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Essays on endowment fund management

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ESSAYS ON ENDOWMENT FUND MANAGEMENT

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by
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Dedication

This work is dedicated to my father, Kurtul Ogunc, and
my grandmother, Safiye ‘Sophia’ Sendir,
who inspire me to delve into perfection.

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Abstract

The debate around the perpetual nature of endowment funds from the perspective of current versus future obligations is a major problem that I would like to address in two ways: (i) a macro-level treatment of the simultaneous asset allocation and spending rate with subsistence levels (analogous to the habit formation concept); and (ii) a micro-level analysis of one part of the endowment portfolio with a particular emphasis on the currency hedging decision. The purpose of the third chapter is to illustrate the significance of joint determination of appropriate asset allocation and spending rate decisions, and to describe the behavior of the endowment fund portfolio under certain modeling assumptions, including a sensitivity analysis that evaluates, in particular, the relationship between the spending rate and stock allocation over an extended period of time by changing the values of certain parameters in the model. The fourth chapter tackles the issue of international diversification from the point of view of active currency hedging. The ability to control risk with the possibility of return enhancement is the main reason why institutional investors such as university endowments should worry about the international diversification of investment portfolios.

I have concentrated on an area, which has been overlooked by endowment funds for a long time. That is, the introduction of currency hedging in the context of an international portfolio and the provision of some behavioral considerations: first, implicitly, in the framework of the traditional expected utility maximization and then, explicitly, in the disappointment-averse functional context. In both chapters, the discussion is heavily based on the specification of the utility function; i.e., habit formation through the use of a subsistence level in the case of asset allocation and spending rate determination, and behavioral/agency-related formulation of various aversion parameters in the international portfolio management chapter.

Chapter 1

Introduction

Endowments are typically set up as permanent funds for the support of institutions such as private high schools, universities and museums. Through regular fund raising programs and occasional fund drives, these institutions have constant access to financial support from alumni and other interested parties. Alternatively, foundations are created by a monetary gift from an individual, family, or corporation with minimal additional contributions after the initial grant. Unlike foundations, there are no tax penalties for a failure to make an annual distributions to beneficiaries for endowments, but structural constraints related to budgetary planning make it difficult for endowment managers to reduce payouts. Endowments should attempt to educate their beneficiaries about the long-term benefits of controlled spending by introducing prudent levels of spending that would preserve their ability to support the sponsoring institution in both good and bad times. Institutions that seek a proper balance between the needs of current and future recipients may be inclined to focus on the preservation of the corpus. This balance could only be accomplished by stringent limitations on current spending. James Tobin[1] requires that an endowment "preserve equity among generations by supporting the set of activities that it is now supporting."

Since the passage of the Uniform Management of Institutional Funds Act by the National Conference of Commissioners on Uniform State Laws in 1972

[1] Please refer to "What is permanent endowment income?" American Economic Review 64 (May):427-432.

and the adoption by almost all the states to the present time, the concept of "prudence" has been introduced in the management of endowment funds in the United States. This includes the balancing and management of short-term and long-term needs, present and anticipated financial requirements, price level trends, and general economic conditions. Educational institutions accumulate endowments to enhance the autonomy of university operations by not relying on government grants, student tuitions and gifts made by the alumni. Particularly, in the case of universities better-endowed institutions could utilize the incremental income stream to gain a competitive edge over the peer group that is dependent on the very similar revenue sources. Moreover, certain long-term commitments of institutions of higher learning such as awarding tenure to a faculty member fit nicely with the permanent nature of endowment funds. It is very important to understand the purposes of raising funds from alumni and corporations for the better structuring of an investment program. The perpetual nature of the endowment provides some kind of a leverage in terms of maintaining independence from external pressures. A viable route for university trustees to create an independent course of action is the support of the operating budget by endowment funds. In the case of public universities, the government grants for the support of specific research projects necessitate the adherence to specifics regulations and guidelines set forth by the state authorities. The Board of Regents is such an entity, which establishes the guidelines for the management of endowment funds that are created by private donations matched by the state. The board supports the endowed chair and professorships program in universities and colleges of Louisiana. Other endowed funds do not fall into the umbrella of such a program and do provide better financial independence to institutions in terms of managing the portfolio of assets as well as the policies set forth for the distribution of funds on an annual basis.

The debate around the perpetual nature of endowment funds from the perspective of current versus future obligations is a major problem that I would like to address in two ways: (i) a macro-level treatment of the simultaneous asset allocation and spending rate with subsistence levels (analogous to the habit formation concept); and (ii) a micro-level analysis of one part of the endowment portfolio with a particular emphasis on the currency hedging decision. The purpose of the third chapter is to illustrate the significance of joint determination of appropriate asset allocation and spending rate decisions, and to describe the behavior of the endowment fund portfolio under certain modeling assumptions, including a sensitivity analysis that evaluates, in particular, the relationship between the spending rate and stock allocation over an extended period of time by changing the values of certain parameters in the model. The fourth chapter tackles the issue of international diversification from the point of view of active currency hedging. The ability to control risk with the possibility of return enhancement is the main reason why institutional investors such as university endowments should worry about the international diversification of investment portfolios. The pressure of enhancing the market value of an endowment fund, particularly, at times when the risk premia of stock investments are adjusted downward, along with the significance of outperforming peer institutions in terms of portfolio performance leads to the consideration of non-traditional approaches to total portfolio management such as the use of private equity, venture capital and hedge funds. Despite the fact that these alternative ways of managing endowment funds require a high level of governance, indicating significant oversight by the board and serious commitment by the endowment management team, they could be very beneficial in providing a cutting-edge solution for the problem at hand and enabling the institution to create a comfortable cushion that is necessary for the survival of the fund during bad times. My concentration will be on an area that has been overlooked by endowment funds for a long time, that is, the introduction of currency hedging in the context of an international portfolio and the provision of some behavioral considerations: first, implicitly, in the framework of the traditional expected utility maximization and then, explicitly, in the disappointment-averse functional context. In both chapters, the discussion is heavily based on the specification of the utility function; i.e., habit formation through the use of a subsistence level in the case of asset allocation and spending rate determination, and behavioral/agency-related formulation of various aversion parameters in the international portfolio management chapter. The use of currency overlay managers to actively hedge currency exposures of the portfolio necessitates the separation of the risk aversion parameter into two: one for the asset volatility and another for the currency component will be reviewed in particular. The rationale of using different parameters of risk aversion is when the international portfolio has two different incentive structure for managers; one for the ‘so-called’ asset manager(s) and another for the currency overlay manager(s).

The compensation of overlay managers is typically based on a fixed annual management fee plus an incentive scheme which provides performance-based fees to the overlay manager. Since the asymmetric nature of incentive schemes of asset and currency managers dictate how one optimizes the overall endowment fund portfolio, the unusual behavior of an endowment fund should not be called irrational, only because the optimal currency hedging level deviates from the one derived under rational expectations. It only justifies the use of different hedging strategies by various institutional investors. I describe in detail how the level of hedging should be revised downwards because of behavioral factors. My conclusions are in the context of what people would predict to see in the market, if certain investors behave in an irrational way.

Global portfolio management, in its broadest sense, involves the simultaneous evaluation of investment decisions regarding individual security selection such as stocks and bonds, industry weightings, country allocations as well as currency exposure. While exposure resulting from investments in international assets could have a significant impact on realized risk-adjusted returns, it is often overlooked at the total portfolio level. It is common among institutional investors to ignore the issue of hedging in the allocation of funds to various asset classes including international securities, and then involve in the structuring of the strategic hedging policy. Investment managers of many university endowment funds believe that any loss from failing to consider hedging at the initial stage is of second order. Hence, a lower level decision is made on how to hedge the international assets against currency risk, if at all.

It has been argued that the covariance between asset class and currency exposure should be analyzed in the context of portfolio construction to maximize the efficiency of the portfolio allocation decision. The full-blown optimization of currency and asset allocation decisions has been demonstrated empirically and theoretically to be more efficient for it exploits active risk diversification. For institutions that have already decided on the allocation of funds among various asset classes, the only viable option is to consider hedging as a follow-up decision, which is called the two-step or sequential approach to currency hedging. The practicality of the so-called one-step (or full-blown) approach is questionable except for the strategic determination of the optimal hedge ratio, which serves either the passive position or the benchmark level in case of a currency overlay program. The use of independent international equity and bond managers in institutional portfolios poses a challenge when it comes to integrating the country asset allocation decisions with active management of currency exposure. Currency overlay managers provide an opportunity to add an independent source of value in global investment portfolios via tactical currency management. The currency overlay product, which was designed and implemented for external portfolios toward the end of 1980's, is the active and independent management of the foreign exchange exposures associated with international stock and bond investments. Many large investment management firms have been actively managing currency exposure to provide enhanced returns over the specified benchmark in global tactical asset allocation products. On the other hand, the principal source of value-added from a currency overlay strategy has been in the risk control in international portfolios during the early periods of product implementation. The risk-oriented approach to currency overlay attempts to re-distribute currency returns to yield an option-like payoff with the purpose of eliminating the risk of ruin. To this end, it seeks downside protection from adverse currency moves, and at the same time, participate when the currency moves in its favor. Over the years, other styles have emerged in the overlay business with the intent of improving the value-added potential of international assets such as the so-called fundamental and technical styles.

University endowments have been increasing the allocation to international assets during the last two decades. The growing globalization of institutional portfolios over this period has heightened the issue of how to cope with the consequential exposure to currency volatility. The traditional approach to dealing with the added risk arising from foreign exchange exposure has included, but has not been limited to, one of the following three forms: (a) no hedging, (b) passive hedging, (c) active hedging. Certain institutional investors with risk preferences that do not tolerate the volatility arising from unhedged currency positions employ either passive or active hedging policies in the management of international assets. In the case of passive hedging that could take the form of full or partial hedging the investment committee fixes a hedge ratio and then delegates the implementation to either in-house managers or external specialists. The active approach, on the other hand, could be seen as periodic deviation from the chosen benchmark (neutral) position whenever the currency market expectation offers an attractive opportunity. There are three well-known management styles as far as active currency hedging is concerned: (a) Risk-controlling, providing option-like payoffs, also known as “dynamic hedging”; (b) Fundamental, based on predicting macroeconomic elements in international markets; and (c) Technical, based on model-driven strategies using technical analysis.

Contrary to the hedging strategy that separates the portfolio choice and the risk from the hedging decision, I will explicitly consider currency hedging in the context of international portfolio management. In order to deal effectively with currency exposure in a portfolio, one needs to evaluate the impact of foreign exchange movements on both risk and return of the total portfolio as well as evaluate alternatives within the decision-making process and provide guidelines as to how to structure a hedging program. To this end, institutional investors could engage in the active management of hedge ratios for individual currency pairs by hiring currency overlay managers with distinct styles.

Additionally, the active management of currency exposure should be handled by third party specialized managers, who do not have any potential conflicts of interest with the management of the underlying assets. This approach helps changes be made in currency exposure without disrupting the active management of individual securities and country allocations. These changes in broad exposure can be structured in a systematic way through a disciplined process of global asset allocation and active currency management, whereby hedging is implemented using currency forwards or options by the overlay manager. The asset and currency managers can fully concentrate on their strengths by separating the asset allocation decision from the currency exposure decision. Moreover, it is much easier to implement and monitor the attributes of each component in global portfolios. The essence of a thorough long-term investment policy rests on the successful design and implementation of strategic asset allocation, which is done in the asset-only space in case of university endowments (asset-only optimization) and in the asset-liability space in case of pension plans (surplus optimization). Grinold and Meese (2000b) postulate that the usual practice of subordinating currency decisions to a lower level leads to large and predictable biases in the strategic asset allocation both in the asset-only and asset-liability case. The challenge facing the use of currency overlay managers as a non-traditional investment vehicle is due to the bureaucratic nature of endowment fund management. The consensus-building behavior between investment committees and internal investment management teams influences the design of the investment process by thwarting non-traditional activities such as private equity, hedge funds and currency overlay, and imposing shorter-than-optimal time horizons from a performance measurement perspective. The words of Keynes (1936) come to mind: “Worldly wisdom teaches that it is better for reputation to fail conventionally than to succeed unconventionally.”

Chapter 2

Literature Review

2. 1. Modeling Issues in the Management of University Endowment Funds

Merton (1993) is the first to postulate that university endowment funds should be managed according to the principles of diversification and hedging by utilizing a reduced-form model of a university as a utility-maximizing agent. Specifically, he defines the university's objective as the maximization of a von Neumann-Morgenstern, time-separable, and concave utility function of the level of a set of activities $\underline{Q}(t) = \left[\underline{Q}_1(t), \ldots, \underline{Q}_m(t)\right]$ as $\max E_0 \left\{ \sum_{t=0}^{\infty} U\left[\underline{Q}(t), t\right] \right\}$. The price (or net cost) of activity *j* is $\underline{S}_j(t)$. No distinction is made between the marginal and average cost of an activity, leading to a perfectly elastic supply of activities. The vector of activity prices $\underline{S}(t) = \left[\underline{S}_1(t), \ldots, \underline{S}_m(t)\right]$ is an exogenous autoregressive process which Merton models as a continuous-time diffusion process. The endowment capital at the beginning of period *t* is *K(t)*. The nonendowment income at *t* is *Y(t)*, an exogenous stochastic process. Merton models *[S(t), Y(t)]* as a diffusion process, and further assumes that the nonendowment income is spanned by the returns of financial assets. Then, the nonendowment income stream $\left[Y(t), Y(t+1), \ldots\right]$ may be capitalized with value $\hat{Y}(t)$. The university's wealth is defined as $\underline{W}(t) = K(t) + \hat{Y}_t$, the sum of endowment capital and capitalized present and future nonendowment income.

Merton models the joint process of financial asset prices, activity prices, and nonendowment income as a diffusion process. The wealth dynamics is

$$\underline{W}(t+1) = \left[\underline{W}(t) - \underline{Q}'(t)\underline{S}(t)\right]\underline{w}'(t)\underline{R}(t+1).$$

The control variables are the activity levels $\underline{Q}(t)$ and the portfolio weights $\underline{w}(t)$. The university maximizes the expected utility by the sequential choice of activity levels and portfolio weights subject to the sequence of budget constraints and the constraint of nonnegative wealth. Basically, Merton models the university's problem as the standard intertemporal consumption and investment problem, which has been studied extensively in the finance literature.

His primary focus is on the portfolio allocation, whereby the optimal portfolio consists of a mean-variance efficient portfolio of the endowment plus the capitalized nonendowment income, modified by an overlay of hedging portfolios designed to hedge against unanticipated shifts in the state variables. In the special case where the indirect utility of consumption is the sum of the logarithm of consumption and a function of the state variables, a myopic policy is optimal. In this case, the university invests the endowment plus the capitalized nonendowment income in a mean-variance efficient portfolio, without an overlay of hedging portfolios.

Merton believes that the nature and size of a university's nonendowment assets significantly influence optimal policy for spending endowment. He postulates that for a given overall expenditure rate as a fraction of the university's total net worth, the optimal spending rate out of endowment will vary, depending on the fraction of net worth represented by nonendowment assets, the expected growth rate of cash flows generated by those assets and capitalization rates. Hence, neglecting those other assets will generally bias the optimal expenditure policy for the endowment. His analysis suggests that trustees and others who judge the prudence and performance of policies by comparisons across institutions should take account of differences in both the mix of activities of institutions and the capitalized values of their nonendowment sources of cash flows. In addition, he theorizes that universities, in addition to investing in assets to achieve an efficient risk-return trade-off in wealth, should optimally use their endowment to hedge against unanticipated and unfavorable changes in the costs of the various activities that enter into their direct utility functions. Even though it is very appealing to believe in the existence of the university-wide utility function by assuming equal marginal rates of substitution among many agents of the university, the bottom line is a university does not have a clearly defined group of residual claimants. The public university could be thought of a series of explicit and implicit contracts among various economic agents that include a highly fragment faculty, a heterogeneous student body with overlapping generations features, the alumni, the trustees, the officers, and the state legislature.

I would a priori argue that the expected utility of an endowment should be positively related to the cushion over inflation-adjusted corpus in each period. In the asset-liability context, this cushion is called a surplus, if positive. This would enable us provide adequate spending policies for future beneficiaries of the endowment by explicitly keeping the purchasing power of the fund intact. Merton describes the liabilities of universities as state variables, and hence, ends up with a hedge portfolio for the various university costs that is preference-dependent. In this alternative setting that includes the natural stochastic benchmark return, the hedge portfolio would depend not only on the liability structure; i.e., spending requirements, but also how much cushion there is at any point in time.

Dybvig (1995) motivates his model, which is close to modern models of habit formation such as those of Constantinides (1990), Detemple and Zapatero (1991), Shrikhande (1992) or Sunderesan (1989), to provide an alternative to the analysis of Grossman and Zhou (1993). Grossman and Zhou model ratcheting of risky investments by assuming a drawdown constraint that precludes wealth from falling below some proportion of the previous maximum without intermediate consumption withdrawal. One interpretation of Dybvig's model is as a formal justification of the ratcheting behavior of consumption posited by Duesenberry (1949), who proposed a consumption function which accounts for some adjustments for habits or standards of living. Conventionally, if income falls, then consumption should fall proportionally with the marginal propensity to consume. Duesenberry rejected this conventional theory and postulated that once consumption habits are acquired, it is hard to get rid of them. Thus, income shocks should have slightly different effects on consumption. Certain consumption habits are formed at high income levels which are not completely abandoned when income falls. Basically, he says that consumption habits are acquired when income was at its highest influence present consumption decisions. The main difference between Dybvig's model and other models of habit formation is in the rapidity of the habit formation and the severity of the agent's preferences for maintaining a new standard of living. In Dybvig's case, the marginal utility of consumption at a point in time is discontinuous when consumption equals the previous maximum, and in fact, the marginal utility is infinite for any decrease in consumption but finite for any increase.

2. 2. Continuous-time Finance and Martingale Methodology

Harrison and Kreps (1979) and Pliska (1986) build the famous Harrison-Kreps-Pliska martingale methodology that relies on the duality principle due to Cox and Huang (1989) and Karatzas *et al.* (1987). Advantages of this approach as it is applied to the continuous-time consumption-investment problem could be summarized as follows:

(i) It is easy to establish the existence of a solution to an intertemporal optimization problem.

(ii) It is capable of handling optimization problems with constraints on state and choice variables such as consumption portfolio constraints and non-negativity constraints on consumption.

(iii) The constructive idea behind the duality approach is to linearize the optimization problem using the Lagrangian method.

The main innovation of the martingale technique is to transform the dynamic budget constraint into set of equivalent static ones. In other words, it is possible to decompose the nonlinear Hamilton-Jacobi-Bellman equation into a linear partial differential equation.

Cox and Huang (1989) deals with the classical optimal portfolio-consumption problem in continuous time under uncertainty. The problem dates back to Merton (1971), who used a stochastic dynamic approach and found an analytical solution in particular cases by establishing a control and using the verification theorem of dynamic programming to verify its optimality. Although the problem is not new, the focus of Cox and Huang is on the explicit construction of optimal controls using martingale techniques, with the non-negativity constraints on consumption explicitly taken into account. A dynamic consumption-portfolio problem in continuous time with general diffusion price processes and a non-negativity constraint on consumption and final wealth is formulated. Some characterizations of the optimal policies are given, and closed-form solutions are found for some particular cases. In particular, a solution is presented for the hyperbolic absolute risk aversion (HARA) class of utility functions when prices follow a geometric Brownian motion. One advantage of this approach is that in the verification theorem linear partial differential equations arise.

Merton approached the problem from a stochastic dynamic programming perspective that results in a nonlinear Hamilton-Jacob-Bellman (HJB) parabolic partial differential equation. Despite the strength of the HJB technique in general, the Merton's solution has certain limitations: (i) dynamic programming can only be used to find an optimal solution if the derived utility function is continuously differentiable, and (ii) the nonlinear nature of the HJB equation renders it difficult to solve. Recent research by Kim and Omberg (1996) as well as by Brennan, Schwartz and Lagnado (1997) shows that the optimal portfolio weights do indeed depend on the investment horizon when the stock returns are predictable. Using a log-linear approximation, Campbell and Viceira are able to characterize the portfolio demand under a stochastic opportunity set. One of their results is that the ratio of the proportion of bonds to stocks in the optimal portfolio increases with risk aversion. Sundaresan and Zapatero (1997) show that the asset allocation policies in which indexed (stochastic) liability is funded will exhibit systematic time variation depending on how close the market value of the assets are relative to the indexed liability. Liu (2002) derives a closed-form solution for the optimal portfolio weights in a stochastic opportunity setting when the default-free short rate follows the square root diffusion process. Wachter (2002) uses martingale methods to characterize the consumption and portfolio strategies in complete markets when stock returns are predictable. Chacko and Viceira develop portfolio and consumption rules under an incomplete market setting with stochastic volatility. They rely on an approximation scheme to solve the Bellman equation in their general applications. In only one special case are they able to find the exact solution. Kogan and Uppal (1999) provide approximation methods for solving consumption and portfolio problems in a continuous-time setting. They show applications drawn from both partial equilibrium and general equilibrium formulations. More recently, Cvitanic and Wang (2001) show that the modern approach to the problem of maximizing expected utility from terminal wealth in financial markets, namely the martingale/duality methodology, works also in the presence of proportional transactions costs. They demonstrate that the optimal terminal wealth is given as the inverse of marginal utility evaluated at the random variable which is optimal for an appropriately defined dual problem.

In the context of a representative 'robust decision maker,' where robustness implies a focus on worst-case scenarios over a restricted set of appropriately defined relevant model misspecifications, Trojani and Vanini (2002) present a version of a simple two-assets Merton (1969) model, and yield explicit and interpretable expressions for the relevant variables. The reference model is the standard geometric Brownian motion process while the maximal admissible distance there from is measured with a continuous time version of relative entropy. They find that the decompositions of the market price of risk in a standard consumption-based component and a further price for model uncertainty risk is independent of the underlying risk aversion parameter.

The time-variation in the price of risk could be considered as a more promising possibility to explain the variation in stock market volatility. This leads to a model with a representative agent whose utility displays habit-formation. Campbell and Cochrane (1999), building on the work of Abel (1990) and Constantinides (1990), have proposed a simple asset pricing model of this kind. They suggest that assets are priced as if there were a representative agent whose utility is a power function of the difference between consumption and habit, where habit is a slow moving nonlinear average of past aggregate consumption. This function makes the agent more risk-averse in bad times, when consumption is low relative to its past history, than in good times, when consumption is high.

In standard 'internal habit' models such as those in Constantinides (1990) and Sundaresan (1989), habit depends on an agent's own consumption and the agent takes account of this when choosing how much to consume. On the other hand, in ‘external habit’ models such as those in Abel (1990) and Campbell and Cochrane (1999), habit depends on aggregate consumption that is unaffected by any one agent’s decisions. Despite the limitation of Abel’s framework due to the constant risk aversion, the model embeds three formations given certain values for parameters; namely, time-separable, ‘catching up with the Joneses’, and habit formation. It would be interesting to model the endowment fund’s utility function in the footsteps of Abel with a restriction on the parameter value, whereby the preference of the fund contains elements of ‘catching up with the Joneses’ and habit formation. University endowments, in the pursuit of making the institution more competitive in the region as well as nation, need to observe other endowments’ spending policies and habits in addition to the internal pressures to keep the spending streams as smooth as possible in the future. In addition, Abel’s postulate of the habit depending on one lag of consumption fits very well with the functioning of an endowment fund.

Time-variation in the price of risk can also arise from the interaction of heterogeneous agents. Constantinides and Duffie (1996) develop a simple framework with many agents who have identical utility functions but heterogeneous streams of labor income. They show how changes in the cross-sectional distribution of income can generate any desired behavior of the market price of risk. Dumas (1989), Grossman and Zhou (1996), Wang(1996), Sandroni (1997), and Chan and Kogan (2001) move in somewhat different direction by exploring the interactions of agents who have different levels of risk aversion. In particular, Chan and Kogan (2001) discuss the countercyclical nature of expected stock returns due to investor heterogeneity in a setting where individuals have catching-up-with-the-Joneses preferences. While the authors rely on numerical analysis and focus on the dynamics of the conditional moments of stock returns, Kogan and Uppal (2001) derives an explicit asymptotic relation between the level of return volatility and the degree of cross-sectional heterogeneity. Kogan and Uppal develop a method to analyze analytically the equilibrium prices and policies in an economy with a stochastic investment opportunity set and incomplete financial markets, when agents have power utility over both intermediate consumption and terminal wealth. These methods could be used to study general equilibrium economies with portfolio constraints when there are multiple investors who differ in their risk aversion and hence the investment opportunity set evolves endogenously. Their model is flexible enough to be applied to an economy where agents exhibit habit persistence. They further assume that agents have time-additive power utility rather than the more general recursive preferences described in Kreps and Porteus (1978) and Duffie and Epstein (1992). Given that log-utility is a special case of the Kreps-Porteus specification of recursive utility, it is possible to extend the asymptotic method to the case of recursive preferences.

2. 3. Behavioral Finance

Similar ideas have been introduced in the field of behavioral finance, which takes its roots from the experimental work of Kahneman and Tversky (1979)[2]. They argued that agents behave as if their utility function is kinked at a

[2] Prospect theory is a descriptive model of decision making under risk, developed to help explain the numerous violations of the expected utility paradigm.

reference point which is close to the current level of wealth. Benartzi and Thaler (1995) postulated that Kahneman and Tversky's prospect theory could explain the equity premium puzzle if agents frequently evaluate their utility and reset their reference points, so that the kink in utility increases their effective risk aversion. Barberis, Huang and Santos (2001), building on behavioral evidence of Thaler and Johnson (1990), argue that prospect theory should be extended to make agents effectively less risk averse if their wealth has recently risen, very much in the spirit of a habit-formation model. Even though Barberis *et al.* base the model on changing risk aversion just like Campbell and Cochrane, they introduce loss aversion over financial wealth fluctuations and allowing the degree of loss aversion to be affected by prior investment performance. The article by Barberis *et al.* is also related to the literature on first-order risk aversion, as introduced using recursive utility by Epstein and Zin (1990) among others. It is, however, different in that most implementations of first-order risk aversion affectively make the investor loss averse over total wealth fluctuations as opposed to financial wealth fluctuations. They also differ from another related work by Hong and Stein (1999), who suppose that investors are only able to process subsets of available information, by assuming that the investor remains rational and dynamically consistent.

A number of empirical anomalies or puzzles have been discovered in financial markets that are apparently incompatible with what is regarded as rational behavior. Behavioral finance is viewed to be an emerging discipline that represents a collection of different approaches which seek to explain these findings and perhaps refine our notions of rationality. In particular, it draws on the psychology and cognitive science literatures to consider how individual decision-making often deviates from rational choice in systematic ways. This may simply reflect limited intelligence or what economists refer to as bounded rationality, being a human inability to numerically evaluate decisions under uncertainty accurately. More importantly, the observed anomalies give rise to alternative behavioral explanations that are difficult to rationalize in expected utility terms but are systematic and potentially derived from the recognition that the way in which a decision is framed is critical or that social interaction is an important element in any market process. Herd behavior is a classical example whereby a social phenomenon is closely linked to the growth of speculative bubbles as witnessed during the last couple of years particularly in the technology sector. The notion of regret aversion may encourage investor herding behavior to invest in respected corporations as these investments carry implicit insurance against regret. Regret aversion may also impact on the behavior of professional fund managers, who may sell loss-making stocks before the end of a quarter to avoid having to explain to investors why they are holding funds in poorly performing shares of stocks.

As stressed by Conlisk (1996), the failure to incorporate bounded rationality into economic models is just bad economics - the equivalent to presuming the existence of a free lunch. Since we have only so much brainpower, and only so much time, we cannot be expected to solve difficult problems optimally. It is eminently rational for people to adopt rules of thumb as a way to economize on cognitive faculties. Departures from rationality emerge both in judgments (beliefs) and in choice. Some examples of the way in which judgment diverges from rationality include overconfidence, optimism, anchoring, and extrapolation. Many of the departures from rational choice are captured by prospect theory and its key theoretical components that incorporate important features of psychology. There are three features of the prospect theory value function:

(i) It is defined over changes to wealth rather than levels of wealth, as in the expected utility framework, to incorporate the concept of adaptation.

(ii) The loss function is steeper than the gain function to incorporate the notion of loss aversion, the notion that people are more sensitive to decreases in their well being than to increases.

(iii) Both the gain and loss function display diminishing sensitivity to reflect experimental findings, the gain function is concave and the loss function is convex.

Regarding the rationality of investors, Schleifer (2000) describes a wealth of behavioral finance research showing that investors are not Bayesian and that their judgments and decisions are systematically influenced by how a problem is framed. Further, he provides evidence that these individual investors are systematic, thereby laying the groundwork for his development of a theory of investor sentiment, which he argues is one of the two essential inputs to behavioral finance theory. Schleifer also reviews the evidence suggesting that under- and overreaction to information are due to the behavioral heuristics of representativeness and conservatism. To fully describe choices, however, prospect theory often needs to be combined with an understanding of 'mental accounting.' One needs to understand when individuals faced with separate gambles treat them as separate gains and losses; and when they treat them as one, pooling them to produce one gain or loss. Mental accounting could be viewed as a heuristics for reducing complexity. Tversky and Kahneman (1981) introduced the concept of 'mental account', according to which people keep not only a mental tab on the totality of all projects and their consequences but also a separate mental account in respect of each of their plans. They postulate that people focus on one account in particular when weighing things up; relationships with other commitments or accounts are usually ignored.

Interestingly, the notion of different 'mental accounts' could help explain the difference between the regret aversion and loss aversion that is considered to be a fuzzy difference, to say the least. One could pose the following question; if regret aversion signifies the fear of regretting decisions after the event[3], then what is the difference between compared with loss aversion? The difference becomes clear when one remembers that regret may also occur when a particular decision has not been made. Had we, for instance, not bought a particular share, against the advice of a friend, and the share then turns out to be a winner, then regret kicks in, even though there was no actual loss. Not acting is a decision – you choose to not act. On the one hand, there are 'payment affect' mental accounts to which the

[3] See Loomes and Sugden (1982 and 1987) for details on regret aversion and related issues.

actual money is booked. On the other hand, people also keep 'non-payment effect' mental accounts, which record those sums that might have been received if a particular decision had not been made, which means that these payments do not affect the actual state of the capital. Gains foregone take the place of a loss in the case of evaluation of a non-payment effect project; the relative gain, on the other hand, will be replaced by the loss not incurred. Dissonance occurs when the decision against the project is regretted because it would have produced a profit. The degree of regret aversion, as with loss aversion, can be seen from the increase in the non-payment effect 'regret function' in the vicinity of the reference point. The dissonance is great and the regret aversion pronounced if the decision against the project was accompanied by considerable commitment. As with loss aversion, a decision maker anticipates the dissonance when considering the realization of a particular project. If there is agreement that a commitment a priori leads to distortions and therefore that all decisions must be made free from any commitment as far as possible, then any problems arising from regret aversion will be resolved of their own accord. I would argue that people should try to keep their own commitments to a minimum to be able to make decisions free from regret or loss aversion. Moreover, the perception of a possible deficit results in risk aversion from a 'need for control' perspective as regret or loss aversion is from a 'need to be free from dissonance' perspective.

Shefrin and Statman (2000) developed 'behavioral portfolio theory (BPT)[4]' as an alternative to the descriptive version of the Markowitz mean-

[4] BPT is a positive portfolio theory built on the foundations of Security Potential/Aspiration (SP/A) theory due to Lopes (1987) and prospect theory. Formally,

variance portfolio. Mean-variance investors evaluate portfolios as a whole; they consider covariance between assets as they construct their portfolios. Mean-variance investors also have consistent attitudes toward risk; they are always averse to risk. Behavioral investors build portfolios as pyramids of assets, layer by layer. The layers are associated with particular goals and particular attitudes toward risk. Some money is in the downside-protection layer, designed to avoid poverty; other money is in the upside-potential layer, designed for a shot at being rich. Alternatively, some money could be in the regret-minimization layer invested in index funds; and others in the diversification-maximization layer invested in absolute return strategies. Specifically BPT investors choose portfolios by considering expected wealth, desire for security and potential, aspiration levels, and probabilities of achieving aspiration levels.

2. 4. Compensation of Investment Managers

There has been a growing body of literature that studies mutual fund tournaments both theoretically and empirically. On the empirical side, Brown, Harlow, and Starks (1996), Chevalier and Ellison (1997) and Kosky and Pontiff (1999) provide evidence that the mutual fund tournament generates incentives for managers not to act in the best interest of investors. Assuming that managers are evaluated on a calendar year basis, Brown *et al.* demonstrate that mid-year losers increase fund volatility in the latter part of the year relative to the mid-year winners. Chevalier and Ellison study how relative performance after three quarters of the year influence investment strategy in the last quarter. Considering

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the mechanics underlying the optimization in SP/A theory can be viewed as an adaptation of the Arzac-Bawa (1977) characterization of the safety-first theory, where the major two-year-old funds, they show that funds that are somewhat behind the market increase risk to a greater extent than funds that are ahead of the market. Kosky and Pontiff (1999) show that changes in risk are less severe for funds that use derivatives.

These studies assume that changes in risk in the last part of the year depend on the difference between the realized return and a benchmark return over the first part of the year. Therefore, these studies differentiate the behavior of funds ahead of the benchmark from those behind the benchmark after the first part of the year. Closely related theoretical papers studying relative performance evaluation in financial markets are those of Huddart (1999), Hvide (1999), and Palomino (1999) who consider a game played by several fund managers. Hvide and Palomino study the consequences of relative performance objective in the context of a single investment decision. Hvide shows that in a situation with moral hazard on both effort and risk, standard tournament rewards induce excessive risk and lack of effort. Palomino assumes that managers with different levels of information compete in oligopolistic markets and aim at maximizing their relative performance against the average performance in their category. He shows that despite the objective function being linear in performances, managers have incentives to choose overly-risky strategies. Huddart considers a two-period model in which interim performances are observable. He shows that asset-based compensation schemes generate incentives for managers to invest in overly-risky portfolio in the first period, and that performance fees align managers' incentives with those of investors.

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difference is in the interpretation of the variables.

Das and Sundaram (1998) consider a model in which fund managers use fee structures to signal their higher ability. They provide conditions under which investors are better off under an incentive fee regime than a ‘fulcrum’ fee regime, which refers to the fund manager’s fee being symmetric around a chosen index. Contrary to the belief that option-like incentive fee structures hurt investors by inducing managers to take ‘excessive’ amounts of risk, they find that, in many circumstances, investors can be made better off in welfare terms by requiring that only asymmetric fees be used. Their model is akin to a principal-agent game in which the agent (the manager) sets the compensation contract, and the principal (the investor) responds by deciding on the amount of resources (funds) to be invested with the agent.

Grinblatt and Titman (1989) assume that managers can risklessly capture the value of any option implicit in their payoff structure by hedging in their personal portfolios. This enables the use of results from option pricing theory in characterizing the fee maximizing level of risk for any given contract structure. Among other things, they demonstrate that for certain classes of portfolio strategies, adverse risk-sharing incentives are avoided when the penalties for poor performance outweigh the rewards for good performance. Goetzmann, Ingersoll, and Ross (2001) are concerned with ‘high watermark’ contracts of the sort frequently used by hedge funds in which the manager receives a proportion of the fund return each year in excess of the portfolio’s previous high water mark, which is defined as the maximum share value since the inception of the fund. The authors provide a closed-form solution for the value of such contracts and show that such contracts are valuable to hedge fund managers and represent a claim on a significant portion of investor wealth.

Heinkel and Stoughton (1994) employ a two-period model with moral hazard and adverse selection. They show that the equilibrium set of contracts in their model features a smaller performance-based fee in the first period than in a first-best contract. They furthermore suggest that this reduced emphasis on the performance component in the first period is analogous to the lack of a performance-based fee in many parts of the asset management industry. Admati and Pfleiderer (1997) consider a scenario where the fund manager has superior information to the investor and faces a fulcrum fee structure. They examine if there are any conditions under which the manager would pick the investor's most desired portfolio. Nevertheless, they are not concerned with determining equilibrium fee structures and portfolios.

Contrary to the tournament hypothesis, Chen and Pennacchi (1999) provide a theoretical model for risk-taking assuming that fund managers are evaluated with respect to an exogenous benchmark index and show that poor performing funds do not necessarily increase the volatility of their fund's returns.

2. 5. Alternative Forms of Utility Functions

Despite the widespread popularity and analytical tractability of the power utility function, the fact that the elasticity of intertemporal substitution is the reciprocal of the coefficient of relative risk aversion; i.e., there exists a one-to-one mapping between the parameters, inspired many to think of alternative functional forms. Epstein and Zin (1989, 1991) and Weil (1989), using the theoretical framework of Kreps and Porteus (1978), have proposed a more general utility specification, called recursive utility, that preserves the scale-invariance of power utility but breaks the tight link between the coefficient of relative risk aversion and the elasticity of intertemporal substitution. Risk aversion in our setting describes the university endowment's reluctance to substitute spending across states of the world, whereas the elasticity of intertemporal substitution describes the institution's willingness to substitute consumption over time (low values imply dislike for consumption growth).

Epstein and Zin investigate to what extent first order risk aversion, based on Kreps-Porteus preferences, can explain the size and predictability of risk premia typically observed in securities markets. In the case of second order risk aversion, approximating von Neumann-Morgenstern utility by a Taylor series, the risk premium is approximately linear in the variance of small risks. The utility functional is said to exhibit first order risk aversion if the utility functional is approximately linear in the standard deviation of small risks. Before Epstein-Zin's introduction of recursive utility into the literature the benchmark was in the form of $U_t = E_t\left[\sum_{t=1}^{\infty} b^t u(C_t)\right]$, which satisfies $U_t = u(C_t) + b\,E_t\left[U_{t+1}\right]$.

Epstein and Zin propose the following:

$$U_t = u(C_t) + b\, f\left(E_t\left[\, f^{-1}\left(U_{t+1}\right)\right]\right).$$

If f is linear, the Epstein-Zin framework collapses to the original benchmark equation. The habit formation could be reconciled within the framework of Epstein-Zin by replacing $u(C_t)$ with $u(C_t - X_t)$,where the habit level is given by

$X_t = a\sum_{j=0}^{\infty}\Phi^j C_{t-1-j}$, where $0 \le \Phi < 1$ and $0 \le a$ (with a further constraint that $C_t - X_t > 0$). If $\Phi = 0$, $u(C_t)$ becomes $u(C_t - a\,C_{t-1})$.

A variation of the habit formation utility function is the idea, that people care mostly about how rich they are relative to their neighbors rather than their own past. Abel (1990) introduced the so-called Catching-up-with-the-Joneses framework to capture this belief. He basically assumes external habit formation, whereby agents care about the ratio of consumption to habit, rather than the difference. As a result, risk aversion is constant and risk premia do not vary through time. Formally, this is captured by the same utility function as the one used for defining habit formation, except that X_t is now a function of (present and) past aggregate consumption rather than past own consumption.

Expected utility theory assumes that the investor can compute expectations with respect to the return distribution, which requires that the agent knows the parametric structure of the return distribution and either knows its parameters or can form Bayesian beliefs about them. The investor is only exposed to the risk inherent in the returns and trades off this risk against expected rewards through the expected utility maximization. Knight (1921) and Ellsberg (1961) argue, however, that the investor may not have all of the information required to form such expectations. An agent may not be able or willing to assign probabilities to a set of alternative parameterizations of the return distribution. Thus, the investor faces additional ambiguity that is not captured in the expected utility framework. Ambiguity aversion preferences formalize the idea that investor dislikes not only risk but also this more vague uncertainty about the world, called Knightian uncertainty. Nonadditive probabilities or Choquet capacities are alternative formalizations of Knightian uncertainty or ambiguity that are closely related to maxmin expected utility. In this case, following Dow and Werlang (1992), the investor's portfolio choice problem is given by:

$\max_{a_t} \min_{p \in \wp} E_p \left[v\left(W_t \left(a_t' R_{t+1}\right)\right) \mid Z_t \right]$, where $v(.)$ is the typical CRRA utility function. The interpretation of this maxmin criterion is as follows: Given the complete ambiguity about the return distribution, the investor considers the worst case outcome through the interior minimization. The exterior maximization achieves the usual risk versus expected reward trade-off. The implementation of the ambiguity aversion requires the need for the characterization of a set of possible return distributions. Ellsberg (1961) introduced the so-called ε-contamination parametrization:

$$P = \left\{ (1-e)\,\overline{p} + e\, p : p \in \wp \right\}, \text{ where}$$

$\wp$ denotes the σ-algebra generated by the support of the return distribution, and $\overline{p}$ could be the empirical return distribution. The parameter e reflects the investor's degree of ambiguity. If $e = 0$, the investor's objective function reduces to that with standard CRRA preferences and the empirical return distribution.

There is an extensive experimental literature confirming that individuals dislike ambiguity in financial markets; e.g., Camerer and Kunreuther (1989) or Sarin and Weber (1993). The crucial difference between ambiguity aversion and expected utility theory with model uncertainty is that with ambiguity aversion the investor cannot or does not want to assign probabilities to the set of alternative return distributions. Agents are uncertain about the true model and are unable or unwilling to assign probabilities to the set of alternative models. Ambiguity aversion is said to be related to the recent literature on robustness[5] such as Anderson, Hansen and Sargent (2000) and Maenhout (2001).

Kahneman and Tversky (1979) argue that humans systematically violate the axioms of expected utility theory in two important ways. First, experimental subjects tend to overweight outcomes that are considered certain, relative to outcomes that are merely probable, which is referred to as the certainty effect. In financial markets, this certainty effect make an investor risk averse in the case of gains, as a small certain gain is preferred to a probable risky gain, but risk seeking in the case of losses, as a probable risky loss is preferred to a small certain loss. In addition, subjects tend to simplify decisions by disregarding components common to the alternative choices and focusing on components that differentiate the choices, which is called the cancellation effect. They formulate prospect theory based on this experimental evidence. This theory consists of an editing stage, where alternatives are put into perspective, and a choice stage. Utility is defined over gains and losses relative to a reference point rather than over the level of wealth as in expected utility theory. Tversky and Kahneman (1992) propose the following objective function for the choice stage in order to capture the differential risk preferences over gains and losses generated by the certainty effect:

[5] Robustness was pioneered in economics by Hansen and Sargent (1995).

$$v\left(W_{t+1}\right) = -l\left(\overline{W} - W_{t+1}\right)^{b} \text{ if } \overline{W} > W_{t+1}$$

$$v\left(W_{t+1}\right) = \left(W_{t+1} - \overline{W}\right)^{b} \text{, otherwise.}$$

$\overline{W}$ is a reference wealth level determined in the editing stage. It could be the initial wealth or its future value, depending on the investor's perspective. The parameter *l* measures the investor's loss aversion and the parameter b captures the degree of risk seeking over losses and risk aversion over gains. The kink at the origin introduced by $l > 1$ makes losses (relatively) more painful than gains are pleasurable.

In addition, the investor in the Tversky and Kahneman framework does not evaluate outcomes on the basis of true probabilities, but on the basis of distorted probabilities. Instead of maximizing the true expectation of the objective function, the investor maximizes:

$$E\left[v\left(W_{t+1}\right)\frac{p\left(p\left(W_{t+1} \mid Z_t\right)\right)}{p\left(W_{t+1} \mid Z_t\right)} \mid Z_t\right] = \int_{-\infty}^{+\infty} v\left(W_{t+1}\right) p\left(p\left(W_{t+1} \mid Z_t\right)\right) dW_{t+1},$$

where $p\left(.\right)$ represents a subjective distortion of the objective probabilities $p\left(.\right)$. The authors suggest parameterizing this probability distortion as:

$$p\left(p\left(W_{t+1} \mid Z_t\right)\right) = \frac{p\left(W_{t+1} \mid Z_t\right)^{c}}{\left\{p\left(W_{t+1} \mid Z_t\right)^{c} + \left(1 - p\left(W_{t+1} \mid Z_t\right)\right)^{c}\right\}^{\frac{1}{c}}}$$

c determines the degree of irrationality. When c=1, the decision weights $p\left(.\right)$ reduce to the objective probabilities *p*(.).

A special case of prospect theory is loss aversion, when b=1, c=1 and l>1. In this case, the investor is risk neutral over gains and is risk neutral over losses, but realizes a greater incremental utility penalty for a loss than for an equally large gain. This results in unconditional risk aversion. Furthermore, since with c=1 the decision weights reduce to the objective probabilities, this investor simply maximizes expected utility. Benartzi and Thaler (1995) find that the main aspect of prospect theory relevant for portfolio choice is loss aversion and that the concavity (convexity) of the value function on the upside (downside), as well as the subjectivity of the probability distortions, are only of second-order importance. Sharpe (1998) argues that the local risk-neutrality property of loss aversion results in portfolio choices that are too extreme due to the fact that the iso-expected utility curves are straight lines in mean versus standard deviation of returns space. Furthermore, Benartzi and Thaler explain that the more often a loss-averse investor evaluates his or her portfolio, the less attractive are high expected return but high variance investments because losses of these investments are realized more often at short horizons than at long horizons. Thus, loss aversion causes short-term investors to be extremely risk averse, since the return distribution straddles the kink of the utility function, but long-term investors to be almost risk neutral, as the mass of the return distribution moves away from the kink.

2. 6. International Diversification

Reflecting the trend toward a greater integration of world capital markets, international diversification of investment portfolios has, for the last several years, received widespread attention at both the academic and practitioner levels. Recent ex ante international portfolio selection studies, including Jorion (1985) and Eun and Resnick (1988) have shown: (i) that it is important to control parameter uncertainty in order to capture the potential gains from international diversification; and (ii) that hedging foreign exchange risk can increase the gains from international stock portfolio diversification. In other words, investors can substantially benefit from international equity diversification when they properly control foreign exchange and parameter uncertainties. When neither of these uncertainties are controlled, however, investors may not be able to realize enough of the potential benefits to justify international investment. Recently, Eun and Resnick (1994) extended the work on international portfolio selection to portfolios of stocks, bonds, and stocks and bonds. They show that when exchange rate risk is hedged with forward contracts, the risk-return relationship is very much improved over unhedged international portfolio investment for bond portfolios and stock and bond portfolios, but only minimal improvement is obtained for stock portfolios. Glen and Jorion (1993) compare portfolios with an optimal combination of forward contracts versus a fully hedged strategy and portfolios hedged using the ‘universal’ hedge ratio of Black (1990). Using a world bond index and a world stock index, currency exposure is conditionally managed using a strategy for taking a long or short position in forward contracts based only on prior information. They find statistically significant improvement over unhedged investment in the world bond index, but not for the world stock index. For neither the world bond index nor stock index do they obtain significant improvement over a simple fully hedged strategy. The hedging strategies employed in most previous studies can be viewed as ‘passive’ because exchange rate uncertainty was unconditionally hedged with forward contracts, without regard to a forecast of the home currency depreciation or appreciating versus the foreign currencies. Early empirical studies (Frenkel, 1981) concluded the forward premium was an unbiased predictor of the future change of the spot exchange rate. Indeed, Levich (1982) found the forward rate to be a more accurate predictor of the direction of spot rate changes than the forecast provided by most forecasting services. Fama (1984) discovered that the forward rate to contain a risk premium that is time varying as well as the expectations component. Since his work, it has been generally agreed that forward prices are not unbiased predictors. Nevertheless, Shapiro (1992) states that the risk premium appears to change sings, being positive at some times and negative at other times, and averages near zero. Therefore, he thinks that it would not be stretching things to treat the forward rate as an unbiased forecast of the future spot rate.

An alternative model is that spot exchange rates follow a pure random walk. According to this model, the best estimate of next period’s rate is the current spot exchange rate. In empirical tests, Meese and Rogoff (1983) find that the random walk model performs as well as any forecasting model for 1-2 month horizons for certain major currencies, which suggests a possible conditional trading strategy. That is, sell the foreign currency proceeds forward only when the forward rate is at a premium to the spot rate. The intuition is that the investor will be ‘locking-in’ a higher sales price at the forward rate than the expected spot rate. Eaker and Grant (1990) provide some preliminary evidence suggesting that this selective hedging strategy, coupled with international equity investment made according to fixed allocation rules, is superior to always hedging the currency risk in international equity portfolios using forward contracts.

This classical hedging approach used an OLS procedure which assumes that the joint distribution of cash and futures price changes remains constant over time. However, a substantial body of evidence indicates that the covariance matrix of cash and futures is time-varying. Recent empirical work has focused on utilizing that dynamic covariance structure to derive time-varying hedge ratios. For example, Cecchetti *et al.* (1988) proposed a hedging model for financial futures based on the univariate autoregressive conditional heteroscedastic (ARCH) model introduced by Engle (1982). They assumed that the conditional correlation between cash and futures prices is constant, and report significant evidence of time-variation in the optimal hedge ratios. Myers (1991), Baillie and Myers (1991), and Sephton (1993) used the bivariate generalized ARCH (GARCH) model of Bollerslev (1986) to capture the time-variation of hedge ratios for agricultural commodities and report improved hedging performance using this approach. Kroner and Claessens (1991) illustrate the usefulness of the GARCH technique for hedging exchange-rate risk associated with external debt. Kroner and Sultan (1993) apply the bivariate GARCH framework to minimize exchange-rate risk using currency futures. These studies offer strong evidence of time variation in hedge ratios and show that, in at least some cases, substantial improvements in hedging performance can be realized by following a dynamic hedging strategy.

I postulate that optimal hedge ratio analysis needs to be updated whenever the assumptions underlying its construction change. Thus, intertemporal management of the overall portfolio construction process seems to be the reasonable approach. On the other hand, it is the degree of certainty in the optimal hedge ratio that increases with a higher allocation into international assets not the hedge ratio itself.

2. 7. Currency Hedging

Perold and Schulman (1988) advocated the view that currency hedging is a free lunch implying that 100% of foreign currency exposures should be optimally hedged. The argument is that hedging results in expected returns of zero with a reduction in risk of a position. The analysis is based on quarterly real (CPI inflation adjusted) returns over 1978-87, and shows that the risk reduction is still large when domestic purchasing power is taken into account. Their recommendation does not necessarily provide maximal risk reduction, as there is no consideration of cross-hedging or optimal hedge ratio analysis. Finally, the authors also note the lack of precision of hedge ratio estimates, which are essentially regression coefficients, a theme we return to below. Eun and Resnick (1988) find empirical evidence that the performance of international stock portfolios is increased if 100% of the exposure is hedged against currency risk. These two studies are instrumental for the first currency overlay mandate to have a 100% hedged benchmark for the performance measurement. An opposite view is proposed by Black (1990) who demonstrated by assuming the validity of Siegel's paradox and homogenous preferences of investors that investors worldwide share 'universal' currency hedge ratios, which he demonstrated in a general equilibrium framework to be strictly below 100%. Black postulates that currencies need to be considered as asset classes in international portfolios that provide correlation structures with impact on diversification. The theoretical framework that underpins Black's analysis is a highly stylised real international capital asset pricing model. In the Black framework, all investors (regardless of domicile) have the same hedge ratio, and every investor holds the same diversified portfolio of world equities. Since there must be a borrower for every lender and currency longs for every short, in equilibrium, asset returns (and their volatilities and correlations) will adjust until all available equities are willingly held and some investor is willing to take each side of all currency contracts. Adler and Prasad (1992) weaken some of the underlying assumptions and generalize Black's result by substituting universal hedges with regression hedges. Glen and Jorion (1993), and Kritzman (1993a) are examples of the use of mean-variance optimization as a cost-benefit analysis for currency hedging. The risk-minimizing hedge ratio becomes 100% plus or minus a component that is determined by the covariances of the forward currency returns with excess returns from the domestic and international parts of the portfolio. The cost of hedging includes the cost of forward contracts and the costs associated with the periodic portfolio rebalancing that is necessary to maintain the hedge. Indirect costs include the small return that one can expect from bearing currency risk (Black, 1990), which is lost when all currency exposure is hedged. Finally, the clear benefit of hedging is risk reduction. Furthermore, increasing the cost of hedging or decreasing the risk aversion will always make the optimal hedge ratio lower. If a portfolio holds high proportion of domestic equity, and these assets perform poorly in times when the domestic currency is strong, then it is particularly dangerous to be exposed to risk from currency exposure for this will further the negative returns when the domestic currency is strong. Since the forward returns are for a long international currency and short domestic currency exposure, however, the correlation between currency and assets have the opposite effect on the optimal hedge ratio for domestic and international asset exposure. A common argument for using a low or zero hedge ratio for an equity portfolio has been that the strength of an equity market is likely to be negatively correlated with the strength of the currency. If the fund is predominately invested in domestic equity assets, then this correlation argument actually leads to a higher hedge ratio. That is, if international currency strength is positively correlated with international asset strength, then the optimal hedge ratio will increase, while a positive correlation between domestic currency strength and domestic asset strength will decrease the optimal hedge ratio.

Froot (1993) argues for a zero hedge ratio over long investment horizons. His analysis is based on estimates of asset return second moments over a 200 year sample. For international equity and exchange rate data that span the modern floating exchange rate period (1973-present), there are only five to six non-overlapping periods of 5-year returns. This makes any analysis based on the estimation of means and variances of long-dated asset returns over the last thirty years of post Bretton Woods data suspect.

Gardner and Stone (1995) argue, on the basis of stochastic simulations, that the estimation of portfolio shares from mean-variance optimization problems, whereby they have such wide confidence intervals that they are suspect for policy analysis (i.e. confidence bands for the hedge ratio for each currency can often be above 100 per cent or below 0 per cent). They find that the sampling variability issues are most acute for estimates of expected asset returns. In a specialized example, where only minimum variance hedging is considered, the authors' estimated hedge ratios are precise enough for hedging policy purposes. In a related paper, Gardner and Wuilloud (1995) argue that, if the investment horizon is short, say one to two years, and the investor has moderate to low risk aversion, there is a substantial probability that the optimal hedge ratio will underperform another portfolio that makes use of a simple alternative hedging strategy. In other words, the international investor will frequently experience regret, which could be large in magnitude. Since the authors set up the optimal hedge ratio to involve expected currency return, it is not surprising that regret can be quite large. The authors then recommend a 50 per cent hedge ratio, as it minimizes the maximal expected regret relative to both a completely hedged portfolio and a completely unhedged portfolio.

Chapter 3

Optimal Asset Allocation and Spending Rules

3. 1. Introduction

Managers of endowment funds are particularly concerned about the downside risks of their investments because of their fiduciary responsibilities to balance today's spending and future growth of the portfolio. They have to make sure they generate enough return to cover both the inflation and spending rate. The main objective of university endowments could be stated as providing adequate spending for current and future beneficiaries while not eroding the principal base (corpus) of the endowment. Litvack, Malkiel, and Quandt (1974) suggest that endowment funds should separate the investment decisions from the spending decisions of the university, protect the real value of the endowment fund and stabilize spendable income.

The perceptions of myopic and long-term investors toward asset allocation could have striking differences that outline the true meaning and purpose of investments. An investment entity with a long-term investment horizon (infinite in theory) and a goal based on spending stream, such as a university endowment, should recognize the ability of the fixed-income investments to hedge against future changes in the investment opportunity set or to provide consistent income stream for spending requirements particularly during bear markets, whereas a myopic investor treats a long-term fixed-income investment as simply another risky asset[6].

This chapter tackles the following question: "What is the optimal asset allocation strategy, which will maximize the expected utility of beneficiaries?" By the principles of prudence and fairness, fiduciaries of endowment funds have the obligation not to discriminate between generations. This dual goal of treating current and future beneficiaries of the endowment fairly and equally entails the endowment manager to take portfolio positions that would provide adequate spending today without jeopardizing the growth of the portfolio going forward. The main issue is to determine what the appropriate benchmark is that incorporates the trade-off between current spending and the prudent growth of the corpus (spending later for the benefit of future generations).

Major and minor needs of 'current' and 'future' beneficiaries of the endowment are to get a smooth spending stream (major for both), to grow the spending stream at least by inflation (major for both) and to outperform a given benchmark (minor for current, but major for future). The third point illustrates a significant conflict of interest, where endowments end up maximizing suboptimal objectives. We have to remember the fact that being impatient for the current spending would be detrimental to the chances of the future generations receiving the same 'real' income. On the other hand, it might be necessary to treat the donation streams as a stochastic process for they might justify higher spending today given a reasonable approximation. Altshuler (2000) argues for increased spending on academic programs and faculty today to turn the accumulated

[6] Refer to Brennan *et al.* (1997) for further details.

endowment wealth into intellectual capital for the university. Moreover, Hansmann (1998)[7], citing the evidence of significant stock price appreciation during the 1990's, questions the low spending rates offered to academic programs and postulates that endowment managers are giving more attention to portfolio management and growth of the endowment fund at the expense of prudent spending rate policies. He, further, argues that financial gift flows are much smoother eliminating the need for a big endowment to smooth out the bulkiness of financial gifts. The distribution of spending equitably across generations[8] involves an implicit assumption of the productivity of academic creativity and research growing linearly at the minimum. Large buffer levels in endowment funds despite the mean reversion witnessed in the stock market during the last year makes a case for higher spending rates at least in the next couple of years. Nevertheless, I think the simultaneous dynamic management of spending rates and portfolio allocation rules should be the preferred approach in conjunction with the flow of donations[9].

This chapter deals with the construction of a continuous-time model integrating the unique properties of endowment management process. Merton (1993) has developed the first most significant continuous-time model to address the complicated problem of optimal investment strategies for endowment funds. The derivations are based on an intertemporal consumption and portfolio selection model and optimal expenditures and allocation rules are derived in this very

[7] Hansmann (1990) is the first to address this issue for universities.
[8] This principle is referred to as 'intergenerational equity.'
[9] The inclusion of exogenous donation stream is an intended area of further research and theoretical development.

general framework that includes nonendowed funds as part of the university's available resources. Thaler and Williamson (1994) proposed an asymmetric rebalancing strategy, whereby an endowment fund would rebalance back to the initial 60/40 mix of stocks and bonds if the return on stocks has been less than the return on bonds at the end of the year. However, if stocks outperform bonds there is no rebalancing triggered. In either case, the strategy is consistent with a higher return expectation for stocks over bonds. It is a momentum strategy only if the stocks are outperforming bonds (could be called a 'conditional momentum' strategy). The authors, by providing historical evidence, argue that the rewards of this strategy could be substantial. Dybvig (1999), using the results in Dybvig (1995), postulates that withdrawals from an endowment fund could be sustained by using TIPS (Treasury Inflation Protection Securities) in conjunction with a risky investment.

In this chapter, I use a simple continuous-time model and try to capture the essential characteristics of dynamic spending and allocation rules of endowment funds. This endowment fund is characterized by an HARA utility function with a minimum subsistence level that increases with the inflation rate. The dynamic framework for the stochastic prices is a Black/Scholes type. The optimal spending and portfolio allocation rules are derived explicitly which allows extensive comparative static interpretations and easy numerical simulations.

3. 2. Framework

The rate of return on the risk-free asset "cash" (or bill) is a constant *r*; that is, $\frac{dB_t}{B_t} = rdt$ (or $B_t = \exp\left\{\int_0^t rds\right\}$). For any date $t > 0$, the period return on the bill, B_t, is given by e^{rt}. On the other hand, the stock index price process follows a geometric Brownian motion, with drift m and variance s^2; i.e., the traditional Black and Scholes framework[10]:

$$\text{(3.1)} \qquad \frac{dS_t}{S_t} = m\,dt + s\,dW_t$$

where $\frac{dS_t}{S_t}$ is the instantaneous change in the stock price, and W_t is a standard Brownian motion. The return from holding the risky asset is a pure capital gain.

The key properties of Brownian motion are the following:

(i) Continuity: Brownian motion has continuous paths.

(ii) Independent, normally distributed increments: If $0 \le t_0 < t_1 < \ldots < t_k$, then the increments $W(t_1) - W(t_0), W(t_2) - W(t_1), \ldots, W(t_k) - W(t_{k-1})$ are independent, and each increment $W(t_j) - W(t_{j-1})$ is a normal variable with mean zero and variance $t_j - t_{j-1}$.

(iii) Markov property: If $0 \le s < t$ are given, then conditioned on the information obtained by observing the Brownian motion until time *s*, the conditional distribution of *W(t)* depends only on the value of *W(s)*.

[10] This model implies that the value of the stock obeys a lognormal distribution at all times. In other words, $\ln \frac{S_t}{S_0} \sim Normal\left[\left(m - \frac{s^2}{2}\right)t, s\sqrt{t}\right]$.

(iv) Martingale property: If $0 \le s < t$ are given, then conditioned on the information obtained by observing the Brownian motion until time *s*, the conditional expectation of *W(t)* is *W(s)*.

(v) Quadratic variation: If $t > 0$ is given, and one partitions the interval $[0, t]$ by choosing partition points $0 = t_0 \le t_1 \le \ldots \le t_k = t$, then as the partition becomes finer, and k, the number of partition points, approaches infinity), $\sum_{j=1}^{k} \left[W(t_j) - W(t_{j-1}) \right]^2$ approaches t almost surely.

