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A Study of Two Methods of Teaching Problem-Solving in Eighth-Grade Mathematics.

Lawrence Houston Davis

Louisiana State University and Agricultural & Mechanical College

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PROBLEM SOLVING IN EIGHTH

GRADE MATHEMATICS

A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Doctor of Philosophy

in

The Department of Education

by

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B.S., Northwestern State University of Louisiana, 1953
M.S.Ed., Northwestern State University of Louisiana, 1958
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ABSTRACT

The purpose of this study was to compare the effectiveness of the presentations of two problem-solving procedures. These presentations were used to teach a unit on solving linear equations over integers to eighth grade students. The experimental procedure employed encoding and decoding skills. The control procedure employed the traditional approach of the textbook.

The subjects were pupils in two eighth grade mathematics classes at Perrin Junior High School, Ponchatoula, Louisiana. Each class contained thirty students and was taught one of the procedures by the investigator. An I.Q. score and a pre-test score on the objectives of the unit content were obtained prior to the beginning of the two weeks period used to teach the unit. During the time the study was in effect the Experimental Group was taught exclusively by a method which utilized encoding and decoding skills to solve linear equations and the Control Group was taught the traditional (textbook) method which utilized axioms of addition, subtraction, multiplication and division to solve linear equations. At the conclusion of the study, a post-test score on the objectives of the unit was obtained for each student in both control and experimental groups. Since appropriate achievement tests were not available, the pre-test and post-test
were constructed by the investigator and were validated by a jury of experts in the teaching of elementary mathematics.

The results of the post-test were analyzed by a t-test to determine whether significant differences in achievement existed within each of the two groups. An analysis of covariance was used to analyze the results of the post-test to determine whether a significant difference in achievement existed between the two groups. The I.Q. scores were correlated with gains in problem-solving performance for each group and the resulting correlation coefficients were tested for a significant difference in order to determine if mental ability was more closely identified with student performance under one of the two treatments.

Consideration of the data compiled during this study warranted the following conclusions:

1. Gains in problem solving by the subjects of the Control Group, all of whom received the textbook treatment for solving linear equations over the integers, were significant at the .01 level of confidence.

2. Gains in problem solving by the subjects of the Experimental Group, all of whom received the encoding and decoding treatment for solving linear equations over integers, were significant at the .01 level of confidence.

3. There was no significant difference between the achievement of the unit objectives by students in the Control
Group and students in the Experimental Group.

4. There was a difference in favor of the Control Group, significant at the .05 level, between the correlation coefficients for mental ability and student performance under the two treatments.
Chapter 1

INTRODUCTION

The development of a student's ability to solve problems continues to be one of the most important general instructional objectives of modern mathematics. George Polya (1962:118) describes solving problems as the most characteristically human activity. Each individual in his lifetime is confronted with problem-solving situations. Therefore, the value of developing problem-solving abilities can hardly be overemphasized and the mathematics classroom cannot be neglected as a proving ground for developing and testing methods of instruction designed to increase the student's problem-solving ability.

Authors (Butler, Wren, and Banks, 1970:229-231) of texts dealing with methods of teaching mathematics tend to agree that pupil difficulties in mathematical problem solving fall into one or more of the following categories: (1) vocabulary; (2) failure to see relationships; (3) interpretation of the problem; (4) computational skills; (5) lack of command of fundamental processes; reading difficulties; and (7) use of poor techniques. Obviously this list is incomplete and contains overlapping items. However, if one examines the list carefully, he finds that many of the items belong to the broader category of communication skills.
Communication skills are essential to problem solving and in mathematics these skills involve the ability to encode and decode mathematical statements using the precise and concise coding systems of the "language of mathematics." When students learn to encode mathematical statements into symbolic form, they learn to write mathematics. When they learn to decode mathematical messages, being careful to note each part of the original message, they learn to read mathematics. And when students learn to read and write mathematics they may be able to overcome some of their problem-solving difficulties.

An illustration will serve to show how the ability to encode and decode mathematical statements may simplify problem solving:

Problem: Solve over the integers, \[2x + 3 = 11.\]

The student could read (decode), write (encode) and think as follows:

Write: \[2x + 3 = 11\]

Read: The sum of \(2x\) and 3 is 11.

Think: What number is represented by \(2x\)?
Answer: 8 because \(8 + 3 = 11\)

Write: \[2x = 8\]

Read: The product of 2 and \(x\) is 8.

Think: What number is represented by \(x\)?
Answer: 4 because \(2 \times 4 = 8\)

Write: \[x = 4\]
Read:  x is the same as 4.

Think:  What number is represented by x?
Answer:  4 because $4 = 4$

Write:

One of the characteristics of modern programs in elementary mathematics is the early introduction, development, and use of the equation or open sentence. Part of the rationale for this early introduction is the possibility that the use of equations would aid in developing the student's ability to solve problems. However, the approach to solving equations that is found in most eighth grade mathematics textbooks does not emphasize reading the equation to determine the content of the message but rather uses what is often called a traditional approach or formal analysis using the structure properties of number systems.

STATEMENT OF THE PROBLEM

This study compared the problem-solving performance of eighth grade students who received instruction in encoding and decoding mathematical statements as an approach to problem solving with the problem-solving performance of eighth grade students who studied a traditional approach to problem solving.

This study attempted to answer the following questions:

1. Will there be significant gains in problem solving within each of two groups of eighth grade students, one group instructed in a particular technique that emphasized encoding and
decoding skills and the other group instructed in a traditional technique that emphasized the structure properties of number systems?

2. Will there be a significant difference in achievement in problem solving between the two groups?

3. Will student ability (I.Q.) be more closely identified with student performance under one of the two treatments than the other?

IMPORTANCE OF THE STUDY

Problem solving seems to be an attribute of man. According to George Polya (1962:118):

The most characteristically human activity is problem solving, thinking for a purpose, devising means to some desired end. Our aim is to understand this activity—it seems to me that this aim deserves a good deal of interest.

Dienes and Golding (1971:46) made the following comment about problem solving in mathematics:

... in examinations when certain types of questions are asked, certain code symbols are used whose properties have been learned and which can be transformed in certain admissible ways leading to certain end results which are known as "correct answers" by examiners. When this trickery has been learned, it is assumed that the candidate for such an examination knows mathematics because he has passed the examination. This is hardly ever true because the large majority of candidates who pass mathematical examinations do not know any mathematics whatsoever. They do not know exactly what kinds of mathematical situations the code systems used by them
are conveying. There is ample evidence of this when students go to college and are unable to use their coding system.

