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Flow Field and Heat Transfer Analyses of Mechanical Seals

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FLOW FIELD AND HEAT TRANSFER ANALYSES OF MECHANICAL SEALS

A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Doctor of Philosophy

in

The Department of Mechanical Engineering

by

Zhaogao Luan
B.S., Sichuan University, 2000
M.S., Sichuan University, 2003
December 2007
Dedication

To my father, Shangqin
and to my mother, Qinghua
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Abstract

Mechanical seals are widely used in rotating equipment such as pumps, turbines and agitators to prevent leakage. One crucial factor for keeping seals reliable is the thermal behavior of the seal rings. The existing literature on thermal effects in seals primarily concentrates on the conduction of heat into the rings. Yet, a more complete analysis requires understanding of the flow field around the rings.

In this dissertation, both the laminar and turbulent flows within the seal chamber are studied and their effects on the heat transfer in the seal rings are evaluated. Based on extensive sets of numerical simulations, heat transfer correlations for calculating the heat convection coefficients at the wetted outer surfaces of the seal rings are developed. In addition, using a commercial software FLUENT, a 3-D computational model is developed for predicting the flow and thermal behavior of conventional mechanical seal. The numerical results are verified by the experimental measurements.

Numerical models are useful for predicting the thermal behavior of mechanical seals. However, numerical computation is time-consuming and the result is hard to be generalized. Therefore, a series of analytical models are developed for rapid evaluation of the heat transfer in the rings of mechanical seals. Using the separation of variable method, the 2-D heat conduction equations in cylindrical coordinates are solved simultaneously for the mating and primary rings with considering the heat generation between them. In addition, a simple and efficient method for estimating the average seal contact face temperature, surface temperature, and heat partitioning factor between the rings of a mechanical seal is presented.

In the analysis of mechanical seals, it is necessary and crucial to understand the heat transfer and tribological behaviors of the lubrication film at the sealing gap between the seal rings. To this end, ThermoElastoHydroDynamic (TEHD) models are developed for the lubrication film at the seal interface. Roughness at the seal faces is considered and its effects on the lubrication film thickness at the sealing gap, power dissipation, and leakage are analyzed and discussed.
1 Introduction

1.1 Flow Field and Thermal Behavior of Mechanical Seals

Mechanical face seals are widely used in rotating equipment such as pumps, turbines and agitators to prevent leakage. Failure of mechanical seals can cause significant environmental damage, force shut down of a plant, and results in a loss of productivity. Therefore, industrial users of mechanical seals require good performance and reliability. To achieve these objectives, careful attention must be directed to the factors that affect the performance and behavior of a mechanical seal including friction, wear, and heat generation between the seal rings. To this end, proper cooling of conventional mechanical seals is of paramount importance.

In practice, it is mainly the distortion of the seal face that create critical leakage path in a mechanical seal. It is well known that one of the causes of seal face distortion is thermal gradients in the rings and the other is mechanical loading. Compared with mechanical loading, the parameters that create the thermal gradient are much more difficult to quantify [1]. Heat conduction in the seal rings and heat convection on the wetted surface of the seal rings are most influential. Many notable publications have devoted significant research toward understanding the nature of the thermal aspects of mechanical seals. For example, Buck [2, 3] provided a simplified approach for determining the seal temperature based on an analytical model that treats a seal as a fin. Pascovici & Etsion [4] studied the thermo-hydrodynamic behavior of a mechanical face seal. Lebeck [5] considered many of the important effects caused by the thermal environment such as thermally induced radial taper, thermally induced waviness, heat checking and hot spotting, and blistering. Jang and Khonsari [6] extended the theory of thermoelastic analysis to predict the critical speed at which hot spots can occur on the surface of a seal.

In this dissertation, the following contributions to the analyses of mechanical seals have been made.

1) Heat transfer correlations for laminar and turbulent flow within a mechanical seal chamber.

The flow field within a seal chamber is influential in the heat transfer behavior of the seal rings. Yet, only a few researchers have focused their attention to the examination of flow in a seal chamber. A review of some pertinent publications is as follows. Verzicco et al. [7, 8] investigated the flow in an impeller stirred tank using an immersed boundary method. Lopez et al. [9] and Shen [10] came up with a numerical scheme for solving the three-dimensional Navier-Stokes equations using primitive variables in cylindrical coordinates. Their scheme is based on a spectral-Galerkin approximation for the space variables and a second-order projection scheme for time. Some numerical results with moderate rotating $Re$ numbers were presented in that paper. These publications concentrated in the flow induced by one direction of flow either axially or azimuthally. Rarely is the flow induced by radial flux studied in the open literature. In mechanical
seals, this kind of flow plays a major role. In addition, pertinent publications that examine the thermal field in a seal chamber are lacking. In this dissertation, both the laminar and turbulent flows within the seal chamber are studied and their effects on the heat transfer in the seal rings are evaluated. Based on extensive sets of numerical simulations, heat transfer correlations for calculating the heat convection coefficients at the wetted outer surfaces of the seal rings are developed.

2) Flow and thermal behavior in an actual conventional seal.

Most of the literatures studying the thermal behavior of seals such as Refs. [1-6] assumed two dimensional (2-D) mechanical seals. In this dissertation, a three dimensional (3-D) computational model for flow and thermal analysis of an actual mechanical seal with experimental verification is presented without invoking the assumption of axisymmetry. The computational model is developed for predicting the flow field in the seal chamber and temperature field within components of the mechanical seal. This model is also used to determine convection heat transfer coefficients on the wetted surfaces of the seal components.

3) Analytical solution to the heat transfer equations of the seal rings.

In 1978, Morariu and Pascovici [11] considered heat conduction in the fluid and the rings and convection by the coolant around the rings. In 1991, Doane et. al [12] performed experimental measurements in order to establish the boundary conditions of the numerical computations and obtained local and average Nusselt number for the wetted area of the mating ring. Many other publications have studied the heat transfer behavior of seal rings. See for example Refs. [1-5]. However, all these studies are based on experimental or numerical analyses which are generally difficult to generalize. Therefore, in this dissertation a series of analytical models are developed for rapid evaluation of the heat transfer in the rings of mechanical seals. Using the separation of variable method, the 2-D heat conduction equations in cylindrical coordinates are solved simultaneously for the mating and primary rings with considering the heat generation between them.

4) Heat transfer analysis in mechanical seals using fin theory.

Thermal effects play an important role on the performance and reliability of mechanical seals. The source of heat generation is frictional in nature and occurs at the interface between the primary (rotating) and the mating (stationary) rings. Heat is distributed axially by conduction into rings and by convection out to the flush fluid that enters through the gland and flows over the outer surfaces of the rings. To select a seal for a given application and operating conditions, the engineer needs to have a reliable method to rapidly estimate the temperature at the contact face between the rings to assure that the average contact temperature is acceptable. For example, when dealing with light hydrocarbon, flashing of the fluid to vapor across the face can cause the seal to run dry and fail. In the dissertation, a simple and efficient method for estimating the average seal contact face temperature, surface temperature, and heat partitioning factor between the
rings of a mechanical seal is presented. The method takes into account both heat conduction and convection. Design charts applicable to a variety of ring shapes are developed, and a series of examples are presented to illustrate their validity.

5) ThermoElastoHydroDynamic (TEHD) behavior of mechanical seals

Luxford [13] developed a simplified fluid film model to simulate the seal interface lubrication which can predict film thickness and the leakage rate. This model was based on the assumption of a parallel seal face. However, research of Green and Etsion [14] shows that stable hydrostatic lubrication requires a convergent gap in the leakage direction. Dumbrava and Morariu [15] presented a thermohydrodynamic (THD) analysis for mechanical face seal by considering heat conduction in the rings and fluid film and the convection by the cooling fluid around the rings. Doust and Parmar [16] used a finite element computer code to make a THD analysis of mechanical seals. In their study, transient thermoelastic effects were investigated. More recently, Pascovici and Etsion [17] introduced a method of approximation of the heat flow path within seal rings, based on which they performed a thermo-hydrodynamic analysis for a face-to-face double seal configuration. Tournerie et al. [18] developed a 3-D model of THD lubrication in Face seals. In their study, general equations of the THD lubrication and the heat transfer through rings were derived and solved numerically. Salant and Cao [19] presented an unsteady numerical model of a mechanical seal, with mixed lubrication applying Duhamel’s method. The model has been used to predict the performance of a mechanical seal during startup and shutdown and the results compared well with those from a finite element analysis. Most of the literatures [13-19] are concentrated on the study of the lubrication film between the rings. A more complicated analysis including the heat transfer of the rings and the interface between them is needed. In this dissertation, simplified Navier-Stokes equations, Reynolds equation and energy equation are established for the lubrication film at the sealing gap considering the heat transfer in the rotating and stationary rings. Rough and smooth seal faces are considered. Heat generation at the sealing gap is calculated and compared with the heat with leakage.

1.2 Outline of the Dissertation

This dissertation focuses on the flow field and heat transfer analysis on mechanical face seals including the rotating ring, stationary ring and the lubrication film at the contact face. Six topics are presented, each of which is treated as a separate chapter and written in the form of a journal paper. Three of these papers [20-22] have already been published.

Chapter 2 and Chapter 3 deal with the effects of laminar and turbulent flow in the seal chambers on the heat transfer in the seal rings, respectively. These two chapters show how the heat convection coefficients at the outer surface of the seal rings are affected by the flow field in the seal chamber. By numerically solving the flow field in the seal chamber and performing conjugate heat transfer analyses for the seals, correlations are developed for predicting the heat convection coefficients and temperature distribution in the rings. The analyses in these chapters are two dimensional.
Chapter 4 extends the 2-D analyses of chapters 2 and 3 to 3-D. In this chapter, a computational model for flow and thermal analysis is developed for a mechanical seal with experimental verification, without assuming axisymmetry. The computational model can predict the flow field in the seal chamber and temperature field within components of the mechanical seal. This model is also used to determine the convection heat transfer coefficients and the Nusselt numbers on the wetted surfaces of the seal components.

Numerical models are useful for predicting the thermal behavior of mechanical seals. However, they are typically time-consuming and the predicted results cannot be easily generalized. In this dissertation, simple and efficient analytical models are developed for predicting the heat transfer in the seal rings. These are presented in Chapters 5 and 6. The coverage of these chapters is described next.

Chapter 5 presents an analytical solution to the heat transfer equations in the seal rings. In this study, a simplified mathematical model of heat transfer in the rotating and mating (stationary) rings of a mechanical seal is developed. The equations are analytically solved using separation of variable method. The heat generation between the rings is considered by solving two set of heat conduction equations simultaneously. Temperature profiles in the rings and temperature distribution at the contact face are presented. The partitioning coefficient of heat transfer at the contact face is evaluated. To verify the results, numerical solution is compared with the analytical solution. Based on the results of the simplified model, a more realistic model of the mechanical seal rings is developed.

Chapter 6 develops a simple and effective method for estimating the average seal contact face temperature, surface temperature, and heat partitioning factor between the rings of a mechanical seal. Design charts are developed for four types of seal shapes commonly used in industry. A series of examples to illustrate their validity is also presented.

In the analysis of mechanical seals, it is necessary and crucial to understand the heat transfer and tribological behaviors of the lubrication film at the sealing gap between the seal rings. Chapter 7 deals with the TEHD behavior of the lubrication film at the sealing gap between the seal rings. In this chapter, several models are developed for estimating the film thickness, pressure distribution, and temperature distribution in the lubrication film. The model takes into account surface roughness and the contact pressure of asperities at the seal faces.

Finally the results of all the work are summarized in the conclusion section in Chapter 8. Possible future research topics are also discussed in Chapter 8.
2 Heat Transfer Correlations for Laminar Flow within Seal Chambers

This chapter presents a numerical investigation of conjugate heat transfer associated with laminar flow within the chamber of a mechanical seal. The appropriate governing equations consisting of the Navier-stokes equations and energy equations are solved simultaneously. The computational model takes into account the temperature distribution within the rotating and mating rings. A series of simulation results are presented for predicting the performance of a mechanical seal used in a pump. Expressions are developed for predicting the convection heat transfer coefficients on the outer seal face.

2.1 Introduction

The frictional heat generation at the interface of the rotating and mating rings can significantly affect the mechanical seal performance and reliability. It is, therefore, not surprising that many researchers have applied advanced experimental and analytical tools to study the nature of heat transfer in seals. For example, Li [23] assumed a constant heat input in the sealing interfaces for treating the energy equation with uniform fluid properties using the finite element method. Morariu and Pascoveci [11] considered heat conduction in the seal ring, and convection by the coolant around the rings. Buck [2] provided a simplified approach for determining the seal temperature based on an analytical model that treats the seal as a fin. The method was recently extended to take into account the heat partitioning between the two rings [22].

In another paper, Buck [3] presented a method for estimating heat generation and face temperature field on representative values of the convective heat transfer coefficient at the outer surface of the seal rings based on experimental results. Doane et al. [12] performed experimental measurements that primarily focused on determining the boundary conditions for the numerical computations. They obtained local and average Nusselt number for the wetted area of the mating ring, and reported measurements of the interface temperature distribution between the rings.

Lebeck [5] considered many of the important effects caused by the thermal environment such as thermally induced radial taper, thermally induced waviness, heat checking and hot spotting, and blistering. Jang and Khonsari [6] extended the theory of thermoelastic analysis to predict the critical speed at which hot spots can occur on the surface of a seal. Salant and Cao [19] recently developed an unsteady numerical model of a mechanical seal and performed the thermal analysis using Duhamel’s method to predict the performance of a mechanical seal during startup and shutdown. These publications concentrated in the thermal analysis of mechanical seals. Yet, the effect of the flow field in the seal chambers and its influence on thermal behavior of seal requires further attention.

Research by Merati et al. [1] predicted the turbulence flow field in a seal chamber by applying FLUENT, a commercially available software package. In their research, the temperature distribution was also predicted within the stationary ring of a mechanical seal.
Clark et al. [24] also used FLUENT to simulate the turbulence flow field in the barrier fluid domain of a dual seal in a centrifugal pump. By visualizing the flow fluid field and performing thermal analysis of the seal rings using the finite element method, they proposed design features favoring axial circulation include larger radial gaps between rotating and stationary components, as well as axially-tapered surfaces which act to propel cooler fluid toward the heat-producing interfaces of the seal.

More recently, Luan and Khonsari [20, 21] numerically solved the laminar and turbulent flow within a seal chamber. In the present study, the CFD analysis of laminar flow is extended to include a heat transfer analysis. To this end, the appropriate governing equations are derived and numerical solution algorithms are developed to solve the Navier-Stokes equations and the energy equations. Thus, the flow and temperature fields within the seal chamber as well as the rotating ring and the stationary ring are predicted simultaneously. Also, a mathematical model of heat generation at the interface between the rotating and mating rings—where the largest magnitude of the heat flux is known to occur—is developed. The results of this paper are restricted to laminar flows in the seal chamber that are known to occur in many applications, especially when dealing with high-viscosity process fluids such as oils. For these fluids, the viscosity tends to drop appreciably with the rise in temperature. The analysis developed in this paper considers the viscosity-temperature behavior of the process fluid.

2.2 Governing Equations

Figure 2.1 shows the schematic of a mechanical seal. The installment includes a gland, rotating ring, and mating ring. The flush fluid enters into the seal chamber radially through the flush inlet port drilled into the gland. Figure 2.2 shows the computation domain of interest. Making use of the axis-symmetric nature of the geometry and assuming laminar flow, the governing equations in cylindrical coordinates reduce to the following:

\[
\frac{u_r}{r} + \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} = 0 \tag{2.1}
\]

\[
\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z} = -\frac{1}{\rho_f} \frac{\partial p}{\partial r} + \nu_f \left( \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2} \right) \tag{2.2}
\]

\[
\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho_f} \frac{\partial p}{\partial z} + \nu_f \left( \frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} + \frac{\partial^2 u_z}{\partial z^2} \right) \tag{2.3}
\]

\[
\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + u_z \frac{\partial u_\theta}{\partial z} = \nu_f \left( \frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial u_\theta}{\partial r} + \frac{\partial^2 u_\theta}{\partial z^2} - \frac{u_\theta}{r^2} \right) \tag{2.4}
\]

The flow boundary conditions for the Navier-Stokes equations are [20]:

At the inlet boundary I1
\[
u \quad u_z = 0, u_r = u_{in}, u_\theta = 0 \tag{2.5}
\]

\[
\frac{\partial p}{\partial r} = 0 \tag{2.6}
\]
where $u_{rin}$ is the radial velocity of the flush fluid.

At the outlet boundary O1

$$\frac{\partial u_z}{\partial z} = 0, \quad \frac{\partial u_r}{\partial z} = 0, \quad \frac{\partial u_{\theta}}{\partial z} = 0$$

$p = 0.0$ \hfill (2.7)

At the surface of the mating ring and the gland

$u_z = 0, u_r = 0, u_{\theta} = 0$ \hfill (2.8)

$$\frac{\partial p}{\partial n} = 0$$

where $n$ denotes the normal to the surface, i.e., $r$ or $z$.

At the surface of the rotating ring,

$u_z = 0, u_r = 0, u_{\theta} = u_{rin}$ \hfill (2.9)

$$\frac{\partial p}{\partial r} = 0$$

\hfill (2.10)

\hfill (2.11)

\hfill (2.12)

Figure 2.1 Drawing of the mechanical seal installation
The energy equation for the flow within the seal chamber is:

\[
\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + u_z \frac{\partial T}{\partial z} = \frac{k_f}{\rho_f c_f} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] + \frac{2 v_f}{c_f} \left[ \left( \frac{\partial u_r}{\partial r} \right)^2 + \left( \frac{\partial u_z}{\partial z} \right)^2 \right]
\]

\[+ \left( \frac{\partial u_z}{\partial z} \right)^2 \]

(2.13)

Heat conduction equation in solid rotating ring is:

\[
\frac{\partial T}{\partial t} = \frac{k_r}{\rho_r c_r} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right]
\]

(2.14)

Heat conduction equation in solid mating ring is:

\[
\frac{\partial T}{\partial t} = \frac{k_m}{\rho_m c_m} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right]
\]

(2.15)

Note that the kinetic viscosity \( v_f \) in the above governing equations is applied as Eq. (2.16).

\[
v_f(T) = v_0 e^{-\beta(T-T_0)} \quad \text{for} \quad 27 \, ^\circ\text{C} < T < 157 \, ^\circ\text{C}
\]

(2.16)
where $\nu_0$ is the kinetic viscosity at 27°C, $\beta$ is the viscosity-temperature coefficient.

Turning our attention to the interfacial frictional heat generation, we note that many references (for example, [1]-[3]) assumed that the entire heat at the seal face is distributed uniformly on each side of the contact face of WR and WS. References [3] and [27] calculated the total heat generation by considering the mean pressure-velocity product and friction coefficient, while others determined the total heat generation from measurements of the temperatures at the inlet and outlet of the seal chamber along with the flow rate [12]. In this chapter, we developed a method to calculate the heat generation profile and the different heat flux flowing into each side of the contact face.

The heat generated by friction is calculated by

$$E_p = PVA_f f$$  \hspace{1cm} (2.17)

where $E_p$ is the heat generation, $P$ is the pressure at the contact face, $V$ represents speed, $A_f$ denotes friction area, and $f$ is friction coefficient. The pressure distribution at the contact face is the so-called pressure wedge. Experimental evidence shows that it is reasonable to assume a linear pressure wedge for practical purposes [27]. The mean pressure is given by the following equation [3]

$$P_m = \Delta P(b - k) + P_{sp}$$  \hspace{1cm} (2.18)

where $\Delta P$ is the pressure differential, $P_{sp}$ is the spring pressure, $b$ represents the seal balance ratio, and $k$ denotes the pressure gradient factor.

The expression for the pressure wedge in terms of the mean pressure is:

$$P = 2P_m \left( \frac{r - r_i}{r_o - r_i} \right)$$  \hspace{1cm} (2.19)

where $r_i$ and $r_o$ are the inner and outer radii of the rotating ring, respectively. Substituting Eqs. (2.18) and (2.19) into Eq. (2.17) yields an expression for the heat generations at the interface of the rotating and mating ring:

$$H = 4[\Delta P(b - k) + P_{sp}] \omega \pi r^2 \left( \frac{r - r_i}{r_o - r_i} \right)$$  \hspace{1cm} (2.20)

### 2.3 Numerical Computations

The Navier-Stokes equations and the energy equations are coupled through the viscosity—a nonlinear function given in Eq (2.16). Therefore, Eqs. (2.1)-(2.4) and Eqs. (2.13)-(2.16) must be solved simultaneously.

Equations (2.2)-(2.4) can be written in the following form.

$$\frac{\partial u}{\partial t} = -\nabla p + C + D$$  \hspace{1cm} (2.21)

where $C$ and $D$ are the convection and diffusion terms, respectively.

Similarly, Eqs. (2.13) - (2.15) are written in the following form.
\[ \frac{\partial T}{\partial t} = C_f + D_f \quad (2.22) \]
\[ \frac{\partial T}{\partial t} = C_r + D_r \quad (2.23) \]
\[ \frac{\partial T}{\partial t} = C_m + D_m \quad (2.24) \]

where the subscripts \( f, r \) or \( m \) denote the working fluid, rotating ring, and mating ring, respectively.

In this chapter, the fractional step method (FSM) was used to solve this problem. This method consists of solving the N-S equations in three steps. First, the equations are solved without the pressure terms. Then, the continuity equation is solved using the fractional velocity. Finally, the velocity is explicitly corrected applying the new pressure obtained in the previous step [20]. Thus, in this approach three major steps are:

Step 1: predictor step
\[ \frac{u' - u^n}{\Delta t} = C^n + D^n \quad (2.25) \]
where \( u' \) is the predictor value of \( u \).

\[ \frac{T^{n+1} - T^n}{\Delta t} = C_f^n + D_f^n \quad (2.26) \]
\[ \frac{T^{n+1} - T^n}{\Delta t} = C_r^n + D_r^n \quad (2.27) \]
\[ \frac{T^{n+1} - T^n}{\Delta t} = C_m^n + D_m^n \quad (2.28) \]

Step 2: pressure Poisson equation
\[ \Delta p = \frac{\nabla u'}{\Delta t} \quad (2.29) \]

In this step, the system of equations is solved implicitly for pressure.

Step 3: corrector step
\[ \frac{u^{n+1} - u}{\Delta t} = -\nabla p \quad (2.30) \]

The important operations in the order of execution are:

1) Guess the values of flow field \( u^n(u_r^n, u_\theta^n, u_z^n) \) and temperature \( T^n \).
2) Calculate \( \nu_f \) using Eq. (2.16).
3) Solve the momentum equations without pressure terms Eq. (2.25) to obtain \( u' \).
4) Solve equations Eqs.(2.26)-(2.28) to get \( T^n \).
5) Calculate \( p^n \) by solving the pressure Poisson equation.
6) Correct flow field from Eq. (2.30).
7) Treat the corrected flow field as a new guessed flow field, return to step 2) and repeat the whole procedure until a converged solution is obtained.

For the simulations presented in the chapter, a staggered grid system was applied with uniform spacing. The values of $\Delta r$, $\Delta z$ and $\Delta t$ are: $\Delta r = 0.03$ cm, $\Delta z = 0.02$ cm, and $\Delta t = 1 \times 10^{-6}$. The value of $\Delta t$ was chosen by trial and error. If $\Delta t$ is too large, the results may not be stable. A smaller value of $\Delta t$ did not affect the results, but required a significantly greater computational time.

Once the temperature profile in the computation domain is determined, the local heat flux distribution at a given face can be obtained using the Fourier’s heat conduction equation:

$$ q_n = k_n (T_i - T_{i+1}) / \Delta n $$

(2.31)

where $q$ is the local heat flux, $n$ represents the normal to the desired heat transfer face, and $T_i$ is the temperature on the face. The parameter $T_{i+1}$ represents the temperature at the normal next node beyond the face and $\Delta n$ the distance between the $i$th node on the face and the $(i+1)$ node.

The local heat transfer coefficients $h$ along the outer wetted surface of the seal rings is calculated by applying

$$ q_n = h(T_s - T_\infty) $$

(2.32)

where $q_n$ is the local heat flux computed from Eq. (2.31). $T_s$ is the temperature of the surface of the seal ring which is varying along the length and $T_\infty$ is the temperature of the flush in fluid.

The local Nusselt number for the surface is calculated using

$$ Nu = \frac{hD_p}{k_f} $$

(2.33)

where $r_o$ is the outer radius of the rotating ring and $k_f$ is the conductivity of the sealed fluid.

The average Nusselt number is evaluated using the following expression.

$$ \overline{Nu} = \frac{1}{L} \int_0^L Nu \, dz $$

(2.34)

where $L$ is the integral range, $Nu$ is the local Nusselt number.

### 2.4 Results and Discussion

Figure 2.2 presents the schematic of the model with notations describing the boundary conditions. A summary of the boundary conditions is shown in Table 2.1. Material properties of the rings and working fluid are shown in Table 2.2. Table 2.3 shows the
values used in the simulations. Table 2.4 presents the pressure and friction values used in the calculation of friction heat.

In this research, the flow and heat transfer of a mechanical seal rotating at 1800 rpm, 3600 rpm and 7200 rpm are studied with flush rate of 63.1 cm$^3$/s (1gpm). The working fluid in this study is engine oil SAE10W. Because the process fluid is oil, the viscosity is high, and the flow is laminar. When dealing with oil, the viscosity can change dramatically with temperature. To take this into consideration, Eq. (2.16) was implemented. In the equation, the kinetic viscosity $\nu = 664 \times 10^{-6}$ m$^2$/s at $T_o=27$ °C, $\beta = 0.0474$ was determined for engine oil SAE10W by fitting a least squares line through the data of Ref. [25].

Table 2.1 Boundary conditions

<table>
<thead>
<tr>
<th>Type</th>
<th>Thermal Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>Velocity Inlet</td>
<td>$T = 298$ K</td>
</tr>
<tr>
<td>O1</td>
<td>Pressure Outlet</td>
<td></td>
</tr>
<tr>
<td>W1</td>
<td>Wall, constant $T$</td>
<td>$T = 298$ K</td>
</tr>
<tr>
<td>W2</td>
<td>Wall, constant $T$</td>
<td>$T = 298$ K</td>
</tr>
<tr>
<td>W3</td>
<td>Wall, insulation</td>
<td>$\partial T / \partial z = 0$</td>
</tr>
<tr>
<td>W4</td>
<td>Wall, convection to air</td>
<td>$h = 36.5$ W/m$^2$K</td>
</tr>
<tr>
<td>W5</td>
<td>Wall, convection to air</td>
<td>$h = 37.4$ W/m$^2$K</td>
</tr>
<tr>
<td>W6</td>
<td>Wall, convection to air</td>
<td>$h = 38.3$ W/m$^2$K</td>
</tr>
<tr>
<td>W7</td>
<td>Wall, insulation</td>
<td>$\partial T / \partial z = 0$</td>
</tr>
</tbody>
</table>

Table 2.2 Material properties of the rings and working fluid (at 298K)

<table>
<thead>
<tr>
<th></th>
<th>$k$ (W/m.K)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$c$ (J/kg.K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine Oil (SAE10W)</td>
<td>$k_f = 0.145$</td>
<td>$\rho_f = 885$</td>
<td>$c_f = 1900$</td>
</tr>
<tr>
<td>Mating ring (Carbon Graphite)</td>
<td>$k_m = 14$</td>
<td>$\rho_m = 1825$</td>
<td>$c_m = 783$</td>
</tr>
<tr>
<td>Rotating ring (Silicon Carbide)</td>
<td>$k_r = 118$</td>
<td>$\rho_r = 3100$</td>
<td>$c_r = 670$</td>
</tr>
</tbody>
</table>

The temperature values of W1 and W2 are assumed to be 298 K since these boundaries are far from the dominant source of heat generation at the interface of the stationary ring and the rotating ring. On faces W4, W5 and W6 heat transfer occurs by convection. Their heat transfer coefficients are based on Ref. [21]. Boundaries W3 and W7 are assumed to be insulated since experiment and experience show that the heat generated from the friction of the rotating ring and stationary ring are mostly taken away by heat convection at the wetted outer surface (W4-W6) of the rings [3]. At the wetted surface of the rings, it is assumed that there is no fluid motion and energy transfer occurs by conduction only [25]. Thus, Fourier’s law of heat conduction is applied to both the rings and the working fluid at the interfaces.
Table 2.3 Dimension of the mechanical seal (in cm)

<table>
<thead>
<tr>
<th>Name</th>
<th>L₁</th>
<th>L₂</th>
<th>RI₁</th>
<th>RO₁</th>
<th>RI₂</th>
<th>RO₂</th>
<th>RO₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension</td>
<td>0.43</td>
<td>0.35</td>
<td>4.14</td>
<td>4.48</td>
<td>3.68</td>
<td>5.08</td>
<td>5.46</td>
</tr>
</tbody>
</table>

Table 2.4 Pressure and friction values of the seal

<table>
<thead>
<tr>
<th>Name</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔP</td>
<td>Pressure differential</td>
</tr>
<tr>
<td>$P_{sp}$</td>
<td>Spring pressure</td>
</tr>
<tr>
<td>k</td>
<td>Pressure gradient factor</td>
</tr>
<tr>
<td>$b$</td>
<td>Seal balance ratio</td>
</tr>
<tr>
<td>$f$</td>
<td>Friction coefficient</td>
</tr>
</tbody>
</table>

Table 2.5 Difference between the results of Correlation and Simulation

<table>
<thead>
<tr>
<th>Case</th>
<th>$\overline{h}_{\text{Rotor}}$ (W/m².K)</th>
<th>$\overline{h}_{\text{Stator}}$ (W/m².K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLUENT</td>
<td>Correlation Equation</td>
<td>Present Model</td>
</tr>
<tr>
<td>1800 rpm</td>
<td>1476.8</td>
<td>1407.3</td>
</tr>
<tr>
<td>3600 rpm</td>
<td>1620.1</td>
<td>1555.9</td>
</tr>
<tr>
<td>7200 rpm</td>
<td>1686.9</td>
<td>1930.0</td>
</tr>
</tbody>
</table>

2.4.1 Flow Field

Figure 2.3 shows the vector plots within the seal chamber at 1800 rpm with 1 gpm of flush rate. It can be observed that upon entering the inlet port, the flush fluid tends toward the rotating ring. In the vicinity of the rotating ring, the axial component of the velocity is much greater than those around the inlet of the gland, which is indicative of a favorable condition as far as cooling the surface of the rings is concerned. Computations predict that there exists some back flow of the process fluid in the positive $z$-direction which after traveling toward the mating ring, loops back around, and exits the fluid domain. Back flow occurs as a result of high rotational speed of the primary ring which tends to create a low pressure region within the seal chamber, sucking the flow into the chamber. Since back flow emerges from the process fluid and not directly from the flush, it is less effective in cooling the rings. The flow pattern presented in this paper is consistent with the results presented in Ref. [20].
2.4.2 Temperature Field

Temperature contours for the seal chamber, rotating ring and stationary ring with rotational speed of 1800 rpm are shown in Fig. 2.4. The maximum temperature within the fluid region (i.e. in the seal chamber) is in the corner close to the interface between the rings. The maximum temperature is estimated at 34.2 °C. This shows that most of the heat generated in the interface is transferred into the fluid at the corner of the rotating and stationary rings. The temperature within the rings in the interface is considerably higher. The interfacial heat generated is transferred to the sealed fluid primarily by convection in the vicinity of the outer surface of the rotating ring and stationary ring. In the close vicinity of the surface of the rotating ring, the temperature is generally above 34.2 °C. Further away from the ring in the region where back flow occurs the temperature is approximately 29.6 °C, which is higher than the inlet flush temperature.

Fig. 2.3 Vector plots with rotation speed of 1800 rpm with flush rate of 1gpm (63.1 cm³/s)

Figure 2.5 illustrates the effect of the shaft speed on the temperature distribution along the contact face. In this figure, the temperature is plotted against the dimensionless radius, \((r - r_o)/(r_i - r_o)\), where \(r_i\) and \(r_o\) are the inner and outer radius of the rotating ring, respectively. For the 1800 rpm case, the temperature increases with increasing radius until it reaches its maximum value at a dimensionless radius of about \(r = 0.9\). This maximum value is about 19.13 °C above that at the \(r_o\) location. Beyond \(r = 0.9\), the temperature drops approximately 1.5 °C comparing to its maximum value at the outer radius of the rings (wetted side). For 3600 rpm shaft speeds, the temperature also reaches its maximum at around \(r = 0.9\) with their maximum being 40.30 °C above that at the \(r_o\) location. The temperature decays rapidly after the maximum point. Note that the
temperature distributions for the three shaft speeds are similar in trend. For the case of 7200 rpm, the temperature approaches its maximum at $r = 0.85$. And the maximum temperature is 51.56 $^\circ$C above that at the $r_0$ location.

