 (1/year)</td>
<td>Average Flow</td>
</tr>
</tbody>
</table>

where  \(S_8\) = concentration of animal biomass in gram organic matter/m\(^3\).

\(S_9\) = concentration of phytoplankton biomass in gram organic matter/m\(^3\).

\(C_5\) = concentration of organic nitrogen in gram/m\(^3\).

\(C_6\) = concentration of ammonia nitrogen in gram/m\(^3\).

\(A_8 = 33.3 \left(\frac{\text{gram organic matter in animal biomass}}{\text{gram nitrogen in animal biomass}}\right)\)  
(Ref. 3.25, 3.26)

\(A_9 = 25.0 \left(\frac{\text{gram organic matter in phytoplankton biomass}}{\text{gram nitrogen in phytoplankton biomass}}\right)\)  
(Ref. 3.25, 3.26)

\(A_{ph}\) = coefficient of photosynthesis for phytoplankton biomass--given in Table 3.12.
nitrogen to coarse and fine detritus and to dissolved ni­
trite and nitrate nitrogen. The last term represents the
consumption by phytoplankton. It was assumed that this
consumption of dissolved ammonia nitrogen is proportional
to gross photosynthesis to phytoplankton.

Values of the rate coefficients were not available in
the literature. The discussion given in the case of dis­
solved organic nitrogen for the evaluation of model parame­
ters is applicable here as well.

Dissolved Nitrite and Nitrate Nitrogen: In Table 3.10,
the rate expression for dissolved nitrite and nitrate nitro­
gen and the rate coefficients that appear in the rate equa­
tion are given. Thomann et al. (Ref. 3.51) have also de­
veloped a steady-state, one-dimensional material transport
model for dissolved nitrite and nitrate nitrogen. For the
rate equation, they have used a first order kinetic equation
successfully. Here also, a first order kinetic equation was
selected. Different terms in the rate expression directly
follow from the biological mass flow diagram (Figure 3.4).
The first term represents the formation from dissolved am­
monia nitrogen and the second term represents denitrifi­
cation.

Values of the rate coefficients were not available in
the literature. The coefficient occurring in the first
term (representing the formation) was previously evaluated
as discussed in the case of dissolved ammonia nitrogen.
TABLE 3.10 Rate Expression for Dissolved Nitrite and Nitrate Nitrogen

The Rate Expression:

\[ \bar{r}_7 = k_{67} \cdot C_6 - k_{dn} \cdot C_7 \quad (3.9) \]

The Model Parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Method of Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>k_{67}</td>
<td>4.0 (1/year)</td>
<td>Average Flow + Parametric Study</td>
</tr>
<tr>
<td>k_{dn}</td>
<td>324.0 (1/year)</td>
<td>Average Flow</td>
</tr>
</tbody>
</table>

where \( C_6 \) = concentration of ammonia nitrogen in gram/m³.

\( C_7 \) = concentration of nitrite and nitrate nitrogen in gram/m³.
The coefficient $k_{dn}$, occurring in the second term representing denitrification, was obtained from the qualitative information available (Ref. 3.24, 3.26) for dissolved nitrite and nitrate nitrogen. As mentioned earlier, these concentrations are very low in the system and it was decided to use the value of $k_{dn}$ equal to 324.0 per year. This reduced 90 percent of the existing concentrations of nitrate nitrogen to gaseous nitrogen.

**Animal Biomass:** In Table 3.11, the rate expression for animal biomass and the rate coefficients that appear in the rate equation are given. As in the case of dissolved organic nitrogen, dissolved ammonia nitrogen, and dissolved nitrite and nitrate nitrogen, here also a first order kinetic expression was selected to represent the dynamics of the animal biomass. The first three terms in the rate expression contribute to the formation of the animal biomass and the next six terms contribute to the loss of animal biomass. Unfortunately, there was not sufficient information to explicitly include the effect of available nitrogen on the growth rate.

Values of the rate coefficients were not available in the literature. The yearly averaged data (Ref. 3.75) were used to evaluate the parameters.
TABLE 3.11 Rate Expression for Animal Biomass

The Rate Expression:

\[ \bar{r}_8 = k_{48} S_4 + k_{38} S_3 + k_{98} S_9 - k_f S_8 - k_{8R} S_8 \\
- k_{84} S_8 - k_{85} S_8 - k_{86} S_8 \]  

(3.10)

The Model Parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Method of Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{48} )</td>
<td>19.5 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{38} )</td>
<td>9.68 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{98} )</td>
<td>53.72 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_f )</td>
<td>0.164 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{8R} )</td>
<td>20.87 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{83} )</td>
<td>9.087 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{84} )</td>
<td>20.41 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{85} )</td>
<td>0.14 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>( k_{86} )</td>
<td>0.08 (1/year)</td>
<td>Average Flow</td>
</tr>
</tbody>
</table>
Phytoplankton Biomass: In Table 3.12, the rate expression for phytoplankton biomass and the rate coefficients that appear in the rate equation are given. This rate equation is essentially similar to the one developed by Riley (Ref. 3.74). The first term represents the gross photosynthesis of phytoplankton biomass. The gross photosynthesis is a function of solar radiation and amount of available nutrients. \( A_n \) represents the moderation caused by nutrient concentration. As shown in Table 3.12, if the concentration of dissolved ammonia nitrogen drops below 0.2 gm/m\(^3\) (Ref. 3.74), the value of \( A_n \) is less than one (to be exact, \( C_6/0.2 \) where \( C_6 \) is the concentration of dissolved ammonia nitrogen); and the gross photosynthesis is moderated by that amount. If the concentration of dissolved ammonia nitrogen is greater than or equal to 0.2 gm/m\(^3\), a sufficient amount of this nutrient was available and nutrient concentrations above this amount did not influence the gross photosynthesis.

The second term represents the respiration of phytoplankton biomass. A temperature dependent respiration coefficient proposed by Riley (Ref. 3.74) was used. The next two terms represent grazing and natural death of phytoplankton. Calculated yearly averaged data (Ref. 3.7) were used to adjust the coefficients occurring in these terms.
TABLE 3.12 Rate Expression for Phytoplankton Biomass

The Rate Expression:

\[
\bar{r}_g = A_{ph} \cdot S_g - k_{9R} \cdot S_g - k_{98} \cdot S_g - k_{94} \cdot S_g \tag{3.11}
\]

\[A_{ph} = A_n \cdot k_g \cdot SS\]

\[A_n = \begin{cases} 
\frac{C_6}{0.2} & \text{for } C_6 < 0.2 \text{ gm/m}^3 \\
1.0 & \text{for } C_6 \geq 0.2 \text{ gm/m}^3
\end{cases}\]

The Model Parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Method of Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_9)</td>
<td>1.0 (day/year\cdot langley)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>(k_{9R})</td>
<td>0.0175 (e^{0.069T}) (1/day)</td>
<td>Literature (Ref. 3.75)</td>
</tr>
<tr>
<td>(k_{98})</td>
<td>53.72 (1/year)</td>
<td>Average Flow</td>
</tr>
<tr>
<td>(k_{94})</td>
<td>2.82 (1/year)</td>
<td>Average Flow</td>
</tr>
</tbody>
</table>

where \(SS\) = daily averaged solar radiation (langley/day) -- given in Table 3.13.

\(T\) = temperature in degrees centigrade -- given in Table 3.13.
Specifications of Natural Forcing Functions

There are two external forcing functions, namely solar radiation and temperature. These two variables are functions of time.

Incident solar radiation has a maximum value during the summer time. The variation in the daily averaged solar radiation was modeled as a sinusoidal function of time with a period of one year. The average value of solar radiation and the amplitude of the sine wave were fixed using solar radiation incident on Barataria Bay (Ref. 3.26). The expression for incident solar radiation is given in Table 3.13. This expression was used in the rate equations, Eq. 3.3 and Eq. 3.9.

The daily air and water temperature for Barataria Bay have been measured. Vora (Ref. 3.5) averaged twelve years (1958-1969) of temperature data to obtain monthly averaged air and water temperatures. These temperatures were fitted with a Fourier series. In Table 3.13, this functional relationship is given. This was used in the rate equations, Eqs. 3.3, 3.4, 3.5, and 3.11.

Summary

In this chapter, an attempt has been made to explain the working of the Barataria Bay ecosystem and the cycling of nitrogen that occurs in it. Rate equations were established to analyze the above developed information quantitatively, and procedures to evaluate the model parameters
TABLE 3.13 The Natural Forcing Functions of Solar Radiation and Average of Water and Air Temperature

**Daily Averaged Solar Radiation**

\[
SS = 346.76 \left[ 1 + 0.40845 \cos\{2\pi(t - 0.472)\} \right] \quad (3.12)
\]

where \(SS\) = solar radiation in langley/day

\(t\) = time in years \((0 \leq t \leq 1)\)

**Temperature (Average of Air and Water)**

\[
T = a_0 + \sum_{i=1}^{4} [a_i \sin(2\pi it) + b_i \cos(2\pi it)] \quad (3.13)
\]

\[
\begin{align*}
    a_0 & = 22.175 \\
    a_1 & = -3.1109 & b_1 & = -8.0186 \\
    a_2 & = -0.4061 & b_2 & = -0.93931 \\
    a_3 & = -0.008777 & b_3 & = 0.096233 \\
    a_4 & = 0.24337 & b_4 & = 0.13399
\end{align*}
\]

where \(T\) = average of air and water temperature in degrees centigrade

\(t\) = time in years \((0 \leq t \leq 1)\)
were given. It should be noted that only one rate equation explicitly has the effect of available nitrogen included in it. The rate equations developed in this chapter are summarized in Table 3.14 and the values of all the rate coefficients are given in Table 3.15 for convenience.
### TABLE 3.14 Summary of Rate Expressions

<table>
<thead>
<tr>
<th>Rate Expression</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. Live Standing Crop of Spartina</strong></td>
<td>( \overline{r}<em>1 = k_1 \cdot SS \cdot S_1 - k_2 \cdot S_1^2 - k</em>{12} \cdot S_1 - k_{1R} \cdot S_1 ) (3.3)</td>
</tr>
<tr>
<td><strong>2. Dead Standing Crop of Spartina</strong></td>
<td>( \overline{r}<em>2 = k</em>{12} \cdot S_1 - k_{23} \cdot S_2 - k_{2R} \cdot S_2 ) (3.4)</td>
</tr>
<tr>
<td><strong>3. Settled Coarse Detritus</strong></td>
<td>( \overline{r}<em>3 = k</em>{23} \cdot \frac{1}{D} \cdot S_2 + k_{83} \cdot S_8 + k_{63} \cdot A_3 \cdot C_6 - k_{3R} \cdot S_3 - k_{38} \cdot S_3 \ - k_{34} \cdot S_3 ) (3.5)</td>
</tr>
<tr>
<td><strong>4. Fine Suspended Detritus</strong></td>
<td>( \overline{r}<em>4 = k</em>{34} \cdot S_3 + k_{94} \cdot S_9 + k_{84} \cdot S_8 + k_{64} \cdot A_4 \cdot C_6 - k_{45} \cdot S_4 \ - k_{48} \cdot S_4 ) (3.6)</td>
</tr>
<tr>
<td><strong>5. Dissolved Organic Nitrogen</strong></td>
<td>( \overline{r}<em>5 = k</em>{85} \cdot \frac{1}{A_8} \cdot S_8 + k_{45} \cdot \frac{1}{A_4} \cdot S_4 - k_{56} \cdot C_5 ) (3.7)</td>
</tr>
<tr>
<td><strong>6. Dissolved Inorganic Nitrogen (NH(_4^+))</strong></td>
<td>( \overline{r}<em>6 = k</em>{86} \cdot \frac{1}{A_8} \cdot S_8 + k_{56} \cdot C_5 - k_{67} \cdot C_6 - k_{63} \cdot C_6 \ - k_{64} \cdot C_6 - A_{ph} \cdot \frac{1}{A_9} ) (3.8)</td>
</tr>
<tr>
<td><strong>7. Dissolved Inorganic Nitrogen (NO(_3^-))</strong></td>
<td>( \overline{r}<em>7 = k</em>{67} \cdot C_6 - ADN \cdot C_7 ) (3.9)</td>
</tr>
</tbody>
</table>
TABLE 3.14 (continued)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{r}<em>8 = k</em>{48} \cdot S_4 + k_{38} \cdot S_3 + k_{98} \cdot S_9 - k_f \cdot S_8 - k_{8R} \cdot S_8 )</td>
<td>8. Animal Biomass</td>
</tr>
<tr>
<td>( - k_{83} \cdot S_8 - k_{84} \cdot S_8 - k_{85} \cdot S_8 - k_{86} \cdot S_8 )</td>
<td>8. Animal Biomass</td>
</tr>
<tr>
<td>( \bar{r}<em>9 = A</em>{\text{ph}} \cdot S_9 - k_{9R} \cdot S_9 - k_{98} \cdot S_9 - k_{94} \cdot S_9 )</td>
<td>9. Phytoplankton Biomass</td>
</tr>
<tr>
<td>( A_{\text{ph}} = A_n \cdot k_9 \cdot SS )</td>
<td>9. Phytoplankton Biomass</td>
</tr>
<tr>
<td>( A_n = C_6 / 0.2 ) \text{ for } C_6 &lt; 0.2 \text{ gm/m}^3 )</td>
<td>9. Phytoplankton Biomass</td>
</tr>
<tr>
<td>( = 1.0 ) \text{ for } C_6 \geq 0.2 \text{ gm/m}^3 )</td>
<td>9. Phytoplankton Biomass</td>
</tr>
</tbody>
</table>
TABLE 3.15 Values of the Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Method of Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (k_{1R})</td>
<td>(0.4693 \cdot 3^{T/10})</td>
<td>1/year</td>
<td>Literature (Ref. 3.71)</td>
</tr>
<tr>
<td>2 (k_{9R})</td>
<td>(0.0175 \cdot e^{0.069 \cdot T})</td>
<td>1/day</td>
<td>Literature (Ref. 3.69)</td>
</tr>
<tr>
<td>3 (k_1)</td>
<td>0.045</td>
<td>day/(year\cdot Langley)</td>
<td>Least Squares</td>
</tr>
<tr>
<td>4 (k_2)</td>
<td>0.0112</td>
<td>1/(year \cdot \frac{gram, organic}{m^2, marsh, area})</td>
<td>Least Squares</td>
</tr>
<tr>
<td>5 (k_{12})</td>
<td>(10.5 \left(\frac{58.33 - T}{58.33}\right))</td>
<td>1/year</td>
<td>Least Squares</td>
</tr>
<tr>
<td>6 (k_{23})</td>
<td>(0.4 \times 2.25^{T/10})</td>
<td>1/year</td>
<td>Least Squares</td>
</tr>
<tr>
<td>7 (k_{2R})</td>
<td>(0.25 \times 2.25^{T/10})</td>
<td>1/year</td>
<td>Least Squares</td>
</tr>
<tr>
<td>8 (k_{3R})</td>
<td>17.07</td>
<td>1/year</td>
<td>Average Flow</td>
</tr>
<tr>
<td>9 (k_{38})</td>
<td>9.68</td>
<td>1/year</td>
<td>Average Flow</td>
</tr>
<tr>
<td>10 (k_{34})</td>
<td>50.25</td>
<td>1/year</td>
<td>Average Flow</td>
</tr>
<tr>
<td>11 (k_{45})</td>
<td>0.41</td>
<td>1/year</td>
<td>Average Flow + Parametric Study</td>
</tr>
<tr>
<td>12 (k_{4R})</td>
<td>21.26</td>
<td>1/year</td>
<td>Average Flow</td>
</tr>
<tr>
<td>13 (k_{48})</td>
<td>19.50</td>
<td>1/year</td>
<td>Average Flow</td>
</tr>
<tr>
<td>14 (k_{56})</td>
<td>2.0</td>
<td>1/year</td>
<td>Average Flow + Parametric Study</td>
</tr>
</tbody>
</table>
TABLE 3.15 (continued)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Method of Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>$k_{64}$</td>
<td>0.21</td>
<td>1/year</td>
</tr>
<tr>
<td>16</td>
<td>$k_{63}$</td>
<td>0.09</td>
<td>1/year</td>
</tr>
<tr>
<td>17</td>
<td>$k_{67}$</td>
<td>4.0</td>
<td>1/year</td>
</tr>
<tr>
<td>18</td>
<td>$k_{dn}$</td>
<td>324.0</td>
<td>1/year</td>
</tr>
<tr>
<td>19</td>
<td>$k_{83}$</td>
<td>9.087</td>
<td>1/year</td>
</tr>
<tr>
<td>20</td>
<td>$k_{84}$</td>
<td>20.41</td>
<td>1/year</td>
</tr>
<tr>
<td>21</td>
<td>$k_{85}$</td>
<td>0.14</td>
<td>1/year</td>
</tr>
<tr>
<td>22</td>
<td>$k_{86}$</td>
<td>0.08</td>
<td>1/year</td>
</tr>
<tr>
<td>23</td>
<td>$k_{8R}$</td>
<td>20.87</td>
<td>1/year</td>
</tr>
<tr>
<td>24</td>
<td>$k_f$</td>
<td>0.164</td>
<td>1/year</td>
</tr>
<tr>
<td>25</td>
<td>$k_{94}$</td>
<td>2.82</td>
<td>1/year</td>
</tr>
<tr>
<td>26</td>
<td>$k_{98}$</td>
<td>53.72</td>
<td>1/year</td>
</tr>
<tr>
<td>27</td>
<td>$k_9$</td>
<td>1.0</td>
<td>day/(year•Langley)</td>
</tr>
</tbody>
</table>
REFERENCES


3.3 Odum, E. P., op. cit., Chapter 3.

3.4 Odum, E. P., op. cit., p. 8.


3.24 Ho, Clara, Personal Communication (1972).


3.34 Ibid., p. 15.


3.41 Kormondy, E. J., op. cit., p. 46.


3.60 Brezonik, P. L., op. cit., p. 22.


CHAPTER IV
DERIVATION OF EQUATIONS FOR THE TRANSPORT PHENOMENA
OF SHALLOW ESTUARINE BAY SYSTEMS

Introduction

The purpose of this chapter is to derive the vertically averaged equations that describe hydrodynamics and material transport processes in a shallow, vertically mixed estuarine bay. The vertical integration of the general equations of continuity and motion results in the hydrodynamic model. The same mathematical procedure, when applied to the general equation of species continuity, produces the material transport model.

The chapter is divided into six sections. The first section consists of the derivation of the hydrodynamic model. The second section consists of the derivation of the material transport model. The third section describes the time-scale and its importance and the fourth section consists of derivation of time-averaged equations of the hydrodynamic model and the material transport model. The fifth section is divided into two parts. In the first part, the approach to the solution of the hydrodynamic model is described. The second part gives the results obtained using this approach. In the last section, the method of
solution of the material transport model is described.

**Hydrodynamic Model**

The hydrodynamic model is obtained by vertically integrating the equations of continuity and motion. Hacker (Ref. 4.1) has given vigorous derivation of the hydrodynamic model. In order to transform these equations, Hacker has made a certain number of approximations. In Table 4.1 these approximations are summarized. In the next section important steps from Hacker's derivation of the hydrodynamic model are given.

**Equation of Continuity:** Hacker has transformed the general equation of continuity given by Dronkers (Ref. 4.2). For a turbulent incompressible fluid, the equation is given as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (4.1)
\]

From the experimental data, Hacker has concluded that the most effects are two dimensional and in the horizontal plane. Hence the general equation is transformed into two dimensions by vertical integration. In other words, the average values of the variables in the vertical direction are obtained so as to produce a two dimensional model to describe the physical system.

Integrating Eq. (4.1) in the z-direction, from the bottom, \(z_b\), to the surface, \(z_s\), we obtain:
<table>
<thead>
<tr>
<th>Approximation</th>
<th>Terms Neglected</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incompressible Flow: Constant density is assumed due to negligible changes in density from temperature and salinity variations.</td>
<td>$\nabla \cdot \rho$</td>
</tr>
<tr>
<td>Two-Dimensional Effects: The important effects occur in the horizontal plane.</td>
<td>Inertia and stress terms in the $z$-direction</td>
</tr>
<tr>
<td>No Underground Seepage: Flow of water through the bottom is negligible.</td>
<td>$w(x,y,h,t)$</td>
</tr>
<tr>
<td>No Gravity Effects: Flow is in the horizontal plane.</td>
<td>$g_x$</td>
</tr>
<tr>
<td>No Diffusive Transport of Momentum: This is due to the low velocities found in the system.</td>
<td>$2 \left( - \mu_{laminar} + \mu_{eddy} \frac{\partial^2 u}{\partial x^2} \right)$</td>
</tr>
<tr>
<td>No Bottom Slip: The velocities at the bottom are zero.</td>
<td>$u(z_B)$, $v(z_B)$</td>
</tr>
<tr>
<td>No Momentum Transfer with Rain: Momentum transfer with rainfall is negligible.</td>
<td>$wu(z_s)$, $wv(z_s)$</td>
</tr>
<tr>
<td>Uniform Velocity Profiles in the Vertical Direction: A uniform velocity is assumed.</td>
<td>$u(z)$, $v(z)$</td>
</tr>
</tbody>
</table>
TABLE 4.1 (continued)

<table>
<thead>
<tr>
<th>Approximation</th>
<th>Terms Neglected</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Bottom Variation with Time: The bottom profile does not change with time.</td>
<td>$\frac{\partial h}{\partial t}$</td>
</tr>
<tr>
<td>No PV Work: The fluid is incompressible.</td>
<td>$\frac{Dp}{Dt}$</td>
</tr>
<tr>
<td>Binary Diffusion: Due to low concentrations, species diffuse independently</td>
<td>$D_A^{<em>}$, multicomponent = $D_A^{</em>}$, binary</td>
</tr>
<tr>
<td>from each other.</td>
<td></td>
</tr>
</tbody>
</table>
\[
\int_{z_b}^{z_s} \frac{\partial u}{\partial x} \, dz + \int_{z_b}^{z_s} \frac{\partial v}{\partial y} \, dz + w(x,y,z_s,t) - w(x,y,z_b,t) = 0
\] (4.2)

If \( F_n(x,y,t) = 0 \) is defined as the equation representing a surface, e.g., the bay air-water surface or water-bottom surface, then at every point on either surface the substantial derivative of \( F_n \) can be written as:

\[
\frac{DF_n}{dt} = \frac{\partial F_n}{\partial t} + u \frac{\partial F_n}{\partial x} + v \frac{\partial F_n}{\partial y} = 0
\] (4.3)

In Figure (4.1), \( h \) is the distance from the bottom to the given reference plane (mean water level of the estuary) and \( L \) is the corresponding distance to the water surface from the reference plane.

If the Leibnitz integral rule is applied to the first two terms of Eq. (4.2), after rearranging, this equation becomes:

\[
\frac{\partial}{\partial x} \int_{z_b}^{z_s} u \, dz + \frac{\partial}{\partial y} \int_{z_b}^{z_s} v \, dz - \left[ u(z_s) \frac{dz_s}{dx} + v(z_s) \frac{dz_s}{dy} \right] \\
+ \left[ u(z_b) \frac{dz_b}{dx} + v(z_b) \frac{dz_b}{dy} \right] + w(z,y,z_s,t) - w(x,y,z_b,t) = 0
\] (4.4)

Applying the definition of the substantial derivative at the bottom and at the surface, incorporating in the above equation and using the mean water level as the reference plane results in:
Figure 4.1. Definition of variables for shallow estuarine body of water
\[
\frac{\partial}{\partial x} \int_{z_b}^{z_s} u \, dz + \frac{\partial}{\partial y} \int_{z_b}^{z_s} v \, dz + \frac{\partial L}{\partial t} + w(x, y, z_s, t) - w(x, y, z_b, t) = 0 \tag{4.5}
\]

Defining:

\[
U = \frac{1}{D} \int_{z_b}^{z_s} u \, dz \tag{4.6}
\]

\[
V = \frac{1}{D} \int_{z_b}^{z_s} v \, dz \tag{4.7}
\]

Then Eq. (4.5) becomes:

\[
\frac{\partial (DU)}{\partial x} + \frac{\partial (DV)}{\partial y} + \frac{\partial L}{\partial t} + w(x, y, z_s, t) - w(x, y, z_b, t) = 0 \tag{4.8}
\]

Hacker has assumed no underground seepage, hence,

\[
w(x, y, z_b, t) = 0 \tag{4.9}
\]

The \(w\) velocity at the surface is the net of the rainfall rate and the evaporation rate.

\[
w(x, y, z_s, t) = - (R - E_v) \tag{4.10}
\]

The minus sign for \(R\) in Eq. (4.10) is due to the fact that the direction of the rain is in the negative z-direction. Thus, Eq. (4.8) can be written as:

\[
\frac{\partial (DU)}{\partial x} + \frac{\partial (DV)}{\partial y} + \frac{\partial L}{\partial t} = R - E_v \tag{4.11}
\]
Eq. (4.11) is the vertically integrated continuity equation, and it forms a part of the hydrodynamic model. The other part of the hydrodynamic model consists of the components of the vertically integrated equation of motion. In Eq. (4.11) the terms $DU$ and $DV$ can be thought of as an average discharge rate,

$$DU = Q_x \quad (4.12)$$

$$DV = Q_y \quad (4.13)$$

Using the above definitions, Eq. (4.11) can be written as:

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial L}{\partial t} = R - E_v \quad (4.14)$$

**Equations of Motion:** Hacker has transformed the general equation of motion given by Bird, Stewart, and Lightfoot (Ref. 4.3). The time-average turbulent equation of motion is given as:

$$\rho \frac{D\mathbf{v}}{Dt} = -\mathbf{v}_p - \mathbf{v}_t(l+t) + \rho \mathbf{g} \quad (4.15)$$

Eq. (4.15) can be expanded in a rectangular coordinate system. The $x$-$y$ plane of this coordinate system is on the surface of the earth, and the Coriolis force term is included. Moreover, since the $x$-$y$ plane is parallel to the horizontal plane, the effect of gravity is eliminated. Expanding Eq. (4.15) in this fashion for the $x$-component results in:
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - F_v = - \frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{1}{\rho} \left[ \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right] \]

(4.16)

Integrating in the vertical direction, from the bottom to the surface, gives:

\[ \int_{z_b}^{z_s} \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] dz - \int_{z_b}^{z_s} F_v dz = - \frac{1}{\rho} \int_{z_b}^{z_s} \frac{\partial p}{\partial x} dz \]

(4.17)

Hacker has used the expressions for shear and normal stresses \( \tau_{xy} \) and \( \tau_{xx} \) as given by Bird (Ref. 4.3). Substituting these in terms of viscosity and velocity gradients:

\[ \int_{z_b}^{z_s} \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] dz - \int_{z_b}^{z_s} F_v dz = - \frac{1}{\rho} \int_{z_b}^{z_s} \frac{\partial p}{\partial x} dz \]

\[ - \frac{1}{\rho} \int_{z_b}^{z_s} \left[ 2 \left( \mu \text{laminar} + \mu \text{eddy} \right) \frac{\partial^2 u}{\partial x^2} + \left( \mu \text{laminar} + \mu \text{eddy} \right) \left( \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial \tau_{xz}}{\partial z} \right) \right] dz \]

(4.18)

In order to evaluate the inertial terms, Hacker has assumed that the velocity is uniform in the vertical direction. Due to the shallowness of the bay under study, this approximation is reasonable. He has also given some experimental
measurements confirming the above approximation. As a result of the low velocities existing in the system, the diffusive transport of momentum plays a negligible role. Consequently, all terms except \( \partial x / \partial x \) in brackets in the right hand side of Eq. (4.18) are neglected. Using Leibnitz's rule, Eq. (4.18) transforms, after neglecting the above mentioned terms and rearranging, to:

\[
\frac{\partial}{\partial t} \int_{z_b}^{z_s} u \, dz + u \frac{\partial}{\partial x} \int_{z_b}^{z_s} u \, dz + \frac{\partial}{\partial y} \int_{z_b}^{z_s} u \, dz + \left[ w_u(z_s) - w_u(z_b) \right] \\
- u(x, y, z_s, t) \left[ \frac{\partial z_s}{\partial t} + U(z_s) \frac{\partial z_s}{\partial x} + V(z_s) \frac{\partial z_s}{\partial y} \right] \\
+ u(x, y, z_b, t) \left[ \frac{\partial z_b}{\partial t} + U(z_b) \frac{\partial z_b}{\partial x} + V(z_b) \frac{\partial z_b}{\partial y} \right] \\
- F \int_{z_b}^{z_s} v \, dz = - \frac{1}{\rho} \frac{\partial p}{\partial x} \int_{z_b}^{z_s} \, dz - \frac{1}{\rho} \int_{z_b}^{z_s} \frac{\partial \tau_{xz}}{\partial z} \, dz
\]

(4.19)

The terms in the second and third brackets are equal to zero from the definition of substantial derivative applied to the bottom and top surfaces. Also \( [w_u(z_s) - w_u(z_b)] \) is zero because \( u(z_b) \) is zero (no bottom slip) and \( u(z_s) \) is the net of rainfall rate and evaporation rate which is negligible as far as momentum added to the system is concerned. Therefore, Eq. (4.19) transforms to:
Vertically averaged velocities were defined by Eqs. (4.6) and (4.7); using these definitions and approximation of uniform vertical profiles, Eq. (4.20) can be written as:

\[
\frac{\partial(U)}{\partial t} + U \frac{\partial(U)}{\partial x} + V \frac{\partial(V)}{\partial y} - FV = - \frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{1}{D} \left[ \tau_{xz}(z_s) - \tau_{xz}(z_b) \right]
\]  

Expanding the derivative terms in the left-hand side of Eq. (4.21) and using the definition of substantial derivative, after rearrangement, Eq. (4.21) can be written as:

\[
\frac{\partial(U)}{\partial t} + U \frac{\partial(U)}{\partial x} + V \frac{\partial(U)}{\partial x} - FV = - \frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{1}{D} \left[ \tau_{xz}(z_s) - \tau_{xz}(z_b) \right]
\]

Using the z-component of the equation of motion, Hacker obtained the expression for the pressure gradient term in the above equation as:

\[
\frac{\partial p}{\partial x} = \rho g \frac{\partial L}{\partial x}
\]  

Substituting Eq. (4.23) in Eq. (4.22) results in:
The above equation is the vertically integrated x-component of the equation of motion. The y-component of the equation of motion can be derived in the same fashion. To be able to solve this equation, the stress term must be evaluated. Hacker has used the following empirical relationships available in the literature.

The empirical relationship used for the bottom stresses is:

\[
\frac{\tau_{xz}(z_b)}{D\rho} = \frac{g(U^2 + V^2)\frac{\lambda}{2}U}{Dc^2}
\] (4.25)

The chezy coefficient, c, is calculated as (Ref. 4.4):

\[
c = \frac{1.49}{n} \cdot D^{1/6}
\] (4.26)

The bottom roughness, n, is given in the literature (Ref. 4.4) and its most common value given is 0.026.

The empirical relationship for the surface stresses used is:

\[
\frac{\tau_{xz}(z_s)}{D\rho} = K_1 w^2 \cos \theta = X
\] (4.27)

\[
\frac{\tau_{yz}(z_s)}{D\rho} = K_1 w^2 \sin \theta = Y
\] (4.28)

where:

\[
K_1 = 0.0026 \rho_a
\] (4.29)
The above relationship has been used by many previous studies (Ref. 2.10, 2.11, 2.12, 2.15) successfully and will be used in the present work.