The greater part of our conscious thinking is concerned with problems and efforts directed toward developing problem-solving abilities are important and worthwhile. Kilpatrick (1969:530) cites the need for finding methods and devices that would improve problem solving without putting the child in the kind of strait jacket provided by formal analysis and other prescriptive techniques.

DEFINITION OF TERMS

Decoding: Decoding is translating from code into ordinary language.

Encoding: Encoding is translating from ordinary language into code.

Linear equation: A linear equation is any equation which can be put in the form \(mx + b = 0\), where \(m \neq 0\).

Problem: A problem is a question proposed for solution or consideration.

Problem solving: Problem solving is finding a way out of a difficulty, a way around an obstacle, attaining an aim which was not immediately attainable.
Content

A unit on solving linear equations over the integers was chosen as the content to be taught in the study. This selection was made because the students would have had prior experience adding, subtracting, multiplying, and dividing with integers but no experience with solving linear equations over the integers.

The problems to be solved were those linear equations found in the textbook supplemented by additional lists of problems of the same type as those found in the text.

Population and Sample

The population was defined as the eighth grade mathematics students in the regular classes of the Tangipahoa Parish Public School System during the academic year 1975-1976.

The sample consisted of two classes selected from among those of the population. Since it was necessary to use pre-formed groups, the selection of the classes for use in the study was made so that these groups were as nearly representative of the entire population as possible.

Treatments

Two treatment approaches were devised to achieve the objectives of the unit of content. The Control Group received
the treatment approach to solving linear equations found in the textbook series. The Experimental Group received instruction in encoding and decoding mathematical sentences with emphasis on meaning rather than mechanics. These skills were used to solve the same problems as the Control Group. A unit on encoding and decoding was written for the Experimental Group.

**Instrumentation**

It was necessary to construct instruments to measure the achievement of the objectives of the unit. Validation of these instruments was accomplished through obtaining a consensus among a committee of experts in elementary mathematics. Reliability coefficients were computed by the method of split-halves (odd versus even items). The resulting coefficients were then corrected using the Spearman-Brown Prophecy Formula.

The *Beta Test (New Edition: Form Em)* of the *Otis Quick-Scoring Mental Ability Tests* was administered to obtain a measure of mental ability for each student.

**Procedures for Gathering and Analyzing Data**

Each student was given a mental ability test and a pre-test prior to the presentation of the two treatments. A post-test on the objectives of the unit was administered at the conclusion of each treatment. An analysis of covariance was used to correct for initial differences among the two classes. Gains in problem
solving within each of the two groups and achievement in problem solving between the two groups were analyzed for statistical significance. Correlation coefficients for mental ability and achievement in problem solving were computed within each of the two groups.
Chapter 2

REVIEW OF RELATED LITERATURE

Problem solving in elementary school mathematics is an area in which the literature has been reviewed extensively. Alan Riedesel (1969:54-55) reviewed 83 outstanding articles and research reports from the past fifty years and offered the following problem-solving suggestions:

1. The improvement of computation is important to problem-solving ability but the improvement of computation alone has little, if any, measurable effect upon reasoning and problem solving.

2. To assure optimal achievement pupils must be interested in the problem-solving situation. Pupils react well to a variety of problem settings.

3. Children are receptive to "puzzle type" or enrichment problems.

4. The use of a formal approach of requiring pupils to answer: (a) What is given? (b) What is to be found? (c) What operations are used? (d) What is an estimate of the answer? does not produce superior results in problem solving. However, using one of these questions as a focal point for a lesson does improve problem solving.
5. The following practices of teachers improve problem solving:

(a) Provide problems of appropriate difficulty.
(b) Help pupils analyze the given information.
(c) Encourage and praise pupils when they perform processes correctly.
(d) Help pupils check final solutions.
(e) Start with easy problems.


7. A variety of computational types should be part of most problem-solving lessons.

8. Tape recordings can be used effectively with pupils with reading problems.

9. No best technique for problem solving has been found. However, the following techniques increase problem-solving ability:

(a) Make use of mathematical sentences in solving problems.
(b) Make use of drawings and diagrams to help pupils solve problems.
(c) Make use of orally presented problems. They are representative of out-of-school problem solving situations.

The review of the more recent related literature is presented in three parts: (1) problem-solving ability, (2)
problem-solving tasks, and (3) problem-solving processes.

PROBLEM-SOLVING ABILITY

Although the ability to solve mathematical problems is not necessarily a universal phenomenon, one can learn something about its nature by examining the relationships between an individual's success in problem solving and other characteristics of his thinking and personality. Studies of problem-solving ability range from straightforward comparisons of group performance to intricate factor analyses.

Tate and Stanier (1964:371-376) analyzed the performance of good and poor problem solvers using tests of critical thinking and practical judgment. The subjects were junior high school students. On the critical thinking tests, it was found that poor problem solvers tended to avoid the judgment "not enough facts" and made unqualified "true" or "false" judgments. On the practical judgment test they tended to select answers having a high affective component. Tate and Stanier argued that students' errors may have a temperamental rather than an intellectual basis.

The relation of sex differences to problem-solving ability was studied by Sheehan (1968:84-87). The subjects were high school freshmen enrolled in algebra. A criterion test designed to measure high and low level cognitive processes was administered. The superior performance of girls on the lower level processes
disappeared after adjustments were made for their initial superiority using analysis of covariance. Boys showed superior performance after adjustments were made. Koopman (1964:3398) found girls to be less confident of their problem solutions than boys.

Success in solving word problems in mathematics clearly depends upon skills in reading and computation but the relative contributions of these skills is not clear. Martin (1963:4547-4548) found that each of the factors of reading comprehension, computation, abstract verbal reasoning, and arithmetic correlated positively with problem solving among fourth graders. Martin suggests that the relationship between problem-solving ability and its underlying skills, particularly high ordered verbal skills, is more complex than had been supposed.

Certain affective factors have been shown to be related to problem-solving ability. Jonsson (1966:i3757-3758), working with a sample of sixth graders, showed interaction of test anxiety and test difficulty, especially for girls, to the detriment of the performance of highly anxious subjects taking the more difficult version of the testing instrument.