Fig. 2.4 Temperature contour with rotation speed of 1800 rpm with flush rate of 1 gpm (63.1 cm$^3$/s). (All temperatures are in $^\circ$C)

Fig. 2.5 Temperature distribution along the contact face between rotating ring and stationary ring
2.4.3 Heat Transfer Analysis at the Contact Interface

Heat flux generated due to friction at the contacting face dissipates into both the rotating and the stationary rings by conduction and to the surrounding fluid in the chamber by convection. Figures 2.6 and 2.7 show the heat transfer behavior at the stationary and the rotating rings. The difference in the heat transfer arises from the difference in the materials and the thermal capacity of the rings. Figures 2.6-2.7 show the variation of the interfacial heat flux along the radial direction for different rotational speeds. Figure 2.6 shows that the heat flux entering the rotating ring increases with increasing the radius until it attains its maximum values at about $r = 0.72$. After this point, it tends to drop. Figure 2.7 shows that the heat flux entering the stationary ring from the friction contact face. For the three rotational speeds simulated, the heat flux into the stationary ring first increases with increasing radius until it reaches the maximum at $r = 1.0$. Combining Figs. 2.6 and 2.7, it can be found that the maximum overall heat flux occurs at the region close to outer radius of the rotating ring. This is consistent with the conclusion made in section 2.4.2 that most of the heat generated in the interface is transferred into the fluid at the corner. Note that in Fig. 2.7, the heat flux is predicted to rapidly rise at $r = 0.9$. The reason can be explained as following. Examining Fig. 2.4, it can be found that the temperature at the stationary ring near the contact face is as low as about 34.2 °C while the temperature at the contact face at $r = 1.0$ is about 100 °C (see Fig. 2.5 for case of 1800 rpm). Therefore, the temperature gradient at the stationary ring near the contact region is large, resulting in a rise in the heat flux after $r = 0.9$.

2.4.4 Heat Transfer Analysis at the Surface of the Rings

Let us now turn our attention to the heat transfer along the outer surfaces of the rotating ring and stationary ring. Figures 2.8-2.10 present the variation of the heat transfer coefficient as a function of the axial or radial directions. Surfaces 1, 2 and 3 denote the outer surface of the rotating ring and stationary ring (see Fig. 2.2). The results presented in Fig. 2.8 show that the heat transfer coefficient along face 1 increases with $z$ increasing. This is reasonable. The heat source is at the interface (corresponding to $z = 1.0$ in Fig. 2.8) between the rings. Therefore, the heat flux transferred to the process fluid is increasing from $z = 0$ to $z = 1$, and the heat transfer coefficient varies in the same fashion.

Figure 2.9 shows that the heat transfer coefficient along face 2 increases with increasing $r$ until it approaches its second maximum values, then drops to their minimum value at around $r = 0.7$, and finally reaches its maximum value. Heat transfer coefficient at $r = 1.0$ approaching its maximum is because that $r = 1.0$ is corresponding to the corner at the stationary ring. Thus, the heat can be much efficiently removed.

Figure 2.10 shows the plots of the heat transfer coefficient number along face 3. Face 3 is referring to the outer surface of the stationary ring. The values of Face 3 for the three rotational speed are decreasing with $z$ increasing until they attain their minimum values at $z = 0.9$. The reason is that $z = 0$ is at the corner at the stationary ring which has a higher heat transfer coefficient as discussed in the above paragraph. It can be noted that after $z = 0.9$, the heat transfer coefficient increases slightly because $z = 1.0$ is at boundary wall W2.
(see Fig.2.2). Note that a constant temperature of 298 K was specified at the boundary of wall W2. The relatively low temperature of wall W2 can improve the heat transfer slightly.

Fig. 2.6 Friction heat flux flowing into the rotating ring along the contact face

Fig. 2.7 Friction heat flux flowing into the stationary ring along the contact face
2.4.5 Heat Transfer Correlations

Eighteen cases were simulated with rotational speed from 1800 to 7200 rpm and with flush rate from 63.1 to 315.5 cm$^3$/s (1-5 gpm). Based on the results the average Nusselt
numbers for the rotating ring and stationary ring were calculated using Eqs. (2.33) and (2.34). The Nusselt numbers for the rotating ring is taken to have the following form

\[ \overline{Nu_{\text{rotor}}} = C_1 \Pr^{C_2} \Re_{Dp}^{C_3} \]  

(2.35)

Note that the form of Eq. (2.35) is based on Hilpert’s empirical correlation of convection heat transfer for flow over cylinders [25]. The same expression is also recommended by Becker [26] for predicting the convective heat transfer from a horizontal cylinder rotating in a tank of water. The parameters \(C_1, C_2\) and \(C_3\) are constants to be determined. There are three unknowns and 18 equations for the rotating ring. Values of \(C_1, C_2\) and \(C_3\) are determined by applying the Least Square method. Similarly, the correlation for stationary ring is derived. For stationary ring, the following equation is proposed

\[ \overline{Nu_{\text{stator}}} = C_1 \Pr^{C_2} \Re_{Dp}^{C_3} \Re_{Dm}^{C_4} \]  

(2.36)

Eq. (2.36) is based on the combination of Hibert empirical correlation for flow over cylinder and Martin’s [25] form of the heat transfer correlation for impinging jet cooling. The determination of \(C_1, C_2, C_3\) and \(C_4\) are done following the same procedure as in the rotating ring.

\[ \overline{Nu_{\text{Rotor}}} = 1.33 \times 10^5 \Pr^{-0.76} \Re_{Dp}^{0.20} \]  

(2.37)  

\[ \overline{Nu_{\text{Stator}}} = 1.01 \times 10^3 \Pr^{-0.30} \Re_{Dp}^{0.23} \Re_{Dm}^{0.07} \]  

(2.38)

where,
\[ \text{Re}_{dp} = \frac{U_{\text{flush}}D_p}{v} \quad (2.39) \]
\[ \text{Re}_{dm} = \frac{U_{\text{flush}}D_m}{v} \quad (2.40) \]
\[ U_{\text{flush}} = \frac{\omega D_p}{2} \quad (2.41) \]

where \( U_{\text{flush}} \) is the rotation velocity at the surface of the rotating ring, \( U_{\text{flush}} \) is the radial flush in flow velocity, \( D_p \) is the outer diameter of the rotating ring and \( D_m \) is the outer diameter of the mating ring. \( Pr \) is evaluated at the surface of the rings in deducing the two equations above. Note that Eqs. (2.37) and (2.38) apply to laminar flows, which satisfy:

\[ \text{Re}_{dp} < 2500, \text{Re}_{dm} < 4500, Pr > 4000 \quad (2.42) \]

### 2.4.6 Illustrative Example

Consider a pump with rotational speed of 1800 rpm, the process fluid is assumed to be engine oil and the outer radius of the rotating ring is 4.48 cm. The outer radius of the mating ring is 5.08 cm. The flush rate is 1 gpm. Determine the heat convection coefficient over the rotating and stationary rings.

From Eq. (2.41)

\[ U_{\text{flush}} = \frac{\omega D_p}{2} = \frac{(1800 \times 2\pi/60) \times (4.48 \times 10^{-2} \times 2)}{2} = 8.44 \text{ m/s} \]

From Eq. (2.39)

\[ \text{Re}_{dp} = \frac{U_{\text{flush}}D_p}{v} = \frac{8.44 \times (4.48 \times 10^{-2} \times 2)}{664 \times 10^{-6}} = 1151.6 \]

The Prandtl number, \( Pr \), is estimated at the temperature of the outer surface of the rotating ring, which is 4785.5. Using Eq. (2.37), the average Nusselt number for the rotating ring is

\[ \overline{\text{Nu}}_{\text{Rotor}} = 1.33 \times 10^5 \text{ Pr}^{-0.76} \quad \text{Re}_{dp}^{0.20} = 1.33 \times 10^5 \times (4785.5)^{-0.76} \times (1151.1)^{0.20} = 869.62 \]

From Eq. (2.33)

\[ \overline{h}_{\text{rotor}} = \frac{\overline{\text{Nu}}_{\text{rotor}}k_f}{D_p} = 1407.3 \text{ W/m}^2\text{K} \]

Next, we calculate the heat transfer coefficient for the mating ring with an outer radius of 5.08 cm and flush rate of 63.1 cm$^3$/s (1.0 gpm) entering a flush hole with diameter of 0.35 cm.

\[ U_{\text{flush}} = \frac{Q_{\text{flush}}}{\pi D_{\text{flush}}^2/4} = \frac{63.1 \times 10^{-6}}{\pi \times (0.35 \times 10^{-2})^2/4} = 6.56 \text{ m/s} \]
From Eq. (40)

\[
\text{Re}_{Dm} = \frac{U_{\text{flush}} D_m}{v} = \frac{6.56 \times (5.08 \times 10^{-2} \times 2)}{664 \times 10^{-6}} = 1003.8
\]

\(Pr\) is estimated at the temperature of the outer surface of the stationary ring as 4304.8. Then the average Nusselt number for the mating ring

\[
\overline{\text{Nu}_{\text{Stator}}} = 1.01 \times 10^3 \text{ Pr}^{-0.30} \text{ Re}_{Dp}^{0.23} \text{ Re}_{Dm}^{-0.07} = 1.01 \times 10^3 \times (4304.8)^{-0.30} \\
\times (1151.6)^{0.23} \times (1003.8)^{0.07} = 673.52
\]

From Eq. (2.33)

\[
\overline{h}_{\text{stator}} = \frac{\overline{\text{Nu}_{\text{stator}}} k_f}{D_p} = 1090.0 \text{ W/m}^2\text{K}
\]

To check the validity of the results, the computer program was run. The energy equations for the fluid in the seal chamber and in the rings were numerically solved and then temperature distribution for the entire computation domain shown in Fig. 2.2 was obtained. Having obtained the temperature field, one can combine Eqs. (2.31) and (2.32) and determine the heat convection coefficient. The results are: \(\overline{h}_{\text{rotor}} = 1440.2 \text{ W/m}^2\text{K}\) and \(\overline{h}_{\text{stator}} = 1031.7 \text{ W/m}^2\text{K}\). Comparing with the correlations, the error is less than 5% for the rotor ring and less than 8% for the stationary ring.

The above procedure was repeated for 3600 rpm and 7200 rpm. The results along with those of 1800 rpm are shown in Table 2.5. Also shown in Table 2.5 are the results of 2-D simulations using a commercial software package FLUENT; see [22] for details. It can be seen that the results are in reasonable agreement with the FLUENT predictions. Therefore, the correlations are useful for assessment of heat transfer coefficient.

2.5 Conclusions

In this chapter, the Navier-Stokes equations and the energy equation within the flow of a seal chamber, rotating ring and stationary ring were solved simultaneously using the fractional step method (FSM) combined with staggered grid. The flow in seal chamber is laminar and the variation of viscosity with temperature was taken into account.

The model is capable of predicting the flow field and temperature field within the seal chamber as well as within the seal components. Temperature distributions and local and average heat transfer results were calculated for the shaft speeds 1800 rpm, 3600 rpm and 7200 rpm. The heat flux distribution flowing into the stationary ring and the rotating ring were also determined.

The results of the computed temperature field were also used to determine the local heat flux distributions and the local and average heat convection coefficient of the outer surface of the rotating ring and stationary ring. Useful heat transfer correlation equations for average Nusselt numbers for both the rotating ring and the stationary ring are also presented.
2.6 Nomenclature

\[ A_f = \text{surface area of friction at seal contact face} \]
\[ b = \text{mechanical seal balance ratio} \]
\[ c_f = \text{constant-pressure specific heat of process fluid} \]
\[ c_m = \text{constant-pressure specific heat of stationary ring} \]
\[ c_r = \text{constant-pressure specific heat of rotating ring} \]
\[ D_p = \text{outer diameter of the rotating ring} \]
\[ D_m = \text{outer diameter of the mating ring} \]
\[ D_{\text{flush}} = \text{diameter of flush hole} \]
\[ E_p = \text{heat generated at the seal contact face} \]
\[ f = \text{friction coefficient} \]
\[ h = \text{local heat transfer coefficient} \]
\[ \bar{h} = \text{average heat transfer coefficient} \]
\[ k_f = \text{conductivity of water} \]
\[ k_m = \text{conductivity of stationary ring} \]
\[ k_r = \text{conductivity of rotating ring} \]
\[ Nu = \text{Local Nusselt number} \]
\[ \overline{Nu} = \text{average Nusselt number} \]
\[ p = \text{local pressure at seal contact face} \]
\[ p_m = \text{average pressure at seal contact face} \]
\[ p_{\text{sp}} = \text{spring pressure} \]
\[ q = \text{heat flux at wetted outer surface of rotating ring and stationary ring} \]
\[ Q_{\text{flush}} = \text{flow rate of flush flow} \]
\[ r = \text{radial coordinate} \]
\[ Re_{Dp} = \text{Reynolds number of primary ring} \]
\[ Re_{Dm} = \text{Reynolds number of mating ring} \]
\[ RI_1 = \text{inner radius of rotating ring} \]
\[ RI_2 = \text{inner radius of mating ring} \]
\[ RO_1 = \text{outer radius of rotating ring} \]
\[ RO_2 = \text{outer radius of mating ring} \]
\[ t = \text{time} \]
\[ T(r,z) = \text{cross-sectional temperature in rotating ring, stationary ring and flow} \]
\[ T_s = \text{temperature at outer surface of rotating ring and stationary ring} \]
\[ T_\infty = \text{ambient temperature} \]
\[ u_r = \text{radial velocity component of seal chamber flow} \]
\[ u_z = \text{axial velocity component of seal chamber flow} \]
\[ u_{rin} = \text{flush in velocity} \]
\[ u_\theta = \text{angular velocity component} \]
\[ u_{\text{rn}} = \text{rotating speed of primary ring} \]
\( V = \) local velocity of rotating ring  
\( z = \) axial coordinate  
\( \Delta T_{AV} = \) average temperature difference  
\( \Delta p = \) pressure differential  
\( \beta = \) exponential coefficient of the kinematic viscosity function  
\( \rho_f = \) density of process fluid  
\( \rho_m = \) density of mating ring  
\( \rho_r = \) density of rotating ring  
\( \nu_f = \) kinematic viscosity of process fluid  
\( \omega = \) shaft speed
3 Conjugate Heat Transfer Analysis of Mechanical Seals with Turbulent Flow

This chapter consists of two parts. The first part deals with the turbulent flow in the seal chamber. In the second part, a conjugate heat transfer model is developed for estimating the effects of the turbulent flow on the heat transfer in the seal rings. In this chapter, numerical investigation of conjugate heat transfer of turbulent flow within a mechanical seal chamber is made. A mathematical model of heat generation at the contact face between rotor and stator is developed. The computational model predicts the temperature distribution within the rings. The local average heat flux, heat transfer coefficient and Nusselt number on the outer wetted seal face are presented. The largest magnitude of the heat flux occurs on the rotating ring near the interface between the rings. The correlations for calculating the average Nusselt numbers for rotor and stator are also presented.

3.1 Introduction

The most practical and common method for preventing process fluid (sometimes hazardous) from leaking out of the pump into the surroundings is to isolate the rotating shaft and its housing with a mechanical seal. It, therefore, comes as no surprise that a mechanical seal is often considered to be one of the most vital components of an industrial pumps, mixers, etc. During normal operations, these devices often generate considerable heat as a result of sliding friction between the rotating and mating rings. In conventional mechanical seals, heat is normally removed by forced convection cooling induced by an influx of the flush fluid. Experiments and experience show that a cooler operating temperature can enhance the seal performance and reliability [25]. Providing an effective and adequate flush rate is, therefore, crucial for removing heat in order to sustain proper functionality and to prevent premature seal failure.

There are a number of published papers concerning thermal analysis of conventional mechanical seals. Most of the existing publications in the open literature have devoted their attention to the modeling of thermal aspects of seals at the interface between the primary (rotating) and stationary (mating) rings. Pertinent studies relevant to the discussion of the present paper include the work of Li [23] who treated the energy equation by assuming a constant heat input at the sealing interface between the rotating and mating rings. He used the finite element method to predict the seal temperature distribution. Morariu and Pascovici [11] considered the heat conduction in the fluid and the rings and convection by the coolant around the rings. Buck [2, 3] provided a simplified approach for determining the seal temperature based on an analytical model that treats a seal as a fin. Doane et. al [12] performed a series of experimental measurements to determine the interface temperature distribution. He also predicted the local and the average Nusselt number for the wetted area of the mating ring. They measured the interface temperature distribution between the rings. This information served as the boundary conditions for the numerical computations. Lebeck [5] considered many of the important effects caused by the thermal environment such as thermally
induced radial taper, thermally induced waviness, heat checking and hot spotting, and blistering. Jang and Khonsari [6] extended the theory of thermoelastic analysis to predict the critical speed at which hot spots can occur on the surface of a seal. These publications concentrated in the thermal analysis of mechanical seals but neglected the effect of the flow field in the seal chamber.

In turning our attention to the flow analysis in mechanical seals, we refer to a noteworthy publication by Merati et al. [1] who conducted a study of the turbulence flow field in a seal chamber by applying the FLUENT computer code. In their research, the temperature distribution within the stator of a mechanical seal was presented. Clark et al. [24] also used FLUENT to simulate the turbulence flow field in the barrier fluid domain of a dual seal in a centrifugal pump. They contended that design features favoring axial circulation include larger radial gaps between rotating and stationary components, as well as axially-tapered surfaces that would propel cooler fluid toward the heat-producing interfaces of the seal. Nevertheless, experience shows that larger radial gap can also increase the risk of leakage. Parker and Merati [28] investigated turbulent Taylor Couette flow with high-quality, flow visualization photographs that clearly displayed the modes of vortex formation. Their study can be beneficial for enhancing the design of mechanical seal chamber.

In a companion paper (Luan & Khonsari [20]), the results of the flow characteristics of flush fluid was presented which concentrated solely on laminar flows dealing with a relatively viscous process fluid such as oil. In this chapter we extend the analysis to include turbulence; hence covering a great range of operating speeds and types of process fluids. For this purpose, the $k-\varepsilon$ turbulence model posed in cylindrical coordinates was implemented to simulate the turbulent flow in the mechanical seal chamber. Moreover, the characteristic of flow with very high Reynolds number is reported. Applying the turbulent flow field, a mathematical model of heat generation at the interface between rotating and mating rings is developed. The largest magnitude of the heat flux occurs on the rotating ring near the interface between the rings. These numerical results provide a good understanding of what effects the flow field has on the heat transfer of the mechanical seals and thus can have meaningful guidance in the operation and the design of the mechanical seals.

### 3.2 Turbulent Flow Field in the Seal Chamber

To predict the heat transfer in the seal chamber, it is necessary to know the flow field in it. Therefore, in this section, we focus on the turbulent flow field in the seal chamber. The $k-\varepsilon$ turbulence model posed in cylindrical coordinates was applied for this purpose. Simulations are performed using the Fractional Approach Method. The results of the computer code have been verified by using the FLUENT and by comparing to published results for turbulent Taylor Couette flow. Numerical results of four cases including two rotational speeds with four flush rates are reported. The behavior of the turbulent flows

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with very high Reynolds number was also investigated. The physical and practical implications of the results are discussed.

### 3.2.1 Governing Equations

The schematic of the mechanical seal and the associated gland structure is shown in Fig. 3.1. The computational domain encompassing the appropriate region of the flush and the rings is Fig. 3.2, where we have taken advantage of the axisymmetric nature of the problem. Accordingly, the governing equations written in dimensionless form are:

**Conservation of mass:**

\[
\frac{\partial u_r}{\partial z} + \frac{u_r}{r} + \frac{\partial u_r}{\partial r} = 0 \tag{3.1}
\]

**Navier-Stokes equations:**

\[
\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial z} + u_r \frac{\partial u_r}{\partial r} + \frac{u_r^2}{r} = -\frac{\partial p}{\partial r} + \frac{1}{\text{Re}} \left( \frac{\partial}{\partial z} \left[ 2(1+\nu) \frac{\partial u_r}{\partial z} \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[ r(1+\nu) \left( \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right) \right] - \frac{2}{3} \frac{\partial k}{\partial z} \right) \tag{3.2}
\]

\[
\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial z} + u_r \frac{\partial u_r}{\partial r} - \frac{u_\theta^2}{r} = -\frac{\partial p}{\partial r} + \frac{1}{\text{Re}} \left( \frac{\partial}{\partial r} \left[ (1+\nu) \left( \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial r} \right) \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[ 2r(1+\nu) \frac{\partial u_r}{\partial r} \right] - (1+\nu) \frac{u_r}{r^2} \right) \tag{3.3}
\]

\[
\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial z} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_r u_\theta}{r} = \frac{1}{\text{Re}} \left( \frac{\partial}{\partial z} \left[ (1+\nu) \frac{\partial u_\theta}{\partial z} \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[ r(1+\nu) \frac{\partial u_\theta}{\partial r} \right] - (1+\nu) \frac{u_\theta}{r^2} \right) \tag{3.4}
\]

**\( k-\varepsilon \) equations:**

\[
\frac{\partial k}{\partial t} + u_r \frac{\partial k}{\partial r} + u_z \frac{\partial k}{\partial z} = \frac{1}{\text{Re}} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{v}{\sigma_k} \frac{\partial k}{\partial r} \right) + \frac{1}{\sigma_k} \frac{\partial}{\partial z} \left( \frac{v}{\sigma_k} \frac{\partial k}{\partial z} \right) \right) + G - \varepsilon \tag{3.5}
\]

\[
\frac{\partial \varepsilon}{\partial t} + u_r \frac{\partial \varepsilon}{\partial r} + u_z \frac{\partial \varepsilon}{\partial z} = \frac{1}{\text{Re}} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{v}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial r} \right) + \frac{1}{\sigma_\varepsilon} \frac{\partial}{\partial z} \left( \frac{v}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial z} \right) \right) + c_{1e} \frac{\varepsilon}{k} G - c_{2e} \frac{\varepsilon^2}{k} \tag{3.6}
\]

where

\[
u = \frac{u_r}{u_{cin}}, u_z = \frac{u_z}{u_{cin}}, u_\theta = \frac{u_\theta}{u_{cin}}, r = \frac{r^*}{r_{cin}}, z = \frac{z^*}{r_{cin}}, p = \frac{p^*}{\rho u_{cin}^2}, t = \frac{t^*}{u_{cin}}, k = \frac{k^*}{u_{cin}^2}, \varepsilon = \frac{\varepsilon^*}{u_{cin}}, v = c_\mu \text{Re} \frac{k^2}{\varepsilon} \tag{3.7}
\]

The parameters bearing a superscript “*” are dimensional and those without are dimensionless.
The parameter $G$ is given by:

$$
G = \frac{v}{Re} \left\{ 2 \left[ \left( \frac{\partial u_r}{\partial r} \right)^2 + \left( \frac{u_z}{r} \right)^2 + \left( \frac{\partial u_z}{\partial z} \right)^2 \right] + \left( \frac{\partial u_\theta}{\partial r} \right)^2 + \left( \frac{\partial u_r}{\partial r} + \frac{u_z}{r} \right)^2 + \left( \frac{\partial u_\theta}{\partial r} + \frac{u_z}{r} \right)^2 \right\} \quad (3.9)
$$

Parameters $u_r$, $u_z$ and $u_\theta$ are the radial, axial and azimuthal velocity components, respectively. These components as well as $p$ are all average values. The parameter $k$ represents the turbulent kinetic energy and $\varepsilon$ is its dissipation. The parameter $\nu$ denotes the turbulent eddy viscosity, and $Re$ is Reynolds number given by:

$$
Re = \frac{\rho u_{cin} r_{ol}}{\mu} \quad (3.10)
$$

where $u_{cin}$ is the azimuthal velocity of the surface of the rotating ring, $\rho$ is the density, and $\mu$ is the viscosity of the working fluid.

The constants appearing in Eqs. (3.1)-(3.7) take the standard values recommended by Luo et al. [29] and Biswas et al. [30], i.e.

$c_p = 0.09$, $c_{1c} = 1.44$, $c_{2c} = 1.92$, $\sigma_k = 1.0$, $\sigma_\varepsilon = 1.3$, and $\sigma_t = 0.9$

### 3.2.2 Boundary and Initial Conditions

At the inlet boundary:

$$
\begin{align*}
& u_z = 0.0, u_r = u_{rin}, u_\theta = 0.0 \\
& \frac{\partial p}{\partial r} = 0.0
\end{align*} \quad (3.11, 3.12)
$$

where $u_{rin}$ is the radial velocity of the flush fluid.

![Fig. 3.1 Drawing of the mechanical seal installation](image)
At the surface of the mating ring and the gland:

\[ u_z = 0.0, u_r = 0.0, u_\theta = 0.0 \]  
\[ \frac{\partial p}{\partial n} = 0.0 \]  
where \( n \) denotes the normal to the surface, i.e., \( r \) or \( z \).

At the surface of the rotating ring:

\[ u_z = 0.0, u_r = 0.0, u_\theta = u_{cin} \]  
\[ \frac{\partial p}{\partial r} = 0.0 \]  

For \( k \) and \( \varepsilon \), the inlet boundary conditions are:

\[ k = \frac{1.5 I^2}{u_{cin}^2} \]  
\[ \varepsilon = \left( \frac{k}{\mu} \right)^{0.75} \]  
where \( I \) represents the turbulence intensity factor, which commonly varies in the range 2~15\% [31]. In this paper, \( I = 3\% \). The Von Karmann Constant is \( \chi = 0.42 \) [30].

The governing equations for \( k \) and \( \varepsilon \) are derived based on the assumption that the Reynolds number is large. These equations must be modified in the laminar sublayer close to a solid wall, where viscous effects become dominant. It is well known that the flow field calculated from \( k-\varepsilon \) equations without considering the laminar sublayer is unsatisfactory. To include the low-Reynolds number effects in the near-the-wall region,
the typical approach of using a “wall function” proposed by Launder and Spalding [32] is applied. This approach integrates the governing equations for the flow and turbulence from the wall upward to a distance \( y = y_n \) away from the wall. The low-Reynolds number region \( 0 < y < y_n \) is removed from the turbulence computational domain and the flow in this region is assumed to obey the law of the wall as described by the following equation.

\[
\frac{U}{U_t} = \frac{1}{\chi} \ln(ReU_t y) + B
\]  

(3.19)

where \( U \) is the mean velocity parallel to the wall at \( y = y_n \), \( U_t \) represents the friction velocity, \( B \) denotes the roughness constant (\( B = 5.5 \) for smooth walls). Note that Eq. (3.19) is also dimensionless. Through the implicit solution of Eq. (3.19), one can obtain the friction velocity \( U_t \). Thus, the Dirichlet wall boundary conditions for \( k \) and \( \varepsilon \) are [31]:

\[
k_w = \frac{U_t^2}{\sqrt{c_\mu}}
\]  

(3.20)

\[
\varepsilon_w = \frac{U_t^3}{\chi y_n}
\]  

(3.21)

By definition

\[
\tau_w = \rho U_t^2
\]  

(3.22)

Writing Eq. (3.22) in dimensionless form, we obtain:

\[
\frac{\partial U}{\partial y} = ReU_t^2
\]  

(3.23)

Equation (3.23) is solved for \( U \) and employed as boundary condition on the mean flow equations. Luo et al [29] suggested that the point \( y = y_n \) is very close to the wall so that it is reasonable to apply the Law of the wall right at the wall surface. This results in a wall slip velocity \( U = U(y_n) \).

It is worthwhile to mention that the literature contains alternative approaches to the law of the wall in conjunction with the \( k - \varepsilon \) model. For example, Demartini et al. [33] imposed the boundary conditions as \( \frac{\partial k}{\partial n} = 0 \) and \( \varepsilon_p = c_\mu \frac{k^{0.75}}{\chi y_p^{1.5}} \), where \( n \) is the coordinate normal to the wall, and \( p \) denotes the volume adjacent to the wall. They verified their results by experimental measurement. In the present paper, the law of the wall was applied as proposed by [29]. Note that the the law-of-the-wall relations are accurate only for two-dimensional, near-wall turbulent flows where local equilibrium prevails. We shall, however, make use of these relationships in a more general sense because of the lack of a more comprehensive theory with comparative simplicity.
As for the initial guess, $k$ and $\varepsilon$ are assigned inlet values. $u_z$, $u_r$, $u_\theta$ and $p$ are assumed to be zero.

### 3.2.3 Numerical Method

Equations (3.2)-(3.4) can be written in the following form.

$$\frac{\partial u}{\partial t} = -\nabla p + C + D + T$$  \hspace{1cm} (3.24)  

where C, D, and T represent the convection, diffusion, and turbulence terms, respectively.

Similarly, Eqs. (3.5) and (3.6) are written in the following form.

$$\frac{\partial k}{\partial t} = C_k + D_k$$  \hspace{1cm} (3.25)  

$$\frac{\partial \varepsilon}{\partial t} = C_\varepsilon + D_\varepsilon$$  \hspace{1cm} (3.26)

where C and D with subscript $k$ or $\varepsilon$ are also convection and diffusion terms, respectively.

In this chapter, the fractional step method (FSM) was used to solve this problem. This method consists of solving the N-S equations in three steps [34]. First, the equations are solved without the pressure terms. Then the continuity equation is solved, using the fractional velocity. Finally, the velocity is explicitly corrected by applying the new pressure obtained in the previous step. The following equations summarize the three major steps:

**Step 1: predictor step**

$$\frac{u' - u^n}{\Delta t} = C^n + D^n + T^n$$  \hspace{1cm} (3.27)

where $u'$ is the predictor value of $u$.

$$\frac{k^{n+1} - k^n}{\Delta t} = C_k^n + D_k^n$$  \hspace{1cm} (3.28)

$$\frac{\varepsilon^{n+1} - \varepsilon^n}{\Delta t} = C_\varepsilon^n + D_\varepsilon^n$$  \hspace{1cm} (3.29)

**Step 2: pressure Poisson equation**

$$\Delta p = \frac{\nabla u'}{\Delta t}$$  \hspace{1cm} (3.30)

In this step, the system of equations is solved implicitly for pressure.

**Step 3: corrector step**

$$\frac{u^{n+1} - u'}{\Delta t} = -\nabla p$$  \hspace{1cm} (3.31)
The important operations in the order of execution are:

1) Guess the values of flow field \( u^n (u_r^n, u_\theta^n, u_z^n) \) and turbulence terms \( k^n \) and \( \varepsilon^n \).
2) Solve the momentum equations without pressure terms Eq. (3.27) to obtain \( u' \).
3) Solve \( k - \varepsilon \) equations Eqs. (3.28)-(3.29) to get \( k^{n+1} \) and \( \varepsilon^{n+1} \).
4) Calculate \( p \) by solving the pressure Poisson equation.
5) Correct flow field from Eq. (3.31).
6) Treat the corrected flow field as a new guessed flow field, return to step 2) and repeat the entire procedure until a converged solution is obtained.

A FORTRAN computer code was developed to simulate the governing equations. In the simulations presented in the paper, a staggered grid system was applied with uniform spacing. The values of \( \Delta r \), \( \Delta z \) and \( \Delta t \) are: \( \Delta r = 0.0022 \), \( \Delta z = 0.0022 \) and \( \Delta t = 1 \times 10^{-7} \). The value of \( \Delta t \) was chosen by trial and error. If \( \Delta t \) is too large, the results may not be stable. A smaller value of \( \Delta t \) did not affect the results, but require a significantly greater computational time. A computer with Pentium(R) 4 CPU 3.2 GHz and 2.0 GB of RAM was used to execute the code. Typically it takes about 5 hours to complete the computation for a specified set of input variables.

### 3.2.4 Discussions for Turbulent Flow

#### 3.2.4.1 Validation

In order to verify the validity of our computer code, it was applied to predict the behavior of turbulent Taylor Couette flow and compare to the experimental results reported in [28]. The test apparatus consisted of two vertically aligned concentric cylinders and two end walls. The working fluid, sodium iodide, was placed in the annulus region confined by the two cylinders of radii 30.96 mm and 46.04 mm respectively, providing a gap width of 15.08 mm. The annulus length was 60.32 mm. The inner cylinder rotational speed was 1750 rpm. Figure 3.3 shows the comparison between the predicted results and the experimental results of [28]. In the figure, the rotating inner surface is shown on the left side. The results consistently indicate that there exist two vortexes as induced by the rotating inner cylinder. The trend predicted by numerical results and the measurements are in agreement.

In addition, the commercial finite volume code (FLUENT) was used to simulate three cases of 1800 rpm and 3600 rpm with flush in rate of 1 gpm (63.1 cm³/s). The same boundary conditions and material properties as in the FORTRAN code were applied. To make quantitative comparison, three velocity components at the outlet were plotted in Figs. 3.4-3.6. In these figures, the outlet velocity is plotted against the dimensionless radius, \( (r - r_{i2})/(r_{o1} - r_{i2}) \), where \( r \) is the radial location, and \( r_{i2} \) and \( r_{o1} \) are the inner radius and the outer radius of the rotor ring, respectively. Figure 3.7 shows the vector plots at the outlet with the flush rate of 1.0 gpm (63.1 cm³/s) predicted using the FLUENT software package as well as the FORTRAN code based on the analysis presented earlier. The figures show good quantitative agreement between the simulations developed based in the present analysis and those predicted using the FLUENT software package.
Fig. 3.3 Comparison of the numerical and experimental results of turbulent taylor-couette Flow: (a) vector plot from present model (b) experimental measurement [10]

Fig. 3.4 Validation for axial velocity at the outlet with 1.0 gpm (63.1 cm³/s) of flush rate
3.2.4.2 Flow Field Results for a Pump Seal

In this section, we present the results of a series of simulations on a mechanical seal used in a pump. In these simulations, the process fluid was water at 25°C.
The flow regime within the seal chamber was predicted by calculating the Reynolds number and Taylor number [35]. At the rotational speed of 1,800 rpm, for example, we have

\[
Re = \frac{u_{cin} r_{o1}}{\nu} \approx 3.7 \times 10^5 \tag{3.32}
\]

\[
Ta = \frac{2\eta^2 d^4}{1 - \eta^2} \left(\frac{\Omega}{\nu}\right)^2 \tag{3.33}
\]

where
\[
\eta = r_{o3} / r_{o1}, \text{ is the radius ratio}
\]
\[
d = r_{o3} - r_{o1}, \text{ represents the gap width}
\]
\[
\Omega = \text{ inner cylinder angular velocity, and}
\]
\[
\nu = \text{kinematic viscosity of the fluid}
\]

The parameters used in the simulations are summarized in Table 3.1. Using these values, Eq. (3.33) predicts that the Taylor number is: \( Ta = 1.34 \times 10^9 \). Given that the critical Taylor number [18] is \( Ta_c = 2279 \), the flow is in the turbulent regime. The ratio of the operating Taylor number to the critical Taylor number is:

\[
\frac{Ta}{Ta_c} = 5.9 \times 10^5 \tag{3.34}
\]

According to Koschmieder’s study [36] turbulent vortices form at Taylor number greater than 1000 \( Ta_c \). Thus, turbulent state exists even at rotating speed as low as 1800 rpm. The rotating speed simulated in this paper ranges from 1800 rpm-3600 rpm, hence turbulent.

