2005

Geostatistical integration of geophysical, well bore and outcrop data for flow modeling of a deltaic reservoir analogue

Hong Tang
Louisiana State University and Agricultural and Mechanical College

Follow this and additional works at: https://digitalcommons.lsu.edu/gradschool_dissertations

Part of the Petroleum Engineering Commons

Recommended Citation
https://digitalcommons.lsu.edu/gradschool_dissertations/2860

This Dissertation is brought to you for free and open access by the Graduate School at LSU Digital Commons. It has been accepted for inclusion in LSU Doctoral Dissertations by an authorized graduate school editor of LSU Digital Commons. For more information, please contact gradetd@lsu.edu.
GEOSTATISTICAL INTEGRATION OF GEOPHYSICAL, WELL BORE AND OUTCROP DATA FOR FLOW MODELING OF A DELTAIC RESERVOIR ANALOG

A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College
In partial fulfillment of the Requirements for the degree of Doctor of Philosophy

in

The Department of Petroleum Engineering

by

Hong Tang

B.Sc in Petrophysics, Southwest Petroleum Institute, 1995
M.S. in Geology, University of Petroleum, China, 2002
August 2005
ACKNOWLEDGEMENTS

At this opportunity the author wishes to express his most sincere gratitude and appreciation to Dr. Christopher D. White, associate professor, for his valuable guidance and genuine interest as research advisor and chairman of the examination committee. Deep appreciation is also extended to other members of the committee, Dr. Julius Langlinais, Dr. Anuj Gupta, Dr. Frank T-C Tsai and Dr. John Wrenn for their support and constructive suggestions.

The constructive discussion and cooperation with researchers from University of Texas at Dallas are highly appreciated. They are Royhan Gani and Keumsuk Lee under the direction of Dr. Janok Bhattacharya and Dr. George McMechan in the Department of Geology and Geophysics.

In addition, great appreciation is extended to my wife Na Deng and my parents for all their supports and love.

Finally, the author is also indebted to the Petroleum Engineering Department, for providing the financial support. This research was funded by the U.S. Department of Energy under contract DEFG0301ER15166.
# TABLE OF CONTENTS

ACKNOWLEDGEMENTS............................................................................................................ ii

LIST OF TABLES......................................................................................................................... x

LIST OF FIGURES ...................................................................................................................... vii

ABSTRACT................................................................................................................................... ix

CHAPTER 1 INTRODUCTION .................................................................................................... 1
  1.1 Significance of the Research......................................................................................... 1
  1.2 Problem Statement ........................................................................................................ 2
  1.3 Method Preparation ....................................................................................................... 5
  1.4 Literature Review .......................................................................................................... 6
    1.4.1 Trend Modeling ..................................................................................................... 6
    1.4.2 Data Integration ..................................................................................................... 7
    1.4.3 Upscaling ............................................................................................................... 9

CHAPTER 2 RESERVOIR DESCRIPTION AND DATA SETTING........................................ 12
  2.1 Geologic Background and Regional Geological Setting ............................................ 12
  2.2 Data Available ............................................................................................................ 15
    2.2.1 Sedimentology ..................................................................................................... 16
    2.2.2 GPR (Ground Penetrating Radar) ........................................................................ 16
    2.2.3 Borehole Data ...................................................................................................... 22
    2.2.4 Photomosaic Data ................................................................................................ 23
  2.3 Concretion Occurrence and Petrophysical Character ................................................. 24
    2.3.1 Shape and Dimension Estimation ........................................................................ 24
    2.3.2 Concretion Distribution ...................................................................................... 26
    2.3.3 Petrophysical Character of the Concretions ......................................................... 27

CHAPTER 3 WELL BORE FACIES CLASSIFICATION ......................................................... 29
  3.1 Multivariate Facies Classification............................................................................... 29
    3.1.1 Beta-Bayesian Method (BBM) Introduction ....................................................... 29
    3.1.2 Multinomial Logistic Regression......................................................................... 31
    3.1.3 Discriminant Analysis ......................................................................................... 32
  3.2 Results and Analysis ................................................................................................... 33
    3.2.1 Model Comparison Using West African Data..................................................... 33
    3.2.2 Classification Procedure ...................................................................................... 33
    3.2.3 Model Comparison and Confidence Measurements ............................................ 35
    3.2.4 Application in Raptor Ridge Outcrop .................................................................. 37
  3.3 Summary ..................................................................................................................... 39

CHAPTER 4 GEOSTATISTICAL MODELING ........................................................................ 41
  4.1 Introduction of SGBSIM and Background ..................................................................... 41
    4.1.1 Bayes Rule for Radar Integration ...................................................................... 41
LIST OF FIGURES

Figure 1-1. Flow chart of research approach ................................................................. 5

Figure 2-1. Regional paleogeography ............................................................................ 13

Figure 2-2. Regional stratigraphy ................................................................................. 14

Figure 2-3. 2D and 3D GPR grid locations from ............................................................ 17

Figure 2-4. Horizontal resolution defined by Fresnel zone width ................................. 20

Figure 2-5. Geological facies and concretion section in well 8 ....................................... 22

Figure 2-6. Dip direction photomosaic of Raptor Ridge outcrop, Frontier Formation .... 24

Figure 2-7. Cumulative probability for concretion dimension ....................................... 25

Figure 2-8. Cumulative probability of concretions length ............................................. 26

Figure 2-9. Proportion of cement in both horizontal and vertical profiles ....................... 27

Figure 3-1. Fitting conditional probability with beta function ....................................... 35

Figure 3-2. The confidence measurements of beta-Bayesian Method ............................ 37

Figure 3-3. Influence of sample size and models on prediction accuracy ....................... 38

Figure 3-4. Comparison of prediction accuracy for different methods .......................... 38

Figure 3-5. Facies prediction profile of raptor ridge data ............................................. 40

Figure 4-1. Flow chart of SGBSIM algorithm ............................................................... 45

Figure 4-2. Indicator correlogram and equivalent Gaussian correlogram ...................... 47

Figure 4-3. Fitted semivariogram model for Gaussian semivariogram ......................... 48

Figure 4-4. GPR responses for channel and bar sediments .......................................... 50

Figure 4-5. Concretion realization from cutoff-based method ....................................... 51

Figure 4-6. Comparison of the geostatistical realizations ............................................. 54

Figure 4-7. Comparison of concretion prediction and its influences on fluid flow ........... 54

Figure 4-8. Comparison of the realization of SGBSIM with different fraction trend ....... 55
Figure 4-9. Comparison of Radar Update Effects ................................................................. 56
Figure 5-1. Outcrop facies, concretions, and flow ............................................................... 66
Figure 5-2. The variances distribution of flow responses ....................................................... 68
Figure 5-3. The response surfaces of interests ...................................................................... 75
Figure 6-1. The location of subregion for grid selection ....................................................... 78
Figure 6-2. The cornerpoint flow grid at Raptor Ridge .......................................................... 79
Figure 6-3. The tracer injection behavior in x, y and z- directions ........................................ 80
Figure 6-4. Flow grids comparison ....................................................................................... 81
Figure 6-5. Comparison of the flow responses and CPU time of different grid sizes .......... 83
Figure 6-6. Simplification of a 3D cube for averaging .......................................................... 84
Figure 6-7. Relative error and sum of squared error of upscaling ........................................ 88
Figure 6-8. The responses of 3D flow simulation ................................................................. 93
Figure 7-1. The flow responses of concretion fraction .......................................................... 96
Figure 7-2. Comparison of concretion fraction on flow response ......................................... 96
Figure 7-3 Comparison of radar updates and its influence on sweep efficiency ................. 98
Figure 7-4. The effective pore volume tracer bypassed after 1 pore volume injection .......... 99
Figure 7-5. Three concretion realizations and their sweep efficiency .................................. 100
ABSTRACT

Significant world oil and gas reserves occur in deltaic reservoirs. Characterization of deltaic reservoirs requires understanding sedimentary and diagenetic heterogeneity at the submeter scale in three dimensions. However, deltaic facies architecture is complex and poorly understood. Moreover, precipitation of extensive calcite cement during diagenesis can modify the depositional permeability of sandstone reservoir and affect fluid flow. Heterogeneity contributes to trapping a significant portion of mobile oil in deltaic reservoirs analogous of Cretaceous Frontier Formation, Powder River Basin, Wyoming.

This dissertation focuses on 3D characterization of an ancient deltaic lobe. The Turonian Wall Creek Member in central Wyoming has been selected for the present study, which integrates outcrop digitized image analysis, 2D and 3D interpreted ground penetrating radar surveys, outcrop gamma ray measurements, well logs, permeameter logs and transects, and other data for 3D reservoir characterization and flow modeling. Well log data are used to predict the geological facies using beta-Bayes method and classic multivariate statistic methods, and predictions are compared with the outcrop description. Geostatistical models are constructed for the size, orientation, and shape of the concretions using interpreted GPR, well, and outcrop data. The spatial continuity of concretions is quantified using photomosaic derived variogram analysis.

Relationships among GRP attributes, well data, and outcrop data are investigated, including calcite concretion occurrence and permeability measurements from outcrop. A combination of truncated Gaussian simulation and Bayes rule predicts 3D concretion distributions. Comparisons between 2D flow simulations based on outcrop observations and an ensemble of geostatistical models indicates that the proposed approach can reproduce essential aspects of flow behavior in this system.
Experimental design, analysis of variance, and flow simulations examine the effects of geological variability on breakthrough time, sweep efficiency and upscaled permeability. The proposed geostatistical and statistical methods can improve prediction of flow behavior even if conditioning data are sparse and radar data are noisy. The derived geostatistical models of stratigraphy, facies and diagenesis are appropriate for analogous deltaic reservoirs. Furthermore, the results can guide data acquisition, improve performance prediction, and help to upscale models.
CHAPTER 1 INTRODUCTION

1.1 Significance of the Research

Deltas are an important reservoir type. For example, the deltaic East Texas Field had initial oil in place of about 5 billion barrels, making it the largest field in the 48 states (Gibson Consulting, website). The Niger Delta province is the twelfth richest in petroleum resources, with 2.2% of the world’s discovered oil and 1.4% of the world’s discovered gas (Tuttle, 1999). Recovery from these reservoirs may be affected by interwell-scale heterogeneity that cannot be characterized solely from subsurface data. Outcrop data can provide the necessary information. However, there are few 2D outcrop characterizations of deltaic deposits and even fewer 3D studies that integrate outcrop with shallow subsurface data (Li and White 2002; Novakovic and others, 2002).

A multiple-university, Department of Energy-sponsored study of the Wall Creek Member integrates probe and core permeameter measurements, wireline logs, outcrop digitized images, and 2D and 3D high resolution GPR surveys. Study components include geological, geophysical, and reservoir engineering. This dissertation focuses on data integration, geostatistics, and reservoir engineering.

The Wall Creek member of the Frontier Formation is a Cretaceous (Turonian, 93.5 million years before present.) fluvial-deltaic sandstone exposed in central Wyoming. The Wall Creek is an ideal candidate for an integrated study because it is an excellent analog to reservoirs in the Power River Basin and elsewhere; it has been studied at regional and intermediate scales; its outcrops are accessible; and bedding planes are at depths accessible to ground penetrating radar (GPR). Furthermore, the Wall Creek member has significant depositional variability.

In this study, GPR datasets are used to create strata-conforming reservoir simulation
grids. Reservoir simulation models estimate effective properties and predict recovery behavior. A designed set of simulation models identifies and quantifies geologic and geophysical factors that control flow behavior for this reservoir type. Use of experimental design also increases the usefulness of this dataset as an analog for similar reservoir.

1.2 Problem Statement

Tyler (1988) estimated that conventional development of heterolithic fluvial-deltaic reservoirs bypasses 24 to 69 percent of the mobile oil originally present. These reservoirs have been estimated to contain 15 billion barrels of mobile oil that is unrecovered due to reservoir heterogeneity in the USA alone. More sophisticated reservoir models may make development strategies more economical and increase recovery efficiency.

To model subsurface fluid flow and to predict the efficiency of different recovery process, we must understand the geological control of reservoir heterogeneities (Begg and others, 1993; Flint and Bryant, 1993; Rosvoll and others, 1997). Tyler (1988) illustrates the relationship between different scales of heterogeneities with drained and uncontacted reservoir compartments. For example, reservoirs with complex architectures such as fluvial and fluvial-dominated deltaic reservoirs have low to moderate recoveries (10-70 percent) due to poor lateral continuity of facies. In general, a geological model has structural, depositional-stratigraphic, and diagenetic-geochemical components. Many depositional models have been rather qualitative. Recently, geologic models and petrophysical data are commonly combined to create reservoir models. Combining the sedimentary and facies framework with petrophysics models unifies and quantifies geological components and reservoir fluid flow properties.

Some descriptive geological models lack quantitative data required for engineering studies:
Geomodels may be at inappropriate scales for reservoir exploitation, or may lack a critical assessment of the relative importance of various heterogeneous elements on fluid flow. Engineering factors (such as well productivity, well geometry, and injection rate) may not be considered by geoscientists but may be important in determining recovery behavior. Therefore, reservoir modeling teams recognize important sensitivities, uncertainties, constraints, and goals of reservoir performance prediction. – (White, 2001)

Moreover, “Poor understanding of the distribution of internal barriers to effective reservoir drainage may cause a significant part if the hydrocarbon resource to be uncontacted” (Tyler and Finley, 1991) and “Field scale engineering models are often unable to represent the effects of small scale heterogeneity” (Haldorsen and Damsleth, 1993). Finally, these effects are obscured by oversimplified geologic models, inaccurate upscaling, or over-coarse field-scale models.

Improved reservoir models must quantify internal reservoir architecture and accurately predict the spatial variation of permeability in a way that is useful to reservoir engineers. Useful representations include geostatistical models, effective properties or pseudofunctions and recovery efficiency predictions. (White, 2001)

Outcrop data provide a continuous image of facies continuity and architecture at scales shorter than typical well spacing. Unfortunately, there are few true 3D modeling studies. Most studies of facies architecture are based on variably oriented vertical cliff faces (e.g., numerous examples in Miall and Tyler, 1992; White and Barton, 1999; Willis and others, 1999; White and Willis, 2000; Dutton and others, 2000), but even variably oriented outcrop exposures provide only multiple two-dimensional samples and are not truly three-dimensional. Moreover, the effects of reservoir heterogeneity can be different in three dimensions compared to two dimensions, because of the greater opportunity for flow to circumvent flow baffles and greater spatial variability of displacement velocities in 3D compared to 2D. Three-dimensional GPR
data sets provide a unique data source to examine the effects of fine-scale geologic variability in three dimensions.

There are several true 3D geostatistical modeling and flow simulation researches concerning shale or calcite concretions of outcrop (Li and White, 2002 and Novakovic and others, 2002). In the Raptor Ridge outcrop, in which concretions may affect fluid flow; the facies and concretions have different distribution characters from previous studies. Applying geostatistical methods to integrate these factors into our geological model is the principal goal of this research.

Compared with stratigraphic and sedimentary architecture, diagenetic influences on fluid flow are less commonly modeled. Diagenetic overprints (such as calcite concretions and secondary porosity) are controlled by factors including lithology, subsurface temperature and pressure history, pore fluids (hydrocarbon and formation water), and advection rates. Diagenetic overprints may be highly variable and are hard to predict. Precipitation of extensive cement during diagenesis can modify the depositional permeability and affect fluid flow during production. For example, two-dimensional studies of the Frewens and Wall Creek sandstone outcrops indicate that the calcite concretions reduce the upscaled permeability by almost 50 percent and alter the displacement front geometry (Dutton, 1999; White and Willis, 2000; Dutton and others, 2002).

In comparison with shale (Li and White, 2001) or facies (Xu and Journel, 1993; Marathon, 1987), radar responses for concretions are noisy, which introduces uncertainty into concretion characterization. To overcome this difficulty, many different data should be integrated into concretion characterization. Furthermore, the concretion distribution at Raptor Ridge has a vertical trend, which cannot be modeled by geostatistical methods that assume
stationary. Data integration and trend modeling are two motivations for this research.

In summary, three-dimensional studies of delta front sandstones are necessary because of their economic significance as oil and gas reservoir. Their complex internal architecture and property distributions in the three dimensions are not yet well understood. These complexities may affect the recovery behavior of this important reservoir class. Three dimensional reservoir models can improve our ability to estimate the properties and behavior of analogous reservoirs. Concretion characterization in the study area requires a new geostatistical method to impose vertical trends and integrate many types of data.

1.3 Method Preparation

![Flow chart of research approach]

Figure 1-1. Flow chart of research approach

This research focuses on facies modeling including diagenetic overprint—that is, calcite concretion modeling, permeability structure, and quantification of geologic and model effects using designed fluid flow simulation.

The work flow includes data preparation, GPR surfaces and flow grid construction, facies-derived permeability structure construction and designed flow simulation. The geostatistical and GPR modeled concretion distribution will change the permeability structure. The algorithm of permeability upscaling is required for designed simulation.
1.4 Literature Review

This study includes three related parts: 1) imposing local geological character (nonstationarity or trends) in modeling spatially variable properties; 2) integrating different types of data such as geological, geophysical, engineering data; 3) upscaling and building reasonable and accurate flow grids.

1.4.1 Trend Modeling

Trends include locally varying directions of continuity and locally varying means. “Reservoir facies and petrophysical properties may exhibit changing directions of continuity; that is, the principle direction of continuity may depend on location (Deutsch, 2002)”. A locally varying mean implies that the expected value of the attribute being estimated varies spatially. For example, if the hydraulic energy of a sedimentary environment decreases upward, the grain size of sediments fines upward down within a sedimentary succession. The variation of grain size causes is directly related to the permeability trend.

Trends occur at different observation scales and vary by data type (Deustch, 2002). Trend analysis relies on data density: “Most variation appears stochastic in the presence of sparse data; as more data becomes available, the more refined the trend model and less of intrinsic variation is left to geostatistical modeling (Deustch, 2002). Deterministic and stochastic methods have been developed to model trends.

Trend decomposition is the direct removal of a trend using prior knowledge or observation. Li and White (2003) removed the elevation trend from surface data. Then, geostatistical methods such as kriging or simulation are applied to residuals, which are assumed to be stationary. After kriging or simulation, the trends are added to the residuals to get the restored surface. Alternative methods are based on modifications of simple kriging; these
methods include kriging with a prior trend model (KT), ordinary kriging (OK), kriging with external drift (KED) and Bayesian kriging (Journel and Rossi, 1989; Goovaerts, 1997; Deutsch and Journel, 1998). Hudson and Wackernagel (1994) used kriging with an external drift to map January mean temperature for Scotland. This KED model combined data from 145 climate stations with a digital elevation model. The trend kriging model assumes a random function (RF) model in which \( Z(\mathbf{u}) \) is the sum of the trend, \( m(\mathbf{u}) \), plus a residual \( R(\mathbf{u}) \):

\[
Z(\mathbf{u}) = m(\mathbf{u}) + R(\mathbf{u})
\]

The trends \( m(\mathbf{u}) \) can be modeled using polynomial functions of the coordinates \( \mathbf{u} \).

These two methods are simple and straightforward, but have some disadvantages. The direct trend decomposition method arbitrarily divides components of the original data into trends and residuals, which can cause artifacts. A good practice is to check the covariance of trend and residual \( C_{r-m}(0) \). If the covariance is close to 0, it implies that the trend models and residuals are independent. There are few artifacts if the trend and residual are independent (Deutsch, 2002). Alternatively, Li and White (2003) computed the semivariogram of residuals. If the variogram reaches constant variance beyond a certain range, the Gaussian assumption of residuals is satisfied.

Cokriging and cosimulation methods model locally varying mean \( m(\mathbf{u}) \) as the sum of primary, \( Z(\mathbf{u}) \), and secondary data \( Z_2(\mathbf{u}) \). Cokriging and cosimulation of residuals \( R(\mathbf{u}) \) or primary data \( Z(\mathbf{u}) \) are used (Journel and Huijbregts, 1978; Goovaerts, 1994, 1997).

1.4.2 Data Integration

Xu and others (1992) used collocated kriging to integrate seismic data into reservoir modeling. Mosey and others used cosimulation to integrate well data with GPR data. Rahman and others (2004) used cosimulation and pseudocrossvariogram to integrate noncollocated
secondary data to improve estimates of hydraulic conductivity. Compared with cokriging, cosimulation methods better reproduce high and low values and allow stochastic assessment of flow model uncertainty (Isaaks and Srivastava, 1989). However cokriging and cosimulation require crossvariograms, which are difficult to compute and do not necessarily improve prediction (Goovaerts, 1999).

Corregionalized kriging and simulation are not the only ways to integrate data. Other methods include Bayes method, optimization method and nonparametric transformations.

Bayes theorem was developed by an 18th Century English mathematician, logician--Thomas Bayes (1701-1761). He developed the theorem that bears his name in the study of logic and inductive reasoning. Bayes theorem was published posthumously in 1763. Later, in 1774, the theorem was proved independently by Laplace. Bayes method decomposes posterior probability into the likelihood and the prior. It is widely used in petroleum engineering for data integration, decision-making and reservoir modeling.

Goovaerts (1999) integrated spatial coordinates with supervised classification of hyperspectral data. The conditional probability derived from discriminant analysis and the prior probability computed from indicator kriging is combined using Bayes rule to improve the habitat mapping. The Bayesian approach will also be applied in this study. As will be borne out in the development of the method and its application (section 4.1), the advantages of the Bayesian method include simplicity and computational efficiency.

Data can also be integrated using optimization. Kirkpatrick and others (1983) and Deustch and others (1992, 1994) developed simulation annealing (SA) and annealing cosimulation (ACS) for data integration. This approach combines simulated annealing with sequential Gaussian simulation and cosimulation, and provides a tool for integrating data,
including core, well test, permeability-porosity cross plots and spatial variability using objective functions. However, like many optimization algorithms, it may be computationally expensive; moreover, annealing algorithms tend to converge to local extremes.

### 1.4.3 Upscaling

“Upscaling is a technique that transforms a detailed geologic model to a coarse grid flow simulation model so that fluid flow behavior in the two systems is the same” (Li and others, 2001). Upscaling includes two steps: controlling the global geological features (or gridding) and preserving the local geologic details within a coarse gridlock (or averaging). For gridding, methods like cornerpoint grids, unstructured PEBI meshes, control volume finite elements, and curvilinear grid are commonly more efficient and accurate than block-centered grids (Hirasaki and O’Dell, 1970; Leventhal and others, 1985; Peaceman, 1996; King and Mansfield, 1999). Within this research, cornerpoint grids are used to represent the inclined stratal geometries accurately. The other aspect of upscaling is averaging. Finer grids capture more geologic details (heterogeneity). However, fine grids also demand much more computation. For the resources available in this study, the upper limit of grid size is about 1 to 10 million. Upscaling balances the computational costs and prediction accuracy.

Cardwell and Parson (1945) proved that the harmonic and arithmetic average provided the lower and upper bounds, respectively, of upscaled effective permeability. This method is useful and easy to apply, because it uses simple summation formulas. However, the assumption of low spatial correlation of permeability is restrictive.