Existence and uniqueness of risk-neutral probability Q are characterized by Radon-Nikodym derivatives, $Z_t \,.\, Z_t = \frac{dQ}{dP}$

$$(3.2) \quad \frac{dZ_t}{Z_t} = -l\, dW_t \Rightarrow dZ_t = -l\, Z_t dW_t$$

$$(3.3) \quad Z_t = e^{-\frac{1}{2} l^2 t} e^{-l W_t}$$

where λ is the instantaneous price of risk: $l = \frac{m - r}{s}$

The state space of uncertainty is denoted by Ω, whose elements *w* are states of nature which specify the complete realization of *w* on *[0,T]*. The collection of distinguishable events is the Borel sigma field of Ω, denoted by $\Im$, and the probability belief is the Wiener measure on $(\Omega, \Im)$ called *P*. The martingale approach requires the transformation of $(\Omega, \Im, P)$ into $(\Omega, \Im, \mathrm{Q})$.

The assumed utility function belongs to the family of hyperbolic absolute risk aversion (HARA) functions. It could be interpreted as a utility function with a subsistence level $\underline{Y_t}$ that is a deterministic function of time. In addition, it is in the class of homothetic functions[11] with a time varying habit or subsistence level, similar to the habit formation structure articulated by Constantinides (1990) and Campbell and Cochrane (1999).

$$\text{(3.4)} \quad U_t\left(Y_t\right) = \frac{1}{1-g}\left(Y_t - \underline{Y_t}\right)^{1-g}$$

This representation could be thought of as a restriction to a particular class of HARA functions with decreasing absolute risk aversion; i.e., $g > 0$ [12].

The subsistence level is the minimum spending level a given endowment needs to sustain and assumed to be:

$$\underline{Y_t} = aV_0 e^{pt},$$

where the growth rate is $p < r$, indicating a constant inflation rate that is always lower than the constant risk-free rate. $0 < a < 1$ represents the spending rate based on inflation-adjusted initial endowment value, V_0. In particular, $\underline{Y_t}$ is positive, implying a decreasing relative risk aversion (DRRA).

3. 3. Optimization Program and Optimal Spending Stream

Assume a university endowment fund with a self-financing strategy (constraint)[13] is maximizing a HARA utility function in an infinite time horizon

[11] There is no one-to-one mapping between the ratio of marginal utilities and the rate of time preference (or the intertemporal elasticity of substitution) in the case of homothetic functions, which do not include the quadratic and the CARA class.

framework. For endowments, the self-financing constraint could be re-written under the assumption of complete markets, which is satisfied under the given framework of one Brownian motion, two non-redundant securities and continuous rebalancing of the portfolio[14]:

$$\underset{(Y_t)}{Max}\int_0^{+\infty} e^{-rt} E\left[U_t\left(Y_t\right)\right]dt$$

$$\text{subject to } E^*\left[\int_0^{+\infty} e^{-rt} Y_t Z_t dt\right] = V_0$$

where V_0 is the initial endowment value, $r > 0$ is the time-preference for spending today[15], Y_t are the control variables, and E^* denotes expectation under risk-neutral probability.

We should note that in the case of endowment funds, the spending Y_t should be understood as a spending stream. An application of the Euler's theorem is required to solve the optimization program:

$$L = \int_0^{+\infty} e^{-rt} E\left[U_t\left(Y_t\right)\right]dt - k\left[E^*\left(e^{-rt}\int_0^{+\infty} Y_t Z_t dt\right) - V_0\right]$$

$$\frac{\partial L}{\partial Y_t} = 0 \Leftrightarrow e^{-rt}\left(Y_t - \underline{Y_t}\right)^{-g} - k\, Z_t e^{-rt} = 0$$

$$e^{-rt}\left(Y_t - \underline{Y_t}\right)^{-g} = k\, Z_t e^{-rt}$$

$$\left(Y_t - \underline{Y_t}\right)^{-g} = k\, Z_t e^{(r-r)t}$$

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[12] The case of $g < 0$ implies increasing absolute risk aversion, which is found to be unrealistic in practical situations. We postulate that the rise in the endowment size would be accompanied by more allocation in the risky asset, and not vice versa.

[13] $\frac{dV_t}{V_t} = x_t \frac{dS_t}{S_t} + \left(1 - x_t\right)\frac{dB_t}{B_t} - Y_t dt$

[14] See Cox and Huang (1989) and Karatzas, Lehoczky and Shreve (1987) for details.

[15] Tobin (1974) argues that the university trustees should have a zero subjective rate of time preference.

$$\left[\left(Y_t - \underline{Y_t}\right)^{-g}\right]^{-\frac{1}{g}} = \left[k\, Z_t e^{(r-r)t}\right]^{-\frac{1}{g}}$$

$$Y_t - \underline{Y_t} = k^{-\frac{1}{g}} Z_t^{-\frac{1}{g}} e^{-\frac{r-r}{g}}$$

Lemma 1:

$$\int_0^{+\infty} e^{at}\, dt = -\frac{1}{a}$$

For any constant a negative ($a < 0$), to insure the convergence of the integral.

Lemma 2:

$$E\left(e^{an}\right) = e^{\frac{1}{2}a^2}$$

where a is a constant and $n \sim N(0,1)$.

Define $k = k^{-\frac{1}{g}}$, then the optimal spending process is:

$$(3.5) \qquad Y_t = \underline{Y_t} + kZ_t^{-\frac{1}{g}} e^{-\frac{r-r}{g}t}$$

This is a constant ($\underline{Y_t}$) plus a lognormal distribution with the following parameters $k - \dfrac{r-r}{g}t + \dfrac{1}{2}\dfrac{l^2}{g}t, \dfrac{l}{g}\sqrt{t}$ [16].

The constant k is found from the constraint:

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[16] $\log\left(kZ_t^{-\frac{1}{g}} e^{-\frac{r-r}{g}t}\right) = k - \dfrac{r-r}{g}t - \dfrac{1}{g}\log Z_t$

$$\int_0^{+\infty} e^{-rt}\underline{Y_t}dt + k^{-\frac{1}{g}}\int_0^{+\infty} e^{\frac{r-r}{g}t} E\left(Z_t^{-\frac{1}{g}+1}\right)dt = V_0$$

$$(3.6)\qquad k^{-\frac{1}{g}} = k = \frac{V_0 - \int_0^{+\infty} e^{-rt}\underline{Y_t}dt}{\int_0^{+\infty} e^{-\frac{r-r}{g}t} e^{-rt} E\left(Z_t^{1-\frac{1}{g}}\right)dt}$$

There are technical conditions for the optimization program to converge. The first technical condition implies that the initial wealth allocation should be greater than the discounted future spending under the risk neutral probability, which corresponds to the price at date 0 of the spending process.

$$\begin{aligned} V_0 > E^*\left[\int_0^{+\infty} e^{-rt}\underline{Y_t}dt\right] \\ = \int_0^{+\infty} e^{-rt} a V_0 e^{pt}\, dt \\ = \int_0^{+\infty} a V_0 e^{-(r-p)t}\, dt \\ = a\,\frac{V_0}{r-p} \end{aligned}$$

Thus, $V_0 > a\,\dfrac{V_0}{r-p} \Rightarrow (r-p)V_0 > aV_0$

$$(3.7)\qquad a < r - p$$

(3.7) is the first technical condition of the problem. Then, the constant k is given by:

$$(3.8)\qquad k = \frac{V_0\left(1 - \frac{a}{r-p}\right)\left[rg^2 + (r-r)g + \frac{1}{2}l^2(g-1)\right]}{g^2}$$

$$= k - \frac{r-r}{g}t - \frac{1}{g}\left(-\frac{1}{2}l^2 t - lW_t\right) \qquad = k - \frac{r-r}{g}t + \frac{1}{2}\frac{l^2}{g}t + \frac{l}{g}W_t$$

Proof:

$$k = \frac{V_0 - \int_0^{+\infty} e^{-rt} \underline{Y_t} dt}{\int_0^{+\infty} e^{-\frac{r-r}{g}t} e^{-rt} E\left(Z_t^{1-\frac{1}{g}} \right) dt}$$

First, let's look at the numerator:

$$\begin{aligned} V_0 - \int_0^{+\infty} e^{-rt} \underline{Y_t} dt &= V_0 - \int_0^{+\infty} e^{-rt} a V_0 e^{pt} dt \\ &= V_0 - V_0 a \frac{1}{r-p} \\ &= V_0 \left(1 - \frac{a}{r-p} \right) \end{aligned}$$

Now, we will tackle the expectation term in the denominator.

First, we will rearrange the term inside the expectation; $Z_t^{1-\frac{1}{g}} = Z_t^{-\frac{1-g}{g}}$.

We know from the definition of Z_t, from (3.3), that $Z_t = e^{-\frac{1}{2}l^2 t} e^{-lW_t}$.

Then,

$$\begin{aligned} E\left(Z_t^{1-\frac{1}{g}} \right) &= E\left(Z_t^{-\frac{1-g}{g}} \right) \\ &= E\left[\left(e^{\frac{1}{2}l^2 \left(\frac{1-g}{g} \right) t} e^{l \frac{1-g}{g} W_t} \right) \right] \end{aligned}$$

Note: Since $W_t = \sqrt{t}\, n$, where n ~ *N(0,1)*, Lemma 2 is applicable in $E\left(e^{l \frac{1-g}{g} W_t} \right)$.

Thus, $E\left(e^{l \frac{1-g}{g} W_t} \right) = E\left(e^{l \frac{1-g}{g} \sqrt{t} n} \right) = e^{\frac{1}{2} l^2 \left(\frac{1-g}{g} \right)^2 t}$

Then, $$E\left[\left(e^{\frac{1}{2}l^2\left(\frac{1-g}{g}\right)t}e^{l\frac{1-g}{g}W_t}\right)\right]=e^{\frac{1}{2}l^2\left(\frac{1-g}{g}\right)t}e^{\frac{1}{2}l^2\left(\frac{1-g}{g}\right)^2 t}$$

$$=e^{\frac{1}{2}l^2 t\left[\left(\frac{1-g}{g}\right)+\left(\frac{1-g}{g}\right)^2\right]}$$

$$=e^{\frac{1}{2}l^2 t\left[\frac{(1-g)g+1+g^2-2g}{g^2}\right]}$$

$$=e^{\frac{1}{2}l^2 t\left[\frac{(1-g)}{g^2}\right]}$$

Now, we will rewrite the denominator after having determined $E\left(Z_t^{1-\frac{1}{g}}\right)=e^{\frac{1}{2}l^2 t\left[\frac{(1-g)}{g^2}\right]}$.

$$\int_0^{+\infty}e^{-\frac{r-r}{g}t}e^{-rt}E\left(Z_t^{1-\frac{1}{g}}\right)dt=\int_0^{+\infty}e^{-\frac{r-r}{g}t}e^{-rt}e^{\frac{1}{2}l^2\left[\frac{(1-g)}{g^2}\right]t}dt$$

$$=\int_0^{+\infty}e^{\left[-r-\frac{r-r}{g}+\frac{1}{2}l^2\frac{(1-g)}{g}\right]t}dt$$

Using Lemma 1 and assuming that $-r-\frac{r-r}{g}+\frac{1}{2}l^2\frac{(1-g)}{g^2}<0$:

$$\frac{1}{\left[r+\frac{r-r}{g}-\frac{1}{2}l^2\frac{(1-g)}{g^2}\right]}=\frac{1}{\left[r+\frac{r-r}{g}+\frac{1}{2}l^2\frac{(g-1)}{g^2}\right]}=\frac{g^2}{rg^2+(r-r)g+\frac{1}{2}l^2(g-1)}$$

Then, $k=\dfrac{V_0\left(1-\dfrac{a}{r-p}\right)\left[rg^2+(r-r)g+\dfrac{1}{2}l^2(g-1)\right]}{g^2}$ Q.E.D.

Moreover, the assumption mode for the convergence of the indefinite integral in the derivation of *k* introduces the second technical condition.

Lemma 1 provides the following condition for the integral in the derivation of *k* to converge.

$$-r-\frac{r-r}{g}+\frac{1}{2}l^{2}\frac{(1-g)}{g^{2}}<0 \Leftrightarrow -rg^{2}-rg+rg+\frac{1}{2}l^{2}-\frac{1}{2}l^{2}g<0$$

This further reduces to: $r>(1-g)\left[r+\frac{1}{2}\frac{l^{2}}{g}\right]$.

As r should also be positive, the technical condition 2 could be written as:

$$\text{(TC 2)} \qquad (3.9) \qquad r>Max\left\{0,(1-g)\left[r+\frac{1}{2}\frac{l^{2}}{g}\right]\right\}$$

Note that this condition is also known as 'transversality condition' as shown in Ingersoll (1987), p. 275.

3. 4. Optimal Endowment Value

Using again the risk-neutral probability and the martingale property of the discounted (by the risk-free rate) processes under the risk-neutral probability defined by its Radon-Nikodym derivative.

$$V_t=\int_t^{+\infty}E_t\left[e^{-rs}Y_sZ_{s-t}\right]ds$$

where Z_{s-t} is the Radon-Nikodym at date t of the change of probability at date s and E_t is the conditional expectation at date t. This further implies the derivation of the optimal endowment value that writes:

$$(3.10)\ \ V_t=V_0\left[a\,\frac{e^{-(r-p)t}}{r-p}+\left(1-\frac{a}{r-p}\right)Z_t^{-\frac{1}{g}}\right]$$

The optimal endowment value is a constant, which could be interpreted as an "insurance-type" lower bound, plus a lognormal distribution.

Proof:

$$V_t = \int_t^{+\infty} e^{-rs}\underline{Y_s}ds + k\int_t^{+\infty} e^{-rs} e^{-\frac{r-r}{g}s} E_t\left[Z_s^{-\frac{1}{g}} Z_{s-t} \right] ds$$

$$Z_s = Z_t Z_{s-t}$$

$$V_t = \int_t^{+\infty} e^{-rs}\underline{Y_s}ds + kZ_t^{-\frac{1}{g}}\int_t^{+\infty} e^{-rs} e^{-\frac{r-r}{g}s} E_t\left[Z_{s-t}^{1-\frac{1}{g}} \right] ds$$

$$\int_t^{+\infty} e^{-rs}\underline{Y_s}ds = aV_0 e^{-rt} e^{pt} \int_t^{+\infty} e^{-(r-p)(s-t)}ds$$

$$= aV_0 \frac{e^{-(r-p)t}}{r-p}$$

We will then tackle $\int_t^{+\infty} e^{-rs} e^{-\frac{r-r}{g}s} E_t\left[Z_{s-t}^{1-\frac{1}{g}} \right] ds$.

We will apply the change of variable $s' = s-t$.

$$\int_t^{+\infty} e^{-rs} e^{-\frac{r-r}{g}s} E_t\left[Z_{s-t}^{1-\frac{1}{g}} \right] ds = e^{-rt} e^{-\frac{r-r}{g}t} \int_t^{+\infty} e^{-r(s-t)} e^{-\frac{r-r}{g}(s-t)} E_t\left[Z_{s-t}^{1-\frac{1}{g}} \right] ds$$

$$= e^{-rt} e^{-\frac{r-r}{g}t} \int_0^{+\infty} e^{-rs} e^{-\frac{r-r}{g}s} E\left[Z_s^{1-\frac{1}{g}} \right] ds$$

We know that $E\left(Z_s^{1-\frac{1}{g}} \right) = E\left[e^{\frac{1}{2}l^2\left(\frac{1-g}{g}\right)s} e^{l\left(\frac{1-g}{g}\right)W_s} \right]$.

Thus,

$$\int_0^{+\infty} e^{-rs} e^{-\frac{r-r}{g}s} E\left[Z_s^{1-\frac{1}{g}} \right] ds = \int_0^{+\infty} e^{-rs} e^{-\frac{r-r}{g}s} E\left[e^{\frac{1}{2}l^2\left(\frac{1-g}{g}\right)s} e^{l\left(\frac{1-g}{g}\right)W_s} \right] ds$$

$$= \frac{g^2}{rg^2 + (r-r)g + \frac{1}{2}l^2(g-1)}$$

$$V_t = aV_0 \frac{e^{-(r-p)t}}{r-p} + kZ_t^{-\frac{1}{g}} \frac{g^2}{rg^2 + (r-r)g + \frac{1}{2}l^2(g-1)}$$

This is the lognormal distribution of endowment value. After simplifying with (3.8), one gets the following:

$$V_t = aV_0 \frac{e^{-(r-p)t}}{r-p} + V_0\left(1-\frac{a}{r-p}\right)Z_t^{-\frac{1}{g}}$$

or

$$V_t = V_0\left[a\,\frac{e^{-(r-p)t}}{r-p} + \left(1-\frac{a}{r-p}\right)Z_t^{-\frac{1}{g}}\right] \qquad \text{Q.E.D.}$$

3. 5. Optimal Spending Rate

Define the spending rate, y_t , to be the dollar amount consumed from the endowment at time *t* divided by the value of the endowment fund at time *t*. Under this definition of the spending rate, one could make the following observations using (3.5) and (3.10):

$$\begin{aligned} y_t &= \frac{Y_t}{V_t} \\ &= \frac{\underline{Y}_t + kZ_t^{-\frac{1}{g}} e^{-\frac{r-r}{g}t}}{V_0\left[a\,\frac{e^{-(r-p)t}}{r-p} + \left(1-\frac{a}{r-p}\right)Z_t^{-\frac{1}{g}}\right]} \end{aligned}$$

$$\text{(3.11)} \quad y_t = \frac{a\,e^{pt} + \left(1-\frac{a}{r-p}\right)\left[rg^2 + (r-r)g + \frac{1}{2}l^2(g-1)\right]\frac{1}{g^2}Z_t^{-\frac{1}{g}} e^{-\frac{r-r}{g}t}}{\left[a\,\frac{e^{-(r-p)t}}{r-p} + \left(1-\frac{a}{r-p}\right)Z_t^{-\frac{1}{g}}\right]}$$

At *t*=0, the initial spending rate, turns out to be greater than the spending rate used in the subsistence level of Y_t:

$$y_0 = \frac{a + \left(1 - \frac{a}{r-p}\right)\left[rg^2 + (r - r)g + \frac{1}{2} l^2 (g-1)\right]\frac{1}{g^2}}{\left[\frac{a}{r-p} + 1 - \frac{a}{r-p}\right]}$$

$$y_0 = a + \left(1 - \frac{a}{r-p}\right)\left[rg^2 + (r - r)g + \frac{1}{2} l^2 (g-1)\right]\frac{1}{g^2}$$

We know that $\left(1 - \frac{a}{r-p}\right)$ and $\left[rg^2 + (r - r)g + \frac{1}{2} l^2 (g-1)\right]$ are both greater than zero from TC1 and TC2, respectively. Then, $y_0 > a$, which means that one would start with an actual spending rate greater than the minimum level defined in $\underline{Y_t}$.

In addition, y_t is independent of the initial endowment value, V_0. This property is linked to the assumption made about $\underline{Y_t}$ and the assumed utility function. For t large enough, y_t is decreasing in $Z_t^{-\frac{1}{g}}$ [17] due to the fact that if S_t is increasing, $Z_t^{-\frac{1}{g}}$ will be increasing as well. This implies that as the stock prices go up, the spending rate should be decreased. On the other hand, a bearish environment could be particularly seen beneficial for the managers of foundations, which are obligated to spend at least 5% of their assets to operate

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$$\begin{vmatrix} \left(r + \frac{r-r}{g} + .5 l^2 \frac{g-1}{g^2}\right) e^{-\frac{r-r}{g}t} & e^{pt} \\ 1 & \frac{e^{-(r-p)t}}{r-p} \end{vmatrix}$$

under the tax-exempt status[18]. The rise in the ex-ante real rates of return during sustained bear markets helps fiduciaries of these institutions not to erode the principal value of donations, and at the same time, provide the required minimum spending level to beneficiaries.

3. 6. Optimal Portfolio Strategy

Define x_t to represent the proportion of market value of the portfolio allocated to the stock index. Then, the self-financing constraint could be written as follows:

$$\frac{dV_t}{V_t} = \left[r + x_t \left(m - r \right) - Y_t \right] dt + x_t s \, dW_t$$

We know from equation (3.2) that $dZ_t = -l \, Z_t dW_t$. In addition, from equation (3.10) we know that V_t is a function of t and Z_t: $V_t = f\left(t, Z_t\right)$. By applying Ito’s lemma, one would get

$$\begin{aligned} dV_t &= [\ldots] dt + \frac{\partial V_t}{\partial Z_t} dZ_t \\ &= [\ldots] dt - l \, Z_t \frac{\partial V_t}{\partial Z_t} dW_t \end{aligned}$$

Then,

$$\begin{aligned} x_t s &= -l \, \frac{Z_t}{V_t} \frac{\partial V_t}{\partial Z_t} \\ x_t &= -\frac{l}{s} \frac{Z_t}{V_t} \frac{\partial V_t}{\partial Z_t} \end{aligned}$$

$= -\dfrac{e^{pt}}{r - p} \left[r - p - \left(r + \dfrac{r - r}{g} + .5 l^2 \dfrac{g - 1}{g^2} \right) e^{-rt} e^{-\frac{r - r}{g} t} \right]$, which implies that for t “large enough”, it is negative and the function is decreasing.

$$\frac{\partial V_t}{\partial Z_t} = -\frac{1}{g} V_0 \left(1 - \frac{a}{r-p}\right) Z_t^{-\frac{1}{g}-1}$$

$$x_t = \frac{l}{sg}\left(1 - \frac{a}{r-p}\right) V_0 \frac{Z_t^{-\frac{1}{g}}}{V_t}$$

$$= \frac{l}{sg} \frac{\left(1 - \frac{a}{r-p}\right) Z_t^{-\frac{1}{g}}}{a\frac{e^{-(r-p)t}}{r-p} + \left(1 - \frac{a}{r-p}\right) Z_t^{-\frac{1}{g}}}$$

So, the optimal portfolio strategy is:

$$(3.12) \quad x_t = \frac{l}{sg} \frac{1}{1 + a\frac{e^{-(r-p)t}}{r-p-a} Z_t^{\frac{1}{g}}}$$

Note that the optimal portfolio strategy defined by the weight x_t invested in the risky security is independent of the initial level of the endowment value, V_0. This is a wealth-independence effect. Note also that $r-p-a>0$, if S_t is increasing, Z_t will be decreasing, implying an increase in x_t. Moreover, as t goes to infinity, $Z_t^{\frac{1}{g}}$ goes to zero (on most paths) at an exponential rate, leading to a constant weight (concave) strategy. If both S_t and t are increasing, x_t increases at a decreasing rate with larger t until it approaches the constant weight strategy, $x_t = \frac{l}{sg}$. Coincidentally, the maximum for the spending rate function happens as the portfolio strategy approaches the concave limit, indicating a decrease in the spending rate.

[18] Certain conclusions derived from the particular model used in this paper supports the arguments raised by Arnott and Bernstein (1997).

We can also show this in terms of dollar investments in stocks at t as defined by $X_t = x_t V_t$:

$$X_t = \frac{l}{sg} V_0 \left(1 - \frac{a}{r-p}\right) Z_t^{-\frac{1}{g}}$$

$$(3.13) \quad X_t = \frac{l}{sg} V_0 \left(\frac{r-p-a}{r-p}\right) Z_t^{-\frac{1}{g}}$$

This further implies that if S_t is increasing, $Z_t^{-\frac{1}{g}}$ will be increasing, implying an increase in X_t, indicating a higher dollar allocation to stocks when the stock market is going up. In addition, when the risk aversion parameter rises (a more risk-averse endowment), the dollar amount invested in stocks declines.

The resulting is a convex (momentum) strategy regarding the weights as well as dollar investments. In case of spending levels set at an exogenous resistance level, endowment funds should be managed in the spirit of the constant proportion portfolio insurance (CPPI) strategy due to Black and Perold (1992).

The management of endowment funds is challenging due to the need for a balance between providing adequate and stable spending for current and future beneficiaries and the growth of the investment portfolio. By maximizing the expected utility function of the HARA type for the optimal spending stream under the self-financing constraint and the geometric Brownian motion assumption for the stock price process, the following conclusions have resulted:

(i) The spending rate is independent of the initial endowment value.

(ii) For t “large enough”, a bullish market results in the spending rate being lowered as the strategy approaches a constant weight, so-called ‘concave’, strategy.

(iii) The optimal allocation to stocks Is not dependent on the initial endowment value, yielding the so-called ‘wealth-independence’ effect.

(iv) As the stock price goes up, the allocation to stocks increases both in terms of weight and dollar amount. The optimal strategy, thus, could be called a momentum or convex strategy.

(v) There is an inverse relationship between the stock index and optimal spending rate while the dollar spending value is positively related to the value of the stock index.

3. 7. Simulation Results

I use the following relationship between S_t and Z_t to derive values of the standard normal variable that would yield bullish and bearish paths:

$$S_t = e^{\left(m-\frac{1}{2}s^2\right)t} e^{s W_t} \Leftrightarrow S_t^{\frac{1}{s}} = e^{\left(\frac{m-\frac{1}{2}s^2}{s}\right)t} e^{W_t}$$

$$Z_t = e^{-\frac{1}{2}l^2 t} e^{-l W_t}$$

$$= e^{-\frac{1}{2}l^2 t}\left[S_t^{\frac{1}{s}} e^{-\left(\frac{m-\frac{1}{2}s^2}{s}\right)t}\right]^{-l}$$

$$= e^{\frac{l}{s}\left(m-\frac{1}{2}s^2\right)t-\frac{1}{2}l^2 t} S_t^{-\frac{l}{s}}$$

Simulating paths of W_t will allow one to obtain corresponding stock price process as well as Radon-Nikodym process. Given the following assumed values for the stock price process parameters; m=12% and s=20%, a value of –2 for the standard normal variate would provide a bearish path. In addition, I look at values of 0 and 2 for median and bullish paths, respectively[19]. Due to the positive drift in the stock price process, the median path also results in a bullish market environment. The other derived parameter value for the simulation experiment is the one associated with the risk aversion parameter. Assuming m=12% and s=20% as well as α=2%, π=2%, r=5%, γ should be around 0.97 for an endowment fund to have an initial allocation of 60% into stocks. Moreover, I set the time preference parameter for spending at 5%.

Tables 1 through 3 show results in terms of spending rate, stock allocation and endowment value for a 30-year period for bullish, median and bearish paths, respectively. The chosen bullish path seems to be a very bullish environment since the stock weight approaches the constant weight limit of 180.4% {[(12%-5%)/20%]÷(20%*0.97)} in almost 20 years. Despite the reluctance of endowment fund managers to use leverage in the management of the portfolio, the assumption of geometric Brownian motion along with a subsistence level requirement (in the spirit of habit formation) indicates otherwise. The spending rate reaches a limit of about 4.8% after the first 10-year period of the bullish path. Table 5 indicates that the average endowment annually spent between 4.7% and 4.9% of the portfolio

[19] A constant value of the standard normal distributions defining the brownian motion generating the stock index is used for the purpose of these simple numerical simulations, which means that if $W_t = n\sqrt{t}$, n takes constant values equal to –2, 0 or 2.

value during the four years ending in 2000. The results do suggest that spending rate depends on the age of the endowment fund.

Long-term spending goals of large and small university endowment funds, as depicted in Table 4, have been reduced during the 1997-2000 period, whereas the medium-sized endowments did not post any significant changing behavior. The absolute changes are identical for larger and smaller endowments in the universe, whereas the medium-sized endowments do not change their long-term spending goals much during the same period. I further conjecture based on the findings in Table 4 that the significant drop witness in 1998 could be as a result of the incredible bull period and its psychological toll on endowment managers and trustees of the board. The analysis done by deMarche Associates shows that endowments should not have annual spending rates of 6% or more of the market value of the portfolio.

Table 5 gives the distribution of the spending rate of endowments for 1997 through 2000. The percentage of universities that has gone over this empirical limit stayed at 7% in 2000 after reaching a high of 11% in 1998. On the other hand, the percentage of universities that spent 3% or lower of the portfolio market value has more than doubled from a low of 6% in 1997 to a period-high of 13% in 2000. Larger endowments have spent less over the period moving from a high of 4.8% in 1997 to a low of 4.2% in 2000 (see Table 6).

Table 1: Simulation scenario - Bullish path

Year	Spending rate	Stock allocation	Endowment value (in millions)
0	3.60%	60.14%	500
1	4.03%	95.69%	689
5	4.61%	144.33%	1,434
10	4.82%	166.98%	3,317
15	4.85%	175.12%	7,244
20	4.85%	178.22%	15,040
25	4.83%	179.46%	29,975
30	4.82%	179.99%	57,833

Table 2: Simulation scenario - Median path

Year	Spending rate	Stock allocation	Endowment value (in millions)
0	3.60%	60.14%	500
1	3.74%	63.93%	501
5	4.27%	79.99%	515
10	4.87%	100.90%	560
15	5.32%	120.71%	642
20	5.60%	137.67%	772
25	5.73%	150.99%	965
30	5.75%	160.75%	1,244

Table 3: Simulation scenario - Bearish path

Year	Spending rate	Stock allocation	Endowment value (in millions)
0	3.60%	60.14%	500
1	3.50%	37.98%	410
5	3.98%	24.70%	332
10	4.93%	20.69%	279
15	6.18%	19.84%	239
20	7.78%	20.44%	206
25	9.78%	22.03%	179
30	12.27%	24.48%	157

Table 4: Long-term spending goals by endowment size (1997 – 2000)

	1997	**1998**	**1999**	**2000**
Over $1 billion	4.8%	4.6%	4.7%	4.7%
$250 million - $1 billion	4.7%	4.7%	4.6%	4.7%
$250 million and under	4.9%	4.7%	4.8%	4.8%
All	4.8%	4.7%	4.7%	4.8%

Source: Greenwich Associates

Table 5: Spending rate distribution (1997 – 2000)

	1997	1998	1999	2000
1%	0%	1%	1%	0%
2%	1%	1%	1%	1%
3%	5%	7%	7%	12%
4.0-4.3%	19%	20%	22%	19%
4.4-4.7%	14%	17%	21%	18%
4.8-5.0%	30%	28%	21%	24%
5.1-5.3%	9%	7%	8%	9%
5.4-5.7%	11%	10%	9%	8%
5.8-5.9%	2%	0%	2%	3%
6%	6%	6%	2%	4%
7%	1%	3%	6%	1%
8%	2%	0%	0%	1%
9%	0%	0%	0%	0%
10%	0%	1%	0%	0%
Over 10%	1%	1%	1%	1%
Mean	**4.9**	**4.8**	**4.8**	**4.7**

Source: Greenwich Associates

Table 6: Mean spending rates by endowment size (1997 – 2000)

	1997	1998	1999	2000
Over $1 billion	4.8%	4.5%	4.4%	4.2%
$250 million - $1 billion	4.8%	4.8%	4.7%	4.7%
$250 million and under	5.1%	5.0%	5.0%	4.9%
All	4.9%	4.8%	4.8%	4.7%

Source: Greenwich Associates

3. 8. Sensitivity Analysis

The effects of changing parameter values such as mean stock return, stock volatility, time-preference of spending and interest rates on the spending rate and stock allocation are analyzed in this section. The following table summarizes the range of values used for each parameter under five paths; i.e., *n* =-2, -1, 0, +1, +2, only the negative ones indicating a bearish path:

μ	8%, 10%, 14%, 16%
σ	16%, 18%, 22%, 24%
ρ	1%, 3%, 7%, 9%
r	4.5%, 5.5%, 6%

As could be seen in the *Appendix*, with the exception of the time-preference parameter, any change in the remaining parameters led to the re-setting of the risk aversion parameter. This is necessary to set the initial stock and bond allocations at 60% and 40%, respectively. Moreover, in the case of the interest rate, the time preference parameter has to be used at 6%, instead of the base case of 5%. The resulting effects on the spending rate and stock allocation are summarized below with the simulation period covering 50 years:

Under very bullish (*n* = +2) and medium bullish (*n* = +1) states, only when μ is less than 10%, the relationship between spending rate and stock allocation ends up being inverse. In the slight bullish (*n* = 0) and medium bearish (*n* = -1) paths, they are positively related as μ is changes, all things being equal . Under the very bearish (n = -2) scenario, on the other hand, the spending rate rises in a linear fashion to satisfy the subsistence level requirement embedded in the utility function, whereas the stock allocation follows a U-shaped pattern. The timing of the trough depends on the value of the stock return parameter, whereby the 8% case does not begin to rise even after 50 years.

An increase in the volatility of the stock index results in a slightly inverse relationship between spending rate and stock allocation when the stock index process is assumed to follow a bullish path. Lower volatility and/or higher return of the stock process are required to have a positive relationship between spending and stock weight in the portfolio.

In the bullish scenarios, as ρ falls below 5%, the (otherwise-direct) relationship between spending rate and stock allocation becomes significantly inverse. Under bearish states, the spending rate goes up as time increases, whereas the stock allocation drops initially the first 5 to 10 years, then increases significantly.

When the interest rate parameter takes on the value of 4.5%, the resulting relationship could not be generalized in any scenario. In all other cases, the positive relationship between the spending rate and stock allocation is insensitive to the changes in r. Higher values of r result in a positive relationship.

3. 9. Conclusion

The management of endowment funds is challenging due to the need to find a balance between providing adequate and stable spending for beneficiaries and growth of the portfolio. By maximizing the expected utility function of the

HARA type for the optimal spending stream under the self-financing constraint and the geometric Brownian motion assumption for the stock price process, the spending rate is independent of the initial endowment value. Moreover, for t "large enough", a bullish market results in the spending rate being lowered as the strategy approaches a constant weight strategy.

The optimal portfolio allocation to stocks is not dependent on the initial endowment value, yielding the so-called wealth-independence effect. As the stock price goes up, the allocation to stocks increases both in terms of weight and dollar amount. The optimal strategy can then be classified as a clear momentum or convex strategy. With regard to the optimal spending rate, we find an inverse relation between the stock index and the optimal spending rate while the dollar spending value is positively related to the value of the stock index.

In a probability-maximizing framework, Browne (1999) postulates that extensive use of leveraging could be risk-reducing when there is no finite deadline or when the investor has an external source of income. Thus, it is imperative to include a stochastic model for the donation stream when deriving intertemporal solutions for university endowments. Moreover, additional donations would lower management fees as well as provide an opportunity to invest in more successful money management firms. Certain economies of scale result from the growth of the endowment portfolio that could be a crucial determinant in the allocation and spending of endowment funds.

The sharing of the realized endowment portfolio value in terms of annual spending to beneficiaries at various colleges and departments of a particular university is typically done in a linear fashion on a pro rata basis. Similar restrictions are forced upon the member's payoff profile in pension funds and participating life insurance contracts. Linear sharing rules are Pareto optimal as long as the degrees of relative risk aversion of academic units are identical in the case of CRRA utility functions. Nevertheless, I would argue that beneficiaries of university endowment pools have CRRA utility functions with different degrees of relative risk aversion or even completely different utility functions that might include parameters to satisfy the aversion of participants toward loss or disappointment. The apparent heterogeneity among beneficiaries of the endowment pool could be attributed to the following facts:

(i) Different degrees of relative risk aversion

(ii) Different forms of utility functions

(iii) Dispersion among portfolio values due to differences in fund-raising success

(iv) Different intertemporal rate of substitution

Given the fact that total dollars available for spending are distributed on a pro rata basis, the management of the endowment portfolio by the Treasurer or CFO of the foundation (in line with preferences of board members) induces utility losses for particular members of the pool. In this setting, the endowment fund manager ends up investing in the same manner as would an investor with some CRRA utility function with a relative risk aversion parameter that is different from that of the college or academic unit under consideration. Moreover, the minimum spending rate policy to provide a guaranteed allocation is considered to be a useful practice for it makes the budgetary planning more predictable. In particular, if an account is dedicated to support the salary of a faculty member in the form of a chair or professorship, such rules make it possible to hire and keep excellent educators at colleges and universities.

One should analyze the effects of the suboptimal asset allocation that is a result of the utility loss, which is a function of the difference between the level of relative risk aversion of a college and the one used by the endowment fund manager to derive the investment policy for the endowed portfolio. Some colleges are more aggressive than others when it comes to aversion to risk or to various behavioral factors, and the compromise between preferences made by the fund manager results in utility loss for members of the endowment pool. Another issue surrounding the suboptimal management of endowment funds arises from the fact that certain colleges would like to spend more today than tomorrow. The stock market rally witnessed during the 90s helped those who favor increased spending in the short-term over the growth of the corpus for long-term purposes to make a case for the distribution of the unrealized gains to academic units. Proponents of this view might aim at investing in the infrastructure of the college and development efforts.

From the standpoint of increasing the productivity and the attractiveness of a given college, it could be argued that putting more dollars back into the college might yield more return over the long-run than relying on the endowment fund manager. On the other hand, other colleges that do not need immediate resources for the expansion of the academic programs would be perfectly fine with spending less today. I would a priori postulate that compromising between preferences among colleges would eventually lead to suboptimal asset allocation and spending decisions. When there is a minimum spending rate guarantee in place, colleges with high levels of relative risk aversion are compensated partly for the loss induced by an aggressive investment policy, whereas others with a low level of relative risk aversion are suffering a further utility loss relative to the loss induced by a conservative investment policy.

In the previous section, I have eluded into the fact that the desire of some endowment pool participants to capture a serious portion of the excess return on an annual basis might create long-lasting internal conflicts of interest between the managers of the fund and the beneficiaries. To minimize the tension between various parties involved in the management of university endowments, a surplus distribution mechanism similar to the ones used in many life insurance products could provide a creative way to better manage endowment funds. Moreover, the effective combination of minimum spending rates and the sharing of the positive excess return might yield a fair distribution of available funds to the colleges and academic units. I realize the significance of the learning curve necessary to achieve these drastic changes, but the simple fact that universities are complex entities only justifies the development of more efficient structures.

It would not be prudent to distribute a large portion of the excess return for it acts as a buffer during prolonged bear markets. The minimum spending rate requirement as outlined in the investment policy of the endowment portfolio falls into the category of annual/multi-period return guarantees, whereby the endowment promises to yield a certain rate of return to the beneficiaries on an annual basis. Colleges could use the surplus (or excess return) distribution mechanism to increase the level of spending in a given year for the benefit of the faculty and student body. At other times when the stock market is not able to cover the minimum spending rate and the purchasing power, there might be a need to tap into the surplus account. The decentralization of certain functions of endowment fund management could eliminate some problems and eventually lead to more effectively managed institutions of higher learning. The portion of the excess return that is reinvested back into the endowment portfolio is vital to capitalize on the compounding principle as well as to lower external fees paid to outside portfolio managers. The similarities between this mechanism and the one used by some life insurance products of the participating nature end with the notion that there are no contract maturities as far as endowments are concerned. Colleges that do not use their portion of the excess return during good years would find it cost effective to replace the computers used by the faculty and the students at times when most technology companies are losing value in the stock market. These companies typically offer significant discounts during bad times to boost revenue streams. If there is no discretion given to individual academic units, one could question the efficiency of the system from a holistic point of view. Risk and return sharing arrangements could be established between the endowment fund and the colleges as well as among colleges.

Chapter 4

Currency Hedging for International Portfolios

4. 1. Introduction

During the last decade we have witnessed a serious attempt by many investment management firms to push an allocation by institutional investors into alternative investments. To some, the complexity of non-traditional investment strategies by far outweighs the theoretical justification in the risk/return dimension, which is mainly due to their low correlations with traditional asset classes such as global stocks and bonds, resulting in more efficient financial structures. In the mean time, due to the perpetual nature of endowments, fiduciaries of university foundations with large asset sizes have been more responsive to the notion of diversifying the broad asset classes with such alternative investments as private equity, hedge funds, and real estate, just to name a few.

The recent downward spiral of financial markets and the apparent slowing of the U.S. economy hurt portfolios that are heavily exposed to the systematic risk of domestic stocks. The technology sector was hit the hardest because of the incredible overvaluations witnessed in stocks of tech companies. Some investment professionals as well as serious thinkers in the financial industry embraced the idea of a completely new economic structure for the U.S. There was not one day in 2000 when investors did not hear a conversation about the differences between the “New Economy’ and ‘Old Economy’ stocks, and implications for portfolio management. Over the last year, we all rediscovered painfully that the business ‘cycle’ is still alive, and more importantly, diversification into various asset classes is crucial to smoothing out these tough rides of the recent past.

Strategic asset allocation and asset liability modeling are believed to provide the most value added to institutional portfolios. Brinson, Hood and Beebower (1986) are the first to provide evidence of the importance of asset allocation decision in the overall institutional fund management process. The selection of asset classes in the initial phase of constructing an efficient portfolio becomes crucial to achieving long-term goals of pension plans. Moreover, since the remaining pieces of the puzzle flow directly from the initial analysis, we cannot say enough about the challenges the investment management and consulting community face in terms of definitions and purposes of asset class categories, and assumptions about distributional characteristics of each. The impact of changing economic conditions as well as the introduction of new asset categories on modeling assumptions needs to be monitored continuously, and appropriate methodologies and techniques should be implemented in response to these developments to keep the asset allocation as efficient and prudent as possible. Any asset allocation exercise that fails to dynamically model the interaction of assets both public and private and liabilities such as the spending requirements is unlikely to produce solutions where risk, return, benefit security and cost are balanced most appropriately.

The return in an active global portfolio comes from three sources: individual security selection, country/region bets, and (embedded) currency exposure. Many institutional investors have not been giving the necessary attention to the effects of foreign exchange movements on the overall portfolio. Others, usually very large institutional investors with a high allocation into international investments, have realized the relevance of currency hedging on the overall fund management process. Over the years, different approaches have been utilized to address the issue within the guidelines of the investment policy to make the 'embedded' currency gain/loss an explicit part of the portfolio management process. Some endowment fund managers did not like the fact that currencies, in the long-term, do not provide an ex-ante return, but increase the volatility of the total portfolio. This thought process resulted in the creation of the 'risk-controlled' currency overlay product, which attracted a great deal of interest from volatility-sensitive institutional investors in the late 80s. Since then, currency managers and investors have discovered the value-added potential of active currency management due to the widely published inefficiencies that are very unique to the foreign exchange markets and expected to stay for the years to come. Through the end of 2000, the unhedged mandates in U.S. dollar-based accounts produced the following one-, three-, and five-year information ratios[20], 0.56, 0.47, and 0.69, respectively. Annualized information ratios for the 50% hedged composite, the currency managers, on average, report 0.26, 0.23 and 0.46 for the respective time periods. The average annualized information ratios, which are computed using a time-weighted scheme since the inception of each account, are 0.82 and 0.60 for the unhedged and 50% hedged benchmarks, respectively. I

[20] Information ratio is defined as the ratio of the excess return over a pre-determined benchmark divided by its standard deviation, which is called the tracking error.

would argue that the discrepancy between the information ratios of unhedged and 50% hedged benchmarks could be due to the forward rate premium bias, first reported by Fama (1984) and then made popular by Kritzman (1993a), that is widely utilized by currency overlay managers. For a U.S. institutional investor in an EAFE mandate, hedging more of the low-interest rate currency exposure; i.e., the U.S. dollar, would result in the unhedged mandates outperforming the 50% hedged counterparts as far as risk-adjusted excess returns are concerned. Given the nature of global financial markets, the ability of most currency managers generating excess returns, and risk preferences of many institutional investors including factors related to governance budgets, the two-step procedure of selecting securities and countries/regions first, and then letting an overlay expert handle the currency exposure from both a risk-controlling and alpha generation perspective seems to be a more viable route for pension plans[21].

The asymmetric nature of the polar benchmarks limits the long and short capabilities of active currency managers, who argue that having a symmetrical mandate would benefit the investor a great deal due to the expansion of the opportunity set in the long-term. The fundamental law of active management[22] provides a framework to understand why an expanded opportunity set; i.e., larger breadth, would translate into immediate increase in the information ratios for

[21] See Grinold and Meese (2000b) for a different point of view. I would argue that, given the points raised in this article and the fact that one uses the same kind of risk aversion as the one used to derive the optimal portfolio in an asset allocation or Asset Liability Management (ALM) framework, the two-step procedure would yield close-to optimal and fairly sensible solutions for the problem at hand.

[22] See Grinold and Kahn (2000a) for details of the concept.

overlay managers, ceteris paribus, because of the constant skill level[23]. In polar benchmarks such as unhedged and fully hedged, the base currency's movements have a large impact on the excess return generation potential of the currency manager, assuming there is no short sale allowed on the base currency. Moreover, the argument for an unhedged benchmark using a diversified currency basket does not translate into the smoothing of currency fluctuations due to high return correlations between major currencies both in strong- and weak-dollar economic environments. Black (1990), in a fairly provocative article, argued that a portion of foreign equity investments should be permanently unhedged, which is basically postulating that one should take a buy-and-hold position in currency with a fraction of the capital.

The purpose of this chapter is to review some of the currency arithmetic from an optimal hedge ratio perspective as well as provide interesting insights in the use of different risk preferences for asset and currency volatilities as well as their interactions. I propose the incorporation of a risk aversion parameter for the covariability of asset and currency positions. The point is that one should be able to understand the contribution of an active position to worsening or improving the benchmark risk properties. The use of this extra risk aversion parameter might help currency overlay managers derive more meaningful and profitable hedge ratios that are used for the management of the foreign exchange exposures in institutional portfolios. Furthermore, one could make the argument that the

[23] For other styles of portfolio management, increasing the breadth of the portfolio may not translate into immediate increase in information ratios due to the problems of keeping the skill level constant.

traditional performance measures including the information ratio are not incorporating this notion in the ranking of active portfolio managers. The skill in choosing the optimal bet size by any active portfolio manager; i.e., the magnitude of the active positions around the benchmark; would be captured effectively via the proposed methodology.

The chapter discusses the case for an endowment fund, which views the (embedded) currency exposure of the international assets in the portfolio as an opportunity to generate additional excess returns while reducing the risk of significant cash outflows that could result from settling forward contracts with a loss. I will follow the general procedure of balancing a particular hedging strategy's effects on the portfolio's expected risk/return profile. The process of effective hedge ratio management entails the combination of expected utility maximization with a comprehensive view as it pertains to the strength or weakness of the U.S. dollar vis-à-vis other currencies used by portfolio managers of foreign assets.

4. 2. Framework

I will analyze the effects of dual optimization in a portfolio of international assets. Let r_H, r_{FA}, r_f represent the rates of return on the hedged foreign assets, unhedged foreign assets (in US dollars), and the forward contract, respectively. Note that r_{FA} could be further broken down into two components: the return on the foreign asset in local terms, r_L, and the currency return, r_C, via $r_{FA} = r_L + r_C$. The significance of this relationship could be seen when the analyst uses the variance of foreign asset returns in local terms as opposed to US dollar terms in an efficient frontier analysis to justify the allocation into international investments. The variance of the foreign asset returns in local terms comes out to be less than the variance of returns in US dollars terms, unless the covariance of returns in local terms with the embedded currency returns is highly negative. This could easily be witnessed if one looks at the variance equation of r_{FA}. W stands for the weighting of the forward contract; i.e., the hedge ratio. The relationship between them can be explained via the following equation:

$$(4.1) \quad r_H = r_{FA} + r_f W$$

The variability of returns on the hedged foreign assets could be given as follows:

$$Var\left(r_H\right) = Var\left(r_{FA} + r_f W\right) = s_H^2$$

$$(4.2) \quad s_H^2 = s_{FA}^2 + s_f^2 W^2 + 2 s_{FA} s_f r_{FA,f} W$$

Then, the minimum variance (or risk minimizing) hedge ratio, W_{MV}, becomes –1 times the asset's beta with respect to the forward contract.

$$\frac{\partial s_H^2}{\partial W} = 2 s_f^2 W + 2 s_{FA} s_f r_{FA,f} = 0$$

$$(4.3) \quad W_{MV} = -\frac{s_{FA} r_{FA,f}}{s_f} = -b$$

One can conclude that W_{MV} will be different from zero in all cases unless the foreign asset's return in dollar terms and the forward return are near to being uncorrelated, $r_{FA,f} \approx 0$, which makes a strong case for an hedged position even for a risk minimizing investor. In order to evaluate the statistical properties of the risk minimizing hedge ratio, one could run a simple regression, $r_{FA} = a + b r_f + e$, and perform hypothesis tests on the estimates of beta. Moreover, the values of the coefficient of determination, $R^2 = \left[r_{FA,f} \right]^2$, from the estimated equation would gauge the effectiveness of the model, and hence, the ex-post validity of the derived risk minimizing hedge ratio[24].

Positive (negative) values of $r_{FA,f}$ result in W_{MV} being less (greater) than zero. Positive W_{MV} indicates shorting the U.S. dollar, which is not allowed in almost all overlay accounts. On the other hand, negative values represent various levels of hedging activity with the extreme case of –1 indicating full hedging back to the U.S. dollar, given the total portfolio is international.

Different levels of risk aversion and cost of hedging would (and should) affect the amount of hedging employed by the overlay manager. If we let m_H represent the return on hedged foreign assets and l the level of risk aversion, then the expected utility derived from currency hedging could be described as follows:

$$E(U) = m_H - l s_H^2 \tag{4.4}$$

The return on hedged foreign assets could be written as:

$$\begin{aligned} m_H &= W\left[m_f + m_L - c_f\right] + (1-W)(m_C + m_L) \\ &= W m_f + W m_L - W c_f + m_C - W m_C + m_L - W m_L \\ &= W\left[m_f - m_C - c_f\right] + m_C + m_L \end{aligned}$$

where

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[24] Note that low values of R-squared might be coupled with high hedge ratios in case when $s_{FA} >> s_f$. In these cases, the t-statistic of the estimated beta should be evaluated carefully from a statistical point of view.

$$m_f = \ln\left(\frac{F_0}{S_0}\right)$$

$$m_C = E\left[\ln\left(\frac{S_1}{S_0}\right)\right]$$

m_L … local return

c_f … transaction cost (bid-ask spread, which is approximately 5 basis points in currencies of developed countries)

When W=0, indicating no hedging, $m_H = m_C + m_L$. That is, the expected return on foreign assets becomes the sum of the currency return and local return. When W=1, indicating full hedging, $m_H = m_f + m_L - c_f$. It becomes the sum of forward return and local return after subtracting the cost of the bid-ask spread in the particular foreign exchange pair.

Let's plug the detailed definition of m_H into the expected utility relationship:

$$\begin{aligned}
E(U) &= W\left[m_f - m_C - c_f\right] + m_C + m_L - l\,s_H^2 \\
&= W\left[m_f - m_C - c_f\right] + m_C + m_L - l\left[s_{FA}^2 + s_f^2 W^2 + 2s_{FA} s_f r_{FA,f} W\right] \\
&= W\left[m_f - m_C - c_f\right] + m_C + m_L - l\,s_{FA}^2 - l\,s_f^2 W^2 - 2l\,s_{FA} s_f r_{FA,f} W
\end{aligned}$$

Then, the optimal hedge ratio for a utility maximizing agent can be derived as:

$$\begin{aligned}
&\frac{\partial E(U)}{\partial W} = \left[m_f - m_C - c_f\right] - 2l\,s_f^2 W - 2l\,s_{FA} s_f r_{FA,f} = 0 \\
&m_f - m_C - c_f - 2l\,s_{FA} s_f r_{FA,f} = 2l\,s_f^2 W_{UMV}
\end{aligned}$$

$$(4.5) \quad W_{UMV} = \frac{\mu_f - \mu_C - c_f - 2\lambda\sigma_{FA}\sigma_f\rho_{FA,f}}{2\lambda\sigma_f^2}$$

Now, simplify the resulting utility maximizing hedge ratio:

$$\begin{aligned} W_{UMV} &= \frac{\mu_f - \mu_C - c_f - 2\lambda\sigma_{FA}\sigma_f\rho_{FA,f}}{2\lambda\sigma_f^2} \\ &= \frac{\mu_f - \mu_C - c_f}{2\lambda\sigma_f^2} - \frac{2\lambda\sigma_{FA}\sigma_f\rho_{FA,f}}{2\lambda\sigma_f^2} \\ &= \frac{\mu_f - \mu_C - c_f}{2\lambda\sigma_f^2} - \frac{\sigma_{FA}\rho_{FA,f}}{\sigma_f} \\ &= \frac{\mu_f - \mu_C - c_f}{2\lambda\sigma_f^2} - \beta \end{aligned}$$

$$(4.6) \quad W_{UMV} = \frac{\mu_f - \mu_C - c_f}{2\lambda\sigma_f^2} + W_{MV}$$

The utility maximizing hedge ratio will not be too different from the minimum variance ratio as long as the risk aversion parameter and the volatility in the currency markets are significantly high. Suppose the minimum variance hedge ratio comes out to be –0.8. That is, $W_{MV} = -0.8$, indicating a hedge ratio of 80%, which implies the sale of the forward contract in an amount equal to 80% of the value of the underlying portfolio. This is by itself a significant bet both in a 50% hedged and, particularly, an unhedged benchmark, which the currency overlay manager is hired to beat by being on the right side of the currency movements.

One could define $W_U = \frac{\mu_f - \mu_C - c_f}{2\lambda\sigma_f^2}$ to represent the part of the optimal hedge ratio that includes risk preferences of the investor as well as the cost of hedging. Then,

$$(4.7) \quad W_{UMV} = W_U + W_{MV}$$

W_U will be less than zero, indicating some level of hedging of the foreign currency into the U.S. dollars, whenever $(m_f - m_C) < c_f$.

$$\begin{aligned} m_f - m_C &= \ln\left(\frac{F_0}{S_0}\right) - \ln\left(\frac{S_1}{S_0}\right) \\ &= \ln F_0 - \ln S_0 - \left[\ln S_1 - \ln S_0\right] \\ &= \ln F_0 - \ln S_1 \\ &= \ln\left(\frac{F_0}{S_1}\right) \end{aligned}$$

In other words, when this period's forward rate is less than the next period's expected spot rate, $F_0 < S_1$, indicating that F_0 underestimates the level of depreciation or overestimates the level of appreciation, W_U will be negative. For instance, if one believes the current forward rate is underestimating the level of depreciation or overestimating the level of appreciation in the Japanese yen against the U.S. dollar, she would sell the yen forward contract, and thus, increase the level of hedging. In this specific case, whatever value the risk aversion parameter or volatility of the forward contract takes on, the optimal hedge ratio, W_{UMV}, ends up being greater than the risk minimizing hedge ratio. It takes the portfolio manager farther away from an unhedged benchmark position, whereas the effect in a 50% hedged mandate depends on the level of W_{MV} [25].

[25] When $W_{MV} < -0.5$, e.g., $W_{MV} = -0.6$, W_{UMV} takes the hedge position farther from the 50% hedged benchmark. On the other hand, $W_{MV} > -0.5$, e.g., $W_{MV} = -0.4$, one gets closer to the symmetrical mandate position, resulting in smaller tracking error.

In addition, if the investor is more risk averse or the volatility of the forward contract is high, the currency manager is inclined to hedge even more from a utility maximization perspective. This theoretical result would be hard to justify in an unhedged mandate if there is a high probability (strong belief) that the U.S. dollar will depreciate in the next period. There should be a platform for the inclusion of probability statements about the strength of the dollar derived from the information set up to time t-1 that includes fundamental as well as technical factors. This would enable the currency manager to adjust (update) the expected utility maximizing hedge ratio in a Bayesian context. For instance, it would be interesting to include a prior distribution on the hedge ratio that reflects the knowledge regarding the strength of the dollar. Ideally, depending on the specific expectations in the foreign exchange markets, the prior could take the form of a uniform distribution.

Originally, l is defined as the risk aversion with respect to the variability in hedged asset return, which includes the return on unhedged foreign asset as well as the return on the forward contract. One might argue that the risk aversion parameter for the variability of the unhedged foreign asset, s_{FA}^2, should be different than the risk aversion parameter for the variability of the forward contract, s_f^2. In the above analysis, one implicit assumption is that the risk aversion parameter is the same for both types of risks. Within the context of portfolio construction, the risk aversion parameter is defined to balance the trade-off between risk reduction and return enhancement. It simply measures how many units of expected return one is willing to give up for each unit reduction in the variability of the return series. The assumption of equal risk aversion for foreign asset return volatility and forward contract return volatility does not seem reasonable due to different perceptions of risk that are due to differences in incentive structures of currency ovarlay managers and international equity managers. Despite the fact that agency problems might lead to irrational decision-making on the part of the endowment manager, my objective is to provide a framework that captures the inherent asymmetric nature of managerial incentives utilized in the management of international portfolios with active currency hedging component attached as a separate mandate.

Chow (1995) introduced the notion of mean-variance-tracking error (MVTE) optimization, which maximizes an objective function that includes different parameter values for absolute and relative risk aversion. He basically extended the objective function of [Expected Return – Risk Aversion x Risk] to include the tracking error, which is the standard deviation of relative return with respect to a benchmark. In his optimization algorithm, the objective function takes the form [Expected Return – Risk Aversion x Risk – Tracking Aversion x Tracking Error^2], which produces an efficient surface in three dimensions. The only way in which the dimensional expansion of the efficient frontier would fail to improve upon a constrained mean-variance optimization due to Markowitz (1952) is if the investor knew in advance what constraints were optimal. This ironically becomes the solution of the three-dimensional problem. There is also a behavioral argument to be made about the MVTE optimization, which takes care of two simultaneous fears of investors, namely being wrong (absolute performance) and alone (relative performance).

My belief is that within the context of active currency management and expected utility maximization, s^2_{FA} could be viewed as the asset volatility estimator and s^2_f as the currency volatility estimator; i.e., the tracking error. Moreover, one could actually incorporate different risk aversion parameters for absolute and relative volatilities instead of imposing ad hoc constraints in the optimization process. To this end, I will define l_a and l_c to represent the asset and currency risk aversion parameters, respectively. At this point, I would like to come back to the previous discussion of using a framework that would use additional information about the strength of the dollar in the setting I am proposing. If we suppose that additional information gathered through analysis of patterns in the price charts of various currencies leads to a strong dollar in a trending environment, a position near an unhedged benchmark would jeopardize the success of the overlay program. The overall risk would be very high, whereas the risk measured against devations from the benchmark position; i.e., tracking error, would be non-existing. In technical terms, if a manager puts a larger weighting on the asset risk parameter versus the currency counterpart in an unhedged mandate; i.e., $l_a < l_c$, the strength of the dollar would result in unacceptable performance in the absolute dollars generated by the currency overlay program. The extended utility maximization framework helps the endowment fund manager incorporate the additional information on the dollar by assuming $l_a >> l_c$, which would, in return, increase the financial efficiency of the overall portfolio.

If we relax the assumption that $r_{FA,f} = 0$, we could introduce l_m as the risk aversion parameter for the covariability of foreign asset and currency returns. The use of a different aversion parameter for the covariance in the expected utility framework could be justified in terms of completely different dynamics of asset versus currency markets as well as the descriptive nature of the approach used here whereby certain irrationalities could be modeled more effectively. The assumption of one common risk aversion parameter for both markets and their interaction could yield misleading hedging decisions. In this setting, the following optimal hedge ratio would result:

$$(4.8) \qquad W^*_{UMV} = \frac{m_f - m_C - c_f}{2 l_c s_f^2} + \frac{l_m}{l_c} W_{MV}$$

Proof:

$$E(U) = W\left[m_f - m_C - c_f\right] + m_C + m_L - l_a s_{FA}^2 - l_c s_f^2 W^2 - 2 l_m s_{FA} s_f r_{FA,f} W$$

$$\frac{\partial E(U)}{\partial W} = m_f - m_C - c_f - 2 l_c s_f^2 W - 2 l_m s_{FA} s_f r_{FA,f} = 0$$

$$m_f - m_C - c_f - 2 l_m s_{FA} s_f r_{FA,f} = 2 l_c s_f^2 W^*_{UMV}$$

$$W^*_{UMV} = \frac{m_f - m_C - c_f - 2 l_m s_{FA} s_f r_{FA,f}}{2 l_c s_f^2}$$

Further simplification of this relationship provides interesting insights:

$$W_{UMV}^{*} = \frac{m_f - m_C - c_f}{2 l_c s_f^2} - \frac{2 l_m s_{FA} s_f r_{FA,f}}{2 l_c s_f^2}$$

$$= \frac{m_f - m_C - c_f}{2 l_c s_f^2} - \frac{l_m}{l_c} \frac{s_{FA} r_{FA,f}}{s_f}$$

$$= \frac{m_f - m_C - c_f}{2 l_c s_f^2} - \frac{l_m}{l_c} b$$

$$W_{UMV}^{*} = \frac{m_f - m_C - c_f}{2 l_c s_f^2} + \frac{l_m}{l_c} W_{MV} \qquad \text{Q.E.D.}$$

One could argue that $l_c > l_m$ in many cases due to the fact that currency managers are regret minimizers with the objective of not being farther away from the benchmark as well as concerns regarding the possibility of large cash outflows at times of forward contract settlements. In most cases, this behavior is a by-product of the attitude of institutional investors toward currency overlay as a fund management vehicle. Please note that I am not using the term ‘as an investment management vehicle’, because active currency hedging programs in the traditional sense do not require additional investments by the endowment fund unless there is an explicit intent to use them as portable alpha engines[26] as part of the absolute return part of the portfolio. Given any risk aversion parameter for currency volatility, the argument results in an optimal hedge ratio that is lower than the hedge ratio, which is computed without l_m. That is, $W_{UMV}^{*} > W_{UMV}$ given $l_c > l_m \rightarrow \frac{l_m}{l_c} < 1$.