Figures 3.8-3.9 show the streamline plots of the fluid in the seal chamber subject to different flush rates ranging from 0.5 gpm (31.55 cm\(^3\)/s) to 1.0 gpm (63.1 cm\(^3\)/s). These figures show the flow behavior of the fluid in the chamber as the influx of flush emerges from entry point and flows over to the stationary and rotating rings. It can be seen that the flush fluid goes down to the stator ring then merges with the backflow which sweeps the rotating ring and then exits the seal chamber along the vicinity of the gland surface.

The dense streamline flow is the merging of the backflow and the flush influx as it exit out of the seal chamber. Generally, such dense streamline flow can perform better surface heat transfer because their high velocity. Nevertheless, the gland is not the main heat source within the seal chamber.

The heat source is at the interface between the rotating and mating rings. Therefore, the backflow around the rotating ring helps to remove heat from the interface between the rings, whereas the flush fluid does not reach the rotating ring and exits out the chamber along the outer diameter.
Fig. 3.7 Validation for vector plots at the outlet with rotational speed of 1800 rpm and flush rate of 1.0 gpm (63.1 cm³/s) (a) FLUENT results (b) FORTRAN code results
Fig. 3.8 Streamline plot with rotation of 1800 rpm and flush in rate (a) 0.5 gpm (31.55 cm³/s) (b) 1.0 gpm (63.1 cm³/s)
Fig. 3.9 Streamline plot with rotation of 3600 rpm and flush in rate (a) 0.5 gpm (31.55 cm³/s) (b) 1.0 gpm (63.1 cm³/s)
Table 3.1 Parameters used in Eq.(3.33)

<table>
<thead>
<tr>
<th>Name</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$ the radius ration</td>
<td>0.82</td>
</tr>
<tr>
<td>$d$ the gap width</td>
<td>0.98 cm</td>
</tr>
<tr>
<td>$\Omega$ inner cylinder angular velocity</td>
<td>188.4 s$^{-1}$</td>
</tr>
<tr>
<td>$\nu$ kinematic viscosity of the fluid</td>
<td>$1 \times 10^{-6}$ m$^2$/s</td>
</tr>
</tbody>
</table>

The simulation results predict that when the flow is turbulent flow, the mating ring is cooled by the flush but the rotating ring is cooled by the backflow. Thus, increasing the flush rate help cool the mating ring more effectively than the rotating ring. Comparing Figs. (a) and (b) in Figs. 3.8-3.9, we can see that at the same rotational speed, increasing the flush rate, the dense streamline flow layer becomes thicker. Specifically, the layer for 1.0 gpm flush rate is 0.1 cm thicker than 0.5 gpm at rotation speed of 1800 rpm and 0.025 cm thicker at 3600 rpm. We can also see that the rotational speed can decrease the layer thickness at the same flush rate.

3.2.4.3 Turbulence Intensity

In general turbulence intensity is a quantity that characterizes the intensity of turbulence disturbance in the flow. In the present study, the turbulence intensity is defined as follows \[31\],

$$
I = \frac{\sqrt{2k/3}}{u_{ref}}
$$

where $u_{ref}$ is the reference velocity which is the local mean velocity magnitude. $u_{ref}$ can be chosen in different ways. $u_{\theta}$ was chosen as the reference velocity here.

Figure 3.10 shows the turbulence intensity contour plot with rotation of 1800 rpm and flush rate of 1.0 gpm. The simulations show that the turbulent intensity is as low as about 3% at the region close to the inlet and outlet while in the vicinity of the rotating ring, the intensity can be as high as about 50%. Figure 3.10 also shows that the intensity decreases when $r$ increases. The intensity close to the surface of the rotating ring has the greatest values. High intensity turbulence is generally beneficial for heat removal.

3.2.4.4 Turbulent Flow Field with a Very High Reynolds Number

To investigate the effect of the high rotational speed on the turbulence field in the seal chamber, a high rotation speed of 18,000 rpm with 1 gpm flush in rate was calculated. In this case, the Reynolds number is about $3.23 \times 10^6$. Fig. 3.11 shows the streamline plot of this flow. The simulations predict the formation of a large vortex in the region around the stationary ring.
The flush influx is so small compared to the backflow induced by the rotating ring that it can not be shown clearly on the streamline plot. The vortex near the stationary ring region is induced by the backflow in the rotating ring region. Since the flush in flow is comparatively very small, the inlet can be seen as a stationary wall boundary. Thus the
stationary region is similar to the so-called driven cavity type flow where the flush fluid forms a vortex.

From Fig. 3.11, it can be seen that the flush flow still sweeps and then cools the stationary ring despite its small velocity magnitude at the inlet. Due to the large amount of backflow, the flush fluid in the vortex flows much faster than that in turbulent cases without forming vortex, thus more effectively cool the stationary ring. However, two facts should be noticed. First, at high rotational speed vortex formation is beneficial for heat transfer but at the same time the frictional heat generated at the interface of the rings is much greater. Second, in the middle of the vortex, the “closed” flow streamlines cannot exit out of the seal chamber (see Fig. 3.11), and therefore it is not effective for removal of heat. In the situations like in Figs. 3.8-3.9, i.e. without formation of a vortex, it can be found that all the streamlines in the mating ring region finally exits out of the seal chamber, thus allowing fresh cooling flush fluid to remove heat and exit the chamber.

3.2.5 Conclusions for Flow Results

In this chapter, the development of a computer program to investigate the turbulent flow mechanism in mechanical seal chambers applying the \( k - \varepsilon \), two-equation turbulence model is reported. A CFD code was developed using the fraction step method. It is found that at a given operating speed, cooling is not much enhanced if the flush rate is increased. Simulations for the turbulent flow with very high rotational speed (Re = \( 3.23 \times 10^6 \)) reveal that the flush influx is so small compared to the backflow induced by the rotating ring. However, the flush flow still sweeps so that it can cool the mating ring despite its small velocity magnitude at the inlet.

3.3 Conjugate Heat Transfer

In this section, numerical investigation of conjugate heat transfer associated with the turbulent flow predicted in the last section 3.2 is made. A mathematical model of heat generation at the contact face between the rotor and stator ring is developed. The computational model can predict the temperature distribution within rings and the heat convection transfer coefficient at the wetted outer surface of the rings.

3.3.1 Governing Equations for Heat Transfer

Taking advantage of the axisymmetric property of the problem, the energy equations of the seal chamber and the rings in cylindrical coordinates can be written as below.

\[
\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + u_z \frac{\partial T}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \frac{k_f}{\rho_f c_f} + \frac{\nu_t}{\sigma_t} \right) \frac{\partial T}{\partial r} \right] + \frac{\partial}{\partial z} \left[ \left( \frac{k_f}{\rho_f c_f} + \frac{\nu_t}{\sigma_t} \right) \frac{\partial T}{\partial z} \right] \quad (3.36)
\]

\[
\frac{\partial T}{\partial t} = \frac{k_r}{\rho_r c_r} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] \quad (3.37)
\]

\[
\frac{\partial T}{\partial t} = \frac{k_m}{\rho_m c_m} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] \quad (3.38)
\]
Equation (3.36) is the convection heat transfer equation in the turbulent flow field within the seal chamber, where \( \nu = c_\mu \frac{k^2}{\varepsilon} \) with \( c_\mu = 0.09 \) , and \( \sigma_t = 0.9 \). These parameters are used in the \( k-\varepsilon \) turbulence model (see Ref. [11] for details). The parameters \( k \) and \( \varepsilon \) represent the turbulence kinetic energy and the turbulence kinetic energy dissipation, respectively. Equations (3.37) and (3.38) are the conduction heat transfer equations for rotating (primary) and mating (stator) rings, respectively. Note that the velocity components, \( u_r, u_z \), and \( k \) and \( \varepsilon \) are necessary to determine the temperature field. This requires solving the \( k-\varepsilon \) turbulence model as described in [20].

3.3.2 Boundary Conditions

Figure 3.12 presents the schematic of the model with notations describing the boundary conditions. A summary of the boundary conditions is shown in Table 3.2. The temperature values of W1 and W2 are assumed to be uniform temperature. Boundaries W4, W5 and W6 exchange heat to the surrounding by convection. Boundaries W3 and W7 are assumed to be adiabatic since experiment and experience shows that the heat generated from the friction of the rotor and stator are mostly taken away by the working fluid [5]. At the wetted surface of the rings, we assumed that there is no fluid motion and energy transfer occurs only by conduction [25]. Thus, the Fourier’s heat-conduction law can be applied to both the rings and the working fluid at the interfaces.

Fig. 3.12 Schematic of the 2-D model with boundary condition notations (All dimensions are in cm)
Table 3.2 Boundary conditions

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Type</th>
<th>Thermal Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>Velocity Inlet</td>
<td>$T = T_\infty$</td>
<td>Velocity of flush</td>
</tr>
<tr>
<td>O1</td>
<td>Pressure Outlet</td>
<td>$T = T_\infty$</td>
<td>Seal chamber wall</td>
</tr>
<tr>
<td>W1</td>
<td>Wall, constant $T$</td>
<td>$T = T_\infty$</td>
<td>Seal chamber wall</td>
</tr>
<tr>
<td>W2</td>
<td>Wall, constant $T$</td>
<td>$T = T_\infty$</td>
<td>Seal chamber wall</td>
</tr>
<tr>
<td>W3</td>
<td>Wall, insulation</td>
<td>$\frac{\partial T_r}{\partial z} = 0$</td>
<td></td>
</tr>
<tr>
<td>W4</td>
<td>Wall, convection to air</td>
<td>$k_m \frac{\partial T_m}{\partial r} = h_1(T_s - T_\infty)$</td>
<td>Stator-sleeve gap</td>
</tr>
<tr>
<td>W5</td>
<td>Wall, convection to air</td>
<td>$k_m \frac{\partial T_m}{\partial z} = h_2(T_s - T_\infty)$</td>
<td>Stator-sleeve gap</td>
</tr>
<tr>
<td>W6</td>
<td>Wall, convection to air</td>
<td>$k_r \frac{\partial T_r}{\partial r} = h_3(T_s - T_\infty)$</td>
<td>Rotor-sleeve gap</td>
</tr>
<tr>
<td>W7</td>
<td>Wall, insulation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WR/WS</td>
<td>Interfacial wall</td>
<td>$k_r \frac{\partial T_r}{\partial r} - k_m \frac{\partial T_m}{\partial r} = E_p$</td>
<td>Rotor-stator interface</td>
</tr>
</tbody>
</table>

Table 3.3 Material properties of the rings and working fluid (at 298K)

<table>
<thead>
<tr>
<th></th>
<th>$K$ (W/m.K)</th>
<th>$\rho$ (Kg/m$^3$)</th>
<th>$c_p$ (J/Kg.K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working fluid (water)</td>
<td>$k_f = 0.613$</td>
<td>$\rho_f = 997$</td>
<td>$c_f = 4179$</td>
</tr>
<tr>
<td>Mating ring (Carbon Graphite)</td>
<td>$k_m = 14$</td>
<td>$\rho_m = 1825$</td>
<td>$c_m = 783$</td>
</tr>
<tr>
<td>Rotating ring (Silicon Carbide)</td>
<td>$k_r = 118$</td>
<td>$\rho_r = 3100$</td>
<td>$c_r = 670$</td>
</tr>
</tbody>
</table>

Let us now turn our attention to the interfacial heat generation at the interface WR and WS as shown in Fig. 3.12. Many references such as [1], [2] and [3], assumed that all the heat at the seal face is distributed uniformly on each side of the contact face of WR and WS. References [3] and [27] calculated the total heat generation by considering the mean pressure velocity product and friction coefficient. Other references, see for example [12], determined the total heat generation experimentally by measuring the temperatures at the inlet and outlet of the seal chamber along with the flow rate. However, it was found that the temperature drop is very small due to the relatively large flow rate, which resulted in large uncertainty in the computation of the heat generation [1].

In this chapter, the heat generated by friction is estimated using

$E_p = PVA_f f$  \hfill (3.39)

where $E_p$ is heat, $P$ is pressure at the contact face, $V$ represents the rotational speed, $A_f$ is friction area and $f$ denotes the friction coefficient. The pressure distribution at the contact face is often referred to by the pressure wedge. Experimental evidence shows that
it is reasonable to assume a linear pressure wedge for practical purposes [27]. The mean pressure is given by the following expression [3]

\[ P_m = \Delta P (b - k) + P_{sp} \]  (3.40)

where \( \Delta P \) represents the pressure differential, \( P_{sp} \) is the spring pressure pushing the rotating ring to the mating ring, \( b \) denotes the seal balance ratio, and \( k \) is the pressure gradient factor.

\[ P = 2P_m \left( \frac{r - r_i}{r_o - r_i} \right) \]  (3.41)

Equation (3.41) is the expression of the pressure wedge, where \( r_i \) and \( r_o \) are the inner and outer radius of the rotor, respectively. Substituting Eqs. (3.40) and (3.41) into Eq. (3.39) yields the following expression for determining the interfacial heat generation.

\[ E_p = 4[\Delta P (b - k) + P_{sp}] \omega \pi f r^2 \left( \frac{r - r_i}{r_o - r_i} \right) \]  (3.42)

### Table 3.4 Pressure and friction values of the seal

<table>
<thead>
<tr>
<th>Name</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta P )</td>
<td>Pressure differential</td>
</tr>
<tr>
<td>( P_{sp} )</td>
<td>Spring pressure</td>
</tr>
<tr>
<td>( k )</td>
<td>Pressure gradient factor</td>
</tr>
<tr>
<td>( b )</td>
<td>Seal balance ratio</td>
</tr>
<tr>
<td>( f )</td>
<td>Friction coefficient</td>
</tr>
<tr>
<td>( T_a )</td>
<td>Ambient temperature</td>
</tr>
<tr>
<td>( h_1 )</td>
<td>Heat transfer coefficient</td>
</tr>
<tr>
<td>( h_2 )</td>
<td>Heat transfer coefficient</td>
</tr>
<tr>
<td>( h_3 )</td>
<td>Heat transfer coefficient</td>
</tr>
</tbody>
</table>

### 3.3.3 Numerical Simulation

Equations (3.36), (3.37) and (3.38) are solved simultaneously using FDM (Finite Difference Method). Details of the method are discussed in Refs. [20, 37].

Note that in Eq. (3.36), the energy equation for working fluid, the convective terms involve the turbulence flow field within the seal chamber. In this study, we assume the viscosity of the working fluid does not change with temperature. Therefore, the calculation of the energy equation for the working fluid can be decoupled from the calculation of the Navier-Stokes equations (in this case, \( k-\varepsilon \) turbulence model). In other words, the turbulent flow field including the average velocities and the turbulence parameters such as \( k \) and \( \varepsilon \) can be solved first. Then these values can be treated as knowns in the later calculation of the energy equation. Note that the flow field for the seal chamber in this study was previously solved by Luan and Khonsari [20].
With a heat generation as shown in Eq. (3.42), the heat source of the whole computation domain is located at the interface of the rotor and the stator. The amount of heat conducted into each ring is not equal because of the difference in the material properties (particularly the thermal conductivity) and ring size. In many analytical studies, a heat partitioning coefficient is used to take this effect into account. However, in this study, we place a control volume with a heat generation rate (as shown in Eq. (3.42)) at the interface and iteratively solve the pertinent heat transfer problem where the heat flux continuity is satisfied at this interface. Therefore, a heat partitioning coefficient is not needed in this study.

Once the temperature profile in the computation domain is determined, the local heat flux distribution at a given face can be obtained by

\[
q_n = k_s (T(i)) \frac{(T(i) - T(i + 1))}{\Delta n}
\]  

(3.43)

where \( q \) is the local heat flux and \( n \) represents the normal to the desired heat transfer face and \( T(i) \) is the temperature at the face, \( T(i + 1) \) is the temperature at the normal next node beyond the face and \( \Delta n \) the distance between the face and the \( (i + 1) \) node.

The local heat transfer coefficients \( h \) along the outer wetted surface of the seal rings was calculated by applying

\[
q_n = h(T_s - T_w)
\]  

(3.44)

where \( q_n \) is the local heat flux computed from Eq. (3.43), \( T_s \) represents the temperature of the surface of the seal rings, and \( T_w \) is the temperature of the flush fluid. Once the local heat transfer coefficients \( h \) is known, the local Nusselt number for the surface can be calculated using

\[
Nu = \frac{hD_p}{k_f}
\]  

(3.45)

where \( D_p \) is the outer diameter of the rotating ring (rotor) and \( k_f \) is the sealed water thermal conductivity. The average Nusselt number is given by the following equation.

\[
\overline{Nu} = \frac{1}{L} \int_0^L Nu \, dz
\]  

(3.46)

where \( L \) is the integral range, \( Nu \) is the local Nusselt number.

### 3.3.4 Results and Discussions for Conjugate Heat Transfer

The working fluid in this study is water at 25°C (298 K). The materials of the mating ring and rotating ring are Carbon Graphite and Silicon Carbide, respectively. The properties of the seal rings and the working fluid are shown in Table 2. Table 3 shows the values of the parameters of the seal appeared in Eq. (7) and Table 1. Note that the heat transfer coefficients for W4-W6 are estimated based on the results of Ref. [1].
3.3.4.1 Validation

To verify the results of the present computational model, FLUENT, a commercial CFD software, is used. The same computation domain is considered including the seal rings and the flow filed in the seal chamber as shown in Fig. 3.12. The boundary conditions are treated as shown in Table 3.2. To take the heat generation (shown in Eq. (3.42)) at the sealing interface into account, a heat source in a very thin element is applied at the contact region between the rotor and the stator. To obtain a reliable heat partitioning between the seal rings, the source element is used as thin as $10^{-5}$ m and has a good heat conductivity of 1000 W/m.K. High heat conductivity of the heat source element can insure that the generated heat can be transferred into the seal rings very quickly, therefore, the heat transfer behavior of the source elements themselves have little effect on the heat partitioning. As long as the heat conductivity is big enough and the film thickness is small enough, the heat transfer results in the problem do not change with theses values.

Figure 3.13 illustrates the effect of the shaft speed on the temperature distribution along the contact face verified by the FLUNET result. It can be found that temperature distribution calculated from the FLUENT is very close to the present model, which suggests that the present computational model is reliable.

![Graph](image.png)

**Fig. 3.13 Validation of the numerical results for the interfacial temperature distribution at contact face between the seal rings**

In Fig. 3.13, temperature is plotted against the dimensionless radius, $(r - r_i)/(r_o - r_i)$, where $r$, $r_i$, and $r_o$ are the radial location of interest, the inner and outer radius of the rotor ring, respectively. Three speeds are considered: 1800, 3600, and 7200 rpm.
Temperature increases with increasing shaft speed, with about 8 °C temperature rise differential occurring between 1800 rpm and 3600 rpm and 18 °C increasing between 3600 rpm and 7200 rpm. The temperature rise between 3600 rpm and 7200 rpm is approximately twice of that between 1800 rpm and 3600 rpm.

For all three cases, the interfacial temperature increases with \( r \) increasing before reaching the maximum. This is because of the heat generation (Eq.(3.42)) we applied at the interface between the rings. Examining Eq. (3.42), it can be recognized that the heat generation is a function of \( r^3 \) which means that the heat generation is increasing with increasing \( r \). However, the temperature decays rapidly after the maximum point since that at \( r=r_o \), i.e. the outer surface of the seal rings, the flow within the seal chamber can remove the heat very quickly, which brings down the temperature.

For the 1800 rpm case, the temperature increases with increasing radius until it reaches its maximum value at a dimensionless radius of about 0.9 with this maximum being about 2.38 °C above that at the zero location. Beyond 0.9, the temperature drops 1.40 °C comparing to its maximum value at a location corresponding to the outer radius of the rings (wetted side). For 3600 rpm and 7200 rpm shaft speeds, the temperature reaches its maximum at around 0.9 with their maximum being above 4.84 °C and 13.15 °C, respectively. Note that the temperature distributions for the three shaft speeds are similar in trend.

### 3.3.4.2 Temperature Results

Temperature contours for the seal chamber, rotor and stator with rotational speed of 1800 rpm are shown in Fig. 3.14. The flow in the seal chamber is mainly at a constant temperature 25 °C (the temperature of inlet flow), which is consistent with the measurement results of Ref. [39].

The heat generated from the friction at the contact face is mainly convected to the seal chamber fluid in the vicinity of the interface between the rotor and stator. From Fig. 3.14 we can see that in the vicinity of the contact face the temperature is about 31 °C, while a short distant away from the contact face of the stator, the temperature is almost 25 °C.

---

![Temperature contour plot with rotation speed of 1800 rpm](image)

**Fig. 3.14** Temperature contour plot with rotation speed of 1800 rpm
3.3.4.3 Flow Field

Figure 3.15 shows the streamline plot of the fluid in the seal chamber subject to flush rate of 63.1 cm$^3$/s (1.0 gpm). The figure shows the flow behavior of the fluid in the chamber as the influx of flush emerges from entry point and flows over to the mating and rotating rings. The flush fluid flows down toward the mating ring then merges with the back flow which sweeps the rotating ring and then exits the seal chamber along the vicinity of the gland surface. Note that a modest increase in the flush rate does not affect the back flow around the rotating ring and does not improve heat transfer. Increasing the flush rate, however, can improve the heat transfer around the mating ring.

3.3.4.4 Heat Transfer Results

The rotating ring and mating ring are of different size, shape, and often made of different materials. Therefore, the amount of heat entering into the each ring is different as shown in Figs.3.16-3.17. Consistent with the contact face temperature in Figs. 3.13, Figs. 3.16 and 3.17 reveal that the heat fluxes increases along the radius and that a greater amount of heat flows into rotor than into stator for all three rotational speeds.

Referring to Fig. 3.16, for all three cases, the heat conducting into the rotor increases along the radial direction until it reach its maximum values at about 0.97. After this point, the heat flux drops slightly. Figure 3.17 shows that heat conducted into the stator, also increases in the radial direction, but at a lower rate compared with the rotor until it attains its maximum values at 1.0.

Let us now turn our attention to the heat transfer along the outer surface of the rotor and stator exposed to flush fluid. Figures 3.18-3.20 show the heat transfer coefficients over the wetted surfaces of the rotor and stator. Along face 1, i.e. the wetted surface of rotor, $h_{rotor}$ decreases gradually. This is due to the sweeping back flow of the process fluid from the outlet of the seal chamber over the outer surface of the rotating ring (see Fig. 3.15).

Figures 3.19-3.20 show that along the outer surface of the stationary ring (Faces 2 and 3), increasing the rotation speed from 1800 rpm to 7200 rpm results in an increase in $h_{stator}$. The heat transfer coefficient for face 2 increases much more than that for face 3 since face 2 is cooled by back flow which has high velocity with high rotational speed while face 3 is mostly affected by the flush in flow.

Results presented in Figs. 3.18-3.20 also reveal that the heat transfer coefficient at the surface of the rotor is generally greater than that at the surface of the stator and that the heat transfer coefficient increases with rotational speed increasing because the higher rotating speed can induce more back flow with high velocity sweeping the rotor and part of the stator. These results are consistent with turbulent flow field discussed in Ref. [20]. According to the flow characteristics, the mating ring is primarily cooled by the influx of the flush fluid.
Table 3.5 Difference between the results of correlation and simulation

<table>
<thead>
<tr>
<th>Case</th>
<th>( \tilde{h}_{\text{Rotor}} ) (W/m(^2).K)</th>
<th></th>
<th>( \tilde{h}_{\text{Stator}} ) (W/m(^2).K)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FLUENT Correlation Error</td>
<td>Present Model</td>
<td>Error</td>
</tr>
<tr>
<td>1800 rpm</td>
<td>2.85 \times 10^4 2.73 \times 10^4 2.77 \times 10^4 2%</td>
<td>6738.8</td>
<td>6623.4</td>
</tr>
<tr>
<td>3600 rpm</td>
<td>5.20 \times 10^4 5.00 \times 10^4 5.10 \times 10^4 1.8%</td>
<td>1.01 \times 10^4</td>
<td>9657.3</td>
</tr>
<tr>
<td>7200 rpm</td>
<td>9.55 \times 10^4 9.18 \times 10^4 9.41 \times 10^4 3%</td>
<td>1.54 \times 10^4</td>
<td>1.41 \times 10^4</td>
</tr>
</tbody>
</table>

Fig. 3.15 Streamline plot with rotation of 7200 rpm and flush in rate of 63.1 cm\(^3\)/s (1.0 gpm)

3.3.4.5 Correlation Results

A series of numerical simulations covering the cases of 900 rpm-7200 rpm with flush rate from 63.1-315.5 cm\(^3\)/s (1-5 gpm) were run to determine the heat transfer correlations. The Nusselt numbers for the rotating ring is taken to have the following form

\[
\overline{Nu}_{\text{rotor}} = C_1 Pr^{C_2} Re_{\text{dp}}^{C_3}
\]

(3.47)
Note that the form of Eq. (3.47) is based on Hilpert’s empirical correlation of convection heat transfer for flow over cylinders [39]. This expression is also used by Becker [26] in arriving at an expression for the convective heat transfer from a horizontal cylinder rotating in a tank of water. The parameters $C_1$, $C_2$ and $C_3$ are determined by applying the Least Square method. Similarly, the correlation for stationary ring is derived.

For stationary ring, the following equation is proposed
\[
\overline{Nu}_{\text{Stator}} = C_1 \Pr^{C_2} \Re_{Dp}^{C_3} \Re_{Dm}^{C_4}
\]  
(3.48)

Equation (3.48) is based on the combination of Hibert empirical correlation for flow over cylinder and Martin’s [25] form of the heat transfer correlation for impinging jet cooling. The determination of $C_1$, $C_2$, $C_3$ and $C_4$ are done following the same procedure as in the rotating ring.

\[
\overline{Nu}_{\text{Rotor}} = 0.028 \Pr^{-0.038} \Re_{Dp}^{0.875}
\]  
(3.49)

\[
\overline{Nu}_{\text{Stator}} = 0.363 \Pr^{-0.056} \Re_{Dp}^{0.545} \Re_{Dm}^{0.041}
\]  
(3.50)

where,
\[
\Re_{Dp} = \frac{U_{\text{flush}} D_p}{\nu}
\]  
(3.51)

\[
\Re_{Dm} = \frac{U_{\text{flush}} D_m}{\nu}
\]  
(3.52)

\[
U_{\text{flush}} = \frac{\alpha D_p}{2}
\]  
(3.53)

where $U_{\text{flush}}$ is the rotation velocity at the surface of the rotating ring, $U_{\text{flux}}$ is the radial flush in flow velocity, $D_p$ is the outer diameter of the rotating ring and $D_m$ is the outer diameter of the mating ring. $Pr$ is evaluated at the surface of the rings in deducing the two equations above. Note that Eqs. (3.49) and (3.50) apply to turbulent flows, which satisfy: $4.21 \times 10^3 < \Re_{Dp} < 3.36 \times 10^6$  
(3.54)

### 3.3.5 Illustrative Example

Consider a pump with rotational speed of 4800 rpm, the process fluid is assumed to be water at 25 °C and the outer radius of the rotating ring is 4.48 cm. the outer radius of the mating ring is 5.08 cm. The flush rate is 1 gpm. Determine the heat convection coefficient over the rotating and stationary rings. From Eq. (3.53), we have

\[
\omega = \frac{4800 \times 2\pi}{60} = 502.65 \text{ rad/s}
\]

\[
U_{\text{flush}} = \frac{\omega D_p}{2} = \frac{(502.65) \times (4.48 \times 10^{-2} \times 2)}{2} = 22.52 \text{ m/s}
\]
Fig. 3.16 Friction heat flux flowing into the rotor along the contact face

Fig. 3.17 Friction heat flux flowing into the stator along the contact face
Fig. 3.18 Heat transfer coefficient along face 1

Fig. 3.19 Heat transfer coefficient along face 2
From Eq. (3.51)

\[ \text{Re}_{D_p} = \frac{U_{\text{in}} D_p}{\nu} = \frac{22.52 \times (4.48 \times 10^{-2} \times 2)}{9.0 \times 10^{-7}} = 2.24 \times 10^6 \]

\( Pr \) is estimated at the inlet temperature which is 6.02. Using Eq. (3.49), the average Nusselt number for rotating ring is

\[ \overline{\text{Nu}}_{\text{Rotor}} = 0.028 \Pr^{-0.038} \text{Re}_{D_p}^{0.875} = 0.028 \times (6.02)^{-0.038} (2.24 \times 10^6)^{0.875} = 9418.9 \]

From Eq. (3.45)

\[ \overline{\text{h}}_{\text{Rotor}} = \frac{\overline{\text{Nu}}_{\text{Rotor}} k_f}{D_p} = \frac{9418.9 \times 0.613}{4.48 \times 10^{-2} \times 2} = 6.44 \times 10^4 \text{ W/m}^2\cdot\text{K} \]

Next, we calculate the heat transfer coefficient for the mating ring with an outer radius of 5.08 cm and flush rate of 63.1 cm\(^3\)/s (1.0 gpm) entering a flush hole with diameter of 0.35 cm.

\[ U_{\text{flash}} = \frac{Q_{\text{flash}}}{\pi D_{\text{flash}}^2 / 4} = \frac{63.1 \times 10^{-6}}{\pi \times (0.35 \times 10^{-2})^2 / 4} = 6.56 \text{ m/s} \]

From Eq. (3.52)

\[ \text{Re}_{D_m} = \frac{U_{\text{flash}} D_m}{\nu} = \frac{6.56 \times (5.08 \times 10^{-2} \times 2)}{9.0 \times 10^{-7}} = 7.41 \times 10^5 \]
$Pr$ is estimated at the inlet temperature as $Pr=6.02$. Then the average Nusselt number for mating ring can be obtained.

$$\overline{Nu}_{\text{Stator}} = 0.363 \cdot Pr^{-0.056} \cdot Re_{Dp}^{0.545} \cdot Re_{Dm}^{0.041} = 0.363 \times (6.02)^{-0.056} \times (2.24 \times 10^6)^{0.545}$$

$$\times \left(7.41 \times 10^5\right)^{0.041} = 1651.2$$

From Eq. (3.45)

$$h_{\text{stator}} = \frac{\overline{Nu}_{\text{stator}} \cdot k_f}{D_p} = \frac{1651.2 \times 0.613}{4.48 \times 10^{-2} \times 2} = 1.13 \times 10^4 \text{ W/m}^2\cdot\text{K}$$

To check the validity of the results, the computer program was run to directly simulate this case without the use of the correlation equations. The energy equations for the fluid in the seal chamber and in the rings were numerically solved and then temperature distribution for the whole computation domain shown in Fig. 3.12 can be obtained. Once the temperature field is predicted, then the convection heat transfer coefficient was calculated by combining Eqs. (3.43) and (3.44). The results are: $h_{\text{rotor}} = 6.63 \times 10^4$ W/m$^2$·K and $h_{\text{stator}} = 1.18 \times 10^4$ W/m$^2$·K. Comparing with the correlations, the error is about 3% for the rotating ring and 4% for the mating ring as shown in Table 3.5.

Also shown in Table 3.5, the difference between the results of correlations and simulations for cases of 1800 rpm, 3600 rpm and 7200 rpm along with the average heat transfer coefficient estimated by FLUENT. It can be seen that the differences between our model and FLUENT results are small.

### 3.3.6 Conclusions

A CFD analysis of turbulent flows with consideration of heat transfer in a mechanical seal is presented. The flow in the seal chamber is turbulent. The turbulent flow field was solved in Ref. [11] and was applied directly in the present study. The energy equation within the flow of the seal chamber, and heat conduction equations in the rotor and the stator are solved simultaneously by applying FDM (Finite Difference Method) combined with staggered grid.

Heat generation due to friction at the contact face between the rotor and the stator is considered. The model applies the appropriate boundary conditions and predicts the temperature fields for the flow field, the rotor and the stator. Temperature distributions and local and average heat transfer are calculated for the shaft speeds of 1800 rpm, 3600 rpm, and 7200 rpm. The heat flux distribution flowing into the stator and the rotor is also determined.