Pressure solver methods upscale permeability in first solve the fine grid pressure distributions then calculate the averaged effective permeability from the pressure drop and flow rate. Pressure solver averaged results are close to those obtained by history matching (Begg and
A periodic boundary condition and full tensor analysis may generate more accurate predictions (Durlofsky, 1994, Pickup, 1992). Renormalization and power law average methods are also widely used. Renormalization uses multiple-step calculations with an equivalent resistor-network approach (King 1989, Malick 1995). These methods use unrealistic boundary conditions and may be less accurate (Peaceman, 1996). However, Gauiter and Noetingier (1997) stated that periodic boundary conditions may improve its performance. Power law averaging was developed by Journel and others (1986). This method requires empirical determination of the power law average exponent using fine grid simulations. Although power-law averaging is generally faster than pressure solver approaches, the power law method assumes that the same empirical averaging exponent applies to all coarse grids.

Carrera and Neuman (1986) and Sun and others (1998) reported a zonation method, which is the simplest parameterization method and widely used in oil and gas industry and hydrology to characterize reservoir or aquifer heterogeneity. Zonation uses two steps to upscale and handle crossflow. First, before building a model, geologic layers are grouped. Zones are selected to minimize variation within zones and maximize the variation between zones, then layer mean permeability is compared for each well. Zones are correlated or grouped if the difference of layers means is less than or equal to a certain statistics criteria derived from the reservoir permeability variation (Testerman, 1962).

Other upscaling methods such as global upscaling and experimental designed upscaling have been described. Global upscaling uses a special gridding algorithm that can decrease errors. However, this method uses a power law average in each coarse gridblock; the assumption of a uniform exponent between fine and coarse grids is not clearly correct.
The modeling (including trend modeling and data integration) and upscaling methods discussed above provide a foundation for this research. A new algorithm imposes trends and integrates diverse data (Section 4.1). An upscaling algorithm simplifies large geostatistical model and assign permeability to the flow grids (section 6.3).
2.1 Geologic Background and Regional Geological Setting

This study of the Wall Creek Member focuses on a deltaic lobe in the “Raptor Ridge” locality within the Frontier Formation outcrop belt. The outcrop belt of Frontier Formation in central Wyoming provides a valuable larger-scale context (Figure 2-1).

The Frontier Formation (Figure 2-2) is an upper Cretaceous clastic wedge deposited in an eastward migrating foreland basin. This foreland basin, stretching roughly north-south, developed between the Cordilleran volcanic arc in the west and hinterland region (cratonic North America) in the east (Dickinson, 1981; Lawton, 1994). The basin is asymmetric due to thrust loading at the western margin. The Cretaceous Western Interior Seaway occupied this asymmetric foreland basin. The clastic wedge of the Frontier Formation prograded to the southeast into the seaway (Willis and others, 1999; Bhattacharya and Willis, 2001). The coeval proximal deposits were deposited farther west at the present-day boundary between Wyoming and Utah. These deposits comprise a thick succession of nonmarine facies including conglomeratic fluvial deposits cut locally into marine shoreface deposits (Hamlin, 1996). The Frontier formation was sourced from uplifted strata in the west (Wiltschko and Dorr, 1983; Cobban and others, 1994).

The Frontier formation contains at least three unconformity-bounded members. From oldest to youngest, these are the Cenomanian Belle Fourche Member, the Emigrant Gap Member of middle Turonian age, and the late Turonian to early Coniacian Wall Creek Member (Merewether and others, 1979; Merewether, 1980). In addition to the sandstones, the Frontier contains several isochronous bentonite beds that can be correlated regionally.

Recent research indicates that the Wall Creek Member can be explained by a forced
Figure 2-1. Regional paleogeography

(a) Paleogeography of western interior seaway during upper Cretaceous. (b) Study area showing frontier outcrop in central Wyoming. (c) Location of raptor ridge in the frontier outcrop belt (from Bhattacharya and Willis, 2001)
Figure 2-2. Regional stratigraphy

The present research investigates the Wall Creek member on the top of the Frontier formation. This is the type log through the Frontier Formation showing stratigraphy and major bentonites. From (Bhattacharya and Willis, 1999).

<table>
<thead>
<tr>
<th>Facies</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>Marine Mudstone</td>
</tr>
<tr>
<td>F2</td>
<td>Thin bedded siltstone and mudstone</td>
</tr>
<tr>
<td>F3</td>
<td>Decimeter to meter-thick sandstone beds</td>
</tr>
<tr>
<td>F4</td>
<td>Bioturbated siltstone and sandstone</td>
</tr>
<tr>
<td>F5</td>
<td>Intensely bioturbated gray-white sandstone</td>
</tr>
<tr>
<td>F6</td>
<td>Cross-bedded pebbly sandstone</td>
</tr>
<tr>
<td>F7</td>
<td>Amalgamated flat-stratified sandstone</td>
</tr>
<tr>
<td>F8</td>
<td>Unidirectional cross-bedded sandstone</td>
</tr>
<tr>
<td>F9</td>
<td>Tidally-influenced cross-bedded sandstone</td>
</tr>
<tr>
<td>F10</td>
<td>Wavy parallel centimeter-scale bedded sandstone</td>
</tr>
</tbody>
</table>
regression model (Plint and Walker, 1987a, b; Plint and others, 1986; Posamentier and others, 1992; Van Wagoner and others, 1990; Walker and Plint, 1992, Bhattacharya and Willis, 2001; Gani, 2004).

The Raptor Ridge sandstone 6 is the uppermost parasequence of the Wall Creek Member (Figure 2-2). Sandstone isolith maps indicate that it is a shore-parallel elongated delta lobe. River- and tide-dominated facies with a subordinate wave dominated facies are observed. Sand 6 at Raptor Ridge is interpreted as an asymmetric wave influenced delta (Bhattacharya and Giosan, 2003)

The Wall Creek Member is composed of a multiple upward-coarsening sandstone bodies containing or capped by pebble conglomerate beds and separated by bentonitic mudstones. Ten facies have been defined as the building blocks of the Wall Creek Member (Table 2-1). The facies succession of the topmost parasequence grades upward from laminated sandy mud to heterolithic strata, then to flat stratified sandstones cut by channelized sandstone and locally overlain by heterolithic cross-stratified sandstones (HCS), and finally to crossbedded sandstone. It is interpreted as a mixed-influenced, top-truncated delta(Gani, 2004).

Based on the stratigraphic and sedimentary models, local stratigraphic surfaces are interpreted at the Raptor Ridge locality from 3D GPR data. The flow model honors geologic and geophysical data. The following section introduces GPR physics and interpretation, borehole data and interpretation, and photomosaic data and interpretation.

2.2 Data Available

Facies-controlled petrophysical modeling is applied within a stratigraphic and sedimentologic framework. The following sections describe data used in this research.
2.2.1 Sedimentology

Facies were identified for the sedimentary rocks exposed at the Raptor Ridge and surrounding locations. Approximately 20 sedimentologic logs with total gamma ray (measured by handheld gamma ray scintillometer) were measured (Bhattacharya et al., 2003). These profiles are correlated with a regional subsurface database over the entire Power River Basin (Bhattacharya, 2003) and used for facies modeling locally. Vertical logs and a bedding diagram of 250 m long cliff facies were collected (Bhattacharya and others, 2003). Bedding diagrams of the cliff faces (both perpendicular and parallel to depositional strike) extend down to the centimeter scale; these include bedding maps, facies maps, cement/concretion maps, and shale maps.

2.2.2 GPR (Ground Penetrating Radar)

“GPR is a noninvasive geophysical technique that detects electrical discontinuity in a shallow surfaces (<50m)” (Neal, 2004). Similarly to seismic surveying, GPR propagates waves into the earth; for GPR, they are electromagnetic rather than sound waves. These signals reflect and refract in the earth, and the time series of signal returns can be used to characterize geologic units.

The GPR survey is reconciled with outcrop and borehole measurements to correlate depositional surfaces, which are then transformed into flow grids. Furthermore, two GPR attributes, instantaneous frequency (ω) and instantaneous amplitude (A) are correlated with lithology and concretion observations from cores from on-site well bores. Deterministic and statistical relationships are then used to predict the concretion distribution. Proper interpretation requires understanding of GPR geophysics fundamentals and assumptions.

Several one-, two- and three-dimensional GPR surveys have been obtained in the Raptor
Ridge area (Table 2-2; Figure 2-3). The 2D surveys were $100 \times 300$ m and $160 \times 200$ m at a nominal frequency of 50 MHz. The 3D surveys were $30 \times 80$ m, and $12.5 \times 12.5$ m at 50 MHz and 100 MHz, respectively. The 100 MHz survey was collected specifically to image cemented zones at top of outcrop.

### Table 2-2. GPR grid parameters in the study area.

<table>
<thead>
<tr>
<th>Survey</th>
<th>Nom. Freq.</th>
<th>Location of Southwest Corner</th>
<th>Dip Extent</th>
<th>Strike Extent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large 3D</td>
<td>100 MHz</td>
<td>354595.327</td>
<td>4794913.723</td>
<td>80 m</td>
</tr>
<tr>
<td>Small 3D</td>
<td>100 MHz</td>
<td>354523.292</td>
<td>4794998.187</td>
<td>12.5 m</td>
</tr>
<tr>
<td>Rect. 2D(West)</td>
<td>50 MHz</td>
<td>354446.866</td>
<td>4794571.247</td>
<td>160 m</td>
</tr>
<tr>
<td>2D Line (East)</td>
<td>50 MHz</td>
<td>354583.246</td>
<td>4794956.852</td>
<td>100 m</td>
</tr>
<tr>
<td>Number of dip-parallel lines for large 3D</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resolution (m)</td>
<td>Horizontal</td>
<td>0.2</td>
<td>Vertical</td>
<td>0.1</td>
</tr>
</tbody>
</table>

* a Position of GPS using UTM coordinate system
b Large 3D at Raptor Ridge is our focus area

**Figure 2-3. 2D and 3D GPR grid locations from**
(Bhattacharya, 2004).

For sedimentary interpretation, the basic assumption is that “at the resolution of the survey and after appropriate data processing, reflection profiles will contain accurate information regarding the nature of a sediment body’s primary depositional structure” (Neal, 2004). This
requires that the original depositional structures can be restored within the GPR resolution accuracy and processing and interpretation methods remove noise. Knapp (1990) discussed two common definitions of seismic resolution, which can be applied in GPR. The first definition relates to the ability to determine reflector position in space or time. GPR resolution is a function of wavelet sharpness or frequency. As frequency increases, vertical resolution improves in inverse proportion (i.e., doubling frequency makes it possible to resolve features half as large).

The second definition is “the ability to resolve two closely spaced features”. This is controlled by wavelength (Knapp, 1990). Wavelength ($\lambda$) is defined as ratio of velocity ($v$) to frequency ($f$):

$$\lambda = \frac{v}{f} \quad \text{(2-1)}$$

Eqn. 2-1 indicates that, as the frequency increases, the wavelength decreases. Thus GPR resolution increases with frequency, although the transmitted radar energy (and depth of penetration) decreases with increasing frequency. In practice, the balance of resolution and depth of penetration is determined by the objective of research. Sedimentary surface correlation needs relatively low resolution but moderate depth of penetration, justifying low frequency. Identification of small-scale geobodies like concretions or subsurface pipelines requires higher resolution, and thus high frequencies and an implied lower depth of penetration. In the study area, a 50 MHz 2D radar grid is used to build local sedimentary surfaces. A 100 MHz 3D radar grid helps to identify the effects of calcite concretions on GPR.

The theoretical vertical resolution is determined by wavelength. “Wave theory indicates that best vertical resolution can be achieved in one-quarter of dominant wavelength. Within that vertical distance any reflection will interfere in a constructive manner and result in a single, observed reflection” (Sheriff, 1977; Neal 2004). Neal (2003) recorded the best GPR vertical
resolution, between 0.02 to 0.08 m, within low-loss materials like sand and gravels with high frequency antennae (900 MHz).

In the study area, if the concretion dimension is larger than the radar resolution, then concretions will be detected by radar (Table 2-2). If concretions can be resolved, a direct or deterministic interpretation of concretion occurrence is possible. Concretions below radar resolution must be predicted stochastically (chapter 3).

Similarly, wave theory indicates that the horizontal resolution is determined by Fresnel zone width (Figure 2-4). The Fresnel zone width is a function of wavelength and depth to a particular reflector. With the increase of depth, more energy is expended laterally; thus the horizontal resolution is poorer. Other factors like the shape of radar front and horizontal spacing between traces on the radar reflection profile complicate radar horizontal resolution (Neal 2004).

In summary, the horizontal and vertical resolution improves with the increase of radar frequency and decreases depth; it is complicated by the geometry and electromagnetic properties of media radar propagating through.

GPR uses electromagnetic energy to detect underground objects. The behavior of subsurface energy propagation is controlled by three material properties: dielectric permeability ($\varepsilon$), electrical conductivity ($\sigma$) and magnetic permeability ($\mu$) (Olhoeft, 1998). An alternating electric field is applied to the material to be characterized by the radar antenna. Within the material (here, sediments), the electrical charges are bonded and unable to move freely; they respond to the applied field by undergoing a small displacement. When the resulting internal electric field balances the external electric field, the charges stop moving. This charge separation is called polarization. Polarization stores electric field energy in the medium being investigated.
Figure 2-4. Horizontal resolution defined by Fresnel zone width.  

a) Electromagnetic waves propagate through ground with a cone shape front; all reflection within ¼ of dominant wavelength will interfere constructively to form a single reflection. b) Waves closest to the Fermat path contribute most to reflection amplitude c) Fresnel zone width is a function of reflectors’ depth and frequency dependent wavelength. The higher frequency (or the shorter the wavelength), the higher the horizontal resolution will be (Neal, 2004).

by radar. The amount of energy stored is determined by the real dielectric permittivity ($\varepsilon$) at the frequency of measurement (Powers, 1997). Dielectric permeability ($\mu$) measures capacity to transport charge in a static electric field. “The most important conduction based energy losses occur due to ionic charge transport in water and electrochemical process associated with cation exchange on clay minerals” (Olhoeft, 1998). Magnetic permeability ($\mu$) describes the magnetic field energy stored and lost through induced magnetism.

The amplitude ($A$) declines exponentially from its initial value ($A_0$) as it travels a distance $z$,
\[ A = A_0 e^{-\alpha z} \]  \hspace{1cm} (2-2)

where \( \alpha \) is the attenuation constant (Theimer and others, 1994; Neal, 2004). For low-loss material such as sand and gravel, this constant is frequency independent.

\[ \alpha = \frac{\sigma}{2} \sqrt{\frac{\mu}{\varepsilon}} \]  \hspace{1cm} (2-3)

Eqn. 2-3 indicates that the conductivity (\( \sigma \)) exerts the greatest influence over attenuation constant. By assuming the magnetic responses at radar frequencies are negligible, the velocity of an electromagnetic wave is a function of the host medium’s relative dielectric permeability (\( \varepsilon_r \))

\[ v = \frac{c_0}{\sqrt{\varepsilon_r}} \]  \hspace{1cm} (2-4)

where \( c_0 \) is the electromagnetic wave velocity in a vacuum (\( 3 \times 10^8 \text{ ms}^{-1} \)).

From Eqn. 2-3, 2-4, materials with low conductivity have small attenuation constants. Gas and unsaturated sediments have low attenuation constants, whereas seawater and saturated sediments have high attenuation constants. Fresh water has a high \( \varepsilon \) compared to air and typical rock-forming minerals; fresh water thus controls dielectric properties of common geologic materials. Generally, materials with lower electromagnetic permeability such as unsaturated sediments or gas have low radar velocity, whereas saturated sediments and water have high velocity. Possible exceptions include high conductivity materials like seawater or certain types of clays, or magnetic materials like magnetite.

In the study area, the radar amplitude attenuates quickly with increase of depth and is weakly correlated with concretion occurrence. Instantaneous frequency (\( \omega \)) is used for sedimentary boundary correlation.
2.2.3 Borehole Data

Core plugs and gamma ray logs (or gamma ray spectrum) are used to calibrate the radar surfaces interpretation, and to build radar-log relationship for facies and concretion prediction (Figure 2-5). Core plugs (about 10 cm long) were taken at 20 cm intervals from each vertical measured section for permeability measurement. Plug permeability was measured using a computerized profile permeameter.

Figure 2-5. Geological facies and concretion section in well 8
Gamma ray (spectrum) logs show an increasing radioactivity (or increasing clay volume) downward. The GPR amplitude has higher values in the top 4 m, and attenuates quickly downward. The radar attributes are weakly correlated concretion occurrence. (The red rectangles in the concretion column represent concretions; rectangles with different color mean different facies observed in well 8)
Ten wells were drilled with an average depth of 10 m within the focus area; two wells are within the 3D GPR survey area. Full core was taken from each well for sedimentologic and petrophysical analysis. The data are used to correlate the outcrop mapping with the subsurface GPR data.
The gamma log measures the intensity of gamma ray, and the gamma spectrum tool records the intensity of several radioactive elements (uranium, thorium and potassium). The gamma and gamma spectrum logs can be calibrated to classify lithofacies. Shale or clay is assumed to have higher radioactivity than clean sand. In Raptor Ridge, the shale content increases downward. Consequently, the GR increases downward. Different gamma spectrum reflects different mineralogical and sedimentary information. Uranium (mainly $^{238}\text{U}$) in clastic rock comes from depositional minerals, organic material absorption, and uranium dissolution in rock matrix (Tan and others, 1998). Compared with the uranium, radioactive thorium (mainly $^{208}\text{Th}$) is more stable. Absorption on clay minerals is the main factor that controls the thorium distribution. Based on measurements of radioactivity in various types of rocks on earth, the ratio of Th to U is used as a good indicator of rock type. Potassium (mainly $^{40}\text{K}$) is abundant in the shale or clay, which is another good indicator of lithology. This study uses gamma spectrum for facies classification, which is then cross validated with geologic facies observations and predictions from other statistical methods (chapter 3).

2.2.4 Photomosaic Data

Photomosaics show the inclined beds dipping southeast (at about 4 degrees) in the same direction as paleocurrents, supporting the interpretation that they are delta front clinoforms, after removing structural dip. The photomosaic along depositional strike shows bidirectional offlapping bedforms. Both of these bedding packages are well imaged in GPR data just behind the cliffs. The bedding diagram (Figure 2-6) shows the interpretation of a terminal distributary channel (TC) intimately associated with seaward dipping bar growth (BG) elements, front splay (FS) elements and tidally reworked elements. These bedding maps, the concretion zone interpreted from GPR data, the borehole petrophysics data, and the cliff-measured permeability
are integrated into the flow model to determine the flow behavior of mix-influenced, top truncated delta fronts with complex facies structure.

Figure 2-6. Dip direction photomosaic of Raptor Ridge outcrop, Frontier Formation. Outcrop photomosaic illustrates that most concretions occur in facies 4a and 5b. Orientations of concretion align with bedding surfaces, with 4 degree basinward structural dip (Modified from Bhattacharya, 2003).

2.3 Concretion Occurrence and Petrophysical Character

2.3.1 Shape and Dimension Estimation

The concretions have various sizes and shapes. Based on measurements of dip directional cliff face, the major concretions range from 0.7 m to 5.5 m in length, and from 0.2 m to 0.6 m in height. Two concretions are greater than 10 m long. The concretions have various shapes ranging from “almond shape (nearly spherical but with flattened edges), to long, thin ellipsoids, to short, thick ellipsoids, to coalesce” (Nyman, 2004).

Accurate estimation of concretion dimensions must consider measurement biases. Inference of concretion dimensions from finite-extent samples can be approached using geometric probability (White and others, 2004). Because the concretion observations are lower-
dimensional than the population (e.g. thin sections, outcrops), objects with large extents normal to the exposure tend to be over-represented (normal count bias). Lateral truncation causes observed object length to be less than true length (lateral length bias). The normal biases are removed using the Abel integral equation (Wicksell 1925) and lateral length bias can be removed using an Erlang model (White and Willis, 2000).

Wicksell and Krumbein methods are used to debias the concretion dimensions observed from outcrop and photomosaic (Figure 2-7).

![Figure 2-7. Cumulative probability for concretion dimension](image)

(a) Length distribution and (b) thickness distribution. Based on Wicksell (the preferred method), eighty percent of concretions are between 0.4 and 2.7 m in length and 0.08 and 0.60 m in thickness.

White and others (2004) claimed the Wicksell method is more realistic for bodies like concretions (no compensation in packing, as required by the Krumbein model). The debiased cumulative probability distribution has 80 percent of concretions between 0.4 and 2.7 m in length and between 0.08 and 0.6 m in thickness. If one considers the fraction of cement occurring in concretions of various sizes (rather than the frequency of occurrence of a certain size of concretion, then the curves appears much different. By assuming a linear correlation among length, width and thickness, 80 percent of cement is in concretions between approximately between 2.5 and 15.5 m long (Figure 2-8). Concretions with these dimensions may influence the
flow behavior, which is proved by flow simulation discussed in chapter 4. To capture flow responses of the concretions, the geostatistical modeling grid dimension should be smaller than the minimum significant (in terms of flow) concretion sizes.

![3D Length Distribution](image)

**Figure 2-8. Cumulative probability of concretions length.**
The volume based method indicates that 80 percent concretion range between 2.5 and 15.5 m in length.

Because of computational cost, it is important to balance flow grid size and prediction accuracy. A process of gradually upscaling geostatistical grids and examining the flow response helps determine the appropriate flow grid for 3D flow simulations.

### 2.3.2 Concretion Distribution

Outcrop profiles in this study are from southwest facing (approximately dip aligned) ([Figure 2-6](image)) and southeast facing (approximately strike) cliffs. Gani and others (2003) presented interpreted concretion, facies, and beddings photomosaic profile of the dip directional cliff. The concretions are outlined by polygons. The interpreted drawing is then transformed to a dense pixel grid, where cement occurrence is assigned 1 and 0 if no cement occurs. Similarly, different facies polygons are converted into a sequence of grids. The grid size is 1000 horizontally by 800 vertically, and individual pixels are approximately 8 cm long by 2.5 cm thick. The concretion proportion is almost stationary horizontally in contrast with vertical trend ([Figure 2-9](image)). The cement fraction increases vertically from 0-3 percent in the bottom 5 meters to 18-20 percent in
the middle 4 meters; and no concretion occurs in the top 2 meters. The trend of cement abundance is obvious from photomosaic.

2.3.3 Petrophysical Character of the Concretions

Routine plug permeability, pulse decay permeability, and mini-permeameter profiles from well cores have been measured. Permeability values in most concretions are reported as 0~0.2 md, which is considered “tight” cement. Cement zones with comparatively higher permeability are probably partially cemented and called “light” cement (Bhattacharya, 2004). The average cement permeability values are about 1-2 md for facies 5 and facies 4.

![Proportion of cement in both horizontal and vertical profiles](image)

**Figure 2-9. Proportion of cement in both horizontal and vertical profiles**
(a) The vertical cement proportion shows an obvious high peak between 7.2 and 10.5 m (b) There is no obvious trend (stationary) in horizontal direction.

Depositional and facies environments suggest that sandstone 4b should provide the best reservoir quality (Nyman, 2004). However, concretions in facies 4b decrease reservoir quality. As a result, the best reservoir quality is in facies 4a. The regional average sandstone permeability is 132 md. Reservoir quality in sandstone 6 is fair; its average permeability is 59.5 md.