[26] Kritzman (1993b), in the context of a risk-minimizing investor, shows that one should hedge the currency risk of investment portfolios even if they contain no foreign assets. This illustrates the importance of the currency as an asset class and a significant component of the risk/return profile.

Qualitatively, the implication of this result is that assigning different risk aversion parameters for the foreign asset variance and currency variance in the expected utility framework along with the hesitancy toward taking too much bets in the currency markets results in lower hedge ratios, ceteris paribus. In addition, the asymetric nature of currency overlay manager's incentive structure encourages aggressive active management of hedge ratios with the belief that higher volatility would result in better compensation for the manager.

4. 3. Active Currency Hedging for a Disappointment-Averse Investor

Proponents of the fairly new, but rapidly evolving, field of behavioral finance postulate that portfolio selection, with or without international assets, is much more complicated than simply making a choice from the efficient frontier that optimally balances expected return against volatility as measured by the standard deviation of returns. The expected utility maximization framework utilized in the previous section for the computation of optimal hedge ratios consider behavioral aspects of decision-making by board members of university endowment funds. I had motivated the use of different risk aversion parameters for asset and currency volatilities in terms of differences in the managerial incentives for asset and currency managers as well as agency problems that I had phrased as 'governance issues.' It is a well-known fact that people are guided by emotions, which often reflect a certain kind of benchmarking in the process. Benchmarks could take various forms, for instance, an explicit one could be the performance of a particular index or a more implicit one is the problem-specific reference point/subsistence level used to measure gains or losses. Many scholars from economics and psychology studied the manner in which regret influences decisions and people's emotional disposition. Regret could be defined as the pain relative to not having taken a better action, whereas disappointment is the pain from comparing the actual outcome with a better one. In other words, regret captures the difference between the performance of the selected portfolio and the performance of any other foregone portfolio; whereas disappointment captures the discrepancy between actual and expected performance.

Shefrin (2000) presents an example from financial markets that would help us differentiate these closely related concepts, both of which is based on the act of comparison: It was late July of 1998 when financial markets were jittery about economic problems in Asia and the market had just fallen by 20 percent. Imagine a conversation between you and two of your friends, George and Paul. George had a lot of his portfolio in stocks and was fretting about a severe market decline. In the end, he decided to sell his stocks and buy CDs instead. Paul, instead, had been holding CDs which had just matured. He thought that the market would rebound and considered buying mutual fund shares. However, he renewed his CDs. Thereafter, the market appreciated by over 25 per cent. Both investors held CD portfolios during this period. Both would have been better off by holding stocks. The question is; which one feels worse about the situation? Most people would say George is not only disappointed about the outcome but also experiences regret stemming from the action he took. So, he seems to be worse off emotionally. Interestingly, this example proves the point that behavioral aspects are typically path-dependent, meaning where you start, what you think and when all lead to distinct emotions at the end.

Regret and disappointment are important factors when it comes managing investment portfolios that are connected to a certain kind of benchmarking mechanism. We could say that comparison is the psychological basis for benchmarking. I agree with Shefrin that people are hard-wired to engage in comparison, and measure themselves against some benchmark. I would further argue that the challenge becomes how to come up with the most relevant benchmark in any given situation so that whatever actions we might take would not result in regret and disappointment. Those who are fearful of experiencing regret may fear taking an action that will leave them vulnerable to regret.

People are especially prone to feeling the regret of a decision that turned out badly when they feel responsible for that decision. Institutional investors such as pension plans and endowments transfer responsibility when they engage the services of money managers. In addition to transferring the responsibility of managing the funds at the institutions, board members hire consultants for advice on which money managers to choose for the institutional portfolio. It could be conjectured that trustees at various pension funds and endowments create a psychological option for themselves by taking these actions on behalf of the institution for which they serve as fiduciaries. When the portfolios perform well, they can take the credit, otherwise, they can shift the blame to the money managers and consultants. Very recently, Unilever (U.K.) even sued Merrill Lynch Asset Management for negligence and reportedly settled the case outside the court system at the expense of the money management firm. So, we started witnessing legal actions against money managers that perform below par for not having adhered to the guidelines set forth by the client. There is also one more reason why institutions find the (partial) transfer of responsibility appealing: cognitive limitations. I would argue that some fiduciaries are unable to differentiate between payoff-irrelevant information (also called ‘noise’) and payoff-relevant information, mostly due to cognitive biases in processing information. Lastly, the fact that investors have the tendency to evaluate gains and losses frequently leads to second-guessing exercises and attempting to resolve the regret and disappointment issue.

Here, I will concentrate on the disappointment aversion framework, introduced by Gul (1991)[27], and investigate the optimal hedging behavior for a disappointment-averse hedger. Unlike the notion of risk aversion, feelings of disappointment violate the separability axioms that impose that preferences are independent across states; that is, outcomes in events that did not occur affect attitudes towards outcomes that did. Regret, on the other hand, involves comparing outcomes in a given event with those that would have occurred in the same event had the agent chosen a different act or lottery or portfolio for that matter. Disappointment involves comparing outcomes from different events in the same act or lottery. In principle, one could be disappointed without ever having choices to make. The preferences will be a one parameter extension of standard

[27] Grant and Kajii (1998) and Skiadas (1997) provided two other notions of disappointment aversion. Grant *et al.* (2001) demonstrate how different formalizations lead to different notions of disappointment aversion by comparing the models of Gul, Grant & Kajii, and Skiadas.

iso-elastic preferences in the usual expected utility framework. They have the characteristic that good outcomes that are above the certainty equivalent are downweighted relative to bad outcomes. The use of disappointment averse preferences is particularly beneficial in the case of currency hedging due to the complexity of the issue and resulting behavioral concerns of fiduciaries. It is shown that disappointment aversion utility displays first order risk aversion, where the risk premium is proportional to standard deviation, as opposed to variance in the case of expected utility. This feature helps one to account for the phenomenon that individuals are risk averse with respect to gambles which yield a large loss with small probability (as in the stock market) but risk loving with respect to gambles that involve winning a large prize with small probability (as in lottery gambles).

Both disappointment aversion and loss aversion, according to the prospect theory of Kahneman and Tversly (1979), define the utility function asymmetrically over gains and losses relative to a reference point. For a loss-aversion utility function, the reference point is arbitrarily exogenously chosen, whereas disappointment-averse utility function determines the reference point endogenously that could be updated over time. The second appealing aspect of this kind of framework is that it is fully axiomatic and provides a normative theory, eliminating the need for ad hoc techniques witnessed in the descriptive theoretical frameworks. Lastly, the fact that standard preferences are a special case of disappointment averse preferences with the loss aversion parameter put equal to one. Thus, one could capture many of the asymmetric affects of loss aversion without resorting to behavioral theory[28].

The preferences of a disappointment averse agent could be summarized by $\left[U(R), b\right]$, where U is a conventional utility function describing the utility of earning the rate of return R from a given investment, and $b \geq 0$ is a parameter that measures the degree of disappointment aversion. In the absence of disappointment aversion, the agent's utility level is simply $\left[U(R)\right]$. Now, I will define the expected utility of a disappointment-averse agent as $V(b)$ with b representing the degree of disappointment aversion. Suppose that the agent faces uncertain rates of return, R, in n states of nature. Let m denote the certain return that yields the same utility level as the uncertain return: $V(b) = U(m)$. This means, the investor is indifferent between the prospect of a safe return and risky return in n states of nature. The agent reveals disappointment aversion if she attaches extra disutility to circumstances where the realized return is below m. The disappointment-averse utility function could be defined as:

$$V(b) = E\left[U(R)\right] - b\,E\left[U(m) - U(R) \mid R < m\right]$$

$E\left[U(m) - U(R) \mid R < m\right]$ is the expected value of $U(m) - U(R)$, conditional on the realized return being below the certainty equivalent return. In other words, the term $E\left[U(m) - U(R) \mid R < m\right]$ measures the average disappointment. It is the expected discrepancy between the certainty equivalence utility and the actual

[28] A different treatment of an investor's asymmetric response to gains and losses is given by Roy (1952), Browne (1999), Stutzer (2000), and Maenhout (2001), who model agents with the objective of minimizing the possibility of undesirable outcomes.

utility in states of nature where the realized return is below the certainty-equivalent return. Basically, the disappointment-averse expected utility equals the conventional expected utility, adjusted downwards by a measure of disappointment aversion, b, times the "conditional expected disappointment."

I will now define two states of nature, whereby the agent earns the return *R* in state 1 or 2, and $R_1 > R_2$ with probabilities $(a, 1-a)$, respectively. Now, we are in a position to redefine $V(b)$:

$$V(b) = aU(R_1) + (1-a)U(R_2) - b(1-a)\left[V(b) - U(R_2)\right]$$

Further rearranging of the terms helps separate the utilities of earning R_1 and R_2:

$$V(b) = a\left[1-(1-a)d\right]U(R_1) + (1-a)(1+ad)U(R_2)$$

where $d = \dfrac{b}{1+(1-a)b}$

If the agent is disappointment-averse, that is $b > 0$, he attaches extra weight $(1-a)ad$ to bad states; i.e., in the case of R_2, when is disappointed (relative to the probability weight used in the conventional utility), and attaches a lesser weight $-a(1-a)d$ to good states. Note that when $b = 0$, *V(.)* simplifies to the conventional expected utility.

I will define R_1 and R_2 as follows:

$$R_1 = W\left(m_f - m_{c1} - c_f\right) + m_{c1} + m_L$$

$$R_2 = W\left(m_f - m_{c2} - c_f\right) + m_{c2} + m_L$$

where $m_{c1} = \ln\left(\frac{S_{11}}{S_0}\right)$ and $m_{c2} = \ln\left(\frac{S_{12}}{S_0}\right)$ and $S_{11} < S_{12}$. Stated differently, in the case of an international portfolio, the good state refers to the spot currency rate being smaller than the one in the bad state. This refers to the fact that a smaller spot rate indicates appreciation of the foreign currency against the U.S. dollar, providing higher return on invested capital due to currency movements. All the remaining variables are as defined before.

The objective is to find an optimal hedging behavior by maximizing the disappointment-averse utility function, *V(b)*. On taking partial derivative with respect to *W*, we have

$$\frac{\partial V(b)}{\partial W} = a\left[1-(1-a)d\right]U'(R_1)\left(m_f - m_{c1} - c_f\right) + (1-a)(1+ad)U'(R_2)\left(m_f - m_{c2} - c_f\right)$$

The optimal value of *W*, W^*, must satisfy the following condition: $\frac{\partial V(b)}{\partial W} = 0$.

The optimal forward position should equate the following two terms in the above equation:

$$a\left[1-(1-a)d\right]U'(R_1)\left(m_f - m_{c1} - c_f\right) = -(1-a)(1+ad)U'(R_2)\left(m_f - m_{c2} - c_f\right)$$

If we assign the same probability to states 1 and 2, this relationship could be simplified further. Given the fact that $a = (1-a)$;

$$a\left[1-(1-a)d\right]U'(R_1)\left(m_f - m_{c1} - c_f\right) = -(1-a)(1+ad)U'(R_2)\left(m_f - m_{c2} - c_f\right)$$
$$a(1-ad)U'(R_1)\left(m_f - m_{c1} - c_f\right) = -a(1+ad)U'(R_2)\left(m_f - m_{c2} - c_f\right)$$
$$\left(a - a^2 d\right)U'(R_1)\left(m_f - m_{c1} - c_f\right) = \left(a + a^2 d\right)U'(R_2)\left(m_{c2} - m_f - c_f\right)$$

When a increases, the RHS increases more than the LHS. As a result, $U'(R_1)$ must increase to restore the equality. That is, R_1 must decrease, or equivalently, W^* must decrease. In other words, an increase in the probability of the good state occurring reduces the optimal forward position, which is an intuitive conclusion based on the definition of the good state. That is, the higher the probability of the foreign currency appreciating , the less likely it is to hedge these currency positions. In other words, the endowment benefit more by not hedging any significant portion of the foreign exchange exposure.

It could also be shown that $\frac{\partial W^*}{\partial b} < 0$. As the hedger becomes more disappointment-averse, a smaller forward position will be held. This result is in line with the conclusion derived in the previous section, which introduced behavioral arguments into the traditional expected utility framework by defining different risk aversion parameters for asset and currency volatilities.

4. 4. Conclusion

The purpose of this chapter was to incorporate behavioral issues as it relates to the active management of currency hedging of international portfolios in the context of traditional expected utility maximization as well as the axiomatic disappointment aversion frameworks. I have introduced separate risk aversion parameters for asset and currency markets, and due to the asymmetric nature of the compensation structure of currency managers, concluded that lower hedge ratios would arise, ceteris paribus. Alternatively, I evaluated the same problem in the disappointment averse utility function setting, and concluded that the more the endowment fund manager gets disappointment averse, the less likely it is to have higher hedge ratios.

Whatever the style of problem solving might be, the right approach for designing currency overlay program including the choice of the appropriate benchmark, the combination of an effectively diversified group of managers as well as performance monitoring should entail the analysis of the effects of various outcomes on the fund's asset liability structure given financial objectives and governance constraints.

As a further research inquiry I would a priori argue that the level of surplus, as defined by assets minus liabilities or excess return distribution, could be a significant determinant for the modeling of the active currency hedging issue.

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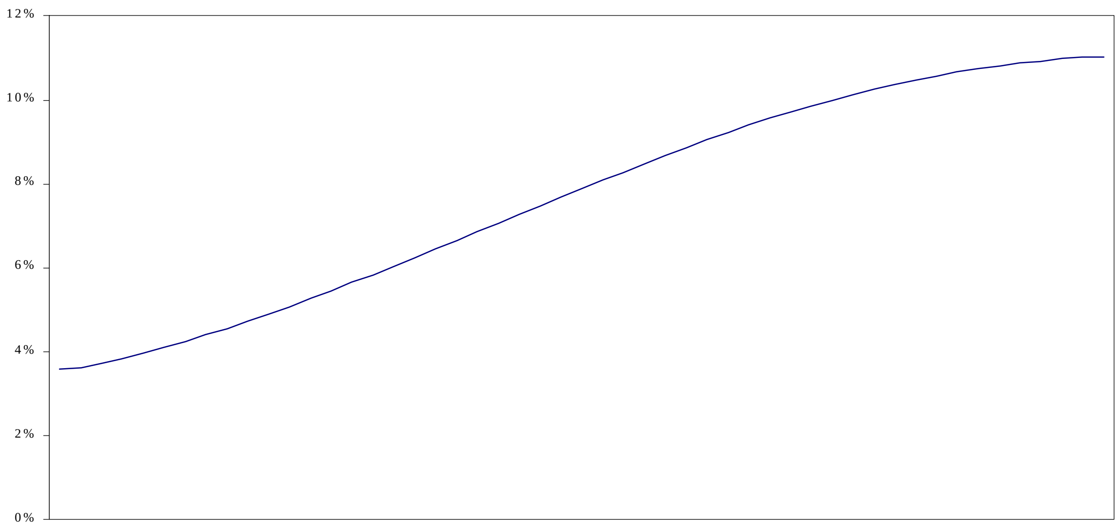
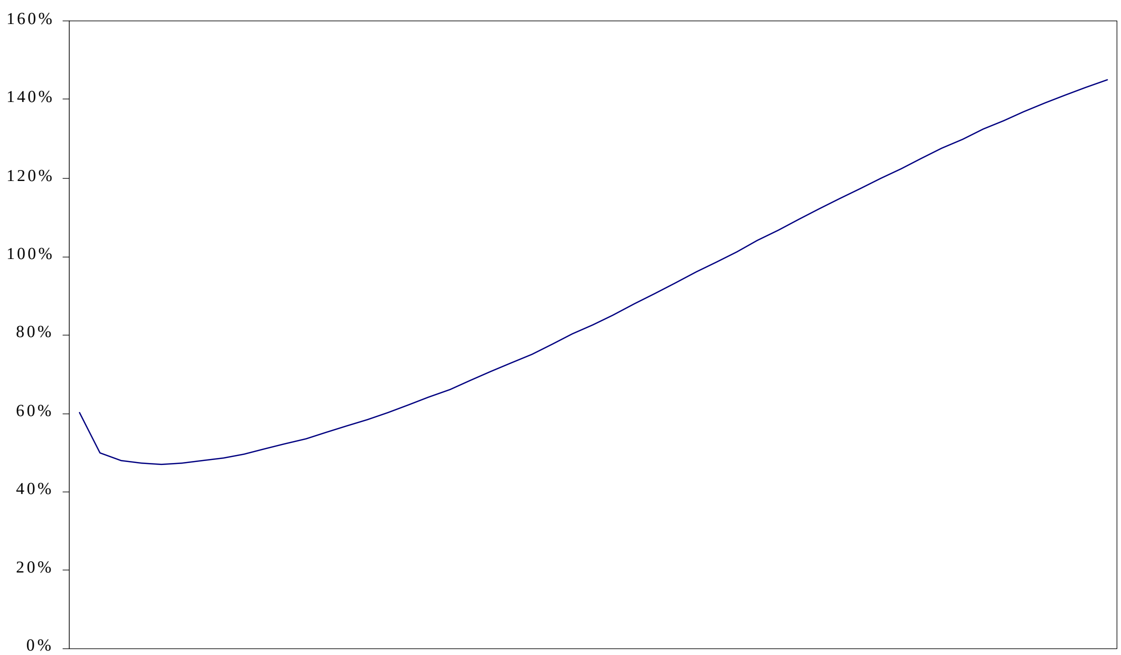
Appendix

Sensitivity Analysis

Spending rate



Stock allocation



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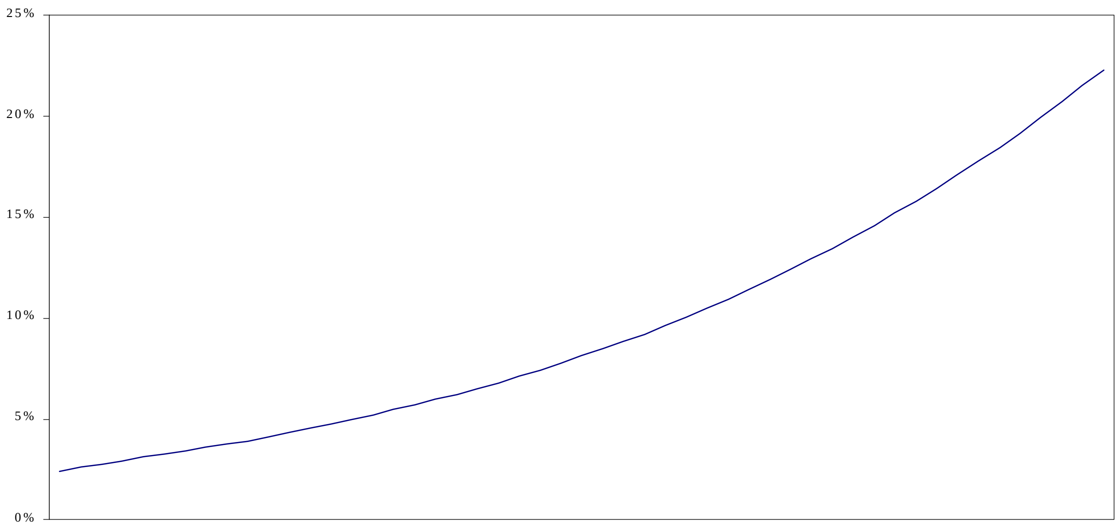
Spending rate



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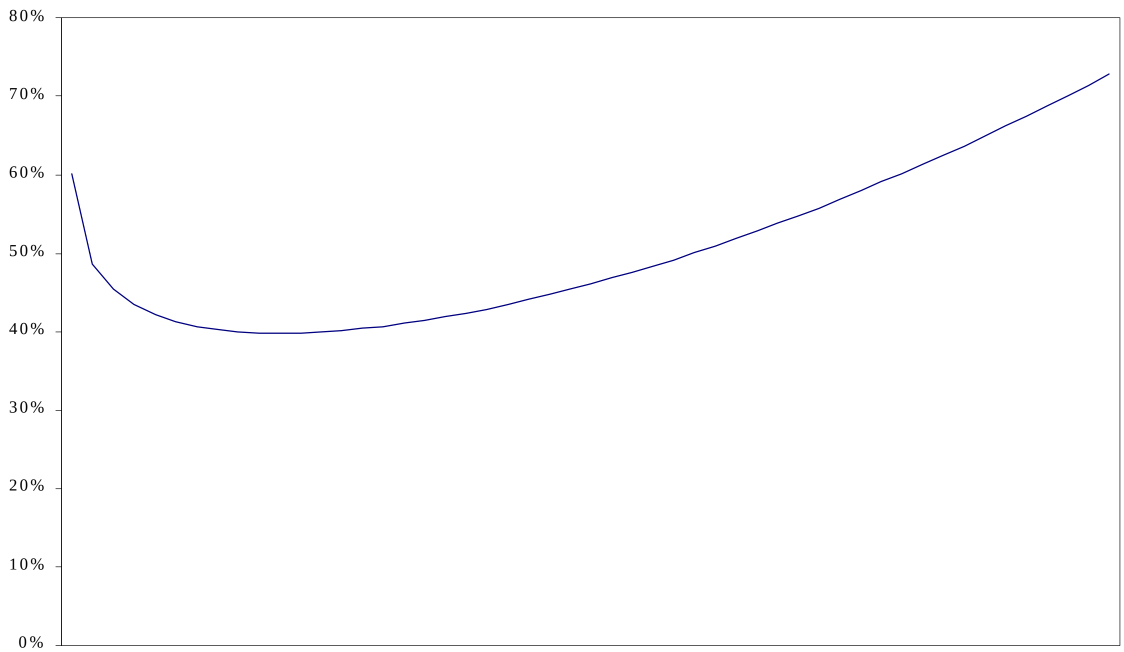
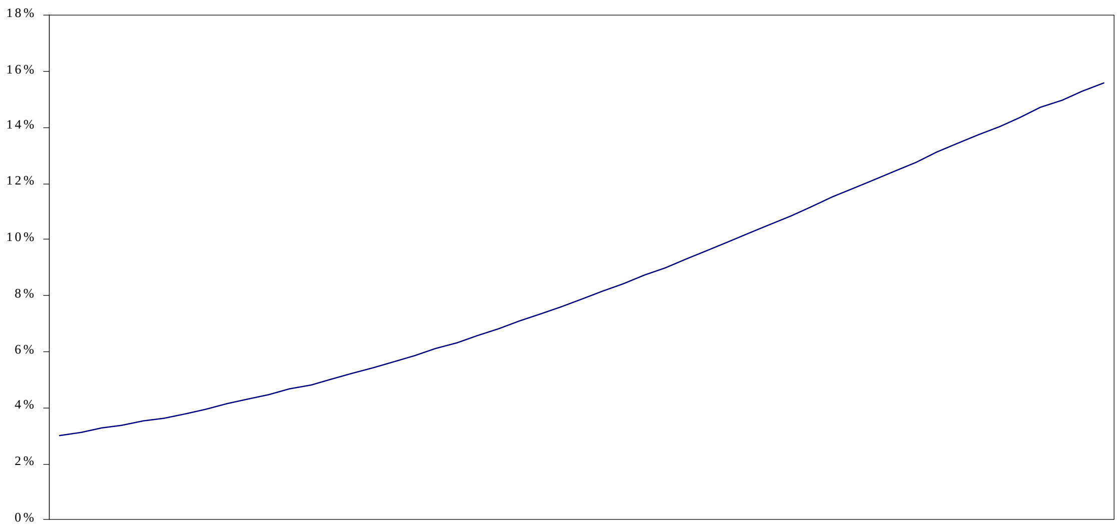


***m*=8%, *s*=20%, α=2%, π=2%, *r*=5%, γ=0.42, ρ=5%, n=-1**

Spending rate



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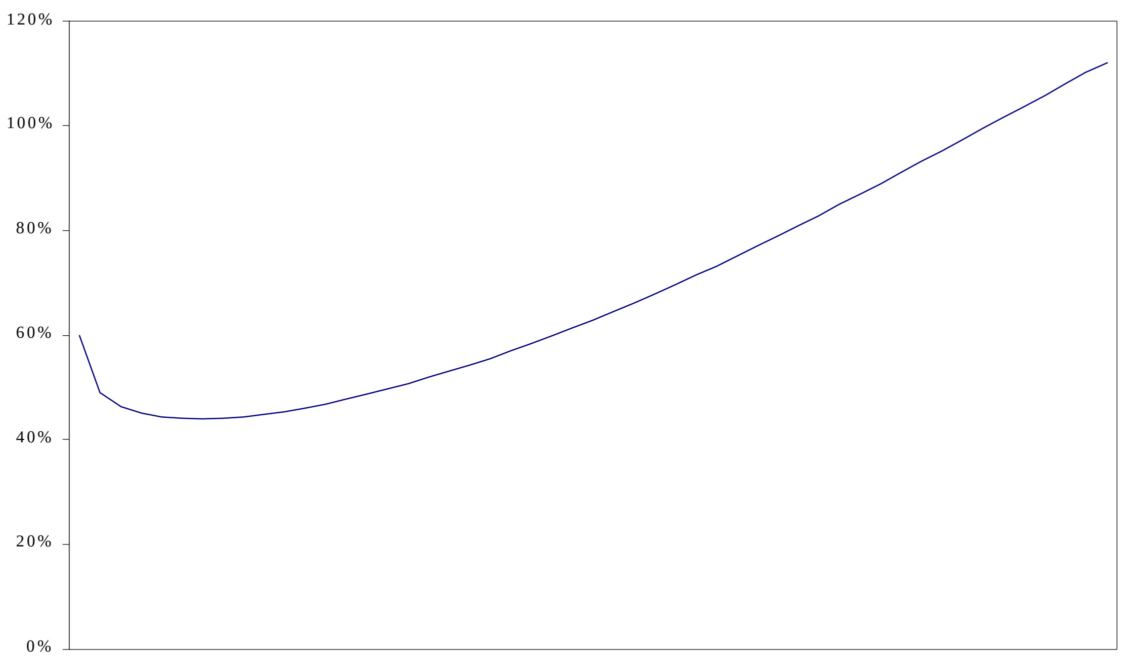

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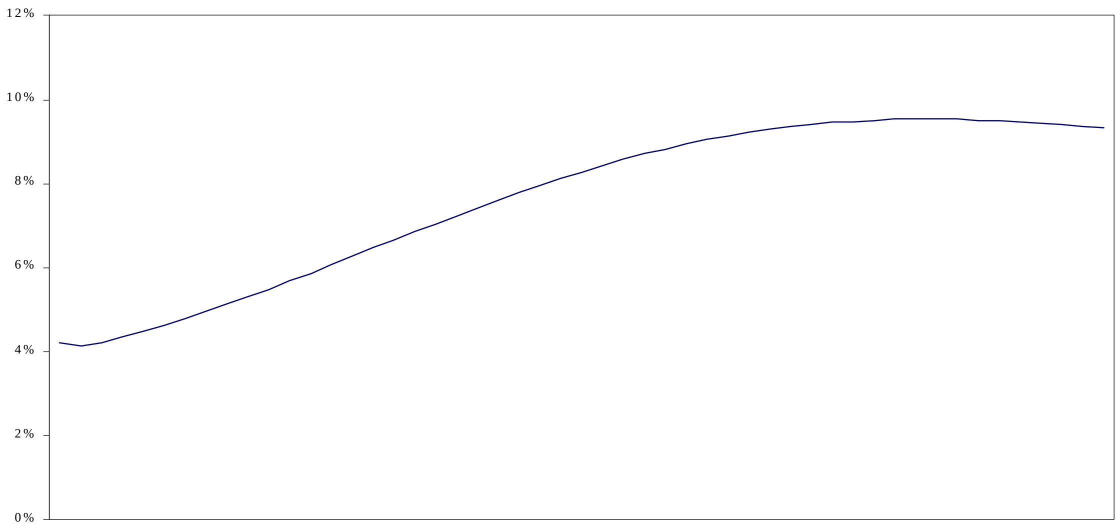
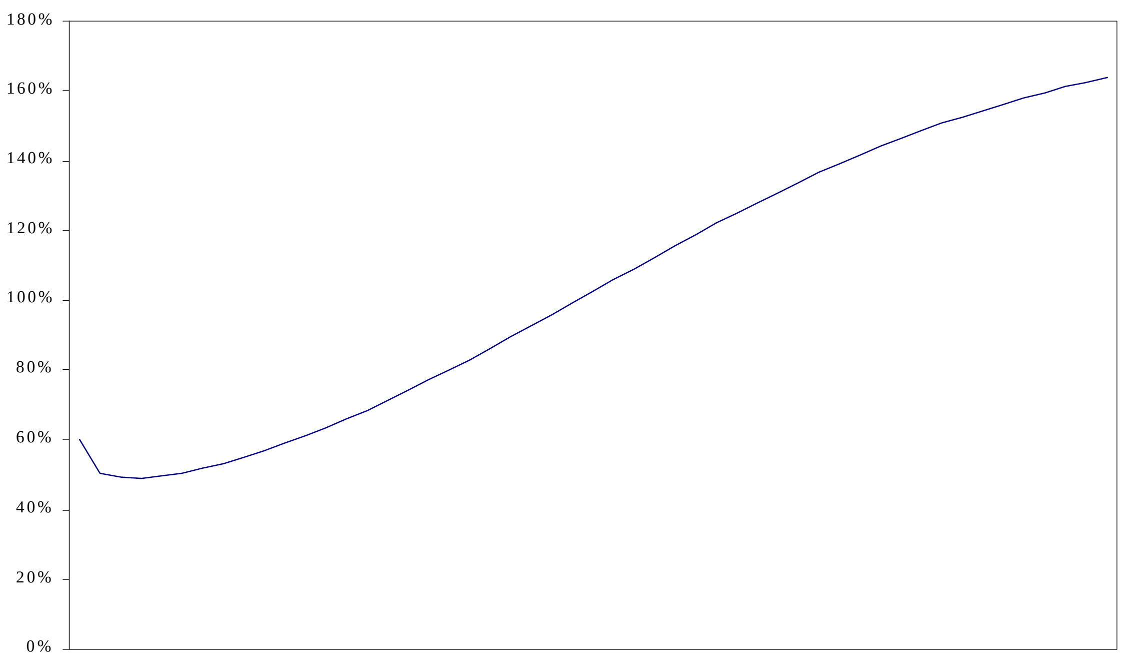
Spending rate



Stock allocation



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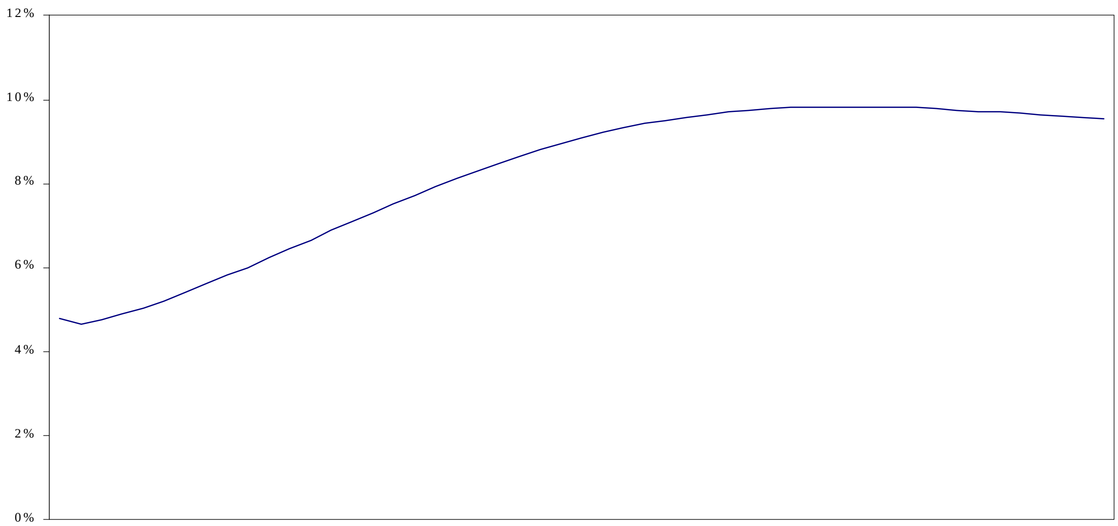
Spending rate



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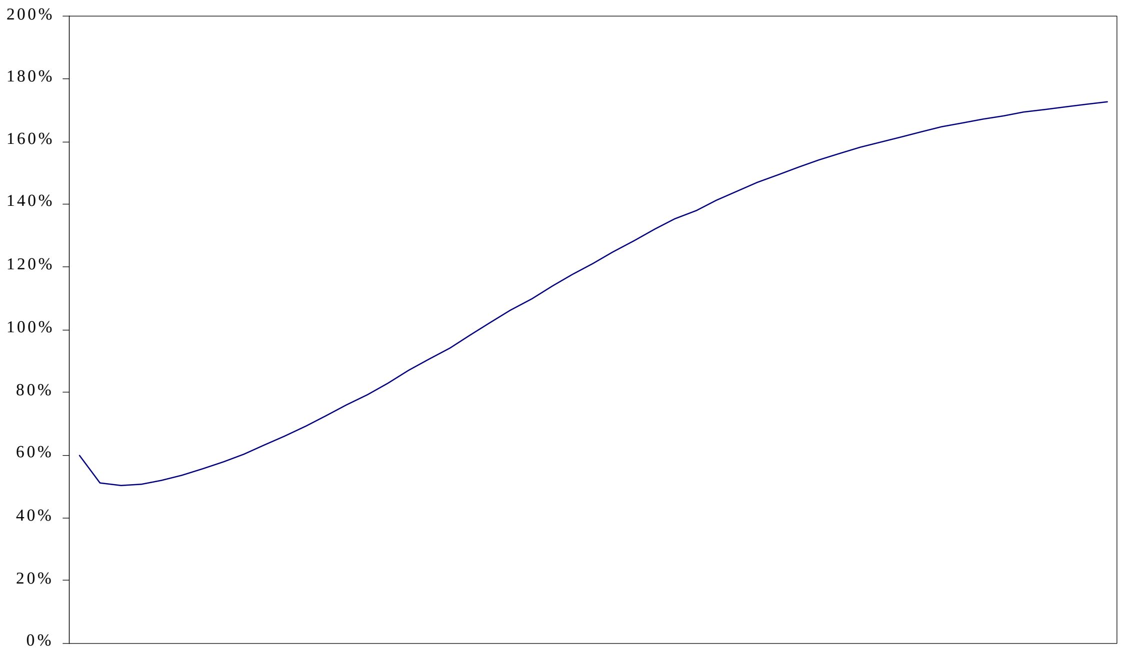

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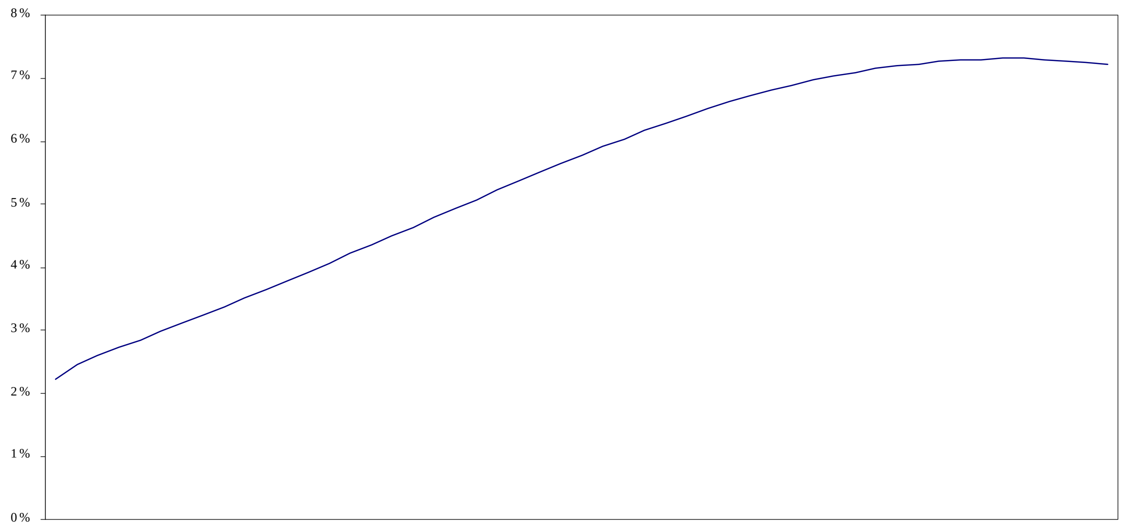
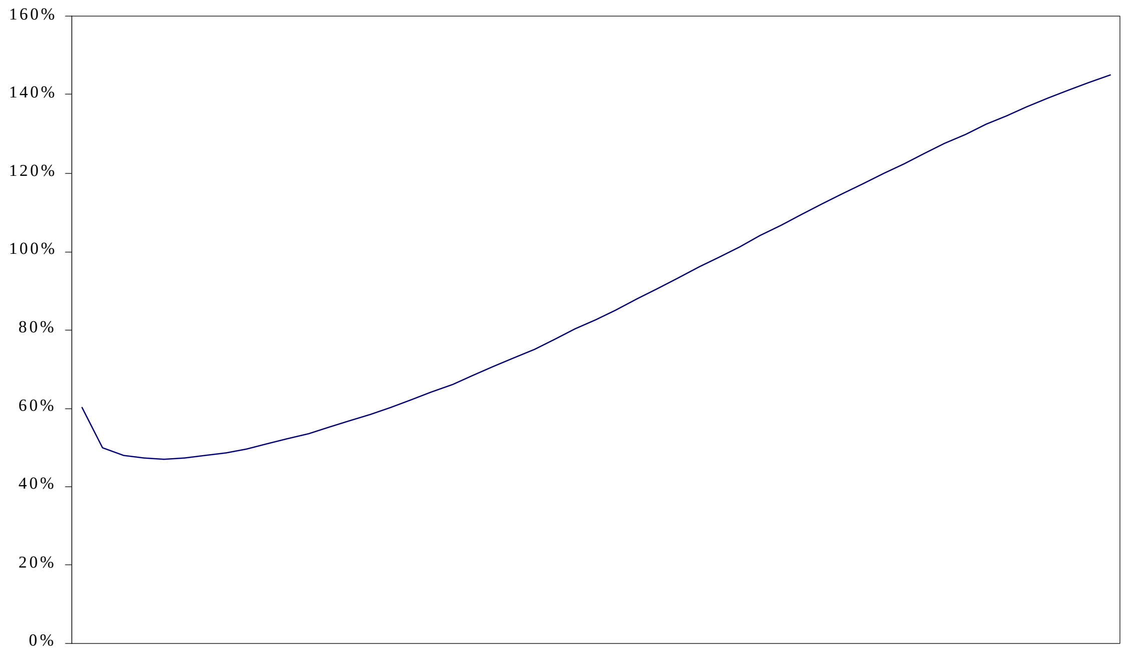
Spending rate

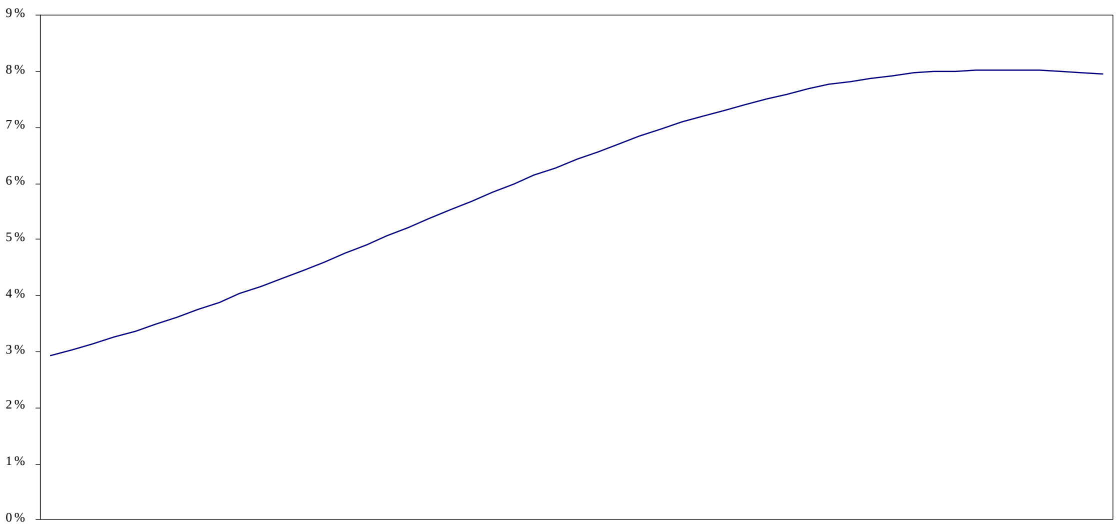


Stock allocation

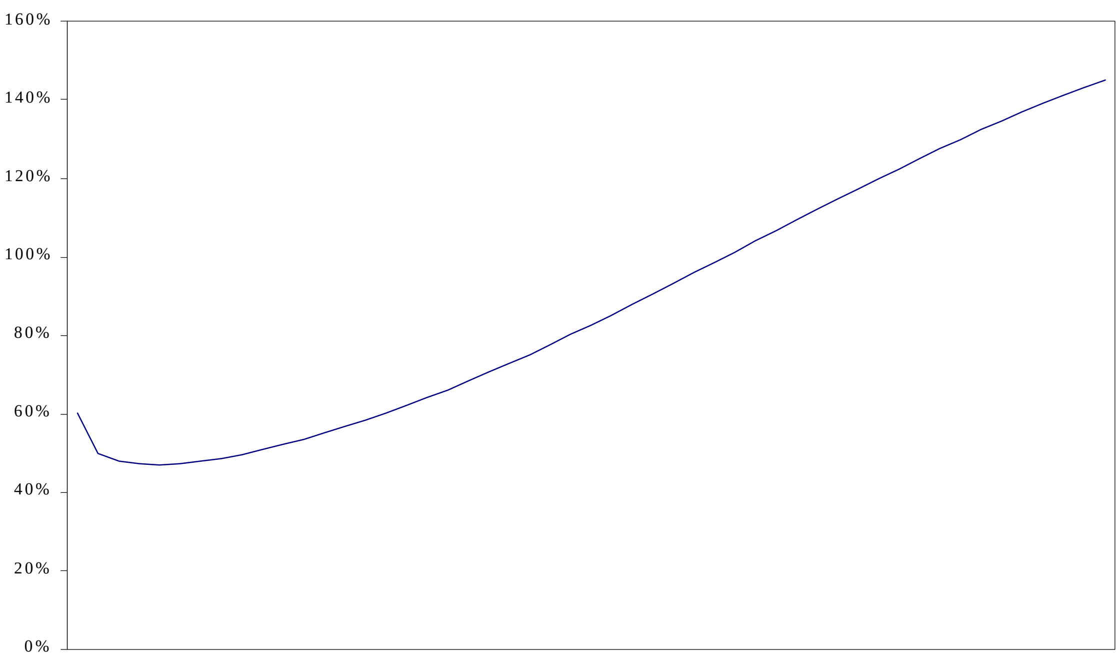


m=12%, s=20%, α=2%, π=2%, r=5%, γ=0.97, ρ=1%, n=-1

Spending rate



Stock allocation



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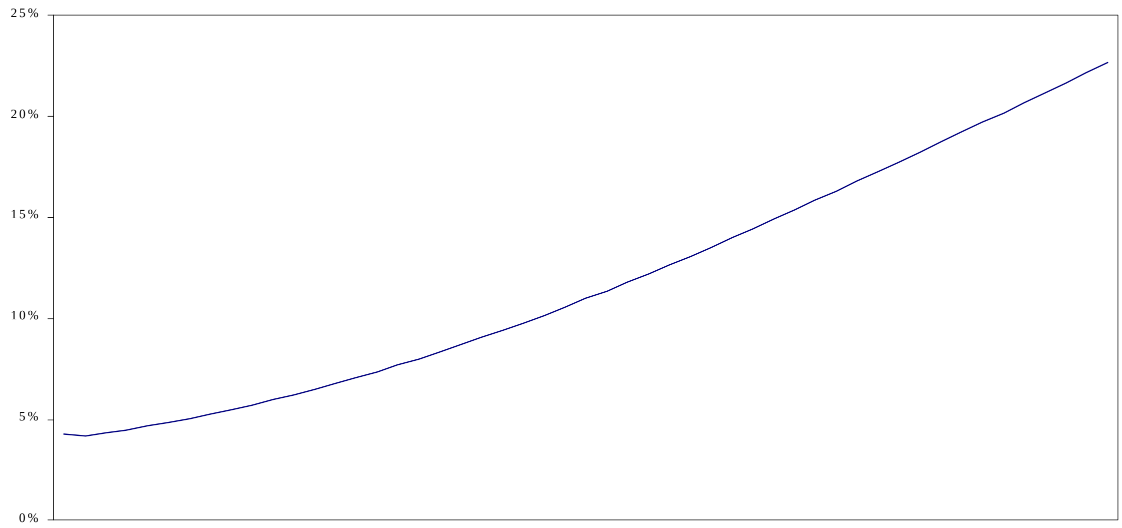
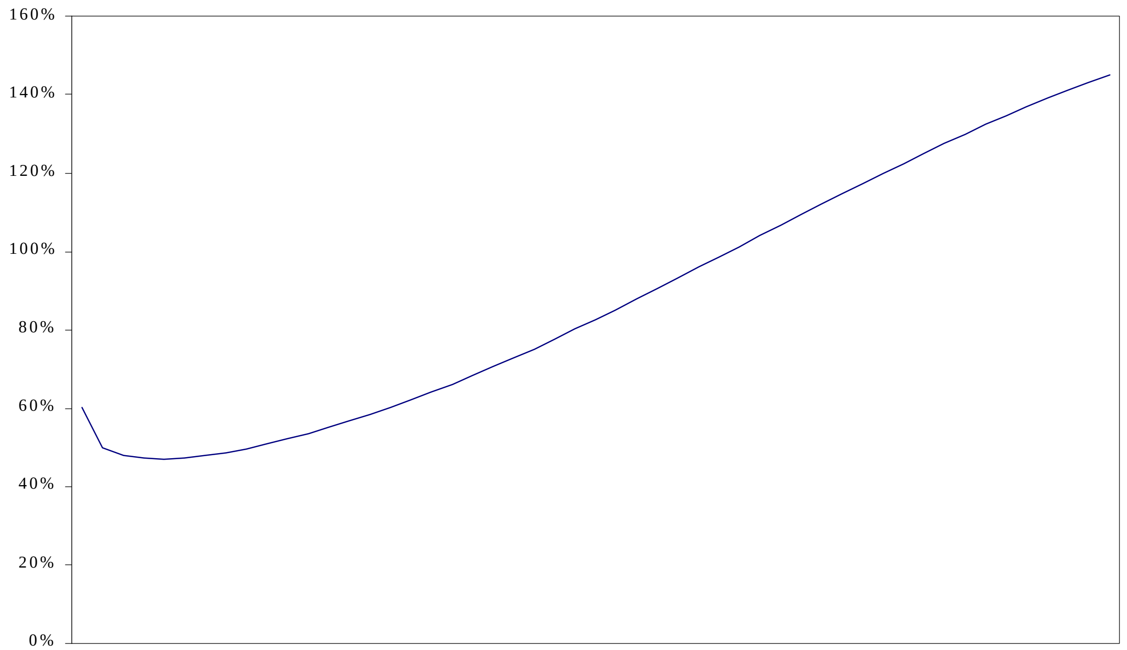
Spending rate

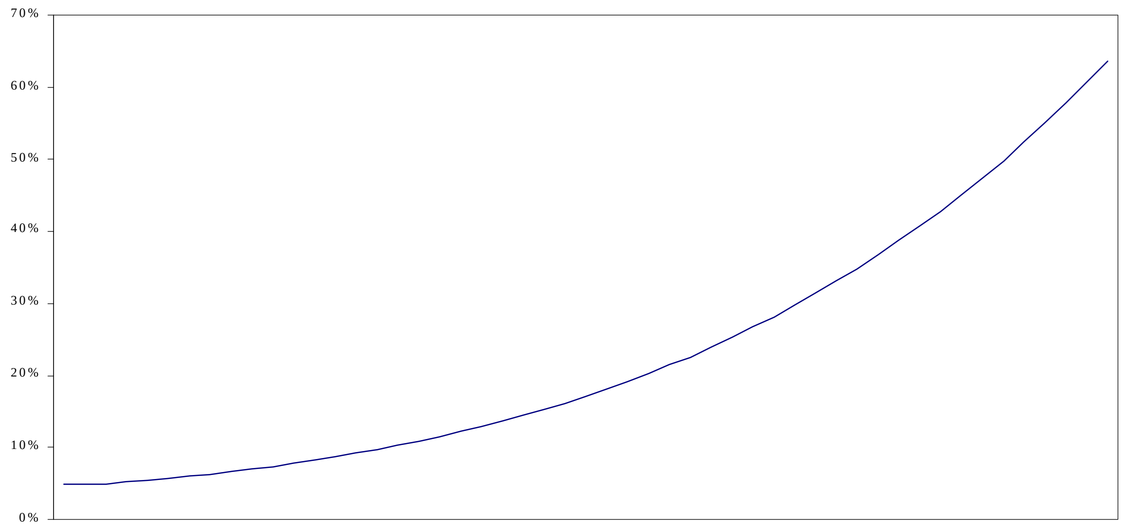


Stock allocation

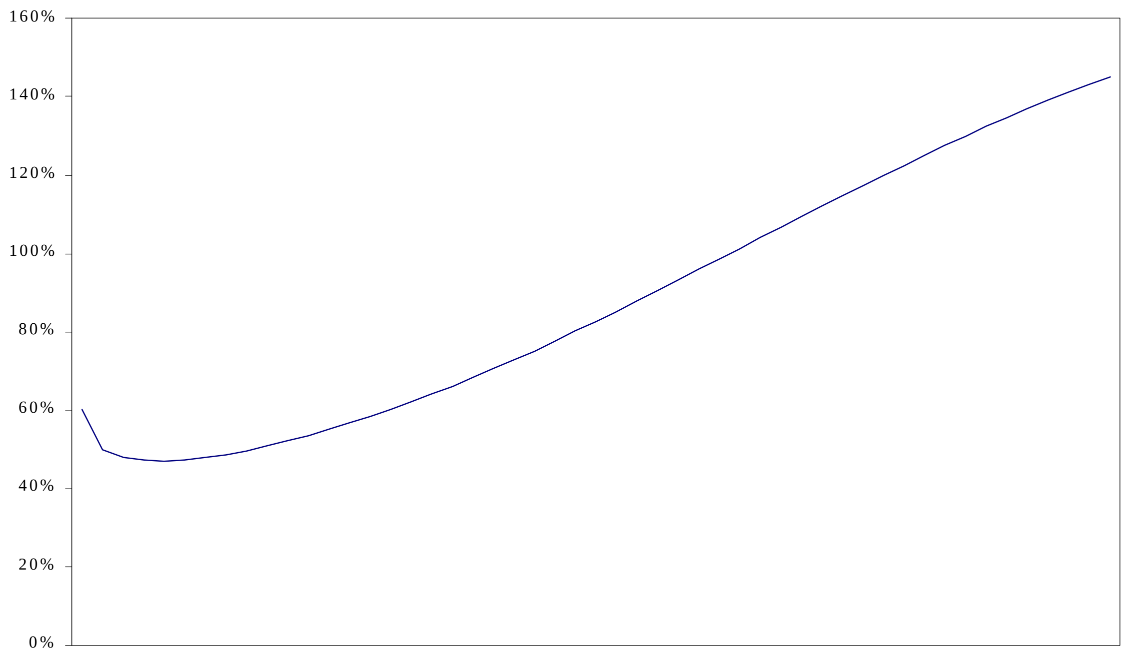


***m*=12%, *s*=20%, α=2%, π=2%, *r*=5%,γ=0.97, ρ=7%, n=-1**

Spending rate



Stock allocation



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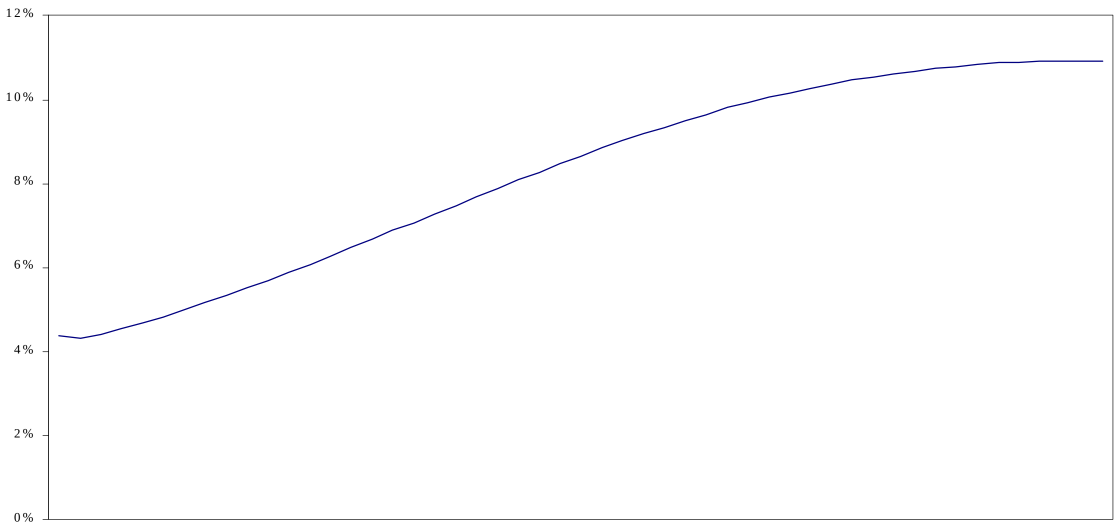
Spending rate



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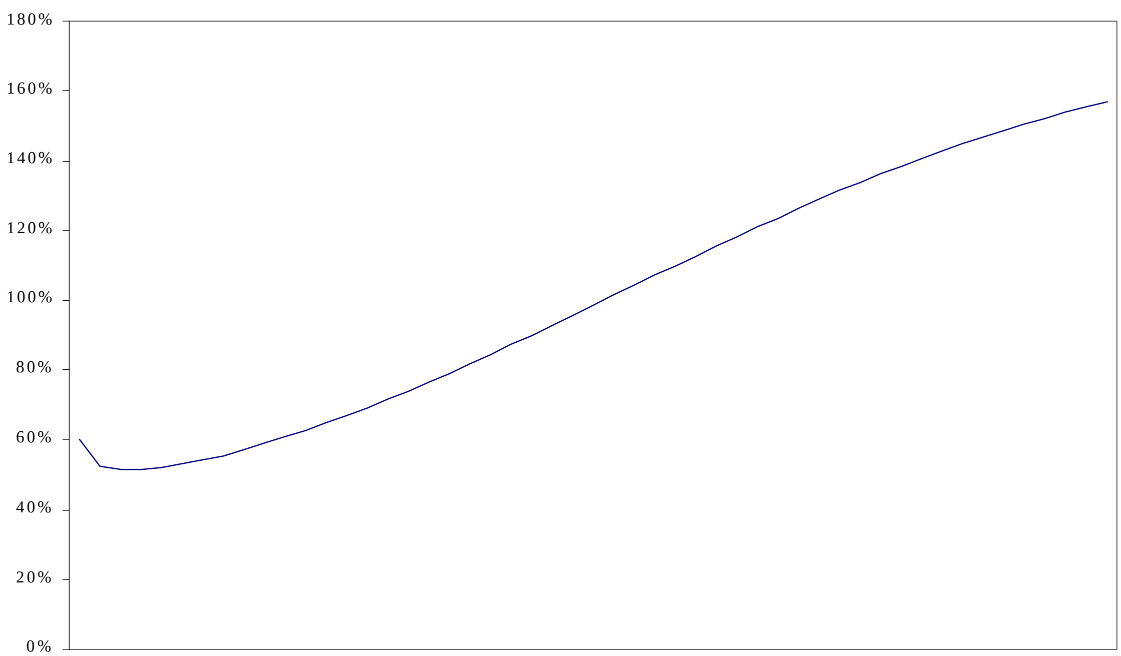

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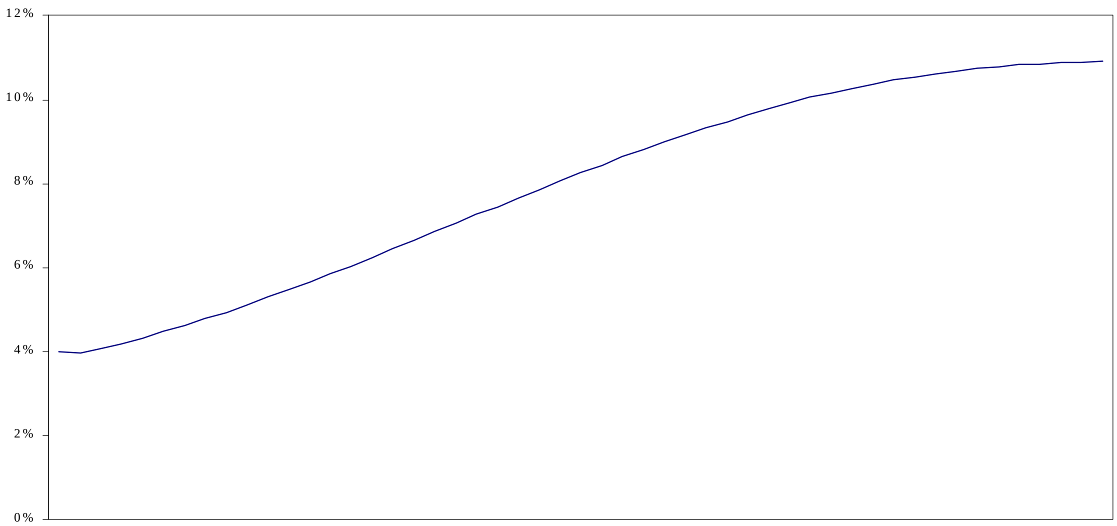
Spending rate



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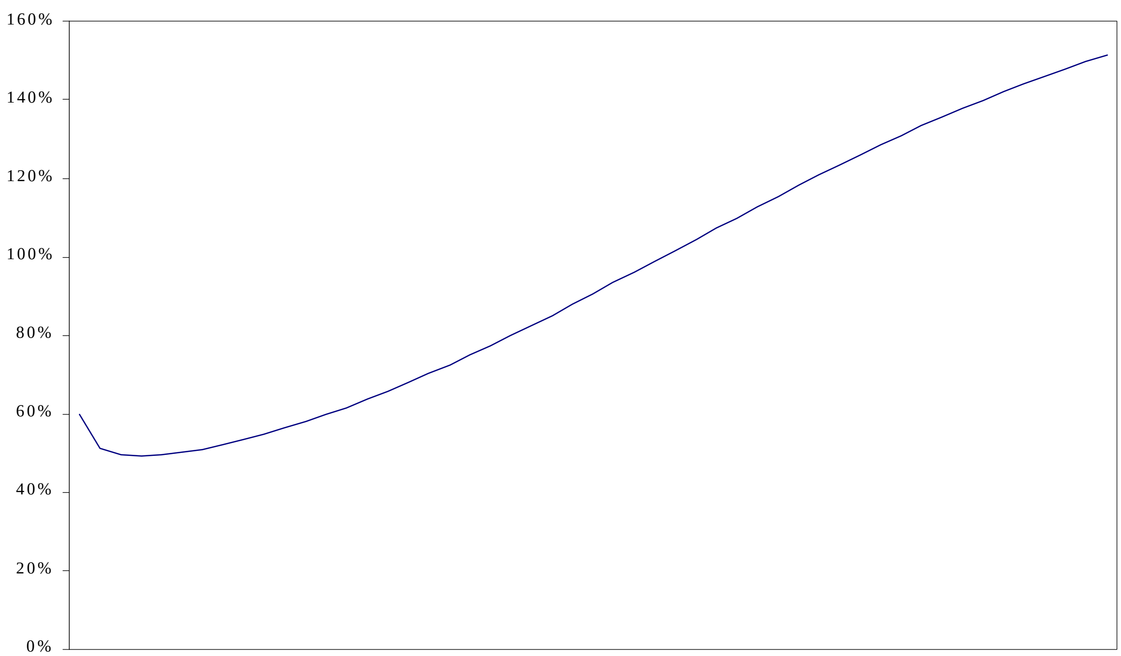