The calculated temperature is used to determine the heat flux distributions and the average Nusselt numbers at the outer surfaces of the rotor and the stator. In this chapter, the correlation equations for calculating the average Nusselt numbers at the outer surfaces of the rotor and the stator are presented. An illustrative example is presented to show the utility of the correlation equations. The correlations can be used to predict the heat transfer coefficients.
transfer coefficient at the seal surfaces which is critically important in analyzing the thermal behavior of mechanical seals [2, 3].

3.4 Nomenclature

\( A_f = \) surface area of friction at seal contact face
\( b = \) mechanical seal balance ratio
\( c_f = \) constant-pressure specific heat of water
\( c_m = \) constant-pressure specific heat of stationary ring
\( c_r = \) constant-pressure specific heat of rotating ring
\( D_p = \) outer diameter of the rotating ring
\( D_m = \) outer diameter of the mating ring
\( D_{flush} = \) diameter of flush hole
\( E_p = \) heat generated at the seal contact face
\( f = \) friction coefficient
\( h = \) local heat transfer coefficient
\( \bar{h} = \) average heat transfer coefficient
\( k_f = \) conductivity of water
\( k_m = \) conductivity of stationary ring
\( k_r = \) conductivity of rotating ring
\( \text{Nu} = \) Local Nusselt number
\( \bar{\text{Nu}} = \) average Nusselt number
\( p = \) local pressure at seal contact face
\( p_m = \) average pressure at seal contact face
\( p_{sp} = \) spring pressure
\( q = \) heat flux at wetted outer surface of rotating ring and stationary ring
\( Q_{flush} = \) flow rate of flush flow
\( r = \) radial coordinate
\( Re_{D_p} = \) Reynolds number of primary ring
\( Re_{D_m} = \) Reynolds number of mating ring
\( R1_1 = \) inner radius of rotating ring
\( R1_2 = \) inner radius of Mating ring
\( RO_1 = \) outer radius of rotating ring
\( RO_2 = \) outer radius of Mating ring
\( t = \) time
\( T(r, z) = \) cross-sectional temperature in rotating ring, stationary ring and flow
\( T_s = \) temperature at outer surface of rotating ring and stationary ring
\( T_e = \) ambient temperature
\( u_r = \) radial velocity component of seal chamber flow
\( u_z = \) axial velocity component of seal chamber flow
\( u_{rin} = \) flush in velocity
\( u_\theta \) = angular velocity component
\( u_{\theta n} \) = rotating speed of primary ring
\( V \) = local velocity of rotating ring
\( z \) = axial coordinate
\( \Delta T_{AV} \) = average temperature difference
\( \Delta p \) = pressure differential
\( \rho_f \) = density of process fluid
\( \rho_m \) = density of mating ring
\( \rho_r \) = density of rotating ring
\( \nu \) = kinematic viscosity of process fluid
\( \omega \) = shaft speed
\( c'_u = k - \varepsilon \) turbulent model constant
\( \sigma_f = k - \varepsilon \) turbulent model constant
\( k \) = turbulent kinematic energy
\( \varepsilon \) = turbulent kinematic energy dissipation
Chapter 4 presents a 3D computational fluid dynamics (CFD) model for flow and thermal analysis of a mechanical seal with experimental verification. The computational model is capable of predicting the flow field in the seal chamber and temperature field for the seal rings including the rotating and the stationary rings. This model is also used to determine convection heat transfer coefficients on the wetted surfaces of the seal rings. Comparison with experimental tests conducted in a laboratory is presented.

4.1 Introduction

Mechanical seals are widely used in industry to seal working liquid or gas from leaking. Failure of mechanical seals can have major environmental impact on chemical and process industries. Therefore, end-users of mechanical seals demand good performance and reliability. To achieve this goal, researchers find it is necessary to apply advanced experimental and analytical tools to better understand and quantify those parameters which affect the seal performance and reliability.

In practice, the distortion of the seal face is known to create a critical leakage path in a mechanical seal. There are two sources that cause distortion: One is thermal gradients in the rings and the other mechanical loading. Compared with mechanical loading, the parameters that create the thermal gradient are much more difficult to quantify [1]. Heat conduction in the seal rings and heat convection on the wetted surface of the seal rings are most influential parameters that affect the thermal gradients. In Chapters 2 and 3, we developed 2D numerical models for both laminar and turbulent flows and presented correlations for determining the Nusselt numbers on the rings in a mechanical seal. In this chapter, the analysis of flow and heat transfer in a mechanical seal is extended to three-dimension. Also, the results of computation are compared to laboratory tests.

A brief literature survey of pertinent research is as follows. Doane et. al [12] performed experimental measurements to better understand the nature of the boundary conditions of the numerical computations and obtained local and average Nusselt number for the wetted area of the mating ring. Lebeck [5] studied the thermal environment of the seals such as thermally induced radial taper, thermally induced waviness, heat checking and hot spotting, and blistering. Jang and Khonsari [6] extended the theory of thermoelastic analysis to predict the critical speed at which hot spots can occur on the surface of a seal. These publications concentrated in the thermal analysis of mechanical seals but neglected the effect of the flow field in the seal chambers. Research by Merati et al. [1] well predicted the turbulence flow field in a seal chamber and the temperature distribution within the stator of a mechanical seal. More recently, Luan and Khonsari [20, 21] developed CFD models for analyzing the turbulent and laminar flows in a seal chamber and discussed the effects of the flow field on the heat transfer behavior of seal rings.
4.2 Experimental Test Apparatus

The experimental apparatus is shown in Fig. 4.1. The apparatus consists of an instrumental mechanical seal, associated pumping loop, coolant (flush) loop, reservoir, and an inlet and discharge water pipe to maintain the temperature in the reservoir at steady state. The temperature of the water in the reservoir will rise as long as the experiment continues if fresh water at ambient temperature is not added to it. For this purpose, water is piped to the reservoir and at the same time some of the water in the pumping cycle is discharged at a rate that maintains the water in the reservoir at steady state. The pump is driven by a 3 hp motor operating at 3450 rpm. Type J thermocouples are used to measure the temperature and their sensitivity was about 62 μV/°C. The system pressure is maintained at 40 psi. Four temperature measurements are taken in the mating ring and another one is used to measure the temperature of the gland. The procedure of these measurements is described below.

The thermocouples are cemented in the proper holes of the gland and the stator ring. The pump is insulated using fiberglass insulation. The Webdaq instrumentation system is programmed so that it can acquire and store temperature reading over the duration of the test.
4.3 Experiment Procedure

The experiment is started by turning on the pump, setting the shaft speed at 3450 rpm, and waiting 2 hours to let the temperatures reach steady state. Five sets of temperatures are recorded at 1 minute interval including four in the mating ring (stator) and one in a small inlet near the top of the gland. These readings are averaged to get a single temperature value for each location. After recording these data, the pump is stopped and the test chamber allowed to be cooled. Then the experiment is repeated three times at the same shaft speed and operating pressure to ensure that the results are reproducible.
4.4 Computational Models and Boundary Conditions

The commercial CFD software, FLUENT, is used to create a model and simulate the 3D flow in the seal chamber. The results of the flow field are then used to determine the thermal behavior of the rotating and mating rings of a mechanical seal.
Figure 4.2 shows the seal installment of interest. Note there are two flush inlet ports in the gland. Figure 4.3 shows the notations describing the boundary conditions. The summary of the thermal boundary conditions is shown in Table 4.1. In reality, there exists air space between the shaft and the seal rings. Therefore, heat convection should happen in W4, W5 and W6. At the contact face (WR and WS) of the two rings, friction heat is generated. The heat generated at the seal contact face is estimated using the method introduced in chapters 2 and 3.

Table 4.1 Boundary conditions

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Type</th>
<th>Thermal Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>Velocity Inlet</td>
<td>$T = T_{\infty}$</td>
<td>Velocity of flush</td>
</tr>
<tr>
<td>I2</td>
<td>Velocity Inlet</td>
<td>$T = T_{\infty}$</td>
<td>Velocity of flush</td>
</tr>
<tr>
<td>O1</td>
<td>Pressure Outlet</td>
<td>$T = T_{gland}$</td>
<td>Seal chamber wall</td>
</tr>
<tr>
<td>W1</td>
<td>Wall, constant $T$</td>
<td>$T = T_{gland}$</td>
<td>Seal chamber wall</td>
</tr>
<tr>
<td>W2</td>
<td>Wall, insulation</td>
<td>$\frac{\partial T_m}{\partial z} = 0$</td>
<td>Stator-sleeve gap</td>
</tr>
<tr>
<td>W3</td>
<td>Wall, convection to air</td>
<td>$k_m \frac{\partial T_m}{\partial r} = h_1(T_s - T_{air})$</td>
<td>Stator-sleeve gap</td>
</tr>
<tr>
<td>W4</td>
<td>Wall, convection to air</td>
<td>$k_m \frac{\partial T_m}{\partial z} = h_2(T_s - T_{air})$</td>
<td>Stator-sleeve gap</td>
</tr>
<tr>
<td>W5</td>
<td>Wall, convection to air</td>
<td>$k_m \frac{\partial T_s}{\partial r} = h_3(T_s - T_{air})$</td>
<td>Rotor-sleeve gap</td>
</tr>
<tr>
<td>W6</td>
<td>Wall, insulation</td>
<td>$\frac{\partial T_r}{\partial z} = 0$</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2 Material properties of rotor and stator

<table>
<thead>
<tr>
<th></th>
<th>$K$ (W/m.K)</th>
<th>$\rho$ (Kg/m$^3$)</th>
<th>$c$ (J/Kg.K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>working fluid (water)</td>
<td>$k_f = 0.6$</td>
<td>$\rho_f = 997$</td>
<td>$c_f = 4179$</td>
</tr>
<tr>
<td>Rotor (Carbon Graphite)</td>
<td>$k_m = 24$</td>
<td>$\rho_m = 2250$</td>
<td>$c_m = 699$</td>
</tr>
<tr>
<td>Stator (Stainless Steel 316)</td>
<td>$k_r = 21.4$</td>
<td>$\rho_r = 7990$</td>
<td>$c_r = 500$</td>
</tr>
</tbody>
</table>

4.5 Results and Discussions

The basic dimension of the mating ring (stator) and the rotating ring are shown in Figs. 4.4 - 4.5. Three thermocouples are located at the same depth from the contact interface at 0.0025 cm. However, each of these thermocouples is at a different radial distance of 2.05 cm, 2.2 cm and 2.3 cm, respectively. Another thermocouple is located at a radial distance of 2.2 cm but at a different angular position.
The 3D element of Tet/Hybrid available in FLUENT was used. The grid is uniform, and no effort of optimizing the grid is attempted.

Table 4.3 Pressure and friction values of the seal

<table>
<thead>
<tr>
<th>Name</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta P$</td>
<td>Pressure differential</td>
</tr>
<tr>
<td>$P_{sp}$</td>
<td>Spring pressure</td>
</tr>
<tr>
<td>$k^3$</td>
<td>Pressure gradient factor</td>
</tr>
<tr>
<td>$b$</td>
<td>Seal balance ratio</td>
</tr>
<tr>
<td>$f$</td>
<td>Friction coefficient</td>
</tr>
<tr>
<td>$T_{air}$</td>
<td>Air temperature</td>
</tr>
<tr>
<td>$T_{gland}$</td>
<td>Gland temperature</td>
</tr>
<tr>
<td>$T_{in}$</td>
<td>Inlet water temperature</td>
</tr>
<tr>
<td>$h_1$</td>
<td>Heat transfer coefficient</td>
</tr>
<tr>
<td>$h_2$</td>
<td>Heat transfer coefficient</td>
</tr>
<tr>
<td>$h_3$</td>
<td>Heat transfer coefficient</td>
</tr>
</tbody>
</table>

The material properties used for the silicon carbide rotor and graphite stator are shown in Table 4.2. Temperatures at W1, W2, W3 and W7 are assumed to be the measured gland temperature which is around 314 K measured by a thermocouple (see Table 4.3). Axial $U_z$ and radial $U_r$ velocity components are used as inlet velocity boundary conditions at I1 and I2,

$U_z=1.72 \text{ m/s}$                                                                                                                  (4.1)
$U_r=9.74 \text{ m/s}$                                                                                                                  (4.2)

Values in Eqs. (4.1) and (4.2) are calculated with 1 gpm (63.1 cm³/s) flush flux.

As discussed in Chapters 2 and 3, to take account the heat generation at the sealing interface, a heat source in a very thin element, at the contact region between the rotor and the stator, is applied. To obtain a reliable heat partitioning between the seal rings, the source element is used as thin as $10^{-5}$ m and has a good heat conductivity of 1000 W/m.K. As long as the heat conductivity of the fluid film is big enough and the film thickness is small enough, theses values do not affect the heat transfer problem much.

We used about 100,000 elements in the computation. A computer with Pentium(R) C CPU 3.2 GHz and 2.0 GB of RAM was used to execute the code. Typically it takes about 2 hours to complete the computation. The convergence criteria we used for velocities and the temperature are $1 \times 10^{-6}$ and $1 \times 10^{-8}$, respectively.

4.5.1 Velocity Field

Figure 4.6(a) shows the vector plot at the flush inlet region ($\theta = 0^\circ$). It can be seen that at the flush inlet region, the flush fluid entering the right inlet port flows onto the outer
surface of the mating ring, then joins the flush entering through the left port, and flows out of the seal chamber. It can be expected that, at the flush inlet region the flush in flow can cool the mating ring very efficiently.

Figure 4.6 (b) shows the vector plot at the seal chamber with $\theta = 180^\circ$. The region represents the flow field at those regions far away from the flush inlet. Examining the figure, it can be recognized that there exists a fairly big vortex at the region around the mating ring. At the region around the rotating ring, back flow occurs at the outlet. It can be noticed that the back flow sweeps over the outer surface of the rotating ring and part of the surface of the mating ring (vertical surface), thus it can cool the two rings effectively.

4.5.2 Temperature, Heat Flux, Heat Transfer Coefficients

Constant temperature contours for the seal chamber, rotor and stator at $\theta = 180^\circ$ are shown in Fig. 4.7. The temperature in the seal chamber is maintained at a constant temperature of about 310 K, which is consistent with Ref. [1]. Highest temperatures occur near the contact face between the stator and rotor rings, as expected.

As discussed in section 4.5, temperatures were measured at three radial locations at the same axial distance of $z = 0.0025$ m from the contact face within the stator. The measurement results are shown in Fig. 4.8. The temperatures rise during the transient period. The temperatures become steady after about 110 minutes. Examining Fig. 4.8, it can be found that the temperatures of positions 2 and 4 are very close, which suggests that the temperature at the interface between the seal rings is axisymmetric since positions 2 and 4 are located in the same radial position, but at a different angular position.

Comparisons of the temperature magnitudes with the computational results are shown in Fig. 4.9. The trend and magnitude of the measurements and computational results are in agreement. The measured values are consistently smaller than the computational results. The maximum discrepancy between the measurement and prediction is 1.5 °C. The temperature profile drops to minimum after it reaches its maximum at around $r=2.1$ cm.

Based on the temperature distribution predicted by FLUENT, one can use the Newton’s Law of cooling to calculate the heat transfer coefficients at the outer surfaces of the seal rings. Figure 4.10 shows the heat transfer coefficients at the surfaces of the rotor.

In Fig. 4.10, $h$ decreases gradually along the axial direction, $z$. This trend can be explained by examining the flow field within the seal chamber shown in Fig.4.6 (b). Back flow from the outlet of the seal chamber sweeps the outer surface of the rotating ring and helps to remove the heat there. Thus, it can be expected that the heat transfer is more efficient at the outlet region, and that with the flow carrying more heat along the outer surface of the rotating ring, the heat transfer efficiency will decay gradually.
Fig. 4.6 Computational vector plots: (a) at $\theta = 0^\circ$  (b) $\theta = 180^\circ$
Fig. 4.7 Contours of constant temperature

<table>
<thead>
<tr>
<th>Level</th>
<th>Temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>318.34</td>
</tr>
<tr>
<td>2</td>
<td>316.92</td>
</tr>
<tr>
<td>3</td>
<td>317.50</td>
</tr>
<tr>
<td>4</td>
<td>318.07</td>
</tr>
<tr>
<td>5</td>
<td>318.64</td>
</tr>
<tr>
<td>6</td>
<td>319.22</td>
</tr>
<tr>
<td>7</td>
<td>319.79</td>
</tr>
<tr>
<td>8</td>
<td>320.37</td>
</tr>
<tr>
<td>9</td>
<td>320.84</td>
</tr>
<tr>
<td>10</td>
<td>321.52</td>
</tr>
<tr>
<td>11</td>
<td>322.10</td>
</tr>
<tr>
<td>12</td>
<td>322.87</td>
</tr>
</tbody>
</table>

Fig. 4.8 Experimental measurements of temperatures
Fig. 4.9 Comparison of temperature magnitudes with computational results within the stator at \( Z = 0.0025 \) m

Fig. 4.10 Heat transfer coefficient for the horizontal surface of the rotor at \( \theta = 180^\circ \)
In chapter 3, we developed heat transfer correlations based on 2D numerical results. Now, we use those correlations to calculate the average heat transfer coefficients for the two rings in the 3D mechanical seal of the current interest. Following the procedures presented in chapter 3, we have

$$\omega = \frac{3450 \times 2\pi}{60} = 361.1 \text{ rad/s}$$
\[
U_{\text{in}} = \frac{\omega D_p}{2} = \frac{(361.1) \times (2.24 \times 10^{-2} \times 2)}{2} = 8.1 \text{ m/s}
\]

\[
\text{Re}_{D_p} = \frac{U_{\text{in}} D_p}{\nu} = \frac{8.1 \times (2.24 \times 10^{-2} \times 2)}{9 \times 10^{-7}} = 4.03 \times 10^5
\]

\(Pr\) is estimated as 4.6 for water at inlet temperature of 310 K. Using the average Nusselt number for the rotating ring is

\[
\overline{Nu}_{\text{Rotor}} = 0.028 \Pr^{-0.038} \text{Re}^{0.875}_{D_p} = 0.028 \times (4.6)^{-0.038} (4.03 \times 10^5)^{0.875} = 2121.4
\]

\[
\overline{h}_{\text{rotor}} = \frac{\overline{Nu}_{\text{Rotor}} k_f}{D_p} = \frac{2121.4 \times 0.6}{2.24 \times 10^{-2} \times 2} = 28412 \text{ W/m}^2\text{K}
\]

\(U_{\text{flush}} = 9.74 \text{ m/s}\)

\[
\text{Re}_{D_m} = \frac{U_{\text{flush}} D_m}{\nu} = \frac{9.74 \times (2.54 \times 10^{-2} \times 2)}{9 \times 10^{-7}} = 5.5 \times 10^5
\]

\(Pr\) is estimated as 4.6 at inlet temperature. Then the average Nusselt number for the mating ring

\[
\overline{Nu}_{\text{Stator}} = 0.363 \Pr^{-0.056} \text{Re}^{0.545}_{D_p} \text{Re}^{0.041}_{D_m} = 0.363 \times (4.6)^{-0.056} \times (4.03 \times 10^5)^{0.545} \times (5.5 \times 10^5)^{0.041} = 650.2
\]

From Eq. (10)

\[
\overline{h}_{\text{stator}} = \frac{\overline{Nu}_{\text{Stator}} k_f}{D_p} = \frac{650.2 \times 0.6}{4.48 \times 10^{-2}} = 8708.0 \text{ W/m}^2\text{K}
\]

Taking the average values of Figs. 4.10-4.12, we can calculate the average heat transfer coefficients for the outer surfaces of the rotating and the mating ring, respectively. The results are: \(\overline{h}_{\text{rotor}} = 28611.7 \text{ W/m}^2\text{K}\) and \(\overline{h}_{\text{stator}} = 11099.0 \text{ W/m}^2\text{K}\). Comparing these results calculated from 3D FLUENT simulations to the correlation results, it can be found that the difference for the rotating ring is very small, while for the mating ring the 3D FLUENT result is about a bit higher than the correlation result. This is due to the two flush inlet ports in the 3D model. Note that the 2D correlations are developed based on a mechanical seal with only one flush inlet.

### 4.6 Conclusions

1. A 3D flow model with heat transfer is developed applying proper boundary conditions.
2. Flow filed in the seal chamber is not axisymmetric but the temperature field within the seal components is axisymmetric.
3. The seal face generated heat is mainly convected to the seal chamber flow by the rotor and the largest magnitude of heat flux occurs on the rotor surface near the interface between the rotor and the stator.
4. The temperature comparison between the experimental and computational investigations at the some locations within the stator insured the reliability of our model.
5. By comparison of the average heat transfer coefficients calculated from 3D FLUENT simulations and 2D heat transfer correlations, we can find that the 2D correlations presented in chapter 3 can be applied in the 3D cases, especially for the rotating ring. Even for the mating ring, the difference due to different flush inlets is still as small as 13%.

4.7 Nomenclature

\( B \) = balance ratio
\( c_f \) = heat capacity of water
\( c_m \) = heat capacity of the mating ring (the stator)
\( c_r \) = heat capacity of the rotating ring (the rotor)
\( f \) = friction coefficient
\( h \) = heat transfer coefficient
\( h_1 \) = heat transfer coefficient
\( h_2 \) = heat transfer coefficient
\( h_3 \) = heat transfer coefficient
\( k_f \) = heat conductivity of water
\( k_m \) = heat conductivity of the mating ring
\( k_r \) = heat conductivity of the rotating ring
\( R \) = radial coordinate
\( T \) = temperature
\( T_{\text{air}} \) = air temperature
\( T_{\text{gland}} \) = gland temperature
\( U_r \) = radial velocity
\( U_z \) = axial velocity
\( \rho_f \) = density of water
\( \rho_m \) = density of the mating ring
\( \rho_r \) = density of the rotating ring
Chapter 5 presents an analytical solution to the heat transfer in the seal rings. The 2D heat transfer equations in cylindrical coordinates for the mating ring and rotating ring are solved simultaneously considering the heat generation at the contact face between the rings. For this purpose, the separation of variable method is applied. The analytical solution is compared with the numerical one predicted by CFD software, FLUENT. The partitioning factor of the heat transfer at the interface between the rings is also discussed.

5.1 Introduction

Many researchers have applied advanced experimental and analytical tools to understand and quantify the nature of heat transfer in mechanical seals. A brief review of publications to the present study is stated as follows. Li [23] assumed a constant heat input in the sealing interfaces for the energy equation and applied the finite element method (FEM) to predict the temperature distribution of mechanical seals. Morariu and Pascovici [11] studied the heat conduction in the rings and convection by the coolant around the rings. Buck [2, 3] provided a simplified approach for determining the seal temperature based on an analytical model that treats a seal as a fin. Recently Luan and Khonsari [22] extended this method to consider the heat partitioning at the sealing interface. Doane et. al [12] performed experimental measurements as the boundary conditions of the numerical computations and obtained local and average Nusselt number for the wetted area of the mating ring. They measured the interfacial temperature distribution between the rings. Lebeck [5] considered many of the important effects caused by the thermal environment such as thermally induced radial taper, thermally induced waviness, heat checking and hot spotting, and blistering. Jang and Khonsari [6] developed the theory of thermoelastic analysis to predict the critical speed at which hot spots can occur on the surface of a seal.

In this study, a simplified mathematical model of heat transfer in the rotating and mating rings of a mechanical seal is developed and solution is attained using the separation of variable method. Temperature profiles in the rings and temperature distribution at the contact face are presented. The partitioning factor of heat transfer at the contact face is considered. To verify the results, the analytical solution is compared with the numerical one estimated by software package, FLUENT.

5.2 Problem Formulation

5.2.1 Mechanical Seals with Rings of the Same Inner and Outer Radius: Case 1

Figure 5.1 shows the schematic of the rotating and stationary rings with the same inner and outer radius in a mechanical seal. Taking advantage of axisymmetric nature of the problem and assuming uniform heat convection coefficients $h_1$ and $h_2$ at the outer surface of the rotating ring and the stationary ring, respectively, allows one to formulate a two-dimensional problem. Generally, the materials of the rotating and stationary rings are
different. The interfacial frictional heat at the contact face is conducted into the two rings and then transferred to the free stream at the seal chamber by convection. The heat conduction in the two rings is determined simultaneously considering the heat generation at their contact face as matching conditions since the heat partitioning factor is not known.

Assuming uniform material properties, the governing equation for the steady state heat conduction in the rotating ring is:

$$\frac{\partial^2 \theta_1}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_1}{\partial r} + \frac{\partial^2 \theta_1}{\partial z^2} = 0 \quad \text{for } r_i < r < r_o, \quad -L < z < 0$$  \hspace{1cm} (5.1)

The heat diffusion equation in the stationary ring is:

$$\frac{\partial^2 \theta_2}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_2}{\partial r} + \frac{\partial^2 \theta_2}{\partial z^2} = 0 \quad \text{for } r_i < r < r_o, \quad 0 < z < L$$  \hspace{1cm} (5.2)

where $\theta_1 = T_1 - T_\infty$ and $\theta_2 = T_2 - T_\infty$. Note that under the assumptions of steady state and axisymmetry, the effect of rotation of the rotating ring is not reflected in the above heat conduction equations but in the interfacial friction heat generation at the contact face between the two rings.

Equations (5.1) and (5.2) are subject to the following boundary conditions.

$$\frac{\partial \theta_1}{\partial r} \bigg|_{r=r_i} = 0, \quad \frac{\partial \theta_2}{\partial r} \bigg|_{r=r_i} = 0$$  \hspace{1cm} (5.3)

$$k_r \frac{\partial \theta_1}{\partial r} \bigg|_{r=r_o} = -h_1 \theta_1 \bigg|_{r=r_o}, \quad k_m \frac{\partial \theta_2}{\partial r} \bigg|_{r=r_o} = -h_2 \theta_2 \bigg|_{r=r_o}$$  \hspace{1cm} (5.4)

$$k_r \frac{\partial \theta_1}{\partial z} \bigg|_{z=0} = q, -k_m \frac{\partial \theta_2}{\partial z} \bigg|_{z=0} = q_m$$  \hspace{1cm} (5.5)

$$\frac{\partial \theta_1}{\partial z} \bigg|_{z=-L} = 0, \quad \frac{\partial \theta_2}{\partial z} \bigg|_{z=L} = 0$$  \hspace{1cm} (5.6)
In the above equations, \( k_r \) and \( k_m \) are the heat conductivity for the rotating and stationary rings, respectively. Parameters \( q_r \) and \( q_m \) are the heat flux flowing into the rotating and stationary rings, respectively.

We seek a solution with form of \( \theta(r, z) = R(r)Z(z) \) which upon substitution into the governing equation (Eq. (5.1)).

\[
\frac{1}{R \cdot r} \left[ \frac{d}{dr} \left( r \frac{dR}{dr} \right) \right] = -\frac{1}{Z_1} \left[ \frac{d^2 Z_1}{dz^2} \right] = -\lambda_1^2 \tag{5.7}
\]

Accordingly, \( R_1(r) \) should satisfy

\[
\frac{d}{dr} \left( r \frac{dR_1}{dr} \right) + \lambda_1^2 r R_1 = 0 \tag{5.8}
\]

with

\[
R_1(r_i) = 0 \tag{5.9}
\]

\[
k_r R_1'(r_o) + h_i R_1(r_o) = 0 \tag{5.10}
\]

Likewise, \( Z_1(z) \) must satisfy

\[
\frac{d^2 Z_1}{dz^2} - \lambda_1^2 Z_1 = 0 \tag{5.11}
\]

with

\[
Z_1(-L) = 0 \tag{5.12}
\]

The second-order ordinary differential equations for \( R_1(r) \) and \( Z_1(z) \) with applying the boundary conditions can be analytically solved. The results are:

\[
R_1(r) = C_1 \left[ J_0(\lambda_1 r) - \frac{J_{-1}(\lambda_1 r)}{Y_{-1}(\lambda_1 r)} Y_0(\lambda_1 r) \right], \quad C_1 \text{ is constant} \tag{5.13}
\]

\[
Z_1(z) = C_2 \left[ \sinh(\lambda_1 z) + \frac{\cosh(\lambda_1 L)}{\sinh(\lambda_1 L)} \cosh(\lambda_1 z) \right], \quad C_2 \text{ is constant} \tag{5.14}
\]

Thus,

\[
\theta_1(r, z) = \sum_{n=1}^{\infty} A_n \left[ J_0(\lambda_{1n} r) - \frac{J_{-1}(\lambda_{1n} r)}{Y_{-1}(\lambda_{1n} r)} Y_0(\lambda_{1n} r) \right] \left[ \sinh(\lambda_{1n} z) + \frac{\cosh(\lambda_{1n} L)}{\sinh(\lambda_{1n} L)} \cosh(\lambda_{1n} z) \right] \tag{5.15}
\]

where \( A_n \) is constant and \( \lambda_{1n} \) is eigenvalue. The eigenfunction is determined from the boundary conditions (Eqs.(5.9)-(5.10))

\[
J_{-1}(\lambda_{1n} r)[k_r \lambda_{1n} Y_{-1}(\lambda_{1n} r_o) + h_i Y_0(\lambda_{1n} r_o)] - Y_{-1}(\lambda_{1n} r)[k_r \lambda_{1n} J_{-1}(\lambda_{1n} r_o) + h_i J_0(\lambda_{1n} r_o)] = 0 \tag{5.16}
\]

Using the same method, the temperature distribution in the stationary ring can be found. The result is:
\[ \theta_2(r,z) = \sum_{n=1}^{\infty} B_n \left[ J_0(\lambda_{2n} r) - \frac{J_{-1}(\lambda_{2n} r)}{Y_{-1}(\lambda_{2n} r)} Y_0(\lambda_{2n} r) \right] \sinh(\lambda_{2n} z) = -\frac{\cosh(\lambda_{2n} L)}{\sinh(\lambda_{2n} L)} \cosh(\lambda_{2n} z) \]  

(5.17)

where \( B_n \) is constant and \( \lambda_{2n} \) is eigenvalue which is determined by the eigenfunction

\[ J_{-1}(\lambda_{2n} r) \left[ k_m \lambda_{2n}^2 Y_{-1}(\lambda_{2n} r) + h_2 Y_0(\lambda_{2n} r) \right] - Y_{-1}(\lambda_{2n} r) \left[ k_m \lambda_{2n}^2 J_{-1}(\lambda_{2n} r) + h_2 J_0(\lambda_{2n} r) \right] = 0 \]  

(5.18)

At \( z=0 \), i.e. at the contact face, the temperature of the two rings is equal

\[ \theta_2 \bigg|_{z=0} = \theta_1 \bigg|_{z=0} \]  

(5.19)

Assuming \( q \) is the total heat generation at the contact face, then

\[ q = q_r + q_m \]  

(5.20)

where \( q_r \) and \( q_m \) are the heat flux transferred into the rotating ring and the mating ring, respectively.

Substituting Eqs. (5.5), (5.15) and (5.17) into Eq. (5.19) and (5.20)

\[ \sum_{n=1}^{\infty} A_n \left[ J_0(\lambda_{1n} r) - \frac{J_{-1}(\lambda_{1n} r)}{Y_{-1}(\lambda_{1n} r)} Y_0(\lambda_{1n} r) \right] \cosh(\lambda_{1n} L) = \sum_{n=1}^{\infty} B_n \left[ J_0(\lambda_{2n} r) - \frac{J_{-1}(\lambda_{2n} r)}{Y_{-1}(\lambda_{2n} r)} Y_0(\lambda_{2n} r) \right] \]  

\[ \frac{\cosh(\lambda_{2n} L)}{\sinh(\lambda_{2n} L)} \]  

\[ k_r \sum_{n=1}^{\infty} A_n \lambda_{1n} \left[ J_0(\lambda_{1n} r) - \frac{J_{-1}(\lambda_{1n} r)}{Y_{-1}(\lambda_{1n} r)} Y_0(\lambda_{1n} r) \right] - k_m \sum_{n=1}^{\infty} B_n \lambda_{2n} \left[ J_0(\lambda_{2n} r) - \frac{J_{-1}(\lambda_{2n} r)}{Y_{-1}(\lambda_{2n} r)} Y_0(\lambda_{2n} r) \right] = q \]  

(5.21)

Combining the two equations above, Eqs. (5.23) and (5.24) are derived

\[ \sum_{n=1}^{\infty} A_n \left[ J_0(\lambda_{1n} r) - \frac{J_{-1}(\lambda_{1n} r)}{Y_{-1}(\lambda_{1n} r)} Y_0(\lambda_{1n} r) \right] \frac{\cosh(\lambda_{1n} L)}{\sinh(\lambda_{1n} L) \cosh(\lambda_{2n} L)} \]  

\[ k_r \lambda_{1n} \frac{\cosh(\lambda_{1n} L) \sinh(\lambda_{2n} L)}{\sinh(\lambda_{1n} L) \cosh(\lambda_{2n} L)} \]  

\[ J_0(\lambda_{1n} r) - \frac{J_{-1}(\lambda_{1n} r)}{Y_{-1}(\lambda_{1n} r)} Y_0(\lambda_{1n} r) \]  

\[ = q \]  

(5.23)

\[ - \sum_{n=1}^{\infty} B_n \left[ k_m \lambda_{2n} + k_r \lambda_{1n} \frac{\cosh(\lambda_{2n} L) \sinh(\lambda_{1n} L)}{\sinh(\lambda_{2n} L) \cosh(\lambda_{1n} L)} \right] \]  

\[ J_0(\lambda_{2n} r) - \frac{J_{-1}(\lambda_{2n} r)}{Y_{-1}(\lambda_{2n} r)} Y_0(\lambda_{2n} r) \]  

\[ = q \]  

(5.24)

Using the orthogonality of the Bessel functions, \( A_n \) and \( B_n \) can be solved. The results are as follows:

\[ A_n = \frac{1}{k_r \lambda_{1n} + k_m \lambda_{2n} \frac{\cosh(\lambda_{1n} L) \sinh(\lambda_{2n} L)}{\sinh(\lambda_{1n} L) \cosh(\lambda_{2n} L)}} \int_0^r \left[ J_0(\lambda_{1n} r) - \frac{J_{-1}(\lambda_{1n} r)}{Y_{-1}(\lambda_{1n} r)} Y_0(\lambda_{1n} r) \right] d r \]  

(5.25)

\[ B_n = -\frac{1}{k_m \lambda_{2n} + k_r \lambda_{1n} \frac{\cosh(\lambda_{2n} L) \sinh(\lambda_{1n} L)}{\sinh(\lambda_{2n} L) \cosh(\lambda_{1n} L)}} \int_0^r \left[ J_0(\lambda_{2n} r) - \frac{J_{-1}(\lambda_{2n} r)}{Y_{-1}(\lambda_{2n} r)} Y_0(\lambda_{2n} r) \right] d r \]  

(5.26)
Since the coefficients $A_n$ and $B_n$ are solved, the temperature distribution can be obtained using Eqs. (5.15) and (5.17). Then the heat flux $q_r$ and $q_m$ can be solved thus the ratio of the heat flux into the rotating ring and the stationary ring can be calculated:

\[
\gamma = \frac{\int q_r (2\pi dr)}{\int q_m (2\pi dr)} = -\frac{\int (2\pi dr)k_r \sum_{n=1}^{\infty} \lambda_{1n} A_n \left[ J_0(\lambda_{1n} r) - \frac{J_{-1}(\lambda_{1n} r)}{Y_{-1}(\lambda_{1n} r)} Y_0(\lambda_{1n} r) \right]}{\int (2\pi dr)k_m \sum_{n=1}^{\infty} \lambda_{2n} B_n \left[ J_0(\lambda_{2n} r) - \frac{J_{-1}(\lambda_{2n} r)}{Y_{-1}(\lambda_{2n} r)} Y_0(\lambda_{2n} r) \right]} \tag{5.27}
\]

Also, the temperature distribution at the interface between the rings is calculated using

\[
\theta_{int}(r) = \frac{1}{2}(\theta_1(r,0) + \theta_2(r,0)) \tag{5.28}
\]

### 5.2.2 Mechanical Seals with Rings of Different Inner and Outer Radius: Case 2

In section 5.2.1, the seal rings with the same inner and outer radius are considered. In practice, most rotating and stationary rings have different radius. Figure 5.2 shows the schematic of the mechanical seal of case 2. It can be seen from this figure that the outer radius of the stationary ring is larger and the axial length of the two rings are also different.