Nyman (2004) proposed that calcite concretion precipitation is inhibited in by detrital muds facies 4b, and thereby a higher porosity and permeability are preserved. Low detrital mud content in facies 4a led to a greater reduction in reservoir quality. This differs from the nearby Murphy Reservoir locality: there, in sandstone 6, reservoir quality is reduced by detrital muds.
“Reservoir quality between the sandstones reflects differences in depositional environments and resulting diagenetic histories” (Nyman, 2004).

**Table 2-3 Permeability statistics for all facies and cement observed at outcrop faces**

<table>
<thead>
<tr>
<th>Number</th>
<th>Facies Description</th>
<th>Permeability statistics (md)</th>
<th>Number of samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sandy mudstone with paper thin laminations</td>
<td>1.1  1.2  110</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Interbedded mudstones and rippled sandstones</td>
<td>74.6 89.6 117</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Massive to parallel laminated sandstone</td>
<td>9.2 19.0 48</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Channelized sandstone with numerous mud chips</td>
<td>132.1 110.2 273</td>
<td></td>
</tr>
<tr>
<td>5b</td>
<td>Dune scale to bar scale cross bedded sandstones</td>
<td>114.2 127. 377</td>
<td></td>
</tr>
<tr>
<td>5a</td>
<td>5a ebb dominated; 5b flood dominated</td>
<td>142.5 126.5 647</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Hummocky sandstones (fine to medium)</td>
<td>59.5 81.0 12</td>
<td></td>
</tr>
<tr>
<td>Cement</td>
<td>Cement in facies 5</td>
<td>1.4  2.6  195</td>
<td></td>
</tr>
<tr>
<td>Cement</td>
<td>Cement in facies 4</td>
<td>1.9  3.7  42</td>
<td></td>
</tr>
</tbody>
</table>

(Bhattacharya, 2004)
CHAPTER 3 WELL BORE FACIES CLASSIFICATION

Sedimentary facies classifications can support reservoir characterization because flow properties are commonly assigned using facies-specific correlations. "A facies is a body of rock with specified characteristics... A facies should ideally be a distinctive rock that forms under certain conditions of sedimentation, reflecting a particular process or environment.” (Reading, 1996) In uncored wells, sedimentary facies cannot be observed directly, and facies are inferred from wireline log data. Some well log data, which are sensitive to lithology such as gamma spectrum, spontaneous potential, density and neutron logs, are good indicators of facies. Well log assisted facies classifications are extensively reported (Baldwin and others, 1990, Bhatt and Helle, 2002; Saggaf and Nebrija, 2000; Tang, 1998, Tang and Yi, 2002).

3.1 Multivariate Facies Classification

Three multivariate statistical methods (beta-Bayesian, multinomial logistic regression, and discriminant analysis) are examined in this study. The techniques are illustrated using log and facies data from a western African sandstone reservoir. Selected methods are applied at the study area.

3.1.1 Beta-Bayesian Method (BBM) Introduction

Multivariate facies classification requires data to train the model. Typical samples from reservoirs pose several challenges: the cored interval selection is commonly biased, reservoir facies usually have more samples than the non-reservoir facies, and the core sample size is limited because coring is expensive and technically difficult. One objective of this study is to estimate the necessary sample size.

Bin-dependent classification methods may introduce errors in selection of bin size. “If too few bins are selected, the FOP (Facies Occurrence Probability, here referred to as the
conditional probability) lacks the ability to discriminate between the adjacent log readings; if there are too many bins, the FOP will not be estimated precisely” (Kapur and Lake, 2000).

Several methods can minimize those errors. Sampling bias can be decreased by adjusting facies prior $P(F_i)$, where $P$ is the probability of facies $F_i$. The correct estimation of prior probability can be obtained by observations or geologic experience. Direct or indirect relationships between facies and log data, which usually are densely sampled (order 10 cm), helps establish the conditional probability model. Finally, to minimize the error of bin selection, the beta-Bayesian method uses beta distributions (Weisstein, 1999) to model the conditional probabilities $P(x | F_i)$; $x$ is the vector of logs such as gamma ray, density and neutron. A nonlinear regression method (Person, 1997) adjusts the beta distribution parameters to maximize prediction accuracy.

A Bayesian method combines prior information on fraction of facies with the wireline log data. In our case, Bayes formula is:

$$P(F_i | x) = \frac{P(x | F_i)P(F_i)}{P(x)}$$

Eqn. 3-1 is used to classify facies $F_i$ from given log data $x$. $x$ is a vector of any log data such as gamma ray, neutron, or density. $P(F_i)$ is the proportion (or probability) of facies $i$ in training set, which is also called the prior or from analog data. $\frac{P(x | F_i)}{P(x)}$ is the ratio of probability a log reading for specific facies to the probability of log reading for all facies, which is called likelihood. The left side of this Eqn.3-1 $P(F_i | x)$ is the posterior, or probability of a facies for given log data. The posterior probability and likelihood guide facies classification.

If the log reading is independent from facies, or uninformative for facies prediction, then $P(x | F_i) = P(x)$. Eqn.3-1 changes to $P(F_i | x) = P(F_i)$, which means posterior equals to prior and is
not informative. Otherwise, if $P(x|F_i)$ is larger (or smaller) than $P(x)$, then $P(F_i|x)$ should be larger (or smaller) than the $P(F_i)$.

Assuming multiple logs’ probability distribution are independent, Eqn. 3-1 changes to following equation:

$$P(F_i|x_1,x_2,x_3) = \frac{P(x_1,x_2,x_3 | F_i)}{P(x_1,x_2,x_3)} P(F_i) \propto P(x_1|F_i)P(x_2|F_i)P(x_3|F_i)P(F_i) \text{ ........... (3-2)}$$

in which $P(F_i|x_1,x_2,x_3)$ is the probability of facies $F_i$, given readings of logs $x_1,x_2$ and $x_3$.

$P(x_1,x_2,x_3 | F_i)$ is the conditional probability of log readings range with in $x_1,x_2$ and $x_3$, if in facies $F$.

From Eqn.3-2, including new logs may or may not affect the posterior $P(F_i|x_1,x_2,x_3)$. The posterior will differ from the prior only if the conditional probabilities of log responses given facies differ from the unconditional distributions.

### 3.1.2 Multinomial Logistic Regression

Multinomial logistic regression (MLR) assumes all samples are from one of $n$ populations (in our study, $n$ facies). MLR uses the logit transform of category probability as response to regress on; the logit is the log of probability ratios (Eqn.3-3). This method then estimates odds ratio of one outcome to a reference outcome (here, facies 8). MLR estimates $n-1$ probabilities; the $n^{th}$ probability is determined from $\sum_{i=1}^{n} P_i = 1$. Using a linear model,

$$\text{Logit} \left( \frac{P_i}{P_n} \right) = \ln \left( \frac{P_i}{P_n} \right) = \beta_0 + \sum_{i=1}^{n-1} \beta_i x_i + \varepsilon \text{ ......................... (3-3)}$$

in which $P_i$ is the probability of occurrence of facies $i$, given log data $x_1,\ldots,x_i$. $\beta_0,\ldots,\beta_i$ are the coefficients of regression and $\varepsilon$ is the error between model and prediction (SAS Institute, 1989). A maximum likelihood method computes the regression coefficients, $\mathbf{b}$, (the estimation of $\boldsymbol{\beta}$).
The regression coefficients $\beta$ quantify the effects of unit changes in independent variables (i.e., the log data) on probability of each facies. These effects on category probability are, by the nature of logistic function, nonlinear. The estimated category probabilities can be computed from a matrix equation:

$$\begin{bmatrix}
1 & -C_1 & \cdots & -C_{n-1} & P_1 \\
1 & -C_2 & \cdots & -C_{n-1} & P_2 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
1 & 1 & \cdots & 1 & 1
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2 \\
\vdots \\
P_{n-1} \\
P_n
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
1
\end{bmatrix}

\hspace{1cm}\text{(3-4)}$$

in which $C_i = e^{\sum_{j=1}^{i} \beta_j x_j}$. Probability for each facies is then

$$P_i = \frac{C_i}{1 + \sum_{k=1}^{n} C_k} \quad (i = 1, \ldots, n-1)$$

$$P_n = \frac{1}{1 + \sum_{k=1}^{n} C_k}$$

\hspace{1cm}\text{(3-5)}$$

The summary of probability at each depth equals to 1. At each depth, the rock is assigned to the facies with the maximum probability. A confidence measurement can evaluate the selection (section 3.2.3).

**3.1.3 Discriminant Analysis**

Discriminant Analysis (DA) is a multivariate method to group samples into categories. Like MLR, DA assumes that all samples are from one of $n$ populations ($F=(F_1, \ldots, F_n)$).

Probability density functions (pdf’s) for groups are expressed $f_1(x), \ldots, f_n(x)$. The groups are assumed to be normally distributed; however, DA is robust with respect to the normality assumption. For linear discriminant functions, a common variance is used for all facies. Fortunately, DA is also robust to this assumption (so long as variances and covariances of all
variables are within a factor of four of one another). Normalized data (range from 0-1) satisfy this requirement. The DA classification boundaries between groups are linear in form. According to Bayes theorem and maximum likelihood theory (Hand, D.J. 1981) observations should be assigned to the facies with maximum likelihood.

\[ F_i^* = \{ x : P_j f_j(x) = \max(P_j f_j(x)) \} \quad j = 1, \ldots, n \quad \text{.....................} \quad (3-6) \]

The maximum likelihood method (Eqn.3-6) is same as minimizing the pairwise generalized squared distances within groups.

3.2 Results and Analysis

3.2.1 Model Comparison Using West African Data

To compare the effects of the three types of classification, these methods are applied to Karpur and Lake’s (2000) published data from a West Africa sandstone reservoir. Wireline log data, core petrophysics, and core-based facies classifications are available. There are 8 sedimentary facies: turbidite, debris, lagoon/marsh, abandoned channel, sand flat, shallow marine, shoreface, and lower shoreface (Table 3-1).

3.2.2 Classification Procedure

Prior distributions give the expected fractions facies in the section to be analyzed. Prior distributions are computed from the fraction of the subsampled training sets in this research. Alternatively, some other empirical facies proportion can approximate the prior distribution to correct for bias in a cored interval.

The second step in the beta-Bayesian classification is to fit conditional probability models for wireline responses given facies to beta distributions, for each lithology and log. The conditional probability distributions \( P(x|F_i) \) may have different shapes, skewness, mean and variance which are not easily fit by normal or lognormal distributions. The beta distribution
(Figure 3-1) can fit these conditional distributions.

The conditional cumulative distribution functions (ccdf) are computed using a randomly selected subsample. Nonlinear regression is used to fit ccdf’s for each facies using the least squares criterion. The conditional probabilities are used with a simplified form of Bayes rule that assumes that the petrophysical property distributions are conditionally independent. This conditional independence can be expressed mathematically by stipulating that $P(x_i \mid F_j)$ does not depend on $x_k, k \neq i$.

Finally, the priors, conditional probabilities, and log measurements are combined using Eqn. 3-2. The measurement point is classified as belonging to the facies with the highest posterior probability.

The beta-Bayesian method is checked by predicting data not in the training data set. Classification errors can then be tabulated. Other prediction methods (MLR and DA) are checked in the same way, using a subsample for model computations and the full sample for checking.

Multinomial logistic regression method and discriminant analysis use the CATMOD and DISCRIM procedures of SAS software (SAS Institute Inc., 1989). Using the same training set, MLR uses maximum likelihood method to predict the regression coefficient. A discriminant function was developed from the training set and used to classify all 900 observations.

| Table 3-1. Facies classification and petrophysical properties |
|---|---|---|---|
| Facies Number | Facies Name | $P(F)$ | Average Porosity(%) | Average Permeability(md) |
| 1 | Turbidite | 0.14 | 31.6 | 2334.7 |
| 2 | Debris | 0.01 | 17.9 | 338.5 |
| 3 | Lagoon/Marsh | 0.07 | 23.5 | 1098.5 |
| 5 | Abandoned Channel | 0.13 | 27.5 | 5381.4 |
| 7 | Sand Flat | 0.01 | 22.8 | 1796.4 |
| 8 | Shallow Marine | 0.22 | 30.9 | 3419.1 |
| 9 | Shoreface | 0.21 | 26.8 | 908.9 |
| 10 | Lower Shoreface | 0.21 | 21.1 | 782.8 |

(Karpur and Lake 2000)
3.2.3 Model Comparison and Confidence Measurements

Statistical classification methods can quantify the prediction uncertainty (or confidence measurement); these methods estimate probability logs for all facies. The uncertainty in these predictions can be compared quantitatively using two derived logs to evaluate prediction uncertainty. The probability of the most probable facies is called overall confidence. Higher overall probability indicates higher confidence of such classification, and vice versa. The distinguishing ability is defined as the ability to differentiate one facies from other facies, which is estimated by the highest probability minus the next highest probability.

Confidence measurements compare the prediction uncertainty of one method. The effects of log combinations on prediction ability are examined by the confidence measurements and overall prediction accuracy. Different logs combinations may also affect facies classification ability and prediction accuracy differently. This study compared different combination of wireline logs using randomly selected subsamples (20 percent of total sample) to construct conditional probability distribution function (cpdf). The prediction accuracy increases with more logs included (Figure 3-2). The enclosed area (surrounded by overall confidence and
distinguishing ability curves) increases as uncertainty increases. Including more logs that are sensitive to lithology into the training set increases prediction accuracy for all models.

Seven combinations of 3 logs are tested in the beta-Bayesian method, MLR and DA and 350 randomly selected points are used to establish models (sample size will be discussed later). Many random samples were drawn to construct estimation errors for each method (Table 3-2). The models have different prediction abilities; adding more logs into the model improves prediction accuracy. MLR with 3 logs has the best prediction accuracy (83%) for this test case.

With the beta-Bayesian method, overall confidence and distinguishing ability are quite high, near 100 percent at cored sections. For intervals with less core information, our overall confidence and interval confidence can be either small or high, which depends on the different classifying model and log response. Otherwise, for those intervals with similar log responses, distinguishing ability can fall to zero – the most likely and best alternative classification is nearly equiprobable.

Training set size and estimation method both affect the prediction performance. A reasonable way to compare the influences of the sample size and method is to examine prediction accuracy using two-way analysis of variance (ANOVA). For each model and sample size, 6 training sets were randomly selected (Figure 3-2, Tables 3-3, 3-4).

Table 3-2. Accuracy comparison of all methods with different log combinations.

<table>
<thead>
<tr>
<th>Log Combination</th>
<th>Beta-bayesian method</th>
<th>Multinomial Logistic Reg</th>
<th>Discriminant Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamma Ray</td>
<td>56.50</td>
<td>52.63</td>
<td>55.25</td>
</tr>
<tr>
<td>Neutron</td>
<td>44.87</td>
<td>39.32</td>
<td>24.63</td>
</tr>
<tr>
<td>Density</td>
<td>72.10</td>
<td>73.02</td>
<td>73.13</td>
</tr>
<tr>
<td>Gamma Ray/Neutron</td>
<td>71.00</td>
<td>77.41</td>
<td>62.88</td>
</tr>
<tr>
<td>Gamma ray/Density</td>
<td>78.50</td>
<td>78.88</td>
<td>73.38</td>
</tr>
<tr>
<td>Neutron/Density</td>
<td>79.50</td>
<td>78.14</td>
<td>71.50</td>
</tr>
<tr>
<td>All three logs</td>
<td>80.63</td>
<td>83.03</td>
<td>77.50</td>
</tr>
</tbody>
</table>

For all methods, additional logs increase prediction accuracy.
Figure 3-2. The confidence measurements of beta-Bayesian Method
(a-c) illustrate the confidence measurements of BBM within 5850-5950(m) for 3 log combinations between Gamma, Neutron and Density. (a) Comparison of facies prediction using GR only with observed facies classification; (b) Comparison of facies prediction using gamma and neutron logs; (c) Comparison of facies prediction using all three logs. Decrease of the enclosed area indicates that prediction accuracy increases from left to right.

Increased sample size increases the prediction accuracy. Prediction accuracy increases rapidly with sample size and is significantly higher when sample size increases to 200 (about 25% of total dataset; Figure 3-3). Moreover, a Tukey pairwise comparison (Figure 3-4, Table 3-3) shows there are no significant differences among these methods.

3.2.4 Application in Raptor Ridge Outcrop

Because of the limited training set, conditional probability computed from beta-Bayesian method is less representative. For the same reason, the training model for logistic regression does not converge. Discriminant analysis is used to classify the outcrop facies at Raptor Ridge.
Figure 3-3. Influence of sample size and models on prediction accuracy

A random subsample of 25% of the total sample data is used as training set for all methods. Overall confidence and distinguishing ability curves are used to evaluate the prediction accuracy. The enclosed area (surrounded by overall confidence and distinguishing ability curves) indicates the uncertainty of prediction.

Table 3-3. Tukey pair wise comparison of model influence on prediction accuracy

<table>
<thead>
<tr>
<th>Tukey</th>
<th>Grouping</th>
<th>Mean</th>
<th>N</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>.824</td>
<td>48</td>
<td>MLR</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>.815</td>
<td>48</td>
<td>DA</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>.81</td>
<td>48</td>
<td>BBM</td>
<td></td>
</tr>
</tbody>
</table>

(Same as Table 3-3)
About fifty facies samples are used to build training set. Thirty-one facies classification and gamma ray log from well bore are used to validate the facies. Prediction accuracy, geological accuracy, and overall accuracy are calculated with the following equation,

\[
A_p = \frac{n_{ii}}{n_{i.}} \quad \text{(3-7)}
\]

\[
A_G = \frac{n_{ii}}{n_{.j}} \quad \text{(3-8)}
\]

\[
A_T = \frac{n_{ii}}{n_{..}} \quad \text{(3-9)}
\]

where \( n_{ii} \) is the classification count in \( i \) row and \( j \) column of Table 3-4. 

\( A_p \) is the fraction of core facies that agree with wireline facies, and \( A_G \) the fraction of wireline facies that really are that core facies.

The prediction accuracy is high (87.1 percent) and the main reservoir component, facies 5, has 100 percent prediction accuracy and geological accuracy (Table 3-4). Facies profiles for the wells drilled on Raptor Ridge outcrop were predicted using the discriminant function (Figure 3-5). A similar statistical method is also used for GPR concretion classification (section 4.3).

Table 3-4. Error matrix for discriminant analysis using 31 samples

<table>
<thead>
<tr>
<th>Facies from logs</th>
<th>Facies from core</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>6</td>
</tr>
</tbody>
</table>

* Accuracy 100.0% 62.5% 83.3% 100.0% 100.0% 87.1%

*: Facies 5, the main reservoir component, has 100 percent geological and prediction accuracy. Overall accuracy is 87.1 percent.

3.3 Summary

(1) The beta-Bayesian method uses empirical beta distributions to model the distribution of petrophysical properties conditional to facies, eliminating difficulties in bin selection.
Petrophysical property distributions are assumed conditionally independent to simplify the use of Bayes rule.

Enclosed area is an indicator of uncertainty; refer to Figure 3-2.

Figure 3-5. Facies prediction profile of raptor ridge data

(2) Confidence, discrimination ability, and probability logs compare the prediction performance of the statistical methods as well as illustrating influences of log combinations and sample size. Two-way analysis of variance compares prediction accuracy of the models.

(3) For a given dataset, there are no significant differences (with 90 percent confidence) in predictions by the three methods. Additional logs improve prediction accuracy from 30 to above 80 percent. Final prediction accuracy is 82 to 90 percent for these three algorithms.

(4) Including 10 to 20 percent of the complete core and facies data in model construction provides accurate predictions; models were validated against the data not used in model construction.

(5) The classification models can generate three-dimensional log-derived facies distributions for geologic modeling and reservoir simulation.
CHAPTER 4 GEOSTATISTICAL MODELING

4.1 Introduction of SGBSIM and Background

The Sequential Gaussian Bayesian SIMulation (SGBSIM) algorithm integrates (1) borehole and outcrop observations, (2) spatial statistics from outcrops, including variograms and trends, and (3) dense spatial data from a 3D ground-penetrating radar survey. Two challenges must be overcome to accomplish this integration. First, imposition of trends requires modification of stationary geostatistical methods, which assume that the expected value and variance of estimated attributes do not vary spatially. Second, the radar data (amplitudes and frequency) must be related to an attribute of interest, concretion occurrence. As will be shown later, radar and petrologic attributes are correlated only weakly; combined with noise inherent in the radar survey, this complicates data integration.

SGBSIM uses a truncated Gaussian to impose trends and the observed spatial correlation, and a Bayes method for geophysical-petrophysical integration via conditional probabilities estimated from a clustering algorithm.

4.1.1 Bayes Rule for Radar Integration

Bayes rule and conditional probabilities are used to compute probabilities of category occurrence (chapter 3; Zhu and Journel, 1992; Goovaerts, 2002)

\[
P(C | x) = \frac{p(x | C)P(C)}{p(x | C)P(C) + p(x | \overline{C})P(\overline{C})} \quad \text{.................................. (4-1)}
\]

Here, \( x \) is the collocated geophysical data, and \( c \) is the classification of current point. \( C \) and \( \overline{C} \) indicate concretion and no concretion, respectively.

The conditional probability of particular attributes values given facies classification \( p(x | C) \) can be computed from statistical methods such as clustering analysis, logistic regression and discriminant analysis (Goovaerts, 2002; Tang and others, 2004).
Observed trends from outcrop characterization can be directly input as the prior probability; in a subsurface context, well data can be used for vertical trends and possibly for lateral trends. Many wireline logs can detect concretions (Dutton and others, 2002).

4.1.2 Truncated Gaussian Simulation to Impose Trends

Truncated Gaussian simulation can impose observed concretion trends. The indicator semivariogram of concretion occurrence is computed from a digitized outcrop interpretation. The indicator semivariogram is transformed to the equivalent Gaussian variogram to enforce the trend. Gaussian variograms are used within a sequential Gaussian simulation algorithm (Deutsch and Journel, 1998). The trend model truncates simulated Gaussian variables into indicators:

\[
i \left( u^i \right) = \begin{cases} 
1, & y \left( u^i \right) \leq t \left( u^i \right) \\
0, & y \left( u^i \right) > t \left( u^i \right)
\end{cases}
\]  

(4-2)

where \( t \left( u^i \right) \), \( i = 1 \ldots n_{xyz} \) is the vertical Gaussian transform of the concretion proportion trend and \( y \left( u^i \right) \) is the simulated standard Gaussian variable. In SGBSIM, this truncation approach is modified to yield a probability (i.e., an indicator in the range 0 to 1) rather than a binary indicator (a value of either 0 or 1).

4.1.3 Sequential Gaussian Bayes Simulation (SGBSIM)

To integrate different data types, the truncated Gaussian simulation procedure is modified to incorporate geophysical data. SGBSIM workflow includes: determining the equivalent Gaussian semivariogram, computing the concretion trend curve, computing conditional probabilities relating radar observations to concretion occurrence, and modified truncated Gaussian sequential simulation to predict the concretion occurrence.

1. **Determine equivalent Gaussian semivariogram.** Indicator semivariograms are transformed
to equivalent truncated Gaussian variograms (Matheron et al, 1987). The Gaussian variograms is found point by point by finding the covariance function $C_y(h)$ (section 4.2).