Hydrodynamic Model Equations

Substituting Eq. (4.25) and Eq. (4.27) in Eq. (4.24) will result in the vertically integrated x-component of the equation of motion for the hydrodynamic model as given by Eq. (4.30). A similar derivation to the one shown will result in the equation for the y-component, as shown in Eq. (4.31). Thus, the hydrodynamic model equations are:

\[
\frac{\partial (DU)}{\partial x} + \frac{\partial (DV)}{\partial y} + \frac{\partial L}{\partial t} = R - E_v \quad (4.11)
\]

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} - F_V + g \frac{\partial L}{\partial x} = X - g \frac{U(U^2 + V^2)^{1/2}}{Dc^2} \quad (4.30)
\]

\[
\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + F_U + g \frac{\partial L}{\partial y} = Y - g \frac{V(U^2 + V^2)^{1/2}}{Dc^2} \quad (4.31)
\]

Materials Transport Model

The materials transport model is obtained by vertical integration of the species continuity equation. In Bird, Stewart and Lightfoot (Ref. 4.5) the general turbulent species continuity equation is given for a binary system as:

\[
\frac{\partial \rho_A}{\partial t} + (\bar{\mathbf{v}} \cdot \rho_A \mathbf{\bar{v}}) + (\bar{\mathbf{v}} \cdot \mathbf{J}_A^t) - r_A = 0 \quad (4.32)
\]

where the turbulent diffusion term is modeled by the following equation
\[ J_A' = -D_{AB}^* \cdot \rho \vec{v}_A \] (4.33)

where

\[ B_{AB}^* = B_{AB}^{\text{laminar}} + B_{AB}^{\text{eddy}} \] (4.34)

Assuming that constant density and diffusivity apply, Eq. (4.33) transforms to:

\[ J_A' = - B_{AB}^* \cdot \vec{v} \rho_A \] (4.35)

Writing Eq. (4.32) in terms of constant density and diffusivity by substituting Eq. (4.35) the result is:

\[ \frac{\partial \rho_A}{\partial t} + (\vec{v} \cdot \rho_A \vec{v}) - \vec{v} \cdot B_{AB}^* \vec{v} \rho_A - r_A = 0 \] (4.36)

Eq. (4.36) is the general, binary diffusion, species continuity equation. The concentration of a species in the estuarine system can be described by a binary diffusion. The reason for this is that the concentration of most of the species found in estuarine water is relatively low and each species diffuses independently from the others. Consequently, a binary diffusion coefficient can be used, where water is one component and the species in question is the other.

Transposing the diffusion term and rate term of Eq. (4.36) to the right hand side and expanding gives:

\[ \frac{\partial \rho_A}{\partial t} + \frac{\partial}{\partial x} (\rho_A u) + \frac{\partial}{\partial y} (\rho_A v) + \frac{\partial}{\partial z} (\rho_A w) = \frac{\partial}{\partial x} \left( B_{AB}^* \frac{\partial \rho_A}{\partial x} \right) \]

\[ + \frac{\partial}{\partial y} \left( B_{AB}^* \frac{\partial \rho_A}{\partial y} \right) + \frac{\partial}{\partial z} \left( B_{AB}^* \frac{\partial \rho_A}{\partial z} \right) + r_A \] (4.37)
Eq. (4.37) is the general species continuity equation and can be applied to a shallow estuarine bay. Integrating this equation in the vertical direction, in order to obtain a two-dimensional equation, results in:

\[
\int_{z_b}^{z_s} \left[ \frac{\partial \rho_A}{\partial t} + \frac{\partial}{\partial x} (\rho_A u) + \frac{\partial}{\partial y} (\rho_A v) + \frac{\partial}{\partial z} (\rho_A w) \right] \, dz = \int_{z_b}^{z_s} \left[ \frac{\partial}{\partial x} \left( B_{AB}^* \frac{\partial \rho_A}{\partial x} \right) \right] \, dz + \int_{z_b}^{z_s} r_A \, dz \quad (4.38)
\]

Assuming a vertically uniform velocity profile, and applying Leibnitz's rule to each term in Eq. (4.38), gives:

\[
\int_{z_b}^{z_s} \frac{\partial \rho_A}{\partial t} \, dz = \frac{\partial}{\partial t} \int_{z_b}^{z_s} \rho_A \, dz - \rho_A (x, y, z_s, t) \frac{\partial z_s}{\partial t} + \rho_A (x, y, z_b, t) \frac{\partial z_b}{\partial t} \quad (4.39)
\]

\[
\int_{z_b}^{z_s} \frac{\partial}{\partial x} (\rho_A u) \, dz = \frac{\partial}{\partial x} \int_{z_b}^{z_s} \rho_A \, dz - \rho_A (x, y, z_s, t) \frac{\partial z_s}{\partial x} + \rho_A (x, y, z_b, t) \frac{\partial z_b}{\partial x} + U \rho_A (x, y, z_b, t) \frac{\partial z_b}{\partial x} \quad (4.40)
\]

\[
\int_{z_b}^{z_s} \frac{\partial}{\partial y} (\rho_A v) \, dz = \frac{\partial}{\partial y} \int_{z_b}^{z_s} \rho_A \, dz - \rho_A (x, y, z_s, t) \frac{\partial z_s}{\partial y} + \rho_A (x, y, z_b, t) \frac{\partial z_b}{\partial y} + V \rho_A (x, y, z_b, t) \frac{\partial z_b}{\partial y} \quad (4.41)
\]
\[
\int_{z_s}^{z_b} \frac{\partial}{\partial z} (\rho_A w) \, dz = \rho_A w(z_s) - \rho_A w(z_b) \tag{4.42}
\]

\[
\int_{z_s}^{z_b} \frac{\partial}{\partial x} \left( P_{AB} \frac{\partial \rho_A}{\partial x} \right) \, dz = \frac{\partial}{\partial x} \int_{z_s}^{z_b} P_{AB} \frac{\partial \rho_A}{\partial x} \, dz - P_{AB} \frac{\partial \rho_A}{\partial x} (x, y, z_s, t) \frac{\partial z_s}{\partial x}
+ P_{AB} \frac{\partial \rho_A}{\partial x} (x, y, z_b, t) \frac{\partial z_b}{\partial x} \tag{4.43}
\]

Developing the first term on the right hand side of Eq. (4.43) by Leibnitz's rule:

\[
\int_{z_s}^{z_b} P_{AB} \frac{\partial \rho_A}{\partial x} \, dz = \frac{\partial}{\partial x} \int_{z_s}^{z_b} P_{AB} \rho_A \, dz - P_{AB} \rho_A (x, y, z_s, t) \frac{\partial z_s}{\partial x}
+ P_{AB} \rho_A (x, y, z_b, t) \frac{\partial z_b}{\partial x} \tag{4.44}
\]

\[
\int_{z_s}^{z_b} \frac{\partial}{\partial y} \left( P_{AB} \frac{\partial \rho_A}{\partial y} \right) \, dz = \frac{\partial}{\partial y} \int_{z_s}^{z_b} P_{AB} \frac{\partial \rho_A}{\partial y} \, dz - P_{AB} \frac{\partial \rho_A}{\partial y} (x, y, z_s, t) \frac{\partial z_s}{\partial y}
+ P_{AB} \frac{\partial \rho_A}{\partial y} (x, y, z_b, t) \frac{\partial z_b}{\partial y} \tag{4.45}
\]

Developing the first term on the right hand side of Eq. (4.45) by Leibnitz's rule:

\[
\int_{z_s}^{z_b} P_{AB} \frac{\partial \rho_A}{\partial y} \, dz = \frac{\partial}{\partial y} \int_{z_s}^{z_b} P_{AB} \rho_A \, dz - P_{AB} \rho_A (x, y, z_s, t) \frac{\partial z_s}{\partial y}
+ P_{AB} \rho_A (x, y, z_b, t) \frac{\partial z_b}{\partial y} \tag{4.46}
\]
It can be noted here that when the diffusion coefficient \(D^*\) is taken to be independent of depth, it becomes a dispersion coefficient \(D\) by definition. Substituting Eqs. (4.39) through (4.48) into Eq. (4.37) results in:

\[
\begin{align*}
\frac{\partial}{\partial t} & \int_{z_b}^{z_s} \rho_A \, dz + \frac{\partial}{\partial x} \int_{z_b}^{z_s} U \rho_A \, dz + \frac{\partial}{\partial y} \int_{z_b}^{z_s} V \rho_A \, dz \\
- \rho_A(x,y,z_s,t) & \left[ \frac{\partial z_s}{\partial t} + U \frac{\partial z_s}{\partial x} + V \frac{\partial z_s}{\partial y} \right] \\
+ \rho_A(x,y,z_b,t) & \left[ \frac{\partial z_b}{\partial t} + U \frac{\partial z_b}{\partial x} + V \frac{\partial z_b}{\partial y} \right] + \rho_A w(z_s) - \rho_A w(z_b) \\
= \frac{\partial}{\partial x} & \left[ \int_{z_b}^{z_s} \rho_A \, dz - D^* \frac{\partial^2 \rho_A}{\partial x^2} + \frac{\partial^2 \rho_A}{\partial z^2} - D^* \frac{\partial^2 \rho_A}{\partial z^2} + \frac{\partial \rho_A}{\partial z} \int_{z_b}^{z_s} \rho_A \, dz \right] \\
+ D^* \frac{\partial \rho_A}{\partial x} & \left( \frac{\partial z_b}{\partial x} \right) + \frac{\partial}{\partial y} \left[ \int_{z_b}^{z_s} \rho_A \, dz \right] \\
- D^* \rho_A(x,y,z_s,t) & \frac{\partial z_s}{\partial y} + D^* \frac{\partial \rho_A}{\partial y} \left( \frac{\partial z_b}{\partial y} \right) \\
& - D^* \frac{\partial \rho_A}{\partial y} \frac{\partial z_s}{\partial y} + D^* \frac{\partial \rho_A}{\partial y} \frac{\partial z_b}{\partial y}
\end{align*}
\]
Defining a vertically averaged concentration:

\[ S_A = \frac{1}{D} \int_{z_b}^{z_s} \rho_A \, dz \]  \hspace{1cm} (4.50)

and rearranging, Eq. (4.49) becomes:

\[
\frac{\partial}{\partial t} (D S_A) + \frac{\partial}{\partial x} (U D S_A) + \frac{\partial}{\partial y} (V D S_A) - \rho_A(x, y, z_s, t) \\
- \rho_A(x, y, z_s, t) \left[ \frac{\partial z_s}{\partial t} + U \frac{\partial z_s}{\partial x} + V \frac{\partial z_s}{\partial y} \right] \\
+ \rho_A(x, y, z_b, t) \left[ \frac{\partial z_b}{\partial t} + U \frac{\partial z_b}{\partial x} + V \frac{\partial z_b}{\partial y} \right] + \rho_A w(z_s) \\
- \rho_A w(z_b) = \frac{\partial}{\partial x} \left( p_{AB} \frac{\partial (DS_A)}{\partial x} \right) + \frac{\partial}{\partial y} \left( p_{AB} \frac{\partial (DS_A)}{\partial y} \right) \\
- \frac{\partial}{\partial x} \left( p_{AB}^* \rho_A(x, y, z_s, t) \frac{\partial z_s}{\partial x} \right) + \frac{\partial}{\partial x} \left( p_{AB}^* \rho_A(x, y, z_b, t) \frac{\partial z_b}{\partial x} \right) \\
- p_{AB}^* \frac{\partial \rho_A(x, y, z_s, t)}{\partial x} + p_{AB}^* \frac{\partial \rho_A(x, y, z_b, t)}{\partial x} \\
- \frac{\partial}{\partial y} \left( p_{AB}^* \rho_A(x, y, z_s, t) \frac{\partial z_s}{\partial y} \right) + \frac{\partial}{\partial y} \left( p_{AB}^* \rho_A(x, y, z_b, t) \frac{\partial z_b}{\partial y} \right) \\
- \frac{\partial}{\partial y} \left( p_{AB}^* \rho_A(x, y, z_s, t) \frac{\partial z_s}{\partial y} \right) + \frac{\partial}{\partial y} \left( p_{AB}^* \rho_A(x, y, z_b, t) \frac{\partial z_b}{\partial y} \right) \\
- p_{AB}^* \frac{\partial \rho_A(x, y, z_s, t)}{\partial y} + p_{AB}^* \frac{\partial \rho_A(x, y, z_b, t)}{\partial y} \\
+ \frac{\partial}{\partial z} \left( D_{AB} \frac{\partial z_s}{\partial z} \right) - D_{AB} \frac{\partial \rho_A(z_b)}{\partial z} + \bar{r}_A D 
\]  \hspace{1cm} (4.51)
Substituting the definition of substantial derivative into Eq. (4.51) and neglecting the higher order diffusive terms gives:

\[
\frac{\partial (DS_A)}{\partial t} + \frac{\partial (UDS_A)}{\partial x} + \frac{\partial (VDS_A)}{\partial y} = \frac{\partial}{\partial x}\left( D_{AB} \frac{\partial S_A}{\partial x} \right) + \frac{\partial}{\partial y}\left( D_{AB} \frac{\partial S_A}{\partial y} \right) \\
+ \left[ \frac{\partial \rho_A}{\partial z}(z_s) - D_{AB} \frac{\partial \rho_A}{\partial z}(z_b) - \rho_A w(z_s) + \rho_A w(z_b) + \bar{r}_A \right] D_{AB}^{*} S_A
\]

(4.52)

The terms in the brackets on the right hand side of the above equation represent the sinks and sources of species A. These sinks and sources take into account convection and diffusion through the air and bottom surfaces, and chemical reactions. Eq. (4.52) can be written as:

\[
\frac{\partial (DS_A)}{\partial t} + \frac{\partial (UDS_A)}{\partial x} + \frac{\partial (VDS_A)}{\partial y} = \frac{\partial}{\partial x}\left( D_{AB} \frac{\partial S_A}{\partial x} \right) \\
+ \frac{\partial}{\partial y}\left( D_{AB} \frac{\partial S_A}{\partial y} \right) + \rho_A D_{AB}^{*}
\]

(4.53)

Eq. (4.53) is the vertically-averaged, materials transport equation.

**Time-Scales and Their Representation**

The general equation of continuity, Eq. (4.1), and the general equation of motion, Eq. (4.15), and their space-averaged forms, Eq. (4.11) and Eq. (4.30) represent the effective instantaneous variation in velocities and water depth. The general turbulent species continuity equation, Eq. (4.32), and its space-averaged form, Eq. (4.53),
represent the effective instantaneous variation of concentration. In these cases, some small time period averaging is used, such that turbulent fluctuations in the variables have been averaged out. In many applications, where a long term behavior of concentration of a species is to be observed, such temporal refinement requires astronomically high computation time, and consequently cannot be used. The alternative is to simplify the equations of change either by eliminating or reducing temporal variation. This can be done in two possible ways: (1) by considering steady-state conditions and (2) by appropriate long-term averaging.

In the space-averaged forms of the equations of change, various quantities such as $S_A$, $U$, $V$, and $P_A$ are functions of time. The approximation by steady-state condition assumes that various parameters exhibit no variation with respect to time. Thus it assumes that:

$$\frac{\partial S_A}{\partial t} = 0$$  \hspace{1cm} (4.54)

$$\frac{\partial U}{\partial t} = 0$$  \hspace{1cm} (4.55)

$$\frac{\partial V}{\partial t} = 0$$  \hspace{1cm} (4.56)

and thus the above variables are functions of position only. Application of steady-state conditions has limitations. While predicting behavior of different parameters in an estuarine system, a true "steady-state" cannot be attained due to the tidal oscillations. It is not possible to justify
a steady-state material transport model because continuously reacting species are present, and the reaction rates always contribute to changing the concentration of an individual species with time.

In the second approach, the equations of change are modified by long term averaging. This requires integration of the equations over a tidal period or over several tidal periods. By integrating equations, harmonic variation is averaged out. The remaining time variation, which is usually much slower, is much more important biologically. The tidal averaged equations can be used for the prediction of slowly varying biological variables. Hence a model developed from tidal-averaged equations is referred to as a "slowly varying dynamic" or "long-term dynamic" model. In the next section, the important aspects of the time averaging of the previously derived hydrodynamic model and materials transport model are given. The complete derivations of time averaged equations of hydrodynamic model and materials transport model are given in Appendix A.

Time Averaging of the Hydrodynamic Model and the Material Transport Model

Hydrodynamic Model: The hydrodynamic model consists of the continuity equation and the momentum equation. The continuity equation for estuarine flow is:

\[
\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial L}{\partial t} = R - E_v
\]  

(4.14)
Integrating the above equation over a time period \( \Delta t \), from \( t_o \) to \( t_f \) (i.e., \( t_f - t_o = \Delta t \)), gives:

\[
\frac{1}{\Delta t} \int_{t_o}^{t_f} \left[ \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial L}{\partial t} \right] dt = \frac{1}{\Delta t} \int_{t_o}^{t_f} [R - E_v] dt \quad (4.57)
\]

Defining the time averaged variables as:

\[
Q_x = \overline{Q}_x + Q'_x
\quad (4.58)
\]

where

\[
\overline{Q}_x = \frac{1}{\Delta t} \int_{t_o}^{t_f} Q_x dt
\quad (4.59)
\]

and similarly

\[
Q_y = \overline{Q}_y + Q'_y
\quad (4.60)
\]

\[
L = \overline{L} + L'
\quad (4.61)
\]

\[
R = \overline{R} + R'
\quad (4.62)
\]

\[
E_v = \overline{E}_v + E'_v
\quad (4.63)
\]

The bar (\( \overline{\ } \)) indicates the average over the period and the prime ('') indicates the instantaneous deviation from the average. Thus, Eq. (4.57) can be written as:
\[
\frac{1}{\Delta t} \int_{t_0}^{t_f} \left[ \frac{\partial (\overline{Q}_x + Q_x')}{\partial x} + \frac{\partial (\overline{Q}_y + Q_y')}{\partial y} + \frac{\partial (\overline{L} + L')}{\partial t} \right] \, dt
\]

\[
= \frac{1}{\Delta t} \int_{t_0}^{t_f} [\overline{R} + R' - \overline{E}_v - E_v'] \, dt
\]

(4.64)

Evaluating each term separately:

\[
t_f \int_{t_0}^{t_f} \frac{\partial \overline{Q}_x}{\partial x} \, dt = \frac{\partial}{\partial x} \left( \overline{Q}_x \right) \bigg|_{t_0}^{t_f} + \overline{Q}_x \left( \frac{\partial t_f}{\partial t} + \frac{\partial t_0}{\partial t} \right) = \frac{\partial \overline{Q}_x}{\partial x} \Delta t
\]

(4.65)

\[
t_f \int_{t_0}^{t_f} \frac{\partial \overline{Q}_x}{\partial x} \, dt = \frac{\partial}{\partial x} \left( \overline{Q}_x \right) \bigg|_{t_0}^{t_f} + \overline{Q}_x \left( \frac{\partial t_f}{\partial t} + \frac{\partial t_0}{\partial t} \right) = 0
\]

(4.66)

Since by definition the time average of the fluctuation, \(Q'_x\), is zero. Similarly:

\[
t_f \int_{t_0}^{t_f} \frac{\partial \overline{Q}_y}{\partial y} \, dt = \frac{\partial \overline{Q}_y}{\partial y} \Delta t
\]

(4.67)

\[
t_f \int_{t_0}^{t_f} \frac{\partial Q'_y}{\partial y} \, dt = 0
\]

(4.68)

\[
t_f \int_{t_0}^{t_f} \frac{\partial \overline{L}}{\partial t} \, dt = \frac{\partial \overline{L}}{\partial t} \Delta t
\]

(4.69)
Substituting Eqs. (4.64) through (4.74) into Eq. (4.57) results in the time-averaged continuity equation:

\[
\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial L}{\partial t} = \bar{R} - \bar{E}_v
\]  

(4.75)

**X-Component of the Momentum Equation:** The momentum equation in the x-direction is:

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} - FV = g \frac{\partial L}{\partial x} = \tau_s^x - \tau_x^b
\]  

(4.76)
Integrating over a time period $\Delta t$ gives:

$$\frac{1}{\Delta t} \int_{t_0}^{t_f} \left[ \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} - FV + \frac{\partial L}{\partial x} \right] dt = \frac{1}{\Delta t} \int_{t_0}^{t_f} \left[ \tau^s_x - \tau^b_x \right] dt \tag{4.77}$$

Define:

$$U = \bar{U} + U'$$ \hfill (4.78)

$$V = \bar{V} + V'$$ \hfill (4.79)

$$D = \bar{D} + D'$$ \hfill (4.80)

$$\tau^s_x = \tau^s_x + \tau'^s_x$$ \hfill (4.81)

$$\tau^b_x = \tau^b_x + \tau'^b_x$$ \hfill (4.82)

Substitute the above definitions in Eq. (4.77). Evaluating each term and putting the time average of the fluctuations equal to zero, and rearranging, gives the $x$-component of the time averaged equation of motion:

$$\frac{\partial \bar{U}}{\partial t} + \bar{U} \frac{\partial \bar{U}}{\partial x} + \frac{1}{\Delta t} \int_{t_0}^{t_f} U' \frac{\partial U'}{\partial t} dt + \bar{V} \frac{\partial \bar{U}}{\partial y} + \frac{1}{\Delta t} \int_{t_0}^{t_f} \frac{\partial \bar{L}}{\partial x} - FV + \frac{\partial \bar{L}}{\partial x}$$

$$= \tau^s_x - \tau^b_x$$ \hfill (4.83)

The $y$-component of the equation of motion can be similarly derived:

$$\frac{\partial \bar{V}}{\partial t} + \bar{V} \frac{\partial \bar{V}}{\partial y} + \frac{1}{\Delta t} \int_{t_0}^{t_f} V' \frac{\partial V'}{\partial t} dt + \bar{U} \frac{\partial \bar{V}}{\partial x} + \frac{1}{\Delta t} \int_{t_0}^{t_f} \frac{\partial \bar{L}}{\partial x} + F\bar{U} + \frac{\partial \bar{L}}{\partial y}$$

$$= \tau^s_y - \tau^b_y$$ \hfill (4.84)
Materials Transport Model: The materials transport model is given as:

$$\frac{\partial (D S_A)}{\partial t} + \frac{\partial (U D S_A)}{\partial x} + \frac{\partial (V D S_A)}{\partial y} - \frac{\partial}{\partial x} \left( D \frac{D S_A}{D x} \right) - \frac{\partial}{\partial y} \left( D \frac{D S_A}{D y} \right) - P_A$$

(4.85)

Eq. (4.85) can be integrated as:

$$\frac{1}{\Delta t} \int_{t_0}^{t_f} \left[ \frac{\partial (D S_A)}{\partial t} + \frac{\partial (U D S_A)}{\partial x} + \frac{\partial (V D S_A)}{\partial y} - \frac{\partial}{\partial x} \left( D \frac{D S_A}{D x} \right) - \frac{\partial}{\partial y} \left( D \frac{D S_A}{D y} \right) - P_A \right] dt = 0$$

(4.86)

Define:

$$S = \overline{S}_A + S'_A$$

(4.87)

$$P = \overline{P}_A + P'_A$$

(4.88)

Following the procedure used with the hydrodynamic model gives the time integrated species transport model.

$$\frac{\partial (D \overline{S}_A)}{\partial t} + \frac{1}{\Delta t} \int_{t_0}^{t_f} \frac{\partial}{\partial t} (D' S'_A) dt + \frac{\partial}{\partial x} (D' \overline{U}_x \overline{S}_A) + \frac{1}{\Delta t} \frac{\partial}{\partial x} \overline{S}_A \int_{t_0}^{t_f} D' U' dt$$

$$+ \frac{1}{\Delta t} \frac{\partial}{\partial x} \overline{D} \int_{t_0}^{t_f} U' S'_A dt + \frac{1}{\Delta t} \frac{\partial}{\partial x} \overline{U} \int_{t_0}^{t_f} D' S'_A dt + \frac{1}{\Delta t} \frac{\partial}{\partial x} \int_{t_0}^{t_f} D' U' S'_A dt$$
Approach to the Solution of the Hydrodynamic Model

Inspecting the time average equations of motion, Eq. (4.83) and Eq. (4.84), and time average material transport model, Eq. (4.89), we can see that certain additional terms occur because of the averaging procedure. These terms must be evaluated in order to solve the time averaged equations. Since these additional terms are the functions of fluctuating components of the variables, they cannot be evaluated directly. The rest of this section describes the methods by which the hydrodynamic model and the material transport model can be solved.
Hydrodynamic Model: Comparing the space-averaged form of the hydrodynamic model, Eqs. (4.14, 4.30, 4.31) with the time-averaged form of the hydrodynamic model Eqs. (4.75, 4.83, 4.86), we see that the form of the equation of continuity is the same in both types of model. The only difference is that the variables in the time-averaged equation, Eq. (4.75) represent the value of every variable averaged over time period $\Delta t$. Comparing the space-averaged equation of motion with the time-averaged equation of motion, we see that the time-averaged equation, Eq. (4.83) and Eq. (4.84) contain two additional terms. In order to solve Eq. (4.83) and Eq. (4.84), we must evaluate these additional terms. These terms are the functions of fluctuating component of the tidal time averaged velocities and cannot be evaluated directly. Hence, to solve the hydrodynamic model, a simplified but valid approach is used which is described below.

For the hydrodynamic model a quasi-steady solution of the hydrodynamic model is obtained. In a natural estuary, as stated earlier, the inflows and the boundary values for the tides vary with respect to time. Hence, average values of a tidal period continuously change. However, these average changes are very slow and, for all practical purposes, the boundary conditions could be assumed constant for several tidal cycles. If these constant boundary conditions are input for the solution of the hydrodynamic model, after some time, various variables attract
quasi-steady state. This required time duration may range from a few to several tidal cycles depending on the nature of the system and initial conditions of the various parameters. In the quasi-steady state, the different variables become periodic in the sense that:

$$u(t) = u(t + T'')$$  \hspace{1cm} (4.90)

$$v(t) = v(t + T'')$$  \hspace{1cm} (4.91)

$$L(t) = L(t + T'')$$  \hspace{1cm} (4.92)

Thus on the time span of one tidal cycle, the values of different parameters vary and hence they are in dynamic conditions but since these values repeat at the same instance of time in the next tidal cycle, they are in a dynamic steady-state condition. This is also known as the quasi-steady state. Once the system attains a quasi-steady state, the net velocities (also called time-averaged velocities) are computed by obtaining the tidal-mean of these repetitive values:

$$\bar{U} = \frac{1}{T''} \int_{t}^{t+T''} u(t) \, dt$$  \hspace{1cm} (4.93)

$$\bar{V} = \frac{1}{T''} \int_{t}^{t+T''} v(t) \, dt$$  \hspace{1cm} (4.94)

$$\bar{L} = \frac{1}{T''} \int_{t}^{t+T''} L(t) \, dt$$  \hspace{1cm} (4.95)
The computations for the above parameters are repeated whenever there is significant change in the boundary conditions. Since boundary conditions are very slowly varying, the true values of the above parameters do vary slightly from one tidal cycle to the next tidal cycle. These values are obtained by interpolating the values of variables between two different computed values of variables with two different boundary conditions. In the following section, results obtained by the quasi-steady solution of the hydrodynamic model are given.

**Quasi-Steady State Solution of the Hydrodynamic Model:** To solve the hydrodynamic model for the current distribution over the bay, the following types of information were needed: tidal heights history of the passes, fresh water runoff, atmospheric conditions, bottom friction coefficients and bathymetric data. Tidal history at the passes was obtained from Tide Tables published by the U.S. Department of Commerce (Ref. 4.6). Fresh water runoff data and atmospheric conditions were obtained from the Louisiana State University Sea Grant Program (Ref. 4.7). The bottom friction coefficients were obtained from the studies of Hacker (Ref. 4.1) for the Barataria Bay region. The bathymetric data for Barataria Bay were also taken from Hacker's study.

The bottom friction coefficients for the conditions found in Barataria Bay have been reported by a number of
investigators (Ref. 2.11, 2.13). The value used in this case was a Manning friction factor of 0.026, which was also used by Hacker (Ref. 4.1). This value was used in the Tracor study (Ref. 4.8) on Galveston Bay and also used by Leendertse (Ref. 4.4).

Tidal variation at the passes was modeled by fitting a sinusoidal curve to the tidal range. Since tidal fluctuations generally follow a sinusoidal variation, this procedure represents a good approximation of this variation within the accuracy of computations. Tidal ranges at the different passes into Barataria Bay are the same but the times of high and low tide at Barataria Pass are ahead of the other three passes. Barataria Pass was taken as reference, Caminada Pass lags by 1.358 hours, and Quatre Bayou and Pass Abel are found to lag 0.875 hours as reported in the Tide Tables (Ref. 4.6). These lags were included in the tidal simulation.

The grid system originally designed by Hacker (Ref. 4.1) for the Barataria Bay region was used. This grid system was placed on the area of interest in a fashion that all tidal passes lined up with the bottom row of the grid system. Grid size was chosen to best represent the widths of the passes. With the grid size chosen, only one grid point was assigned for each pass. A smaller grid size than the one used would be desirable as it would give more accurate results. However, a smaller grid size would require more computer storage than was available. The Barataria Bay
estuary was modeled using a finite difference network of 1300 yards square mesh size.

It was found that if constant boundary conditions were maintained, the solution for Barataria Bay reached quasi-steady state conditions after four tidal cycles. In this case, the boundary conditions at the passes were time-dependent over a tidal cycle but the same time history was used for the following tidal cycles. The values of the $x$-component of velocity, $U$, the $y$-component of velocity, $V$, and the depth above mean sea level, $L$, at the fourth and fifth tidal cycles were exactly the same at a fixed location in the system.

Figures 4.2, 4.3, and 4.4 show the behavior of $U$, $V$, and $L$ as they attain quasi-steady state conditions from initial conditions. Once these parameters attain quasi-steady state conditions, they are averaged over the tidal cycle following the Eqs. (4.93, 4.94, 4.95). A total of 12 such runs were made which represented all 12 months of the year. The detailed computer program and the user's manual including a flow diagram for the program are given in Appendix B.