For the rotating ring, the computation is the same as case 1 in section 5.2.1. The temperature distribution can be calculated by Eqs. (5.15), (5.16) and (5.25). The computation of the stationary ring is more involved. The complexity lies in the boundary ($z = 0$). The boundary consisting of two parts are:

\[
-k_m \frac{\partial \theta_m}{\partial z} \bigg|_{z=0} = q_m, \quad r_1 < r < r_{o1} \tag{5.29}
\]

\[
k_m \frac{\partial \theta_m}{\partial z} \bigg|_{z=0} = h_2 \theta_m \bigg|_{z=0}, \quad r_{o1} < r < r_{o2} \tag{5.30}
\]

The other boundary conditions in Fig.5.2 are the same as in Fig.5.1. It is assumed that the heat generated at the interface between the rings is removed by heat convection at the outer surface of the two rings, and that the inner surfaces and the axial ends are insulated.

To solve the temperature distribution within it, the stationary ring is divided into two parts: stationary ring 1 and 2 (see Fig.5.3). Note that there exist two matching conditions at $r=r_{o1}$ for stationary ring 1 and 2

\[
\theta_{m1} \bigg|_{r=r_{o1}} = \theta_{m2} \bigg|_{r=r_{o1}} \tag{5.31}
\]

\[
\frac{\partial \theta_{m1}}{\partial r} \bigg|_{r=r_{o1}} = \frac{\partial \theta_{m2}}{\partial r} \bigg|_{r=r_{o1}} \tag{5.32}
\]
The stationary ring 1 can be solved using separation of variables since it has homogeneous boundary conditions in the $z$ direction. However, there are no homogeneous boundary conditions for stationary ring 2. To tackle this problem, stationary ring 2 is decomposed into two parts: stationary ring 2 (a) and stationary ring 2 (b) as shown in Fig. 5.4. In the axial direction $z$, the boundary condition is homogeneous for stationary ring 2 (a). In the $r$ direction, the boundary condition for stationary ring 2(b) is homogeneous. The superposition of the temperature in stationary ring 2 (a) and (b) is the temperature of stationary ring 2. Note that stationary ring 2 (b) can be solved following the same procedure of solving the stationary ring in case 1 (see Fig. 5.1). The temperature distribution in stationary ring 2(b) can be expressed as

$$\theta_{m2b}(r,z) = \sum_{n=1}^{\infty} B_n \left[ J_0(\lambda_{2n}r) - \frac{J_{-1}(\lambda_{2n}r_{11})}{Y_{-1}(\lambda_{2n}r_{11})} Y_0(\lambda_{2n}r) \right] \sinh(\lambda_{2n}z) - \frac{\cosh(\lambda_{2n}L_2)}{\sinh(\lambda_{2n}L_2)} \cosh(\lambda_{2n}z)$$

(5.33)

where eigenvalues $\lambda_{2n}$ are determined by

$$J_{-1}(\lambda_{2n}r_{11}) Y_0(\lambda_{2n}r_{01}) - Y_{-1}(\lambda_{2n}r_{11}) J_0(\lambda_{2n}r_{01}) = 0$$

(5.34)

And $B_n$ can be determined by

$$B_n = \frac{\int_{r_i}^{r_o} \left[ \int_{r_{o1}}^{r_{o2}} q_u(r) J_0(\lambda_{2n}r) - \frac{J_{-1}(\lambda_{2n}r_{11})}{Y_{-1}(\lambda_{2n}r_{11})} Y_0(\lambda_{2n}r) \right] dr \right]^2}{\int_{r_i}^{r_o} \left[ \int_{r_{o1}}^{r_{o2}} q_u(r) J_0(\lambda_{2n}r) - \frac{J_{-1}(\lambda_{2n}r_{11})}{Y_{-1}(\lambda_{2n}r_{11})} Y_0(\lambda_{2n}r) \right] dr}$$

(5.35)
For stationary ring 2 (a), the homogeneous boundary condition is along the $z$ direction. Thus, the temperature distribution can be expressed as

$$\theta_{m2a}(r, z) = C_0 + \sum_{n=1}^{\infty} C_n (I_0(\lambda_{3n}r) + \frac{I_{-1}(\lambda_{1n}r_1)}{K_{-1}(\lambda_{1n}r_1)}K_0(\lambda_{3n}r)) \cos(\lambda_{3n}z)$$ (5.36)

where

$$\lambda_{3n} = \frac{n\pi}{L_2}$$ (5.37)

$$C_0 = \frac{1}{L_2} \int_{-L_2}^{L_2} \theta_0(z)dz, \quad C_n = \frac{2}{L_2} \int_{-L_2}^{L_2} \theta_0(z) \cos(\lambda_{3n}z)dz$$ (5.38)

Note that $C_0$ and $C_n$ are not known at present since $\theta_0(z)$ is unknown.

The homogeneous boundary condition for stationary ring 1 is in the $z$ direction therefore the solution is

$$\theta_{m1}(r, z) = \sum_{n=1}^{\infty} D_n (I_0(\lambda_{4n}r) + \frac{k_m \lambda_{4n} I_{-1}(\lambda_{4n}r_{o2}) + h_2 I_0(\lambda_{4n}r_{o2})}{k_m \lambda_{4n} K_{-1}(\lambda_{4n}r_{o2}) - h_2 K_0(\lambda_{4n}r_{o2})}K_0(\lambda_{4n}r)) \times (\sin(\lambda_{4n}z) + \frac{k_m \lambda_{4n}}{h_2} \cos(\lambda_{4n}z))$$ (5.39)

where $\lambda_{4n}$ is calculated by

$$-k_m \lambda_{4n} \sin(\lambda_{4n}L_2) + h_2 \cos(\lambda_{4n}L_2) = 0$$ (5.40)

Applying the matching condition shown in Eq. (5.32), we can obtain

$$D_n = \frac{1}{\lambda_{4n}(I_{-1}(\lambda_{4n}r_0)) - \frac{k_m \lambda_{4n} I_{-1}(\lambda_{4n}r_{o2}) + h_2 I_0(\lambda_{4n}r_{o2})}{k_m \lambda_{4n} K_{-1}(\lambda_{4n}r_{o2}) - h_2 K_0(\lambda_{4n}r_{o2})}K_{-1}(\lambda_{4n}r_0))}$$

$$\int_{0}^{L_2} \frac{\partial \theta_{m2b}(r,z)}{\partial r} \bigg|_{r=r_{o1}} \left( \sin(\lambda_{4n}z) + \frac{k_m \lambda_{4n}}{h_2} \cos(\lambda_{4n}z) \right) dz$$ (5.41)

Now that $\theta_{m1}$ is solved in Eq. (5.39), then $\theta_0(z)$ in Eq. (5.38) can be treated using the matching condition of Eq. (5.31). Therefore $C_0$ and $C_n$ can be calculated now.

Finally, the temperature distribution in the entire stationary ring can be calculated as below

$$\theta_m(r, z) = \theta_{m1}(r, z) \quad \text{at} \quad r_{o1} < r < r_{o2}$$ (5.42)

$$\theta_m(r, z) = \theta_{m2a}(r, z) + \theta_{m2b}(r, z) \quad \text{at} \quad r_{o1} < r < r_{o2}$$ (5.43)
5.2.3 Partitioning Factor of the Heat Generation at the Interface between Rings

The computations involved in determining the heat partitioning factor is complicated and time-consuming (see Eq. (5.27)). In this section, we present a simple and effective approximate method which can be used to calculate the partitioning factor of the interfacial heat generation.
Fig. 5.5 Thermal resistance network

It is assumed that the temperature $T_m$ for the rotating and stationary rings at the interface boundary is equal. The free stream temperature around the two rings is $T_\infty$. Examining Fig. 5.2, the friction heat is bifurcated into two heat flows which are going through the rotating and stationary rings, and finally into the cooling free stream by heat convection at their wetted outer surface. This heat flow process can be described by the thermal resistance network shown in Fig. 5.5. The heat conduction and convection part is expressed as the thermal conduction resistance as $R_{\text{cond}}$ and $R_{\text{conv}}$, respectively. The subscript 1 and 2 refers to the rotating and stationary rings, respectively.

The thermal convection resistance is defined by

$$R_{\text{conv},1} = \frac{1}{h_1 A_1}, \quad R_{\text{conv},2} = \frac{1}{h_2 A_2}$$  \hspace{1cm} (5.44)

where $A_{1,2}$ is the heat convection area of the rotating and stationary rings, respectively.

The thermal conduction resistance is defined by

$$R_{\text{cond},1} = \frac{1}{S_1 K_r}, \quad R_{\text{cond},2} = \frac{1}{S_2 K_s}$$  \hspace{1cm} (5.45)

Table 5.1 Parameters for case 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>11,000 W/m$^2$</td>
</tr>
<tr>
<td>$T_\infty$</td>
<td>300 K</td>
</tr>
<tr>
<td>$h_1$</td>
<td>10,000 W/m$^2$.K</td>
</tr>
<tr>
<td>$h_2$</td>
<td>11,000 W/m$^2$.K</td>
</tr>
<tr>
<td>$k_r$</td>
<td>24 W/m.K</td>
</tr>
<tr>
<td>$k_m$</td>
<td>16 W/m.K</td>
</tr>
<tr>
<td>$r_i$</td>
<td>2.07 cm</td>
</tr>
<tr>
<td>$r_o$</td>
<td>2.24 cm</td>
</tr>
<tr>
<td>$L$</td>
<td>0.43 cm</td>
</tr>
</tbody>
</table>

In the equation above, $S_{1,2}$ is the heat conduction shape factor which is used to calculate two-dimensional conduction resistance. Note that subscripts 1 and 2 represent the rotating ring and the mating ring, respectively. Now that all the thermal resistances are known, the partitioning factor of the friction heat can be solved by
5.3 Results and Discussion

In this study, we considered two cases: case 1 and case 2. Their dimensions and material properties used in the calculation are shown in Table 5.1 and Table 5.2.

5.3.1 Results for Case 1

The temperature distribution for the seal rings of case 1 is shown in Fig. 5.6 (a). The analytical results are obtained using Eqs. (5.15) and (5.17). Five leading terms (i.e., $n=5$) are applied in the series solution. In this case, a uniform heat generation $q$ is applied for simplicity.

To verify the analytical solution, the temperature distribution for the seal rings of case 1 is calculated using FLUENT. The numerical result is shown in Fig. 5.6 (b). Using FLUENT, the same computation domain as that of the analytical calculation is considered with appropriate boundary conditions (see Fig. 5.1). To consider the interfacial heat portioning, a heat source in a very thin element, at the contact region between the rotor (rotating ring) and the stator (mating or stationary ring), is applied. To obtain a reliable heat partitioning between the seal rings, the source element is used as thin as $10^{-5}$ m and has a good heat conductivity of 1000 W/m.K.

By comparison of Figs 5.6 (a) and 5.6 (b), it can be seen that the temperature pattern is the same. Figures 5.6 (a) and (b) suggests that the temperature gradient at the region near the interface is much higher than that of the other region far away from the interface, which shows that the heat generated at the interface is mostly removed by the outer surface in the vicinity of the interface.

Figure 5.7 shows the temperature distribution at the interface. In this figure, the temperature is plotted against the dimensionless radius, $(r - r_i)/(r_o - r_i)$, where $r_o$ and $r_i$ are the inner and outer radius of the rotating ring, respectively. The interface temperature is calculated by Eq. (5.28). Also, the difference between the analytical and numerical solution is very small.

Examining Fig. 5.7, we can find that the temperature is at its maximum value of about 306.2 K at the inner radius of the ring and decreasing gradually to its minimum value of about 304.5 K.

5.3.2 Results for Case 2

We apply five leading terms to calculate the series solutions to the rings of case 2. The analytical solution for case 2 is obtained by using Eqs. (5.15), (5.33), (5.36), (5.39), (5.42)
and (5.43). The parameters used in case 2 are shown in Table 5.2. Figure 5.8 shows the temperature contour plots of the analytical and numerical solution. The pattern is the same and the temperature magnitude is close (less than 1 °C). Note that at \( r = r_{oi} \) and \( 0 < z < L_2 \), several temperature contour lines are discontinuous. This is due to the limited accuracy of approximation of the analytical solution since only five terms is used to calculate the temperature distribution for the rings.

Figure 5.9 shows the interface temperature distribution for case 2. The figure shows very small difference between the analytical and numerical solution. The maximum temperature is about 306.5 K being above about 3 °C of the temperature at the outer radius. The temperature difference is about 3 °C. This is normal for a small ring with a higher heat convection transfer coefficient (if using water as process fluid) of about 10,000 W/m².K. Look at the interfacial temperature distribution (Fig. 3.13) in the chapter 3 and Fig. 4.9 in chapter 4. The temperature difference for those is less than 5 °C.

### 5.3.3 Partitioning Factor of the Heat Generation

In this section, two cases are considered: Case A with the same configuration as case 2 in the previous section and Case B also with the same parameters as case 2 (see Table 2) excepting \( r_i = 4.14 \) cm, \( r_{oi} = 4.48 \) cm and \( r_{o2} = 5.08 \) cm. In calculating the partitioning factor \( \gamma \), the shape factor is the only unknown (see Eq. (5.46)).

In order to calculate the shape factor, the partitioning factors of 16 cases are calculated. Heat convection coefficient for the stationary ring \( h_2 \) is fixed as 10,000 W/m²K, \( k_m = 24 \) W/m.K, and \( k_r = 16 \) W/m.K. Heat convection coefficient for the rotating ring \( h_1 \) is varying from 5,000 to 30,000 W/m².K with 1000 as the increasing step. Substituting 16 partition factors \( \gamma \) into Eq. (46), two unknowns \( S_1 \) and \( S_2 \) and 16 equations are obtained. Using the Least Square method, we obtain

\[
S_1 = 0.275 \text{ m}, S_2 = 0.219 \text{ m for case A} \tag{5.47}
\]

\[
S_1 = 0.526 \text{ m}, S_2 = 0.321 \text{ m for case B} \tag{5.48}
\]

Figure 5.10 shows the partitioning factor comparison between the approximate solution by Eq. (5.46) and the exact solution by Eq. (5.27). From this figure, it can be seen that the difference is very small. The partitioning factor is increasing with increasing the convection coefficient for the rotating ring, \( h_1 \). This is reasonable because that more heat is entering the rotating ring when \( h_1 \) is greater.

In order to examine the accuracy of the conduction shape factor calculated in Eq. (5.47), a range of \( k_r \) varying from 10-400 W/m.K was simulated. In these simulations \( k_r = 24 \) W/m.K, \( h_1 = h_2 = 10,000 \) W/m²K were fixed. The results are shown in Fig. 5.11. The difference between the exact and approximate solution are small. Examining Fig. 5.11, one can find that the partitioning factor is increasing since \( k_r \) increases which implies that the heat flux into the rotating ring increases.
Fig. 5.6 Contour plots of temperature distribution for case 1: (a) analytical (b) FLUENT
5.4 Conclusions

In the present chapter, using the separation of variables method, heat transfer within two cases of mechanical seal rings is analytically solved. In the analysis, the heat generation at the interface between the rings are taken account and treated as a coupling condition of the two rings. Temperature profiles are presented. The analytical solution are compared and verified with the numerical solution estimated by FLUENT.
The partitioning factor of the friction heat is also studied. A method for determining the heat partitioning factor is introduced using the thermal resistance network theory. In addition, a simple but effective approximate method to calculate the partitioning factor is presented.

![Contour plots of temperature distribution for case 2: (a) analytical (b) FLUENT](image)

Fig. 5.8 Contour plots of temperature distribution for case 2: (a) analytical (b) FLUENT
Fig. 5.9 Temperature distribution at the contact face for case 2

Fig. 5.10 Comparison between the exact and approximate solution of the partition factor with varying $h_1$
Fig. 5.11 Comparison between the exact and approximate solution of the partition factor with varying $K_m$

5.5 Nomenclature

$h_{1,2} = \text{heat convection coefficient at the outer surface of the seal rings}$

$k_r = \text{heat conductivity of the rotating ring}$

$k_m = \text{heat conductivity of the stationary ring}$

$L = \text{axial length of the seal ring}$

$q = \text{heat flux}$

$q_r = \text{heat flux into the rotating ring}$

$q_m = \text{heat flux into the stationary ring}$

$r = \text{radial coordinate}$

$r_i = \text{inner radius of the seal ring}$

$R_{1,2} = \text{function of } r$

$R_{\text{cond}} = \text{thermal resistance of heat conduction}$

$R_{\text{conv}} = \text{thermal resistance of heat convection}$

$r_o = \text{outer radius of the seal ring}$

$S_{1,2} = \text{heat conduction shape factor}$

$T = \text{temperature of the seal ring}$

$T_\infty = \text{ambient temperature}$

$z = \text{axial coordinate}$

$Z_{1,2} = \text{function of } z$

$\gamma = \text{heat partitioning factor}$

$\lambda_{1,2} = \text{eigenvalue}$
6 Heat Transfer Analysis in Mechanical Seals Using Fin Theory

In this chapter, a simple and efficient method for estimating the average seal contact face temperature, surface temperature, and heat partitioning factor between the rings of a mechanical seal is presented. The method takes into account both heat conduction and convection. Design charts applicable to a variety of ring shapes are developed, and a series of examples are presented to illustrate their validity.

6.1 Introduction

Thermal effects play an important role on the performance and reliability of mechanical seals. The source of heat generation is frictional in nature and occurs at the interface between the primary (rotating) and the mating (stationary) rings. Heat is distributed axially by conduction into the rings and by convection out to the flush fluid that enters through the gland and flows over the outer surfaces of the rings. To select a seal for a given application and operating conditions, the engineer needs to have a reliable method to estimate the temperature at the contact face between the rings to assure that the average contact face temperature is acceptable. For example, when dealing with light hydrocarbon, flashing of the fluid to vapor across the face can cause the seal to run dry and fail.

Many researchers have applied advanced experimental and analytical tools to predict the flow field and to quantify the extent of heat conduction and convection heat transfer process involved. For example, Li [23] used FEM to predict the temperature distribution in a mechanical seal. Morariu and Pasovici [11] considered heat conduction in the fluid and the rings and convection by the coolant around the rings. Buck [2, 3] developed a simplified approach for determining the seal temperature based on an analytical model that treats a seal as a fin. Doane [12] performed experimental measurements and used the results as the boundary conditions of the numerical computations to predict the local and average Nusselt number for the wetted area of the mating ring. Lebeck [5] considered many of the important effects caused by the thermal environment such as thermally induced radial taper, thermally induced waviness, heat checking and hot spotting, and blistering. Blasbalg and Salant [40] theoretically studied the axial dynamic stability of two-phase mechanical seals to predict the transient behavior of seals by developing a numerical model featuring thermoelastic deformations, asperity contact and squeeze film effects. Cicone et al [41] developed a complex model which solves the transient TEHD problem of mechanical face seals. They showed that unfavorable thermal distortions may lead to face contact and result in a significant dissipation of heat, elevated temperature and wear. Jang and Khonsari [6] extended the theory of thermoelastic analysis to predict the critical speed at which hot spots can occur on the surface of a seal. These publications concentrated on the thermal analysis of mechanical seals but neglected the effect of the flow field in the seal chambers which were discussed and analyzed in Luan & Khonsari [20,21].

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In the present study, using the idea of representing the seal ring as a fin [2, 3], we present a simple method for analyzing the heat transfer in mechanical seals. This method provides an efficient method for estimating the heat transfer in mechanical seals including the seal contact face temperature, and the heat partitioning factor. Design charts are developed for four types of seal shapes used in accompanying examples to illustrate their validity.

6.2 Cylindrical Fin Model

Figure 6.1 shows a typical mechanical seal with heat transfer paths. To analyze this heat transfer process, Buck [2] assumed that the seal ring can be considered as a one-dimensional rectangular fin. In this paper, the seal ring is treated as a hollow cylindrical fin and the method is extended to predict the heat partitioning factor between the rotating and mating rings. The geometry is shown in Fig. 6.2. The base of the fin (the left side of the rectangle) is subject to interfacial heat represented by uniform heat flux, while its tip (the right side of the rectangle) is assumed to be insulated for simplicity. This is reasonable since most of the heat is removed by the heat convection at the outer surface of the ring.

It can be shown that the rate of heat transfer through such a fin is [25]:

\[
 q_f = hP kA_c (T_b - T_\infty) \tanh(mL) \quad (6.1)
\]

where \( h \) is the heat convection coefficient, \( k \) is the heat conductivity of the fin, \( T_b \) is the average temperature at the base, \( P \) is the circumference of the outer surface, \( A_c \) is the area of the axial cross section of the fin and \( m \) is expressed as follows [25].

\[
 m = \sqrt{\frac{hP}{kA_c}} \quad (6.2)
\]
It can be shown that the efficiency of a fin—defined as the ratio of actual heat transferred and the heat which would be transferred if entire fin area were at base temperature—with the above-mentioned boundary conditions (Fig. 6.2) is [25]:

\[ \eta = \frac{Q_{actual}}{Q_{ideal}} \]
\[ \eta = \frac{\tanh(mL)}{mL} \] (6.3)

Figure 6.3 shows the variation of efficiency as a function of \( mL \). At very low value of \( mL \), the fin efficiency is approximately 100\%. For specified dimensions and thermal conductivity, a low value of \( mL \) implies that the heat convection coefficient is low.

For a cylindrical seal shape with \( A_c = \pi(r_o^2 - r_i^2) \), Eq. (6.2) becomes

\[ mL = \left( \frac{L}{W} \right) \sqrt{\frac{2hr_o W}{k(r_o + r_i)}} \] (6.4)

where \( W \) is the difference of the inner and outer radius of the cylindrical fin. Equation (6.4) suggests that the efficiency of a seal fin varies with length, width, thermal conductivity and the convective heat transfer coefficient.

Using the definition of fin efficiency, one can write:

\[ \eta = \frac{q_f}{hA_b(T_b - T_x)} \] (6.5)

where \( q_f \) is the heat flux at the base of the seal fin, and \( A_b \) is the heat convection area. The heat taken away by convection is: \( hA_b(T_b - T_x) \). For a specified geometry and given heat flux \( q_f \), one can calculate the fin efficiency if the base temperature \( T_b \) is known. From Fig. 6.2, it can be recognized that the temperature at the base varies radially, in the \( r \) direction. Thus, in order to calculate Eq. (6.5), a two-dimensional heat transfer problem must be solved. Then, having obtained the temperature distribution, one can predict an averaged base temperature. For a two-dimensional problem, a closed-form analytical solution is developed for a simple shape shown in Fig. 6.2 (see Appendix A). However, numerical solution is more convenient when the geometric shape is more complicated.

### 6.3 Heat Partitioning Factor

An important consideration in determining the thermal behavior of contacting bodies is the heat partitioning factor. In the case of mechanical seals, the rotating ring is usually made of carbon and the stationary ring is typically silicon carbide [2]. The amount of heat conducted into each of these rings can be calculated by determining an approximate heat partitioning factor.

Let \( \eta_1 \) and \( \eta_2 \) represent the efficiency of the rotating ring and stationary ring respectively. Using Eq. (6.5), we have:

\[ q_1 = \eta_1 h_1 A_{h1} (T_b - T_x) \] (6.6)

\[ q_2 = \eta_2 h_2 A_{h2} (T_b - T_x) \] (6.7)

The partitioning factor can be estimated using equations (6.6) and (6.7).

\[ \gamma = \frac{q_1}{q_2} = \frac{\eta_1 h_1 A_{h1}}{\eta_2 h_2 A_{h2}} \] (6.8)
Eq. (6.8) reveals that once the efficiencies and heat convection coefficients are known, the heat partitioning factor can be easily predicted.

The total frictional heat is:

\[ E_p = pVA_f f = q_1 + q_2 \]  

\[ (6.9) \]

where \( p \) is pressure, \( V \) represents the rotation speed, \( A_f \) is the friction area, and \( f \) is the friction coefficient. The method of calculating the total friction heat as shown in Eq. (6.9) is used in Refs. [2,3], which provides a good estimate for the heat generation, \( E_p \), at the interface between the rings. To calculate \( E_p \), the coefficient of friction, \( f \), must be specified. Using the definition of the partitioning factor \( \gamma \), equation (6.9) can be written as follows:

\[ E_p = q_2 + \gamma q_2 \]  

\[ (6.10) \]

where

\[ q_2 = \frac{E_p}{1 + \gamma}, \quad q_1 = \frac{\gamma E_p}{1 + \gamma} \]  

\[ (6.11) \]

### 6.4 Temperature at the Surface of Seal Rings

Using the concept of fin efficiency, the average temperature at the contact face and the outer surface of the seal can be easily predicted as described in this section.

Combining Eqs. (6.6), (6.7) and (6.9),

\[ E_p = q_1 + q_2 = \left( \eta_1h_1A_{h1} + \eta_2h_2A_{h2} \right) (T_b - T_x) \]

\[ (6.12) \]

Solving Eq. (6.12) for the average base temperature \( T_b \) yields:

\[ T_b = \frac{E_p}{\eta_1h_1A_{h1} + \eta_2h_2A_{h2}} + T_x \]  

\[ (6.13) \]

To determine the surface temperature of the rings the heat transferred by convection must be considered. The appropriate equations are:

Heat loss from the primary ring is given by:

\[ q_1 = h_1A_{h1}(T_{s1} - T_\infty) \]  

\[ (6.14) \]

Heat from the mating ring is given by:

\[ q_2 = h_2A_{h2}(T_{s2} - T_\infty) \]  

\[ (6.15) \]

where \( T_{s1} \) and \( T_{s2} \) are the average temperature at the outer surface of the primary and mating ring respectively. The heat transferred by the rings can be calculated from Eq. (6.11). Thus,

\[ q_1 = \frac{\gamma E_p}{1 + \gamma} = h_1A_{h1}(T_{s1} - T_\infty) \]  

\[ (6.14) \]

\[ q_2 = \frac{E_p}{1 + \gamma} = h_2A_{h2}(T_{s2} - T_\infty) \]  

\[ (6.15) \]
From Eqs. (6.14)-(6.15)

\[ T_{x1} = \frac{\gamma E_p}{h_1 A_{h1}(1 + \gamma)} + T_{\infty} \]  \hspace{1cm} (6.16)

\[ T_{x2} = \frac{E_p}{h_2 A_{h2}(1 + \gamma)} + T_{\infty} \]  \hspace{1cm} (6.17)

Fig. 6.4 Plots of seal shapes

6.5 Results and Discussions

Seal rings take on many different shapes and configurations, and are often more complex than the simple hollow cylinder shown in Fig. 6.2. Lebeck et al. [41] and Buck [2, 3] made use of the concept of fin efficiency to take into account the shape variation. In their research, using a rectangular-shape fin model, three basic shapes were considered (see Fig. 6.4 shapes 1 to 3). In this paper, we consider four different shapes to cover a wide range of configurations. Shapes 1, 3 and 4 pertain to the rotating ring, while Shape 2 is primarily for modeling the mating ring since its radius is normally greater than that of the rotating ring.
Fig. 6.5 Heat transfer efficiency of a seal for shape 1

Fig. 6.6 Heat transfer efficiency of a seal for shape 2
Fig. 6.7 heat transfer efficiency of a seal for shape 3

Fig. 6.8 heat transfer efficiency of a seal for shape 4
Shape 1 in Fig. 6.4 was analyzed using the method in the Appendix. The heat flux is uniformly applied over the base of the seal fin. The width \( W \) is the outer radius minus the inner radius of the ring. For other shapes, the two-dimensional heat transfer analyses were performed by solving the heat conduction problem in the fin numerically using the FLUENT package. Once the temperature distribution is available, the average base temperature \( T_b \), and the fin efficiency can be conveniently evaluated.

Figures 6.5-6.8 show the heat transfer efficiencies for shapes 1-4. Note that the cases considered in shapes 2-4 all have \( l = W = W' \). Since \( W = r_o - r_i \), once the radii \( r_o \) and \( r_i \) of the rubbing region and the length of the ring \( L \) are specified (see Fig. 6.4), the dimensions of entire fins of shapes 2-4 can be established. Therefore, the heat transfer efficiency for shapes 1-4 can be calculated based on the values of \( L, r_o \), and \( r_i \) as shown in Eq. (6.4). Figures 6.5-6.8 reveal the difference between rectangular fin model [2,3] and cylindrical fin model. It can be seen that the heat transfer efficiency of a cylindrical fin is generally greater than that of a square fin at the same ratio of \( L/W \). At low values of \( mL \ (mL \leq 1) \), the difference between the two models is fairly large while at high values of \( mL \ (mL \geq 10) \), the difference is very small. Typically, the product of \( mL \) is less than 10. Note that in shape 1-4, the length \( L \) represents the axial length of the wetted outer surface of the rotating or stationary ring. \( W \) is the radial width of the seal face where heat flux is applied to.

Figure 6.5 shows how efficiency varies with \( mL \), as defined by Eq. (6.2). It can be seen that increasing \( mL \) results in a reduction of the efficiency. Also, efficiency gradually increases as the length-to-width ratio, \( L/W \), increases and in the limit approaches the ideal fin. In other words, the greater the length-to-width ratio, the better the heat transfer.

Figure 6.6 shows the heat transfer efficiencies for shape 2. Shape 2 represents a seal shape whose outer radius is greater than the friction face width \( W \) (with \( W' = W \)), which presents the mating ring. Shape 2 can be seen as a modified shape 1 created by adding a hollow cylinder of width of \( W' \). This addition increases the area of heat convection, but this is offset by an increase in the heat conduction resistance. The combined result is the heat efficiency of shape 2 is lower than that of shape 1, which can be seen by comparing Figs. 6.5 and 6.6. It can be also noted that shape 2 can be reduced to be shape 1 by decreasing \( W' \) to 0.

Figure 6.7 shows the heat transfer efficiency for shape 3. Shape 3 can represent a rotating ring with a “wear nose” with \( l = W' = W \). It can be made from shape 2 by removing a hollow cylinder of length \( l \) and width \( W' \). Thus it can be expected that the heat transfer efficiency of shape 3 is higher than shape 2 but lower than shape 1. The comparison of Figs. 6.5-6.7 meets this expectation. With \( l \) decreasing to 0, shape 3 becomes shape 2.

The heat transfer efficiency results for shapes 4—another pertinent configuration of a rotating ring—are presented in Fig. 6.8. It can be constructed from shape 1 by attaching a cylinder shell with width \( W' \) and length \( (L-l) \) into inside surface of shape 1. It can be seen from the comparison of Figs. 6.8 and 6.5 that the heat transfer efficiency of shape 4 is slightly lower than that of shape 1, which means that the addition of the shell to the inner
surface of shape 1 does not deteriorate heat transfer to a great extent. This is because the inner surface is assumed to be insulation thus there is not much heat transferred to this added “cell”.

