Modeling and Optimal Control of Insect Pest Population Dynamics.

Norman Frank Marsolan Jr
Louisiana State University and Agricultural & Mechanical College
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by

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ABSTRACT

Pest management is required to decrease the insect damage inflicted on agricultural crops such as soybeans. Pesticides are used to control these pests, but to frequently without evaluation of pest population and crop dynamics. It is necessary to consider these dynamics when seeking the best time or times to apply pesticides. The objective of this research is to develop a modeling/optimization technique that permits development of optimal control policies which minimize the combined cost of insect pest control by pesticides and economic yield loss from pest damage. A procedure from variational calculus and a non-linear programming technique are used independently for the numerical optimization of the model. Optimal control policies are developed which are distributed both in time and in time and species age.

The soybean pests modeled by distributed parameter dynamic models are the Southern green stink bug, *Nezara viridula* (Linnaeus), and the velvet bean caterpillar, *Anticarsia gemmatalis* Hubner. The stink bug and caterpillar alter the soybean plant model's yield by depodding and defoliation. The assumed predator-parasite complex is represented by the logistic equation and is controlled by insecticide use. This biological complex contributes to the mortality of the defoliating caterpillar and, therefore, has the capability of influencing pesticide policies.
The predator-parasite complex is indeed found to significantly alter the optimal timing of pesticide application. In fact, there does exist conditions in which the number of discrete pesticide applications is increased by the complex. This is especially true in soybean varieties of extended maturation periods.

Levels of infestations are found to significantly depend on the stage of soybean development, as well as the pest population's levels. In general, the crop can tolerate much higher levels of caterpillar infestation than is allowed in current practice.

The continuous policies of pesticide application, evaluated with the distributed minimum principle, offer valuable information regarding the dynamics of the crop ecosystem components. The logical pattern search procedure provides the best discrete times for single and multiple pesticide applications providing results useful for field application.
CHAPTER I

INTRODUCTION

The ever-increasing demand for more food by a growing world population creates a need for the efficient production of agricultural crops. In the last several decades we have achieved some success by relying heavily on the use of fertilizers, plant growth regulators, herbicides and insecticides to increase yield. But, we rely on the use of these chemical tactics without having fully researched and developed the potential for cultural manipulation and biological control. The object of current active research by investigators in many disciplines is to remedy this position.

Cultural manipulation of agricultural crops includes the use of such methods as varying crop row spacings, irrigation practices and plant breeding. In addition to this approach, there is increasing emphasis on biological controls, such as the manipulation of predator species populations to enhance natural pest control. These techniques tend to be preferable economically and environmentally in comparison to the use of chemicals to improve yields. To date the use of insecticides is by far the most prevalent means of pest management. Extensive research programs are needed to provide growers with alternatives to this.

As long as the use of pesticides continues to be of such importance,
we must attempt to use it as effectively as possible, while minimizing economic and environmental costs. It is the purpose of our research to study this problem.

The crop ecosystem presents a very complex system, one well-suited for study by systems analysis. Components of the system include the plants and their resulting yield, the insects and their damage potential and man and his ability to tune the system. Figure 1-1 illustrates such a system.

![Diagram](image)

**Figure 1-1. Major Crop Ecosystem Components**

The soybean ecosystem is an example of a complex crop system worthy of study. The soybean is becoming an important source of protein for the world and an important cash crop of the southern United States.
Last year in Louisiana alone, 2 million acres were planted in soybeans. To increase soybean yield without detrimental effects on the ecosystem would be of great benefit.

The soybean ecosystem includes a rather large complex of insects and their natural controls. No single insect species has yet emerged as the primary pest because of the relatively short period of time since the crop became commercially important. Soybean insect pests include foliage, pod, stem and root feeders. Many of them are harmless or even beneficial, for they may be predators or parasites of harmful pests. Some insects may even stimulate the plant to release hormones which may aid the plant in becoming resistant to other pests. However, some insects cause yield loss and possible crop failure if not controlled.

Insect pest populations vary greatly each season and from region to region. Soybeans must, therefore, be scouted for pests at intervals throughout the season. Two particular species that must be monitored in the Gulf-South are the velvet bean caterpillar (Anticarsia gemmatalis Hubner) and the Southern green stink bug (Nezara viridula, Linnaeus). The velvet bean caterpillar is a defoliator and in large numbers can completely strip a field of vegetation in three days time. In 1975 in Georgia this caterpillar is estimated to have caused $25 million in damage. A pod feeder, the Southern green stink bug is estimated to have aggravated $5 million worth of damage during the same period (Bramblett, 1976). In Louisiana the stink bug is probably the major insect pest of soybeans.

In the research described herein, we simulate the population
dynamics of the stink bug and velvet bean caterpillar, along with a pesticide-sensitive caterpillar predator-parasite (PP) complex. This complex accounts for the natural biological control that exists in the field. We also simulate soybean plant growth dynamics in order to obtain seed yields and losses due to insect damage. Simulated are two varieties which are seasonally planted in the South. One, Dare is an early maturing variety, while Lee matures later and is, therefore, more acceptable to damage by late season defoliators.

An optimal pesticide application policy for pest management could be developed either for a region of the country or for an individual grower. A regional policy would require a thorough knowledge of pest populations over a large area, as well as a large amount of economic data. However, such data is not yet available. Because acceptance is a major aspect of a successful management program, and the grower is an integral part of any pest management policy, we concentrate our efforts on the developing of policies for individual growers.

The grower's economic concerns include the cost of production, pest management and pest damage. We account for dollars lost from soybean quantity and quality reduction due to the stink bug and velvet bean caterpillar. The dollar return from the sale of beans and the cost of pesticide is also included. Such non-quantitative factors as pesticide pollution and social costs are not discussed in this research. These factors can be treated only qualitatively given the present data.

We employ two mathematical techniques in our pest management policy studies. A logical search procedure is used to determine the
best day or days for discrete pesticide application. Also, as a matter of theoretical and practical interest, we seek the optimal continuous pest control policy. A technique based in variational calculus and developed in modern control theory provides the method for determining this latter policy. The development of continuous policies is becoming of greater importance as more is learned about the continuously acting biological control agents. We begin our research report with a review of past work in population and plant modeling and optimization.

Population Modeling

Two forms of mathematical models are used in describing biological species dynamics—stochastic and deterministic. Stochastic simulation techniques were developed in the 1940's, but do not promise widespread use in describing complex systems because of the large number of experiments required to obtain sufficient statistics. Stochastic population dynamic models include those of Mann (1971), Jaquette (1970), Becker (1970), and Menke (1973).

Deterministic models have found greater acceptability since they are frequently easier to solve and often offer an analytical solution. Verhulst (1838) is often credited with initiating the application of model building to life processes with his derivation of the logistic equation to describe birth and death of population members. This work was virtually ignored until 1920 when Pearl and Reed (1920) rediscovered it in their demographic work. This initial research led to modeling efforts based on a similar foundation of assumptions. The intrinsic factors of populations were used to describe variations
in population magnitude, but such extrinsic factors as climate, habi-
tate, food sources and competition were ignored. Even in recent models
these same basic assumptions are often made with reasonable success.
During the 1920's and 1930's a number of new models were developed.
The mathematicians Lotka (1925), Voltera (1926) and Bailey (1931),
were apparently stimulated or encouraged by the work of biologists
Thompson (1922), D'Ancona and Nicholson, respectively, to engage in
population model building.

Bailey (1931), Lewis (1942) and Leslie (1945) incorporated the age-
dependent structure of a population's dynamics into their modeling.
Von Foerster (1959) then Hoyle (1963) were perhaps the first to
introduce use of partial differential equations to describe age-size
population growth. The partial differential species equation of change
is called the von Foerster equation in the life sciences. Streifer
(1974) generalized the von Foerster equation to various species and
introduced such affects as species interaction.

Insect species normally progress through a series of distinct
physiological stages with the progression being described better as a
probabilistic distribution than as a single discrete time delay. Rudd
(1976) introduced this concept into population modeling with use of
reservoir theory (Eriksson, 1971) or the impulse response function. The
technique involves convoluting the flow of population numbers from a
previous stage with a probability distribution to yield a distributed
stage output. The "state of the art" for single species population
dynamic's models rests with this concept. Ruesink (1975) accomplishes
this effect with a discrete "box-car" model, but with little flexibil-
ity in the choice of probability distributions.

The age-dependent models of Bailey (1931) and von Foerster (1959) take developmental stage dependent data into account by dividing the age distribution deterministically into mean lengths of survival for each stage. Marsolan and Rudd (1975) use the impulse response function and calculate a stage probability distribution for their partial differential equations.

Even with these sophistications, many intrinsic and extrinsic variables are often assumed to have negligible effects. Temperature is known to have a significant effect on developmental rates in some regions on some species of insects. This effect is being actively studied by several researchers (Gutierrez, 1975; Stinner, 1975; Ruesink 1975). Humidity, light reception, rainfall and other extrinsic factors affect intrinsic behavior, but little quantitative information is available concerning their effects.

Species Interaction

Species interactions are very difficult to describe, primarily due to the lack of data to support theories. Only the simplest conclusions are amendable to mathematical description. Lokta (1925) and Volterra (1926) independently developed the first model for predator-prey relations. The assumptions which are incorporated include density-dependent birth and death rates. These rates, which are based on the probability of collision or encounter of individuals is the product of each density (as in chemical kinetics). Climactic effects are neglected. While biologically simple, these models are mathematically tractable and of some academic interest. Nicholson and Bailey (1935)
describe in a similar fashion parasite-prey interactions, Lee (1975) accounts for parasitism in his convective model. Hollings (1959, 1966) developed component models in which effects of hunger, prey species learning and feeding are taken into account.

**Plant Models**

To accurately account for crop growth and its response to insect pest infestation we must have a plant model. Several complex descriptive programs have been written to simulate various crops. The development of forest and crop simulation models has been an extremely active area of research.

Cotton models (Baker, et. al., 1974; Gutierrez, et. al., 1975) have been developed which simulate growth in root structure, above-ground foliage, dry matter production, nutrient concentration, leaf and boll count and location. Other crop models, developed principally by agronomists, include those of corn (deWit, et. al., 1970; Duncan, 1974), alfalfa (Fick, 1975; Bula, et. al., 1974; Miles, et. al., 1973), soybeans (Rudd, 1975; Weaver, 1975; Curry, 1974; Johnson, et. al., 1974) and others. These models are all deterministic.

Several plant models (Gutierrez, et. al., 1974, 1975; Jones, et. al., 1974) incorporate insect pest feeding. It is, however only a recent advance to incorporate the plant's response to pest injury.

Data have been collected to discern the impact of the depodding and defoliation on soybean production. Studies of mechanical defoliation include Kalton, et. al. (1949) and Thomas, et. al. (1974). Depodding has been investigated by Thomas, Newsom et. al. (1974) and Todd and Turnipseed (1974). Many workers are now attempting to determine
the effects of insect damage on crop yield.

Optimization

The use of mathematics to determine the best policies for control of insect pest populations in order to obtain greater crop productivity has advanced through the use of two approaches—simulation and optimization theory. Simulation requires the repeated simulation of the ecosystem with various policies of control from a set of assumed tactics. Watt (1962) discusses its role in pest management. One disadvantage is that it yields only the best policy of those tested. Optimization is a better approach to pest management, since by logical or mathematical techniques it produces the best control policy of all possible.

Watt (1963) was one of the first to apply optimization to determine the "best" pest management program out of a set of alternations. Jaquette (1970), Mann (1971) and Becker (1970) applied a variational approach to their Markov models. Using deterministic ordinary differential equation models, Watt (1963) and Shoemaker (1973) used dynamic programming, while Vincent (1972) and Leitmann (1972) employed variational calculus. Gupta and Rink (1973) applied similar techniques to epidemic control. Marsolan and Rudd (1975) applied the variational calculus to a partial differential equation model describing an insect pest population.
Summary

We proceed with our analysis of the soybean ecosystem by first introducing the mathematical and numerical techniques employed in this research. The form of the system's equations and the two optimization methods employed are presented. Chapter III discusses the major soybean and pest components which are modeled. The specific equations used are outlined and solved and where possible, verified.

The optimal soybean pest management policies obtained via the calculus of variations and non-linear programming are presented in Chapter IV. Results are given in this chapter, while final conclusions follow in Chapter V.
CHAPTER II

THEORY

In this chapter we present the general mathematical and numerical methods for the modeling and optimization of the plant growth and pest population dynamics.

We wish to model the population dynamics of a species with a model which will fully utilize the available data. An age-dependent partial differential equation model is used for we believe it to offer the greatest power while being mathematically tractable. In this chapter we formally present representative equations and the numerical techniques used in obtaining their solutions.

Because most biological species progress through observable stages of physiological development, most developmental data is stage-dependent. We employ a technique used in chemical engineering stage processes to probabilistically distribute the individuals of a continuous age model, as presented by the population dynamics equation, among a series of discrete stages. The approach is based on the concept of the impulse response function.

The criteria for an optimal continuous pesticide policy is derived based on the set of ordinary and partial differential equations used to represent the system dynamics. The numerical techniques used to solve this set of equations is discussed. We continue with a description of a
non-linear programming technique used to obtain the optimal timing of discrete control applications.

**POPULATION DYNAMICS**

Many systems can be represented by first order hyperbolic partial differential equations of the form

\[ g(t, \mathbf{y}, \mathbf{z}, \frac{\partial \mathbf{z}}{\partial t}, \frac{\partial \mathbf{z}}{\partial \mathbf{y}}, \mathbf{u}) = 0 \]  

2-1

where \( \mathbf{y} = (y_1, y_2, \ldots, y_q) \), to be referred to as the spatial coordinate, is a point belonging to a subspace \( D \) of a Euclidean \( q \)-dimensional space. Within the context of the population dynamics model the elements of the independent variable \( \mathbf{y} \) could represent species age, mass and spatial coordinate, for example. The time variable \( t \) is defined on \( 0 \leq t \leq 1 \). \( \mathbf{u}(t, \mathbf{y}) \) is a vector function describing the control trajectories from a set of possible trajectories, \( V \). \( V \) is a \( j \)-dimensional set of continuous and piecewise continuous functions contained in \( D \) satisfying a set of control inequality constraints, \( C_j \). Constraining the pesticide control tactic to inflicting between a zero and 95 percent mortality is an example of an inequality constraint. The dependent variable \( \mathbf{z}(t, \mathbf{y}) = (z_1(t, \mathbf{y}), z_2(t, \mathbf{y}), \ldots, z_n(t, \mathbf{y})) \) characterizes the state of the system for any \( t \) and \( \mathbf{y} \). The initial and boundary conditions are \( \mathbf{z}(0, \mathbf{y}) \) and \( \mathbf{z}(t, \mathbf{y} = \mathbf{y}_0) \), respectively.

In addition to the population dynamics partial differential equations we simultaneously employ a set of ordinary differential equations for the plant model and biological control complex. Though we may consider these ordinary differentials as a special case of the partial differential equations, we find it useful in practice to derive the necessary conditions for the optimum in terms of both forms.
The lumped parameter state variable $z(t)$ may be described as

$$\frac{dz}{dt} g(t, z, u) = 0$$  \hspace{1cm} \text{(2-2)}

with initial conditions $z(t = 0)$.

**System Numerical Solution Techniques**

Analytic solution of the partial differential equations is precluded by the assumed system non-linearity and the unknown control function. We therefore discretize the continuous equations and solve the resulting difference equations on the digital computer.

Various discretization schemes are possible (Sage, 1968); temporal or spatial discretization yields a simultaneous set of ordinary differential equations, while temporal and spatial discretization yields a simultaneous set of algebraic difference equations. The latter method is incorporated here. First-order linear hyperbolic equations of the form of Eq. 2-1 can be approximated with good stability and little truncation error by the explicit centered-difference analog (Von Rosenberg, 1969). With this technique we average about the midpoint of three known and one unknown discrete values of the state variable. Figure 2-1 displays the rectangular grid of the discretization schemes.

$$
\begin{array}{cccc}
X & 0 & 0 & 0 \\
X & X & 0 & 0 \\
X & X & X & X \\
X & X & X & X \\
y-k/2 & y & \text{} & \\
y-k & \text{} & \text{} & \\
\end{array}
$$

**Figure 2-1. Discretization Scheme**

(0 are unknown points, X are known)
We may construct the forward difference equation correct to second order from the Taylor series expansion in one independent variable

\[
\begin{align*}
z(t + h, y) &= z(t, y) + \frac{dz}{dt}(t, y) h + \frac{d^2z}{dt^2}(t, y) \frac{h^2}{2} \\
&\quad + \frac{d^3z}{dt^3}(t, y) \frac{h^3}{6} + \cdots 
\end{align*}
\]

The backward difference analog is derived from

\[
\begin{align*}
z(t - h, y) &= z(t, y) - \frac{dz}{dt}(t, y) h + \frac{d^2z}{dt^2}(t, y) \frac{h^2}{2} \\
&\quad - \frac{d^3z}{dt^3}(t, y) \frac{h^3}{6} + \cdots
\end{align*}
\]

Where \( h \) and \( k \) are finite perturbations in each of the respective independent variables.

Subtracting Eq. 2-3 from Eq. 2-4 yields the central difference analog,

\[
\frac{dz}{dt}(t, y) = \frac{z(t + h, y) - z(t - h, y)}{2h} - O(h^2)
\]

The \( O(h^i) \) indicates an error of the \( i \)th order in \( h \). As in the Crank-Nicolson method we average the derivatives at two times or space levels for the spatial and temporal derivatives, respectively. The centered-difference time derivative correct to second order for a two-dimensional problem is

\[
\frac{dz}{dt}(t + \frac{h}{2}, y - \frac{k}{2}) = \frac{1}{2} \left[ \frac{z(t + h, y) - z(t, y)}{h} \right] \\
+ \frac{z(t + h, y - k) - z(t, y - k)}{h}
\]

2-5
The spatial derivative is derived analogously.

The value of \( z(t + \frac{h}{2}, y - \frac{k}{2}) \) is then approximated as the average of the adjacent four grid points. The centered-difference approximation is

\[
z(t + \frac{h}{2}, y - \frac{k}{2}) = \frac{1}{4} (z(t, y - k) + z(t, y) + z(t + h, y - k) + z(t + h, y))
\]

In cases where perturbations in the boundary conditions occur, oscillations can propagate in the neighborhood of the discontinuity. This is the result of computing \( z(t + h, y) \) from the two widely different values, \( z(t + h, y - k) \) and \( z(t, y - k) \). To eliminate the oscillations we use the diagonal average

\[
z(t + h, y) = \frac{1}{2} (z(t, y - k) + z(t + h, y))
\]

Von Rosenberg (1969) displays success with application of this diagonal average to his first-order hyperbolic partial differential equation describing a countercurrent heat exchanger under step changes in temperature.

It can be shown (Smith, 1965) that when the ratio of increments \( (h/k) \) equals the species velocity there is no truncation error. The ordinary differential equations are also numerically solved but with a first-order method. The Taylor series expansion for one independent variable, time, is as in Eq. 2-3.

**Impulse Response Function**

Many physical and biological processes move unit quantities sequentially through a series of stages. Each of these unit quantities possess a different stage residence-time distribution, or resides in a particular stage for a period of time describable by a probabilistic distribution. To describe this probabilistical affect of the stage we use the
concept of an impulse response function (Eriksson, 1971). This function describes the time-density distribution of a pulse introduced into a system (Figure 2-2).

![Figure 2-2. Impulse Response Function, $\phi$.](image)

The effect is also observed in insect species (Rudd, 1976) as the insects progress through physiological stages. We calculate a continuous, species age-dependent probability distribution to describe the stagewise progression of a species.

It is convenient to describe the development of a stage probability distribution by assuming the introduction of one unit into the first process stage. We may draw an analogy to the laying of eggs by adult insects.
With successive increments in time the unit ages and is simultaneously subjected to a probability of exiting the stage. When a unit leaves this stage it immediately enters the next process stage and encounters the second stage impulse response function.

We express (Rudd, 1976) a unit pulse input at age \( y \) equal zero as

\[
q_{0,1}(y) = \delta(y)
\]

where \( \delta \) is the Dirac delta function,

\[
\delta(y) = 0 \quad \text{on } [-\infty, 0) \text{ and } (0, \infty]
\]

and

\[
\int_{-\infty}^{\infty} \delta(y) \, dy = 1
\]

The population of units \( N_i \) in the first process stage at any age is then

\[
\frac{dN_1(y)}{dy} = - \int_{-\infty}^{\infty} q_{0,1}(r) \varphi_{0,1}(y-r) \, dr \quad N_1(0) = 1
\]

The impulse response function \( \varphi_i, i+1 \) is defined on \( (-\infty, \infty) \) and \( \varphi_i, i+1 \, (y) = 0 \quad \forall \, y < 0 \).

Expanding for multiple stages we obtain.

\[
\frac{dN(t)}{dt} = \begin{bmatrix}
-q_{1,2}(y) \\
q_{1,2}(y) - q_{2,3}(y) \\
q_{n-1, n}(y)
\end{bmatrix}
\]

2-8
for the interstage flux from stage \( i \) to stage \( i + 1 \), \( q_i, i + 1 \), described by the convolution integral

\[
q_{i - 1, i}(y) = \int_{-\infty}^{\infty} q_{i - 2, i - 1}(r) \theta_{i - 1, i}(y - r) \, dr \quad 2-9
\]

The initial flux into the system \( q_{0,1}(y) \) is described as above. The initial conditions for the \( n \) population variables are \( N(0) = [1 \ 0 \ 0 \ \ldots \ 0]^T \).

The functions \( [N_i \mid i = 1, 2 \ldots n] \) are the time-dependent probabilities that a traced unit is in process stage \( i \).

**MINIMUM PRINCIPLE DERIVATION**

We wish to formulate an optimal control problem for the system described by the aforementioned equality constraints, Eq. 2-1 and 2-2, and inequality constraints, \( C_1 \). To find the \( u \in S \) which minimizes the given functional

\[
IP = IP(t, y, z, \frac{\partial z}{\partial t}, \frac{\partial z}{\partial y}, u) \quad 2-10
\]

on \( t \in [0,1] \) and \( y \in D \) we derive the necessary condition for an extremum through variational calculus.

The development of the distributed parameter system minimum principle to follow is a modification of Sage's (1968) method.

It is assumed that the initial time, initial manifold and terminal time, \( t = 1 \), are specified. The state variables at the terminal time are assumed free. The assumptions allow for a simplified derivation of the minimum principle which satisfy the conditions of a great many physical and biological systems.