In Table 4.2, the ranges of the average values of velocities and depths are given. The values of average velocities and depths thus obtained are used as input to solve the material transport model. The following section describes the approach used to solve the material transport model.
Figure 4.2. Behavior of U velocity as it attains quasi-steady state at point (16,16), February
Figure 4.3. Behavior of V velocity as it attains quasi-steady state at point (16,16), February.
Figure 4.4. Behavior of fluctuating depth (L) as it attains quasi-steady state value at point (16,16), February
### TABLE 4.2 Range of Monthly Average Velocities in x and y Directions U, V and Height above Mean Sea Level L, from Quasi-Steady State Solution

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity in x direction</td>
<td>U</td>
</tr>
<tr>
<td></td>
<td>0.001-0.1 ft/sec</td>
</tr>
<tr>
<td>Velocity in y direction</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>0.0009-0.019 ft/sec</td>
</tr>
<tr>
<td>Height above mean sea level</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>0.2-0.9 ft</td>
</tr>
</tbody>
</table>
Material Transport Model: Comparing the space-averaged equation, Eq. (4.53), and time-averaged equation, Eq. (4.89), we see additional terms. These additional terms are approximated by diffusion type terms. Since the diffusion terms are already present in Eq. (4.89), the approximation resulting from modeling additional terms is added to already existing diffusion terms and Eq. (4.89) reduces to the following form:

\[
\frac{\partial (\bar{D} \bar{S}_A)}{\partial t} + \frac{\partial (\bar{D} \bar{U} \bar{S}_A)}{\partial x} + \frac{\partial (\bar{D} \bar{V} \bar{S}_A)}{\partial y} - \frac{\partial}{\partial x} \left( D_{AB}^{**} \bar{D} \frac{\partial \bar{S}_A}{\partial x} \right) - \frac{\partial}{\partial y} \left( D_{AB}^{**} \bar{D} \frac{\partial \bar{S}_A}{\partial y} \right) - \bar{P}_A = 0
\]

(4.96)

where

\[
D_{AB}^{**} \frac{\partial \bar{S}_A}{\partial x} = \frac{1}{\Delta t} \bar{D} \left[ \bar{S}_A \int_{t_0}^{t_f} D' U' dt + \bar{D} \int_{t_0}^{t_f} U' S'_A dt + \bar{U} \int_{t_0}^{t_f} D' S'_A dt \right.
\]

\[
+ \left. \int_{t_0}^{t_f} D' U' S'_A dt - D_{AB} \int_{t_0}^{t_f} D' \frac{\partial S'_A}{\partial x} dt - \Delta t \bar{D} D_{AB} \frac{\partial \bar{S}_A}{\partial x} \right]
\]

(4.97)

and

\[
D_{AB}^{*} \frac{\partial \bar{S}_A}{\partial y} = \frac{1}{\Delta t} \bar{D} \left[ \bar{S}_A \int_{t_0}^{t_f} D' V' dt + \bar{D} \int_{t_0}^{t_f} V' S'_A dt + \bar{V} \int_{t_0}^{t_f} D' S'_A dt \right.
\]

\[
+ \left. \int_{t_0}^{t_f} D' V' S'_A dt - D_{AB} \int_{t_0}^{t_f} D' \frac{\partial S'_A}{\partial y} dt - \Delta t \bar{D} D_{AB} \frac{\partial \bar{S}_A}{\partial y} \right]
\]

(4.98)
There is only historical justification from turbulence theory for approximating the effect of tidal mixing in terms of the product of a diffusion coefficient and a time averaged concentration gradient. The dispersion terms on the left hand sides of the above equations are not truly diffusive and also represent effect of advection. They arise out of a reformulation of an averaged equation, which replaces the advective terms with similar expressions involving only mean variables. The dispersion terms in Eq. (4.96), as can be seen from Eq. (4.97) and Eq. (4.98), represent three different effects. One is due to turbulent fluctuations, one is due to vertical gradient in concentration, and one is due to tidal oscillations in mean velocities and concentrations. The magnitude of the dispersion not only varies from estuary to estuary, but may vary from point to point and from time to time in the same estuary depending upon the behavior of the various contributors to the dispersion terms (Ref. 4.8). At present, there does not exist a general technique for the a priori calculation of dispersion coefficients and the values must be selected based on experimental data.

Once the dispersion coefficient is selected, the material transport model, Eq. (4.96), is readily solved. The velocities required to solve the material transport model are obtained from the quasi-steady solution of the hydrodynamic model. The sink and source terms P are used as given in Chapter III. The results obtained by solving the material transport model are discussed in Chapter VI.
Summary

In this chapter, the hydrodynamic model and the material transport model are derived. Important steps in the derivation of their time-averaged form are also given. Certain additional terms occur in the time averaged form of the above models. Different approaches to handle these terms are also given. In addition, results obtained by solving the hydrodynamic model for the quasi-steady state are also given.
REFERENCES


4.7 Data Management System, Louisiana State University Sea Grant Program, Baton Rouge, La.

CHAPTER V

NUMERICAL IMPLEMENTATION OF TRANSPORT PHENOMENA

EQUATIONS OF SHALLOW ESTUARINE BAY SYSTEMS

Introduction

In order to solve the equations derived in the previous chapter, the alternating directions implicit technique (ADI) and explicit technique are used. The ADI technique is used to obtain the solution of the hydrodynamic model. The explicit technique is used to obtain the solution of the material transport model.

The first part of this chapter will present the numerical implementation of the hydrodynamic model. The second part will present the numerical implementation of the material transport model. The last part deals with numerical operations used to describe special conditions inside the calculation grid.

Finite Difference Approximation of the Hydrodynamic Model

To implement the mathematical models, a means of relating the models to a physical system is required. This is accomplished by superimposing a grid system over the area to be modeled. The variables are assigned values within the cell structure produced by superimposing a grid system. In
Figure 5.1, the variables are placed on a space staggered grid. The water levels, $L$, are located at integer values of $j$ and $k$, the depths, $h$, are stored at half integer values of $j$ and $k$. The U velocities are located at half integer values of $j$ and integer values of $k$, and the V velocities are located at integer values of $j$ and half integer values of $k$. The differential equations of the hydrodynamic model are finite differenced using this grid. The ADI technique, successfully used by Leendertse (Ref. 5.1) for Jamaica Bay and by Hacker (Ref. 5.2) for Barataria Bay, is used to solve the hydrodynamic model. This method is advantageous as it allows the numerical problem to be placed in a tridiagonal matrix form.

The ADI technique works over the grid in the following fashion: The x-component of the equation of motion and the continuity equation are applied on a given row. The resulting equations are solved implicitly for the U velocities and the water level $L$, in the given row. The same procedure is followed in the next row, and so on until the whole field under the grid is covered. The next step alternates in direction, and the y-component of the equation of motion and the continuity equations are applied on a given column and the resulting equations are solved implicitly for the V velocities and the water levels, $L$, in the given column. The same procedure is followed in the next column, and so on, until the whole field under the grid is covered.
Figure 5.1 Space-Staggered Scheme
When the two operations mentioned above are combined, all the unknowns (U velocities, V velocities and water levels, L) have been calculated implicitly. These two operations combine to form a time step. The solution for the next time step is found by repeating this procedure.

**First Half-Time Step:** As was previously mentioned, the first half-time step is used to calculate the U velocities and the water levels. Hacker (Ref. 5.2) has written the x-component of the equation of motion of the hydrodynamic model,

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} - FV + g \frac{\partial L}{\partial x} + g \frac{U^2 + V^2} {DC^2} = 0 \quad (4.30)
\]

and the continuity equation,

\[
\frac{\partial L}{\partial t} + \frac{\partial (DU)}{\partial x} + \frac{\partial (DV)}{\partial y} = R - E_v \quad (4.11)
\]

in the finite difference forms using the notations given in his dissertation. As mentioned earlier, this procedure puts the above equations in the tridiagonal matrix forms. Hacker (Ref. 5.2) has given detailed manipulation of the above equations. The x-component of the momentum equation is transformed into the following form.

\[
- r_{j} L_{j,k}^{n+\frac{1}{2}} + r_{j+\frac{1}{2}} U_{j+\frac{1}{2},k}^{n+\frac{1}{2}} + r_{j+1} L_{j+1,k}^{n+\frac{1}{2}} = B_{j+\frac{1}{2},k}^{n} \quad (5.1)
\]

where

\[
r_{j} = r_{j+1} = \frac{g \Delta t}{2 \Delta x} \quad (5.2)
\]
\[ r_{j+\frac{1}{2}} = 1 + \frac{\Delta t}{2 \Delta x} \left( u_{j+\frac{1}{2},k}^{n-\frac{1}{2}} - u_{j-\frac{1}{2},k}^{n-\frac{1}{2}} \right) \]

\[ + \frac{g}{2} \frac{\Delta t}{2} \left[ (u_{j+\frac{1}{2},k}^{n-\frac{1}{2}})^2 + (v)^2 \right]^{1/2} \frac{1}{(h^2 + \Delta x^2)} (C^2) \]

\[ (5.3) \]

\[ B_{n,j+\frac{1}{2},k} = u_{n,j+\frac{1}{2},k}^{n-\frac{1}{2}} + \left[ F \Delta t \right] \left[ u_{n,j+\frac{1}{2},k+1}^{n-\frac{1}{2}} - u_{n,j+\frac{1}{2},k-1}^{n-\frac{1}{2}} \right] \]

\[ - \frac{g}{2} \frac{\Delta t}{2} \left( L_{j+1,k}^{n-\frac{1}{2}} - L_{j,k}^{n-\frac{1}{2}} \right) + \frac{\tau_s}{\rho(h^2 + \Delta x^2)} \frac{\Delta t}{(h^2 + \Delta x^2)} \]

\[ \left[ u_{n,j+\frac{1}{2},k}^{n-\frac{1}{2}} \right]^2 + (V^+) \right]^{1/2} \frac{1}{(h^2 + \Delta x^2)} (C^2) \]

\[ (5.4) \]

Applying the same procedure to the continuity equation, it is transformed into the following form:

\[ -r_{j-\frac{1}{2}} u_{j-\frac{1}{2},k}^{n+\frac{1}{2}} + L_{j,k}^{n+\frac{1}{2}} + r_{j+\frac{1}{2}} u_{j+\frac{1}{2},k}^{n+\frac{1}{2}} = A_j^n \]

\[ (5.5) \]

where

\[ r_{j-\frac{1}{2}} = (h_{j-\frac{1}{2},k+\frac{1}{2}} + h_{j-\frac{1}{2},k-\frac{1}{2}} + L_{j,k}^{n-1} + L_{j,k}^n) \frac{\Delta t}{4 \Delta x} \]

\[ (5.6) \]

\[ r_{j+\frac{1}{2}} = (h_{j+\frac{1}{2},k+\frac{1}{2}} + h_{j+\frac{1}{2},k-\frac{1}{2}} + L_{j,k}^{n} + L_{j,k}^n) \frac{\Delta t}{4 \Delta x} \]

\[ (5.7) \]
\[ A^n_j = L^n_j,k + (h_{j+\frac{1}{2},k-\frac{1}{2}} + h_{j-\frac{1}{2},k-\frac{1}{2}} + L^n_{j,k} + L^n_{j,k-1}) v^n_{j,k-\frac{1}{2}} \]

\[ \left( \frac{\Delta t}{4 \Delta x} \right) - (h_{j+\frac{1}{2},k+\frac{1}{2}} + h_{j-\frac{1}{2},k+\frac{1}{2}} + L^n_{j,k+1} + L^n_{j,k}) v^n_{j,k+\frac{1}{2}} \]

(5.8)

Eqs. (5.1) and (5.5) can now be solved for the unknown values of the \( V \) velocities and water levels on the \( k \)th row if boundary conditions are specified at both ends of the row. These boundary conditions can be: the \( U \) velocities at both ends, the \( U \) velocity at one end and the water level at the other end or the water levels at both ends. If there are \( j \) grid points in the given row \( (j=1,2,3,...N) \), Eqs. (5.1) and (5.3) can be placed in matrix form in which the matrix will be tridiagonal and the unknown vector will have \( (2N-2) \) elements. For example, let the known boundary conditions be \( L_1,k \) and \( U_{n+\frac{1}{2},k} \); then the system of equations that result can be written in matrix form as:
If the boundary conditions are $U_{1+\frac{1}{2},k}$ and $U_{n+\frac{1}{2},k}$, then the resulting matrix system is:

$$
\begin{bmatrix}
-r_{1} & r_{2} \\
0 & 1 \\
-r_{2} & r_{2+\frac{1}{2}} & r_{3} \\
-r_{2+\frac{1}{2}} & 1 & r_{3+\frac{1}{2}} \\
\end{bmatrix}
\begin{bmatrix}
L_{1,k} \\
L_{2,k} \\
L_{2+\frac{1}{2},k} \\
L_{3,k} \\
\end{bmatrix}
= 
\begin{bmatrix}
B_{1+\frac{1}{2},k} - r_{1+\frac{1}{2}} U_{1+\frac{1}{2},k} \\
B_{2+\frac{1}{2},k} \\
B_{n-\frac{1}{2},k} \\
A_{n-\frac{1}{2}} U_{n+\frac{1}{2},k} \\
\end{bmatrix}
$$

(5.9)

$$
\begin{bmatrix}
-r_{n-1} & r_{n-\frac{1}{2}} & r_{n} \\
-r_{n-\frac{1}{2}} & 1 \\
\end{bmatrix}
\begin{bmatrix}
U_{n-\frac{1}{2},k} \\
L_{n,k} \\
\end{bmatrix}
$$

(5.10)
Thus, by knowing the values of water levels at the time level \( n \), the \( V \) velocities at time level \( n \), the \( U \) velocities at time level \( n-\frac{1}{2} \) and the boundary conditions at time level \( n+\frac{3}{2} \), the value of water levels and \( U \) velocities for the \( n+\frac{1}{2} \) level can be calculated by applying the Thomas Algorithm to the tridiagonal matrix. The values are obtained for the whole grid system as the above calculation is repeated for all the rows.

Once all the rows have been swept, the first half-time step for the hydrodynamics has been completed. The second half-time step follows.

**Second Half-Time Step:** In the second half-time step, \((n+\frac{1}{2})t\) to \((n+1)t\), the \( V \) velocities and the water levels are calculated. Applying the same technique, Hacker (Ref. 5.2) has written the \( y \)-component of the equation of motion of the hydrodynamic model,

\[
\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + FU + g \frac{\partial L}{\partial y} + \frac{g(U^2 + V^2)^{1/2}}{DC^2} - y = 0
\]

(4.31)

and the continuity equation,

\[
\frac{\partial L}{\partial t} + \frac{\partial (DU)}{\partial x} + \frac{\partial (DV)}{\partial y} = R - E_v
\]

(4.11)

Rearranging the finite difference form of the \( y \)-component of the momentum equation, it is transformed into:

\[
-r_k L_{j,k}^{n+\frac{1}{2}} + r_{k+\frac{1}{2}} V_{j,k+\frac{1}{2}}^{n+1} + r_{k+1} L_{j,k+1}^{n+1} = B_{j,k+\frac{1}{2}}^{n+\frac{1}{2}}
\]

(5.11)

where:

\[
r_k = r_{k+1} = \frac{g \Delta t}{4 \Delta x}
\]

(5.12)
The continuity equation for the second half-time step takes the form:

\[ \begin{align*}
- \Delta r_{k-\frac{1}{2}} & = \frac{\Delta t}{2} \left[ \frac{(v_{j,k-\frac{1}{2}}^n - v_{j,k-\frac{1}{2}}^n)}{\Delta x} \right] + g \frac{\Delta t}{2} \left[ \frac{(U^+)^2 + (v_{j,k+\frac{1}{2}}^n)^2}{(H^X + L^Y)(C^Y)^2} \right]^{1/2} \\

B_{n+\frac{3}{2}}^{n+\frac{3}{2}} & = v_{j,k+\frac{3}{2}}^n + \left[ \frac{-F \Delta t}{2} \left( \frac{v_{j+\frac{1}{2},k+\frac{3}{2}}^n - v_{j+1,k+\frac{3}{2}}^n}{} \right) \right] U^+ \\
& - \frac{g \Delta t}{2} \left[ \frac{L_{j,k+\frac{3}{2}}^n - L_{j,k}^n}{\Delta x} \right] + \frac{\tau_s \Delta t}{\rho (H^X + L^Y)} \\
& - \frac{g \Delta t}{2} \left[ \frac{(U^+)^2 + (v_{j,k+\frac{3}{2}}^n)^2}{(H^X + L^Y)(C^Y)^2} \right]^{1/2}
\end{align*} \]

(5.13)

(5.14)

\[ \begin{align*}
- \Delta r_{k-\frac{1}{2}} & = \frac{\Delta t}{2} \left[ \frac{(v_{j,k-\frac{1}{2}}^n - v_{j,k-\frac{1}{2}}^n)}{\Delta x} \right] + g \frac{\Delta t}{2} \left[ \frac{(U^+)^2 + (v_{j,k+\frac{1}{2}}^n)^2}{(H^X + L^Y)(C^Y)^2} \right]^{1/2} \\

A_{n+\frac{3}{2}}^{n+\frac{3}{2}} & = L_{j,k}^{n+\frac{3}{2}} + \left( h_{j+\frac{1}{2},k+\frac{3}{2}}^n + h_{j-\frac{1}{2},k-\frac{3}{2}}^n + L_{j,k}^{n+\frac{3}{2}} + L_{j+1,k}^{n+\frac{3}{2}} \right) \frac{\Delta t}{4} \Delta x
\end{align*} \]

(5.15)

(5.16)

(5.17)

(5.18)
Eqs. (5.11) and (5.15) can now be solved for the unknown values of the $V$ velocities and water levels on the $j$th column if boundary conditions are specified at both ends of the column. Similar to the first-half step, these boundary conditions can be the $V$ velocities at both ends, the water level at one end the $V$ velocity at the other end, or the water levels at both ends. Again, the resulting system of equations can be put in matrix form. For example: in a column with $N$ grid points and boundary conditions of $L_{j,1}$ and $V_{j,n+\frac{1}{2}}$, the matrix is:

\[
\begin{bmatrix}
 r_{1+\frac{1}{2}} & r_2 \\
 -r_{1+\frac{1}{2}} & 1 & r_{2+\frac{1}{2}} \\
 -r_2 & r_{2+\frac{1}{2}} & r_3 \\
 -r_{2+\frac{1}{2}} & 1 & r_{3+\frac{1}{2}} \\
 -r_{n-\frac{1}{2}} & r_{n-\frac{1}{2}} & r_n \\
 -r_{n-\frac{1}{2}} & 1 & r_{n+\frac{1}{2}} \\
\end{bmatrix}
\begin{bmatrix}
 V_{j,1+\frac{1}{2}} \\
 V_{j,2+\frac{1}{2}} \\
 V_{j,3+\frac{1}{2}} \\
 V_{j,n-\frac{1}{2}} \\
 V_{j,n+\frac{1}{2}} \\
\end{bmatrix}
= 
\begin{bmatrix}
 B_{1+\frac{1}{2}} L_{j,1} \\
 B_{2+\frac{1}{2}} L_{j,2} \\
 B_{3+\frac{1}{2}} L_{j,3} \\
 B_{n-\frac{1}{2}} L_{j,n-\frac{1}{2}} \\
 A_n - r_{n+\frac{1}{2}} V_{j,n+\frac{1}{2}} \\
\end{bmatrix}
\]

(5.19)

Thus, by knowing the values of water levels at the time level $n+\frac{1}{2}$, the $V$ velocities at time level $n$ and the appropriate boundary conditions at time level $n+1$, the values of
water levels and V velocities for the n+1 level can be calculated by applying the Thomas Algorithm to the tridiagonal matrix. The values are obtained for the whole grid system as the above calculation is repeated for all the columns.

The combination of the first half-time step and the second half-time step result in the implicit calculation of the velocities and water levels in a whole time step. The calculation proceeds as the above mentioned steps are repeated. These velocities are input to the material transport model after proper modifications.

Open End Boundary Conditions: The field of computation has tidal entrances as boundaries. The values of the water levels in these tidal entrances are known as a function of time. However, as the open sea is beyond this field of computation, the horizontal velocities of the incoming waters is not known. In order to circumvent this problem, it is assumed that the advective terms are zero at the boundaries. Thus, if the entrance is in a row in the field of computation, the x-component of the equation of motion, Eq. (4.30), reduces to:

\[
\frac{3U}{\partial t} + V \frac{3U}{\partial y} - g \frac{\partial L}{\partial x} + g \frac{U(U^2 + V^2)^{1/2}}{\partial x} = 0 \quad (5.20)
\]

This equation applies to the grid points representing a tidal entrance in the x-direction. The same reasoning applies to tidal entrances in the y-direction; with the y-component of the equation of motion, Eq. (4.31), reducing to:
\[
\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + FU + g \frac{\partial L}{\partial y} + g \frac{V(U^2 + V^2)^{1/2}}{DC^2} - \frac{Y}{k} = 0 \tag{5.21}
\]

Eq. (5.20) along with the continuity equation is placed in the system of equations in which one of the known boundary conditions is the tidal level. Such a system is exemplified by the matrix (5.9). Only one change is necessary in the matrix and that is a change in the coefficient for the equation in question. The appropriate value of \( r_{j+\frac{1}{2}} \) becomes:

\[
r_{j+\frac{1}{2}} = 1 + \frac{g \Delta t}{2} \left[ (U_{j+\frac{1}{2},k})^2 + (V^+) \right]^{1/2} \frac{1}{(h^y + l^x)(c^x)^2} \tag{5.22}
\]

The same procedure applies to tidal entrances in the \( y \)-direction. Eq. (5.21) along with the continuity equation is placed in the system of equations in which one of the known boundary conditions is the tidal level. Such a system is exemplified by the matrix (5.19). Again, the coefficient \( r_{k+\frac{1}{2}} \) is modified and becomes

\[
r_{k+\frac{1}{2}} = 1 + \frac{g \Delta t}{2} \left[ (U^+)^2 + (V_{j,k+\frac{1}{2}}^n) \right]^{1/2} \frac{1}{(h^x + l^y)(c^y)^2} \tag{5.23}
\]

**Finite Difference Approximation of the Material Transport Model**

The space staggered scheme (Figure 5.1) used for the implicit scheme is also used for the explicit scheme. The results obtained from the hydrodynamic model are used for
inputs to the explicit solution of the material transport model.

The material transport model can be written as:

\[
\frac{\partial (D\bar{S}_A)}{\partial t} + \frac{\partial (UD\bar{S}_A)}{\partial x} + \frac{\partial (VD\bar{S}_A)}{\partial y} = \frac{\partial }{\partial x} \left( D B^{**} \frac{\partial \bar{S}_A}{\partial x} \right) \\
+ \frac{\partial }{\partial y} \left( D B^{**} \frac{\partial \bar{S}_A}{\partial y} \right) + \bar{P}_A
\]  

(4.96)

In the above equation, \( \bar{P}_A \) represents sink and source terms which are given by reaction rate terms given in Chapter III.

The above equation can be expanded as:

\[
\frac{\partial (D\bar{S}_A)}{\partial t} + \frac{\partial (UD\bar{S}_A)}{\partial x} + \frac{\partial (VD\bar{S}_A)}{\partial y} = \left[ \frac{\partial (D B^{**})}{\partial x} \cdot \left( \frac{\partial \bar{S}_A}{\partial x} \right) + D B^{**} \frac{\partial^2 \bar{S}_A}{\partial x^2} \right] \\
+ \left[ \frac{\partial (D B^{**})}{\partial y} \cdot \left( \frac{\partial \bar{S}_A}{\partial y} \right) + D B^{**} \frac{\partial^2 \bar{S}_A}{\partial y^2} \right] - \bar{P}_A
\]  

(5.24)

The above equation can be put in a finite difference form using the forward difference approximation. The resulting equation is:

\[
\frac{\bar{S}^{n+1}_{j,k} - \bar{S}^n_{j,k}}{\Delta t} + \frac{\left[ \bar{S}^{n+1}_{j+1,k} - \bar{S}^n_{j+1,k} \right]}{\Delta x} + \frac{\left[ \bar{S}^{n+1}_{j,k+1} - \bar{S}^n_{j,k+1} \right]}{\Delta y} \\
+ \frac{\left[ \bar{S}^{n+1}_{j+1,k} - \bar{S}^n_{j+1,k} \right]}{\Delta x^2} + D B^{**} \frac{\partial \bar{S}_A}{\partial x} \\
+ \frac{\left[ \bar{S}^{n+1}_{j+1,k} - \bar{S}^n_{j+1,k} \right]}{\Delta x^2} + D B^{**} \frac{\partial \bar{S}_A}{\partial y}
\]
\[ + D_{j,k}^{n+1} \cdot p_{AB}^{**n+1} \left( \frac{S_{j+1,k}^n - 2 S_{j,k}^n + S_{j-1,k}^n}{\Delta x^2} \right) + \frac{\left( p_{j,k+1}^{n+1} \cdot D_{AB}^{**n+1} - p_{AB}^{**n+1} \cdot p_{j,k}^{n+1} \right) \left( S_{j,k+1}^n - S_{j,k}^n \right)}{\Delta y^2} \]

\[ + D_{j,k}^{n+1} \cdot p_{AB}^{**n+1} \left( \frac{S_{j,k+1}^n - 2 S_{j,k}^n + S_{j,k-1}^n}{\Delta y^2} \right) + \frac{p_{AB}^{n+1}}{A_{j,k}} \]

(5.25)

Rearranging and solving for \( S_{j,k}^{n+1} \) at the latest time interval results in:

\[ S_{j,k}^{n+1} = \frac{S_{j,k}^n \cdot D_j^{n+1} - \Delta t}{D_j^{n+1}} - \frac{\Delta t}{D_j^{n+1}} \left[ \frac{D_{j+1,k}^{n+1} \cdot v_{j+1,k}^{n+1} \cdot S_{j+1,k}^n - D_{j,k}^{n+1} \cdot v_{j,k}^{n+1} \cdot S_{j,k}^n}{\Delta x} \right] \]

\[ + \frac{\left( D_{j,k+1}^{n+1} \cdot v_{j,k+1}^{n+1} \cdot S_{j,k+1}^n - D_{j,k}^{n+1} \cdot v_{j,k}^{n+1} \cdot S_{j,k}^n \right)}{\Delta y} \]

\[ - \frac{D_{j,k}^{n+1} \cdot p_{AB}^{**n+1}}{A_{j,k}} \left( \frac{S_{j+1,k}^n - 2 S_{j,k}^n + S_{j-1,k}^n}{\Delta x^2} \right) + \frac{\left( p_{j,k+1}^{n+1} \cdot D_{AB}^{**n+1} - p_{AB}^{**n+1} \cdot p_{j,k}^{n+1} \right) \left( S_{j,k+1}^n - S_{j,k}^n \right)}{\Delta y} \]

\[ - \frac{D_{j,k}^{n+1} \cdot p_{AB}^{**n+1}}{y^2} \left( \frac{S_{j+1,k}^n - 2 S_{j,k}^n + S_{j,k-1}^n}{\Delta y^2} \right) + \frac{p_{AB}^{n+1}}{A_{j,k}} \]

(5.26)
The explicit solution of Eq. (5.26) is limited by the following stability criterion reported by TRACOR (Ref. 5.3); the size step is limited by:

$$\Delta x = \Delta y \leq \min \left[ \frac{8 \ B_{\text{min}}^{**}}{|U_{\text{max}}|}, \frac{8 \ B_{\text{min}}^{**}}{|V_{\text{max}}|} \right]$$

(5.27)

and the time step is limited by:

$$\Delta t \leq \min \left[ \frac{\Delta x}{2 |U_{\text{max}}|}, \frac{\Delta y}{2 |V_{\text{max}}|} \right]$$

(5.28)

and/or

$$\Delta t \leq \min \left[ \frac{(\Delta x)^2}{4 \ B_{\text{max}}^{**}}, \frac{(\Delta y)^2}{4 \ B_{\text{max}}^{**}} \right]$$

(5.29)

The computer program and users' manual for the material transport model applied to Barataria Bay are given in Appendix E.

**Numerical Operations for Special Conditions**

In order to properly simulate bay conditions, the geometry of the bay has to be included. Internal barriers such as islands create changes in the transport phenomena. Therefore, special numerical operations have to be performed to account for these conditions. In this section, the special numerical operations will be discussed for momentum transfer, and the details of these methods are reported by Hacker (Ref. 5.2).

There are two types of special situations which must
be taken care of in the case of the hydrodynamic model. The first one is the presence of barriers. For a barrier against flow in the $x$-direction, the $U$ velocity is known to be zero along the $j = m + \frac{5}{2}$ line. To simulate these conditions the matrix should be properly modified. Hacker (Ref. 5.2) has given the modified matrix and has also described the procedure to solve the modified matrix. The same procedure is followed for barriers against the flow in the $y$-direction. This procedure is also applied when islands are included in the field of calculation, in which case the water level is set to zero if the conditions are appropriate.

Another operation needed is the one to allow for flooding of the "dry" grid points. After each sweep, the "dry" points are checked for flooding. If the average water level around the "dry" point is such that the "dry" point is under water, then the water level at the "dry" point is set by the following equation:

$$L_{j,k}^{n+\frac{1}{2}} = \frac{1}{4} \left( L_{j-1,k-1}^{n+\frac{1}{2}} + L_{j-1,k-1}^{n+\frac{1}{2}} + L_{j+1,k-1}^{n+\frac{1}{2}} + L_{j+1,k+1}^{n+\frac{1}{2}} \right) \quad (5.30)$$

To insure conservation of mass, the water added to the previously "dry" point is subtracted from three adjacent grid points:

$$L_{j-1,k-1}^{n+\frac{1}{2}} = L_{j-1,k-1}^{n+\frac{1}{2}} - \frac{L_{j,k}^{n+\frac{1}{2}}}{3} \quad (5.31)$$

$$L_{j+1,k-1}^{n+\frac{1}{2}} = L_{j+1,k-1}^{n+\frac{1}{2}} - \frac{L_{j,k}^{n+\frac{1}{2}}}{3} \quad (5.32)$$
\[ L_{j-1,k+1}^{n+\frac{1}{2}} = L_{j-1,k+1}^{n+\frac{1}{2}} - \frac{L_{j-1,k}^{n+\frac{1}{2}}}{3} \]  

(5.33)

The same procedure is followed for calculations in the y-direction.

**Summary**

In this chapter the hydrodynamic model and material transport model were put in numerical form. An implicit technique was developed for the hydrodynamic model, and an explicit technique was developed for the material transport model. The next chapter discusses the results obtained with the numerical techniques presented here.
REFERENCES


TRANSPORT PHENOMENA IN A BAY-MARSH SYSTEM

VOLUME II

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Chemical Engineering

by

Hasitkumar Kantilal Trivedi
B.Chem.E., University of Bombay, 1969
M.E. in Chem.E., Lamar University, 1970
August, 1976
CHAPTER VI
RESULTS OF THE MATERIALS TRANSPORT MODEL
OF BARATARIA BAY

Introduction

Solutions of the material transport models of shallow estuarine bay systems were obtained for the Barataria Bay estuary. The purpose of this chapter is to present the results of these solutions and give comparisons with field data. The first part of this chapter will present a description of the simulation and specific data used for Barataria Bay. The second part will present the results of the cases that were done: typical conditions, increased marsh area, decreased marsh area, and increased nutrient (nitrogen) concentration. The third part consists of comparisons of results obtained with field measurements. The last part consists of a discussion of numerical considerations in the computer solution.

Simulation of Barataria Bay

To simulate the above described cases for the Barataria Bay estuary, the following types of information were needed: velocity distributions in the bay, air and water temperatures, dispersion coefficients, bathymetric data, and
boundary conditions at the Gulf and the north boundaries. The velocity distributions were obtained from the hydrodynamic model developed by Hacker (Ref. 6.1) for Barataria Bay. Solar radiation data were obtained from the LSU Sea Grant Program (Ref. 6.2). Air and water temperature data were obtained from Vora (Ref. 6.3). The bathymetric data were taken from Hacker's study (Ref. 6.1).

In Table 6.1, the conditions in the Gulf obtained from the LSU Sea Grant Program (Ref. 6.2) are given as the boundary conditions for the material transport model. In Table 6.2, the conditions at the northern boundary are given which were obtained from the LSU Sea Grant Project (Ref. 6.2). These were the best estimates of the concentrations that would be found at the northern and the southern boundaries of the Barataria Bay during the year 1969.

In Table 6.3 are given the initial conditions for the material transport model. These values were obtained from the LSU Sea Grant Project (Ref. 6.2). Except for live standing crop of *Spartina alterniflora* and dead standing crop of *Spartina alterniflora* for which accurate experimental data were available, all the values of the initial conditions were the best estimates of the concentrations that would be found in the bay during the year 1969.