6.6 Illustrating Examples

In this section, examples are presented to illustrate the procedure for using the results presented in this paper for predicting heat partitioning factor, average contact face temperature and average outer surface temperatures of the rotating and mating rings. Also the results are compared with the computation by 2D finite volume method (FLUENT).

6.6.1 Example 1

Estimate the heat partition factor for a seal using a rotating and stationary ring pair with dimensions shown in Fig. 6.9. Assume that the heat convection coefficients for the two rings are \( h_1 = h_2 = 1.0 \times 10^4 \text{ W/m}^2\text{K} \). The rotating ring is made of carbon graphite with heat conductivity \( k_1 = 8.6 \text{ W/mK} \) and the stationary ring is silicon carbide with heat conductivity \( k_2 = 138.5\text{ W/mK} \). The process fluid is water at 300 K.

![Fig. 6.9 Dimension of the seal rings for the illustration example 1 (in cm)](image)

Shape 3 and shape 2 should be applied to the rotating and stationary rings respectively shown in Fig. 6.9.

For the rotating ring with \( r_i = 3.0 \text{ cm} \) and \( r_o = 3.5 \text{ cm} \),

\[
\frac{L}{W} = \frac{L}{r_o - r_i} = 2
\]

Using Eq. (4),

\[
mL = \left( \frac{L}{W} \right) \sqrt{\frac{2h_i r_3 W}{k (r_o + r_i)}} = 5.0
\]

Using the values of \( mL \) and \( L/W \), the heat transfer efficiency can be calculated from Fig. 6.7 as:
\[ \eta_i \approx 0.072 \]

Next consider the stationary ring which can be represented by shape 2. Similarly,
\[ \frac{L}{W} = \frac{L}{r_o - r_i} = 2 \]

Note that the width \( W \) is always referring to the rubbing or contact face of the two rings.
\[ mL = \left( \frac{L}{W} \right) \sqrt{\frac{2hzwW}{k(r_o + r_i)}} = 1.25 \]

Using the values of \( mL \) and \( L/W \), the heat transfer efficiency can be calculated from Fig. 6 as:
\[ \eta_2 \approx 0.43 \]

The heat convection areas for the rotating and stationary rings are:
\[ A_{h1} = 35.33 \text{ cm}^2 \]
\[ A_{h2} = 36.90 \text{ cm}^2 \]

Now, Eq. (6.8) can be used to calculated the partition factor,
\[ \gamma = \frac{q_1}{q_2} = \frac{\eta_1 h_1 A_{h1}}{\eta_2 h_2 A_{h2}} = 0.1603 \]
To verify the results, 2D finite volume method (FLUENT) was used to solve exactly the same problem. Figure 6.10 shows the heat fluxes at the contacting faces obtained in this way. Integrating these fluxes over the contacting surfaces leads to a value of 0.1723 for the partitioning factor, which is close to that obtained from the fin model. This suggests that the fin theory method to calculate heat partition factor is reliable.

### 6.6.2 Example 2

Estimate the average temperature at the outer surface of the rotating and stationary rings for a seal with dimensions shown in Fig. 6.11. The heat generation rate is assumed to be \( q = 1.18 \times 10^5 \text{ W/m}^2 \) and the heat convection coefficients for the two rings are \( h_1 = h_2 = 1.0 \times 10^4 \text{ W/m}^2\text{K} \).

Fig. 6.11 Dimension of the seal rings for the illustration example 2 (in cm)

The rotating ring is made of carbon graphite with heat conductivity \( k_1 = 8.6 \text{ W/mK} \) and the stationary ring is silicon carbide with heat conductivity \( k_2 = 138.5 \text{ W/mK} \). The process fluid is water at 300 K. From Equation (6.9) the heat generation rate per unit area is calculated [3],

\[
q = pVf
\]

where \( p \) is the average seal face pressure at the contact face, \( V \) is the velocity at the mean face diameter and \( f \) is the friction coefficient. Here we take \( p = 1.79 \times 10^5 \text{ N/m}^2 \) (26 psi) and \( f = 0.1 \). Assuming the rotating speed \( N \) is 1800 rpm, \( V \) can be determined by

\[
V = \left( \frac{2\pi N}{60} \right) \left( \frac{r_i + r_o}{2} \right) = 6.60 \text{ m/s}
\]

Thus

\[
q = pVf = 1.18 \times 10^5 \text{ W/m}^2
\]

Shape 1 can be applied to the two rings in Fig. 6.11.

First following the procedure of example 1, the heat partition factor can be solved. For the rotating ring with \( r_i = 3.0 \text{ cm} \) and \( r_o = 4.0 \text{ cm} \),

\[
\frac{L}{W} = \frac{L}{r_o - r_i} = 1
\]
Using Eq. (4),
\[ mL = \left( \frac{L}{W} \right) \sqrt{\frac{2h_r r_w W}{k(r_o + r_i)}} = 3.65 \]

Using the values of \( mL \) and \( L/W \), the heat transfer efficiency can be calculated from Fig. 5 as:
\[ \eta_i \approx 0.13 \]

Next consider the stationary ring which can be represented by shape 1. Similarly,
\[ \frac{L}{W} = \frac{L}{r_o - r_i} = 1.5 \]

Note that the width \( W \) is always referring to the rubbing or contact face of the two rings.
\[ mL = \left( \frac{L}{W} \right) \sqrt{\frac{2h_r r_w W}{k(r_o + r_i)}} = 1.36 \]

Using the values of \( mL \) and \( L/W \), the heat transfer efficiency can be calculated from Fig. 6.5 as:
\[ \eta_{2-1} \approx 0.47 \quad \text{at} \quad L/W=1 \]
\[ \eta_{2-2} \approx 0.59 \quad \text{at} \quad L/W=2 \]

By interpolation,
\[ \eta_2 = \frac{\eta_{2-1} + \eta_{2-2}}{2} = 0.53 \quad \text{at} \quad L/W=1.5 \]

The heat convection areas for the rotating and stationary rings are:
\[ A_{h1} = 2\pi (1.0 \times 4.0) = 25.13 \text{ cm}^2 \]
\[ A_{h2} = 2\pi (1.5 \times 4.0) = 37.70 \text{ cm}^2 \]

Now, Eq. (6.8) can be used to calculated the partition factor,
\[ \gamma = \frac{q_1}{q_2} = \frac{\eta_1 h_{1} A_{h1}}{\eta_2 h_{2} A_{h2}} = 0.164 \]

The total heat generation is:
\[ E_p = qA_f = 1.18 \times 10^5 \times \pi \left( \left( 4 \times 10^{-2} \right)^2 - \left( 3 \times 10^{-2} \right)^2 \right) = 259.50 \text{ W} \]

Using Eq. (6.16), the average temperature at the outer surface of the rotating ring is:
\[ T_{s1} = \frac{\gamma E_p}{h_1 A_{h1} (1 + \gamma)} + T_s = \frac{0.164 \times 259.50}{1 \times 10^4 \times 25.13 \times 10^{-4} \times (1 + 0.164)} + 300 = 301.45 \text{ K} = 28.45 \text{ °C} \]
Fig. 6.12 Temperature Distribution at the outer surface of rotating ring

Fig. 6.13 Temperature Distribution at the outer surface of stationary ring
Similarly, using Eq. (6.17), the average temperature at the outer surface of the stationary ring is:

\[
T_{s2} = \frac{E_p}{h_2 A_{h2}(1 + \gamma)} + T_{\infty} = \frac{259.50}{1 \times 10^4 \times 37.70 \times 10^{-4} \times (1 + 0.164)} + 300 = 305.91 \text{ K} = 32.91 ^\circ \text{C}
\]

Using 2D finite volume method (FLUENT), the temperature distribution in the rings can be obtained. Figure 6.12 and 6.13 show the temperature distribution at the outer surface of the two rings respectively. \(T_{s1}\) and \(T_{s2}\) are calculated by taking the average values. In Figs. 6.12-6.13, \(z_0\) and \(z_1\) denotes the left end and the right end of the ring. The FLUENT results are \(T_{s1} = 28.81 ^\circ \text{C} (302.01 \text{ K})\) and \(T_{s1} = 32.75 ^\circ \text{C} (305.95 \text{ K})\), respectively. The difference is also very small.

Using Eq. (6.13), the average temperature at the contact face between the rings can be estimated,

\[
T_h = \frac{E_p}{\eta_1 h_1 A_{h1} + \eta_2 h_2 A_{h2}} + T_{\infty} = \frac{259.50}{0.13 \times 1 \times 10^4 \times 25.13 \times 10^{-4} + 0.53 \times 1 \times 10^4 \times 37.7 \times 10^{-4}} + 300
\]

\[
= 311.16 \text{ K} = 38.16 ^\circ \text{C}
\]

Using FLUENT, the average temperature at the contact face is found to be \(T_h = 37.32 ^\circ \text{C} (310.52 \text{ K})\). Again the difference is little. The temperature distribution at the contact face calculated by FLUENT is plotted in Fig. 6.14.

![Fig. 6.14 Temperature Distribution at the contact face](image-url)
6.7 Conclusions

In the present study, using the idea of representing the seal ring as a fin [2, 3], we present a simple but realistic method for analyzing the heat transfer in mechanical seals. This method provides an efficient method for estimating the heat transfer in mechanical seals including seal contact face temperature, heat partition factor. Illustrating examples are presented to show the process of applying the heat efficiency charts to analyze the heat transfer in seals. The results of the simulation were verified using FLUENT.

6.8 Nomenclature

\( A_c \) = area of the cross section of fin  
\( A_f \) = area of the frictional interface  
\( A_h \) = area of the convection surface  
\( E_p \) = total frictional heat generation  
\( f \) = friction coefficient  
\( k \) = heat conductivity  
\( L \) = length of the ring  
\( P \) = pressure at the contact face  
\( r_i \) = inner radius  
\( r_o \) = outer radius  
\( T_b \) = average temperature at the contact face  
\( T_{s1} \) = average temperature at the outer surface of the primary ring  
\( T_{s2} \) = average temperature at the outer surface of the mating ring  
\( T_\infty \) = temperature of ambient fluid  
\( V \) = rotating speed of the primary ring  
\( \eta \) = heat transfer efficiency  
\( \gamma \) = heat portioning factor
7 Analyses of the TEHD Behavior of Mechanical Seals

In this chapter, models are developed for investigating thermoelastohydrodynamic (TEHD) behavior of mechanical seals. Roughness at the seal faces is considered and its effects on the lubrication film thickness at the sealing gap, power dissipation, and leakage are analyzed and discussed. Heat generation at the sealing gap is calculated and compared with the heat loss removed by leakage. It shows that the heat loss with leakage is negligible compared with the total heat generation which suggests that the heat generation is mostly transferred into the seal rings and then removed by heat convection at the outer surface of seal rings.

7.1 Introduction

The most important indicator of seal performance is leakage rate. It determines the success or failure of a seal. In a mechanical seal, the leakage rate is directly related to the lubrication fluid formed between the rotating and stationary rings. Hence, numerous authors have studied the lubrication characteristics of these rings to better understand the behavior of mechanical seals. Brunetiere et al. [43] developed a TEHD model to determine mean face temperature, power loss, leakage rate and fluid film thickness. Lebeck [44] formulated a mixed-lubrication friction model and developed appropriate relationships for predicting the minimum film thickness and leakage in a mechanical seal. Using this model the total power consumption of seals can be calculated in which both the fluid and mechanical contact between asperities are properly accounted for. Pascovici and Etsion [4] performed a thermohydrodynamic (THD) analysis for a face-to-face double seal configuration considering temperature and viscosity variation along the sealing gap. More recently, Salant and Cao [19] came up with an unsteady numerical model of a mechanical seal with mixed lubrication using Duhamel’s method. The model can be used to predict the thermal behavior of the lubrication fluid film of the seals during startup and shutdown. Tournerie et al. [18, 45] developed 2-D and 3-D models to analyze the thermal behavior of the lubrication film of a seal. In their study, the Reynolds equation and energy equation were solved numerically using the finite difference method (FDM) and the method of influence coefficient to determine the seal face distortion.

This chapter is organized as follows:

In Section 7.2, a simple hydrostatic model of lubrication film with rough and smooth surfaces for mechanical face seal is presented that allows for the determination of mean pressure distribution, fluid film thickness, power loss and leakage rate with provision for surface roughness. A realistic analytical solution of the fluid pressure distribution is obtained by solving the axis-symmetric Reynolds equation which takes into account the wedge developed between the seal rings due to thermal gradients. This relaxes the assumption of constant fluid film thickness, which is commonly used in literatures [5, 43, 44, 46]. Comparison of the results shows that the two methods yield significantly different results especially when the difference between the inner and the outer radius of the ring is fairly large. It is demonstrated that surface roughness considerably increases the mean fluid film thickness. Based on the results of film thickness, the effect of
roughness on the leakage rate and dissipated power are also discussed for both rough and smooth seal face.

In Section 7.3, a THD model is presented to analyze the lubrication film between seal faces with provision for surface roughness. A simplified energy equation is derived and solved analytically. The variation of temperature in both radial and axial directions is considered. The effect of various parameters on the thermal behavior of sealing gap is explored. It is shown that the temperature variation across the sealing gap can be significant, raising questions about the validity of constant viscosity solution often made in analyses.

In Section 7.4, we provide numerical and analytical models for analyzing the TEHD behavior of a mechanical seal. With proper boundary conditions, simplified Navier-Stokes equations, Reynolds equation and energy equation are derived for the lubrication film at the sealing gap between the rotating and stationary rings. We analyze and discuss the effects of seal face roughness on the thermal behavior in the lubrication film. The heat power dissipation is discussed as well.

### 7.2 A Note on the Lubrication Film in Hydrostatic Mechanical Face Seals

#### 7.2.1 Model

Figure 7.1 shows a schematic of the model showing seal faces with roughness. Based on the study of Doust and Parmar [16], the seal faces present themselves at an angle $\beta$ due to the thermal gradients in the seal rings. The lubrication fluid film at the seal gap is of thickness $h(r)$ with $h_i$ at the inlet to $h_o$ at the outlet. The corresponding fluid pressures at the inner and outer pressure (at the seal chamber) are $p_i$ and $p_o$, respectively.

![Fig. 7.1 Schematic of the model](image)

This model is intended for an outer pressurized seal but the method can be easily extended to an inner pressurized seal. The pressure between the two seal faces includes both hydraulic and rough surface contact pressure [44]. In the present study, the following assumptions are made:
1. The problem is steady-state and axis-symmetric;
2. The fluid is Newtonian with constant density and viscosity across the seal gap;
3. The radial taper is due to thermal effects and does not depend on the details of the contact or hydraulic pressure [43, 44]; and
4. Inner pressure is zero [43, 44].

In order to analyze leakage rate, first it is necessary to predict the film thickness. The film thickness is a function of \( r \) and can be determined by considering the axial force balance between the closing force acting on the back of the rings and the opening force—caused by the fluid pressure in the lubricating film—and the contact force at the seal gap. In the section that follows, the film thickness is predicted with provision for surface roughness.

### 7.2.1.1 Seal Face with Roughness

For seal face with roughness, the hydrodynamic pressure is given by the Reynolds equation of the following form [46]:

\[
\frac{\partial}{\partial r}\left( r \phi_s \frac{h^3}{12 \mu} \frac{\partial p_s}{\partial r} \right) = \frac{\omega_1 - \omega_2}{2} \frac{\sigma}{\phi_s} \frac{\partial p_s}{\partial r} \tag{7.1}
\]

where \( \phi_s \) and \( \phi_s \) is the pressure flow factor and shear flow factor respectively, \( \omega_{1,2} \) are the rotating speeds of the surfaces, and \( \sigma \) is the roughness variance. The pressure flow factor is defined as [47]

\[
\phi_s = 1 - Ce^{-\gamma r}, \quad \text{for } \gamma \leq 1 \tag{7.2}
\]

\[
\phi_s = 1 + CH^{-\gamma r}, \quad \text{for } \gamma > 1 \tag{7.3}
\]

where \( C \) and \( r \) are constants whose values can be found in Refs. [46, 47] and \( H \) is the dimensionless film thickness defined as: \( H = h / \sigma \). In Eqs. (7.2)-(7.3), the parameter \( \gamma \) is the length-to-width ratio of a representative asperity [45]. \( \gamma > 1 \), \( \gamma = 1 \) and \( \gamma < 1 \) correspond to the roughness pattern of longitudinal orientation, isotropic, and transverse orientation, respectively. The shear flow factor, \( \phi_s \), is defined as [47]:

\[
\phi_s = V_{r1} \Phi_s (H, \gamma_1) - V_{r2} \Phi_s (H, \gamma_2) \tag{7.4}
\]

where \( V_{r1} \) and \( V_{r2} \) are the variance ratios given by

\[
V_{r1} = \left( \frac{\sigma_1}{\sigma} \right)^2 \quad \text{and} \quad V_{r2} = \left( \frac{\sigma_2}{\sigma} \right)^2 = 1 - V_{r1} \tag{7.5}
\]

The parameter \( \Phi_s \) is a function of \( H \) and the roughness pattern parameter \( \gamma \). In this study, it is assumed that the surfaces of the seal rings have the same roughness pattern \( \gamma \) and roughness variance, \( \sigma \), i.e.

\[
\sigma_1 = \sigma_2 \quad \text{and} \quad \gamma_1 = \gamma_2 \tag{7.6}
\]

Therefore, the shear flow factor \( \phi_s = 0 \) and Eq. (7.1) reduces to
\[ \frac{d}{dr} \left( r^2 \phi \frac{dp_\sigma}{dr} \right) = 0 \]  
\[ \text{(7.7)} \]

where the film thickness \( h \) is a function of the radius, \( r \).

\[ h(r) = (h_i - r \beta) + \beta r \]  
\[ \text{(7.8)} \]

The pressure boundary conditions are:
\[ p_\sigma (r_o) = p_o \quad \text{and} \quad p_\sigma (r_i) = 0 \]  
\[ \text{(7.9)} \]

In the case of rough seal faces, the opening force results from a combination of the fluid pressure governed by Eq. (7.7) and the contact force \( F_{\text{contact}} \), which is determined by contact mechanics of the contacting asperities. If assuming that the asperities’ height distribution is Gaussian [48], the contact pressure at the seal gap can be expressed as [19]

\[ p_c = S \int_{h}^{\infty} \frac{1}{\sigma \sqrt{2\pi}} e^{-(z^2/2\sigma^2)} dz \]  
\[ \text{(7.10)} \]

where \( S \) is the flow pressure or flow stress. Halling [49] indicated that \( S \) is approximately three times the tensile yield strength for a metal, i.e. essentially the indentation hardness.

The contact force is:
\[ F_{\text{contact}} = 2\pi \int_{ri}^{ro} p_c r dr \]  
\[ \text{(7.11)} \]

Therefore, the opening force is
\[ F_\sigma = 2\pi \int_{ri}^{ro} p_\sigma r dr + F_{\text{contact}} \]  
\[ \text{(7.12)} \]

By equaling the opening force and the closing force, we have
\[ F_\sigma = F_c \]  
\[ \text{(7.13)} \]

where \( F_c \) is the closing force which can be calculated from the following Eq. (7.14).
\[ F_c = \pi (r_o^2 - r_i^2) p_o B_t \]  
\[ \text{(7.14)} \]

\( B_t \) is the total balance ratio and is defined as [43]
\[ B_t = \frac{F_{sp}}{\pi (r_o^2 - r_i^2) p_o} + B \]  
\[ \text{(7.15)} \]

where \( F_{sp} \) is spring force and \( B \) is the seal balance ratio.

Having obtained an expression for the fluid pressure calculated from Eq. (7.7), one can determine the mean fluid film thickness by balancing the axial closing force and opening force acting on the seal ring. Therefore, the mean film thickness of rough seal face can be predicted by numerically solving Eq. (7.13).
Ref. [19] introduced an approximate method to calculate the opening force for the seal faces with roughness in the following form.

\[ F'_{\sigma} = 2\pi \int_{r_i}^{r_o} pr\,dr + F_{\text{contact}} \quad (7.16) \]

where \( p \) is the hydraulic pressure in the lubrication film for smooth seal faces which is discussed in later section of smooth seal face.

Once the mean film thickness is determined by balancing the axial force acting on the seal ring, leakage rate \( Q \) for rough seal faces can be expressed as [5]

\[ Q = r_o^2 \int_{0}^{2\pi} \left( -\frac{h^3}{12\mu} \frac{\partial p_{\sigma}}{\partial r} \right)_{r=r_o} d\theta = 2\pi r_o \left( -\frac{h^3}{12\mu} \frac{\partial p_{\sigma}}{\partial r} \right)_{r=r_o} \quad (7.17) \]

where \( \mu \) is the fluid viscosity at the seal gap. Since the pressure distribution for rough seal faces is known, Eq. (7.17) can be solved.

The dissipated power due to a combination of the fluid friction and the contact friction can be calculated using the following expression [19, 43].

\[ E_c = \pi \mu \omega^2 \frac{r_o^4 - r_i^4}{2h_m^2} + 2\pi \int_{r_i}^{r_o} f_c \omega p_c r^2 dr \quad (7.18) \]

where \( f_c \) is the contact friction coefficient and \( \omega \) is the rotation velocity of the rotor.

### 7.2.1.2 Smooth Seal Face

The problem described above can be simplified by assuming smooth seal faces [4, 18, 43, 44, 45]. As we show presently, in this case, it is possible to obtain a closed-form analytical solution for the pressure distribution.

For smooth seal faces \( \sigma = 0 \), \( H \) approaches infinity and then \( \phi_s = 1 \); see Eqs. (7.2)-(7.3). Substituting \( \phi_s = 1 \) into Eq. (7.7), the Reynolds equation for smooth seal faces can be obtained. The result is:

\[ \frac{d}{dr} \left( rh^3 \frac{dp}{dr} \right) = 0 \quad (7.19) \]

with the same boundary conditions as Eq. (7.8).

Solution to Eq. (7.19) is available when \( h \) is constant [5, 44, 46]. Equation (7.19) can be solved analytically when \( h = h(r) \) as a function of \( r \) as prescribed by Eq. (7.9). The solution is:

\[ p(r) = C_1 \left( -\frac{1}{h_i - r_i \beta} \ln \left( \frac{h}{r} \right) + \frac{1}{h} + \frac{h_i - r_i \beta}{2} \frac{1}{h^2} \right) + C_2 \quad (7.20) \]

where \( C_1 \) and \( C_2 \) are constants determined by boundary conditions.
\[ C_1 = \left( \frac{1}{h_i - r_i \beta} \ln \frac{r_i h_i}{r_o h_o} + \left( \frac{1}{h_o} - \frac{1}{h_i} \right) + \frac{h_i - r_i \beta}{2} \left( \frac{1}{h_o^2} - \frac{1}{h_i^2} \right) \right) \frac{p_o}{P_o} \]  

\[ C_2 = -C_1 \left( \frac{1}{h_i - r_i \beta} \ln \frac{h_i}{r_i} + \frac{1}{h_i} + \frac{h_i - r_i \beta}{2} \left( \frac{1}{h_i^2} \right) \right) \]  

(7.21)\quad(7.22)

and referring to [43]:

\[ h_i = h_m - 0.5 \beta (r_o - r_i) \]  

(7.23)

\[ h_o = h_m + 0.5 \beta (r_o - r_i) \]  

(7.24)

where \( h_m \) is the mean value of the lubrication film thickness.

Lebeck [44] obtained the following approximate solution to the fluid pressure by assuming a narrow seal radius difference

\[ p(r) = \frac{p_o}{1 - \frac{1}{h_o^2} - \frac{1}{h_i^2}} \left( \frac{1}{h^2} - \frac{1}{h_i^2} \right) \]  

(7.25)

This expression is valid and yields very close results to the exact solution when \( (r_o - r_i) \) is small.

The opening force for the smooth seal face is the integration of the fluid pressure in the seal gap. For the approximate pressure as Eq. (7.25), the opening force is [43, 44]:

\[ F_o = \pi (r_o^2 - r_i^2) p_o \frac{h_o}{h_o + h_i} \]  

(7.26)

For the exact pressure from the expression given in Eq. (7.20), the opening force is:

\[ F_o = -\frac{2 \pi C_1}{h_i - r_i \beta} \left( \frac{h^2}{2} \left( \ln h - \frac{1}{4} \right) - \frac{h_i - r_i \beta}{\beta^2} \left( h \ln h - h \right) - r^2 \left( \ln r - \frac{1}{4} \right) \right) \bigg|_{r_i}^{r_o} \]

\[ + 2 \pi \left( \frac{r}{\beta} - \frac{h_i - r_i \beta}{\beta^2} \right) \ln h \bigg|_{r_i}^{r_o} + \pi C_1 \left( h_i - r_i \beta \right) \left( \frac{h_i - r_i \beta}{\beta^2 h} + \frac{1}{\beta^2} \ln h \right) \bigg|_{r_i}^{r_o} + \pi C_2 \left( \right) \bigg|_{r_i}^{r_o} \]  

(7.27)

The closing force can be calculated using Eq. (7.14). By equalizing the closing and opening force, the mean lubrication fluid film thickness can be determined. Using the approximate opening force in Eq. (7.26), the approximate thickness can be explicitly expressed as [43]

\[ h_m = \frac{(r_o - r_i) \beta}{4(B_i - 0.5)} \]  

(7.28)

Using the exact opening force in Eq. (7.27) and
The mean film thickness is implicitly included in the above Eq. (7.29) but it can be easily solved using some simple root finding method such as the Bisection method.

Once the mean film thickness is solved by balancing the axial force acting on the seal ring, the leakage rate $Q$ for smooth seal faces can be expressed as [43].

\[
Q = \frac{\pi r_o p_o h_o^2 h_i^2}{12 \mu (r_o - r_i) h_m} \tag{7.30}
\]

where $\mu$ is the fluid viscosity at the seal gap.

Finally, the dissipated power can be calculated [19, 43] using the following expression.

\[
E_c = \frac{\pi \mu \omega^2 (r_o^4 - r_i^4)}{2 h_m} \tag{7.31}
\]

Note that Eq. (7.31) is obtained by eliminating the contact friction term in Eq. (7.18).

### 7.2.2 Results and Discussion

In this section, we predict the performance of a mechanical seal using the model described in Section 7.2.1. Specifically, three sets of inner and outer radii shown in Table 7.1 are considered for comparison purposes. Additional parameters used in the simulations are given in Table 7.2. The values in Tables 7.1 and 7.2 are chosen based on the results of Ref. [5, 43, 44] so that direct comparison can be made.

<table>
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<td>0.03302</td>
</tr>
<tr>
<td>$r_o$</td>
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<td>0.05302</td>
<td>0.05302</td>
</tr>
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<td>0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
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<td>314 N</td>
</tr>
<tr>
<td>$P_o$</td>
<td>$3.45 \times 10^6$ pa</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$6.82 \times 10^{-4}$ pa.s</td>
</tr>
<tr>
<td>$\omega$</td>
<td>188.5 s$^{-1}$</td>
</tr>
<tr>
<td>$f_c$</td>
<td>0.1</td>
</tr>
</tbody>
</table>
7.2.2.1 Effect of Roughness Pattern, $\gamma$

Figure 7.2 shows the effect of roughness pattern factor $\gamma$ on the pressure distribution. Note that $\gamma > 1$, $\gamma = 1$, and $\gamma < 1$ correspond to the roughness pattern of longitudinal orientation, isotropic, and transverse orientation, respectively. Compared with that of the smooth seal face, the pressure in a rough seal gap is considerably greater. According to the definition of the roughness pattern [46], it can be recognized that with $\gamma$ increasing from 1/3 (transverse) to 9 (longitudinal), the roughness pattern is becoming more difficult for the lubrication fluid to flow through, which results in that the hydraulic pressure is increasing with decreasing $\gamma$.

From Fig. 7.2, it can be seen that due to the constraints imposed by the boundary conditions, the pressure distribution for the smooth and rough surfaces are very close at the regions near $r_i$ and $r_o$, however.

Figure 7.3 shows the effects of the roughness variance on the pressure distribution. The results predict that, in the range of $\sigma = 0.1 \times 10^{-6}$ m to $\sigma = 1 \times 10^{-6}$ m, the pressure distribution does not change appreciably, hence the effect of roughness is relatively small.

Turning our attention to the film thickness of rough sealing gap, we present results for two methods. One is using the opening force shown in Eq. (7.12), the other is using Eq. (7.16) introduced by [19]. Prediction of Eq. (7.12) is obviously more realistic since it uses the fluid pressure distribution calculated from the Reynolds equation with provision for surface roughness.

![Graph showing hydraulic pressure distribution for rough seal face along r direction](image)

Fig. 7.2 Hydraulic pressure distribution for rough seal face along $r$ direction with $\Delta r = 0.02$ m, $\sigma = 0.5 \times 10^{-6}$ m and $\beta = 100 \times 10^{-6}$
Fig. 7.3 Hydraulic pressure distribution for rough seal face along $r$ direction with $\Delta r = 0.02 \text{ m}$, $\gamma = 1.0$ and $\beta = 100 \times 10^{-6}$

Fig. 7.4 Comparison of mean film thickness of rough seal face with $\Delta r = 0.01 \text{ m}$, $S = 250 \text{ Mpa}$ and $\sigma = 0.5 \times 10^{-6} \text{ m}$ by using Eq. (7.12) and Eq. (7.16)
The comparison is shown in Fig. 7.4. It can be seen that the difference increases with increasing the taper angle, $\beta$. For a small taper angle such as $\beta \leq 10 \times 10^{-6}$, the difference is as small as approximately 5%, which means the approximate method of Eq. (7.16) may be satisfactory. However, for larger taper angles, this approximate method is not valid since the difference is large. For example, at $\beta = 500 \times 10^{-6}$, the difference is as much as 60%. Nevertheless, the mean film thickness predicted by Eq. (7.12) is always conservative.

In Fig. 7.5 the mean film thickness is plotted against the taper angle $\beta$ for smooth surface and isotropic rough surface with $\gamma = 1$. It can be seen that the mean film thickness of the rough face is greater compared with that of the smooth seal face.

7.2.2.2 Effects of Roughness Variance, $\sigma$

Figure 7.6 shows the mean film thickness with various values of $\sigma$ ranging from $0.1 \times 10^{-6}$ m to $1.0 \times 10^{-6}$ m. It shows that the film thickness is increasing with increasing $\sigma$ for small taper angle. While for large taper angle, the thickness does not change appreciably with $\sigma$. This can be explained by examining the results of the mean film thickness shown in Fig. 7.4. The mean film thickness increases, with increasing the taper angle. For a given roughness, the thicker the lubrication film is, the less the effect of roughness.

Therefore, for large taper angle (corresponding to thick film thickness), the thickness does not change appreciably with $\sigma$. Figure 7.7 shows the mean film thickness with
various flow stress parameter, \( S \). It shows that when \( \beta \geq 10 \times 10^{-6} \), the flow stress does not have much effect on the film thickness, which is in agreement with Lebeck’s argument [5]. Physically, when \( S \) is increasing, the contact pressure increases (see Eq. (7.10)) and that brings about an increase in average film thickness. However, since the contact pressure calculated from Eq. (7.10) is small compared with the hydraulic pressure at the sealing gap, the film thickness does not appreciably increase the opening force. Referring to Fig. 7.7, it can be observed that for \( \beta \leq 10 \times 10^{-6} \) the mean film thickness increases very little with increasing \( S \).

### 7.2.2.3 Smooth Seal Face

Figure 7.8 shows that the comparison between the exact solution calculated from Eq. (20) and the approximate solution calculated from Eq. (7.25). These results are obtained at a fixed taper angle \( \beta \) with \( \Delta r \) values ranging from 0.005 to 0.02 m. Constrained by the boundary conditions, it can be seen that the approximate solution is very close to the exact expression (Eq. 7.25) only in regions close to the inner and outer radii. Elsewhere, the discrepancy is greater and tends to grow with increasing \( \Delta r \). This suggests that the approximate solution becomes inaccurate for large values of \( \Delta r \). For example, at \( \Delta r = 0.01 \text{ m} \), the maximum error is 5%; while at \( \Delta r = 0.02 \text{ m} \), the error increases to nearly 8.5%. Figure 7.9 shows the effect of the taper angle \( \beta \) on the difference between the exact and approximate pressure. It suggests that the difference is increasing with decreasing \( \beta \). The values of the exact pressure are generally greater than the approximate ones.

![Fig. 7.6 Mean film thickness with \( \Delta r = 0.01 \text{ m}, S = 250 \text{ Mpa and } \gamma = 1.0 \)](image-url)
Fig. 7.7 Mean film thickness with $\Delta r = 0.01\,\text{m}$, $\gamma = 1.0$ and $\sigma = 0.5 \times 10^{-6}\,\text{m}$

Fig. 7.8 Pressure distribution along $r$ direction at the seal gap with $\beta = 500 \times 10^{-6}$
The mean film thickness was predicted using Eq. (7.29) by balancing the axial opening force and closing force acting on the seal ring. Equation (7.29) was solved using a root finding method. This solution is compared with the approximate method of Brunetiere [42] as shown in Fig. 7.10 where the mean film thickness is plotted against taper angle. The results show that the difference increases with increasing $\Delta r$, as expected. The mean film thickness linearly varies with the taper angle, which is reasonable since the seal faces are assumed to be smooth. While at the first glance it may appear that approximate solution yields a reasonable prediction for film thickness, careful examination of Fig. 7.10 reveals that this is not always the case. For example for $\beta \leq 10 \times 10^{-6}$ the error caused by the simplified Eq. (7.28) can reach to about 40%, while for $\beta \geq 100 \times 10^{-6}$ the maximum error is approximately 5%.