The index of performance \( IP \) will be assumed of the form
\[ IP = 0 (z_m(1), t=1) + \int_0^T W(t, z, \frac{dz}{dt}, u) \, dt + \int_0^T S(t, z, \frac{\partial z}{\partial t}, \frac{\partial z}{\partial y}, u) \, dD \, dt \]

We employ the following conventions for ease of manipulation; \( z_k = (z_i \mid i = 1, 2, ..., k) \) are the state variables described by

\[ \frac{\partial z}{\partial t} = f_k(t, z, \frac{\partial z}{\partial y}, u) \]

where

\[ f_k = [f_1(\ ), f_2(\ ), ..., f_k(\ )]^T \]

\( z_j \) defined by \( (z_i \mid i = k + 1, k + 2, ..., k + j) \) is given by

\[ \frac{\partial z_j}{\partial y} = f_j(t, z, u) \]

and \( z_m \) is the set of ordinary differential equations where \( (z_i \mid i = k + j + 1, k + j + 2, ..., k + j + m) \) and

\[ \frac{dz_m}{dt} = f_m(t, z, u) \]

With the assumptions that \( \Theta \) and \( Q \) are spatially dependent and

\[ \frac{\partial \Theta}{\partial y} = 0, \quad \frac{\partial Q}{\partial y} = 0 \quad \text{and} \quad M = \frac{q}{\pi} (y_{i_{\text{max}}} - y_{i_{\text{min}}}) \]
and dropping the arguments for brevity we write IP as,

\[
IP = \int \left( \frac{\mathbf{Q}}{M} + \mathbf{W} \right) d\mathbf{D} + \int \int \left( \frac{\mathbf{Q}}{M} + \mathbf{S} \right) d\mathbf{D} d\mathbf{t} \quad 2-15
\]

In mathematical programming techniques we redefine the constrained optimization problem by introducing a set of constant multipliers which adjoin the equality constraints to the objective function. In variational calculus we approach the optimization of functionals in much the same manner, but the multiplier is a function rather than a constant. Here we use the multiplier, or co-state variables \( P(t, y) \), to adjoin the system equality constraints, Eq. 2-12 to 2-14, to Eq. 2-15 yielding.

\[
IP = \int \left( \frac{\mathbf{Q}}{M} + \mathbf{W} \right) \bigg|_{t=1} d\mathbf{D} + \int \int \left[ \frac{\mathbf{Q}}{M} + \mathbf{S} + \mathbf{P}_m^T (\mathbf{f}_m - \frac{d\mathbf{z}_m}{dt}) \right. \\
+ \mathbf{P}_k^T (\mathbf{f}_k - \frac{\partial \mathbf{z}_k}{\partial \mathbf{t}}) + \mathbf{P}_j^T (\mathbf{f}_j - \frac{\partial \mathbf{z}_j}{\partial \mathbf{y}}) \bigg] d\mathbf{D} d\mathbf{t} \quad 2-16
\]

We arrange the above to obtain

\[
IP = \int \left( \frac{\mathbf{Q}}{M} + \mathbf{W} \right) \bigg|_{t=1} d\mathbf{D} + \int \int \left[ \frac{\mathbf{H}}{M} \frac{d\mathbf{z}_m}{dt} - \mathbf{P}_m^T \frac{\partial \mathbf{z}_k}{\partial \mathbf{t}} \right. \\
- \mathbf{P}_k^T \frac{\partial \mathbf{z}_j}{\partial \mathbf{y}} \bigg] d\mathbf{D} d\mathbf{t} \quad 2-17
\]

The scalar Hamiltonian \( \mathbf{H} \) is

\[
\mathbf{H} = \frac{\mathbf{Q}}{M} + \mathbf{S} + \mathbf{P}_m^T \mathbf{f}_m + \mathbf{P}_k^T \mathbf{f}_k + \mathbf{P}_j^T \mathbf{f}_j \quad 2-18
\]
To obtain the first necessary condition for a minimum we take the first variation of the adjoined objective function

\[
\delta(IP) = \int \left[ (\delta z_m)^T \frac{a(\theta)}{\delta z_m} + (\delta z)^T \frac{aW}{\delta z} \right] dD + \int \left[ \frac{aH}{a\bar{u}} + (\delta z)^T \frac{aH}{a\bar{y}} \right] \frac{d(\delta z_m)}{dt} - \int \left[ \frac{a(\delta z_k)}{\delta t} - \frac{a(\delta z_j)}{\delta y} \right] dD dt
\]

We may note from the chain rule that

\[
\frac{p^T a(\delta z)}{\delta t} = a \left( p^T \delta z \right) - (\delta z)^T \left( a_p \right)
\]

and

\[
(\delta \frac{a\bar{z}}{a\bar{y}})^T \frac{aH}{a\bar{z}} = (\bar{a} \frac{a\bar{z}}{a\bar{y}} (\delta z))^T \frac{aH}{a\bar{z}} - (\delta z)^T \bar{a} \frac{aH}{a\bar{y}} \frac{a\bar{z}}{a\bar{y}}
\]

Substituting Eqs. 2-20 and 2-21 into 2-19 we obtain
\[ \delta (IP) = \int \left[ (\delta z_m)^T \frac{\partial \Theta}{\partial z_m} + (\delta z)^T \frac{\partial W}{\partial z} \right] \, dt + \int \left[ (\delta z_k)^T \frac{\partial H}{\partial z_k} \right] \, dt = 1 \]

\[ + (\delta u)^T \frac{\partial H}{\partial u} + (\frac{\partial}{\partial y} (\delta z_k))^T \frac{\partial H}{\partial z_k} - (\delta z_k)^T \left( \frac{\partial}{\partial y} \frac{\partial H}{\partial z_k} \right) \]

\[ + \left( \delta z_m \right)^T \frac{dp_m}{dt} - \frac{dp_m}{dt} (P_m \delta z_m) + (\delta z_k)^T \frac{\partial p_k}{\partial t} - \frac{\partial p_k}{\partial t} (P_k \delta z_k) \]

\[ + \left( \delta z_j \right)^T \frac{\partial p_j}{\partial y} - \frac{\partial p_j}{\partial y} (P_j \delta z_j) \, dt \, dt \]

Integrating and noting that \( \delta z (t = 0) = 0 \) we obtain

\[ \delta (IP) = \int \left[ \left( \delta z_m \right)^T \left( \frac{\partial W}{\partial z_m} - p_m + \frac{\partial W}{\partial z_m} \right) + (\delta z_k)^T \left( \frac{\partial W}{\partial z_k} - p_k \right) \right] \, dt \]

\[ + \left( \delta z_j \right)^T \frac{\partial p_j}{\partial y} \right] \, dt + \int \left[ - \left( P_j \delta z_j \right) + (\delta z_k)^T \frac{\partial H}{\partial z_k} \right] \, dt = 1 \]

\[ + \int \left[ (\delta z_k)^T \left( \frac{\partial H}{\partial z_k} + \frac{\partial p_k}{\partial t} + \frac{\partial p_k}{\partial t} \right) + (\delta z_m)^T \left( \frac{\partial H}{\partial z_m} + \frac{dp_m}{dt} \right) \]

\[ + \left( \delta z_j \right)^T \left( \frac{\partial H}{\partial z_j} + \frac{\partial p_j}{\partial y} \right) + (\delta u)^T \frac{\partial H}{\partial u} \right] \, dt \, dt \]

If \( IP \) is to be at a minimum \( IP^* \), the first variation of \( IP \) must be zero.
Then for any arbitrary variations $\delta z$ and $\delta u$ we argue that the components of Eq. 2-23 must equal zero for $\delta(I_P)$ to be zero. We, therefore, have the co-state equations

\[
\frac{\partial P_k}{\partial t} = -\frac{\partial H}{\partial z_k} + \frac{\partial}{\partial y} \frac{\partial H}{\partial (\frac{\partial z_k}{\partial y})}
\]

\[2-24a\]

\[
\frac{\partial P_j}{\partial y} = -\frac{\partial H}{\partial z_j}
\]

\[2-24b\]

\[
\frac{dP_m}{dt} = -\frac{\partial H}{\partial z_m}
\]

and transversality conditions

\[
P_k = \frac{\partial W}{\partial z_k} \quad t = 1 \neq D
\]

\[2-24b\]

\[
\frac{\partial H}{\partial (\frac{\partial z_k}{\partial y})} = 0 \quad y = 1 \neq t
\]

\[
P_L = 0 \quad y = 1 \neq t
\]

\[
P_m = \frac{\partial W}{\partial z_m} + \frac{\partial (-\frac{\partial}{\partial M})}{\partial z_m} \quad t = 1 \neq y
\]

The state equations (Eq. 2-1 and 2-2) and boundary conditions along with the co-state equations and transversality conditions (Eq. 2-24) describe the first necessary conditions for an extremum. We essentially
redefine the constrained optimization problem (Eq. 2-10) into a statement of an equivalent unconstrained problem.

The necessary condition for the optimum with \( u \in S \) is \( \frac{\partial H}{\partial u} = 0 \) when there is no inequality constraint \( C_1 \). When \( C_1 \) exists, then we simply drive the Hamiltonian as small as possible with respect to the control trajectory (Sage, 1968).

It can be shown (Butkovskiy, 1969) that for linear systems the necessary conditions, as stated above, are also sufficient. In problems requiring a non-linear system we can use variational calculus to obtain the second variation in IP. This yields an additional necessary condition.

**Numerical Solution Of Minimum Principle Equations**

The numerical solution of the optimal control trajectory \( u \) requires the solution to a two-point boundary-value problem. Solution techniques are divided into direct and indirect. We use the direct approximation to the solution technique in this research, but before its development, we describe other applicable procedures.

One indirect method is the "approximation to the problem" or approximation in function space (Dreyfus, 1962). In this technique an optimal trajectory is obtained in each iteration of the state and co-state equations, but the constraints and terminal manifold equations might not be satisfied. This is an optimal solution to a different problem. We therefore iterate upon the model equations until we obtain the correct problem. As we are dealing with a partial differential equation, we are required to estimate two functions as the unknown transversality conditions. Even with the assumption of continuity, this effort is extremely time consuming.
A direct method is invariant imbedding (Bellman, et. al., 1960). In this procedure we solve for a missing initial or terminal condition, thereby developing two initial value problems. The missing state boundary conditions are assumed a function of a general class of initial conditions in which is imbedded the specific co-state transversality conditions. If \( n \) ordinary differential equations were obtained from discretizing the partial differential, our invariant imbedding problem would require solution to \( n \times (n + 1) \) simultaneous equations, a sizeable concession for large systems of equations. In addition, if the system equations are non-linear, it may prove necessary to assume a second or higher ordered functional relationship for the unknown.

The "approximation to the solution" or approximation in policy space (Dreyfus, 1962), which we use in this study, is another direct method in which the state and co-state trajectories satisfy the transversality conditions on each iteration of the system equations, but the control trajectory is probably not the extremum. Iteration is continued until the optimal control is obtained. In this method we evaluate the necessary condition, \( \partial H / \partial u \), and adjust the previous estimate of the control trajectory in the direction of the steepest gradient. We continue this iteration until the necessary condition approaches zero and the index of performance remains stationary, an indication of an extremum. The technique used is based on finding first-order effects of controls upon the cost function and is, therefore, referred to as a first variation technique.

To account for inequality constraints \( C_1 \) we add a penalty function to the objective function. As the infeasible region is entered, a weighted penalty is added to the IP. When the unconstrained optimum is in the
infeasible region the penalty will cause the necessary condition to approach zero and alter sign at the boundary. This approach is accredited to Bryson and Kelly (Bryson and Denham, 1962; Kelly, 1960) who developed it independently. Second order methods (Merriam, 1964), as well as conjugate gradient techniques, may be incorporated in seeking the optimum.

The incremental first-order change in the objective function for perturbations of control \( \Delta u \) \((t,y)\) is

\[
\Delta (IP) = \int \int \frac{\partial H}{\partial u} \Delta u \, dD \, dt
\]

To obtain the greatest change \( \Delta (IP) \) we alter the estimated control \( u \) \((t,y)\) by a multiple of the non-zero \( \frac{\partial H}{\partial u} \),

\[
\Delta u = - k \frac{\partial H}{\partial u}
\]

where \( k \) is a positive non-zero constant. The new control is calculated as the old estimate plus the change in the control which would yield the greatest improvement in the index of performance.

Steps of computation for the technique are:

1. Assume an initial control trajectory, \( u^0 \)
2. Calculate the state variables by integrating forward in time
3. Calculate the adjoint variables by backward integration from \( t_{\text{max}} \) to \( t_{\text{min}} \)
4. Compute \( \frac{\partial H}{\partial u} \), the necessary condition of an extremum, and \( IP^0 \)
(5) Evaluate improved estimate of control trajectory
\[ u^1 = u^0 + \Delta u^0 \]

(6) Calculate the state variables

(7) Evaluate \( IP^1 \), if \( IP^1 > IP^0 \) (when minimizing), utilize \( u^0 \) and calculate a new \( u^1 \) with a different \( k \)

(8) Proceed from (2) until the index of performance varies within a specified tolerance or \( \Delta u^0 \) is minimal

The first variation method quickly converges to the region of the optimal control. This is observed even when the initial guesses are very poor. As with any gradient technique oscillations appear in the solution as the extremum is approached. Continued refinement of the gradient multiplier helps in avoiding this condition.

An additional problem is in the selection of the slope's multiplier magnitude. Though techniques are available for selection (Eveleigh, 1967) we use the one-dimensional golden section search to evaluate the minimum along the gradient. Sufficiency of unimodality is not considered a problem in our effort to seek improvement.

The technique is limited only by the accuracy of the linear approximation to the slope of the functional with respect to changes in control action.

**NONLINEAR PROGRAMMING—PATTERN SEARCH**

To obtain a point we use a method developed by Hooke and Jeeves (1961) called pattern search. This is a procedure which follows a logical pattern for searching a multidimensional response surface for an extremum.

Pattern search is based on the underlying assumption that if any move on the response surface yields a better result then continuation
in that direction will produce even better functional evaluations. If successive experimentations in the same direction produce better results we can increase the step size and proceed to an extremum with a step of larger norm. But, if failure is encountered, we retreat to the last success, destroying the pattern and reducing our step size, returning to a more cautious search for improvement in each of the independent variables.

We express this procedure mathematically after Wilde and Beightler (1967), by referring to the two-dimensional surface of Figure 2-3. Consider an initial or base point $b_1$ which in general is an n-dimensional vector in Euclidean space. We define $\Delta_i$, a vector described by the unit vector $e_i = (0, 0, ... 0, 1, 0 ... 0)$ and specified step size of each independent variables $\delta_i$, as $\Delta_i = \delta_i e_i$. The search begins by perturbing from the base point, first adding then subtracting $\Delta_i$. If improvement in a functional evaluation is obtained over that of $b_1$ then we accept its domain point as temporary base $t_{11}$. Perturbation about each subsequent independent variable is then undertaken accepting a new temporary base $t_{11}$ upon finding any improvement. The best temporary base is then defined as the second base point $b_2$. With this the first pattern is established. Reasoning that improvement is on the ray containing $b_1$ and $b_2$, we extend from $b_1$ a distance twice the Euclidean norm \| $b_2 - b_1$ \|. This new base point is temporary and defined as $t_{20}$. It is from this point that we once again perturb eventually defining the best point as $b_3$. We again define our next temporary base $t_{30}$ as twice the distance from the previous two base points, $b_3$ and $b_2$. We should note that if improvement in the range continues the \| $b_i - b_{i-1}$ \| increases and the search accelerates.
Figure 2-3
Establishing and Destroying a Pattern
(after Wilde and Beightler, 1967)
If perturbation about a temporary base yields no better value in the range we accept the temporary base as the base point and proceed. If, however, we find that upon stepping none of the temporary bases yield an improvement we must return to the last base point and either halt the search or decrease the step size and begin a new pattern.

As in gradient techniques and many other logical methods the pattern search has the disadvantage of halting the search at a local minimum. To circumvent this we initialize the procedure at various initial conditions. The method does not require implicit calculations of the slope and its speed of convergence and accuracy are good. In addition, pattern search, unlike the gradient technique, is a good ridge follower. It is because of this feature that the computational time only linearly increases with increasing dimensionality (Wilde and Beightler, 1967).

SUMMARY

In this chapter we have concentrated on presenting general mathematical and numerical techniques employed in this research. Chapter III and IV follow with specific application to the soybean ecosystem.
CHAPTER III

SYSTEMS DYNAMICS

In this chapter we describe the model used to simulate soybean plant growth and insect pest population dynamics. Each of the studied components are described and assumptions outlined. Where possible we validate each of the subsystems.

The insect pests are the Southern green stink bug (SGSB), a pod feeder, and the velvet bean caterpillar (VBC), a defoliator. The Dare and Lee varieties of the soybean are simulated to account for the plant's dynamic response to pest damage.

The stink bug is not believed to be strongly influenced by predation and parasitism (Herzog, Jensen, Newsom, 1975), but the velvet bean caterpillar is thought to be significantly affected by natural biological control. To date we cannot confidently identify the population species which belong to this predator-parasite (PP) complex. But, because this complex plays an important role in the dynamics of the velvet bean caterpillar we simulate it with a simple dynamic model. Pesticide application affects the stink bug, velvet bean caterpillar and PP complex. A diagram of the simulated system is displayed in Figure 3-1.

Our system model simulates plant growth dynamics from the time bean pods begin development until harvest. The plant growth and pop-
Figure 3-1
Components of Simulated System
ulation equations are assumed spatially homogeneous. Plant mass and insect population are computed on a per foot of row basis.

We assume no additional immigration or inflights after podset, except where deterministically described. Climatic effects are ignored for lack of detailed information and because the insects of the Gulf-South region appear insensitive to environmental conditions during the simulation period.

**BASIC POPULATION EQUATION**

Chapter II introduced the first-order linear hyperbolic equation as a potential insect species model. We approach the population modeling utilizing this age-dependent model because of the precision it offers and the ease with which it can be treated by variational calculus. We use the convolution integral to determine the partial differential equation's age-dependent coefficients. We now develop the specific system equations.

The population model is based on the concept of the conservation of species numbers, and to derive the equations we consider an open system about a species of age $Y$ at time $T$, Figure 3-2. Input into the Euler control volume around age $Y$ is the flux of the number of species of age $Y - \Delta Y$ and inflight. Inflight or immigration is the flux of adult insects into the field during the time period of the simulation. Output is mortality and species aging to age $Y + \Delta Y$. 
The appropriate discrete equation for the time dependent state is then,

$$ \frac{3(Z(T,Y)\Delta Y)}{\Delta t} = -Z(T,Y + \Delta Y) + Z(T,Y) - M(T,Y)\Delta Y + I(T,Y)\Delta Y $$

where $M$ is mortality and $I$ the rate of inflight. Assuming $\Delta Y$ independent of time, we divide Eq. 3-1 by $\Delta Y$ and allowing $\Delta Y$ to approach 0 we obtain

$$ \frac{3Z(T,Y)}{\Delta t} + \frac{3Z(T,Y)}{\Delta Y} + M(T,Y) = I(T,Y) $$

Mortality and inflight can be suitably represented as first order exponential decays and growths, respectively, giving
\[ M(T, Y) = k(Y)Z(T, Y) \]
\[ + k_u(Y) (1 - k(Y)) u(T, Y)Z(T, Y) \]
\[ + k_p(Y) \frac{Z(T, Y)Z_g(T)}{1 + \frac{Z(T, Y)}{c_p}} \]

and

\[ I(T, Y) = q(Y) [Z_{ref}(Y) - Z(T, Y)] \]

where

- \( k(Y), k_u(Y), k_p(Y) \) are, respectively, coefficients of natural mortality, pesticide mortality, and PP feeding mortality fraction
- \( c_p \) is the maximum feeding capacity of the PP species population
- \( Z_g(T) \) is the PP population
- \( q(Y) \) is the species immigration coefficient

The control variable \( u(T, Y) \) is both age- and time-dependent and indicates the fraction of the population of a given age instantaneously killed at a given time. The assumption of instantaneous control deviates from reality in very few cases. Some pesticides will be effective for a short period of time in the field, decaying exponentially in effectiveness. However, present government regulations and research has provided pesticides with only short time spans of effectiveness.
The pesticide mortality \( k_u \) is an age-dependent distribution describing the insect developmental stages at which pesticide is effective. We write

\[
k_u(Y) = \sum N_i(Y) c_i
\]

where \( N_i \), defined by Eq. 2-8, is the stage probability distribution. \( c_i \) is zero if the \( i \)th stage is not controllable and one if it is controllable. We may, therefore, increase mortality of stages susceptible to control by a specific control tactic.

The velvet bean caterpillar population dynamics are modified by the PP complex. We simulate predation and parasitism by first normalizing the PP population and distributing its effect on the prey with the PP feeding mortality fraction \( k_p \). Again, \( k_p \) is stage- and therefore age-dependent. We obtain its value \( k_p(Y) \) from an equation similar to Eq. 3-5, but now \( c_i \) may be a real number on \([0,1]\).

Describing the PP feeding capacity \( c_p \) is easier if we study its limits (Figure 3-3). As \( c_p \) approaches infinity the PP feeds on a constant proportion of the prey. If instead, we allow \( c_p \) to approach zero, then PP feeding approaches a constant value. This argument is justifiable because the PP population can only feed on the numbers of prey required to satisfy their needs (Watt, 1959). Such factors as hunting, attacking and consuming a prey limit feeding rates.
The driving force for immigration $Z_{\text{max}}(T,Y) - Z(T,Y)$ is the unattained portion of an estimated maximum population. The immigration coefficient $q(Y)$ is assumed a fraction of the initial pest inflight.

We combine Eqs. 3-2, 3-3 and 3-4 to form the population model. In this research we normalize the equation assuming a fixed length of the simulation time period $t_m$, a maximum species age $y_m$, and maximum $Z_{\text{max}}$ and minimum $Z_{\text{min}}$ population levels. The normalized variables are then,

$$z = \frac{Z - Z_{\text{min}}}{Z_{\text{max}} - Z_{\text{min}}} \quad t = \frac{T}{t_m} \quad y = \frac{y}{y_m}$$
where we define the initial age and time equal zero. The dimensionless equivalent to the population model (Eqs. 3-1, 3-2, -3 and 3-4), with $Z_{\text{min}}$ equal zero is

$$\frac{Z_{\text{max}}}{t_m} \frac{\partial z}{\partial t} + \frac{Z_{\text{max}}}{y_m} \frac{\partial z}{\partial y} + (k + k_u (1 - k) u) Z_{\text{max}} z$$

$$+ \frac{k_p z g z_{\text{max}}}{1 + \frac{Z_{\text{max}} z}{C_p}} = q Z_{\text{max}} (z_{\text{ref}} - z)$$

3-6

The initial condition is simply a specified age-dependent in-flight, $Z(0,y)$. The state boundary condition represents the eggs laid at a given time. This is accomplished by the normalized integral boundary condition

$$z(t,0) = y_m \int_0^1 B(y) z(t,y) \, dy$$

3-7

where $B(y)$ is the normalized oviposition rate. These equations are solved numerically on the digital computer using the centered difference algorithms of Chapter II. Appendix II demonstrates their usage.

**SOUTHERN GREEN STINK BUGS**

Overwintering stink bugs begin feeding on clover, weeds, and early commercial crops. They produce the next generation which eventually migrates into the soybean field at blossom and podset (Newsom, et. al. 1974). It is believed that those stink bugs which enter the field prior to podset either die or leave for lack of food. This
generation is the second of the year and creates the greatest amount of damage in the early maturing varieties of soybeans. The third generation is usually laid in the soybean field and damage the maturing seedpods. The third and perhaps a fourth generation survive the winter to continue the life cycle.

The adult is capable of flight and easily migrates from field to field. The female lays approximately 400 eggs in masses of 85 over a period of one month. The eggs hatch in about six days and after passing through five nymphal stages reach adulthood. The five stages are referred to as consecutive instar periods. The nymphs are mobile, but since they cannot fly, often remain near their egg mass.

Adult and larval stinkbugs feed on beans by piercing the pod hull. They are, therefore, external feeders and have adapted by evolving piercing-sucking mouthparts. The stylus pierces the bean and sucks out juices destroying or reducing the bean in quality and quantity. There is also some evidence that because of this feeding process, the stink bug is a vector of plant diseases.

**Stink Bug Damage**

Losses in soybean quality and quantity have been studied by a number of investigators. Miner (1961), Blichenstaff and Huggans (1962), Todd and Turnipseed (1974), Thomas and Newsom (1974), Duncan and Walker (1974) and Jensen and Newsom (1972) have demonstrated the pest's potential for damage. Miner, Todd and Turnipseed described the oil and protein content of the soybean before and after stink bug feeding. Duncan and Walker found that small field tests with one stink bug per 1.32, 3.1 and 4.93 feet of row over a period of seven weeks yield
62.1, 45.6 and 33.1 percent damaged seeds, respectively. The most significant quantitative work comes from Thomas and Newsom. Here, stink bugs of each damage-producing stage were caged on soybean plant racemes of three different stages of plant development; pods 1/2 to 3/4 filled, pods 3/4 to completely filled and pods completely filled and beginning to yellow. The average number of stink bug punctures and bean weight reductions were recorded. This data, as an average of the three tests, is presented in Table 3-1.

Table 3-1

Average Number of SGSB Pod Punctures and Bean Weight Reductions From SGSB Feeding.

<table>
<thead>
<tr>
<th>SGSB Stage</th>
<th>Number Punctures Per Day</th>
<th>Grams Reduction Per Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
<td>.0733</td>
</tr>
<tr>
<td>4</td>
<td>3.4</td>
<td>.0703</td>
</tr>
<tr>
<td>5</td>
<td>4.5</td>
<td>.0797</td>
</tr>
<tr>
<td>Average Adult</td>
<td>4.1</td>
<td>.0870</td>
</tr>
</tbody>
</table>

We used the data of Table 3-1 to describe the quantity loss due to the SGSB. The daily potential for gram weight reduction $z_3$ is

$$\frac{1}{y_m} \frac{\partial z_3(t,y)}{\partial y} = \frac{f_3}{y_m} = c_1(y)z_1(t,y)$$
where the SGSB feeding rate \( c_i \) is given by

\[
c_i(y) = \sum_{\text{stages}} \text{Feeding rate (grams per day)} \times N_i
\]

\( N_i \) is defined in Eq. 2-8 as the stage probability function.

During early pod development, feeding by the stink bug may promote abscission of pods (Newsom, et. al., 1974). However, this effect is difficult to measure since the plant initially produces more pods than can be filled. We assume this to be a compensatory process on the part of the plant. Jensen and Newsom found that damage was more a function of where the bean was punctured than the number of times. If the radicle-hypocotyl axis of the seed is punctured during early development then heavy damage is almost inevitable.

Quality damage is more difficult to describe mathematically. Though some data exists, a clear description of the damage is lacking due to our inability to predict puncture location and number.

To derive an expression for quality damage we first attempt to discern a relationship between the number of seeds not damaged of a 100 seed sample and the yield from that field (Herzog, 1974). Three fields are analyzed by linear regression and in each case we conclude independency with a confidence interval of 95 percent.

We turn to the caged field data of Todd et.al. (Table 3-2) for SGSB quality damage data. Because no data for intermediate population levels are given we rely on simulating the population magnitudes. Utilizing the data of Thomas et.al., we calculate the number of bug punctures per average foot of row. Correlating these data with the percent damaged seeds of 100 seed samples produces a correlation co-
efficient of .951, significant at the 99% level. The linear statistical fit shown in Figure 3-4 with zero intercept has a slope of .0719.

Table 3-2

Caged Field Data Used for Determination of the Quality Damage - SGSB Population Relationship

<table>
<thead>
<tr>
<th>Dates of Population Infestation</th>
<th>Stage SGSB Introduced</th>
<th>Initial Population Levels (Number Per Foot Row)</th>
<th>Mean Percent Damaged</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment IIa</td>
<td>4th Instar</td>
<td>1/3</td>
<td>5.4</td>
</tr>
<tr>
<td>8-18 to 9-23</td>
<td>1</td>
<td>14.4</td>
<td></td>
</tr>
<tr>
<td>9-25 to 11-7</td>
<td>3</td>
<td>51.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>55.8</td>
<td></td>
</tr>
<tr>
<td>Experiment IIIa</td>
<td>Adults</td>
<td>1/3</td>
<td>20.5</td>
</tr>
<tr>
<td>8-17 to 9-17</td>
<td>1</td>
<td>64.1</td>
<td></td>
</tr>
<tr>
<td>9-18 to 11-10</td>
<td>3</td>
<td>99.3</td>
<td></td>
</tr>
</tbody>
</table>

Stink Bug Model Parameters

(1) Stage Distribution - As data available on mortality and fecundity are based on physiological stages we must be able to convert them to age-dependent distributions for use with the population equation. The impulse response function introduced in Chapter II, Eq. 2-8, provides the theory necessary to determine these age-dependent parameters.

