The effects of self-assessment on student learning of mathematics

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THE EFFECTS OF SELF-ASSESSMENT ON STUDENT LEARNING
OF MATHEMATICS

A Thesis

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ABSTRACT

Self-assessment can be defined as students judging their own work, based on evidence and explicit criteria, for the purpose of improving future performance. Many researchers agree that implementation of student self-assessment in the classroom is an important tool for improving academic performance and self-efficacy.

In this thesis, I investigated the effects of student self-assessment on learning gains of 81 alternative high school Algebra I students. This study took place over the course of two units of instruction. Classes were divided into two groups. One group of students utilized rubrics to self-assess their own work for the first unit, while the other group did not. During the second unit, treatments for both groups were switched. Pretest and posttest data for both units was collected. Student learning gains under each treatment were compared using a paired t-test. Mean learning gains under both treatments were also examined using a two-sample t-test for comparing two population means. In all cases, no significant difference was found.
CHAPTER 1. SELF-ASSESSMENT: THEORY & PRACTICE

This thesis concerns the effects of self-assessment on student learning of mathematics. In this chapter, self-assessment is defined, and theoretical arguments for the use of self-assessment are presented. In addition, several studies on the effectiveness of self-assessment are analyzed, and suggested techniques for using self-assessment in the classroom are examined. Challenges to implementing self-assessment effectively are also discussed, as well as methods for overcoming these obstacles.

1.1 Formative vs. Summative Assessment

In education, there are two types of assessment: formative assessment, also called assessment-for-learning, and summative assessment, also known as assessment-of-learning (Stiggins, Arter, Chappuis, & Chappuis, 2007). Black, Harrison, Lee, Marshall & Wiliam (2004) explain the distinction as follows:

Assessment for learning is any assessment for which the first priority in its design and practice is to serve the purpose of promoting students’ learning. It thus differs from assessment designed primarily to serve the purposes of accountability, or of ranking, or of certifying competence. An assessment activity can help learning if it provides information that teachers and their students can use as feedback in assessing themselves and one another and in modifying the teaching and learning activities in which they are engaged. Such assessment becomes “formative assessment” when the evidence is actually used to adapt the teaching work to meet learning needs. (p. 10)

This thesis concerns assessment-for-learning.

Stiggins, et al. (2007) state that there are three questions students must answer when dealing with assessment for learning: (1) “Where am I going?” (2) “Where am I now?” and (3) “How can I close the gap?” (p. 41). Teachers can help students answer the first question by revealing learning expectations to students in easy-to-understand language and by providing a range of poor- to good-quality work to students. To answer the second question, teachers should
provide practice relying on descriptive feedback students can use to improve their work as well as encouraging students to monitor their progress day-to-day. To assist students with closing the gap between where they are now and where they are going, teachers should show students how to improve one attribute at a time and how to reflect on changes in their work and their academic capabilities (Stiggins, 2008, p. 220). Stiggins (2008) argues for the emotional benefits of assessment for learning in terms of students feeling in control of their own learning success.

However, despite the arguments for good use of formative assessment, Black, et al. (2004) stressed the need for improvement in the use of such assessment. Black and his coauthors identify three main problems: “1) the assessment methods that teachers use are not effective in promoting good learning, 2) grading practices tend to emphasize competition rather than personal improvement, and 3) assessment feedback often has a negative impact, particularly on low-achieving students, who are led to believe that they lack “ability” and so are not able to learn” (p. 9). In response to these problems, these authors propose improvements under four headings: “questioning, feedback through grading, peer- and self-assessment, and the formative use of summative tests” (p.11). These improvements, according to their studies, produced an effect size of approximately 0.3 standard deviations. Black, et al. concluded that “such improvements, produced across a school, would raise a school in the lower quartile of the national performance tables to well above average” (p. 11). One of these headings, self-assessment, is of particular interest to many researchers. According to Stiggins, et al. (2007), “Periodic articulation about their understanding of quality and about their own strengths and weaknesses is essential to students’ ability to improve. Self-assessment is a necessary part of learning, not an add-on that we do if we have time or the “right” students. Struggling students are the right students” (p. 242).
1.2 Theoretical Arguments for Self-Assessment

Rolheiser and Ross (2001) have this to say about the current educational environment:

[O]ne of the most challenging shifts in conceptions of assessment is related to the changing role of the teacher and the changing educational environment. The context for educators is changing rapidly and dramatically. It is more complex and volatile...In such a shifting context our outcomes for students have sufficiently changed and traditional assessment practices are no longer adequate. (Rolheiser and Ross, 2001)

In today’s educational environment, many students lack what researchers refer to as self-regulation. Zimmerman (1989) defines self-regulation as “the self-directive process by which learners transform their mental abilities into academic skills. Learning is viewed as an activity that students do for themselves in a proactive way rather than as a covert event that happens to them in reaction to teaching” (p. 2). Zimmerman describes the self-regulatory process in three phases: the forethought phase (before the learning effort), the performance phase (during the learning), and the self-reflection phase (after the learning effort) (p. 4). It is in this self-reflection phase that students make self-judgments and form opinions about the cause of their performance (Zimmerman, 1989, p. 5). According to Zimmerman,

Attributing a poor score to limitations in fixed ability can be very damaging motivationally because it implies that efforts to improve on a future test will not be effective. In contrast, attributing a poor math score to controllable processes, such as the use of the wrong solution strategy, will sustain motivation because it implies that a different strategy may lead to success. (p. 5)

Many researchers are turning to self-assessment as a means of harnessing this self-reflection phase to improve student learning. Researchers generally agree that self-assessment, or self-evaluation, can be defined as students judging their own work, based on evidence and explicit criteria, for the purpose of improving future performance (McMillan and Hearn, 2008, p. 40; Rolheiser and Ross, 2001). Zimmerman states that close self-monitoring, along with other
high-quality self-regulatory processes, can enhance motivation of beginners in a new discipline (p. 3). Further, Zimmerman asserts the goal of a teacher should be to empower students to become self-aware of their own strengths and limitations in learning (p. 2).

Rolheiser and Ross (2001) provide a detailed layout of the effects of self-assessment on student achievement:

When students evaluate their performance positively, self-evaluations encourage students to set higher goals…and commit more personal resources or effort…to them. The combination of goals…and effort…equals achievement... A student's achievement results in self-judgment…, such as a student contemplating the question, "Were my goals met?" The result of the self-judgment is self-reaction…, or a student responding to the judgment with the question, "How do I feel about that?" Goals, effort, achievement, self-judgment, and self-reaction all can combine to impact self-confidence…in a positive way.

Rolheiser and Ross base their confidence in self-evaluation on four arguments. They claim student learning will improve because “(i) self-evaluation will focus student attention on the objectives measured, (ii) the assessment provides teachers with information they would otherwise lack, (iii) students will pay more attention to the assessment, and (iv) student motivation will be enhanced.” Black and Wiliam (1998) also emphasize the importance of self-assessment, claiming that it allows students to “understand the main purposes of their learning and thereby grasp what they need to do to achieve” (p. 143). McMillan and Hearn (2008) agree, claiming that student self-assessment promotes “intrinsic motivation, internally controlled effort…and more meaningful learning” (p. 40). Also, McMillan and Hearn assert that self-assessment promotes a mastery goal orientation in which the focus is on improving knowledge, understanding, and skill, as opposed to a performance goal orientation in which the greater focus is on the final score or grade (p. 43). When used with other self-regulatory skills such as goal-setting and generating strategies for more learning, McMillan and Hearn claim that self-
assessment will lead to improved student performance with meaningful motivation (p. 48).

Stiggins (2008) also promotes self- and peer-assessment as a means of better engaging students.

Bring students into the actual scoring process, thus spreading the work over more shoulders! Form scoring teams, one for each exercise on a test. Have them develop scoring criteria under your watchful eye. Offer them advice as needed to generate appropriate criteria. Then have them actually score some essays, which you double check. Discuss differences in scores assigned. Students find this kind of workshop format very engaging. (p. 151)

Interestingly, Stiggins, et al. (2007) claim that while all students benefit from self-assessment, “the largest gains accrue to the lowest achievers” (p. 37).