2. **Compute the concretion trend curve** $t_k(u_i)$. At any location $u_i$, the probability of occurrence of concretion is computed from digitized outcrop profile. The non-concretions probability is $f_c(u_i) = 1 - f_e(u_i)$. The proportion curve is transformed into a standard Gaussian variable

$$t_k(u_i) = G^{-1}(f_c(u_i)) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldOTS

where $G^{-1}(\cdot)$ is the inverse standard normal Gaussian function.

3. **Transforming the radar attributes into conditional probability.** Clustering analysis transforms the radar attributes (instantaneous amplitude, $A$ and frequency, $\omega$) into conditional probabilities. From Bayes rule, if radar attributes are not informative, the posterior probability equals prior $P(C \mid x) = P(C)$. If the radar attributes are informative, the conditional probability will adjust the prior model. In consequence, the vertical concretion proportion trends of the updated realizations may deviate from imposed trends. An iterative method can reconcile concretion proportions implied by the radar with well bore concretion observations by adjusting cluster center means and variances. This strategy was used in this study, and implemented with a nonlinear optimization (section 4.3).

4. **Modified simulation of the Gaussian field (Figure 4-1).** Simulation is a sequential Bayes Gaussian Simulation program, modified from SGSIM (Deutsch and Journel, 1998). Like any sequential simulation, a random path along the grid is generated.

- At each node $u_i$ along the random path, the mean and standard deviation of the Gaussian surrogate $z_i$ for the concretion indicator $(\mu, \sigma)$ are computed by solving
an ordinary kriging system, conditioned on the original data and all neighborhood
points that have been simulated so far.

\[ G((Y - \hat{\mu}, \hat{\sigma})) = \text{Pr} \{Y(u_i) \leq y \mid (n)\} \] ................................... (4-4)

where \((n)\) indicates the data neighborhood used for kriging. The cumulative
distribution function is determined by two parameters: the estimated mean \((\hat{\mu})\)
and variance \((\hat{\sigma})\) for each node. Also the normality assumption of simulated
variables is required. \(Y(u_i)\) denotes a random variable at \(u_i\). \(y\) is a kriging
predicted variable.

- Convert the Gaussian variable to a probability using the trend. The predicted CDF
  transformed from input trend \(t_k(u)\) is used to convert Gaussian variables into
  probabilities at the location \(u_i\).

\[ P(C; u_i) = P\{ z_i < t_k(u_i) \mid z_i \sim G(\hat{\mu}(u_i), \hat{\sigma}(u_i)) \} \] ...................................(4-5)

or \[ P(C; u_i) = G(t_k - \hat{\mu}, \hat{\sigma}) \] ............................................... (4-6)

where \(\hat{\mu}\) and \(\hat{\sigma}\) are the kriged mean and standard deviation for the Gaussian
surrogate at \(u_i\) and \(z_i \sim G(\hat{\mu}(u_i), \hat{\sigma}(u_i))\) indicates that \(z\) at \(u_i\) is Gaussian with the
stipulated mean and variance. \(P(C; u_i)\) is the estimate of concretion probability
prior to including the radar data. The prior \(P(C)\) is combined with the conditional
probability of radar attributes given concretion classification \(p(x \mid C; u_i)\) (or
simplified, with a point implicit in the definition, as \(p(x \mid C)\)) using Bayes rule:

\[ P(C \mid x) = \frac{p(x \mid C)p(C)}{p(x \mid C)P(C) + p(x \mid \overline{C})P(\overline{C})} \] ........................................ (4-7)
Appendix A lists these steps and a few lines of the GTSIM (from GSLIB; Deutsch and Journel, 1998) code surrounding the novel steps is quoted. The parameter file, which is similar to GTSIM, is also attached.

![Flow chart of SGBSIM algorithm](image)

**Figure 4-1. Flow chart of SGBSIM algorithm**

- Draw a random number \((\rho)\) uniformly distributed in \([0, 1]\). The interval in which \(\rho\) falls determines the simulated category at the location \(u_i\). Then the indicators are transformed to Gaussian variables. To enforce the specified or observed trend, another random uniform variable \(\rho'\) is drawn in following way based on above determined categories:

\[
\rho' = \begin{cases} 
\text{drawn from } [0, t_k(u)], & \text{if } \text{concretion} \\
\text{drawn from } [t_k(u), 1], & \text{if } \text{nonconcretion}
\end{cases} \quad \text{(4-8)}
\]

- The Gaussian value \(z(u_i)\) is assigned to current node \(u_i\), assuming following kriging mean and variance \((\hat{\mu}, \hat{\sigma})\):

\[
z(u_i) = G^{-1}(\rho', 1) \times \hat{\sigma} + \hat{\mu} \quad \text{................................. (4-9)}
\]
• This step provides a value for the Gaussian surrogate, $z$, to krig at nodes visited later in the sequential simulation.

• Then visit the next node, till all nodes have been visited.

4.2 Semivariograms and Semivariogram Models

4.2.1 The Indicator Semivariogram

Indicator semivariogram is a graphical tool of measuring spatial continuity. The indicator variogram with separation vector $h$ is computed as

$$
\gamma_I(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [i(u_a) - i(u_a+h)]^2
$$

where $N(h)$ is the data points separated by vector $h$; $i(u_a)$ is the indicator. The indicator semivariogram represents how often two indicator values separated by a vector $h$. If $\gamma_I(h)$ is smaller, spatial continuity is higher. The equivalent Gaussian variograms are found point by point by finding the covariance function $C_y(h)$ that satisfies:

$$
\gamma_I(h; P_c) = P_c (1 - P_c) \frac{1}{2\pi} \int_{0}^{2\pi} \exp(-\frac{y_p^2}{1 + \sin \theta}) d\theta
$$

Where $P_c$ is the cutoff proportion, $y_p$ is the corresponding inverse of the standard Gaussian distribution for a cumulative probability of $p$, $C_y$ is the covariance function for the standard Gaussian variable $y_p$, and $\gamma_I(.)$ is the indicator semivariogram at cutoff $P_c$ (Deutsch and Journel, 1998; White and Novakovic, 2002. In Eqn. (4-11), $P_c (1 - P_c)$ is the sill of indicator variogram, which is the indicator variance; $C_y$ is the covariance function for the standard Gaussian variable $y_p$. For a stationary process, $P_c$ corresponds to $t_c(u_i)$ for all $u_i$. The mean fraction over the outcrop areas was used in these calculations, $P_c = 0.12$. 

46
Two FORTRAN subroutines GAM and GAMV of GSLIB (Deustch and Journel, 1998) are used to compute directional indicator semivariograms. A point-by-point nonlinear solver computes the equivalent Gaussian correlogram from the experimental indicator variogram (Figure 4-2).

Figure 4-2. Indicator correlogram and equivalent Gaussian correlogram
Left is the dip directional indicator correlogram and equivalent Gaussian correlogram. Right plot is for strike direction.

4.2.2 Semivariogram Model

The computed semivariograms are fit by several models (Figure 4-3). A model is used instead of the experimental data because semivariograms \( \gamma(h) \) input into kriging or simulation algorithm require for all distances and directions, but the semivariogram was calculated only for specific distances and directions. A model also provides a convenient, parametric form to incorporate geological trends and anisotropy, minimize sampling errors, and filter artifacts. Finally, appropriate variogram models ensure a positive definite covariance matrix, which is required to solve the kriging system.

Exhaustive semivariogram models are computed to determine the directions of main axis of continuities in both dip and strike directions. Three nested models fit the Raptor Ridge semivariograms, exponential, spherical, and dampened hole effect:
Spherical: $\gamma(h) = c \cdot Sph\left(\frac{h}{a}\right) = \begin{cases} \begin{aligned} & c \cdot \left[1.5 \frac{h}{a} - 0.5 \left(\frac{h}{a}\right)^3\right] \quad \text{if } h < a \\ & c \quad \text{if } h \geq a \end{aligned}\end{cases}$ .......................................... (4-12)

Gaussian: $\gamma(h) = c \cdot \left[1 - \exp\left(-\frac{(3h)^2}{a^2}\right)\right]$ .................................................. (4-13)

Dampened hole effect: $\gamma(h) = c \cdot \left[1 - \exp\left(-\frac{3h}{d}\right) \cdot \cos\left(\frac{h}{a} \cdot \pi\right)\right]$ .............................................. (4-14)

where $h = |\mathbf{h}|$ is the lag distance, $a$ is the range, $c$ is the sill, and $d$ is the distance which 95 percent of the hole effect is dampened out. The variance magnitude of the periodic component is less than 5 percent of $c$ beyond $d$.

![Figure 4-3. Fitted semivariogram model for Gaussian semivariogram](image)

(a) Three structures model the semivariograms in the dip and strike directions. Geometric and zonal heterogeneities are introduced. (b) Vertical data and model.
In both directions, the nugget effect is zero, which indicates that concretions are continuous at small lag distance (Table 4.1), because of the high-quality continuous outcrop images. The hole effect ranges are 3.5-4 m, which characterizes the cyclicity of concretion occurrence in both directions; this structure is damped beyond $h = 10$ m. The largest-scale variance structure has a significant a zonal heterogeneity.

### Table 4-1. Truncated Gaussian semivariogram model

<table>
<thead>
<tr>
<th>Structure</th>
<th>Dip Direction Range (m)</th>
<th>Strike Direction Range (m)</th>
<th>Vertical Range (m)</th>
<th>Dip (degrees)</th>
<th>Variance component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>1000.0</td>
<td>8.0</td>
<td>0.5</td>
<td>4.0</td>
<td>0.2</td>
</tr>
<tr>
<td>Spherical</td>
<td>2.0</td>
<td>13.0</td>
<td>0.03</td>
<td>4.0</td>
<td>0.4</td>
</tr>
<tr>
<td>Hole</td>
<td>4.0</td>
<td>3.5</td>
<td>0.1</td>
<td>4.0</td>
<td>0.4</td>
</tr>
</tbody>
</table>

The nugget effects for both directions are 0; the internal ranges from semivariogram models are about 3.6 m. The dampened distance for hole effect is 10 m. The dip angle of major axis is measured right-oriented downward to major axis. And minor axis is perpendicular to major axis.

### 4.3 GPR Data Interpretation

#### 4.3.1 GPR Detection of Concretions

Instantaneous amplitude decreases sharply in the deeper channel deposits (Figure 2-2). We roughly separate calibrations for the bars (the shallow part), and for the channels (the deeper part). The boundary between these is deterministically defined by the sedimentologic faces interpretation of the GPR data, constrained by the cores.

The concretions responses have broad distributions; concretions and uncemented rock have wide and overlapping ranges of radar attributes (Figure 4-4). There is a correlation between the thickness and composition of the concretions with GPR responses, which produces clusters in the amplitude-frequency cross plots.

The frequency content is controlled mainly by contrasts in petrophysical properties, including clay content, porosity, permeability, water saturation, and thickness of beds. Amplitude is mostly a function of the polarity and magnitude of the impedance
contrasts at bed interfaces and also of some geometric properties of the layers, such as the composite interfaces and thickness (Corbeanu and others, 2002). The dispersion of concretion clusters and their overlap with the nonconcretion clusters pose modeling challenges (Figure 4-4, Table 4-2). The reason is that the concretions may cover large range of petrophysical properties (such as porosity and permeability) and occur in many

Figure 4-4. GPR responses for channel and bar sediments. Concretions and non-concretions have large overlap because they cover large ranges of lithofacies and transport properties. The concretions in bar deposits (left) are more scatter than those in channel deposition (right). Because of attenuation, amplitude of bar deposition is larger than amplitude in channel deposition. The 95 percent posterior areas for concretion and nonconcretion are computed by adjusting the clusters’ parameters to maximize the well bore prediction.

Table 4-2 Cluster center and covariances matrix

<table>
<thead>
<tr>
<th>Bar Deposites Covariance Matrix</th>
<th>Cluster Center A</th>
<th>Cluster Center F</th>
<th>Channel Deposites Covariance Matrix</th>
<th>Cluster Center A</th>
<th>Cluster Center F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conc A F</td>
<td>0.40 0.08</td>
<td>8.13 8.20</td>
<td>Conc A F</td>
<td>0.87 0.01</td>
<td>6.84 8.01</td>
</tr>
<tr>
<td>Non-Conc A F</td>
<td>4.00 0.05</td>
<td>8.97 8.36</td>
<td>Non-Conc A F</td>
<td>4.00 -0.33</td>
<td>5.60 9.02</td>
</tr>
</tbody>
</table>

lithofacies. The variability of petrophysical properties is associated with tight to light cement. Furthermore, concretions have a broader range of radar responses than nonconcretions facies because the concretions are variously sized and hosted in facies with different electromagnetic
properties; therefore their radar attributes vary widely. The different amount of mud nuclei in the concretions is another reason for the scattering of the concretions distribution.

Despite these sources of scatter, many concretions have similar geometry and petrophysical properties; thus, their radar responses do cluster, at least weakly.

4.3.2 Deterministic Cutoff Method for Concretion Prediction

First, concretion classification is based on the cross-plots of the logarithm of instantaneous frequency ($\omega$) and instantaneous amplitude ($A$) separately for the bars and for the channels. The training set of the concretions and non-concretions are from the cores. The clusters modes and one standard deviation range define the different criteria. These criteria were then applied to the GPR volume (with $\omega$ and $A$ of each data point), using the bar criterion in the bar region and the channel criterion in the channel region. Criterion boundaries are adjusted until so that the volume fraction of concretions predicted by radar matches the outcrop exposures and the cores (Figure 4-5).

![Figure 4-5. Concretion realization from cutoff-based method](image)

The volume percentage of concretions at wells and outcrop exposures are honored (by Gani and Lee., 2004).

However, this method cannot evaluate the uncertainty caused by overlapping of data, which is prominent in this data set. In addition, this approach cannot predict concretions below GPR
resolution. Furthermore, it is difficult to integrate this deterministic concretion prediction with other types of data.

4.3.3 Probabilistic Clustering for Concretion Classification

Because instantaneous GPR amplitude and frequency are similar for all concretions and somewhat different for nonconcretions, these groups can be considered as clusters and modeled using cluster analysis. The centers and variances of clusters are computed to maximize the prediction accuracy of well bore classification.

First, the distance from clusters center are computed

\[ D_c^2 = (x - \bar{x})^T \Sigma^{-1} (x - \bar{x}) \] .............................. (4-15)

Where \( x = (\omega, A) \) and \( \bar{x} = (\bar{\omega}, \bar{A}) \) are radar attribute vector and cluster center attribute vector; \( \bar{\omega} \) is the centroid frequency for the cluster. \( \Sigma \) is the covariance matrix. By assuming multivariate Gaussian distribution, the conditional probability of a datum belonging to group \( C \) is

\[
p(x | C) = \left( \frac{1}{2\pi} \right)^{\rho/2} \frac{1}{\sqrt{\Sigma_C}} \exp \left\{ -\frac{1}{2} (x - \bar{x}_C)^T \Sigma_C^{-1} (x - \bar{x}_C) \right\} \] .......................... (4-16)

where \( p(x | C) \) conditional probability density of the radar data \( r \) if a datum belongs to the concretions group. Nonconcretion conditional probability can be calculated in a similar way.

Non-linear regression is used to maximize the overall accuracy. In choosing the clusters center and variances, 55 percent and 67 percent overall prediction accuracy are obtained for channel and bar sediments. These are relatively poor accuracies, which motivates integration with geospatial data (i.e., the indictor variograms and trend curves).
4.4 Geostatistical Modeling Results

4.4.1 2D Geostatistical Modeling

The concretions are modeled using SGBSIM. By adjusting modeling choices such as variogram parameters, conditional wells, secondary data and factors of trends, different scenarios can be modeled. Geostatistical realizations combined with designed flow simulation can be used to analyze uncertainty or sensitivity of flow responses caused by different modeling factors.

Furthermore, understanding the relationship between flow responses and modeling factors can guide the production history match and help validate the geologic models. For example, by understanding the outcrop flow responses modeled with dense data, we can develop theories to help predict geological factors of underground reservoir from flow responses.

The main strength of outcrop analog study is validation. Using the SGBSIM algorithm, realizations with different parameters can be simulated and compared with observations. The comparisons can be divided into two categories: static and dynamic.

First, static features such as dip, trend, range, and fraction are compared. A designed flow simulation is used to validate flow responses for geostatistical models against flow models for the observed 2D concretion distribution (chapter 5).

Geostatistically simulated concretion distributions appear similar to observations from cliff faces (Figure 4-6). The geostatistical simulation appears to capture the main characteristics of concretion distribution, such as concretion range, dip angle, proportion and trends. However, the vertical trend is not aligned with bedding. This may be caused by using a simplified step function to model the vertical trend. A more detailed trend, using a moving average of the observed vertical trend, may improve the results. Furthermore, some post-processing algorithms such as simulated annealing or multipoint statistics can be utilized to improve visualization.
Figure 4-6. Comparison of the geostatistical realizations
(a) A geostatistical realization has the same trend, range and proportion of observed concretion. (b)-(e) the realizations use the same set of modeling parameters as (1), and change one parameter at a time. (b) No trend is imposed; (c) the concretions dip angles are twice larger than (a); (d) the realization has one and half more concretion than the observation; (e) No conditional wells are used; (f) the concretion range is half of the observed range.

These topics are beyond the focus of this research and are discussed by White and others (2003).

Dynamic comparison (using tracer flow simulations) indicates that concretion proportion can change the fluid flow front at breakthrough (Figure 4-7; chapter 5).

Figure 4-7. Comparison of concretion prediction and its influences on fluid flow
(a), (b) are the concretion observation. (a) is the facies and concretion structure; the blue blocks represent concretions, different colors represent different permeability. (b) is the tracer flow front at breakthrough time. Red represents injected tracer, blue represents original tracer. (c) is a geostatistical prediction, with 1.5 times more concretion than the observation. the simulation captured the characters of the trend, range, dip angle, and proportion of concretion. (b) and (d) are flow fronts for the two scenarios. Injection direction is from right to left. The concretion proportion changes
the tracer breakthrough front. These simulations cannot test integration of radar data, because no radar data are available at the outcrop face.

4.4.2 3D Geostatistical Modeling

Three-dimensional stochastic realizations can be generated by SGBSIM. Unlike 2D realizations, no 3D deterministic data set can be used to validate the geostatistical model. However, the dense 3D GPR data, well bore data and pseudo wells from cliff condition the models. Different types of 3D models are shown in Figure 4-8.

The radar Bayes update is informative even though the radar concretion response is weak. The radar conditional probability is calibrated with well bore observation. The conditional probability of concretion decreases downward, which may enhance the trend imposing (Figure 4-9 (e)). In particular, the high conditional probability blocks in the lower channel deposits have reduced occurrence of concretions, which honors the outcrop observation and may affect flow responses. In the Figure 4-9 (a), the concretion prediction uses no radar information and no vertical trend. It is equivalent to the results of truncated Gaussian simulation (GTSIM), which is a prior input for SGBSIM in this study. Then the Bayes update integrates radar information
illustrated in Figure 4-9 (b). Compared with Figure 4-9 (a), the radar updates combine the prior prediction (shape, dimension, and fraction) with radar data (distribution, petrophysics). The

Figure 4-9. Comparison of Radar Update Effects
(a) Prediction without trend and 1.5 times fraction of the observation. (b) concretion prediction using same parameters as (a) and with Bayes radar update. (c) (d) are predictions imposing trend, without and with radar update. (e) the noisy radar conditional probability. The radar update integrates the radar information and has the similar effect of imposing trend.

Furthermore, the artifacts of low (“zero”) concretion deposits caused by the simple step function trend model were offset by the radar conditional probability. The large concretions in the lower part of the model are reduced as well. The low concretion fraction combined with high
permeability channel facies can affect flow response, for example increasing effective permeability.

The large concretions predicted by variogram-only model are not so obvious in the updated prediction, possibly because the collocated radar conditional probability is less continuous. This may be more realistic because radar attributes reflect the physical properties of concretion such as “tightness” (density). However, the photomosaic description mainly reflects the color contrast or shapes of concretion on the cliff faces. As a result, the updated prediction using conditional probability derived from radar attributes could be closer to the real permeability structure than using the prior probability only. Of course, it also depends on the radar data quality and processing technology. Furthermore, postprocessing such as MAPS (Deutsch, 2002) can be used to improve the visualization.

4.5 Summary

1. A sequential Gaussian Bayes simulation has been developed. The new algorithm SGBSIM can impose trends and integrate secondary data with little extra computational cost.

2. The indicator semivariograms of concretion occurrence on the outcrops (in the dip and strike directions) are transformed into equivalent Gaussian semivariograms. Exponential, Gaussian and dampened hole-effect models are used to interpret the concretion spatial heterogeneity and fit the variogram models.

3. Radar responses to concretions are noisy because the instantaneous amplitude attenuates quickly downward, and the radar responses of concretion are confounded with complex lithologic, geologic, and geometric characters.

4. To include the uncertainty in modeling, a cluster analysis method is used to determine the conditional probability model. Compared with deterministic method, this statistic method has
several advantages. It can evaluate the uncertainty caused by overlapping of data, which is dominant in this data set. It can predict the concretions under the resolution of GPR resolution. Furthermore, it integrates geophysical concretion prediction with other types of data to reduce uncertainty. Geostatistical modeling shows that SGBSIM can reproduce outcrop observations of correlation, fraction, and trend. An iteration process is combined with the clustering analysis to assure that radar update will match the well bore concretion observations.
CHAPTER 5. FLOW MODEL VALIDATION USING OUTCROP DATA

5.1 Introduction of Experimental Design and Response Surface Method

5.1.1 Experimental Design

Experimental design and response surface methods have been used in a variety of reservoir engineering studies of uncertainty (Damsleth and others, 1992; White and others, 2000), performance prediction (Chu, 1990), upscaling (Narayanan and others, 1999), sensitivity (Jones and others, 1995; Willis and White, 2000), and inversion (White and others, 2000). Experimental design “selects efficient and meaningful reservoir simulation models from the space of all feasible models. The selected models span the factor space and yield low-error response surfaces” (Wang and White, 2002).

Experimental design is closely related to the response surfaces method (RSM); response models are empirical models relating multiple recovery responses to geologic and engineering factors. Designed simulations have following advantages over less formal techniques:

1. They identify interactions between factors.
2. They often have desirable properties such as orthogonality and rotatability.
3. They often yield more accurate and less biased estimates of system behavior.
4. They can be statistically tested to identify and quantify the factors most influential in determining effective properties or production behaviors.
5. Critically, the designed approach can detect and model factor interactions, which are not commonly done in ‘one-at-a-time’ sensitivity analyses (Narayanan and others, 1999).
6. Furthermore, accurate records of all factor settings for all runs are easy to record automatically, and variable and uncertain factors must be identified and assessed a priori by the modeling team (White and Royer, 2003)

Experimental designs can be classified depending on the levels of design factors. A two-level design can estimate the main and interaction effects. The two-level full factorial design needs $2^k$ samples; $k$ is the number of factors. If $k$ is large, it is too expensive to use this design.
For many factors, two-level fractional factorial designs can be used. Fractional factorials require fewer experiments, but confound some factor interactions. Response models should have more designs points than model coefficients \(k\). Central composite designs (CCD) combine two-level fractional factorials, axial points and the center point. Box-Behnken designs (BBD) are efficient for fitting second order response surfaces. However, it does not estimate the response accurately at extreme values near the vertices of the factor space hypercube. D-optimal designs maximize the trace of the factor covariance matrix; G-optimal design minimize the maximum predict variance. Designs of these types may require fewer runs. However, these designs require (generally nonlinear) optimization, prior specification of the model to be fit, candidate points, and the number of experiments. In addition, they are not necessarily orthogonal or rotatable. As an alternative, confounded mixed-level factorial designs can be constructed using orthogonal array (OA) or nearly orthogonal array (NOA) designs. If a multilevel design has different levels for different factors, it is called a mixed-level design.