It should be noted that only one value of the concentration for each species was used throughout the bay. In order to distribute the species over the bay and to allow for the system to reach that state where its response
**TABLE 6.1 Boundary Conditions at the Gulf**

<table>
<thead>
<tr>
<th>Month</th>
<th>Coarse Detritus gm/m³</th>
<th>Fine Detritus gm/m³</th>
<th>Dissolved Organic Nitrogen gm/m³</th>
<th>Dissolved Ammonia Nitrogen gm/m³</th>
<th>Dissolved Nitrite-Nitrate Nitrogen gm/m³</th>
<th>Animal Biomass gm/m³</th>
<th>Phytoplankton Biomass gm/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1.0</td>
<td>3.0</td>
<td>0.27</td>
<td>0.35</td>
<td>0.013</td>
<td>2.0</td>
<td>0.0009</td>
</tr>
<tr>
<td>February</td>
<td>1.0</td>
<td>3.0</td>
<td>0.29</td>
<td>0.45</td>
<td>0.012</td>
<td>2.0</td>
<td>0.00095</td>
</tr>
<tr>
<td>March</td>
<td>1.0</td>
<td>3.0</td>
<td>0.31</td>
<td>0.41</td>
<td>0.011</td>
<td>3.0</td>
<td>0.00095</td>
</tr>
<tr>
<td>April</td>
<td>2.0</td>
<td>4.0</td>
<td>0.38</td>
<td>0.35</td>
<td>0.011</td>
<td>4.0</td>
<td>0.00095</td>
</tr>
<tr>
<td>May</td>
<td>2.0</td>
<td>4.0</td>
<td>0.45</td>
<td>0.30</td>
<td>0.01</td>
<td>5.0</td>
<td>0.0011</td>
</tr>
<tr>
<td>June</td>
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<td>5.0</td>
<td>0.5</td>
<td>0.15</td>
<td>0.011</td>
<td>6.0</td>
<td>0.0011</td>
</tr>
<tr>
<td>July</td>
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<td>5.0</td>
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<td>0.08</td>
<td>0.011</td>
<td>5.0</td>
<td>0.0011</td>
</tr>
<tr>
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<td>0.09</td>
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<td>4.0</td>
<td>0.00095</td>
</tr>
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<td>0.15</td>
<td>0.012</td>
<td>4.0</td>
<td>0.00095</td>
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<td>0.00095</td>
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<td>1.0</td>
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<td>0.41</td>
<td>0.25</td>
<td>0.013</td>
<td>3.0</td>
<td>0.0009</td>
</tr>
<tr>
<td>December</td>
<td>1.0</td>
<td>3.0</td>
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<td>0.30</td>
<td>0.015</td>
<td>2.0</td>
<td>0.0009</td>
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</table>
TABLE 6.2 Boundary Conditions at the Northern Boundary

<table>
<thead>
<tr>
<th>Month</th>
<th>Species</th>
<th>Coarse Detritus gm/m³</th>
<th>Fine Detritus gm/m³</th>
<th>Dissolved Organic Nitrogen gm/m³</th>
<th>Dissolved Ammonia Nitrogen gm/m³</th>
<th>Dissolved Nitrite-Nitrate Nitrogen gm/m³</th>
<th>Animal Biomass gm/m³</th>
<th>Phytoplankton Biomass gm/m³</th>
</tr>
</thead>
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<tr>
<td>January</td>
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<td>0.9</td>
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<td>0.35</td>
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<td>0.0009</td>
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<td>0.9</td>
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<td>0.45</td>
<td>0.012</td>
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<td>0.41</td>
<td>0.011</td>
<td>1.95</td>
<td>0.00095</td>
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<tr>
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<td>0.38</td>
<td>0.35</td>
<td>0.011</td>
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<td>0.00095</td>
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<td>2.0</td>
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<tr>
<td>July</td>
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<td>1.0</td>
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<td>0.013</td>
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<td>0.9</td>
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<td>0.015</td>
<td>1.8</td>
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</table>
TABLE 6.3 Initial Conditions Used in the Materials Transport Models, January 1969

<table>
<thead>
<tr>
<th>Species</th>
<th>Value</th>
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<tbody>
<tr>
<td>Live standing crop of marsh grass</td>
<td>253.3 gm organic/m² of marsh</td>
</tr>
<tr>
<td>Dead standing crop of marsh grass</td>
<td>1011.0 gm organic/m² of marsh</td>
</tr>
<tr>
<td>Coarse detritus</td>
<td>9.36 gm organic/m³</td>
</tr>
<tr>
<td>Fine detritus</td>
<td>16.68 gm organic/m³</td>
</tr>
<tr>
<td>Dissolved organic nitrogen</td>
<td>1.18 gm/m³</td>
</tr>
<tr>
<td>Dissolved ammonia nitrogen</td>
<td>1.0 gm/m³</td>
</tr>
<tr>
<td>Dissolved nitrite-nitrate nitrogen</td>
<td>0.001 gm/m³</td>
</tr>
<tr>
<td>Animal biomass</td>
<td>8.655 gm organic/m³</td>
</tr>
<tr>
<td>Phytoplankton biomass</td>
<td>0.001 gm organic/m³</td>
</tr>
</tbody>
</table>
reflected only the effect of forcing functions, the computations were allowed to proceed for three months of real time before they were used to prepare the results presented in this chapter. All the forcing functions (solar radiation, temperature, boundary conditions) used in this work were cyclic and their period was one year. The computational scheme should, therefore, converge to the same value of the concentrations after one year of computations. This was found in this research. This was similar to quasi-steady solution obtained for velocities in Chapter IV.

The dispersion coefficients were obtained from the numerical experiments. The details are given in a later section.

Results of the Barataria Bay Simulation

Solutions of the materials transport model were obtained for a number of important cases which would have an ecological impact on the Barataria Bay estuary. These include: a simulation during a typical year, an increase in the marsh area which simulates the effect of planting Spartina alterniflora, a decrease in the marsh area which simulates the loss of marsh from dredging, an increase in the concentration of available nutrient (nitrogen) which simulates the addition of fertilizers.

Typical Conditions: The year 1969 was selected for the simulation because it is typical of the conditions that have been encountered in Barataria Bay for a number of years
in the past. Fresh water flows through the selected area show that this year has the average flow of fresh water, hence it does not fall into the category of a "wet year" or a "dry year."

In Figure 6.1, variation of the standing crop of marsh grass is given for a period of one year. It can be seen from the figure that the maximum production of marsh grass is predicted during June and July. This is the time when the intensity of solar radiation reaches its highest value. At this time, because of overcrowding, the effect of self-shading increases. The reduction in the standing crop of marsh grass concentration is also affected by continuous respiration loss and conversion of marsh grass to the dead standing crop. This, coupled with the reduction in the intensity of solar radiation, results in a decrease in the production of marsh grass. This trend continues until the end of January when the lowest standing crop of marsh grass is predicted. The net yearly production of marsh grass is predicted to be 2785.0 gm/m². This is comparable to the field results, as will be discussed later. Also, these results are the same for all places in the bay that have marsh grass growing. The reason is that the growth rate of marsh grass in the model is not affected by the availability of nitrogen or the transport of nitrogen in the bay.

In Figure 6.2, the predicted production of the dead standing crop of marsh grass is given. It can be seen from the figure that the highest production is predicted during
Figure 6.1. Live standing crop of *Spartina alterniflora*
Figure 6.2. Dead standing crop of **Spartina alterniflora**
February and the lowest production is predicted during September. The dead standing crop of marsh grass is formed from the live standing crop of marsh grass, and there is approximately a lag of eight months between the maximum and the minimum production of live standing crop and the dead standing crop. This large lag is due to the temperature dependent bacterial activity. The net yearly production of dead marsh grass is predicted to be about 2660.0 $gm/m^2$. This is comparable to field results, as will be discussed later. Also, these results are the same for all places in the bay that have dead standing crops of marsh grass.

In Figure 6.3, on the map of the region modeled, isopleths for the concentrations of coarse detritus are given for February and June. These months were selected because they represented typical winter and summer months. These are the months when significant biological changes are observed. The highest concentrations are predicted in the marsh area. Concentrations decrease, moving from the marsh to the water area. Comparing the isopleths for February with those of June, we can see that during June the concentrations at every point in the bay-marsh system have increased considerably. However, the overall trend of the isopleths is relatively unchanged. June is the month when the highest concentrations of the coarse detritus are predicted. This is reasonable since coarse detritus is formed by the microbial decomposition of dead standing crops of marsh grass. The microbial decomposition activity
Figure 6.3. Isopleths of concentrations for coarse detritus, typical case (concentrations in gm/m$^3$)
increases as the temperature increases and reaches the highest value during May and June.

In Figure 6.4, isopleths for the concentrations of fine detritus are given for February and June. Again, the highest concentrations are predicted for the marsh area and the concentrations decrease, moving from the marsh to the water area. Comparing the isopleths for February with those of June we can see that during June all the isopleths have moved toward the water area with a net increase in the concentrations throughout the bay. July is the month during which the maximum concentration of fine detritus is found. Fine detritus is produced by decomposition of coarse detritus, and there is a lag of about one month between the maximum of coarse detritus and the maximum of fine detritus.

In Figure 6.5, the isopleths for the concentrations of dissolved organic nitrogen are given for February and June. It can be seen that higher concentrations are observed in the marsh area. Concentrations decrease, moving from the marsh to the water area. February is the month during which the concentration of dissolved organic nitrogen reached its lowest value. Dissolved organic nitrogen is formed by the microbial decomposition of fine detritus and also by animal excretions. The maximum of animal biomass occurs in August, and that of fine detritus in July. There is a lag of about two months between the maximum of animal biomass and the maximum of organic nitrogen concentration. Thus, the maximum concentrations of organic nitrogen are predicted in
Figure 6.4. Isopleths of concentrations for fine detritus, typical case (concentrations in gm/m$^3$)
Figure 6.5. Isopleths of concentrations for dissolved organic nitrogen, typical case (concentrations in gm/m$^3$)
November. Comparing the isopleths for February with those of June, we can see that all the isopleths have moved a considerable distance toward the water area.

In Figure 6.6, the isopleths for the concentrations of dissolved ammonia nitrogen are given for February and June. Here also higher concentrations are predicted in the marsh area and lower concentrations are predicted in the water area. The lowest concentrations of ammonia nitrogen are predicted during June. At this time, the concentration isopleths have moved back toward the marsh area. Ammonia nitrogen is being incorporated into coarse and fine detritus as a result of microbial activity. In addition, it is also being consumed by phytoplankton. Summer is the period during which all these species reach their maximum values and contribute to the reduction of the concentration of dissolved ammonia nitrogen.

In Figure 6.7, the isopleths for the concentrations of nitrite and nitrate nitrogen are given for February and June. It was pointed out in Chapter III that in this bay-marsh system, nitrite and nitrate nitrogen are virtually absent. Due to the presence of reduced zones, nitrate is denitrified as soon as it is formed. However, the concentration of nitrite and nitrate nitrogen is high in the Gulf (Table 6.1). This nitrogen is transported into the bay-marsh system from the Gulf through the passes. Hence, as we go toward the passes the concentrations increase. Higher concentrations are also predicted at the northern boundary where there is
Figure 6.6. Isopleths of concentrations for dissolved ammonia nitrogen, typical case (concentrations in gm/m³)
Figure 6.7. Isopleths of concentrations for dissolved nitrite and nitrate nitrogen, typical case (concentrations in $10^2$ gm/m$^3$)
an inflow of fresh water to the system. Water entering at the northern boundary has higher concentrations of nitrite and nitrate nitrogen, and hence higher concentrations are predicted at these entrances. Due to the low boundary concentrations during June, the isopleths have moved back toward the Gulf as well as toward the northern boundary.

In Figure 6.8, the isopleths for the concentrations of animal biomass are given for February and June. Higher concentrations are predicted toward the marsh area. The concentrations decrease toward the water area. Maximum concentrations of animal biomass are predicted during August. This is reasonable. Animals ingest coarse as well as fine detritus. Coarse detritus concentrations are maximum during July. There is a lag of about two months between the maximum of coarse detritus and fine detritus and the maximum of animal biomass. Using the model results, the amount of animal biomass caught (fishery production) over a year is predicted to be about 144 lb/acre. These are comparable with Day's (Ref. 6.4) calculated value of 170 lb/acre.

In Figure 6.9, the isopleths for the concentrations of phytoplankton biomass are given for February and June. It can be seen that the concentrations are slightly higher toward the marsh area and they decrease as we move toward the water area. The maximum concentrations of phytoplankton biomass are predicted during June. Phytoplankton is a primary producer. Due to the highest intensity of solar
Figure 6.8. Isopleths of concentrations for animal biomass, typical case (concentrations in gm/m$^3$)
Figure 6.9. Isopleths of concentrations for phytoplankton, typical case (concentrations in $10^2$ gm/m$^3$)
radiation during this period, the phytoplankton concentrations are also high. After this period, the intensity of solar radiation decreases. In addition, due to the lowest concentrations of ammonia nitrogen, the nutrient moderation effect will contribute to the reduction in phytoplankton biomass.

In summary, this section gives the results of the Barataria Bay simulation for the typical conditions which were found during the year 1969. Figures are presented which show the distribution of various species in the bay-marsh system.

The ecosystem of the Barataria Bay consists of a relatively shallow area designated as the marsh area, which is periodically inundated and a relatively deep area, designated as the bay area. Live standing crops of marsh grass are rooted plants on the marsh surface and are not found in the bay.

Distributions of coarse detritus, fine detritus, animal biomass, organic nitrogen, ammonia nitrogen and phytoplankton biomass showed generally similar trends in which concentrations decreased going from marsh area to the bay area. Detritus, which is an important food source for animal biomass, is formed from the dead standing crops of marsh grass on the marsh surface. Due to the advective forces, the detrital material gets distributed in the bay, giving rise to the gradients of concentrations with the highest concentrations in the marsh area. Since the
concentrations of animal biomass are proportional to the amount of detrital material, they also follow the same trends. The detrital material and the animal biomass play an important role in the cycling of organic nitrogen and ammonia nitrogen. Hence, concentrations of these two species also follow the predicted trends of detritus and animal biomass. The phytoplankton biomass depends on the availability of ammonia nitrogen, hence it follows the same trend.

However, in the case of nitrite and nitrate nitrogen, the concentrations decrease from the Gulf passes to the marsh area. This is because the Mississippi River brings nitrite and nitrate nitrogen to the vicinity of the passes at the southern boundary of Barataria Bay. This nitrogen is transported into the bay-marsh system by the tidal movement. As mentioned in Chapter III, there is an absence of the oxidized zone in the system and the nitrite and nitrate nitrogen is rapidly denitrified and lost from the system.

Comparison of Results with Field Data

In order to examine the validity of the materials transport model for Barataria Bay, the results for the typical conditions were compared with experimental data. For the species for which there were field data, the comparisons show that the material transport model predicted the concentration variations within the accuracy of the data. For the other species, qualitative variations were known and
this analysis coincided with these variations. These comparisons are presented below for each species.

**Spartina alterniflora:** In the case of *Spartina alterniflora*, the available field data were used to obtain rate coefficients using the technique of least squares. It was shown in Table 3.3 that the overall correlation coefficient for the analysis was 0.7977. The value of 95% confidence limit for this case was 0.602. Thus, a reasonably good fit between the field data and the values given by the model equation was obtained. It was also pointed out in Chapter III that for biological systems, this correlation is considered very good. Since the live marsh grass is stationary, it supplies input to dead marsh grass which in turn supplies inputs to the species that are affected by tidal motion.

**Dead Standing Crop of Marsh Grass:** In the case of dead standing crop of marsh grass also, the available field data were used to evaluate rate coefficients. It was shown in Table 3.5 that the overall correlation coefficient for the analysis was 0.7496. The value of 95% confidence limit in this case was 0.553. Thus a reasonably good fit between the field data and the values given by the model equation was obtained. Since the dead marsh grass is also stationary, it supplies inputs to the species that are affected by tidal motion.
Coarse Detritus and Fine Detritus: Day (Ref. 6.4) has estimated the average yearly flow of detritus in the system. He has also given a qualitative picture of the seasonal variation. The average value used by Day was used to evaluate rate coefficients. The model shows similar seasonal trends as discussed by Day. It is believed that the highest concentrations should be obtained during summer and the lowest concentrations should be obtained during winter. Examining Figures 6.3 and 6.4, we can see that such behavior is predicted by the materials transport model.

The model results were also compared with the measured data recently published by Happ (Ref. 6.5). Happ studied the distribution of organic carbon in Barataria Bay. Organic carbon content is a direct measure of the amount of detritus present in an area. She has observed a gradient in the organic carbon concentration which decreased from the marsh to the coast. Such behavior is predicted by this present study.

Happ measured dissolved organic carbon and total organic carbon. She has defined dissolved organic carbon as the organic carbon associated with detritus which could pass through a 0.45 μ filter and total organic carbon as the organic carbon associated with a total of organic detritus and living organisms.

The comparison of the material transport model results and the data of Happ are given in Table 6.4. Referring to the table, the deviation varies from about -72% to 110%.

<table>
<thead>
<tr>
<th>Month</th>
<th>Dissolved Organic Carbon</th>
<th>Percent Deviation</th>
<th>Total Organic Carbon</th>
<th>Percent Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted</td>
<td>Observed</td>
<td></td>
<td>Predicted</td>
<td>Observed</td>
</tr>
<tr>
<td>January</td>
<td>5.1</td>
<td>11.2</td>
<td>-72.3</td>
<td>5.3</td>
</tr>
<tr>
<td>February</td>
<td>5.2</td>
<td>5.0</td>
<td>-37.5</td>
<td>6.2</td>
</tr>
<tr>
<td>March</td>
<td>3.3</td>
<td>3.8</td>
<td>-13.2</td>
<td>6.5</td>
</tr>
<tr>
<td>April</td>
<td>4.5</td>
<td>4.2</td>
<td>7.1</td>
<td>8.0</td>
</tr>
<tr>
<td>May</td>
<td>5.4</td>
<td>5.0</td>
<td>8.0</td>
<td>10.0</td>
</tr>
<tr>
<td>June</td>
<td>6.4</td>
<td>5.5</td>
<td>16.4</td>
<td>12.0</td>
</tr>
<tr>
<td>July</td>
<td>7.2</td>
<td>5.0</td>
<td>44.0</td>
<td>13.3</td>
</tr>
<tr>
<td>August</td>
<td>6.8</td>
<td>5.8</td>
<td>17.2</td>
<td>13.7</td>
</tr>
<tr>
<td>September</td>
<td>5.4</td>
<td>5.8</td>
<td>-6.9</td>
<td>11.5</td>
</tr>
<tr>
<td>October</td>
<td>4.5</td>
<td>5.6</td>
<td>-19.6</td>
<td>9.3</td>
</tr>
<tr>
<td>November</td>
<td>4.0</td>
<td>5.5</td>
<td>-27.3</td>
<td>7.7</td>
</tr>
<tr>
<td>December</td>
<td>3.6</td>
<td>10.2</td>
<td>-64.7</td>
<td>6.5</td>
</tr>
</tbody>
</table>
The differences in the predicted and experimental values are due to the following reasons. The experimental data were taken from the time period of November 1972 to November 1973, while the model simulation was performed for the conditions of the year 1969. The experimental results for January and December appear to be in error since the concentration of total organic carbon is equal to or less than the dissolved organic carbon for these two months. According to Happ's above given definition of total organic carbon, the concentration of total organic carbon must be higher than dissolved organic carbon.

It can be seen from Table 6.4 that for dissolved organic carbon, the percent deviation for seven out of twelve months (58.3%) is less than 25 percent. The agreement is not as good in the case of total organic carbon. The percent deviation is less than 40 percent for six out of twelve months (50%) here. The results obtained in this research are independent of the data collected by Happ.

**Dissolved Organic Nitrogen:** In the case of dissolved organic nitrogen, the model results were compared with the measurements made by Ho (Ref. 6.6) in Airplane Lake of Barataria Bay. This lake is in the southwestern region of the bay. The data were reported for the months given in Table 6.5 and are compared with the predicted values in this table. It can be seen that the maximum percent deviation is 21 percent. The differences in the predicted and the experimental
TABLE 6.5 Comparison of Predicted and Observed Values of Dissolved Organic Nitrogen at Airplane Lake in Barataria Bay. Model data are for the year 1969 and experimental data are for the year 1973.

<table>
<thead>
<tr>
<th>Month</th>
<th>Predicted Value gm/m³</th>
<th>Observed Value gm/m³</th>
<th>Percent Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>0.7</td>
<td>0.85</td>
<td>21.0</td>
</tr>
<tr>
<td>February</td>
<td>0.48</td>
<td>0.51</td>
<td>6.25</td>
</tr>
<tr>
<td>March</td>
<td>0.41</td>
<td>0.46</td>
<td>12.50</td>
</tr>
<tr>
<td>July</td>
<td>0.79</td>
<td>0.94</td>
<td>18.75</td>
</tr>
</tbody>
</table>
values occur for the following reasons. The experimental data were taken for the year 1973, while the model simulation was performed for the year 1969. In addition, the model is independent of the experimental data in the sense that they were not used to estimate the rate coefficients. Considering these, it can be concluded that the results obtained in this research are in good agreement with the experimental data.

**Dissolved Ammonia Nitrogen:** No experimental data were available for dissolved ammonia nitrogen in Barataria Bay. However, qualitative behavior of the variation of the concentrations of ammonia nitrogen is thought to be understood (Ref. 6.2). According to this, ammonium concentration should be the highest during the spring and lowest during summer. Such behavior is predicted by the material transport model for the dissolved ammonia nitrogen, and this was shown in Figure 6.6.

**Dissolved Nitrite and Nitrate Nitrogen:** No field data were available for the seasonal variations of nitrite and nitrate nitrogen in the Barataria Bay region. However, as in the case of ammonia nitrogen, here also qualitative behavior is thought to be understood (Ref. 6.2). As was mentioned in Chapter III, because of the presence of a reduced zone, nitrite and nitrate nitrogen exist in very low concentrations in the Barataria Bay region. Such reduced zones do not exist in the Gulf, which is the southern boundary of the area.
Hence, concentrations of nitrite and nitrate nitrogen are transported into the bay region from the passes. The concentration decreases from the passes to the marsh. This behavior was predicted by the material transport model and was illustrated in Figure 6.7.

**Animal Biomass:** No field data were available for the seasonal variations of the total animal biomass in Barataria Bay. However, Day (Ref. 6.4) has estimated total yearly averaged animal biomass. He has also described the qualitative behavior of the biomass for the seasonal variation over the year. Using the value of the yearly averaged biomass given by Day (Ref. 6.4), the rate coefficients were evaluated.

The model predicted the expected qualitative behavior. According to the analysis, the highest concentrations are predicted during summer and the lowest concentrations are predicted during spring, and this was shown in Figure 6.8.

**Phytoplankton Biomass:** No field data were available for the seasonal variations of phytoplankton biomass in Barataria Bay. Stowe et al. (Ref. 6.7) have estimated the yearly averaged production of phytoplankton in Airplane Lake. This value was used to estimate the rate coefficients in the phytoplankton model. They have also given the qualitative behavior of the seasonal variations of phytoplankton biomass and this analysis predicted this behavior. According to the analysis, the highest concentrations are predicted during
June and the lowest concentrations are predicted during late fall as shown in Figure 6.9.

**Transport of Detritus and Nutrients from Barataria Bay**

In Chapter III, the biological energy flow diagram and the biological mass flow diagram were developed. While developing these diagrams, it was pointed out that due to tidal currents, coarse detritus, fine detritus, dissolved organic nitrogen, and ammonia nitrogen are exported from the bay-marsh system to the Gulf and nitrite-nitrate nitrogen is imported to the system from the Gulf. Using the model results, the amounts of these exchanges were calculated. The results are summarized in Table 6.6.

Examining Table 6.6, we can see that quite a large amount of detritus ($6.30 \times 10^5$ tons/yr), dissolved organic nitrogen ($1.91 \times 10^2$ tons/yr) and dissolved ammonia nitrogen ($1.33 \times 10^2$ tons/yr) are lost from the system. This has a significant impact on the near-shore ecosystem. It was calculated that the total of coarse detritus and fine detritus exported from the system was about 48 percent of the total organic matter annually produced in the system. This value agrees well with Day's (Ref. 6.4) calculated value of 42.4 percent. This exported material supports the near-shore ecosystem.

The commercially important species in the near-shore region are menhaden, shrimp, and other fish. The biological energy flow diagram of Vora (Ref. 6.3) includes the
TABLE 6.6 Net Export of Various Species from Barataria Bay to the Gulf Through All Passes

<table>
<thead>
<tr>
<th>Species</th>
<th>Flow Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>gm/year</td>
</tr>
<tr>
<td>Coarse detritus</td>
<td>$1.86 \times 10^{11}$</td>
</tr>
<tr>
<td>Fine detritus</td>
<td>$3.85 \times 10^{11}$</td>
</tr>
<tr>
<td>Dissolved organic nitrogen</td>
<td>$1.73 \times 10^{8}$</td>
</tr>
<tr>
<td>Dissolved ammonia nitrogen</td>
<td>$1.21 \times 10^{8}$</td>
</tr>
<tr>
<td>Dissolved nitrite-nitrate</td>
<td>$-8.65 \times 10^6^*$</td>
</tr>
</tbody>
</table>

*Import
near-shore system. He has calculated that the annual catch of this species in the near-shore region is about $1.54 \times 10^{11}$ grams or $1.7 \times 10^5$ tons/yr which is comparable to the numbers given in Table 6.6. The dockside value of this catch is about $11.9$ million annually.

It can also be seen from Table 6.6 that about $9.52$ tons/yr of nitrite-nitrate nitrogen are transported into the bay-marsh system from the Gulf. The data in Table 6.1 show that the concentration of this species is higher in the Gulf than in the bay-marsh system. The net transport is therefore into the system. However, as explained in Chapter III, this species is denitrified and lost from the system.

**Variation in Marsh Area**

Two cases were simulated to study the effect of variation in the marsh area. In the first case, the marsh area was increased to simulate the effect of planting of marsh grass. In the second case, the marsh area was decreased to simulate the effect of dredging and other activities that can permanently change marsh grass area to water area. These additions and deletions in the marsh grass area were logically distributed such that the changes would be reasonable. In these cases, it was assumed that the productivity of new marsh area was the same as that of the existing marsh area. For both cases the marsh grass area was changed by five percent from the base case. This change represented a
small but distinguishable perturbation on the system which is potentially realizable also.

Based on the above, increasing the marsh grass area would not have any effect on the concentration of the marsh grass and its dead standing crop. However, the total production would increase in proportion to increased marsh area. This in turn increases the production of the rest of the species.

In Figures 6.10 and 6.11, the isopleths of the concentrations for coarse detritus and fine detritus are given for the month of June comparing the base case with the case for increased marsh. We can see from these figures that the isopleths in the case of increased marsh area follow the same general trend as observed in the base case. However, increase in concentration is observed throughout the bay-marsh system.

The isopleths for the rest of the species are essentially the same as the typical case and are given in Appendix C. For comparison the isopleths of the concentrations for a typical case are also given in these figures. Comparing these, we can see the increase in the concentrations of each species. This increase ranged from one percent to eight percent. Total amount of detritus exported from the bay-marsh system increased by 3.9 percent.

Total yearly production of animals also increased. This in turn increased the total fisheries catch. The total increase in revenue due to this increase in catch was
Figure 6.10. Isopleths of concentrations for coarse detritus, higher marsh area, June (concentrations in gm/m$^3$)
Figure 6.11. Isopleths of concentrations for fine detritus, higher marsh area, June (concentrations in gm/m$^3$)
calculated using the values given by Vora (Ref. 6.3). The increase in annual revenue was found to be about two dollars per acre. Thus, planting of marsh grass would be economically feasible only if the cost of planting marsh grass can be covered by the annual increase of two dollars per acre if the justification is based on increased fisheries production.

The isopleths of the concentrations for all the species for the month of February are given in Appendix C. They showed similar trends.

In the second case, the available marsh grass area was decreased by five percent. This simulated the effects of dredging and other activities that can result in a permanent change in marsh grass area to water area. As mentioned earlier, there were no changes in the concentrations of marsh grass and its dead standing crop. However, in proportion to decreased marsh grass area, the total production would decrease. This in turn decreased the production of the rest of the species.

In Figures 6.12 and 6.13, the isopleths of the concentrations for coarse detritus and fine detritus are given for the month of June. The isopleths for the rest of the species are essentially the same as the typical case and are given in Appendix C. Comparing these, we can see the decrease in the concentrations of each species. This decrease ranges from one percent to nine percent as compared to the typical case.
Figure 6.12. Isopleths of concentrations for coarse detritus, lower marsh area, June (concentrations in gm/m$^3$)
Figure 6.13. Isopleths of concentrations for fine detritus; lower marsh area, June (concentrations in gm/m³)
Total amount of detritus exported from the bay-marsh system decreased by 3.6 percent from the typical case. Total yearly production of animals also decreased. This in turn decreased their total catch. Total decrease in the annual revenue due to this was calculated to be about $1.80 per acre. Thus the loss in annual income from the fisheries sector would have to be balanced with the gain from the activity which caused the loss of marsh grass area.

The isopleths of the concentrations for all the species for the month of February are given in Appendix C. They showed similar trends.

The results obtained above appear to be in contradiction to the results obtained by Vora (Ref. 6.3). Vora has studied the optimization of productivity in Barataria Bay. His sensitivity analysis showed that as the marsh area increased by 25 percent, the landings of commercially important species (blue crab, oyster, white shrimp, brown shrimp, and menhaden) decreased between 5.2 and 15 percent. As the marsh area decreased by 25 percent, the landings increased in the same range. In Vora's study as well as in this study, the biological energy flow in the system was the key to the analysis. In this research, it was assumed that the feed to the animal biomass was proportional to the concentration of total available organic matter in the system. In Vora's detailed biological energy flow diagram, the major portion of the food (89 percent) of commercially important species consisted of organic matter either produced in the water column
or obtained from the sediment. In addition to this, the biological energy flow diagram used in this present research applies only to Barataria Bay while Vora's biological energy flow diagram applies to both the bay and the near-shore region. Vora has therefore included menhaden, a species of great importance commercially (he has shown that the landings of these species are 89 percent of the total landings in terms of weight and 41 percent of the total landings in terms of dollar value) and harvested in the near-shore region, in his biological flow diagram. Considering such differences in the formulation of the biological energy flow diagrams of the two studies, deviations in the results are relatively small (about 15 percent).

**Addition of Nutrients**

In order to simulate the effect of addition of fertilizers on the marsh area, the rate constant which represents the effect of crowding and self-shading in the rate equation for marsh grass dynamic ($k_2$) was changed. In Appendix D, it is shown that this rate coefficient is inversely proportional to the nutrient concentration. In this simulation, the value of the rate constant was decreased by five percent. This change represented a small but distinguishable perturbation on the system which is potentially realizable also.

In Figure 6.14, the variation of marsh grass is given for the period of one year which was obtained using the
Figure 6.14. Comparison of live standing crop of *Spartina alterniflora* for high nutrients case and typical case.
modified rate constants. Comparing this case with that for the typical case of the standing crop of marsh grass, we can see on Figure 6.14 that similar concentration profiles are obtained. In both cases, the highest concentrations are predicted during June and July and the lowest concentrations are predicted during February. However, the highest concentration in this case has about 9.5 percent higher value and the lowest concentration has about 14 percent higher value than that in the typical case.

In Figure 6.15, the predicted production of the dead standing crop of marsh grass is given using the modified rate constant and the typical case. Again, similar concentration profiles are obtained. In both cases, the highest concentrations are predicted during February and the lowest concentrations are predicted during September. However, the highest concentration in this case has about eight percent higher value and the lowest concentration has about 13 percent higher value than that in the typical case.

Thus as shown above, the total production of marsh grass as well as that of the dead standing crop of marsh grass increased. In this respect, this case resembles the one previously discussed for increased marsh grass area. Here also, increase in the concentrations of the rest of the species was predicted. In Figures 6.16 and 6.17 the isopleths of the concentrations for coarse detritus and fine detritus are compared with the typical case concentrations for the month of June.
Figure 6.15. Comparison of dead standing crop of Spartina alterniflora for high nutrients case and typical case
Figure 6.16. Isopleths of concentrations for coarse detritus, higher nutrients case, June (concentrations in gm/m$^3$)
Figure 6.17. Isopleths of concentrations for fine detritus, higher nutrients case, June (concentrations in gm/m$^3$)
Comparing these figures with those of the typical case for the same month, we can see the increase in the concentrations of each species. This increase ranged from one percent to twelve percent. Increase in yearly production of animals increased the yearly amount of catch and hence the annual revenue. Using the values given by Vora (Ref. 6.3) the increase in annual revenue derived by addition of nutrients was about $2.85 per acre. Thus, the addition of nutrients will be economically feasible only when the cost per acre is less than $2.85 if the justification is based on increased fisheries production above.