1.3 Studies of Self-Assessment and Rubric Use

Having examined the theoretical arguments, the question of empirical evidence arises. Several studies of the effectiveness of student self-assessment have been undertaken. One such study by Olina and Sullivan (2002) investigated the effect of teacher- and self-evaluation on student learning and self-efficacy of 189 Latvian high school students and their 6 teachers in science classrooms. Using ratings of student projects, posttest scores, student attitude surveys, and teacher attitude surveys, Olina and Sullivan compared the learning gains of three groups of students: a teacher-evaluation condition, a self-plus-teacher evaluation condition, and a no-evaluation condition. The results were interesting. Students in the teacher- and self-plus-teacher-evaluation groups outperformed students in the no-evaluation group, as expected. However, the effect produced on student attitudes provides great evidence for the benefits of self-assessment. Although students in the no-evaluation group reportedly enjoyed conducting experiments more than the other students, Olina and Sullivan report that “students in the self-plus-teacher evaluation group had greater confidence about their ability to independently conduct
experiments in the future than the other two groups” (p. 19). This lends credibility to theories of self-assessment leading to greater student self-efficacy.

Another notable study was conducted by Andrade, Wang, Du, and Akawi (2009) on the relation between long- and short-term rubric use (including self-assessment) and self-efficacy for writing by 268 elementary and middle school students. Although average self-efficacy ratings of all condition groups increased over time, Andrade, et al. reported a greater increase in self-efficacy of girls in the treatment group (using rubrics to check their work) than in the control group. The effect of the rubrics on self-efficacy of boys appeared to be statistically insignificant. To explain these results, the authors relied on research on attribution theory stating:

> [G]irls tend to be more concerned with mastering a writing task than do boys, who, on average, tend to be more concerned with showing someone else that they are capable. Our findings regarding the differences in increases in self-efficacy after self-assessment may reflect these different achievement goals: Girls may derive more satisfaction and confidence from self-generated evidence of progress on a writing assignment than do boys, who seek confirmation of progress from others, including perhaps their teachers and peers. (Andrade, et al., 2009, p. 296)

Thus, the researchers concluded, self-assessment shielded female students from “potentially debilitating effects of negative adult feedback” (p. 296). This may have led girls to attribute shortcomings to effort, not inherent ability, which allowed them to see how they could improve. This, in turn, led to increased self-efficacy ratings. “The boys in the study, however, might have been less influenced by the presence of the rubric because they placed less value on their own feedback” (p. 296).

### 1.4 How to use Self-Assessment (Techniques and Difficulties)

Several researchers offer advice on how teachers can involve students in the assessment process. Stiggins (2008) offers an array of ways students can be involved in this. According to Stiggins, students can:
Take the test and receive the grade.
Be invited to offer the teacher comments on how to improve the test.
Suggest possible assessment exercises.
Actually develop assessment exercises.
Assist the teacher in devising scoring criteria.
Create the scoring criteria on their own.
Apply scoring criteria to the evaluation of their own performance.
Come to understand how assessment and evaluation affect their own academic success.
Come to see how their own self-assessment relates to the teacher’s assessment and to their own academic success. (p. 23)

Rolheiser and Ross (2001) claim that self-evaluation must be taught to students in four stages. First, students should be involved in defining the criteria by which their performance will be judged. Second, students should be taught how to apply those criteria to their own work. Third, students should be provided with feedback on their own self-evaluations. Finally, teachers should work with students to develop goals and action plans (Rolheiser & Ross, 2001).

Although by its very definition self-assessment places a great deal of the assessment work load on the students’ shoulders, experts agree there are responsibilities of the teacher in this process. McMillan and Hearn (2008) claim that in order to use self-assessment effectively, teachers must “pass the evaluative responsibilities to their students by scaffolding and modeling goal setting, evaluation, strategy adjustment, and reflection” (p. 44). Rolheiser and Ross (2001) agree, stating that “[s]tudents harbor misconceptions about the self-evaluation process (e.g., the role that evidence plays).” Because of this, Rolheiser and Ross argue that students should be taught how to accurately evaluate their own work. “Simply requiring self-evaluation is unlikely to have an effect on achievement.”

For all of the cited benefits to student learning afforded by self-assessment, there are new difficulties introduced into the classroom with it. Many teachers first attempting self-assessment
may be tempted to believe the greatest hardship will be one of student academic dishonesty. However, this is very often not the case. Rolheiser and Ross (2001) argue that:

Many teachers, parents, and students believe that if students have a chance to mark their own work they will take advantage, giving themselves higher scores regardless of the quality of their performance. We have found that students, especially older ones, may do this if left to their own devices. But, when students are taught systematic self-evaluation procedures, the accuracy of their judgment improves.

Instead, Black, et al. (2004) states that when teaching students to effectively self-assess, “the first and most difficult task is to get students to think of their work in terms of a set of goals” (p. 14). Once students convert to this way of thinking, “they begin to develop an overview of that work that allows them to manage and control it for themselves” (p. 14). Another difficulty researchers cite is that “pupils can assess themselves only when they have a sufficiently clear picture of the targets that their learning is meant to attain” (Black and Wiliam, 1998, p. 143). Thus, if students do not understand learning objectives, the effectiveness of self-assessment is diminished. A further complication of self-assessment is that it takes time for students, in particular low-achieving students, to develop the skill (Black, et al., 2004, p.14). A teacher in one of the schools Black, et al. (2004) studied had the following to say about self-assessment:

The kids are not skilled in what I am trying to get them to do. I think the process is more effective long term. If you invest time in it, it will pay off big dividends, this process of getting the students to be more independent in the way that they learn and to take the responsibility themselves. (p. 14)

In order to overcome the obstacles and effectively implement self-assessment in their classrooms, teachers must utilize what Rolheiser and Ross (2001) describe as “[t]houghtfully designed self-evaluation procedures that provide students with explicit criteria at an appropriate level of generality, that provide for student involvement in assessment decision-making, that elicit student cognitions about their performance, that ground student goal setting in accurate
data, and that are integrated with sensitive instruction.” Further, the assessments themselves should be of high quality. According to Stiggins (2008),

Quality classroom assessments display four significant design features: (1) they rely on assessment methods capable of reflecting the targets in question; (2) they are built of quality ingredients (test items, scoring schemes, etc); (3) they sample achievement with enough tasks to lead to a confident conclusion about student mastery; and (4) they are constructed and used in ways that minimize distortion in results due to bias (p. 75).

Another important aspect of quality self-assessment classroom practice is usually the presence of a well-written rubric. According to Andrade, et al., a rubric is “a document that articulates the expectations for an assignment by listing the criteria, or what counts, and describing levels of quality from excellent to poor” (p. 287). McMillan and Hearn (2008) claim that “[p]roviding evaluation criteria through rubrics, models, or anonymous exemplars helps students concretely understand outcomes and expectations. They then begin to understand and internalize the steps necessary to meet the goals” (p. 45). However McMillan and Hearn also warn that “not all rubrics are equal: to promote learning they should indicate levels of proficiency, not just scores for grades” (p. 45). In addition, McMillan and Hearn claim that in order for self-assessment to be fully effective, students should be given the opportunity to make adjustments to their work before it is graded (p. 46). Finally, teachers should be wise to avoid conditions that drive out intrinsic motivation and instead create classroom climates that promote it. According to Stiggins, et al. (2007),

Intrinsic motivation to learn is supported when the learner meets the following criteria:
- Has a sense of control and choice
- Gets frequent and specific feedback on performance
- Encounters tasks that are challenging, but not threatening
- Is able to self-assess accurately
- Encounters learning tasks related to everyday life
The following conditions tend to drive out intrinsic motivation:
- Coercion
- Intimidation
- Rewards or punishments linked to evaluative judgments
- Comparing one student to another
- Infrequent or vague feedback
- Limitation of personal control
- Responsibility without authority (p. 39)

Stiggins (2008) cites five possible factors in grades: Achievement, Aptitude, Effort, Compliance, and Attitude (p. 271). Students need to know which of these influences their grades and what they can do to improve performance. Also, Stiggins lists five possible reasons why a student might not have learned the necessary material in the first place: 1. The student was lacking in prerequisites for the learning objectives. 2. The teacher did not adequately understand the learning target, and so could not convey it well. 3. Instructional methods and/or materials were inadequate. 4. The student lacked the confidence and/or motivation to strive for success. 5. Something beyond teacher and student control (such as a death in the family) interfered with learning (p. 48). When one or more of these situations occur, students (not just teachers) must be prepared to diagnose the problem and work to fix it. According to Zimmerman (1989), “Although teachers also need to know a student's strengths and limitations in learning, their goal should be to empower their students to become self-aware of these differences. If a student fails to understand some aspect of a lesson in class, he or she must possess the self-awareness and strategic knowledge to take corrective action” (p. 2). Student self-assessment is a vehicle to this self-awareness.