5.1.2 Orthogonal Array Designs

Like other designs, OA designs span design spaces as few runs as possible, and also simplify response model calculations. The OA design is the most efficient designs for a particular number of runs (Kalla, 2005). The efficiency of an OA design can be characterized by two properties. First, for a given number of runs, an OA design fills the factor space uniformly so that it is more efficient in exploring the factor space than other designs. Second, the D-optimal criterion, which is related to the regression estimators’ variances, indicates that OA designs have the smallest (or best) D-optimality compared to other designs. This means the OA design has the least estimator variance, and the resultant model minimally confounds regression coefficient estimates. OA designs have several advantages:
(1) Large number of factors can be studied, and conclusions valid over the entire region spanned by the control factors.
(2) Large saving in the experimental effort by decreasing number of runs.
(3) Analysis is easy as columns are orthogonal.
(4) OA is an adaptive design. Changing the values of the factors can refine OA design, columns (factors) can be eliminated, and new columns can be added.
(5) Wide range of factors and levels especially by the use of NOA.
(6) Perfect for screening by the use of two level supersaturated designs.
(7) OA designs can be called hierarchical designs. Quadratic and interaction effects can be evaluated by augmenting another design with same levels and factors as the linear effects design.
(8) OA is a mix-level design, which is more flexible than fix-level designs such as BBD. 

Kalla (2005)

However, for some sets of factors and level combinations, OA designs require many runs or the desired OA may not exist. In such situations, a nearly orthogonal array (NOA) can be used (Xu, 2002). Furthermore, OA designs require manipulation of matrices and a predetermined number of runs. When the number of factors and levels are large, the iterative algorithm is slow. However, because design cost is still small compared with simulation cost, OA design is an attractive approach for reducing total study costs.

5.2 Response Surface Method (RSM)

In spite of the rapid improvement of computer capacity and speed, the complexity of numerical reservoir simulation makes it impossible to rely exclusively on simulation itself. An alternative is to use approximate models or a “model of model” (metamodel; Kleijnen, 1987) to replace the expensive simulation.

Kriging can be used to build metamodels (Kalla, 2005). It is extremely flexible because of its wide range of correlation functions. However, model construction is time consuming and the correlation matrix is singular if multiple experiments coincide. Radius basis functions (RBF) (Simpson, 1999) and multivariate adaptive regression splines (MARS) (Jin and others, 2000) are also used to build metamodels.
In this study, second-order polynomial response surface models (RSM) are used. The normalized regression coefficients evaluate factor significance and sensitivity. Stepwise regression narrows a large regressor set to consider only significant terms. The stepwise-fitted surface is simpler because it has fewer coefficients, and the model’s smoothness facilitates quick evaluation. However, low-order polynomial models cannot model highly nonlinear behavior or abrupt transitions due to thresholding phenomena (Jin and others, 2000). The regression coefficient $R^2$ for second order polynomial regression may be low. On the other hand, high order polynomials may oscillate and be unreliable because too few degrees of freedom remain unused.

A weighted least square regression accounts for heteroscedasticity to improve the BLUE estimation (Best Linear Unbiased Estimation). Heteroscedacity is the dependence of the variance on the mean, and is a violation of the assumptions in basic linear least squares. In this approach, a secondary response surface of variance (the inverse of weight) is modeled, which describes the response variance across the factor space (Wang and White, 2002).

**5.3 Designed Simulation**

**5.3.1 Factors**

Factors are input parameters that are varied during experimental design (White and Royer, 2003). Six factors are examined in this study (Table 5-1). The semivariogram range ($R$) describes the spatial continuity of concretion occurrence. In this study, the experimental variogram was modeled by three types of variogram model (exponential, spherical and dampened hole effect model). The higher the level of range is, more continuous the concretion is. Dip direction of concretion ($D$) characterizes the dip angle of concretions. Concretion observations from the outcrop are parameterized into a step concretion curve. The concretion trend ($T$) is an indicator whether trend curves are used not. Conditioning sections ($C$) is the
number of sections used in conditioning the stochastic simulations. Three measured sections from the outcrop are used as conditioning data. The one-section model uses one section located at the middle column of the grid; the two-section model uses the left and right most columns; the three-section model uses columns located at two ends and middle of the outcrop.

<table>
<thead>
<tr>
<th><strong>Factors</strong></th>
<th><strong>Levels</strong></th>
<th><strong>Number of Variables</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Semivariogram range&lt;sup&gt;a&lt;/sup&gt; (R)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>Cement Dip angle&lt;sup&gt;a&lt;/sup&gt; (D)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Cement fraction&lt;sup&gt;a&lt;/sup&gt; (F)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>1.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Cement trend (T)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Conditioning wells (C)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Flow direction (O)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>South to North</td>
<td>North to South</td>
</tr>
<tr>
<td>Total</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> the multiplier of observed concretion parameters. Similarly, the dip angle and the cement fraction are less, equal and larger than the observed values.

<sup>b</sup> the symbols of factors and levels are used in experimental design

The concretion fraction ($F$) is the average concretion coverage at a given depth.

### 5.3.2 Responses

Responses are the experiments or the model output, based on which decision is made. Three responses -- upscaled permeability, breakthrough time and sweep efficiency -- are examined.

1. Upscaled permeability ($\bar{k}$) is defined as the ratio of flow rate to pressure computed from simulation results. If the upscaled permeability were assigned to all cells in the fine-grid model, that homogeneous model would give the same flow single phase rate-pressure relation as the heterogeneous fine-grid model (for the same boundary conditions).
2. Breakthrough time is a dimensionless time in pore volumes. It is the total tracer injection volume when the outlet tracer concentration exceeds 1 percent. The dimensionless breakthrough time \( \tau_{bt} \) is

\[
\tau_{bt} = \frac{1}{V_p} \int_0^{t(c_o=1/100)} q(t)dt
\]

where \( t \) is the time, \( q \) is the volumetric flow rate at reservoir conditions, \( V_p \) is the total pore volume, and \( t(c_o = 1/100) \) is the time that the outlet tracer concentration first reaches one percent of the injected concentration. The normalized concentration \( c_o \) is the tracer concentration at the outlet divided by the injected concentration. For uniform linear flow, \( \tau_{bt} = 1 \).

3. Sweep efficiency \( N_{pD} \) is the fraction of the initial tracer free water recovered after 1 pore volume of injection:

\[
N_{pD} = \int_0^1 [1 - c_o(\tau)]d\tau
\]

For uniform linear flow, \( N_{pD} = 1 \).

These responses can be extracted from simulation results automatically.

5.3.3 OA Design

The selection of designed runs is determined by several factors such as experiment scales (Jin and others, 2000). Experimental scales (large or small) are the variables included in experiments. Large-scale problems (with more than 10 variables; Jin and others, 2000) require more runs to understand; small-scale problems (with 2-3 variables) require fewer runs.

For designs, second order polynomial has \( k = \frac{(n + 1)(n + 2)}{2} \) coefficients to estimate and the number of experiments should be at least \( k \). Giunta et al., (1994) and Kaufmann, et al., (1996) found that for a reasonably accurate regression model it is better to run 1.5\( k \) runs for 5-10 variables, 3\( k \) runs for 10-20 variables, and 5.5\( k \) runs for a 20-30 variables. (Jin and others, 2000)
In this study, the number of polynomial coefficients $k$ is $(6+1)(6+2)/2=28$. Thus, the number of ruleset small; magnitudes to 84 for large sets (3 × 28; medium set rule). Available OA designs are 36, 72 and 144. A reasonably conservative 72-run OA design is selected. The OA design of 6 factors uses 3 levels for the continuous factors (range, dip, and cement fraction) and conditioning wells and 2 levels for the binary factors trend (on or off) and direction (Table 5-2).

Table 5-2. Orthogonal Array (OA) design with 72 runs

<table>
<thead>
<tr>
<th>Runs</th>
<th>condition Wells</th>
<th>Range</th>
<th>Dip Angle</th>
<th>Cement Fraction</th>
<th>Trend</th>
<th>Injection Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 0 2</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0 1 0</td>
<td>0 1</td>
<td>1 0</td>
<td>0 1</td>
<td>0 1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0 1 2</td>
<td>1 0</td>
<td>1 2</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0 2 0</td>
<td>1 1</td>
<td>1 1</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1 0 0</td>
<td>1 1</td>
<td>1 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0 0 2</td>
<td>2 1</td>
<td>1 1</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0 0 2</td>
<td>2 0</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0 0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0 2 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0 1 0</td>
<td>1 0</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0 1 0</td>
<td>1 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0 1 0</td>
<td>1 0</td>
<td>1 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>0 1 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>0 0 0</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
</tbody>
</table>

*: The first three factors are of 3 levels; and the last two factors are 2 levels. 0, 1, 2 represent different levels of factors. Corresponding levels are listed in table 5-1.

5.4 Flow Simulation Description

Fluid flow is simulated with a commercial simulator (Schlumberger Technology Co., 1997). The displacements are ideal tracer flow or two-phase waterflood displacement. For tracer
flow, there is no buoyancy, capillary pressure, viscous contrast and relative permeability effect (Calhoun and Tittle, 1968). The tracer displacements are less time consuming, and the tracer simulation isolates the effects of permeability heterogeneity and makes the results less complex to interpret. Finally, truncation error can be reduced for fully miscible systems (Rubin and Blunt, 1991).

The flow grids are upscaled from fine geostatistical grids. The grid size is $781 \times 541$ in the $y$ and $z$-directions, corresponding to block dimensions of $17.3 \times 3$ cm, which are smaller than most concretions. Because the total block count of 4,422,521 is computationally prohibitive, a static upscaling method is used to upscale to a grid 3 times coarser than the original grid in both directions (Li and others, 2001; section 6.2). This upscaling yields a flow grid with 46,800 blocks, which is manageable for sensitivity analysis (Table 5-3; Figure 5-1).

(a)

(b)

**Figure 5-1. Outcrop facies, concretions, and flow**
Outcrop of dip direction is about 120 m long and 15 m high; different colors represent different permeability (top); the blue blocks are low-permeability concretions. Tracer (red) are simulated to inject from left (north) to right (south) and displace the water (blue).

Flow simulations use the same procedure as White and others (2003). For the single-phase tracer simulation, models are initially saturated with 0 percent tracer at uniform hydraulic potential. Flow is injected from one face (or column) to opposite face (or column) with other 4 faces assumed to be impermeable.
Water is injected at a constant rate of 0.001 pore volume per day. For the both injection and production wells, the boundary condition is infinite conductivity and nonuniform flux. To ensure that the effects of non-horizontal flow are included correctly in effective property calculations, the injection and production wells have the same, explicitly specified pressure datum (Novakovic and others, 2002).

Table 5-3. 2D Flow Model Summary

<table>
<thead>
<tr>
<th>Pore Volume (RB)</th>
<th>1201</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Grids</td>
<td>46,800</td>
</tr>
<tr>
<td>X</td>
<td>135</td>
</tr>
<tr>
<td>Y</td>
<td>15</td>
</tr>
<tr>
<td>Block dimension(m)</td>
<td>0.52 0.08</td>
</tr>
<tr>
<td>Counts</td>
<td>260 180</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Execution times for 1 pore volume (s) a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Minimum</td>
</tr>
<tr>
<td>Maximum</td>
</tr>
<tr>
<td>All Runs</td>
</tr>
<tr>
<td>Count</td>
</tr>
</tbody>
</table>

Assumptions

Ideal Tracer
Single Phase flow
Constant viscosity
Small and constant compressibility

1.4G-MHZ Pentium IV processor

5.5 Flow Response Analysis

Based on the 432 designed simulations (72 model types with 6 realizations each), a weighted least square method builds a second order polynomial response surfaces. The fitted response surface model relates flow responses and input geological, geostatistical, and engineering factors. The factors sensitivity analysis and model comparison are based on the response surfaces model.

5.5.1 Response Surfaces Model Using Weighted Least Square Method

One assumption of ordinary regression is that the variance of regression residual is constant, or homogeneous, across observations. If this assumption is violated, the errors are "heteroscedastic." Heteroscedasticity often arises in the analysis of cross-sectional data. For
example, the response variances vary with trend and fraction (Figure 5-2). A common solution for heteroscedasticity is transformation such as a logarithm transformation of permeability.

Figure 5-2. The variances distribution of flow responses.
For variances of fraction and trend are not constant for all three responses. This heteroscedasticity justifies use of WLS for response surface building.
Alternatively, a weighted least square method (WLS) is used to build the RSM model.

Unlike linear and nonlinear least squares regression, weighted least squares regression is not associated with a particular type of function used to describe the relationship between the process variables. Instead, weighted least squares reflect the behavior of the random errors in the model; and it can be used with functions that are either linear or nonlinear in the parameters. It works by incorporating extra nonnegative constants, or weights, associated with each data point, into the fitting criterion. The size of the weight indicates the precision of the information contained in the associated observation. Optimizing the weighted fitting criterion to find the parameter estimates allows the weights to determine the contribution of each observation to the final parameter estimates. (NIST/SEMATECH e-Handbook of Statistical Methods, 2004).

Based on the 432 flow simulations, the inverse of the variance is computed and used as regression weights. The variance is computed at each factor combination using multiple realizations; the variation of the variance over the factor space is being investigated. The WLS method uses SAS RSREG procedure (SAS Institute, 1998) to compute second order polynomial fits for the factor space of 6 factors.

WLS provides an RSM model that is more accurate than ordinary least squares. The $R^2$ and adjusted $R^2$ show that more variance is explained by the WSL model than ordinary least square method with $R^2$ increasing (0.07-0.14) – these statistics refer to the models of variance, not models for the responses themselves.

5.5.2 Response Surface Models of Variance

Because the response variances are not constant, a response surface was used to examine its variability. The prediction results help understand the uncertainty distribution and assist sensitivity and uncertainty analysis.

An $F$-test of the model variance compared to error variance indicates that the variance captured by the polynomial model is significant. But all regression coefficients ($R^2$) are low 0.37-0.51 (Table 5-4).
Table 5-4. Response Surfaces Model of Variance.

<table>
<thead>
<tr>
<th>Models</th>
<th>Flow Responses</th>
<th>Models</th>
<th>Se</th>
<th>BT</th>
<th>Kbar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>0.17 &lt;.0001</td>
<td>0.20</td>
<td>&lt;.0001</td>
<td>0.16</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Quadratic</td>
<td>0.19 &lt;.0001</td>
<td>0.23</td>
<td>&lt;.0001</td>
<td>0.09</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Interaction</td>
<td>0.03 0.0449</td>
<td>0.07</td>
<td>&lt;.0001</td>
<td>0.12</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Total R²</td>
<td>0.40 &lt;.0001</td>
<td>0.51</td>
<td>&lt;.0001</td>
<td>0.37</td>
<td>&lt;.0001</td>
</tr>
</tbody>
</table>

LOF test P val. of F test <.0001 | <.0001 | <.0001

<table>
<thead>
<tr>
<th>Factors</th>
<th>Coeff.</th>
<th>P value</th>
<th>Coeff.</th>
<th>P value</th>
<th>Coeff.</th>
<th>P value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>22.12</td>
<td>&lt;.0001</td>
<td>63.61</td>
<td>&lt;.0001</td>
<td>2.39</td>
<td>0.0082</td>
</tr>
<tr>
<td>F</td>
<td>5.31</td>
<td>0.0012</td>
<td>21.24</td>
<td>&lt;.0001</td>
<td>1.20</td>
<td>0.0067</td>
</tr>
<tr>
<td>O</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R*R</td>
<td>-11.74</td>
<td>&lt;.0001</td>
<td>-20.80</td>
<td>0.0003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D*R</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D*D</td>
<td>-26.32</td>
<td>&lt;.0001</td>
<td>-69.45</td>
<td>&lt;.0001</td>
<td>-3.68</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>F*R</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F*D</td>
<td>-26.32</td>
<td>&lt;.0001</td>
<td>-69.45</td>
<td>&lt;.0001</td>
<td>-3.68</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>F*F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T*R</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T*D</td>
<td>-11.63</td>
<td>0.0005</td>
<td>-2.27</td>
<td>&lt;.0001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T*F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O*R</td>
<td>-4.08</td>
<td>0.0092</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O*D</td>
<td></td>
<td></td>
<td>-1.98</td>
<td>&lt;.0001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>O*F</td>
<td></td>
<td></td>
<td>1.21</td>
<td>0.0067</td>
<td></td>
<td></td>
</tr>
<tr>
<td>O*T</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>-2.37</td>
<td>0.0021</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*a* is the $R^2$ of linear, quadratic and interactions; and the total $R^2$. The significance is evaluated by $P$ value of F test.
*b* the coefficients are normalized by the intercept, which relates to the base case. Bold factors are significant with $P <0.001$; Bold and italic are significant with value within $(0.0001<P<0.005)$; blank are $P$ value $> 0.05$

A lack-of-fit (L.O.F) test explains the above error variances can be further divided in L.O.F. error and stochastic fluctuation (randomness). The L.O.F. test compares the L.O.F. variance with pure error variance and illustrates a significant L.O.F. error. This implies that the response variances are due to nonlinearity; and the second order polynomial model used is not adequate.
5.5.3 Sensitivity Analysis of Variance Models

There is evidence for non-stationarity but that we cannot currently model it. Because the existing RSM can only explain about half of the total variances, only the highly significant factors are considered. The cement fractions and trends are significantly affect variances of all responses. For all responses the fraction and/or the quadratic term of fraction ($FF$) are the most significant factors.

For example, The $F$ and $FF$ term of $\tau_{bt}$ (63.61 and –69.45) indicates that with the decrease of fraction (or increase of $F$), the variance decreases, which mean the random distribution of concretion may increase the uncertainty of flow responses. Similarly, imposing the trend (increasing $T$) the uncertainty increases.

5.6 Response Surface Model Analysis and Sensitivity analysis

Here, RSM model are constructed for the means responses rather than the variance. Several scenarios compare effects factors like fraction, trend and injection direction, and factor sensitivity is examined.

5.6.1 Model Comparison

The orthogonal array design chooses 72 parameter combinations to relate flow responses to the factors and factor interactions. Because concretions are predicted stochastically, 6 realizations are used for each model to test their variances. The number of simulation required is $3^4 \times 2^2 \times 6 = 1944$ for a full mixed-level factorial but decreases to a more manageable $72 \times 6 = 432$ runs for the OA design.

Four particular models are selected (Table 5-5) to examine factors of interests. For the flow responses ($K$, $\tau_{bt}$, $N_{ph}$), these models use nonparametric pairwise comparisons to test the
influences of concretion fraction and inclusion of cement trends. Analysis of variance (ANOVA) and sensitivity analysis examine factor effects.

Table 5-5. Designed models for comparison

<table>
<thead>
<tr>
<th>Model</th>
<th>C</th>
<th>R</th>
<th>D</th>
<th>F</th>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

*: Bold characters represent model differences

Models A and B compare the effect of concretion fraction proportion. Models C and D compare the effect of concretion trend. The model factors and corresponding levels of the four models are listed in Table 5-5.

The Tukey test (Table 5-6) indicates that, in model A and B, the concretion fraction will significantly decrease $k$, $\tau_{bt}$ and $N_{pD}$. Increasing the fraction from 8 to 15 percent, the flow responses $(k, \tau_{bt}$ and $N_{pD})$ decrease about (17.1, 6.8 and 3.1 percent). Comparison of C to D indicates that concretion trend has significant effects on flow response. Imposing the concretion trend will significantly decrease the $k$, $\tau_{bt}$ and $N_{pD}$ by (11.1, 5.6 and 1.5 percent); the trend
forces the concretion accumulates within the middle high permeability zones. Even though the overall fraction is low (8-11%), the trend forms a high concretion zone (20-30%), which hinders flow though these zones and decreases permeability (chapter 7). The concretion fraction has slightly larger effects on flow responses than imposing a trend, because the fraction baffles flow.

Finally, the base case with all parameters the same as concretion observation is not significantly different from outcrop observation. The differences between the responses of the flow model and outcrop observation are within the 95 percent confidence intervals of the model realizations, which means with 95 percent confidence the model realizations is not significantly different from observation. The validation is meaningful, because (as discussed above) flow responses are sensitive to the geostatistical model. If responses did not change significantly when geostatistical parameters were varied, the validation would not be meaningful.

This validation is important for the following three dimensional reservoir analog modeling and real reservoir modeling, where the predicted model cannot be directly validated.

5.6.2 Analysis of Variance and Sensitivity Analysis

Factor sensitivity is examined using the response model. Cement trend, injection direction and cement fraction are highly significant and affect all three responses (Table 5-7, Figure 5-3). Increasing concretion fraction and imposing a vertical trend both decrease the reservoir quality ($k$, $\tau_{bi}$ and $N_{pD}$ (chapter 7). Concretion range ($R$) significantly affects the sweep efficiency but does not affect breakthrough time or upscaled permeability much. Other factors such as direction ($O$) and dip angle ($D$) are insignificant. For each response, the RSM coefficients are good indicators of the factors relative importance. The cement fraction ($F$), range ($R$) and cement trend ($T$) have more influence than other factors. Upscaled permeability is more
sensitive to \((F)\), \((T)\) and their interactions. The significant factor interactions are illustrated in response surfaces (Figure 5-3). Table 5-7 also indicates all influential factors with coefficients

**Table 5-7. Analysis of variance and sensitivity of flow response**

<table>
<thead>
<tr>
<th>Factors</th>
<th>Flow Response</th>
<th>Upscaled Permeability (md)</th>
<th>Breakthrough Time (PV)</th>
<th>Sweep Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Coeff.</td>
<td>Significance</td>
<td>Coeff.</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.984</td>
<td>0.861</td>
<td>0.959</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>0.005</td>
<td>*</td>
<td>-0.113</td>
<td>0.123</td>
</tr>
<tr>
<td>D</td>
<td>0.012</td>
<td>*</td>
<td>0.096</td>
<td>0.033</td>
</tr>
<tr>
<td>F</td>
<td>-0.148</td>
<td>***</td>
<td>-0.338</td>
<td>***</td>
</tr>
<tr>
<td>T</td>
<td>-0.061</td>
<td>***</td>
<td>-0.112</td>
<td>***</td>
</tr>
<tr>
<td>R*C</td>
<td>0.030</td>
<td>***</td>
<td>-0.180</td>
<td>0.054</td>
</tr>
<tr>
<td>R*R</td>
<td>0.000</td>
<td>***</td>
<td>0.212</td>
<td>*</td>
</tr>
<tr>
<td>D*C</td>
<td>0.014</td>
<td>***</td>
<td>0.166</td>
<td>o</td>
</tr>
<tr>
<td>D*R</td>
<td>-0.001</td>
<td>**</td>
<td>-0.143</td>
<td>0.018</td>
</tr>
<tr>
<td>F*C</td>
<td>0.024</td>
<td>0.033</td>
<td>o</td>
<td>0.025</td>
</tr>
<tr>
<td>F*R</td>
<td>0.022</td>
<td>***</td>
<td>0.180</td>
<td>*</td>
</tr>
<tr>
<td>F*D</td>
<td>-0.008</td>
<td>*</td>
<td>-0.078</td>
<td>0.040</td>
</tr>
<tr>
<td>F*F</td>
<td>-0.007</td>
<td>o</td>
<td>0.074</td>
<td>-0.018</td>
</tr>
<tr>
<td>T*D</td>
<td>0.021</td>
<td>**</td>
<td>-0.084</td>
<td>0.004</td>
</tr>
<tr>
<td>T*F</td>
<td>-0.144</td>
<td>***</td>
<td>-0.354</td>
<td>***</td>
</tr>
<tr>
<td>R²</td>
<td>0.815</td>
<td>0.951</td>
<td>0.833</td>
<td>0.877</td>
</tr>
<tr>
<td>R²_adj</td>
<td>0.812</td>
<td>0.950</td>
<td>0.830</td>
<td>0.874</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>Code</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.001</td>
<td>***</td>
<td>Highly Significant</td>
</tr>
<tr>
<td>0.001</td>
<td>0.01</td>
<td>**</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.05</td>
<td>*</td>
<td>Significant</td>
</tr>
<tr>
<td>0.05</td>
<td>0.1</td>
<td>o</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>1</td>
<td>blank</td>
<td>Insignificant</td>
</tr>
</tbody>
</table>

larger than mean, identified by the RSM. Twenty-eight coefficients in the polynomial are screened to obtain seven highly significant (\(P<0.001\)) main and interaction factors. Because the 3D flow model has more practical importance and is usually computationally expensive, those most significant factors (fraction \((F)\), trend\((T)\) ) will be modeled and examined in the following 3D flow simulation.