Gangler (1967:2157) reported finding evidence of the influence of motivational factors. College students who were told that their work on a series of learning tasks in logic would count toward their course grade performed less well on learning and
problem-solving tasks than students who were not so informed. The effect was greater for students of higher intelligence than those of lower intelligence.

Kellmer Pringle and McKenzie (1965:50-59) argued that a less competitive school environment may reduce frustration and stress among low ability students in problem-solving situations. Robert Soar (1975) recently stated that evidence exists that shows elementary school children perform better at problem solving when their classroom management allows for considerable freedom of behavior.

PROBLEM-SOLVING TASKS

Problem materials vary from proofs to simple puzzles, and variations in problems with respect to content and structure have some effect in problem-solving performance. Several studies have assessed the effect of such variation.

Travers (1969:9-18) asked a sample of high school freshmen to choose and solve one of two problems that were identical in structure but placed in different settings. The subjects showed strong preferences for "social-economic" situations compared with "mechanical-scientific" situations and abstract situations.

Scott and Lighthall (1967:61-67) tested the hypothesis that disadvantaged children would perform better on a problem whose content dealt with food and shelter needs, than on a problem whose
content dealt with higher needs, such as mastery and education. Need content was not related to degree of disadvantage among third and fourth graders.

The effects of two variations of the language used in a problem on its difficulty were investigated by Steffe (1967). Ninety first graders in individual interviews were asked to combine two sets containing elements with the same name and then they were asked to combine two sets containing elements with different names. The problems dealing with sets having the same name were significantly easier than the problems dealing with sets whose elements had different names.

In another study dealing with the structure of the problems, Williams and McCreight (1965:418-421) found that placing the questions at the beginning rather than the end of a problem statement did not significantly improve the performance of fifth and sixth graders who were asked to solve problems of both types.

The relative contribution to problem difficulty of six variables: (1) operations, the minimum number of different operations needed for a solution; (2) steps, the minimum number of applications of operations; (3) length, the problem length in words; (4) sequential, whether or not the problem could be solved by the same operations as the preceding one; (5) verbal cues, whether or not the problem contained a verbal clue to the operations
needed; and (6) **conversion**, whether or not conversion of units was required—was studied by Suppes, Loftus, and Jerman (1969:1-15). The six variables accounted for 45 percent of the variance in performance on the problems. Sequential, conversion, and operation variables made the greatest contribution in that order. Data were obtained from twenty-seven bright fifth graders who were asked to solve sixty-eight word problems in a computer assisted program.

In a later study Jerman (1973:109-123) found that the length variable (number of words in the problem statement) was apparently more important in the upper grades than in the lower grades in a sample taken from grades 4-9.

**PROBLEM-SOLVING PROCESSES**

In order to investigate problem-solving processes, studies have been designed so that the subjects generate observable sequences of behavior for study. Such studies are referred to as developmental studies and Piaget's theories on the growth of logical thinking have served as both a focus and touchstone for such studies.

Freyberg (1966:164-168) used an objective test designed to measure the development of Piagetian concepts in a two-year study. Scores on the concept test were as predictive of arithmetic computation and problem-solving ability as was Primary Mental
Abilities Test.

Polya (1965) is responsible for the modern interest in the study of methods and rules of discovery and invention—the heuristic. Polya set forth maxims for problem solving which he axiomatizes correspond to mental actions. According to Kilpatrick (1968:4380) evidence for the validity of Polya's observations on the problem-solving process has come from work on computer simulation of human behavior.

SUMMARY

One can learn something about the nature of problem solving by examining the relationship between an individual's success in problem solving and certain characteristics of his thinking and personality. Success in problem solving depends upon skills in reading, reasoning, and computation, but the relative contribution of these skills is not clear.

Motivation, anxiety, and school environment are a few factors that have been shown to be related to problem-solving ability.

Problem language, content, context, and structure have a significant effect on problem-solving performance.

Problem-solving processes are being examined by studies designed to generate observable behavior.
The majority of problem-solving studies are evaluations of a single device or technique. The investigator found no evidence of studies concerned with developing encoding and decoding skills as aids to problem solving.
Chapter 3

PROCEDURES USED IN THE STUDY

The study was conducted in Perrin Junior High School, Ponchatoula, Louisiana. The school had an enrollment of approximately 500 students and a faculty of 25. Parents of students in the school represented a cross section of socio-economic levels, but the majority fell into the lower middle income group. The school enrollment was about 70 percent white and 30 percent black.

SELECTION OF SAMPLE

A number of factors influenced the selection of the classes used in the study. Since all teaching was done by the investigator, it was necessary to arrange the teaching schedule to allow for travel to the school, while coordinating it with reduced teaching responsibilities at Southeastern Louisiana University in Hammond, Louisiana. The assistance of the Superintendent of the Tangipahoa Parish Public School System was requested in selecting classes which would be representative of the overall population of the schools in the system. Within these constraints, two eighth grade mathematics classes in Perrin
Junior High School were selected. The membership of the classes was the result of regular placement of the students during the fall semester of the 1975-1976 school year. Each class consisted of thirty students.

ASSIGNMENT OF TREATMENTS

One of the selected classes was randomly assigned the designation of Control Group and the other class was designated the Experimental Group. The Control Group received the textbook treatment for solving linear equations over integers while the Experimental Group received a treatment that used encoding and decoding skills to solve the same problem sets of linear equations.

INSTRUMENTATION

In the absence of adequate testing instruments, it was necessary to construct and validate tests to measure achievement of the unit objectives. A pre-test and a post-test were developed. Each consisted of thirty items of varying difficulty and measured achievement in solving linear equations over integers.

Validity

According to Best (1959:176), a test is valid if it measures what it claims to measure. Tate (1965:183) said that if an achievement test is in agreement with the content which is
taught, it is valid in the given situation. According to the

Educational Testing Service Handbook for Cooperative Mathematics

Tests (1964:7), content validity is best insured by entrusting test
collection to persons well qualified to judge the relationship of
test content to teaching objectives.