### 7.2.2.4 Leakage Rate

Using Eqs. (7.17) and (7.30), the leakage rate is calculated for smooth and rough seal face, respectively. The results are shown in Fig. 7.11. It can be seen that the leakage rate is increasing with increasing the taper angle and that the leakage rate of rough seal face is much greater than that of smooth seal face, suggesting that a smoother seal face performs better than rough one in terms of reducing leakage rate.

### 7.2.2.5 Power Loss

The power dissipated at the seal gap can be predicted using Eq (7.18) for rough seal faces and Eq. (7.31) for smooth faces. The results are presented in Fig. 7.12. It is shown
that the power is decreasing with increasing taper angle for both rough and smooth seal face. Because the additional term involving the frictional heat associated with contact asperities, the dissipated power for rough seal face is considerably larger than for the smooth seal face when \( \beta \leq 15 \times 10^{-6} \). However, this is not true for \( \beta > 15 \times 10^{-6} \). In that case, the dissipated power for the smooth surface is slightly greater compared that for smooth surface. Eq. (7.18) reveals that the reason is that the heat from both contact friction and fluid film friction is decreasing with average film thickness increasing. For \( \beta > 15 \times 10^{-6} \), the film thickness of smooth seal face is smaller than that of rough face. This results in a greater power loss associate with smooth face greater even though there is an additional contact friction term for rough seal face.

### 7.2.3 Conclusions

This section presents a simple hydrostatic model of lubrication film with rough and smooth seal faces for mechanical face seal that allows for the determination of mean pressure distribution, fluid film thickness, power loss and leakage rate. By solving the axis-symmetric Reynolds equation, a more realistic analytical solution of the pressure distribution is obtained that accounts for the taper angle formed between the seal surfaces because of thermal gradients. Comparison of the results with and without the assumption of constant fluid film thickness shows that the difference can not be ignored especially when the inner and outer difference of the rings is large. The effects of surface roughness on the mean film thickness is also analyzed and discussed. It is demonstrated that surface roughness considerably increases the mean fluid film thickness. Based on the results of film thickness, the leakage rate and dissipated power is also discussed. The model and results presented in this chapter can be used as an adjunct analysis for more complicated TEHD analysis of mechanical seals.

![Fig. 7.10 Mean film thickness of smooth seal face](image-url)
Fig. 7.11 Comparison of leakage rate of smooth and rough seal face with $\Delta r = 0.01 \text{m}, \gamma = 1.0, S = 250 \text{ Mpa}$ and $\sigma = 0.5 \times 10^{-6} \text{m}$

Fig. 7.12 Comparison of dissipated power of smooth and rough seal face with $\Delta r = 0.01 \text{m}, \gamma = 1.0, S = 250 \text{ Mpa}$ and $\sigma = 0.5 \times 10^{-6} \text{m}$
7.2.4 Nomenclature

\[ B = \text{conventional balance ratio} \]
\[ B_t = \text{total balance ratio} \]
\[ C, C_{1}, C_{2} = \text{constants} \]
\[ E_c = \text{dissipated power} \]
\[ f_c = \text{contact friction coefficient} \]
\[ F_c = \text{closing force} \]
\[ F_{\text{contact}} = \text{contact force} \]
\[ F_o = \text{open force of smooth seal face} \]
\[ F_{o}' = \text{approximate open force of smooth face} \]
\[ F_{sp} = \text{spring force} \]
\[ F_\sigma = \text{open force of rough seal face} \]
\[ F_{\sigma}' = \text{approximate open force of rough seal face} \]
\[ H = \text{lubrication film thickness} \]
\[ H = \text{dimensionless film thickness} \]
\[ h_i = \text{inner film thickness} \]
\[ h_m = \text{mean film thickness} \]
\[ h_o = \text{outer film thickness} \]
\[ p = \text{fluid pressure of smooth seal face} \]
\[ p' = \text{approximate fluid pressure of smooth seal face} \]
\[ p_i = \text{inner pressure} \]
\[ p_o = \text{outer pressure} \]
\[ Q = \text{leakage rate} \]
\[ R = \text{radius} \]
\[ r_i = \text{inner radius} \]
\[ r_o = \text{outer radius} \]
\[ S = \text{flow stress} \]
\[ V_{r1}, V_{r2} = \text{roughness variance ratio} \]
\[ \beta = \text{Taper angle} \]
\[ \gamma = \text{roughness pattern factor} \]
\[ \mu = \text{fluid viscosity} \]
\[ \sigma = \text{roughness variance} \]
\[ \phi_s = \text{shear flow factor} \]
\[ \Phi_s = \text{function of } \gamma \text{ and } H \]
\[ \phi_i = \text{pressure flow factor} \]
\[ \omega_1, \omega_2 = \text{rotational speed} \]

7.3 A Thermohydrodynamic Analysis of a Lubrication Film between Rough Seal Faces

In the present section, we extend the work of Pascovici and Etsion [4, 17] to include surface roughness effect and heat transfer into the stationary ring. The results should provide a more realistic model for prediction of thermal behavior of mechanical seals.
7.3.1 Model

7.3.1.1 Seal Face Roughness

A schematic of the seal model is shown in Fig. 7.13. The seal consists of two rings: rotor and stator. In reality, the contact faces for the two rings are rough. In the figure, \( h_T \) is the actual film thickness and \( h \) is the nominal film thickness. Assuming that the asperity height distribution for the seal faces is Gaussian [48], the average actual film thickness, \( h_T \), is [46]:

\[
h_T = \frac{1}{2} h \left[ 1 + \text{erf} \left( \frac{h}{\sqrt{2} \sigma} \right) \right] + \frac{\sigma}{\sqrt{2 \pi}} e^{-k^2/2\sigma^2}
\]

(7.32)

where \( \sigma \) is the roughness variance.

At the sealing gap, the nominal lubrication film is formed with thickness \( h \) which is a function of \( r \).

\[
h = h_i + \phi(r - r_i)
\]

(7.33)
where \( h_i \) is the inner nominal film thickness which can be determined by a force balance between the closing force and opening force acting on the sealing gap [4, 43], and \( \phi \) is taper angle of the seal rings which is defined as [44]

\[
\phi = \frac{h_o - h_i}{r_o - r_i}
\]

(7.34)

Under the assumption of axis-symmetry, the heat energy equation in lubrication film is expressed as [4]

\[
k_f \frac{\partial^2 T}{\partial z^2} + \mu \left( \frac{\partial u_\theta}{\partial z} \right)^2 = 0
\]

(7.35)

According to Ref. [4], for an aligned seal the rotating velocity \( u_\theta \) in the lubrication film changes linearly with film thickness, thus

\[
\frac{\partial u_\theta}{\partial z} = -\frac{\omega r}{h_T}
\]

(7.36)

where \( \omega \) is the rotating speed of rotor ring.

Substituting Eq. (7.36) into Eq. (7.35) and integrating twice, we have

\[
T(r, z) = -\frac{\mu}{2k_f} \left( \frac{\omega r}{h_T} \right)^2 z^2 + C_1 z + C_2
\]

(7.37)

where \( C_1 \) and \( C_2 \) are functions of \( r \) and can be determined by boundary conditions as following.

At the seal face of the rotor:

\[
T\big|_{z=0} = T_1(r)
\]

(7.38)

At the seal face of the stator:

\[
T\big|_{z=h} = T_2(r)
\]

(7.39)

Substituting Eqs. (7.38) and (7.39) into Eq. (7.37), \( C_1 \) and \( C_2 \) can be obtained.

\[
C_1 = \left( \frac{T_2 - T_1 + \frac{\mu}{2k_f} \omega^2 r^2}{h_T} \right)
\]

(7.40)

\[
C_2 = T_1
\]

(7.41)

In Ref. [4], Pascovici and Etsion neglected the heat partitioning at the contact face of the seal rings by assuming that the stator is subject to insulation boundary conditions thus no heat transferred into it. In this study, to account for the heat transfer into the stator, we
introduce the heat partitioning factor $\gamma$, which is defined as the ratio of the heat flux into rotor over the heat flux into stator. The detailed derivation of $T_1$ and $T_2$ is shown in Appendix B. The final results are given below.

$$T_1 = T_f + \frac{\gamma}{1 + \gamma \sin 2\phi_1} \left[ \frac{\cos^2 \phi_1}{H_1 r_o} + \frac{\ln \left( \frac{r_o}{r} \right)}{k_1} \right]$$

$$T_2 = T_f + \frac{1}{1 + \gamma \sin 2\phi_2} \left[ \frac{\cos^2 \phi_2}{H_2 r_o} + \frac{\ln \left( \frac{r_o}{r} \right)}{k_2} \right]$$

(7.42)

(7.43)

where $\phi_{1,2}$ are the angles giving the direction of the heat flux path in the rotor and stator, respectively. The parameter $E_c$ is the power dissipation in the lubrication film. While for a smooth seal face, the heat generation results solely from the viscous dissipation within the lubrication film, in the case of rough seal faces the contact friction due to the surface asperities is also a contributing factor to temperature rise. Hence, the total heat dissipation power is expressed as [19]:

$$E_c = \frac{\mu \omega^2 r^2}{h_r} + f \omega p_c r$$

(7.44)

Where $f$ is the contact friction coefficient and $p_c$ is the contact pressure at the sealing gap.

Using the Gaussian height distribution, the contact pressure is [19]

$$p_c = S \int_0^\infty \frac{1}{\sigma \sqrt{2\pi}} e^{-\left( z - \frac{h}{2\sigma} \right)^2} dz$$

(7.45)

where $S$ is the flow stress [5, 19].

The mean radial temperature, $T_m$, is defined as

$$T_m = \frac{1}{h_r} \int_0^{h_r} T dz$$

(7.46)

Substituting Eq. (7.37) into Eq. (7.46), we have

$$T_m = \frac{\mu \omega^2 r^2}{12 k_f} + \frac{1}{2} \left( T_1 + T_2 \right)$$

(7.47)

Substituting Eqs. (7.42), (7.43) and (7.44) into Eq. (7.47), the mean radial temperature for the lubrication film with rough seal faces can be determined.

$$T_m = \frac{\mu \omega^2 r^2}{12 k_f} + T_f + \frac{\gamma}{1 + \gamma \sin 2\phi_1} \left( \frac{\mu \omega^2 r^2}{h_r} + f \omega p_c r \right) \left[ \frac{\cos^2 \phi_1}{H_1 r_o} + \frac{\ln \left( \frac{r_o}{r} \right)}{k_1} \right]$$
The viscosity $\mu$ at radial location is expressed as a function of the radial average temperature $T_m$ in the form \[ \mu = \mu_f e^{-\beta(T - T_f)} \] (7.49)

where $\mu_f$ is the reference viscosity of the process fluid at $T_f$, which is taken to be the temperature of the sealed fluid around the outer surface of the rings.

Introducing the following dimensionless parameters $\overline{T} = \frac{T}{T_f}$, $\overline{r} = \frac{r - r_o}{h_i}$ and $\overline{h_r} = \frac{h_r}{h_i}$, the dimensionless form of Eq. (7.48) is

\[
\frac{T_m}{T_f} = 1 + D \left( \frac{1}{1 + \gamma} \left( \frac{2 \cos^2 \phi_2}{Nu_1} - K_1 \ln \left( \frac{r_o}{r} \right) \frac{r_o^3}{h_r^2} + \lambda r^2 \right) G_1 + \right) \]

\[ e^{\frac{-\beta(T_m - 1)}{T_f}} \] (7.50)

In the above Eq. (7.50), the parameter $D$ is heat dissipation number; $K_{1,2}$ is the thermal conductivity ratio, $Nu_{1,2}$ is the Nusselt number, $\lambda$ is the heat generation ratio and $G_{1,2}$ is the geometric parameter. These parameters are defined as follows:

\[
D = \frac{\mu_f \omega^2 r_o^2}{k_f T_f}, \quad K_1 = \frac{k_L}{k_1}, K_2 = \frac{k_L}{k_2}, Nu_1 = \frac{2r_o H_1}{k_f}, \quad Nu_2 = \frac{2r_o H_2}{k_f}, \quad G_1 = \frac{r_o}{h_i \sin 2\phi_1}, \quad G_2 = \frac{r_o}{h_i \sin 2\phi_2}, \quad \lambda = \frac{h_i f p_e}{r_o \mu \omega} \] (7.51)

Note that the heat generation ratio $\lambda$ describes the ratio of viscous dissipation to friction heat. If $\lambda >> 1$, it suggests that the friction heat is dominant in the total power dissipation. If $\lambda << 1$, it shows that the viscous dissipation is dominant.

### 7.3.1.2 Smooth Seal Face

In this section, the seal faces are assumed to be smooth, which is reasonable only when the roughness at the seal faces is very small and thus the contact friction due to asperities has little effect on the lubrication film. In the last section of rough seal face, Eq. (7.32) is applied to determine the lubrication film thickness. The smooth seal face can be treated
as a special case of rough seal face with zero roughness. Thus, with \( \sigma = 0 \), Eq. (7.32) becomes

\[
h_r = \frac{1}{2} h \left[ 1 + \text{erf} \left( \frac{h}{\sqrt{2} \sigma} \right) \right] + \frac{\sigma}{\sqrt{2} \pi} e^{-h^2/(2\sigma^2)} = \frac{1}{2} h [1 + \text{erf}(\infty)] + 0 = h
\]

(7.52)

Note that in the above equation, \( \text{erf}(\infty) = 1 \). Equation (7.52) shows that the lubrication film thickness for smooth seal face is equal to its nominal lubrication film \( h \) which is determined by Eq. (7.33).

Substituting Eq. (7.52) into Eqs. (7.35) and (7.36), the heat energy equation in lubrication film for smooth seal face is expressed as [4]

\[
k_f \frac{\partial^2 T}{\partial z^2} + \mu \left( \frac{\partial u_{\theta}}{\partial z} \right)^2 = 0
\]

(7.53)

with

\[
\frac{\partial u_{\theta}}{\partial z} = -\frac{\omega r}{h}
\]

(7.54)

Using the boundary conditions shown in Eqs. (7.38) and (7.39) and the same method as the case with rough seal face, Eq. (7.53) can be solved.

\[
T(r, z) = -\frac{\mu}{2k_f} \left( \frac{\omega r}{h} \right)^2 z^2 + C_1 z + C_2
\]

(7.55)

where \( C_1 \) and \( C_2 \) are determined by Eqs. (7.41) and (7.42) with \( h_r = h \).

For smooth seal face, there is friction heat. Therefore, Eq. (7.44) becomes

\[
E_c = \frac{\mu \omega^2 r^2}{h}
\]

(7.56)

Equation (7.56) is the heat generation rate at the sealing gap for smooth seal face. Substituting Eqs. (7.42) and (7.43) into Eq. (7.47) with \( E_c = \frac{\mu \omega^2 r^2}{h} \), the mean radial temperature at the lubrication film, \( T_m \), for smooth seal face can be obtained.

\[
T_m = \frac{\mu \omega^2 r^2}{12k_f} + T_f + \frac{\gamma}{1 + \gamma} \frac{\mu \omega^2 r^3}{h \sin 2\varphi_1} \left[ \cos^2 \varphi_1 \frac{\ln \left( \frac{r_o}{r} \right)}{k_1} + \frac{\cos^2 \varphi_2}{H z r^2} \frac{\ln \left( \frac{r_o'}{r} \right)}{k_2} \right]
\]

(7.57)
Using dimensionless parameters \( \bar{T} = \frac{T}{T_f} \), \( \bar{r} = \frac{r}{r_o} \) and \( \bar{h} = \frac{h}{h_i} \), the mean temperature becomes

\[
\bar{T}_m = 1 + D \left\{ \gamma \left( \frac{2 \cos^2 \varphi_1 - K_1 \ln \bar{r}}{Nu_1} \right) G_1 \frac{\bar{r}^3}{\bar{h}} + \frac{1}{1 + \gamma} \left[ \frac{2 \cos^2 \varphi_2 - K_2 \ln \left( \frac{r_o}{r_i} \right)}{Nu_2} \right] G_2 \frac{\bar{r}^2}{\bar{h}} \right\} e^{-\bar{h}(\bar{r} - 1)} \quad (7.58)
\]

In the above Eq. (7.58), the parameter \( D \) is heat dissipation number; \( K_{1,2} \) is the thermal conductivity ratio, \( Nu_{1,2} \) is Nusselt number, and \( G_{1,2} \) is geometric parameter. These parameters are defined in Eq. (7.51).

Pasovici and Etsion [4] imposed the same boundary conditions at the seal face of the rotor shown in Eq. (7.38) but considered seal face of stator to be insulated, i.e.

\[
\frac{\partial \bar{T}}{\partial z} \bigg|_{z=h} = 0 \quad (7.59)
\]

Using the same nomenclature, Pasovici and Etsion’s result for insulated stator is:

\[
\bar{T}_m = 1 + D \left\{ 2 \left( \frac{2 \cos^2 \varphi_1 - K_1 \ln \bar{r}}{Nu_1} \right) G_1 \frac{\bar{r}^3}{\bar{h}} + \frac{1}{1 + \gamma} \left[ \frac{2 \cos^2 \varphi_2 - K_2 \ln \left( \frac{r_o}{r_i} \right)}{Nu_2} \right] G_2 \frac{\bar{r}^2}{\bar{h}} \right\} e^{-\bar{h}(\bar{r} - 1)} \quad (7.60)
\]

In the above equations, the dimensionless film thickness is \( \bar{h} = 1 + \Phi \left( \bar{r} - 1 \right) \), where \( \Phi = \frac{h_i}{h_o} \) is the dimensionless taper angle of the ring. Note that if we set \( \gamma = \infty \)

### 7.3.2 Results and Discussion

Table 7.3 shows the seal parameters and appropriate property values used in the simulations. Two cases are considered in the present paper. One is for water seal the other is for oil.

Consider a mechanical seal with the outer radii of the rotor and stator of 45 mm and 46 mm, respectively. The inner radius of the rotor is of 40.05 mm. The rotor with rotational speed of 300 rad/s and stator have thermal conductivity \( k_1 = 45 \text{ W/m.K} \) and \( k_2 = 16.3 \text{ W/m.K} \), respectively.

A reference value of \( \varphi = 45^0 \) is chosen as the heat flux path angle in both rings and also a reference value of \( h_i = 1 \mu \text{m} \) is selected for the sealing gap [4]. In addition, based on the theory of influence coefficient method [18, 19], the value of heat partitioning factor \( \gamma \) is determined using a commercial software FLUENT with appropriate boundary conditions. The details of calculating \( \gamma \) are shown in Appendix C.
Table 7.3 Properties and corresponding parameters [3]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Water seal</th>
<th>Oil seal</th>
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<tr>
<td>$T_f$ (°C)</td>
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</tr>
<tr>
<td>$\mu_f$ (Pa.s)</td>
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<td>$k_f$ (w/m°C)</td>
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<tr>
<td>$H_{1,2}$ (w/m²°C)</td>
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<td>3,000</td>
</tr>
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</tr>
<tr>
<td>$\bar{\beta}$</td>
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<td>1.52</td>
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<tr>
<td>$D$</td>
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<td>$K_2$</td>
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<td>$\nu_2$</td>
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</tr>
<tr>
<td>$\gamma$</td>
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<td>1.25</td>
</tr>
</tbody>
</table>

7.3.2.1 Effects of Roughness Variance $\sigma$

Figures 7.14 (a), (b) show the mean radial temperature in the lubrication film of water and oil with smooth $\sigma = 0$, $\sigma = 0.25\mu$m and $0.3\mu$m. Note that the mean radial temperature distribution is calculated by Eq. (7.50). Since $\bar{T}_m$ is implicitly included in Eq. (7.50), an iterative solution is required.

From Fig. 7.14, it can be seen that for both water and oil, the radial average temperature increases slightly compared with $\sigma = 0$. However, increasing roughness to $\sigma = 0.3\mu$m, results in a considerable rise in the average temperature. It can also be found that in this case, the effect of $\sigma$ is restricted to the region close to the inner radius $r/r_o < 0.96$ while for the region with $r/r_o > 0.96$, roughness has little effect on the temperature distribution.

The effect of roughness on the axial temperature distribution at $r = 0.5(r_o + r_i)$ is investigated shown in Fig.7.15 which is calculated from Eq.(7.37). For water, the axial temperature is increasing from $\sigma = 0$ to $\sigma = 0.3\mu$m.

For oil, the temperature increases slightly from $\sigma = 0$ to $\sigma = 0.25\mu$m thus no obvious difference is observed. However, with roughness increasing to $\sigma = 0.3\mu$m, the temperature increases about 0.996 °C, but to a lesser extent than 1.58 °C of water.
Fig. 7.14 Effects of roughness variance $\sigma$ on the radial average temperature with $\Phi = 4.5$ and $S = 250$ Mpa: (a) water (b) oil
Fig. 7.15 Effects of roughness variance $\sigma$ on the axial temperature at $r = 0.5(r_o + r_i)$ with $\Phi = 4.5$ and $S = 250$ Mpa: (a) water (b) oil
7.3.2.2 Effects of Flow Stress \( S \)

Figure 7.16 presents the effects of flow stress \( S \) on the radial average temperature distribution in the lubrication film. The flow stress is the compressible strength of the weaker material [5]. Halling [49] indicated that \( S \) is approximately three times the tensile yield strength for a metal, i.e. essentially the indentation hardness. From Fig. 7.16, it can be seen that for both water and oil the temperature increases with increasing the flow stress. Similar to the roughness variance, the effect of flow stress is also limited to the region close to the inner radius. This suggests that the larger sealing gap at the region close to the outer radius reduces the influence of the flow stress on the temperature distribution.

7.3.2.3 Effects of Film Thickness

The comparison of the effects of film thickness \( h_i \) on average temperature at \( r = 0.5(r_o + r_i) \) for rough and smooth seal face is shown in Fig. 7.17. The film thickness is calculated from Eqs. (7.50) and (7.58), respectively. Note that in this figure the horizontal axis is \( G^{-1} \) since \( h_i \) is expressed in the dimensionless parameter \( G \). For instance, a value of \( G^{-1} = 10^{-5} \) corresponds to an inner film thickness \( h_i = 0.45 \mu m \). From this figure, it can be recognized that the average temperature changes abruptly for small sealing gap. When the film thickness increases to \( G^{-1} = 10^{-4} \) corresponding to \( h_i = 4.5 \mu m \), the effect of film thickness on the temperature becomes very small. From Fig. 7.17 it can be seen that for both cases of water and oil, the temperature for smooth surface is lower than for rough seal face due to the additional contact friction heat generation. However, the difference between them is decreasing with increasing the film thickness \( h_i \), which means that a larger sealing gap can reduce the effects of the roughness at the seal faces, but this adversely affects the leakage.

7.3.2.4 Smooth Seal Face

Figure 7.18 presents the comparison of the radial average temperature at sealing gap calculated from Pascovici and Etsion’s model [4] and the present model for both oil and water. Note that the radial average temperature is calculated by Eq. (7.58). Figure 7.18 reveals that the maximum temperature occurs at the inner radius for both models. The results calculated from the present model are lower than those from Pascovici and Etsion’s model [4]. This is reasonable since the latter model [4] assumes that the rotor is insulated.

Figure 7.19 shows the temperature distribution in the axial direction at \( r = 0.5(r_o + r_i) \). It can be seen that the axial temperature does not change appreciably in the lubrication film, which does not necessarily mean the axial temperature gradient is small. As a matter of the fact, the axial temperature gradient across the film gap is fairly large, considering the minute size of the film thickness. For example, when the lubrication fluid is oil, the temperature across the 1.25 \( \mu m \) gap varies from 3.19\( T_f \) to 3.33\( T_f \), signifying heat transfer to both the rotator ring and the stator ring.
Fig. 7.16 Effects of flow stress $S$ on the radial average temperature with $\Phi = 4.5$ and $\sigma = 0.3 \mu m$: (a) water (b) oil
Fig. 7.17 Comparison of the axial temperature at $r = 0.5(r_o + r_i)$ for smooth and rough seal face with $\Phi = 4.5$, $\sigma = 0.3 \, \mu\text{m}$ and $S = 250 \, \text{Mpa}$
Fig. 7.18 Comparison of the radial average temperature at sealing gap calculated from Pascovici and Etsion's model [3] and the present model with $\Phi = 4.5$

Fig. 7.19 Comparison of the axial temperature at $r = 0.5(r_o + r_i)$ calculated from Pascovici and Etsion's model [4] and the present model with $\Phi = 4.5$
Fig. 7.20 Effects of taper angle \( \Phi \) on the average temperature at \( r \) direction

Figure 7.20 presents the effects of taper angle on the radial temperature distribution. Three cases of \( \Phi = 0.45, 4.5, \) and \( 45 \) are calculated. The corresponding taper angle is given as \( \phi = 10 \times 10^{-6} \text{ rad}, 100 \times 10^{-6} \text{ rad}, \text{ and } 1000 \times 10^{-6} \text{ rad} \). For small taper angle of \( \Phi = 0.45 \), for both water and oil, the temperature decreases slightly in the radial direction. For a very large taper angle of \( \Phi = 45 \) and \( 4.5 \), \( 66.6 \times 10^{-6} \) and \( 100 \times 10^{-6} \text{ rad} \), the reduction of the radial temperature is considerable due to the relatively large sealing gap which reduces the viscous heat generation in the lubrication film. Under the conditions simulated, with \( \Phi = 45 \) the radial maximum and minimum temperature difference for water and oil is 19.18 \( ^\circ \text{C} \) and 42.02 \( ^\circ \text{C} \), respectively. Note that at the inner radius, the temperature is the same for all the three cases of \( \Phi \).

7.3.3 Conclusions

A THD model is presented for a lubrication film between smooth and rough seal faces. The present analytical solution is an extension of the work of Pacovici and Etsion [4]. The extension includes taking surface roughness into account and relaxing the assumptions of the insulated boundary conditions for the stator. An approximate heat flow path described by Ref. [4] is applied to simplify the analysis, based on which, a simplified energy equation is derived and solved analytically for predicting the radial and axial temperature variation. The equations are nondimensionalized and an analytical parametric simulation is performed to study the effect of various parameters on the thermal behavior of the lubrication film at the sealing gap. The results show that the radial temperature variation can vary significantly and that the commonly assumed constant viscosity is not valid at the sealing gap for case of high viscosity fluids such as
oils. In the present study, the viscous and contact heat generation at the interface of the rings is considered to be transferred into both rings. The heat portioning factor is numerically calculated by applying software FLUENT. Roughness at the seal faces are also considered in the analysis. This present work can be a good adjunct analysis for more complicated TEHD analysis on seals.

7.3.4 Nomenclature

\( C_{1,2} \) = integration constants
\( D \) = dissipation number
\( E_c \) = dissipation power
\( f \) = friction coefficient
\( G_{1,2} \) = geometric parameter
\( h \) = film thickness (\( \mu m \))
\( \bar{h} \) = dimensionless film thickness \( h / h_i \)
\( h_i \) = inner film thickness (\( \mu m \))
\( h_f \) = film thickness with rough seal faces (\( \mu m \))
\( \bar{h}_f \) = dimensionless film thickness with rough seal faces
\( k_f \) = thermal conductivity of fluid (W/k.m)
\( k_{1,2} \) = thermal conductivity of seal rings (W/k.m)
\( \bar{K}_{1,2} \) = conductivity ratio \( k_f / k_{1,2} \)
\( Nu_{1,2} \) = Nusselt number
\( p_c \) = contact pressure (N/m\(^2\))
\( q_1 \) = heat flux into rotor (W/m\(^2\))
\( q_2 \) = heat flux into stator (W/m\(^2\))
\( r \) = radius (mm)
\( \bar{r} \) = dimensionless radius \( r / r_o \)
\( r_i \) = inner radius (mm)
\( r_o \) = outer radius for rotor (mm)
\( r_o' \) = outer radius for stator(mm)
\( S \) = flow stress (Mpa)
\( T \) = temperature (\( ^\circ C \))
\( \bar{T} \) = dimensionless temperature
\( T_f \) = process fluid temperature (\( ^\circ C \))
\( T_m \) = radial average temperature (\( ^\circ C \))
\( u_o \) = rotating velocity (m/s)
\( z \) = axial coordinate (\( \mu m \))
\( \beta \) = temperature-viscosity coefficient (1/\( ^\circ C \))
\[ \beta = \text{dimensionless coefficient} \, \beta T_f \]
\[ \phi = \text{taper angle (rad)} \]
\[ \Phi = \text{dimensionless taper angle} \]
\[ \varphi = \text{angle of heat flux path in the rings} \]
\[ \gamma = \text{heat partitioning factor} \]
\[ \lambda = \text{ratio of viscous dissipation to friction heat} \]
\[ \mu = \text{viscosity of fluid (Pa.s)} \]
\[ \sigma = \text{roughness variance (\(\mu m\))} \]
\[ \omega = \text{rotating speed of rotor (1/rad)} \]

### 7.4 Numerical Analysis of TEHD Behavior of Mechanical Seals

In this section, numerical analysis of TEHD behavior of mechanical seals is presented. With proper boundary conditions, simplified Navier-Stokes equations, Reynolds equation and energy equation are derived for the lubrication film at the sealing gap between the rotating and stationary rings. The model is applied to both rough and smooth seal faces.

#### 7.4.1 Mathematical Modeling and Numerical Solution

##### 7.4.1.1 Film Thickness of the Interfacial Lubrication Film

Figure 7.21 shows the schematic of the seal installation. It can be seen that the floating rotating ring is subjected to the pressure from the interfacial film, the sealed fluid, the springs. Figure 7.22 shows the dimensions of the seal rings and the interfacial lubrication film. Note that in this study, we consider surface roughness. The results are based on steady state operation and we take advantage of axisymmetric nature of the problem.

Fig. 7.21 Schematic of a seal installation
The film thickness can be determined by balancing the opening force and closing force acting on the rotating rings. The opening force $F_o$ and closing force $F_c$ are defined as follows [19, 42].

$$F_o = \pi (r_{o1}^2 - r_{i1}^2)p_o \frac{h_o}{h_o + h_i} + F_{\text{contact}}$$  \hspace{1cm} (7.61)

$$F_c = \pi (r_{o1}^2 - r_{i1}^2)p_o B_i$$  \hspace{1cm} (7.62)

where $r_{o1}$ and $r_{i1}$ are the inner and outer radius of the rotating ring, respectively as shown in Fig.7.22; $p_o$ is the outer pressure ($p_i$ is assumed to be 0 in this study); and $h_o$ and $h_i$ are the inner and outer film thickness respectively. $B_i$ is the total balance ratio [43],

$$B_i = \frac{F_{sp}}{\pi (r_{o1}^2 - r_{i1}^2)p_o} + B$$  \hspace{1cm} (7.63)

where $F_{sp}$ is the spring load and $B$ is the conventional balance ratio.

Fig. 7.22 Schematic of seal rings: (a) seal rings (b) fluid film between seal rings
The contact force $F_{\text{contact}}$ can be determined by the contact mechanics of the contacting asperities.

$$F_{\text{contact}} = 2\pi \int_{r_i}^{r_e} p_c r dr$$

(7.64)

where $p_c$ is the contact pressure distribution. Using a simple plastic asperity deformation model and assuming a Gaussian asperity distribution function [19, 48], the contact pressure is:

$$p_c = S \int_{h}^{\infty} \frac{1}{\sigma \sqrt{2\pi}} e^{-z^2/2\sigma^2} dz$$

(7.65)

where $S$ is the flow pressure or flow stress and $\sigma$ is the roughness variance.

According to the fluid film geometry and considering the taper angle $\phi$ presented in Fig.22 (b), the film thickness can be expressed as,

$$h = h_i + \phi(r - r_i)$$

(7.66)

where $h_i$ can be determined by solving

$$F_o = F_c$$

(7.67)

The calculation of seal face taper angle $\phi$ in Eq.(7.66) is shown in Appendix D.

### 7.4.1.2 Flow Field in the Interfacial Lubrication Film

The flow field in the lubrication film is governed by the Navier-Stokes equations in cylindrical coordinates. According to Hughes [51], the Navier-Stokes equations can be simplified into [52]

$$\mu \frac{\partial^2 u_r}{\partial z^2} = \frac{\partial p}{\partial r}$$

(7.68)

$$\frac{\partial^2 u_\theta}{\partial z^2} = 0$$

(7.69)

with boundary conditions as shown in Fig.7.22,

$$u_r = 0, u_\theta = 0 \quad \text{at} \quad z = 0$$

(7.70)

$$u_r = 0, u_\theta = \omega r \quad \text{at} \quad z = -h$$

(7.71)

Equations (7.68) and (7.69) can be solved easily to determine the velocity distribution. The results are:

$$u_r = \left( \frac{1}{2\mu} \right) \frac{dp}{dr} \left( z^2 + hz \right)$$

(7.72)

$$u_\theta = -\frac{\omega z}{h}$$

(7.73)

In Eq. (7.72), the pressure differential $dp/dr$ can be expressed as [44]
Under the assumptions of axis-symmetry and steady state, the energy equations in the rotating and stationary rings can be simplified as given below

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_{1,2}}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{\partial T_{1,2}}{\partial z} \right) = 0
\]

(7.75)

For the lubrication film, in addition to the assumptions of axis-symmetry and steady state, is assumed to be neglected. Since the film thickness is small compared with the radius of the seal rings and the rotational speed of the rotor is high, the flow field in the sealing gap is mainly dominated by \( u_\phi \) and \( u_r \). Under these assumptions, the energy equation in the film can be simplified into

\[
\rho c_r u_r \frac{\partial T_3}{\partial r} = k_f \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_3}{\partial r} \right) + \frac{\partial^2 T_3}{\partial z^2} \right) + 2 \mu \left( \left( \frac{\partial u_\phi}{\partial r} \right)^2 + \left( \frac{u_r}{r} \right)^2 \right)
\]

\[
+ \mu \left( \frac{\partial^2 u_\phi}{\partial z^2} + \left( \frac{\partial u_r}{\partial z} \right)^2 + \left( r \frac{\partial u_\phi}{\partial r} \right)^2 \right)
\]

(7.76)

Note that subscripts 1, 2 and 3 represent the rotating ring, the mating ring, and the fluid film, respectively. As shown in Fig. 7.22 (a), the boundary conditions at the outer surface of the rotor and stator rings are assumed to be heat convection. The other boundaries for the rings are assumed to be insulated. Considering the coordinates in Fig. 7.22 (b), the boundary for the interfaces between the seal rings and the fluid film are as follow.