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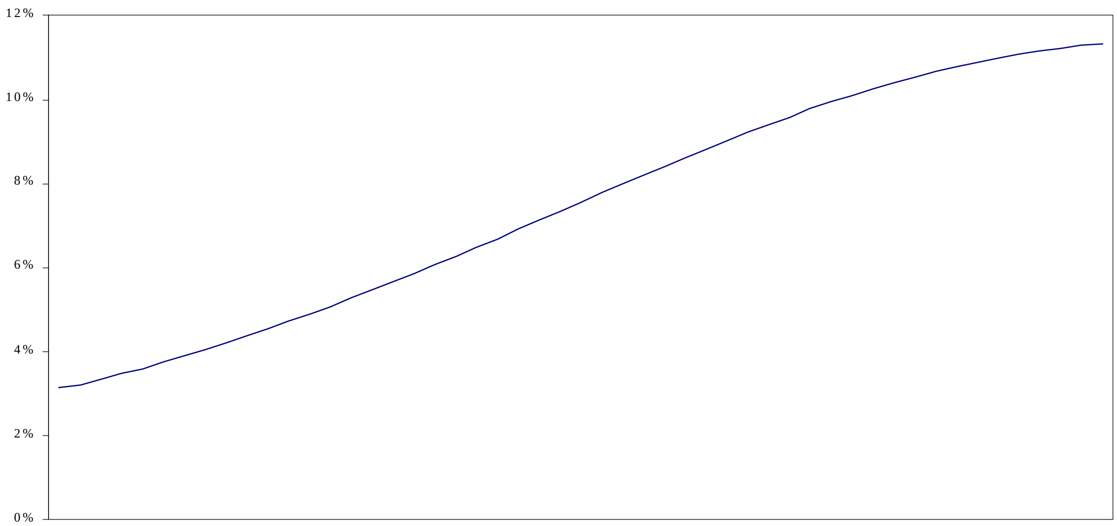

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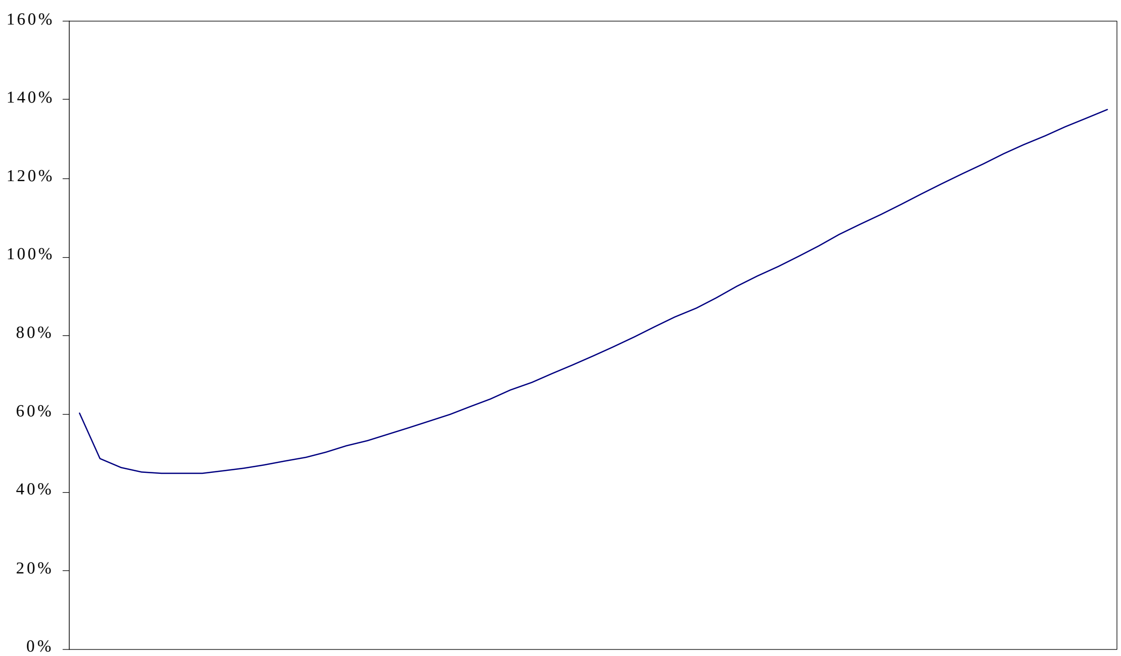

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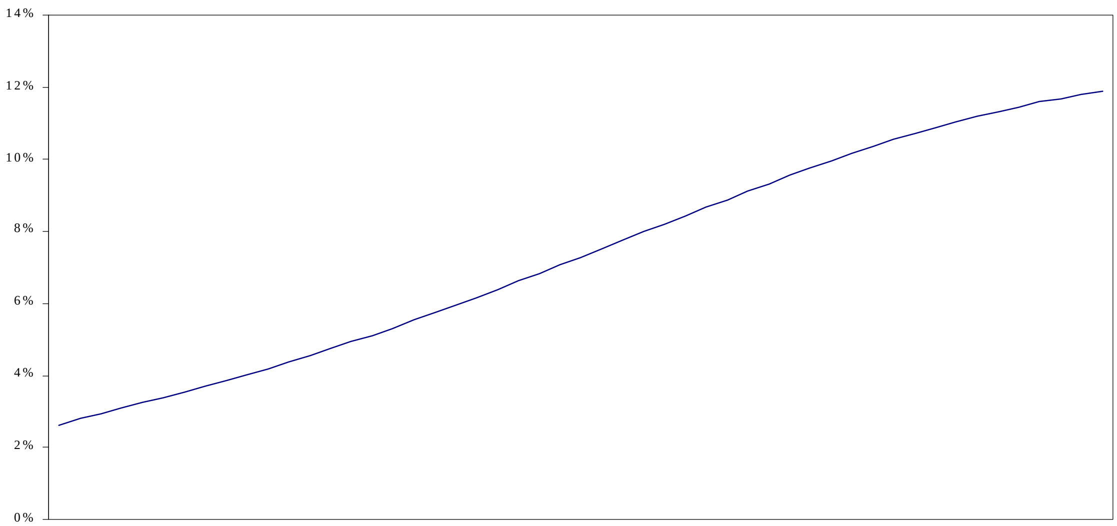
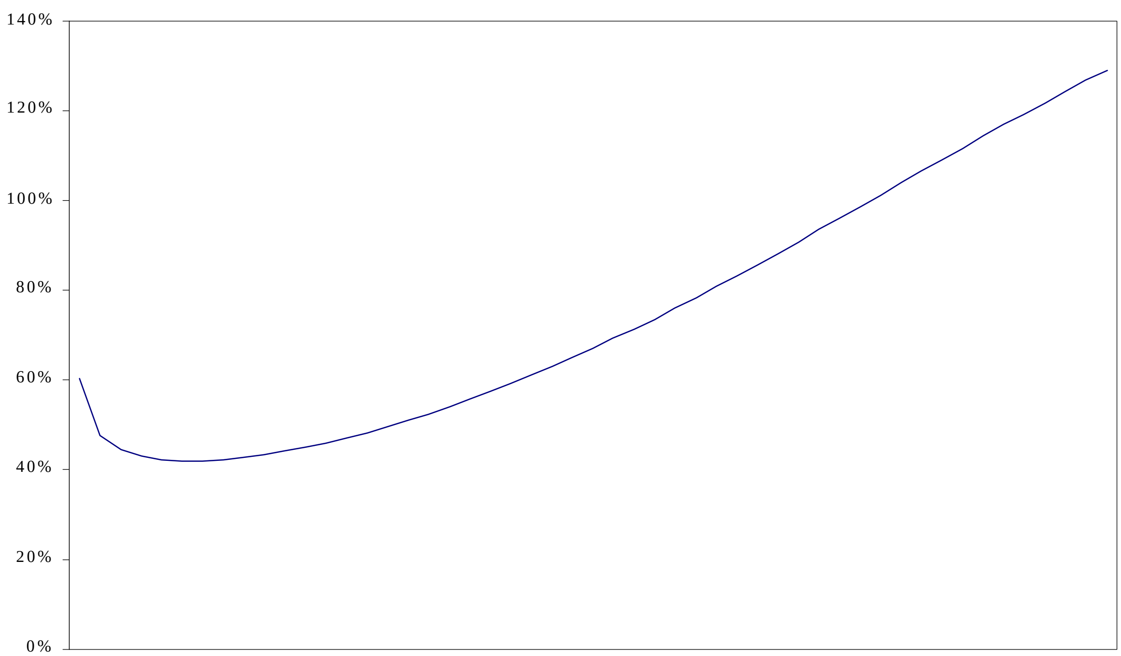
Spending rate



Stock allocation



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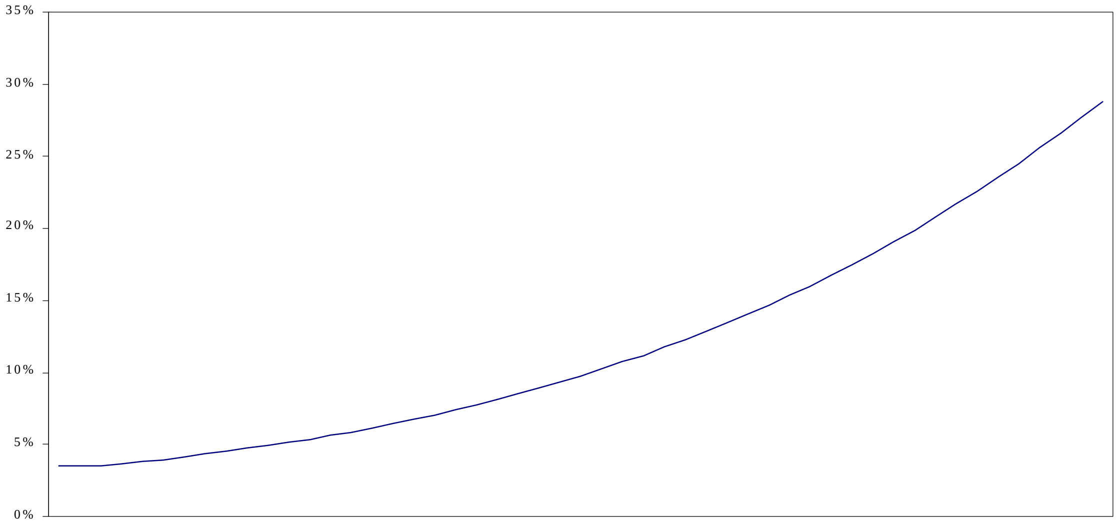
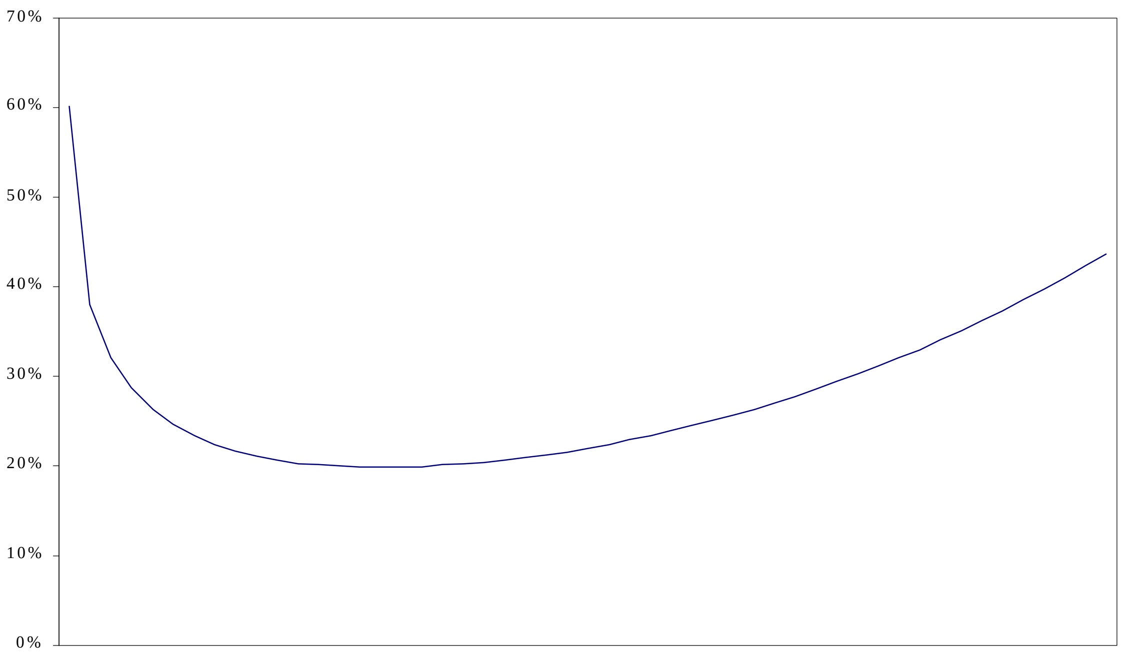


Stock allocation



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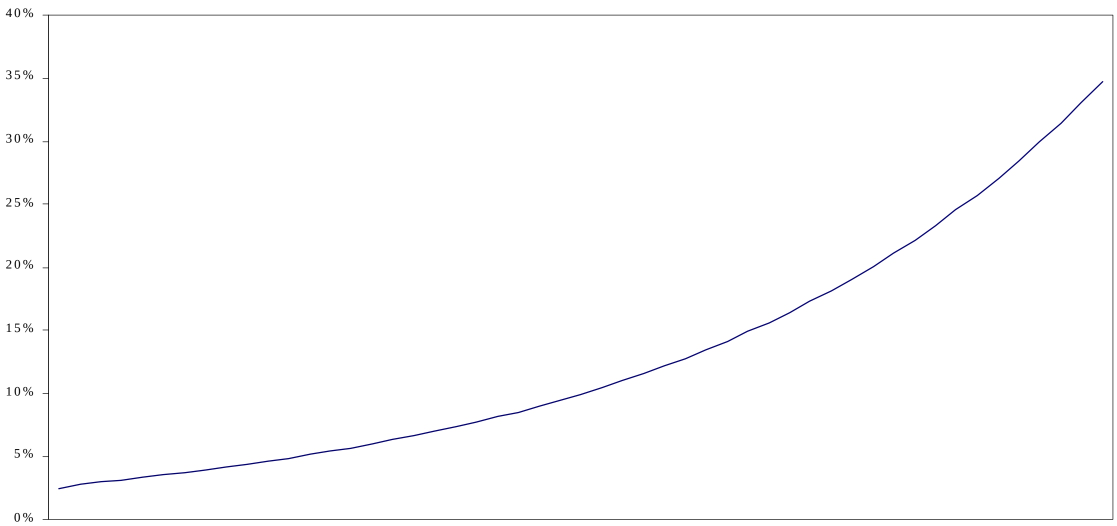

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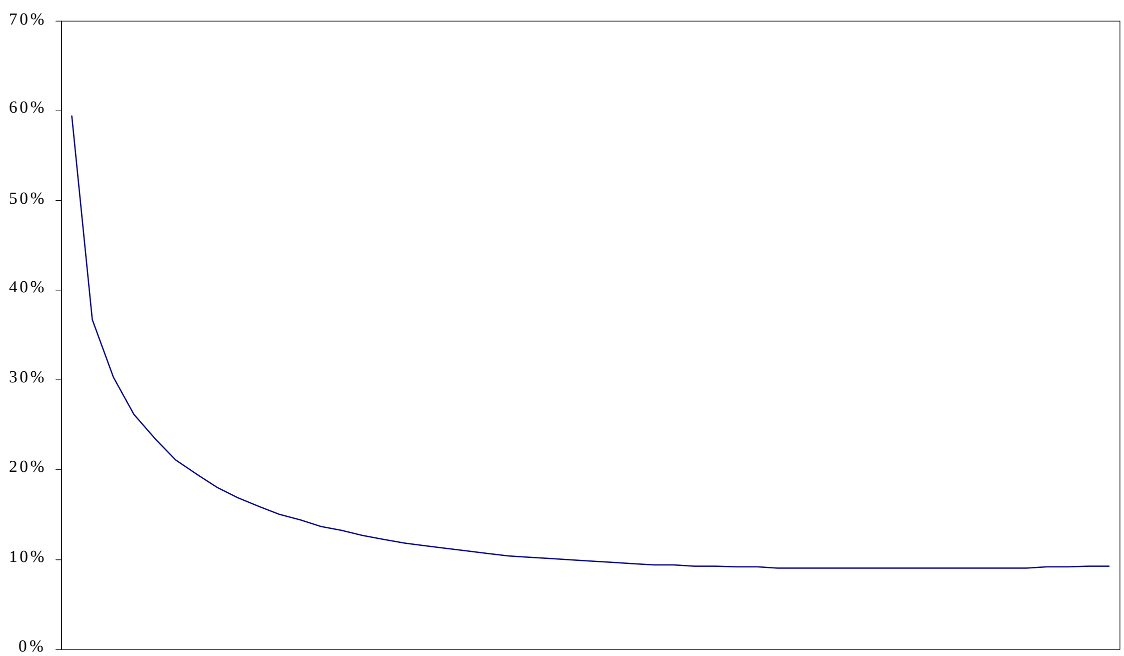

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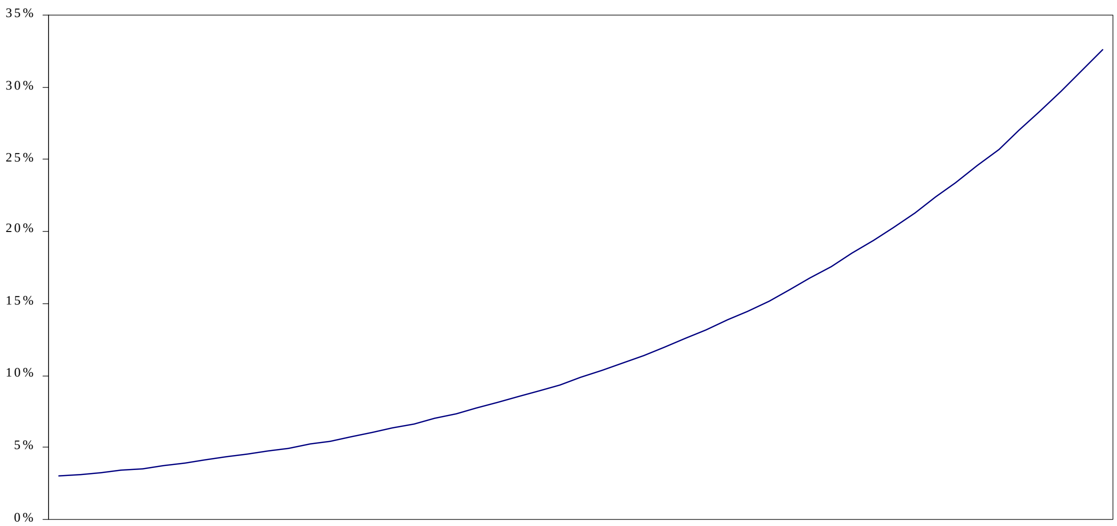
Spending rate



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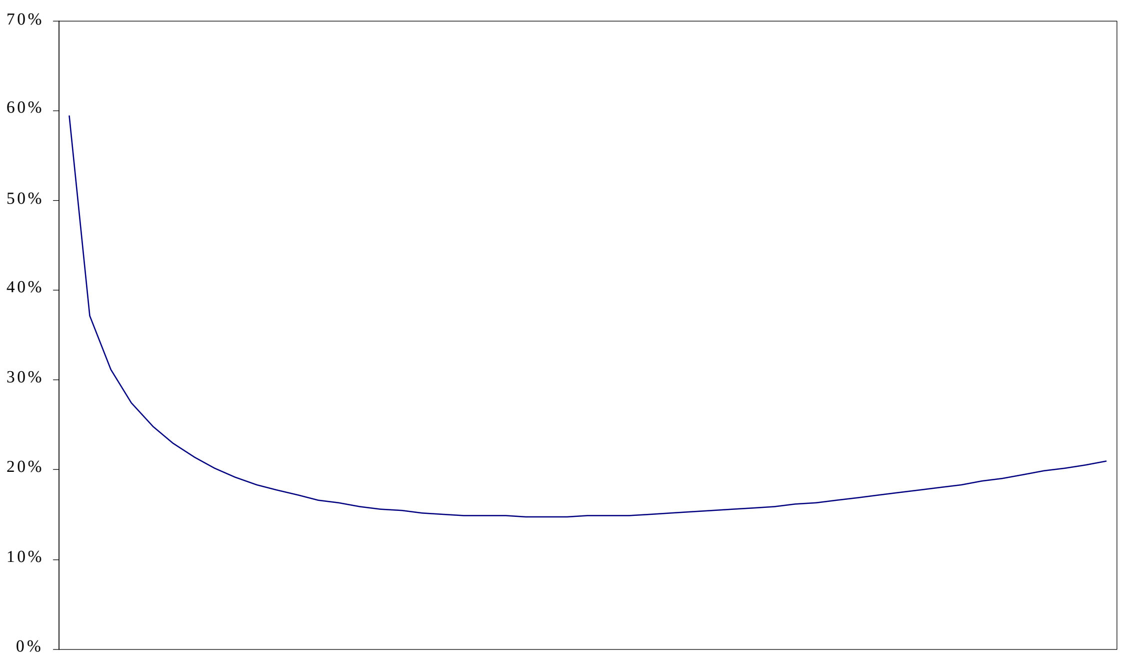
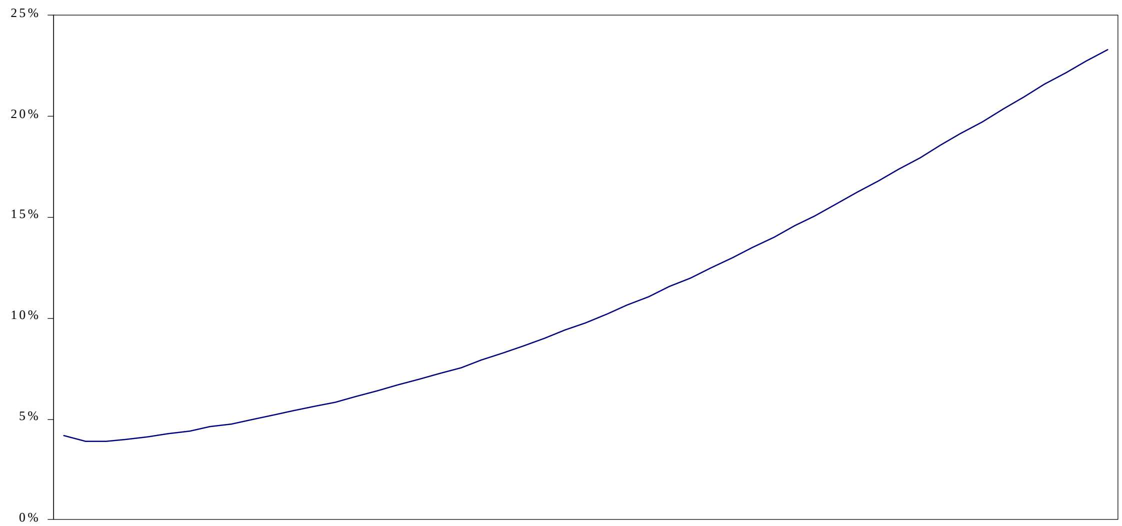


m=10%, s=20%, α=2%, π=2%, r=5%, γ=0.7, ρ=5%, n=-2

Spending rate



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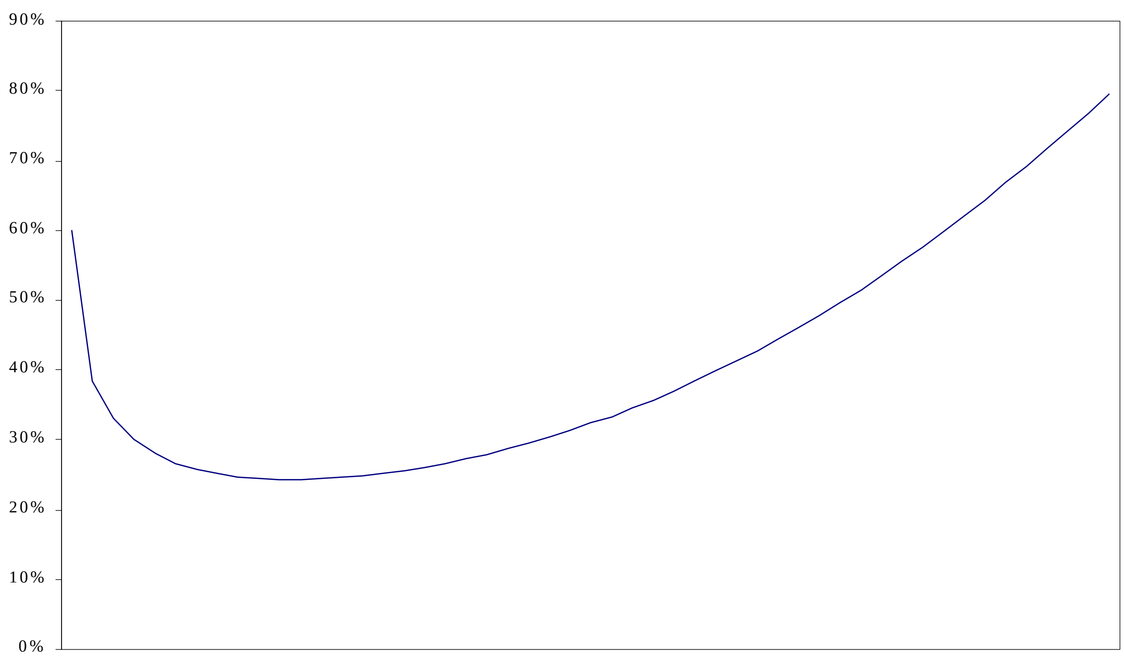

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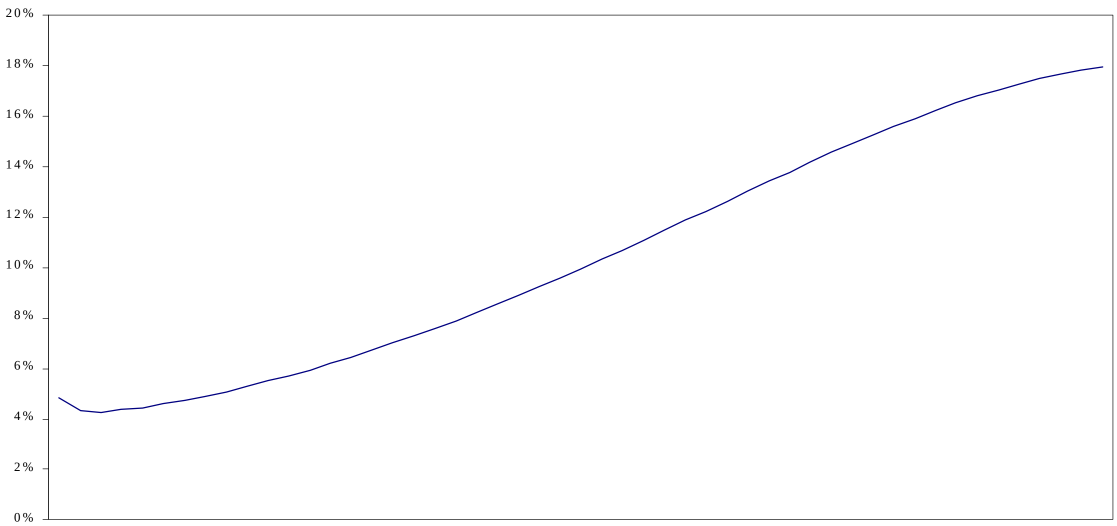
Spending rate



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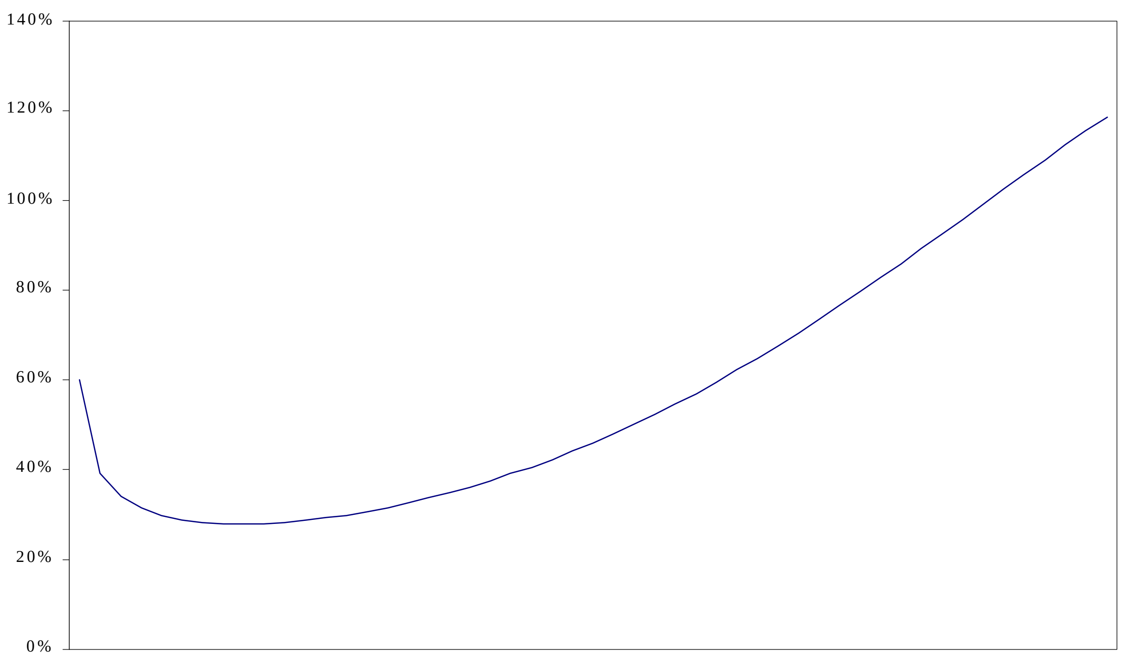

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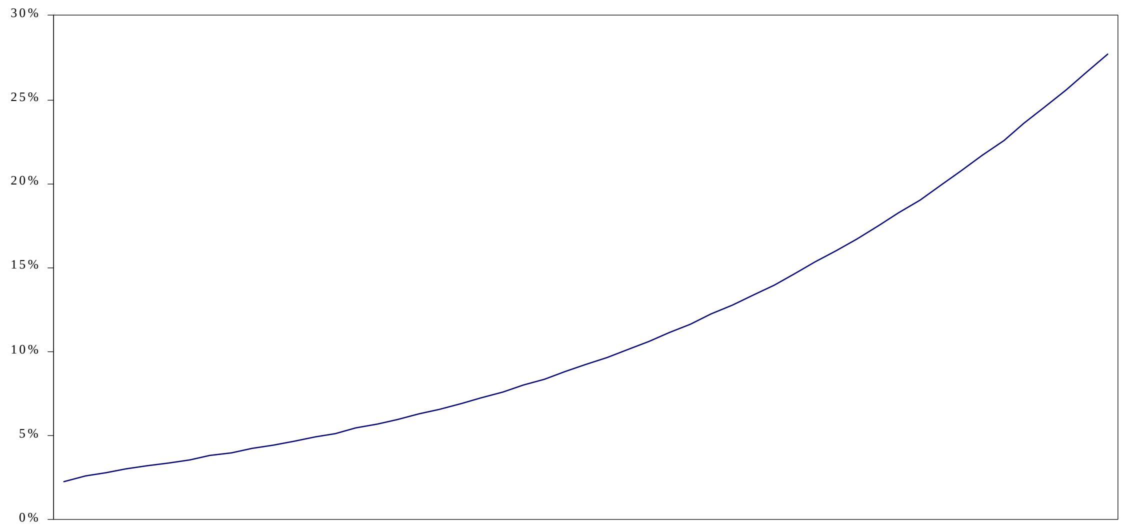
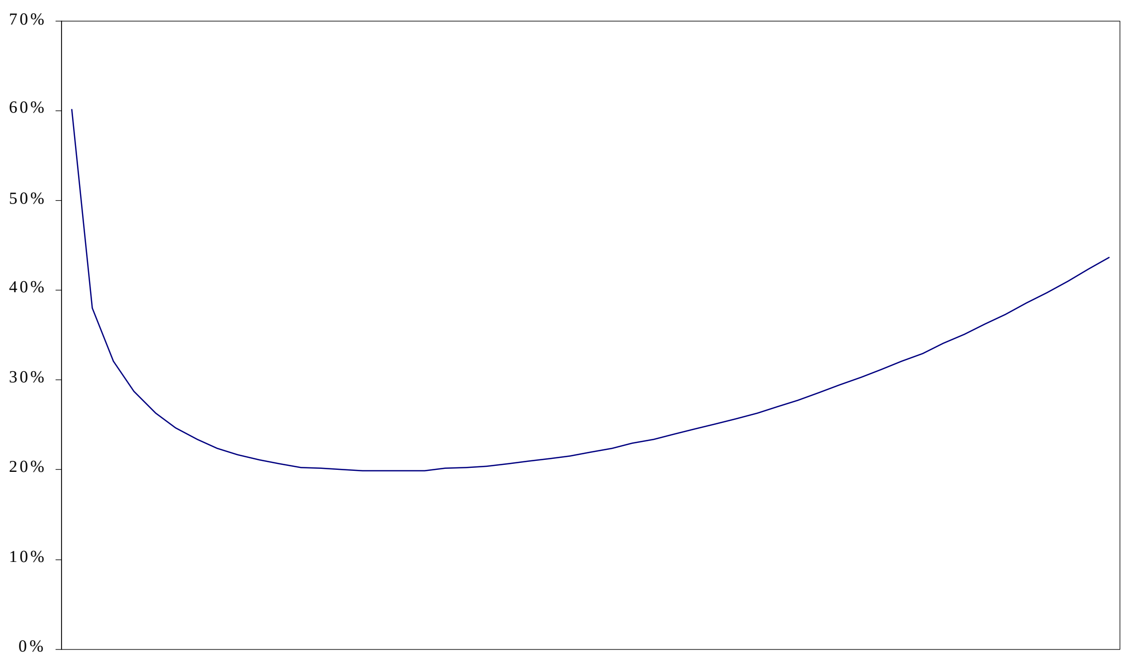
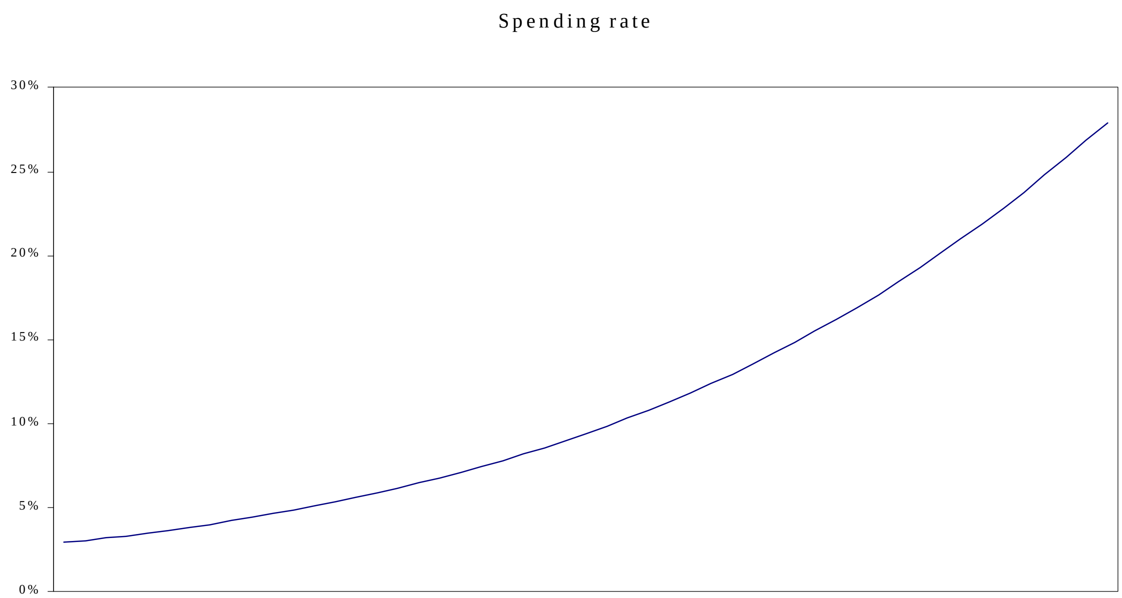
Spending rate

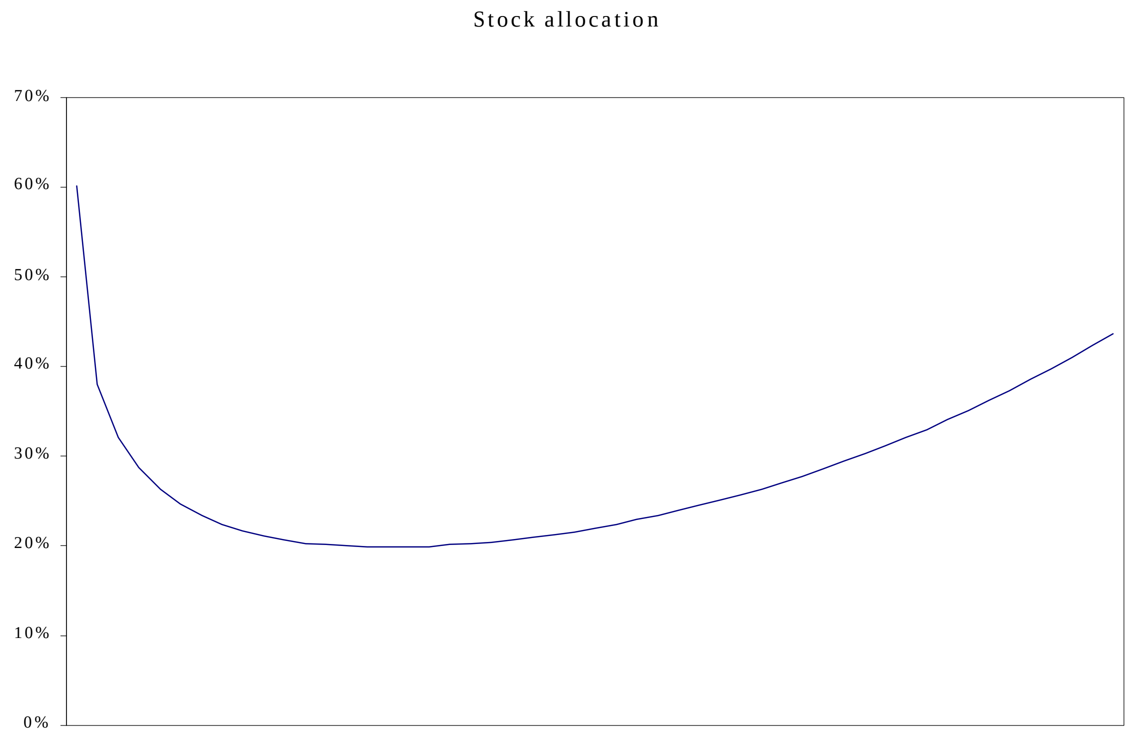


Stock allocation



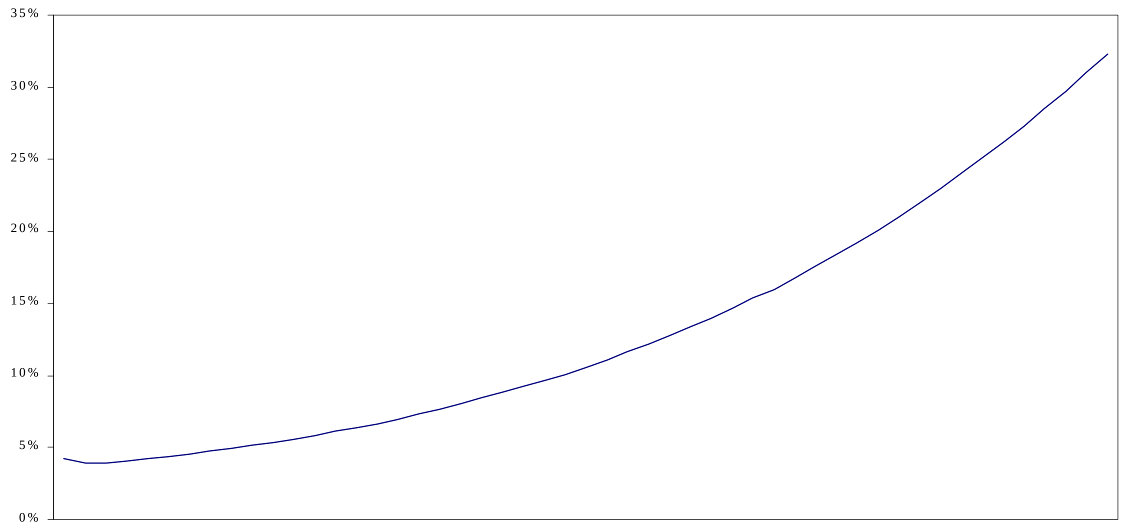
m=12%, s=20%, α=2%, π=2%, r=5%,γ=0.97, ρ=1%, n=-2



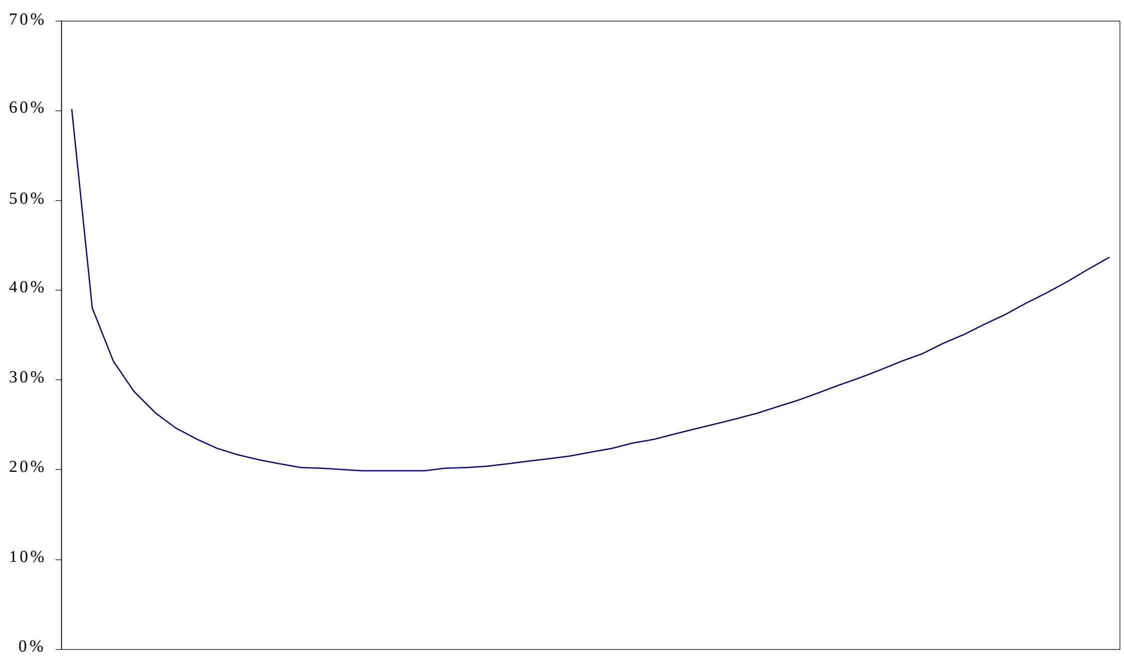


m=12%, s=20%, α=2%, π=2%, r=5%,γ=0.97, ρ=3%, n=-2

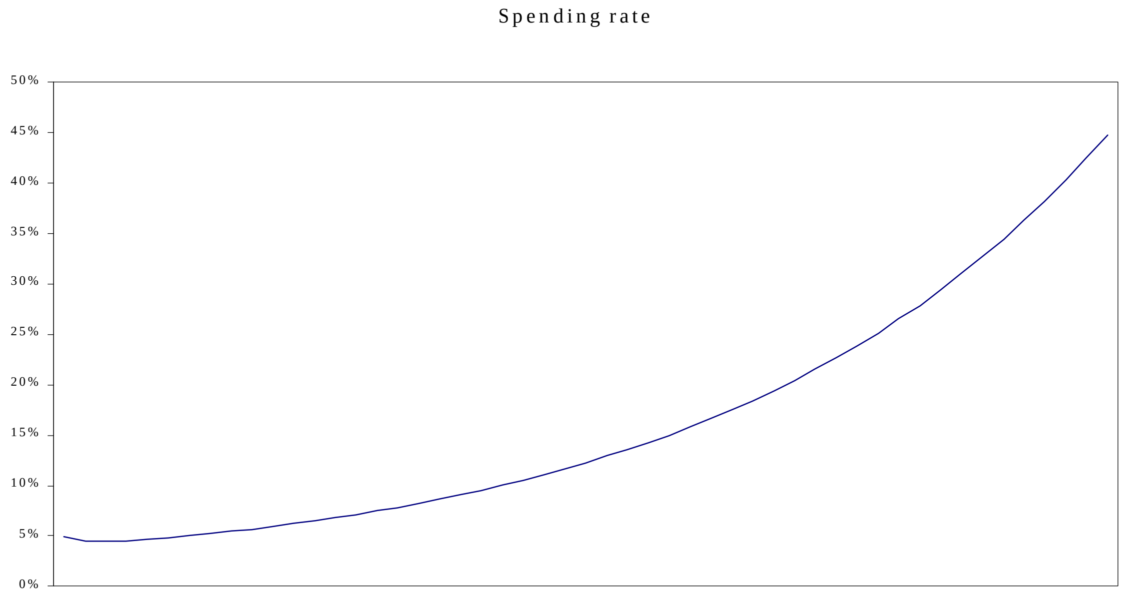
Spending rate

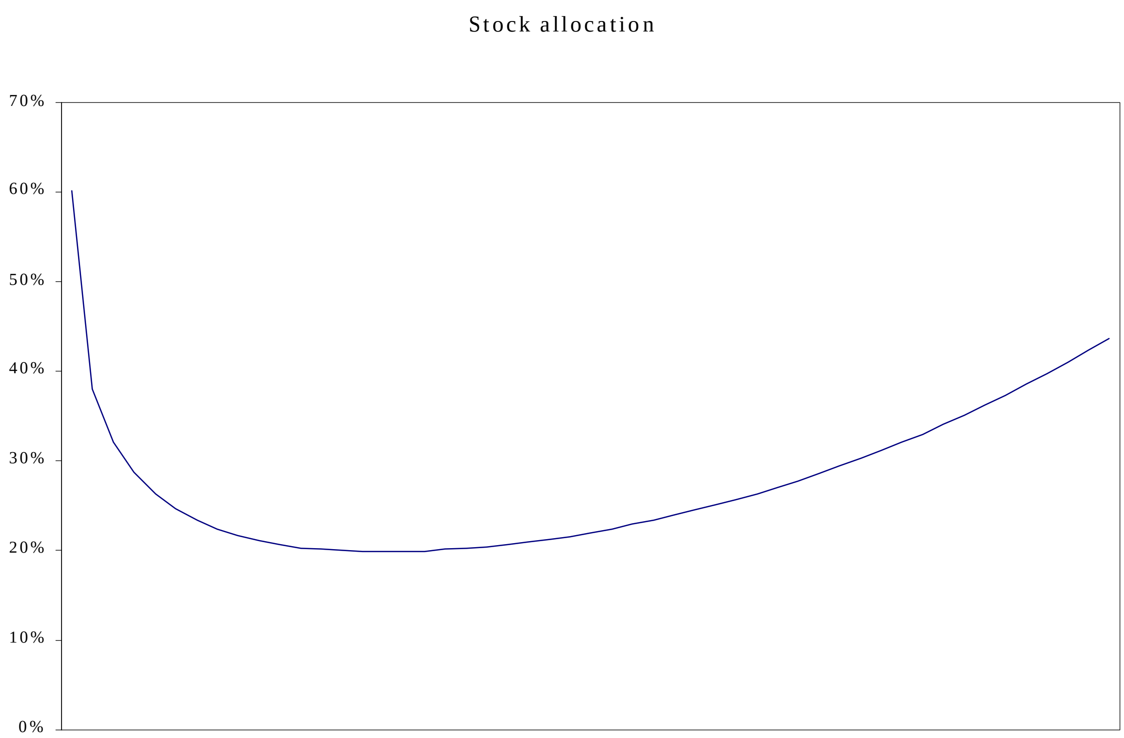


Stock allocation



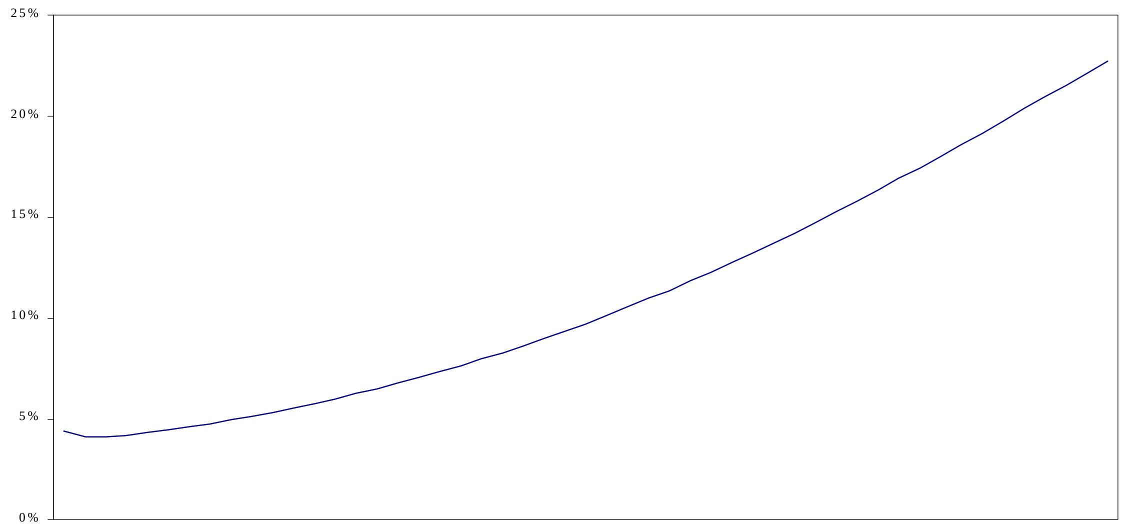
***m*=12%, *s*=20%, α=2%, π=2%, *r*=5%,γ=0.97, ρ=7%, n=-2**





m**=12%,** s**=20%,** α**=2%,** π**=2%,** r**=5%,**γ**=0.97,** ρ**=9%, n=-2**

Spending rate



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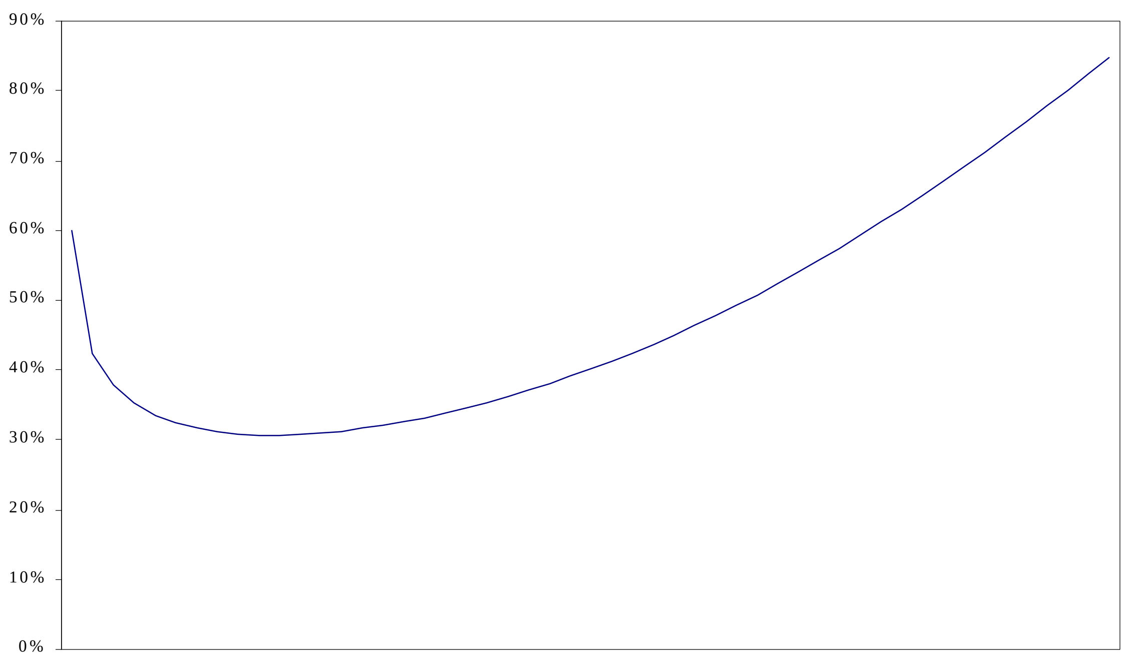
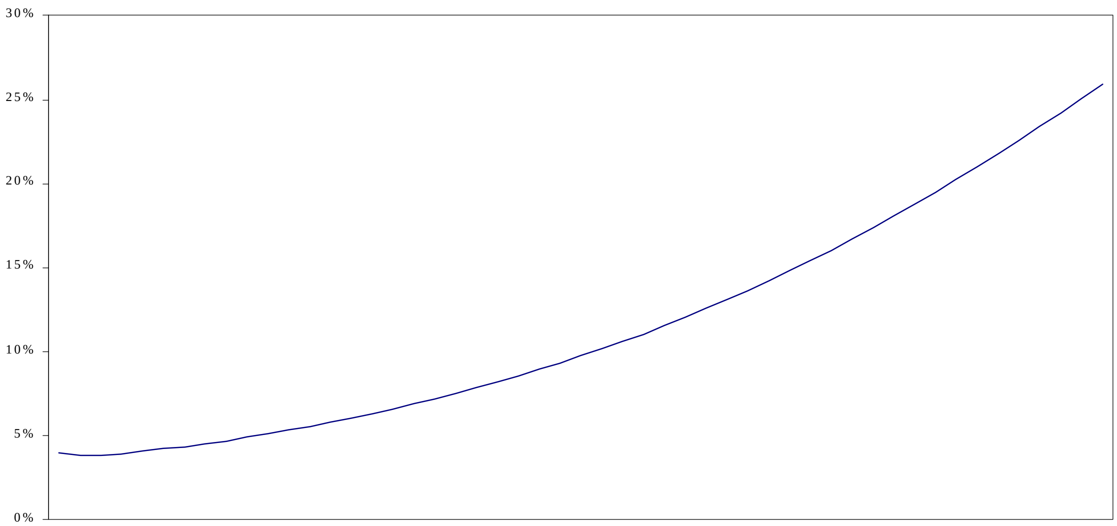


m=12%, s=16%, α=2%, π=2%, r=5%,γ=1.52, ρ=5%, n=-2

Spending rate



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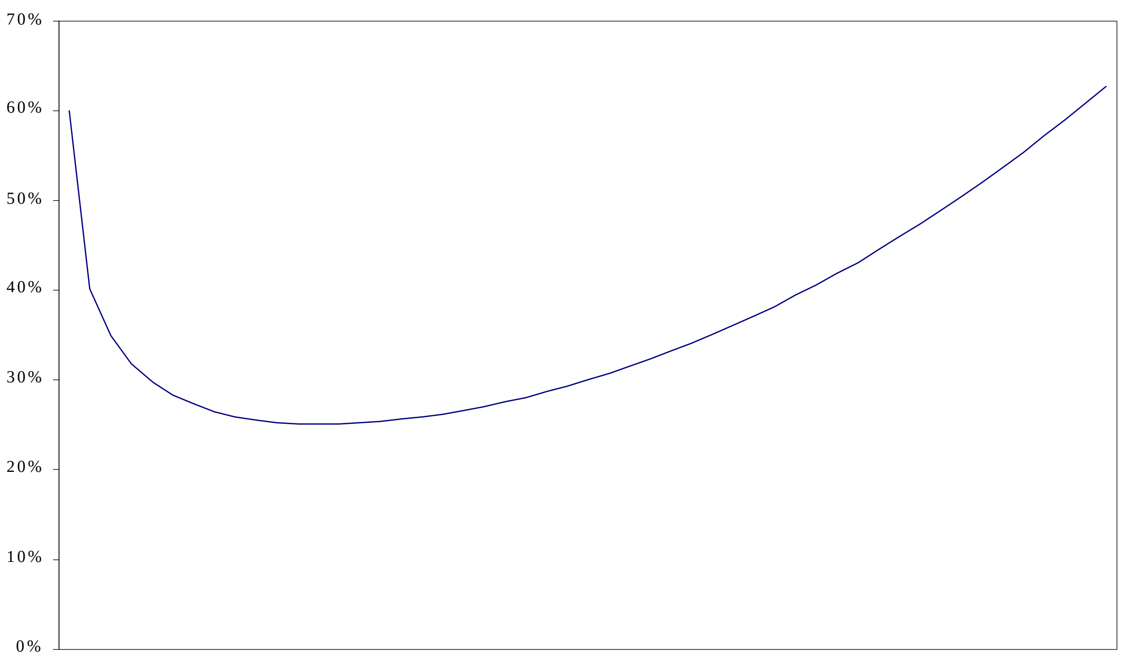
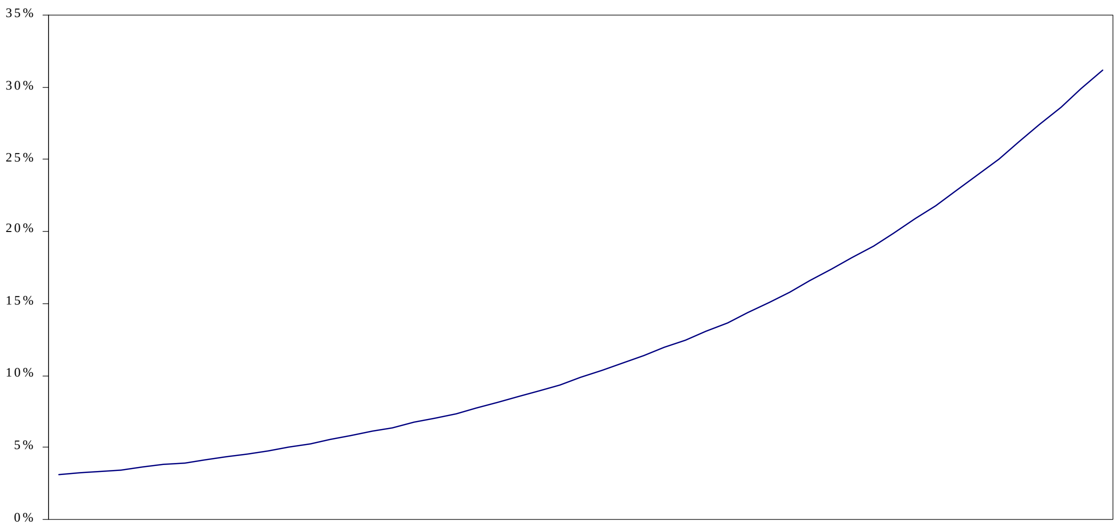