The isopleths for the rest of the species are essentially the same as the typical case and are given in Appendix C. The isopleths of the concentrations for all the species for the month of February are given in Appendix C. They showed similar trends.

### Numerical Considerations in the Computer Solution

This part of the chapter is divided into two sections. In the first section, numerical experiments related to finite-difference techniques are discussed. In the second section, numerical experiments related to the selection of dispersion coefficients are discussed.

**Numerical Considerations for the Finite-Difference Techniques:** The partial differential equations that describe the transport of materials in the bay are of such complexity
that a numerical solution was required. Two finite-difference techniques, Alternating Direction Implicit (ADI) and Explicit methods, were used for the solutions. The materials transport model was solved using the explicit technique and the hydrodynamic model was solved using the ADI technique. Of these two methods, only the explicit technique requires that stability criteria be met. These stability criteria were presented by TRACOR (Ref. 5.3) and were shown in Chapter V as Eqs. (5.27), (5.28), and (5.29). These criteria were met for the results shown for Barataria Bay.

To perform the numerical solution, a finite difference grid was placed on the area of interest in a fashion that insured that all tidal passes were lined up with the bottom row of the grid system. The boundaries for the study were chosen to be the same as those of Hacker (Ref. 6.1), who developed the hydrodynamic model. Grid size was chosen with as many points as possible to fit on the computer available for the solution, an IBM 360/65. To aid in establishing the accuracy of the numerical solution, two finite difference networks were used. One had a 1300 yard square grid, shown in Figure 6.18; and the other had an 1800 yard square grid, shown in Figure 6.19. In Figure 6.20 a comparison is presented for dissolved organic nitrogen at one grid point for the two grid sizes. The maximum difference of seven percent was found between the two grid sizes. Comparable results were obtained for the other species and other grid points.
Figure 6.18. Finite difference grid (1300 yards square) on Barataria Bay (depth shown in feet)
Figure 6.19. Finite difference grid (1800 yards square) on Barataria Bay (depth shown in feet)
Figure 6.20. Comparison of dissolved organic nitrogen concentrations between a 1300 yards grid and an 1800 yards grid, grid location (10,3)
The results are close but not equal and this can be explained by the fact that when using these two different grid sizes (1300 yards and 1800 yards), in actuality two different systems were being modeled. This is because the bathymetry is not exactly the same in the two systems. The only way to reproduce the bathymetry exactly was to make the grid one half of the grid size used in the numerical solutions. This was impossible to do due to computer storage limitations. In using the 1300 yard grid, about 50 percent of the computer's high speed storage was used. If the grid size were to be halved, the computer storage requirements would have to be quadrupled, and this was impossible on the system available. Also, doubling the size of the grid (from 1300 yards to 2600 yards) will give inaccurate results as all the passes will be represented by just one half grid. Consequently it was necessary to compromise and the grid size of 1800 yards was used to establish the convergence and the accuracy. The section on "Typical Conditions" employed the 1300 yard grid.

An attempt was made to apply the ADI technique to the time-averaged equations of the hydrodynamic model (Eqs. 4.75, 4.83, 4.84) for use with the solution of the material transport model. Failing to obtain meaningful results from this analysis, another method was used to obtain the velocity distribution required for the solution of the material transport model. A quasi-steady state solution of the hydrodynamic model was obtained using the method discussed in
Chapter IV. This was used for the velocities in the materials transport model and the explicit technique was used to solve the materials transport model. This is a satisfactory method since the fluid dynamics affects the biology but the biology does not affect the fluid dynamics.

From his numerical experiments, Hacker (Ref. 6.1) has found that the implicit solution of the hydrodynamic model and the explicit solution of the species continuity equation worked best for Barataria Bay. As explained in Chapter V, the explicit scheme is relatively simple and straightforward.

From the numerical experiments, it was decided to use $\Delta t$ of three hours and Eqs. (5.27), (5.28), and (5.29) gave the stability criteria for the explicit technique used. Eqs. (5.28) and (5.29), which restrict the size of the time step, are given below.

\[
\Delta t \leq \min \left[ \frac{\Delta x}{2|u_{\text{max}}|}, \frac{\Delta y}{2|v_{\text{max}}|} \right] \tag{5.28}
\]

\[
\Delta t \leq \min \left[ \frac{(\Delta x)^2}{4 b_{\text{max}}}, \frac{(\Delta y)^2}{4 b_{\text{max}}} \right] \tag{5.29}
\]

It can be seen from the above equations that in order to select the size of the time step $\Delta t$, the magnitude of velocities and dispersion coefficients were required. However, as explained later, a priori values of the dispersion coefficients were not available. Hence it was decided to
fix the $\Delta t$ using the restriction of Eq. (5.28). The maximum $\Delta t$ that can be used for the bay-marsh system was found to be 5.42 hours. However, none of the above criteria are really adequate for practical purposes. The typical and prudent practice is to use some fraction of this analytically-indicated maximum time step (Ref. 6.8). In this study 60 percent of the maximum allowable time step was selected.

In applying the ADI technique to the time-averaged equations of the hydrodynamic model, the size of the time step ($\Delta t$) was slowly increased from the value of two minutes. Increasing $\Delta t$ the accuracy of the solution started deteriorating. At $\Delta t$ of 12 minutes absolutely unacceptable results were obtained. However, the numerical solution remained stable. Hess and White (Ref. 6.9) have recently published the results of their study on the hydrodynamic model using the ADI technique applied to Narragansett Bay. According to their study, highly inaccurate results are obtained if $\Delta t$ used is greater than the following condition:

$$\Delta t \leq \frac{5(\Delta x)}{\sqrt{gh}}$$

(6.1)

Applying the above criterion in this case, the maximum size of the time step is about ten minutes.

It was not feasible to solve the time-averaged hydrodynamic model using a time step of two minutes in this study.
The study was aimed at developing a predictive tool which would characterize the behavior of different parameters over the period of a year. Use of a time step of two minutes would require an astronomical amount of computation time. Consequently it was necessary to proceed as previously described.

**Numerical Considerations in the Selection of Dispersion Coefficients:** The dispersion terms in the material transport equation (Eq. 4.96) are not truly diffusive. They arise out of a reformulation of an averaged equation. The dispersion terms represent three different effects. One is due to turbulent fluctuations, one is due to vertical gradient in concentration, and one is due to tidal oscillations in mean velocities and concentrations. The magnitude of the dispersion not only varies from estuary to estuary, but may vary from point to point and from time to time in the same estuary depending upon the behavior of the various contributors to the dispersion terms. At present, there does not exist a general technique for the _a priori_ calculation of dispersion coefficients as defined by Eqs. (4.97) and (4.98), and the values must be selected based on experimental data.

Unfortunately in the case of Barataria Bay, sufficient data were not available to evaluate dispersion coefficients. Also, the values for the species studied in this research were not reported in the literature. Due to these reasons,
it was decided to select the dispersion coefficients which gave stable and acceptable solution to the material transport model. In Table 6.7, the values used in this study are reported. Using these values of dispersion coefficients and Eq. (5.29), the maximum $\Delta t$ that can be used for the bay-marsh system was found to be 5.0 hours. Thus the selected value of the size of the time step (3 hours) is about 60 percent of the maximum allowable $\Delta t$ in this case. Using these selected values of the dispersion coefficients and the size of the time step, the accuracy and the convergence of the numerical scheme were checked using half the size of the time step, i.e., $\Delta t$ of 90 minutes. In Figure 6.21, this comparison is presented for dissolved organic nitrogen. The maximum difference of three percent was found between the two solutions. Comparable results were obtained for the other species.

We can see from Table 6.7 that in order to describe seven different species only two values of the dispersion coefficient were necessary. One value is necessary in order to account for species which are not dissolved in water and hence are not truly diffusive. Four different species for which such dispersion coefficient was selected are coarse and fine detritus, animal biomass, and phytoplankton biomass. The second value of the dispersion coefficient was selected for the species which are dissolved in water and are truly diffusive. Three species for which such dispersion coefficient was selected are dissolved organic
TABLE 6.7 Dispersion Coefficients

<table>
<thead>
<tr>
<th>Species</th>
<th>Values in ft²/sec</th>
<th>Molecular Diffusivity in ft²/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse detritus</td>
<td>62.0</td>
<td>-</td>
</tr>
<tr>
<td>Fine detritus</td>
<td>62.0</td>
<td>-</td>
</tr>
<tr>
<td>Dissolved organic nitrogen</td>
<td>212.0</td>
<td>2.8 x 10⁻⁸</td>
</tr>
<tr>
<td>Dissolved ammonia nitrogen</td>
<td>212.0</td>
<td>2.8 x 10⁻⁸</td>
</tr>
<tr>
<td>Dissolved nitrite-nitrate nitrogen</td>
<td>212.0</td>
<td>2.8 x 10⁻⁸</td>
</tr>
<tr>
<td>Animal biomass</td>
<td>62.0</td>
<td>-</td>
</tr>
<tr>
<td>Phytoplankton biomass</td>
<td>62.0</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 6.21. Comparison of concentrations of dissolved organic nitrogen between a time step of 1.5 hours and 3.0 hours.
nitrogen, dissolved ammonia nitrogen, and dissolved nitrite and nitrate nitrogen. It should be noted here that the dispersion coefficient for marsh grass and the dead standing crop of marsh grass are zero. This is so because both the species are plants rooted in the marsh surface and hence do not move.

To show the sensitivity of the solution to differing values of the dispersion coefficient, Figure 6.22 is presented for coarse detritus using the typical case conditions for February. For the sensitivity analysis, the dispersion coefficient was changed by ±10% and the results were compared with that of the typical case. We can see that when the dispersion coefficient increased by ten percent, the isopleth of 13 gms/m³ concentration has moved slightly backward, predicting lower concentrations at the original 13 gms/m³ isopleth. The isopleth for 9 gms/m³ concentration has moved very slightly toward the bay. Thus concentrations have increased as we go toward the bay. Higher concentrations are also predicted in the bay. Thus when the dispersion coefficient increased, more spreading of coarse detritus is predicted. When the dispersion coefficient is decreased by ten percent, the isopleths on the marsh surface have moved a bit toward the bay. Thus on the marsh surface coarse detritus tends to accumulate. As we go toward the bay area the isopleths for all the concentrations have moved backward, predicting lower concentrations in the bay area.
Figure 6.22. Isopleths of concentrations for coarse detritus, typical case with different dispersion coefficients (concentrations in gm/m$^3$)
Summary

In this chapter, solutions of the material transport model are presented for the bay-marsh system of Barataria Bay. The results consist of four different cases that were run: typical conditions, increased marsh area, decreased marsh area, and increased nutrient concentration. The results are compared with the experimental data. The comparisons show that the model predicted the species distributions within the accuracy of the data. The amounts of organic matters and nutrients that are transported to the Gulf are calculated. The importance of this transport to the near-shore ecosystem is explained. The results for the variations of marsh and water area show that the increase in marsh area increases the organic matter and hence the animal biomass in the system and vice versa. Addition of nutrients also increases the animal biomass. Numerical considerations in the computer solution are discussed. Specifically, the numerical considerations for the finite-difference techniques and the selection of dispersion coefficients are given. In the next chapter the conclusions and recommendations of this study are given.
REFERENCES


6.6 Ho, Clara, Personal Communication (1972).


Conclusions

Based on the results of this research the following conclusions are drawn:

1. The biogeochemical cycling of nitrogen as it seems to occur in the bay-marsh system of Barataria Bay was quantitatively described. The cycling of nitrogen was quantitatively represented by the series of rate equations. All the rate constants that appear in the rate equations were evaluated or estimated from the experimental data available for the bay-marsh system.

2. The materials transport model predicted the time-varying concentrations of three different species of nitrogen such as dissolved organic nitrogen, dissolved ammonia nitrogen, dissolved nitrite and nitrate nitrogen and that of Spartina alterniflora, dead standing crop of Spartina alterniflora, detritus, animal biomass, and phytoplankton biomass in the bay-marsh system. Verification of the analysis was made by comparing daily-average concentrations with available experimental data for the bay-marsh system. It was found that the predicted results compared with the
experimental data within the accuracy of the available data when experimental data were available, and when they were not available the qualitative behavior was consistent with the way biologists understand the system to function.

3. Transport of detritus and nutrients to the Gulf was calculated. It was pointed out that this has a significant impact on the near-shore ecosystem since it supports the near-shore ecosystem. The commercially important species in the near-shore region are menhaden, shrimp, and other fish. The annual catch of these species is about $11.9 million.

4. Analysis of the effects of planting *Spartina alterniflora* was studied by increasing the marsh area by five percent. Results were obtained that showed the increase in the total productivity of the bay-marsh system. This in turn increased the total fisheries catch. The total increase in revenue due to this increase in catch was calculated to be about two dollars per acre. It was, therefore, concluded that the planting of marsh grass would be economically feasible only if the cost of planting marsh grass can be covered by the annual increase of two dollars per acre if the justification is based on increased fisheries production.

5. Analysis of the effects of dredging and other activities that can permanently change marsh grass area to the water area was studied by decreasing the marsh area by five
percent. Results were obtained that showed the decrease in the total productivity of the bay-marsh system. This in turn decreased the total fisheries catch. Total decrease in the annual revenue due to this was calculated to be about $1.80 per acre. Hence it was concluded that the disadvantage of the loss in annual income from the fisheries sector would have to be balanced with the gain from the activity which caused the loss of marsh grass area.

6. Analysis of the effects of the addition of fertilizers in the marsh area was studied by decreasing the rate constant representing the effect of crowding and self-shading by five percent. Results were obtained that showed the increase in the total productivity of the bay-marsh system. The increase in annual revenue derived by addition of nutrients was about $2.85 per acre. Hence it was concluded that the addition of nutrients will be economically feasible only when the cost per acre is less than $2.85 if the justification is based on increased fisheries production.

7. Starting with the worst possible initial conditions, the hydrodynamic model reached a quasi-steady state in five tidal cycles. Twelve such quasi-steady state solutions were obtained which represented each month of the year. The results were interpolated to obtain daily-average velocities.

8. The materials transport model was time averaged to obtain an equation that can be solved using a larger time step. New terms appeared in the time averaged form of the
materials transport model, which were approximated by a diffusion type of term.

9. The computer programs of the models are in a form that can be readily used by engineers and scientists for studies of ecological design, e.g., planting of marsh grass, dredging in the bay-marsh system, addition of fertilizers, etc. Users' manuals are included with the program for ease in making their application.

Recommendations

Based upon the above mentioned conclusions, the following recommendations are made:

1. Research should continue in order to update the values of rate coefficients. This should be done by taking more experimental data for all the species. The experimental stations should be set up at various locations in the bay-marsh system. The data should be collected throughout the year.

2. Research should continue in the area of finite difference numerical solutions of the materials transport model. Research should continue to modify ADI technique so that it can be used to solve the materials transport model.
NOMENCLATURE

A = intercompartmental energy flow

\[
A_3 = 100.0 \left( \frac{\text{gram organic matter in coarse detritus}}{\text{gram nitrogen in coarse detritus}} \right)
\]

\[
A_4 = 58.8 \left( \frac{\text{gram organic matter in fine detritus}}{\text{gram nitrogen in fine detritus}} \right)
\]

\[
A_8 = 33.3 \left( \frac{\text{gram organic matter in animal biomass}}{\text{gram nitrogen in animal biomass}} \right)
\]

\[
A_9 = 25.0 \left( \frac{\text{gram organic matter in phytoplankton biomass}}{\text{gram nitrogen in phytoplankton biomass}} \right)
\]

\[
A_{\text{ph}} = \text{coefficient of photosynthesis for phytoplankton biomass (l/year)}
\]

C = Chezy coefficient (L^{1/2}/t)

CC = Fishing, Catch

\[
C_p = \text{heat capacity of water (L}^2/t^2 T)\]

\[
C_i = \text{concentration of species } i, \text{ gram/m}^3\]

\[
B_{AB} = \text{binary diffusivity of species } A \text{ through species } B (L^2/t)\]

\[
B_{\text{AB}}^{\text{laminar}} = \text{binary diffusivity of species } A \text{ through species } B \text{ in laminar flow (L}^2/t)\]

\[
B_{\text{AB}}^{\text{eddy}} = \text{binary diffusivity of species } A \text{ through species } B \text{ in turbulent flow (L}^2/t)\]

\[
B_{AB} = \text{space average dispersion coefficient of species } A \text{ through species } B (L^2/t)\]

\[
B_{AB}^{**} = \text{time and space average dispersion coefficient of species } A \text{ through species } B (L^2/t)\]

\[
B_{AB}^{**,\text{min}} = \text{minimum value of time and space average dispersion coefficient of species } A \text{ through species } B (L^2/t)\]

\[
B_{AB}^{**,\text{max}} = \text{maximum value of time and space average dispersion coefficient of species } A \text{ through species } B (L^2/t)\]
\[ D = \text{depth of water, L} \]
\[ \frac{D}{Dt} = \text{substantial derivative} \]
\[ E_v = \text{rate of evaporation, L/t} \]
\[ E = \text{export to Gulf} \]
\[ F = \text{Coriolis force parameter, dimensionless} \]
\[ F_n = \text{function of} \]
\[ g = \text{gravitational acceleration (L/t}^2) \]
\[ \Delta G^0 = \text{change in internal energy (calories)} \]
\[ h = \text{distance between reference plane and bottom, L} \]
\[ i \text{ or } j = \text{counter for different species (i,j = 1 to 9)} \]
\[ J_A, J'_A = \text{mass flux rate of A (M/t L}^2) \]
\[ K_x = \text{space average thermal dispersion coefficient in x-direction (L}^2/t) \]
\[ K_y = \text{space average thermal dispersion coefficient in y-direction (L}^2/t) \]
\[ K, K_1, K^* = \text{wind friction coefficient, dimensionless} \]
\[ k_1 = 0.045, \text{coefficient of photosynthesis for marsh grass (day/year-langley)} \]
\[ k_2 = 0.0112, \text{coefficient of self-shading and nutrient availability (1/year-gram organic/m}^2 \text{ of marsh area)} \]
\[ k_{iR} = \text{coefficient of respiration for species i (1/year)} \]
\[ k_{ij} = \text{coefficient of loss of species i to species j (1/year)} \]
\[ k_{dn} = 324.0, \text{coefficient of denitrification (1/year)} \]
\[ k_f = 0.164, \text{coefficient of fishing (1/year)} \]
\[ L = \text{water level above mean sea level, L} \]
\[ n = \text{roughness factor, dimensionless} \]
\[ P_A = \text{space average sink and source term for species A which includes convection and diffusion through the surface and bottom of the system (M/t L}^2) \]
\[ P_E = \text{sink and source term for energy (M/t L}^2) \]
\[ p, p_a = \text{air pressure} \ (M/L^2) \]
\[ r_A = \text{sink and source term for species A} \ (M/t \ L^2) \]
\[ \bar{r}_i = \text{space average sink and source term for species i} \ (M/t \ L^2) \]
\[ R = \text{rainfall rate, } L/t \]
\[ RR = \text{respiration} \]
\[ S_i = \text{space average concentration of species i} \ (M/L^3) \]
\[ \bar{S}_i = \text{space and time average concentration of species i} \ (M/L^3) \]
\[ SS = \text{solar radiation} \]
\[ t = \text{time} \ (t) \]
\[ \Delta t = \text{incremental change in time} \ (t) \]
\[ T = \text{temperature} \ (T) \]
\[ T'' = \text{time for a tidal cycle} \ (t) \]
\[ t_0, t_f = \text{integration limits from initial time } t_0 \text{ to final time } t_f \ (t) \]
\[ Q_x = \text{magnitude of average discharge rate in x direction} \ (L^3/tL) \]
\[ Q_y = \text{magnitude of average discharge rate in y direction} \ (L^3/tL) \]
\[ u, v, w = \text{mass average velocities in x, y, and z directions} \ (L/t) \]
\[ U, V = \text{space average velocities in x and y directions} \ (L/t) \]
\[ U_{\text{max}}, V_{\text{max}} = \text{maximum value of space average velocities in x and y directions} \ (L/t) \]
\[ W = \text{wind velocity} \ (L/t) \]
\[ x, y, z = \text{coordinate axis, } L \]
\[ \Delta x, \Delta y = \text{incremental changes in x and y, } L \]
\[ X, Y = \text{wind friction force in x and y directions} \ (M/L^2) \]
\[ z_b, z_s = \text{integration limits from bottom to surface in z direction} \ (L) \]
\( a = \) bottom friction factor, dimensionless
\( \beta = \) bottom friction correction factor, dimensionless
\( \Delta = \) forward difference operator
\( \eta = \) wind friction factor, dimensionless
\( \theta = \) angle of wind velocity vector and x-axis (degrees)
\( \mu = \) viscosity (M/Lt)
\( \rho = \) density of a species (M/L\(^3\))
\( \tau = \) viscous stress (M/t\(^2\) L)
\( \overline{\tau} = \) viscous stress tensor (M/t\(^2\) L)
\( \omega = \) mass fraction, dimensionless

**Overlines**
- \( \rightarrow = \) vector quantity

- \( \bar{\cdot} = \) averages

- \( \cdot' = \) instantaneous deviation from the average

**Superscripts**
- \( s = \) surface

- \( b = \) bottom
APPENDIX A

TIME AVERAGING OF THE TRANSPORT PHENOMENA EQUATIONS
APPENDIX A

TIME AVERAGING OF THE TRANSPORT PHENOMENA EQUATIONS

In Chapter IV, important steps in the time averaging of the transport phenomena equations are given. The complete derivations of time averaged equations are given in this appendix.

Time Averaging of the Hydrodynamic Model

The hydrodynamic model consists of the continuity equation and the momentum equation. The continuity equation for estuarine flow is given in Chapter IV and is:

\[
\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial L}{\partial t} = R - E_v \tag{4.14}
\]

Integrating the above equation over a time period \(\Delta t\), from \(t_o\) to \(t_f\) (i.e., \(t_f - t_o = \Delta t\)), gives:

\[
\frac{1}{\Delta t} \int_{t_o}^{t_f} \left[ \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial L}{\partial t} \right] dt = \frac{1}{\Delta t} \int_{t_o}^{t_f} \left[ R - E_v \right] dt \tag{A.1}
\]

Defining the time averaged variables as:

\[
Q_x = \overline{Q}_x + Q_x' \tag{A.2}
\]

where:

\[
\overline{Q}_x
\]
\[ \overline{Q_x} = \frac{1}{\Delta t} \int_{t_0}^{t_f} Q_x \, dt \] (A.3)

and similarly:

\[ Q_y = \overline{Q_y} + Q'_y \] (A.4)

\[ L = \overline{L} + L' \] (A.5)

\[ R = \overline{R} + R' \] (A.6)

\[ E_v = \overline{E_v} + E'_v \] (A.7)

The bar (—) indicates the average over the period and the prime (') indicates the instantaneous deviation from the average. Thus Eq. (A.1) can be written as:

\[ \frac{1}{\Delta t} \int_{t_0}^{t_f} \left[ \frac{\partial (\overline{Q_x} + Q'_x)}{\partial x} + \frac{\partial (\overline{Q_y} + Q'_y)}{\partial y} + \frac{\partial (\overline{L} + L')}{\partial t} \right] \, dt \]

\[ = \frac{1}{\Delta t} \int_{t_0}^{t_f} [\overline{R} + R' - \overline{E_v} - E'_v] \, dt \] (A.8)

Evaluating each term separately:

\[ \int_{t_0}^{t_f} \frac{\partial \overline{Q_x}}{\partial x} \, dt = \int_{t_0}^{t_f} \frac{\partial \overline{Q_x}}{\partial x} \, dt - \int_{t_0}^{t_f} \overline{Q_x} \, dt + \overline{Q_x} \int_{t_0}^{t_f} \frac{\partial t_f}{\partial t} \, dt - \int_{t_0}^{t_f} \frac{\partial t_o}{\partial t} \, dt = \frac{\partial \overline{Q_x}}{\partial x} \Delta t \] (A.9)
\[
\int_{t_0}^{t_f} \frac{\partial Q_x'}{\partial x} \, dt = \frac{\partial}{\partial x} \left[ Q_x' \, dt - Q_x' \right]_{t_0}^{t_f} \frac{\partial t_f'}{\partial t} + Q_x' \left. \frac{\partial t_o'}{\partial t} \right| = 0 \quad (A.10)
\]

since by definition the time average of the fluctuation, \(\bar{Q}_x\), is zero.

Similarly:

\[
\int_{t_0}^{t_f} \frac{\partial Q_y'}{\partial y} \, dt = \frac{\partial Q_y'}{\partial y} \Delta t \quad (A.11)
\]

\[
\int_{t_0}^{t_f} \frac{\partial Q_y'}{\partial y} \, dt = 0 \quad (A.12)
\]

\[
\int_{t_0}^{t_f} \frac{\partial L'}{\partial t} \, dt = \frac{\partial L'}{\partial t} \Delta t \quad (A.13)
\]

\[
\int_{t_0}^{t_f} \frac{\partial L'}{\partial t} \, dt = 0 \quad (A.14)
\]

\[
\int_{t_0}^{t_f} R' \, dt = R' \Delta t \quad (A.15)
\]

\[
\int_{t_0}^{t_f} R' \, dt = 0 \quad (A.16)
\]
\[
\int_{t_0}^{t_f} \overline{E_v} \, dt = \overline{E_v} \Delta t \tag{A.17}
\]

\[
\int_{t_0}^{t_f} E_v' \, dt = 0 \tag{A.18}
\]

Substituting Eqs. (A.8) through (A.18) into Eq. (A.1) results in the time-averaged continuity equation given in Chapter IV:

\[
\frac{\partial \overline{Q_x}}{\partial x} + \frac{\partial \overline{Q_y}}{\partial y} + \frac{\partial \overline{L}}{\partial t} = \overline{R} - \overline{E_v} \tag{4.76}
\]

**X-Component of the Momentum Equation:** The momentum equation in the x-direction given in Chapter IV is:

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} - FV + g \frac{\partial L}{\partial x} = \tau_x^s - \tau_x^b \tag{4.30}
\]

Integrating over a time period \( t \) gives:

\[
\frac{1}{\Delta t} \int_{t_0}^{t_f} \left[ \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} - FV + g \frac{\partial L}{\partial x} \right] \, dt = \frac{1}{\Delta t} \int_{t_0}^{t_f} \left[ \tau_x^s - \tau_x^b \right] \, dt \tag{A.19}
\]

defining:

\[
U = \overline{U} + U' \tag{A.20}
\]

\[
V = \overline{V} + V' \tag{A.21}
\]

\[
D = \overline{D} + D' \tag{A.22}
\]

\[
\tau_x^s = \overline{\tau_x^s} + \tau_x'^s \tag{A.23}
\]

\[
\tau_x^b = \overline{\tau_x^b} + \tau_x'^b \tag{A.24}
\]
Substituting Eqs. (A.20) through (A.24) into Eq. (A.19):

\[
\frac{1}{\Delta t} \int_{t_0}^{t_f} \left[ \frac{\partial (\bar{U} + U')}{\partial t} + (\bar{U} + U') \frac{\partial (\bar{U} + U')}{\partial x} + (\bar{V} + V') \frac{\partial (\bar{U} + U')}{\partial y} 
- F(\bar{V} + V') + g \frac{\partial (\bar{L} + L')}{\partial x} \right] \, dt = \frac{1}{\Delta t} \int_{t_0}^{t_f} \left[ (\tau_x^s + \tau_x^{b'}) \right] \, dt
\]

(E.25)

Evaluating each term separately:

\[
\frac{\partial \bar{U}}{\partial t} = \frac{\partial U}{\partial t} \Delta t
\]

(A.26)

\[
\frac{\partial U'}{\partial t} = 0
\]

(A.27)

\[
\int_{t_0}^{t_f} \bar{U} \frac{\partial \bar{U}}{\partial x} \, dt = \bar{U} \int_{t_0}^{t_f} \frac{\partial \bar{U}}{\partial x} \, dt = \bar{U} \frac{\partial \bar{U}}{\partial x} \Delta t
\]

(A.28)

\[
\int_{t_0}^{t_f} \bar{U} \frac{\partial U'}{\partial x} \, dt = \bar{U} \int_{t_0}^{t_f} \frac{\partial U'}{\partial x} \, dt = 0
\]

(A.29)

\[
\int_{t_0}^{t_f} U' \frac{\partial U'}{\partial x} \, dt = \int_{t_0}^{t_f} U' \frac{\partial U'}{\partial x} \, dt
\]

(A.30)

\[
\int_{t_0}^{t_f} U' \frac{\partial \bar{U}}{\partial x} \, dt = 0
\]

(A.31)
\[
\int_{t_0}^{t_f} V \frac{\partial U}{\partial y} \, dt = \int_{t_0}^{t_f} V' \frac{\partial U'}{\partial y} \, dt
\]
(A.32)

\[
\int_{t_0}^{t_f} V' \frac{\partial U'}{\partial y} \, dt = 0
\]
(A.33)

\[
\int_{t_0}^{t_f} \frac{\partial U}{\partial y} \, dt = 0
\]
(A.34)

\[
\int_{t_0}^{t_f} \frac{\partial U'}{\partial y} \, dt = \int_{t_0}^{t_f} \frac{\partial U'}{\partial y} \, dt
\]
(A.35)

\[
\int_{t_0}^{t_f} F \overline{V} \, dt = F \overline{V} \, dt
\]
(A.36)

\[
\int_{t_0}^{t_f} F \, V' \, dt = 0
\]
(A.37)

\[
\int_{t_0}^{t_f} \frac{\partial L}{\partial x} \, dt = \frac{\partial L}{\partial x} \Delta t
\]
(A.38)

\[
\int_{t_0}^{t_f} \frac{\partial L'}{\partial x} \, dt = 0
\]
(A.39)

\[
\int_{t_0}^{t_f} \tau_x^b \, dt = \tau_x^b \Delta t
\]
(A.40)
Substituting Eqs. (A.26) through (A.43) into Eq. (A.25) gives the x-component of the time-averaged equation of motion:

\[
\frac{1}{\Delta t} \left[ \frac{\partial \overline{U}}{\partial t} \Delta t + \overline{U} \frac{\partial \overline{U}}{\partial x} \Delta t + \int_{t_0}^{t_f} U' \frac{\partial U'}{\partial x} \, dt + \overline{V} \frac{\partial \overline{U}}{\partial y} \Delta t + \int_{t_0}^{t_f} V' \frac{\partial U'}{\partial y} \, dt \right] \\
- F \overline{V} \Delta t + g \frac{\partial \overline{L}}{\partial x} \Delta t = \frac{1}{\Delta t} \left[ \overline{\tau}_x^s \Delta t - \overline{\tau}_x^b \Delta t \right]
\]  

(A.44)

Rearranging, we obtain Eq. (4.84) of Chapter IV:

\[
\frac{\partial \overline{U}}{\partial t} + \overline{U} \frac{\partial \overline{U}}{\partial x} + \frac{1}{\Delta t} \int_{t_0}^{t_f} U' \frac{\partial U'}{\partial x} \, dt + \overline{V} \frac{\partial \overline{U}}{\partial y} + \frac{1}{\Delta t} \int_{t_0}^{t_f} V' \frac{\partial U'}{\partial y} \, dt - F \overline{V} + g \frac{\partial \overline{L}}{\partial x}
\]

\[
= \overline{\tau}_x^s - \overline{\tau}_x^b
\]