The cumulative response of a unit impulse into an insect stage
Fraction Damaged Seeds

Damage Potential (number of punctures)

Figure 3-4

Linear Regressive Fit of Quality Damage by the Southern Green Stink Bug
yields the emergence function $E(t)$ (Figure 3-5). The derivative with
time of this function, $dE(t)/dt$, is the impulse response function
(Rudd, 1976).

Figure 3-5. The Emergence function $E(t)$ and
Impulse Response Function, $\varnothing$.

With the emergence data of Kiritani (1964), Table 3-3, and the theory
of Chapter II we obtain a stage probability distribution function,
Figure 3-6.
Figure 1-6
Distribution of Southern Green Stink Bug in Developmental Stages
Table 3-3

Means and Standard Deviations of Residence Times in Developmental Stages for Nezara Viridula

<table>
<thead>
<tr>
<th>Stage</th>
<th>Mean (Days)</th>
<th>Standard Deviation (Days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Egg</td>
<td>6</td>
<td>1.0</td>
</tr>
<tr>
<td>Instars</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First</td>
<td>3</td>
<td>1.8</td>
</tr>
<tr>
<td>Second</td>
<td>8</td>
<td>3.8</td>
</tr>
<tr>
<td>Third</td>
<td>7</td>
<td>3.1</td>
</tr>
<tr>
<td>Fourth</td>
<td>8</td>
<td>4.0</td>
</tr>
<tr>
<td>Fifth</td>
<td>8</td>
<td>3.9</td>
</tr>
</tbody>
</table>

We must note that if the population of any single stage is abruptly altered the other stages which share probabilities of existence at that age will also be effected. However, we have determined through simulation that the model is not overly sensitive to perturbations in the population at any specific age.

(2) Natural Mortality - Natural mortality is calculated from laboratory stink bug survival data of Corpuz (1969), in which it is shown that adults live an average of 33 days with a standard deviation of 3.4 days. The uncontrolled nymphal mortality rate is nearly constant throughout the nymphal stages, and is about 94 percent net according to cage-study data of Kiritani, Hokyo and Kimura (1963). We assume egg mortality to be 25 percent (Kiritani, Hokyo and Kirura,
1967). This includes the eggs which are unfertile and those that fail to hatch.

The age-dependent coefficients for egg mortality are calculated simply as the product of the impulse response function for eggs and a constant stage mortality. Nymphal mortality is assumed to take effect upon the insects entering or exiting a nymphal stage. Mortality of adults is calculated from the distribution of longevity, keeping track of each nymph as they mature to an adult. The resulting mortality distribution is displayed in Figure 3-7.

(3) **Fecundity** - The adult Southern green stink bug of the second generation experiences a preoviposition period of approximately twenty days (Herzog, 1974). Egg masses are then laid with a mean frequency of 6.91 days and standard deviation 2.6 days (Kiritani and Hokyo, 1965). Approximately 370 eggs are laid by each fertilized female, but 32% of the females are sterile or have died before laying (Kiritani and Kimura, 1963). Because the sex ratio is approximately 1:1 (Corpuz, 1969) the adult lays an average of 127 eggs.

We once again keep track of each emerging adult and allow for a constant preoviposition period and a series of six normally distributed oviposition periods. Adding the laying ability of each emerging adult yields the age-dependent fecundity coefficient distribution of Figure 3-8.

(4) **Seasonal Inflights** - Inflights other than the initial migration are simulated using a linear potential for field populations. The driving force presented above, Eq. 3-4, includes an age-dependent coefficient that permits only adults to fly into the field. We assume
Figure 3-7
Mortality Function of the Southern Green Stink Bug
Figure 3-8
Southern Green Stink Bug Fecundity
that new inflights both suffer normal mortality and lay eggs the day on which they arrive. Control does not affect new arrivals.

(5) Control - Control affects only the last two nymphal and adult stages (Jensen, 1974). Control is accomplished by multiplying the percent control by the sum of controllable stage probabilities at any age, Eq. 3-5.

Results and Validation

Empirically we frequently find an initial population inflight of 1/3 stink bug per foot of row (Newsom, 1974). Since the inflights are composed only of adults we use as our model initial conditions 1/3 bug per foot row normally distributed in the adult ages (Figure 3-9). The model describes the population dynamics over a 60-day period, beginning 60 days prior to harvest when bean pods of the deterministic Dare varieties begin podset. The Lee variety encounters podset approximately 90 days prior to harvest.

Figure 3-10 shows the population dynamics without control and immigration in Dare, while Figure 3-11 displays the population dynamics during Lee's ninety day period. Here and in following figures we sum the five nymphal populations. For validation, we superimpose the combined nymphal and adult field population data taken independently by Herzog (1974) in Dare. The confidence intervals shown are calculated from the negative binomial distribution (Rudd, et al., 1976), which describes the distribution of clustered individuals. The stink bug is found to group in the field, primarily due to the large egg masses and limited mobility of the immature bugs.

Figure 3-12 presents the results with the assumption that inflights
Figure 3-9

Initial Conditions for Southern Green Stink Bug Simulation
Uncontrolled Southern Green Stink Bug Population Assuming No Immigration.

Data Shown are Field-Survey Data of Herzog (1974)
Figure 3-11

Uncontrolled Population of the Southern Green Stink Bug
Uncontrolled Southern Green Stink Bug Population Assuming $q(y) = .01Z_1(0, y)$

Data Shown are Field-Survey Data of Herzog (1974).
occur continuously at a reduced level after the initial arrival of the stink bugs. The coefficient of immigration is assumed one-hundredth the initial inflight of adults. Naturally, the population trends are sensitive to the rate of later inflight; what is shown is one of several assumed rates we tried. The results are still too tenuous to present any substantial conclusions on seasonal inflights, as a result the bulk of our experimental studies assume zero immigration.

We must point out that the data is from only one season with a specific environment. Even so, the satisfactory agreement between model results and field-survey data leads us to conclude that the model offers a reasonable predictive tool with which to simulate real systems. This agreement is particularly striking when one recalls that the parameters in the model are taken unmodified from laboratory and field-cage studies, most of which were done in Japan.

VELVET BEAN CATERPILLAR

The velvet bean caterpillar is perhaps the most damaging defoliator in the soybean field today (Newsom, et. al., 1974). Each year the overwintering populations of moths which survive in the warm region of Florida and in the Caribbean begin a migration which may take the species as far north as Kentucky and as far west as Texas. The moths may reach Kentucky in June or July. The population is killed by the winter cold in all regions except southern Florida.

The small nocturnal moths lay their eggs singly on the undersurface of leaves within just nine days after copulating (Watson, 1916). Eggs hatch in about four days and the defoliating caterpillars then progress through six larval stages. The caterpillar then pupates two
inches below the soil surface, eventually emerging as the moth. Approximately three generations exist each year. Like the stink bug, they appear to be attracted to the field in large numbers near podset.

Two models of the velvet bean caterpillar are published; a stochastically driven model by Menke (1973) and a deterministic convolution model of Rudd (1976). We compare our simulation results with these published results.

**Velvet Bean Caterpillar Damage**

Though the caterpillar has been studied for years, very little data are available on the rates at which the nymphs consume soybean foliage. The data we used were obtained by Greene (1971) and communicated by Menke (1973).

The first and second larval stages consume minimal amounts, whereas the third and fourth can eat 14.5 and 29.1 mg., respectively, each day. The major damage producers are the fifth and sixth instars which can daily consume 218 mg. and 262 mg. respectively. The sixth instar exhibits the greatest damage potential because of its longer stage residence time. We assume that the caterpillar does not attack the beans directly.

We express the daily feeding potential as

\[
\frac{1}{ym} \frac{\partial z_4(t,y)}{\partial y} = \frac{f_4(y)}{ym} = \frac{c_2(y)z_2(t,y)}{ym}
\]

where the VBC feeding rate \(c_2\) is given by
The effect of leaf area reduction on yield is studied in following sections.

**Velvet Bean Caterpillar Model Parameters**

1. **Stage Distribution** - The population stage dynamics of the caterpillar relative to that of the SGSB's are rapid. As a result, the use of constant stage residence times does not appear to greatly alter the precision of the model. Rudd (1976) employs the following stage durations in days for the egg, first through sixth instar, pupa and moth, respectively, 3, 2, 4, 3, 4, 3, 12, 8 and 14. The result is a uniform stage distribution over each period (Figure 3-13).

2. **Mortality** - Mortality coefficients are easily calculated when we assume an exponential decay

\[
k_2(Y) = -\frac{1}{T_{stage}} \ln(1 - k_{stage})
\]

and have a constant stage mortality \(k_{stage}\) and residence time \(T_{stage}\). A rate of mortality of 30 percent, which accounts for control by a PP complex, is assumed for all stages (Rudd, 1976) In accounting explicitly for the biological influence, we assume a 13 percent control per stage as the natural mortality and an additional 17 percent as the PP effect. This allows a twenty-fold increase in larva population levels when biological control measures are available and an 100-fold increase after pesticide has eliminated many predators and
Figure 3-13

Stage-Age Distribution of the Velvet Bean Caterpillar in Days
parasites.

The PP effect on the caterpillar, which is non-linear because of the feeding capacity, $c_p$ of Eq. 3-3, is here assumed to be 10 insects. In order to offset the affect of $c_p$ on reducing the biological mortality even at low levels of caterpillar populations, we must increase the PP mortality from 17 to 20 percent.

(3) **Fecundity** - Approximately 700 eggs are produced by each female moth. We assume a sex ratio of 1:1 and distribute the rate of egg laying by the positively skewed distribution given by Greene (Figure 3-14).

(4) **Control** - Insecticide control is assumed effective only for the larval stages (Herzog, 1975). It is indefinite whether the eggs and moths are effected, but it is felt that pupa are not because of their location underground.

The results for the numerical solution of the VBC population equation is shown in Figure 3-15 along with the results of published models using a similar data base. Because the PP interaction results in a non-linear difference equation it is difficult to determine numerical stability except by simulation with various discretized time and age increments. We can, however, conclude stability based on the comparison of results with the other models (Figure 3-15). These other models do not incorporate species interactions. Figure 3-16 represents model results with initial conditions (Figure 3-17) that satisfy VBC population data in the Ransom variety of soybean (Herzog, 1975). Two levels of the PP growth rate are simulated.
Figure 3-14

Number of Eggs Laid By The Adult Velvet Bean Caterpillar After Mating
Figure 3-15

Comparison of Velvet Bean Caterpillar Model with Published Model

Results: (1-1-1) Stochastic Model Results of Menke (1973);
(-///-///) Model Results of Rudd (1976). Note Scale Change at Day 43.
Population (number larva per foot row)

Figure 3-16

Population Velvet Bean Caterpillar After Two Discrete Controls

Data are Field-Survey Results of Herzog (1975)
Initial Conditions for Velvet Bean Caterpillar Simulation
Estimation of appropriate initial conditions is difficult because of sampling noise and the use of laboratory rate coefficients. Figure 3-18 shows the uncontrolled population with the initial conditions of Figure 3-17. The density of the caterpillar in later periods of the season may be unrealistic since diminishing food sources and decreasing temperature reduces fecundity and increases mortality. But, as to be pointed out in the development of the plant model, this large population does not affect predicted yields.

**PREDATOR-PARASITE (PP) COMPLEX**

The soybean ecosystem is a highly complex system of interactions between species. The velvet bean caterpillar is strongly influenced by biological control agents in the field. Though we can't define all the PP components we simulate a dynamic complex. We assume that the PP complex is not a function of the VBC population, because of a large number of alternative hosts available to the PP complex. Our main interest is in the effect of insecticide uses on the PP complex and the resulting effect on the VBC population. The SGSB is assumed independent of the PP complex.

To represent the PP complex we elect to use the logistic equation,

\[ \frac{dZ_8(T)}{dT} = rZ_8(T) \left( \frac{N - Z_8(T)}{N} \right) - u(T) Z_8(T) \]

which when normalized yields,

\[ \tau_m \frac{dz_8(t)}{dt} = rz_8(t) (1-z_8(t)) - u(t) z_8(t) \]
Figure 3-18

Population Velvet Bean Caterpillar with Initial Conditions of Figure 3-17. (note scale change)
where \( r \) is the net rate of population growth, \( N \) is the maximum population the environment can sustain and \( u \) is the fraction of population controlled by insecticides. We reason that if insecticide is applied, we diminish the potential for biological control and the mortality rate for the velvet bean caterpillar decreases (Eq. 3-3).

The rate at which the PP complex recovers from chemical control is dependent on \( r \). Because this value is essentially unknown quantitatively, we experiment with limiting values. Figures 3-16 and 3-19 show the effect of two PP growth rates on the VBC population after application of 99% control. With these growth rates we simulate a PP complex which can either recover rapidly after control, \( r \) equal 1.3, or cannot recover at all, \( r \) equal zero.

SOYBEAN PLANT MODELS

It is felt that the use of a soybean plant model is important because of the compensatory ability of the plant when insect pest feeding occurs. The soybean plant is found in many varieties where each differ in such basic characteristics as flower, pubesence and hilum color, bloom and maturity date, seed size and quality, and the general spatial dimensions of a mature plant. Varieties also differ in physiological stage progression. Table 3-4 lists various stages of soybean plant development. For instance, the determinate varieties proceed stage-wise through vegetative, bloom, podset and podfill. The indeterminate varieties fill pods and bloom throughout the podfilling stage of development. Only the determinate varieties are grown in the Gulf-South region.
Population (number larva per foot row)

Figure 3-19

Population Velvet Bean Caterpillar After Discrete Control on Day 1
(Note scale change)
Table 3-4

Description of Plant
Developmental Stages (Fehr, et. al., 1971)

**Vegatative Stages**

**V1**  Unrolled leaf at unifoliolate node
**V2 - VN**  2 to N nodes on plant's main stem

**Reproductive Stages**

**R1**  1 flower at any node
**R2**  Flower at node below uppermost node with a completely unrolled leaf.
**R3**  Pods appear--pod .5cm long at one of four uppermost nodes (completely unrolled leaf)
**R5**  Beans beginning to develop
**R6**  Pod of the four uppermost nodes with completely unrolled leaf contains a full size green bean
**R7**  Pods yellowing and half of leaves yellow.
   Physiological maturity
**R8**  95% pods brown. Harvest maturity
In the South the determinate varieties are planted in the spring. After planting there is a short period before plant emergence, which is followed by a two month period of vegetative development. Blossoms develop at the end of this period. Podset begins approximately sixty days prior to harvest in Dare and ninety in Lee. When podset begins, the pod's rate of accumulation of matter is exponential while new leaf development almost halts. The stem weight peaks and when the pods are full the leaves senesce and drop from the plant. The beans are normally allowed to dry in the field, facilitating harvesting and eliminating the cost of drying the beans with heaters.

Jensen (1974) monitored the growth of the Dare variety by collecting pod weight, stem weight, leaf weight and leaf area throughout the season. The data is presented in Figure 3-20.

It is interesting to note the abrupt manner in which the plant responds in time. It is as if there exists a mechanism within the plant which almost instantaneously signals the progression from one developmental stage to the next, such as the progression from leaf to pod development. Length of daylight is commonly assumed to be the plant's guide. The plant is apparently aware of the point of diminishing return in the production and maintenance of a leaf canopy. Emphasis on dry matter development shifts toward the production of pods and, therefore, the beans. Stem weight reaches a peak and then decreases as the rate of pod growth reaches its peak. If defoliation occurs it reduces the rate of photosynthesis and, therefore, the assimilate available to the pods. W.G. Rudd (1975), with the data collected by Jensen, is able to simulate the growth of several determinate varieties
Figure 3-20
Soybean Plant Model and Dare Variety Data.
Day Zero is Podset
of soybean. The soybean dynamics model is deterministically based on one year's growth data. Though such external factors as water stress, sunlight, temperature and rainfall will alter productivity and perhaps stage timing, we neglect these factors because of the lack of data and theory.

The original plant model is a discrete discontinuous model describable only by a series of logical steps and arithmetic manipulations. Appendix III presents this model. Because this model is not phrased in continuous terms, we modify the equations into a simultaneous set of continuous ordinary differential equations.

At least two approaches can be used to obtain a tractable model. One is the approximation in the limit approach where finite corners are described by infinite functions. Instead, we employ the Fermi function,

$$ F^+(w) = \frac{1}{1 + \exp(aw)} $$

where \( a \) is a constant which determines the approximated step function's slope. The unit step is used as a multiplier of the plant dynamics at every point.

Since we simulate the other components of the system from podset to harvest, we write our plant model for the same period of time. The dry stem weight \( z_5 \) in grams per foot of row is calculated by,

$$ \frac{1}{z_m} \frac{dz_5}{dt} = \frac{f_5}{z_m} = F^-(t_m(t_1 - t)) D_1 - F^-(t_m(t_1 - t)) \cdot $$
\[ F^-(z_5 - z_{5\text{max}}) \cdot \]
\[ (F^-(D_2 z_6 - D)(D_1(1 - D_3) + D_2 D_3 z_6)) \]
\[ + F^+(D_2 z_6 - D_1 D_2 z_6) - F^-(t_m(t_1 - t)) F^+(z_5 - z_{5\text{max}}) D_1 \]

The combined bean and hull dry gram weight is,

\[ \frac{1}{t_m} \frac{dz_6}{dt} = f_6 = \frac{F^-(t_m(t_1 - t)) F^-(z_5 - z_{5\text{max}})}{F^-(D_2 z_6 - D_1)} \]
\[ [D_1(1 - D_3) + D_2 D_3 z_6] + F^+(D_2 z_6 - D_1) D_2 z_6 \]
\[ + F^-(t_m(t_1 - t)) F^+(z_5 - z_{5\text{max}}) D_1 - z_3 \bigg|_{y = 1} \]

and the dry leaf weight in grams per foot of row is given by,

\[ \frac{1}{t_m} \frac{dz_7}{dt} = f_7 = -D_5 \frac{A(t_m t E \times P(-A(t_m(t_1 - t)))}{(1 + E \times P(-A(t_m(t_1 - t))))^2} \]
\[ + F^-(t_m(t_1 - t)) - z_4 \bigg|_{y = 1} \]

We define the variables in Table 3-5. Because the pod weight beyond the pod growth stopping time \( t_1 \) is independent of leaf area we essentially ignore its dynamics past that point.

To solve the simultaneous equations we employ a first-order difference approximation. Upon integration we obtain the results presented
Table 3-5

Definition of Terms in Soybean Plant Model

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Coefficient of Fermi approximation</td>
</tr>
<tr>
<td>$c_1, c_2$</td>
<td>Gram/day SGSB and VBC feeding</td>
</tr>
</tbody>
</table>
| $D_1$ | Dry weight production  
$= F^-(D_7 - z_7)(E_1 + E_2 z_7 + E_3 z_7^2)$  
$+ F^+(D_7 - z_7)E_4(1 - \exp \left(-E_5 z_7\right))$ |
| $D_2$ | Rate at which pods can fill |
| $D_3$ | Constant which controls the rate at which dry matter can leave the stem |
| $D_5$ | Product of initial ($t = 0$) leaf weight, and rate at which the leaves senesce. |
| $D_7$ | A constant leaf dry weight at which dry weight production changes from an exponential to quadratic function of leaf weight |
| $E_1, E_2, E_3$ | Quadratic coefficients for dry weight production  
$E_1 = 0.054 \times E_4$  
$E_2 = 0.4778 \times E_4 \times \frac{F}{G A R E A}$  
$E_3 = -0.0623 \times E_4 \times \left(\frac{F}{G A R E A}\right)^2$ |
| $E_4$ | Maximum photosynthetic rate |
| $E_5$ | Coefficient for fit of exponential dry weight production |
Table 3-5 (Continued)

\[ E_5 = 0.91435 \times \frac{F}{\text{GAREA}} \]

- **F**: Surface area per gram leaf
  - \( F = 215 \text{ cm}^2/\text{g} \)

- **GAREA**: Ground area per foot row, cm\(^2\)

- **\( t_1 \)**: Time of growth lapse

- **\( z_{5\text{max}} \)**: Maximum stem weight
in Figure 3-20, along with the data. The Lee variety model is fitted to the same data after a necessary scaling of the time axis.

Because the purpose of employing the soybean plant dynamics is to evaluate the crop yield more exactly as a result of pest damage, we validate the model for the form of pest damage encountered. In this research we look only at insects which affect the plant by feeding on the foliage or pods.

**Plant Response to Depodding**

When pods are removed, the photosynthate and nitrification products intended for them from neighboring leaves are translocated to other nearby pods, usually those lower on the stem (Caldwell, 1973). Data (Herzog, 1974) demonstrates this effect for field plots. Where pod damage is high the remaining beans are heavier than in plots where no damage occurs. This is just one way in which the plant compensates for loss.

The results of mechanical depodding by Thomas, Ignoff, et. al., (1974). The soybean variety with which Thomas et. al., removed pods is Clark 63, an indeterminate variety of maturity group IV, an early maturer. We remove from our Dare and Lee varieties pod weight state variable, weight equivalent to the fractional depodding presented in the field experiment. Figure 3-21 presents the field and model results for Dare. The greatest deviation is for early season complete depodding, an unrealistic situation when dealing with the insect species studied.

**Plant Response to Defoliation**
Figure 3-21

Model Results (M) and Field Data (D) for the Effect of a Specified Percent Depodding ( ).
Defoliation is even harder to describe in terms of quantitative loss than is depodding because of its indirect effect on bean production. Photosynthesis produces a product which is translocated to the roots, stems and pods. Until the leaf is greater than half its mature size it demands dry matter. After then, the leaves produce a net gain in assimilate and begin supporting the needs of other plant parts.

Leaf defoliation is assumed independent of location. We remove weight equivalent to leaf area as experimentally done by Kalton, Weber and Eldrige (1949). Figure 3-22 presents the percent of expected yield after defoliation. In almost all cases the model appears less sensitive than the plant to defoliation. This is believed to be the result of the different variety planted by Kalton. The leaf area is much greater in the Southern varieties, and as a result, the reduction in potential yield is less sensitive to the magnitude of defoliation.

SUMMARY

In this chapter we develop and where possible validate the models of particular components of the soybean ecosystem. The complexity of the real system is thwarting, however, for the purpose of studying the dominate system components we demonstrate the adequacy of the models.
Figure 3-22

Model Results (M) and Field Data (D) for the Effect of a Specified Percent Defoliation ( ).
CHAPTER IV

SYSTEM OPTIMIZATION AND CONTROL

Presently, decisions for the control by pesticides of insect pest populations are based on a recommended "economic threshold". This empirically obtained level is a population per unit area which is believed to represent a significant threat to crop yield. There is difficulty in its use without consideration of plant dynamics. For instance, near and after the time the plant's yield is independent of leaf area we should ignore defoliators. Also, if any damaging species maintain a population just below the economic threshold during the season, then the damage may be greater than if there is only a temporary population peak above that level. Required for better pest management policies is a procedure for estimating the time-dependent damage potential a measured field population may be capable of inflicting.

We approach the pest management problem through the use of simulation and optimization. Given early-season field data, we hope to be able to determine the appropriate pesticide policies for the remainder of a season.

In this chapter we obtain the continuous and discrete pesticide policies for the two species, the Southern green stink bug (SGSB) and velvet bean caterpillar (VBC), in the early maturing Dare and the later maturing Lee soybean varieties. Best and worst cases for the predator-
parasite (PP) complex growth rate $r$ and soybean quality damage are studied. We begin by discussing the objective function.

**OBJECTIVE FUNCTION**

In general, the total quantitative description of economic objectives for a crop ecosystem is beyond present capabilities. The difficulty lies in such ill-defined areas as the cost of the effect of pesticide on the environment, which includes such factors as the "cost" of chemical runoff into waterways, the poisonous properties of pesticide and the deposition of the chemicals in animal tissue.

The question of appropriate costs is just as complex if we consider the agricultural sector a closed system. Considering only insect pests, we must account for the possible loss of effectiveness of natural control by predators, parasites and pathogens when insecticides are applied.

Another problem is in determining for whom to optimize the soybean production—the individual grower who desires a seasonal profit, growers of a region wherein a few would suffer for the good of the majority, for the nation, or perhaps for the consumer. If the optimization of crop yield is considered on a regional basis we must account for product supply and demand and any resulting fluctuations in crop value. In no case will satisfying any one of the above satisfy all.

Since the ultimate objective of this work is a study of the field applicability of the methods, we believe that our results must be acceptable to both the grower and the grower's pest management consultant. It is our approach, therefore, to define
our objective for the benefit of the individual grower. We account for those properties of the system for which we have data; the cost of pesticide and its application, and the value of beans produced.

**Soybean Quantity Reduction**

Two approaches are used to describe the soybean quantity reduction or loss. In the first, we calculate the quantity loss in weight by integrating the product of the insect population and its pod feeding rate. This yields the cumulative weight reduction potential of the depodder. This approach can also be successful when dealing with defoliators when a quantitative correlation between feeding rate and yield reduction is available. The mathematical expression for the pod feeding process is given by

$$
\int \int D(y) z_{t}(t, y) \, dy \, dt
$$

The percent of bugs per stage is given by the N_i's of Eq. 2-8, while the bean weight reductions are compiled in Table 3-1. We call this cumulative feeding potential D the damage function. The assumption that daily damage by insects at a given age is simply proportional to the population at that age is based on available feeding data and is deterministically justified. We also assume that the soybean plant does not compensate for seed damage by adding more pods or growing larger beans.