Jordan (1953:15-16) stated:

If our objectives were to make the most
valid test for an elementary algebra class,
the teacher would be the best one to do it.
He would know exactly the areas he had taught,
the objectives he had in mind. He might analyze
the areas into the processes employed and then
construct a test which contained samples of
all the algebraic processes with each process
represented at three or four different levels
of difficulty. If such a test were carefully
constructed it would reflect accurately progress
in the mastery of the algebraic processes
studied and the defined objectives. In such a
test the curricular or internal validity would
be satisfactory. For curricular validity for
this particular subject, this process has no
rival.

One way of determining the validity of an achievement
test, according to Best (1959:176), is to seek the opinion of
experts in the field. The pre-test and post-test and a statement
of the unit objectives were shown to Professors Henry E. Corkern
and Harold R. Moore of Southeastern Louisiana University, Depart-
ment of Mathematics and to two eighth grade mathematics teachers,
Mrs. Dixie Moore of Southwood Academy, Hammond, Louisiana and
Eulon Alford of Hammond Junior High School. After carefully
examining the tests, they agreed that, in the light of the statement
by Best, the tests were valid.

Reliability

According to Wert, Neidt, and Ahmann (1955:328) test reliability is the degree of consistency with which the test measures whatever it does measure. One acceptable method of computing the reliability coefficient of an achievement test is the split halves method. In using this method, the test is divided into two parts of equal length, scores are obtained on each half-test and a coefficient of correlation is computed for the two sets of scores. This gives a reliability coefficient of a test half as long as the original test. Using this coefficient and the Spearman-Brown prophecy formula, a reliability coefficient for the original test can be computed.

The pre-test and the post-test, each consisting of thirty items, were administered to two Hammond Junior High School eighth grade mathematics classes containing twenty-four and twenty-six students respectively. Using the technique of split halves (odd numbered items versus even numbered items) and the Spearman-Brown prophecy formula, the reliability coefficients for the thirty item pre-test and the thirty item post-test were found to be .96 and .80 respectively.

The Beta Test (New Edition:Form Em) of the Otis Quick-Scoring Mental Ability Tests, published by World Book Company, was administered to each student to obtain a measure of that student's
mental ability (I.Q.).

INSTRUCTIONAL PROCEDURES

A preliminary conference was held with the principal of the school and with the teacher of the selected classes. It was agreed that the investigator would be allowed to serve as a teacher aide during the months prior to the beginning of the study. This provided opportunities for the investigator to become acquainted with each student. The length of the instructional period for each group was fifty-five minutes. The classroom teacher was asked to avoid giving any instruction in solving linear equations and he agreed not to use the results of the pre-test to determine any student's grade. A unit on adding, subtracting, multiplying, and dividing with integers had been completed just prior to the beginning of this study and therefore a review of these operations was not considered necessary.

The Control Group devoted the entire class period to textbook materials and supplementary exercises. Axioms of addition, subtraction, multiplication, and division for equations were used as advocated by the textbook to solve linear equations over integers. All exercises were collected, graded, and returned to the students.

The Experimental Group devoted the entire class period to materials designed to produce skills in encoding and decoding
mathematical statements conveyed by linear equations. Emphasis was placed on meaning rather than mechanics. Linear equations over integers were solved by thinking of them as English statements about operations on integers. The Experimental Group worked the same sets of exercises as the Control Group and these exercises were also collected, graded, and returned to the students.

The investigator was the instructor for each of the two groups so that teacher variability was essentially eliminated. The mental ability test was administered to both groups on November 12, 1975. The pre-test was given on November 19, 1975. On December 1, 1975, the treatments began and the post-test was administered to both groups on December 12, 1975.

Subjects were allowed to proceed independently in recording their responses on the post-test. They were permitted to take as much time as was needed. All students completed the post-test within forty minutes.

STATISTICAL PROCEDURES

The design of this investigation is the "Nonequivalent Control Group Design." Campbell and Stanley (1963:47) stated that one of the most widespread experimental designs in educational research involves a control group and an experimental group, each given a pre-test and a post-test, but in which the control group and the experimental group do not have pre-experimental equivalence.
Rather, the groups consist of naturally assembled collections such as classrooms, as similar as availability permits, yet not so similar that one can dispense with the pre-test. The assignment of a given subject to one group or the other group is assumed to be random and under the control of the experimenter.

According to Tate (1955:515), it is possible to introduce control in two or more classes of experimental data by making allowance for initial differences among the classes which may have prejudiced the results of the treatment. Such control is possible in situations where there is available an associated measure for each of the final experimental measures. Analysis of covariance is one method of analyzing differences existing among classes of final experimental data, taking into account differences existing in the associated initial data.

A t-test was performed on differences between post-test and pre-test scores within each group. An analysis of covariance procedure was used based upon adjusted results on the post-tests. A correlation coefficient between mental ability measures and post-test scores was calculated for each group. Calculations were performed on a statistical calculator provided by the Department of Mathematics at Southeastern Louisiana University, Hammond, Louisiana.
Chapter 4

PRESENTATION AND ANALYSIS OF DATA

In this chapter the data generated by the study are reported and analyzed.

The number of subjects in each of the treatment groups was thirty. For purposes of analysis of the data, each student was assigned a number within his group and his pre-test, post-test, and I.Q. scores were paired with this number.

The data presented in Table 1 show the scores achieved by each subject in the Control Group on the mental ability test, pre-test, post-test and the difference between the post-test and pre-test scores. The data presented in Table 2 show the corresponding scores of the Experimental Group subjects.

ACHIEVEMENT WITHIN GROUPS

In order to determine whether there were significant gains in problem solving within each of the two groups, the mean difference between pre-test and post-test score was tested for significance using a t-test.