(4.84)

The y-component of the equation of motion can be similarly derived, which is given in Chapter IV:
Time Averaging of Material Transport Model

The material transport model is given as:

\[
\frac{\partial (DS_A)}{\partial t} + \frac{\partial (UDS_A)}{\partial x} + \frac{\partial (VDS_A)}{\partial y} - \frac{\partial}{\partial x} \left( D \frac{\partial S_A}{\partial x} \right) - \frac{\partial}{\partial y} \left( D \frac{\partial S_A}{\partial y} \right) - P_A = 0
\]  

Integrating over a time period \( \Delta t \) gives:

\[
\frac{1}{\Delta t} \int_{t_0}^{t_f} \left[ \frac{\partial (DS_A)}{\partial t} + \frac{\partial (UDS_A)}{\partial x} + \frac{\partial (VDS_A)}{\partial y} - \frac{\partial}{\partial x} \left( D \frac{\partial S_A}{\partial x} \right) - \frac{\partial}{\partial y} \left( D \frac{\partial S_A}{\partial y} \right) - P_A \right] dt = 0
\]  

Defining:

\[
S_A = \bar{S}_A + S'_A
\]

\[
P = \bar{P} + P'
\]

Substituting Eqs. (A.20), (A.21), (A.22), (A.46) and (A.47) into Eq. (A.45) gives:
\[ \frac{1}{\Delta t} \int_{t_0}^{t_f} \left[ \frac{\partial}{\partial t} \left( \overline{D} \overline{S}_A \right) + \frac{\partial}{\partial x} \left( \overline{U} \overline{S}_A \right) \right] + \frac{\partial}{\partial y} \left( \overline{V} \overline{S}_A \right) + \frac{\partial}{\partial z} \left( \overline{W} \overline{S}_A \right) \right] dt = 0 \] (A.48)

Evaluating each term of Eq. (A.48) individually:

\[ \int_{t_0}^{t_f} \frac{\partial}{\partial t} \left( \overline{D} \overline{S}_A \right) dt = \frac{\partial (\overline{D} \overline{S}_A)}{\partial t} \Delta t \] (A.49)

\[ \int_{t_0}^{t_f} \frac{\partial}{\partial t} \left( \overline{D} \overline{S}_A' \right) dt = 0 \] (A.50)

\[ \int_{t_0}^{t_f} \frac{\partial}{\partial t} \left( \overline{D}' \overline{S}_A' \right) dt = 0 \] (A.51)

\[ \int_{t_0}^{t_f} \frac{\partial}{\partial t} \left( \overline{D}' \overline{S}_A' \right) dt = \int_{t_0}^{t_f} \frac{(\overline{D}' \overline{S}_A')}{\partial t} dt \] (A.52)

\[ \int_{t_0}^{t_f} \frac{\partial}{\partial x} \left( \overline{D} \overline{U} \overline{S}_A \right) dt = \frac{\partial (\overline{D} \overline{U} \overline{S}_A)}{\partial x} \Delta t \] (A.53)

\[ \int_{t_0}^{t_f} \frac{\partial}{\partial x} \left( \overline{D} \overline{U} \overline{S}_A' \right) dt = \frac{\partial}{\partial x} \overline{D} \overline{U} \int_{t_0}^{t_f} S_A' dt = 0 \] (A.54)
\[ \int_{t_0}^{t_f} \frac{\partial}{\partial x} (\overline{D} \ u' \ \overline{S}_A) \ dt = 0 \]  
\hspace{2cm} (A.55)

\[ \int_{t_0}^{t_f} \frac{\partial}{\partial x} (D' \ \overline{U} \ \overline{S}_A) \ dt = 0 \]  
\hspace{2cm} (A.56)

\[ \int_{t_0}^{t_f} \frac{\partial}{\partial x} (D' \ u' \ \overline{S}_A) \ dt = \frac{\partial}{\partial x} \overline{S}_A \int_{t_0}^{t_f} D' \ u' \ dt \]  
\hspace{2cm} (A.57)

\[ \int_{t_0}^{t_f} \frac{\partial}{\partial x} (D' \ \overline{U}' \ \overline{S}_A') \ dt = \frac{\partial}{\partial x} \overline{S}_A' \int_{t_0}^{t_f} D' \ \overline{S}_A' \ dt \]  
\hspace{2cm} (A.58)

\[ \int_{t_0}^{t_f} \frac{\partial}{\partial x} (D' \ \overline{U}' \ \overline{S}_A') \ dt = \frac{\partial}{\partial x} \overline{S}_A' \int_{t_0}^{t_f} D' \ \overline{U}' \ \overline{S}_A' \ dt \]  
\hspace{2cm} (A.59)

\[ \int_{t_0}^{t_f} \frac{\partial}{\partial x} (D' \ u' \ \overline{S}_A') \ dt = \frac{\partial}{\partial x} \overline{D} \int_{t_0}^{t_f} D' \ u' \ \overline{S}_A' \ dt \]  
\hspace{2cm} (A.60)

\[ \int_{t_0}^{t_f} \frac{\partial}{\partial y} (\overline{D} \ \overline{V} \ \overline{S}_A) \ dt = \frac{\partial}{\partial y} (\overline{D} \ \overline{V} \ \overline{S}_A) \Delta t \]  
\hspace{2cm} (A.61)

\[ \int_{t_0}^{t_f} \frac{\partial}{\partial y} (\overline{D} \ \overline{V}' \ \overline{S}_A') \ dt = 0 \]  
\hspace{2cm} (A.62)

\[ \int_{t_0}^{t_f} \frac{\partial}{\partial y} (\overline{D} \ \overline{V}' \ \overline{S}_A') \ dt = 0 \]  
\hspace{2cm} (A.63)
\[
\int_{t_0}^{t_f} \frac{\partial}{\partial y} \left( \mathbf{D}' \cdot \overline{\mathbf{V}} \cdot \overline{S}_A \right) \, dt = 0 \quad \text{(A.64)}
\]

\[
\int_{t_0}^{t_f} \frac{\partial}{\partial y} \left( \mathbf{D}' \cdot \mathbf{V}' \cdot \overline{S}'_A \right) \, dt = \frac{\partial}{\partial y} \mathbf{D} \int_{t_0}^{t_f} \mathbf{V}' \cdot \overline{S}'_A \, dt \quad \text{(A.65)}
\]

\[
\int_{t_0}^{t_f} \frac{\partial}{\partial y} \left( \mathbf{D}' \cdot \mathbf{V}' \cdot \overline{S}'_A \right) \, dt = \frac{\partial}{\partial y} \mathbf{V} \int_{t_0}^{t_f} \mathbf{D}' \cdot \mathbf{V}' \, dt \quad \text{(A.66)}
\]

\[
\int_{t_0}^{t_f} \frac{\partial}{\partial y} \left( \mathbf{D}' \cdot \mathbf{V}' \cdot \overline{S}'_A \right) \, dt = \frac{\partial}{\partial y} \mathbf{S}'_A \int_{t_0}^{t_f} \mathbf{D}' \cdot \mathbf{V}' \, dt \quad \text{(A.67)}
\]

\[
\int_{t_0}^{t_f} \frac{\partial}{\partial y} \left( \mathbf{D}' \cdot \mathbf{V}' \cdot \overline{S}'_A \right) \, dt = \frac{\partial}{\partial y} \int_{t_0}^{t_f} \mathbf{D}' \cdot \mathbf{V}' \cdot \overline{S}'_A \, dt \quad \text{(A.68)}
\]

\[
\int_{t_0}^{t_f} \frac{\partial}{\partial x} \left( \mathbf{D} \cdot \mathbf{P}_{AB} \cdot \frac{\partial \overline{S}_A}{\partial x} \right) \, dt = \left[ \frac{\partial}{\partial x} \left( \mathbf{D} \cdot \mathbf{P}_{AB} \cdot \frac{\partial \overline{S}_A}{\partial x} \right) \right] \Delta t \quad \text{(A.69)}
\]

\[
\int_{t_0}^{t_f} \frac{\partial}{\partial x} \left( \mathbf{D} \cdot \mathbf{P}_{AB} \cdot \frac{\partial S'_A}{\partial x} \right) \, dt = \frac{\partial}{\partial x} \mathbf{D} \cdot \mathbf{P}_{AB} \int_{t_0}^{t_f} \frac{\partial S'_A}{\partial x} \, dt = 0 \quad \text{(A.70)}
\]

\[
\int_{t_0}^{t_f} \frac{\partial}{\partial x} \left( \mathbf{D}' \cdot \mathbf{P}_{AB} \cdot \frac{\partial S'_A}{\partial x} \right) \, dt = 0 \quad \text{(A.71)}
\]

\[
\int_{t_0}^{t_f} \frac{\partial}{\partial x} \left( \mathbf{D}' \cdot \mathbf{P}_{AB} \cdot \frac{\partial S'_A}{\partial x} \right) \, dt = \frac{\partial}{\partial x} \left( \mathbf{P}_{AB} \int_{t_0}^{t_f} \mathbf{D}' \cdot \frac{\partial S'_A}{\partial x} \, dt \right) \quad \text{(A.72)}
\]
\[
\int_{t_0}^{t_f} \frac{\partial}{\partial y} \left( \bar{D} \frac{\partial S_A}{\partial y} \right) dt = \frac{\partial}{\partial y} \left( \bar{D} \frac{\partial S_A}{\partial y} \right) \Delta t \tag{A.73}
\]

\[
\int_{t_0}^{t_f} \frac{\partial}{\partial y} \left( \bar{D} \frac{\partial S_A'}{\partial y} \right) dt = 0 \tag{A.74}
\]

\[
\int_{t_0}^{t_f} \frac{\partial}{\partial y} \left( D' \frac{\partial S_A}{\partial y} \right) dt = 0 \tag{A.75}
\]

\[
\int_{t_0}^{t_f} \frac{\partial}{\partial y} \left( D' \frac{\partial S_A'}{\partial y} \right) dt = \frac{\partial}{\partial y} \left( D_A \int_{t_0}^{t_f} D' \frac{\partial S_A'}{\partial y} dt \right) \tag{A.76}
\]

\[
\int_{t_0}^{t_f} p_A' dt = \bar{p}_A \Delta t \tag{A.77}
\]

\[
\int_{t_0}^{t_f} p_A' dt = 0 \tag{A.78}
\]

Substituting Eqs. (A.49) through (A.78) into Eq. (A.48) gives the time averaged material transport model Eq. (4.90) of Chapter IV:

\[
\frac{\partial (\bar{D} \bar{S}_A)}{\partial t} + \frac{1}{\Delta t} \int_{t_0}^{t_f} \frac{\partial}{\partial t} \left( D' S_A' \right) dt + \frac{1}{\Delta t} \frac{\partial}{\partial x} \left( \bar{D} \bar{U} \bar{S}_A \right) + \frac{1}{\Delta t} \frac{\partial}{\partial x} \bar{S}_A \int_{t_0}^{t_f} D' U' dt
\]

\[
+ \frac{1}{\Delta t} \frac{\partial}{\partial x} \bar{D} \int_{t_0}^{t_f} U' S_A' dt + \frac{1}{\Delta t} \frac{\partial}{\partial x} \bar{U} \int_{t_0}^{t_f} D' S_A' dt + \frac{1}{\Delta t} \frac{\partial}{\partial x} \int_{t_0}^{t_f} D' U' S_A' dt
\]
\begin{equation}
\frac{\partial}{\partial t} (\overline{D} \nabla \overline{S}_A) + \frac{1}{\Delta t} \frac{\partial}{\partial y} \overline{D} \int_{t_0}^{t_f} V' S_A' \, dt + \frac{1}{\Delta t} \frac{\partial}{\partial y} \overline{V} \int_{t_0}^{t_f} D' S_A' \, dt
\end{equation}

\begin{equation}
+ \frac{1}{\Delta t} \frac{\partial}{\partial y} \overline{S}_A \int_{t_0}^{t_f} D' V' \, dt + \frac{1}{\Delta t} \frac{\partial}{\partial y} \overline{D} \overline{V} \int_{t_0}^{t_f} D' V' S_A' \, dt - \frac{\partial}{\partial x} \left( \rho_{AB} \overline{D} \frac{\partial \overline{S}_A}{\partial x} \right)
\end{equation}

\begin{equation}
- \frac{1}{\Delta t} \frac{\partial}{\partial x} \rho_{AB} \int_{t_0}^{t_f} D' \frac{\partial \overline{S}_A}{\partial x} \, dt - \frac{\partial}{\partial y} \left( \rho_{AB} \overline{D} \frac{\partial \overline{S}_A}{\partial y} \right) - \frac{1}{\Delta t} \frac{\partial}{\partial y} \rho_{AB} \int_{t_0}^{t_f} D' \frac{\partial \overline{S}_A}{\partial y} \, dt
\end{equation}

= 0 \quad (4.90)
APPENDIX B

USER'S MANUAL, FLOW DIAGRAM AND COMPUTER PROGRAM
FOR THE HYDRODYNAMIC MODEL
APPENDIX B

USER'S MANUAL, FLOW DIAGRAM AND COMPUTER PROGRAM
FOR THE HYDRODYNAMIC MODEL

This appendix gives user's manual, flow diagram and computer program for the hydrodynamic model developed by Hacker. The generalized computer flow diagram is given in Figure B.1 and the expanded flow diagram is given in Figure B.2. Table B.1, Figure B.3, Table B.2, and Table B.3 give a description of subroutines, program deck assembly, all the default values, and description of variables respectively. A complete listing of the computer program is given at the end.
Figure B.1. Generalized computer flow diagram for implicit solution of the hydrodynamic model.
Figure B.2. Expanded computer flow diagram for implicit solution of the hydrodynamic model.
1. CALCULATE FRESH WATER INPUT
   - CALL RAIN
   - CALL CPASS
   - CALL QBPASS
   - CALL BPASS

2. PLACE VELOCITIES AND LEVELS CALCULATED IN PROPER LOCATION

3. ADJUST WATER LEVEL OF FLOODING GRID POINTS

MARCH TO NEXT ROW

THE GRID COVERED?

YES

NO

CALL TRID

CALL TRIDIAGONAL MATRIX AND CONSTANT VECTOR FOR HYDRODYNAMICS OF GIVEN ROW
3

ADVANCE TIME 1/2 TIME STEP

CALLULATE FRESHWATER INPUT

CALL CPASS

CALL QBPASS

CALL BPASS

MARCH TO THE NEXT COLUMN

PLACE VELOCITIES AND LEVELS CALCULATED IN PROPER LOCATION

CALL TRID

ADJUST WATER LEVELS OF FLOODING GRID POINTS

MODIFY ELEMENTS IF COLUMN HAS TIDAL ENTRANCE

YES

THE GRID COVERED?

NO
2F CYCLE FOR QUASI STEady STATE AVERAGE U,V,L,LP

ADVANCE TIME BY 1/2 STEP

OUTPUT?

WRITE BAY CONDITIONS

TIDAL CYCLE COMPLETE?

QUASI STEADY STATE?

PUNCH AVERAGED BAY CONDITIONS

STOP
<table>
<thead>
<tr>
<th>Subroutine</th>
<th>Description</th>
<th>Additional Subroutines Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>INIT</td>
<td>Initializes all values used in the calculation</td>
<td>none</td>
</tr>
<tr>
<td>TRID</td>
<td>Solves system of simultaneous equations with tridiagonal matrices. Has been modified for grid points in which tidal levels and velocities are known (barriers, islands, etc.).</td>
<td>none</td>
</tr>
<tr>
<td>BPASS</td>
<td>Calculates tidal level variations at Barataria Pass. Gives a sine wave using specified ranges and times of low and high tide (i.e., period).</td>
<td>none</td>
</tr>
<tr>
<td>QBPASS</td>
<td>Calculates tidal level variations at Quatre Bayou Pass. Delays tide by 0.875 hours as compared with the Barataria Pass tide.</td>
<td>none</td>
</tr>
<tr>
<td>CPASS</td>
<td>Calculates tidal level variations at Caminada Pass. Delays tide by 1.358 hours as compared with the Barataria Pass tide.</td>
<td>none</td>
</tr>
<tr>
<td>BLOCK DATA</td>
<td>Data package containing all default data values.</td>
<td>none</td>
</tr>
<tr>
<td>RAIN</td>
<td>Calculates the rise in water levels due to rainfall.</td>
<td>none</td>
</tr>
</tbody>
</table>
Figure B.3. Program deck assemblage
The following chart shows the format of the card input:

<table>
<thead>
<tr>
<th>Card</th>
<th>Variables</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bathymetry Data</td>
<td>H(J,K)</td>
<td>17F4.1</td>
</tr>
<tr>
<td>Starting Bay Conditions</td>
<td>V(J,K), V(J,K)</td>
<td>7X, 4E12.4</td>
</tr>
<tr>
<td>Title Card</td>
<td>L(J,K), LP(J,K)</td>
<td>20A4</td>
</tr>
<tr>
<td>Namelist Option Cards</td>
<td>W, WANG, RHO</td>
<td>Namelist</td>
</tr>
<tr>
<td></td>
<td>CMANN, TLOW, THIGH, RANGEL, RANGEH, FWR, RR, TRB, TRE, CORR</td>
<td></td>
</tr>
</tbody>
</table>

The variables contained in the cards following the title card form the NAMELIST/SHADOW/ option. All these variables have values set internally in the program. These default values are shown in Table B.2. If any of these values are to be changed, they are entered using the namelist option as cards following the title card. For example: if ranges of high and low tides and the fresh water runoff are to be changed, it can be done by the following card after the title card:

```
b &SHADOW &RANGEH=1.2, RANGEL=.5, FWR=1800.0 b &END
```

All the other variables in the namelist option will have default values. Note must be made that even when all variables are to have default values, at least one must be entered in the namelist format to insure that no end of data file error terminates the execution of the program.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Default Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>0.0 (ft/sec)</td>
</tr>
<tr>
<td>WANG</td>
<td>0.0 (rads)</td>
</tr>
<tr>
<td>RHO</td>
<td>62.4 (lbs/ft³)</td>
</tr>
<tr>
<td>CMANN</td>
<td>0.026 (dimensionless)</td>
</tr>
<tr>
<td>TLOW</td>
<td>12.0 (hr)</td>
</tr>
<tr>
<td>THIGH</td>
<td>24.0 (hr)</td>
</tr>
<tr>
<td>RANGEL</td>
<td>1.1 (ft)</td>
</tr>
<tr>
<td>RANGEH</td>
<td>1.1 (ft)</td>
</tr>
<tr>
<td>FWR</td>
<td>1000.0 (ft³/sec)</td>
</tr>
<tr>
<td>RR</td>
<td>0.0 (ft/hr)</td>
</tr>
<tr>
<td>TRB</td>
<td>0.0 (hr)</td>
</tr>
<tr>
<td>TRE</td>
<td>0.0 (hr)</td>
</tr>
<tr>
<td>CORR</td>
<td>0.0 (hr)</td>
</tr>
</tbody>
</table>
TABLE B.3 Description of Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>H(J,K)</td>
<td>Depth, distance from the mean sea level to the bottom, at grid point J,K. (feet)</td>
</tr>
<tr>
<td>U(J,K)</td>
<td>Velocity in the x-direction at grid point J,K. (ft/sec)</td>
</tr>
<tr>
<td>V(J,K)</td>
<td>Velocity in the y-direction at grid point J,K. (ft/sec)</td>
</tr>
<tr>
<td>LP(J,K)</td>
<td>Water level, height above mean sea level, at grid point J,K; calculated at the first half-time step. (feet)</td>
</tr>
<tr>
<td>L(J,K)</td>
<td>Water level at grid point J,K; calculated at the second half-time step. (feet)</td>
</tr>
<tr>
<td>TITLE</td>
<td>Title for identification of run.</td>
</tr>
<tr>
<td>WANG</td>
<td>Wing angle with respect to the x-direction (radians).</td>
</tr>
<tr>
<td>RHO</td>
<td>Water density (lbs/ft³)</td>
</tr>
<tr>
<td>CMANN</td>
<td>Manning friction factor coefficient (dimensionless)</td>
</tr>
<tr>
<td>TLOW</td>
<td>Time after start of run when low tide occurs (hours)</td>
</tr>
<tr>
<td>THIGH</td>
<td>Time after start of run when high tide occurs (hours)</td>
</tr>
<tr>
<td>RANGEL</td>
<td>Range of low tide (ft)</td>
</tr>
<tr>
<td>RANGEH</td>
<td>Range of high tide (ft)</td>
</tr>
<tr>
<td>FWR</td>
<td>Fresh water runoff entering at the top of the bay at pre-established points (ft³/sec)</td>
</tr>
<tr>
<td>RR</td>
<td>Rainfall rate (ft/hr)</td>
</tr>
<tr>
<td>TRB</td>
<td>Time after start of run when rainfall begins (hours)</td>
</tr>
<tr>
<td>TRE</td>
<td>Time after start of run when rainfall stops (hours)</td>
</tr>
<tr>
<td>CORR</td>
<td>Time after start of run when sun rises (a negative number if the sun is up at the start of the run) (hours).</td>
</tr>
</tbody>
</table>
REAL*8 SUB, SUP, DIAG, E
REAL*8 U, V, L, LP, H
REAL LL
REAL N
INTEGER E1, E2, E3, E4
INTEGER C01, C02, C03
DIMENSION KB(40), RD(40), RP(40), A(40), CS(40), JD(40)
DIMENSION AL(40), D(40), SUP(40), SUB(40), DIAG(80), B(63)
DIMENSION TITLE(20)
COMMON/A7/SSEA
COMMON/A1/C(40, 20), DE(40, 20)
COMMON/A3/TIME, TMAX, TPRINT, TTHIR, DELT
COMMON/A4/DDELX, SETL, CMANN, SETC
COMMON/A5/TW3, TAIR, TSEA, N, CP, EPS, SIG, T2RAD
COMMON/A6/PAIR
COMMON/A8/W, WANG
COMMON/A9/RHOAIR, RHO, G
COMMON/B1/THRI1, THRI2, THRI3, THRI4
COMMON/B2/COSR
COMMON/B3/MIN, MAX, NAX, LMAX
COMMON/B4/MINA, MAX1, MAX11, MAXA
COMMON/B5/MINY, MINYA, MAX1, NAX1, NAXA
COMMON/B6/N
COMMON/B9/L(L40, 20), LP(L40, 20), H(L40, 20), U(L40, 20), V(L40, 20)
COMMON/C1/FWP, PW
COMMON/C3/DP, TBB, TEE
COMMON/C4/E1, E2, E3, E4
COMMON/C5/TLOW, THIGH, RANGEL, RANGEL, TIMEIN
COMMON/D1/AVG/L(L40, 20), AVG/L(40, 20), AVG/L(40, 20), AVG/LP(40, 20)
COMMON/S/CAMN, THIGH, RANGEL, RANGEL, CMANN, TLOW, THIGH, RANGEL, RANGEL, COS, FWP, E, TBF, TRE
**VELOCITY IN THE X-DIRECTION (FETE/SEC)**  MAIN 320
**VELOCITY IN THE Y-DIRECTION (FETE/SEC)  MAIN 340**
**LEVEL ABOVE MEAN SEA LEVEL (FEET)  MAIN 350**
**LEVEL ABOVE MEAN SEA LEVEL (FEET)  MAIN 360**
**DEPTH BELOW MEAN SEA LEVEL (FETE)  MAIN 370**
**SALINITY (U/JO)  MAIN 390**
**TEMPERATURE (DEGS. F)  MAIN 390**
**GENERATION OF ENERGY (DEGS. F/SEC)  MAIN 400**
**DEPTH (FEET)  MAIN 410**
**TEMPERATURE DISPERSION/X-DIR./ (FETE/SEC)  MAIN 430**
**TEMPERATURE DISPERSION/Y-DIR./ (FETE/SEC)  MAIN 440**
**SALINITY DISPERSION/X-DIR./ (FETE/SEC)  MAIN 450**
**SALINITY DISPERSION/Y-DIR./ (FETE/SEC)  MAIN 460**
**GRID STEPSIZE (FEET)  MAIN 480**
**MINIMUM DEPTH ALLOWED IN THE GRID (FEET)  MAIN 490**
**MANNING COEFFICIENT DIMENSIONLESS  MAIN 500**
**TIME STEPSIZE (SCCOND)  MAIN 510**
**SYSTEM TIME (SECONDS)  MAIN 520**
**TIME AT WHICH LOW TIDE OCCURS (HOURS)  MAIN 530**
**TIME AT WHICH HIGH TIDE OCCURS (HOURS)  MAIN 540**
**PROGRAM STOPS AT HIGH TIDE  MAIN 550**
**RANGE OF LOW TIDE (FEET)  MAIN 560**
**RANGE OF HIGH TIDE (FEET)  MAIN 570**
**PRINTING TIME INTERVAL (NO. IT OF HYDO.)  MAIN 580**
**PRINTING CLOCK (HOURS)  MAIN 590**
**WET BULB TEMPERATURE (DEGS. C)  MAIN 600**
**TEMPERATURE OF AIR (DEGS. C)  MAIN 610**
**TEMPERATURE OF OPEN SEA (DEGS. F)  MAIN 620**
**EVAPORATION COEFFICIENT DIMENSIONLESS  MAIN 630**
**HEAT CAPACITY OF WATER (BTU/L3)  MAIN 640**
**WATER DENSITY (LBS/FT**3)  MAIN 650**
**AIR DENSITY (LBS/FT**3)  MAIN 660**
**ACCEL. DUE TO GRAVITY (FETE/SEC**2)  MAIN 670**
**EMISIVITY DIMENSIONLESS  MAIN 680**
**STEFAN/BOLTZMAN CONSTANT (KCAL/FT**2*SLC*DEG.K)  MAIN 690**
CALL ERRSET(208,256,-1,1)

ITMR=0
ITIDAL=0
MINA=MIN-1
MIND=MIN-2
MIN1=MIN+1
MIN2=MIN+2
MAXA=MAX-1
MAXB=MAX-2
MAX1=MAX+1
MAX11=MAX+2
MAXC=(MAX1-MIN)*2
MAXD=MAXC-1
NAXA=NAX-1
NAXB=NAX-2
NAX11=NAX+2
NAXC=(NAX1-MIN)*2
NAXD=NAXC-1
CO1=MIN*2+1
CO2=MIN*2
CO3=MIN*2-1

C-***--*** INITIALIZE DATA   ALL VALUES EQUAL ZERO
CALL INIT

C READ DATA
READ (5,535)H (9,2)
READ (5,535)H (18,2)
READ (5,535)H (23,2)
READ (5,535)H (27,2)
535 FORMAT (F10.4)
DO 8 J=MIN,MAX1
     READ(5,100) (H(J,K),K=MIN,NAX1)
8 CONTINUE
C ALWAYS INITIAL CONDITIONS ARE ZERO
DO 7 J=MIN,NAX1
DO 7 K=MINA,NAX1
U(J,K)=0.0
V(J,K)=0.0
L(J,K)=.633
LP(J,K)=.633
7 CONTINUE
READ(5,108) TITLE
READ(5,SHADOW)
END FILE 5
WRITE(6,SHADOW) MAIN1160
WRITE(6,109) TITLE
ITPK=TPRINT+0.01
ITLOW=TLow+0.01
ITHIGH=THIGH+C.01

C

C*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-* [vjAlDi 1270
C MAIN 1280
C MAIN 1310
C******** CALCULATE CHEZY COEFFICIENTS *******************************^j jj-]| 320
DO 4 J=MINB,MAX11 MAIN133C
DO 4 K=MINE,MAX 11 HAIN134C
DE (J,K) = H (J,K) + L (J,K) MAIN1350
IF (DE (J,K) .LE.O. 1) DE (J,K) =0.1 MAIN 1360
C(J,K) = (1.49/CMAAN) * (DE (J,K) ** (0.1 66 7) ) MAIN137C
4 CONTINUE MAIN1380
C*-*-*-* CALCULATE WIND STR ESS -*-*-*-*-*-*-*-*-*-*-*-*-*-*_*-*-*-*_*;.i
A
TWINX=0.00 26*RHOAI
T
T* (W**2.0)*COS (WANG) /,AI'-i 140C
THINY=C.0C26*EHOAIR*{W**2.0) *SIN (WANG) MAIN1-4 1C
C*-*—*—*-*-*-*-*-*-*-*-*-*—*—*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*_*_¥-*p< A IN 1 4 20
EBC=G*DELT/(2.0*DELX) MAIN1450
KRITE(6,1G2) TLCW, RANGEL
WRITE{6,103) THIGH,RANGEH
BRITE(6,104) TSEA,SSEA
WRITE(6, 1G5) TAIR,TV.E,PAIR
WRITE (6, 106) W,WANG
WRITE(6, 107) CORR
99 ITIDAL=ITIDAL+1
THR=0.0 MAIN 990
TIME=0.0
TIME=THR*3600.0
TIMEIN=TIME
C TLCPI=L ( 9,3)
C TLBFI=L (18,3)
C TLPAl=L(23,3)
C TLCBPI=L (27,3)
DO 24 K=MIN,NAX1
C SET UP THE UPPER DIAGONAL -SUP-
C DO 10 J=MIN,MAX1
KB(J)=RBC
10 CONTINUE
DO 11 J=MIN,MAX1
RD(J)=(H(J,K)+H(J,K-1)+I(J+1,K)+L(J,K))*(DEL)/(4.0*DELX)
11 CONTINUE
DO 12 A=MINI,MAX
I=J*2-C01
SUP(I)=KB(J)
12 CONTINUE
DO 13 J=MIN,MAX
I=J*2-C02
SUP(I)=RD(J)
13 CONTINUE
C SET UP THE MAIN DIAGONAL -DIAG-
C DO 14 J=MIN,MAX
RP(J)=1.0+DEL/(2.0*DELX)*(U(J+1,K)-U(J-1,K))+(DEL/2.0)*(DSOR(U)(MIN<=160)
*2+(U.25*(V(J+1,K)+V(J,K)+V(J,K-1)+V(J+1,K-1)))*DELX)/(U.5*MIN<=170)
*(H(J,K)+H(J,K-1)+L(J+1,K)+L(J,K))+(1.5*(C(J+1,K)+C(J,K)))*2)
14 CONTINUE
IF(J.EQ.MIN) GO TO 14
I=J*2-C03
DIAG(I)=RP(J)
14 CONTINUE
DIAG(I)=-RBC
DO 15 L=2,MAXC,2
DIAG(I)=1.0
15 CONTINUE
C SET UP LOWER DIAGONAL -SUB-
C
C
RD (MIN) = 0.0
DO 16 J = MIN, MAX
   I = J*2 - C03
   SUB (I) = -RD (J)
16 CONTINUE
DO 17 J = MIN1, MAX
   I = J*2 - C02
   SUB (I) = -RB (J)
17 CONTINUE
C
SET UP THE CONSTANT VECTOR -B-
DO 18 J = MIN, MAX
   BB (J) = U (J, K) + DELT/(2.0*DELX) * (U (J, K+1) -U (J, K-1)) * (0.25* (V (J+1, K) - V (J, K)))
   1+V (J, K) +V (J, K-1) +V (J+1, K-1)) -G*DELT/ (2.0*DELX) * (I.P (J+1, K) - LP (J, K))
   2-G*DELT/2.*U (J, K)*DSQRT((U (J, K) **2 + 0.25* (V (J+1, K) + V (J, K) + V (J, K-1)) **2)
   3) +V (J+1, K-1)) **2) /(5.0* (H (J, K) + H (J, K-1) + L (J+1, K) + L (J, K)) * (0.5*APIN2460
   4*C (J+1, K) +C (J, K)) **2) + DELT=TWINX/(RHO*0.5* (H (J, K) * H (J, K-1) + L (J+1, K) + L (J, K)))
   5, K) + L (J, K)))
I = J*2 - C03
B (I) = BB (J)
18 CONTINUE
B (1) = BB (MIN) - RP (MIN) * U (MIN, K)
DO 19 J = MIN1, MAX1
   A (J) = L (J, K) + (H (J, K-1) + H (J-1, K-1) + L (J, K) + L (J, K-1)) * V (J, K-1) * (DELT/ (MAIN254C
   14.0*DELX)) - (H (J, K) + H (J-1, K) + L (J, K+1) + L (J, K)) * V (J, K) * (DELT/ (4.0*DELX))
   2)
I = J*2 - C02
B (I) = A (J)
19 CONTINUE
E (2) = A (MIN1) + RD (MIN1) * U (MIN, K)
B (MAXC) = A (MAX1) - RD (MAX1) * U (MAX1, K)
DO 201 I = 1, MAXC
C 201 WRITE (6, 90) I, SUB (I), SUP (I), DIAG (I), B (I)
SET MATRIX ELEMENTS TO SIMULATE HAMMERS