To include the compensating response of the soybean plant we em-
ploy the Rudd-Jensen plant model described in Chapter III. Here the objective function depends upon the final yield from the simulated plant. Loss due to pod feeding is partially compensated for by the plant through continued photosynthesis or the use of stored dry matter, depending on the stage of plant development. The yield per foot of row may be simply stated as the product of this final bean weight $z_6$ and the dollar value of soybean by weight. We express this as

$$\beta \left. z_6(t) \right|_{t=1}$$

where $\beta$ is the price per gram dry pod weight

$$\beta = \frac{\text{Bean weight}}{\text{Total Pod Weight}} \times \frac{\text{Bushels}}{\text{Gram Beans}} \times \frac{\text{Price}}{\text{Bushel}}$$

For this calculation we use the commonly found factor of .7 for the ratio of bean weight to pod weight.

**Quality Reduction**

Damage by pod feeders directly affects the quality of soybean (Duncan, 1968). Because of this damage potential the prices paid for soybeans are discounted at a rate proportional to $1/4$ of the actual percentage of damaged seed. Several states follow this pricing procedure, but even within a state all soybean buyers don't uniformly deduct for quality damage. For instance, in Louisiana two years ago all buyers deducted, but last year only one did so. We feel that to analyze the cost of pest damage to a grower we must study this possibility.

The relation between population level and quality reduction we use is the one developed in Chapter III, Figure 3-4. The contribution to the objective function in terms of the normalized variables is then
\[ \beta Q_r z_0(t) \int_0^y E(y)z_1(t, y) \, dy \, dt \]

Or we can restate this expression as

\[ \beta Q_r z_0(t) z_0(t) \bigg|_{t = 1} \]

where

\[ \frac{dz_0(t)}{dt} = f_0 = t_m z_{10}(t, y) \bigg|_{y = 1} \]
\[ z_0(0) = 0 \]

\[ \frac{\partial z_{10}(t, y)}{\partial y} = f_{10} = y_m E(y)z_1(t, y) \]
\[ z_{10}(t, 0) = 0 \]

\[ Q_r \] is the fractional discount due to seed quality damage. We express \( E(y) \), the age-dependent puncture rate, as

\[ E(y) = \sum_{\text{stages}} \frac{\text{average number of punctures per } SGSB \text{ by stage}}{N_1 \times \text{punctures per stage}} \]

Data for \( E(y) \) are listed in Table 3-1.

**Control Costs**

Control costs include the cost of pesticide and its application. Methyl parathion is a common pesticide for application to the SGSB and VBC. For a continuous pesticide policy we must be able to describe on a continuous basis the amount of pesticide required for a specific fraction killed. We accomplish this through an approximation of data (Jensen, 1972) by a quadratic fit between fractional kill and the quantity of pesticide (Figure 4-1). Approximately one-half pound of
methyl parathion is required per acre for a 99% control of SGSB and VBC. The normal recommendation for the control of VBC is 1/4 pound of methyl parathion per acre. Because this data is laboratory obtained and no field data is available on the relative susceptibility of the SGSB and VBC to pesticide, we assume the same effective control rate for both species (Figure 4-1). Significant difference in control policies is not believed to be introduced as a result. The cost of control is determined by

![Graph showing the relationship between pounds of pesticide and percent insect kill.](image)

Approximation to Insecticide Effect
where

\[
\int \int R(y)u^2(t,y) \, dy \, dt
\]

The cost of application, a discontinuous step function, can be approximated by the Fermi equation,

\[
\int \frac{A_p}{1 + A_c \exp(-A_2 U(t))} \, dt
\]

where \(A_p\) is the cost of application. \(A_c\) and \(A_2\) are constants which adjust the slopes of the step function approximation. We can consider

\[
\int \frac{A_p}{1 + A_c} \, dt
\]

as the cost of seasonal pest surveys. Typical control costs are $2.5 per acre of which $1 is for pesticide and $1.5 is the fixed cost of crop dusting/spraying.

**RESULTS**

Since the SGSB is perhaps the most damaging pest of soybean production we begin our pest management study by determining the optimal continuous pesticide policies for this insect alone. The system equation, Eq. 3-5, can be optimized while minimizing the combined costs of
potential pod punctures and pest control by pesticide. The objective function is then

$$IP = \int_0^1 \int_0^1 (D(y)z_1(t,y) + R(y) u^2(t,y)) \, dy \, dt$$

where the variables are as previously defined in Eqs. 4-1 and 4-6. Applying the derived equations for the minimum principle, Eqs. 2-24, we obtain the state and costate equations

$$\frac{\partial z_1}{\partial t} = - \frac{t_m \partial z}{\partial y} - t_m k_1 z_1 - t_m (1 - k_1) z_1 u + t_m q(z_{1\text{ref}} - z_1)$$

$$\frac{\partial P_1}{\partial t} = - \frac{t_m \partial P_1}{\partial y} - t_m (k_1(u - 1) - q - u)P_1 - D$$

and transversality conditions

$$z_1(0,y) = z_1(y_0)$$

$$z_1(t,0) = y_m \int_0^1 Bz_1 \, dy$$

$$P_1(1,y) = 0$$

$$P_1(t,1) = 0$$

The control policy derived from the necessary condition, $\partial H/\partial u$, is then

$$u(t,y) = \frac{t_m}{2R(y)} (1 - k_1) z_1(t,y) P_1(t,y)$$

This control policy is proportional to the number of controllable
insects, $z_1(t,y)$, and inversely related to the unit cost of pesticide $R$. For the SGSB we obtain the control policies in Figures 4-2 and 4-3, for immigration coefficients of 0 and $0.01z_1(0,y)$, respectively. The policies, which do not account for the cost of application, result in SGSB populations in Figures 4-4 and 4-5. Pesticide and soybean costs of $4 per pound and $8.5 bushel, respectively, are used. We assume the control cost $R(y)$ is evenly distributed over insect developmental stages.

In both cases the optimal policy is to control the adults already in the field, then take action against the newly laid eggs, which can later become destructive insects. We then control the remaining nymphs. We note that toward the end of the season all controls are relaxed, since young nymphs pose little future threat to the crop which will soon be harvested. When we allow immigration, the control levels to an equilibrium value. This equilibrium is a result of the driving force for immigration being a function of the field population density.

To better represent a practical control policy we restrict our control efforts to the 4th and 5th instars and adult stage. Therefore, the evaluated control trajectory is a function of time only. Control action is distributed to the controllable stages by the age dependent coefficient $k_u(y)$, Eq. 3-3. The new objective function is

$$IP = \int_0^1 Ru^2(t) \, dt + \int_0^1 \int_0^1 D(y) \, z_1(t,y) \, dy \, dt$$

or

$$IP = \int_0^1 \int_0^1 (\frac{Ru^2(t)}{M} + D(y)z_1(t,y)) \, dy \, dt$$
Continuous Control of SGSB with Control a Function of Time and Species Age
Continuous Control Policy of SGSB with Immigration When Control is a Function of Time and Species Age
Figure 4-4

Controlled Population SGSB when Control is a Function of Time and Species Age
Controlled Population SGSB with Immigration when Control is a Function of Time and Species Age

\[ q(y) = 0.01 z_1(0, y) \]
Since \( M \) is defined as (Eq. 2-15)

\[
M = y_{\text{max}} - y_{\text{min}} = 1
\]

then

\[
IP = \int_0^1 \int_0^1 (R u^2(t) + D(y)z(t, y))dydt \quad \text{4-11}
\]

The resulting control policy and population are displayed in Figures 4-6 and 4-7, respectively, for no immigration. The results are not much different from those anticipated given the trajectories of \( u(t, y) \). They provide for control of the initial inflight then of the damage producers once they reach a controllable stage. Once again, the control policy suggests a decrease in control effort as harvest is approached. Table 4-1 lists the economic results for the above cases.

To include application costs we introduce a step change in the objective function for any control action. We do this by introducing Eq. 4-6. Upon numerical calculation of the optimum we find the control policy to tend to a constant value of mortality between 0 and one percent. The characteristics of the Fermi equation determine this value which is established near the root of the approximated step function. The weight of the application cost does, therefore, significantly effect the continuous control.

Approaching the optimization problem without a plant model disregards the compensatory ability of the soybean plant. We now introduce the plant dynamics into the optimization problem.
Fraction Killed

\$8.5/\text{bushel}

\$4/\text{lb. pesticide}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure46.png}
\caption{Continuous Optimal Control of Stink Bug in Dare Variety}
\end{figure}
Population (number per foot row)

---

Eggs
Nymphs
Adults

$8.5/bushel
$4/lb pesticide

Figure 4-7

Controlled Population Southern Green Stink Bug
**Table 4-1**

Damage By and Control of the SGSB With the Objective Function of Eq. 4-11

($8.5/bushel, $4/lb pesticide, assuming a 47.45 bushel/acre yield)

<table>
<thead>
<tr>
<th>Variety</th>
<th>IP $/acre</th>
<th>Yield Bushesl/acre</th>
<th>Damage $/acre</th>
<th>Control $/acre</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No Damage</strong></td>
<td>403.33</td>
<td>47.45</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td><strong>No Control (q = 0)</strong></td>
<td>376.14</td>
<td>44.25</td>
<td>27.19</td>
<td>---</td>
</tr>
<tr>
<td><strong>Continuous Control (q = 0, u(t,y))</strong></td>
<td>402.22</td>
<td>47.38</td>
<td>.59</td>
<td>.52</td>
</tr>
<tr>
<td><strong>No Control (q = .01z(t,y))</strong></td>
<td>370.11</td>
<td>43.54</td>
<td>33.22</td>
<td>---</td>
</tr>
<tr>
<td><strong>Continuous Control (q = .01z(t,y), u(t,y))</strong></td>
<td>401.25</td>
<td>47.29</td>
<td>1.36</td>
<td>.72</td>
</tr>
<tr>
<td><strong>No Control (q = 0)</strong></td>
<td>376.14</td>
<td>44.25</td>
<td>27.19</td>
<td>---</td>
</tr>
<tr>
<td><strong>Continuous Control (q = 0, u(t))</strong></td>
<td>396.82</td>
<td>46.92</td>
<td>4.53</td>
<td>1.98</td>
</tr>
</tbody>
</table>

**Addition of Soybean Plant Model**

When the plant model is introduced, the yield is determined in absolute terms and the objective function is expressed as a combination of final pod weight (Eq. 4-2), soybean quality (Eq. 4-3) and control efforts (Eqs. 4-6 and 4-7):

\[
\text{IP} = - \beta z_6(t) \bigg|_{t=1} + \beta Q_z z_6(t) z_9(t) \bigg|_{t=1}^{1} + \int_{0}^{1} (R u^2(t)) dt \\
+ A_p \left(1 + \frac{1}{A_c \exp (-A_2 u(t))} \right) dt
\]
or with the previous assumptions,

\[
IP = - \beta z_6(t)(1 - Qr z_9(t)) + \int_0^1 \int_0^1 (R u(t)) dy dt + A_p \frac{1}{1 + A_c (\exp(-A_2 u(t)))} dy dt
\]

4-12

The equality constraints

\[
\begin{align*}
\frac{az_1}{at} &= f_1 \\
\frac{dz_5}{dt} &= f_5 \\
\frac{dz_8}{dt} &= f_8 \\
\frac{az_2}{at} &= f_2 \\
\frac{dz_6}{dt} &= f_6 \\
\frac{dz_9}{dt} &= f_9 \\
\frac{az_3}{ay} &= f_3 \\
\frac{dz_7}{dt} &= f_7 \\
\frac{az_{10}}{ay} &= f_{10} \\
\frac{az_4}{ay} &= f_4
\end{align*}
\]

and objective function, Eq. 4-12, are substituted into Eq. 4-14, yielding the costate equations and normalized costate equations,

\[
\begin{align*}
\frac{ap_1}{at} &= - \frac{tm}{ym} \frac{ap_1}{ay} + p_1(t_m(k_1 + q_1) + t_m(1 - k_1)k_{u_1} u) \\
&- y_m c_1 p_3 - y_m^e p_{10} \\
\frac{ap_2}{at} &= - \frac{tm}{ym} \frac{ap_2}{ay} + p_2(t_m(k_2 + q_2) + t_m(1 - k_2)k_{u_2} u)
\end{align*}
\]
\[
\begin{align*}
\frac{\partial \theta_1}{\partial y} &= t_m p_6 \\
\frac{\partial \theta_2}{\partial y} &= t_m p_7 \\
\frac{\partial \theta_3}{\partial y} &= t_m p_8 f'((t_1 - t)t_m) \\
\frac{\partial \theta_4}{\partial y} &= t_m \left( \frac{1}{\sqrt{1 + z_2^2}} \right) dy - t_m p_8 (r - u - 2rz_8) \\
\frac{\partial \theta_5}{\partial y} &= 0 \\
\frac{\partial \theta_6}{\partial y} &= -t_m p_9
\end{align*}
\]

and transversality conditions,
\begin{align*}
P_1(1, y) &= 0 \\
P_2(1, y) &= 0 \\
P_3(t, 1) &= 0 \\
P_4(t, 1) &= 0 \\
P_5(1) &= 0 \\
P_6(1) &= \beta(Q_r z_9 - 1) \\
P_7(1) &= 0 \\
P_8(1) &= 0 \\
P_9(1) &= aQ_r z_6 \\
P_{10}(t, 1) &= 0
\end{align*}

where along with the definitions from Table 3-4, the \( \delta \)'s are defined as

\begin{align*}
\delta_1 &= F^-(t_m(t_1 - t)) \frac{\partial(F^-(z_5 - z_{5\text{max}}))}{\partial z_5} [F^-(D_2z_6 - D_1)((1 - D_3)D_1 + D_2D_3z_6)] \\
&\quad + F^+(D_2z_6 - D_1)D_2z_6 + F^-(t_m(t_1 - t)) \frac{\partial(F^+(z_5 - z_{5\text{max}}))}{\partial z_5} D_1 \\
\delta_2 &= F^-(t_m(t_1 - t))F^-(z_5 - z_{5\text{max}}) \left[ \frac{\partial(F^-(D_2z_6 - D_1))}{\partial z_6} \right] \\
&\quad \left[ (1 - D_3)D_1 + D_2D_3z_6 \right] \\
&\quad + F^+(D_2z_6 - D_1)D_2 + D_2D_3 F^-(D_2z_6 - D_1) \\
&\quad + D_2z_6 \frac{\partial(F^+(D_2z_6 - D_1))}{\partial z_6} \\
\delta_3 &= F^-(t_m(t_1 - t)) F^-(z_5 - z_{5\text{max}}) \left[ \frac{\partial(F^-(D_2z_6 - D_1))}{\partial z_7} \right]
\end{align*}
\[
((1 - D_3)D_1 + D_2D_3z_6) \\
+ (1 - D_3) \frac{aD_1}{az_7} + D_2z_6 \frac{a(F(D_2z_6 - D_1))}{az_7} \\
+ F'(t_m(t_1 - t)) F(z_5 - z_{5_{\text{max}}}) \frac{aD_1}{az_7}
\]

The general expression of the necessary condition for an optimum is then

\[
\frac{\partial H}{\partial u} = 0 = 2Ru + P_1 (-t_m(1-k_1)k_{u_1}z_1) \\
+ P_2 (-t_m(1-k_2)k_{u_2}z_2) \\
- t_mP_8z_8 + A_pA_cA_2 \text{Exp}(-A_2u)/(1+A_c\text{Exp}(-A_2u))^2
\]

Here the control function is proportional to the controllable SGSB, VBC and PP populations. For the remainder of the research we assume \(A_p\) equal zero.

The set of equations as stated above represent the general case studied in this research. We divide the studies into a set of cases which are studied independently. These are: (1) the VBC with PP complex, (2) the SGSB alone, and (3) the SGSB, VBC and PP complex. We also consider two varieties of soybean, Dare and Lee, for each of the three cases above. Two levels of quality reduction, 0 and .25, are studied. Different PP complex growth rate values \(r\) and ratios of bean price to pesticide cost are used. Initial conditions of the populations are those outlined in Figures 3-9 and 3-17. We include the control policies and several resulting population levels in this chapter, but most controlled populations are presented in Appendix I.
Effect of Southern Green Stink Bug - Continuous Control

To obtain the equations which express just the SGSB effect, we set the VBC feeding parameter $c_2$ equal zero, thus eliminating its influence on crop productivity. The effect of the SGSB on the crop yield for various parameter values are listed in Table 4-2. We now discuss this table in connection with the appropriate figures.

Figure 4-8 displays the continuous SGSB control trajectory. It is interesting to compare this figure with Figure 4-6, where the damage function rather than the plant model is used. With the plant simulated the initial control action is less, but later season SGSB control greater. The variation in early season action is due to the compensating ability of the plant after early depodding. Without the plant model, early season feeding is as damaging as any later season feeding. We, therefore, require a higher level of control near harvest because of the linear effect of damage on yield.

With both objective functions we find a minimum in the control policy around day ten. This is because of the cycle in which the SGSB is found, the eggs and early nymphs, which are uncontrollable, are the bulk of the field population.

The control policy when the SGSB is on Lee (Figure 4-9) is much the same as when on the Dare variety, however, the initial control in Lee is higher in order to eliminate later season population peaks which do not occur in the Dare. The later season control stays at a significant level in order to suppress the late season population (see Appendix I).
Table 4-2

Damage by and Control of the SGSB

($8.5/bushel, $4/lb pesticide, 1/4 quality reduction, no application cost)

<table>
<thead>
<tr>
<th></th>
<th>Dare</th>
<th>Lee</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IP $/acre</td>
<td>Yield bushel $/acre</td>
</tr>
<tr>
<td>No Damage</td>
<td>403.33</td>
<td>47.45</td>
</tr>
<tr>
<td>No Control</td>
<td>372.18</td>
<td>45.5</td>
</tr>
<tr>
<td>Continuous Control</td>
<td>391.95</td>
<td>47.1</td>
</tr>
<tr>
<td>Single Pesticide Application</td>
<td>399.41</td>
<td>47.28</td>
</tr>
<tr>
<td>Two Pesticide Applications</td>
<td>397.64</td>
<td>47.28</td>
</tr>
</tbody>
</table>

|                                |               |                | Time of Application (days since podset) |
| No Damage                      |               |                |                                           |
| No Control                     |               |                |                                           |
| Continuous Control             |               |                |                                           |
| Single Pesticide Application   |               |                |                                           |
| Two Pesticide Applications     |               |                |                                           |
Southern Green Stink Bug Control Policy - Dare Variety
Figure 4-9

Southern Green Stink Bug Control Policies - Lee Variety

- $8.5/bushel
- $4/lb. pesticide
- 0 quality reduction
- 1/4 quality reduction

Time (days)
When we add quality reduction to the objective function we obtain the control policies of Figures 4-10 and 4-9. The controlled population in Dare is shown in Figure 4-11.

There is little alteration in the control timing when the potential of quality reduction is added. There is, however, some alteration in the magnitude of the control effort. In both varieties the economic impact of the SGSB on quality is extremely significant (Table 4-2). It becomes, therefore, important to eliminate the SGSB population as soon as possible. This explains the magnitude increase in control on the first day.

Similarities in the controlled population for the two varieties, where in Dare we account for quality damage and in Lee we ignore quality damage, is striking. Even without accounting for quality loss, we control more in Lee because of the possible influence of later generations of the SGSB.

Effect of Southern Green Stink Bug - Discrete Control

The discrete control, at the rate of 99% mortality per application, is economically applied only once during the growing season (Table 4-2). This application, as inferred from the continuous results, is on the first day past podset. The resulting SGSB population is shown in Figure 4-12.

In the Dare variety, when two applications are made, the optimal times for application are on days 1 and 39. In Lee, they are on days 1 and 45. These times correspond well to the continuous control pol-
Figure 4-10

Optimal Control Policies for the Southern Green Stink Bug and Velvet Bean Caterpillar, individually, on Dare Variety
Population Southern Green Stink Bug after Continuous Control (Figure 4-10). Damage to Dace by Stink Bug
Figure 4-12

Population Southern Green Stink Bug after Discrete Control on Day 1.

Damage to Lee by Stink Bug
icy peaks of Figures 4-10 and 4-9. But, as indicated earlier, two applications do not produce a better objective function value. To account for application cost in the discrete case we simply subtract $1.5 per acre from the objective function for each application.

**Effect of Velvet Bean Caterpillar - Continuous Control**

To obtain the influence of the VBC on the soybean varieties we set the SGSB feeding rate $c_1$ equal zero. The economic results of the various experiments are outlined in Table 4-3.

Figures 4-10 and 4-13 display the VBC continuous control policies. In both cases the magnitude of control is indicative of the magnitude of damage the VBC is capable of (Table 4-3). The control policy tapers to zero as day 35 is approached in Dare and day 50 in Lee. This is because, as presented in Chapter III, the dependency of pod development on leaf area ends at these times. It is also for this reason that we plot the VBC populations in Lee only through the period of 50 days for which they can alter yield.

The VBC populations in Dare and Lee after continuous control demonstrate the PP effect on the VBC when pesticide control is used. A lower level of control at day one for a PP growth rate of zero (Figure 4-13) is a result of the ability of pesticide to decrease the mortality of the VBC by controlling the PP population. Even though a greater pesticide usage near the final days of VBC influence on Lee could control its exponential growth, this emerging population is the early larval stages, the non-damaging proportion of the population. It is the population on the days immediately following day 50 in Lee which have a large defoliating potential. This emphasizes the advantage of planting early maturing varieties of soybean.
Table 4-3

Damage by and Control of the VBC
($8.5/bushel, $4/lb pesticide, no application cost)

<table>
<thead>
<tr>
<th>IP $/acre</th>
<th>Yield bushel/acre</th>
<th>Control $/acre</th>
<th>PP Growth r</th>
<th>Defoliation %</th>
<th>Times Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dare</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Damage</td>
<td>403.33</td>
<td>47.45</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>No Control</td>
<td>402.36</td>
<td>47.2</td>
<td>---</td>
<td>---</td>
<td>10</td>
</tr>
<tr>
<td>Continuous Control</td>
<td>402.49</td>
<td>47.38</td>
<td>.15</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Continuous Control</td>
<td>402.67</td>
<td>47.40</td>
<td>.14</td>
<td>1.3</td>
<td>.4</td>
</tr>
<tr>
<td>Single Application</td>
<td>401.02</td>
<td>47.41</td>
<td>1.95</td>
<td>0</td>
<td>5, 15</td>
</tr>
<tr>
<td>Two Applications</td>
<td>399.25</td>
<td>47.43</td>
<td>3.90</td>
<td>0</td>
<td>3, 6, 17</td>
</tr>
<tr>
<td>Single Application</td>
<td>401.06</td>
<td>47.42</td>
<td>1.95</td>
<td>1.3</td>
<td>4, 14</td>
</tr>
<tr>
<td>Two Applications</td>
<td>399.27</td>
<td>47.44</td>
<td>3.90</td>
<td>1.3</td>
<td>1, 5, 17</td>
</tr>
<tr>
<td>Lee</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Damage</td>
<td>403.50</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>No Control</td>
<td>401.26</td>
<td>47.2</td>
<td>---</td>
<td>---</td>
<td>19</td>
</tr>
<tr>
<td>Continuous Control</td>
<td>401.82</td>
<td>47.3</td>
<td>.42</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>Continuous Control</td>
<td>402.44</td>
<td>47.4</td>
<td>.46</td>
<td>1.3</td>
<td>3</td>
</tr>
<tr>
<td>Single Application</td>
<td>400.53</td>
<td>47.35</td>
<td>1.95</td>
<td>0</td>
<td>25, 20</td>
</tr>
<tr>
<td>Two Applications</td>
<td>399.34</td>
<td>47.44</td>
<td>3.90</td>
<td>0</td>
<td>3, 12, 23</td>
</tr>
<tr>
<td>Single Application</td>
<td>400.86</td>
<td>47.39</td>
<td>1.95</td>
<td>1.3</td>
<td>9, 17</td>
</tr>
<tr>
<td>Two Applications</td>
<td>399.43</td>
<td>47.45</td>
<td>3.90</td>
<td>1.3</td>
<td>1, 7, 21</td>
</tr>
</tbody>
</table>
Figure 4-13

Optimal Continuous Control of Velvet Bean Caterpillar Population in Lee
Effect of Velvet Bean Caterpillar - Discrete Control

The discrete control of 99\% of the VBC larva occurs on days determined by the variety and the PP growth rate r. In Dare the best application times for the PP growth rates of zero and 1.3 are days 15 and 14, respectively. In Lee they are days 21 and 18, respectively. Later application times are required when r is zero because of the potentially larger population after control due to a decreased natural mortality. When two applications of pesticide are made the times for Dare are days 6 and 17 and days 5 and 17 for r equal zero and 1.3, respectively. For Lee, days 12 and 23 and days 7 and 21, respectively. The later times for application in Lee are a result of the longer period of time that defoliation can produce yield damage.

Again, we find that when application cost is considered, the use of two discrete applications does not produce a greater profit.
Effect of SGSB and VBC on Dare - Continuous Control

The combined damage potential results in the control policy of Figure 4-14. It is interesting to compare this policy with those of the individual component policies of Figure 4-10. As a result of controlling the initial in flight of the damaging SGSB we decrease the PP population and, therefore, the VBC mortality rate. We must then control the VBC more strongly on days 10 through 30. Later season season control is for the SGSB.

The variation in the magnitude of initial control due to the value of the PP growth rate is significant. We must control the SGSB, but we find a balance between the decrease in SGSB damage and the decrease in VBC mortality due to the effect of pesticide on the PP complex. When the PP complex recovers from control, \( r = 1.3 \), we can afford a higher initial SGSB control. Figure 4-15 shows the resulting PP populations after continuous control. We obtain a balance between pesticide control and the PP population level. Figures 4-14 and 4-16 display a similar control response to the change in the PP growth rate as we observed earlier. For a 1.3 growth rate the second control peak is later.