1.5 Summary

This thesis concerns the use of self-assessment as a type of formative assessment or assessment for learning. Researchers claim that self assessment enhances motivation of
beginners in a new area of study (Zimmerman, 1989, p. 3), focuses student attention on learning objectives and the assessments used to measure them (Rolheiser and Ross, 2001), and promotes a mastery goal orientation in which the focus is on improving knowledge, understanding, and skill (McMillan and Hearn, 2008, p.43). Several studies on self-assessment and rubric use document positive effects on student self-efficacy (Olina and Sullivan, 2002), (Andrade, et al., 2009). Challenges to the effective implementation of self-assessment include changing students’ views of their work to that of a set of goals (Black, et al., 2004, p.14), ensuring students have a clear understanding of the learning objectives (Black and Wiliam, 1998, p.143), and allowing enough time for students to adequately develop self-assessment abilities (Black, et al., 2004, p.14). In order to effectively implement self-assessment in their classrooms, teachers must develop self-assessment procedures that provide students with explicit criteria for evaluating their work, that involve students in the decision-making process, that encourage students to think about the quality of their performance, that manifest realistic goal setting, and that are integrated with the instruction (Rolheiser and Ross, 2001).
CHAPTER 2. RESEARCH QUESTIONS AND SETTING

Research findings on the positive benefits of self-assessment prompted me to investigate the potential benefits of self-assessment on student learning in my own Algebra I classroom. Poor student academic performance, as well as the current national focus on standardized testing led me to the following research question for this study: Does the use of self-assessment enhance the short-term learning gains of students in mathematics, as measured by standardized multiple-choice and constructed-response tests?

During the period of this study, I worked in an alternative high school for over-aged students. This school was established in 2008 to serve students who were at least two years behind grade level. By providing students the opportunity to take more classes and earn more credits each school year than they would at a regular high school, this school offered students the opportunity to graduate earlier than what would have otherwise been possible. Total school enrollment at the time of this study numbered fewer than 250 students. Over 95% of the students in the school (and 100% of the students in my classes) were African-American.

Challenges I faced with my students prior to this study included limited student motivation to achieve academic success and a lack of attention to assignment completion. While most students did attempt class assignments, I noticed that they were much more concerned with getting a passing grade than with mastering the content. As stated in the previous chapter, it is precisely these mindsets and habits that many researchers claim can be overcome by the use of self-assessment. Thus I sought to teach my students to utilize rubrics to self-assess their own work. In doing so, I hoped that student attention and motivation would increase, which would, in turn, lead to higher test scores on district-mandated Edusoft Unit Tests.
2.1 The Mathematics: What Was Taught

This research project took place over the course of two units of study in Algebra I. The first unit, Unit 5 of the Louisiana Comprehensive Curriculum, introduced students to systems of equations, operations on matrices, and systems of inequalities. The second unit—Unit 6—dealt with measurement topics. Students were taught how to identify the correct unit of measurement for a given situation. They were also introduced to the concepts of accuracy, precision, absolute error, relative error, and significant digits.

2.1.1 Unit 5: Systems of Equations, Matrices, and Systems of Inequalities

Necessary prerequisites to successful completion of this unit were proficiency in evaluating expressions using the correct order of operations, graphing on a coordinate plane, solving linear equations and inequalities in one and two variables, identifying and analyzing slope of a line and the y-intercept of a line, and evaluating expressions and functions written in function notation using substitution (Comprehensive Curriculum Algebra I, p.216). Students had been instructed in these topics prior to the start of this unit.

The first day of instruction began with the following real-world situation:

Sam left for work at 7:00 a.m. walking at a rate of 1.5 miles per hour. One hour later, his brother, James, noticed that he had forgotten his lunch. He leaves home walking at a rate of 2.5 miles per hour. When will he catch up with Sam to give him his lunch? (Comprehensive Curriculum Algebra I, p.230)

Students had different ideas on how to solve this problem, but the method devised by most students (with some coaching from the teacher) was to set up a table and graph of the situation. This was used to begin a discussion of systems of two linear equations, in which the solution is the point of intersection of the two lines. On the first day, graphing was emphasized. In subsequent classes, students were taught to solve problems using substitution and elimination (linear combination) methods. All lessons were introduced and supplemented with real-world
problems. Students were also given step-by-step instruction on how to solve problems using each solution method.

After students had sufficient time to practice solving systems of two linear equations, they were then introduced to matrices. The ultimate goal in the use of matrices in this unit was to use technology (specifically, graphing calculators) to solve systems of equations by using multiplication of inverse matrices. Unfortunately, due to time constraints, none of the classes were able to complete this lesson. However, students were still taught how to model data in a matrix as well as how to perform addition, subtraction, scalar multiplication (multiplying a matrix by a number), and matrix multiplication (multiplying a matrix by a matrix). Again, all topics were introduced with real-world examples.

Finally students were taught to graph systems of linear inequalities. Students were instructed in shading the region of the graph that satisfies all constraints—the solution to the system of inequalities.

Overall, students seemed to like the real-world scenarios used to introduce each topic. These scenarios made the mathematics seem relevant to them. The most common complaint students had was the amount of work needed to find the solutions to the systems of equations problems. Many students commented that the easiest parts of the unit were the lessons on matrices.

2.1.1 Unit 6: Measurement

Prerequisites needed to understand topics in this unit included proficiency in the use of rulers, clocks, scales, and other measurement instruments; conversion factors; rational number computation; formulas for perimeter, area, volume, and circumference of geometric shapes; and rounding to a given place value (Comprehensive Curriculum Algebra I, p.265). Unlike in Unit 5,
prerequisites for this unit had not been taught previously in this course due to the different nature of the topics. However, students had been taught these topics in years prior to enrollment in Algebra I. Prerequisites for this unit were therefore reviewed quickly in class.

During this unit, students were first taught how to choose the correct unit of measurement for a given situation. This was done initially by having the students measure different objects in the classroom using different types of rulers and meter sticks. Other types of measurements were then discussed. This led students to the understanding that the size of the unit must be comparable to the size of the object being measured.

Much of the remaining portion of the unit dealt with terminology. Students worked with their teacher to define accuracy, precision, absolute error, relative error, and significant digits. Actual measurements and kinesthetic activities were then used to enhance understanding. For example, during the lesson on accuracy, students recorded the time shown on their watches when directed, while the teacher recorded the official U.S. time shown on the website www.time.gov. Recorded times were then compared to determine which watch was the most accurate. Computation problems were also utilized in most lessons.

Although paper and pencil assignments were used in both units, due to the nature of the topics being taught, instruction in the second unit utilized more hands-on activities than in the prior unit. Nearly every lesson in Unit 6 incorporated some type of kinesthetic measurement activity for the students. Thus, some students seemed to be more engaged in the activities utilized in this unit than in Unit 5. Also, many students who had formerly been struggling with algebraic concepts in prior units felt as though they had a fresh start in the measurement unit.
CHAPTER 3. METHODS AND MATERIALS

3.1 Subjects

Five classes of eighty-one over-aged alternative high school students participated in the study. All of the students were enrolled in Algebra I and were taught by the same teacher. Some students had taken and failed Algebra I previously, while others were enrolled in the course for the first time. Many of the students in this study had a high-absentee rate. Discipline problems were frequent, and several students were placed in In-School-Suspension during portions of the study. Class size ranged from five students to 21 students.

3.2 Materials

The pre- and post-tests administered during the study were district-mandated Edusoft Unit Tests for Units 5 and 6 of Algebra I. Edusoft is a standards-based testing system designed and operated by the Riverside Publishing Company (Riverside Publishing). Each test was written by teachers and content trainers in the district. The Unit 5 test consisted of 10 multiple-choice items and one 4-point constructed-response item, for a total of 14 possible points. The test assessed students’ ability to solve and graph systems of equations, graph systems of inequalities, and perform operations on matrices. The Unit 6 test consisted of 14 multiple-choice questions and one 4-point constructed-response item. However, due to a mistake in the wording of one of the multiple-choice items, this multiple-choice item was declared invalid and given a 0-point value. Thus, students could score a maximum of 17 points on this test. The Unit 6 Test assessed students’ knowledge of accuracy and precision, absolute and relative error, and significant digits, as well as their ability to choose the most appropriate unit of measurement for a given situation. For the constructed-response items on both tests, the test-specific district rubrics were used to score student responses. These tests were chosen as the measurement
instruments for this study primarily for reasons of convenience: Edusoft testing was required by the district, and no suitably better measurement instrument was available.