Means for the pre-test were 6.966 for the Control Group and 12.960 for the Experimental Group. The mean score for the Control Group on the post-test was 15.566, while that for the
Table 1

Basic Data for Subjects in Control Group (Textbook Treatment)

<table>
<thead>
<tr>
<th>Student Number</th>
<th>I.Q.</th>
<th>Pre-test</th>
<th>Post-test</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>96</td>
<td>2</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>2.</td>
<td>100</td>
<td>5</td>
<td>27</td>
<td>22</td>
</tr>
<tr>
<td>3.</td>
<td>117</td>
<td>20</td>
<td>28</td>
<td>8</td>
</tr>
<tr>
<td>4.</td>
<td>90</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>5.</td>
<td>103</td>
<td>22</td>
<td>29</td>
<td>7</td>
</tr>
<tr>
<td>6.</td>
<td>91</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>7.</td>
<td>84</td>
<td>3</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>8.</td>
<td>78</td>
<td>8</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td>9.</td>
<td>111</td>
<td>3</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>10.</td>
<td>121</td>
<td>12</td>
<td>30</td>
<td>18</td>
</tr>
<tr>
<td>11.</td>
<td>100</td>
<td>15</td>
<td>23</td>
<td>8</td>
</tr>
<tr>
<td>12.</td>
<td>98</td>
<td>13</td>
<td>21</td>
<td>8</td>
</tr>
<tr>
<td>13.</td>
<td>80</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>14.</td>
<td>82</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>15.</td>
<td>90</td>
<td>2</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>16.</td>
<td>103</td>
<td>6</td>
<td>24</td>
<td>18</td>
</tr>
<tr>
<td>17.</td>
<td>108</td>
<td>21</td>
<td>29</td>
<td>8</td>
</tr>
<tr>
<td>18.</td>
<td>91</td>
<td>2</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>19.</td>
<td>114</td>
<td>3</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>20.</td>
<td>114</td>
<td>8</td>
<td>24</td>
<td>16</td>
</tr>
<tr>
<td>21.</td>
<td>84</td>
<td>4</td>
<td>18</td>
<td>14</td>
</tr>
<tr>
<td>22.</td>
<td>96</td>
<td>8</td>
<td>22</td>
<td>14</td>
</tr>
<tr>
<td>23.</td>
<td>122</td>
<td>12</td>
<td>26</td>
<td>14</td>
</tr>
<tr>
<td>24.</td>
<td>98</td>
<td>6</td>
<td>13</td>
<td>7</td>
</tr>
<tr>
<td>25.</td>
<td>101</td>
<td>11</td>
<td>22</td>
<td>11</td>
</tr>
<tr>
<td>26.</td>
<td>88</td>
<td>2</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>27.</td>
<td>104</td>
<td>0</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>28.</td>
<td>93</td>
<td>7</td>
<td>26</td>
<td>19</td>
</tr>
<tr>
<td>29.</td>
<td>94</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>30.</td>
<td>102</td>
<td>8</td>
<td>12</td>
<td>4</td>
</tr>
</tbody>
</table>

Means: 98.4  6.966  15.566
Table 2

Basic Data for Subjects in
Experimental Group (Coding Treatment)

<table>
<thead>
<tr>
<th>Student Number</th>
<th>I.Q.</th>
<th>Pre-test</th>
<th>Post-test</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>114</td>
<td>23</td>
<td>30</td>
<td>7</td>
</tr>
<tr>
<td>2.</td>
<td>98</td>
<td>2</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>3.</td>
<td>103</td>
<td>10</td>
<td>16</td>
<td>6</td>
</tr>
<tr>
<td>4.</td>
<td>97</td>
<td>22</td>
<td>30</td>
<td>8</td>
</tr>
<tr>
<td>5.</td>
<td>99</td>
<td>19</td>
<td>25</td>
<td>6</td>
</tr>
<tr>
<td>6.</td>
<td>108</td>
<td>13</td>
<td>29</td>
<td>16</td>
</tr>
<tr>
<td>7.</td>
<td>121</td>
<td>21</td>
<td>29</td>
<td>8</td>
</tr>
<tr>
<td>8.</td>
<td>111</td>
<td>24</td>
<td>29</td>
<td>5</td>
</tr>
<tr>
<td>9.</td>
<td>106</td>
<td>19</td>
<td>30</td>
<td>11</td>
</tr>
<tr>
<td>10.</td>
<td>126</td>
<td>25</td>
<td>27</td>
<td>2</td>
</tr>
<tr>
<td>11.</td>
<td>109</td>
<td>0</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>12.</td>
<td>80</td>
<td>4</td>
<td>17</td>
<td>13</td>
</tr>
<tr>
<td>13.</td>
<td>90</td>
<td>9</td>
<td>15</td>
<td>6</td>
</tr>
<tr>
<td>14.</td>
<td>108</td>
<td>8</td>
<td>18</td>
<td>10</td>
</tr>
<tr>
<td>15.</td>
<td>100</td>
<td>15</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>16.</td>
<td>76</td>
<td>4</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>17.</td>
<td>81</td>
<td>5</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>18.</td>
<td>105</td>
<td>4</td>
<td>25</td>
<td>21</td>
</tr>
<tr>
<td>19.</td>
<td>110</td>
<td>5</td>
<td>26</td>
<td>21</td>
</tr>
<tr>
<td>20.</td>
<td>97</td>
<td>7</td>
<td>30</td>
<td>23</td>
</tr>
<tr>
<td>21.</td>
<td>121</td>
<td>23</td>
<td>29</td>
<td>6</td>
</tr>
<tr>
<td>22.</td>
<td>93</td>
<td>1</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>23.</td>
<td>92</td>
<td>18</td>
<td>28</td>
<td>10</td>
</tr>
<tr>
<td>24.</td>
<td>83</td>
<td>7</td>
<td>13</td>
<td>6</td>
</tr>
<tr>
<td>25.</td>
<td>96</td>
<td>10</td>
<td>24</td>
<td>14</td>
</tr>
<tr>
<td>26.</td>
<td>96</td>
<td>9</td>
<td>16</td>
<td>7</td>
</tr>
<tr>
<td>27.</td>
<td>106</td>
<td>7</td>
<td>26</td>
<td>19</td>
</tr>
<tr>
<td>28.</td>
<td>116</td>
<td>24</td>
<td>29</td>
<td>5</td>
</tr>
<tr>
<td>29.</td>
<td>114</td>
<td>21</td>
<td>27</td>
<td>6</td>
</tr>
<tr>
<td>30.</td>
<td>122</td>
<td>23</td>
<td>30</td>
<td>7</td>
</tr>
</tbody>
</table>

Means . . . . . . . . 102.5  12.960  22.133
Experimental Group was 22.133.

The difference between the pre-test mean and the post-test mean for each of the two groups was tested for significance with a t-test. The results of this procedure are shown in Table 3 and Table 4.

Under the heading of "df" are given the degrees of freedom for the pre-test and post-test. The column headed "SD" shows the standard deviation of the means.