The WLS model improves the response surface fit. The largest improvement is for effective permeability. \(R^2_{adj}\) increases from 0.81 to 0.92.
categories: (1) significant factors for all responses (cement fraction, $F$ and trend, $T$); (2) marginally significant factors for some responses (conditioning sections, $C$, and range, $R$) and some interactions; and (3) factors are not significant for any responses (dip, $D$ and direction, $O$).

![Figure 5-3. The response surfaces of interests](image)

(a-c) imposing trend and increase concretion fraction will decrease flow responses. (d) The concretion range will also affect sweep efficiency. The models with longer range have higher sweep efficiency.

In summary, in Raptor Ridge system designed factors can be divided into three

5.7 Summary and Conclusions

1. A designed flow simulation examines three flow responses (breakthrough time, upscaled permeability and sweep efficiency) with six geologic and engineering factors, which are the concretion trend, fraction, range, dip angle, injection direction and conditional wells.
2. An orthogonal array design can efficiently span the factor spaces. In this design, 432 simulation runs replace a 1,944 experiment full factorial design.

3. Because of the nonuniform response variances, a weighted least squares method is used to build the response models and improves the model fitting. The adjusted $R^2$ increases for breakthrough time, upscaled permeability and sweep efficiency. Dual models and lack of fit test indicate that the variance is nonlinearly related to the factors.

4. The outcrop cliff face on the dip direction is digitized; and the equivalent flow grid is 781x541 in $y$ and $z$-direction with corresponding $17.3 \times 3$ cm block size, which are smaller than most concretions dimension and can represent the observed concretions.

5. The base case geostatistical and flow models (with all factors at the observed values) are not significantly different from simulations based directly on the digitized outcrop interpretation. This validates the stochastic model. The new algorithm can reproduce the variogram, vertical trend and the two dimensional outcrop flow behavior.

6. The concretion trend significantly decreases $k$ (11 percent), $\tau_{bt}$ (5.6 percent), and $N_{pD}$ (1.4 percent). Compared with low fraction models, the high fraction models decrease $k$ (17.1 percent), $\tau_{bt}$ (6.8 percent), and $N_{pD}$ (3.1 percent). A long concretion range may improve the recovery. The number of conditioning wells has no effect, probably because correlation range is large compared to well spacing.
CHAPTER 6. THREE DIMENSIONAL FLOW SIMULATIONS

Designed 3D flow simulations examine the hydraulic effects of concretions. A 3D flow grid is established using 14 radar-interpreted surfaces. Several flow simulations on selected subgrids guide the choice of the appropriate flow grid size, upscaling subgrid size and the number of stochastic realizations. A nearly orthogonal array design spans the factor space and decreases the required number of simulation runs. ANOVA and response surface models examine the flow responses and analyze factor importance.

6.1 Flow Grid Description

The flow grid uses cornerpoint geometry (Eclipse, Schlumberger Technology Co., 2004). The cornerpoint grid preserves the complex geological geometry more accurately and requires fewer grid blocks than a Cartesian grid, which saves computation time (Li and White, 2002).

The flow model should resolve the concretions; that is, the flow grid sizes in x, y, and z-direction should be equal to or smaller than major concretion sizes. On the other hand, several factors restrict the number of gridblocks. Computational costs increase quickly with the block count, which makes the suites of flow simulations less viable. Because the permeability model is derived from radar data, there is no benefit to modeling flow properties at resolutions finer than the radar resolution. Practically, small concretions may not “detour,” “baffle,” or “seal” the flow path as significantly as larger concretions, so that failure to resolve the smallest concretions should not make much difference in flow responses.

Keeping these points in mind, three flow models with different grid dimensions examine the relationship between flow responses and grids dimensions to guide selection of a reasonable grid size. To allow consideration of very high resolutions, block dimensions are as small as 0.5 × 0.08 × 0.61 m, in x × y × z. The subregion (Figure 6-1) was selected to be representative of the
overall volume. This region is large enough to be representative, and small enough to make very high resolution flow models feasible. Furthermore, the subgrid has top truncation bedding, which is typical of the survey area, the geometry is proportional to original subgrid, and facies ratios and concretion fractions are typical. Finally, the subgrid has similar flow responses to the whole grid (verified subsequently, section 6.2.2).

![Figure 6-1. The location of subregion for grid selection](image)

Left part is the subregion (subgrid1); it is the subgrid for flow grid comparison and selection; right is the whole grid and corresponding location (shaded area) of the subregion. Two wells are drilled: well 8 and 9. The subgrid 2 is selected around well 8 to examine the algorithm of permeability subsampling.

Fourteen GPR interpreted surfaces are used to build the cornerpoint grid (the GRID package, Eclipse, Schlumberger Technology Co., 2004). The 14 layers are interpolated guided by the geological model. For example, the upper part of the delta sediments is interpreted as bar sediments of a top truncated low-stand delta (Bhattacharya and Willis, 2001), which implies that the interpolated layers should follow the top truncation character. The lower part of the subject sandstone body is interpreted as channel deposits. The interpolation is proportional to the thickness, to reproduce the subparallel, prograding geometry. Based on flow simulation of
otherwise identical models with different resolutions, the chosen intermediate horizontal grid block counts are 60 in $x$-direction and 160 in $y$-direction with corresponding 50 by 50 cm$^2$ grid block sizes (Figure 6-2, 6-3; Table 6-1, section 6.2.2).

Figure 6-2. The cornerpoint flow grid at Raptor Ridge
This is the chosen intermediate grid with block counts $60 \times 160 \times 28$ in $x$, $y$ and $z$-directions

6.2 Flow Model Grid Specification

As discussed previously (section 2.3.1), eighty percent of concretions are about 60-270 cm in length and 8-60 cm in height. Radar horizontal resolutions are 20 cm horizontally and 10 cm vertically. Ideally, a flow grid dimension should have 20-270 cm horizontal resolution and 10-60 cm vertical resolution. Flow simulation on part of GPR survey volume examines flow grid influences on flow responses. Then an optimal grid size is determined from flow responses and computational costs.

6.2.1 Method Description

Grid size effects on flow responses are tested using three flow models with different horizontal resolution. The flow models have the same vertical resolution, but the horizontal resolutions vary from 100 by 100 cm for coarse model to 50 by 50 cm for the intermediate model, and finally 20 by 20 cm for the fine model.
Table 6-1. Parameters of intermediate 3D flow grid

<table>
<thead>
<tr>
<th></th>
<th>Pore Volume</th>
<th></th>
<th>Total Grids</th>
<th>26,880</th>
<th>Active Grids</th>
<th>24,427</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Direction</strong></td>
<td></td>
<td></td>
<td>X</td>
<td>Y</td>
<td>Z</td>
<td></td>
</tr>
<tr>
<td><strong>Geometry (m)</strong></td>
<td></td>
<td></td>
<td>30</td>
<td>80</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td><strong>Block dimension (m)</strong></td>
<td></td>
<td></td>
<td>0.5</td>
<td>0.08</td>
<td>0.61&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
</tr>
<tr>
<td><strong>Counts</strong></td>
<td></td>
<td></td>
<td>60</td>
<td>160</td>
<td>38</td>
<td></td>
</tr>
</tbody>
</table>

Execution times for 1 pore volume injection(s) <sup>b</sup>

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>9129</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>551</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>41895</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Count</td>
<td>54</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Assumptions**

- Ideal Tracer
- Single Phase flow
- Constant viscosity
- Small and constant compressibility

<sup>a</sup> 3.4G-MHZ Pentium IV processor

**Figure 6-3. The tracer injection behavior in x, y and z- directions.**

(a) Tracer injection in x-direction. (b) Tracer injection in y-direction. (c) Tracer injection in z-direction. Grid blocks with tracer concentrations below 0.35 are transparent to show displacement geometry.
Establish the flow grids. The 100 by 100 cm cornerpoint flow grid is uniformly interpolated to build flow grids with the desired horizontal resolution. Because the grid block count of the finest flow model (more than 2 million) is too large to be manageable for the large suite of runs needed for sensitivity analysis, a subgrid with overall dimensions of $7.5 \times 20$ m in dip and strike direction is used for the flow comparison. The subgrid is selected to be proportional to the original grid dimensions to ensure the ratio of flow injection and production areas in three directions are same as the ratio of the whole grid. (Figure 6-4).

![Flow grids comparison](image)

Figure 6-4. Flow grids comparison
Subgrids of the outcrop with different grids number are used to test the flow response of different number of grids (a) (1×1m) (b) (0.5×0.5m);(c) (0.2×0.2 m). Different colors represent different layers.

Establish the permeability grid. The permeability grid is based on two parts: (1) outcrop and well bore permeability observation are extrapolated to 3D space within radar interpreted surfaces. (2) The geostatistical procedure SGBSIM integrates radar, well bore and outcrop data to predict stochastic concretion realizations; the geostatistical simulations have the same resolution as radar data. The concretion realizations are superimposed on the deterministic facies-based permeability model to form the final permeability model, which carries the sedimentary and diagenesis information.
**Upscale and assign permeability to flow grid.** A static upscaling algorithm (Li and others, 1999; section 6.3) computes average (or effective) permeability for the flow simulation grid blocks, incorporating the dense geophysically derived geostatistical permeability simulation.

**Conduct flow simulation.** A tracer flow simulation is conducted on these flow grids. Because the concretions are predicted stochastically, six realizations of SGBSIM examine the random error. To eliminate differences caused by stochastic fluctuation, the same dense permeability models are used for the grids of different sizes.

### 6.2.2 Determine the Appropriate Flow Grid Size Using Flow Simulations.

Analysis of Variance (ANOVA) and Tukey pairwise comparison (SAS Institute, 1998) are used to differentiate the flow responses differences of different group from random errors. Finally, the appropriate subgrid sizes for upscaling are determined.

The Tukey comparison of flow responses indicates that the upscaled permeability of the fine grid is not significantly different from intermediate grid. However, the fine grid breakthrough time and sweep efficiency are higher than results for other grids (Figure 6-4, 6-5; Table 6-2). The relative differences breakthrough and sweep are both about 11 percent. The fine grid sweep efficiency is higher than other models. However, the average computational cost for the intermediate grid (average 51s) is far lower than for the fine grid (468s), and the error bars do overlap for the fine and intermediate grids for the other two responses (average permeability and breakthrough time). Such computational cost differences will have an even more pronounced effect with the increase of grid size in the full survey volume (versus this subgrid). The CPU time increases exponentially with the increase of grid sizes (Figure 6-5). The upper limit of grid size on a PC based commercial simulator (Eclipse or CMG) is between $10^6$ and $10^7$. The full grid size with the fine resolution is 2,280,000, which is hard to manage for sensitivity analysis.
Table 6-2. Comparison of the influences of different grid sizes.

<table>
<thead>
<tr>
<th>Grouping</th>
<th>Grid Sizes</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>Coarse</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>Intermeadia</td>
</tr>
<tr>
<td>A</td>
<td>Fine</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Minimum Significant Difference</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>Coarse</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>Intermeadia</td>
</tr>
<tr>
<td>A</td>
<td>Fine</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Minimum Significant Difference</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>Coarse</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>Intermeadia</td>
</tr>
<tr>
<td>B</td>
<td>Fine</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Minimum Significant Difference</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Groups with the same letter are not significantly different from each other.

Figure 6-5. Comparison of the flow responses and CPU time of different grid sizes
The length of vertical bar is one standard deviation of each grid size. A exponential trend line are fitted for CPU time; it indicate CPU time will increase exponentially with the increase of grid sizes
Although the intermediate grid is also computationally expensive, efficient experimental designs can make suites of flow simulations on the intermediate grid size viable.
6.3 Subgrid Size for Permeability Upscaling and Subsampling

The geophysical data and geostatistical simulation are on a denser spacing than is feasible for flow simulation (section 6.2). The high-resolution geostatistical models must be upsampled onto the flow grids. This section discusses methods and parameters used to perform the permeability upscaling.

6.3.1 Method Description

Li and others (2001) reported that the upscaled effective permeability is between the upper and lower bound that are permuted orderings of harmonic and arithmetic averages. Related but less specific results were presented by Cardwell and Parsons (1945). The horizontal effective permeability is close to the horizontal directional upper bounds, and the vertical effective permeability is close to the vertical directional lower bounds. The effective permeability in the $x$-direction is computed assuming that “a given coarse gridblock is subdivided into planes so that each plan is perpendicular to the $x$-axis and is only one fine-grid thick”. Similarly, the effective permeability in $z$-direction is computed assuming that “a coarse block is subdivided into columns so that each column is vertical to the layering direction and is only one one-fine-grid cell wide” (Li and others, 2001; Figure 6-6, upper right).

![Figure 6-6. Simplification of a 3D cube for averaging](from Li and others, 1999)
Permeability is generally more correlating horizontally direction than vertically; this agrees with the empirical observation that the horizontal and vertical permeabilities average differently.

The effective permeability in $x$- direction for the coarse grid is the harmonic mean of the arithmetic means of the fine-grid $x$-directional permeabilities,

$$\bar{K}_x^+ = \frac{\Delta X}{\Delta Y \sum_{j=1}^{N_y} \sum_{k=1}^{N_z} \Delta y_j \Delta z_i \Delta k_i \cdot k_{x,i,j,k}}$$

where $\bar{K}_x^+$ is called the upper bound of the effective permeability in $x$- direction. Upper case is used for upscaled dimensions and properties whereas lower case indicates the finer grid: $\Delta X, \Delta Y,$ and $\Delta Z$ are the coarse grid lengths in $x$, $y$ and $z$ directions; $\Delta x_i, \Delta y_j,$ and $\Delta z_{i,j,k}$ are the fine gridblock lengths; and $N_x, N_y,$ and $N_z$ are number of fine grid planes within a given coarse in the three directions. The upper bound on the effective permeability in the $y$-direction is treated the same way as in $x$-direction.

The lower bounds of $x$ and $y$- direction are same and defined by Cardwell and Parsons (1945) as:

$$\bar{K}_x^- = \frac{\Delta Y}{\Delta X \Delta Z \sum_{j=1}^{N_y} \sum_{k=1}^{N_z} \Delta y_j \Delta z_i \Delta k_i \cdot k_{x,i,j,k}}$$

For the $z$-direction the coarse grid is equivalent to be divided into columns so that each column is vertical to the layering direction and is only one fine grid cell wide. The flow though this grid can be calculated with its harmonic mean of fine grids within the coarse grid. The lower and upper bounds are (Figure 6-6, lower right).
The horizontal arithmetic average is near the upper bound. In the vertical direction, the area extent of permeability heterogeneity is more significant and the estimated upper bound is less than arithmetic average.

The method is modified for cornerpoint grid and is used to determine the subgrid size of upscaling. The cornerpoint grid is proportionally divided into subgrids. The permeability which is closest to the subgrid center is assigned to the subgrid. Above upscaling methods are used to compute the effective permeability.

6.3.2 Procedure and Application

Flow grid and permeability grid description

The flow grid is subsampled. Each active cornerpoint grid block is divided into \( n_x \times n_y \times n_z \) smaller subgrids. The subsampling assigns within-grid points with the closest permeability value of the dense permeability grid. Then the above upscaling algorithm computes the average permeability in the \( x \), \( y \), and \( z \)-directions for each block. The subsampling densities \((n_x, n_y, n_z)\) are varied to minimize the relative error and sum of squared error between the subgrids and the fine subgrid. The subsample is never used for flow simulation; it is used only to
incorporate subgrid-scale heterogeneity into effective permeabilities for use in flow simulation. Because the method is algebraic rather than requiring solution of differential equations, it is fast and easy to use. Li and others (2001) showed reasonably accurate results can be obtained for the level of upscaling being done in this study.

One subgrid close to the well 8 is selected, where the upscaled permeability can be easily compared with the concretion observations. The coarse flow grid size is $3 \times 3 \times 38$ m; each grid is 1 meter long, 1 meter wide and approximate 0.6 meter thick. The dense permeability grid is $15 \times 30 \times 230$, which is regularly sampled within the corresponding grid block dimension of $0.2 \times 0.2 \times 0.1$ m in the x, y and z- directions.

The object is to find effective, upscaled coarse-grid permeability such that flow behavior in the coarse and fine grids is similar. To determine the subsampling fine grids, the collocated coarse flow grid is gradually divided into a series of subsampling fine grids. The coarsest grid is the flow grid itself; and the finest grid is the geostatistical grid. The effective permeability in the x and z- directions are computed for each fine grid. A reasonable subgrid size is determined (similar approach to section 6.1.1), and upscaled horizontal permeabilities are assigned for flow simulation.

**Determination of subsampling density**

Sum of square errors (SSE) and relative errors for each active coarse grid are computed to evaluate the similarity of the subsampling grids. The relative error $\varepsilon_r$ is defined as:

$$\varepsilon_r = \left( \frac{1}{N_xN_yN_z} \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=1}^{N_z} \left| \frac{k_{i,j,k} - k_{\text{finest},i,j,k}}{k_{\text{finest},i,j,k}} \right| \right)$$

where $k_{\text{finest},i,j,k}$ is the estimation of true permeability with finest subgrid. With the assumption
that permeability is heterogeneous \((k_h/k_v=5)\), the SSE and relative error \(\varepsilon_r\) decrease with increasing subsampling density (Figure 6-7). Because the radar resolution is \(0.2 \times 0.2 \times 0.1\) m, or about \(5 \times 5 \times 6\) sub-blocks, a \(6 \times 6 \times 6\) subsampling grid is satisfactory.

![Figure 6-7. Relative error and sum of squared error of upscaling](image)

The vertical (a) and horizontal permeability (b) the SSE and \(\varepsilon_r\) of effective permeability are higher in the vertical direction than in horizontal directions. The relative error indicates that with increasing subsampling density, the relative error decreases quickly: when the one-dimensional sampling density is larger than 6, the relative error slowly decreases to below 2 percent.

6.4 Determination of the Realization Count

Because the concretions are predicted stochastically, multiple realizations are needed to differentiate the flow effects from stochastic fluctuations (which are analogous to pure error in classic experimental design or regression analysis). An ideal design needs a small error in the mean response at any factor combination, which can be decreased by increasing number of repetitions (for experiments) or realizations (for stochastic properties). However, computational expense limits repetitions. Using 2D flow simulations and variance analysis, the required number of repetitions can be estimated using Student’s \(t\)-tests, which is appropriate for these small-sample tests. The Student \(t\)-test is the ratio of the smallest response mean difference to the stochastic fluctuation. This ratio (or \(t\)-value) can be used to test the hypothesis that two models
are not different from each other. If the \( t \)-values are high, then hypothesis is rejected; the critical \( t \)-value depends on the degrees of freedom.

First, minimum model differences are computed using 2D flow models (section 5.3). Response surface models and lack of fit tests can differentiate the model variance, the lack of fit error and the pure random error. Finally, assuming this minimum flow response, a Student \( t \)-test can help select the sample size (or number of realizations; related to degrees of freedom) for a specified confidence interval.

The lack of fit test shows that the stochastic fluctuations are negligible compared to effects of factors such as concretion fraction, trend and flow directions. For example, the minimum mean difference between models is 0.003, whereas the stochastic fluctuation is \(< 1 \times 10^{-7} \). The \( t \)-test shows that very few runs (two) are enough to quantify flow effects accurately (Table 6-3).

<table>
<thead>
<tr>
<th>Responses</th>
<th>Factors</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fraction</td>
<td>Trend</td>
</tr>
<tr>
<td>( \tau_{br} )</td>
<td>0.032</td>
<td>0.048</td>
</tr>
<tr>
<td>( K )</td>
<td>3.300</td>
<td>5.100</td>
</tr>
<tr>
<td>( N_{p1D} )</td>
<td>0.019</td>
<td>0.016</td>
</tr>
</tbody>
</table>

The student \( t \)-value with one degree of freedom and two-tail significance level is 636; All \( t \)-value are higher than 636, which means the null hypothesis should be rejected and 1 degree of freedom (two realizations) is enough to differentiate the model effects.

The lack of fit test indicates that the model effects are significant and even with few repetitions (2-3) factor effects on responses can be quantified accurately. In this study 3 repetitions are used.
6.5 Nearly Orthogonal Array Design and Selection of Number of Runs.

The responses are the upscaled permeability ($\bar{K}$), breakthrough time ($\tau_b$) and sweep efficiency ($N_{op}$; chapter 5). Four factors are examined: including radar, concretion proportion, trend model, and flow direction (Table 6-4). For this design, the number of second order polynomial coefficients $k$ equals to $(4+1)(4+2)/2=15$.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Levels</th>
<th>Number of Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radar ($R$)</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>($F$)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Cement fraction ($F$)</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Cement trend ($T$)</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Flow direction ($O$)</td>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

a the multiplier of observed concretion parameters. Similarly, the dip angle and the cement fraction are less, equal and larger than the observed values.

b the symbols of factors and levels are used in experimental design

The minimum runs of the design should not be smaller than $\frac{3}{2}k = 6$ (section 5.3). An available NOA (near orthogonal array design) with 18 runs is selected (Kalla, 2005). Three stochastic realizations (or model repetitions) are used to assess stochastic fluctuations. The OA design of 4 factors and 10 total levels is listed in Table 6-5. This NOA design decrease the number of simulation from full factorial $2^4 \times 3^2 \times 3 = 144$ runs to $18 \times 3 = 54$ runs (the final multiplier of 3 is for realizations).
Table 6-5. Nearly Orthogonal design with 18 runs

<table>
<thead>
<tr>
<th>Levels</th>
<th>Factors</th>
<th>Fraction(F)</th>
<th>Direction(O)</th>
<th>Radar(R)</th>
<th>Trend(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Low</td>
<td>x</td>
<td>On</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Middle</td>
<td>y</td>
<td>Off</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>High</td>
<td>z</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Near Orthogonal Array Design

<table>
<thead>
<tr>
<th>Runs</th>
<th>F</th>
<th>O</th>
<th>R</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>17</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>18</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

6.6 Response Surface Model and Sensitivity Analysis

6.6.1 Analysis of Variance

As discussed in 2D modeling, weighted least square regression is used to model the response surfaces and higher $R^2$ are achieved (Table 6-6). All factors except fraction have significant effects on flow responses. The $y$-direction (dip direction) has the highest breakthrough time, sweep efficiency and upscaled permeability, which is followed by the $x$-direction and $z$-direction. Imposing radar raises the upscaled effective permeability and increases the sweep efficiency.