***m*=12%, *s*=18%, α=2%, π=2%, *r*=5%,γ=1.2, ρ=5%, n=-2**

Spending rate



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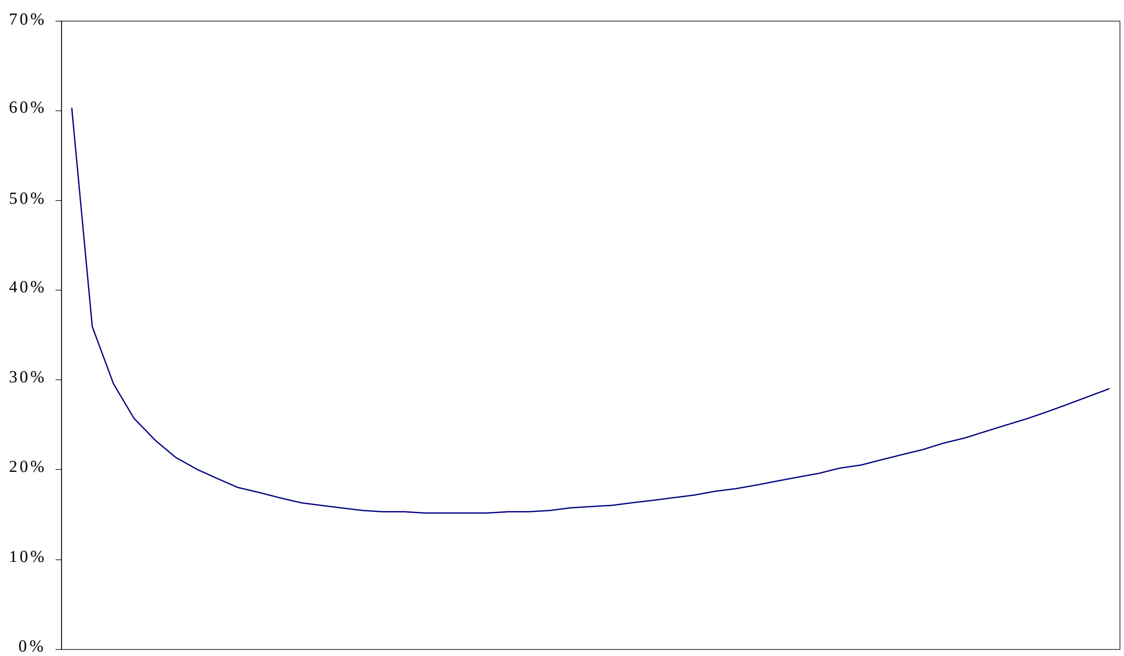
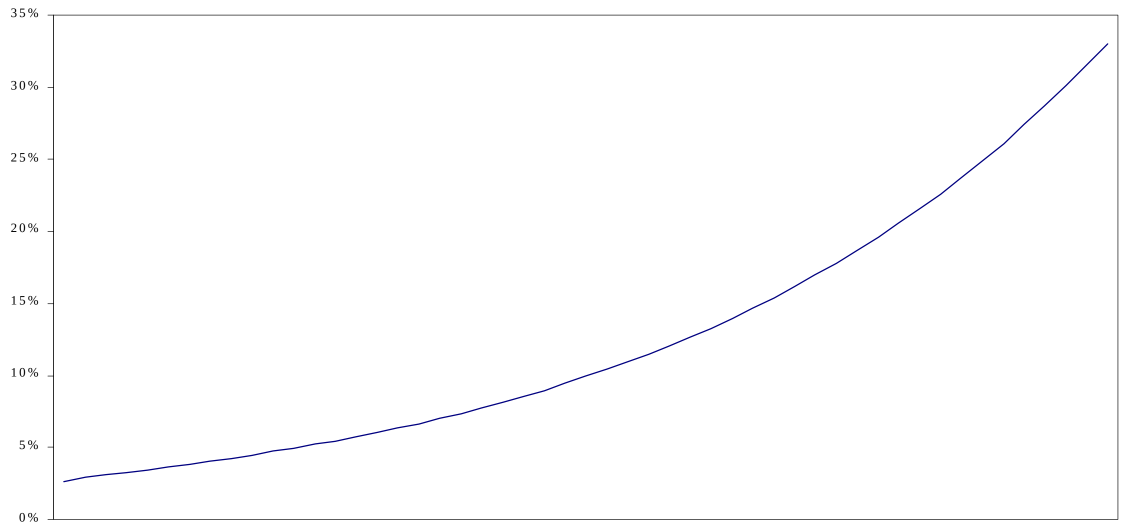


m=12%, s=22%, α=2%, π=2%, r=5%, γ=0.8, ρ=5%, n=-2

Spending rate



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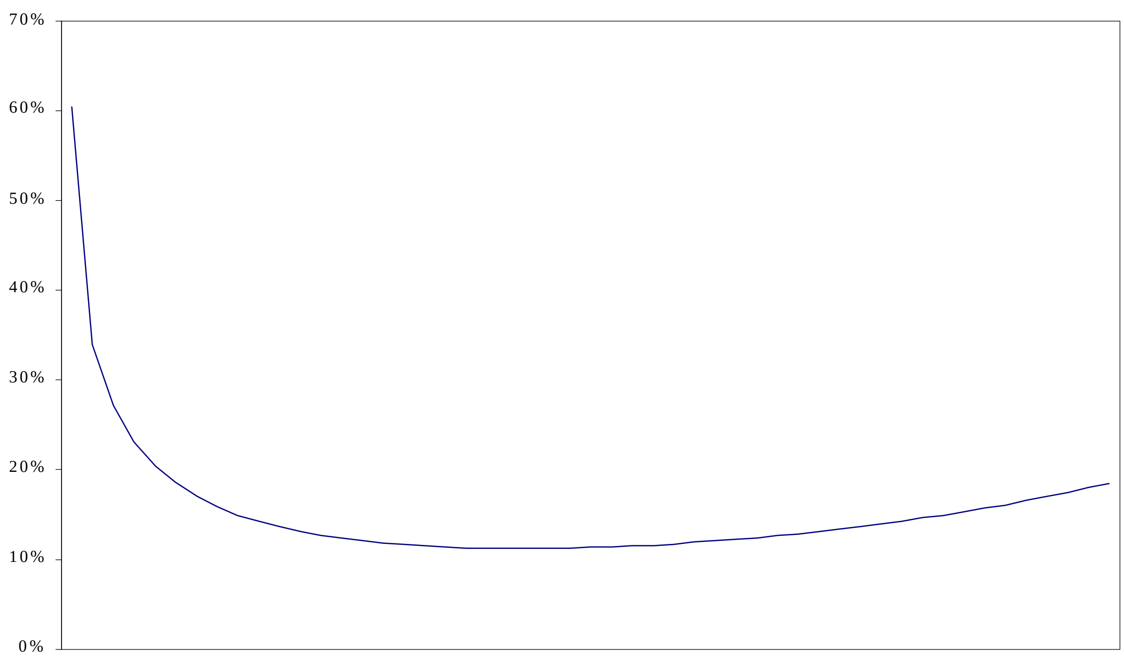
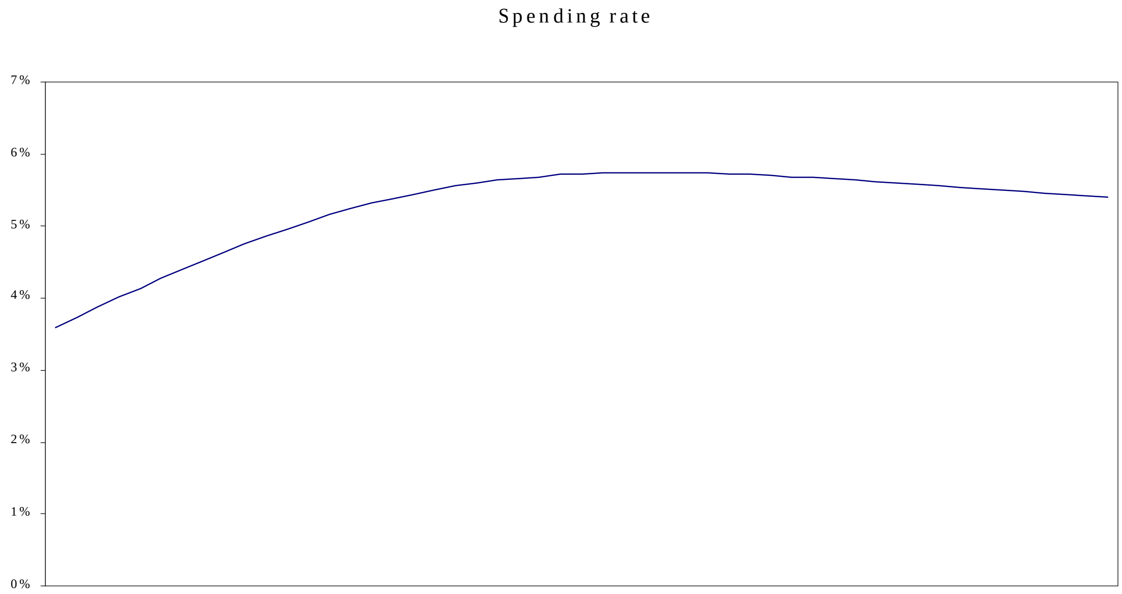
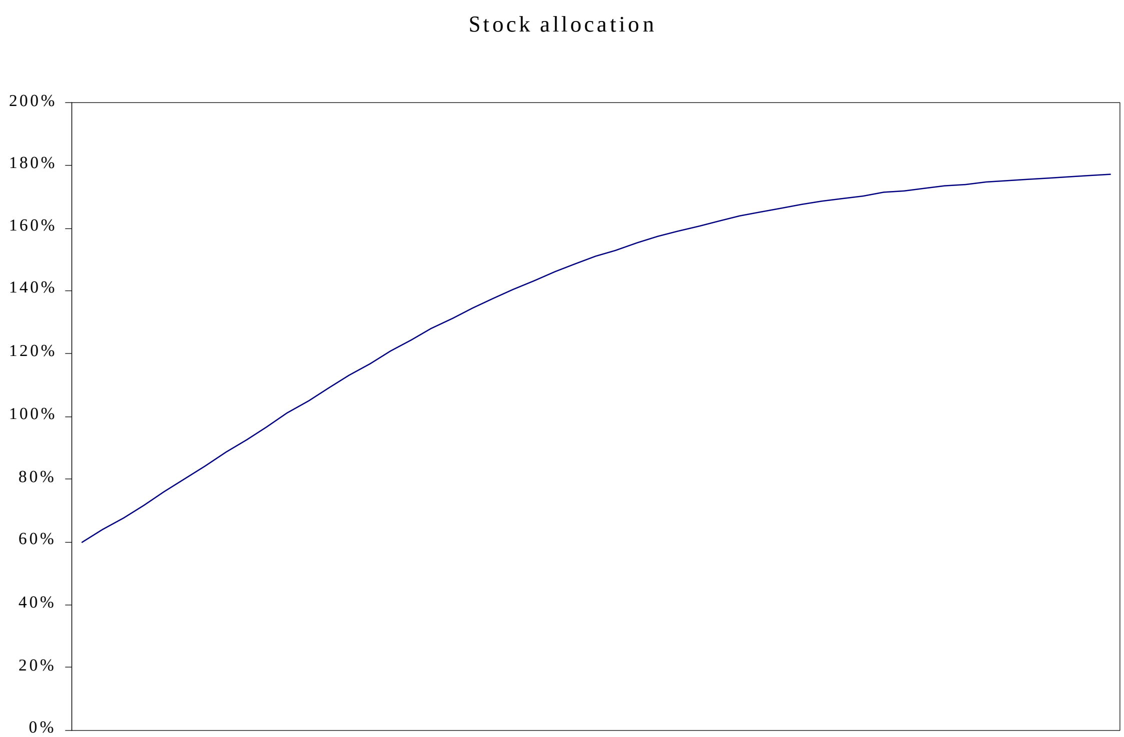


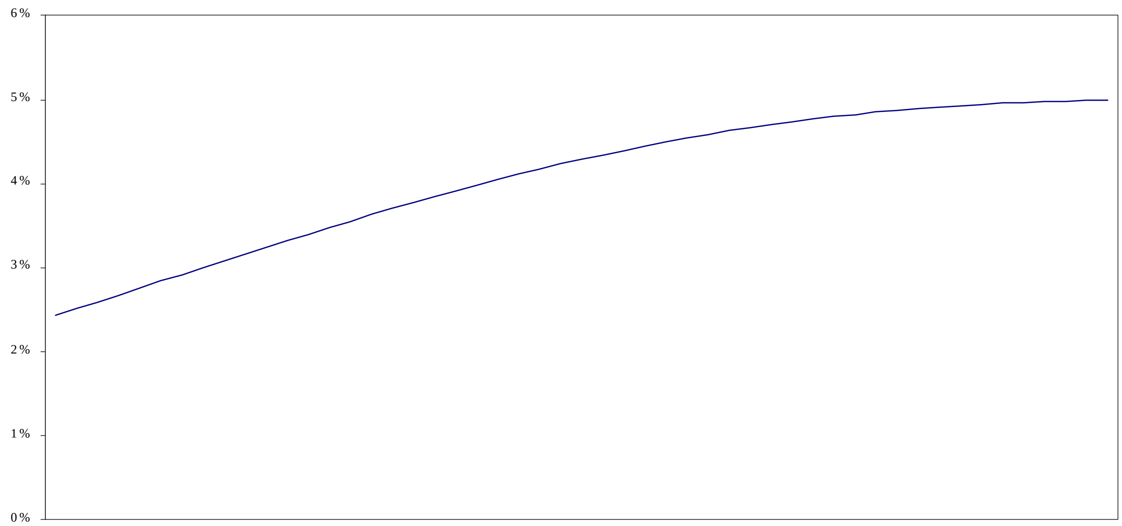
m=12%, s=24%, α=2%, π=2%, r=5%,γ=0.67, ρ=5%, n=-2





m=12%, s=20%, α=2%, π=2%, r=5%,γ=0.97, ρ=5%, n=0

Spending rate



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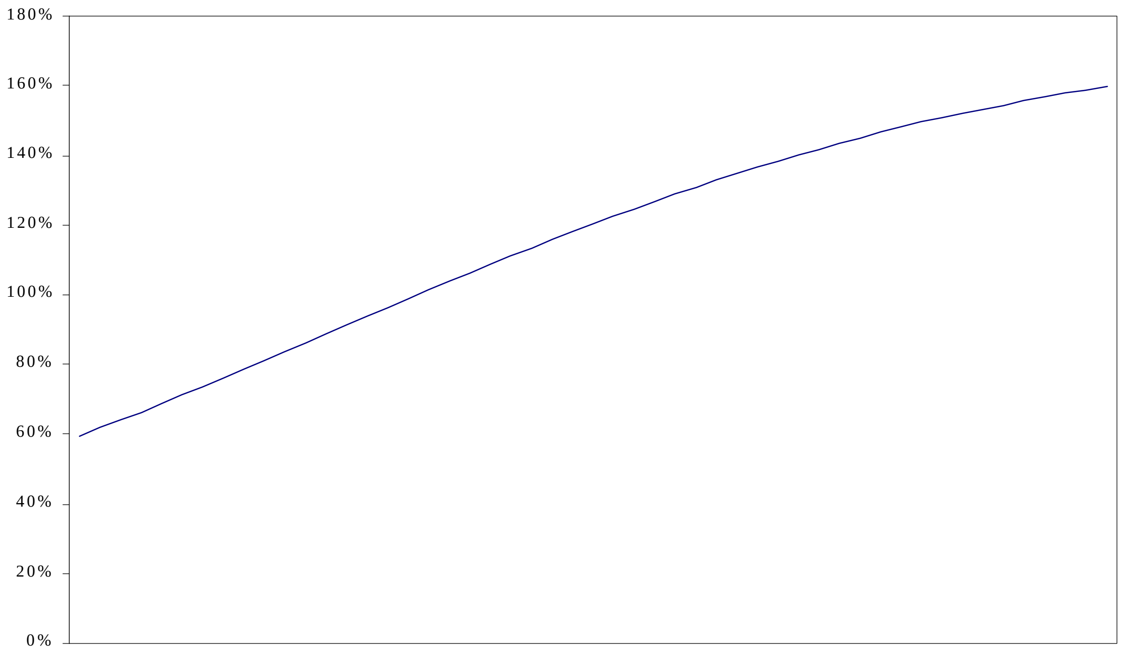
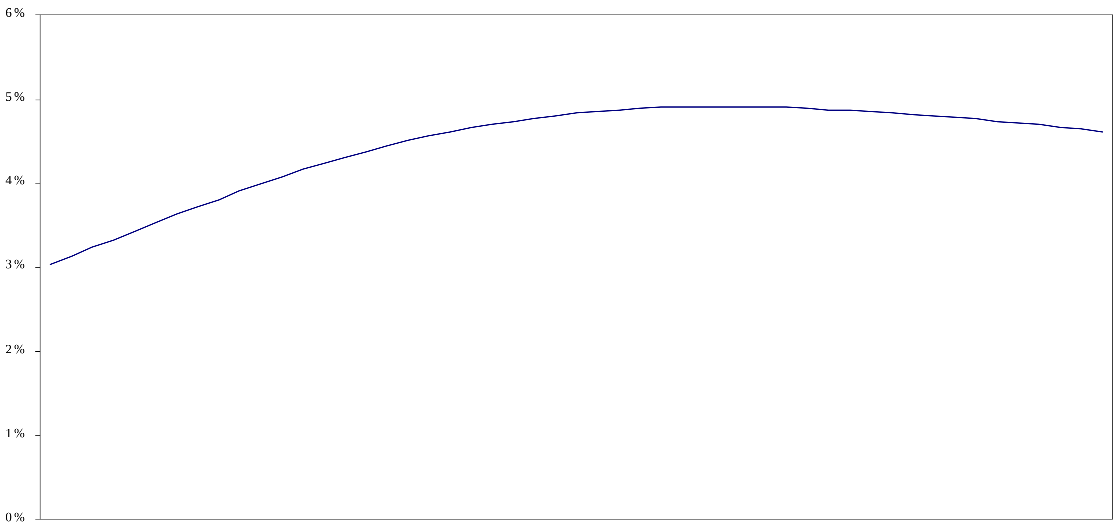
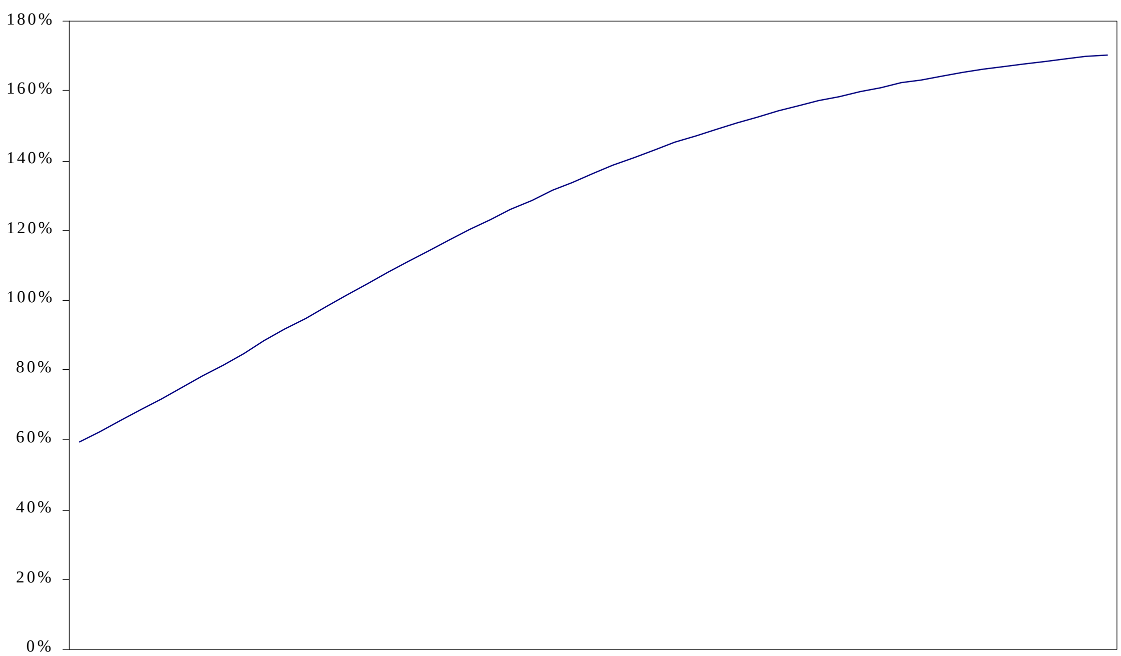


m=8%, s=20%, α=2%, π=2%, r=5%,γ=0.42, ρ=5%, n=0

Spending rate

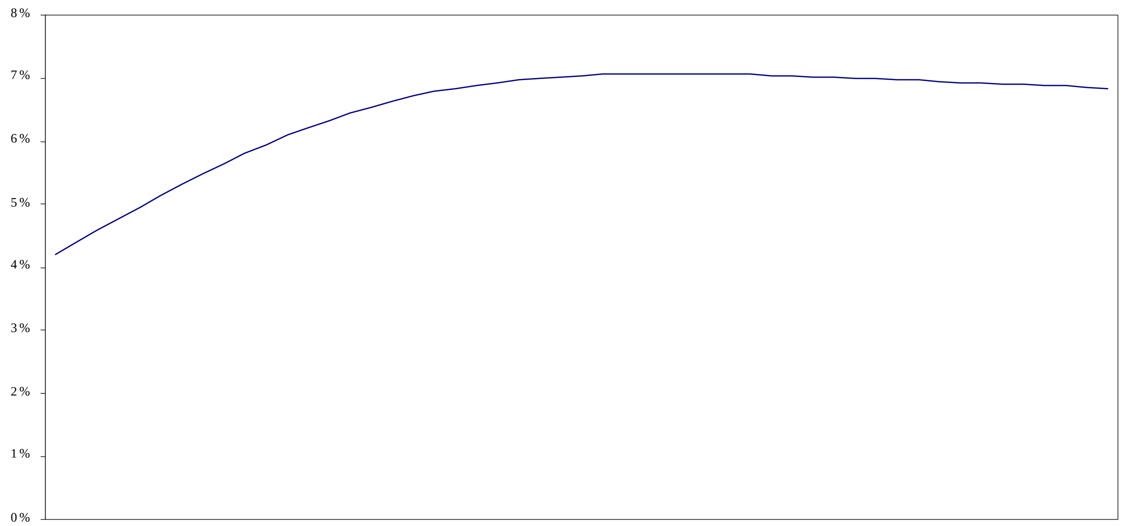


Stock allocation

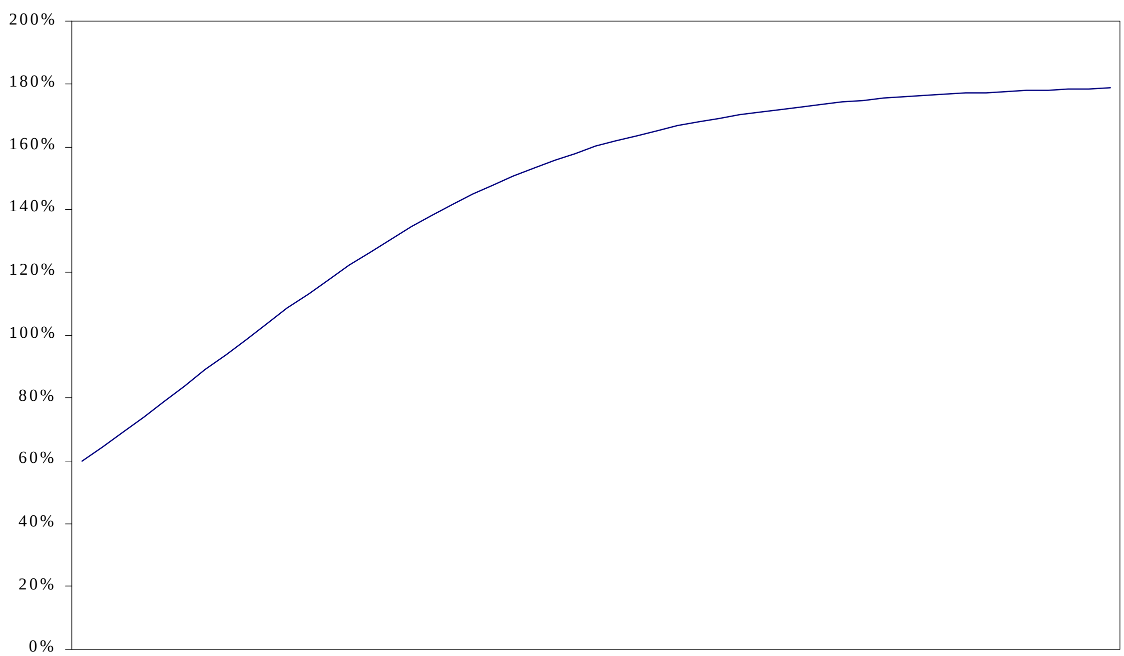


***m*=10%, *s*=20%, α=2%, π=2%, *r*=5%,γ=0.7, ρ=5%, n=0**

Spending rate

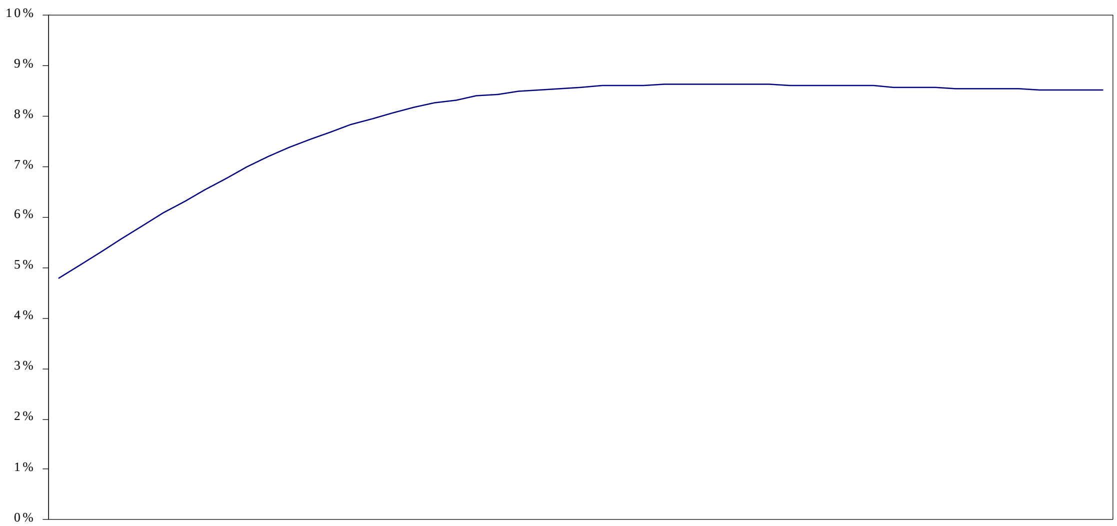


Stock allocation

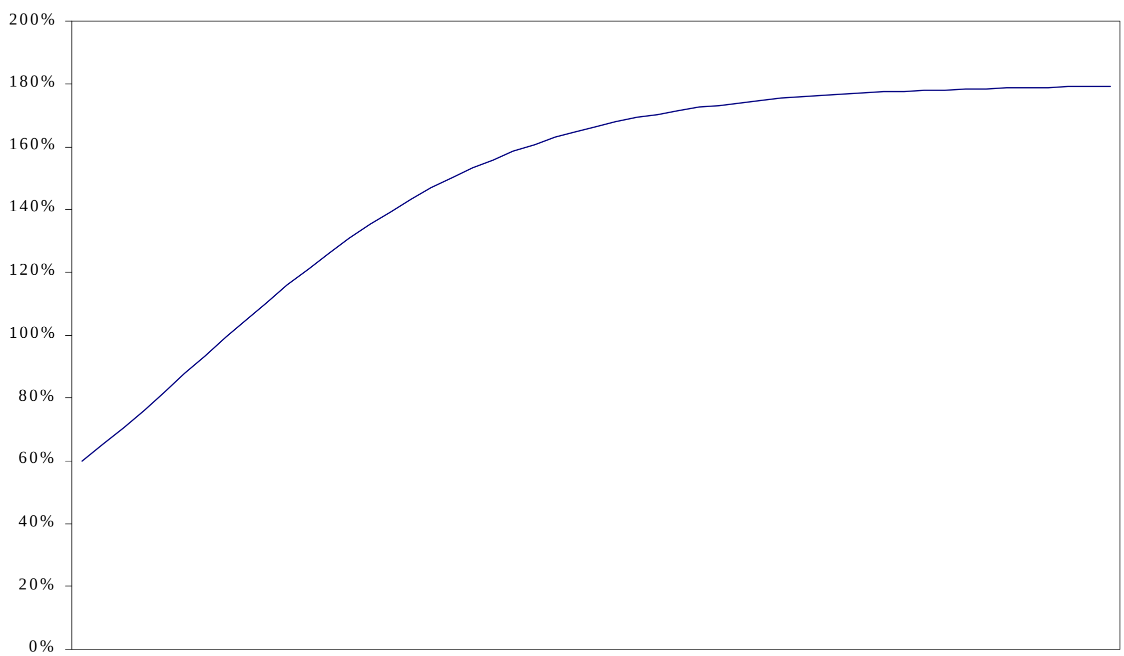


***m*=14%, *s*=20%, α=2%, π=2%, *r*=5%,γ=1.25, ρ=5%, n=0**

Spending rate

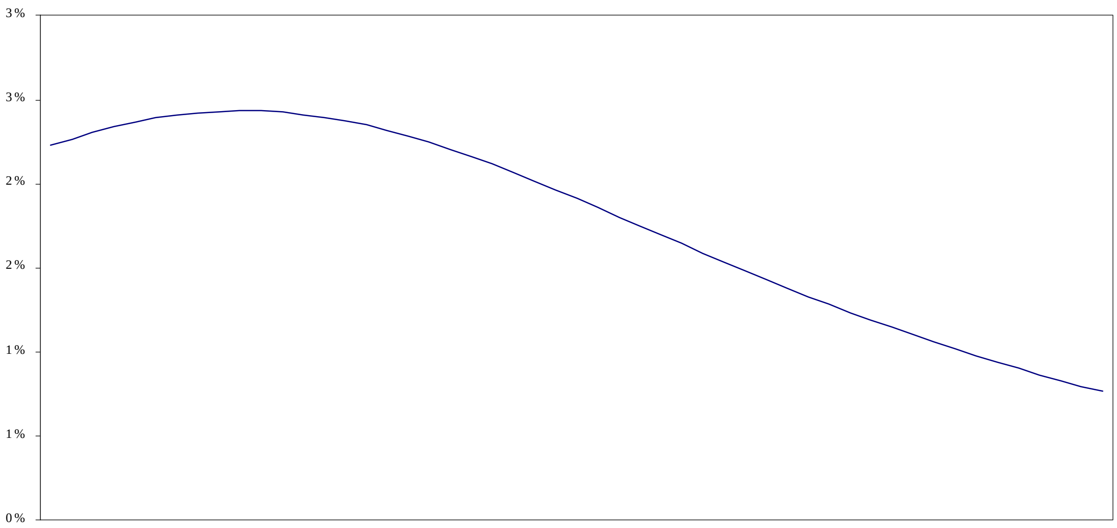


Stock allocation



***m*=16%, *s*=20%, α=2%, π=2%, *r*=5%,γ=1.53, ρ=5%, n=0**

Spending rate



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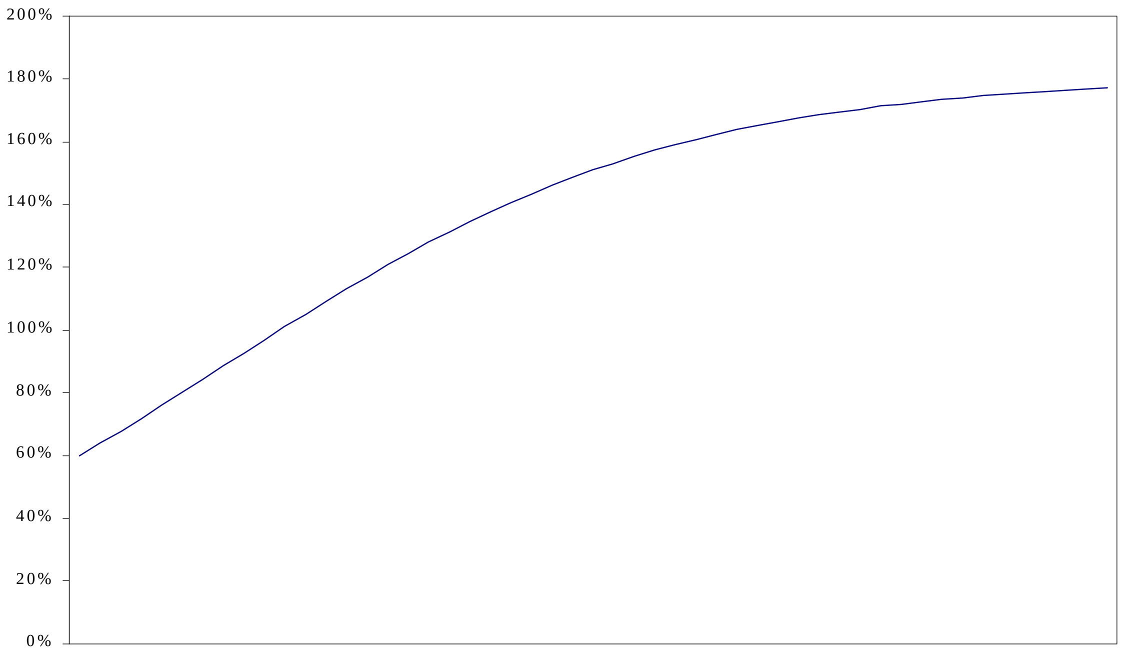
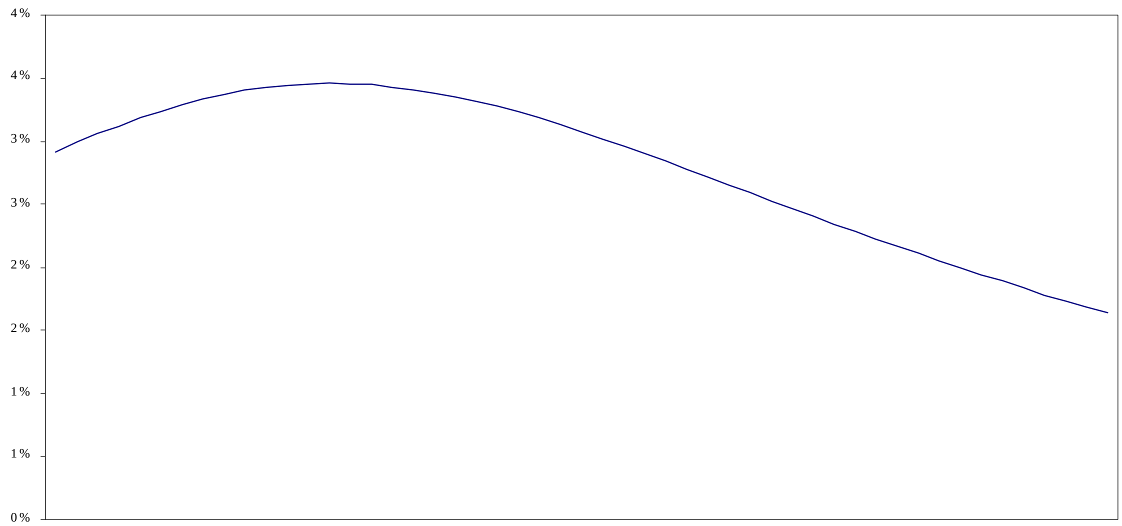
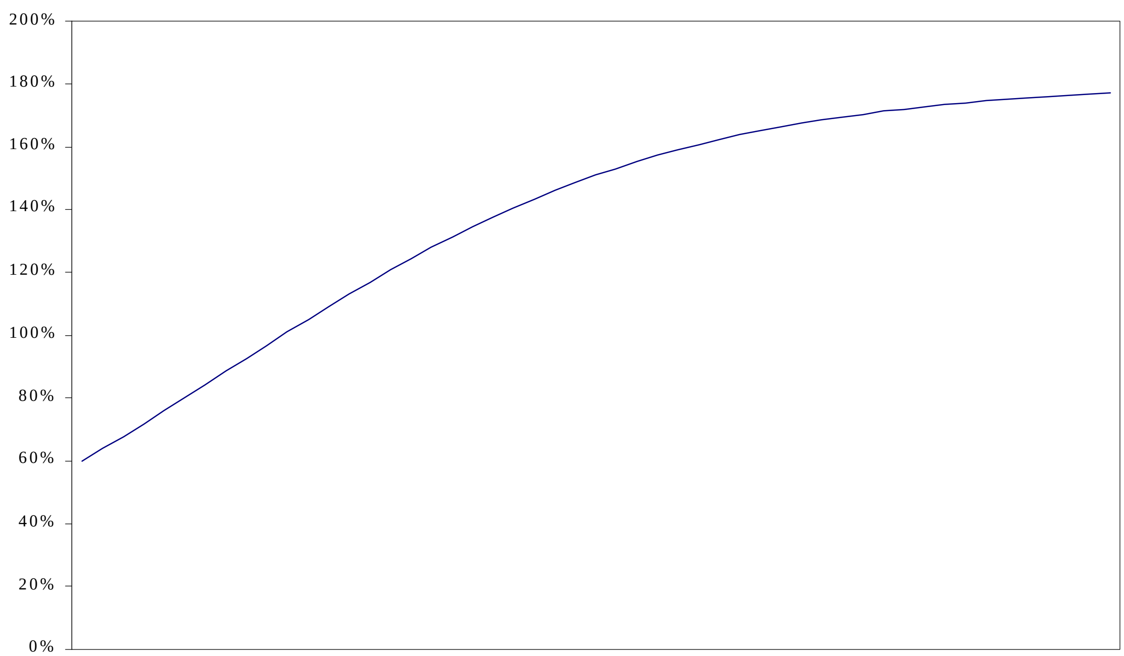


m=12%, s=20%, α=2%, π=2%, r=5%,γ=0.97, ρ=1%, n=0

Spending rate

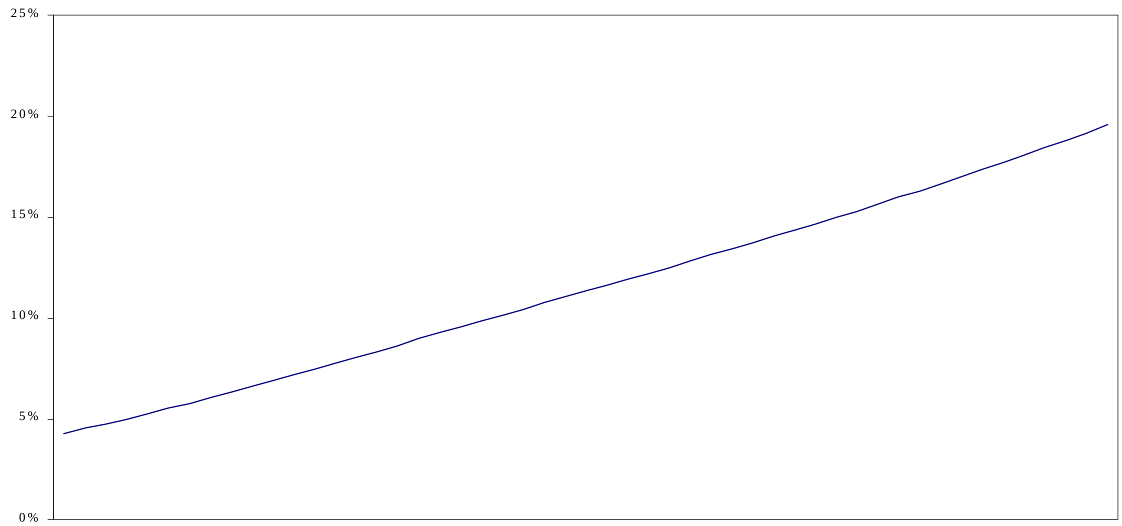


Stock allocation



***m*=12%, *s* =20%, α=2%, π=2%, *r*=5%,γ=0.97, ρ=3%, n=0**

Spending rate



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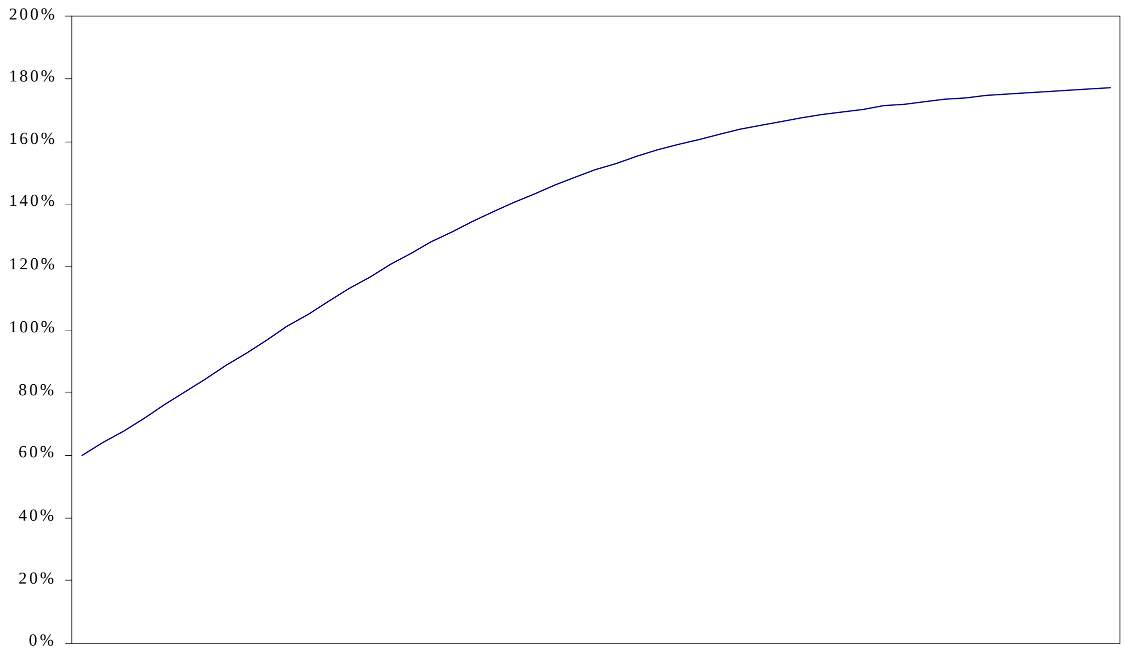
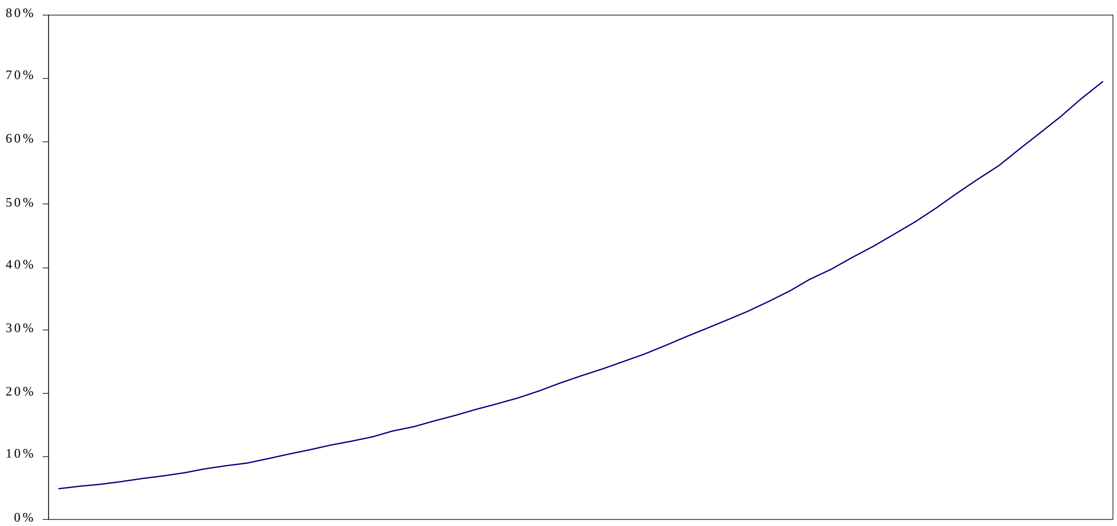


***m*=12%, *s* =20%, α=2%, π=2%, *r*=5%,γ=0.97, ρ=7%, n=0**

Spending rate



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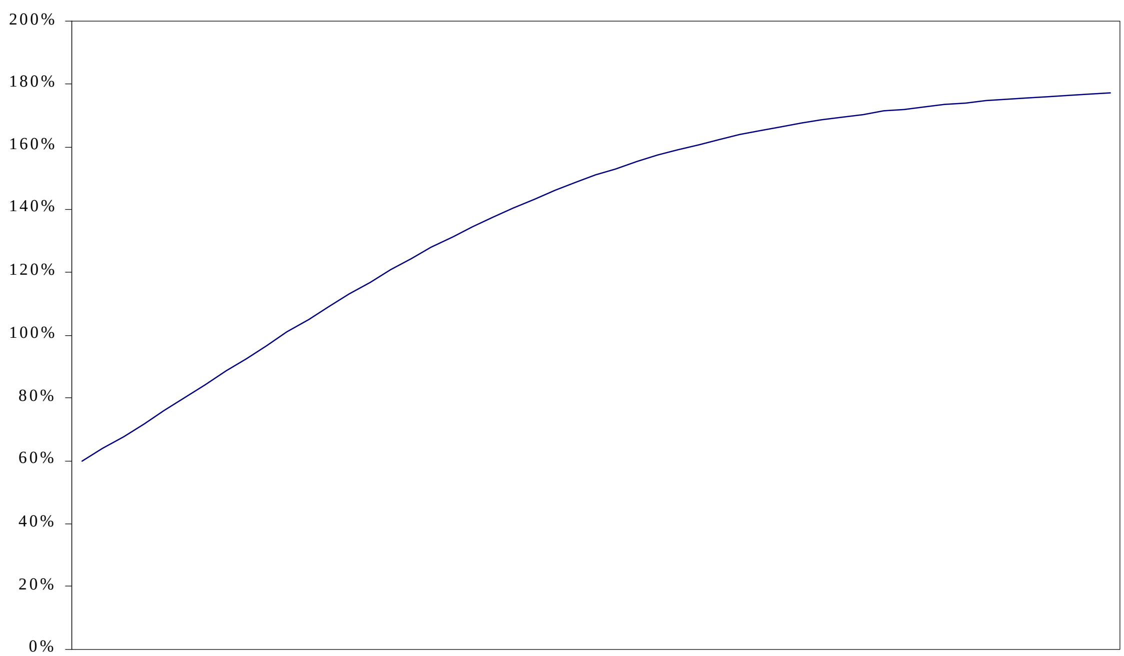
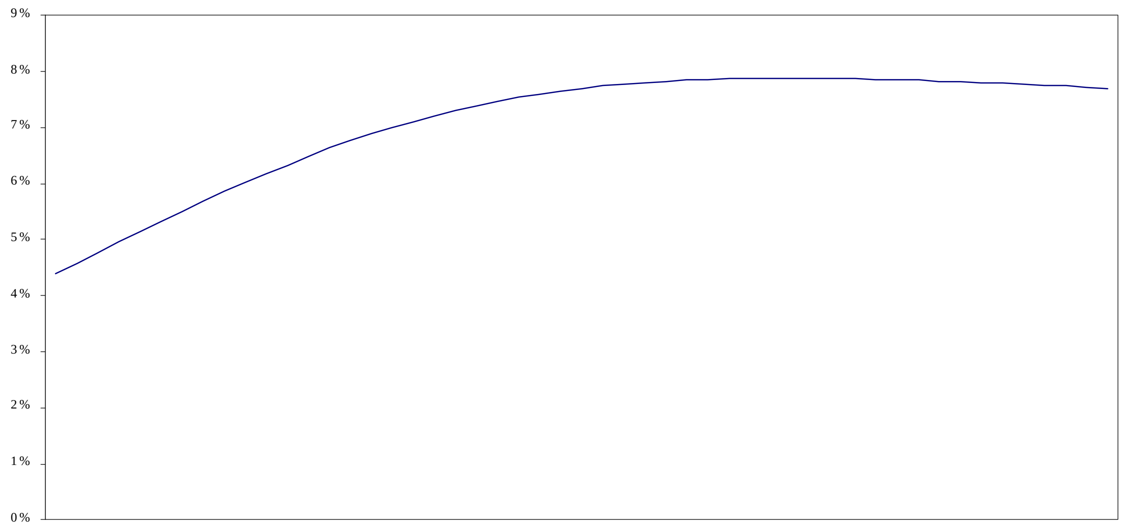


m=12%, s=20%, α=2%, π=2%, r=5%,γ=0.97, ρ=9%, n=0

Spending rate



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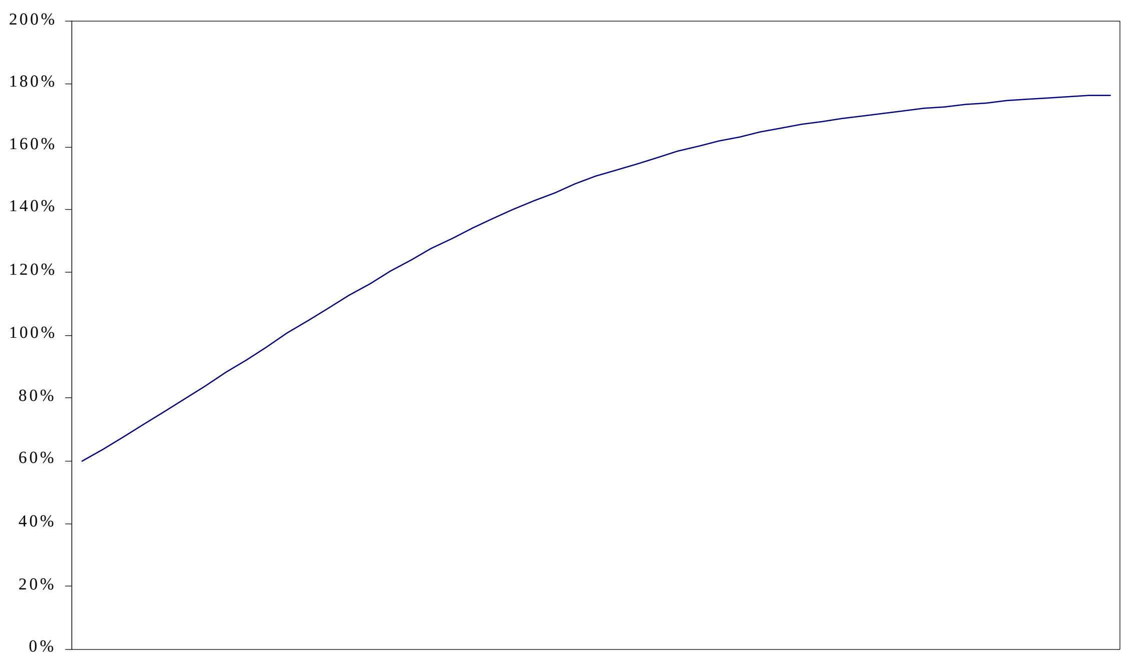
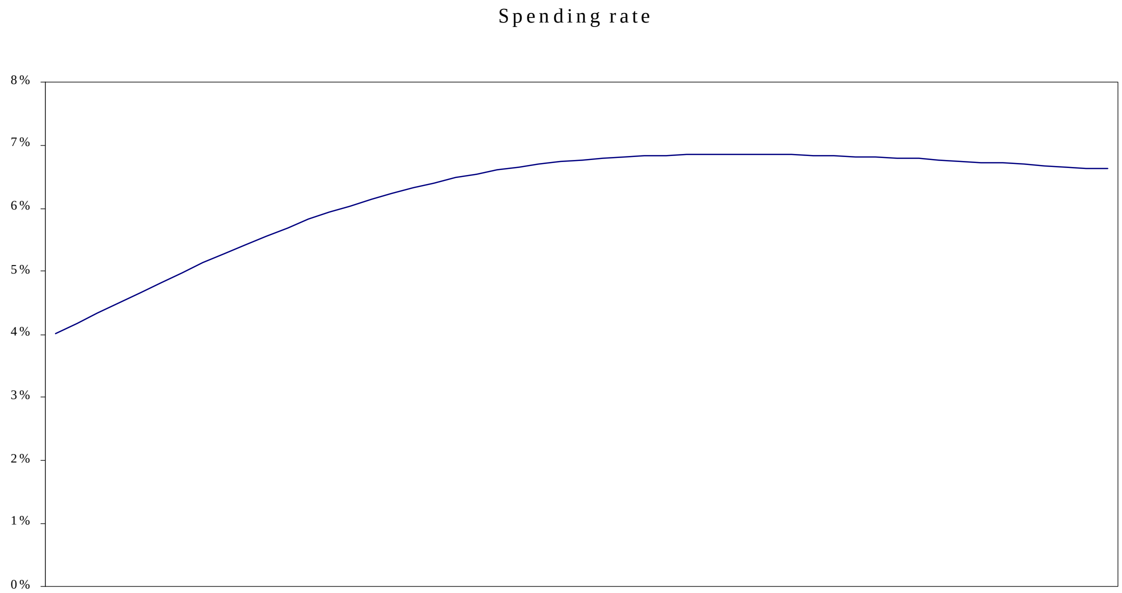
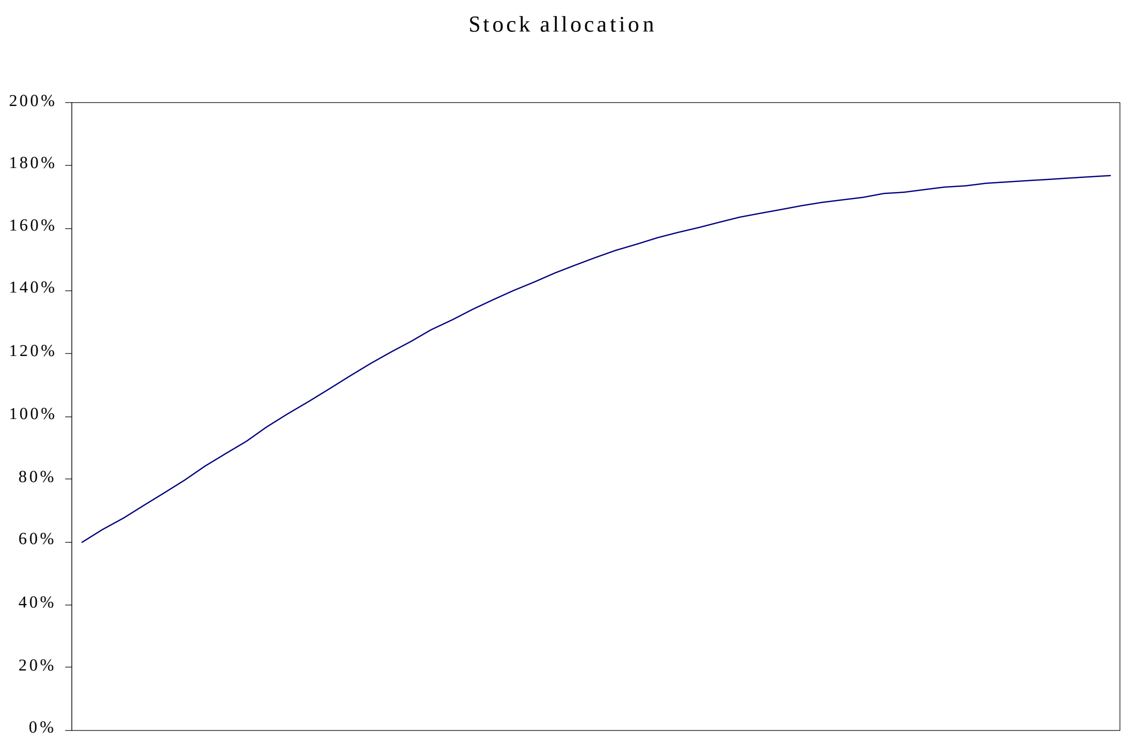


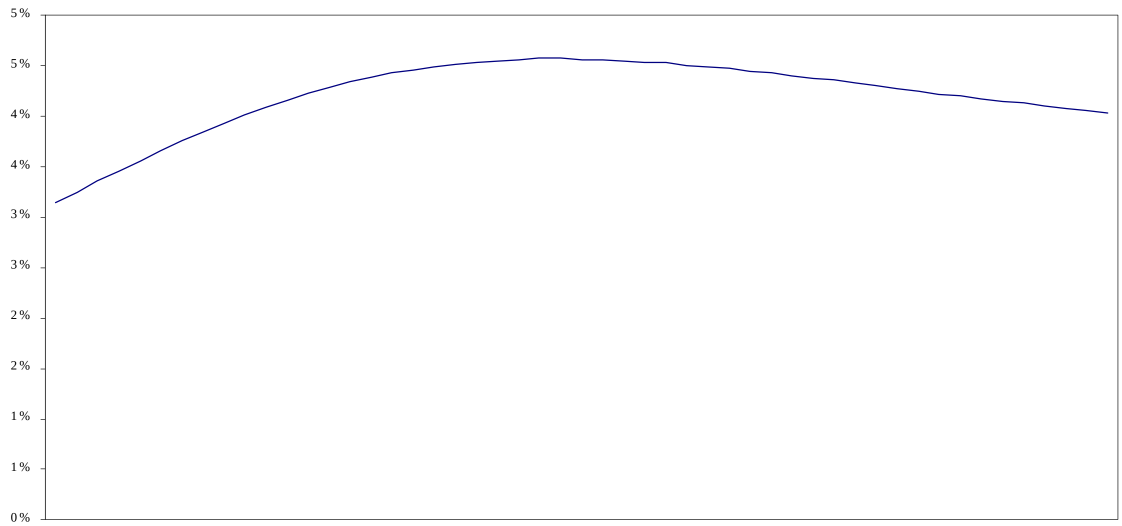
***m*=12%, *s*=16%, α=2%, π=2%, *r*=5%,γ=1.52, ρ=5%, n=0**





m=12%, s=18%, α=2%, π=2%, r=5%, γ=1.2, ρ=5%, n=0

Spending rate



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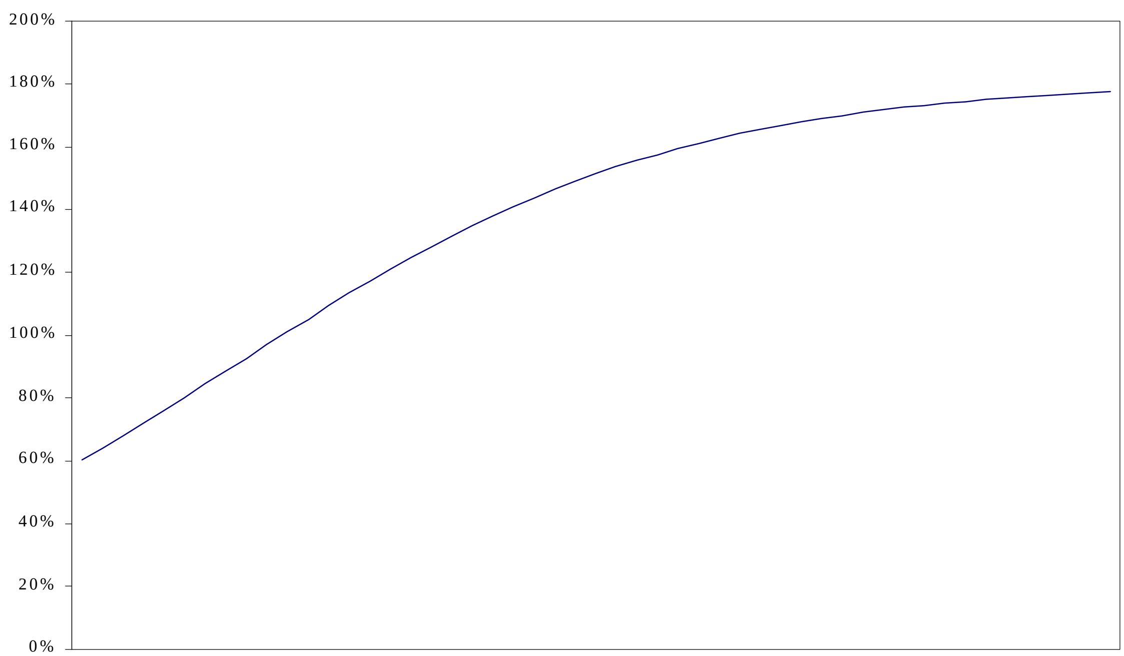
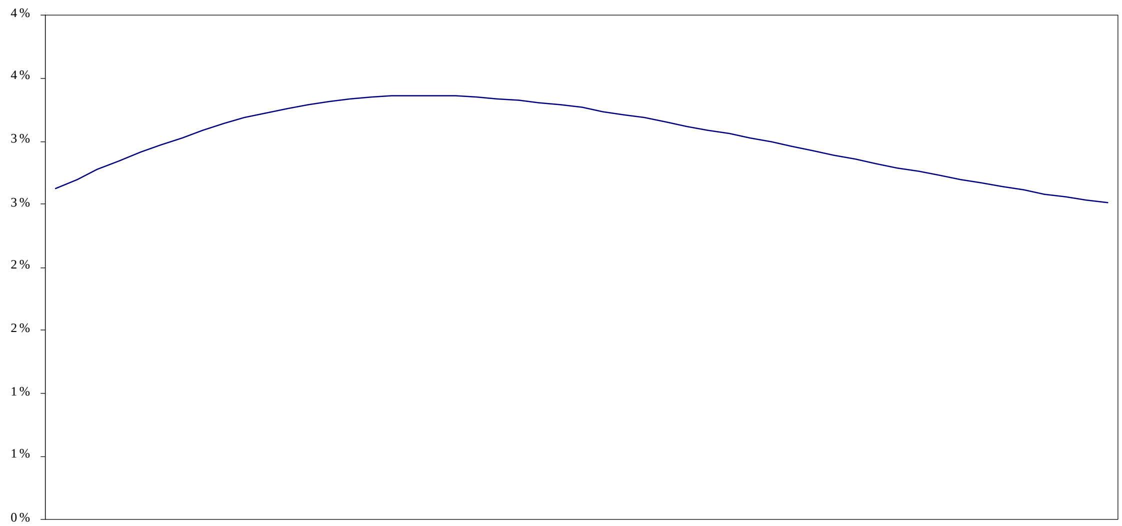