Table 3

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>df</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>30</td>
<td>29</td>
<td>6.966</td>
<td>6.170</td>
</tr>
<tr>
<td>Post-test</td>
<td>30</td>
<td>29</td>
<td>15.566</td>
<td>9.559</td>
</tr>
</tbody>
</table>

\[
CR = \frac{15.566 - 6.966}{2.11} = 4.08
\]

For df = 58

CR at .05 level = 2.00
CR at .01 level = 2.66

The sums of the squares of the deviations taken around the means of the two sets of scores were pooled and SD* was computed as a better estimate of the "true" SD. The standard error of the difference between means, SE, was computed with the "pooled" standard deviation SD*. The critical ratio CR was computed by
dividing the difference of the means by the SE of the difference between the means. This operation yielded a critical ratio of 4.08, which is greater than the critical ratio at the .01 level of confidence. Thus the difference between the pre-test mean and the post-test mean was significant at the .01 level in favor of the post-test mean and the null hypothesis that there existed no significant difference in gains in problem solving within the Control Group was rejected.

Table 4

Analysis of Difference between Pre-test Mean and Post-test Mean within the Experimental Group

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>df</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>30</td>
<td>29</td>
<td>12.960</td>
<td>8.098</td>
</tr>
<tr>
<td>Post-test</td>
<td>30</td>
<td>29</td>
<td>22.133</td>
<td>7.762</td>
</tr>
</tbody>
</table>

\[
CR = \frac{22.133 - 12.960}{2.08} = 4.48 \\
\text{For df = 58} \\
\text{CR at .05 level = 2.00} \\
\text{CR at .01 level = 2.66}
\]

The critical ratio for the Experimental Group was 4.48 as shown in Table 4. Thus the difference between the pre-test mean and the post-test mean was significant at the .01 level in favor of the post-test mean and the null hypothesis that there existed
no significant difference in gains in problem solving within the Experimental Group was rejected.

ACHIEVEMENT BETWEEN GROUPS

In order to determine whether a true difference in achievement in problem solving existed between the two groups, the scores were subjected to an analysis of covariance. The scores from which the calculations were made are shown in Tables 1 and 2.

The total variance of the pre-test and post-test scores were analyzed to identify that portion of the variance attributable to differences among the groups themselves and that portion due to individual differences within the groups. The results of this procedure are shown in Table 5.

Under the heading "df" are given the degrees of freedom available among means, within means, and as a total. The column headed "SS \( x \)" shows the squares of the sums of the pre-test \( (X) \) scores. The squares of the sums of the post-test \( (Y) \) scores are given under the heading "SS \( y \)." The column marked "\( S_{xy} \)" shows the sum found by adding the products of the pre-test \( (X) \) scores and the post-test \( (Y) \) scores of each group. The "SS \( y.x \)" column gives the sum of the squares of the post-test \( (Y) \) scores as adjusted by the pre-test \( (X) \) scores, and the "MS \( y.x \)" column shows the mean squares of the post-test scores as adjusted by the pre-test scores.
The values in this last column represent the adjusted variance of the post-test scores.

### Table 5

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>SS x</th>
<th>SS y</th>
<th>S xy</th>
<th>SS y.x</th>
<th>MS y.x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Among Means</td>
<td>1</td>
<td>499</td>
<td>647</td>
<td>568</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td>Within Groups</td>
<td>57</td>
<td>3125</td>
<td>4549</td>
<td>2849</td>
<td>1952</td>
<td>34</td>
</tr>
<tr>
<td>Total</td>
<td>58</td>
<td>3624</td>
<td>5196</td>
<td>3417</td>
<td>1974</td>
<td>-</td>
</tr>
</tbody>
</table>

\[
F = \frac{22}{34} = 0.6471
\]

For df 1/57
- \(F\) at .05 level = 4.00
- \(F\) at .01 level = 7.08

This operation yielded an \(F\) ratio of 0.6417, which was less than the critical ratio at the .05 level of confidence. Thus the null hypothesis that there existed no significant difference in achievement in problem solving between the Control Group and the Experimental Groups was accepted.

**STUDENT ABILITY (I.Q.) AND STUDENT ACHIEVEMENT IN PROBLEM SOLVING**

In order to determine if student ability (I.Q.) was more closely identified with student performance under one of the two
treatments than the other, correlation coefficients for mental ability (I.Q.) and post-test scores and for mental ability (I.Q.) and the difference between the post-test and pre-test scores were computed for both the Control and Experimental Groups. The correlation coefficients between I.Q. and the difference between post-test and pre-test scores for each of the two groups were converted into corresponding z coefficients by using Fisher's z function. The difference between the two z coefficients was then tested for significance. The data from which these calculations were made are shown in Table 6.

Table 6
Correlation Coefficients for I.Q. and Test Scores

<table>
<thead>
<tr>
<th></th>
<th>Post-test</th>
<th>Post-test - Pre-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.Q. (Control Group)</td>
<td>.63</td>
<td>.50</td>
</tr>
<tr>
<td>I.Q. (Experimental Group)</td>
<td>.63</td>
<td>-.02</td>
</tr>
</tbody>
</table>

$r = .50$ corresponded to Fisher $z = .55$

$r = -.02$ corresponded to Fisher $z = -.02$

SE of the difference between z coefficients $= .27$

$CR = \frac{.55 - -.02}{.27} = 2.11$  
CR at .05 level = 1.96

CR at .01 level = 2.58
The procedure yielded a critical ratio of 2.11 which was greater than the critical ratio at the .05 level of confidence. Thus the null hypothesis that there was no significant difference between the two groups' correlation coefficients between mental ability (I.Q.) and the difference between post-test and pre-test scores was rejected at the .05 level of confidence in favor of the Control Group. The correlation coefficients between mental ability (I.Q.) and the post-test score for each of the two groups were identical.
Chapter 5

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

SUMMARY

The purpose of this study was to compare the effectiveness of the presentations of two problem-solving procedures. These presentations were used to teach a unit on solving linear equations over the integers to eighth grade students. The subjects were pupils in two eighth grade mathematics classes at one of the junior high schools of the Tangipahoa Parish Public School System. Each class was taught by one of the two presentations by the investigator. Each of the subjects provided scores from three instruments, a pre-test and post-test on the objectives of the unit of content, and a mental abilities (I.Q.) test.