Once again, when quality reduction is assumed (Figure 4-16), a larger magnitude in the control effort is required, but timming is altered little (compare with Figure 4-14). The VBC population which reach a peak of larvae per foot row after control when damage to the soybean is by both the SGSB and VBC is much lower than in the case where we control only the VBC. This is a result of the foreground effort of control-
Continuous Optimal Control of Stink Bug and Velvet Bean Caterpillar on Dare Variety
Figure 4-15
Population Predator-Parasite Complex on Dare After Continuous Control (Figure 4-16).
$8.5/\text{bushel}
$4/\text{lb pesticide}
1/4 \text{ quality reduction}

PP Growth Rate

Figure 4-16

Continuous Optimal Control of Stink Bug and Velvet Bean Caterpillar on Dare Variety
ling SGSB. The SGSB population in fields with VBC damage is altered from the case of only SGSB on Dare only during the period of VBC control. The economic values are shown in Table 4-4.

**Effect of SGSB and VBC on Dare - Discrete Control**

As Table 4-4 indicates, and as expected, we apply pesticide the first day after podset, when the SGSB initially infests the field. The period of yield dependency on leaf area in Dare is not long enough for the VBC to significantly recover from a day one application, even when the PP growth rate \( r \) is zero.

When two applications are employed, the timing of the second application is later when quality production is not considered in the objective function. This is especially apparent when the PP complex population recovers after control. We again find it not economically advantageous to apply pesticide twice when an application cost of $1.50 is included.

**Effect of SGSB and VBC on Lee - Continuous Control**

The alteration in the initial days of control for the different PP complex growth rates is even more drastic in Lee (Figure 4-17) than in Dare. In both PP growth rate cases we increase the initial control magnitude when SGSB quality damage is considered (Figure 4-18). This is clearly justified when we consider the amount of quality damage possible in the Lee variety (Table 4-5). The control policy for $4 per bushel beans and one-quarter quality damage is reduced by a negligible fraction in magnitude throughout the season as compared to the $8.5 per bushel beans with no quality damage. The relatively
Table 4-4
SGSB and VBC on Dare Variety
Damage and Control
(8.5/bushel, $4/1b pesticide, 1/4 quality reduction)

<table>
<thead>
<tr>
<th></th>
<th>IP $/acre</th>
<th>Yield bushels/acre</th>
<th>Control $/acre</th>
<th>Quality $/Acre</th>
<th>r</th>
<th>Percent Defoliation</th>
<th>Time Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Damage</td>
<td>403.33</td>
<td>47.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Control</td>
<td>371.34</td>
<td>45.4</td>
<td>14.92</td>
<td></td>
<td></td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Continuous Control</td>
<td>392.11</td>
<td>47.06</td>
<td>6.91</td>
<td>.94</td>
<td>0</td>
<td>.1</td>
<td></td>
</tr>
<tr>
<td>Single Application</td>
<td>397.27</td>
<td>47.02</td>
<td>1.95</td>
<td>.44</td>
<td>0</td>
<td>29</td>
<td>1</td>
</tr>
<tr>
<td>Two Applications</td>
<td>397.49</td>
<td>47.26</td>
<td>3.90</td>
<td>.32</td>
<td>0</td>
<td>5</td>
<td>1, 14</td>
</tr>
<tr>
<td>Continuous Control</td>
<td>394.34</td>
<td>47.1</td>
<td>4.88</td>
<td>.92</td>
<td>1.3</td>
<td>.2</td>
<td></td>
</tr>
<tr>
<td>Single Application</td>
<td>398.68</td>
<td>47.19</td>
<td>1.95</td>
<td>.45</td>
<td>1.3</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>Two Applications</td>
<td>397.65</td>
<td>47.27</td>
<td>3.90</td>
<td>.22</td>
<td>1.3</td>
<td>5</td>
<td>1, 7</td>
</tr>
<tr>
<td>Single Application</td>
<td>397.67</td>
<td>47.02</td>
<td>1.95</td>
<td>0</td>
<td>29</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Two Applications</td>
<td>397.81</td>
<td>47.27</td>
<td>3.90</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>1, 15</td>
</tr>
<tr>
<td>Single Application</td>
<td>399.13</td>
<td>47.19</td>
<td>1.95</td>
<td>1.3</td>
<td>9</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Two Applications</td>
<td>397.89</td>
<td>47.27</td>
<td>3.90</td>
<td>1.3</td>
<td>5</td>
<td>1</td>
<td>1, 14</td>
</tr>
</tbody>
</table>
Daily Fraction Controlled

$8.5/bushel
$4/lb pesticide
PP Growth Rate

\[ r = 0 \]
\[ r = 1.3 \]

Figure 4-17

Continuous Control Policy for Southern Green Stink Bug and Velvet Bean Caterpillar - Lee Variety
Daily Fraction Controlled

$8.5/bushel
$4/lb pesticide
1/4 quality reduction

PP Growth Rate
- r=0
- r=1.3

Figure 4-18
Optimal Control Policy - Damage to Lee by Stink Bug and Velvet Bean Caterpillar
<table>
<thead>
<tr>
<th>IP</th>
<th>Yield bushels/acre</th>
<th>Control $/acre</th>
<th>Quality $/acre</th>
<th>PP Growth</th>
<th>Percent Defoliation</th>
<th>Time Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>403.5</td>
<td>47.47</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>351.12</td>
<td>44.0</td>
<td>22.54 (24%)</td>
<td></td>
<td></td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>391.65</td>
<td>47.0</td>
<td>7.39</td>
<td></td>
<td>0</td>
<td>.4</td>
<td></td>
</tr>
<tr>
<td>382.68</td>
<td>45.25</td>
<td>1.95</td>
<td></td>
<td>0</td>
<td>36</td>
<td>5</td>
</tr>
<tr>
<td>397.13</td>
<td>47.18</td>
<td>3.90</td>
<td></td>
<td>0</td>
<td>5</td>
<td>1, 17</td>
</tr>
<tr>
<td>389.84</td>
<td>47.1</td>
<td>9.65</td>
<td>.55</td>
<td>0</td>
<td>.2</td>
<td></td>
</tr>
<tr>
<td>385.88</td>
<td>45.7</td>
<td>1.95</td>
<td>.62</td>
<td>0</td>
<td>40</td>
<td>1</td>
</tr>
<tr>
<td>396.65</td>
<td>47.18</td>
<td>3.90</td>
<td>.48</td>
<td>0</td>
<td>5</td>
<td>1, 17</td>
</tr>
</tbody>
</table>

Table 4-5
SGSB and VBC on Lee Variety
Damage and Control
($8.5/bushel, $4/lb pesticide, 1/4 quality reduction)
<table>
<thead>
<tr>
<th></th>
<th>IP $/acre</th>
<th>Yield bushel/acre</th>
<th>Control $/acre</th>
<th>Quality $/acre</th>
<th>PP Growth</th>
<th>Percent Defoliation</th>
<th>Time Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous Control</td>
<td>393.73</td>
<td>47.0</td>
<td>5.62</td>
<td>1.3</td>
<td>.2</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>Single Application</td>
<td>397.55</td>
<td>47.0</td>
<td>1.95</td>
<td>1.3</td>
<td>12</td>
<td>1</td>
<td>1, 16</td>
</tr>
<tr>
<td>Two Applications</td>
<td>397.47</td>
<td>47.22</td>
<td>3.90</td>
<td>1.3</td>
<td>2</td>
<td>2</td>
<td>1, 16</td>
</tr>
<tr>
<td>Continuous Control</td>
<td>392.40</td>
<td>47.1</td>
<td>7.56</td>
<td>.55</td>
<td>1.3</td>
<td>.1</td>
<td>1</td>
</tr>
<tr>
<td>Single Application</td>
<td>396.91</td>
<td>47.0</td>
<td>1.95</td>
<td>.64</td>
<td>1.3</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>Two Applications</td>
<td>397.00</td>
<td>47.22</td>
<td>3.90</td>
<td>.47</td>
<td>1.3</td>
<td>2</td>
<td>1, 16</td>
</tr>
</tbody>
</table>
reduced expense of bean damage in both cases lends themselves to similar economic damage potentials.

The SGSB populations controlled by the policies of Figure 4-17 demonstrate once again the influence that the PP complex growth rate has on the control policies and, therefore, the resulting populations. When the growth rate \( r \) equals zero we have less initial control and the resulting population is larger. When the PP complex recovers from control we can afford a more efficient SGSB control policy. When quality damage is included we obtain very similar results, but with slightly lower population levels due to the increased magnitude of early season control.

When the price per bushel of beans is reduced to $4 and quality damage is taken into account we obtain slightly larger controlled SGSB populations as compared to when the beans are $8.5 and without quality reduction.

It is interesting to note collectively the VBC populations after the continuous controls of Figures 4-17 and 4-18. Significant differences in population levels are present when the PP growth rate varies. The peak population levels are greater by a factor of 1.8 when the PP population does not recover after control. Once again, the populations are lower in all cases when compared with the VBC when only they damage Lee. As in the Dare variety, this is the result of emphasizing control of the SGSB, rather than of the VBC population.

The PP complex population is shown in Figure 4-19 (compare to Figure 4-15 for Dare). We again seek the optimal use of this natural VBC control while simultaneously using pesticide.
Predator-Parasite Population on Lee After Continuous Control
(Figure 4-17)
Effect of SGSB and VBC on Lee - Discrete Control

Discrete control of the two pest populations in Lee varies from that in Dare for there do exist cases where two applications of pesticide increases profits. When the PP growth rate is zero, whether or not quality damage is considered, we find it more profitable to control twice. This is because the VBC population increases enough to cause damage of almost 1.5 bushels more than that due to the uncontrolled VBC population. Table 4-5 demonstrates the various cases and their economic results.

We note a change from previous results in the timing of a single application when \( r \) is zero and quality damage is not accounted. Figure 4-20 shows the VBC population after control on day 5.

Figure 4-21 displays the SGSB population after two discrete controls. We obtain a larger population at season end when we control on days one and sixteen, rather than on days one and seventeen. This is because the younger SGSB nymphs are emerging into the controllable stages at this time, so an earlier control doesn't affect the peak stages.

DISCUSSION

In studying Table 4-3 we find that when the VBC is controlled, the continuous control policy yields an extremum superior to that of the discrete policy. We explain this result in terms of the PP complex. When a discrete control of 99% mortality is employed on the VBC population we also control the PP by 99% and as a result, the VBC mortality substantially decreases. However, with the continuous
Population Velvet Bean Caterpillar After Discrete Control on Day 5

Damage to Lee by Stink Bug and Velvet Bean Caterpillar

Figure 4-20

Population (number larva per foot row)

$8.5/bushel
$4/lb pesticide
PP Growth Rate $r$ equal 0

Time (days)
Population (number per foot row)

- Eggs
- Nymphs
- Adults

$8.5/bushel
$4/lb pesticide
1/4 quality reduction
PP Growth Rate r Equal 0

Figure 4-21
Population Southern Green Stink Bug After Discrete Control on Days 1 and 17
Damage to Lee by Stink Bug and Velvet Bean Caterpillar
control we find a balance between the VBC mortality contributions.

If we compare the discrete and continuous policies of the SGSB in Table 4-2 we find that the discrete policy yields a better optimum. The discrete controls offer the necessary population regulation with the least expense, especially when application cost is included. We might conclude that the continuous control trajectory of the SGSB approaches a discrete policy, but that the first variational numerical technique is not able to find this extremum.

It is also interesting to note that the continuous optimum for when quality damage is considered is better than when we don't consider this damage. We can conclude that when quality reduction is included the response surface is more pronounced, driving the numerical technique solution nearer to the optimum.

We are not able to specify, given the results we obtained, a general economic threshold for field use. The present recommendations include a count per foot row of one nymph or adult for the SGSB and eight larvae for the VBC. The results indicate that we can tolerate up to seventy VBC larvae per foot row in Lee if the peak occurs fifty days past podset. In Lee, the times of the peak VBC population and the time when yield becomes independent of leaf area is almost the same. We can conclude from these results that in such soybean varieties as Bragg, where maturation occurs about two weeks later, the VBC could present a greater threat.

Though the two rates of PP growth present widely different levels of populations, the discrete optimal control policies in Lee alters the number of applications in only a few cases. When the PP
complex rapidly recovers after a discrete control, then one pesticide application is always superior to two applications. It is when the PP complex does not recover that we apply twice, once to control the initial SGSB inflight and once for the rapidly increasing VBC population. The most significant effect of the PP complex is in the timing of the discrete applications.

It is interesting to note the ratio of the price per bushel of beans to the cost per pound of insecticide which produces no economic improvement in the objective function when discrete pesticide tactics are implemented. For the Lee variety we find the ratio equal to .2 when we assume quality damage, r equal zero and an application cost of $1.5 per acre. A value of .1 is obtained when application costs are ignored. In this research we used a value of 2.1, a commonly encountered ratio of the costs.
CHAPTER V

CONCLUSIONS

We approached the topic of pest management in an attempt to describe the dynamics of a crop ecosystem and to determine optimal policies for the use of pesticides. We modeled several dominant components of this complex and dynamically sensitive system, and studied their individual and interactive responses to manipulation by pesticides.

Our model includes the soybean plant, a major pod feeder (the Southern green stink bug) and a predator-parasite complex which naturally controls a principle defoliator (the velvet bean caterpillar). The objective function for which we optimize the system model is based on economics as viewed by the individual grower, including yield damage and the cost of pesticide control.

We employed two optimization techniques which offer practical possibilities. The logical pattern search technique finds the best single and multiple times for the application of pesticides. This is directly applicable in field pest management situations. The second technique, the minimum principle, provides a continuous pesticide control policy. This policy would be analogous to a grower dispensing pesticide at arbitrarily specified rates and times. This tactic does not exist, but the policy provides expression of the system's response to an ideal pesticide application program. It is felt that this technique could be more useful once more quantitative data are available on the continuous acting biological control techniques.
With use of the optimization procedures we show the system's sensitivity to the pest life cycles and plant developmental stages. For example, we have shown that early season pesticide applications result in a large defoliating population of velvet bean caterpillar later in the season. This is a result of controlling the natural biological control complex. But the real point of interest is the time at which this peak occurs. In the Lee variety it appears almost simultaneously with the time at which pod development no longer depends on leaf area and, therefore, we can allow the caterpillar to go uncontrolled at this time. As a result, the damage threshold levels of the velvet bean caterpillar must be considered time-dependent. The stages of plant and pest development influence the optimal timings of pesticide application. It is through methods developed herein that we can develop these time dependent threshold levels.

In the Lee and Dare varieties we find the Southern green stink bug to be the dominate pest with most control directed toward its management. If we consider only the velvet bean caterpillar as a threat, we find it uneconomical for a discrete 99% control. If, however, the initial velvet bean caterpillar population is greater in magnitude the discrete technique controls the species at population numbers below the presently recommended threshold levels. After this control we allow the population to increase and reach levels nearly twice the threshold without demands for additional control. As pods develop they become less dependent on leaf area, permitting greater levels of populations before a significant reduction in yield. It is our general conclusion that pest infestations can be tolerated at higher levels than is the current practice.
The predator-parasite complex is an important factor in how populations respond to pesticide, because the complex can affect the optimal pesticide policies. In fact, we find that timing and magnitude of both the discrete and continuous policies are altered depending on the rate at which the biological control complex recovers from a pesticide application. Data must be collected to describe quantitatively in much greater detail this important component of the crop system.

With the simulated predator-parasite complex we can study the pest population's recovery from pesticide applications, and with comparison to similar field conditions determine the actual rate at which the complex can recover. In this way, we can find those insects which demonstrate dynamics appropriate to this response. Though we may not understand the detailed interrelationships between species we should attempt to describe them mathematically, and through experimentation determine the actual responses.

We do not believe that the research tools demonstrated here have reached their peak of usefulness. We have shown that crop system modeling can provide information which can be used to better describe systems and to guide experimentation. As more quantitative information is obtained on crop pests we should incorporate this information into our models and determine the crop's response. The result is bound to be a request for more data specific to the problem relevance. In this way, the modeling-experimentation feedback loop provides a powerful impetus for progress in research in pest management.
Today, pest management efforts are increasing in sophistication with the automation of field population measurements and the use of computers in grower consulting. As these techniques become more reliable and accepted there will be need for practical and efficient population models and optimization techniques. The techniques developed in this research show promise for use in pest management programs of this sort.

Traditionally, engineers have not been active in interdisciplinary research in these areas. But the potential for application of mathematical and systems methods we commonly use is great. This is truly one area where modeling efforts can prove vital in organizing and developing efficient management programs.
Dependent Variables

\( u \) control vector, fraction controlled per day
\( N \) vector of stage probability functions
\( P \) vector of co-state variables
\( z \) vector of normalized dependent state variables
\( z_1, z_1 \) time and age dependent population SGSB, number per foot of row
\( z_2, z_2 \) time and age dependent population VBC, number per foot of row
\( z_3, z_3 \) SGSB damage, grams per foot of row
\( z_4, z_4 \) VBC damage, grams per foot of row
\( z_5, z_5 \) dry stem weight, grams per foot of row
\( z_6, z_6 \) dry pod weight, grams per foot of row
\( z_7, z_7 \) dry leaf weight, grams per foot of row
\( z_8, z_8 \) PP complex, population per foot of row
\( z_9, z_9 \) daily SGSB punctures
\( z_{10}, z_{10} \) age-dependent number of SGSB seed punctures

Independent Variables

\( t, T \) time, days
\( y, Y \) spatial coordinate, days

Functionals

\( g, f \) ordinary and partial differential equations
\( H \) Hamiltonian
\( IP \) objective function, dollars per foot of row
\( \theta, W, Q, S \) functionals in IP
Operators

\( \frac{\partial}{\partial t}, \frac{\partial}{\partial y} \) partial differentials wrt time and the spatial vector

\( \frac{d}{dt} \) ordinary differential wrt time

\( \Delta( ) \) difference operator

\( z^T \) transpose of \( z \)

\( F^\pm \) Fermi approximation

\( \delta \) variational operator

Subspaces and Sets

\( D \) Euclidean subspace

\( V \) set of admissible control functions

\( C_1 \) control inequality constraints

Constants and Parameters

\( A, A_c, A_2 \) constant of Fermi approximation

\( A_p \) application cost per foot of row

\( b_i \) base point in pattern search

\( B_1(y), B_2(y) \) oviposition rate of SGSB and VBC

\( c_i \) defines controllable stages

\( c_1(y), c_2(y) \) SGSB and VBC feeding rates

\( C_p \) PP capacity for VBC feeding

\( D(y) \) SGSB damage function

\( D_1 \) dry weight production

\( D_2, D_3, D_5 \) rates and constants in plant model

\( D_7 \)
Constants and Parameters (cont.)

$E(y)$ puncture rate; emergence function
$E_1, E_2, E_3, E_4, E_5$, constants in plant model

$F$ constant in plant model

$\text{GAREA}$ constant unit area in plant model

$h$ perturbation in $t$

$I(T,Y)$ immigration, number per day

$k$ multiplier; perturbation in spatial coordinate

$k_1(y), k_2(y)$ mortality coefficients SGSB and VBC

$k_{u_1}(y), k_{u_2}(y)$ pesticide mortality coefficients

$k_p(y)$ $\text{PP feeding mortality fraction}$

$k_{\text{stage}}$ stage mortality

$M$ product of ranges of spatial coordinate

$M(T,Y)$ mortality, number per day

$q_1(y), q_2(y)$ immigration coefficient

$q_{i,i+1}(y)$ interstage influx

$Q_r$ fractional discount for quality reduction

$r$ dummy variable; $\text{PP growth rate}$

$R(y)$ control cost per foot of row

$t_i$ time of growth lapse in plant model

$t_{ij}$ temporary base point in pattern search
Constants and Parameters (cont.)

\( T_{stage} \)  stage residence time
\( w \)  dummy variable

\( \alpha_i \)  unit vector
\( \beta \)  price per gram of dry pod weight per foot of row
\( \delta \)  Dirac delta function; variational operator
\( \Delta_i \)  step change vector in pattern search
\( \varnothing \)  impulse response function

\( i,j,k,m,n \)  general subscripts
BIBLIOGRAPHY


APPENDIX I

POPULATIONS RESULTING FROM THE CONTINUOUS AND DISCRETE CONTROL POLICIES
The following figures represent populations of the SGSB and VBC after pesticide control. The figures reference the corresponding control policies of Chapter IV.
Figure I-1

Population Southern Green Stink Bug after Continuous Control (Figure 4-9).

Damage to Lee by the Stink Bug
Population (number per foot row)

- Eggs
- Nymphs
- Adults

$8.5/bushel
$4/lb. pesticide
1/4 quality reduction

Figure 1-2

Population Southern Green Stink Bug after Continuous Control (Figure 4-9).
Damage to Lee by the Stink Bug
$8.5/\text{bushel} \quad \$4/\text{lb. pesticide} \quad \text{PP Growth Rate} \; r = 0

Population (number larva per foot row)

$10$

$8$

$6$

$4$

$2$

$0$

0 10 20 30 40 50

Time (days)

Figure I-3

Population Velvet Bean Caterpillar after Continuous Control (Figure 4-10).

Damage to Dare by Velvet Bean Caterpillar.
Population Velvet Bean Caterpillar After Continuous Control (Figure 4-13)

Damage to Lee by Velvet Bean Caterpillar
Figure I-5

Populations Velvet Bean Caterpillar After Discrete Control. Damage to Dare by the Velvet Bean Caterpillar. (Note scale change)
Figure I-6

Population Velvet Bean Caterpillar After Single Discrete Application.

Damage to Lee by the Velvet Bean Caterpillar
Population Southern Green Stink Bug After Continuous Control

(Figure 4-16). Damage to Dare by Stink Bug and Velvet Bean Caterpillar
Population Southern Green Stink Bug After Continuous Control

(Figure 4-16). Damage to Dare by Stink Bug and Velvet Bean Caterpillar
Figure I-9

Population Velvet Bean Caterpillar After Continuous Control

Damage to Dare by Stink Bug and Velvet Bean Caterpillar
Population Velvet Bean Caterpillar After Continuous Control

(Figure 4-16). Damage to Dare by Stink Bug and Velvet Bean Caterpillar
Population Southern Green Stink Bug After Continuous Control (Figure 4-17). Damage to Lee by Stink Bug and Velvet Bean Caterpillar
Figure I-12
Population Southern Green Stink Bug After Continuous Control (Figure 4-17).
Damage to Lee by the Stink Bug and Velvet Bean Caterpillar
Eggs
Nymphs
Adults

$8.5/bushel
$4/lb pesticide
1/4 quality reduction
PP Growth Rate r Equal 0

Figure I-13
Population Southern Green Stink Bug After Continuous Control (Figure 4-18).
Damage to Lee by Stink Bug and Velvet Bean Caterpillar
- Eggs
- Nymphs
- Adults

$8.5/bushel
$4/lb pesticide
1/4 quality reduction
PP Growth Rate r Equal 1.3

Figure I-14
Population Southern Green Stink Bug After Continuous Control (Figure 4-18)
Damage to Lee by Stink Bug and Velvet Bean Caterpillar
Population Velvet Bean Caterpillar After Continuous Control

(Figure 4-17) Damage to Lee by Stink Bug and Velvet Bean Caterpillar
Population Velvet Bean Caterpillar After Continuous Control (Figure 4-18)
Damage to Lee by Stink Bug and Velvet Bean Caterpillar
Figure T-17
Optimal Southern Green Stink Bug Population After Discrete Control on Day 5
Damage to Lee by Stink Bug and Velvet Bean Caterpillar
Figure 1-18
Population Southern Green Stink Bug After Discrete Control on Days 1 and 16.
Damage to Lee by Stink Bug and Velvet Bean Caterpillar
Population (number of larva per foot row)

- $8.5$/bushel
- $4$/lb pesticide
- $1/4$ quality reduction
- PP Growth Rate $r$
  
  $r=1.3$

**Figure I-19**

Population Velvet Bean Caterpillar After Discrete Control on Days 1 and 17. Damage to Lee by Stink Bug and Velvet Bean Caterpillar
Population Velvet Bean Caterpillar after Discrete Control on Days 1 and 16
Damage to Lee by Stink Bug and Velvet Bean Caterpillar
APPENDIX II

DISCRETIZATION OF POPULATION DYNAMICS MODEL
Presented in Chapter II are the centered difference algorithms which can be used for the numerical solution of the system state and co-state equations. In this appendix we outline the application of the algorithms and present the resulting algebraic equations used.
The normalized species equation of change is

\[
\frac{Z_{\text{max}}}{t_{\text{m}}} \frac{\partial z(t,y)}{\partial t} + \frac{Z_{\text{max}}}{y_{\text{m}}} \frac{\partial z(t,y)}{\partial y} + (k(y) + q(y)) Z_{\text{max}} z(t,y) \\
+ (1 - k(y)) u(t,y) k(y) Z_{\text{max}} z(t,y) \\
+ \frac{z_0(t) k_p(y) Z_{\text{max}} z(t,y)}{1 + \frac{Z_{\text{max}} z(t,y)}{c_p}} = q(y) z_{\text{ref}}
\]

where the variables are defined in Chapter II. We drop the arguments, and without loss of generality assume \( Z_{\text{max}} \) equal 1. Using the centered difference analogs.

\[
z_{i-1/2, n+1/2} = \frac{1}{2} (z_{i, n} + z_{i-1, n})
\]

\[
\frac{\partial z}{\partial t} = \frac{1}{2\Delta t} (z_{i, n} + z_{i-1, n} + z_{i, n+1} - z_{i-1, n+1})
\]

\[
\frac{\partial z}{\partial y} = \frac{1}{2\Delta y} (z_{i, n} + z_{i-1, n+1} + z_{i, n+1} - z_{i-1, n})
\]

where \( \Delta t \) and \( \Delta y \) are the discrete increments in time and species age, respectively. We obtain as our discrete equation,

\[
B_1(z_{i, n+1} - z_{i, n} + z_{i-1, n+1} - z_{i-1, n})
= B_2(z_{i, n+1} - z_{i-1, n+1})
+ \frac{B_4(z_{i, n+1} + z_{i-1, n})}{1 + B_5(z_{i, n+1} + z_{i-1, n})}
+ B_6
\]

\[
B_1(z_{i, n+1} - z_{i, n} + z_{i-1, n} + z_{i, n+1} - z_{i-1, n})
= B_2(z_{i, n+1} - z_{i-1, n+1})
+ \frac{B_4(z_{i, n+1} + z_{i-1, n})}{1 + B_5(z_{i, n+1} + z_{i-1, n})}
+ B_6
\]
with the B's defined as

\[ B_1 = \frac{1}{2} \Delta t \quad B_2 = -\frac{t_m}{(2y_m \Delta y)} \]

\[ B_3 = -\frac{t_m}{2} (k + q + k_u (1 - k) u) \]

\[ B_4 = -\frac{t_m}{2} \ z_0 k_p \]

\[ B_5 = \frac{1}{2} c_p \]

\[ B_6 = t_m q z_{\text{ref}} \]

Solving for the unknown \( z_{i,n+1} \) yields

\[ D_1 z_{i,n}^2 + 1 + D_2 z_{i,n+1} = D_3 \]

where

\[ D_1 = B_1 B_5 - B_2 B_5 - B_3 B_5 \]

\[ D_2 = z_i - 1, n + 1 \ (B_1 B_5 + B_2 B_5) - z_{i,n} (B_1 B_5 + B_2 B_5) \]

\[-2B_3 B_5 z_i - 1, n + (B_1 + B_2 + B_3 - B_4 - B_5 B_6) \]

and

\[ D_3 = (z_{i,n} - z_i - 1, n + 1) (B_1 + B_2) + (B_1 B_5 + B_2 B_5) z_{i-1,n} \]

\[ + z_i - 1, n \ (B_1 + B_2 + B_3 + B_4 + B_5 B_6) \]

\[ + (z_i - 1, n)^2 (B_1 B_5 - B_2 B_5 + B_3 B_5) + B_6 \]

We use the quadratic formula to solve for the value of \( z_{i,n+1} \)

\[ z_{i,n+1} = -\frac{D_2 \pm \sqrt{(D_2 + 4D_1 D_3)^2}}{2D_1} \]
We find the solution numerically using the positive sign, which can be shown to be correct by studying the limits.