We now discuss the intervention, which consisted of assigned self-assessment tasks for students. For each unit of study, 4-point rubrics were developed for each formative assessment (classwork and homework assignments). These rubrics were designed to imitate rubrics used to score students’ constructed-responses on district-mandated Edusoft Unit Tests. Each rubric contained a point total for each problem or activity. Rubrics for assignments where more than 4 points were attainable also contained a scale for converting student scores into a 4-point score. Rubrics for more detailed assignments also contained an answer key. For shorter assignments, the teacher usually wrote the solution on the board for students. A self-scoring sheet was created for each unit of study. These self-scoring sheets listed every classwork and homework assignment, along with a blank area for students to record their scores for each assignment completed. A generic math rubric detailing a description of each score was also used. This was retrieved from the website http://math.about.com/library/blrubric1.htm.

3.3 Procedures

The Algebra I classes in this study were organized into a block schedule with classes meeting every other day. For the first unit of study, three A-day classes were taught to self-assess their work, while two B-day classes were maintained as the control group. This was reversed in the second unit of study, in which B-day classes utilized self-assessment, and A-day classes performed as the control group. This design allowed all classes to complete one unit using self-assessment and one unit with no self-assessment, thus exposing all students to the treatment and allowing the researcher to compare each student’s learning gains under both treatments. This presented a risk of carry-over effect, as students in the A-day classes could have
used self-assessment skills acquired from the first unit of study to assist them in the second unit, in which they were considered the control group. I argue however that this did not happen to a significant degree, as shall be discussed later. A diagram summarizing the experimental design is shown below.

Table 1 Experimental Design

<table>
<thead>
<tr>
<th>A-Day Classes</th>
<th>B-Day Classes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unit 5:</strong> Systems of Equations, Systems of Inequalities, and Matrices</td>
<td><strong>Unit 6:</strong> Measurement</td>
</tr>
<tr>
<td>Self-Assessment (Experimental Group)</td>
<td>No Self-Assessment (Control Group)</td>
</tr>
<tr>
<td>No Self-Assessment (Control Group)</td>
<td>Self-Assessment (Experimental Group)</td>
</tr>
</tbody>
</table>

Students in the control group continued with the same classroom procedures they had been following all year. In the control group, formative assignments—both classwork and homework—were checked for completion by the teacher. If students had any questions about the material after the assignments were checked, the teacher would answer their questions, but students were not asked to score their work or correct their mistakes. Only unit tests and quizzes were checked for correctness. These were graded by the teacher.

Students in the self-assessment group completed the same classwork and homework assignments as students in the control group. However, students in the self-assessment group were given a self-scoring sheet at the beginning of the unit listing the activities and assignments they would be doing. On the due date of each assignment, the teacher would work out the problems and project the rubric for that assignment on the board. Students were asked to score their work based on the 4-point rubric. They were then instructed to record each of their scores on their self-scoring sheet. The teacher repeatedly emphasized the need for students to study and practice the skills where they earned low scores (0 – 2). In addition, a generic rubric explaining
the meaning of each number score was repeatedly shown to students. The teacher also explained
to the students that assignments were still checked for completion and that student scores
recorded on the self-scoring sheets did not influence the students’ grades; these were only to be
used as a guide for studying and self-monitoring.

An important note should be made about the use of rubrics in the study. Often, most (and
sometimes all) of the students in a class would not complete a given assignment. Thus the use of
rubrics to self-assess student work was occasionally delayed until at least a few of the students
finished the assignment. In most cases, this meant that not every assignment on the self-scoring
sheet was completed in every class.

At the beginning of each unit, students in both groups were given the district-mandated
Edusoft test for that unit as a pretest. The teacher explained to students that the purpose of the
pretest was to determine what they did not yet know, and that their grade would not be
negatively impacted by a low score on the pretest. At the end of the unit, students were given the
same test as a post-test. Students were given a grade for this based on how many problems they
answered correctly. In all cases, and the multiple-choice items were scored electronically using
the Edusoft digital scanner, and the constructed-response items were scored by the teacher using
the district-created rubric for that test. Thus, it should be noted that scoring of the constructed-
response items was not blind.

In the first unit of study, Unit 5, students were taught how to solve systems of linear
equations by graphing, using substitution, and using elimination, as well as how to graph and
solve systems of linear inequalities. The unit also included a section on addition, subtraction,
and multiplication of matrices. Unit 6 dealt with measurement topics including accuracy,
precision, absolute error, relative error, computations with significant digits, and choosing the
most appropriate unit of measurement. In short, the two units dealt with completely different topics. Thus it was assumed that students’ success or failure to meet the objectives of one unit would not drastically influence the outcome of the other unit.

All activities in both units were taken from the Louisiana Comprehensive Curriculum. There were no significant differences between the treatment groups as to the time spent on each unit. For all classes, Unit 5 comprised five weeks of study, while Unit 4 was completed in four weeks.

3.4 Data Analysis

Students’ learning gains for each unit were calculated by subtracting students’ pretest scores from students’ posttest scores. Students’ performance data was first graphed and analyzed visually. Further analysis of data from students who completed all four assessments was carried out using a paired t-test to compare each student’s learning gains while utilizing self-assessment to his or her learning gains while not utilizing self-assessment. In addition, a two-sample t-test for comparing two population means was employed. This allowed the experimenter to utilize data from students who completed assessments for at least one unit, thus increasing the sample size. Data was also analyzed using a permutation test. However, the results of the permutation test were the same as the t-test. Thus, because the t-test is more familiar to most readers, only these results were reported.
CHAPTER 4. RESULTS

The mean learning gains, standard deviations, and sample sizes for each treatment group for each of the two units of study are shown in the table below. In all cases the mean self-assessment learning gains were lower than the mean non-self-assessment learning gains.

Table 2 Learning Gains by Treatment Groups

<table>
<thead>
<tr>
<th></th>
<th>Unit 5 Learning Gains</th>
<th>Unit 6 Learning Gains</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M = 8.79%</td>
<td>M = 6.42%</td>
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<tr>
<td></td>
<td>SD = 14.91%</td>
<td>SD = 11.75%</td>
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<td></td>
<td>n = 13</td>
<td>n = 22</td>
</tr>
<tr>
<td>Self-Assessment</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>M = 10.51%</td>
<td>M = 7.84%</td>
</tr>
<tr>
<td></td>
<td>SD = 17.15%</td>
<td>SD = 14.92%</td>
</tr>
<tr>
<td></td>
<td>n = 17</td>
<td>n = 21</td>
</tr>
<tr>
<td>Non-Self-Assessment</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Graphical displays of pretest and posttest data for each unit of study are shown below. It should be noted that on all graphs, larger circles indicate two identical data points. Aside from a few outliers, in both cases there seem to be no major noticeable visual differences between the self-assessment group and the control group.

Figure 1 Unit 5 Pretest and Posttest
A major obstacle to the completion of this study was a very high absentee rate among the students. Of the 81 students enrolled in the studied classes, only 30 completed the pretest and posttest for Unit 5 (13 in the self-assessment group and 17 in the control group), and 43 completed both assessments for Unit 6 (22 in the self-assessment group and 21 in the control group). Of these, only 18 completed pretests and posttests for both units. A graphical display of the learning gains of each of these 18 students under both conditions is shown below.

**Figure 2** Unit 6 Pretest and Posttest

**Figure 3** Students with Complete Data
If both types of instruction had approximately the same effect on student learning, the points on the graph would fall somewhere near the line $y = x$. Likewise, if one type of instruction had been more effective than the other, more of the points would lie on one side of the line than the other. When the line $y = x$ is drawn in the graph (as shown below), more students’ values fall to the left of the line. For these students, instruction without the use of self-assessment was associated with greater gains than instruction utilizing self-assessment.

![Graph showing self-assessment learning gains compared to non-self-assessment learning gains](image)

**Figure 4**  Students with Complete Data and Transposed Line

In order to test whether one type of instruction produced a significantly different result in learning gains, a paired t-test (Peck, Olsen, and Devore, 2009, p.609) was employed to compare each student’s self-assessment learning gain to his or her non-self-assessment learning gain. This yielded a p-value of 0.455088. This means that there is an approximate 45.5% probability of obtaining values at least as far apart as what was obtained in a comparable scenario in which...
there really is no difference between the two treatments. Due to this large p-value, there is no evidence from this sample of 18 students that either method of instruction was more effective.