***m*=12%, *s*=22%, α=2%, π=2%, *r*=5%,γ=0.8, ρ=5%, n=0**

Spending rate



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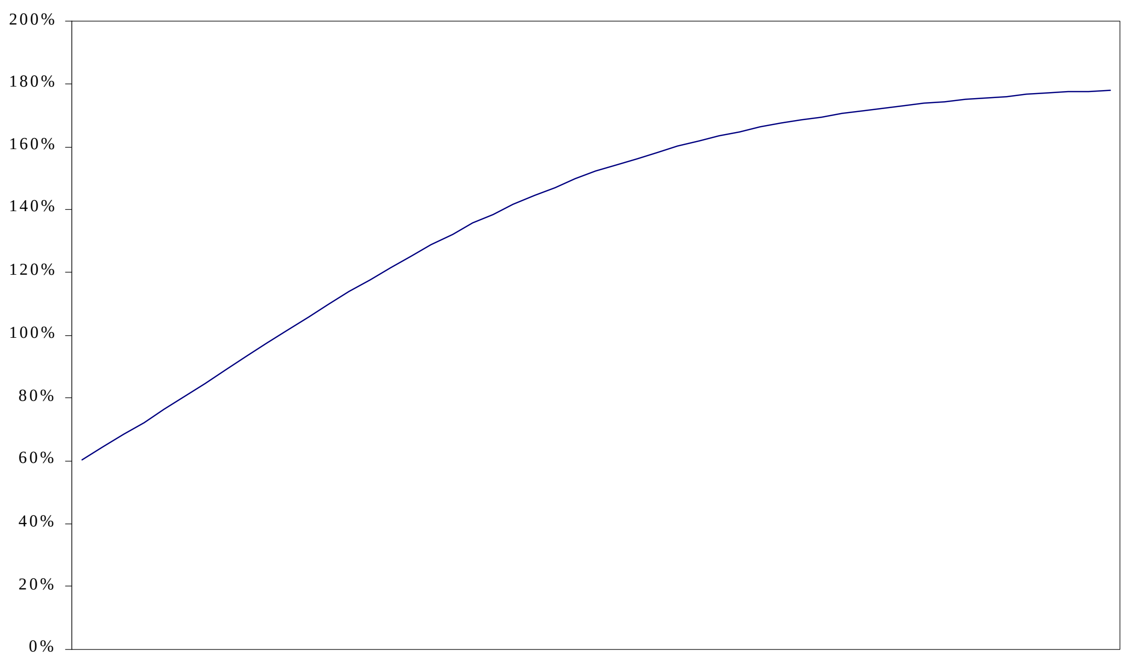
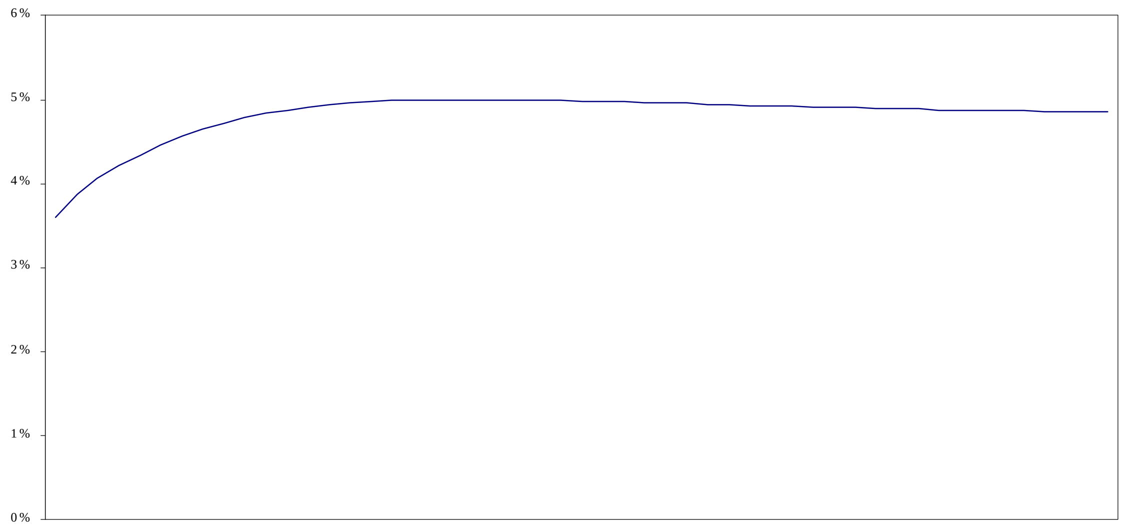
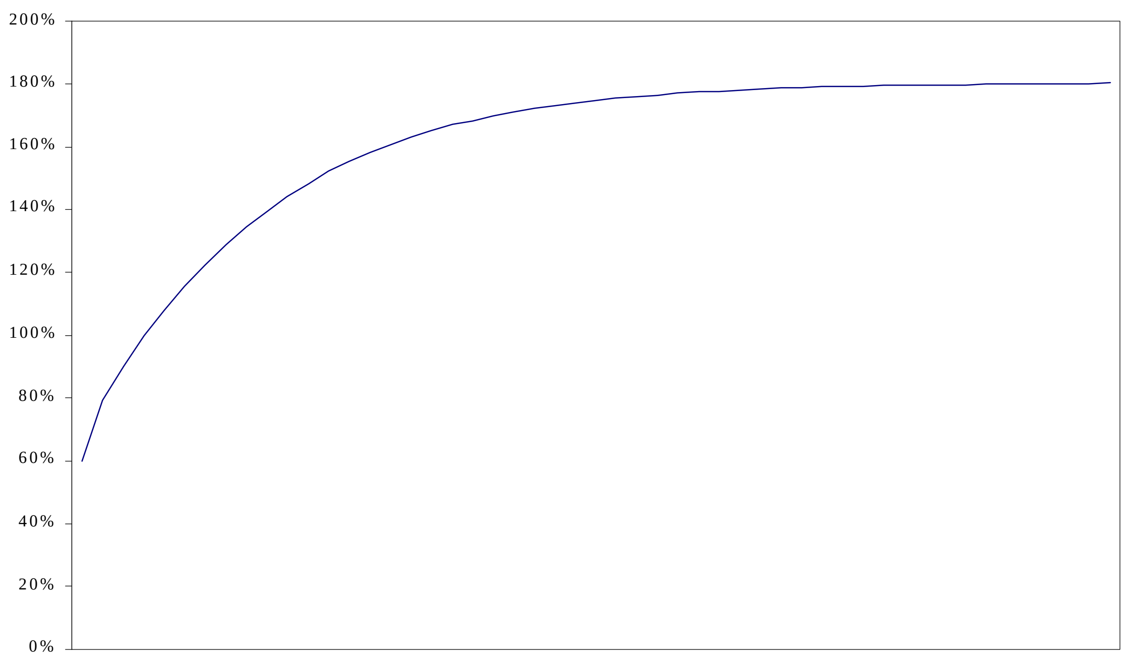


m=12%, s=24%, α=2%, π=2%, r=5%,γ=0.67, ρ=5%, n=0

Spending rate

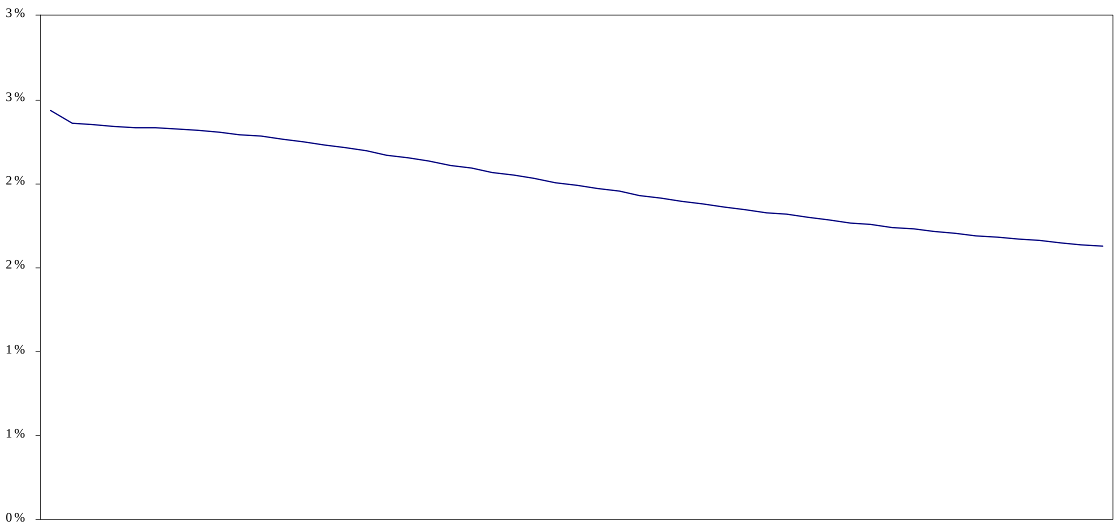


Stock allocation



m**=12%,** s**=20%,** α**=2%,** π**=2%,** r**=5%,**γ**=0.97,** ρ**=5%, n=+1**

Spending rate



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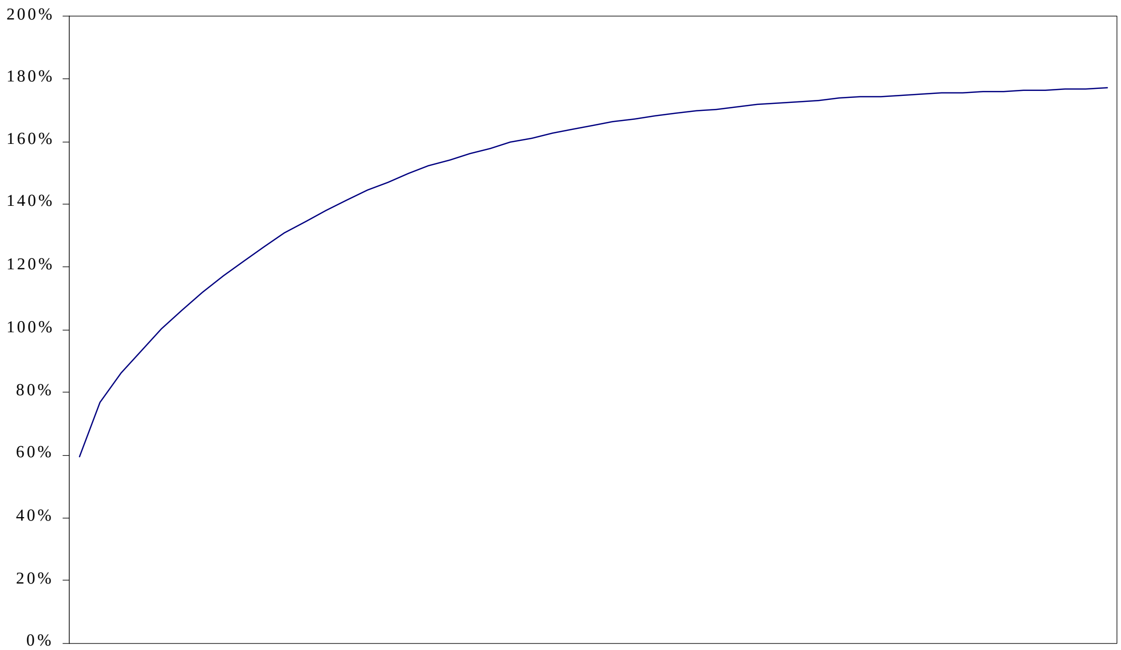
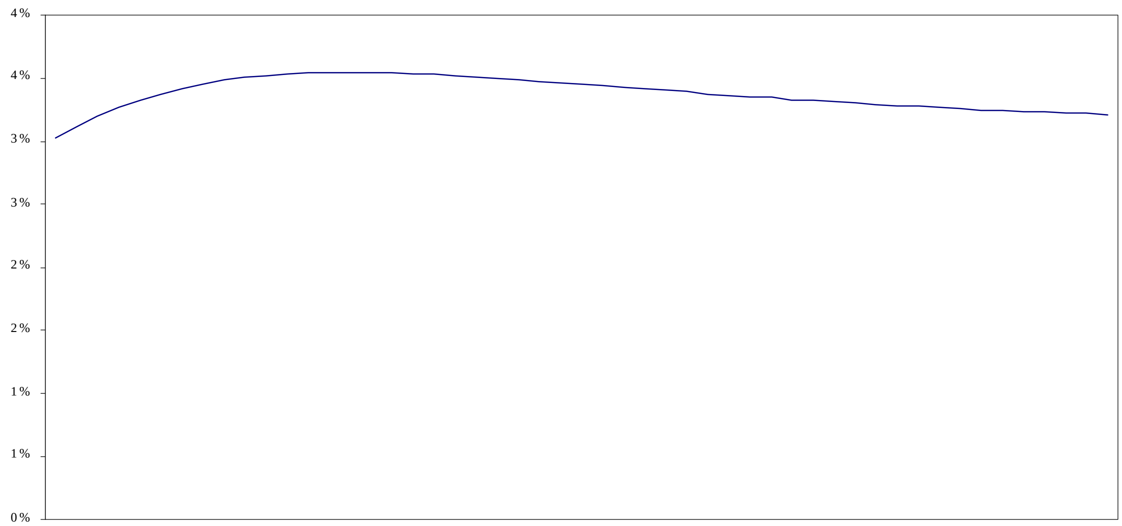
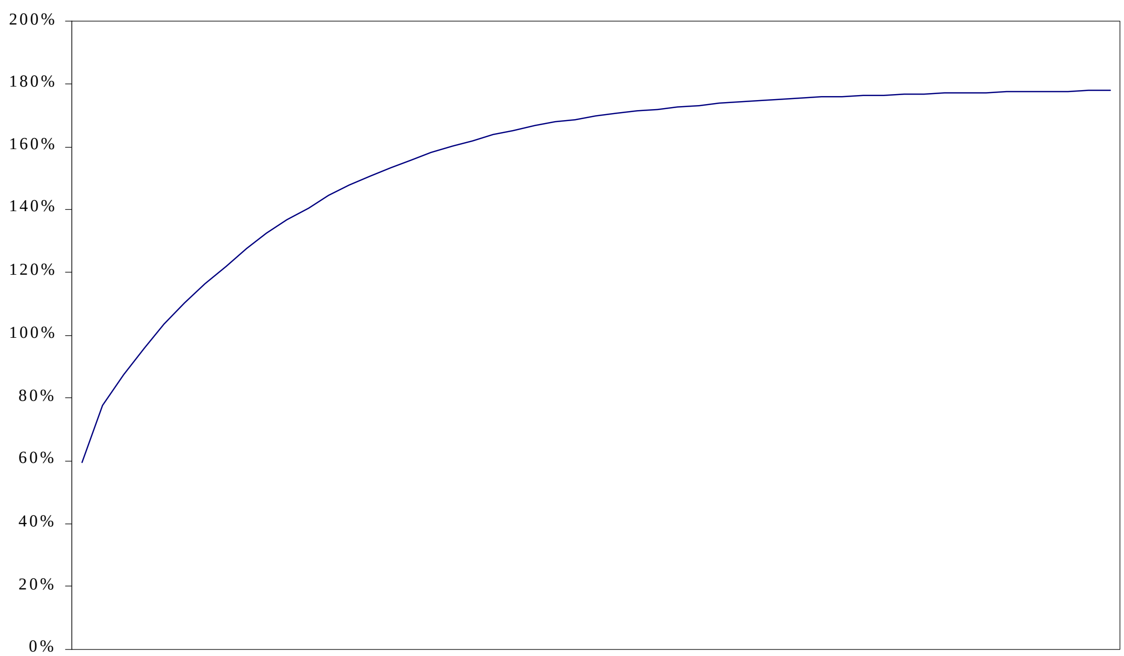


m=8%, s=20%, α=2%, π=2%, r=5%, γ=0.42, ρ=5%, n=+1

Spending rate

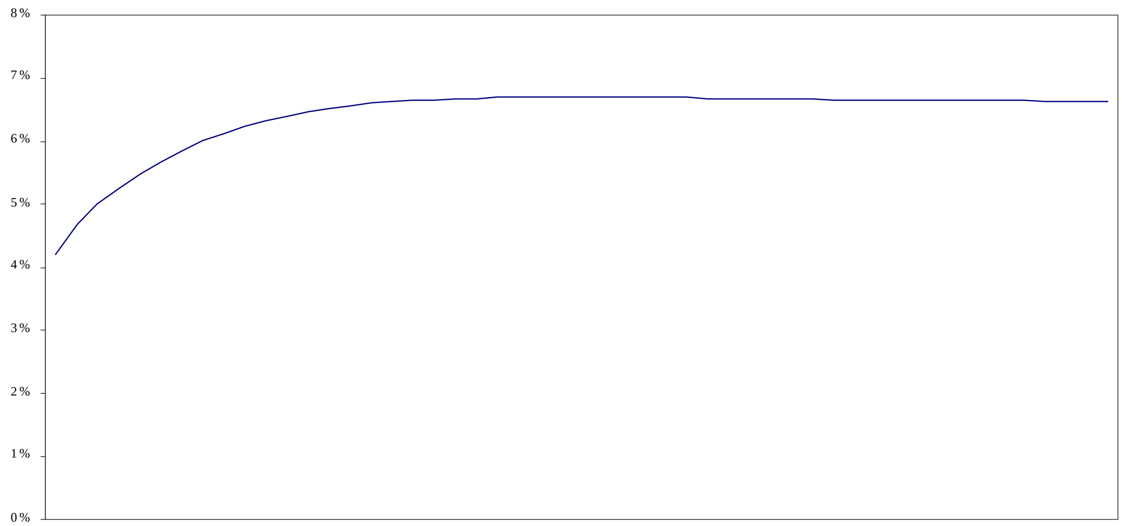


Stock allocation



m=10%, s=20%, α=2%, π=2%, r=5%, γ=0.7, ρ=5%, n=+1

Spending rate



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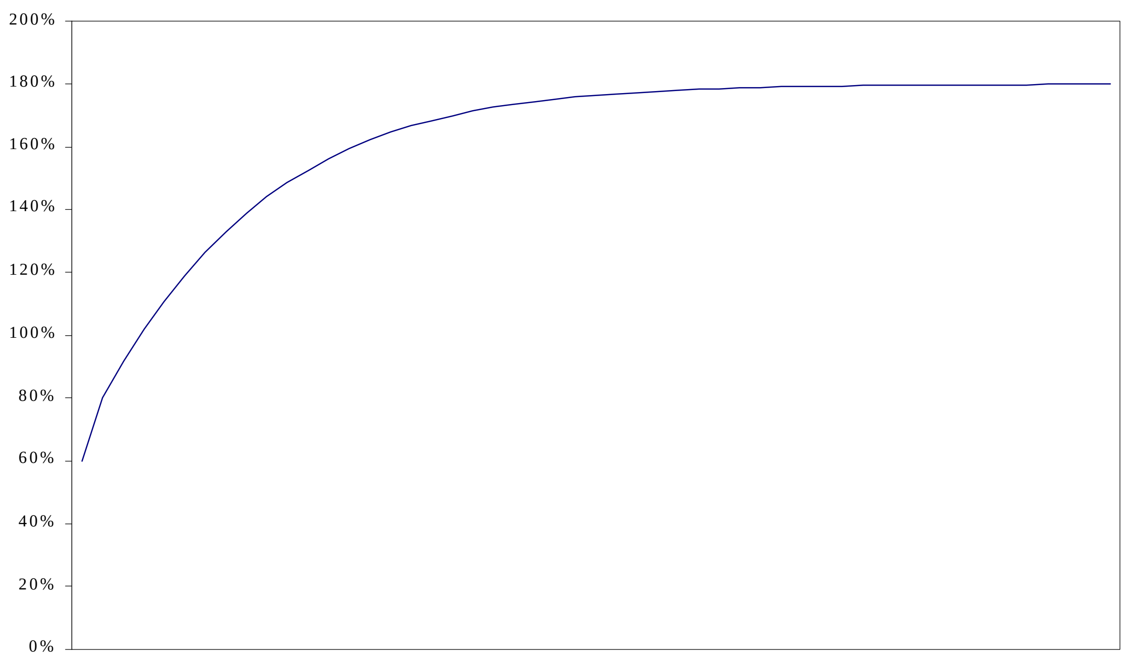
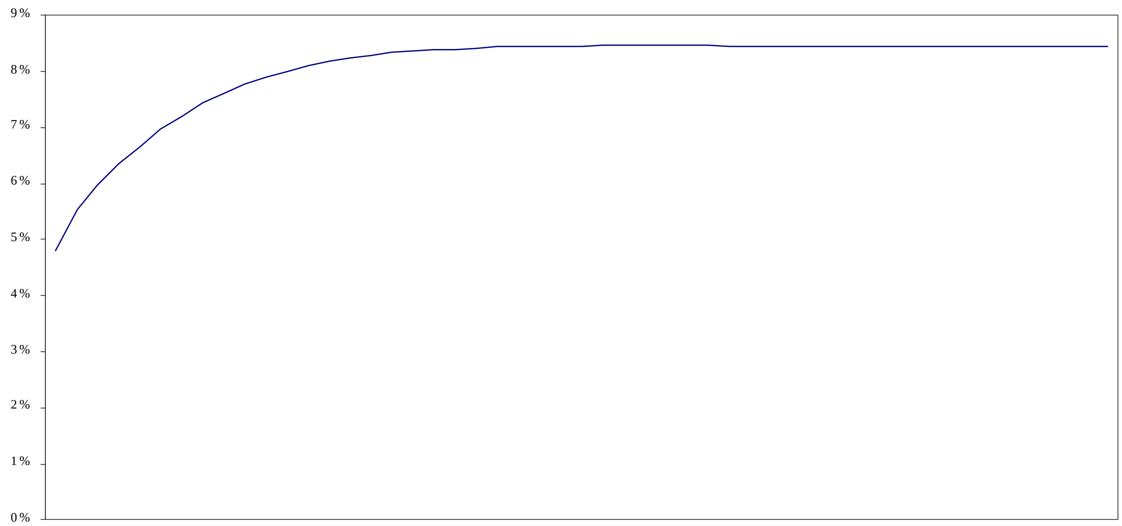


m=14%, s=20%, α=2%, π=2%, r=5%,γ=1.25, ρ=5%, n=+1

Spending rate



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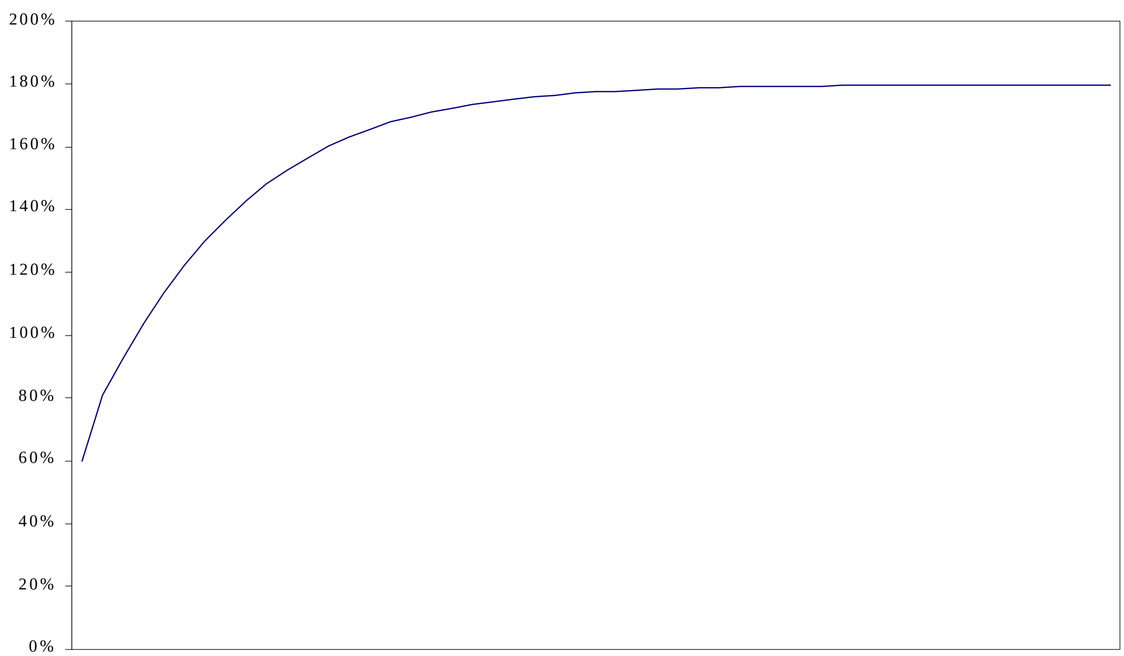
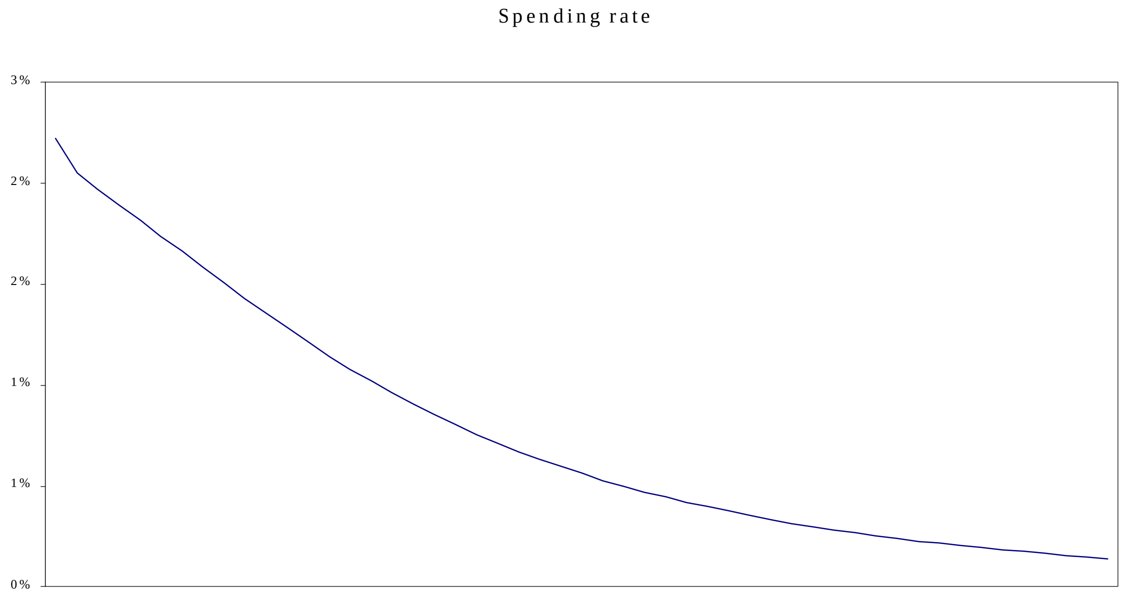
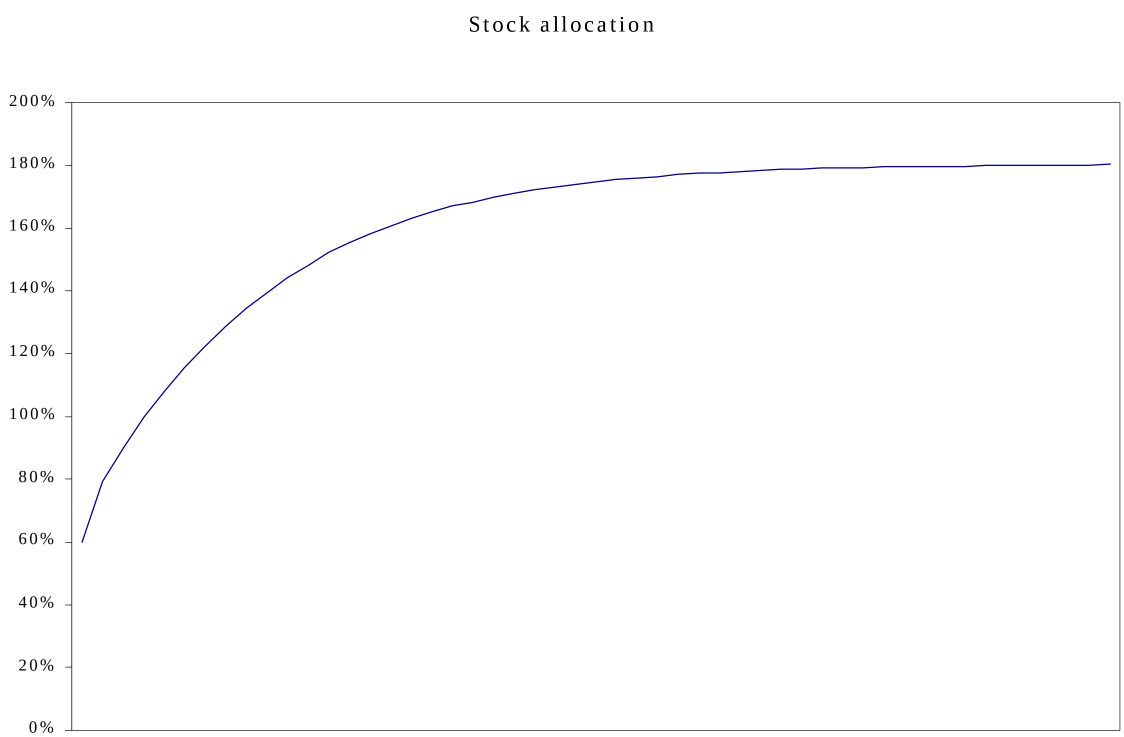
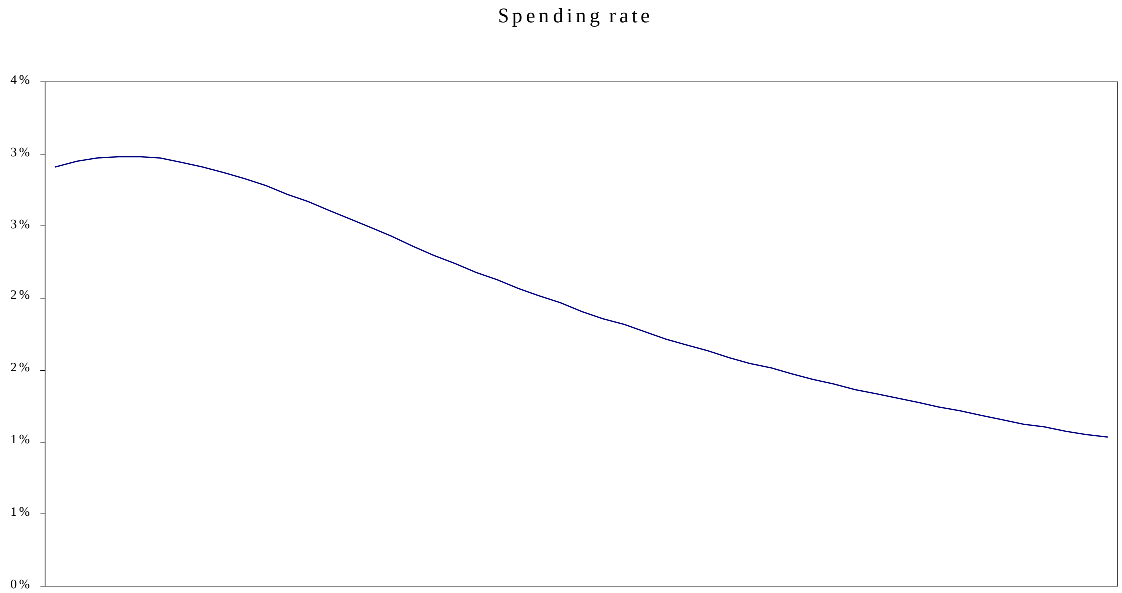


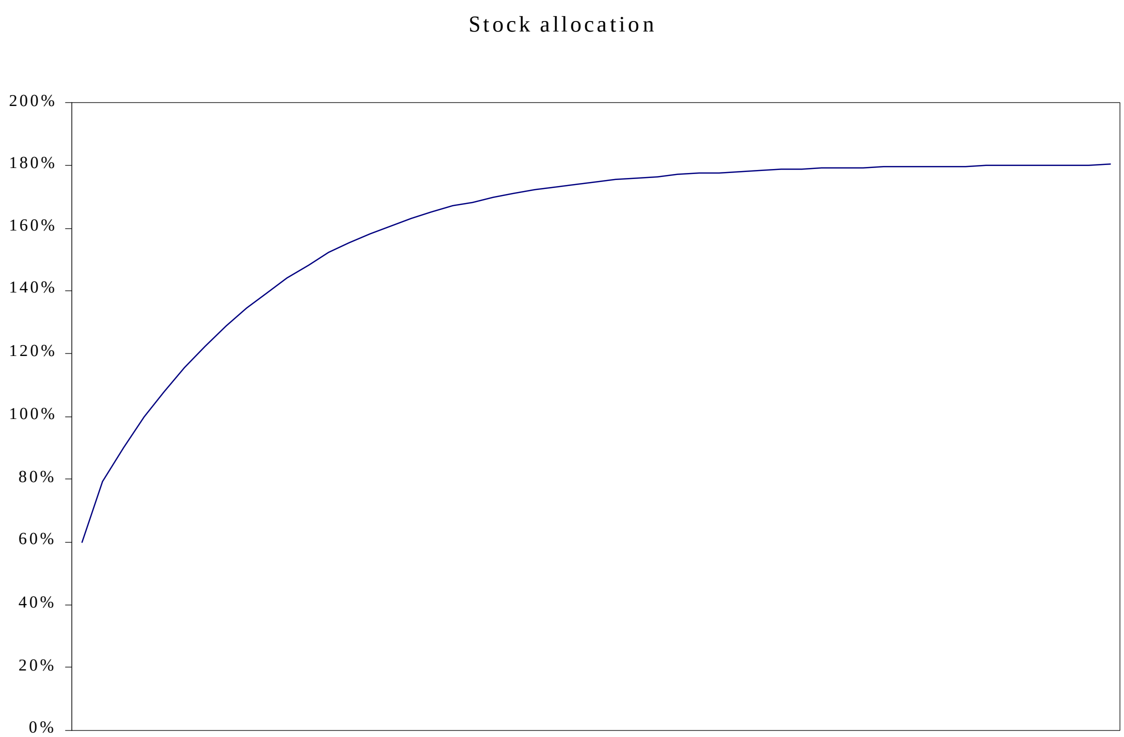
m=16%, s=20%, α=2%, π=2%, r=5%, γ=1.53, ρ=5%, n=+1



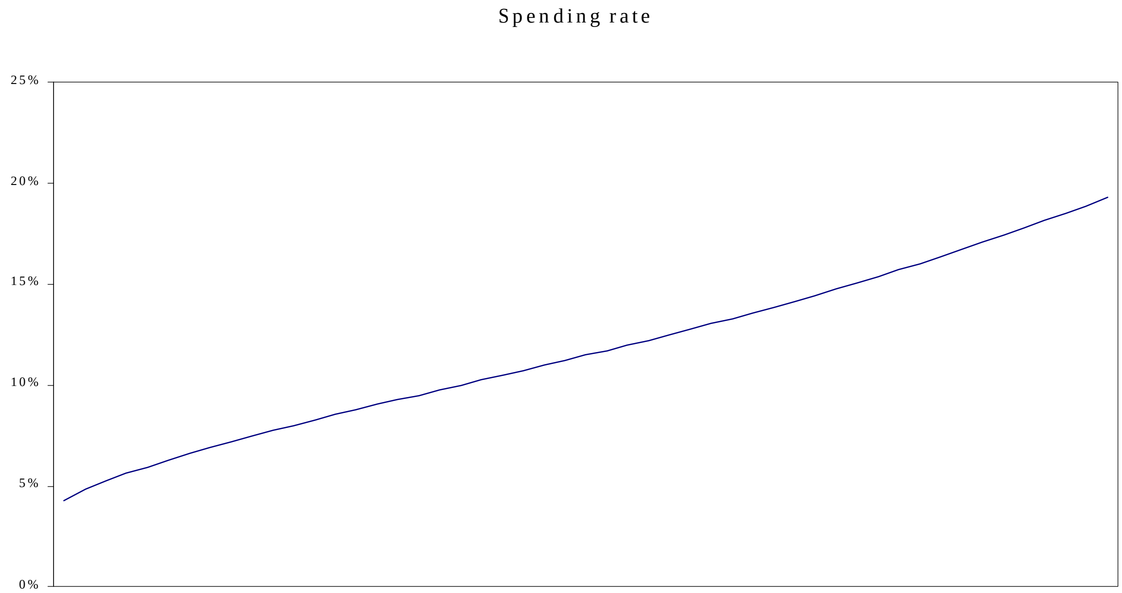


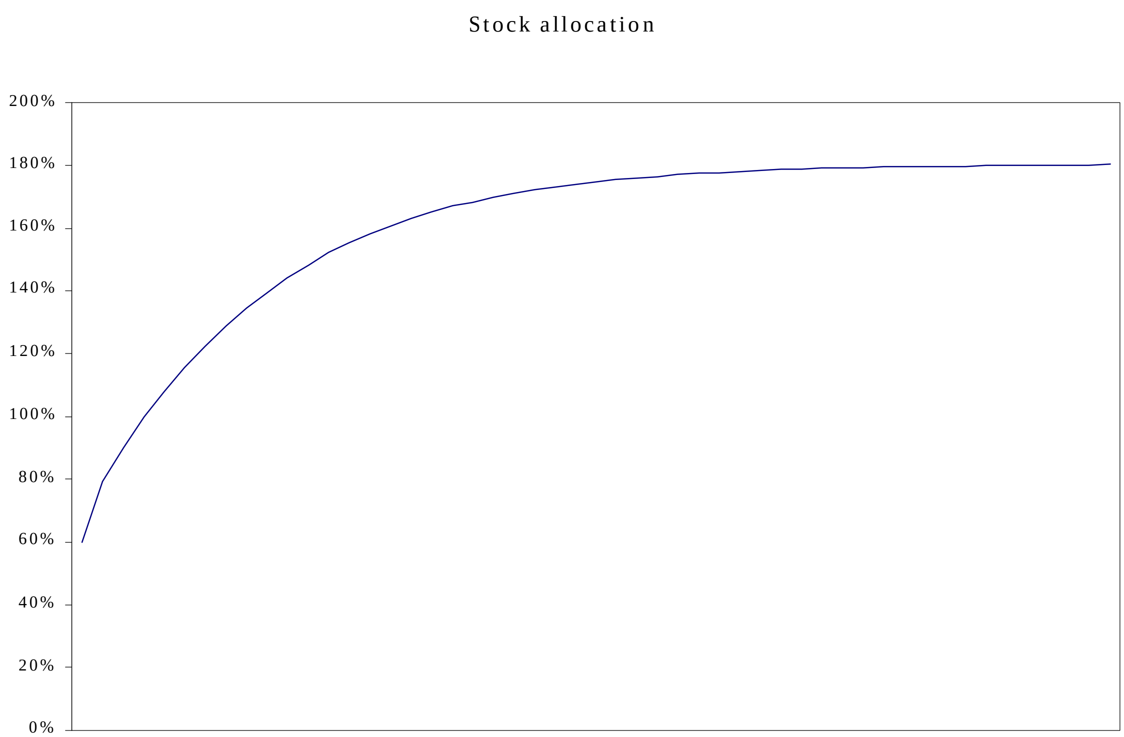
m=12%, s=20%, α=2%, π=2%, r=5%,γ=0.97, ρ=1%, n=+1





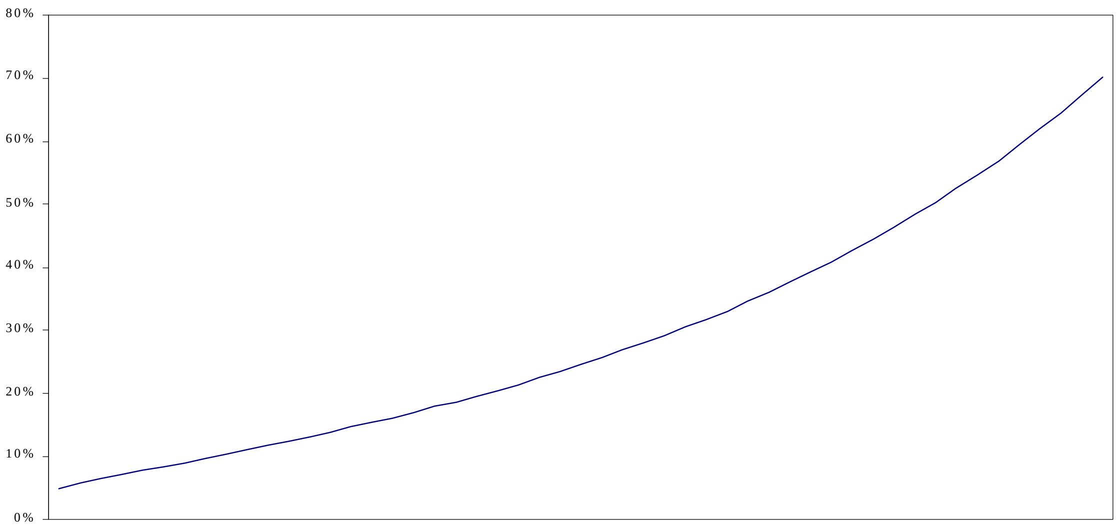
m=12%, s=20%, α=2%, π=2%, r=5%,γ=0.97, ρ=3%, n=+1



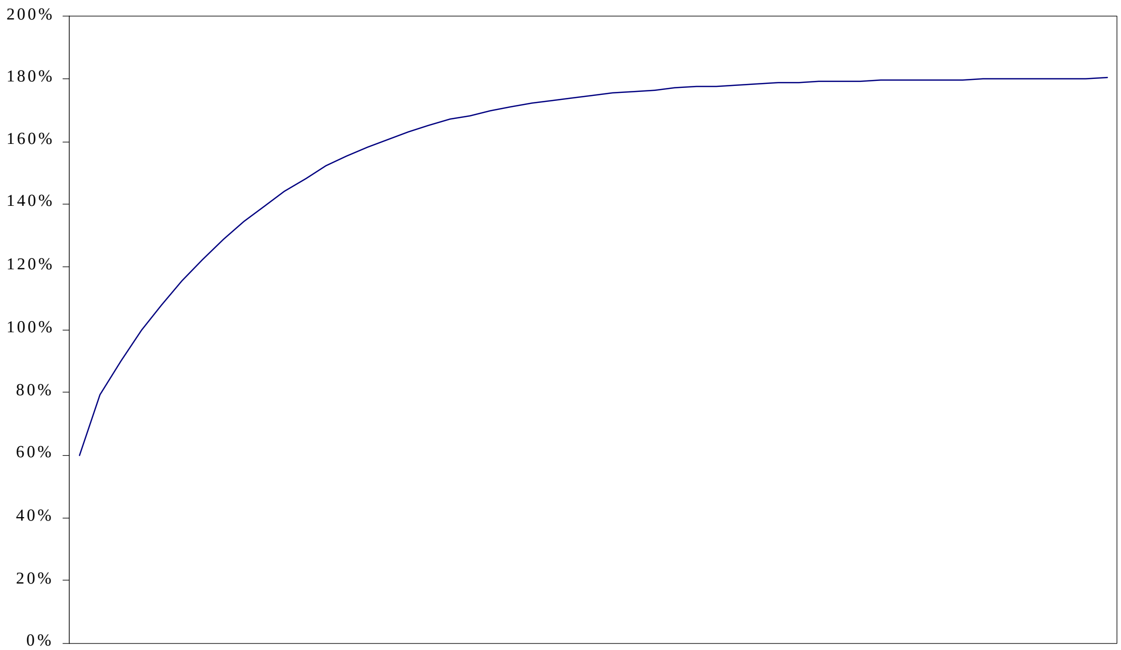


***m*=12%, *s*=20%, α=2%, π=2%, *r*=5%,γ=0.97, ρ=7%, n=+1**

Spending rate

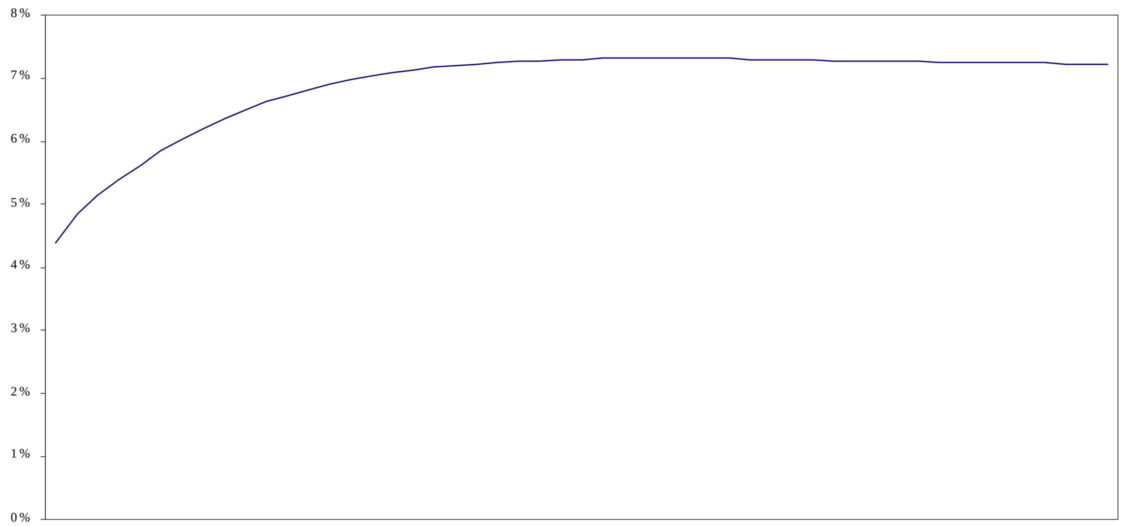


Stock allocation



m=12%, s=20%, α=2%, π=2%, r=5%,γ=0.97, ρ=9%, n=+1

Spending rate



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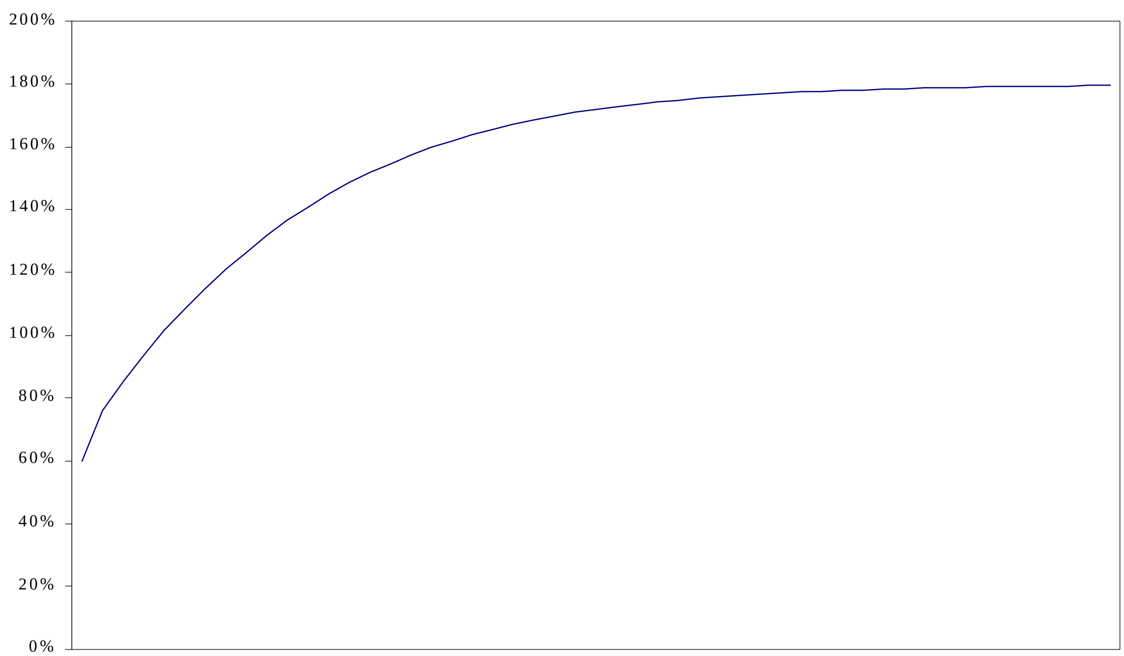
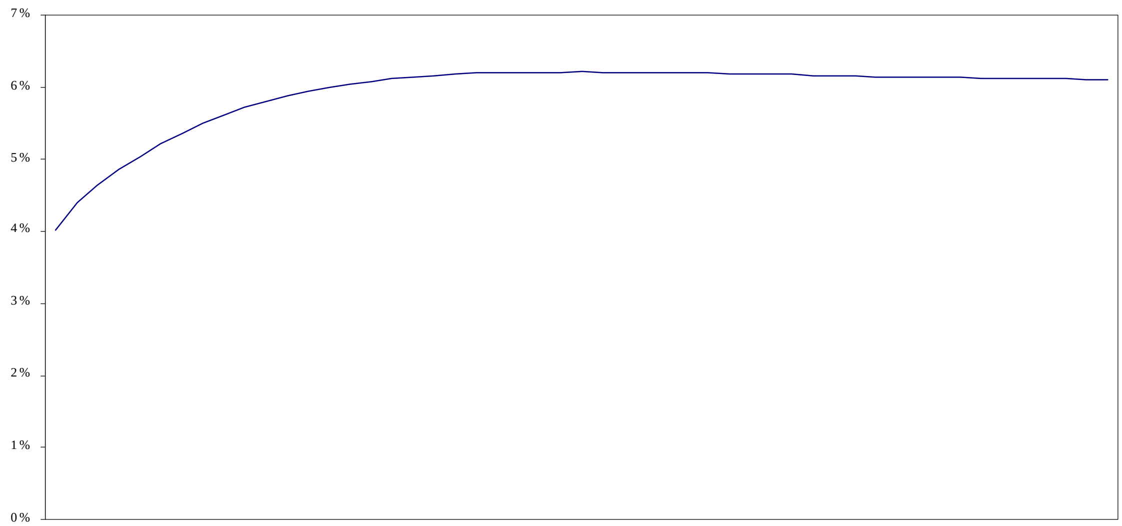


m=12%, s=16%, α=2%, π=2%, r=5%,γ=1.52, ρ=5%, n=+1

Spending rate



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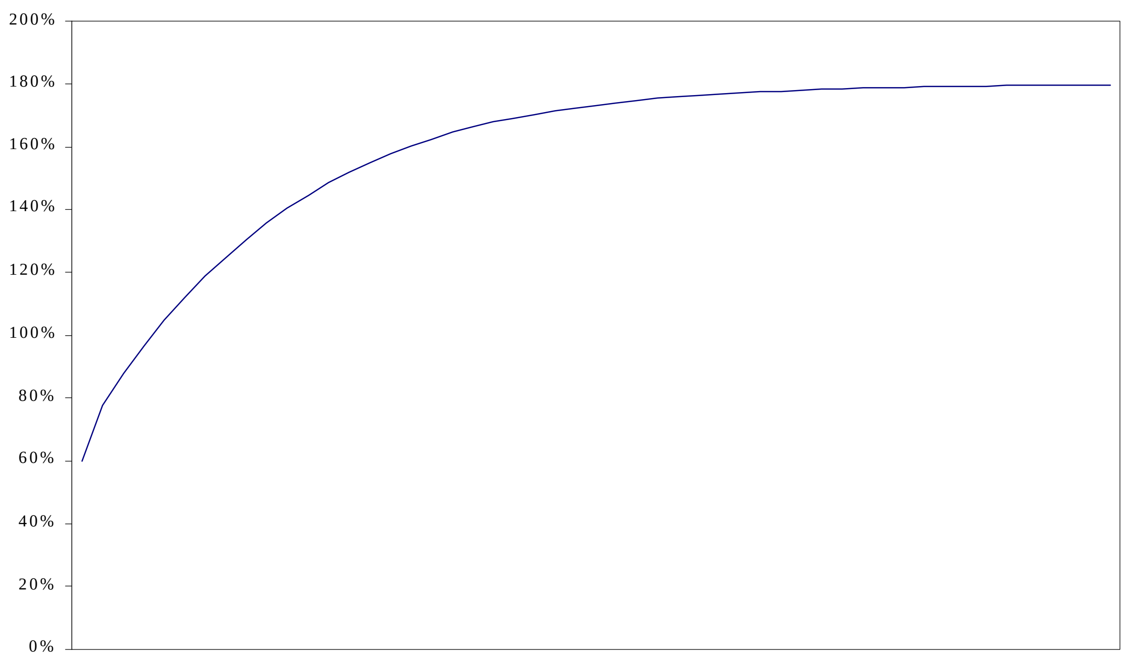
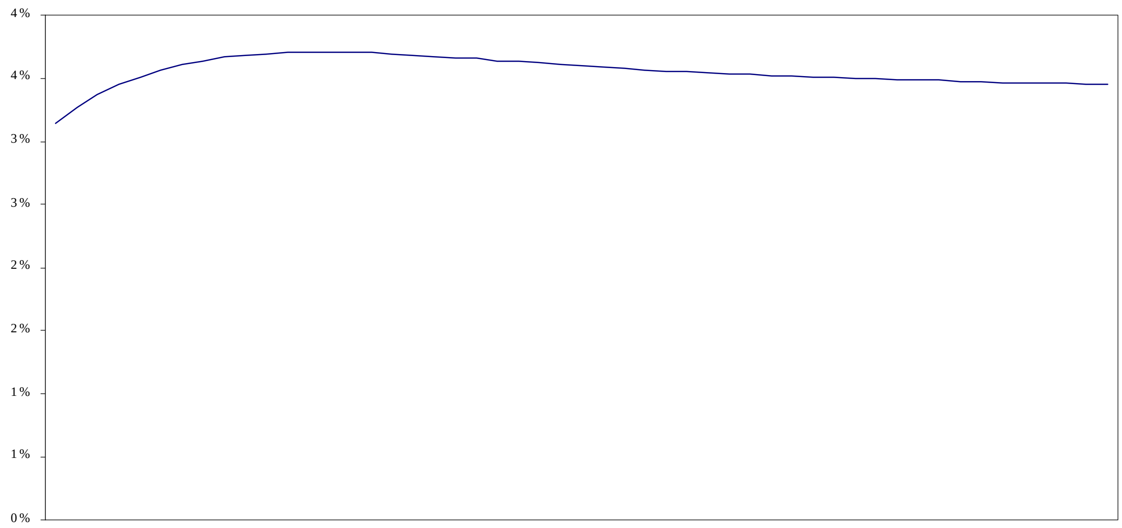
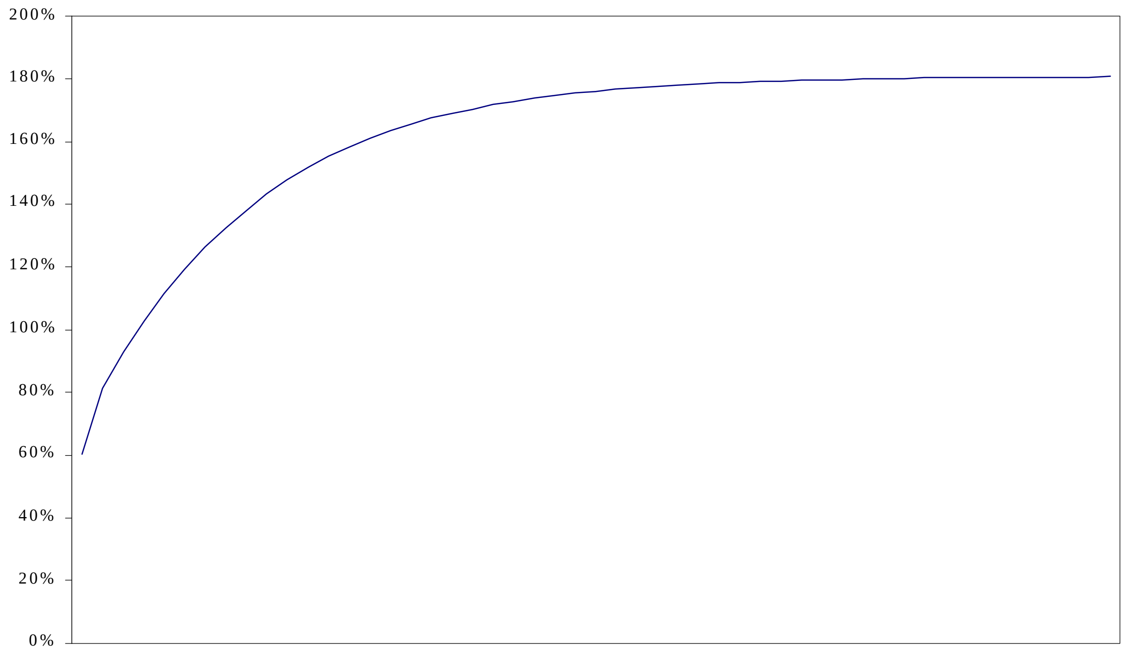