The results of the post test were analyzed by a t-test to determine whether significant differences in achievement (problem-solving performance) existed within each of the two groups. An analysis of covariance procedure was used to analyze the results of the post-tests to determine whether a significant difference in achievement existed among the two groups. The results of the mental abilities (I.Q.) test were correlated with the difference between the post-test and pre-test for each group and the resulting correlation coefficients were tested for a significant difference.
in order to determine if mental ability (I.Q.) was more closely identified with student performance under one of the two treatments than the other.

CONCLUSIONS

Consideration of the data compiled during this study appeared to warrant the following conclusions:

1. Gains in problem solving by the subjects of the Control Group, all of whom received the textbook treatment for solving linear equations over the integers, were significant at the .01 level of confidence.

2. Gains in problem solving by the subjects of the Experimental Group, all of whom received the encoding and decoding treatment for solving linear equations over integers, were significant at the .01 level of confidence.

3. There was no significant difference between the achievement of the unit objectives by students in the Control Group and students in the Experimental Group.

4. There was a difference in favor of the Control Group, significant at the .05 level, between the correlation coefficients for mental ability (I.Q.) and student performance under the two treatments.
RECOMMENDATIONS

Learning to encode and decode mathematics provides an important set of skills for reading and writing the language of mathematics. As a result of conducting this research the investigator believes that there is a need for further study of the utilization of encoding and decoding skills as aids to problem solving and makes the following recommendations:

1. Develop more challenging and diverse techniques for involving students in encoding and decoding mathematics and then replicate this study with a larger sampling of students.

2. Determine the effectiveness of encoding and decoding skills as aids to solving word problems.
BIBLIOGRAPHY


37


Soar, Robert S. "The Development and Validation of Competencies." Address at Louisiana State University and Agriculture and Mechanical College, July 24, 1975.


Williams, Mary H., and Russell W. McCreight. "Shall We Move the Question?," *The Arithmetic Teacher, XII* (October, 1965), 418-421.
Solve each equation.

1. \( x + 5 = 0 \)
   Solution: -5

3. \( 9x = -36 \)
   Solution: -4

5. \( \frac{x - 9}{8} = 1 \)
   Solution: 17

7. \( 2x + 5 = 13 \)
   Solution: 4

9. \( 3(x - 2) = 27 \)
   Solution: 11

2. \( x - 7 = 3 \)
   Solution: 10

4. \( \frac{x}{3} = 11 \)
   Solution: 33

6. \( 3x + 21 = 0 \)
   Solution: -7

8. \( \frac{x}{5} + 6 = 11 \)
   Solution: 25

10. \( 5(x + 3) = 0 \)
    Solution: -3
11. \(23 - 2x = 13\) 
Solution: 5

12. \(4\left(\frac{x}{3}\right) = 8\) 
Solution: 6

13. \(\frac{3x}{2} - 1 = 5\) 
Solution: 4

14. \(\frac{3x + 1}{5} = 2\) 
Solution: 3

15. \(7\left(\frac{x}{2} - \frac{3}{2}\right) = 14\) 
Solution: 10

16. \(-3\left(\frac{x + 7}{2}\right) = -15\) 
Solution: 3

17. \(\frac{3(x + 2)}{4} = 0\) 
Solution: -2

18. \(7(2x - 3) = 7\) 
Solution: 2
19. \(3\left(\frac{2x - 1}{5}\right) = 9\)

Solution: 8

20. \(\frac{5(x - 1)}{3} - 7 = 3\)

Solution: 7

21. \(10\left(\frac{2x + 9}{7}\right) = 110\)

Solution: 7

22. \(\frac{9(2x + 1) - 11}{10} = 7\)

Solution: 4

23. \(\frac{3(x - 4) + 7}{2} = 14\)

Solution: 11

24. \(3(2x + 1) - 1 = 26\)

Solution: 4
25. \[ 6\left( \frac{x}{2} + 1 \right) - 40 = 2 \]

Solution: 30

26. \[ \frac{3(x - 1)}{2} + -6 = 0 \]

Solution: 5

27. \[ \frac{5(x + 1) + 2}{11} = 2 \]

Solution: 3

28. \[ \frac{8(x + 1) + 64}{13} = 0 \]

Solution: -9

29. \[ \frac{5(3x - 1) + 11}{4} = 9 \]

Solution: 2

30. \[ \frac{8(7x - 15)}{3} - 16 = 0 \]

Solution: 3
APPENDIX B

POST-TEST AND KEY

Solve each equation.

1. \( x + 8 = 0 \)  
   Solution: \(-8\)

2. \( x - 9 = 6 \)  
   Solution: \(15\)

3. \( 9x = -54 \)  
   Solution: \(-6\)

4. \( \frac{x}{4} = 9 \)  
   Solution: \(36\)

5. \( \frac{x + 1}{7} = 1 \)  
   Solution: \(6\)

6. \( 4x + 20 = 0 \)  
   Solution: \(-5\)

7. \( 3x - 15 = 12 \)  
   Solution: \(9\)

8. \( \frac{x}{3} - 2 = 5 \)  
   Solution: \(21\)

9. \( 11(x - 3) = 77 \)  
   Solution: \(10\)

10. \( 3(x + 5) = 0 \)  
    Solution: \(-5\)
11. \( 25 - 2x = 5 \)

\[ \text{Solution: 10} \]

12. \( 4\left(\frac{x}{3}\right) = 16 \)

\[ \text{Solution: 12} \]

13. \( \frac{2x}{3} + 4 = 6 \)

\[ \text{Solution: 3} \]

14. \( \frac{5x + 1}{4} = 4 \)

\[ \text{Solution: 3} \]

15. \( 6\left(\frac{x}{3} - 2\right) = 12 \)

\[ \text{Solution: 12} \]

16. \( -4\left(\frac{x + 7}{3}\right) = -12 \)

\[ \text{Solution: 2} \]

17. \( \frac{4(x + 3)}{5} = 0 \)

\[ \text{Solution: -3} \]

18. \( 5(3x - 2) = 5 \)

\[ \text{Solution: 1} \]
19. \( 3\left(\frac{3x + 1}{2}\right) = 15 \)  

Solution: 3

20. \( \frac{3(x - 2)}{2} - 5 = 4 \)  

Solution: 8

21. \( 10\left(\frac{x}{7} + 16\right) = 180 \)  

Solution: 14

22. \( \frac{7(2x - 1) + 11}{10} = 6 \)  

Solution: 4

23. \( \frac{7(x - 2) + 6}{3} = 9 \)  

Solution: 5

24. \( 2(3x + 1) - 1 = 25 \)  

Solution: 4
25. \[5\left(\frac{x}{2} - 2\right) - 39 = 1\]  
Solution: 20

26. \[\frac{2(x - 3)}{3} + -2 = 0\]  
Solution: 6

27. \[\frac{7(x + 3) + 8}{10} = 5\]  
Solution: 3

28. \[\frac{5(x + 4) + 45}{7} = 0\]  
Solution: -13

29. \[\frac{8(4x - 3) + 20}{10} = 6\]  
Solution: 2

30. \[\frac{7(3x - 9)}{3} - 21 = 0\]  
Solution: 6
APPENDIX C

SAMPLE EXERCISES FROM EXPERIMENTAL GROUP

1. **Decode**, that is, write in words, the idea represented by each of the following symbols.