The detailed responses are plotted among the significant factors (Figure 6-8). The models with radar perform differently than models that ignored radar data. A further discussion of these flow responses is presented in chapter 7.
6.6.2 Sensitivity Analysis

The sensitivity analysis indicates that the effective permeability is the most sensitive flow response, which has significant contributions from the most factors (Table 6-6). The radar, trend and direction (G, T, O) have significant effects (with above 95 percent confidence) on the upscaled permeability. Similarly, sweep efficiency is sensitive to radar and injection direction (G, O). The dimensionless breakthrough time is the least sensitive response, which is also sensitive to direction (O, T and F). Among the four designed factors, the injection direction O has the biggest effects on all flow responses.

Table 6-6. Sensitivity analysis of response surface

<table>
<thead>
<tr>
<th>Responses</th>
<th>K_{eff} (md)</th>
<th>Sweep Efficiency (pv)</th>
<th>Breakthrough Time (pv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factors</td>
<td>Coeff. d</td>
<td>P value</td>
<td>Coeff.</td>
</tr>
<tr>
<td>G</td>
<td>-0.07</td>
<td>0.002</td>
<td>-0.20</td>
</tr>
<tr>
<td>T</td>
<td>-0.05</td>
<td>&lt;0.001</td>
<td>-0.04</td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
<td>-0.36</td>
</tr>
<tr>
<td>O</td>
<td>0.92</td>
<td>&lt;0.001</td>
<td>0.26</td>
</tr>
<tr>
<td>G*G</td>
<td>-0.41</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>T*G</td>
<td>-0.11</td>
<td>0.012</td>
<td>0.39</td>
</tr>
<tr>
<td>T*T</td>
<td>-0.09</td>
<td>0.034</td>
<td></td>
</tr>
<tr>
<td>F*G</td>
<td>-0.05</td>
<td>0.011</td>
<td>-0.07</td>
</tr>
<tr>
<td>F*T</td>
<td>-0.04</td>
<td>&lt;0.001</td>
<td>0.03</td>
</tr>
<tr>
<td>O*G</td>
<td>-0.59</td>
<td>&lt;0.001</td>
<td>1.38</td>
</tr>
<tr>
<td>O*T</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O*F</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O*O</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Bold letter represents factors with P value <0.01; italic with P [0.01-0.05], factors with P >0.05 are represented by blanks, in other words, the factors tabulated are significant with 95% of confidence.

Three scenarios with large concretion fraction and range are examined ranges (Table 6-7). Simulation indicates that the large correlation range will decrease the vertical permeability.
Figure 6-8. The responses of 3D flow simulation.
(a) Imposing radar and no trend increase the effective permeability. (b) The effective permeability is highest in y-direction, followed by x-, z-direction. (c) Injection direction in z- and y- has higher sweep efficiency. Imposing radar also causes higher sweep efficiency. (d) Similarly, injection direction in z- and y- has higher dimensionless breakthrough time. The cases with trend have higher breakthrough time.

Table 6-7. Comparison of 3D flow responses

<table>
<thead>
<tr>
<th>Responses</th>
<th>Upscaled Permeability (md)</th>
<th>Sweep Efficiency (pv)</th>
<th>Breakthrough Time (pv)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level Means</td>
<td>Level Means</td>
<td>Level Means</td>
</tr>
<tr>
<td>Factors①</td>
<td>G</td>
<td>15.20</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>14.48</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>14.20</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>O</td>
<td>19.66</td>
<td>0.72</td>
</tr>
<tr>
<td>Increase</td>
<td>Max. K_x/K_y ②</td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>F,R ③</td>
<td>7.68</td>
<td>16.63</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>7.21</td>
<td>15.33</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>8.22</td>
<td>20.69</td>
</tr>
<tr>
<td>Response Difference</td>
<td>X</td>
<td>Y</td>
<td>Z</td>
</tr>
<tr>
<td></td>
<td>F,R ③</td>
<td>1.43</td>
<td>3.38</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>0.33</td>
<td>1.74</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>5.49</td>
<td>3.19</td>
</tr>
</tbody>
</table>

① Reference case FR: increasing fraction and range to 4 times larger than outcrop; case F: fraction is 4 times larger; and range value is same as outcrop; case R: range 4 times larger and fraction percentage is same as outcrop.
② The largest anisotropy of permeability.
and increase the permeability anisotropy \( (k_h/k_v) \). The average anisotropy of upscaled permeabilities is 7.2 and 8.0 in \( x \) and \( y \)-direction. Case \( R \) with range 4 times larger than the
The increasing fraction of concretion increases the tortuosity, so that the sweep efficiency observed value increases the anisotropy to 8.2. The increasing anisotropy effect is occurs because elongated concretions works decrease the vertical permeability more than horizontal permeability. Furthermore, the increasing fraction and range has more effect on permeability in the \( y \)-direction than in \( x \)-direction (Table 6-7, case \( FR, F \)) because elongate concretions are inclined in the dip (\( y \)) direction, which causes more shunting of flow or tortuosity. Thus, the effect is greater for dip-direction flow more than for strike, in which the concretions are approximately horizontal.

is decreased for case \( RF \) and case \( F \). The low fraction case \( R \) has higher sweep efficiency than average (Table 6-7). This effect will be discussed further in chapter 7.

6.6.3 Model Fitting and Lack of Fit Test

The response variability can be adequately interpreted by RSM model. The \( R^2 \) of all responses are above 0.96. The lack of fit test (F-test) indicates that the RSM adequately models response variability; the lack of fit error is not significant.

High \( R^2 \) and insignificant lack of fit error imply that running more models may not improve the overall model fit much.

6.7 Conclusions

(1) A cornerpoint flow grid is established using the 14 radar interpreted stratigraphic surfaces. Stratigraphic surfaces are interpolated in accord with the geologic conceptual model.

(2) By comparing flow responses for several candidate grids, an intermediate flow grid is chosen to balance computational cost and prediction accuracy.
(3) Comparing the upscaled permeability and subgrid numbers led to a choice of $6^3$ subsamples are used to upscale permeability with flow model gridblocks.

(4) Three stochastic realizations (or model replications) are selected based on a Student $t$-distribution.

(5) A nearly orthogonal array (NOA) design is used to decrease the total simulation runs from full factorial 144 to 54 runs.

(6) The flow along $y$-direction has highest flow responses, which is followed by $x$ and $z$, because the concretions are inclined in the $y$- direction and almost parallel in the $x$- direction.

(7) The radar updating increases the upscaled permeability and recovery, and is as significant as imposing trend for many responses. The radar update interacts with many other factors such as direction and fraction (Table 6-7). Those interactions complicate the flow responses and significant effects on effective permeability and sweep efficiency (chapter 7).

(8) Imposing the trend increases upscaled permeability.

(9) The concretions fraction have fewer significant effects on 3D flow than on 2D flow, especially for the factor ranges that appear reasonable at the Raptor Ridge locality.

(10) A second order polynomial RSM model adequately explains flow model variability. The lack of fit test indicates there is no significant lack of fit error. Weighted least square method is used to improve the fitting of the response surfaces model.
CHAPTER 7. DISCUSSION

7.1 2D Hydraulic Response of Concretion

At Raptor Ridge system, upscaled permeability is the most sensitive response to concretion in 2D simulations. According to the 2D sensitivity analysis, increasing the fraction ($F$) from 8 to 15 percent decreases mean upscaled permeability by 17.1 percent (Figure 7-1). Imposing the observed trend rather than assuming uniform concretion frequency decreases upscaled permeability by 11.1 percent (Table 5-6, Figure 7-2).

Figure 7-1. The flow responses of concretion fraction (a) and (b) are the permeability and tracer flow front. (c) (d) are a permeability and tracer flow front with concretion fraction three times larger than (a), (b). Increasing concretion fraction decreases the effective pore volume. However, the concretion modifies the flow front and disperses tracer flows through higher beds, improving the sweep.

Figure 7-2. Comparison of concretion fraction on flow response (a) (b) are the permeability model of concretion realization without trend; and the flow front when tracer breaks through; (c) the permeability model of concretion realization with trend; and other factors are same as (a); (d) the flow front when tracer breakthroughs; compare with (b) the trend deforms the flow front and retard the breakthrough.
Imposing trend \((T)\) decreases upscaled permeability because the concretions are then concentrated in high permeability zones (facies 5a and 4b) and reduce flow through these zones (Nyman, 2004). Large concretions block the flow path and hinder flow, whereas the small concretion may able to significantly change flow paths.

Increasing range \((R)\) increases recovery (Figure 5-3 (d)). Longer correlation ranges further change the flow path (Figure 7-1). Continuous concretions force more flow to go through the upper part of sandstone body and increase the sweep efficiency \((N_{pD})\).

7.2 3D Hydraulic Response of Concretion

As White and others (2002) stated, 2D concretion heterogeneity would be expected to have larger effects on flow responses than 3D. Indeed, 3D concretion effects are smaller than 2D in this study area, and the 3D effects are not very large. The Raptor Ridge system is insensitive to the details of concretion fraction and distribution because of the low concretion fraction and small concretion dimensions compared to the flow domain.

Injection direction, radar and imposing trend have significant effects on flow responses (Table 6-7). The concretion fraction \((F)\) has no significant effect on flow responses. Small coefficients of \(F\) in the response surface model indicate that changing these factors will not affect response much.

Even though the radar concretion responses are noisy, the radar updates alter concretion predictions and affect the flow responses. The radar conditional probability improves the geostatistical concretion prediction. The radar update creates images with fewer small, scattered concretions. This update decreases the overall counts of concretion blocks and improves the reservoir quality (Figure 7-3). Fence plots of transmissibility show radar update decreases the number of low permeability concretion blocks so that reservoir quality is significantly increased.
The update increases average permeability from 14.2 md to 15.2 md and increases sweep efficiency from 0.79 to 0.81. The radar data were analyzed in integrated to ensure that their weak information content was not overstated (section 3.xxxxx). Even so radar data lead to models with significantly different reservoir quality and flow behavior. At Raptor Ridge, the fully integrated models have higher quality and recovery, but this result need not hold for other geologic settings and features.

![Figure 7-3 Comparison of radar updates and its influence on sweep efficiency](image)

(a) (b) (c)

Figure 7-3 Comparison of radar updates and its influence on sweep efficiency (a-c) are properties of radar updated case. (d-f) are properties of the case with same parameters except radar. (a)(d) are the fence plots of transmissibility. Blue blocks in the middle represent low permeability concretions. Low transmissibility blocks at bottom 2 layers are shale. (d) has less concretions than(a). Compared with no radar case (e), radar update cleans the concretion (b). As a result, more tracer is bypassed in (f) (no radar) than in (c) (with radar). Reservoir quality is improved with radar update.

The effects of injection direction are shown in Figure 7-4. Different injection direction has different flow path so that different effective pore volume are bypassed. Injection along the dip direction (the same direction as concretion dipping direction) has the highest sweep efficiency. (Figure7-4, (a)) In x- direction large amount of up-dip pore volume (most channel deposition) are bypassed because of the breakthrough at down-dip (Figure7-4, (a)). The complex facies boundaries, low permeability and concretion distribution might cause the low recovery.
The total amount of concretions is low (about 8-10 percent), and variations around this range still appear to have no affect in 3D. Although the trend may create a local flow barrier (with concretion abundance above 20 percent) and affect flow responses slightly, in this study area the concretions are not abundant enough to affect the overall flow responses significantly. This is different from the 2D model (chapter 5), which has more restricted flow path. This result has important implications for previous, two-dimensional studies of the effects of concretions on flow (e.g., White and others, 2003)

![Image](image.jpg)

**Figure 7-4. The effective pore volume tracer bypassed after 1 pore volume injection.**

The three scenarios use same modeling parameters except the injection direction (O). The different color of blocks represents different tracer concentration in flow grid. Black arrow represents the injection direction. Sweep efficiency in z- direction is highest, which is followed by z- and x- directions.
Three reference scenarios with extreme but plausible factor ranges examine factor influences on flow responses. Case $FR$ has the concretion fraction and range values four times larger than outcrop observations. Case $F$ has the same fraction as case $FR$ and the range is the observed value. Case $R$ has a 4 times larger range and the same fraction as observed value.

As expected, the large fraction decreases the permeability in all directions. A Student $t$-test compares the mean effects of the 144 designed flow responses, the high fraction and range ($FR$) decrease upscaled permeability by 15, 19 and 16 percent in $x$-, $y$- and $z$- direction (Table 6-6). Compared with the case $R$, increasing the concretion fraction decreases the average sweep efficiency about 4-5 percent (Table 6-6, Figure 7-5).

![Figure 7-5. Three concretion realizations and their sweep efficiency](image)

The left blue blocks are concretion realizations on the flow grid. The right green blocks represent the residual volume after 1 pore volume flooding; tracer concentration larger than 0.1 are removed. (a)(b) Case 1, with large fraction and large range, has a large volume upswept. (c),(d) Case 2 with large fraction and low range also has large volume upswept; (e) (f) case 3 with large range and low fraction has little volume upswept. The fraction unswept is related to $1 - N_{pD1}$.
The long correlation range of concretions increases the permeability anisotropy. Increasing concretion fraction affects \( y \)- and \( z \)-directions more because concretions are inclined along the dip (\( y \)) direction and have large cross-sectional area perpendicular to \( z \). As a result, they are more influential on the flow along or perpendicular to the elongation direction. A long concretion range increases the permeability anisotropy, but has less significant effects on sweep efficiency and breakthrough time, because long concretion ranges will baffle the flow path like shales, but do not affect pore volume (Table 6-6, Figure 7-1). The \( k_h/k_v \) is increased from 7.2 to about 8.2.

In general, increasing concretion fraction significantly decreases the upscaled permeability, sweep efficiency, and breakthrough time. Longer concretion ranges increase permeability anisotropy.

### 7.3 Computational Issues

The SGBSIM is modified from GTSIM to integrate secondary data (such as radar or seismic data) with little additional CPU time cost (Appendix B). The extra computational time comes from the Bayes update and probability-\( z \) value transformation. A computational experiment quantifies the computational cost. At each node, the Bayes update is redundantly cycled 1000 times to allow cost estimates. The GTSIM is modified to read in void radar data so that the difference of the computational cost is only caused by the Bayes update and data-transformation. The tested geostatistical grid has 552,000 nodes. The average CPU time for GTSIM is 44 s, whereas the SGBSIM CPU time with 1000 Bayes update per block is 230 s (Athlon 2.8GHz CPU). The average extra additional CPU time per Bayes update is 0.186 s \( [=\frac{(230-44)}{1000}] \), or 0.4 percent of the GTSIM run time. This expense is negligible, as would be expected for a local calculation compared with the searching and linear algebra required for
other components of GTSIM. The extra-time cost compared with GTSIM is negligible (0.93s) even for the largest grid size (more than 2 million geostatistical blocks) in this study.

SGBSIM is very similar to GTSIM. The ability of variogram, conditional well integration and the trend imposing are inherited. The parameters file is also very similar (Appendix B).

A clustering analysis method was also developed for this SGBSIM application. In order to honor the observed proportion after Bayes update, nonlinear regression (Developing Microsoft Excel 95 Solutions, Microsoft Press, 1995) optimizes cluster description to predict categories. There are two common disadvantages of the nonlinear solver. First, the solution depends on the initial guess and easily converges to local minimal values instead of the global minimum. Second, the trial-and-error procedure makes the nonlinear solver computationally expensive. Some more sophisticated algorithm such as simulation annealing (Deustch and others 1994), genetic algorithms (Güyagüler and Horne, 2004) or metropolis algorithm (Aarts and Korst, 1990) are reported to improve the solver performance. In this study, those advanced optimization algorithms have not been applied, because for this problem nonlinear solver performs adequately and it is readily available, convenient and easy to use.

Macros (or spreadsheet programs) read and store radar attributes in memory. These attributes can be repeatedly used until the worksheet is closed. This feature improves the performance for iterative calculation.

The experimental design and related parameters are read by a macro. The parameter files for geostatistical computation and flow simulation are also automatically prepared (APPENDIX C). Parameter files are organized into folders with names reflecting factor settings, so that simulations can be run with simple, standardized batch files. Finally, several computers simultaneously run simulations. The elapsed time is decreased from almost a week (137 hrs) to
one-half day.

Other time-consuming components of this study are stratigraphic surface building, 2D photomosaic profile digitizing and processing, radar data subsampling, upscaling and simulation results processing and analyzing. The tracer flow simulation and stratigraphic surface building are the most time consuming parts.

7.4 Comparisons with Previous Studies

The Raptor Ridge study is different from previous 2D models of the Frewens sandstone (White and others, 2003; Dutton and others 2002). The concretions have different proportions, dimensions and spatial distributions. The Raptor Ridge concretion fraction (8-10 percent) is slightly smaller than Frewens sandstone (average 12 percent). The concretions at Frewens are larger (the median 4.2×5.3×0.6 m in length, width and thickness) than the concretion of this study (median about 1.8×1.8×0.3 m). The Frewens concretion proportion increases upward with mean fraction 11 percent at top 11m. The concretion in our study has high 20 percent mean fraction within middle 3.5 m (most within facies 5a and 4b) and decreases upward and downward. Frewens concretion has a larger influence on upscaled permeability than our case (45 percent compared to 21.1 percent). White and other’s method of 2D validation is applied to the Raptor Ridge system.

Li and White (2002) use GPR data to predict the 3D shale occurrence and its flow responses. Li and White’s model uses a deterministic method to correlate the shale occurrence from well bore and extrapolated to 3D space, whereas we use a probabilistic method because the radar concretion response is noisy. The radar attributes and concretion relationship are based on well bore data, and cluster analysis transforms the 3D radar attribute into conditional probability of concretion occurrence. Because the radar uncertainty is very high at Raptor Ridge, this
method has advantages of not only predicting the occurrence of concretions but also evaluating the uncertainty of radar prediction.

Novakovic and others (2002) used truncated Gaussian simulation method to impose the shale trend and stochastically predict the shale distribution. However, this model only used the semivariogram from outcrop observation for shale modeling and did not use radar information.

SGBSIM directly transforms the radar data and updates other data (semivariogram, trend or well data) within a Bayes framework, which is robust in modeling 3D geological objects. Furthermore, SGBSIM can integrate the observed semivariogram data, trend and radar data whereas GTSIM cannot include radar data.

These three models (current study, Li and White, and Novakovic and others) share many similarities. All of them are three-dimensional, the grids are stratigraphic, and shale or concretion predictions are stochastic.

7.5 Applications

The study of reservoir analogs not only helps us understand the geological and engineering 3D character of low stand, force regression, top truncated deltaic reservoir analogs, but also can provide results or methods to study the analogous reservoirs.

The data integration method in this study can be easily extended to seismic flow barrier interpretation. Using statistical methods to calibrate log and seismic relation, seismic attributes can be transformed into geobody conditional probabilities. The conditional probabilities can be integrated with other data using Bayes rule.

If a reservoir modeling team identifies a trend of the flow barriers (concretions, shales, or low permeability zones with contrasting geophysical properties), then SGBSIM can model non-stationary trends. The trends can be quantified by well bore data or geological interpretation, and
the beta-Bayes method can use a small core training set and multiple logs for facies classification. Second, facies semivariogram estimation is difficult for subsurface modeling. Object-oriented modeling techniques or unconditional simulation might be reasonable options. These methods honor the well bore observation and geological conceptual model. Finally, SGBSIM combines trend, seismic conditional probability and variogram to predict the facies occurrence (or other categories) in 3D space. With the ongoing exploration or production operation, more data are available and can be gradually added and update the existing model within SGBSIM.

Because orthogonal design (OA) or nearly orthogonal array (NOA) effectively span the factor space and minimize the simulation runs, it is useful for simulations of large grid size or simulation designs.

The simplified tracer flow model is used to test the flow behavior of reservoir analogs. Because tracer simulation cannot investigate effects of gravity, relative permeability and capillary, a two or three phase simulation might be useful to study these effects.

**7.6 Future Work**

The concretion prediction uses on well log data, radar data and outcrop photomosaic profile. In this study, using the statistic method to integrate radar data is robust to the uncertainty of noisy concretion radar responses. However, noisy radar attributes of concretions makes the Bayes update less informative. As a result, a detailed seismic processing using amplitude and frequency to create a more accurate and realistic radar-concretion model could improve concretion modeling.

Modeling accuracy might be improved by using resistivity, neutron or density logs, which have good sensitivity to calcite concretions. Shallow unconsolidated outcrops may cause
low core recovery and makes the cement correlation inaccurate, which is critical for radar calibration. Furthermore, partially cemented sandstones are not easy to identify. Wireline logs can help solve these problems.

Facies are deterministically assigned to the stratigraphic grid in this study. Using radar derived facies and permeability models would be more realistic. Several alternative approaches could be compared. First, the traditional way is deterministically interpreting facies (or stratigraphic) boundaries by tracing radar reflection surfaces. Second, statistical methods such as clustering analysis, discriminant analysis or logistic regression could be used to predict the conditional probability of facies and permeability with a training set from well bore data to calibrate facies probabilities using 3D radar data. This approach would be similar to the wireline facies classification discussed in this dissertation. Third, truncated Gaussian simulation can predict several facies (Xu and Journel, 1993, Matheron and others, 1987). SGBSIM could combine the above methods to predict the facies. These alternative facies model could be compared with each other. If there are big differences between models, well calibration and seismic surfaces would have to be checked carefully.

We use permeability upscaling, OA, NOA design and grid size sensitivity analysis to specify reasonable but simpler flow models and decrease the number of simulation runs. Here, we depend on the assumption that different flow grid sizes with similar upscaled permeability should have similar flow response. This assumption needs to be validated, comparing the flow responses of the fine grids with responses of these gradually upscaled grids.
CHAPTER 8. SUMMARY AND CONCLUSIONS

Geostatistical models from this shallow, top truncated, forced regression deltaic reservoir can be used for 3D reservoir characterization of reservoir analogs.

Wireline data can be used to predict facies, using the proposed beta-Bayes method. This method eliminates difficulties in bin selection when computing conditional probabilities. Confidence, discrimination ability, and probability logs compare the prediction performance of the statistical methods. For a given dataset, the new method is comparable to traditional statistical methods. The fitted classification models guide three-dimensional facies and concretion interpretation and prediction and provide error estimates.

The concretions in Raptor Ridge deltaic deposits are characterized by 3D variograms of digitized photomosaic profiles. The concretion distribution is modeled by a vertical trend model and nested variogram models.