When \( C_p \) approaches infinity or \( P(y) = 0 \) \& \( y \), then the second-order polynomial becomes first-order in \( z_{i,n+1} \). The solution technique is then greatly simplified.

When \( t_m \Delta t = y_m \Delta y \), then \( (B_1 + B_2) \) is zero. This greatly simplifies the numerical discretization scheme.
APPENDIX III

FLOW CHART OF SOYBEAN PLANT MODEL
The soybean plant dynamics employed in the system simulation is based upon a logical model written by Rudd (1975) of the LSU Department of Computer Science. The data base was collected by R.L. Jensen. Here we present a flow graph of the model.

The model begins at planting and calculates the pod, stem and leaf weights through crop harvest. In the simulation we assume a 48 inch row spacing. In addition to the definition of terms in the Nomenclature and Table 3-5 we define,

IT as discretized time

t₀ as time at which pod development begins

t₁ as time at which pod development is independent of leaf area

and

SENRTE as the rate of leaf senescing.
Figure III-1
Flow Chart of Soybean Plant Model

TOTWT = \( z_5 + z_6 + z_7 = 0 \)
- \( z_5 (0) = 0 \)
- \( z_6 (0) = 0 \)
- \( z_7 (0) = 0 \)
- \( IT = 1 \)

\[
\begin{align*}
IT &= IT + 1 \\
\text{IT - ITEMRG} &= 0
\end{align*}
\]
A

IT ≥ t₁

Yes

z₇max = z₇

Yes

IT Eq. t₁

No

SEN = 1.5(IT - t₁)² / z₇max

SENI > 1

Yes

z₇ = 0

No

TOTWT = TOTWT - SEN* z₇
z₇ = z₇ - SEN z₇

IT ≤ IFFIN

Yes

C

STOP

No
\[
\begin{align*}
D_1 &= E_1 + E_2 z_7 + E_3 z_7^2 \\
D_1 &= E_4 (1 - \exp(-E_5 z_7)) \quad (z_7 > 3.835 \times \text{GAREA}) \\
\text{TOTWT} &= \text{TOTWT} + D_1 \\
\text{IF } IT > 0 \text{ THEN } z_7 &= z_7 + 0.47 \times D_1 \\
\text{ELSE } z_7 &= z_7 + 0.53 \times D_1 \\
\text{IF } IT = 0 \text{ THEN } z_5 &= z_5 + 0.5 \times D_1 \\
\text{ELSE } z_5 &= z_5 + 0.53 \times D_1 \\
\text{ENDIF } D_5 &= \text{SENRTE} \times z_7 \\
\end{align*}
\]
\[ \text{TOTWT} = \text{TOTWT} - D_1 \]
\[ z_7 = z_7 - D_5 \]
\[ \text{DPW} = D_2 \times z_6 \]

**Flowchart**

1. If \( \text{DPW} > D_1 \) then \( \text{DPW} = D_1 + D_3 \times (\text{DPW} - D_1) \)
2. If \( z_5 < z_{5\text{max}} \) then \( \text{DPW} = D_1 \)
3. If \( z_6 = z_6 + \text{DPW} \)
4. End \( (E) \)
APPENDIX IV

COMPUTER PROGRAM FOR THE NON-LINEAR PROGRAMMING SOLUTION
The program for the determination of the best single and multiple pesticide application times is based on the pattern search technique. The program integrates the SGSB, VBC and PP population equations for each calculated estimate of the control policy.

After reading the number of applications allowed and an estimate of the initial control times POPSB is called to calculate the SGSB population. The POPVBC and PLANT are called to integrate the VBC and soybean plant equations. With these values the objective function is evaluated. Then PATERN is called and the next estimate of the control times is evaluated.

It must be noted that the program listed here was originally compiled on the Sigma V, which has limited storage capacity. We therefore made liberal use of the machines overlaying capacity. If the program is to be executed without this structure several variables may have to be redefined when calling a subroutine. For with the overlay procedure all subroutine DATA are reinitialized when that subroutine is called.

Input requires the time and age limits, dollar cost values, stage probability distributions and an estimate of the control policy. At the end of the source listing is a set of input data, units are described in the source listing.
Lee variety of soybean

Program to integrate the soybean plant, velvet bean, caterpillar, southern green stink bug and predator-parasite complex population models.

Subroutine plant is the soybean plant from pseudost to harvest.

Subroutine popvbc is velvetbean caterpillar and predator complex.

Subroutine popsb is southern green stink bug model.

Subroutine pattern is logical pattern search - the subroutine is made available through .......... LSU Dept. Chemical Engineering

U is the time dependent control vector

Z1 SGSB, number/ft row
Z2 VBC, magnitude / ft row
Z3 Damage by SGSB, gram redug/ft row
Z4 Damage by VBC, gram redug/ft row
Z5 Stem weight, gram/ft row
Z6 Weight of pods, gram/ft row
Z7 Weight of leaves, gram/ft row
Z8 Predator-parasite complex, number/ft row, normalized (0-1)
Z9 Is the sum of bug punctures per foot row

Program objective is to minimize IP, where IP is the objective function

Yield is pod weight at harvest, dry gm/ft row

Bencst is bean price, $/bushel

Concst is pesticide cost per pound, $/lb.

Appcost is per acre cost of pesticide application, $/acre

Qjlrto is quality ratio reduction

DA, DT are normalized age and time increments

Tymax is maximum seasonal time from pseudost

Age Max is maximum age of specifs

NF, NF are number of increments in age and time

SGSPPS, percentage distribution of stages at any age for the southern green bug

VBCPS, percentage distribution of stages at any time for the velvet bean caterpillar.
C RETURN IS $ YIELD/GRAM OF BEANS $8.5/BUshEL, 60 13/BUshEL
C BEANER IS WEIGHT FRACTION BEAN OF POD, 0.7
C PESTICIDE COST IS CONCST = 4.5/2/1089 = .0001837
C FT ROW PER ACRE = 10890, 1/2 LB FOR 99 PERCENT CONTROL
C E6 IS PP COMPLEX GROWTH RATE
C 22INFER IS MULTIPLIER OF INITIAL VBC POPULATION
C N6 IS NUMBER OF APPLICATION TIMES
C A2 AND A2 ARE CONSTANTS OF FERMI APPROXIMATION
C DATA BASED ON TMMAX=60 DAYS
C PRINT POPULATION AND PRINT IF IPRINT = 1
C***********************************************************************
REAL IP
DIMENSION Z3(91),Z4(91),U(91)
DIMENSION SGSBPS(6,101),VBCPS(8,54)
DIMENSION IT1(20)
COMMON TMMAX, AEMAX, IF,NF,IF1,NF1,SGS9PS, VBCPS,?-1NFR, E6, NP
2,IT1,TIMCAST,ICK,IC,IFAIL,ITARR
READ(5,1) TMMAX, AEMAX, IF,NF,IF1,NF1
READ(5,4) BENCF ST, 2ENCF ST, 2PCCST, JULRT9, E6, 21NFR, NP
READ(5,4) A2,A2
PRINT 101,TMMAX, AEMAX, IF,NF, BENCF ST, CONCF ST, APPCST, Q11R7, E6
DATA IPRINT /1/
DATA NRD/1/
ITARR=0
IFAIL=0
IC=0
ICK=0
C DEFINE STAGE PROBABILITY DISTRIBUTIONS
READ(5,2) ((SGS9PS(I,J),I=1,6),J=1,6)
READ(5,3) ((VBCPS(I,J),I=1,8),J=1,39)
DB 5 J=40,101
DB 5 I=1,8
IF(J.LT.60) GB T9 6
IF(I.LE.6) SGSBPS(I,J)=0
IF(J.GT.54) GB T5 5
VBCPS(I,J)=0
Continje
Da=1./Float(IF)
Dt=1./Float(IF)
C Define control vector
Read(5,7) U
D9 31 I=1, NF1
C Correct for offset induced by numerical approximation
31 U(I)=2.*J(I)/(2.*J(I))
C Define IT1 as the optimal storage array
C The day on which we control
Read(5,8) IT1
30 Continje
C Calculate population magnitude
Call Segload (1)
Call Pqpsr (Z3, U, IPRINT, Z9)
Call Segload (2)
Call Pqpvrc (Z4, U, IPRINT)
C Calculate yield
Call Segload (3)
Call Plant (Z3, Z4, YIELD, IPRINT)
C Calculate objective function, IP
V2=0.
V5=0.
D9 20 I=2, NF1
C Application cost
V5=AppCst/(1.+Ac*exp(-Ap*p.*U(I)/(2.*U(I))))/1.980.*
**V5
C Control cost
20 V2=V2+ConCst/21780.*(2.*U(I)/(2.*U(I)))**2
C Quality reduction
V4=79.*00071852*RevCst*.0000257*YIELD*Qulrta
C Yield
V1=-YIELD*RevCst*.0000257
Yield=YIELD*.2799
IP=V1+V2+V4+V5
C Print cost values
PRINT 11,IP,YIELD,V1,V4,V2,V5
IPRINT=0
IF(NRD.NE.1) GO TO 61
C CALL SUBROUTINE FOR PATERN SEARCH
CALL SEGLOAD (4)
CALL PATERN (IP,U,VRD)
IF(NRD.EQ.1) GO TO 30
IPRINT=1
GO TO 30
61 CONTINUE
STOP
1 FORMAT(2F3.0,4I3)
2 FORMAT(3X,6F10.4)
3 FORMAT(3X,8F1.0)
4 FORMAT(1X,4F10.0/1X,2F10.0/I3)
7 FORMAT(40F2.2)
8 FORMAT(2012)
10 FORMAT('CALCULATED YIELD IS',FR.?,13BUSHELS/ACRE'
2/ INDEX OF PERFORMANCE IS ',F10.8,'/FT RED')
11 FORMAT(1X,6E15.4/) 101 FORMAT(1X,'MAXIMUM TIME AND AGE IN DAYS',?FR.1/3X,'AND THE RESPECT
IVE NUMBER OF DISCRETE INCREMENTS',2FR.1/3X,'CERTS: $/BUSHEL BEANS'
,15F5.2/3X,'$/POUND PESTICIDE',F5.2/3X,'$/ACRE APPLICATION',F5.2/3X'
*,PER CENT REJECTION OF $/BUSHEL BY QUALITY',F5.3/1X,'PREDATOR C0'
*,PLEX EIGENVALUE',F6.4)
END
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8.1154.998.2567.1537.4263.3662.2642.0661.611.123.092.066
9.047.0315.0195.012.0075.0045.0037
C G2 = DISTRIBUTED EFFECT OF CONTROL - CONTROL OF ATH, 5TH AND ADULTS
DATA G2/C0.000.00.00.00.00.0001.0002.0005.0011.0026.0052
2.0096.0017.0283.0547.0676.092.1367.1799.2995.3694
3.4427.5068.5893.6575.7197.77381.174.8519.8794.9192.999
4.1.164.1
C C1 IS THE FEEDING RATE - ASSUME THE BEAN ATTACK BEAN WEIGHT = .136 GRAMS
DATA C1S/0.00.00.0733.0703.0797.0877
C ATKRAT IS STING BUGS AVERAGE NUMBER OF PUNCTURES DELIVERED PER DAY PER BUS
DATA ATKRAT/3.0,3.3,3.4,4.5,4.1/BUGPUC/101/0.0/
IF(I=PRINT*EQ.1) PRINT 1
N=0.
C CORRECT FOR A TMAX OTHER THAN 60 DAYS
DB 10 I=1,IF1
G1(I)=G1(I)*TMAX/60
A=0.
C CALCULATE AGE DEPENDENT BUG PUNCTURE COEFFICIENT AND FEEDING RATE
DB 15 J=1,6
A=A+SGSBPS(J,I)
BUGPJC(I)=3JGPJC(I)+SGSBPS(J,I)*ATKRAT(J)
15 C1(I)=C1(I)+C1S(J)+SGSBPS(J,I)
BUGPJC(I)=3JGPJC(I)+ATKRAT(J)*(1-A)
C1(I)=C1(I)+C1S(J)*(1-A)
10 Z1(I,I)=Z1INT(I)
Z9=0.
20 N=N+1
Z3(N)=0
DB 7 J=1,IF1
Z9=Z9+3JGPJC(J)*Z1(I,J)
7 Z3(N)=Z3(N)+C1(J)*Z1(I,J)
IF(I=PRINT*NE.1) DB 10 T3 2
C FIND STAGE POPULATIONS AND TOTAL NYPHES
DB 3 I=1,7
3 PBP(I)=0
5) 4 3 = 1, 1 F 1
A=0
5 1 = 1, 6
A = A + SGSBPS(I, J)
P5(I) = P5P(I) + SGSBPS(I, J)*Z1(I, 1)
P5P(7) = P5P(7) + Z1(1, J)*(1 - A)
PRINT 6, N, P5P
3 CONTINUE
IF(N*EQ*4F1) G9 T9 30
MCASE=1
C NEXT BOUNDARY CONDITION ESTIMATE
Z1( 2, 1) = Z1(1, 1)
70 Z1INT(1) = 0
5 50 I = 2, 1 F 1
3 = TMMAX + S1(I) + (TMMAX/2 - S1(I))*S2(I)*U(N+1)
A = TMMAX - S1(I) - (TMMAX/2 - S1(I))*S2(I)*U(N+1)
Z1( 2, 1) = A + R*Z1(I, 1)
50 Z1INT(1) = Z1INT(1) + SBIRTH(1) + Z1( 2, 1)
Z1( 2, 1) = Z1INT(1)
IF(MCASE*EQ*2) G9 T9 31
MCASE = 2
G9 T9 70
31 G9 12 I = 1, 1 F 1
12 Z1(1, 1) = Z1(2, 1)
G9 TO 20
30 RETURN
1 FORMAT('SOUTHERN GREEN STINK Bug: EGGS, 1-5 INSTARS, ADULTS, SUM OF 2 INSTARS!')
6 FORMAT(I3, 8F8.2)
END
SUBROUTINE POPVBC (74, U, PRINT)
C SUBROUTINE INTEGRATES BY CENTERED DIFFERENCE THE VBC PDF AND BY EULER METHOD
C THE AGE FOR Predator-PARASITE COMPLEX
C*******************************************************************************
C COMMON TMAX, AGEMAX, IF, NF, IF1, NF1, SGSBPS, VRAPS, Z21NFR, E6
DIMENSION Z2(2,54), Z4(91), Z8(91), U(91), SGSBPS(6,101), VRAPS(8,54),
   Z2INT(54), C2(54), C3S(9), POP(10)
C DIMENSION H1(54), H2(54), H3(54), H4(54), H3BIRTH(54)
C H1, H2, H3, AND H4 ARE AGE DEPENDENT COEFFICIENTS FOR THE VELVET BEAN CATERPIL
C DATA BASED ON TMAX=10 DAYS
C H1 = TMAX/2*(MORTALITY + IMMIGRATION COEFFICIENTS)
C H2 = TMAX/2*(DISTRIBUTION OF CONTROL EFFORT)*(1-MORTALITY COEFF.)
C H3 = TMAX/2*(DISTRIBUTION OF PREDATOR EFFECT)
C H4 = TMAX/2*(IMMIGRATION CoEFF)*(REF. POPULATION MAGNITUDE)
C CP IS CAPACITY OF PREDATOR COMPLEX, Z P/(1+(7/CP))
C E6 = (PREDATOR BIRTH MINUS DEATH)
C H3BIRTH IS VBC BIRTH COEFFICIENTS
C ASSUME IMMIGRATION ZERO
DATA H4/54*0.0/ C NATURAL MORTALITY OF 13 PER CENT PER STAGE
DATA H1/3*1.39,1.74,2.09,1.57,3.04,1.22,2*1.39,1.22,3*1.04,
   2*1.22,2*1.39,1.57,3*1.04,1.22,3*1.04, C CONTROL EFFECTIVE ON LARVA STAGES ONLY
DATA H2/3*0.28*1.23*0.3/ C PREDATORY MORTALITY OF 17 PER CENT PER STAGE
DATA H3/3*1.86,3.32,2.85,2*1.31*1.43,1.63,2*1.86,1.63,2*1.31*1.43,
   2*1.86,1.63,1*1.43,1.63,1*1.43, C C2S IS DAILY REDUCTION IN GAMS PER STAGE .25 CM**2/GRAM
DATA C2S/0*0.0,0,0,0145,0.291,0.218,0.2616,0,0/ DATA C2S/54*0.0/ DATA C2/54*0.0/
DATA H3BIRTH/40,0,20,57.5,75.5,62.5,45,40,37.5,15.5,5,0/
DATA Z2INT/3*1.2*1.0034,4*1.0069,0*1.0034,1.1294,3*1.0034,1.0035,1.0034,1.0035,
C 4*0.005,4*0.005,4*0.005,4*0.005,4*0.005,4*0.005,4*0.005,4*0.005,4*0.005,4*0.005,
DATA Z2REF/CP/1.13/ IF(PRINT.EQ.1) PRINT 1
N=0
C CORRECT FOR A TMAX OTHER THAN 60 DAYS
C9 10 I=1,54
H1(I)=H1(I)*TMAX/60*
H2(I)=H2(I)*TMAX/60*
H3(I)=H3(I)*TMAX/60*
H4(I)=H4(I)*TMAX/60*
H2(I)=H2(I)*(30.6-H1(I))
H3(I)=H3(I)*1.2
A=0
D5 15 J=1,8
A=A+VBCPS(J,1)
15 C2(I)=C2(I)+C2S(J)*VBCPS(J,1)
C INITIALIZE AGE-DEPENDENT FEEDING RATE
C2(I)=C2(I)+C2S(9)*(1.-A)
C CALCULATE PREDATOR-PARASITE POPULATION
10 Z2(I,J)=Z2INT(I)*Z2IFR
Z8(I)=1
20 N=N+1
Z4(N)=0
D9 7 J=1,54
7 Z4(N)=Z4(N)+C2(J)*ZP(1,J)
IF(TPRINT*NE*1) GO TO 39
D8 3 I=1,9
3 P9P(I)=0
D9 4 J=1,54
A=0
D9 5 I=1,8
A=A+VBCPS(I,J)
5 P9P(I)=P9P(I)+VBCPS(I,J)*Z2(1,J)
4 P9P(9)=P9P(9)+Z2(1,J)*(1.-A)
PRINT 6,V,P9P,Z8(N)
2 CONTINUE
IF(N*EQ*NE1) GO TO 39
4 CASE=1
A=Z2*U(N+1)/(2*J(N+1))
Z8(N+1)=Z8(N)+E6*R(N)-A*ZR(N)=F6*ZR(N)*Z8
Z2(Z2)=Z2(1,1)
70 Z2INT(I)=0
DB 40 I=2,54
A=1*Z2*/CP*(TMAX+41(I)+42(I)*U(N+1))
C=Z2(I,1=1)**2+(1*/CP*(TMAX-H1(I)+F2(I)*U(N+1))
2 +Z2(I,1=1)*(TMAX-H1(I)+F2(I)*U(N+1)+H3(I)*ZR(N+1)+H4(I)/CP)
3 +2*H4(I)
B=TMAX+41(I)*(1+Z2(I,1=1)/CP)+42(I)*(1+72(I,1=1)/CP)*U(N+1)
2 +H3(I)*Z8(N+1)-1/CP*H4(I)
Z2(Z2)=(-B*(B**2+4*A*C)**(1/2))/(2*A)
IF(Z2(I)>LT=0.) Z2(?,1)=0
40 Z2INT(I)=Z2INT(I)+HRINT(I)*Z2(2,1)
Z2(Z2)=Z2INT(I)
IF(MCASE=EQ=2) 60 T9 31
MCASE=2
60 T9 70
31 D9 12 J=1,54
12 Z2(1,1=1+Z2(2,1)
60 T9 20
30 RETURN
1 FORMAT('3V3C: EG3S, 1-6 LARVA, JPAA, ADULTS, SIM 9F LARVA')
6 FORMAT('13,11F8.3')
END
SUBROUTINE PLANT(Z3,Z4,P,T,IPRINT)
C LEE VARIETY OF SOYBEAN - INTEGRATION BEGINS AT PRESET
C******************************************************************************
C COMMON TMAX,AQMAX,IF,NF,IF1,NF1
DIMENSION Z3(91),Z4(91),PBDWT(91),STMWT(91),WT(91)
DATA D,A,D3,D4,NC,NO,0/216,11057,0.00834,0.0864,96.29/
C 13 IS TIME AT WHICH PBD DEVELOPMENT IS INDEPENDENT OF LEAF AREA
DATA PBDMAX,F,3ARE4,A,T,STM4X,PDRTE,SNKCNR,SEARTE/
  *4.00,215.3716,4.4130,2.0,141.12,0.0011/
C DAY 1 IS DAY 81 IN REAL TIME - PRESET
PBDWT(1)=2.25
*TLF(1)=78*
STMWT(1)=80*
13=13-81
I=0
10
  I=I+1
  IF(I.EQ.91) G9 T0 20
  DW1=DWA+DWC*TF(I)-DWC*TLF(I)*P
  DW2=PBDMAX*(1-EXP(-DWC*TLF(I)))
  CALL FM(0,WTLFI,F3WQ,1)
  CALL FM(D3,TLF(I),FP3W,0)
  TWT=F3W3*DW1+FP3W*DW2
  T1=T3
  T2=1
  CALL FM(T1,T2,T3MT3T,1)
  CALL FM(STMWT(I),STM4X,FMSS,1)
  CALL FM(STMWT(I),STM4X,FQSS,0)
  PBDT=PDRRT*PBDWT(I)
  CALL FM(PBDT,DX,FDX,1)
  CALL FM(PBDT,DY,FPD,0)
  Z4(I)=0.
  PBDT=F1T3T*FMSS*(F3P)*P*(1-SNKCN)+SNKCN*PDRTE*PBDWT(I)+FPD*(PDRTE*P
  *(PBDWT+PBDWT(I)))+F3MT3T*FPSS*DN-Z3(I)
  PBDWT(I+1)=PBDWT(I)+PBDT
  STMWT(I+1)=STMWT(I)+FMT3T*MC=OPDT+Z3(I)
30 PRINT CUMDEF+74(I)
31 RETURN
40 FORMAT(1DAY,9D3,F9.9,T14.14,MTLE)
50 FORMAT(14,F10.2,3F7.2)
25 FORMAT(14,F10.2,3F7.2)
2
3RD AND 1'AF DAMAGE.
SUBROUTINE FM(T1,T2,T4,M)
C PURPOSE TO PREVENT T4 FROM EXCEEDING COMPUTER FLOATING POINT LIMIT AND
C TO EVALUATE EXPONENTIAL
T4=(T1-T2)*4.
IF(T4>3*150.) T4=150.
IF(T4<2*140.) T4=140.
IF(M.EQ.1) T4=EXP(-T4)
IF(M.EQ.2) T4=EXP(T4)
T4=1./(1.+T4)
RETURN
END
SUBROUTINE PATTERN (COST, C9ST, NP, NRD)
INTEGER P, T1, T2, T3, STEP, SJ
DIMENSION P(5), STEP(5), U(91), STBR(1040), T1(5), T2(5), T3(5)
COMMON TMAX, A3EMAX, IF, NF, IF1, N=1, STBR, NP, P, T1, T2, T3, T1CST, IC,
2 IC, IFAIL, IARR
DATA STEP/5*1/