At the interface between the rotating ring and the fluid film:

\[
k_j \frac{\partial T_1}{\partial z} = k_1 \frac{\partial T_1}{\partial z} \quad \text{at} \quad z = -h
\]

(7.77)

At the interface between the stator ring and the fluid film:

\[
k_j \frac{\partial T_1}{\partial z} = k_2 \frac{\partial T_2}{\partial z} \quad \text{at} \quad z = 0
\]

(7.78)

In the above equations, \( k \) is the heat conductivity of the seal ring:

Using \( T^* = \frac{T_{1,2}}{T_f}, r^* = \frac{r}{r_1}, z^* = \frac{z}{L_1} \) to nondimensionalize the above energy equation for seal rings, we arrive at the following equations.
\[
\frac{\partial^2 T_{1,2}^*}{\partial z^2} + \left( \frac{L_1}{r o_1} \right) \left( \frac{\partial^2 T_{1,2}^*}{\partial r^2} + \frac{1}{r^2} \frac{\partial T_{1,2}^*}{\partial r} \right) = 0
\]  

(7.79)

Using \( T_f^* = \frac{T_3}{T_f} \), \( r^* = \frac{r}{r o_1} \), \( z^* = \frac{z}{h_i} \), \( h^* = \frac{h}{h_i} \), \( \frac{u_o}{\omega r o_1} \), \( u_r^* = \frac{u_r}{u_r} \) to nondimensionalize the above energy equation for the lubrication film, we obtain the following equation

\[
u_r^* \frac{\partial T_3^*}{\partial r^*} = E_1 \left( \frac{\partial^2 T_3^*}{\partial z^2} + \left( \frac{h_i}{r o_1} \right)^2 \frac{\partial^2 T_3^*}{\partial r^2} + \frac{1}{r^2} \frac{\partial T_3^*}{\partial r} \right) + E_2 \left( \frac{\partial u_r^*}{\partial r^*} + \frac{u_r^*}{r^*} \right)^2 + E_3 \left( \frac{\partial u_\theta^*}{\partial z^*} + \frac{\partial u_r^*}{\partial \theta} \right)^2 \]

(7.80)

where the expressions of the dimensionless parameters \( E_1, E_2 \) and \( E_3 \) are:

\[
E_1 = \frac{k_f r o_1}{h_i^2 \rho c \nu_r} = \left( \frac{r o_1}{h_i} \right)^2 \frac{1}{P e}
\]

(7.81)

with

\[
P e = \frac{u_r r o_1}{\alpha}
\]

(7.82)

\[
E_2 = 2 \frac{\mu u_r}{r o_1 \rho c \nu_r T_f} = \left( \frac{h_i}{r o_1} \right) \frac{E_c}{Re}
\]

(7.83)

with

\[
E_c = \frac{u_r}{c_v T_f}
\]

(7.84)

\[
E_3 = \frac{\mu \omega \rho r o_1^3}{h_i^2 \rho c \nu_r T_f} = \left( \frac{\omega r o_1}{u_r} \right)^2 \left( \frac{r o_1}{h_i} \right)^2 \frac{E_2}{2}
\]

(7.85)

Examining Eq. (7.80) and noting \( \frac{h_i}{r o_1} \rightarrow 0 \) and \( \frac{u_r}{\omega r o_1} \rightarrow 0 \), Eq. (7.80) can be simplified into,

\[
u_r^* \frac{\partial T_3^*}{\partial r^*} = E_1 \left( \frac{\partial^2 T_3^*}{\partial z^2} \right) + E_2 \left( \frac{\partial u_r^*}{\partial r^*} \right)^2 + E_3 \left( \frac{\partial u_\theta^*}{\partial z^*} \right)^2
\]

(7.86)

In Eq. (7.86), \( E_2 \) and \( E_3 \) are the dimensionless parameter of the viscous heat term thus they are comparable. From Eqs. (7.83) and (7.85), it can be it can be found that
\[ E_3 / E_2 = 0.5 \left( \frac{\omega r_o}{u_r} \right)^2 \left( \frac{r_o}{h_1} \right)^2 \gg 1, \text{ thus } E_2 \ll E_3. \] Therefore, the term with \( E_2 \) can be neglected. Now Eq. (7.86) can be reduced to

\[ u_r \cdot \partial T^*_3 = E_i \left( \frac{\partial^2 T^*_3}{\partial z^2} \right) + E_3 \left( \frac{\partial u^*_\theta}{\partial z^*} \right)^2 \] (7.87)

Pascovici and Etsion [5] assumed that the convective term is negligible, which is reasonable considering the extremely small leakage rate at the sealing gap. Using this assumption, Eq. (7.87) can be simplified into

\[ 0 = E_i \left( \frac{\partial^2 T^*_3}{\partial z^2} \right) + E_3 \left( \frac{\partial u^*_\theta}{\partial z^*} \right)^2 \] (7.88)

The dimensional form of the above Eq. (7.88) is

\[ 0 = k_f \left( \frac{\partial^2 T}{\partial z^2} \right) + \mu \left( \frac{\partial u^*_\theta}{\partial z} \right)^2 \] (7.89)

Equation (7.89) is exactly the same as the energy equation used by Pascovici and Etsion [4]. The derivation above shows that Pascovici and Etsion’s simplification is reasonable. However, in the present study, Eq. (7.80) was solved.

The viscosity \( \mu \) in the lubrication film is assumed to be variant depending on temperature.

\[ \mu = \mu_0 e^{-\beta(T_f - T)} \] (7.90)

where \( T_f \) is the temperature of the process fluid in the seal chamber.

In the present work, two methods are used to calculate the power dissipation at the sealing gap. One method is based on the temperature distribution obtained from our model.

\[ E_c = k_f \left( \pi (r_o - r_i) \right) \frac{\partial T}{\partial z} \bigg|_{z=-h} - k_f \left( \pi (r_o - r_i) \right) \frac{\partial T}{\partial z} \bigg|_{z=0} + c_p \Delta T \left( \rho \sqrt{r_i} \right) 2\pi r_i h_1 \] (7.91)

where \( E_c \) is the total power dissipation. In Eq. (7.91), the first two terms are the heat transferred into the rotor and stator, the third terms is corresponding to the heat loss removed by the leakage.

The other method is

\[ E_c = \pi \mu \omega^2 \frac{r_o^4 - r_i^4}{2h_m} + 2\pi \int_{r_i}^r f_i \omega p_i r^2 \, dr \] (7.92)

In the above equation, the first term in right hand side is the viscous heat, and the second term is friction heat. The viscous heat term in Eq. (7.92) is a simplified analytical formula for predicting the viscous heat generation [19, 43, 44].
7.4.1.4 Numerical Procedure

A computer program was made to calculate the problem in this study. The analysis involves the heat transfer analysis in the lubrication film and the seal rings, and thermal distortion analysis of the seal face. Since the analyses and unknowns are coupled, the numerical analysis requires an iterative scheme.

![Fig. 7.23 Computational flow chart](image)

The flow chart of the calculation procedure is shown in Fig. 7.23. The analysis begins with an initial guess of film thickness, taper angle and temperature distribution. It then calculates the pressure profile including the hydraulic pressure distribution and the contact pressure at the sealing gap. By performing seal load balance, the lubrication film thickness is completed next. Knowing the dimension of the lubrication film, the flow field is obtained using Eqs. (7.72) and (7.73). Then conjugate heat transfer analysis in the lubrication film and seal rings is performed.
A computer with Pentium(R) C CPU 3.2 GHz and 2.0 GB of RAM was used to execute the code. Typically it takes about 30 minutes to complete the computation. After convergence, the outcome of the simulation is the temperature distribution, power dissipation etc.

7.4.2 Results and Discussions

The values of operating parameters and the material properties of the seal are shown in Table 7.4-7.5. The materials of rotor and stator are carbon graphite and silicon carbide, respectively. The process fluid is oil. Its properties are in Table 7.5.

Table 7.4 Dimensions of seal rings in Fig. 7.22

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
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<tr>
<td>$r_{12}$</td>
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<td>$r_{o1}$</td>
<td>5.3 cm</td>
</tr>
<tr>
<td>$r_{o2}$</td>
<td>5.8 cm</td>
</tr>
<tr>
<td>$r_{o3}$</td>
<td>6.2 cm</td>
</tr>
<tr>
<td>$L_1$</td>
<td>0.5 cm</td>
</tr>
<tr>
<td>$L_2$</td>
<td>0.7 cm</td>
</tr>
<tr>
<td>$L_3$</td>
<td>0.5 cm</td>
</tr>
</tbody>
</table>

Table 7.5 Operating parameters and thermal properties of seal rings

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance ratio $B$</td>
<td>0.75</td>
</tr>
<tr>
<td>Spring force $F_{sp}$</td>
<td>314 N</td>
</tr>
<tr>
<td>Outer pressure $p_o$</td>
<td>$3.45 \times 10^5$ Pa</td>
</tr>
<tr>
<td>Inner pressure $p_i$</td>
<td>0</td>
</tr>
<tr>
<td>Fluid inlet temperature $T_f$</td>
<td>40 °C</td>
</tr>
<tr>
<td>Heat transfer coefficient $H_f$</td>
<td>776.0 W/m²-K</td>
</tr>
<tr>
<td>Heat transfer coefficient $H_2$</td>
<td>544.8 W/m²-K</td>
</tr>
<tr>
<td>Oil density $\rho$</td>
<td>850 Kg/m³</td>
</tr>
<tr>
<td>Specific heat $c_p$</td>
<td>2000 W/Kg-K</td>
</tr>
<tr>
<td>Thermal conductivity $k_f$</td>
<td>0.14 W/m-K</td>
</tr>
<tr>
<td>Dynamic Viscosity at 40 °C $\mu_0$</td>
<td>0.08 Pa.s</td>
</tr>
<tr>
<td>Thermal Conductivity for rotor $k_1$</td>
<td>8.6 W/m-K</td>
</tr>
<tr>
<td>Thermal Conductivity for rotor $k_2$</td>
<td>138.4 W/m-K</td>
</tr>
<tr>
<td>Rotating speed $\omega$</td>
<td>1800 rpm</td>
</tr>
</tbody>
</table>
Some statements of the heat convection boundary conditions need to be made here (See Fig.7.22). The heat transfer coefficients for the outer surface of the rotor and stator, $H_1$ and $H_2$ are assumed to be uniform [18]. In the present study, $H_1 = 776.0 \text{ W/m}^2\text{K}$ and $H_2 = 544.81 \text{ W/m}^2\text{K}$ are used. The details of determining $H_1$ and $H_2$ are shown in Appendix E.

Figure 7.24 shows the radial interface temperature for smooth with roughness variance $\sigma = 0$ and rough seal faces with roughness variance $\sigma = 1.0 \times 10^{-6} \mu m$. For the rough seal face, the contact friction coefficient $f = 0.1$ and the flow stress $S = 250 \text{ Mpa}$ are used. From the figure, it can be seen that the interface temperature for the rough seal faces is approximately $27.5 \degree \text{C}$ higher than the smooth seal face due to the contact friction heat in the sealing gap and the temperature is decreasing along the radial direction.

FLUENT was also used to simulate the same problem for smooth and rough seal faces. The comparison is shown in Figure 7.24. It can be recognized that the difference between our model and FLUENT results are very small. To take the viscous heat generation into account, we activated the option in the FLUENT simulations. In this fashion, FLUENT will calculate the viscous generation term in the energy equation of the fluid film. As for the contact friction heat, FLUENT can not calculate this itself. We use the method mentioned in section 7.2 to calculate the contact friction heat generation. And then supply the friction heat as heat generation rate to the fluid film of the FLUENT model.
Figure 7.25 shows the temperature contour in the seal rings for the case of rough seal faces. Because that the heat conductivity of the stationary ring is very high, more heat is conducted into it, resulting in higher temperature in it.

Figures 7.26 and 7.27 show the temperature contour plots in the lubrication film for smooth and rough seal faces. It can be found that the film thickness for the rough seal faces is bigger than that for the smooth seal faces. The reason is that in the rough seal face, the temperature is higher due to the contact friction heat. The higher temperature will induce a greater taper angle in the seal face and then it will cause a larger film thickness [43].

Figure 7.28 shows the power dissipation with rotational speed varying from 200 rpm to 1800 rpm. The figure shows the comparison of the results calculated using Eqs. (7.91) and (7.92). It shows that the difference between them is very small. Especially for the smooth seal face, the difference is less than 5% from rotational speed of 200 rpm to 1800 rpm. For the rough seal face case, the difference is still negligible for lower rotational speed. It can be noted that the difference is increasing with increasing the rotational speed.

In Fig. 7.28, for the case of rough seal face, it can be seen that after rotational speed of 800 rpm, the difference between the results of the formula (Eq. (7.92)) and the present model is fairly large. Generally speaking, the values calculated from our model are more conservative than that from the formula as shown in Eq. (7.92). The power dissipation is increasing gradually with increasing the rotating speed.
From Fig. 7.28, by comparing the results of the smooth seal face and rough seal face, we can find that under the conditions simulated, the power dissipation for rough case is larger even though its film thickness is greater. Note that from Eq. (7.92), it can be noticed that the larger film thickness, the smaller viscous heat generation. Thus, in the current rough seal face case, the contact friction heat makes a fairly big contribution to the total power dissipation at the sealing gap.

![Temperature contour for smooth lubrication film at 1800 rpm](image1)

**Fig. 7.26 Temperature contour for smooth lubrication film at 1800 rpm**

![Temperature contour for rough lubrication film at 1800 rpm](image2)

**Fig. 7.27 Temperature contour for rough lubrication film at 1800 rpm**

It has been argued that [4, 44] most of the heat generated at the sealing gap is removed by the heat convection around the outer surface of the seal rings. To investigate, in the present study, we calculated the heat exiting the sealing gap with leakage and explored its effects on the cooling of the seal rings. The heat removal by leakage is calculated using Eq. (7.93) which is actually the third term in Eq. (7.91).
Fig. 7.28 Dissipation power

\[ E_L = c_p \Delta T \left( \rho \bar{u}_r \right) 2\pi r_i h_i \]  

(7.93)

Figures 7.29 and 7.30 are plotted to explore the heat removed by the leakage at the sealing gap. From the figures, it can be found that the heat loss removed by leakage is increasing with increasing the rotational speed. This is because the lubrication film thickness is increasing. Also it can be noticed that, for rough seal face, the heat removed by leakage is much larger than for the smooth seal face.

Comparing Figs. 7.29-7.30 and Fig. 7.28, it can be concluded that the percentage of \( \frac{E_L}{E_c} \) is small for both smooth and rough seal faces. For smooth seal face, the percentage of \( \frac{E_L}{E_c} \) is less than 1% which means that the heat loss as result of leakage is negligible. Even for the rough seal face, the leakage rate is larger due to thicker lubrication film, the maximum percentage of \( \frac{E_L}{E_c} \) is less than 17%. Therefore, for both cases the heat generation at the sealing gap is mostly transferred into the rings and then removed by the heat convection at the outer surface of the rings.

**7.4.3 Conclusions**

Numerical analysis of TEHD behavior of mechanical seals is made. In this model, thermoelastic deformation and conjugate heat transfer are considered. Simplified Navier-Stokes equations, Reynolds equation and energy equation are solved for the lubrication
film at the sealing gap between the rotating and stationary rings. Rough and smooth seal faces are considered. Heat generation at the sealing gap is calculated and compared with the heat removal by leakage. It shows that the heat removed by leakage flow is negligible compared with the total heat generation which suggests that the heat generation is mostly transferred into the seal rings and then removed by heat convection at the outer surface of seal rings.

![Graph 1](image1.png)

**Fig. 7.29 Power with leakage for rough seal face**

![Graph 2](image2.png)

**Fig. 7.30 Power with leakage for smooth seal face**
7.4.4 Nomenclature

\[ B = \text{conventional balance ratio} \]
\[ B_t = \text{total balance ratio} \]
\[ C_v = \text{heat capacity of fluid} \]
\[ E_{1,2,3} = \text{dimensionless parameter} \]
\[ E_c = \text{dissipated power} \]
\[ E_l = \text{power with leakage} \]
\[ F = \text{contact friction coefficient} \]
\[ F_c = \text{closing force} \]
\[ F_{\text{contact}} = \text{contact force} \]
\[ F_o = \text{open force of smooth seal face} \]
\[ F_{sp} = \text{spring force} \]
\[ h = \text{lubrication film thickness} \]
\[ H_{1,2} = \text{heat convection coefficient} \]
\[ h = \text{film thickness} \]
\[ h_i = \text{inner film thickness} \]
\[ h_o = \text{outer film thickness} \]
\[ k_{1,2} = \text{heat conductivity of rings} \]
\[ k_f = \text{heat conductivity of fluid} \]
\[ L_{1,2} = \text{axial length of rings} \]
\[ P = \text{fluid pressure of smooth seal face} \]
\[ p_i = \text{inner pressure} \]
\[ p_o = \text{outer pressure} \]
\[ q = \text{heat flux} \]
\[ Q = \text{leakage rate} \]
\[ R = \text{radial coordinate} \]
\[ r^* = \text{dimensionless radial coordinate} \]
\[ r_i = \text{inner radius} \]
\[ r_o = \text{outer radius} \]
\[ S = \text{flow stress} \]
\[ T_{1,2,3} = \text{temperature for the rotating ring, the mating ring and the fluid film, respectively} \]
\[ T^* = \text{dimensionless temperature} \]
\[ u_r = \text{radial velocity} \]
\[ u_r^* = \text{dimensionless radial velocity} \]
\[ u_\phi = \text{rotating velocity} \]
\[ u_\phi^* = \text{dimensionless rotating velocity} \]
\[ z = \text{axis} \]
\[ z^* = \text{dimensionless axial coordinate} \]
\[ \beta = \text{viscosity coefficient} \]
\[ \mu = \text{fluid viscosity} \]
\[ \sigma = \text{roughness variance} \]
\[ \phi = \text{taper angle} \]
\[ \omega = \text{rotational speed} \]
8 Conclusions

8.1 Conclusions

An extensive flow and heat transfer analysis of mechanical seals has been implemented in this work. Some important numerical and analytical models have been established. The major contributes to the field of mechanical face seals are in the following aspects:

1. Heat transfer correlations for laminar and turbulent flow within a mechanical seal chamber.

The effects of laminar and turbulent flow in the seal chambers on the heat transfer in the seal rings are studied, respectively. By numerically solving the flow field in the seal chamber and performing conjugate heat transfer analyses for the seals, correlations are developed for predicting the heat convection coefficients and temperature distribution in the rings. The results show that the heat convection coefficients at the outer surface of the seal rings are affected by the flow field in the seal chamber.

2. Flow and thermal behavior in a laboratory seal.

A 3D (three dimensional) computational model for flow and thermal analysis of a laboratory mechanical seal with experimental verification is presented without the use of axisymmetry. The computational model can predict the flow field in the seal chamber and temperature field within components of the mechanical seal. This model is also used to determine convection heat transfer coefficients and Nusselt numbers on the wetted surfaces of the seal components.

3. Analytical solution to the heat transfer equations of the seal rings.

An analytical solution to the heat transfer equations in the seal rings is presented. In this study, a simplified mathematical model of heat transfer in the rotating and mating (stationary) rings of a mechanical seal is developed. This model was analytically solved using separation of variable method. In treating the problem, the heat generation between the rings is considered, therefore the two set of heat conduction equations are solved together. Temperature profiles in the rings and temperature distribution at the contact face were presented. The partition number of heat transfer at the contact face was calculated. To verify the result, numerical solution is compared with the analytical one. Based on the results of the simplified model, a more actual model of the mechanical seal rings is developed and solved.

4. Heat transfer analysis in mechanical seals using fin theory.

A simple and effective method is established for estimating the average seal contact face temperature, surface temperature, and heat partitioning factor between the rings of a
mechanical seal. Design charts developed for four types of seal shapes are used in accompanying examples to illustrate their validity.

5. ThermoElastoHydroDynamic (TEHD) behavior of mechanical seals.

TEHD behavior of the lubrication film at the sealing gap between the seal rings is analyzed. In this chapter, several models are developed for estimating the film thickness, pressure distribution, and temperature distribution in the lubrication film with considering the roughness at the seal faces.

8.2 Suggestions for Possible Future Research

The literature on the effects of waviness and cavitation on the thermal behavior of the mechanical seals is very limited in scope, particularly for considering roughness and contact between the seal rings. It would be interesting to understand the effects of cavitation on the flow field within the lubrication film at the sealing gap with considering two-phase flow. Besides, the development of a complex numerical model including the flow field in the seal chamber and at the sealing gap, heat transfer in the seal rings, thermal and mechanic expansion in the seal rings is challenging future topics and has important significance in practice.

Another area where research is needed is the understanding of the behavior of O-rings. O-rings are used to seal the gap between the shaft (or gland) and the O-rings are crucial components of mechanical seals. Pertinent publications are very few. It would be very challenging but promising to develop mathematical models and test rig to investigate the thermal, structural and even tribological behaviors of the O-rings.
References


Appendix A: Analytical Solution to the Heat Transfer for Cylindrical Fins

For the seal shape shown in Fig. 6.2, the analytical solution of its heat transfer is:

For steady state and axis-symmetric assumption, the heat transfer equation in cylindrical coordinates is reduced to be:

\[
\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} = 0 \tag{A1}
\]

where, \( \theta = T - T_\infty \).

The boundary conditions applying to the above Eq. (A1) are:

\[
\frac{\partial \theta}{\partial r} = 0 \quad \text{at } r=r_i \tag{A2}
\]

\[
\frac{\partial \theta}{\partial r} = -\frac{h}{k} \theta \quad \text{at } r=r_o \tag{A3}
\]

\[
\frac{\partial \theta}{\partial z} = q \quad \text{at } z=0 \tag{A4}
\]

\[
\frac{\partial \theta}{\partial z} = 0 \quad \text{at } z=L \tag{A5}
\]

The solution of this problem is:

\[
\theta(r, z) = \sum_{n=1}^{\infty} B_n \left( J_0(\lambda_n r) - \frac{J_{-1}(\lambda_n r_i)}{Y_{-1}(\lambda_n r_i)} Y_0(\lambda_n r) \right) \left( \sinh(\lambda_n z) - \frac{\cosh(\lambda_n L)}{\sinh(\lambda_n L)} \cosh(\lambda_n z) \right) \tag{A6}
\]

where \( B_n \) are the coefficients of the series which can be determined by:

\[
B_n = \frac{\int_{r_i}^{r_o} \left( \frac{q}{K} r \left( J_0(\lambda_n r) - \frac{J_{-1}(\lambda_n r_i)}{Y_{-1}(\lambda_n r_i)} Y_0(\lambda_n r) \right) \right) dr}{\lambda_n \int_{r_i}^{r_o} q r \left( J_0(\lambda_n r) - \frac{J_{-1}(\lambda_n r_i)}{Y_{-1}(\lambda_n r_i)} Y_0(\lambda_n r) \right) dr} \tag{A7}
\]

where \( \lambda_n \) is determined by:

\[
J_{-1}(\lambda_n r_i)(K\lambda_n Y_{-1}(\lambda_n r_o) + hY_0(\lambda_n r_o)) - Y_{-1}(\lambda_n r_i)(K\lambda_n J_{-1}(\lambda_n r_o) + hJ_0(\lambda_n r_o)) = 0 \tag{A8}
\]
Appendix B: Analytical Solution to the Heat Transfer for Seal Rings

Figure B1 shows the geometry and heat flow path in the seal rings including rotor and stator. Based on the studies of Refs. [4, 15], the heat flux path is approximated by straight lines at a fixed angle $\phi$ to the seal faces as shown in Fig. B2. The angle $\phi$ is given by [15]

$$\tan \phi = \frac{L}{r_o - r_i}$$  \hspace{1cm} (B1)

where $L$ is the axial length of the seal ring, and $r_o$ and $r_i$ are the inner and outer radius of the seal ring, respectively. As shown in Fig. B2, the temperature at contact faces of the rotor and stator is assumed to be $T_1$ and $T_2$. $T_{s1}$ and $T_{s2}$ are the temperature at the outer surface of the rotor and stator, respectively.

At the contact face of the rotor, heat flux transferred into rotor is expressed as [15]

$$q_1 = \frac{\pi k_r (T_1 - T_{s1}) \sin 2\phi_1}{\ln(r_o/r)} \, dr$$  \hspace{1cm} (B2)

At the outer surface of the rotor, the heat flux is

$$q_1 = H_1 (T_{s1} - T_f) 2 \pi r_o \, dz = H_1 (T_{s1} - T_f ) 2 \pi r_o \tan \phi_1 \, dr$$ \hspace{1cm} (B3)
Fig. B2  Approximation of heat flux path in seal rings

Assuming the total power dissipation at the sealing gap is $P$, and $\gamma$ is the heat partitioning factor, then we have:

$$q_1 = \frac{\gamma}{1+\gamma} 2\pi rPdr$$ and $$q_2 = \frac{1}{1+\gamma} 2\pi rPdr$$  \hspace{1cm} (B4)

Combining Eqs. (B2) and (B3) yields:

$$k_1(T_1 - T_{s1})\cos^2 \phi_1 = \frac{n_T}{n_{r_1} r_1}$$  \hspace{1cm} (B5)

Combining Eqs. (B2) and (B4) yields the following equations.

$$k_1(T_1 - T_{s1})\sin 2\phi_1 = \frac{\gamma}{1+\gamma} 2 Pr$$  \hspace{1cm} (B6)

Similarly for stator we arrive at the following equations.

$$k_2(T_2 - T_{s2})\cos^2 \phi_2 = H_2 T_{s2} - T_j$$  \hspace{1cm} (B7)

$$k_2(T_2 - T_{s2})\sin 2\phi_2 = \frac{1}{1+\gamma} 2 Pr$$  \hspace{1cm} (B8)

Solving Eqs. (B5)-(B8), $T_1$ and $T_2$ can be determined. The results are:
\[ T_1 = T_f + \frac{\gamma}{1 + \gamma \sin 2\varphi_1} \frac{2 \Pr}{H_r r_o} \left[ \cos^2 \varphi_1 + \frac{\ln \left( \frac{r_o}{r} \right)}{k_1} \right] \tag{B9} \]

\[ T_2 = T_f + \frac{1}{1 + \gamma \sin 2\varphi_2} \frac{2 \Pr}{H_r r_o} \left[ \cos^2 \varphi_2 + \frac{\ln \left( \frac{r_o}{r} \right)}{k_2} \right] \tag{B10} \]
Appendix C: Computation of Heat Partitioning Factor

Figure C1 shows the schematic of the seal rings of interest in this study with boundary conditions. Heat convection boundary conditions are applied to the outer surface of the rings and the other boundaries are assumed to be insulated. At the interface between the rings, heat is generated and transferred into two rings. The heat portioning factor can be expressed as

\[ \gamma = \frac{q_1}{q_2} \]  \hspace{1cm} (C1)

where \( q_1 \) and \( q_2 \) is the average heat flux transferred into rotor and stator, respectively.

![Fig. C1 Schematic of seal rings with boundary conditions](image)

The total heat generation is:

\[ q = q_1 + q_2 \]  \hspace{1cm} (C2)

The material properties of the rings and the operation conditions are shown in Table 5.1. FLUENT is applied to numerically solve the heat transfer problem in the seals shown in Fig. C1. At the interface, three cases of heat generation are used: \( q = 1 \times 10^4 \) W/m\(^2\), \( q = 1 \times 10^5 \) W/m\(^2\), and \( q = 1 \times 10^6 \) W/m\(^2\).

By applying a total heat generation rate at the interface between the rings and numerically performing the heat transfer analysis using FLUENT, the average heat flux transferred into rotor and stator can be calculated and then \( \gamma \) can be obtained using Eq.
(C1). Table C1 shows the values of heat partitioning factor. From the table, it can be seen that the heat partitioning factor $\gamma$ changes little with different total heat generation $q$, which suggests that for a certain seal rings with proper boundary conditions, the values of $\gamma$ is fixed.

Table C1 Values of heat partitioning factor

<table>
<thead>
<tr>
<th>Total heat generation</th>
<th>$\gamma$ for water seal</th>
<th>$\gamma$ for oil seal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q = 1 \times 10^4$ W/m$^2$</td>
<td>2.1896</td>
<td>1.2470</td>
</tr>
<tr>
<td>$q = 1 \times 10^5$ W/m$^2$</td>
<td>2.1880</td>
<td>1.2459</td>
</tr>
<tr>
<td>$q = 1 \times 10^6$ W/m$^2$</td>
<td>2.1879</td>
<td>1.2458</td>
</tr>
</tbody>
</table>
Appendix D: Calculation of Seal Face Taper Angle $\phi$

In this section, the calculation of seal face taper angle $\phi$ is shown. Temperature gradients in the seal rings are responsible for the thermoelastic deformations of the seal faces. Orcutt [53] found that the initially flat seal face becomes thermal deformed resulting in a convergent gap in the leakage direction. The work of Doust and Parmar [16] showed that thermal distortions may be greater than those due to pressure.

Table D1 Material properties of seal rings

<table>
<thead>
<tr>
<th></th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rotor</strong></td>
<td></td>
</tr>
<tr>
<td>Rotor density $\rho$</td>
<td>1826.1 Kg/m$^3$</td>
</tr>
<tr>
<td>Young’s Modulus $E$</td>
<td>26.9 Gpa</td>
</tr>
<tr>
<td>Thermal expansion Coefficient $\alpha$</td>
<td>$5.04 \times 10^{-6}$ $/°$C</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu$</td>
<td>0.2</td>
</tr>
<tr>
<td><strong>Stator</strong></td>
<td></td>
</tr>
<tr>
<td>Stator density $\rho$</td>
<td>3091.6 Kg/m$^3$</td>
</tr>
<tr>
<td>Young’s Modulus $E$</td>
<td>440 Gpa</td>
</tr>
<tr>
<td>Thermal expansion Coefficient $\alpha$</td>
<td>$4.5 \times 10^{-6}$ $/°$C</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu$</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Fig. D1 Rotation rate for rotor and stator ring

In this study, we considered thermal distortions only and use the rotation rate $b$ to characterize the susceptibility of a ring to deformation [42]. A FEM model (ANSYS
thermal structural coupling analysis) was used to calculate the deformation taper angle at
the rotor and stator, respectively. The material properties are shown in Table D1. Figure
D1 shows the rotation rate $b$ varying with temperature difference $\Delta T_m$. Taper angle $\phi$ can
be calculated using

$$\phi = b \Delta T_m$$ (D1)

where $\Delta T_m$ is determined by

$$\Delta T_m = T_m - T_f$$ (D2)

where $T_m$ is the mean temperature at the seal face.

From Fig. D1, it can be found that under the conditions simulated the rotation rate for
the stator is so small compared with that for the rotor that it can be negligible.
Appendix E: Calculation of Heat Convection Coefficients

For a long time the convection coefficients at the outer surface of the seal rings were determined by empirical or experimental methods [18]. More recently CFD application leads to more accurate calculations of the convection coefficients. In the present work, we applied FLUENT (CFD software) to numerically solve the conjugate heat transfer problem considering heat transfer within seal rings and fluid flow in the seal chamber. The flush rate is assumed to be 1 gpm. At the interface, three cases of heat flux $q = 1.0 \times 10^4$ W/m$^2$, $q = 1.0 \times 10^5$ W/m$^2$ and $q = 1.0 \times 10^6$ W/m$^2$ are applied. The average heat convection coefficients at the outer surfaces of the rotor and rotor are calculated, respectively. It is found that for three cases of heat flux with the same operating conditions, the convection coefficients change little. Therefore, in the present study, $H_1 = 776.0$ W/m$^2$.K and $H_2 = 544.81$ W/m$^2$.K are used.
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Volume Number (Journal): 128
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Vita

Zhaogao Luan was born in Xintai, China. He received Bachelor of Science and Master of Science degrees from Sichuan University in 2000 and in 2003, respectively. In August 2003, he started to pursue his doctoral study in the Department of Mechanical Engineering of Louisiana State University. Since then, he has been a doctoral student under the guidance of Dr. Michael M. Khonsari, Dow Chemical Endowed Chair and Professor in Department of Mechanical Engineering of Louisiana State University. Zhaogao Luan will receive his Doctor of Philosophy degree at the 2007 Fall Commencement.