m=12%, s=18%, α=2%, π=2%, r=5%,γ=1.2, ρ=5%, n=+1

Spending rate

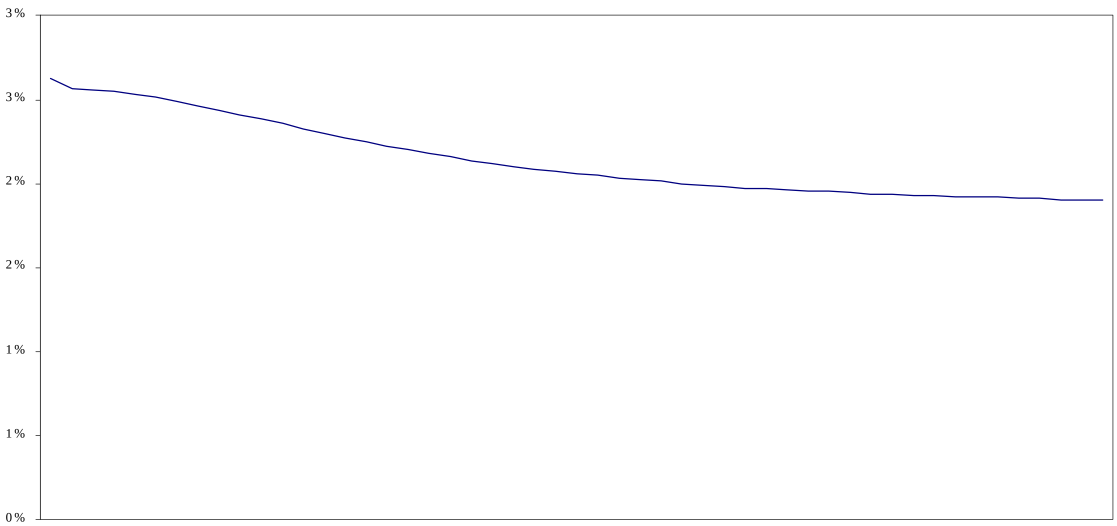


Stock allocation

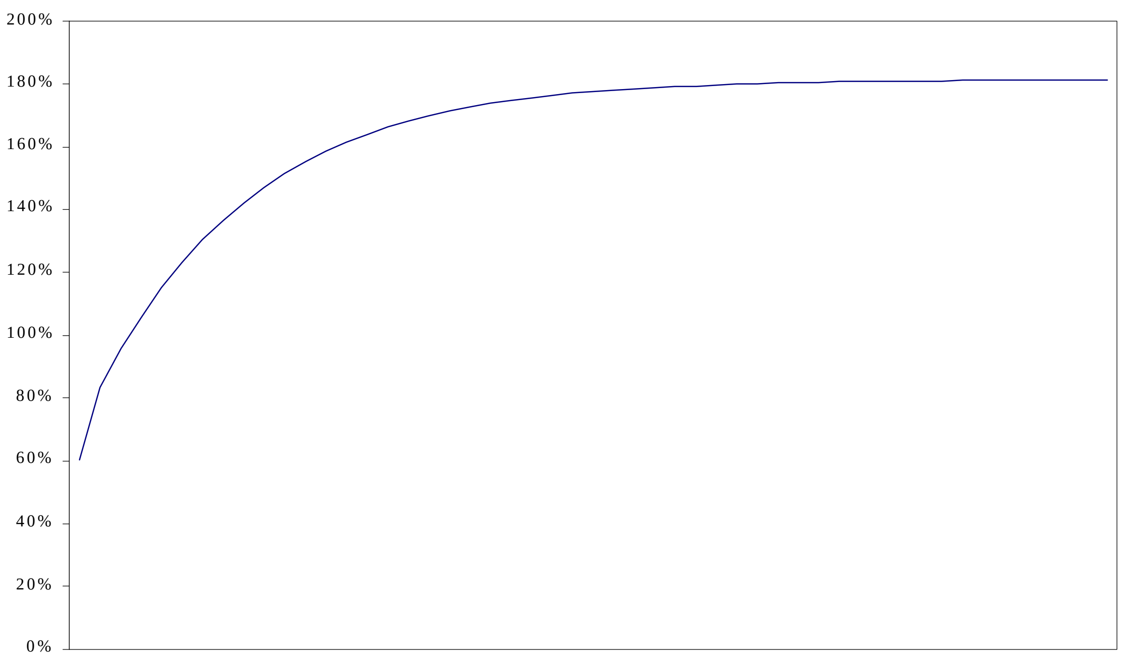


m=12%, s=22%, α=2%, π=2%, r=5%,γ=0.8, ρ=5%, n=+1

Spending rate

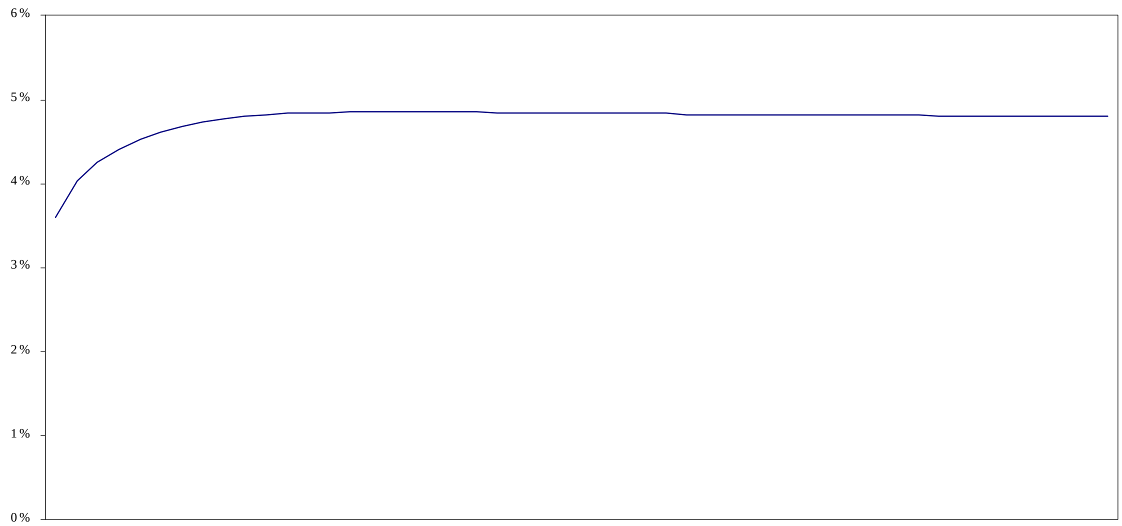


Stock allocation

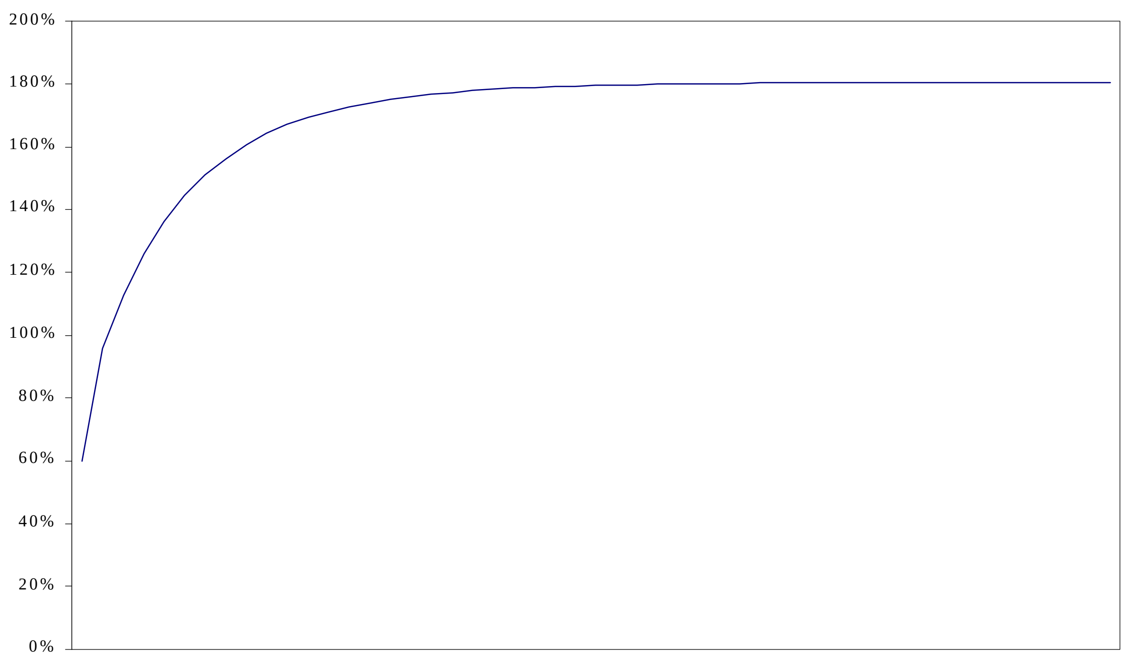


m=12%, s=24%, α=2%, π=2%, r=5%,γ=0.67, ρ=5%, n=+1

Spending rate

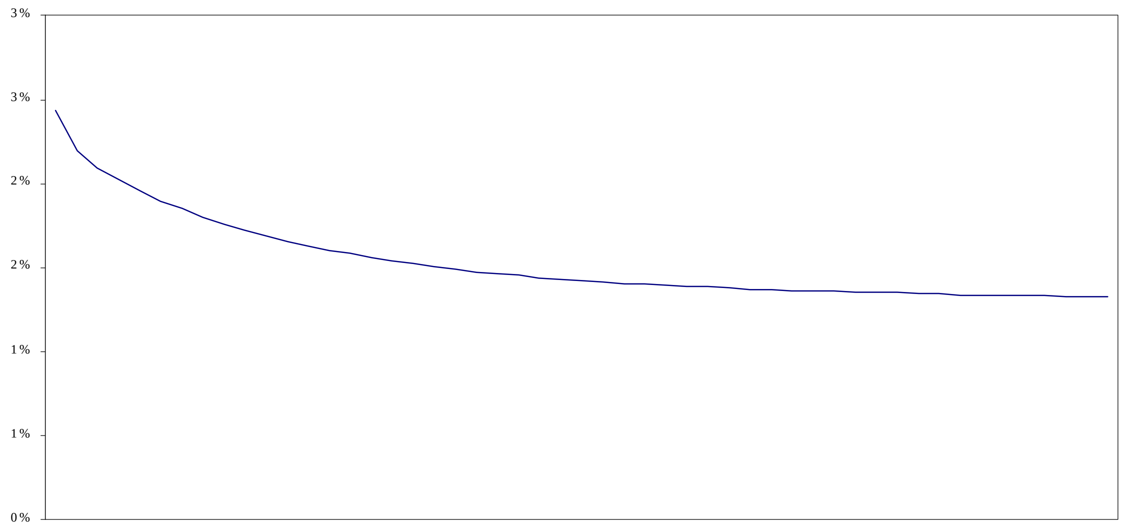


Stock allocation

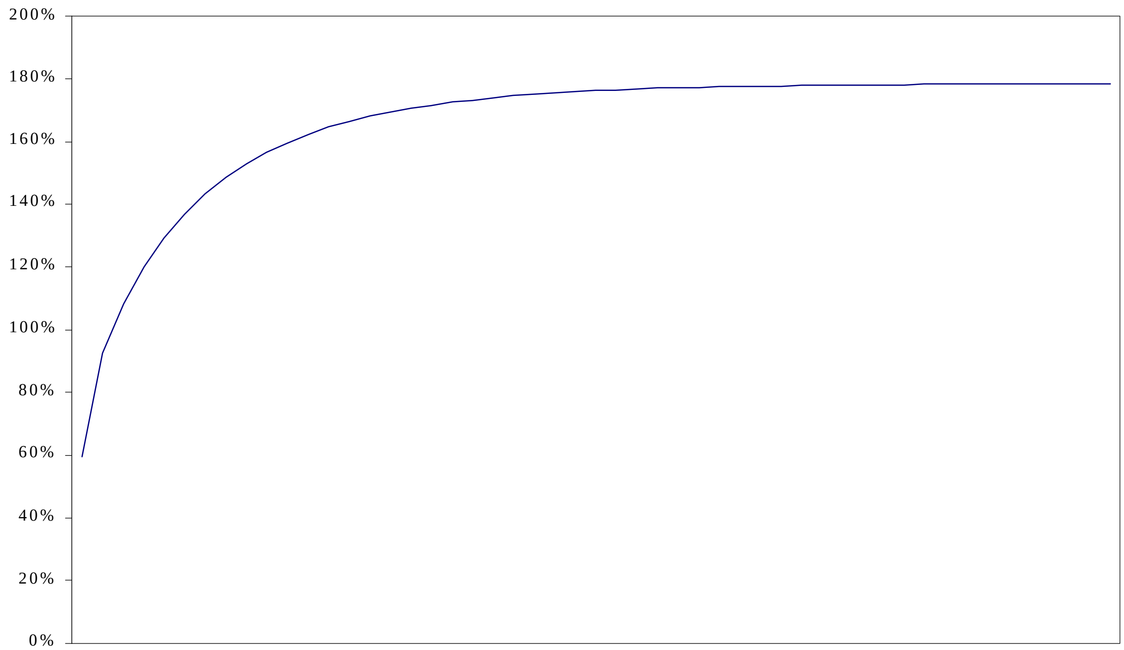


m=12%, s=20%, α=2%, π=2%, r=5%,γ=0.97, ρ=5%, n=+2

Spending rate

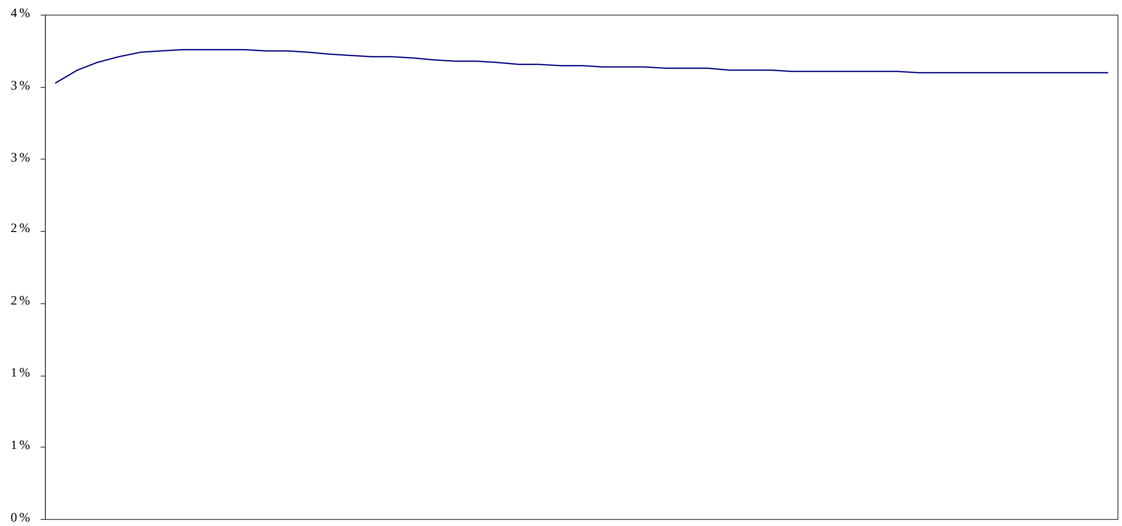


Stock allocation

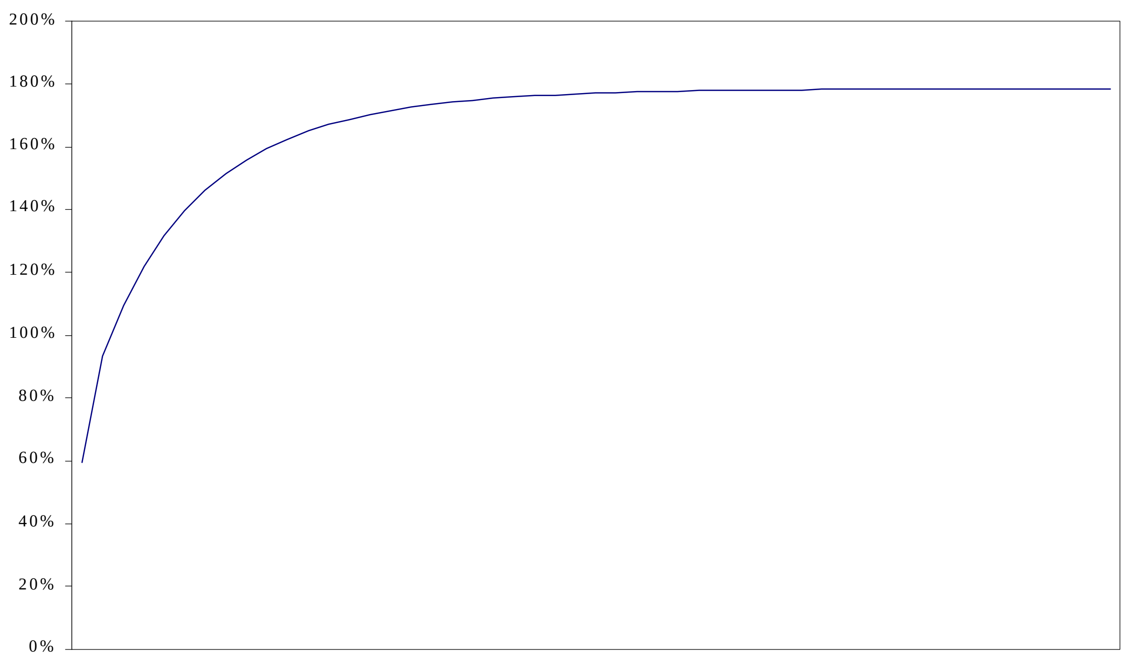


m=8%, s=20%, α=2%, π=2%, r=5%,γ=0.42, ρ=5%, n=+2

Spending rate

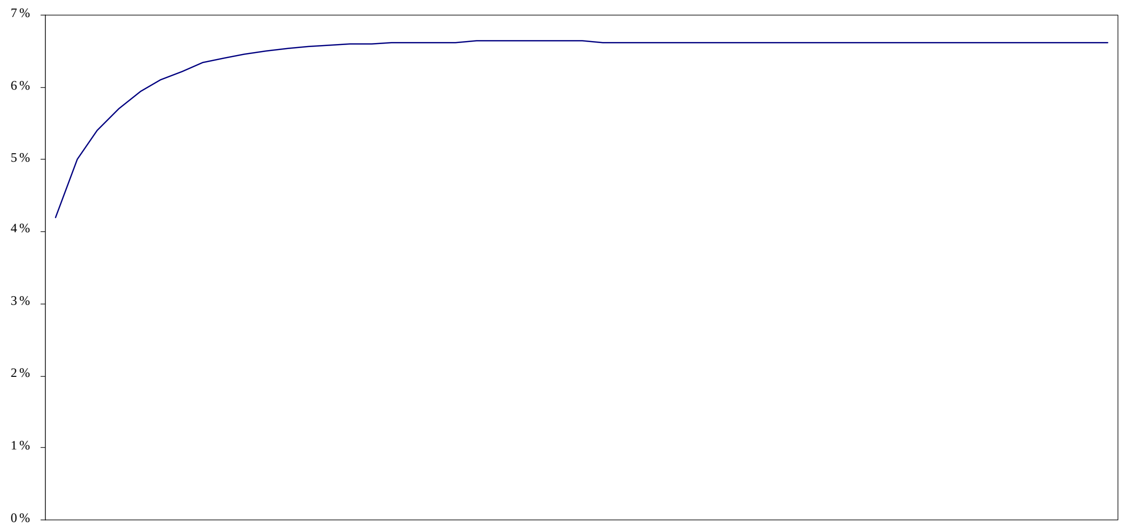


Stock allocation

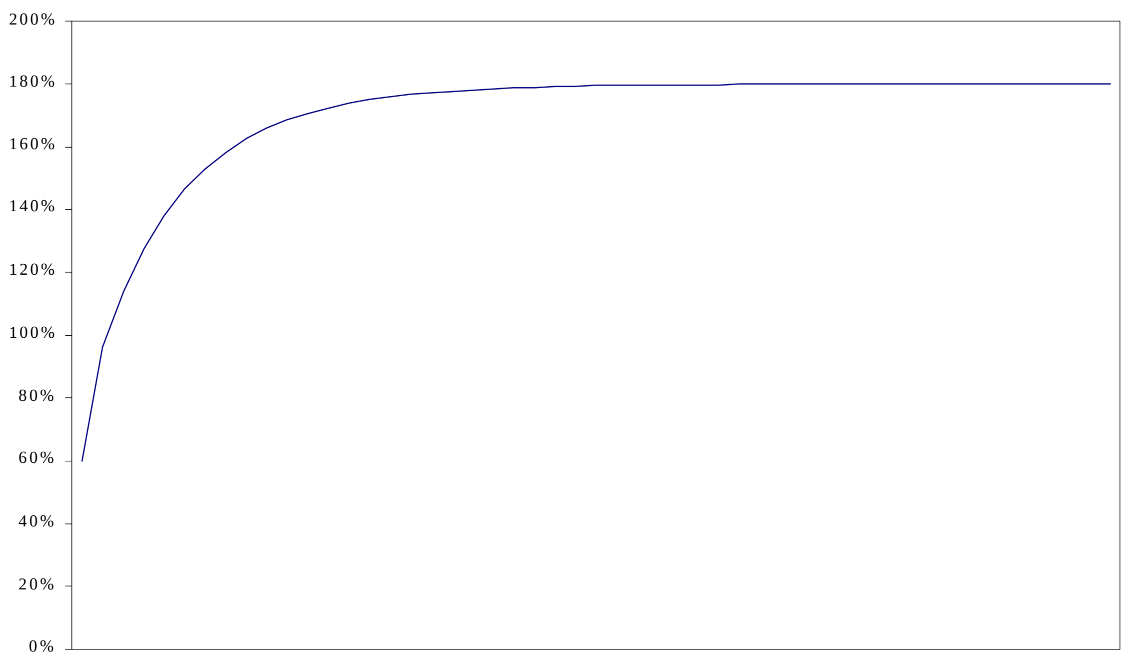


m=10%, s=20%, α=2%, π=2%, r=5%,γ=0.7, ρ=5%, n=+2

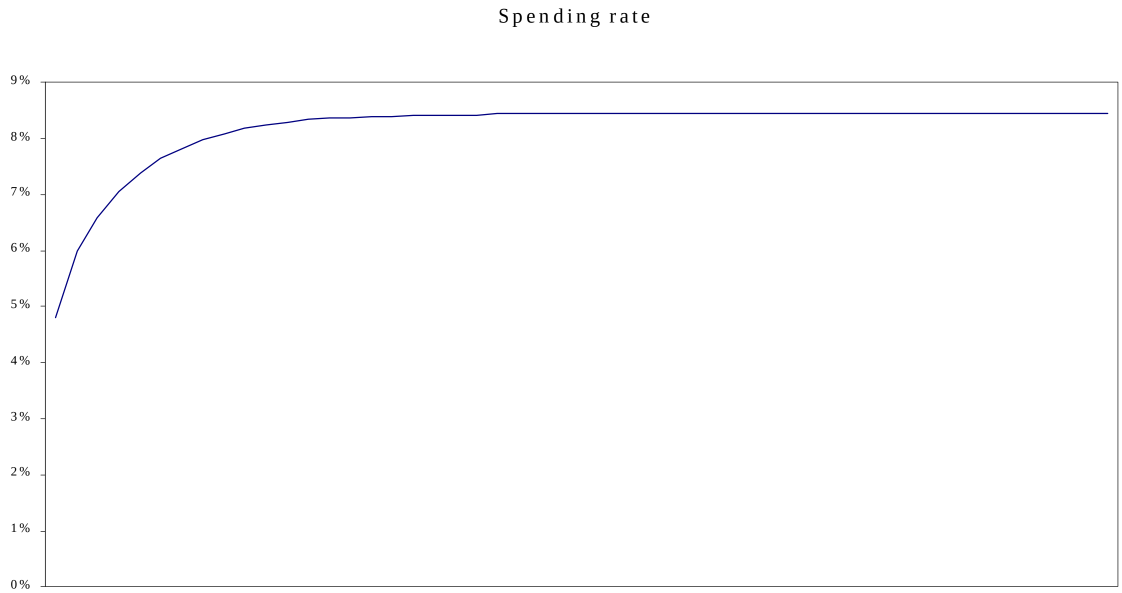
Spending rate



Stock allocation

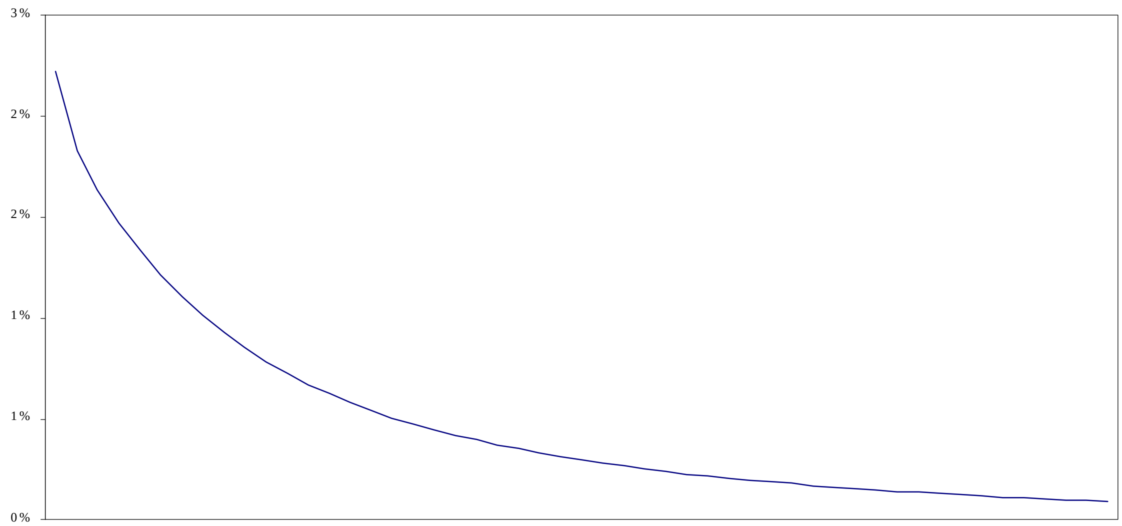


m=14%, s=20%, α=2%, π=2%, r=5%,γ=1.25, ρ=5%, n=+2

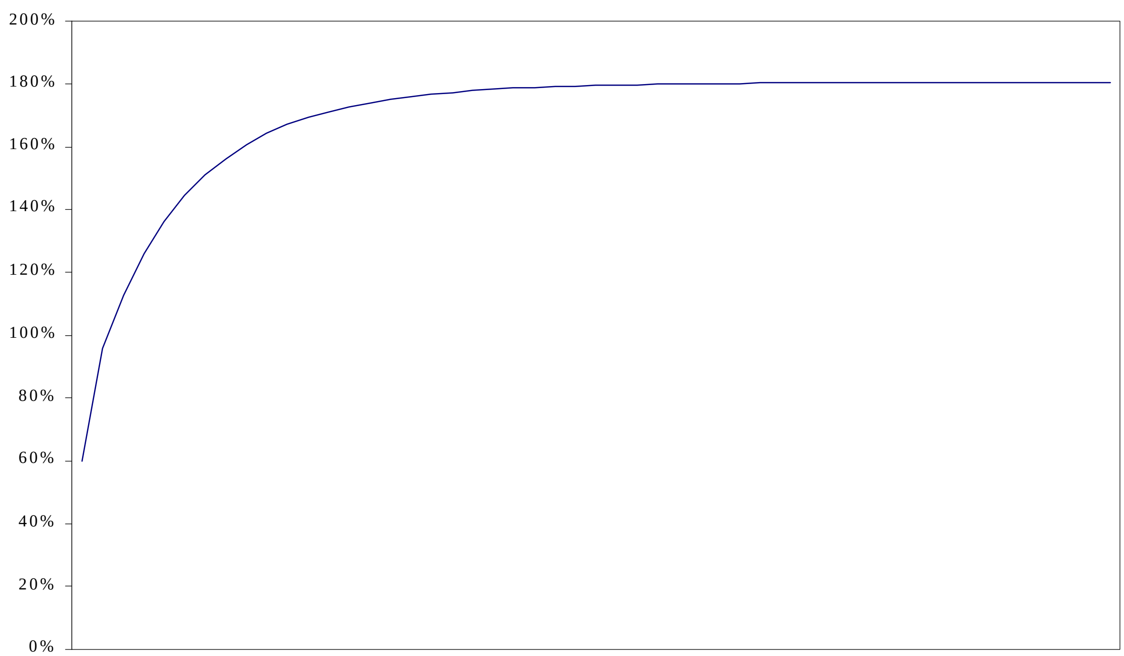


m=16%, s=20%, α=2%, π=2%, r=5%,γ=1.53, ρ=5%, n=+2

Spending rate

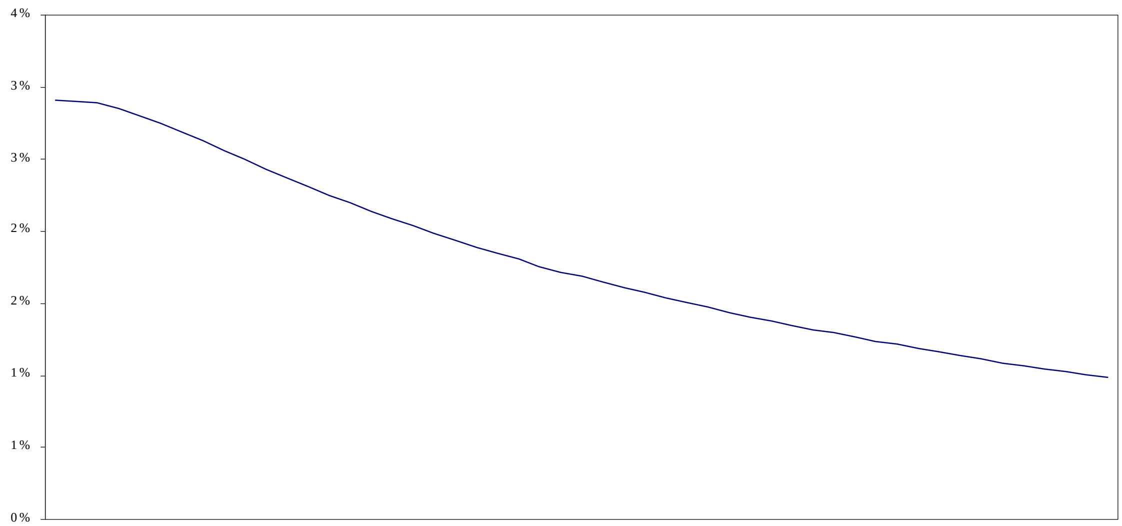


Stock allocation

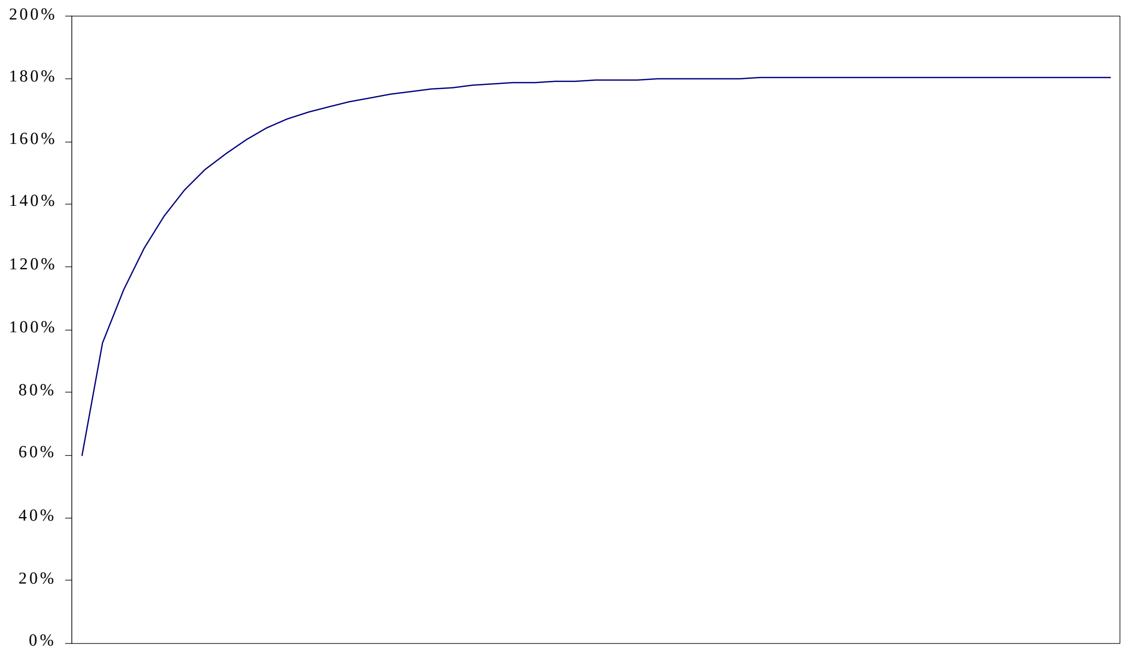


***m*=12%, *s*=20%, α=2%, π=2%, *r*=5%,γ=0.97, ρ=1%, n=+2**

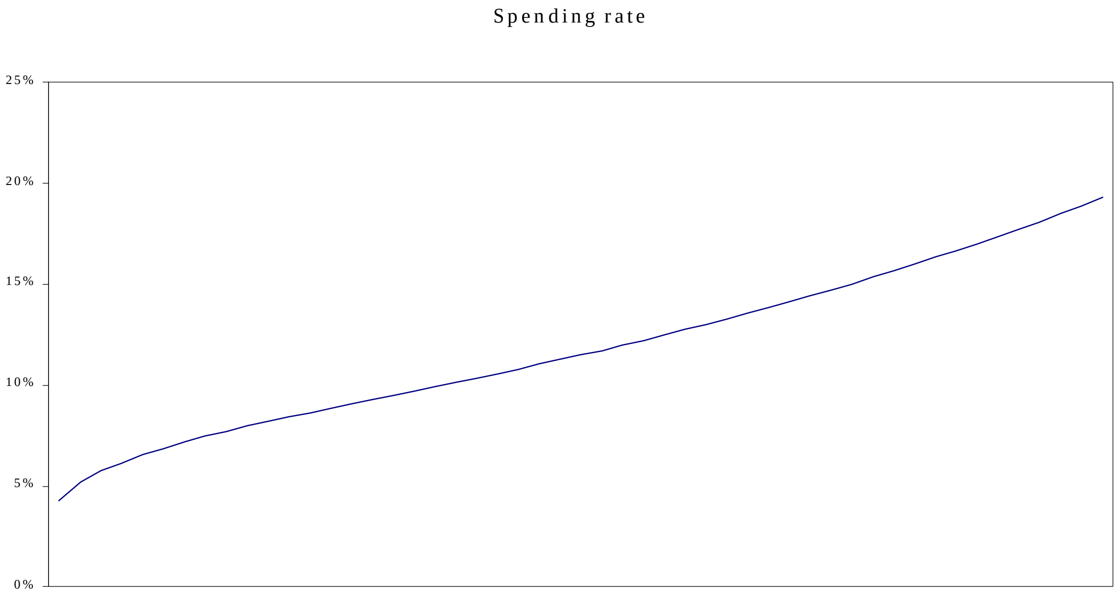
Spending rate

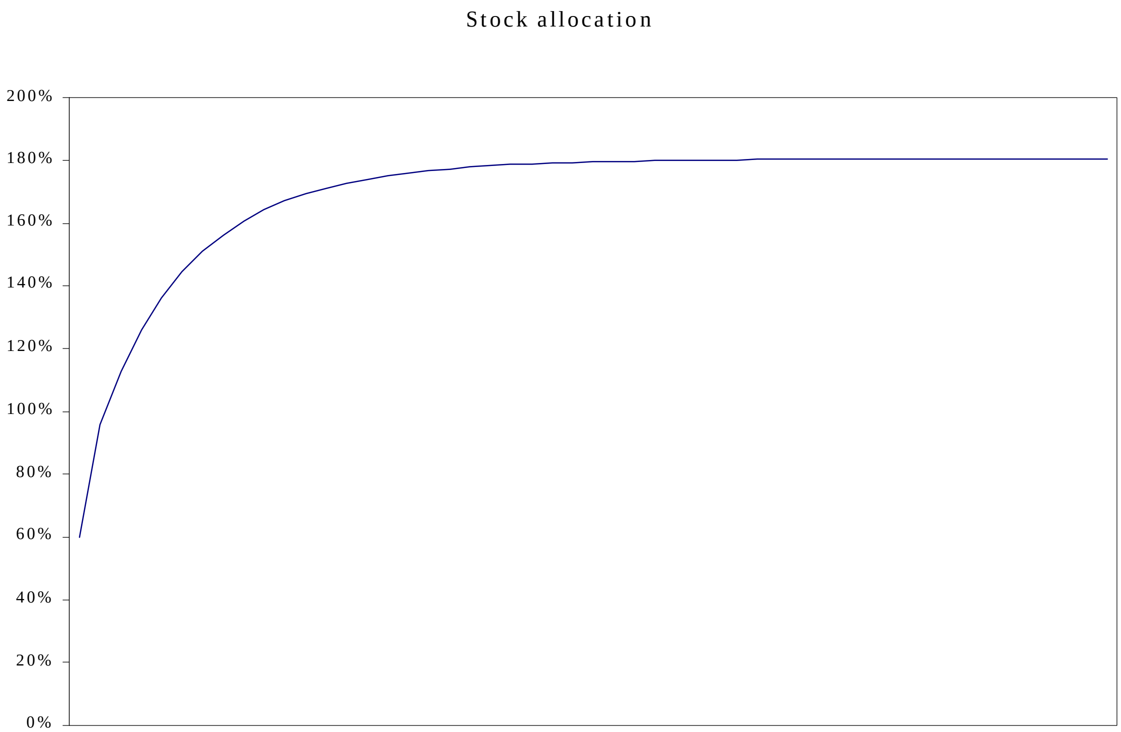


Stock allocation



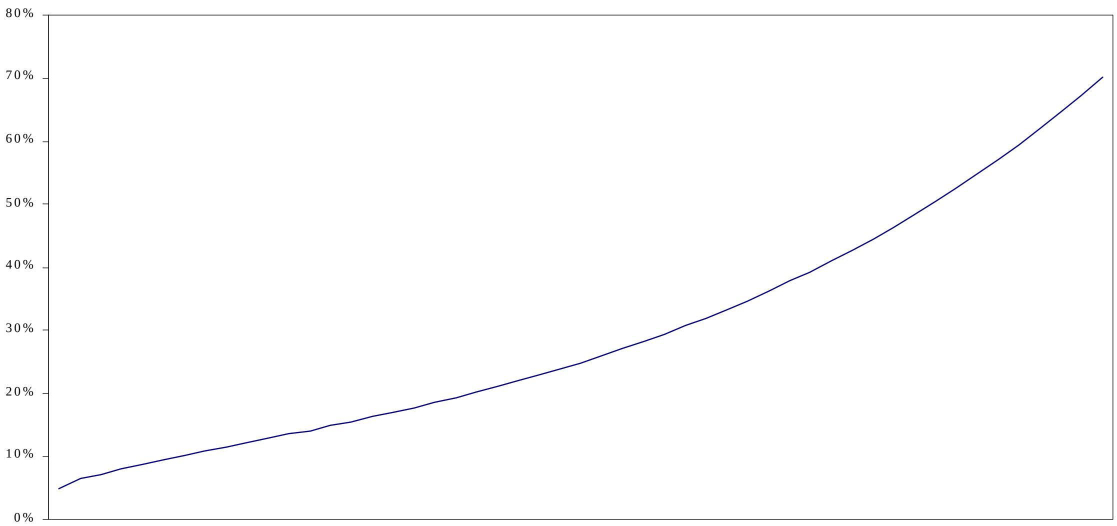
m=12%, s=20%, α=2%, π=2%, r=5%,γ=0.97, ρ=3%, n=+2