I=8
DO 120 J=MIN1,MAX
I=I+1
M=J+2-I
M=J-M
DE(J,K) = H(J,K) + L(J,K)
IF(DE(J,K) .LE.SETD) GO TO 121
GO TO 120
121 SUB(M) = 0.0
DIAG(M) = 0.0
SUP(M-1) = 0.0
SUB(M-1) = 0.0
DIAG(M-1) = 0.0
SUP(M-2) = 0.0
IF((M-3) .EQ. 0) GO TO 120
SUB(M-2) = 0.0
DIAG(M-2) = 0.0
SUP(M-3) = 0.0
120 CONTINUE

THE TRIDIMENSIONAL MATRIX IS SET - SOLVE CALLING TRID

M=MAXC
CALL TRID(SUE, SUP, DIAG, E, M)

PLACE SOLUTION 3 INTO RESPECTIVE VECTORS

LP(MIN,K) = B(1)
LP(MIN1,K) = B(2)
DO 20 J=4,MAXC,2
I=J/2+MIN
LP(I,K) = B(J)
20 CONTINUE
DO 21 J=3,MAXD,2
I = (J+MIN2)/2
U(J,K) = B(J)

21 CONTINUE
CHECK = 0.0

CHECK WATER LEVELS (L) IN THE SYSTEM

DO 200 J = MIN1, MAX1
DE (J,K) = H(J,K) + LP (J,K)
IF (DE (J,K) .LE. SETD) GO TO 201
GO TO 200

201 LL = 0.25 * (LP (J-1, K) + LP (J, K-1) + LP (J+1, K) + LP (J, K+1))
DE (J,K) = H(J,K) + LL
IF (DE (J,K) .LE. SETD) GO TO 200
LP (J,K) = LL
DELL = LL / 3.0

203 LP (J,K-1) = LP (J, K-1) - DELL
LP (J+1,K) = LP (J+1,K) - DELL
LP (J,K+1) = LP (J,K+1) - DFLL
IF (K.EQ. MAX1) LP (J, K-1) = LP (J, K-1) - DFLL

200 CONTINUE

LEVELS HAVE BEEN CHECKED TO SETD AND VELOCITIES SET TO ZERO
WHILE CONSERVING WATER IN THE SYSTEM
KTH ROW VALUES HAVE BEEN CALCULATED

24 CONTINUE

VALUES FOR THE U VELOCITY ARE STORED IN U(J,K)
VALUES FOR THE WATER HEIGHT ARE STORED IN LP (J,K)
NO EXPLICIT VALUES OF V ARE NEEDED

**************************************************************************************************************************
*** 1ST 1/2 STEP VALUES HAVE BEENreiben
***

TIME=TIME/3600.0

THR=THR+1

IF (THR .gt. 32) STOP

ADVANCE 1/2 TIME STEP

CLOCK

CALCULATE FOR THE HYDRODYNAMIC

HALF TIME STEPS HAVE BEEN CONCLUDED

ALL EH:

TIDAL INPUT

FRESH

CALL BRASS(ITHR,DLCP)

CALL DBRASS(3,DLCP)

CALL DBRASS(9,DLCP)

CALL DBRASS(10,DLCP)
LP (18, 3) = DLBP
LP (23, 3) = LTLQBP
LP (27, 3) = DLQBP

C*-----------------------------C
C
C
C
C
C

DO 58J = MIN, MAX1
MINY = MIN
NAXC = (NAX1 - MIN) * 2
NAXD = NAXC - 1
CO1 = MINY * 2 + 1
CO2 = MINY * 2
MINY2 = MINY + 1
CO3 = MINY * 2 - 1
MINY1 = MINY - 1
MINYA = MINY - 1

C
C

SET UP THE UPPER DIAGONAL -SUP-
C

DO 40 K = MINY, NAX1
RD(K) = (H(J, K) + H(J-1, K) + LP(J, K) + LF(J, K+1)) * (DELT / (4.0 * DELX))

40 CONTINUE

DO 41 K = MINY1, NAX1
I = K * 2 - CO1
SUP(I) = RE(K)

41 CONTINUE

DO 42 K = MINY1, NAX
I = K * 2 - CO2
SUP(I) = RD(K)

42 CONTINUE

C
C

SET THE MAIN DIAGONAL -DIAG-
C
DO 46 K=MINY,NAX

SET THE APPROPRIATE CONDITIONS FOR GULF BOUNDARIES

IF(J.EQ.9 .AND.K.EQ.MINY) GO TO 43
IF(J.EQ.18 .AND.K.EQ.MINY) GO TO 43
IF(J.EQ.23 .AND.K.EQ.MINY) GC TO 43
IF(J.EQ.27 .AND.K.EQ.MINY) GO TO 43
IF(K.EQ.MINY) GO TO 45

RP(K)=1.0*DELT/(2.0*DELX)*((V(J,K+1)-V(J,K-1))*G*(DELT/2.0)*DSQRT((.125*(U(J,K)+U(J,K+1)+U(J-1,K)+U(J-1,K+1))**2+(V(J,K)**2)))*(0.5*(C(J,K+1)+C(J,K)))**2)

I=K*2-C03
GO TO 44

CON JITIONS ARE SET FOR SOUTH SIDE GULF BOUNDARIES
ADVECTION OF MOMENTUM IS SET TO ZERO

43 RP(K)=1.0*DELT*(G/2.0)*DSQRT((.25*(U(J,K)+U(J,K+1)+U(J-1,K)+U(J-1,K+1))**2+(V(J,K)**2))/(0.5*(H(J,K)+H(J-1,K)+LP(J,K+1)+LP(J,K)))*(0.5*(C(J,K+1)+C(J,K)))**2)

DIAG(1)=RP(MINY)
GO TO 46

44 DIAG(1)=RP(K)
GO TO 46

45 DIAG(1)=REC
CONTINUE

DO 47 I=2,NAXC,2
DIAG(I)=1.0
CONTINUE

SET UP LOWER DIAGONAL -SUB-

MODIFY SUE FOR ENTRANCE GRID POINTS
48 CONTINUE
   DO 49 K=MINY,NAX
   I=K*2-C03
   SUB(I)=-RD(K)
49 CONTINUE
   IF(J.EQ.9)GO TO 150
   IF(J.EQ.18)GO TO 150
   IF(J.EQ.23)GO TO 150
   IF(J.EQ.27)GO TO 150
   SUB(1)=0.0
150 CONTINUE
   DO 50 K=MINY1,NAX
   I=K*2-C03
   SUB(I)=-RBC
50 CONTINUE
C SET UP THE CONSTANT VECTOR -B-
   DO 51 K=MINY,NAX
   BB(K)=V(J,K)+(-DELT/(2.0*DELX)*((V(J+1,K)-V(J-1,K))*0.5*(U(J,K)+V(J,K))
   1-U(J,K+1)+U(J-1,K)+U(J-1,K+1)))-G*DELT/(2.*DELX)*(L(J,K+1)-L(J,K))
   2-K)-G*DELT/2.0*V(J,K)*DSQRT((0.25*(U(J,K)+U(J,K+1)+U(J-1,K)+U(J-1,K+1))
   3-1,K+1))*2+(V(J,K))*2)/(0.5*(H(J,K)+H(J-1,K)+LP(J,K+1)+LP(J+1,K)+
   4,J,K))*0.5*(C(J,K+1)+C(J,K))*2)+DELT*TWINY/(RHO*(0.5*(H(J,K)+H(J+1,K)+
   5,J-1,K)+LF(J,K)+LP(J,K))))
   I=K*2-C03
   B(I)=BB(K)
51 CONTINUE
C SET THE BOUNDARY CONDITIONS IN THE CONSTANT VECTOR
   B(1)=BB(MINY)-RP(MINY)*V(J,MINY)
C MODIFY B FOR ENTRANCE GRID POINT
C
IF (J.EQ. 9) B(1) = B(B(MINY) + REC*LP(J,:ley))
IF (J.EQ. 18) B(1) = B(B(MINY) + REC*LP(J,:ley))
IF (J.EQ. 23) B(1) = B(B(MINY) + REC*LP(J,:ley))
IF (J.EQ. 27) B(1) = B(B(MINY) + REC*LP(J,:ley))
DO 53 K = MIN(1, NAX)
      A(K) = LP(J,K) + (H(J-1,K) + H(J-1,K-1) + LP(J,K) + LP(J-1,K)) * DEL/
      (4.0*D*LX) - (H(J,K) + H(J,K-1) + LP(J+1,K) + LP(J,K)) * DEL/
      (4.0*D*LX)
      I = K + 2 - CO2
      B(I) = A(K)
      53 CONTINUE

SET THE BOUNDARY CONDITIONS IN THE CONSTANT VECTOR
B(2) = A(MINY1) + RD(MINY) * V(J, MINY)
B(NAX) = A(NAX1) - RD(NAX) * V(J, NAX1)
IF (J.EQ. 9) B(2) = A(MINY1)
IF (J.EQ. 18) B(2) = A(MINY1)
IF (J.EQ. 23) B(2) = A(MINY1)
IF (J.EQ. 27) B(2) = A(MINY1)
WRITE(6,902)
DO 291 I = 1, NAX
      SET MATRIX ELEMENTS TO SIMULATE LATTICES
      I = 8
      DO 151 K = MIN(1, NAX)
      I = I + 1
      M = K + 2 - I
      DE(J,K) = H(J,K) + L(J,K)
      IF (DE(J,K) .LE. SEID) GO TO 152
      GO TO 151
      152 SUB(M) = 0.0
      DIAG(M) = 0.0
      SUP(M-1) = 0.0
SUB(M-1) = 0.0
DIAG(M-1) = 0.0
SUP(M-2) = 0.0
IF ((M-3) . EQ. 0) GO TO 151
SUB(M-2) = 0.0
DIAG(M-2) = 0.0
SUP(M-3) = 0.0
151 CONTINUE

C
C MATRIX IS SET SOLVE CALLING TRID
C
H=NAXC
CALL TRID(SUE, SUP, DIAG, B, M)

C
C PLACE SOLUTION INTO RESPECTIVE VECTORS
C
IF (J . EQ. 9) GO TO 154
IF (J . EQ. 18) GO TO 154
IF (J . EQ. 23) GO TO 154
IF (J . EQ. 27) GO TO 154
L(J, MINY) = B(1)
GO TO 254
154 V(J, MINY) = B(1)
254 CONTINUE
L(J, MINY1) = B(2)
DO 54 K = 4, NAXC, 2
I = K/2 + MINY
L(J, I) = B(K)
54 CONTINUE
DO 55 K = 3, NAXD, 2
I = (K + MINY2)/2
V(J, I) = B(K)
55 CONTINUE
CHECK = 0.0

C
C CHECK WATER LEVELS (L) IN THE BAY
DO 300 K=MIN,NAX1
    DE (J,K) = H (J,K) + L (J,K)
    IF (DE (J,K) .LE. SETD) GO TO 301
    GO TO 300
301    LL = 0.25 * (L (J-1,K) + L (J,K-1) + L (J+1,K) + L (J,K+1))
    DE (J,K) = H (J,K) + LL
    IF (DE (J,K) .LE. SETD) GO TO 300
    L (J,K) = LL
    DELL = LL / 3.0
305    L (J-1,K) = L (J-1,K) - DELL
    L (J,K+1) = L (J,K+1) - DELL
    L (J+1,K) = L (J+1,K) - DELL
    IF (J .EQ. MAX1) L (J-1,K) = L (J-1,K) - DELL
    CONTINUE
C
C    LEVELS HAVE BEEN CHECKED TO SETD AND VELOCITIES SET TO ZERO
C    WHILE CONSERVING WATER IN THE SYSTEM.
C
58 CONTINUE
C
*** SECOND STEP FOR HYDRODYNAMICS ***
*** HAS BEEN COMPLETED ***

VALUES FOR THE V VELOCITY ARE STORED IN V (J,K)
VALUES FOR THE WATER HEIGHT ARE STORED IN L (J,K)

V ( 9, 2) = V ( 9, 3)
V (18, 2) = V (18, 3)
V (23, 2) = V (23, 3)
V (27, 2) = V (27, 3)
IF (ITIDAL .EQ. 6) GO TO 399
GO TO 398
399 DO 717 J=MIN,MAX1
DO 717 K=MINA,NA
AVGU(J,K)=AVGU(J,K)+U(J,K)/600.
AVGV(J,K)=AVGV(J,K)+V(J,K)/600.
AVGL(J,K)=AVGL(J,K)+L(J,K)/600.
AVGLP(J,K)=AVGLP(J,K)+LP(J,K)/600.
717 CONTINUE

C******************************************************************************

C******************************************************************************

398 TIME=TIME+0.5*DELT
THR=TIME/3600.0
ITMR=ITMR+1
IF(THMR.EQ.ITPR)GO TC 78
C******************************************************************************

C******************************************************************************

GO TO 9
78 CONTINUE
C******************************************************************************

C******************************************************************************

ITMR=0
ITHR=THR+0.01
WRITE(6,915)THR
WRITE(6,914)
DO 79 K=MINA,NA
DO 7 91 J = MIN,MAXA
WRITE (6,916) J,K,U(J,K),V(J,K),L(J,K),LP(J,K)
79 CONTINUE
IF(ITHR.EQ.IITHIGH)GO TC 997
GO TO 9
997 IF(ITHIDAL.NE.1)GO TO 99
DO 718 K=MINA,NA
DO 7 18 J = MIN,MAXA
WRITE (6,916) J,K,U(J,K),V(J,K),L(J,K),LP(J,K)
718 CONTINUE
999 CALL EXIT
100 FORMAT(17F4.1)
101 FORMAT(1HC,5X,'DEPTH')
102 FORMAT(1HO,10X,'TIME OF LOW TIDE = ',F10.4,'HOURS',6X,'RANGE OF LOW TIDE = ',F10.4,'FEET')
103 FORMAT(1HO,10X,'TIME OF HIGH TIDE = ',F10.4,'HOURS',5X,'RANGE OF HIGH TIDE = ',F10.4,'FEET')
104 FORMAT(1HO,10X,'SEA TEMPERATURE = ',F10.4,'DEGS F',6X,'SEA SALINITY = ',F10.4,'PPT')
105 FORMAT(1HO,10X,'AIR TEMPERATURE = ',F10.4,'DEGS F',7X,'DEW POINT TEMPERATURE = ',F10.4,'DEGS F',11X,'AIR PRESSURE = ',F10.4,'MILLIBARS')
106 FORMAT(1HO,10X,'WIND SPEED = ',F10.4,5X,'WIND DIRECTION (WRT X-AXIS) = ',F10.4,'DEGREES')
107 FORMAT(1HO,10X,'SUN RISES = ',F10.4,'HOURS WRT TIME OF HIGH TIDE')
108 FORMAT(20A4)
109 FORMAT(1H1,14(/),1CX,2CA4,///)
110 FORMAT(F10.4)
903 FORMAT(1HO,15,4E20.7)
914 FORMAT(1X,///,1X,'GRID POINT',5X,'MAGNITUDE',8X,'U VELOCITY',7X,'V 1 VELOCITY',7X,'WATER LEVEL',17X,'(FT/SEC)',9X,'(FT/SEC)',9X,'(F1/2SEC)',9X,'(FEET)')
915 FORMAT(1H1,40('**'),TIME=';F10.4,'HRS.',4X,30('**))
916 FORMAT(1X,('**'),13,('**'),13,('**'),4(5X,E12.4))
918 FORMAT(1HO,5X,'CHECK = ',F8.2)
919 FORMAT(5F10.4)
920 FORMAT(5(4X,0.0*,3X))
921 FORMAT(7X,4E12.4)
STOP
END
SUBROUTINE INIT

REAL*8 U,V,L,LP,H
COMMON/A 1/C (40,20), DE (40,20)
COMMON/B 9/ L (40,20), LP (40,20), H (40,23), U (40,23), V (40,20)
COMMON/D 1/AVGU (40,20), AVGV (40,20), AVGL (40,20), AVGLP (40,20)
DO 7 J=1,40
DO 7 K=1,20
AVGU (J,K)=0.0
AVGV (J,K)=0.0
AVGL (J,K)=0.0
AVGLP (J,K)=0.0
U (J,K)=0.0
V (J,K)=0.0
LP (J,K)=0.0
L (J,K)=0.0
H (J,K)=2.0
DE (J,K)=0.0
C (J,K)=1.0
CONTINUE
RETURN
END
BLOCK DATA
INTEGER E1, E2, E3, E4
REAL N
COMMON/A3/TIME, TMAX, TPRINT, TIMEB, THR, DELT
COMMON/A4/DELX, SETD, CMANN, SETC
COMMON/A5/TWB, TAIE, TSEA, N, CP, EPS, SIG, QRAD
COMMON/A6/PAIR
COMMON/A7/SSFA
COMMON/A8/W, WANG
COMMON/A9/RHAIR, RHO, G
COMMON/B1/THRI1, THRI2, THRI3, THRI4
COMMON/B2/CORR
COMMON/E3/MIN, MAX, NAX, LMAX
COMMON/C1/FWR, PF
COMMON/C3/RR, TRB, TRE
COMMON/C4/E1, E2, E3, E4
COMMON/C5/TLOW, THIGH, RANGEH, RANGEL, TIMEFIN
DATA SETC /0.5/
DATA CMANN/0.026/
DATA MIN, MAX, NAX, LMAX/3, 37, 18, 9/
DATA RHAIR, W, WANG/0.08, 0.0, 0.0/
DATA RHO, TSEA, SSEA, CP/62.4, 60.0, 23.0, 1.0/
DATA TMAX, TPRINT, DELT, TIME/24.0, 120.0, 120.0, 0.0/
DATA CORR/0.0/
DATA THRI1, THRI2, THRI3, THRI4/-1.0, -1.0, -1.0, -1.0/
DATA G/32.2/
DATA SETD, DELX/0.1, 3900.0/
DATA TWB, TAIR, PAIR/20.1, 20.0, 1000.0/
DATA N, EPS, SIG, QRAD/5.0E-05, 0.97, 1.36, 0.33334/
DATA TIMER/0.0/
DATA TLCW, THIGH, RANGEH, RANGEL/12.0, 24.0, 1.1, 1.1/
DATA E1, E2, E3, E4/9, 18, 23, 27/
DATA PF/8.0/
DATA FWR/100C.0/
DATA RR, TRE, TRF/0.0, 0.0, 0.0/
END
SUBROUTINE TRID (SUP, SUP, DIAG, B)

C**** TRID -- TRIDIAGONAL EQUATION SOLVER (TAKEN FROM C.E.P. 184) ****
C------ ARRANGEMENTS ARE MADE FOR KNOWN D.P. VELOCITIES INSIDE THE MATRIX
C------ DIVISION BY ZERO IS AVOIDED
C SUBROUTINE SOLVES AX = B FOR THE VECTOR X (WHERE A IS TRIDIAGONAL)
C M = ORDER OF SYSTEM
C SUP = SUPER DIAGONAL OF A
C SUB = SUB DIAGONAL OF A
C DIAG = MAIN DIAGONAL OF A
C B = CONSTANT VECTOR
C SUP AND DIAG ARE DESTROYED
C SOLUTION VECTOR IS RETURNED IN B
DIMENSION SUB(80), SUP(80), DIAG(80), B(80)
REAL*8 SUB, SUP, DIAG, B

C
N = M
NN = N - 1
IF (DIAG(1) .EQ. 0.0) GO TO 5
SUP(1) = SUP(1)/DIAG(1)
B(1) = B(1)/DIAG(1)
GO TO 6

5 SUP(1) = 0.0
B(1) = 0.0
GO CONTINUE

6 C CONTINUE
DO 10 I = 2, N
II = I - 1
C DECOMPOSE A TO FORM A = LU WHERE L IS LOWER TRIANGULAR,
C AND U IS UPPER TRIANGULAR
DIAG(I) = DIAG(I) - SUP(II)*SUB(II)
IF (DIAG(I) .EQ. 0.0) GO TO 9
IF (I .EQ. N) GO TO 8
SUP(II) = SUP(II)/DIAG(I)

8 B(I) = (B(I) - SUB(II)*B(II))/DIAG(I)
GO TO 10

C------ COMPUTE Z WHERE LZ = B
B(I) = (B(I) - SUB(II)*B(II))/DIAG(I)
9 SUP(I) = 0.0
   B(I) = 0.0
10 CONTINUE
C-----COMPUTE x BY BACK SUBSTITUTION WHERE UX = Z
   DO 20 K = 1, NN
   I = N - K
20   B(I) = B(I) - SUP(I) * E(I+1)
RETURN
END

TRID 37C
TRID 39C
TRID 40C
TRID 42C
TRID 430
TRID 440
TRID 45C
SUBROUTINE EFAS5 (THR, TLFP)
COMMON/C5/TLCW, TTHGH, RANGEH, PANGEL, TIMEIN
TP=THIGH
TLFP=0.5*(RANGEH+PANGEL) + 0.5*(RANGEH-PANGEL) * COS (2. * 3.141592 * THR / TLFP)
RETURN
END
SUBROUTINE QEPASS (THR, TLQBP)
COMMON/C5/TLOW, THIGH, RANGEH, RANGEL, TIMKIN
  TP = THIGH
  IF (THR .LT. .875) GO TO 103
  TT = THR -.875
  TLQBP = 0.5 * (RANGEH + RANGEL) + 0.5 * (RANGEH - RANGEL) * COS (2. * 3.141592 * TT / T P)
  RETURN
103  TREST = TP -.875 * THR
  TLQBP = 0.5 * (RANGEH + RANGEL) + 0.5 * (RANGEH - RANGEL) * COS (2. * 3.141592 * TRES / TP)
  RETURN
END
SUBROUTINE CASST (THF, TLCP)
COMMON/C5/ ICW, THIGH, RANGEH, RANGEL, TIMEIN
TP=THIGH
IF (THR .LE. 1.356) GO TO 110
TT=THR-1.356
TLCP=0.5* (RANGEH+RANGEL) +0.5* (RANGEH-RANGEL) *COS (2.*3.141592*TT/TP 1)
RETURN
110 T REST=TP-1.358+THR
TLCP=0.5* (RANGEH+RANGEL) +0.5* (RANGEH-RANGEL) *COS (2.*3.141593*T REST 1/TP)
RETURN
END
SUBROUTINE RAIN
REAL*8 U, V, L, LP, H
COMMON/A3/TIME, Tmax, TPRINT, TIMER, THR, DELT
COMMON/B3/MIN, MAX, NAX, LMAX
COMMON/B9/L(40,20), LP(40,20), H(40,20), U(40,20), V(40,20)
COMMON/C3/RR, TREQ, TREF
MAX1 = MAX + 1
NAX1 = NAX + 1
IF (THR.LT. TREQ) GO TO 20
IF (THR.GE. TREQ) GO TO 20
DELL = RR*DELT/(3600.0*2.0)
DO 10 J = MIN, MAX1
    DO 10 K = MIN, NAX1
        L(J,K) = L(J,K) + DELT
10 CONTINUE
20 RETURN
END
***** GAY BAY SECOND TIDAL CYCLE *****
S SHADOW RANGEL=-0.0580, RANGEH=1.27294, TLLOW=10.0, THIGH=20.0,
FWH=3210.7351, DELT=120.0, TFIN=2.0,
&END
* * * * * GAY BAY SECOND TIDAL CYCLE * * * * * * * *

TIME OF LOW TIDE = 10.0000HOURS   RANGE OF LOW TIDE = -0.0058FEET
TIME OF HIGH TIDE = 20.0000HOURS  RANGE OF HIGH TIDE = 1.2722FEET
SEA TEMPERATURE = 60.0000DEGS F  SEA SALINITY = 28.0000PPTH
AIR TEMPERATURE = 20.0000DEGS F  WET BULB TEMPERATURE = 20.1000DEGS F
AIR PRESSURE = 1000.0000MILLIBARS
WIND SPEED = 0.0          WIND DIRECTION (WRT: X-AXIS) = 0.0 DEGREES
SUN RISES = 0.0          HOURS WRT TIME OF HIGH TIDE
APPENDIX C

CONCENTRATION ISOPLETHS
APPENDIX C

CONCENTRATION ISOPLETHS

This appendix gives concentration isopleths for different species for February and June for sensitivity analyses performed in this research. For comparison, isopleths of the typical case are also given. Table C.1 gives a complete list of figures in this appendix.

<table>
<thead>
<tr>
<th>TABLE C.1 List of Figures in Appendix C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Species</td>
</tr>
<tr>
<td>Coarse Detritus</td>
</tr>
<tr>
<td>Fine Detritus</td>
</tr>
<tr>
<td>Organic Nitrogen</td>
</tr>
<tr>
<td>Ammonia Nitrogen</td>
</tr>
<tr>
<td>Nitrite &amp; Nitrate Nitrogen</td>
</tr>
<tr>
<td>Animal Biomass</td>
</tr>
<tr>
<td>Phytoplankton Biomass</td>
</tr>
</tbody>
</table>
Isopleths for Higher Marsh Area
Figure C.1 Isopleths of concentrations for dissolved organic nitrogen, higher marsh area, June (concentrations in gm/m$^3$)
Figure C.2. Isopleths of concentration for dissolved ammonia nitrogen, higher marsh area, June (concentrations in gm/m$^3$)
Figure C.3. Isopleths of concentrations for dissolved nitrite and nitrate nitrogen, higher marsh area (concentrations in $10^2$ gm/m$^3$)
Figure C.4. Isopleths of concentrations for animal biomass, higher marsh area, June (concentrations in gm/m³)
Figure C.5. Isopleths of concentrations for phytoplankton, higher marsh area, June (concentrations in $10^2$ gm/m$^3$)
Figure C.6. Isopleths of concentrations for coarse detritus, higher marsh area, February (concentrations in gm/m$^3$)
Figure C.7. Isopleths of concentrations for fine detritus, higher marsh area, February (concentrations in gm/m³)
Figure C.8. Isopleths of concentrations for dissolved organic nitrogen, higher marsh area, February (concentrations in gm/m$^3$)
Figure C.9. Isopleths of concentrations for dissolved ammonia nitrogen, higher marsh area, February (concentrations in gm/m³)
Figure C.10. Isopleths of concentrations for dissolved nitrite and nitrate nitrogen, higher marsh area, February (concentrations in $10^2$ gm/m$^3$)
Figure C.11. Isopleths of concentrations for animal biomass, higher marsh area, February (concentrations in gm/m$^3$)
Figure C.12. Isopleths of concentrations for phytoplankton, higher marsh area, February (concentrations in $10^2$ gm/m$^3$)
Isopleths for Lower Marsh Area
Figure C.13. Isopleths of concentrations for dissolved organic nitrogen, lower marsh area, June (concentrations in gm/m$^3$)
Figure C.14. Isopleths of concentrations for dissolved ammonia nitrogen, lower marsh area, June (concentrations in gm/m$^3$)
Figure C.15. Isopleths of concentrations for dissolved nitrite and nitrate nitrogen, lower marsh area (concentrations in $10^2$ gm/m$^3$)
Figure C.16. Isopleths of concentrations for animal biomass, lower marsh area, June (concentrations in gm/m$^3$)
Figure C.17. Isopleths of concentrations for phytoplankton, lower marsh area, June (concentrations in $10^2$ gm/m$^3$)
Figure C.18. Isopleths of concentrations for coarse detritus, lower marsh area, February (concentrations in gm/m$^3$)
Figure C.19. Isopleths of concentrations for fine detritus, lower marsh area, February (concentrations in gm/m$^3$)
Figure C.20. Isopleths of concentrations for dissolved organic nitrogen, lower marsh area, February (concentrations in gm/m$^3$)
Figure C.21. Isopleths of concentrations for dissolved ammonia nitrogen, lower marsh area, February (concentrations in gm/m³)
Figure C.22. Isopleths of concentrations for dissolved nitrite and nitrate nitrogen, lower marsh area, February (concentrations in $10^2$ gm/m$^3$)
Figure C.23. Isopleths of concentrations for animal biomass, lower marsh area, February (concentrations in gm/m³)
Figure C.24. Isopleths of concentrations for phytoplankton, lower marsh area, February (concentrations in $10^2$ gm/m$^3$)
Isopleths for Higher Nutrient Case
Figure C.25. Isopleths of concentrations for dissolved organic nitrogen, higher nutrient case, June (concentrations in gm/m$^3$)
Figure C.26. Isopleths of concentrations for dissolved ammonia nitrogen, higher nutrient case, June (concentrations in gm/m$^3$)
Figure C.27. Isopleths of concentrations for dissolved nitrite and nitrate nitrogen, higher nutrient case (concentrations in $10^2 \text{ gm/m}^3$)
Figure C.28. Isopleths of concentrations for animal biomass, higher nutrient case, June (concentrations in gm/m$^3$)
Figure C.29. Isopleths of concentrations for phytoplankton, higher nutrient case, June (concentrations in $10^2$ gm/m$^3$)
Figure C.30. Isopleths of concentrations for coarse detritus, higher nutrient case, February (concentrations in gm/m$^3$)
Figure C.31. Isopleths of concentrations for fine detritus, higher nutrient case, February (concentrations in gm/m^3)
Figure C.32. Isopleths of concentrations for dissolved organic nitrogen, higher nutrient case, February (concentrations in gm/m$^3$)
Figure C.33. Isopleths of concentrations for dissolved ammonia nitrogen, higher nutrient case, February (concentrations in gm/m³)
Figure C.34. Isopleths of concentrations for dissolved nitrite and nitrate nitrogen, higher nutrient case, February (concentrations in $10^2$ gm/m$^3$)
Figure C.35. Isopleths of concentrations for animal biomass, higher nutrient case, February (concentrations in gm/m$^3$)
Figure C.36. Isopleths of concentrations for phytoplankton, higher nutrient case, February (concentrations in $10^2$ gm/m$^3$)
APPENDIX D

RELATIONSHIP BETWEEN NUTRIENT CONCENTRATION AND RATE COEFFICIENT $k_2$ FOR LIVE STANDING CROP OF MARSH GRASS
APPENDIX D

RELATIONSHIP BETWEEN NUTRIENT CONCENTRATION
AND RATE COEFFICIENT \( k_2 \) FOR LIVE STANDING CROP
OF MARSH GRASS

In Chapter VI, it was mentioned that the simulation
of the effect of addition of fertilizers on the marsh area
was performed by changing the rate constant \( (k_2) \) which rep­
resents the effect of crowding and self-shading in the rate
equation for marsh grass dynamic. In this appendix, it is
shown that this rate coefficient is inversely proportional
to the nutrient concentration.