C ***********************************************
C T1CST CONTAINS BEST COST
C COST IS NEWLY CALCULATED VALUE OF OBJECTIVE
C NP IS NUMBER OF PARAMETERS TO BE SEARCHED
C P IS VECTOR OF NP ELEMENTS
C STEP IS INITIAL STEP SIZE OF EACH NP ELEMENTS
C NRD IS NUMBER OF ITERATIONS - USUALLY REDUCE STEP SIZE FOR EACH SUCCESSIVE
C ITERATION
C IARR IS ELEMENTS OF P THAT IS BEING MANIPULATED
C ***********************************************

IF(IARR.EQ.0) GO TO 6
8 7 I=1, NP
T1(I)=P(I)
T2(I)=P(I)
T3(I)=P(I)
11 T1CST=C9ST
12 G9 T9 5
6 IF(T1CST=C9ST) 23, 23, 26
23 IF(IC.GE.2) G9 T9 24
STEP(IARR)=STEP(IARR)
G9 T9 21
24 IFAIL=IFAIL+1
P(IARR)=T1(IARR)
G9 T9 5
25 T1(IARR)=P(IARR)
T1CST=C9ST
IARR=IARR+1
IC=0
IF(IARR.GT. NP) GO TO 50
21 \hspace{1cm} P(IARR) = T1(IARR) \times \text{STEP(IARR)} \\
IC = IC + 1 \\
C \hspace{0.5cm} \text{CHECK BOUNDS ON P} \\
\hspace{1cm} \text{IF(P(IARR) \cdot LT \cdot 2 \cdot 8 \cdot P(IARR), GT \cdot 1) \ S9 \ T0 \ 23} \\
\hspace{4cm} \text{G9 T0 10} \\
50 \hspace{1cm} \text{IF(IFAIL \cdot LT \cdot P) S8 T0 35} \\
\hspace{4cm} \text{IF(ICK \cdot E3 + 2) S9 T0 90} \\
\hspace{4cm} \text{IF(ICK \cdot E3 + 1) S9 T0 35} \\
\hspace{7cm} \text{ICK = 1} \\
\hspace{1cm} \text{D9 33 I=1,NP} \\
\hspace{4cm} T2(I) = T3(I) \\
\hspace{7cm} P(I) = T3(I) \\
33 \hspace{1cm} T1(I) = T3(I) \\
\hspace{1cm} IARR = 0 \\
\hspace{2cm} IC = 0 \\
\hspace{2cm} IFAIL = 0 \\
\hspace{1cm} G9 T0 5 \\
34 \hspace{1cm} T1COST = C9ST \\
35 \hspace{1cm} IB1 = 0 \\
\hspace{1cm} D9 39 I=1,NP \\
\hspace{4cm} T3(I) = T1(I) \\
\hspace{7cm} \text{IF(|ABS(T2(I) - T3(I)) \cdot LT \cdot 1) IB1 = IB1 + 1} \\
39 \hspace{1cm} \text{CONTINUE} \\
\hspace{1cm} \text{IF(IB1 \cdot E3 + NP) S8 T0 30} \\
\hspace{4cm} ICK = 0 \\
C \hspace{0.5cm} \text{ACCELERATION STEP} \\
40 \hspace{1cm} SJ = 3 \\
\hspace{1cm} D9 45 I=1,4 \\
\hspace{4cm} D9 42 I=1,NP \\
\hspace{7cm} T1(I) = T3(I) + SJ \cdot (T3(I) - T2(I)) \\
42 \hspace{1cm} P(I) = T1(I) \\
\hspace{1cm} SJ = SJ - 1 \\
C \hspace{0.5cm} \text{CHECK BOUNDS} \\
C \hspace{0.5cm} \text{IF BOUNDS STIFSIE) S9 T0 46} \\
\hspace{1cm} IB1 = 0 \\
\hspace{1cm} D9 46 I=1,NP
48 IF(T1(I)*LT.1*9R*T1(I)*GT.NF1) 131=131+1
IF(I31.E3*0) G8 T9 46
45 CONTINUE
ICK=1
46 DO 47 I=1,NP
47 T2(I)=T3(I)
IARR=0
IFAIL=0
IC=0
G8 T8 5
90 DO 100 I=1,NP
100 P(I)=T1(I)
NRO=2
10 CONTINUE
C PRINT P VECTOR
PRINT 9,P
C PJT P INTO J VECTOR
DO 8 J=1,NF1
J(I)=0
8 DO 8 J=1,NP
C DEFINE RATE OF CONTROL
C U(ACTUAL)=U*2/(2+J)
8 IF(I*E3*P(J)) J(I)=1.95
RETURN
9 FORMAT (CONTROL VECTOR 1.814)
END
APPENDIX V

COMPUTER PROGRAM FOR MINIMUM PRINCIPLE SOLUTION
The numerical solution of the minimum principle requires the solution to a two-point boundary-value problem. The approximation to the solution technique is used to solve for the optimal control trajectory. Here the boundary conditions are satisfied, but the control is iterated upon. We evaluate the first necessary condition for an extrema and, with use of a multiplier, search along the gradient for a better estimate of the control. The golden section one-dimensional search is used to evaluate the best multiplier value. Once again, the program is based on an overlay structure and care should be taken when converting to different execution schemes.

The input requires only the cost values and initial control. A typical set of initial conditions follows the source listing.

Output on the first and last iteration of the control includes the state and co-state equations values. On intermediate integrations the control and objective function values are printed.
C 3 FINAL INTEGRATION TERMINATION
C KZ IS NUMBER OF ITERATIONS OF CONTROL IMPROVEMENTS

REAL IP, IPC1, IPC2
DIMENSION Z3(I91), Z4(I91), U(I91)
DIMENSION STOR(S106), VAL(I91), IPC1(I91), IPC2(I91), DNUM(I91)

*, VS(10)
COMMON TMMAX, AGEMAX, IF, NF, IF1, NF1, DA, DT, K7, CANCST, BENCST, APPCST,
*, QLRT9, Z2INFR, E6, STOR, UALD, ICASE, SIP, PIP, NEFDI, INC, CHOO
*, AC, A2, VS, NC1
READ(5, 1) TMMAX, AGEMAX, IF, NF, IF1, NF1
READ(5, 5) BENCST, CANCST, APPCST, QLRT9, E6, Z2INFR, K7
READ(5, 5) AC, A2
READ(5, 6) U
PRINT 101, TMMAX, AGEMAX, IF, NF, BENCST, CANCST, APPCST, QLRT9, E6
ICASE = 0
SIP = 1000*
PIP = 1000*
ITNUM = 2
NC = 0
DA = 1./FL9AT(IF)
DT = 1./FL9AT(NF)

C DEFINE CONTROL VECTOR
C 4 1 = 1, NF1
J(1) = 2.*J(1)/(2. = J(1))
4
UALD(1) = J(1)
J(1) = 0.*
30 CONTINUE
C DETERMINE PRINT SCHEME
IPRINT = 1
IF(ICASE.EQ.3.AND.NF3.EQ.0) IPRINT = 3
IF(ITNUM.EQ.3) IPRINT = 3
C CALCULATE POPULATION MAGNITUDE
CALL SGLBAD (1)
CALL PSI2R (Z3, U, IPRINT, 1, PF1, Z9)
CALL SGLBAD (2)
CALL PYPVBC (Z4, U, IPRINT, P2)

C CALCULATE YIELD
   CALL SEGLOAD (3)
   CALL PLANT (Z3, Z4, YIELD, IPRINT, 79)
C NOTE Z3 NB STORES A06
C NOTE Z4 NB STORES A07
C PRINT COST VALUES
   PRINT 10, YIELD
   IF (ITNUM .GE. 1) GO TO 20
C CALL FOR ADJUGNT 2 AND R
   CALL SEGLOAD (4)
   CALL ADJUST2 (Z4, U, IPRINT, IPC2)
C CALL FOR ADJUGNT 1
   CALL SEGLOAD (5)
   CALL ADJUST1 (Z3, U, IPRINT, IPC1, YIELD)
20 CONTINUE
C CALL FOR CALCULATION OF OBJECTIVE AND NEW CONTROL TRAJECTORY
   CALL SEGLOAD (6)
   CALL BRYCEL (U, IPRINT, IPC1, IPC2, PF1, PF2, YIELD, ITNUM, 79)
   IF (ITNUM .GE. 1) GO TO 30
STOP
1 FORMAT (2F3.0, 413)
2 FORMAT (3X, 6F10.4)
3 FORMAT (3X, 6F1.0)
5 FORMAT (1X, 4F10.0/1X, 2F10.0/13)
6 FORMAT (4DF2.2)
10 FORMAT ('LOCALCULATED YIELD IS ', FR2, ' GRAMS')
101 FORMAT ('MAXIMUM TIME AND AGE IN DAYS, 2F1.0/1X AND THE RESPECTIVE NUMBER OF DISCRETE INCREMENTS, 2FR.1/1X, FRCTS: $/BUCKET', F5.2*'
      F5.2/8X, $/POUND PESTICIDE', F5.2/8X, $/ACRE APPLICATION', F5.2/8X
      ', PERCENT REDUCTION OF $/BUCKET BY QUALITY', FR.3/1X, 'PREDATOR COM
      PLEX EIGENVALUE', F6.4)
END
SUBROUTINE PAPSA (Z3U, IPRINT, ITI, ME1, Z9)
C SUBROUTINE INTEGRATES SSSB MODEL EQUATION BY CENTERED DIFFERENCE TECHNIQUE
C******************************************************************************
DIMENSION Z1( 2101), Z2(91), U(91), S(101), R(171), TBIRTH(101),
*C1(101), C1S(7), SGB=5(1,101), Z1NT(101), PAP(8), T11(101)
*, ATKRAT(7), SGBPUC(101)
COMMON TMMAX, AGEMAX, IF, IF1, IF2, IF1, JA, DT, K7, C9NCST, REN CST, APPCST,
* QUILR, Z21NFR, E6, Z11
C GBIRTH IS (BIRTH(I)+BIRTH(I+1))/2
C G2 IS DISTRIBUTE D EFFECT OF CONTROL
C C1 IS FEEDING RATE OF THE STINK BUG
C C1S IS STAGE FEEDING RATE = ATTACK*PER CENT REDUCTION*BEETLE WEIGHT
C G1 IS TMMAX/2*(MORTALITY + IMMIGRATION)
C ZINT IS INITIAL STINK BUG DISTRIBUTION
C ATKRAT IS STINK BUGS AVERAGE NUMBER OF PUNCTURES DELIVERED PER DAY PER BUG
C DATA BASED ON TMMAX = 60 DAYS

DATA Z1INT/52.0, 0.003, 0.006, 0.008, 0.011, 0.015, 0.02, 0.026, 0.033,
2.0042, 0.052, 0.063, 0.076, 0.088, 0.099, 0.109, 0.119, 0.126, 0.133, 0.138,
3.0143, 0.049, 0.055, 0.061, 0.066, 0.071, 0.076, 0.081, 0.086, 0.091, 0.096,
4.001, 0.083, 0.076, 0.056, 0.042, 0.033, 0.026, 0.021, 0.015, 0.017, 0.009,
5.0006, 0.004, 0.003, 0.002, 0.001, 0.005/0.0
DATA Z1REF/1, /C1/101*0.0
DATA GBIRTH/0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
30.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
30.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
4.0, 1281, 1.1675, 2.138, 2.848, 3.642, 4.589, 5.640, 6.74, 7.837, 8.969,
5.0, 105.1, 1.25, 1.055, 1.49, 1.635, 1.785, 1.935, 2.085, 2.23, 2.355, 2.455,
6.0, 495.2, 5.15, 5.32, 5.56, 5.72, 5.89, 6.07, 6.26, 6.45, 6.64, 6.85,
7.0, 72, 7.05, 2.69, 2.665, 2.645, 2.62, 2.675, 2.695, 2.73, 2.77, 2.81,
8.0, 59, 2.59, 2.59/2.59
C ASSUME IMMIGRATION = 0

DATA G1/0.0, 0.033, 0.075, 0.10, 0.259, 0.368, 0.597, 0.748, 0.923, 0.497, 0.969,
2.0, 3.32, 1.349, 2.375, 1.66, 1.498, 1.777, 2.186, 2.261, 2.277, 2.477, 2.621,
3.0, 3.4, 1.22, 1.81, 3.34, 1.70, 1.568, 1.351, 1.194, 1.475, 1.491,
4.0, 707, 1.61, 1.163, 1.214, 0.39, 7.46, 0.94, 7.2, 1.25, 1.27, 0.36, 3.24, 4.43,
C G2 DISTRIBUTED EFFECT OF CONTROL AT 4TH, 5TH AND ADULTS
DATA 3270..0001..0005..0002..0007..0003..0005..0007..0002..0006..0003
2.0096..017..0263..0447..0476..092..1367..1799..2360..2932..3694
3..4427..5461..5893..6575..719..773..8174..8519..8794..9199..999
411..1641..

C C1 IS THE FEEDING RATE ASSUME 2.9 REAN ATTACK. REAN WEIIGHT = 1.106 GRAMS
DATA 3270..0001..0005..0002..0007..0003..0005..0007..0002..0006..0003
2.0096..017..0263..0447..0476..092..1367..1799..2360..2932..3694
3..4427..5461..5893..6575..719..773..8174..8519..8794..9199..999
411..1641..

DATA 3270..0001..0005..0002..0007..0003..0005..0007..0002..0006..0003
2.0096..017..0263..0447..0476..092..1367..1799..2360..2932..3694
3..4427..5461..5893..6575..719..773..8174..8519..8794..9199..999
411..1641..

DATA ATK9AT/30.000..0000..0000..0000..0000..0000..0000..0000..0000..0000..0000
2.0096..017..0263..0447..0476..092..1367..1799..2360..2932..3694
3..4427..5461..5893..6575..719..773..8174..8519..8794..9199..999
411..1641..
N=0
C9 10 I=1; I=F1
C ADJUST DATA TO NEW TMAX
G1(I)=I1(I)*TMAX/60.
A=0
C9 15 J=1.6
A=A+SGS3PS(J.I)
C EVALUATE AGE-DEPENDENT PUNCTURE AND FEEDING RATES
BUJSPJCl)=BUJSPJCl)+SGS3PS(J.I)*ATKRAT(J)
15 C1(I)=C1(I)+C15(J)*SGS3PS(J.I)
C1(I)=C1(I)+C15(7)*(1-A)
BUJSPJCl)=BUJSPJCl)+ATKRAT(7)*(1-A)
C DEFINE INITIAL CONDITIONS
10 Z1(l.;I)=Z1INT(l.)
Z9=O.
20 N=N+1
Z3(V)=O
C9 7 J=1.1F1
C ADD PENALTY FOR NEGATIVE POPULATION
IF(Z1(I,.J)*LT.0.) PF1=PF1+Z1(I,.J)**2
C SJM PUNCTURES
Z9=Z9+BUJSPJCl)*Z1(1.J)
C SJM FEEDING
7 Z3(V)=Z3(V)+C1(J)*Z1(1.J)
8 IF(I*PRINT.LT.1.OR.I*TAM*FC2) GA TO 2
C9 3 I=1.7
2 P9P(I)=O
C9 4 J=1.1F1
A=A
C EVALUATE STAGE POPULATIONS
C9 5 I=1.6
A=A+SGS3PS(I.J)
5 P9P(I)=P9P(I)+SGS3PS(I.J)*Z1(I.J)
C9 4 P9P(1)=P9P(1)+Z1(I.J)*(1-A)
PRINT 6,N,P9P
CONTINUE
IF(N.EQ.1) GO TO 30
MCASE=1
C ASSUME BOUNDARY CONDITION OF BIRTH
Z1( 2 ,1)=Z1(1,1)
70  Z1INT(1)=0
   DO 50 I=2,IF1
     B=TMAX+G1(I)+(TMAX/2-G1(I))*G2(I)*U(N+1)
     A=TMAX-G1(I)-(TMAX/2-G1(I))*G2(I)*U(N+1)
     Z1( 2 ,I)=A/3*71(1,1-1)
50  Z1INT(1)=Z1INT(1)+3BIRTH(I)*Z1( 2 ,I)
C CORRECT BOUNDARY CONDITION
Z1( 2 ,1)=Z1INT(1)
IF(MCASE.EQ.2) GO TO 11
MCASE=2
GO TO 70
11  DO 12 I=1,IF1
     Z1(1,1)=Z1(2,1)
12  Z1(1)=Z1(1,1)
GO TO 20
30  RETURN
1  FORMAT(30,3S13,1X,5I2,8F6.2)
6  FORMAT(13,8F6.2)
END
DATA Z2REF, CP/1:10/ 
C V3CPSS IS DAY BF NEXT STAGE 
DATA V3CPSS/1, 4, 6, 10, 13, 17, 20, 32, 40/ 
IF (IPRINT.GT.1) PRINT 1 
N=0 
K=0 
PE2=0 
D9 10 I=1, 54 
C ADJUST DATA T0 NF=7 TMAX 
H1(N)=H1(N)*TMAX/60* 
H3(N)=H3(N)*TMAX/60* 
H4(N)=H4(N)*TMAX/60* 
H2(N)=H2(N)*(TMAX/2,-H1(N)) 
H3(N)=H3(N)*1+2 
A=0 
D9 16 J=1, 8 
V3CPSS(J, I)=0. 
16 IF (T.EQ.V3CPSS(J)) <=K+1 
16 IF (T.EQ.V3CPSS(J)) <=K+1 
V3CPSS(K, I)=1. 
D9 15 J=1, 8 
A=A*V3CPSS(J, I) 
C EVALUATE AGE-DEPENDENT FEEDING RATE 
15 C2(N)=C2(N)*C2S(J)*V3CPSS(J, I) 
C2(N)=C2(N)*C2S(J)*V3CPSS(J, I) 
C INITIALIZE POPULATION 
10 Z2(N, I)=Z2INT(I)*Z2INF 
Z8(1)=1 
20 N=N+1 
Z4(N)=0 
D9 7 J=1, 54 
C ADD PENALTY FOR NEGATIVE POPULATION 
1F (Z2(N, J).LT.0) PE2=PE2+Z2(N, J)**2 
7 Z4(N)=Z4(N)+C2(J)*Z2(N, J) 
IF (IPRINT.LT.1) G9 T9 2 
D9 3 I=1, 9
*CALCULATE STAGE POPULATIONS*

```plaintext
P9P(J) = 0
DO 4 J = 1, 54
A = 0

C CALCULATE STAGE POPULATIONS
DO 5 I = 1, 8
A = A + VBCPS(I, J)

5 P9P(I) = P9P(I) + VBCPS(I, J) + ZP(N, J)

4 P9P(J) = P9P(J) + Z2(N, I) * (1 - A)

PRINT 6, ZP, P9P, Z8(N)

2 CONTINUE
IF(N + E0 + 1) SI TO 30
MCASE = 1
A = (N + 1) * 2 * (U(N + 1) + 2)

C CALCULATE PREDATOR-PARASITE POPULATION ZR
Z8(N + 1) = Z8(N) + E6*Z8(N) + A7*Z8(N) + E6*Z8(N)*Z

C ASSUME BOUNDARY CONDITION
Z2(N + 1, 1) = Z2(N, 1)

70 ZZINT(1) = 0
DO 40 I = 2, 54
A = 1 + Z2/CP*(TMAX + 2(I + 2(I)*U(N + 1))
C = (N, I - 1) + 2*(1 + P + CP*(TMAX + 2(I) + 2(I)*U(N + 1))
2 + Z2(N, I - 1) + (2(I)*TMAX + 2(I) + 2(I) + 1) + (N + 1) - 2,I) + ZR(N + 1) + 4(I)/CP
3 + Z2 + H4(1)
B = TMAX + 2(I) + (1 + Z2(N, I - 1)/CP) + H2(1) + (N + 1) - 1(I) + 4(I)/CP
2 + H3(I) + Z8(N + 1) - 1*Z2 + 4(I)
Z2(N + 1, 1) = E + (R + 0*P + 4*A + C)**(1 - 2*Z) / (2*Z)
IF(ZZ(N + 1, I) + LT + 1)* Z2(N + 1, I) = 0

40 ZZINT(1) = ZZINT(1) + PRINTH(1) + Z2(N + 1, 1)

C DEFINE BOUNDARY CONDITION
Z2(N + 1, 1) = ZZINT(1)
IF(MCASE = EQ 2) GO TO 20
MCASE = 2
GO TO 70

30 RETURN
1 FORMAT(*-VBC: EGGS, 1-LARVA; CP, PUPA, ADULTS, 2-LARVA*)
```
6      FORMAT (I3,11F8.3)
      END
SUBROUTINE PLANT(Z3,Z4,PX,T,IPRINT,IZ9)

C LEE VARIETY OF SOYBEAN - INTEGRATION BEGINS AT PRDF:

C******************************************************************************
COMM8N TMAX,AGEMAX,IF,IF1,IF1
DIMENSION Z3(91),Z4(91),P9DWT(91),STMWT(91),WTM(91)
DATA DWA,DWB,DNC,DND,0/216,11057,00083,05996,29/
DATA PSTMX,F3,NREA,AT,STMWT,PRTE,SNKCRN,SNRTE/
*4.00,215,3716,44,130,80,141,19*0011/
C DAY 1 IS DAY 81 IN REAL TIME - PRDF:
P9DWT(1)=0.25
WTM(1)=78.*
STMWT(1)=80.*
I=13-81
I=0
10 I=I+1
IF (I.EQ.91) GO TO 20