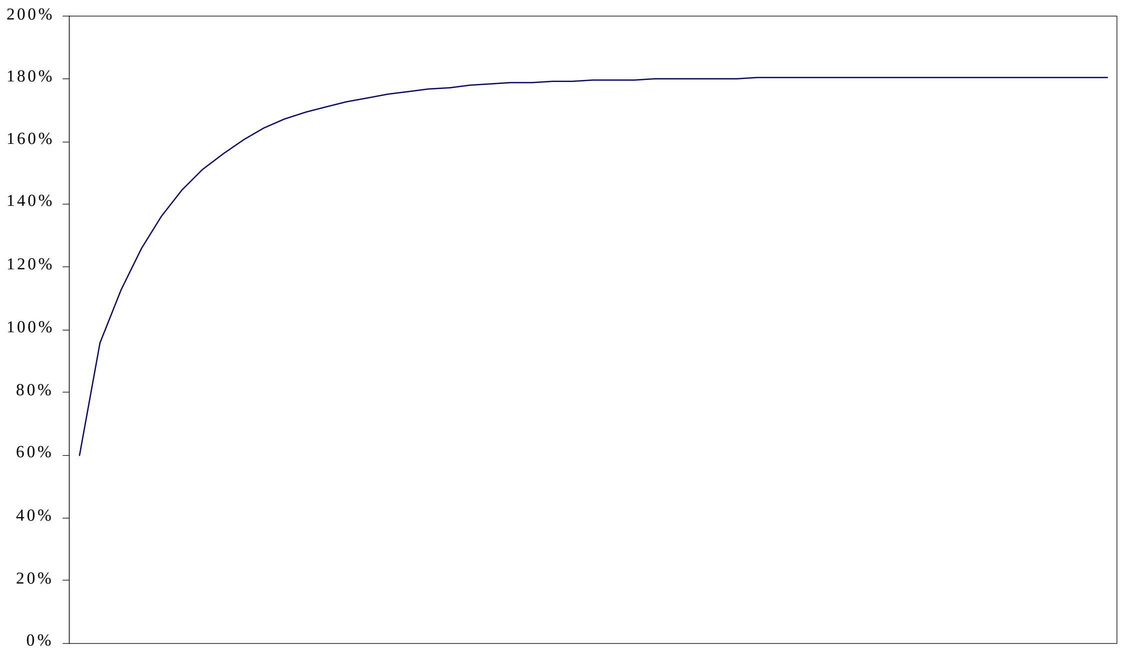


m=12%, s=20%, α=2%, π=2%, r=5%,γ=0.97, ρ=7%, n=+2

Spending rate

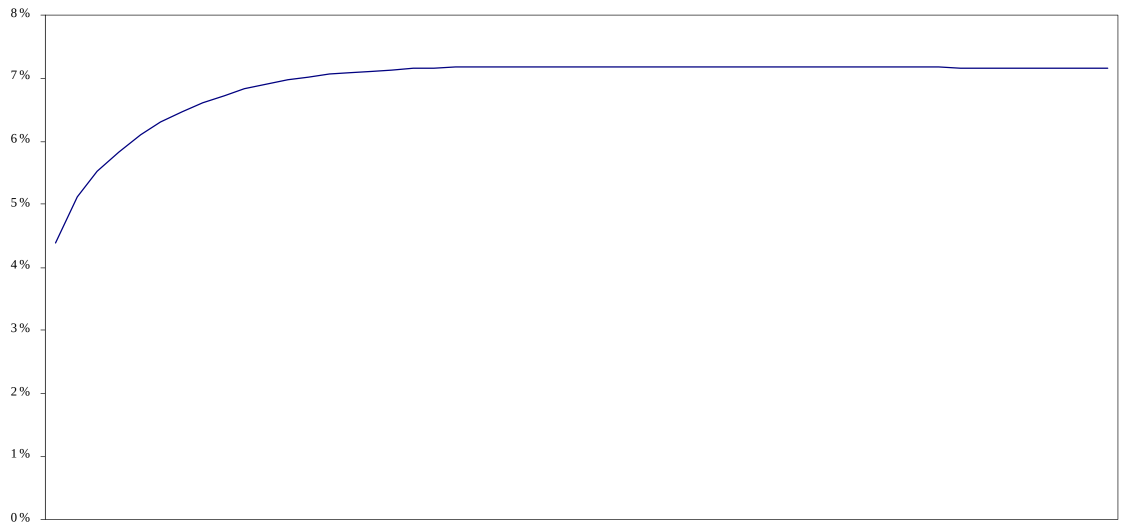


Stock allocation

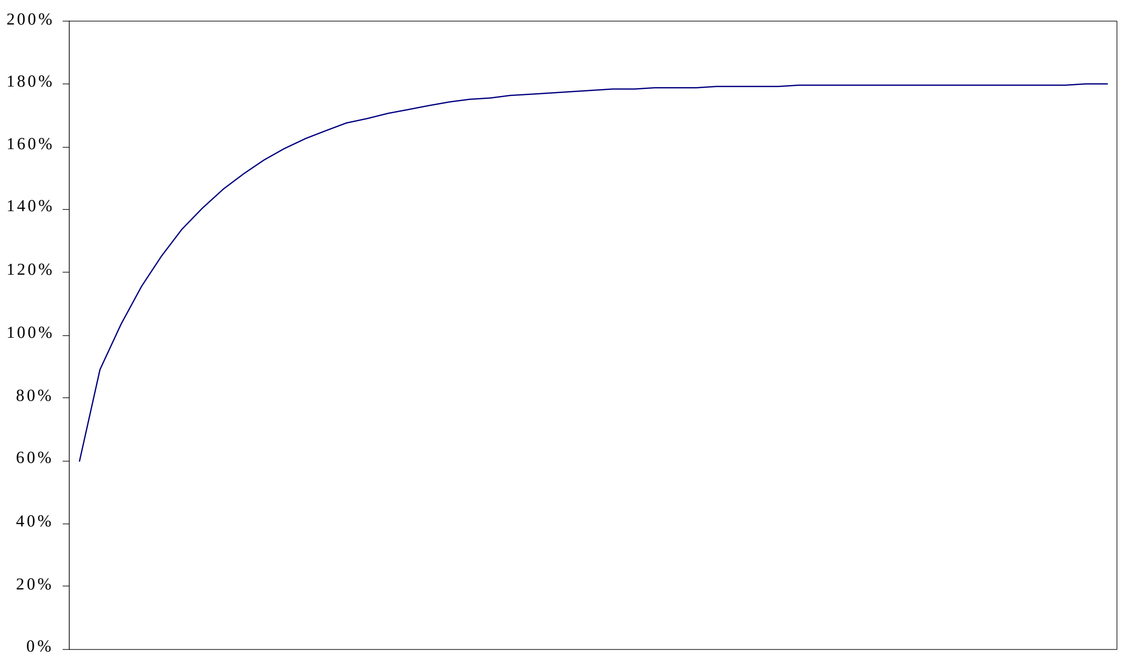


m=12%, s=20%, α=2%, π=2%, r=5%,γ=0.97, ρ=9%, n=+2

Spending rate

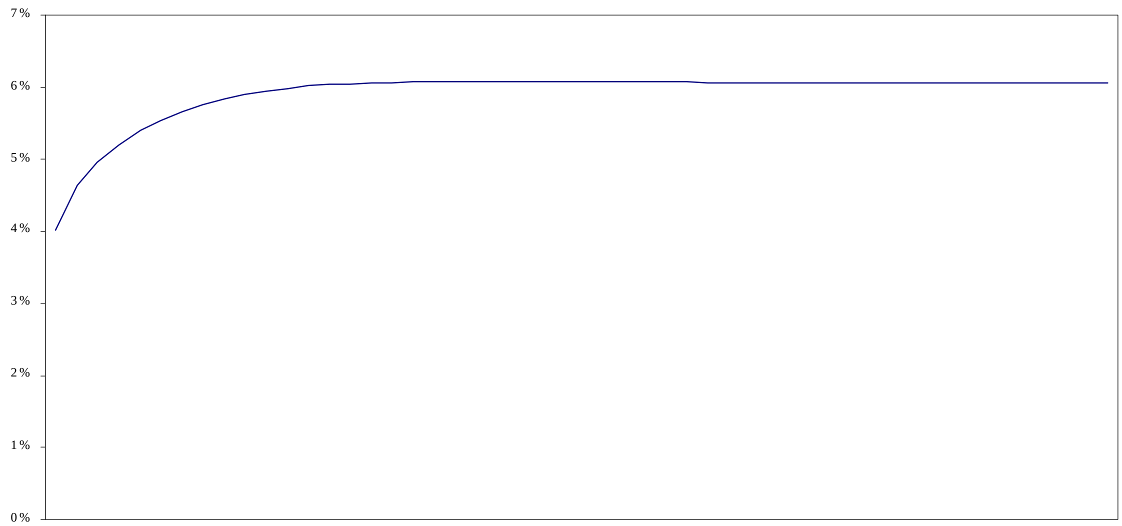


Stock allocation

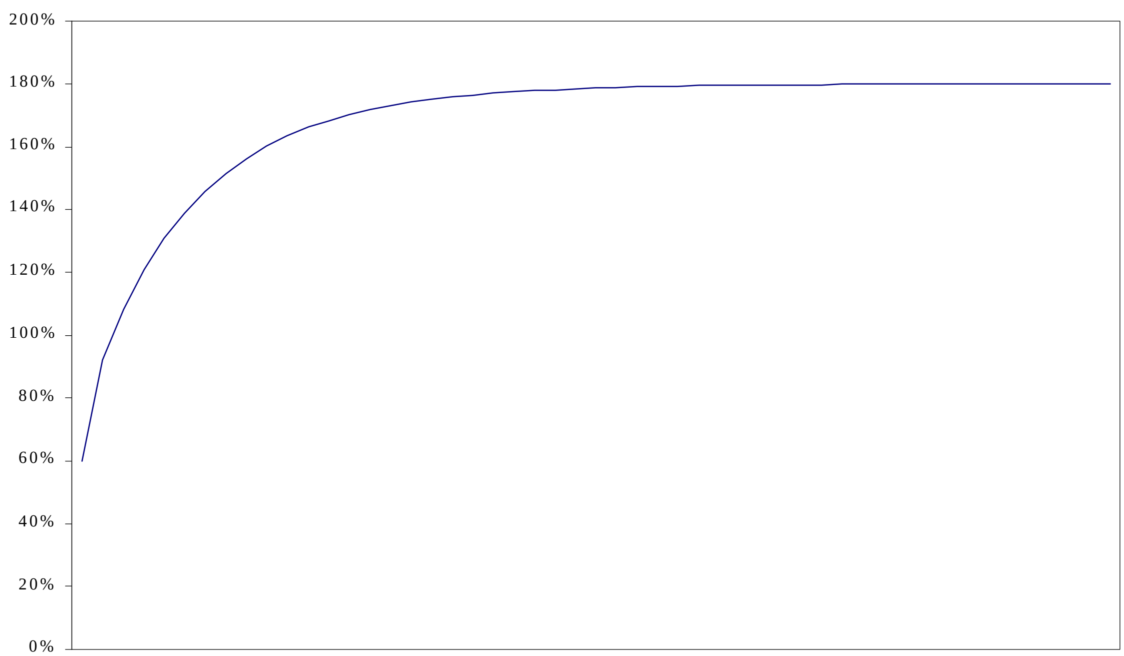


m=12%, s=16%, α=2%, π=2%, r=5%,γ=1.52, ρ=5%, n=+2

Spending rate

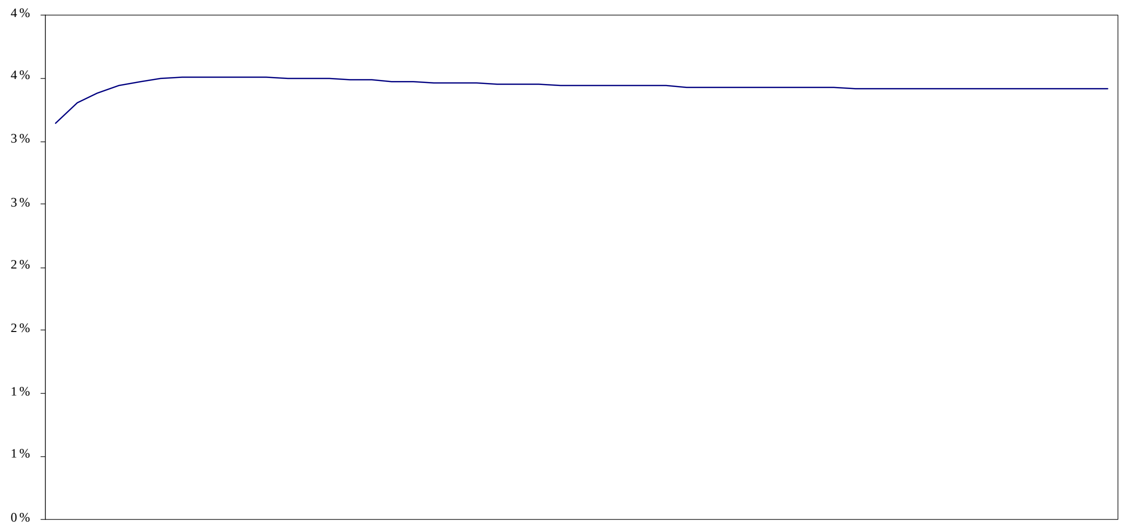


Stock allocation

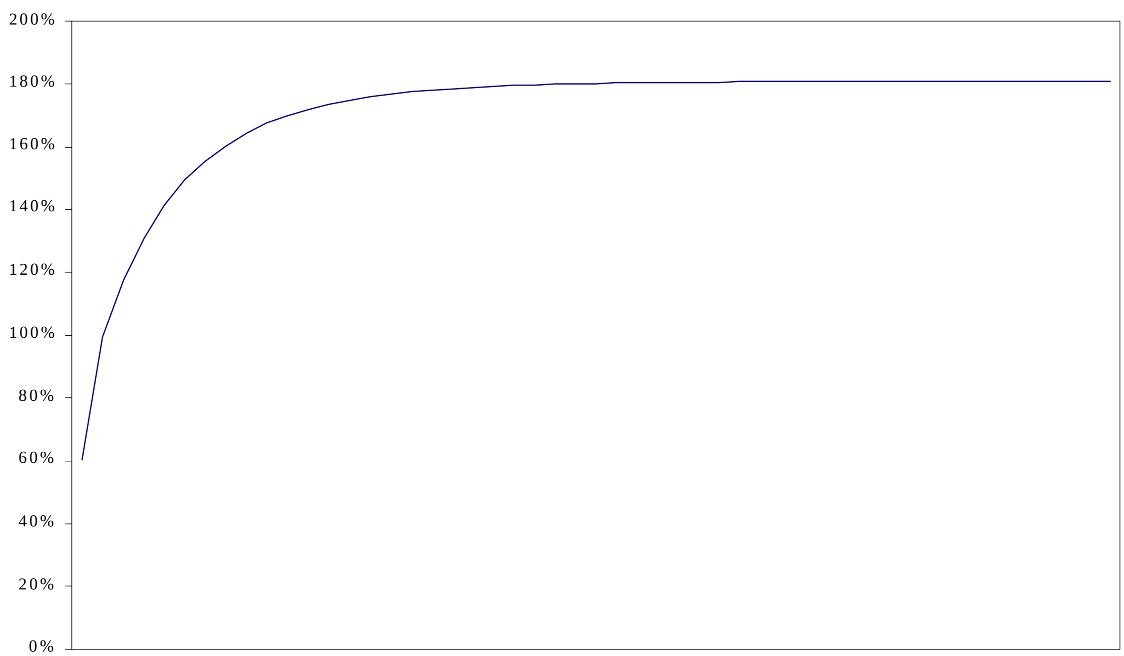


m=12%, s=18%, α=2%, π=2%, r=5%,γ=1.2, ρ=5%, n=+2

Spending rate

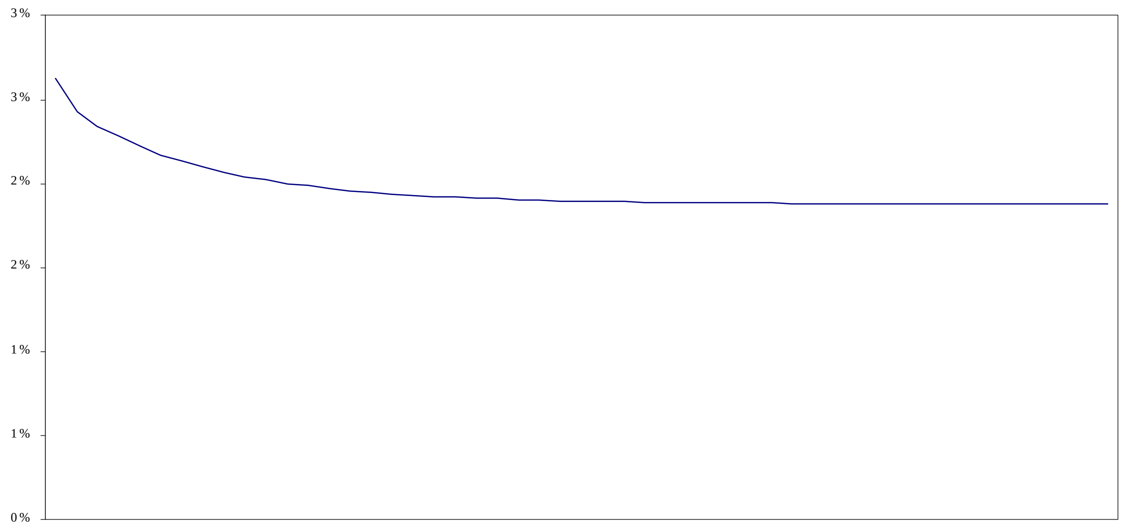


Stock allocation

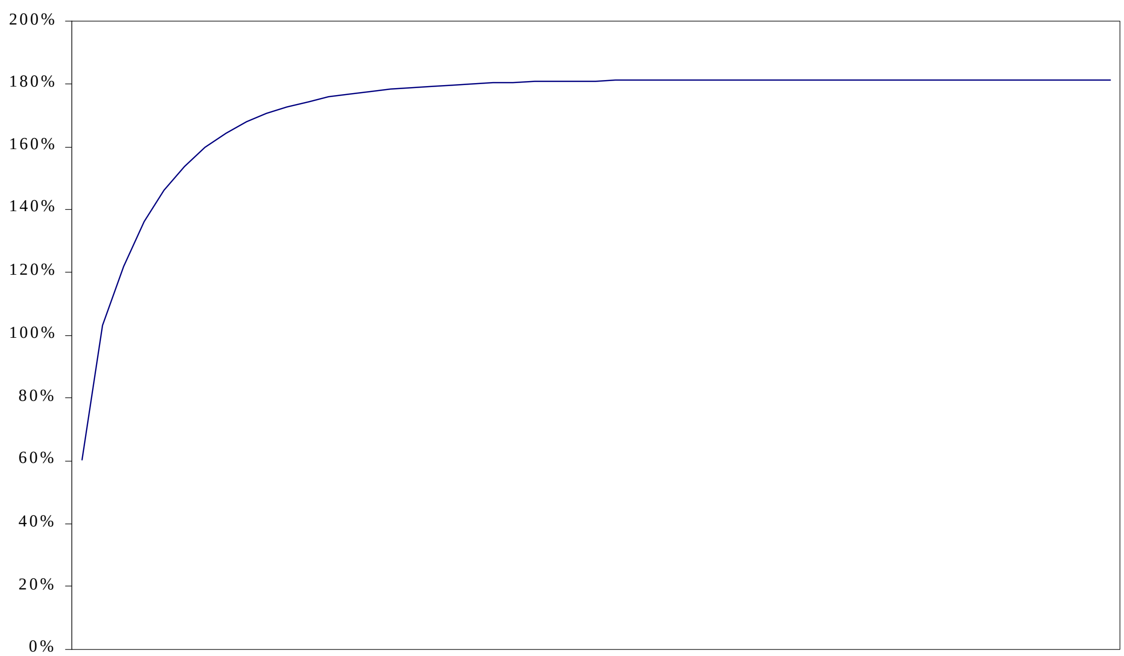


m=12%, s=22%, α=2%, π=2%, r=5%,γ=0.8, ρ=5%, n=+2

Spending rate

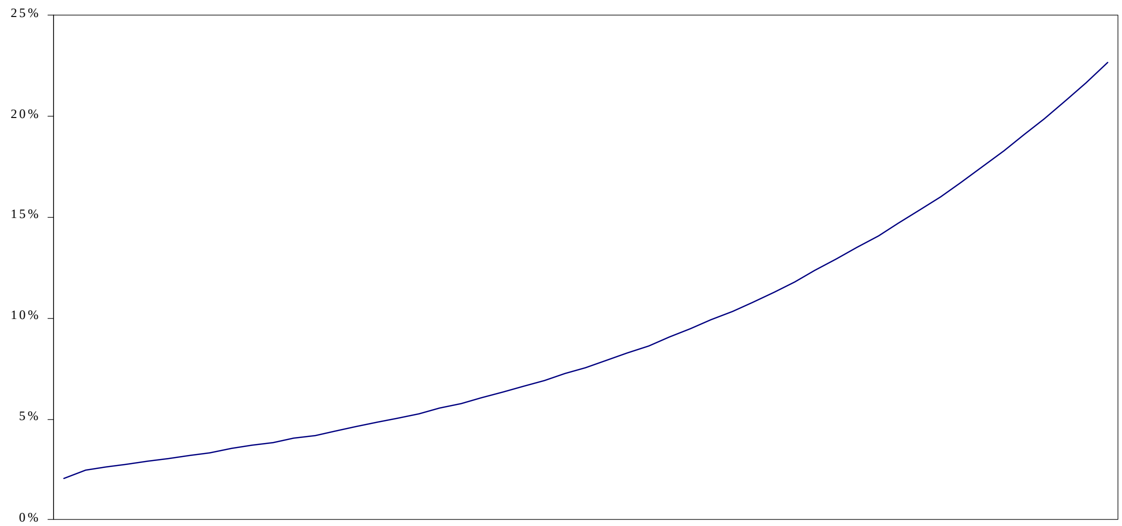


Stock allocation

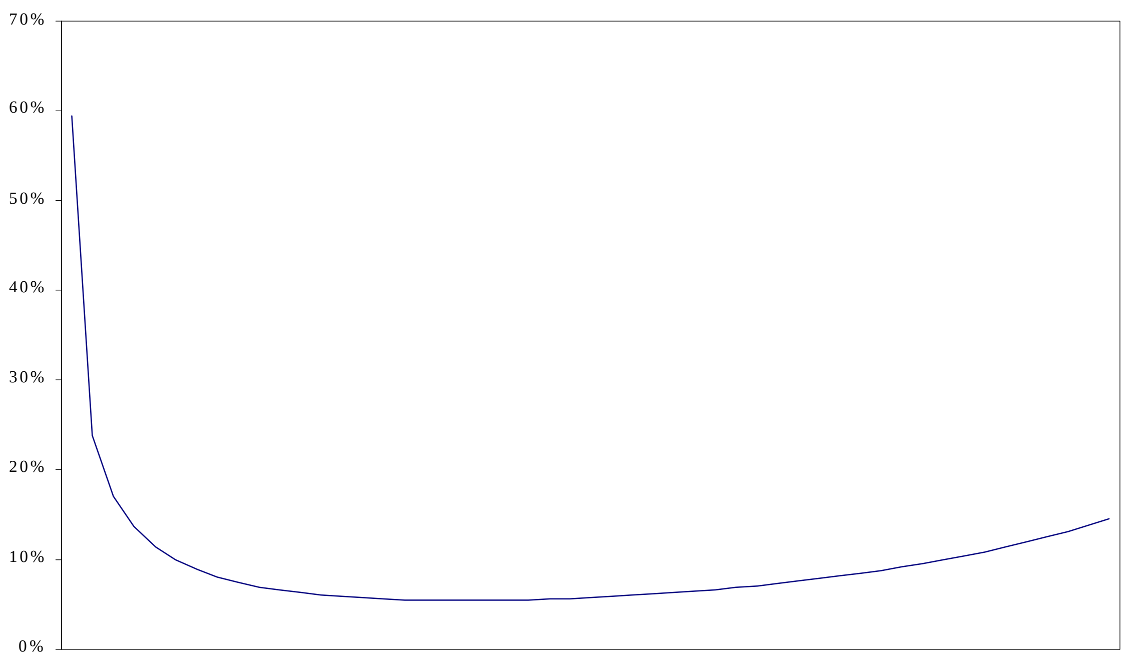


m=12%, s=24%, α=2%, π=2%, r=5%,γ=0.67, ρ=5%, n=+2

Spending rate

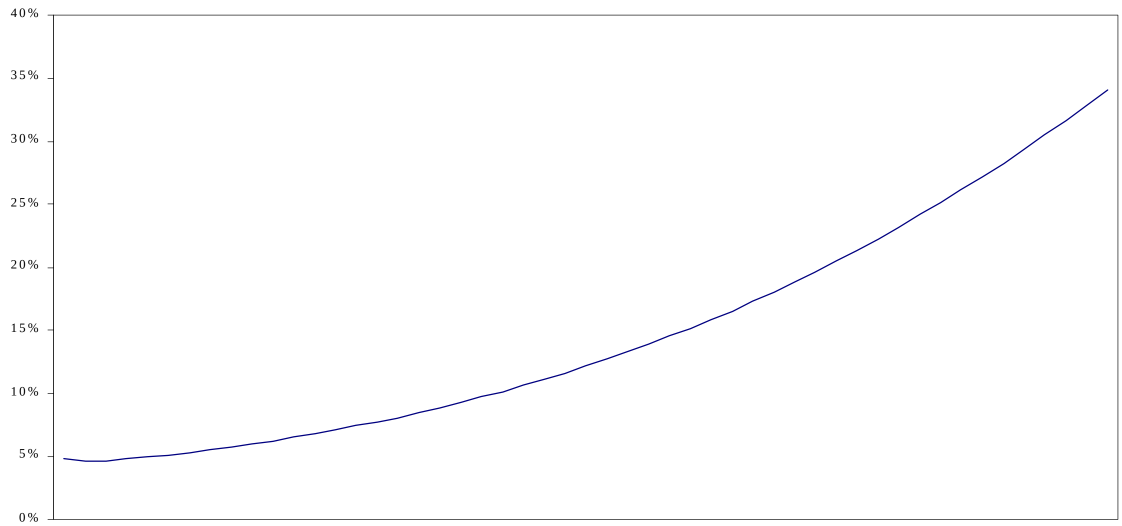


Stock allocation

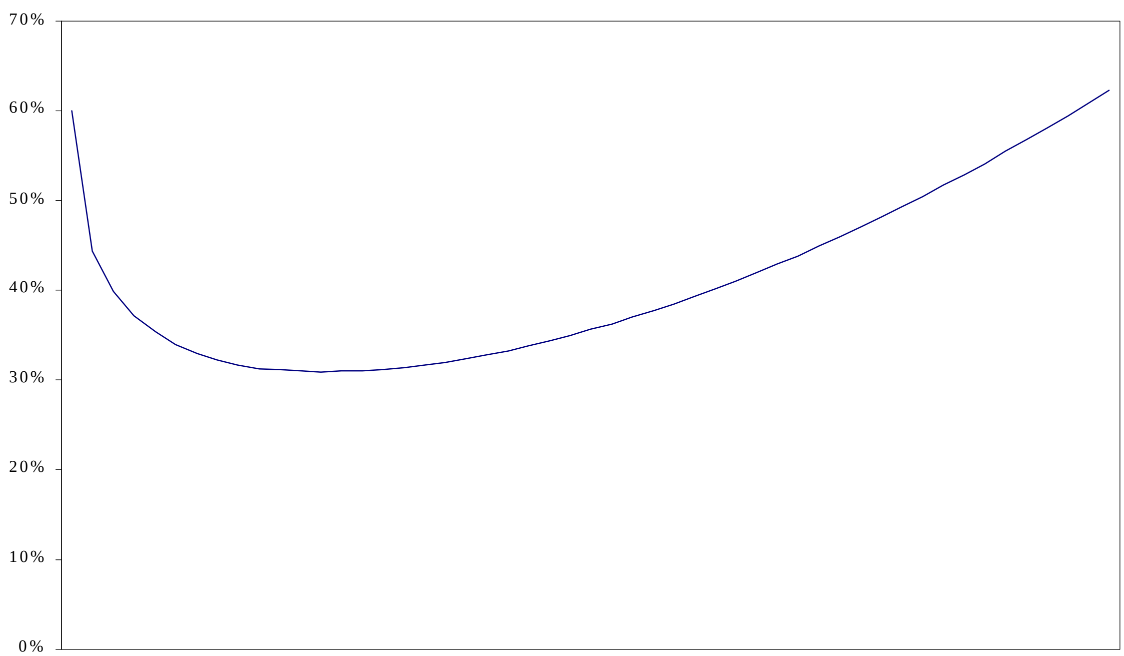


m=12%, s=20%, α=2%, π=2%, r=4.5%,γ=0.63, ρ=6%, n=-2

Spending rate

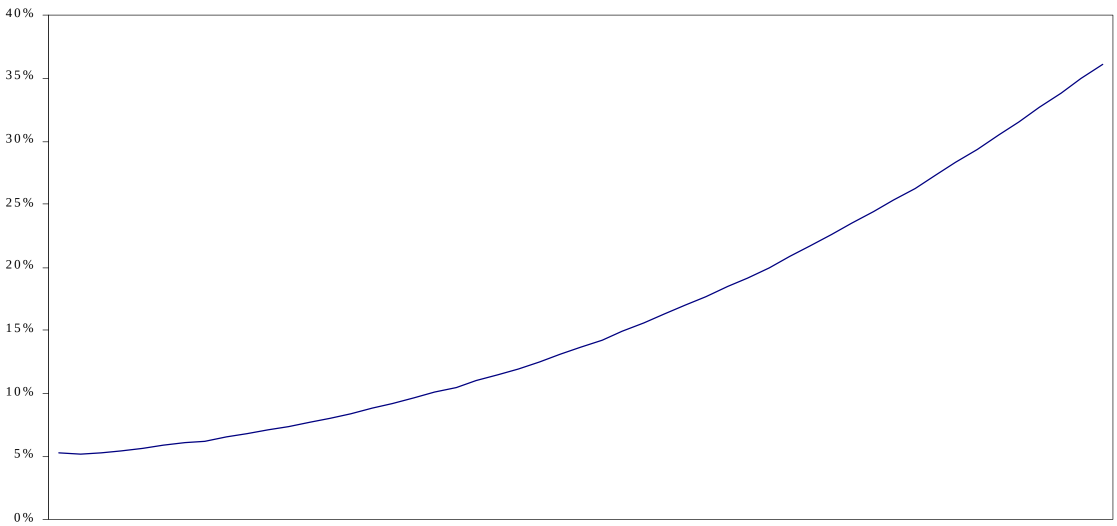


Stock allocation

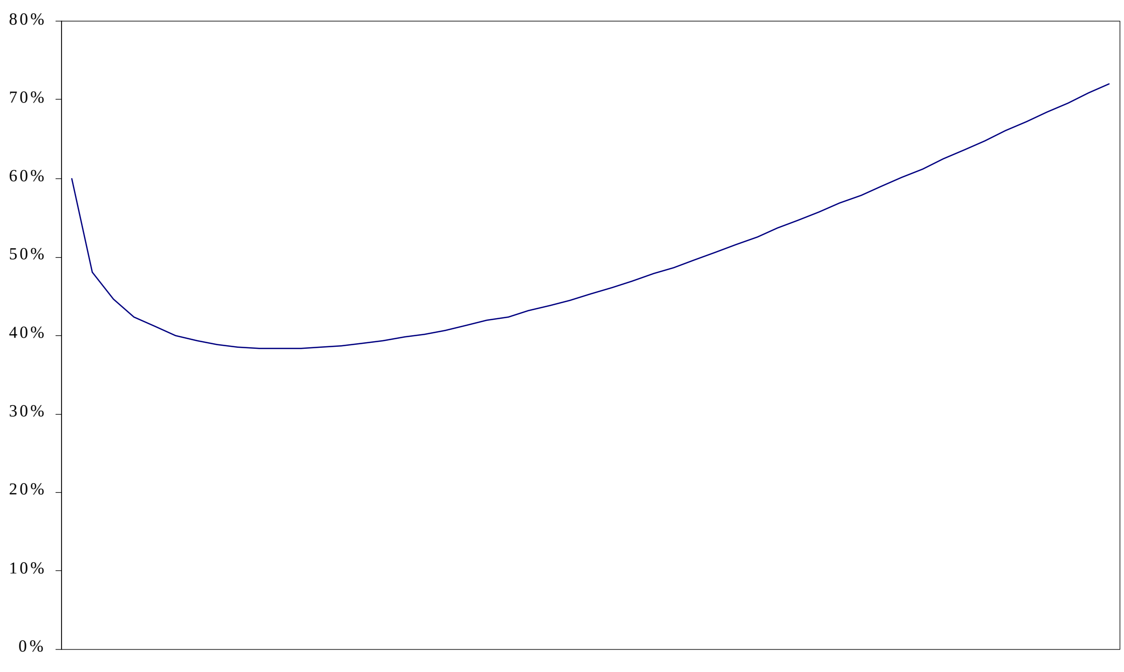


m=12%, s=20%, α=2%, π=2%, r=5.5%, γ=1.16, ρ=6%, n=-2

Spending rate

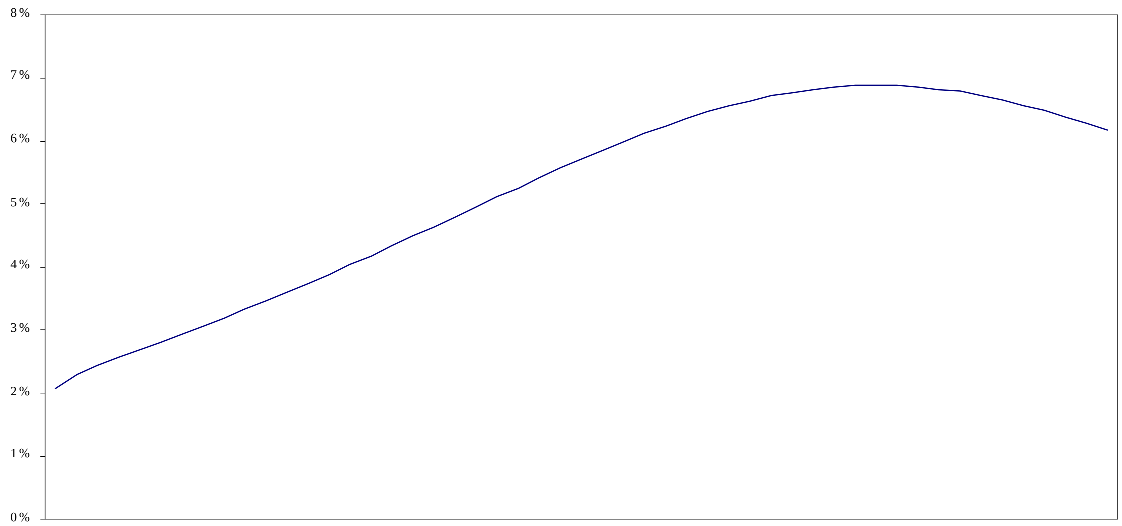


Stock allocation

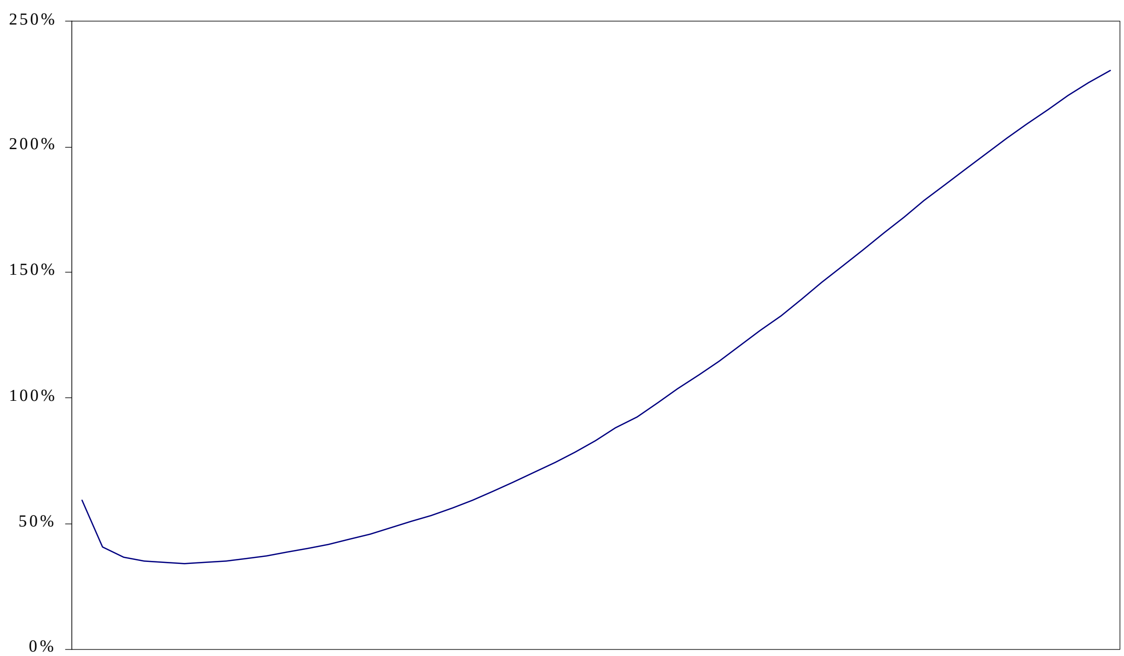


m**=12%,** s**=20%,** α**=2%,** π**=2%,** r**=6%,**γ**=1.25,** ρ**=6%, n=-2**

Spending rate

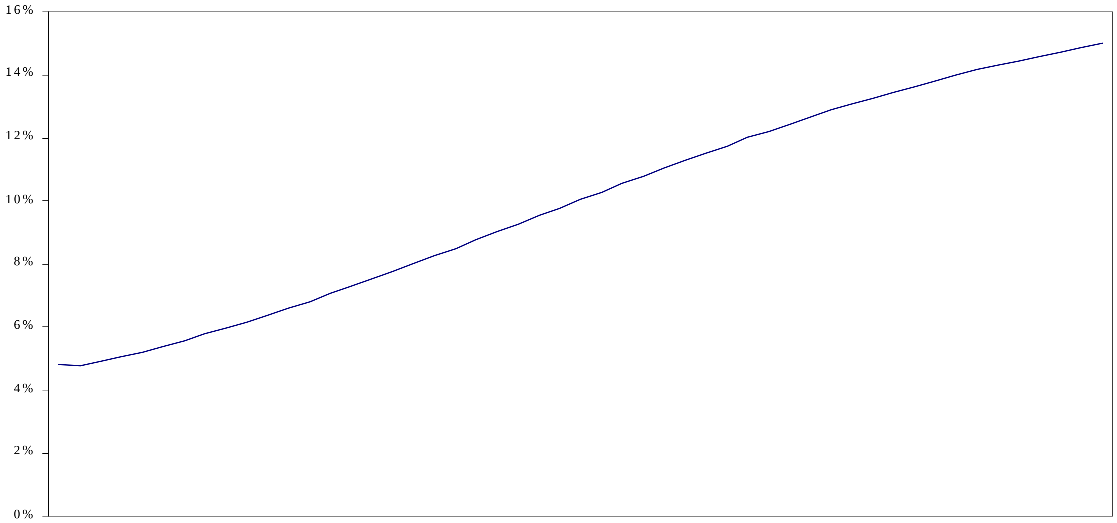


Stock allocation

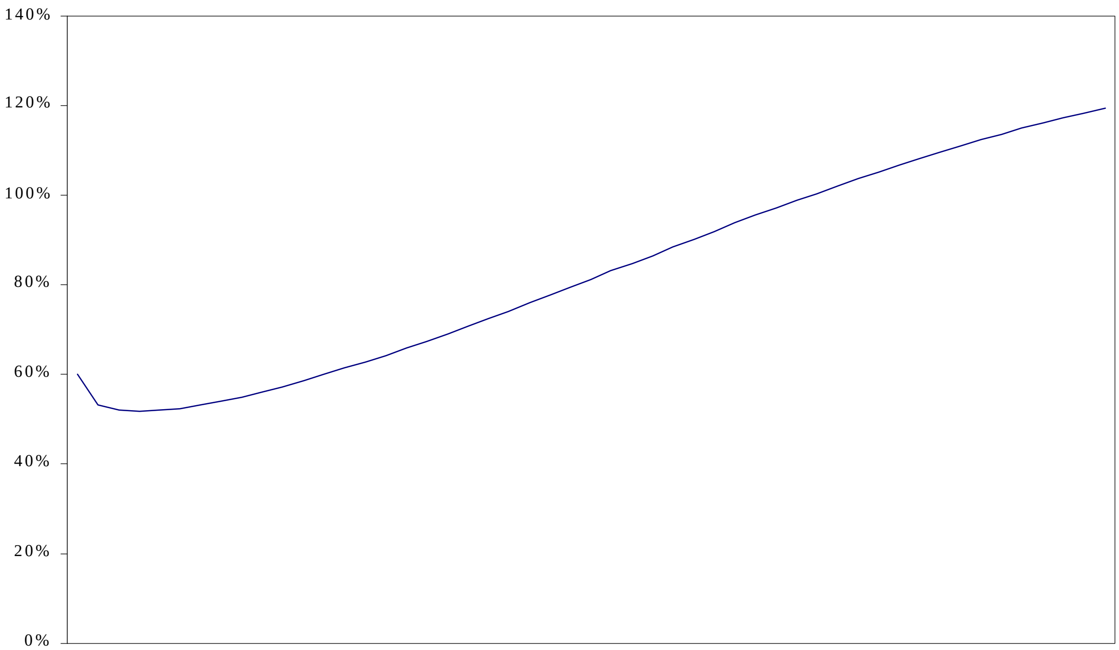


m=12%, s=20%, α=2%, π=2%, r=4.5%,γ=0.63, ρ=6%, n=-1

Spending rate

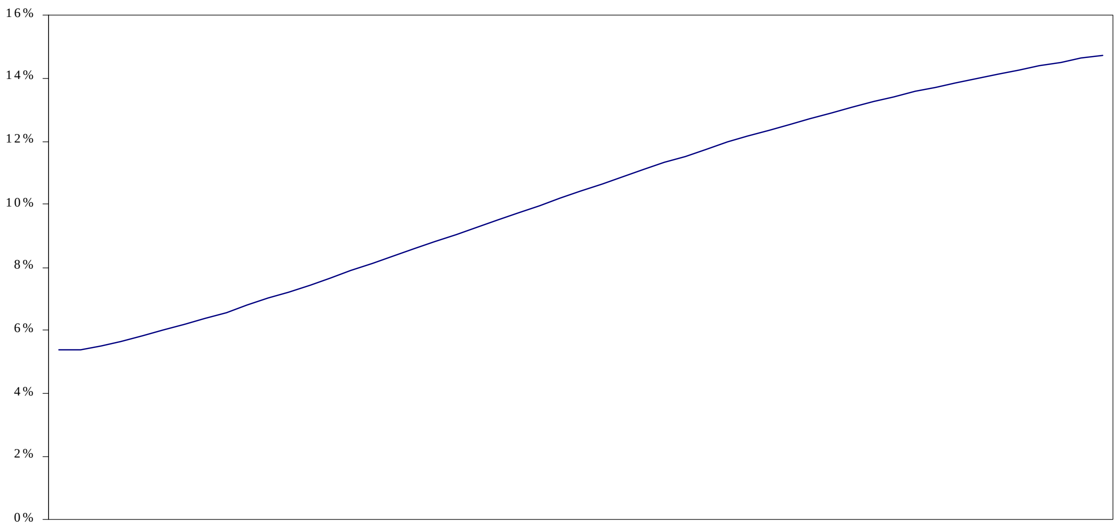


Stock allocation

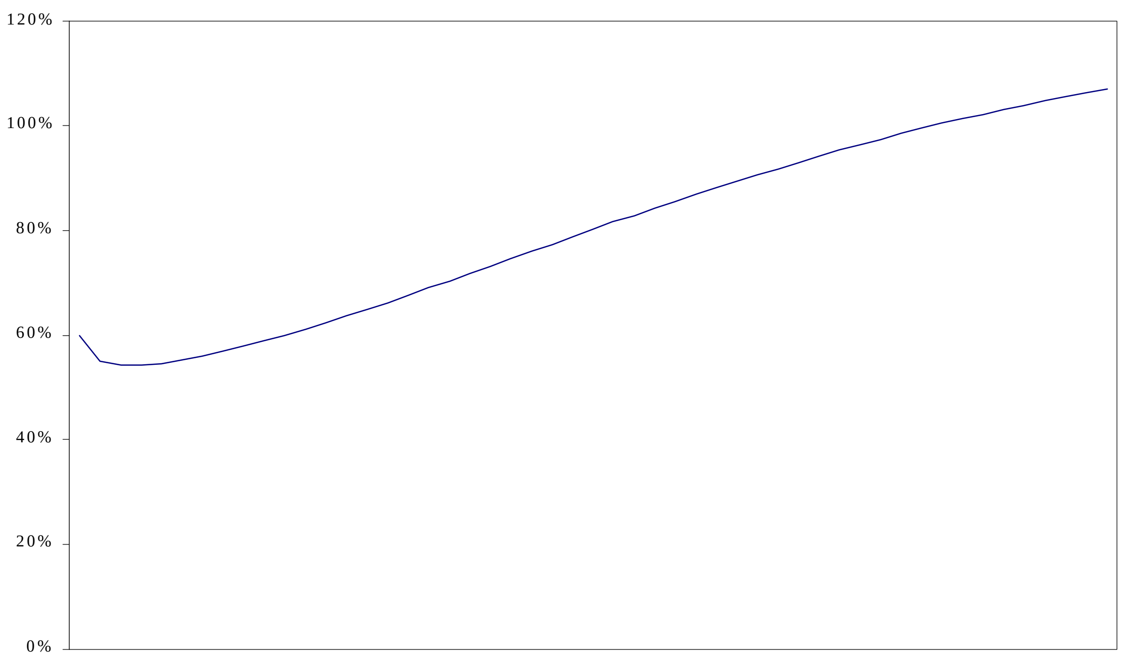


m=12%, s=20%, α=2%, π=2%, r=5.5%,γ=1.16, ρ=6%, n=-1

Spending rate

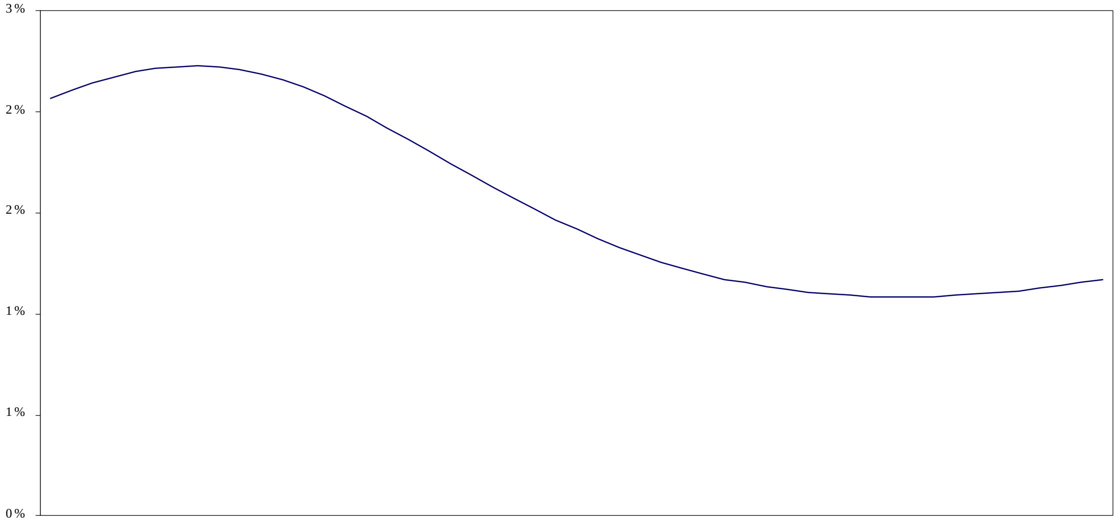


Stock allocation

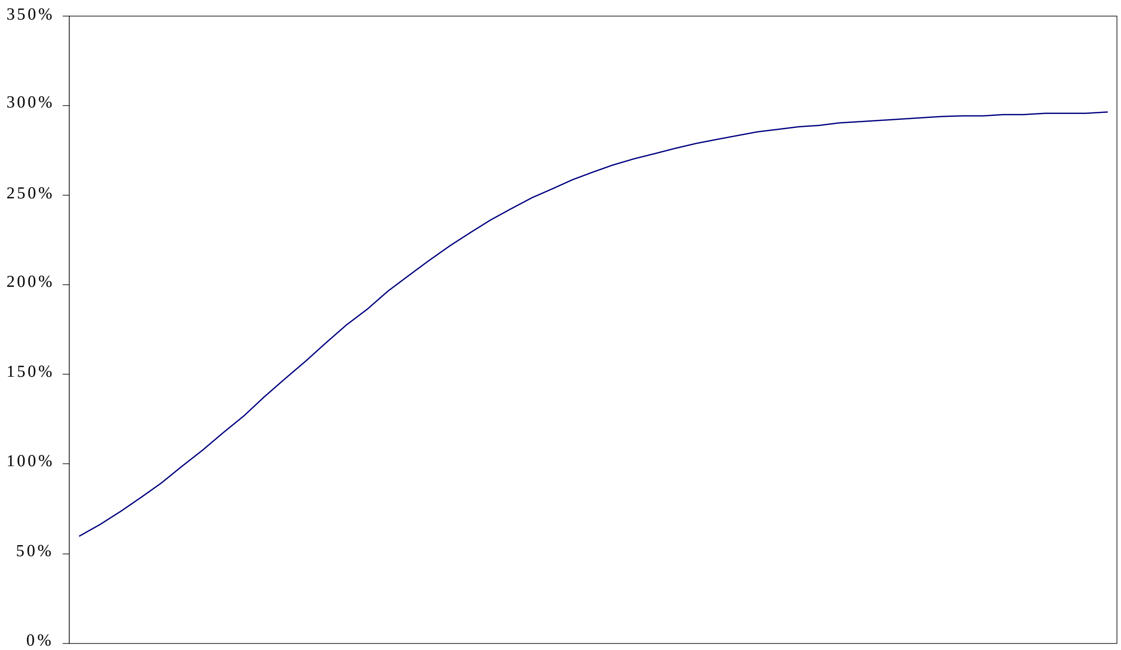


m=12%, s=20%, α=2%, π=2%, r=6%, γ=1.25, ρ=6%, n=-1

Spending rate

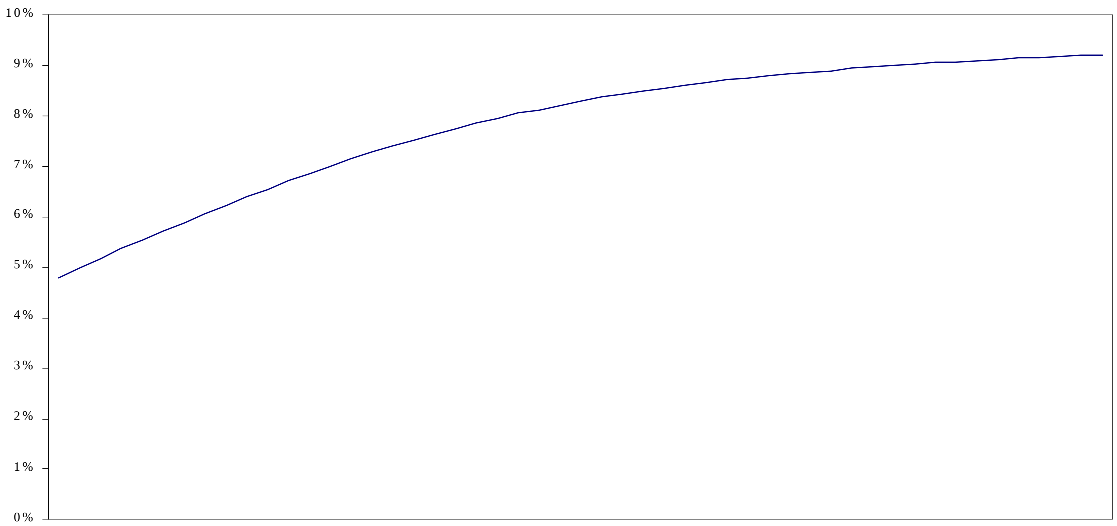


Stock allocation

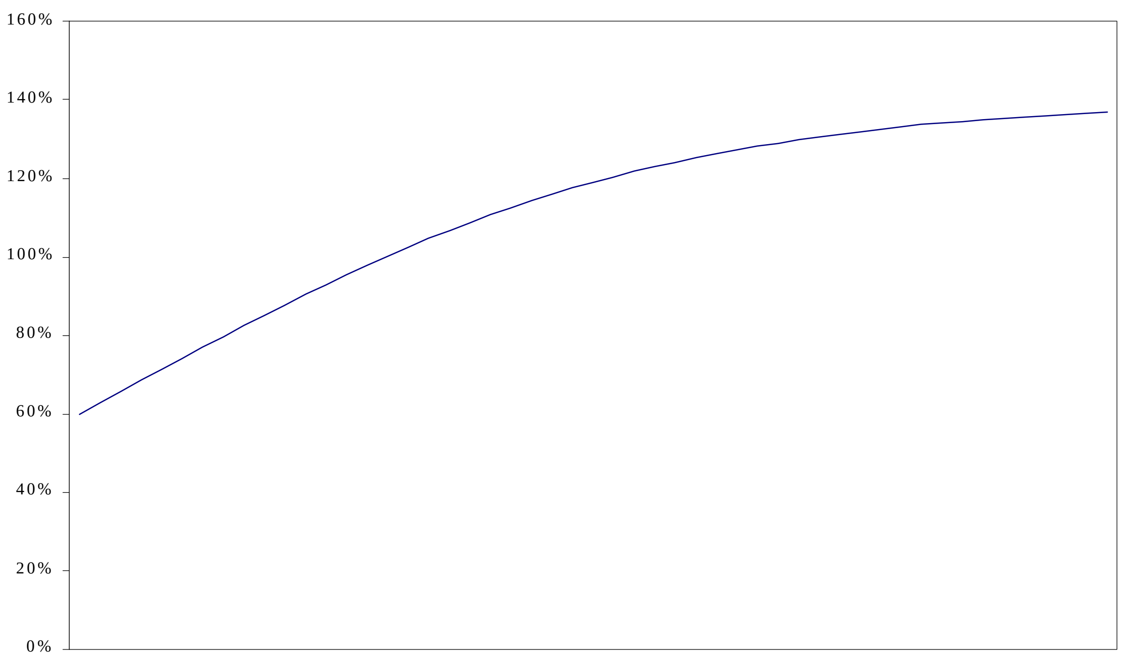


m=12%, s=20%, α=2%, π=2%, r=4.5%,γ=0.63, ρ=6%, n=0

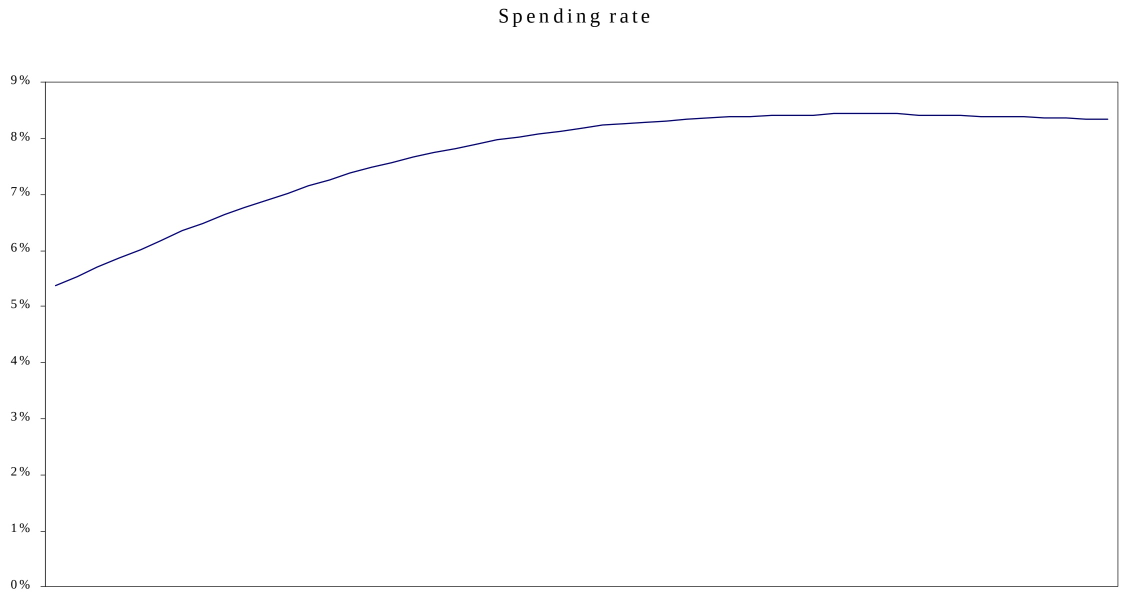
Spending rate

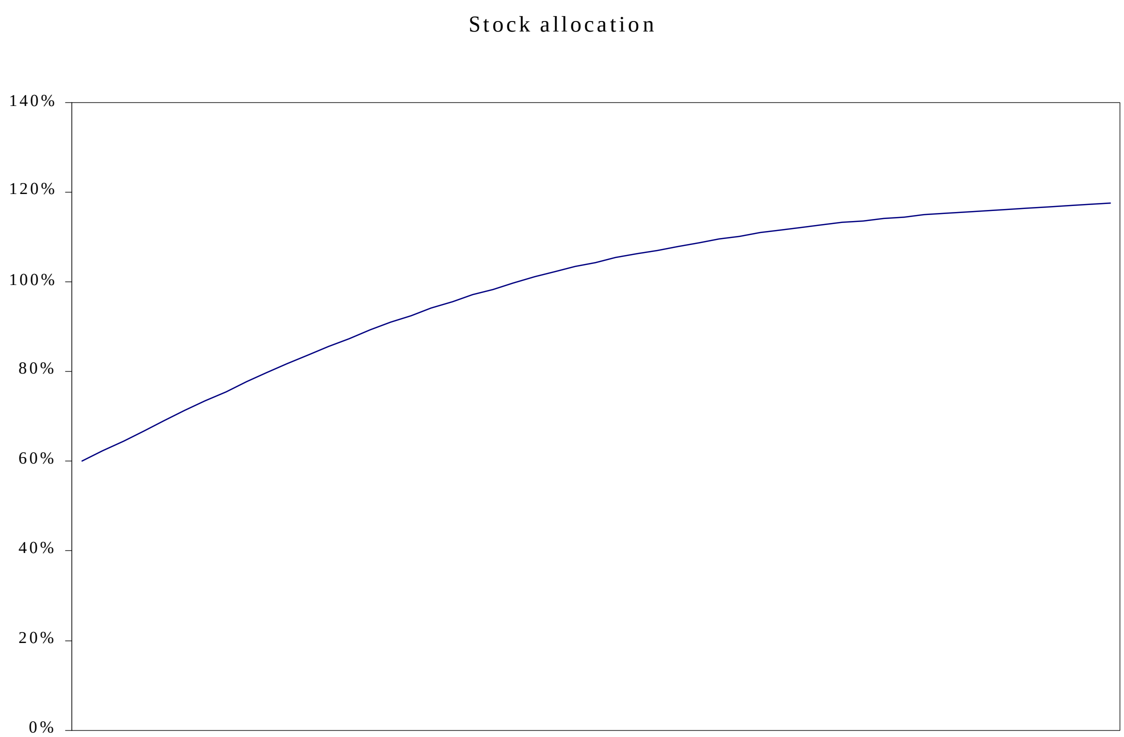


Stock allocation



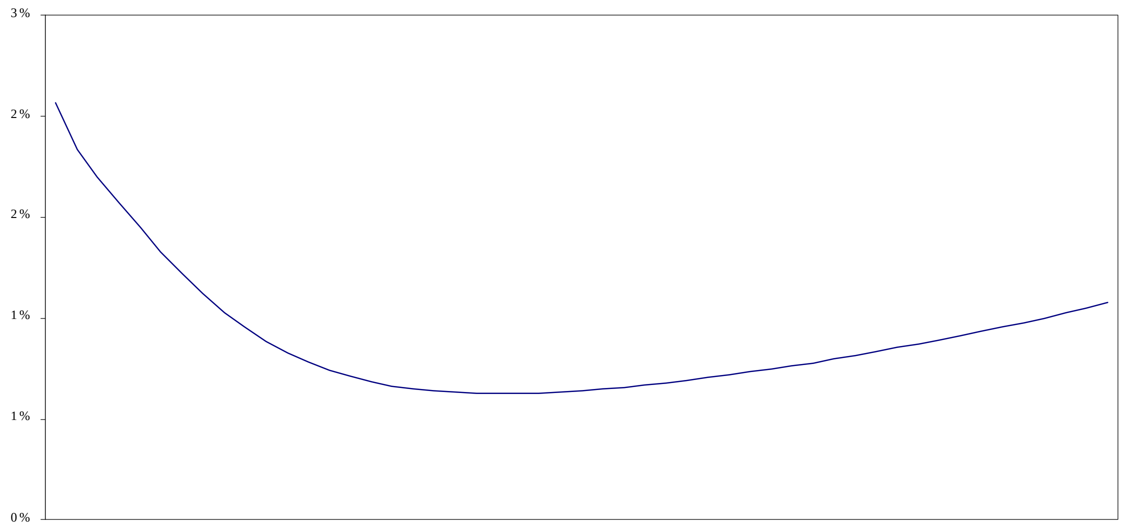
m=12%, s=20%, α=2%, π=2%, r=5.5%, γ=1.16, ρ=6%, n=0



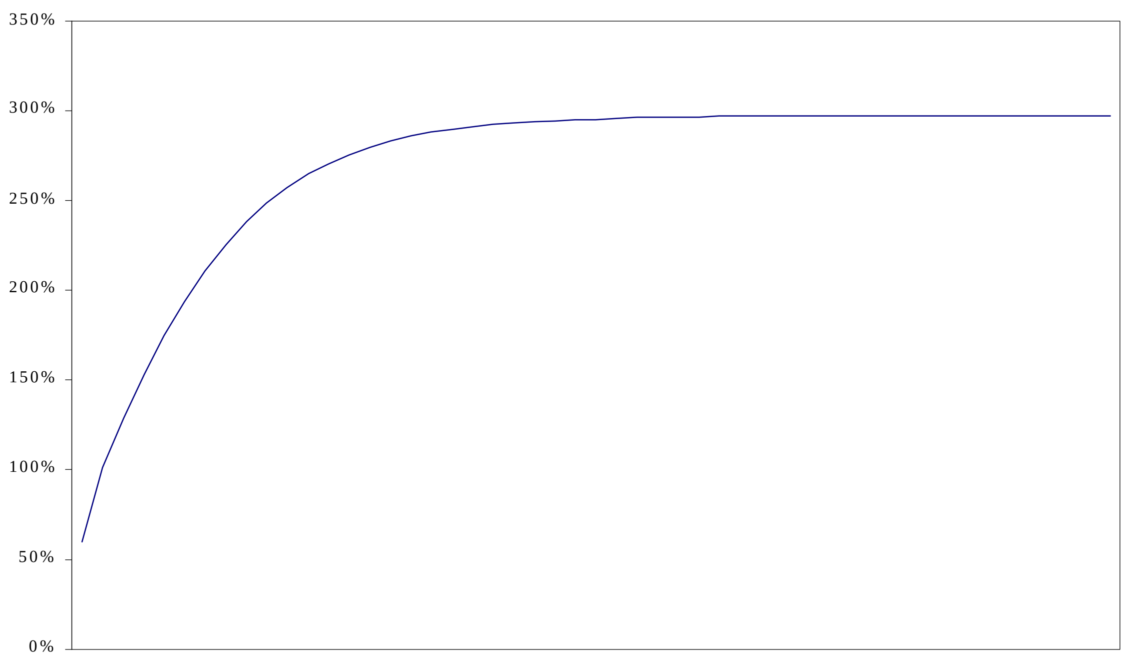


***m*=12%, *s*=20%, α=2%, π=2%, *r*=6%,γ=1.25, ρ=6%, n=0**

Spending rate

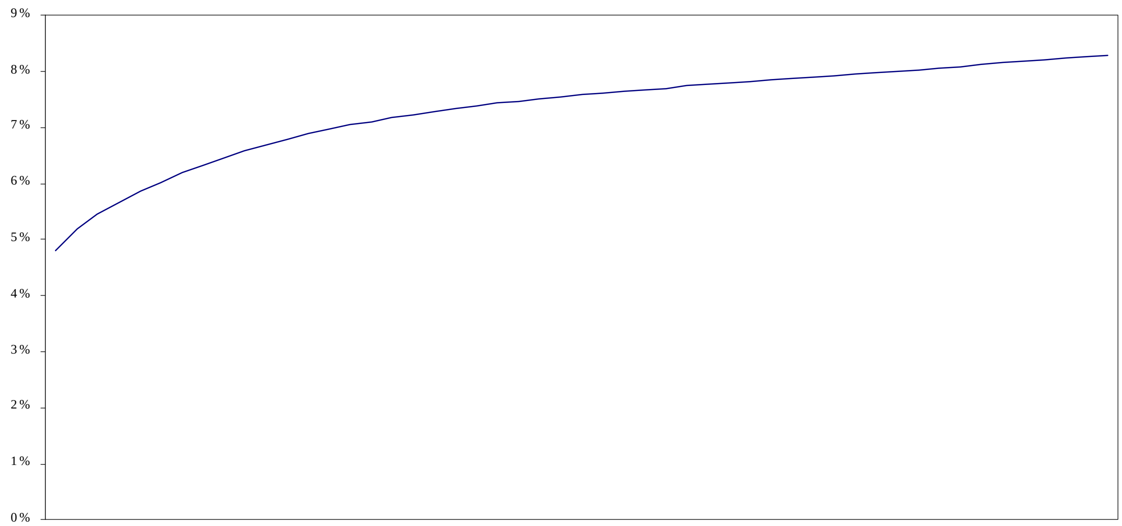


Stock allocation

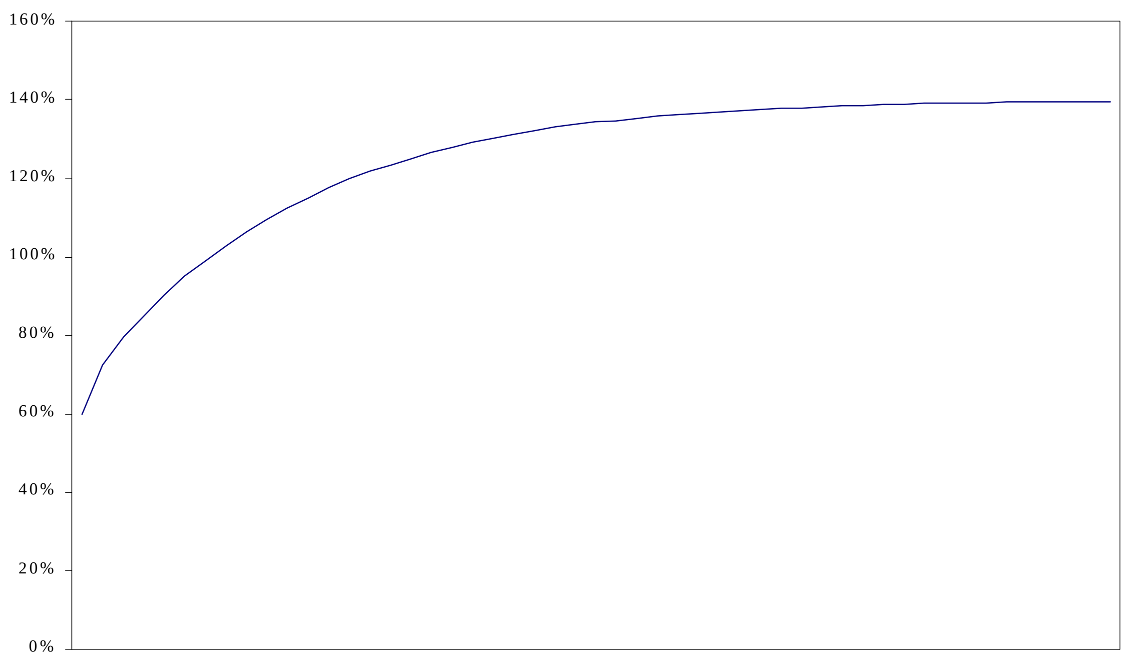


m=12%, s=20%, α=2%, π=2%, r=4.5%,γ=0.63, ρ=6%, n=+1

Spending rate

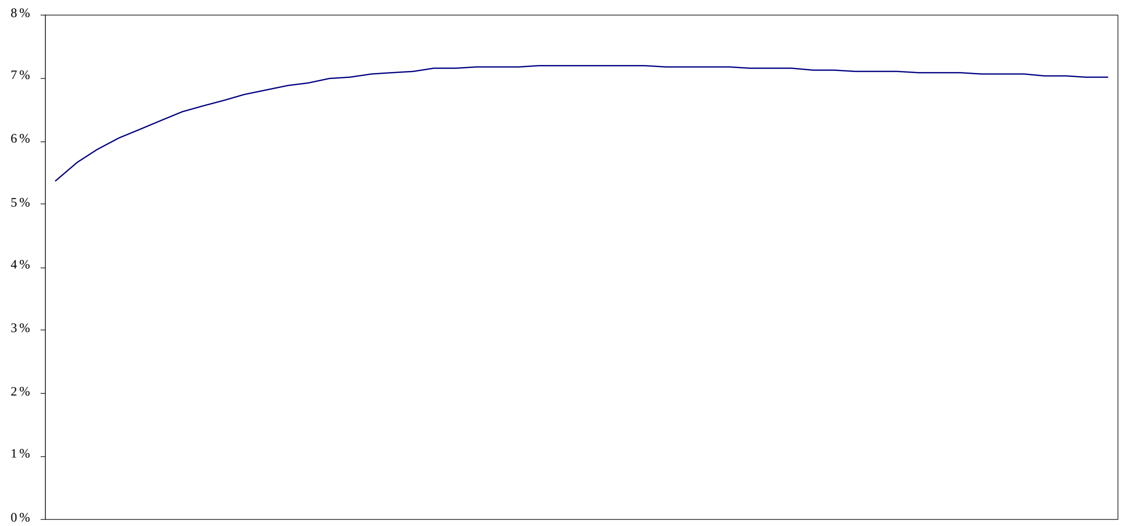


Stock allocation

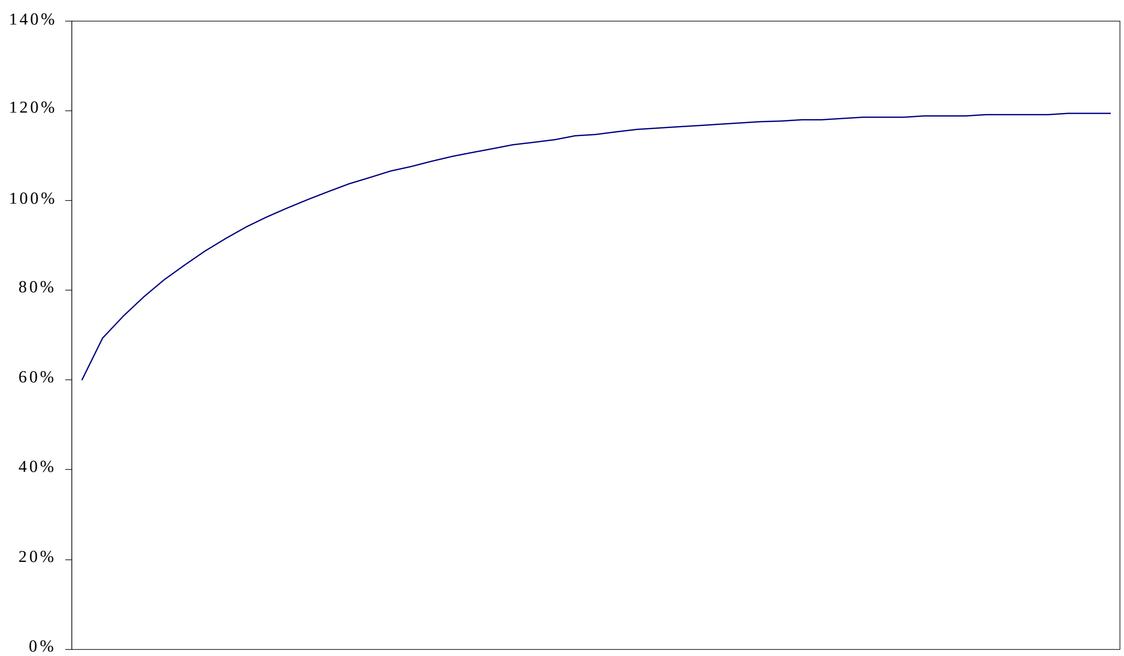


m=12%, s=20%, α=2%, π=2%, r=5.5%, γ=1.16, ρ=6%, n=+1

Spending rate

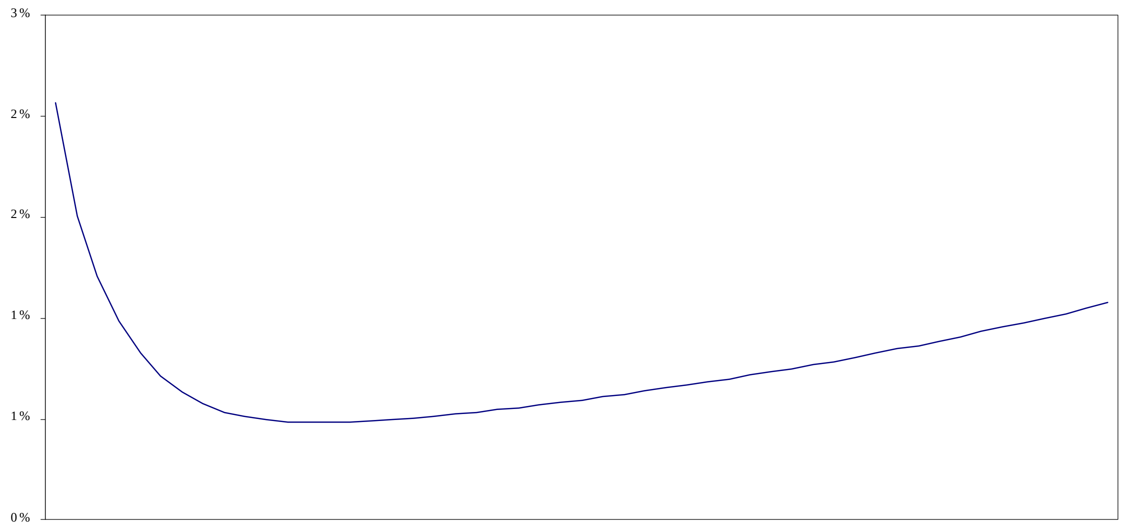


Stock allocation



***m*=12%, *s*=20%, α=2%, π=2%, *r*=6%,γ=1.25, ρ=6%, n=+1**

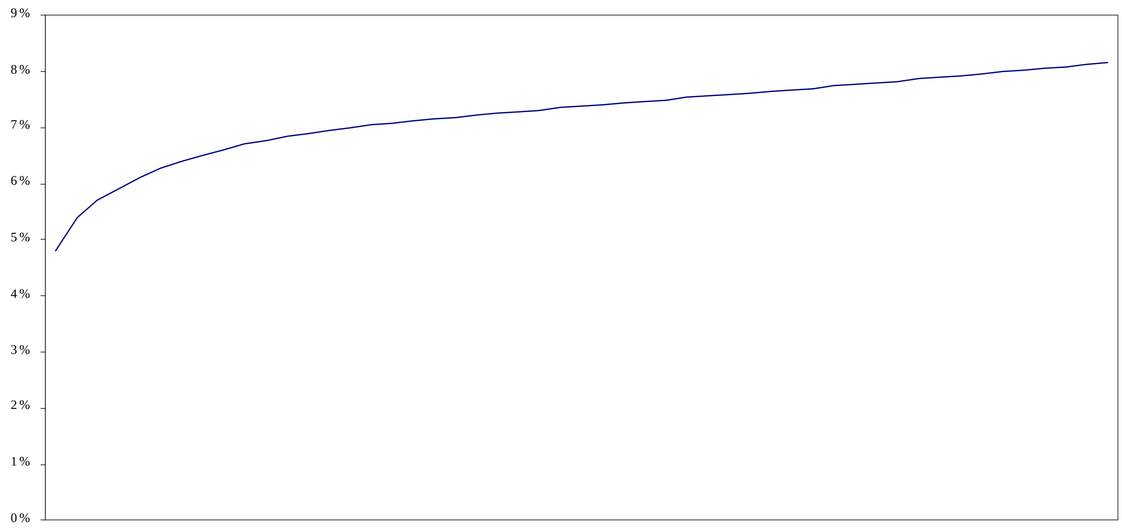
Spending rate



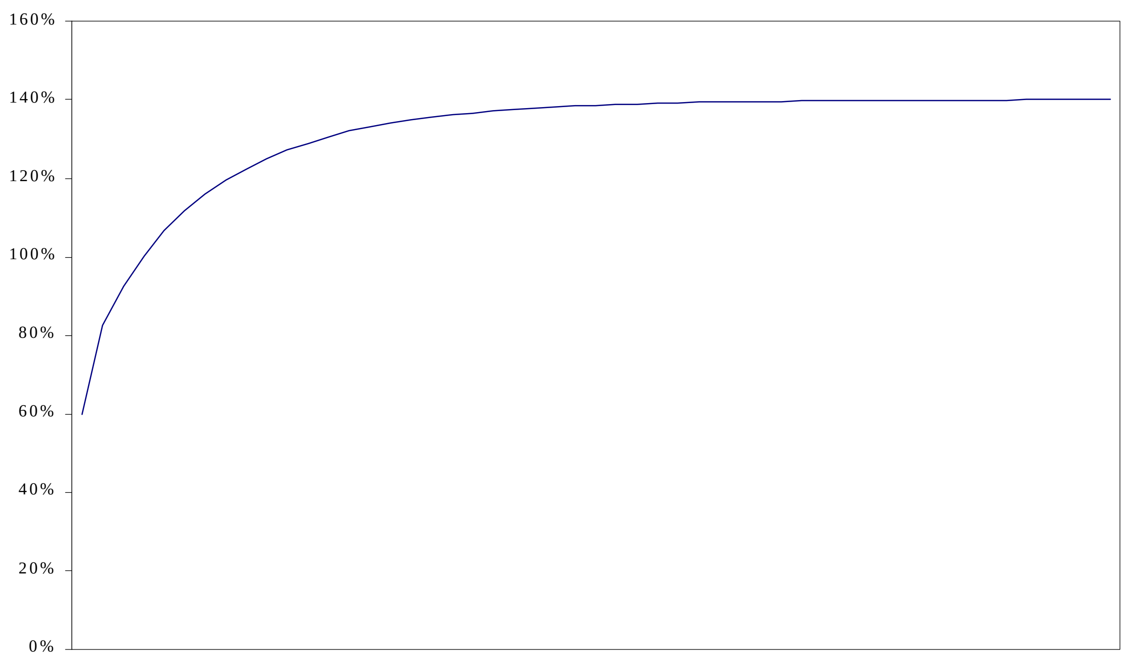
Stock allocation

m=12%, s=20%, α=2%, π=2%, r=4.5%,γ=0.63, ρ=6%, n=+2

Spending rate

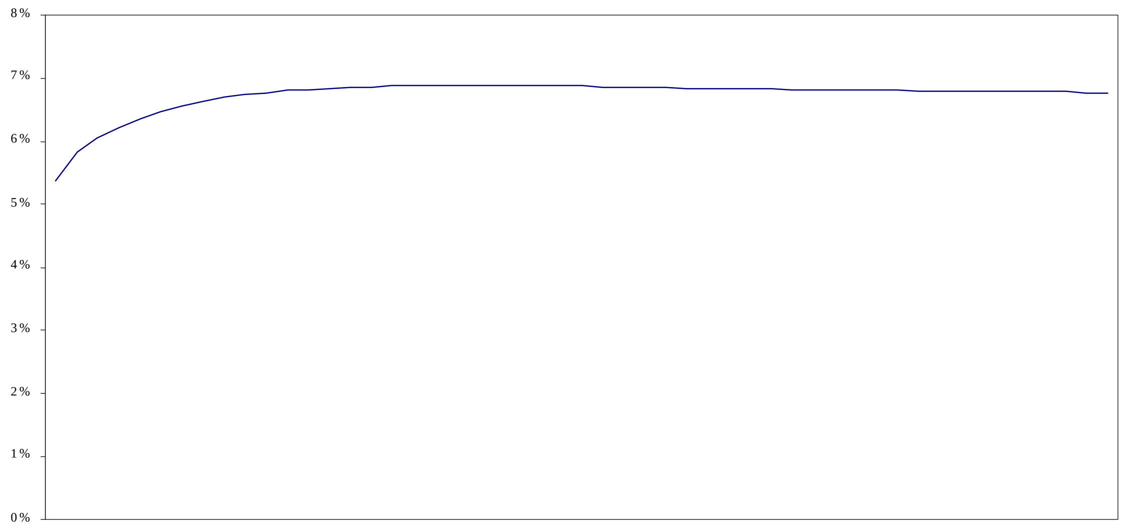


Stock allocation

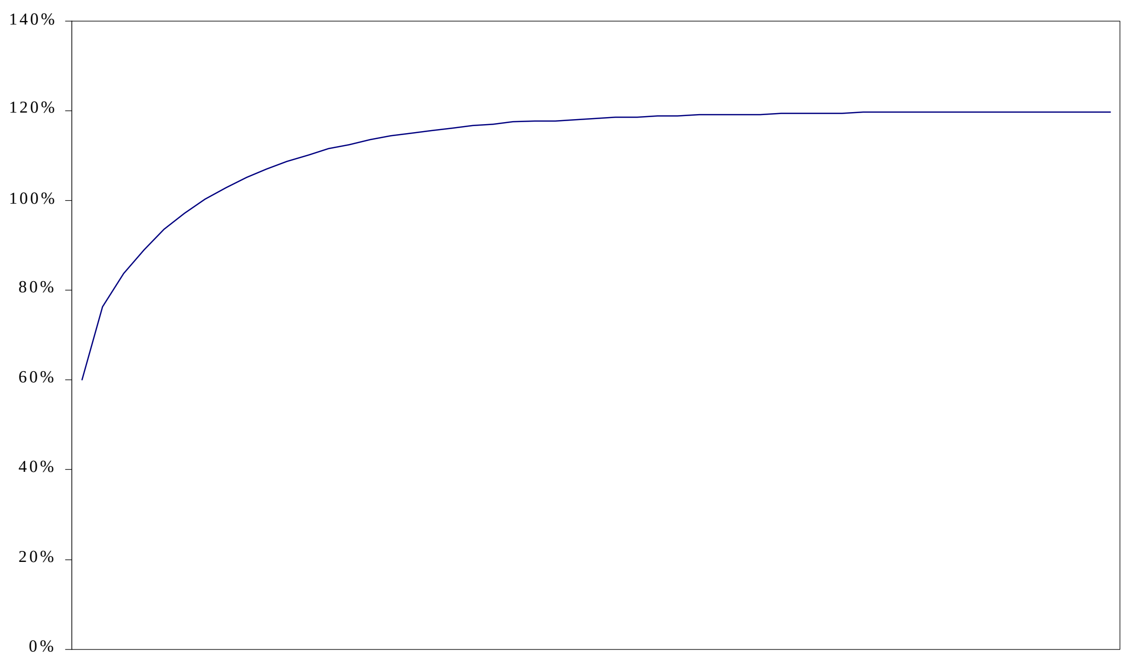


m=12%, s=20%, α=2%, π=2%, r=5.5%, γ=1.16, ρ=6%, n=+2

Spending rate



Stock allocation



m=12%, s=20%, α=2%, π=2%, r=6%,γ=1.25, ρ=6%, n=+2

Vita

Kurtay N. Ogunc was born in Istanbul, Turkey to Aysel and Kurtul Ogunc. After graduating from the Saint George Austrian College with a High School Diploma, he went on to receive a Bachelor of Business Administration degree from Marmara University in 1990, majoring in finance and quantitative methods. He then married her college sweetheart, Asli K. Ogunc, and together, they moved to the U.S. to pursue graduate studies.

Kurtay obtained the Master of Business Administration degree from Western Michigan University in 1992, majoring in finance, and Master of Applied Statistics from Louisiana State University in 1997. He taught various courses in the Department of Information Systems & Decision Sciences and the Department of Finance at the E. J. Ourso College of Business Administration of LSU for 3 ½ years.

Since January of 1994, the semester Kurtay joined LSU, he has worked in the investment operations of the Associate Vice Chancellor for Finance for six years in different roles, the last one being the Investment Manager in charge of the funds managed by the LSU Foundation. In May of 2000, Kurtay moved to Washington, DC to become the research coordinator of the investment consulting division and a specialist consultant in charge of currency overlay managers at Watson Wyatt & Company, a global human capital consulting firm. He will complete the degree of Doctor of Philosophy in May 2002.