However, the small sample size used for the paired t-test leads one to suspect that the results of this test may not be totally conclusive. Thus, in order to obtain a measurement from a larger sample of students, the experimenter utilized data from all students who completed pretests and posttests for at least one unit of study. In total, the experimenter collected data from 35 students who used self-assessment (13 in the first unit of study and 22 in the second) and 38 students who did not (17 in the first unit; 21 in the second). A two-sample t-test for comparing population means (Peck, et al., 2009, p.587) was employed to determine if the mean learning gain for one treatment was significantly higher than the mean learning gain for the other. This yielded a p-value of 0.303458. Again, this means that there is an approximate 30.35% probability of obtaining values at least as far apart as what was obtained in a comparable scenario in which there really is no difference between the two treatments. Thus, because a high p-value was also obtained for this test, there is no evidence that either method of instruction was more effective than the other.
CHAPTER 5. CONCLUSION

This study examined the effects of the use of self-assessment on student learning of mathematics as measured by improvement on multiple-choice and constructed response tests. Despite the positive benefits cited in the research, this study yielded no conclusive evidence to suggest that the use of self-assessment and rubrics enhances student learning of Algebra I topics. Several possible reasons for these results are cited. First of all, prior to this study (and during this study in other classes), students had been taught regularly using non-self-assessment-focused instruction. Thus, the use of self-assessment—as well as the expectations that accompanied it—was a change with which students may not have been totally comfortable. It may be that the short time period allowed for this study (5 weeks for the first unit and 4 weeks for the second) may not have been enough to elicit the full effect of self-assessment on student learning. This would be in agreement with much of the literature presented earlier (Black, et al., 2004, p.14).

Another potentially limiting factor of the effects of self-assessment in this study could have been poor student attendance. Many students in both treatment groups were absent from school or were assigned to In-School-Suspension during portions of instruction. This could have limited the effects of either type of instruction on many students’ learning. This would also account for many of the zero or negative learning gains displayed by some of the students.

A third limitation facing the experimenter in this study was student rate of assignment completion. Often use of rubrics to self-assess was delayed or, in a few instances, some rubrics were not used at all because few (or sometimes none) of the students in a class completed a homework assignment by the due date, or they did not finish a classwork assignment in time to score it with a rubric on the day of completion.
In addition to the dilemmas described above, another possibility remains. It is possible that self-assessment may in fact have had a positive impact on students’ learning. However, instruction in how to use self-assessment and the act of rubric grading took time out of the class period. Thus the non-self-assessment classes had time to work more problems and examples in class which may have balanced out the lack of self-assessment. If this were true, the question of the possibility of carry-over of self-assessment skills from one unit to the next still remains. As stated earlier, it is the opinion of the experimenter that students did not seem to carry effects of one unit of study over into the other (there do not seem to be any statistically significantly different effects to carry over). However, one possible scenario still remains untested: Self-assessment takes time away from instruction while it is being taught, thus potentially negating any positive effects during that unit, but its positive effects—if they exist—may carry over to (and be measurable in) future units. This could be tested in a future similar study in which at least one class uses no self-assessment at all. Other classes could utilize self-assessment on one unit of study, and then none on the next. The learning gains of the two groups on the second unit in which no one was instructed in self-assessment could then be compared. Other possible future studies could examine the effects of self-assessment over a longer period of time with larger and more diverse samples.

A more general conclusion can also be drawn. Teachers are often told by administrators, politicians, researchers, and “experts” in education that if they will only change their teaching methods and incorporate certain instructional techniques then students will learn more and test scores will automatically increase, regardless of their circumstances. It is the finding not only of this thesis but of many documented studies that this is not always true (Barrett, 2009, p16; Ford, 2009, p.19; Richard, 2010, p.36). In all of these studies, the researchers provided theoretical and
anecdotal evidence for treatment benefits. However, in all of these cases, no statistically significant improvement in test scores was found. This is not to suggest that there are not certain “best practices” in teaching, or that it does not matter how a teacher delivers instruction. However, to expect immediate positive results on standardized tests as a result of a change in instructional techniques—and to hold teachers accountable for such result—may not be realistic.

This may be due in part to a failure of standardized testing to detect certain learning gains in students such as motivation, interest in further study, and the desire to apply curriculum content to situations outside of the classroom. It is also conceivable that for some students, areas outside of an individual teacher’s control far outweigh anything the teacher can do in the classroom in terms of gains accrued on a classroom assessment. Most likely, however, is that it may take a great deal of time for positive learning gains to register on test scores as a result of changes in the makeup of a classroom setting.
REFERENCES


APPENDIX A: RUBRICS AND SELF-SCORING SHEETS

Self-Scoring Sheet

Name: __________________

Unit 5: Systems of Equations, Matrices, and Systems of Inequalities:

<table>
<thead>
<tr>
<th>Assignment</th>
<th>How I did (0-4):</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classwork #1: Graphing System of Equations</td>
<td></td>
</tr>
<tr>
<td>Worksheet Problems</td>
<td></td>
</tr>
<tr>
<td>Homework #1: Graphing System of Equations</td>
<td></td>
</tr>
<tr>
<td>(Workbook p.137 #1-3)</td>
<td></td>
</tr>
<tr>
<td>Classwork #2: Battle of the Sexes Questions</td>
<td></td>
</tr>
<tr>
<td>Homework #2: Solving word problems with Systems</td>
<td></td>
</tr>
<tr>
<td>of Equations (Workbook p.138 #5-6)</td>
<td></td>
</tr>
<tr>
<td>Classwork #3: Substitution Problems</td>
<td></td>
</tr>
<tr>
<td>Homework #3: Solving Systems of Equations with</td>
<td></td>
</tr>
<tr>
<td>Substitution (Workbook p.139 #1-3)</td>
<td></td>
</tr>
<tr>
<td>Classwork #4: Elimination Problems</td>
<td></td>
</tr>
<tr>
<td>Homework #4: Solving Systems of Equations with</td>
<td></td>
</tr>
<tr>
<td>Elimination (Workbook p.143 #1-3)</td>
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</tr>
<tr>
<td>Operations on Matrices</td>
<td></td>
</tr>
<tr>
<td>Graphing Systems of Linear Inequalities</td>
<td></td>
</tr>
<tr>
<td>(Workbook p.152 #1-3)</td>
<td></td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
</tr>
</tbody>
</table>
Rubric: Classwork #1: Graphing System of Equations Worksheet Problems

2) Solve each of the following systems of equations by graphing.

A. \[ y = -x + 3 \]
   \[ y = x + 1 \]
   Answer: \((1, 2)\)

B. \[ 2x + 2y = 2 \]
   \[ -3x + 3y = -9 \]
   Answer: \((2, -1)\)

A. (2 possible points)
   1 point if one line is graphed correctly
   2 points if both lines are graphed correctly and point of intersection is correctly labeled.

B. (2 possible points)
   1 point if one line is graphed correctly
   2 points if both lines are graphed correctly and point of intersection is correctly labeled.
Rubric: Homework #1: Graphing System of Equations (Workbook p.137 #1-3)

1. (2 possible points)
   1 point if one line is graphed correctly
   2 points if both lines are graphed correctly and point of intersection is correctly labeled.

2. (2 possible points)
   1 point if one line is graphed correctly
   2 points if both lines are graphed correctly and point of intersection is correctly labeled.

3. (2 possible points)
   1 point if one line is graphed correctly
   2 points if both lines are graphed correctly and point of intersection is correctly labeled.

Scale:

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<td>1</td>
<td>1 point</td>
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<tr>
<td>0</td>
<td>0 points</td>
</tr>
</tbody>
</table>
Rubric: Classwork #2: Battle of the Sexes Questions

Guiding Questions

1. Describe what the point of intersection on the graph tells you.

*It is the year that men and women swim the 100-meter freestyle in the same amount of time.*

2. According to the graph, is there ever a year that women and men swim the 100-meter freestyle in the same time? If so, what year and what time will they swim?