A new sequential Gaussian Bayes based on truncated Gaussian simulation and Bayes rule. SGBSIM integrates diverse outcrop, well bore, and geophysical data. It imposes trends and integrates radar attributes or other secondary data with little extra computational cost. Because radar responses to concretions are noisy, correlation uncertainty is propagated through the simulation process with a cluster analysis method, which estimates conditional probabilities linking radar responses and facies probabilistically. SGBSIM updates kriged indicator estimates with the radar conditional probability model using Bayes rule.

A designed 2D flow simulation verifies that flow responses of the geostatistical models are not significantly different from the responses of observed models. The base model uses the same parameters as the observed value; no parameters are adjusted to obtain the match. This indicates that the proposed geostatistical models are adequate to capture the effects of the concretions observed in the outcrop exposure.
2D sensitivity analysis shows that increasing concretion fraction and imposing vertical trend both decrease the reservoir quality (\( \bar{K} \), \( \tau_{ht} \) and \( N_{pD} \)). Even though the response differences are statistically significant, the differences are not practically significant because of the low fraction and small dimension of concretion at Raptor Ridge locality.

A cornerpoint grid is established from radar interpreted stratigraphic surfaces, in which the \( z \)-surfaces of the grid conform to the geological model. Flow simulations on a representative subgrid examine the flow responses of three different horizontal grid sizes. The intermediate flow grid is selected to balance computational cost and prediction accuracy. Furthermore, an upscaling algorithm compares the differences between coarsened subgrids with the finest subgrid. A \( 6 \times 6 \times 6 \) discrete grid subsamples and averages the dense radar-permeability grid.

The concretions have fewer effects on 3D flow than 2D flow. The flow direction significantly affects the all flow responses. Because of the low concretion fraction in the study area, fraction has no significant effects on flow response. Radar updates, in spite of the weak correlation with the concretion occurrence, can significantly affect concretion prediction. At Raptor Ridge, integrating radar responses yields models with higher effective permeability and sweep efficiency.

The methods of this reservoir analogs study can be extended to subsurface reservoir modeling and simulation. SGBSIM improves the traditional truncated Gaussian simulation by integrating secondary data with small extra cost. Cluster analysis of seismic data can be applied to integrate seismic and geostatistical modeling.

The workflow of designed simulation improves simulation sensitivity analysis, history match and optimization.
REFERENCES


Bari Güyagüler, Roland N. Horne, Uncertainty Assessment of Well-Placement Optimization, SPE 87663, Journal SPE Reservoir Evaluation & Engineering, Issue Volume 7, Number 1, February


Chu, C. F., 1990, Prediction of steamflood performance in heavy oil reservoirs using correlations developed by factorial design method, paper SPE 20020 presented at the 60th California Regional Meeting, Ventura.


Deutsch, C.V and Cockerham,P.W., Geostatistical Modeling of Permeability With Annealing Cosimulation (ACS) SPE 28413 presentation at the SPE 6Sth Annual Technical Conference and Exhibition held in New Orleans, W U.S. September 1994,


Gani, Royan, 3D facies architecture of a delta lobe deposit: example from the Turonian Wall Creek Member, Central Wyoming, USA, Ph.D. Proposal, 2002


Functions”, Paper SPE 4071 presented at 47th SPE-AIME Annual Meeting held in San Antonio, TX, October 8-11, 1972


Kalla, S., 2005, [unpublished MS thesis]: The Louisiana State University


Kyte, J.R., Berry, D.W.: “New Pseudo Functions To Control Numerical Dispersion”, Paper SPE 5105 presented at the 49th SPE-AIME Annual Fall Meeting, held in Houston, TX, October 6-9, 1974


Li, D., Beckner, B., Kumar, A.: "A New Efficient Averaging Technique for Scaleup of Multimillion-Cell Geologic Models", Paper SPE 56554 presented at the 1999 SPE Annual Technical Conference and Exhibition held in Houston, TX, October 3-6, 1999


Li, H and White, C.D., Geostatistical models for shales in distributary channel point bars(Ferron Sandstone, Utah): From ground penetrating radar data to three-dimensional flow modeling, AAPG Bulletin, V87, No.12(December 2003), PP.1851-1868


Peterson, J.A. and Franczyk, K.J., eds., Mesozoic Systems of the Rocky Mountain Region, USA, Rocky Mountain Section, SEPM, p.393-413


Sheriff, R.E. 1977, Limitation on resolution of seismic reflections and geologic detail derivable from them. In Payton, C.E (Ed.), Seismic stratigraphy- Application to hydrocarbon Exploration, AAPG Mem, 16:3-14


Tyler, N., 1988, New oil from old fields, Geotimes, July, p. 8-10.


White, C. D., 3D Sedimentologic and geophysical studies of clastic reservoir analogous: facies architecture, reservoir properties and flow behavior within delta front facies elements of the cretaceous Wall Creek member, Frontier formation, Wyoming, DOE Continuation Research Proposal, 2001


Xu, W., and Journel, A. G., 1993, GTSIM: Gaussian truncated simulations of reservoir units in a west Texas carbonate field: SPE Paper No. 247412. (unsolicited)

Xue, G. etc. “Optimal Transformations for Multiple Regression: Application to Permeability Estimation from Well Logs” SPE/DOE 35412, SPE Improved oil Recovery Symposium, Oklahoma 21 April 1996

APPENDIX A. NOMENCLATURE

$A = \text{instantaneous amplitude}$

$A_0 = \text{initial amplitude}$

$\bar{A} = \text{centroind amplitude for the cluster}$

$c = \text{the indicator of concretion}$

$c_0 = \text{the electromagnetic wave velocity in a vacuum (}3 \times 10^8 \text{ ms}^{-1}\text{)}$

$\bar{c} = \text{the indicator of non-concretion}$

$C = \text{the number of conditioning sections used in conditioning the stochastic simulations}$

$C_y(.) = \text{the covariance function}$

$D = \text{dip direction of concretion}$

$D^2(.) = \text{generalized squared distances from cluster center}$

$n_{ii} = \text{represent number in } i \text{ row and } i \text{ column}$

$n_i = \text{total marginal number of each classified facies}$

$n_j = \text{total marginal number of each geological prior facies}$

$n_{..} = \text{grand total number}$

$f_k(.) = \text{the concretion trend curve}$

$F = \text{the concretion fraction}$

$G = \text{the indicator of imposing radar}$

$G^{-1}(.) = \text{the inverse standard Gaussian distribution}$

$h = \text{distance vector}$

$i(.) = \text{the indicator}$

$\bar{k} = \text{upscaled permeability}$
\( k_h = \) the horizontal permeability
\( k_v = \) the vertical permeability
\( \bar{K}_x^+ = \) the upper bound of the effective permeability in \( x \)- direction
\( \bar{K}_x^- = \) the lower bound of the effective permeability in the \( x \)- direction
\( \bar{K}_z^+ = \) vertical upper bound permeability
\( \bar{K}_z^- = \) vertical lower bound permeability
\( N(.) = \) the number of data points
\( N_{PD} = \) the sweep efficiency
\( N_x = \) number of fine gridblocks in the \( x \)- directions
\( N_y = \) number of fine gridblocks in the \( y \)- directions
\( N_z = \) number of fine gridblocks in the \( z \)- directions
\( P(c) = \) the estimated concretion probability prior to including the radar data.
\( P(r | c) = \) the conditional probability of radar attributes given concretion classification
\( \gamma_I(.) = \) the indicator semivariogram
\( R = \) semivariogram range
\( t(.) = \) vertical pseudo Gaussian value of concretion trend
\( T = \) The concretion trend
\( u_i = \) location vector
\( x = \) collocated geophysical data
\( \Delta X = \) the coarse grid lengths in \( x \)- direction
\( \Delta x = \) the fine grid lengths in \( x \)- direction
\( \Delta Y = \) the coarse grid lengths in \( y \)- direction
\( \Delta y \) = the coarse grid lengths in \( y \)- direction

\( y(.) \) = the simulated standard Gaussian variable

\( \Delta Z \) = the coarse grid lengths in \( z \)- direction

\( z \) = radar travel distance

\( \Delta z \) = the fine grid lengths in \( z \)- direction

\( \Sigma \) = covariance matrix

**Greek Symbols**

\( \alpha \) = the attenuation constant

\( \varepsilon \) = dielectric permeability

\( \varepsilon_r \) = the relative errors

\( \sigma \) = electro conductivity

\( \mu \) = magnetic permeability

\( \hat{\mu} \) = the estimated mean

\( \hat{\delta} \) = the estimated variance

\( \rho \) = a random number

\( \tau_{bt} \) = the breakthrough time

\( \bar{\omega} \) = the centroid frequency for the cluster.
APPENDIX B. SEQUENTIAL GAUSSIAN BAYESIAN SIMULATION SOURCE CODE

As discussed in chapter 4, the sequential Gaussian Bayesian Simulation algorithm proceeds as follows:

- Construct a fine 3D grid with well data assigned to closest grid cells
- Define a random path
- Until each nodes on the random path is visited

Step 1. search for closest nearby well data and previously simulated nodes
Step 2. Kriging estimation based on data found in step 1
Step 3. Trend truncate Gaussian variable to convert to probability
Step 4. Bayes update with the radar derived conditional probability

Step 5. Fire a random number determines the simulated categories at current location; and transform to Gaussian variable.

- End Simulation

C ------------    program main     -----------------------------------------------

c   read in parameters file

c   define a random path

c   Step 2. Perform the kriging.
   call krige(ix,iy,iz,xx,yy,zz,lktype,cmean,cstdev)

c---------------------------------------------------------------------------------

  c   Step 3. truncate Gaussian variable to get probability p(index)
  c   thres( ) is the input trend curve, cmean cstdev is the kriged
  c   mean and standard deviation at current location
  c   zt is the adjusted Gaussian value and p is truncated probability .
  zt=(thres(index)-cmean)/cstdev
  p=gcum(zt)

c---------------------------------------------------------------------------------

  c   Step 4 Bayes update “p” with radar computed conditional probability “cp( )”
updated posterior probability is \( \text{post}() \).
\[
\text{post}(\text{index}) = \frac{\text{cp}(\text{index}) \times \text{p}}{(\text{cp}(\text{index})) \times \text{p} + (1 - \text{cp}(\text{index})) \times (1 - \text{p})}
\]
Step 5. A random number determines the simulated categories at current location;
- Generate random number of two interval \([0 - \text{Pc}]\) if concretion, \((\text{Pc} - 1)\) if non-concretion; \(\text{pp}(\text{index})\) is the probability of posterior determined indicators
- \(\text{p} = \text{acorn}(\text{idum})\)
- if \((\text{p} \leq \text{post}(\text{index}))\) then
  - \(\text{pp}(\text{index}) = \text{p} \times \text{post}(\text{index})\)
- else
  - \(\text{pp}(\text{index}) = \text{p} \times (1 - \text{post}(\text{index})) + \text{post}(\text{index})\)
- endif
- transform to Gaussian variable.
- \(\text{call gauinv(dble(\text{pp}(\text{index}))),xp,ierr)}\)
- \(\text{sim}(\text{index}) = \text{xp} \times \text{cstdev} + \text{cmean}\)
- Goto step 2 Until each nodes on the random path is visited
- Output the indicators of all nodes
- End.
**SGBSIM PARAMETERS FILE “sgbsim.par”**

Parameters for SGBSIM

************************

START OF PARAMETERS:

`./../data/well89conc2gaus.prn`  \ file with data
1 2 3 7 0 0  \ columns for X,Y,Z,vr,wt,sec.var.
`./../data/trend230/trend11.inc`  \ file with trend data
`./../data/pconc.inc`  \ file with radar data
-1.0e21       1.0e21  \ trimming limits
0              \ transform the data (0=no, 1=yes)
sgsim.trn    \ file for output trans table
0              \ consider ref. dist (0=no, 1=yes)
histsmth.out  \ file with ref. dist distribution
1 2  \ columns for vr and wt
0.0    15.0  \ zmin,zmax(tail extrapolation)
1 0.0  \ lower tail option, parameter
1 15.0  \ upper tail option, parameter
0  \ debugging level: 0,1,2,3
sgsim.dbg     \ file for debugging output
sgsim.out     \ file for simulation output
1  \ number of realizations to generate
30 0.19 1  \ nx,xmn,xsiz
80 0.0 1  \ ny,ymn,ysiz
230 0 0.1  \ nz,zn,zniz
69142  \ random number seed
0 8  \ min and max original data for sim
12  \ number of simulated nodes to use
1 3  \ multiple grid search (0=no, 1=yes),num
0  \ maximum data per octant (0=not used)
20.0 10.0 5.0  \ maximum search radii (hmax,hmin,vert)
0.0 0.0 0.0  \ angles for search ellipsoid
0 0.60 1.0  \ ktype: 0=SK,1=OK,2=LVM,3=EXDR,4=COLC
`./../data/nodata`  \ file with LVM, EXDR, or COLC variable
4  \ column for secondary variable
3 0.0  \ nst, nugget effect
2 .2 0.0 -11 0.0  \ it,cc,ang1,ang0g2,ang3
          1000 8 0.5  \ a_hmax, a_hmin, a_vert
1 .4 0.0 -11 0.0  \ it,cc,ang1,ang2,ang3
          2 13 0.03  \ a_hmax, a_hmin, a_vert
5 .4 0.0 -11 0.0  \ it,cc,ang1,ang2,ang3
        4 3.5 0.1  \ a_hmax, a_hmin, a_vert
APPENDIX C. ECLIPSE 3D SIMULATION DATA DECK

-- RAPTORTRACER.DAT By Hong Tang, May, 2005
-- X      Y      I     J    K
-- 30    80     60   160    28

RUNSPEC

NOECHO

TITLE
   Tracer Simulation of Raptor Ridge (case1)

DIMENS
-- NX  NY  NZ
  60 160 28 /

WATER
METRIC

WELLDIMS
-- NWMAXZ NCWMAX NGMXAZ NWGMAX
  5  9600      2   2 /

TRACERS
-- NOTMAX NWTMAX NGTMAX NETMAX NUMDIFF
  0  1  0  0  DIFF /

START
-- DAY MONTH YEAR
  1  JAN  2000 /

--EQULOPTS
--QUIESC /

UNIFIN
UNIFOUT
SAVE /

NSTACK
  200 /

MESSAGES
4*100000000 200 1* 4*100000000 /

NUMRES
  1 /

NUPCOL
  3 /
GRID

--NOGGF
PINCH
  0.01 /

SPECGRID
  60 160 28 1 F /

MINPV
  .001/

NOECHO

--grid
include
GRID3839.GRDECL /
-- include precomputed permeability file
include
permx.out /

equals
'poro' .28 /
/

COPY
PERMX PERMZ /
PERMX PERMY /
/

MULTIPLY
PERMZ .2 /
/

NEWTRAN
INIT

PROPS

---TName Fluid Unit
  RED WAT /
/

TRACTVD

Rock
--P Cr
  68.95  1.02-04 /

GRAVITY
--Oil Water Gas
  50.0  1.05  0.60 /

PVTW
--P  Bw  Cw  Visc
  68.95  1.013  4.E-05  0.4802  2.4E-06 /

SOLUTION

EUQIL
  40  80 /

TBLKFRED
  268800*0.0 /

RPTSOL
  'RESTART=2' 'FIP=2' TRACER /

SUMMARY

WBHP /

GTPRRED
  'p' /

GTPTRED
  'p' /

GTPCRED
  'p' /

FTITRED

TCPU

RUNSUM

SEPARATE

SCHEDULE

RPTSCHED
  'RESTART=2' 'FIP=2' /

RPTRST
  'BASIC=5' -- keep all restarts, output every FREQth reporting period /

NOWARN

-- included well data wldxplus.inc, wldyplus.inc/ wldzplus.inc
-- Example well include file “wldxplus.inc” follows the data deck

Include
  wldxplus.inc/

TUNING
  /
  /  LITMIN LITMIN MXWSIT
  12  1  35  3  16 /

TSTEP
1 4 5 10 10 10 10 10 10 10 10 10 10 10 10 10 /
TSTEP
10*16 /
TSTEP
5*20 /
TSTEP
5*20 /
TSTEP
5*20 /
TSTEP
10*20 /
TSTEP
10*20 /
END

ECLIPSE WELL SPECIFICATION INCLUDE FILES
“Wldxplus.inc”

WELSPECS
--Name Group   I   J  Datum  Phase
--Name Group   x=xmax  y<=ymax  Datum  Phase
'I1' 'I'     1    1 10    'Water' /
'I1' 'I'     2    1 10    'Water' /
'I1' 'I'     3    1 10    'Water' /
'I1' 'I'     4    1 10    'Water' /
.............
'I1' 'I'     52   1 10    'Water' /
'I1' 'I'     53   1 10    'Water' /
'I1' 'I'     54   1 10    'Water' /
'I1' 'I'     55   1 10    'Water' /
'I1' 'I'     56   1 10    'Water' /
'I1' 'I'     57   1 10    'Water' /
'I1' 'I'     58   1 10    'Water' /
'I1' 'I'     59   1 10    'Water' /
'I1' 'I'     60   1 10    'Water' /
'P1' 'P'     1   160 10    'water' /
'P1' 'P'     2   160 10    'water' /
'P1' 'P'     3   160 10    'water' /
'P1' 'P'     4   160 10    'water' /
'P1' 'P'     5   160 10    'water' /
'P1' 'P'     6   160 10    'water' /
'P1' 'P'     7   160 10    'water' /
'P1' 'P'     8   160 10    'water' /
'P1' 'P'     9   160 10    'water' /
'P1' 'P'    10   160 10    'water' /
.............
'P1' 'P'     50  160 10    'water' /
'P1' 'P'     51  160 10    'water' /
'P1' 'P'     52  160 10    'water' /
'P1' 'P'     53  160 10    'water' /
'P1' 'P'     54  160 10    'water' /
'P1' 'P'     55  160 10    'water' /
'P1' 'P'     56  160 10    'water' /
'P1' 'P'     57  160 10    'water' /
'P1' 'P' 58 160 10 'water' /
'P1' 'P' 59 160 10 'water' /
'P1' 'P' 60 160 10 'water' /
/
COMPDAT
--Name I J   K1  K2 State Kr  T   Dw   Kh  S
--Name <xmax <ymax   K1  K2 State Kr  T   Dw   Kh  S
'I1'  1 1  1 28 1* 1* 0.75 1* 0 /
'I1'  2 1  1 28 1* 1* 0.75 1* 0 /
'I1'  3 1  1 28 1* 1* 0.75 1* 0 /
'I1'  4 1  1 28 1* 1* 0.75 1* 0 /
'I1'  5 1  1 28 1* 1* 0.75 1* 0 /
'I1'  6 1  1 28 1* 1* 0.75 1* 0 /
'I1'  7 1  1 28 1* 1* 0.75 1* 0 /
'I1'  8 1  1 28 1* 1* 0.75 1* 0 /
'I1'  9 1  1 28 1* 1* 0.75 1* 0 /
.........
'I1' 51 1 1 28 1* 1* 0.75 1* 0 /
'I1' 52 1 1 28 1* 1* 0.75 1* 0 /
'I1' 53 1 1 28 1* 1* 0.75 1* 0 /
'I1' 54 1 1 28 1* 1* 0.75 1* 0 /
'I1' 55 1 1 28 1* 1* 0.75 1* 0 /
'I1' 56 1 1 28 1* 1* 0.75 1* 0 /
'I1' 57 1 1 28 1* 1* 0.75 1* 0 /
'I1' 58 1 1 28 1* 1* 0.75 1* 0 /
'I1' 59 1 1 28 1* 1* 0.75 1* 0 /
'I1' 60 1 1 28 1* 1* 0.75 1* 0 /
'P1'  1 160 1 28 1* 1* 0.75 1* 0 /
'P1'  2 160 1 28 1* 1* 0.75 1* 0 /
'P1'  3 160 1 28 1* 1* 0.75 1* 0 /
'P1'  4 160 1 28 1* 1* 0.75 1* 0 /
'P1'  5 160 1 28 1* 1* 0.75 1* 0 /
'P1'  6 160 1 28 1* 1* 0.75 1* 0 /
'P1'  7 160 1 28 1* 1* 0.75 1* 0 /
'P1'  8 160 1 28 1* 1* 0.75 1* 0 /
'P1'  9 160 1 28 1* 1* 0.75 1* 0 /
'P1' 10 160 1 28 1* 1* 0.75 1* 0 /
'P1' 11 160 1 28 1* 1* 0.75 1* 0 /
'P1' 12 160 1 28 1* 1* 0.75 1* 0 /
'P1' 13 160 1 28 1* 1* 0.75 1* 0 /
'P1' 14 160 1 28 1* 1* 0.75 1* 0 /
.........
'P1' 50 160 1 28 1* 1* 0.75 1* 0 /
'P1' 51 160 1 28 1* 1* 0.75 1* 0 /
'P1' 52 160 1 28 1* 1* 0.75 1* 0 /
'P1' 53 160 1 28 1* 1* 0.75 1* 0 /
'P1' 54 160 1 28 1* 1* 0.75 1* 0 /
'P1' 55 160 1 28 1* 1* 0.75 1* 0 /
'P1' 56 160 1 28 1* 1* 0.75 1* 0 /
'P1' 57 160 1 28 1* 1* 0.75 1* 0 /
'P1' 58 160 1 28 1* 1* 0.75 1* 0 /
'P1' 59 160 1 28 1* 1* 0.75 1* 0 /
'P1' 60 160 1 28 1* 1* 0.75 1* 0 /
<table>
<thead>
<tr>
<th>Name</th>
<th>Phase</th>
<th>Status</th>
<th>Mode</th>
<th>Qsc, BHP, THP, VFP, VOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>'I1'</td>
<td>WATER</td>
<td>OPEN</td>
<td>RATE</td>
<td>9.722 1* 1* 1* 1* /</td>
</tr>
</tbody>
</table>

WTRACER -- Well Tracer Conc

<table>
<thead>
<tr>
<th>Name</th>
<th>Well</th>
<th>Tracer</th>
<th>Conc</th>
</tr>
</thead>
<tbody>
<tr>
<td>'I1'</td>
<td>'RED'</td>
<td>1.0/</td>
<td></td>
</tr>
</tbody>
</table>

WCONPROD

<table>
<thead>
<tr>
<th>Name</th>
<th>Status</th>
<th>Mode</th>
<th>Rate</th>
<th>BHP</th>
</tr>
</thead>
<tbody>
<tr>
<td>'P1'</td>
<td>OPEN</td>
<td>WRAT</td>
<td>1*</td>
<td>9.722</td>
</tr>
</tbody>
</table>
VITA

Hong Tang, the son of Yaoqun Su and Zhengjiu Tang, was born in NanChong City, Sichuan Province, People’s Republic of China, in November 24, 1973. In July 1995, he received a degree of Bachelor of Science with distinction in petrophysics from the Southwest Petroleum Institute, China. In July 1995, he joined the China National Offshore Oil Company, Tianjin, and worked as a petrophysicist and petroleum engineer. In September 1998, he joined the University of Petroleum (Beijing), China. In July 2002, he received a degree of Master of Science in reservoir geology. In August 2002, he joined the Craft & Hawkins Petroleum Engineering Department at Louisiana State University. He is currently pursuing his study towards a doctorate degree in petroleum engineering. He is also a member of Pi Epsilon Tau, the Society of Petroleum Engineers, American Association of Petroleum Geologists, and Society of Exploration Geophysicists.