The rate expression for the live standing crop of
marsh grass is:

\[
\bar{r}_1 = k_1 \cdot SS \cdot S_1 - k_2 S_1^2 - k_1 R S_1 - k_{12} S_1
\]

\[
= \text{Gross Production - Loss due to respiration and
conversion to dead crop}
\]

where Gross Production (GP) = \( k_1 \cdot SS \cdot S_1 - k_2 S_1^2 \)  \hspace{1cm} \text{(D.1)}

\[
GP = k_1 \cdot SS \cdot S_1 - k_2 S_1^2
\]

\[
\frac{d(GP)}{dP_1} = k_1 \cdot SS - 2 k_2 S_1 \hspace{1cm} \text{(D.2)}
\]

For GP to be maximum

\[
\frac{d(GP)}{dP_1} = k_1 \cdot SS - 2 k_2 S_1 = 0
\]
\[ S_1 = \frac{k_1 \cdot SS}{2 k_2} \quad \text{(D.3)} \]

Substituting (D.3) in (D.1),

\[ (GP)_{\text{max}} = k_1 S \cdot \frac{k_1 \cdot SS}{2 k_2} - 2 k_2 \left( \frac{k_1 \cdot SS}{2 k_2} \right)^2 \quad \text{(D.4)} \]

\[ \therefore (GP)_{\text{max}} = \frac{(k_1 \cdot SS)^2}{4 k_2} \quad \text{(D.5)} \]

Hence,

\[ (GP)_{\text{max}} 1 = \left[ \frac{(k_1 \cdot SS)^2}{4 k_2} \right]_1 \quad \text{(D.6)} \]

and

\[ (GP)_{\text{max}} 2 = \left[ \frac{(k_1 \cdot SS)^2}{4 k_2} \right]_2 \quad \text{(D.7)} \]

Taking the ratio of \((GP)_{\text{max}} 1\) and \((GP)_{\text{max}} 2\):

\[ \frac{(GP)_{\text{max}} 1}{(GP)_{\text{max}} 2} = \frac{\left[ (k_1 \cdot SS)^2 / 4 k_2 \right]_1}{\left[ (k_1 \cdot SS)^2 / 4 k_2 \right]_2} \quad \text{(D.8)} \]

Since \([k_1 \cdot SS]_1 = [k_1 \cdot SS]_2\)

\[ \frac{(GP)_{\text{max}} 1}{(GP)_{\text{max}} 2} = \frac{[k_2]_2}{[k_2]_1} \quad \text{(D.9)} \]

However, since the maximum gross production should be proportional to the amount of nutrient,
where \([N]\) is the concentration of nutrient.

Combining Eqs. (D.9) and (D.10),

\[
\frac{\text{(GP)}_{\text{max} 1}}{\text{(GP)}_{\text{max} 2}} = \frac{[N]_1}{[N]_2}
\]

\[
\text{(D.10)}
\]

Equation (D.11) shows that the rate coefficient \(k_2\) is inversely proportional to the concentration of nutrient.
APPENDIX E

USER'S MANUAL, FLOW DIAGRAM AND COMPUTER PROGRAM
FOR THE MATERIAL TRANSPORT MODEL
APPENDIX E

USER'S MANUAL, FLOW DIAGRAM AND COMPUTER PROGRAM
FOR THE MATERIAL TRANSPORT MODEL

This appendix gives user's manual, flow diagram and computer program for the material transport model developed in this research. The flow diagram for the computational scheme is given in Figure E.1. In Table E.1, a summary is given of the subroutines that were developed for the computer program. In addition to this table, the following details will be helpful in understanding the set-up of the computational scheme.

The main program works as a DRIVER program. In addition, the bathymetry data are read here. When the computations have reached the stage at which printing is desired, the main program does this.

Subroutine PINIT provides the initial conditions for all the species. The initial conditions used for January 1, 1969, are given in Table 6.3. The user can change them here if the starting period is other than the one mentioned above.

Subroutine HYDRO evaluates the forcing functions SOLAR and TEMP. The user may replace these functions with another set here, if the simulation is not being performed for the year 1969. This subroutine also does interpolation to

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obtain the values of velocities and depth.

Subroutine BLOCK DATA is the data package giving values of rate coefficients, dispersion coefficients, time-step $\Delta t$, size-step $\Delta x$, and boundary conditions. The rate coefficients, the dispersion coefficients and the boundary conditions used are given in Tables 3.11, 6.7, and 6.1 and 6.2 respectively. A time-step of three hours and a size-step of 3900 feet were used. Appropriate modifications can be made here if desired.

The assemblage of the program deck, the format of the input cards and the description of the variables used are given in Figure E.2, Table E.2 and Table E.2 respectively. A complete listing of the computer program is given in Table E.4.
Figure E.1. Computer flow diagram for explicit solution of the material transport model
TABLE E.1 Description of Subroutines

<table>
<thead>
<tr>
<th>Subroutine</th>
<th>Description</th>
<th>Additional Subroutines Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>INIT</td>
<td>Initializes all values used in the calculation</td>
<td>none</td>
</tr>
<tr>
<td>PINIT</td>
<td>Gives initial values of all species (concentrations) at the beginning of calculation</td>
<td>none</td>
</tr>
<tr>
<td>HYDRO</td>
<td>Evaluates forcing functions (solar radiation and temperature), reads monthly average velocities and interpolates velocities for a given time</td>
<td>none</td>
</tr>
<tr>
<td>PCXY</td>
<td>Calculates concentrations of species 1 and 2 (live marsh grass and dead marsh grass)</td>
<td>RATE</td>
</tr>
<tr>
<td>BLOCK DATA</td>
<td>Data package containing values of rate coefficients and dispersion coefficients</td>
<td>none</td>
</tr>
<tr>
<td>PC(I)</td>
<td>Calculated concentrations of species 3 through 9</td>
<td>RATE</td>
</tr>
<tr>
<td>RATE</td>
<td>Calculates value of sink and source term</td>
<td>none</td>
</tr>
</tbody>
</table>
Figure E.2. Program deck assemblage
### TABLE E.2 Format of the Input Cards

<table>
<thead>
<tr>
<th>Card</th>
<th>Variables</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bathymetry</td>
<td>H(J,K)</td>
<td>17F4.1</td>
</tr>
<tr>
<td>Monthly average water levels, velocities</td>
<td>U(J,K), V(J,K), RL(J,K)</td>
<td>7X, 3E12.4</td>
</tr>
<tr>
<td>Title card</td>
<td>Title</td>
<td>20A4</td>
</tr>
</tbody>
</table>

### TABLE E.3 Description of Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>H(J,K)</td>
<td>Depth, distance from the mean sea level to the bottom, at grid point J,K (feet)</td>
</tr>
<tr>
<td>RL(J,K)</td>
<td>Water level, height above mean sea level, at grid point J,K (feet)</td>
</tr>
<tr>
<td>D(J,K)</td>
<td>H(J,K) + RL(J,K)</td>
</tr>
<tr>
<td>U(J,K)</td>
<td>Velocity in the x-direction at grid point J,K (ft/sec)</td>
</tr>
<tr>
<td>V(J,K)</td>
<td>Velocity in the y-direction at grid point J,K (ft/sec)</td>
</tr>
<tr>
<td>Title</td>
<td>Title for identification of run</td>
</tr>
<tr>
<td>P(J,K,I)</td>
<td>Concentration of species I at grid point J,K</td>
</tr>
<tr>
<td>DISP(I)</td>
<td>Dispersion coefficient for species I</td>
</tr>
</tbody>
</table>
REAL L
COMMON/D1/P(40,20,9),DISP(9)
COMMON/S2/T,NN,A
COMMON/S5/DEL,T,DELX,DELP,ITEND
COMMON/B3/MIN,NAX,NAX,LMAX
COMMON/B4/MIN1,MAX1,NAX1,MINB
COMMON/B9/L(40,20),H(40,20),U(40,20),V(40,20)
DIMENSION TITLE(20)
100 FORMAT(F10.4)
101 FORMAT(17F4.1)
1000 FORMAT(1X,'('I3','I3','I3','I3'),'9(X,E12.4))
900 FORMAT(20A4)
901 FORMAT(1H1,15(/),46X,20A4)
C
MIN= STARTING GRID POINT FOR COMPUTATIONS IN X AND Y
C DIRECTION
C MAX= MAXIMUM NUMBER OF GRID POINTS IN X- DIATION
C NAX= MAXIMUM NUMBER OF GRID POINTS IN Y- DIATION
C ITT= COUNTER FOR STOPPING THE PROGRAM
C A= STARTING DAY OF THE YEAR - OBTAINED BY MULTIPLYING
C NUMBER OF MONTHS FOR WHICH COMPUTATIONS WERE COMPLETED
C BY 30
C T= NUMBER OF DAYS FOR WHICH COMPUTATIONS WERE COMPLETED
C PRIOR TO THIS RUN
C DELT= SIZE OF THE TIME STEP (DAYS)
C DELX= SIZE OF THE SIZE STEP (FT)
C DELP= SIZE OF THE SIZE STEP (METER)
C ITEND= NUMBER OF DAYS FOR WHICH COMPUTATIONS ARE REQUIRED
C L= HEIGHT ABOVE MEAN SEA LEVEL
C H= HEIGHT BELOW MEAN SEA LEVEL
C U= VELOCITY IN X- DIATION
C V= VELOCITY IN Y- DIATION
C HEAD FATHOMETRY DATA
C PINIT inputs initial values of all species
C HH IS THE DEPTH BELOW MEAN SEA LEVEL. SAME AS H IN THE
C CASE OF TYPICAL RUN, SHOULD BE CHANGED TO MODIFY INCREASE
C OR DECREASE OF MARSH AREA
C PCXY CALCULATES CONCENTRATIONS OF SPECIES 1 AND SPECIES 2
C DISP = DISPERSION COEFFICIENT
C PSEA = BOUNDARY CONDITION AT GULF
C PRESH = BOUNDARY CONDITION AT THE SOUTHERN BOUNDARY
READ(5,90)(TITLE,N)
WRITE(6,901)TITLE
MINA=MIN-1
MAX1=MAX+1
NAX1=UAX+1
MINB=MIN-2
ITT=0
ICOUNT=0
IK=0
A=0.0
T=0.0
NN=1
KKK=0
C INITIALIZE THE DATA
CALL INIT
C READ THE DATA
READ(5,100)H(9,2)
READ(5,100)H(16,2)
READ(5,100)H(23,2)
READ(5,100)H(27,2)
DO 8 J=MIN,MAX1
READ(5,101) (H(J,K),K=MIN,NAX1)
8 CONTINUE
CALL PINIT
C T IS COUNTER FOR NO. OF TIDAL CYCLE
9 T=T+.125
KKK=KKK+1
IK=IK+1
CALL HYDRO
C HYDRO INTERPOLATES HYDROSYNAMICS DATA
IF(KKK.EQ.1)GO TO 9
C CONCENTRATIONS CALCULATIONS
CALL PCXY
DO 555 I=3,LMAX
CALL PC(I)
555 CONTINUE
IF(IK NE. 9)GO TO 9
IK=0
ITT=ITT+1
ICOUNT=ICOUNT+1
IF(ICOUNT NE. 1)GO TO 9
ICOUNT=0
ITT=T
WRITE(6,200)ITT
200 FORMAT(1H1,5(/),50X,'THE DAY IS',5X,I1)
WRITE(6,201)
201 FORMAT(5(/),12X,9HLIVE CROP,4X,9HDEAD CROP,3X,6HCORSE,8X,4HFINE, 17X,9HDISSOLVED,4X,9HDISSOLVED,4X,9HDISSOLVED,5X,6HANIMAL,7X,6HPHYT 20/,12X,11HMARSH GRASS,2X,11HMARSH GRASS,3X,8HDETRITUS,5X,8HDETRITU 3S,5X,7HORG, 6X,6HNH4-N, 7X,7HNO2-NO3,7X,7HICLASS,6X,7HDIAMASS, 4///,1X,9GRID PT, 3X,7HGM/M**2,6X,7HGM/M**3,6X,7HGM/M**3, 6**3)
DO 73 K=MINA,MAX 1
DO 73J=MIN,MAX 1
WRITE(6,1000)J,K,(P(J,K,I),I=MIN,LMAX)
73 CONTINUE
IF(ITT NE. ITEND)GO TO 9
STOP
END
SUBROUTINE INIT
REAL L
COMMON/D1/P (40, 20, 2), D1SP (9)
COMMON/B9/L (40, 20), h (40, 20), u (40, 20), v (40, 20)
COMMON/C0/L (40, 20)
COMMON/C7/D (40, 20)
COMMON/C2/H (40, 20)
COMMON/P1/U1 (40, 20), u1 (40, 20), v1 (40, 20)
COMMON/P2/R1 (40, 20), R12 (40, 20)
COMMON/P3/U (40, 20), VV (40, 20)
DO 7 J = 1, 40
DO 7 K = 1, 20
UU (J, K) = 0.0
VV (J, K) = 0.0
U (J, K) = 0.0
V (J, K) = 0.0
D (J, K) = 0.01
U1 (J, K) = 0.0
V1 (J, K) = 0.0
U2 (J, K) = 0.0
V2 (J, K) = 0.0
R1 (J, K) = 0.0
R12 (J, K) = 0.0
DOLD (J, K) = 0.01
L (J, K) = 0.0
H (J, K) = 2.0
HH (J, K) = 0.0
7 CONTINUE
DO 8 J = 1, 40
DO 8 K = 1, 20
DO 8 I = 1, 9
P (J, K, I) = 0.0
8 CONTINUE
RETURN
END
SUBROUTINE PINIT
COMMON/D1/P(40,20,9),DISP(9)
COMMON/D2/HH(40,20)
COMMON/S8/DEL1,DELP,ITEND
COMMON/B3/MIN,MAX,NAX,LMAX
COMMON/B4/MIN1,MAX1,NAX1,N1N1
101 FORMAT(17F4.1)
DO 6 J=7,MAX
READ(5,101) (HH(J,K),K=MIN,NAX1)
6 CONTINUE
C
PINIT PUTS INITIAL VALUES OF CONCENTRATIONS
DO 50 I = MIN,LMAX
DO 50 J = MIN,MAX1
DO 50 K = MIN,NAX1
GO TO (31,32,33,34,35,36,37,38,39),I
31 IF( HH(J,K) .LT. 0.0) GO TO 51
P(J,K,I) = 253.3
GO TO 50
32 IF ( HH(J,K) .LT. 0.0 ) GO TO 52
P(J,K,I) = 1011.0
GO TO 50
33 IF ( HH(J,K) .LT. 0.0 ) GO TO 53
P(J,K,I) = 9.36
GO TO 50
34 IF ( HH(J,K) .LT. 0.0 ) GO TO 54
P(J,K,I) = 16.68
GO TO 50
35 P(J,K,I) = 1.184
GO TO 50
30 \( P(J, K, I) = 0.9579 \)
GO TO 50
37 \( P(J, K, I) = 0.0005 \)
GO TO 50
38 IF (HH(J, K) .GT. 0.0 ) GC TO 55
\[ P(J, K, I) = 0.655 \]
GO TO 50
55 \( P(J, K, I) = 8.655 \)
GO TO 50
39 IF (HH(J, K) .GT. 0.0 ) GO TO 50
\[ P(J, K, I) = 1.001 \]
GO TO 50
56 P(J, K, I) = 1.001
50 CONTINUE
RETURN
END
SUBROUTINE HYDRO
REAL L
COMMON/Q1/CF1,CF2
COMMON/S2/T,NN,A
COMMON/P1/U1(40,20),V1(40,20),U2(40,20),V2(40,20)
COMMON/F2/RL1(40,20),RL2(40,20)
COMMON/E9/L(40,20),H(40,20),U(40,20),V(40,20)
COMMON/C6/DOLD(40,20)
COMMON/C7/D(40,20)
COMMON/X1/Y,TO
COMMON/S1/SOLAR,TEMP
COMMON/S8/DELT,DELX,DELP,IT END
COMMON/B3/MIN,MAX,NAX,LMAX
COMMON/B4/MIN,MAX,MINA
TYEAR=T/360.
XX=Y*(TYEAR-TO)
X=Y*TYEAR
SOLAR=340.76*(1.+4384.507*COS(XX))
TEMP=22.175-3.1109*SIN(X)-8.0136*COS(X)-.4061*SIN(2*X)
1-.93931*COS(2*X)-.097773*SIN(3*X)+.096233*COS(J*X)+.24437*S
2*SIN(4*X)+.13399*COS(4*X)
DO 10 J=1,40
DO 10 K=1,20
10 DOLD(J,K)=D(J,K)
IF(NN.EQ.0)GO TO 20
DO 11 J=M1,K,NAX1
DO 11 K=M1,NAX1
11 READ(5,100)HI(J,K),V1(J,K),RL1(J,K)
DO 12 J=M1,K,NAX1
DO 12 K=M1,NAX1
12 READ(5,100)U2(J,K),V2(J,K),RL2(J,K)
NN=0
100 FORMAT(7X,3E12.4)
20 CONTINUE
TT=(T-A)/30.
DO 14 J=M1,NAX1
DO 14 K = MINA, NAX 1
U(J,K) = U1(J,K) + (U2(J,K) - U1(J,K)) * TT
V(J,K) = V1(J,K) + (V2(J,K) - V1(J,K)) * TT
U(J,K) = U(J,K) * CF1
V(J,K) = V(J,K) * CF1
L(J,K) = RL1(J,K) + (FL2(J,K) - RL1(J,K)) * TT
D(J,K) = (U * 0.25 * (H(J-1, K-1) + H(J, K-1) + H(J-1, K) + H(J, K)) + L(J,K)) * CF2
14 CONTINUE
END
SUBROUTINE PCXY
REAL I
REAL*8 HK
COMMON/S8/DELT, DELX, DEXP, ITEND
COMMON/B3/ MIN, MAX, NAX, LMAX
COMMON/B4/ MIN1, MAX1, NAX1, MIN6
COMMON/B9/L(40, 20), H(40, 20), U(40, 20), V(40, 20)
COMMON/D1/P(40, 20, 9), DISP(9)
DO 10 I = 1, 2
DO 10 K = MIN, MAX1
DO 10 J = MIN, MAX1
CALL RATE(J, K, I, HK)
P(J, K, I) = P(J, K, I) + DELT*HK
10 CONTINUE
RETURN
END
BLOCK DATA
INTEGER E1,E2,E3,E4
COMMON/C1/CF1,CF2
COMMON/C2/UVLAG3,UVLAG4,UVLAG8,UVLAG9
COMMON/D1/F(40,20,9),DISP(9)
COMMON/D4/PESE(9),PPFSE(9)
COMMON/S8/DLT,DELX,DELP,ITEND
COMMON/E4/MIN,MAX,NAX,LYX
COMMON/C4/E1,E2,E3,E4
COMMON/V1/Y,T0
COMMON/X2/FAC,ALPHA1,ALPHA2
COMMON/X3/AK1,AK2,AK5D
COMMON/X4/AK63,AK63,AK3R
COMMON/X5/AK34,AK4R
COMMON/X6/AK64,AK94,AK84
COMMON/X7/AK48,AK55,AK56
COMMON/X8/AK86,AK67,ADN
COMMON/X9/AKFISH,AK8P,AK98
COMMON/X10/AP,AZ,AK9,AK45
COMMON/A11/A3,A4,Ad,A9
DATA MIN,MAX,NAX,LMAX/3,37,13,9/
DATA UVLAG3,UVLAG4,UVLAG8,UVLAG9/1,5,2,2/
DATA CF1,CF2/2194.56,.3048/
DATA DLT,DELX,DELP,ITEND/0.1250,.3900,0,1188.72,30/
DATA E1,E2,E3,E4/9,18,23,27/
DATA Y,T0/2.284,.472/
DATA FAC,ALPHA1,ALPHA2/2.0,0.0,0.042466/
DATA AK1,AK2,AK5D/0.045,0.011942,8.0/
DATA AKFISH,AK8R,AK3H/.1635115519,20.87177300,15.7238/
DATA AK66,AK67,ADN/0.0,324.3/
DATA AK48,AK55,AK66/1.52022739,.1416548179,2.0/
DATA AK64,AK99,AK34/.20720049,2.8215,20.46893247/
DATA AK38,AK34,AK4R/9.627025,50.2520707,21.2593848/
DATA AK93,AK63,AK3R/9.0862748,.0943499,17.06522608/
DATA AK9,AK45/4.34799,.41/
DATA AP,AZ/2.5,1.5/
DATA A3, A4, A8, A9/100.0, 58.8, 33.3, 25.0/
DATA DISP/2*0.0, 2*52500.00, 2*1750.00, 2*2500.00/
DATA PSEA/2*0.0, 1.0, 3.0, 0.27, 0.9, 0.013, 2.0, 0.009/
DATA PPRESS/2*0.0, 9.9, 9.27, 35.35, 0.13, 1.8, 0.009/
END
SUBROUTINE PC(I)
REAL L
INTEGER E1, E2, E3, E4
REAL*8 HK
REAL*8 DUP, EVP, EFX, EFY, EXY1, DDXY2, TRUE
COMMON/E3/UU (40, 20), VV (40, 20)
COMMON/D4/PSA (9), PFRSH (9)
COMMON/C7/L (40, 20)
COMMON/E4/MINAX, NAX1, NAX2, MINB
COMMON/B3/MAIN, MAX, NAX, IMAX
COMMON/B9/L (40, 20), H (40, 20), U (40, 20), V (40, 20)
COMMON/C4/E1, E2, E3, E4
COMMON/C6/DOLD (40, 20)
COMMON/S8/DLTT, DELX, DELP, ITLND
COMMON/Q2/UVLAG3, UVLAG4, UVLAG5, UVLAG6, UVLAG9
COMMON/D1/P (40, 20, 9), DISP (9)
DO 42 J = MIN, MAX1
DO 42 K = MIN, NAX1
GO TO (31, 32, 33, 34, 35, 36, 37, 38, 39), I
31 GO TO 42
32 GO TO 42
33 UU (J, K) = U (J, K) * UVLAG3
VV (J, K) = V (J, K) * UVLAG3
GO TO 42
34 UU (J, K) = U (J, K) * UVLAG4
VV (J, K) = V (J, K) * UVLAG4
GO TO 42
35 UU (J, K) = U (J, K)
VV (J, K) = V (J, K)
GO TO 42
36 UU (J, K) = U (J, K)
VV (J, K) = V (J, K)
GO TO 42
37 UU (J, K) = U (J, K)
VV (J, K) = V (J, K)
GO TC 42
8 \ UU(J,K) = U(J,K) \cdot UVLAG8
9 \ VV(J,K) = V(J,K) \cdot HVLAG8

GO TO 42

9 \ UU(J,K) = U(J,K) \cdot UVLAG9
10 \ VV(J,K) = V(J,K) \cdot HVLAG9

CONTINUE

P(E1,2,I) = PSEA(I)
P(E2,2,I) = PSEA(I)
P(E3,2,I) = PSEA(I)
P(E4,2,I) = PSEA(I)
P(17,19,I) = PFRESH(I)
P(26,19,I) = PFRESH(I)

DO 71 J = MIN, MAX1
DO 71 K = MIN, MAX1

A = D(J,K)
CALL RATE(J,K,I,HK)

DUP = (D(J+1,K) * U(J+1,K) * P(J+1,K,I) - D(J-1,K) * U(J-1,K)
1 * P(J-1,K,I)) / (2. * DELP)

DVP = (D(J,K+1) * V(J,K+1) * P(J,K+1,I) - D(J,K-1) * V(J,K-1)
1 * P(J,K-1,I)) / (2. * DELP)

DXF = (DISP(I) * (D(J+1,K) - D(J,K))
1 * (P(J+1,K,I) - P(J,K,I)
2 + P(J-1,K,I)) / (DFLE**2)

DFY = (DISP(I) * (D(J,K+1) - D(J,K))
1 * (P(J,K+1,I) - P(J,K,I)
2 + P(J,K-1,I)) / (DFLE**2)

TREE = D(J,K) / OLD(J,K)
TGRAD1 = DOLD(J,K) * UU(J,K) * P(J,K,I)
TGRAD2 = DOLD(J,K) * VV(J,K) * P(J,K,I)
TG1 = ABS(TGRAD1)
TG2 = ABS(TGRAD2)

IF(TG1 .LT. 1.E-6) DUP = 0.0
IF(TG2 .LT. 1.E-6) DVP = 0.0
TGRAD3 = P(J,K,I)
TG3 = ABS(TGRAD3)

IF(TG3 .LT. 1.E-6) DXF = 0.0
IF (TG3 .LT. 1, F-6) CFY = C. 0
IF (J .EQ. 3) GO TO 51
IF (J .EQ. MAX1) GO TO C 51
IF (K .EQ. 19) GO TO 55
IF (K .EQ. 3) GO TO 10 56
GO TO 60
55 IF (J .EQ. 17) GO TO 71
IF (J .EQ. 26) GO TO 71
GO TO C 51
56 IF (J .EQ. E1) GO TO 60
IF (J .EQ. E2) GO TO 60
IF (J .EQ. E3) GO TO 60
IF (J .EQ. E4) GO TO 60
51 DFX = 0.0
DFY = 0.0
DUP = 0.0
DVP = 0.0
CONTINUE
DDXY1 = P(J, K, I) * SFE
DDXY2 = (DELTA/A) * (DUP + DVP - DFX - DFY - H K)
P(J, K, I) = DXY1 - DXY2
71 CONTINUE
RETURN
END
SUBROUTINE RATE(J, K, I, FF)
REAL L
REAL*8 RR
REAL*8 F1
REAL*8 AKR9, BK, AR, BB
COMMON/S8/DELT, DELK, DELP, ITEND
COMMON/S1/SOLAR, TEMP
COMMON/B8/L (40, 20), H (40, 20), U (40, 20), V (40, 20)
COMMON/C6/DOLD (40, 20)
COMMON/D1/P (40, 2C, 9), DISF (9)
COMMON/C7/D (40, 20)
COMMON/X2/FAC, ALPHA1, ALPHA2
COMMON/X3/AK1, AK2, AK5D
COMMON/X4/AEK3, AK63, AK3R
COMMON/X5/AK38, AK34, AK4R
COMMON/X6/AK64, AK94, AK84
COMMON/X7/AK48, AK95, AK56
COMMON/A8/AKE6, AK67, ADN
COMMON/X9/AKISH, AK8R, AK98
COMMON/X10/AR, AK9, AK45
COMMON/X11/A3, A4, A8, A9
F1 = 1.0/360.
AK12 = (3.6*(20.-TEMP)/20. +6.9)
AK1R = .4693*3.0** (TEMP/10.)
AK2R = .025*2.25**(TEMP/10.)
AK23 = .4*2.25**(TEMP/10.)
AK99 = 0.0175*EXP (0.069*TEMP) *3b5.
SOLE = SOLAR/(24.*6C.)
IF (P (J, K, 6) .LT. .2 OR. P (J, K, 0) .EQ. .2) GO TO 41
AN = 1.0
GO TO 42
41 AN = P (J, K, 6) / .2
42 APK = APK * AN * AK9 * SOLE * 365.
GO TO (11, 12, 13, 14, 15, 16, 17, 18, 19), 1
11 RK = 1AK12 * F (J, K, 1) - AK1R * P (J, K, 1) * F1
8B = (AK1 * SOLAR * P (J, K, 1) - (AK2/FAC ) * P (J, K, 1)**2) * F1
12 \( RK = -(AK23 + AK2R) \times F(J, K, 2) \times F1 \)
\( AR = (AK12 \times P(J, K, 1)) \times F1 \)
\( BB = 0.0 \)
\( RR = AR + RK + EE \)
\( \text{RETURN} \)
13 \( RK = (AK83 \times P(J, K, 8) + AK63 \times A3 \times P(J, K, 6) - (AK3R + AK3G) \times P(J, K, 3)) \times F1 \)
\( AR = AK23 \times P(J, K, 2) \times (.5245) \times F1 \)
\( BB = 0.0 \)
\( RR = AR + COLC(J, K) \times (RK + BR) \)
\( \text{RETURN} \)
14 \( RK = (AK94 \times P(J, K, 9) + AK84 \times P(J, K, 8) + AK64 \times A4 \times i(J, K, 0) - (AK4R + AK4G) \times \) 
\( P(J, K, 4)) \times F1 - AK45 \times E(J, K, 4) \times F1 \)
\( AR = AK34 \times E(J, K, 3) \times F1 \)
\( BB = 0.0 \)
\( RR = D(J, K) \times AR + COLC(J, K) \times (RK + BB) \)
\( \text{RETURN} \)
15 \( RK = (AK85 \times P(J, K, 8) / A8 - AK56 \times P(J, K, 6) + AK45 \times F(J, K, 4) / A4) \times F1 \)
\( AR = 0.0 \)
\( BB = 0.0 \)
\( RR = D(J, K) \times AR + COLC(J, K) \times (RK + BB) \)
\( \text{RETURN} \)
16 \( RK = (AK67 \times P(J, K, 6) / A7 - AK58 \times F(J, K, 5) + AK45 \times F(J, K, 4) / A4) \times F1 \)
\( AR = 0.0 \)
\( BB = 0.0 \)
\( RR = D(J, K) \times AR + COLC(J, K) \times (RK + BB) \)
\( \text{RETURN} \)
17 \( RK = (AK67 \times P(J, K, 6) - AK65 \times E(J, K, 7)) \times F1 \)
\( AR = 0.0 \)
\( BB = 0.0 \)
\( FR = D(J, K) \times AR + COLC(J, K) \times (RK + BB) \)
\( \text{RETURN} \)
18 \( RK = (AK95 \times P(J, K, 9) - AK83 \times P(J, K, 8) - (AK83 \times AK85) \times F(J, K, 8) - (AK85 +
AK86) * P(J, K, 8) * F1
AR = (AK48 * P(J, K, 4) + AK86 * P(J, K, J)) * F1
BB = 0.0
RR = D(J, K) * d + DOLD(J, K) * (RK + BB)
RETURN

19 RK = (APH - AKF9 - AK98 - AK94) * P(J, K, 9) * F1
AR = 0.0
BB = 0.0
RR = AR * D(J, K) + DOLD(J, K) * (RK + BB)
RETURN
END
SIMULATION OF TYPICAL CASE - JANUARY 1969
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Note: The table continues with similar entries.
VITA

Hasitkumar Kantilal Trivedi was born on September 4, 1945, in Kapadwanj, Gujarat, India.

After graduating from Sheth M.R. High School, Kathlal, Gujarat, India, in 1962, he attended St. Xavier's College, Ahmedabad, India, for two years. He enrolled in the Bombay University Department of Chemical Technology, Bombay, India, in 1964 and received the degree of Bachelor of Chemical Engineering in 1969. He then entered Lamar University, Beaumont, Texas, and received his Master of Engineering degree in Chemical Engineering in 1970. He came to Louisiana State University in Baton Rouge, Louisiana, in January 1971.

Upon completion of his research work in March 1974, he joined NUS Corporation, a consulting and engineering firm in the field of energy, in Rockville, Maryland. In April 1976, he joined Air Products and Chemicals, Inc., in Pensacola, Florida, where he is presently working with the Technical Department.

Married to the former Miss Mona Morzaria of Bombay, India, the author is currently a candidate for the degree of Doctor of Philosophy in the Department of Chemical Engineering at Louisiana State University.
EXAMINATION AND THESIS REPORT

Candidate: HASITKUMAR KANTILAL TRIVEDI

Major Field: CHEMICAL ENGINEERING

Title of Thesis: TRANSPORT PHENOMENA IN A BAY-MARSH SYSTEM

Approved:

Ralph W. Peters
Major Professor and Chairman

James A. Ingalls
Dean of the Graduate School

EXAMINING COMMITTEE:

Philip A. Bryant
Douglas P. Kerness
Thomas G. Long
Ren Wilkins

Date of Examination:

MARCH 23, 1976