*They will swim the race in 39.1 seconds in the year 2049.*

3. Write the equation in slope intercept form that describes the time it takes men to swim the 100-meter freestyle in a given year. 

\[ y = -0.167x + 64.06 \]

4. Write the equation in slope intercept form that describes the time it takes women to swim the 100-meter freestyle in a given year.

\[ y = -0.255x + 77.23 \]

5. How much faster are the women and the men each year?

*The men are 0.167 seconds faster each year, and the women are 0.255 seconds faster each year.*

1. 1 point for the correct answer

2. (2 points)
   1 point for the correct number of seconds and 1 point for the correct year

3. 1 point for the correct answer

4. 1 point for the correct answer

5. 1 point for the correct answer
<table>
<thead>
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<th>Scale</th>
<th>Points</th>
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<td>2-3 points</td>
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<td>1</td>
<td>1 point</td>
</tr>
<tr>
<td>0</td>
<td>0 points</td>
</tr>
</tbody>
</table>
Rubric: Homework #2: Solving word problems with Systems of Equations (Workbook p.138 #5-6)

5. (2 possible points)
   1 point if one line is graphed correctly
   2 points if both lines are graphed correctly and point of intersection is correctly labeled.

6. (2 possible points)
   1 point if one line is graphed correctly
   2 points if both lines are graphed correctly and point of intersection is correctly labeled.
Rubric: Classwork #3: Substitution Problems

1. $y = x - 4$
   $4x + y = 26$

2. $2c - d = -2$
   $4c + d = 20$

1. (2 possible points)
   1 point if substitution is used correctly and the correct value of one variable is found.
   2 points if the correct values of both variables are found.

2. (2 possible points)
   1 point if substitution is used correctly and the correct value of one variable is found.
   2 points if the correct values of both variables are found.
Rubric: Homework #3: Solving Systems of Equations with Substitution
(Workbook p.139 #1-3)

1. (3 possible points)
   1 point if one equation is correctly changed around to get one variable by itself.
   2 points if substitution is used correctly and the correct value of one variable is found.
   3 points if the correct values of both variables are found.

2. (3 possible points)
   1 point if one equation is correctly changed around to get one variable by itself.
   2 points if substitution is used correctly and the correct value of one variable is found.
   3 points if the correct values of both variables are found.

3. (3 possible points)
   1 point if one equation is correctly changed around to get one variable by itself.
   2 points if substitution is used correctly and the correct value of one variable is found.
   3 points if the correct values of both variables are found.

Scale:

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<td>1-2 points</td>
</tr>
<tr>
<td>0</td>
<td>0 points</td>
</tr>
</tbody>
</table>

37
Rubric: Classwork #4: Elimination Problems

1. \(2x + y = 4\) \quad 2. \(x - y = 0\)
\(x - y = 2\) \quad -3x - y = 2

1. (2 possible points)
   1 point if elimination is used correctly and the correct value of one variable is found.
   2 points if the correct values of both variables are found.

2. (2 possible points)
   1 point if elimination is used correctly and the correct value of one variable is found.
   2 points if the correct values of both variables are found.
Rubric: Homework #4: Solving Systems of Equations with Elimination
(Workbook p.143 #1-3)

1. (2 possible points)
   1 point if elimination is used correctly and the correct value of one variable is found.
   2 points if the correct values of both variables are found.

2. (2 possible points)
   1 point if elimination is used correctly and the correct value of one variable is found.
   2 points if the correct values of both variables are found.

3. (2 possible points)
   1 point if elimination is used correctly and the correct value of one variable is found.
   2 points if the correct values of both variables are found.

Scale:

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<td>1 point</td>
</tr>
<tr>
<td>1</td>
<td>0 points</td>
</tr>
</tbody>
</table>
Rubric: Operations on Matrices

A) Add \[
\begin{bmatrix}
1 & 3 \\
-4 & 5
\end{bmatrix}
\begin{bmatrix}
-7 & 4
\end{bmatrix}
= \begin{bmatrix}
-6 & 7
\end{bmatrix}
\]

B) Subtract \[
\begin{bmatrix}
2 & -5 & 4 \\
4 & 0 & -6 \\
7 & 9 & 12
\end{bmatrix}
\begin{bmatrix}
-3 & 4 & -11
\end{bmatrix}
= \begin{bmatrix}
5 & -9 & 15 \\
13 & -6 & -78 \\
-1 & 21 & -22
\end{bmatrix}
\]

C) \[
\begin{bmatrix}
3 & 23 \\
-7 & 9
\end{bmatrix}
+ \begin{bmatrix}
2 \\
8
\end{bmatrix}
= \text{No Solution}
\]

D) Multiply \[
-5
\begin{bmatrix}
1.2 & 4 & 8 \\
5 & 0 & -4
\end{bmatrix}
= \begin{bmatrix}
-6 & -20 & -40 \\
-1 & 0 & 20
\end{bmatrix}
\]

A) (2 possible points)
2 points if answer is completely correct
1 point for 1 or 2 computational errors

B) (2 possible points)
2 points if answer is completely correct
1 point for 1 or 2 computational errors

C) (1 possible point)
1 point for the correct answer

D) (2 possible points)
2 points if answer is completely correct
1 point for 1 or 2 computational errors

Scale:

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<th>Score</th>
<th>Points</th>
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<td>1-2 points</td>
</tr>
<tr>
<td>0</td>
<td>0 points</td>
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</tbody>
</table>
Rubric: Graphing Systems of Linear Inequalities (Workbook p.152 #1-3)

1) (4 possible points)
   1 point for each line correctly graphed
   1 point for correct shading

2) (3 possible points)
   1 point for each line correctly graphed
   1 point for correct shading

3) (5 possible points)
   1 point for each line correctly graphed
   1 point for correct shading

Scale:

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Measurement Self-Scoring Sheet

Name:________________________

<table>
<thead>
<tr>
<th>Activity:</th>
<th>How I did (0 – 4)</th>
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</thead>
<tbody>
<tr>
<td>Which Unit of Measurement?</td>
<td></td>
</tr>
<tr>
<td>Accuracy Exercises</td>
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<td>Precision Worksheet</td>
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<td>Accuracy vs. Precision</td>
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<td>Calculations with Significant Digits</td>
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<td>Overall (Average Score):</td>
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</table>
Rubric: Which Unit of Measurement?

1 point for each correct answer

Identify the most appropriate unit of measurement to use in order to measure the following:

1. How much water a pan holds \textit{Cups, Pints, Liters}
2. Weight of a crate of apples \textit{Pounds, Kilograms}
3. Distance from New Orleans to Baton Rouge \textit{Miles, Kilometers}
4. How long it takes to run a mile \textit{Minutes}
5. Length of a room \textit{Feet, Yards, Meters}
6. Weight of a Boeing 727 \textit{Tons, Kilograms}
7. Weight of a t-bone steak \textit{Ounces, Grams}
8. Thickness of a pencil \textit{Millimeters, Inches (Fractions)}
9. Weight of a slice of bread \textit{Ounces, Grams}

Scale:

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</table>
Rubric: Accuracy Exercises:

Determine if it is possible to get an accurate measure from the information given.

1 point each

1. Jordan measures a piece of wood to be 4 ½ feet long. Is his measurement accurate?
   Not enough information

2. Jerry bought a 5-pound bag of sugar. When he got home he measured the bag on a scale that he had calibrated with a 5-pound weight. The bag actually weighed 4.75 pounds. Which measurement is more accurate?
   4.75 pounds was more accurate because the scale was calibrated with a 5-pound weight.

3. Alex checked the time on his watch at exactly 3:52:04. The time on the world universal website was exactly the same. Is his watch accurate?
   Yes, his watch is accurate

4. Trevor measured the temperature outside to be 82.67 degrees. Joey also measured the temperature at the same time and got 83.04. Whose measurement is more accurate?
   Not enough information

5. When is it possible to know if a measurement is accurate?
   We can know a measurement is accurate only if we know the true value of the item being measured.

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</tr>
</tbody>
</table>
Rubric: Precision:

1 point each

1. Ruler 1: 4 inches
2. Ruler 2: 4 ½ inches
3. Ruler 3: 4 ¼ inches
4. Ruler 4: 4 3/8 inches
5. Ruler 5: 4 5/16 inches

2 points: (one point for correct answer and one point for correct explanation):

Ruler 5 is the most precise. It has the most subdivisions (tick marks).

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Rubric: Precision vs. Accuracy

1. 2 points: 1 point for correct ranges of Set A and B, and 1 point for correct choice of most precise set.

Data Set A has a range of .06 while Data Set B has a range of .08, thus the more precise data set is Set A.

2. 2 points: 1 point for correct answer and 1 point for correct explanation

There is no way of knowing which is more accurate since in both cases there is no indication of the true measure of the object being measured.