   **Examples:**
<table>
<thead>
<tr>
<th>Symbols</th>
<th>Decoded</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x + 1$</td>
<td>The sum of $x$ and 1</td>
</tr>
<tr>
<td>$n - 3$</td>
<td>The difference of $n$ and 3</td>
</tr>
<tr>
<td>$5x$</td>
<td>The product of 5 and $x$</td>
</tr>
<tr>
<td>$t/3$</td>
<td>The quotient of $t$ and 3</td>
</tr>
</tbody>
</table>

   - a. $y + 6$
   - b. $3 - d$
   - c. $6w$
   - d. $46 + t$
   - e. $g - d - 3$
   - f. $36 + m$

2. **Encode**, that is, write with symbols, each of the following phrases.

   **Examples:**
<table>
<thead>
<tr>
<th>Phrase</th>
<th>Encoded</th>
</tr>
</thead>
</table>
   | The sum of $a$ and 6 | $a + 6$
   | The difference of $c$ and 7 | $c - 7$

   - a. The product of $-4$ and $n$
   - b. The difference of 6 and $n$
   - c. The quotient of $r$ and 3
   - d. The product of 7 and $s$
   - e. The sum of 9 and $w$

3. Let us agree that we will use $S$ to mean **Sum**
D to mean Difference
P to mean Product
Q to mean Quotient

Then we will write

- The sum of x and 2 in shorthand as Sx,2
- The difference of 4 and c as D4,c
- The product of 5 and n as P5,n
- The quotient of 6 and t as Q6,t

Write each of the following in our shorthand form.

a. The product of 5 and n
b. The sum of x and 16
c. The quotient of y and 7
d. The difference of w and 9

4. Write each of the following in traditional symbolic form.

Example: When written in traditional symbolic form

P3,m becomes 3m

a. Dx,6
b. Qx,3
c. P5,n
d. St,l
e. D9,w
f. Px,8
g. Q3,r
h. S4,x
i. D12,y
j. P15,x

5. Encode each of the following phrases.

a. The difference of 2n and 1
b. The sum of $5t$ and $5$

c. The product of $4$ and $t + 3$

d. The quotient of $x - 3$ and $4$

e. The sum of $\frac{x}{4}$ and $8$

f. The difference of $9$ and $3x$.

6. Decode each of the following symbols.

Example: Decoded $5n + 4$ becomes the sum of $5n$ and $4$.

a. $3(x + 1)$

b. $6k - 18$

c. $\frac{5x}{6}$

d. $5p - 2$

e. $\frac{x + 3}{7}$

7. Solve each equation.

Example: Solve over the integers, $x + 7 = 12$.

Write: $x + 7 = 12$

Read: The sum of $x$ and $7$ is $12$.

Think: What number is represented by $x$?
Answer: $5$ because $5 + 7 = 12$.

Write: $x = 5$

Read: $x$ is the same as $5$

Think: What number is represented by $x$?
Answer: $5$, because $5 = 5$

Write: Solution: $5$
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a. \( y + 6 = 28 \)

b. \( t - 31 = 42 \)

c. \( x + 3 = -7 \)

d. \( 2n = 18 \)

e. \( 72 = -8x \)

8. Solve each equation.

a. \( 2n - 1 = 5 \)

b. \( 3t + 5 = 29 \)

c. \( 4(t + 3) = 12 \)

d. \( 9 - 3x = 3 \)

e. \( \frac{n + 7}{5} = 6 \)

f. \(-17t = 0\)

g. \(-13c = -39\)

h. \(\frac{c}{8} = 9\)

i. \(\frac{r}{3} = -6\)

j. \(\frac{n}{5} = 0\)

9. Solve each equation.

a. \( \frac{5(x - 1)}{3} - 7 = 3 \)

b. \( 10 \left(\frac{2x}{7} + 9\right) = 110 \)

c. \( \frac{9(2x + 1) - 11}{10} = 7 \)

d. \( 3(2x + 1) - 1 = 26 \)
VITA

Lawrence Houston Davis was born in Hot Springs, Arkansas on August 18, 1929. His parents were James Lee Davis and Rosa Garrett Davis. His elementary and secondary education was received in the public schools of Garland County, Arkansas, where he was graduated from Hot Springs High School in May, 1948.

A B.S. degree in Chemistry and Mathematics was conferred on him in 1953 by Northwestern State University of Louisiana. In 1958 he received the M.S.Ed. and M.S. degrees in Mathematics from the same institution.

From 1953 to 1956 he served in the U.S. Army and was assigned as a surveyor and instructor of surveying to the 529th Field Artillery Observation Battalion stationed in Karlsruhe, Germany.

In September, 1958 he was employed as a member of the Mathematics Department of Southeastern Louisiana University, Hammond, Louisiana where he presently holds the rank of Associate Professor of Mathematics. He is co-author of Elementary Functions, a college mathematics textbook, published by Prentice-Hall, Inc. in 1969. The Louisiana State University Chapter of Phi Delta Kappa honored him as an outstanding graduate student in Education for 1976.
On December 13, 1953, he married Marie Tilleux. They are the parents of three daughters, Diane Elisabeth, Martha Marie, and Ellen Claire; and three sons, Paul Emerson, James Lee, and Thomas Tilleux.
Candidate: Lawrence Houston Davis

Major Field: Education

Title of Thesis: A STUDY OF TWO METHODS OF TEACHING PROBLEM SOLVING IN EIGHTH GRADE MATHEMATICS

Approved:

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Date of Examination:

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