C******************************************************************************
DWA=DWA+DWC*WTM(F(I))=DWC*WTM(I)**2
DWB=DWB*WTM(I)
IF(DWB.GT.150.) DWB=150.*
IF(DWB.LT.140.) DWB=140.*
DWB=PSTMX*(1.-EXP (?X2))
FM9X=F9X + DX*WTM(I)**2
F9D9=F9D9 + DX*WTC(I)**2
I1=13
T2=I
FMT3T=FMT(T1,T2,1)
FMP2=FMP(*STMW(T(I)),STMWX,1)
FMP3=FMP(*STMWT(I),STMWX,1)
F3MN=PRTE*P3DWT(I)
F3MP=FMP(F3DT,X,1)
F3PPD=FMP(F3DT,D,1)
F3DT=FMT3T*XMXS*(FMP)*(X*(1.*SNKCRN)+SNKCRN*P3DT*P9DWT(I))*.F3PPD/PRTE=*
*(PRTE*P3DWT(I)) + FMT3T*F9PSDZ3(I)}
PBDWT(I+1)=PBDWT(I)+DPDT  
STMWT(I+1)=STMWT(I)+FMT3T*W*DPDT-Z3(I)  
DPDT=EXP(-A*FL9AT(I3=1))  
DPDT=A*DPDT/(1+DPDT)*2  
WTLF(I+1)=WTLF(I)-SFVRE*WTLF(I)*FL9AT(I)*DPDT*FMT3T*74/1  
G9 TO 10
20 IF(IPRINT.LT.2) G9 TO 50
PRINT 25
CJMDEF=0.  
G9 30 I=1,91
J=80+1
CJMDEF=CJMDEF+Z4(I)
30 PRINT 40,J,PBDWT(I),WTLF(I),STMWT(I),Z3(I),Z4(I),CJMDEF
50 CONTINUE
PWT=PBDWT(91)
CALL ADJDT5(Z3,Z4,STMWT,PBDWT,WTLF,IPRINT,731)
RETURN
25 FORMAT(15AY PBDWT WTLF STMWT PRO AND LAE DAMAGE)  
? AND TOTAL DEFOLIATION?)
40 FORMAT(I4,4F10.2,3=F9.2)
END
FUNCTION FM(T1, T2, Y)
C PURPOSE TO PREVENT T4 FROM EXCEEDING COMPUTER FLOATING POINT LIMIT AND
C TO EVALUATE EXPONENTIAL
C****************************************************************************
T4=(T1-T2)*4;
IF(T4*GT*150*) T4=150*;
IF(T4*LT*=-140*) T4=-140*;
IF(M*EQ*1) T4=EXP(-T4);
IF(M*EQ*0) T4=EXP(T4);
T4=1*/(1*+T4);
FM=T4;
RETURN;
END
FUNCTION FMD (T1,T2,4)
FMD=(T1-T2)*4*
IF(FMD*GT*150*) FMD=150*
IF(FMD*LT*-140*) FMD=-140*
IF(M*EQ*1) FMD=EXP(-FMD)
IF(M*EQ*0) FMD=EXP(FMD)
FMD=FMD/(1+FMD)**2
RETURN
END
SUBROUTINE ADJ3T5 (AD, AD7, Z5, Z6, Z7, IPRINT, 79)
C PURPOSE IS TO CALCULATE WITH FIRST-ORDER APPROXIMATION THE PLANT STATE VAR
C
IABLES
C******************************************************************************
DIMENSION AD5(91), AD6(91), AD3(91), AD4(91), AD7(91), Z5(91), Z6(91), Z7
2(91)
EQUIVALENCE (DA, E1), (D3, E2), (D4, E3), (PTS MAX, E4), (DW, E5)
COMMON TMAX, AEG MAX, IF, NE, IF1, IF2, AT, DT, X7, CN CST, FN CST, AP CST,
* JLR T9, Z21 NFR, E6
C B = -2.5698*(10**-5) (J SHELF/G PRO WT)*(8.5/BUSH E)
DATA DA, D3, D4, D5, D6, D7, E1, E2, E3, E4, E5 / 3713.2, 1542.4, 0.01164, 0.529, 0.1929/
DATA PTS MAX, F, 3 AREA, A, I3, STM MX, PORTE, SN C AN, SF NRTF/
* 3.68, 215, 3716.4, 13, 80, 0, 2, 0, 12, 0.0154/
AD5(NE1) = 0
AD6(NE1) = 3 CN CST**0.00025698*(3 JRT9-Z9**0.0071853-1.)
AD7(NE1) = 0
C DAY 1 IS DAY 81 IN REAL TIME - PRO SET
I3 = I3 = 81
I = 91+1
10 I = I-1
IF(I.EQ.1) GO TO 20
D7 = (D7A+CW)* Z7(I)-CW* 77(I)**2
TTS = D7*Z7(I)
IF(TTS*ST*150*) TTS=150*
IF(TTS*LT*140*) TTS=140*
D8 = PTS MAX*(1.-FSP(TTS))
FMQW = FM(I, 0, Z7(I), 1)
FPOQW = FM(I, 3, Z7(I), 0)
CM = FMQW*CM+FPQW*CW
T1 = I3
T2 = I
DPRT = PORTE*Z6(I)
FPRT = FM(T1, T2, 1)
FVSS = FM( Z5(I), STM MX, 1)
FPSS = F(5, Z5(1), STMWMX, 1)
FMPD = F(DDPT, DW, 1)
FPDD = F(DDPT, DW, 0)
DDWDZ7 = FMD(3, Z7(1), 1) * F1 + F2 * Z7(1) * F1 + F3 * Z7(1) * F1
2 * FMD + E4(1) = EXP(TTS) * F(3, Z7(1), 0) + FPD + F4 + F5 * EXP(TTS)
PHII = FMT3T * MD(5(1), STMWMX, 1) * A * FMPD * (DW(1) = SNKCAN + SNKCAN*PDRTF*Z6(1)
2TE + Z6(1) + FPD + PDRTF*76(1) + FMT3T * DW * FMD(76(1), STMWMX, 0) * (-A)
PHI2 = FMT3T * FMSS* (DW(1) = SNKCAN + SNKCAN*PDRTF*76(1) * FPD(DW, 1) * PDRTF +
2 * PDRTF + FPD) * PDRTF + PDRTF * 76(1) * (-A)*PDRTF) *
FMD(DPDT, DW, 0)
PHI3 = FMT3T * FMSS * (DW(1) = SNKCAN + SNKCAN*PDRTF*76(1) * (-A) * DDWDZ7
2 * FMD(DPDT, DW, 1) + (1.0 - SNKCAN) + FPD * DDWDZ7 + PDRTF / 76(1) * (-A) * DDWDZ7
20 IF(I = 1) = 30 THEN RETURN
PRINT 25
I = 1, 91
J = 10 + K
30 PRINT 40, J, AD6(1), AD7(1), AD5(1)
RETURN
25 FORMAT('DAY AND ADJINT VARIABLS: 6, 7, AND 3. DRY WIGHT AND IT
25 DERIVATIVE!')
40 FORMAT('I, 5E12.4')
END
SUBROUTINE ADJOT2 (AD7, IPRINT, IPC2)
INTEGER VBCPSS
REAL IPC2
COMMON TMMAX, AGEMAX, IF, NF, IF1, NF1, DA, DT, K7, PN CST, RNCST, APP CST,
JULRTS, ZP, VFR1, E6, ST9, ZZ, ZR
DIMENSION AD7(91), J(91), ZZ(91, 54), ZR(91), AY(2, 54), IPC(91), H1(54)
?; C2(54), C3(9), H2(54), VBCPSS(5, 54), H3(54), P8(11), AD4(9)
3 + VBCPSS(9), ST9(101)
C VBCPSS IS DAY 8F NEXT STAGE
C IF2 IF(72. LT. 0) P2 = 72. * 2
C H1(54) IS MORTALITY EFFECT * TMMAX/2
C C2(54) IS FEEDING RATE, AGE DEPENDENT
C C3(9) IS DAILY REDUCTION IN GRAMS PER STAGE
C H2(54) IS AGE DEPENDENT CONTROL EFFECT
C H3(54) IS PREDATORY MORTALITY * TMMAX/2
C C0 IS CARRYING CAPACITY OF PREDATOR COMPLEX
C E6 IS BITTH - DEATH FOR PREDATOR
C DATA BASED ON TMMAX = 60 DAYS
DATA H1/3*1.39, 1.74, 2.08, 1.57, 3.5, 1.04, 1.22, 2.32, 1.04, 2.
1*22, 2*39, 3*37, 1*35, 4*4, 5*52, 6*41, 7*143/
DATA H2/3*0.28, 1.32, 2.0/
DATA H3/3*1.86, 2.33, 2*8, 1.34, 1.63, 2*1.86, 1*63, 3*1.4, 1.63,
2 *1.86, 1.16, 1*47, 5*7, 7*549, 1*399/
DATA C2/54*0.0 /
DATA VBCPSS/1, 5, 6, 10, 13, 17, 20, 32, 49, CP/10,
C 7 I = 1, NF1
IPC2(I) = 0
IF (IPRINT.EQ.3) PRINT 1
NEF1
K=0
C 9 10 I = 1, 54
H1(I) = -H1(I) * TMMAX/6
H3(I) = -H3(I) * TMMAX/50
H2(I) = H2(I) * (TMMAX/2 - H1(I))
-3(I) = -3(I)*1*2
A=0
D9 16 J=1,8
VBCPS(J*I)=0
16 IF(1+EQ*VBCPS(J)) <=K+1
IF(1+EQ*VBCPS(J)) <=K+1
VBCPS(J*I)=1
D9 15 J=1,8
A=A+VBCPS(J*I)
15 C2(I)=C2(I)+C2S(J)+V3CP5(J,I)
C2(I)=C2(I)+C2S(J)*(1-A)
10 AD2(2,I)=0
AD8(2)=0
20 N=N+1
IF(IPRINT*LT,3) G9 TO P
D9 3 I=1,9
3 P9P(I)=0
D9 4 J=1,54
A=0

C CALCULATE STAGE PNPJLATIONS
D9 5 I=1,8
A=A+VBCPS(I,J)
5 P9P(I)=P9P(I)+VBCPS(I,J)*AD2(P,I)
4 P9P(J)=P9P(J)+AD2(P,J)*(1-A)
P9P(10)=AD8(2)
P9P(11)=2*(N+1)/(P+1(N+1))
J=N+1
PRINT 4, J, P9P
2 CONTINUE
D9 55 I=1,54
C CALCULATE CONTRIBUTIONS TO NECESSARY CONDITION
55 IPC2(N+1)=IPC2(N+1)+AD2(P,I)*Z2(N+1,I)
IPC2(N+1)=IPC2(N+1)+WMAX+AD3(P)*Z8(N+1)
IF(N*EQ*3) G9 TO 37
C DEFINE BOUNDARY
AD2(2,54)=0
AD4(54)=0
DO 50 I=1,53
J=55-I
AD4(J-1)=DA*TMAX*(AC7(N)+AC7(N+1))/2.0*(A(1,I)+AD4(J-1))
A=1.0+5.0*(Z2(N+1,I)+Z2(N+1,I))/CP
A=H1(J)*H2(J)*U(N+1)*H3(J)/A*ZR(N+1)+H3(J)/CP+5.0*(Z2(N+1,I)+Z2(N,1))
2 J=1)/A*2*Z8(N+1)
50 AD2(1,J-1)=AD2(2,J)*(-TMAX-A)/(-TMAX+A)=1.0/(-TMAX+A)*CP(J)*
2 AGEMAX/2*(AD4(J-1)+AD4(J))
A=0
DO 60 I=1,54
IF(I.EQ.1) GO TO 60
A=A*(AD2(2,I)+AD2(1,I))/2.0*(Z2(N+1,I)+Z2(N+1,I))/CP
2 /1.0+5.0*(Z2(N+1,I)+Z2(N,1))/CP)
60 AD2(2,I)=AD2(1,I)
PBP(1)=2.0*U(N+1)/(2.0+U(N+1))
C EVALUATE PREDATOR-PARASITE ADJINT VARIABLE
AD8(2)=AC8(2)+ AD8(2)=AC8(2)+ A*D+AD8(2)*(E6-PBP(1)=2.0*A*78(N+1))
GO TO 20
30 RETURN
1 FORMAT('ADJINT VARIABLES FOR THE STAGES OF VAR. THEN CONTROL*1')
6 FORMAT(I3,11E8*2)
END
SUBROUTINE ADJ3T1 (AD1, IPRINT, IPC1, YIELD)
REAL IPC1
COMMON TMAX, AGEMAX, IF, IN, IF1, IF1, DA, DT, AD, JS, GCST, BCST, MCAST, ARPST,
* QLRT, Z21FR, E6, Z11
DIMENSION AD1(91), UI(91), Z11(2,101), IPC1(91), T11(101),
2 G1(101), G2(101), C1(101), C1S(7), SGS3PS(6,1-1), PEBR(8), AD1(101)
3, SBIRTH(101), PEBR(8), PEBR(101), ATCRAT(7)
C PF1 IS PENALTY FOR NEGATIVE POPULATION
C PF1 IS IF(Z11.LT.0) PF1=7.**2
C G1 IS MORTALITY COEFF, * TMAX/2
C C1 IS FEEDING RATE, AGE DEPENDENT
C G2 IS AGE DEPENDENT CONTROL BY PESTICIDE
C SGS3PS IS STAGE PROBABILITY DISTRIBUTION
C DATA BASED ON TMAX = 40 DAYS
DATA C1/101&,
DATA Z1REF/1&,
DATA SBIRTH/C0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
20.0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
30.001,0,004,0,017,0,028,0,036,0,041,0,049,0,064,0,075,0,082,0,091
4.1,1241,1,1675,2,183,2,284,3,642,4,539,5,546,6,553,7,657,8,675
5.1,105,1,225,1,355,1,491,1,635,1,785,1,935,2,085,2,232,2,355,2,455
6,2,495,2,515,2,532,2,566,2,605,2,645,2,677,2,717,2,65,2,675
72,7,72,7,705,2,692,2,655,2,645,2,65,2,65,2,65,2,65,2,65,2,65,2,65
82,592,592,592,592
DATA 31/0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
20.0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
31.384,1.532,1,815,1,859,1,933,1,734,1,568,1,351,1,196,1,175,1,191
4,1,707,1,511,1,63,1,724,1,85,1,977,2,185,2,291,2,372,2,462
31.3,84,1.532,1.815,1.859,1.933,1.734,1.568,1.351,1.196,1.175,1.191
4,1,707,1,511,1,63,1,724,1,85,1,977,2,185,2,291,2,372,2,462
5,76,1,589,3,93,3,89,3,93,2,67,2,11,2,21,2,24,1,129,1,156,1,188,2,27
6,276,3,022,3,39,4,23,5,19,6,24,7,34,8,42,9,41,1,028,1,103,1,166
71,2,183,1,268,1,319,1,371,1,424,1,472,1,505,1,511,1,482,1,412,1,299
8,1,154,1,99,8,25,6,7,6,537,4,26,3,36,2,64,2,67,3,161,1,123,0,92,0,66
9,0,47,0,315,0,195,0,12,0,75,0,64,0,037
DATA 32/C0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
2,0096,0,17,0,28,3,447,6,76,0,83,1,367,1,799,2,35,2,998,3,694
34427,5068,5893,6575,719,773,8174,8519,8794,91,92,99
411,1,64,1/
DATA CIS/0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*,0*
15  C1(I)=C1(I)+C1S(J)*SGSRS(I,J,I)
3  BSGPJG(I)=BSGPJC(I)+ATKRAT(7)*(1-A)
C1(I)=C1(I)+C1S(7)*(1-A)
 Z1(2, I)=Z11(I)
10  A01(2, I)=0.
20  N=N-1
 IF(1PRINT.LT.3) GO TO 2
 D9 3  I=1,7
 P9P1(I)=0
3  P9P(I)=0
 D9 4  J=1,1F1
 A=0
 D9 5  I=1,6
 A=A+SGSRS(I,J)
 P9P1(I)=P9P1(I)+SGSRS(I,J)*Z1(2,J)
5  P9P(I)=P9P(I)+SGSRS(I,J)*AD1(2,J)
 P9P1(7)=P9P1(7)+Z1(2,J)*(1-A)
4  P9P(7)=P9P(7)+AD1(2,J)*(1-A)
 P9P(8)=2.*U(N+1)/(2.*U(N+1))
 P9P1(8)=P9P(8)
 J=N+1
 PRINT 6,J,P9P
 PRINT 6,J,P9P1
2  CONTINUE
C  CALCULATE PENALTY FUNCTION CONTRIBUTION
 D9 55  I=1,IF1
55  IPC1(N+1)=IPC1(N+1)+Z1(7, I)*AD1(2, I)*(-2.*(TMAX/2.-G1(I)))*S2(I))
2  IF(N.EQ.0) GO TO 3
 A03(IF1)=0.
 AD1(2, IF1)=0.
 SUM=0
 D9 50  I=1,IF1
 J=IF1+I+1
 AD3(J-1)=AD3(J-1)+TMAX*(AD6(N+1)+AD6(N))/2.*
 A=G1(J)-(TMAX/2.*G1(J))*R2(J)*J(N+1)
AD1(1,J-1)=((-TMAX*A)*AD1(I,J)+S1(J)*AEGMAX*(AD3(I)+AD3(J-1))
1+BGSPJG(I)*AEGMAX*TMAX*0.0071*32*BENCST.00005698*Q;II*RTA*YIELD
2*(I-FLSRT(I)/AEGMAX))
2/(-TMAX*A)
A=TMAX*S1(J)^2+S1(J)*32(J)*U(N+1)
B=TMAX+S1(J)^2+S1(J)*32(J)*U(N+1)
A(1,J-1)=A(2,J)*2/A
SUM=SUM+Z1(I,J-1)*ZBIRTH(J-1)
Z1(I,IF1)=(Z1(1,I)+SJK)/ZBIRTH(IF1)
IF(Z1(I,IF1)>0) Z1(I,IF1)=0.
50 60 1=1,IF1
Z1(2,I)=Z1(1,I)
60 AD2(2,I)=AD1(1,I)
30 TO 20
RETURN
1 FORMAT(10:ADJBTY VARIABLES FOR S.G. STINK BUG AND CONTROL)
6 FORMAT(I3,3E9.2)
END
SUBROUTINE BRYSEL (JNEW, IPRINT, IPC1, IPC2, PF1, PF2, Y1F1, I1, ITNUM, 79)
COMMON TMAX, AEMAX, IF, NF, IF1, NF1, DA, DT, K7, RNCST, RNCST, APPCST,
QLRT8, ZFINFR, E6, STAR, I, ICASE, SIP, PIP, NEDIC, NC, DHDH

* , AC, A2, VS, NC1
REAL IPC1, IPC2
DIMENSION U(91), JNEW(91), DHOU(91), IPC1(91), IPC2(91), STAR(5106)
* TEMP(91), VS(10)
*, APPDH(91)

C $BEANS/FT R=9W=9 P9D/FT RAW=7 BEAN/P9D*LP/454 G*R=4.4/41/61 1 R*RENCST=1.00025*
C BUSHHEL BEANS/ACRE=G/FIT R=9W*LP/454G*BUSHHEL/60LP*109.0FT/ACRE=.7 BEANS/P9D*279
C THERE ARE 48 IN. RAWS = 10, P9D FT RAW PER ACRE
C SLOPE OF PER CENT DAMAGED SIFDS VS Z9 IS .071452
C R=4.5914*(10**-5)(LB/FT RAW)*($4/L)
DATA V1, V2, V3/0.0, 0.0, 0.0/
V# YIELD*.27199
R=C6NCST/21780.
D9 10 I=2, NF1
V#2=JNEW(I)/(2+JNEW(I))
IF(JNEW(I)>0.995JNEW(I)+GT, P) VP=VP+SPI*V#2
C DAILY APPLICATION COST
APPDH(I)=APPCST/(1.+AC*EXP(-A2*V4)))/10890*
V5=V5+APPDH(I)
C CHEMICAL COST
10 V#1=V#1*R*V#2
C QUALITY REDUCTION
V#4=79.00071652*3EVCST.*00025693+QLRT8*YIELD
C OBJECTIVE FUNCTION
V3=V1*YIELD*0.000256398*RENCST*V4+V5
PENNJC=(PF1+PF2)*SIP+VP
VP=V3+PENNJC
D9 80 I=1, NF1
80 TEMP(I)=JNEW(I)*P/(2+JNEW(I))
IF(IPRINT.EQ.-1)PRINT100, VS(2), V#2, V3, V2, V4, V1, V#, PENN IC, PIP, NC
IF(IPRINT.EQ.-1.8R, 1PRINT.EQ.3) PRINT 120, TEMP
IF(1TNJM.EQ.1) G7 T9 31
VS(2)*VP
ICASE=ICASE+1
D9 40 I=1, NF1
40 U(I)=UNEW(I)
50 PRINT 100, VS(2), V3, V2, V4, V1, V5, PENFUNC, PIP
IF(ITNUM*NE.3) G8 T9 36
C ITNUM OF 0 INDICATES TERMINATION OF SEARCH
ITNUM=0
G9 T9 37
36 CONTINUE
IF(ICASE*GT*KZ) G8 T9 990
SIP=SIP*100.
NC1=0
NC=0
C EVALUATE UPPER LIMIT OF INTERVAL FOR PIP
C UPPER LIMIT IS RECIPROCAL OF LARGEST ABSOLUTE SLOPE
37 V2=0.
DB 70 I=2, NF1
V1=0
C ADD PENALTY IF U NOT OF (0, 1)
PENCON=.01
95 IF(U(I)*LT.9.999) V1=2.*U(I)*PENCON
C********UNCONSTRAINED OPTIMA REQUIRES V1=0
UHOU(I)=2.*U(I)/(2.+U(I))
UHOU(I)=APPDFH(I)*AC-A2*EXP(-A2*UHOU(I))/((1.+AC)*EXP(-A2*HOU(I))}
97 UHOU(I)=2.*R*U(I)/(2.+U(I))+IPC1(I)+IPC2(I)+V1+UHOU(I)
PRINT 1003,1,CHOU(I),IPC1(I),IPC2(I),V1
1003 FORMAT(I4,4F20.4)
C********UNCONSTRAINED OPTIMA REQUIRES 39 T9 98
IF(U(I)**LT.1.000) G8 T9 98
IF(U(I)**LT.1.000) G8 T9 98
IF(U(I)**LT.1.000) G8 T9 98
IF(U(I)**LT.1.000) G8 T9 98
IF(U(I)**LT.1.000) G8 T9 98
90 IF(U(I)=2.) 96,94,101
90 IF(U(I))=101,96,98
C IF SLOPE IS POSITIVE AND U NEGATIVE WE INCREASE CENTRAL PENALTY
101  PENC9N= PENC9N*2.
    GO TO 95
C IF SLOPE IS POSITIVE AND U SMALL
96   DHDJU(I)=0.
98   V3=ABS(DHDJU(I))
70   IF(V3*GT*V2) V2=V3
     PRINT 129
     PRINT 130, DHDJ
     IF(ITNUM*EQ*6) RETURN
     IF(V2*EQ*0.) V2=1.
     VS(9)=1./V2
     VS(3)=0.*
     G9 T9 65
C GOLDEN SECTION SEARCH
30   VS(NEEDLE)*VP
    I=6
    IF(NEEDLE*EQ*6) I=8
    IF(NC1*EQ*N6*AND*VP*GT*VS(I)) NC1=NC1+1
    IF(NC1*LT*N6) 39 T9 65
    IF(VP*GE*VS(2)) G9 T9 99R
    ITNUM=2
    RETURN
65   NC=NC+1
    VC1=NC1+1
    IF(NC*LE*2) 39 T0 67
    IF(VS(8)*GE*VS(6)) 39 T9 6R
    G9 63 I=3,6
69   VS(I)=VS(I+2)
    GO TO 74
68   IF(VS(3)*LE*VS(6)) G9 T9 7P
     VS(1)*=VS(8)
     VS(9)=VS(7)
     VS(8)=VS(4)
     VS(7)=VS(5)
     G9 T9 71
72   VS(3)=VS(5)
VS(4)=VS(6)
VS(9)=VS(7)
VS(16)=VS(8)
NC=1
67 IF (NC<EQ.2) G9 TO 74
71 PIP=VS(3)+3E2*(VS(9)-VS(3))
VS(5)=PI
NEEDLE=6
G9 T3 79
74 PIP=VS(3)+618*(VS(9)-VS(3))
VS(7)=PI
NEEDLE=8
79 G9 6) J=2, NF1
60 INEW(I)=J(I)-PI*DELJ(I)
ITNUM=1
RETURN
992 PRINT 106
PRINT 106, VP, V2, V1, V3, PEFHIC, PIP
999 ITNUM=3
G9 997 J=1, NF1
997 INEW(I)=J(I)
RETURN
100 FORMAT(1X,2E12.5,7E10.3,13)
106 FORMAT('**** TERMINATION DUE TO FAILURE TO CONVERGE ****')
120 FORMAT(1X,1PE8.3)
129 FORMAT('NECESSARY CONDITION, DH/DOJ')
130 FORMAT(1X,1PE9.2)
END
DARE VARIETY PLANT MODEL

SUBROUTINE
SUBROUTINE PLANT(Z3,Z4,PD,IT,PRT,PRINT,Z9)
C******************************************************************************************
C DARE VARIETY OF SSYREAN - INTEGRATION BEGINS AT PREDSET
C******************************************************************************************
COMMON TMAX,AGEMAX,IF,RF,IF1,RF1
DIMENSION Z3(61),Z4(61),PDWT(61),STMWT(61),NTF(61)
DATA D1A,D1B,D1C,D1D,D1E /31322,15424,001164,30529,66,29/
DATA PTSMAX,F,AREA,A,T3,STMAXX,IFRT,SNKCN,SNKRTF/
*558,215,3715,44,95,85,0,2,0,12,0,00154/
C DAY 1 IS DAY 61 IN REAL TIME - PREDSET
PDWT(1)=0.25
WTLF(1)=78
STMWT(1)=80
I3=13-61
J=0
10 I=I+1
IF(I*E9+61) 69 TO 20
C ******************************************************************************************
DW1=D1A+D1B*KT_F(I)*D1C*TILF(I)*+2
DW2=0.1D*WTLF(I)
IF(DW2*GT*15C+) DW2=15C+:
IF(DW2*LT*.140: DW2=-140:
DW2=PTSMAK*(1-MEM(DW2))
FMAK=F(4(C,WTLF(I),1))
FP0W=F(4(C,WTLF(I),0)
DW=FM0X+D1.1+FP0X+X
T1=I3
T2=I
FM3=T=F(M(T1,T2,1)
FMSS=F(4(STMWT(I),STMXX,1)
FPSS=F(4(STMWT(I),STMXX,0)
DFPD=PRTF*PDCT(I)
FMPD=F(4(DFPT,c),1)
FPDP=F(4(DFPT,c),0)
DFOT=F(4P3T*FMSS*(EXP*(D1*(1-SCS)+SCS*SNKTXPRIT()PRWT(I)))+FPDP*PRTF*20
*(PODE*POWNT(I))+F*TE*FPSS*PO=Z3(I)
POCW(I+1)=POCW(I)+DPNT
SMNT(I+1)=SMNT(I)+PO*TE*DPNT*Z3(I)
DPNT=EXP(-A*FBAT(I3-I))
DPNT=A**DPNT/(1+DPNT)**2
*TLF(I+1)=*TLF(I)-F*FPORE*F*TLF(I)*F*FBAT(I)*DPNT*F*UT3T)=74(I)
90 TO 10
20 IF(IPRINT.LT.2) 30 TO 50
PRINT 25
CUMDEF=0.
39 39 I=1,61
J=60+1
CUMDEF=CUMDEF+74(I)
30 PRINT 40,J,POCW(I),*TLF(I),SMNT(I),Z3(I),74(I),CUMDEF
50 CONTINUE
POCW=POCW(61)
CALL ADJUST(Z3/4,SMNT,POCW,*TLF,IPRINT,79)
RETURN
25 FORMAT(I)*POWNT
2*1 AND TOTAL DEFLECTION)
40 FORMAT(I4,4F10.2,3F7.2)
END
VITA

Norman F. Marsolan, Jr. is the son of Norman and Jean Noel Marsolan of Covington, Louisiana. He was born in New Orleans on October 23, 1950.

Norman graduated from Covington High School in May 1968, and attended Louisiana State University in Baton Rouge, where he received his Bachelor of Science in Chemical Engineering in the Fall of 1972. He entered Louisiana State Universities Graduate School in the Summer of 1973, and earned his Master of Science in Chemical Engineering in May 1975. He is presently a candidate for the degree of Doctor of Philosophy in Chemical Engineering.

Norman married the former Maureen O'Meara of Detroit, Michigan, in August of 1972. Both Norman and Maureen have been active in Boy and Girl Scouting programs during the last six years. They are presently advisors for a local Explorer Post.
EXAMINATION AND THESIS REPORT

Candidate: NORMAN FRANK MARSOLAN, JR.

Major Field: CHEMICAL ENGINEERING

Title of Thesis: MODELING AND OPTIMAL CONTROL OF INSECT PEST POPULATION DYNAMICS

Approved:

[Signatures]

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination: JULY 14, 1976