3. 2 points: 1 point for correct procedure with minor arithmetic error; 2 points for all correct averages

<table>
<thead>
<tr>
<th>Trial #</th>
<th>Weight on Scale 1</th>
<th>Weight on Scale 2</th>
<th>Weight on Scale 3</th>
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<tbody>
<tr>
<td>1</td>
<td>101.5</td>
<td>100.00</td>
<td>100.10</td>
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<td>2</td>
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<td><strong>101.5g</strong></td>
<td><strong>100.00g</strong></td>
<td><strong>100.00g</strong></td>
</tr>
</tbody>
</table>

4. 2 points: 1 point for correct answer, and 1 point for correct explanation

Scale 1 since the range of values is smaller than in the other scales.

or

Scales 2 and 3 have units to the hundredth of a gram, and so in terms of the measurement device being used, scales 1 and 2 are the most precise.

5. 2 points: 1 point for correct answer, and 1 point for correct explanation

Scale 3 since the range of values is larger.

Or

Scale 1 would be least precise since it is only able to be read to the nearest tenth of a gram as opposed to the nearest hundredth in the other two scales.
6. 2 points: 1 point for correct answer, and 1 point for correct explanation

If we look at the average weights to be the weight given by each scale, then both Scale 2 and Scale 3 are equally accurate.

7. 2 points: 1 point for correct procedure with minor arithmetic error; 2 points for all correct averages

<table>
<thead>
<tr>
<th>Trial #</th>
<th>Group 1 (g)</th>
<th>Group 2 (g)</th>
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<th>Group 4 (g)</th>
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<td><strong>10.13255</strong></td>
<td><strong>1.01</strong></td>
</tr>
</tbody>
</table>

8. 1 point for correct answer

Group 1 data represents both an accurate and precise measurement.

9. 2 points: 1 point for correct answer, and 1 point for correct explanation

Group 3 had the least accurate answer for the weight of the paper clip since its average value is farthest from the actual value of the paper clip.

10. 2 points: 1 point for correct answer, and 1 point for correct explanation

Group 4 had an accurate weight (if the average is used) but was not precise at all because the numbers are not close together.

**Scale:**

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Rubric: Absolute and Relative Error

1. (1 point for each correct answer)
   a. $|500 \text{ cm} - 507 \text{ cm}| = 7 \text{ cm}$
   b. $|150 \text{ calories} - 154.37 \text{ calories}| = 4.37 \text{ calories}$
   c. $|2.4 \text{ MV} - 2.1 \text{ MV}| = 0.3 \text{ MV}$
      or
      $|2.4 \text{ MV} - 2.7 \text{ MV}| = 0.3 \text{ MV}$
   d. Shortest: 72 inches – 2.25 inches = 69.75 inches
      Longest: 72 inches + 2.25 inches = 74.25 inches
   e. $\frac{5}{8} \text{ in} - 1/32 \text{ in} = \frac{19}{32} \text{ in}$
      $5/8 \text{ in} + 1/32 \text{ in} = \frac{21}{32} \text{ in}$

2. (1 point for each correct answer)
   a. $\frac{7 \text{ cm}}{507 \text{ cm}} \times 100 = 1.38\%$
   b. $\frac{4.37 \text{ calories}}{154.37 \text{ calories}} \times 100 = 2.83\%$
   c. $\frac{0.3 \text{ MV}}{2.1 \text{ MV}} \times 100 = 14.29\%$
      And
      $\frac{0.3 \text{ MV}}{2.7 \text{ MV}} \times 100 = 11.11\%$
   d. $\frac{2.25 \text{ in}}{69.75 \text{ in}} \times 100 = 3.23\%$
      And
      $\frac{2.25 \text{ in}}{74.25 \text{ in}} \times 100 = 3.03\%$
   e. $(\frac{1}{32}) / (\frac{19}{32}) \times 100 = 5 \frac{5}{19}\% \text{ or } 5.26\%$
      And
      $(\frac{1}{32}) / (\frac{21}{32}) \times 100 = 4 \frac{16}{21}\% \text{ or } 4.76\%$

3. (1 point for the correct answer and 1 point for a correct explanation)

   Measure A was the best because it had the smallest relative error.

Scale:

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</table>
Rubric: Significant Digits

2 points each—1 point for correct number of significant digits and 1 point for correct explanation

1. 302  Number of Significant Digits: 3
   Why?  The 3 and 2 are digits from 1-9, and the 0 is between significant digits

2. 500  Number of Significant Digits: 1
   Why?  The 5 is significant because it is a digit from 1-9. The zeros are not significant because they are just placeholders.

3. 31.02  Number of Significant Digits: 4
   Why?  The 3, 1, and 2 are digits from 1-9, and the zero is between significant digits
   Or
   The 3, 1, and 2 are digits from 1-9, and the zero is to the right of both the decimal and a significant digit

4. 31.020  Number of Significant Digits: 5
   Why?  The last zero is to the right of both the decimal and a significant digit, and the other digits are significant for the same reason as problem 3.

5. 0.000021  Number of Significant Digits: 2
   Why?  The 2 and 1 are significant because they are digits from 1-9. The zeros are not significant because they are just placeholders.

Scale:

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</tbody>
</table>
Rubric: Calculations with Significant Digits

1 point for each correct answer:

Solve the following. Round answers to the correct number of significant digits.

1. $7.14 \text{ cm} + 8.2 \text{ cm} \approx \textbf{15.3 cm}$ (Round to the least precise decimal place)

2. $23.7 \text{ m} - 13.81 \text{ m} \approx \textbf{9.9 m}$ (Round to the least precise decimal place)

3. $5.8 \text{ mm} \cdot 0.7 \text{ mm} \approx 4 \text{ mm}^2$ (Round to the least number of significant digits)

4. $103 \div 1.2 \approx 86$ (Round to the least number of significant digits)

5. Find the perimeter and area of the rectangle below. Round your answers to the correct number of significant digits.

```
12.14 cm

3.9 cm

12.14 cm

3.9 cm

12.14 cm
```

Perimeter $= 12.14 + 12.14 + 3.9 + 3.9 \approx 32.1 \text{ cm}$

Area $= 12.14 \cdot 3.9 \approx 47 \text{ cm}^2$

Scale:

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## APPENDIX B: PRETEST AND POSTTEST DATA FOR BOTH UNITS

### Table 3 Pretest and Posttest Data for Both Units

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<td>6</td>
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<tr>
<td>76</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>5</td>
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<td>77</td>
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<tr>
<td>80</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>81</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>
1. What is the solution to this system of equations?

\[
\begin{align*}
y &= -3x - 2 \\
6x + 2y &= -4
\end{align*}
\]

A. (6, 2)  
B. (1, -5)  
C. no solution  
D. infinitely many solutions  

2. Which ordered pair is the solution to the system of equations below?

\[
\begin{align*}
x + 3y &= 7 \\
x + 2y &= 10
\end{align*}
\]

A. \( \left( \frac{7}{2}, \frac{13}{4} \right) \)  
B. \( \left( \frac{7}{2}, \frac{17}{5} \right) \)  
C. (-2, 3)  
D. (16, -3)  

3. In the following equations, the value of \( x \) represents the price per pound of gravel and the value of \( y \) represents the price per pound of sand.

\[
\begin{align*}
2y &= 3x + 8 \\
5y - 15x &= -30
\end{align*}
\]

At what price for sand do the two lines intersect?

A. $6.67  
B. $10.00  
C. $12.33  
D. $14.00  

4. Cassie has $20 in her savings account. She plans to save $2 per week from her babysitting job. Mitch, Cassie's brother, has only $8 in his savings account but plans to save $5 per week from his lawn mowing job. How many weeks will it take before the amounts in Cassie's and Mitch's savings accounts are the same?

A. 2  
B. 4  
C. 10  
D. 28
5. Which of these graphs represents the solution to the system of equations $y = 2 - x$ and $y = 2x + 2$?

A. 

B. 

C. 

D. 

PLEASE GO ON TO THE NEXT PAGE
6. What is the difference of the two matrices?

\[
\begin{bmatrix}
5 & -2 & 10 \\
4 & -5 & 20 \\
1 & -3 & 30
\end{bmatrix} -
\begin{bmatrix}
9 & -3 & 15 \\
1 & -4 & 21 \\
8 & -7 & 30
\end{bmatrix}
\]

A. \[
\begin{bmatrix}
4 & 1 & 5 \\
3 & 1 & 1 \\
7 & 4 & 0
\end{bmatrix}
\]

B. \[
\begin{bmatrix}
-4 & 1 & -5 \\
3 & -1 & -1 \\
-7 & 4 & 0
\end{bmatrix}
\]

C. \[
\begin{bmatrix}
14 & -5 & 25 \\
5 & -9 & 41 \\
9 & -10 & 60
\end{bmatrix}
\]

D. \[
\begin{bmatrix}
-4 & 7 & -14 \\
-3 & -1 & -24 \\
2 & 4 & 0
\end{bmatrix}
\]

7. Debra earns $5 an hour baby-sitting and $8 an hour working at the local library. Her goal is to earn at least $80 this week, but she can't work more than 15 hours. Debra models this situation using the graph below.

According to the graph, which combination of hours will allow Debra to meet her goal?

A. 9 hours baby-sitting and 3 hours at the library
B. 12 hours baby-sitting and 4 hours at the library
C. 3 hours baby-sitting and 9 hours at the library
D. 4 hours baby-sitting and 12 hours at the library
8. The Kramer Cellular Phone Company offers two payment plans. Under Plan A, customers pay a monthly fee of $30 and $0.05 per minute. Under Plan B, they pay a monthly fee of $20 and $0.10 per minute. Which graph correctly shows when the cost of these plans will be equal?
9. Kayla works four days a week as a waitress and as a nanny. The hours she works each day are shown.

<table>
<thead>
<tr>
<th>M</th>
<th>T</th>
<th>W</th>
<th>Th</th>
</tr>
</thead>
<tbody>
<tr>
<td>waitress</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>nanny</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

Once school starts, she wants to cut all her hours by one-half. Which matrix represents the new work schedule?

A. \[
\begin{bmatrix}
6 & 5 & 4 & 7 \\
2 & 2 & 3 & 1.5
\end{bmatrix}
\]

B. \[
\begin{bmatrix}
12 & 10 & 8 & 14 \\
8 & 8 & 12 & 6
\end{bmatrix}
\]

C. \[
\begin{bmatrix}
3 & 2.5 & 2 & 3.5 \\
2 & 2 & 3 & 1.5
\end{bmatrix}
\]

D. \[
\begin{bmatrix}
3 & 2.5 & 2 & 3.5 \\
4 & 4 & 6 & 3
\end{bmatrix}
\]

10. Which graph BEST represents the solution to this system of inequalities?

\[
\begin{align*}
2x & \geq y - 1 \\
2x - 5y & \geq 10
\end{align*}
\]

A. [Graph A]  
B. [Graph B]  
C. [Graph C]  
D. [Graph D]
DIRECTIONS: On a separate sheet of paper answer the following. Be sure to label your answers A and B.

11. The manager of a movie theater found that Saturday’s sales were $3675. He knew that a total of 650 tickets were sold Saturday. Adult tickets cost $7.50 and children’s tickets cost $4.50.

A. Let $A$ represent the number of adult tickets sold and $C$ represent the number of children’s tickets sold. Write 2 equations: Write one to represent the number of tickets sold and write one to represent the amount of money collected.

B. How many of each kind of ticket were sold? Show your work. Include all steps.
1. At a carnival, a prize was given to the person who came closest to guessing the correct value of pennies in a jar. Carlos and Heather each attempted two guesses, shown in this table.

<table>
<thead>
<tr>
<th></th>
<th>Guess</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carlos</td>
<td>1</td>
<td>$32.37</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$32.63</td>
</tr>
<tr>
<td>Heather</td>
<td>1</td>
<td>$20</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$22</td>
</tr>
</tbody>
</table>

The actual value of the pennies was $30.00. Which of these statements is true?

A. Carlos was more precise and more accurate.
B. Heather was more precise and more accurate.
C. Carlos was more precise, but Heather was more accurate.
D. Heather was more precise, but Carlos was more accurate.

2. Evette chains her dog Buster to a tree so that he will be able to play outdoors.

Which of these units is most appropriate to measure the space Buster can play in?

A. centimeters
B. meters
C. square centimeters
D. square meters
3. Juan goes to the store to buy some apples. The apples were priced by the pound, and he bought 4.5 pounds. If Juan paid $31.50 total before tax, how many pounds of apples did he buy?

A. 4.5 pounds  
B. 5 pounds  
C. 7 pounds  
D. 141.75 pounds

4. Multiply the following numbers together and give the answer to three significant digits:

13.5 \times 8.7

A. 117  
B. 117.4  
C. 117.45  
D. 120

5. Jenny thinks she has $118.00 on her gift card. When she goes to the mall to spend it, the clerk at the store tells her she actually has $112.80 on the card. What was the absolute error of Jenny’s guess about her gift card?

A. -5.20  
B. 5.20  
C. 6.20  
D. 520

6. The students in Mr. Frazier’s math class want to measure the length, in centimeters, of a piece of plastic. The students will use rulers that measure length to the nearest tenth of a centimeter. Which statement best explains the precision of the rulers?

A. The length will be precise to one decimal place.  
B. The length will be precise to two decimal places.  
C. The length will be precise to four decimal places.  
D. The length will be precise to six decimal places.

7. Ace Moving Company bases its charges on the weight of the items being moved. Which of these units is best for determining the weight of a dining room table?

A. grams  
B. milligrams  
C. ounces  
D. pounds

8. The money slot on a vending machine is wide enough to accept nickels, dimes, and quarters. Which of these units is best for measuring the thickness of one coin?

A. grams  
B. milligrams  
C. millimeters  
D. centimeters
9. After it rains, Lauren checks the water level of a nearby lake. Which unit of measurement is **best** to use when measuring the depth of a small lake?
   A. feet  
   B. miles  
   C. kilometers  
   D. centimeters

10. Steven determined that his car travels 20 miles per gallon, with a 3-mile margin of error. Which of these represents the relative error?
   A. 0.15  
   B. 0.3  
   C. 1.5  
   D. 3

11. The Costume Shop predicted $10,000 for September sales. Actual sales for September were $12,000. Predicted sales for October were $50,000, and actual sales were $52,000. Which of these statements is true?
   A. The absolute error for September's forecast was less than October's.  
   B. The absolute error for September's forecast was greater than October's.  
   C. The relative error for September's forecast was less than October's.  
   D. The relative error for September's forecast was greater than October's.

12. The Olanders have a dog kennel in their yard, and they want to know its volume. Mr. and Mrs. Olander asked their four children to help measure it. The results are shown in this table.

<table>
<thead>
<tr>
<th>Child</th>
<th>Measurement</th>
<th>Volume (ft³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mara</td>
<td>1</td>
<td>108</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>114</td>
</tr>
<tr>
<td>Brian</td>
<td>1</td>
<td>105.00</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>105.50</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>105.25</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>105.00</td>
</tr>
<tr>
<td>Kelsey</td>
<td>1</td>
<td>110</td>
</tr>
<tr>
<td>Nadine</td>
<td>1</td>
<td>114.375</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>115.125</td>
</tr>
</tbody>
</table>

The actual volume of the kennel is 111.125 ft³. Which of the Olander children's measurements is most accurate?

A. Mara  
B. Brian  
C. Kelsey  
D. Nadine
13. At a gymnastics meet, Stacy received the scores shown below from 5 judges.
   8.75  9.0  9.625  8.75  9.25

   What is the average of Stacy's scores when the correct number of significant digits is used?
   
   A. 9
   B. 9.075
   C. 9.08
   D. 9.1

14. A desk drawer is 16.0 inches long, 5.125 inches deep, and 13.75 inches wide. What is the volume of the drawer if the correct number of significant digits is used?

   A. 1,100 cubic inches
   B. 1,127.5 cubic inches
   C. 1,128 cubic inches
   D. 1,130 cubic inches
Constructed Response

Directions: On another sheet of paper answer Parts A, B, and C. Be sure to label your answers A, B, and C. Show all your work.

15. Anna's teacher placed a giant jar of Skittles on her desk. She told the kids that they could each guess how many Skittles were in the jar and at the end of the week, any student whose guess had a relative error of less than 2% would win some of the Skittles. Anna guessed that there were 5,700 Skittles in the jar. There were actually 5,802 Skittles.

A. What was the absolute error of Anna's Guess?

B. What was the relative error of Anna's Guess?

C. What is the range of guesses that would earn a student Skittles?
VITA

Daniel Hotard was born in 1985 in Gretna, Louisiana. He graduated from Louisiana State University with a Bachelor of Science in mathematics with a concentration in secondary education in May 2007. Daniel has taught math in alternative middle and high schools in East Baton Rouge Parish for the past three years. He has been enrolled in the Louisiana State University Master of Natural Sciences program from 2008 to 2010.